Dynamic Intuitionistic Fuzzy Multiattribute Decision Making Based on Evidential Reasoning and Modified Dynamic Intuitionistic Fuzzy Weighted Geometric Operator

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Abstract

The present work is focused on dynamic intuitionistic fuzzy multi-attribute decision making (DIF-MADM) problem, while dynamic means the decision-related information may be collected at different periods, a situation commonly happened in many of real world MADM problems. After the review and analysis of some drawbacks on the existing DIF-MADM methods, on the one hand, we propose a new DIF-MADM methods based on the evidential reasoning algorithm in order to address some of those limits; on the other hand, and a new dynamic intuitionistic fuzzy weighted geometric operator is introduced, named modified dynamic intuitionistic fuzzy weighted geometric (MDIFWG) operator, then a MDIFWG-based DIF-MADM method is also proposed to address some other limits of the existing methods. Some numerical examples are provided to illustrate the practicality and feasibility of the proposed two methods through, the comparative analysis with the existing DIF-MADM methods, along with some sensitivity analyses also carried out to analyse the distinct features of the proposed methods.

Keywords: Dynamic intuitionistic fuzzy multi-attribute decision making (DIF-MADM), Modified dynamic intuitionistic fuzzy weighted geometric (MDIFWG) operator, Evidential reasoning algorithm

1. Introduction

As an important extension of fuzzy set, Intuitionistic Fuzzy Set (IFS) [1, 2, 3] is characterized by three parameters at the same time, namely, a membership degree, a nonmembership degree and an indeterminacy degree are adopted at the same time. Therefore, IFS is considered to be more appropriate to represent and deal with imprecise, uncertain and vague information in some decision making problems. In last few years, some fuzzy multi-attribute decision making

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methods based on IFS have been proposed, e.g., [5, 7, 10, 11, 16, 21, 30, 31, 32, 33, 36, 37, 42, 45, 46], among others. All these studies are focused on the decision making problems where all the decision-related information are provided at the same period, however, those information are usually collected at different periods in many real decision problems. To handle this type of situation, Xu and Yager [39] investigated dynamic intuitionistic fuzzy multi-attribute decision making (DIF-MADM) problems where all the attribute values are expressed as intuitionistic fuzzy numbers (IFNs) collected at different periods.

Regardless of the MADM problem based on IFS or DIF-MADM problem, aggregation of intuitionistic fuzzy information is always one of key research issues. Accordingly, many aggregation operators have been introduced under intuitionistic fuzzy environment and applied to different MADM problems, e.g., as far as IFS is concerned, intuitionistic fuzzy weighted averaging (IFWA) operator [34], intuitionistic fuzzy ordered weighted averaging (IFOWA) operators [34], intuitionistic fuzzy hybrid aggregation (IFHA) operator [40], intuitionistic fuzzy weighted geometric (IFWG) operator [28, 38], intuitionistic fuzzy ordered weighted geometric (IFOWG) operators [38], intuitionistic fuzzy hybrid geometric (IFHA) operators [38] and other induced aggregation operators [17, 20, 29, 35, 43, 44]. In addition, different aggregation operators have been also introduced and applied into different DIF-MADM methods [4, 8, 12, 18, 19, 25, 27, 52], e.g., dynamic intuitionistic fuzzy weighted averaging (DIFWA) operator [39], uncertain dynamic intuitionistic fuzzy weighted averaging (UDIFWA) operator [39], dynamic intuitionistic fuzzy weighted geometric (DIFWG) operator [26, 32, 41], uncertain dynamic intuitionistic fuzzy weighted geometric (UDIFWG) operators [26, 32, 41], dynamic intuitionistic fuzzy weighted averaging Einstein (DIFWA$^{\epsilon}$) operator and dynamic intuitionistic fuzzy weighted geometric Einstein (DIFWG$^{\epsilon}$)operator [15].

Although different aggregation operators have been introduced, they still cannot help to overcome the drawback of some existing DIF-MADM methods which result in unreasonable preference orders of alternatives in some decision situations [15, 26, 32, 39]. Motivated by this limitation in some existing DIF-MADM methods, this paper aims at proposing new DIF-MADM strategy and new aggregation operators and evaluates their feasibility and performance compared with the existing work.

In order to improve the DIF-MADM method, we proposed to use new strategy based on evidential reasoning (ER) methodology. On the basis of Dempster-Shafer Theory [13, 14], Yang and Xu [47, 48] proposed an ER algorithm for MADA under uncertainty. Since then, ER methodology/algorithms have been successfully used in different decision making problems [9, 22, 23, 24, 49, 50, 51, 53]. Specially, Yang et al. [51] presented an ER approach for MADA under both probabilistic and fuzzy uncertainties. Chen et al [10] took the advantage of the ER methodology and the representation capability of IFSs to propose a new fuzzy MADM method based on the ER methodology. Chen et al. [11] also proposed a new method for fuzzy MADM based on the transformation techniques between IFN and rightangled triangular fuzzy numbers along with a new intuitionistic fuzzy geometric averaging operators of IFNs. The ER methodology has shown its potential capability in MADM and the likability to be incorporated with the DIF-MADM method, this is one of main focus of the present work.

Now that aggregation operators plays the key role in DIF-MADM method, in order to overcome the drawbacks of some existing DIF-MADM methods, the second focus of the present work is on introducing and evaluating the new aggregation operators. Accordingly, a new dynamic intuitionistic fuzzy weighted geometric aggregation operators (MDIFWG) is proposed along with the corresponding DIF-MADM method. The remaining of the paper is organized as follows: Section 2 includes preliminary concepts and definitions relevant, such as IFS and intuitionistic fuzzy variable, score function, and evidential reasoning algorithm. In Section 3, we provide the formal description of DIF-MADM problems and review and analyse some drawbacks of existing DIF-MADM methods. In Section 4, a new DIF-MADM methods based on the ER algorithm is proposed first (denoted as Method I) and then a new DIFWG operator named MDIFWG operator introduced along with the MDIFWG-based DIF-MADM method (denoted as Method II). In Section 5 focuses on the evaluation of the feasibility and validity of the proposed DIF-MADM methods through some numerical examples and comparative analysis with some existing DIF-MADM method, along with some sensitivity analysis. This paper is concluded in Section 6.

2. Preliminaries

In this section, firstly some basic concepts related to intuitionistic fuzzy set and dynamic intuitionistic fuzzy set are reviewed, along with an overview of the evidential reasoning algorithm [47, 48, 50], which are the basis of the present work.

2.1. Intuitionistic fuzzy set and intuitionistic fuzzy variable

Definition 1. *[1] Let $X = \{x_1, x_2, \cdots, x_n\}$ be a finite universe of discourse, an intuitionistic fuzzy set (IFS) A in X characterized by a membership function $\mu_A : X \to [0, 1]$ and a non-membership function $\nu_A : X \to [0, 1]$, which satisfies the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. An IFS A can be expressed as*

$$A = \{\langle x, (\mu_A(x), \nu_A(x))\rangle | x \in X\}.$$

$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the degree of indeterminacy, $\pi_A(x)$ represents the degree of hesitance of x to A and is also called intuitionistic index. For convenience, called $(\mu_A(x), \nu_A(x))$ is an intuitionistic fuzzy number (IFN) and denoted by (μ_A, ν_A).

For an IFS A on the universe X, A will be reduced to a fuzzy set under the condition that intuitionistic index $\pi_A(x) = 0$ for any $x \in X$.

Refer to [41], the intuitionistic fuzzy number $(\nu_A(x_i), \mu_A(x_i))$ is the complement of a intuitionistic fuzzy number $(\mu_A(x_i), \nu_A(x_i))$, denoted as $(\mu_A(x_i), \nu_A(x_i))^C = (\nu_A(x_i), \mu_A(x_i))$.

In MADM problem, aggregation operator plays an important role in combining relevant information from multiple sources. Xu and Yager [41] developed some aggregation operators to aggregate IF information. However, these operators can only be used to deal with time independent arguments. If time is taken into account, for example, the argument information may be collected at different periods, then these aggregation operators will not work effectively. Accordingly, Xu and Yager [39] proposed the concept of intuitionistic fuzzy variables, as shown below:

Definition 2. *[39] Let t be a time variable, then $\alpha(t) = (\mu_{\alpha(t)}, \nu_{\alpha(t)})$ is called an intuitionistic fuzzy variable, where $\mu_{\alpha(t)} \in [0, 1], \nu_{\alpha(t)} \in [0, 1]$ and $\mu_{\alpha(t)} + \nu_{\alpha(t)} \in [0, 1]$.*

For an intuitionistic fuzzy variable $\alpha(t)$, if $t = t_1, t_2, \cdots, t_k$, then $\alpha_{t_1}, \cdots, \alpha_{t_k}$ indicate k IFNs collected at p different periods.

