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Pre-design of laterally supported stair steps

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ABSTRACT

It is becoming increasingly common to design monumental staircases and their steps with elevated stiffness and low mass, obtaining high natural frequencies, off the range of frequencies that are excitable by pedestrians. However, this sometimes leads to unacceptable levels of vibration, with impulsive responses. In many cases the dynamic behaviour of steps is almost independent from the rest of the staircase, causing a phenomenon designated as local vibrations, which could be much more severe than the global vibrations of the staircase.

In order to avoid this problem, this paper presents a simplified expression to pre-design stair steps which guarantee that excessive vibrations will not occur, without the need to perform a dynamic analysis. The expression was deduced based on the results of an experimental campaign, several numerical analyses and a theoretical analysis. During this study it was necessary to define an acceptable limit of vibrations specific for this type of vibration, which affects mainly the feet of pedestrians. The expression deduced is easy to apply because it depends only of vertical stiffness of the step. Finally, the pre-design expression is also applied to the staircase used in the experimental campaign, and it was concluded that it would be easy to avoid excessive vibrations, with a negligible cost increase.

1. Introduction

Almost all constructions have staircases to connect floors at different levels. Usually staircases have a straight or spiral form and their material is commonly steel, concrete, timber, aluminium or glass.

In the past, staircases like other constructions were designed to be very robust, so their mass, stiffness and strength were high enough to avoid poor structural behaviour. However, over the decades, as the materials strength and durability increased, architects have started to ask for slender and lighter solutions, often for aesthetic reasons, i.e. monumental staircases. As a consequence engineers have become increasingly aware about the need to design structures with more accuracy and precision. Today, the structural design of staircases includes the verification of ultimate limit states (ULS) and serviceability limit states (SLS). The latter refers to deflections and vibrations. ULS and deflection are usually well known and controlled by designers. Regarding the vibrations applied to stairs, however, scientific knowledge is still scarce.

Usually in the design of steel staircases to deal with vibrations, the objective is (i) to quantify as accurately as possible the force applied during walking and (ii) then to calculate the induced vibrations by performing one of the existing numerical analyses (footfall force time histories, Fourier Series, Steady State analysis and Effective Impulse), (iii) these being finally compared with the acceptable limits proposed by the various design guides.

Regarding the forces applied by an individual using the staircase (ground reaction forces, GRFs), they are commonly obtained through force plate measurements. Some examples of authors who have performed tests on force plates are Kerr and Bishop [1–3], Gonzalez [4] and Kasperski et al. [5].

Kerr [3] demonstrated that GRFs tend to have a greater magnitude as the step frequency increases, both for descents and ascents,and the maximum forces for descents are usually higher than the maximum forces for ascents. Kerr [3] also reported that forces applied on staircases are higher, than the forces applied on flat surfaces and it is furthermore possible to walk on stairs with higher step frequencies.

Bougard [6] measured the maximum loads applied and not the GRFs time histories. The author concluded that the dynamic forces measured were in accordance with the British design codes, as they incorporated an amplification factor of at least 1.6 to be multiplied by the specified loads.

Gaile [7] presented a different methodology to obtain analytical functions of continuous walking force time histories based on inverse dynamics. Although based in a different method, the results obtained were in agreement with was reported by Kerr [3].

Regarding the numerical analysis to calculate human-induced

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vibrations, few authors have conducted studies comparing vibrations measured experimentally with vibrations calculated numerically.

Davis et al. [8], through a comparison between the experimental and numerical results, verified that there is a clear overestimation using *Steady State* analysis and suggested the multiplication of the results obtained using *Steady State* analysis by an adjustment factor equal to 0.35 to consider imperfect resonances.

Zhou et al. [9] evaluated the vibrations of an indoor spiral steel staircase and compared these later with the vibrations obtained numerically using a Fourier Series.

Huntington and Mooney [10] presented a case study carried out on a monumental steel staircase with a span of 12 m. They found that using numerical models and expressions indicated by AISC 11 [11] led to overestimation of the results.

Some authors such as Eid et al. [12] and Setareh [13] only evaluated vibrations on stairs numerically.

Setareh [13] presented a dynamic analysis performed on the design of a monumental metal staircase. The author calculated that between Fourier Series and *Steady State* analysis an adjustment factor of 0.84 was required. This, according to Setareh [13], means that the adjustment factor of 0.35 recommended by Davis et al. [8] may be unconservative, although, it should be mentioned that this factor was obtained based only on numerical analysis and not on experimental tests.

Eid et al. [12] conducted a dynamic analysis on a numerical model of a steel staircase to compare the results of the design guides AISC 11 [11] and SCI P354 [14]. Due to the large difference between the results obtained by these two design guides, the authors concluded that more research is needed in order to obtain a greater consensus among the various design guides.

Various design guides are available that are specifically aimed at assessing the human response to vibrations [11,14–16]. However, as far as staircases are concerned, only SCI P354 [14] directly refers to vibration acceptance criteria for these types of structures.

It is noticed that most of the studies found in literature relate to staircase vibrations where the entire structure moves as a whole, i.e. studies of global vibrations. However, if the connections between the steps of the staircase and the staircase itself have a low rotational stiffness, this may cause the dynamic behaviours of these two structural elements to be almost independent of each other, causing a phenomenon referred to as local vibration (in steps). In fact, in many cases, mainly when the length of the steps is long, it is observed that local vibrations can reach significant levels of response.

Generally, steps are very slender and light elements, and this normally tends to cause excessive impulsive vibrations. These local vibrations affect mainly the feet of pedestrians and not their whole body, so the actual limits of design guides are not the most appropriate for this situation. According to the author's knowledge, there are no studies in the literature about the vibrations on stair steps and the respective tolerable limits of vibration. This research paper has the purpose of reaching a simplified expression to allow engineers to pre-design stair steps which do not have local vibrations, without the need to perform a very time-consuming dynamic numerical analysis that may be unfamiliar to many of them.

To achieve this end, two phases are developed. Firstly, an experimental campaign is carried out to measure vibrations in a staircase with excessive local vibrations. These experimental results are then applied to validate the results of the numerical analysis. Secondly, the pre-design expression is developed, initially based on two different methodologies validated by experimental results, and then choosing the most suitable. Finally, the new pre-design expression is applied to the stair steps used in the experimental campaign.

2. Experimental program

2.1. Staircase description

The steel staircase studied in this paper was conceived to allow

Fig. 1. Studied steel staircase.

pedestrian access between the two floors of a public building located in Funchal, Madeira, as seen in Fig. 1. The steel staircase has been the subject of several adverse comments from pedestrians who use it, due to the high level of vibration felt.

The high level of vibration experienced is associated with the movement of the stair steps and not with the movement of the whole structure; that is, the vibrations are local and not global. Taking into account that the dynamic behaviour of the steps is practically independent from the rest of the structure, it becomes relevant only to describe their properties. The stair steps are each composed of a metal plate with a thickness of 6 mm covered with a thin coating of synthetic rubber sheet and its dimensions are shown in Fig. 2. The connection between each stair step and the rest of the stair structure is made by an auxiliary hollow profile (blue in Fig. 2) that is weakly welded (red in Fig. 2) to the step, due to people's use of the staircase over the years. Therefore, the weld connecting the steps to the staircase stringers has a practically null rotational stiffness and the connection can be considered as pinned, helping to understand why the vibrations are at the local level.