2.2. Score function of decision-making problem

Given a finite set of alternatives, an intuitionistic fuzzy MADM problem is a kind of problem in which the evaluation of each alternative with respect to a set of attributes is expressed by IFNs, and the most desirable alternative is selected based on the degree of suitability to which each alternative satisfies the decision-makers requirements. However, the size relations or the inclusion relations does not exist in IFS under ambient conditions, some comparison technologies of IFNs have been developed to determine the order relations of IFNs. Score function, an important tool to evaluate IFNs in order to obtain the best alternative in decision making problem, is needed to convert IFNs into real numbers in order to become easier to compare with each other.

In the intuitionistic fuzzy MADM problem, as far as the score function is concerned, an effective score function has the following properties [29]: (1) the degree of membership, non-membership and indeterminacy (hesitation) of IFS should be considered; (2) it should have higher precision; and (3) it should also have stronger selection ability.

Wang [29] analysed limitations of existing score functions for IFS, an effective score function is given based on the cross entropy of membership degree from the non-membership degree, it is used to determine the absolute value of influence difference that the membership degree and the non-membership degree responded to the hesitation degree. The cross-entropy [29] of the degree of membership from the non-membership based on IFS is defined as follows.

Definition 3. *[29] Let $\alpha = (\mu, \nu)$ be an IFN of an IFS, the cross-entropy of the degree of membership μ from the degree of no-membership ν is called cross-entropy based on IFS, which measures the divergence between μ and ν:*

$$H(\alpha) = H(\mu, \nu) = \begin{cases} log_2 \frac{2}{2-\nu}, & \mu = 0 \\ log_2 \frac{2}{1+\nu}, & \mu = 1 \\ \mu log_2 \frac{2\mu}{(\mu+\nu)} + (1-\mu) log_2 \frac{2(1-\mu)}{2-(\mu+\nu)}, & 0 < \mu < 1 \end{cases} \tag{1}$$

From Definition 3, it is obvious that $H(\mu, \nu) \neq H(\nu, \mu)$, that is, $H(\mu, \nu)$ is not symmetric. Therefore, Definition 3 should be modified as:

$$H_M(\alpha) = \frac{H(\alpha) + H(\alpha^C)}{2}. \tag{2}$$

Theorem 1. *[29] Let $\alpha = (\mu, \nu)$ be an IFN, then $H_M(\alpha)$ satisfies the following properties:*

(1) $H_M(\alpha) \in [0, 1]$;

(2) $H_M(\alpha) = H_M(\alpha^C)$;

(3) If $\alpha = (1, 0)$ or $\alpha = (0, 1)$, then $H_M(\alpha) = 1$;

(4) If $\alpha = \alpha^C$, then $H_M(\alpha) = 0$.

Entropy is very important for measuring uncertain information. As far as the cross-entropy defined in Eq. (2) is concerned, for a given IFN $\alpha = (\mu, \nu)$, if $H_M(\alpha) = 0$, then the divergence between μ and ν responding to the degree of hesitation π_i is the smallest; if $h_M(\alpha) = 1$, then the divergence between μ and ν responding to the degree of hesitation π_i is the largest.

In order to determine the best alternative in decision making problem, an effective score function is defined as follows to measure the degree of suitability to which the alternative satisfies the DM's requirement.

Definition 4. *Let $\alpha = (\mu, \nu)$ be an IFN. The new score function of α is defined as*

$$S(\alpha) = \begin{cases} \mu - \nu + H_M(\alpha)\pi & \mu > \nu \\ \mu - \nu - H_M(\alpha)\pi & \mu < \nu \\ 0^* & \mu = \nu \end{cases} \tag{3}$$

where $\pi = 1 - \mu - \nu$ and 0^* means that S is close to 0.

For an IFN $\alpha = (\mu, \nu)$, the value of unknown degree $\pi = 1 - \mu - \nu$ is moderate under the condition $\mu = \nu$. As π denotes degree of indeterminacy, hence the degree of accuracy of IFN α will change with π change and indeterminacy of π almost have little influence on score value of α, so the value is close to 0 rather than equal to 0. Only if $\pi = 0$, i.e. $\mu = \nu = 0.5$, the value of score equal to 0, that is, the degree of indeterminacy is the smallest and the value of accuracy is the largest. For example, there are two alternatives: $\alpha_1 = (0.5, 0.5)$ and $\alpha_2 = (0.3, 0.3)$, it is obvious that $\pi_1 < \pi_2$. Therefore, $S(\alpha_2) < S(\alpha_1)$=0, it follows that the alternative α_1 is better than the alternative α_2.

Theorem 2. *Let $\alpha = (\mu, \nu)$ be an IFN. Then $S(\alpha)$ satisfies the following properties:*
(1) $S(\alpha) \in [-1, 1]$;
(2) $S(\alpha) = 1$ if and only if $\alpha = (1, 0)$;
(3) $S(\alpha) = -1$ if and only if $\alpha = (0, 1)$;
(4) If $S(\alpha) = 0$ if and only if $\alpha = (0.5, 0.5)$.

For any two IFNs α_1, α_2,
(1) if $S(\alpha_1) < S(\alpha_2)$, then $\alpha_1 \prec \alpha_2$;
(2) if $S(\alpha_1) > S(\alpha_2)$, then $\alpha_1 \succ \alpha_2$;
(3) if $S(\alpha_1) > S(\alpha_2)$, then $\alpha_1 \sim \alpha_2$.

Example 1. *Let $\alpha_1 = (0.52, 0.2), \alpha_2 = (0.7, 0.3), \alpha_3 = (0.12, 0.68)$ be three IFNs. By Eq. (2), we have*

$$H_M(\alpha_1) = 0.1519, H_M(\alpha_2) = 0.1959, H_M(\alpha_3) = 0.0841,$$

and so

$$S(\alpha_1) = 0.3625, S(\alpha_2) = 0.4, S(\alpha_3) = -0.5432.$$

Therefore $\alpha_3 \prec \alpha_1 \prec \alpha_2$.

2.3. Evidential reasoning algorithm for MADM

In this subsection, we review the ER algorithm for MADM under uncertain environment [50, 49, 51]. Let $X = \{x_1, x_2, \cdots, x_m\}$ be a set of alternatives and $A = \{a_1, a_2, \cdots, a_p\}$ be a set of attributes. Assume that there are N evaluation grades $\theta_1, \theta_2, \cdots, \theta_N$ for assessing the attributes of alternatives and denoted by $\Theta = \{\theta_1, \theta_2, \cdots, \theta_N\}$, w_i refer to the weight of attribute $a_i (i = 1, 2, \cdots, p)$, respectively, with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$. Let $S(a_i(x_j))$ denote the evaluation value of attribute a_i of alternative x_j and be defined as follows:

$$S(a_i(x_j)) = \{(\theta_n, \beta_{n,i})(x_j), n = 1, 2, \cdots, N\}, \tag{4}$$

where $i = 1, 2, \cdots, p$ and $j = 1, 2, \cdots, m$.

The assessments of the attributes of the alternatives are represented by a decision matrix $D = (S(a_i(x_j)))_{p\times m}$. Now we aggregate the assessment values of attributes for all alternatives. According to Eq. (4), the belief of degree $\beta_{\theta_n,i}(x_j)$ regarding to the ith attribute a_i of alternative x_j can be transformed into bps $m_{\theta_n,i}(x_j)$ as follows:

$$m_{n,i}(x_j) = w_i\beta_{n,i}(x_j); \tag{5}$$

$$m_{\Theta,i}(x_j) = 1 - \sum_{n=1}^{N} m_{\theta_n,i}(x_j) = 1 - w_i \sum_{n=1}^{N} \beta_{\theta_n,i}(x_j), \tag{6}$$

where $n = 1, 2, \cdots, N, i = 1, 2, \cdots, p$ and $j = 1, 2, \cdots, m$.

Below is the results aggregating the criteria (or attribute) by combining the basic probability masses generated above, where $m_{n,I(1)}(x_j) = m_{n,1}(x_j)$, $m_{\Theta,I(1)}(x_j) = m_{\Theta,1}(x_j)$,

$$\{\theta_n\}: \quad m_{n,I(i)}(x_j) = K[m_{n,I(i-1)}(x_j)m_{n,i}(x_j) + m_{n,I(i-1)}(x_j)m_{\Theta,i}(x_j) + m_{\Theta,I(i-1)}(x_j)m_{n,i}(x_j)] \tag{7}$$

$$\{\Theta\}: \quad m_{\Theta,I(i)}(x_j) = K[m_{\Theta,I(i-1)}(x_j)m_{\Theta,i}(x_j), \tag{8}$$

$$K = 1 - \sum_{r=1}^{N} \sum_{t=1,t\neq r}^{N} m_{m_r,I(i-1)}(x_j)m_{t,i}(x_j)$$

$$\{\theta_n\}: \quad \beta_n(x_j) = \frac{m_{n,I(p)}(x_j)}{1 - m_{\Theta,I(p)}(x_j)}. \tag{9}$$

From Eq. (6), we can obtain another equivalent form:

$$\beta_n(x_j) = \frac{(1 - \beta_\Theta(x_j))m_{n,I(p)}(x_j)}{1 - m_{\Theta,I(p)}(x_j)}, \tag{10}$$

where $\beta_\Theta(x_j) = \sum_{i=1}^{p} w_i(1 - \sum_{n=1}^{N} \beta_{n,i}(x_j))$.

3. Analysis of the existing DIF-MADM methods

In this section, we will review the formal representation of the typical DIF-MADM problem, and analyse their drawbacks, then in Section 4, we will introduce methods in order to overcome those drawbacks.

3.1. Formal representation of DIF-MADM

In general, MADM has always been used to find the most desirable one from a finite set of alternatives with respect to the predefined attributes. DIF-MADM methods aim at handling the MADM problems under dynamic intuitionistic fuzzy environment, especially on MADM problems with the subjective information and the attitudinal character of the decision makers. A DIF-MAGDM problem can be formally described as follows:

(1) $X = \{x_1, x_2, \cdots, x_m\}$ a set of m alternatives;

(2) $A = \{a_1, a_2, \cdots, a_n\}$ the set of n attributes whose weight vector is $w = (w_1, \cdots, w_n)$ with $w_i > 0$ and $\sum_{i=1}^{n} w_i = 1$;

(3) There are p periods $P = \{t_1, t_2, \cdots, t_p\}$, whose weight vector is $\omega(t) = (\omega(t_1), \cdots, \omega(t_p))$ with $\omega(t_k) > 0 (k = 1, 2, \cdots, p)$ and $\sum_{k=1}^{p} \omega(t_k) = 1$.

(4) The decision makers provide the attribute values of alternative $x_i \in X(i = 1, 2, \cdots, m)$ with respect to attribute $a_j(j = 1, 2, \cdots, n)$ at period $t_k(k = 1, 2, \cdots, p)$ and construct the intuitionistic fuzzy decision making matric

$$D_{t_k} = (\alpha_{ij,t_k})_{m\times n} = \begin{pmatrix} (\mu_{11,t_k}, \nu_{11,t_k}) & (\mu_{12,t_k}, \nu_{12,t_k}) & \cdots & (\mu_{1n,t_k}, \nu_{1n,t_k}) \\ (\mu_{21,t_k}, \nu_{21,t_k} & (\mu_{22,t_k}, \nu_{22,t_k}) & \cdots & (\mu_{2n,t_k}, \nu_{2n,t_k}) \\ \vdots & \vdots & \vdots & \vdots \\ (\mu_{m1,t_k}, \nu_{m1,t_k}) & (\mu_{m2,t_k}, \nu_{m2,t_k}) & \cdots & (\mu_{mn,t_k}, \nu_{mn,t_k}) \end{pmatrix}$$

where $(\mu_{ij,t_k}, \nu_{ij,t_k})$ is an IFN, μ_{ij,t_k} is the degree that alternative x_i should satisfy the attribute a_j at period t_k, ν_{ij,t_k} is the degree that alternative x_i should not satisfy the attribute a_j at period t_k, and $0 \leq \mu_{ij,t_k}, \nu_{ij,t_k} \leq 1$, $0 \leq \mu_{ij,t_k} + \nu_{ij,t_k} \leq 1$.

3.2. Analysis of the existing DIF-MADM methods

Although with some interesting and solid results, there are still some drawbacks found in the existing DIF-MADM methods presented in Gumus [15], Xu [39], Wei [32] and Park [26]. In these DIF-MADM methods, different aggregation operators were introduced. First of all, we recall some operators defined on intuitionistic fuzzy variables [39]

Let $\alpha(t_1) = (\mu_{\alpha(t_1)}, \nu_{\alpha(t_1)})$, $\alpha(t_2) = (\mu_{\alpha(t_2)}, \nu_{\alpha(t_2)})$ be two IFNs, then

(1) $\alpha(t_1) \otimes \alpha(t_2) = (\mu_{\alpha(t_1)}\mu_{\alpha(t_2)}, \nu_{\alpha(t_1)} + \nu_{\alpha(t_2)} - \nu_{\alpha(t_1)}\nu_{\alpha(t_2)})$,

(2) $\alpha(t_1) \oplus \alpha(t_2) = (\mu_{\alpha(t_1)} + \mu_{\alpha(t_2)} - \mu_{\alpha(t_1)}\mu_{\alpha(t_2)}, \nu_{\alpha(t_1)}\nu_{\alpha(t_2)})$,

(3) $\lambda\alpha(t_1) = (1 - (1 - \mu_{\alpha(t_1)})^\lambda, \nu^\lambda_{\alpha(t_1)})$,

(4) $\alpha(t_1)^\lambda = (\mu^\lambda_{\alpha(t_1)}, 1 - (1 - \nu_{\alpha(t_1)})^\lambda)$.

Base on the above definitions, some aggregation operators are defined as follows:

Let $\alpha(t_1), \alpha(t_2), \cdots, \alpha(t_p)$ be a collection of IFNs collected at p different periods t_k $(k = 1, 2, \cdots, p)$, and $\lambda(t) = (\lambda(t_1), \lambda(t_2), \cdots, \lambda(t_p))$ be the weight vector of the periods $t_k(k = 1, 2, \cdots, p)$ with $\lambda(t_i) \geq 0$ and $\sum_{i=1}^p \lambda(t_i) = 1$. Then a dynamic intuitionistic fuzzy weighted averaging (D-IFWA) operator [39] is defined as follows:

$$\begin{aligned} DIFWA(\alpha(t_1), \alpha(t_2), \cdots, \alpha(t_p)) &= \lambda(t_1)\alpha(t_1) \oplus \lambda(t_2)\alpha(t_2) \oplus \cdots \oplus \lambda(t_p)\alpha(t_p) \\ &= (1 - \prod_{i=1}^p (1 - \mu_{\alpha(t_i)})^{\lambda(t_i)}, \prod_{i=1}^p \nu_{\alpha(t_i)})^{\lambda(t_i)}); \end{aligned} \tag{11}$$

A dynamic intuitionistic fuzzy weighted geometric (DIFWG) operator [32, 26] is defined as follows:

$$\begin{aligned} DIFWG(\alpha(t_1), \alpha(t_2), \cdots, \alpha(t_p)) &= \alpha(t_1)^{\lambda(t_1)} \oplus \alpha(t_2)^{\lambda(t_2)} \oplus \cdots \oplus \alpha(t_p)^{\lambda(t_p)} \\ &= (\prod_{i=1}^p \mu^{\lambda(t_i)}_{\alpha(t_i)}, 1 - \prod_{i=1}^p (1 - \nu_{\alpha(t_i)}))^{\lambda(t_i)}). \end{aligned} \tag{12}$$

A dynamic intuitionistic fuzzy Einstein weighted geometric (DIFWG$^\epsilon$) operator [15, 28] is defined as follows:

$$\begin{aligned} & DIFWG^\epsilon(\alpha(t_1), \alpha(t_2), \cdots, \alpha(t_p)) \\ &= (\frac{2\prod_{i=1}^p \mu^{\lambda(t_i)}_{\alpha(t_i)}}{\prod_{i=1}^p (2 - \mu_{\alpha(t_i)})^{\lambda(t_i)} + \prod_{i=1}^p \mu^{\lambda(t_i)}_{\alpha(t_i)}}, \frac{\prod_{i=1}^p (1 + \nu_{\alpha(t_i)}))^{\lambda(t_i)} - \prod_{i=1}^p (1 - \nu_{\alpha(t_i)}))^{\lambda(t_i)}}{\prod_{i=1}^p (1 + \nu_{\alpha(t_i)}))^{\lambda(t_i)} + \prod_{i=1}^p (1 - \nu_{\alpha(t_i)}))^{\lambda(t_i)}}). \end{aligned} \tag{13}$$