2.2. Dynamic characterization of stair steps

Before quantifying the level of vibration or acceleration to which the steps are subjected, it is essential to dynamically characterize them. An ambient modal analysis has been performed to determine the steps vibration modes (natural frequencies and corresponding mode shapes and damping ratios. In these tests two accelerometers MMA8452Q, whose specifications are indicated in Table 1 were used.

Fig. 3 presents the averaged normalized power spectrum density (ANPSD) with the frequencies of the local vibration modes. The frequencies of the first two local modes are respectively 24.0 Hz and 45.6 Hz. The measured frequencies of the local modes are different from the measured frequencies of the global modes, demonstrating that the dynamic response of the steps is independent from the rest of the structure and that their behaviour can be studied as separated parts.

The steps of the sample staircase have high frequencies, indicating that the responses should be impulsive and not in resonance. The shapes of the first two vibration modes obtained experimentally, in their respective modal coordinates, are represented in Fig. 4. As it can be observed, the first mode (24.0 Hz) is vertical with some torsion and the second mode (45.6 Hz) is exclusively of torsion. For both modes, applying the half-power bandwidth method, the damping was

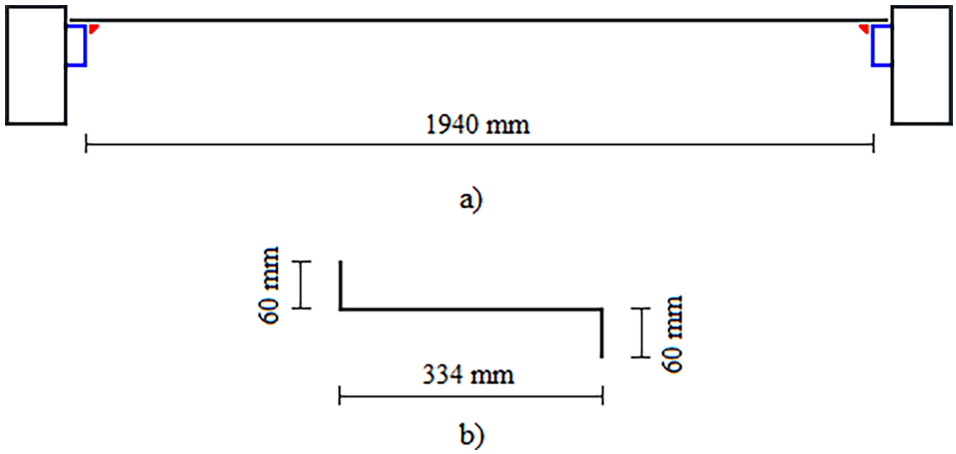

Fig. 2. Step geometry: (a) longitudinal section, (b) cross-section.

Table 1
Accelerometers specifications.

Range	Frequency bandwidth	Sampling frequency	Resolution	Noise
± 20 m/s^2	1,6 Hz – 800 Hz	200 Hz	0.01 m/s^2	1 mm/s^2/$\sqrt{\text{Hz}}$

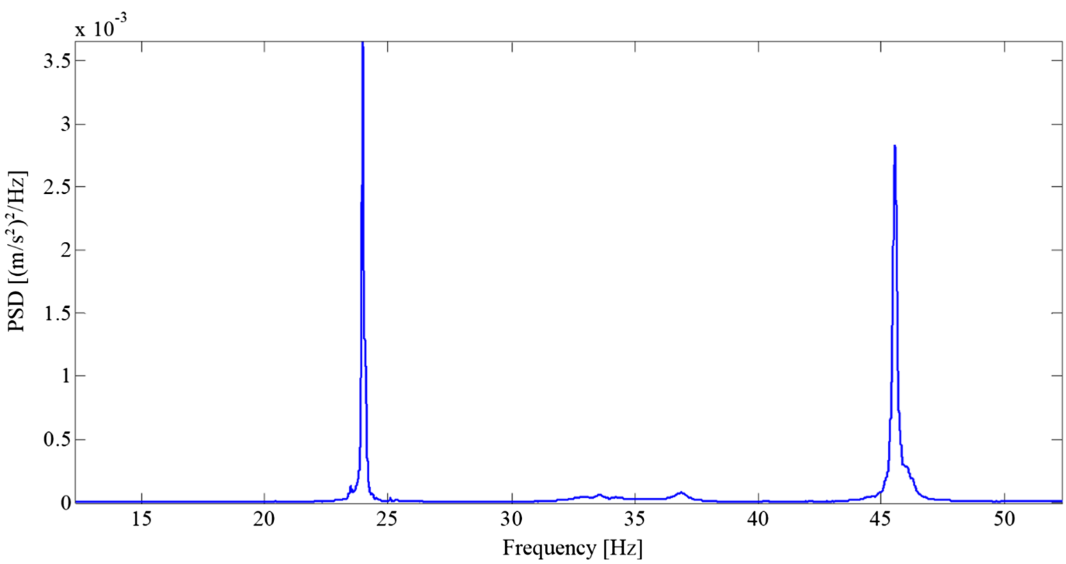

Fig. 3. Power spectrum density.

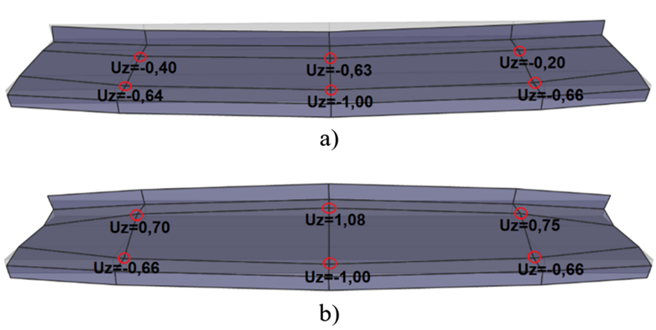

Fig. 4. Modal shapes: (a) first mode, (b) second mode.

consistently estimated to be about 0.82% of critical. Although, this value has been estimated in a steel stair step (local), it is in accordance with the results obtained by various authors [1,4,8], who obtained in complete steel stairs (global) a damping of approximately 1%.

2.3. Walking tests

In order to determine the level of vibration to which the steps of the studied staircase are exposed, several walking tests were performed, placing individuals ascending and descending the staircase with different step frequencies, ranging from a normal walk to a fast run. Firstly, experimental tests were performed with a pedestrian individually walking the staircase, and then having a group of 4 pedestrians (2 + 2) traversing it.

According to several authors [1,5,8], it is possible to walk on staircases with step frequencies varying between approximately 2.0 and 4.5 Hz, therefore in the walking tests conducted step frequencies situated in this interval were used. Since steps have a high fundamental frequency (24.0 Hz), this means that ascending and descending the staircase with step frequencies in the range of 2.0 to 4.5 Hz, will hardly excite it so that a

Table 2
Description of the experimental tests.

	Number of trials	
Step frequency	Isolated pawn	Group of pedestrians
Descent 2.2 Hz	4	4
Descent 3.3 Hz	4	4
Ascent 2.0 Hz	4	4
Ascent 3.0 Hz	4	–

resonant response could be obtained. Furthermore, in each step only one footfall is applied, so there is no possibility of a resonant build-up. For these reasons, in the walking tests performed, step frequencies were chosen which seem to be more plausible for pedestrians when walking the staircase during their daily routine, and not with the objective of obtaining a resonant response. It was decided to use the following step frequencies: 2.0 Hz for a normal ascent, 3.0 Hz for a fast ascent, 2.2 Hz for a normal descent and 3.3 Hz for a fast descent.

Table 2 describes the experimental tests performed for a single pedestrian and a group of pedestrians, respectively.

For the measurement of the accelerations caused by the pedestrians, the two accelerometers described in Table 1 were again used. Both were placed at midspan of a random step, since this is where larger responses are generated and because pedestrians walking on a staircase (if not accompanied) tend to place their feet approximately at that location.

2.4. Experimental results and discussion

Comparing the experimental results of the group of pedestrians with the results of a single pedestrian, it was verified that the accelerations were close, obtaining for both typologies maximum values slightly lower than twice the gravitational acceleration. Therefore, group enhancement can be excluded and only the accelerations relative to single pedestrian were analysed throughout the paper.

Fig. 5 gives some examples of the acceleration graphs measured experimentally for a single pedestrian, respectively for ascents at 2.0 Hz and 3.0 Hz and for descents at 2.2 Hz and 3.3 Hz.

From the graphs of the experimental accelerations presented in the previous figures (Fig. 5(a) to (d)) it is possible to make some observations:

- The responses are close to 24.0 Hz (fundamental frequency of the step), clearly demonstrating that the vibrations in the studied staircase are local and not global.
- Since the staircase steps respond with a high frequency and only one footfall is applied, it is not surprising that all the graphs present an impulsive response and not a resonant response.
- The values of the accelerations when the individual puts the foot on the step in which the accelerometers are placed are much higher than the values of accelerations when the individual puts the foot on the previous steps. This indicates that the contribution of the response of the other steps is low and each step may be treated independently.
- The values of the accelerations are very high, evidencing that structures, i.e. stair steps, with low fundamental frequencies do not always produce the most conditioning response. Structures designed with high frequencies but with a low mass, as in this case (24 Hz and 46,2 kg), will probably also give rise to significant vibrations.
- The accelerations increase as the step frequency increases, being higher for the descents (Fig. 5(c) and d)) than for the ascents (Fig. 5(a) and (b)), which is in agreement with results verified by other authors [1,4,8].
- Lastly, it is important to emphasize that in the worst scenario (descent at 3.3 Hz, Fig. 5d)) the acceleration peaks reached values slightly lower than twice the gravitational acceleration.

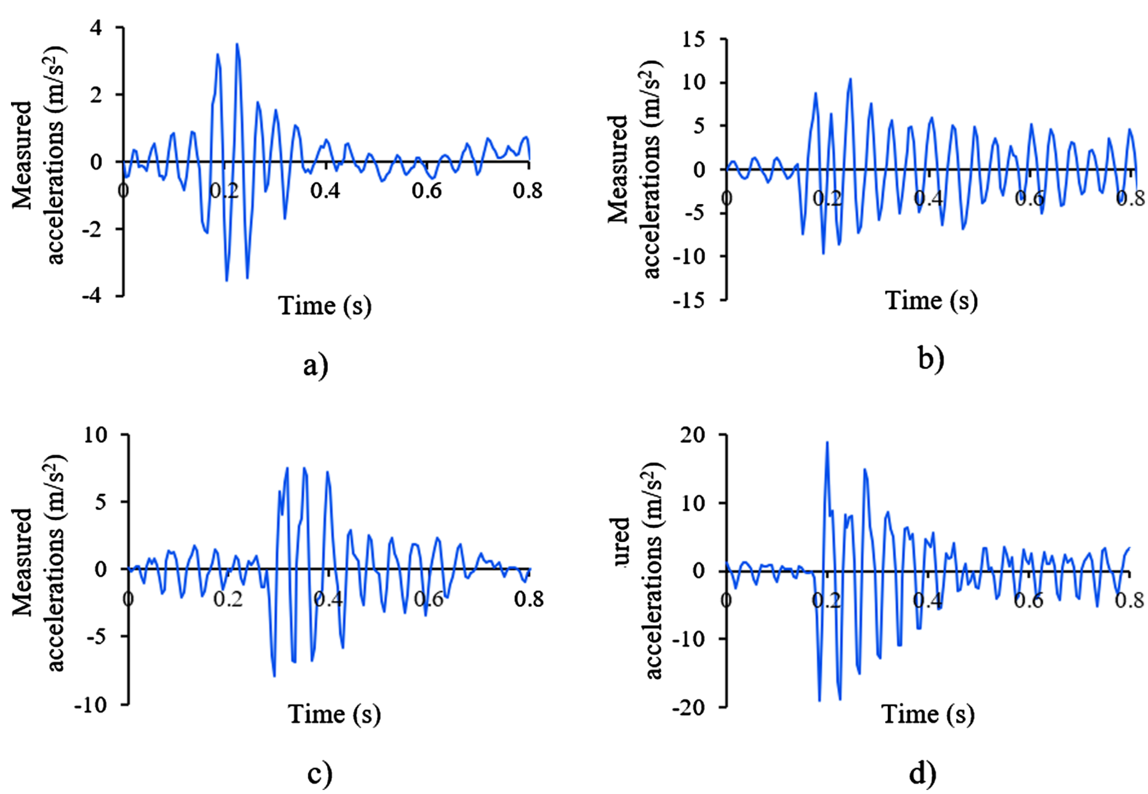

Fig. 5. Measured accelerations: (a) ascent at 2.0 Hz, (b) ascent at 3.0 Hz, (c) descent at 2.2 Hz, (d) descent at 3.3 Hz.

3. Pre-design of stair steps

3.1. Proposed acceptance criteria

The design guides conceived for the analysis of vibrations induced by humans on structures are based essentially on the following three phases: quantification of the walking dynamic forces, obtaining the accelerations numerically, and comparing the expected accelerations with an acceptance criteria. In most of the existing standards the limits of satisfactory acceleration magnitude are expressed in relation to frequency-weighted base curve and a series of multiplying factors, which take into account different vibration environments in buildings and different types of structures (i.e. stairs and footbridges). This base curve is presented in the principal international standards [11,14–16] as shown in Fig. 6, although slightly altered to comprise peak and not r.m.s accelerations.

As far as staircases are concerned, there is not much information available in the different design guides concerning acceptable limits for vibrations. Due to the lack of specific acceptability criteria for staircases in the various design guides, Bishop et al. [1] have proposed their own factors, which should be multiplied by the base curve. The only design guide that directly refers to the acceptance criteria of vibrations in staircases is the SCI P354 [14], but the factors referred to in this design guide are those suggested by Bishop et al. [1]. Other authors who proposed their own acceptable limits were Zhou et al. [9] and Davis et al. [8], being the limits proposed by the latter also based on the work developed by Bishop et al. [1]. Kim et al. [17] and Eid et al. [12] suggested that accelerations should be compared with the base curve of peak accelerations for indoor bridges, given by AISC 11 [11], taking into account the scarcity of information exclusively related to staircases in the various design guides.