In the following, we analyse and illustrate some drawbacks about those aggregation operators:

(**Drawback A.**) For Eq. (11), if there exist $\mu_{\alpha(t_1)} = 1$, and $\mu_{\alpha(t_2)} = \cdots = \mu_{\alpha(t_p)} = 0$, then $1 - \prod_{i=1}^{p}(1-\mu_{\alpha(t_i)})^{\lambda(t_i)} = 1$, which is incorrect because of the impact of the values $\mu_{\alpha(t_2)} = \cdots = \mu_{\alpha(t_p)} = 0$ on the aggregation result is not considered. For example, in a DIF-MADM problem, as far as alternative x_1 is concerned, if $\mu_{\alpha(t_1)} = 1$ for the first attribute at the period t_1, although the degree of memberships and no-memberships of the first attribute regarding to the alternative x_1 at others periods changed, aggregation result of the first attribute regarding to x_1 will be constant. Therefore, DIFWA operator Eq. (11) is not well defined. Accordingly the DIF-MADM method presented in [39] uses this ill-defined DIFWA operator will get an unreasonable preference order of the alternatives in some situations.

(**Drawback B.**) Considering Eq. (12) and Eq. (13), if there is only one membership degree of IFNs is equal to 0, the aggregation membership degree of IFNs is 0 even if the membership degrees of $n-1$ IFNs are not 0, this seems a rather extreme property. Therefore, the DIF-MADM methods in [32] and [15] will again lead to inappropriate preference order of alternatives in some situations.

(**Drawback C.**) besides Drawback B about, there is another drawback in the existing DIF-MADM methods presented in [15, 26, 32, 39]. The example below gives a better illustration.

Example 2. *Suppose there is an investment company, which wants to invest a sum of money in the best option (adapted from Ref.[32]). There is a panel with three possible alternatives to invest the money: (1) x_1 is a car company; (2) x_2 is a food company; (3) x_3 is a computer company. The investment company must take a decision according to the following four attributes: (1) a_1 is the risk analysis; (2) a_2 is the growth analysis; (3) a_3 is the social-political impact analysis and the environmental impact analysis. The three possible alternatives $x_i (i = 1, 2, 3)$ are to be evaluated using the intuitionistic fuzzy information by the decision maker under the above four attributes at the periods $t_k (k = 1, 2, 3)$, as listed in the following matrix, shown as Tabled 1 and 2. Let $\lambda(t) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ be the weight vector of the periods $t_k (k = 1, 2, 3)$, and $w = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ be the weight vector of the attributes $a_j (j = 1, 2, 3, 4)$.*

Table 1: Individual IF decision matrix $D_{t_k} (k = 1, 2, 3)$

years		a_1	a_2	a_3
t_1	x_1	(0.25,0.7)	(0.5, 0.2)	(0.7, 0.2)
	x_2	(0.5, 0.2)	(0.4, 0.5)	(0.4, 0.1)
	x_3	(0.4, 0.3)	(0.5, 0.3)	(0.6, 0.3)
t_2	x_1	(0.6, 0.3)	(0.4, 0.1)	(0.6, 0.1)
	x_2	(0.7, 0.1)	(0.25, 0.7)	(0.5, 0.3)
	x_3	(0.5, 0.2)	(0.7, 0.2)	(0.4, 0.5)
t_3	x_1	(0.4, 0.5)	(0.7, 0.3)	(0.5, 0.3)
	x_2	(0.6, 0.3)	(0.6, 0.3)	(0.7, 0.2)
	x_3	(0.7, 0.1)	(0.6, 0.1)	(0.25, 0.7)

Calculate the distance between the alternative x_i and the intuititonistic fuzzy positive ideal solution (IFPIS) $\alpha^+ = (1, 0)$ and the distance between the alternative x_i and the intuititonistic fuzzy negative ideal solution (IFNIS) $\alpha^- = (0, 1)$ by the equations in [39], respectively, we have

Table 2: Complex intuitionistic fuzzy decision matrix by DIFWA operators

a_1	a_2	a_3	
x_1	(0.4351, 0.4721)	(0.5515, 0.1820)	(0.6081, 0.1820)
x_2	(0.6081, 0.1820)	(0.4351, 0.4721)	(0.5515, 0.1820)
x_3	(0.5515, 0.1820)	(0.6081, 0.1820)	(0.4351, 0.4721)

$$d(x_1, \alpha^+) = d(x_2, \alpha^+) = d(x_3, \alpha^+) = 0.4684,$$
$$d(x_1, \alpha^-) = d(x_2, \alpha^-) = d(x_3, \alpha^-) = 0.7213.$$

According to [39], the closeness coefficient of each alternative is given by

$$c(x_i) = \frac{d(x_i, \alpha^-)}{d(x_i, \alpha^-) + d(x_i, \alpha^+)}.$$

It follows that $c(x_1) = c(x_2) = c(x_3) = 0.6063$. Therefore $x_1 = x_2 = x_3$, which is obviously an incorrect preference orders of alternatives. The same results also can be obtained by using Gumus's [15] and Wei's [32] DIF-MADM method based on the DIFWG operators defined in Eqs. (12) and (13).

In Section 4 bellow, two new methods are proposed to overcome the above mentioned drawbacks of the existing DIF-MADM methods.

4. New methods for DIF-MADM problems

In this section, we propose two kinds of DIF-MADM methods to overcome the drawbacks presented in Section 3. It shows that Method I can overcome the drawbacks A, B and C. And Method II can overcome the drawbacks B and C.

4.1. **Method I:** *New DIF-MADM based on the ER methodology*

Suppose that the alternatives are assessed on each attribute using the following two assessment grades: H_1 and H_2, where H_1 stands for satisfying the fuzzy concept ”excellence”, H_2 stands for not satisfying the fuzzy concept ”excellence”, and $H = \{H_1, H_2\}$ stands for the assessment grade indeterminacy. The proposed method for intuitionistic fuzzy MADM based on IFSs and the ER algorithm is now presented as follows:

Step 1. Determine the belief matrix of decision maker w.r.t. attribute a_j of alternative x_i regarding the evaluation grade H_1, H_2 as follows:

$$D_{t_k} = (\mu_{ij,t_k}, \nu_{ij,t_k})_{m\times n} = (\beta_{1j,t_k}(x_i), \beta_{2j,t_k}(x_i))_{m\times n}$$
$$= \begin{pmatrix} (\beta_{11,t_k}(x_1), \beta_{21,t_k}(x_1)) & (\beta_{12,t_k}(x_1), \beta_{22,t_k}(x_1)) & \cdots & (\beta_{1n,t_k}(x_1), \beta_{2n,t_k}(x_1)) \\ (\beta_{11,t_k}(x_2), \beta_{21,t_k}(x_2)) & (\beta_{12,t_k}(x_2), \beta_{22,t_k}(x_2)) & \cdots & (\beta_{1n,t_k}(x_2), \beta_{2n,t_k}(x_2)) \\ \vdots & \vdots & \vdots & \vdots \\ (\beta_{11,t_k}(x_m), \beta_{21,t_k}(x_m)) & (\beta_{12,t_k}(x_m), \beta_{22,t_k}(x_m)) & \cdots & (\beta_{1n,t_k}(x_m), \beta_{2n,t_k}(x_m)) \end{pmatrix}$$

where $(\mu_{ij,t_k}, \nu_{ij,t_k}) = (\beta_{1j,t_k}(x_i), \beta_{2j,t_k}(x_i))$, $\beta_{1j,t_k}(x_i)$ denotes the degree of belief of decision maker d_l w.r.t. attribute a_j of alternative x_i at period t_k regarding evaluation grade H_1 and $\beta_{2j,t_k}(x_i)$

represents the degree of belief w. r. t. attribute a_j of alternative x_i at period t_k regarding evaluation grade H_2, $0 \le \beta_{1j,t_k}(x_i), \beta_{2j,t_k}(x_i) \le 1$ and $0 \le \beta_{1j,t_k}(x_i) + \beta_{2j,t_k}(x_i) \le 1 (j = 1, 2, \cdots, n; i = , 2, \cdots, m; k = 1, 2, \cdots, p)$.