In order to deduce the pre-design expression that will be presented in this paper, it was necessary to define an acceptable limit of accelerations. However, the limits proposed by the abovementioned authors might not be appropriate to the particular case studied.

This is due to the fact that these limits are exclusively intended for staircases that respond globally, in which the discomfort caused to their users by vibrations felt occurs, in general, on the whole body and during the time individuals take to walk through the totality of the staircase, this being different from the discomfort felt when there are local vibrations. When vibrations are local, the phenomenon is different, the discomfort is mostly felt only in the foot during the few instants in which the individual places it on the stair step, being the level of perceptibility of vibrations lower for these cases.

Furthermore, the multiplication factors proposed by Bishop et al. [1] were suggested based on the analysis of staircases with low natural frequencies, where there is a plausible response in resonance, and not on staircases that respond impulsively, as is the case of the subject developed in this paper.

For the reasons presented previously, it was necessary to define a different acceptable limit of vibrations, which was more suitable for the phenomenon of the discomfort felt only in the foot for a few instants.

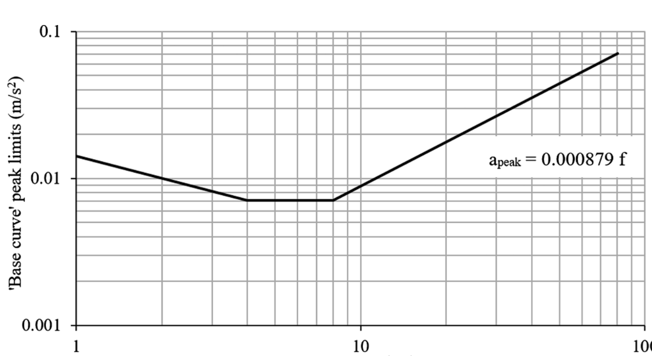

Fig. 6. Frequency-weighted base curve for perception of vibrations.

In standards BS 6472 [15] and ISO 10137 [16] in Tables 5 and C.1 respectively, a multiplication factor of 128 in the base curve is recommended for short duration excitations, for up to 3 occurrences, that must be used in cases of blast-induced vibration. This multiplication factor seems appropriate for the phenomenon studied, since the vibration mechanism (independent short duration occurrences) is similar and more comparable to blast-induced loads than to human-induced loads (continuous vibrations during the stair traversing).

It should be observed that impulsive responses are a characteristic of structures with high frequencies and therefore, to this end, only the third straight line of Fig. 6 is of interest, i.e. natural frequencies higher than 8 Hz. As a consequence, multiplying the equation of this straight line by the factor of 128 it is originated the acceptance criteria considered in this paper (Eq. (1)).

$$a_{peak} = 128 \times 0.000879f = 0.113f = \frac{0.113}{2\pi}\sqrt{\frac{k}{m^*}} \tag{1}$$

where K and m^* are respectively the vertical stiffness (N/m) and the generalized mass (kg) (see definition in Section 3.2.1) of the stair step.

For 24.0 Hz (frequency of the stair step), considering Eq. (1), it is obtained a peak acceleration approximately equal to 2.70 m/s^2. During the walking tests, the participants after each test indicated if they felt discomfort or if no major discomfort was felt. According to the values of the peak accelerations of each test and the answers collected, it was concluded that for these stair steps the proposed acceptance criteria is suitable.

3.2. Pre-design expression

The pre-design approach described here suggests, through simplified expression, that stair steps have certain geometrical characteristics, so in practice excessively high vibrations do not occur. The use of pre-design expressions avoids numerical analysis, which is sometimes very time-consuming, poorly calibrated and not well understood by all designers.

In this paper, two pre-design curves will be presented, which were obtained based on two different methodologies and that are valid for steps with high natural frequencies, responding impulsively, as well as for current step spans (1 to 3 m). The first pre-design curve was obtained based on numerical analyses performed using the structural analysis software SAP2000 [18] and the second was deduced from Duhamel's integral. It should also be noted that the two pre-design curves were determined through peak accelerations, since these are the conditioning accelerations.

3.2.1. Numerical analysis with SAP2000

In order to deduce the pre-design curve presented in this section, the first step was to determine the peak accelerations with the software SAP2000 for several steps with different values of stiffness and generalized mass. To calculate the generalized mass (m^*) of the step, a percentage of 50% of its total mass (m) was used, this being the generalized mass corresponding to a simply supported beam [19–21]. Considering that the structure of the step is relatively simple and its response is mainly influenced by the first vibration mode, it can be assimilated to a simply supported beam. The use of a generalized mass instead of the total mass was in order to apply the same parameter used in Duhamel's integral, making the curves comparable.

A detailed numerical model of the analysed stair step was constructed in SAP2000 with shell elements. The first two numerical frequencies and mode shapes were similar to the experimental results. For the proposed pre-design curve to be deduced with accuracy, it was important that the accelerations initially calculated through numerical simulations were close to the experimentally measured ones. There are

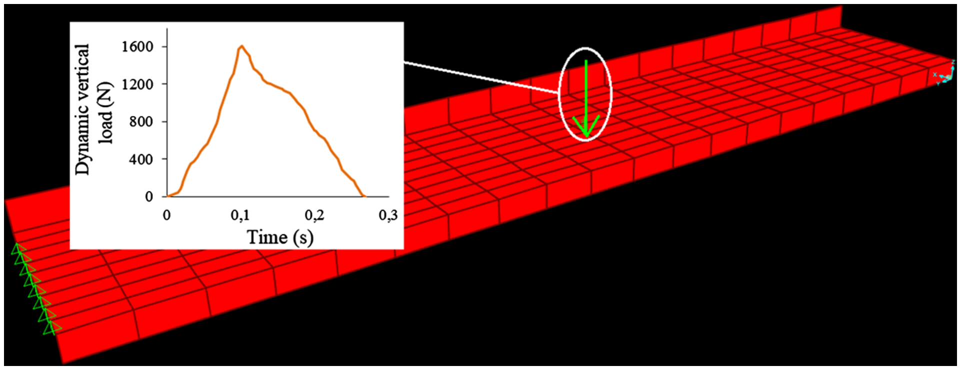

Fig. 7. Application of GRFs to the step in the numerical model.

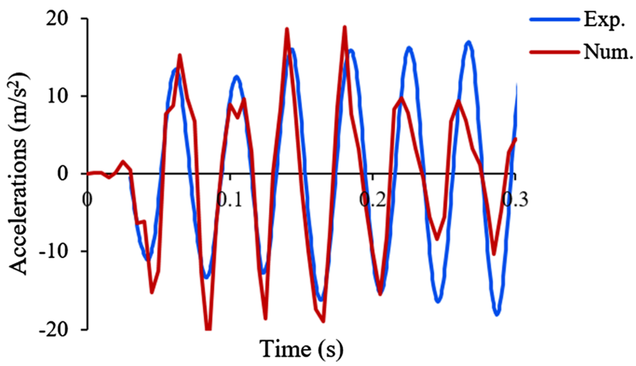

Fig. 8. Comparison of accelerations due to a descent at 3.3 Hz.

several numerical methods to predict human-induced vibrations in the design of flexible staircases: (i) applying footfall force time histories (ground reaction forces, GRFs) and (ii) Fourier Series to Finite Element models, thus simulating the pedestrian's movement, (iii) Steady State analysis, and (iv) Effective Impulse. An extensive number of simulations was done with the previously mentioned methods, in order to simulate with precision the pedestrian's walking, being concluded that the most realistic method of obtaining accelerations numerically is through the application of GRFs measurements directly on FE models of stairs. Hence, this was the method used throughout the paper to determine accelerations numerically.