Step 1.1. Based on the above step, the intuitionistic fuzzy assessment $(\mu_{ij,t_k}, \nu_{ij,t_k})$ can be transformed into the ER belief distribution assessment profiled as

$$\{(H_1, \beta_{1j,t_k}(x_i)), (H_2, \beta_{2j,t_k}(x_i)), (H, \beta_{Hj,t_k}(x_i))\} \tag{14}$$

Transform the degree of belief $\beta_{qj,t_k}(x_i)$ into basic probability mass $\tilde{m}_{qj,t_k}(x_i)$ and $\tilde{m}_{H_j}(x_i)$ by the following formulae [49]:

$$\tilde{m}_{qj,t_k}(x_i) = w_j(t_k)\beta_{qj,t_k}(x_i); \tag{15}$$

$$\tilde{m}_{Hj,t_k}(x_i) = 1 - \sum_{j=1}^{n} \tilde{m}_{qj,t_k}(x_i). \tag{16}$$

We can then obtain the basic probability mass matrix:

$$P_{t_k} = \begin{pmatrix} (\tilde{m}_{11,t_k}(x_1), \tilde{m}_{21,t_k}(x_1)) & (\tilde{m}_{12,t_k}(x_1), \tilde{m}_{22,t_k}(x_1)) & \cdots & (\tilde{m}_{1n,t_k}(x_1), \tilde{m}_{2n,t_k}(x_1)) \\ (\tilde{m}_{11,t_k}(x_2), \tilde{m}_{21,t_k}(x_2)) & (\tilde{m}_{12,t_k}(x_2), \tilde{m}_{22,t_k}(x_2)) & \cdots & (\tilde{m}_{1n,t_k}(x_2), \tilde{m}_{2n,t_k}(x_2)) \\ \vdots & \vdots & \vdots & \vdots \\ (\tilde{m}_{11,t_k}(x_m), \tilde{m}_{21,t_k}(x_m)) & (\tilde{m}_{12,t_k}(x_m), \tilde{m}_{22,t_k}(x_m)) & \cdots & (\tilde{m}_{1n,t_k}(x_m), \tilde{m}_{2n,t_k}(x_m)) \end{pmatrix}$$

where $0 \le \tilde{m}_{1j,t_k}(x_i), \tilde{m}_{2j,t_k}(x_i) \le 1$ and $0 \le \tilde{m}_{1j,t_k}(x_i) + \tilde{m}_{2j,t_k}(x_i) \le 1 (j = 1, 2, \cdots, n; i = 1, 2, \cdots, m; k = 1, 2, \cdots, p)$.

Step 1.2. Let the combined probability mass $\tilde{n}_{q1,t_k}(x_i)$ of the decision maker d_l w.r.t. attribute a_j of alternative x_i at period t_k be equal to $\tilde{m}_{q1,t_k}(x_i)$, that is, $\tilde{n}_{q1,t_k}(x_i) = \tilde{m}_{q1,t_k}(x_i)(q = 1, 2)$. Similarly, $\tilde{n}_{H1,t_k}(x_i) = \tilde{m}_{H1,t_k}(x_i)(q = 1, 2)$. Now, calculate the combined probability mass $\tilde{n}_{qj,t_k}(x_i)$ and $\tilde{n}_{Hj,t_k}(x_i)$ w. r. t. the attribute a_j of alternative x_i at period t_k by the following equations:

$$\begin{aligned} \tilde{n}_{qj,t_k}(x_i) &= \frac{\tilde{n}_{qj-1,t_k}(x_i)\tilde{m}_{qj,t_k}(x_i) + \tilde{n}_{qj-1,t_k}(x_i)\tilde{m}_{Hj,t_k}(x_i) + \tilde{n}_{Hj-1,t_k}(x_i)\tilde{m}_{qj,t_k}(x_i)}{1 - \sum_{r=1}^{2}\sum_{h=1,h\neq r}^{2} \tilde{n}_{rj-1,t_k}(x_i)\tilde{m}_{hj,t_k}(x_i)} \\ \tilde{n}^{g}_{Hj,t_k}(x_i) &= \frac{\tilde{n}_{Hj-1,t_k}(x_i)\tilde{m}_{Hj,t_k}(x_i)}{1 - \sum_{r=1}^{2}\sum_{h=1,h\neq r}^{2} \tilde{n}_{rj-1,t_k}(x_i)\tilde{m}_{hj,t_k}(x_i)} \end{aligned} \tag{17}$$

where $j = 1, 2, \cdots, n; i = 1, 2, \cdots, m; k = 1, 2, \cdots, p$.

Step 1.3. Aggregate the evaluating values of decision makers with respect to attribute a_j of alternative x_i at period t_k to obtain the belief:

$$\beta_{q,t_k}(x_i) = \frac{(1 - \beta_{H,t_k}(x_i))\tilde{n}_{qn,t_k}(x_j)}{1 - \tilde{n}_{Hn,t_k}(x_i)} \tag{18}$$

with $\beta_{Hn,t_k}(x_i) = \sum_{j=1}^{n} w_j(t_k)(1 - \sum_{q=1}^{2} \tilde{n}_{qj,t_k}(x_i))$. where $\sum_{q=1}^{2} \beta_{q,t_k}(x_i) + \beta_{Hn,t_k}(x_i) = 1$. Let the aggregated value obtained by above equations $\beta_{1,t_k}(x_i), \beta_{2,t_k}(x_i)$ form an intuitionistic fuzzy value $(\beta_{1,t_k}(x_i), \beta_{2,t_k}(x_i))$, where $\beta_{1,t_k}(x_i)$ and $\beta_{2,t_k}(x_i)$ are the degree of decision maker d_l w.r.t. alternative x_i at period t_k regarding evaluation grades H_1 and H_2.

Step 2. Based on Step 1, construct the aggregated decision making matrix Q as follows

$$
\begin{aligned}
Q &= (\beta_{1,t_k}(x_i), \beta_{2,t_k}(x_i))_{m\times p}\\
&= \begin{pmatrix}
(\beta_{1,t_1}(x_1), \beta_{2,t_1}(x_1)) & (\beta_{1,t_2}(x_1), \beta_{2,t_2}(x_1)) & \cdots & (\beta_{1,t_p}(x_1), \beta_{2,t_p}(x_1))\\
(\beta_{1,t_1}(x_2), \beta_{2,t_1}(x_2)) & (\beta_{1,t_2}(x_2), \beta_{2,t_2}(x_2)) & \cdots & (\beta_{1,t_p}(x_2), \beta_{2,t_p}(x_2))\\
\vdots & \vdots & \vdots & \vdots\\
(\beta_{1,t_1}(x_m), \beta_{2,t_1}(x_m)) & (\beta_{1,t_2}(x_m), \beta_{2,t_2}(x_m)) & \cdots & (\beta_{1,t_p}(x_m), \beta_{2,t_p}(x_m))
\end{pmatrix}
\end{aligned}
$$

where $(\beta_{1,t_k}(x_i), \beta_{2,t_k}(x_i))$ is an IFN, $\beta_{1,t_k}(x_i)$ is the degree of belief with respect to alternative x_i at period t_k regarding evaluation grade H_1 and $\beta_{2,t_k}(x_i)$ represents the degree of belief w. r. t. alternative x_i at period t_k regarding evaluation grade H_2, $0 \le \beta_{1,t_k}(x_i), \beta_{2,t_k}(x_i) \le 1$ and $0 \le \beta_{1,t_k}(x_i) + \beta_{2,t_k}(x_i) \le 1 (i = 1, 2, \cdots, m; k = 1, 2, \cdots, p)$.