In the experimental program (Section 2.4), the conditioning accelerations were obtained for descents with step frequencies close to 3.3 Hz, therefore GRFs traces with frequencies close to this were chosen from the literature. After applying the GRFs to the model (see Fig. 7), for each one a time history analysis was performed to determine the accelerations. GRFs of various authors [2–5,22] were used, being the GRF obtained by Kerr [2,3] the one that originated accelerations closest to the measurements, that is, the difference between the two curves, during the instants in which the footfall force is applied, is minimal (see Fig. 8). Accordingly, it was decided to use the GRF trace measured by this author in the calculation of the peak accelerations for the different simulated steps, where the values of stiffness and generalized mass were changed.

Initially, the step had a vertical stiffness at midspan of 526 kN/m and a generalized mass of 23.1 kg, and these were increased until the acceleration values began to be substantially lower than the above-mentioned acceptance criteria (Section 3.1). In total, 189 peak acceleration values were calculated for as many combinations of generalized mass and stiffness. Fig. 9 shows the points regarding the peak accelerations calculated with SAP2000. Fig. 9 also displays the surface defined by a 4th degree polynomial function ($R^2 \approx 0.90$) that best adjusts to the values of peak accelerations obtained for the different stiffnesses and generalized masses.

The acceptance criteria (Eq. (1)) can also be represented by a 3D surface, as shown in Fig. 10. Intercepting the two surfaces (Figs. 9 and 10), a 2D graph is obtained with the values of stiffness and generalized mass necessary for accelerations not to exceed these limits, as presented in Fig. 11.

The values of stiffness and generalized mass above the line

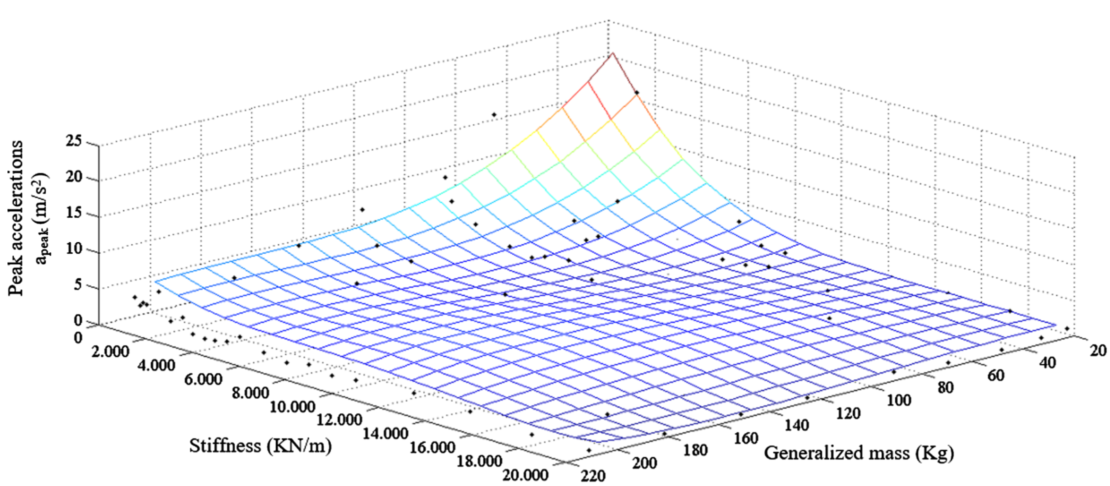

Fig. 9. Numerical peak accelerations and regression polynomial surface.

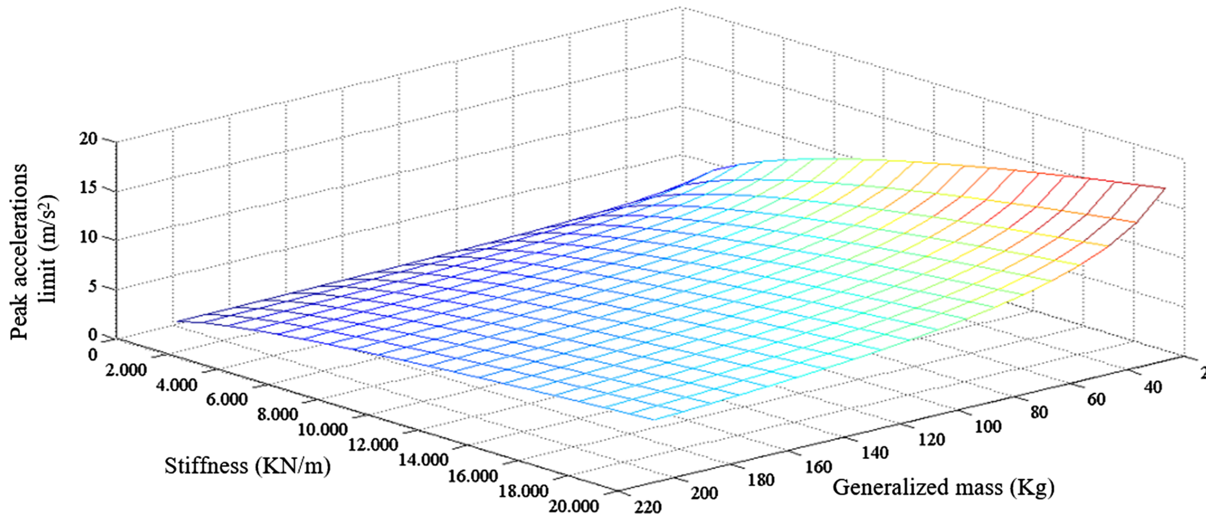

Fig. 10. 3D surface of acceptance criteria.

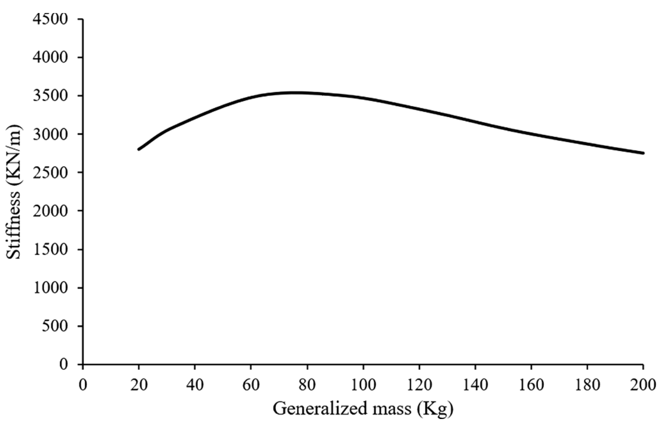

Fig. 11. First pre-design curve, based on numerical simulations.

represented in Fig. 11 give rise to peak accelerations lower than the acceptance criteria. Hence, the pre-design expression derived from the numerical analysis can be considered valid and suitable for stair steps responding impulsively with other natural frequencies, and not only for the studied case of 24 Hz.