Step 2.1. Based on the above step, the intuitionistic fuzzy assessment $(\beta_{1,t_k}(x_i), \beta_{2,t_k}(x_i))$ can be transformed into the ER belief distribution assessment profiled by

$$\{(H_1, \beta_1(x_i)), (H_2, \beta_2(x_i)), (H, \beta_H(x_i))\}. \tag{19}$$

Transform the degree of belief $\beta_{q,t_k}(x_i)$ into basic probability mass $\tilde{m}_{q,t_k}(x_i)$ and $\tilde{m}_{H,t_k}(x_i)$ by the following formulae:

$$\tilde{m}_{q,t_k}(x_i) = w_k \beta_{q,t_k}(x_i); \tag{20}$$

$$\tilde{m}_{H,t_k}(x_i) = 1 - \sum_{k=1}^{p} \tilde{m}_{q,t_k}(x_i). \tag{21}$$

We can obtain the basic probability mass matrix

$$
Q = \begin{pmatrix}
(\tilde{m}_{1,t_1}(x_1), \tilde{m}_{2,t_1}(x_1)) & (\tilde{m}_{1,t_2}(x_1), \tilde{m}_{2,t_2}(x_1)) & \cdots & (\tilde{m}_{1,t_p}(x_1), \tilde{m}_{2,t_p}(x_1))\\
(\tilde{m}_{1,t_1}(x_2), \tilde{m}_{2,t_1}(x_2)) & (\tilde{m}_{1,t_2}(x_2), \tilde{m}_{2,t_2}(x_2)) & \cdots & (\tilde{m}_{1,t_p}(x_2), \tilde{m}_{2,t_p}(x_2))\\
\vdots & \vdots & \vdots & \vdots\\
(\tilde{m}_{1,t_1}(x_m), \tilde{m}_{2,t_1}(x_m)) & (\tilde{m}_{1,t_2}(x_m), \tilde{m}_{2,t_2}(x_m)) & \cdots & (\tilde{m}_{1,t_p}(x_m), \tilde{m}_{2,t_p}(x_m))
\end{pmatrix}
$$

where $0 \le \tilde{m}_{1,t_k}(x_i), \tilde{m}_{2,t_k}(x_i) \le 1$ and $0 \le \tilde{m}_{1,t_k}(x_i) + \tilde{m}_{2,t_k}(x_i) \le 1 (i = 1, 2, \cdots, m; k = 1, 2, \cdots, p)$.

Step 2.2. Let the combined probability mass $\tilde{n}_{q,t_1}(x_i)$ w.r.t. alternative x_i at period t_1 be equal to $\tilde{m}_{q,t_1}(x_i)$, that is, $n_{q,t_1}(x_i) = \tilde{m}_{q,t_1}(x_i)(q = 1, 2)$. Similarly, $n_{C,t_1}(x_i) = \tilde{m}_{C,t_1}(x_i)(q = 1, 2)$. Now, calculate the combined probability mass $\tilde{n}_{q,t_k}(x_i)$ and $\tilde{n}_{C,t_k}(x_i)$ w. r. t. alternative x_i at period t_k by the following equations:

$$
\begin{aligned}
\tilde{n}_{q,t_k}(x_i) &= \frac{\tilde{n}_{q,t_{k-1}}(x_i)\tilde{m}_{q,t_k}(x_i) + \tilde{n}_{q,t_{k-1}}(x_i)\tilde{m}_{H,t_k}(x_i) + \tilde{n}_{H,t_{k-1}}(x_i)\tilde{m}_{q,t_k}(x_i)}{1 - \sum_{r=1}^{2}\sum_{h=1,h\neq r}^{2} \tilde{n}_{r,t_{k-1}}(x_i)\tilde{m}_{h,t_k}(x_i)}\\
\tilde{n}_{H,t_k}(x_i) &= \frac{\tilde{n}_{H,t_{k-1}}(x_i)\tilde{m}_{H,t_k}(x_i)}{1 - \sum_{r=1}^{2}\sum_{h=1,h\neq r}^{2} \tilde{n}_{r,t_{k-1}}(x_i)\tilde{m}_{h,t_k}(x_i)},
\end{aligned} \tag{22}
$$

where $k = 2, \cdots, p; i = 1, 2, \cdots, m$.

Step 2.3. Aggregate the evaluating values of decision makers with respect to alternative x_i to obtain the belief:

$$\beta_q(x_i) = \frac{(1 - \beta_H(x_i))\tilde{n}_q(x_j)}{1 - \tilde{n}_H(x_i)}, q = 1, 2. \tag{23}$$

with $\beta_H(x_i) = \sum_{k=1}^{p} w_k(1 - \sum_{q=1}^{2} \tilde{n}_{q,t_k}(x_i))$. where $\sum_{q=1}^{2} \beta_q(x_i) + \beta_H(x_i) = 1$. Let the aggregated value be obtained by above equations $\beta_1(x_i)$,$\beta_2(x_i)$ form an IFN ($\beta_1(x_i)$,$\beta_2(x_i)$), where $\beta_1(x_i)$ and $\beta_2(x_i)$ are the degree decision makers of alternative x_j regarding evaluation grades H_1 and H_2, respectively.

Step 3 Calculate the scores of IFN ($\beta_1(x_i)$,$\beta_2(x_i)$) obtained by the aggregation result of Step 3. Let $\alpha_i = (\beta_1(x_i),\beta_2(x_i))(i = 1, 2, \cdots, m)$.

Step 3.1 According to Eq. (1), calculate $H(\alpha_i)$ and $H(\alpha_i^C)$, where $\alpha_i^C = (\beta_2(x_i),\beta_1(x_i)),(i = 1, 2, \cdots, m)$;

Step 3.2 Calculate the score of α_i and denote as $S(\alpha_i)$;

Step 4 Determine the ranking of alternatives according to Step 3.2. The larger the value $S(\alpha_i)$, the better the order of alternative $x_i(i = 1, 2, \cdots, m)$.

Step 5 End.

4.2. **Method II:** *New DIF-MADM method based on new aggregation operators*

In this section, we propose new operators in IFNs and further propose the new DIFWG operator, and then introduce a new DIF-MADM method which can overcome some drawbacks analysed in Section 3.

4.2.1. *New aggregation operators for DIF-MADM problems*

Before the new operator of IFNs are given, we firstly introduce a new definition of operation on intuitionistic fuzzy variables.

Definition 5. *Let $\alpha(t_1) = (\mu_{\alpha(t_1)}, \nu_{\alpha(t_1)}), \alpha(t_2) = (\mu_{\alpha(t_2)}, \nu_{\alpha(t_2)})$ be two IFNs. Then*

(1) $\alpha(t_1) \otimes \alpha(t_2) = (\mu_{\alpha(t_1)} + \mu_{\alpha(t_2)} - \mu_{\alpha(t_1)}\mu_{\alpha(t_2)}, \nu_{\alpha(t_1)}(1 - \mu_{\alpha(t_2)} - \nu_{\alpha(t_2)}) + \nu_{\alpha(t_2)}(1 - \mu_{\alpha(t_1)}))$,

(2) $\alpha(t_1)^{\lambda} = (1 - (1 - \mu_{\alpha(t_1)})^{\lambda}, (1 - \mu_{\alpha(t_1)})^{\lambda} - (1 - \mu_{\alpha(t_1)} - \nu_{\alpha(t_1)})^{\lambda})$.

Base on the above operators, a modified DIFWG aggregation operator is defined as follows:

Definition 6. *Let $\alpha(t_1), \alpha(t_2), \cdots, \alpha(t_p)$ be a collection of IFNs collected at p different periods $t_k(k = 1, 2, \cdots, p)$, and $\lambda(t) = (\lambda(t_1), \lambda(t_2), \cdots, \lambda(t_p))$ be the weight vector of the periods $t_k(k = 1, 2, \cdots, p)$ with $\lambda(t_i) \geq 0$ and $\sum_{i=1}^{p} \lambda(t_i) = 1$. Then a modified dynamic intuitionistic fuzzy weighted geometric (MDIFWG) operator is defined as follows:*

$$MDIFWG(\alpha(t_1), \alpha(t_2), \cdots, \alpha(t_p)) = \alpha(t_1)^{\lambda(t_1)} \oplus \alpha(t_2)^{\lambda(t_2)} \oplus \cdots \oplus \alpha(t_p)^{\lambda(t_p)}. \tag{24}$$

Based on (1), (2) in Definition 3, we have:

$$\begin{aligned} &MDIFWG(\alpha(t_1), \alpha(t_2), \cdots, \alpha(t_p)) \\ &= (1 - \prod_{i=1}^{p}(1 - \mu_{\alpha(t_i)}))^{\lambda(t_i)}, \prod_{i=1}^{p}(1 - \mu_{\alpha(t_i)})^{\lambda(t_i)} - \prod_{i=1}^{p}(1 - \mu_{\alpha(t_i)} - \nu_{\alpha(t_i)})^{\lambda(t_i)}). \end{aligned} \tag{25}$$

4.3. *New DIF-MADM method based on MDIFWG operators*

In this section, we design a new method for DIF-MADM based on the proposed MDIFWG operator presented in Section 4.1.1. The details of this method are described as follows:

Step 1. Utilize the MDIFWG operator to aggregate all the intuitionistic fuzzy decision matrices $D_{t_k} = (\alpha_{ij,t_k})_{m\times n}(k = 1, 2, \cdots, p)$ into a complex intuitionistic fuzzy decision matrix $D = (\alpha_{ij})_{m\times n}$:

$$\begin{aligned}\alpha_{ij} &= MDIFWG((\mu_{ij,t_1}, \nu_{ij,t_1}), (\mu_{ij,t_2}, \nu_{ij,t_2}), \cdots, (\mu_{ij,t_p}, \nu_{ij,t_p})) \\ &= (1 - \prod_{k=1}^{p}(1 - \mu_{ij,t_k}))^{\lambda(t_k)}, \prod_{k=1}^{p}(1 - \mu_{ij,t_k})^{\lambda(t_k)} - \prod_{k=1}^{p}(1 - \mu_{ij,t_k} - \nu_{ij,t_k})^{\lambda(t_k)}),\end{aligned} \tag{26}$$

where $\alpha_{ij} = (\mu_{ij}, \nu_{ij})$ is an IFN obtained by Eq. (30) or Eq. (31).

Step 2- Step 6 are the same as Xu's method [39].

Step 2. Define the intuitionistic fuzzy ideal solution (IFIS) $\alpha^+ = (\alpha_1^+, \cdots, \alpha_m^+)$ and the intuitionistic fuzzy negative ideal solution (IFNIS) $\alpha^- = (\alpha_1^-, \cdots, \alpha_m^-)$, respectively, where $\alpha_i^+ = (1, 0)(i = 1, 2, \cdots, n)$ are the n largest IFNs and $\alpha_i^+ = (0, 1)(i = 1, 2, \cdots, n)$ are the n smallest IFNs. Furthermore, for convenience, we denote the alternatives $x_i(i = 1, 2, \cdots, m)$ by $x_i = (\alpha_{i1}, \alpha_{i2}, \cdots, \alpha_{in}), i = 1, 2, ..., m$.

Step 3. Calculate the distance between the alternative x_i and the IFIS α^+ and the distance between the alternative x_i and the IFNIS α^-, respectively:

$d(x_i, \alpha^+) = \sum_{j=1}^{n} w_j(1 - \mu_{ij})$,

$d(x_i, \alpha^-) = \sum_{j=1}^{n} w_j(1 - \nu_{ij})$.

Step 4. Calculate the closeness coefficient of each alternative

$$c(x_i) = \frac{d(x_i, \alpha^-)}{d(x_i, \alpha^+) + d(x_i, \alpha^-)}.$$

Step 5. Determine the preference orders of all the alternatives $x_i(i = 1, 2, \cdots, m)$ according to the closeness coefficients $c(x_i)(i = 1, 2, \cdots, n)$, the greater the value $c(x_i)$, the better the alternative x_i.

Step 6. End.

5. Case study

In this section, we use some examples to illustrate and compare the proposed methods with some existing DIF-MADM methods.

5.1. Examples and comparative analysis

Example 3. *A problem of evaluating university faculty for tenure and promotion (adapted from Bryson and Mobolurin [26]) is used to illustrate the developed approach. A practical use of the proposed approach involves the evaluation of university faculty for tenure and promotion. The attributes at some university are A_1: teaching, A_2: research, and A_3: service. The committee evaluates the performance of five faculty candidates (alternatives) $x_i(i = 1, 2, 3, 4, 5)$ in the three years $t_k(j = 1, 2, 3)$. According to the attribute $G_j(j = 1, 2, 3)$, and construct, respectively, the intuitionistic fuzzy decision matrices $D_{t_k}(k = 1, 2, 3)$. Let $\lambda(t) = (0.2, 0.3, 0.5)$ be the weight vector of the years t_k and $w = (0.3, 0.4, 0.3)$ be the weight vector of the attributes $A_j(j = 1, 2, 3)$*

(1) **Method I:** We utilize the ER algorithm.

Table 3: Individual IF decision matrix $D_{t_k}(k = 1, 2, 3)$

years		a_1	a_2	a_3
	x_1	(0.8, 0.1)	(0.9, 0.1)	(0.7, 0.2)
	x_2	(0.7, 0.3)	(0.6, 0.2)	(0.6, 0.3)
t_1	x_3	(0.5, 0.4)	(0.7, 0.3)	(0.6, 0.3)
	x_4	(0.9, 0.1)	(0.7, 0.2)	(0.8, 0.2)
	x_5	(0.6, 0.1)	(0.8, 0.2)	(0.5 0.1)
	x_1	(0.9, 0.1)	(0.8, 0.2)	(0.8, 0.1)
	x_2	(0.8, 0.2)	(0.5, 0.1)	(0.7, 0.2)
t_2	x_3	(0.5, 0.5)	(0.7, 0.2)	(0.8, 0.2)
	x_4	(0.9, 0.1)	(0.9, 0.1)	(0.7, 0.3)
	x_5	(0.5, 0.2)	(0.6, 0.3)	(0.6, 0.2)
	x_1	(0.7, 0.1)	(0.9, 0.1)	(0.9, 0.1)
	x_2	(0.9, 0.1)	(0.6, 0.2)	(0.5, 0.2)
t_3	x_3	(0.4, 0.5)	(0.8, 0.1)	(0.7, 0.1)
	x_4	(0.8, 0.1)	(0.7, 0.2)	(0.9, 0.1)
	x_5	(0.6, 0.3)	(0.8, 0.2)	(0.7, 0.2)

Step 1.1: Based on the decision matrices D_{t_1}, D_{t_2} and D_{t_3}. Based on the weight of attributes and Eq. (15), Eq. (16), we can obtain the basic probability mass P_{t_1}, P_{t_2} and P_{t_3}:

$$P_{t_1} = \begin{pmatrix} (0.24, 0.03) & (0.36, 0.04) & (0.21, 0.06) \\ (0.21, 0.09) & (0.24, 0.08) & (0.18, 0.09) \\ (0.15, 0.12) & (0.28, 0.12) & (0.18, 0.09) \\ (0.27, 0.03) & (0.28, 0.08) & (0.24, 0.06) \\ (0.18, 0.03) & (0.32, 0.08) & (0.15, 0.03) \end{pmatrix}$$

$$P_{t_2} = \begin{pmatrix} (0.27, 0.03) & (0.32, 0.08) & (0.24, 0.03) \\ (0.24, 0.06) & (0.2, 0.04) & (0.21, 0.06) \\ (0.15, 0.15) & (0.28, 0.08) & (0.24, 0.06) \\ (0.27, 0.03) & (0.36, 0.04) & (0.21, 0.09) \\ (0.15, 0.06) & (0.24, 0.12) & (0.18, 0.06) \end{pmatrix}$$

$$P_{t_3} = \begin{pmatrix} (0.21, 0.03) & (0.36, 0.04) & (0.27, 0.03) \\ (0.27, 0.03) & (0.24, 0.08) & (0.15, 0.06) \\ (0.12, 0.15) & (0.32, 0.04) & (0.21, 0.03) \\ (0.24, 0.03) & (0.28, 0.08) & (0.27, 0.03) \\ (0.18, 0.09) & (0.32, 0.08) & (0.21, 0.06) \end{pmatrix}$$

Step 1.2 We can obtain the combined probability based on Eq. (17)

$$\begin{aligned}
&\tilde{n}_{13,t_1}(x_1) = 0.5911, \tilde{n}_{23,t_1}(x_1) = 0.0686, \tilde{n}_{13,t_1}(x_2) = 0.457, \tilde{n}_{23,t_1}(x_2) = 0.1598,\\
&\tilde{n}_{13,t_1}(x_3) = 0.4341, \tilde{n}_{23,t_1}(x_3) = 0.2053, \tilde{n}_{13,t_1}(x_4) = 0.568, \tilde{n}_{23,t_1}(x_4) = 0.0928,\\
&\tilde{n}_{13,t_1}(x_5) = 0.5016, \tilde{n}_{23,t_1}(x_5) = 0.0896, \tilde{n}_{13,t_2}(x_1) = 0.5971, \tilde{n}_{23,t_2}(x_1) = 0.0755,\\
&\tilde{n}_{13,t_2}(x_2) = 0.4891, \tilde{n}_{23,t_2}(x_2) = 0.0978, \tilde{n}_{13,t_2}(x_3) = 0.4754, \tilde{n}_{23,t_2}(x_3) = 0.1709,\\
&\tilde{n}_{13,t_2}(x_4) = 0.5998, \tilde{n}_{23,t_4}(x_4) = 0.0814, \tilde{n}_{13,t_2}(x_5) = 0.4261, \tilde{n}_{23,t_2}(x_5) = 0.1576,\\
&\tilde{n}_{13,t_3}(x_1) = 0.6128, \tilde{n}_{23,t_3}(x_1) = 0.0523, \tilde{n}_{13,t_3}(x_2) = 0.4953, \tilde{n}_{23,t_3}(x_2) = 0.1022,\\
&\tilde{n}_{13,t_3}(x_3) = 0.48, \tilde{n}_{23,t_3}(x_3) = 0.1294, \tilde{n}_{13,t_3}(x_4) = 0.5742, \tilde{n}_{23,t_4}(x_2) = 0.0772,\\
&\tilde{n}_{13,t_5}(x_3) = 0.5147, \tilde{n}_{23,t_3}(x_5) = 0.133
\end{aligned}$$