3.2.2. Duhamel's integral

Since the response of the step is mainly influenced by the first vibration mode, this means that it can also be described using Duhamel's integral, applicable to single degree-of-freedom systems.

Having in mind that only the peak acceleration is of interest and the same occurs in the initial instants, since the step responds impulsively, in this case, the contribution of the damping can be considered negligible. As such, Duhamel's integral can be described as follows:

$$u(t) = \frac{1}{wm^*}\int_0^t p(\tau)senw(t-\tau)d\tau \tag{2}$$

where $u(t)$, m^* and w have already been defined previously, t is the instant at which the response $u(t)$ is being calculated and τ is the instant that load $p(\tau)$ is acting and the variable that is integrated ($\tau < t$).

First, it was necessary to define the load function $p(\tau)$ to be used in Duhamel's integral. According to Fig. 7, in the first instants (where the maximum accelerations occur) the GRF trace is almost linear. Hence, in a simplified way, it was decided to define the load function $p(\tau)$ by a linear function, given by Eq. (3), and subsequently to find the proportionality constant value (a) that gives rise to the maximum acceleration obtained in the experimental tests.

$$p(\tau) = a\tau \tag{3}$$

Substituting the load function $p(\tau)$ described by Eq. (3) in Duhamel's integral (Eq. (2)) and then deriving it in order at t (time) twice, gives the expression for the acceleration, which is described by Eq. (4):

$$u(t)'' = \frac{a\sin(tw)}{wm^*} \tag{4}$$

Taking into consideration that the maximum acceleration occurs when $\sin(tw) = 1$ and that the angular frequency, in structures with a single degree of freedom, is given by $w = \sqrt{\frac{K}{m^*}}$, it is possible to achieve Eq. (5):

$$\ddot{u}_{máx} = \frac{a}{wm^*} = \frac{a}{\sqrt{Km^*}} \tag{5}$$

Replacing m^* and K respectively with the initial values of the generalized mass and stiffness of the stair step (23.1 kg and 526 000 N/m) and then equalizing Eq. (5) with the maximum experimental acceleration value, the parameter $a = 62700$ N/s of the load function $p(\tau)$ was obtained.

Eq. (5) can be equalized to the acceptance criteria (Eq. (1)) and be rewritten in order to obtain a pre-design expression, described as below:

$$K \geq 3487 \text{ kN/m} \tag{6}$$

where K is the vertical stiffness of the stair step.

3.2.3. Comparison of the two pre-design methodologies

The two pre-design curves (Fig. 11 and Eq. (6)) are presented graphically as shown in Fig. 12 to be more easily and conveniently compared.

As it is possible to observe, the curve derived from Duhamel's integral is not dependent on the generalized mass and is defined by a constant straight line with a vertical stiffness value of 3487 kN/m.

The curve derived from numerical analysis with SAP2000 does not substantially depends on the generalized mass, since the values of vertical stiffness do not vary significantly in the considered interval of 20 to 200 kg, being the minimum value approximately equal to 2697 kN/m and the maximum value close to 3523 kN/m.

Taking into account that the two methodologies do not give rise to very different results and that the same are almost constant (depending only on vertical stiffness), a final pre-design expression is proposed (Eq. (7)) with a constant value, which is slightly higher than the obtained by the two methodologies, i.e. to be more conservative and to ensure a lower probability of adverse comments by pedestrians.

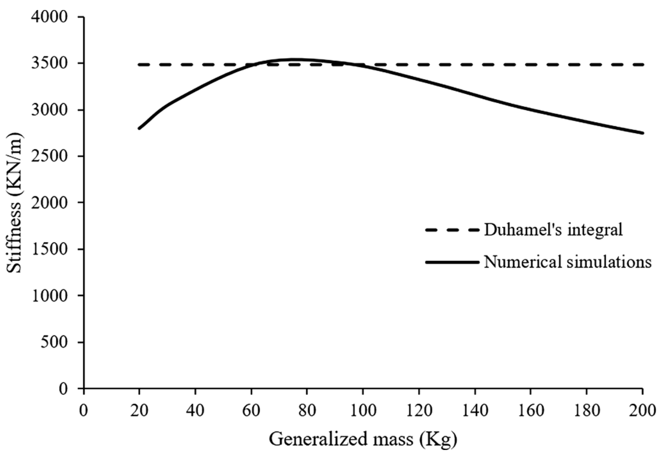

Fig. 12. Comparison of the pre-design curves.

$$K \geq 3600 \text{ kN/m} \quad (7)$$

It is well-known that higher the structure mass, lower its impulsive response. However, the presented pre-design expression does not depend on the step's mass. This occurs because, according to the acceptance criteria defined by Eq. (1), the acceleration peak limit increases as the natural frequency increases. This means that even if accelerations are larger, for steps with lower masses and a vertical stiffness higher than 3600 kN/m, their perceptibility will not be high enough to cause discomfort to pedestrians, since it reduces for higher natural frequencies.

3.3. Application field and extension for different support conditions

Eq. (7), as with most pre-design expressions, should be used with caution and having in mind some considerations.

Before anything, it should be noted that normally step structures are relatively simple, such that their responses are mostly influenced by the first vibration mode, therefore Eq. (7) can be used with confidence in most pre-design situations.

It is recommended that Eq. (7) be applied only in steps with a free span up to 3 m. For spans greater than 3 m, it is suggested that an analysis be carried out using one of the numerical methods mentioned above (GRFs, Fourier Series, Steady State or Effective Impulse), in order to verify that the accelerations do not exceed the defined acceptable limit.

This Eq. (7) is valid for a total mass between 20 kg and 300 kg. The utilization of steps with a total mass lower than 20 kg and higher than 300 kg is an unfeasible solution in practice, this being the reason for the stated interval of total mass in which the pre-design expression should be employed.

In general, steps can have different support conditions, but Eq. (7) continues to be valid for other steps with different types of support, being only necessary to change where the value of the vertical stiffness K (kN/m) is evaluated. Fig. 13 represents the most typical support conditions used in steps of staircases. Note that for the pinned and fixed solutions the vertical stiffness K is evaluated in the mid-span, while for the cantilever solution the stiffness is evaluated in the free end.

Considering that, in some cases, the stiffness of steps with fixed supports tend to decrease over time due to its use, as observed in the analysed staircase, it is recommended, for fixed solutions, to reduce the full stiffness of steps when calculating the stiffness to compare with limit of Eq. (7) and assure that connections are correctly built on site.

3.4. Additional condition to avoid resonances

Structures with low natural frequencies can be assumed to develop a resonant response, while structures with high natural frequencies act impulsively, with a transient response [23]. The two curves presented were developed to be applied to steps with high natural frequencies, for which the predominant response is impulsive. Consequently, it becomes important to understand the limit or boundary from which a resonant build-up is no longer possible and the structure begins to respond impulsively.

In the case of floors, the frequency of the 4th harmonic amplitude of the walking force is normally taken as the boundary between low and high frequency, setting the cut-off frequency as approximately 10 Hz [23]. However, for staircases there is not much information in the literature regarding which boundary should be employed. The only design guide found in the literature that refers to a cut-off frequency for staircases is SCI P354 [14], which indicates that staircases up to 12 Hz should be treated as low-frequency. It is believed that this boundary may not be the most appropriate. Assuming that from the 4th harmonic amplitude, the higher harmonics do not present enough energy to cause

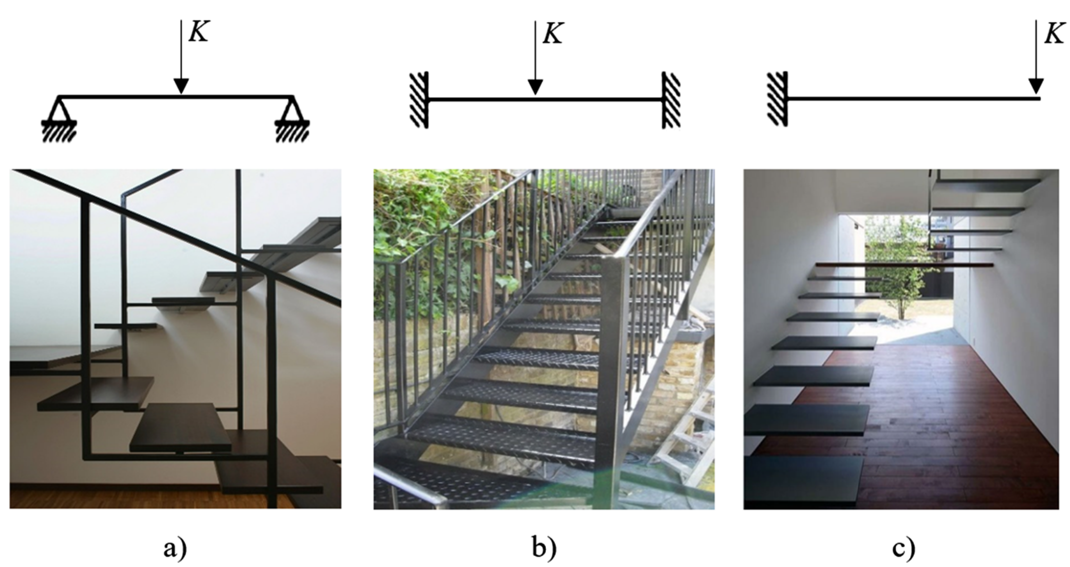

Fig. 13. Typical support conditions of steps: (a) pinned, (b) fixed, (c) cantilever.

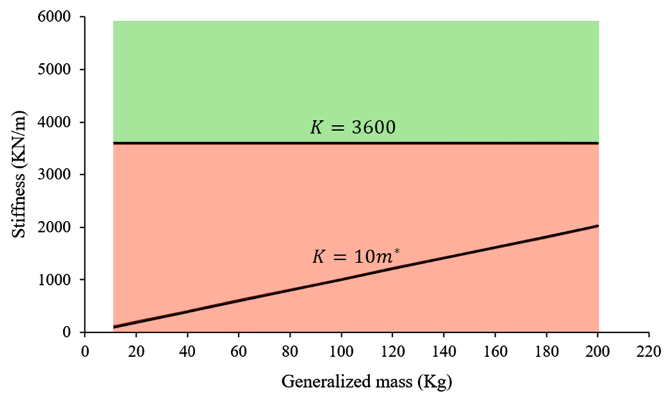

Fig. 14. Safe (green) and unsafe (red) zones.

resonant effects, and that in staircases there is the possibility of crossing them with step frequencies of about 4.0 Hz, which correspond to the 4th harmonic, the cut-off frequency is equal to 16.0 Hz.

Considering that the combination between stiffness and generalized mass should be greater than 16.0 Hz and the angular frequency is given by $w = \sqrt{\frac{K}{m^*}}$, the following equation is obtained:

$$K \geq 10\ \ m^* \tag{8}$$

where K and m^* are respectively the vertical stiffness (kN/m) and the generalized mass (kg) of the stair step.

Considering the limits defined by Eqs. (7) and (8) is possible to define a safe zone and an unsafe zone, which is represented at green and at red in Fig. 14, respectively. If a determined stair step is designed within the safe zone, the probability of significant vibrations becomes much reduced.

From Fig. 14 it is noticed that Eq. (8) will never be conditioning, so employing Eq. (7) will give rise to step's geometries with natural frequencies higher than 16 Hz, hence avoiding the arise of resonant responses.

4. Discussion and application

4.1. Discussion

According to Middleton and Brownjohn [24], the use of lightweight materials in the construction of floors with high stiffness, causing high natural frequencies to avoid resonant effects, may not be the most suitable. In fact, an impulsive response can sometimes generate excessive acceleration values, as was verified by the experimental results of the analysed staircase.

During the experimental program the maximum accelerations measured were higher than the gravitational acceleration, inclusively reaching values almost twice this acceleration. That is, during some instants the ascendant force created by the step in the pedestrian is almost twice his dead weight. This means that if pedestrians were a completely rigid body and the acting time of this acceleration level was longer, they could be pushed out of the staircase. This is one reason which helps to explain why structures that respond impulsively can also produce unacceptable levels of vibration.

Clearly, the level of accelerations verified in the stair steps is unacceptable. However, when dealing with vibrations only felt in the foot, the tolerance of individuals to this type of vibrations is higher, the discomfort felt being much reduced when compared with whole body vibrations. In these cases, it seems appropriate to consider a higher acceptable limit of accelerations, and it was therefore decided in the deduction of the pre-design expression to use the frequency-weighted base curve for vibrations perceptibility with a multiplying factor of 128.

As explained in Section 3.4, staircases should be designed with natural frequencies higher than 16.0 Hz in order to avoid resonant effects of arising. However, using Eq. (7), this is no longer a conditioning factor, which makes the same more suitable and convenient to be applied in the pre-design of stair steps.

As Eq. (7) was obtained based only on numerical analyses adjusted from measurements performed uniquely on the steel steps tested in this work, it should be verified experimentally for other spans and types of sections. This expression was developed based on numerical models and tests performed on metal steps, but theoretically it can be used for other types of materials, since the fundamental principles of dynamics used to deduce it are independent of the type of material. However, by only performing experimental tests on steps composed of other types of materials and spans, it is possible to confirm it.

4.2. Application of the pre-design expression to the stair step analysed

This section describes an example of application where the dimensions of the studied step were changed with the purpose of verifying the geometric characteristics needed to satisfy the stiffness condition imposed by Eq. (7). In this example, two different solutions were tested: the first solution consisted in increasing the step height (H), maintaining the remaining dimensions (width, length and thickness) and the second solution consisted in increasing the step thickness (t), maintaining the remaining dimensions (width, length and height). Both solutions were tested in two distinct models of the step: with fixed supports and pinned supports, which amounted to a total of 4 different simulations. It should be emphasized that the numerical model with pinned supports is representative of the actual step and it is calibrated with the real model as specified in Section 3.2.1. Fig. 15 demonstrates the results of the simulations performed.

As expected, it is verified that increasing only the step height (H) does not practically increase the step mass, but increasing the thickness (t) increases the mass almost proportionally.

Fig. 16 represents the steps sections that fulfil Eq. (7), corresponding to the intersection of the simulations cases with the pre-design constant straight line in Fig. 15. As predictable, increasing the step height is much more efficient than increasing the step thickness. In the studied steps, it can be concluded that it would be easier to avoid excessive vibrations if the webs were twice the height. The cost increase for the whole staircase would be minimal.

The minimum dimensions of the various steps shown in Fig. 16 were obtained by increasing the thickness and height separately. In practice, when designing stair steps, it is more feasible to combine the increase of the height with some increase of the thickness as well, although always giving more emphasis to the step height.

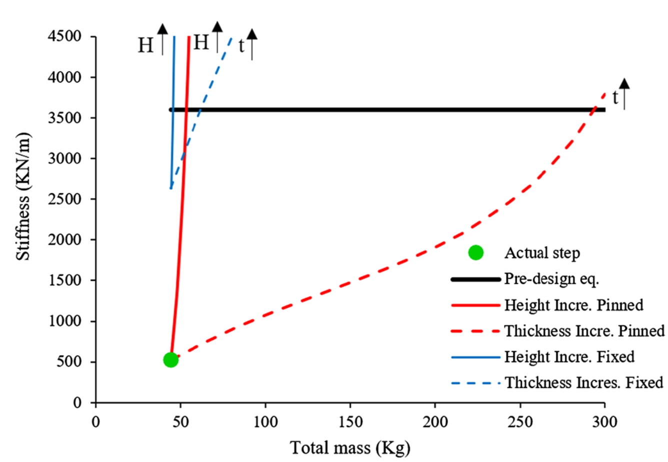

Fig. 15. Application of the pre-design expression to the stair step analysed.

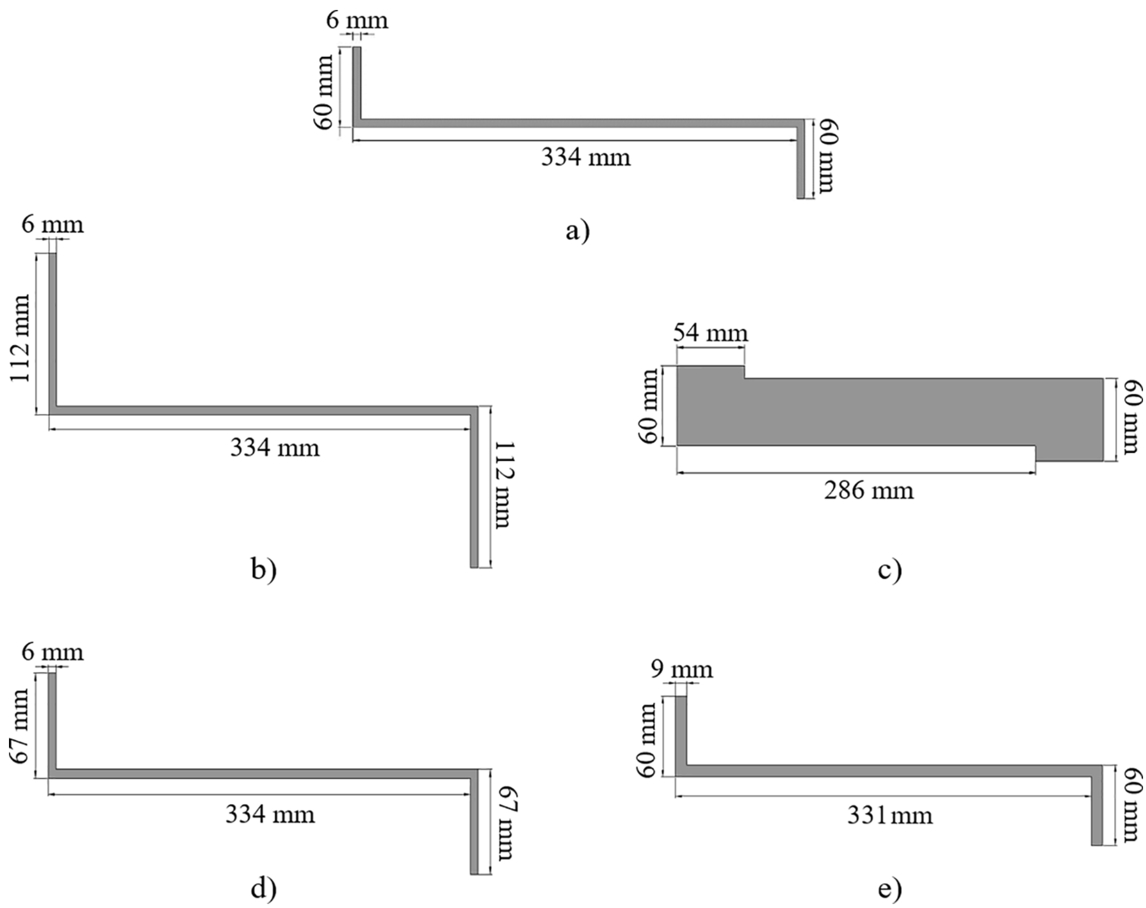

Fig. 16. Step sections that satisfy the pre-design expression: (a) actual step, (b) height increment pinned, (c) thickness increment pinned, (d) height increment fixed, (e) thickness increment fixed.

5. Summary and conclusions

In staircase design, one of the major difficulties that can arise for designers is how to define the geometric characteristics of the staircase and its steps in order to avoid the occurrence of significant vibrations.

To overcome this difficulty, this paper presents a simplified expression (Eq. (7)) to be used in the pre-design of stair steps, to avoid excessive vibrations. This expression depends only on the vertical stiffness of the steps, so it can be easily applied. As it is a pre-design approach, it differs from classic design because, instead of the need to calculate accelerations by complex numerical analysis, it suggests the application of a simplified expression, with the purpose of conferring certain geometric characteristics to the steps, which will not give rise in the future to vibration problems.

It should be emphasized that the presented pre-design expression also implies that the projected steps will have a natural frequency higher than 16.0 Hz, thus avoiding the occurrence of resonance. The pre-design expression for stair steps can be applied to several materials (like steel, aluminium and timber), in different structural systems, and for spans between 1.0 m and 3.0 m. Therefore, it can be potentially useful during the design of staircases of any construction. The application example has shown that it would be easier to avoid excessive vibrations, with a negligible cost increase, if the pre-design expression were applied.

This paper presents a study of a steel staircase, whose steps have a large level of vibrations, causing a significant amount of discomfort to its users. The steps have a high natural frequency (24.0 Hz), which means that their response was impulsive and not in resonance. The maximum accelerations measured were slightly less than twice the gravitational acceleration.

Kerr [3] indicated that “Any staircase having a natural frequency less than 10 Hz will likely be susceptible to human induced resonance which would probably produce unacceptable levels of vibration”. It should be added to this affirmation that “designing structures solely with a high natural frequency in order to avoid the occurrence of resonant effects, not considering the actual value of stiffness and mass, may not be the most adequate either”. In fact, structures that respond impulsively have a high probability of being subjected to significant vibration problems if the vertical stiffness is not high enough, as is the case of the stair steps under consideration in this paper.

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