We can obtain the remaining combined probability based on Eq. (18)

$$\begin{aligned}
&\tilde{n}_{H3,t_1}(x_1) = 0.34, \tilde{n}_{H3,t_1}(x_2) = 0.3832, \tilde{n}_{H3,t_1}(x_3) = 0.3606, \tilde{n}_{H3,t_1}(x_4) = 0.3391, \tilde{n}_{H3,t_1}(x_5) = 0.4087;\\
&\tilde{n}_{H3,t_2}(x_1) = 0.3275, \tilde{n}_{H3,t_2}(x_2) = 0.4131, \tilde{n}_{H3,t_2}(x_3) = 0.3537, \tilde{n}_{H3,t_2}(x_4) = 0.3187, \tilde{n}_{H3,t_2}(x_5) = 0.4163;\\
&\tilde{n}_{H3,t_3}(x_1) = 0.3349, \tilde{n}_{H3,t_3}(x_2) = 0.4024, \tilde{n}_{H3,t_3}(x_3) = 0.3905, \tilde{n}_{H3,t_3}(x_4) = 0.3486, \tilde{n}_{H3,t_3}(x_5) = 0.3522.
\end{aligned}$$

Step 1.3 Aggregate the evaluating values of with respect to attribute a_1, a_2, a_3 of alternative x_1, x_2, x_3 at the periods t_1, t_2, t_3 to obtain the belief distributions based on Eq. (19) as follows:

$$\begin{aligned}
&\beta_{1,t_1}(x_1) = 0.8422, \beta_{2,t_1}(x_1) = 0.0978, \beta_{1,t_1}(x_2) = 0.6595, \beta_{2,t_1}(x_2) = 0.2305,\\
&\beta_{1,t_1}(x_3) = 0.6381, \beta_{2,t_1}(x_3) = 0.3019, \beta_{1,t_1}(x_4) = 0.8251, \beta_{2,t_1}(x_4) = 0.1349,\\
&\beta_{1,t_1}(x_5) = 0.6702, \beta_{2,t_1}(x_5) = 0.1198; \beta_{1,t_2}(x_1) = 0.8611, \beta_{1,t_2}(x_1) = 0.1089,\\
&\beta_{1,t_2}(x_2) = 0.675, \beta_{1,t_2}(x_2) = 0.135, \beta_{1,t_2}(x_3) = 0.7061, \beta_{1,t_2}(x_3) = 0.2539,\\
&\beta_{1,t_2}(x_4) = 0.8805, \beta_{1,t_2}(x_4) = 0.1195, \beta_{1,t_2}(x_5) = 0.5913, \beta_{1,t_2}(x_5) = 0.2187;\\
&\beta_{1,t_3}(x_1) = 0.866, \beta_{1,t_3}(x_1) = 0.074, \beta_{1,t_3}(x_2) = 0.688, \beta_{1,t_3}(x_2) = 0.142,\\
&\beta_{1,t_3}(x_3) = 0.6853, \beta_{1,t_3}(x_3) = 0.1847, \beta_{1,t_3}(x_4) = 0.82, \beta_{1,t_3}(x_4) = 0.1102\\
&\beta_{1,t_3}(x_5) = 0.7469, \beta_{1,t_3}(x_5) = 0.1931
\end{aligned}$$

Let the aggregated value obtained by the belief distributions $\beta_{1,t_k}(x_i), \beta_{2,t_k}(x_i)$ form the intuitionistic fuzzy values $(\beta_{1,t_k}(x_i), \beta_{2,t_k}(x_i))$ as follows:

$$\begin{aligned}
&(\beta_{1,t_1}(x_1), \beta_{2,t_1}(x_1)) = (0.8422, 0.0978), (\beta_{1,t_1}(x_2), \beta_{2,t_1}(x_2)) = (0.6595, 0.2305),\\
&(\beta_{1,t_1}(x_3), \beta_{2,t_1}(x_3)) = (0.6381, 0.3019), (\beta_{1,t_1}(x_4), \beta_{2,t_1}(x_4)) = (0.8251, 0.1349),\\
&(\beta_{1,t_1}(x_5), \beta_{2,t_1}(x_5)) = (0.6702, 0.1198); (\beta_{1,t_2}(x_1), \beta_{1,t_2}(x_1)) = (0.8611, 0.1089),\\
&(\beta_{1,t_2}(x_2), \beta_{1,t_2}(x_2)) = (0.675, 0.135), (\beta_{1,t_2}(x_3), \beta_{1,t_2}(x_3)) = (0.7061, 0.2539),\\
&(\beta_{1,t_2}(x_4), \beta_{1,t_2}(x_4)) = (0.8805, 0.1195), (\beta_{1,t_2}(x_5), \beta_{1,t_2}(x_5)) = (= 0.5913, 0.2187);\\
&(\beta_{1,t_3}(x_1), \beta_{1,t_3}(x_1)) = (0.866, 0.074), (\beta_{1,t_3}(x_2), \beta_{1,t_3}(x_2)) = (0.688, 0.142),\\
&(\beta_{1,t_3}(x_3), \beta_{1,t_3}(x_3)) = (0.6853, 0.1847), (\beta_{1,t_3}(x_4), \beta_{1,t_3}(x_4)) = (0.82, 0.1102)\\
&(\beta_{1,t_3}(x_5), \beta_{1,t_3}(x_5)) = (0.7469, 0.1931)
\end{aligned}$$

Step 2. Based on Step 1, construct the aggregation decision making matrix Q as follows

$$Q = (\beta_{1,t_k}(x_i), \beta_{2,t_k}(x_i))_{3\times 3}$$
$$= \begin{pmatrix} (\beta_{1,t_1}(x_1), \beta_{2,t_1}(x_1)) & (\beta_{1,t_2}(x_1), \beta_{2,t_2}(x_1)) & (\beta_{1,t_3}(x_1), \beta_{2,t_3}(x_1)) \\ (\beta_{1,t_1}(x_2), \beta_{2,t_1}(x_2)) & (\beta_{1,t_2}(x_2), \beta_{2,t_2}(x_2)) & (\beta_{1,t_3}(x_2), \beta_{2,t_3}(x_2)) \\ (\beta_{1,t_1}(x_3), \beta_{2,t_1}(x_3)) & (\beta_{1,t_2}(x_3), \beta_{2,t_2}(x_3)) & (\beta_{1,t_3}(x_3), \beta_{2,t_3}(x_3)) \\ (\beta_{1,t_1}(x_4), \beta_{2,t_1}(x_4)) & (\beta_{1,t_2}(x_4), \beta_{2,t_2}(x_4)) & (\beta_{1,t_3}(x_4), \beta_{2,t_3}(x_4)) \\ (\beta_{1,t_1}(x_5), \beta_{2,t_1}(x_5)) & (\beta_{1,t_2}(x_5), \beta_{2,t_2}(x_5)) & (\beta_{1,t_3}(x_5), \beta_{2,t_3}(x_5)) \end{pmatrix}$$
$$= \begin{pmatrix} (0.8422, 0.0978) & (0.8611, 0.1089) & (0.866, 0.074) \\ (0.6595, 0.2305) & (0.675, 0.135) & (0.688, 0.142) \\ (0.6381, 0.3019) & (0.7061, 0.2536) & (0.6853, 0.1847) \\ (0.8251, 0.1349) & (0.8805, 0.1195) & (0.8198, 0.1102) \\ (0.6702, 0.1198) & (0.5913, 0.2187) & (0.7469, 0.1931) \end{pmatrix}$$

Step 2.1. Based on Step 2 and Eq. (21), we can obtain the basic probability mass matrix

$$Q = \begin{pmatrix} (0.1684, 0.0196) & (0.2583, 0.0327) & (0.433, 0.037) \\ (0.132, 0.04611) & (0.2025, 0.0405) & (0.344, 0.071) \\ (0.1276, 0.06037) & (0.2118, 0.0762) & (0.3426, 0.0924) \\ (0.165, 0.027) & (0.26416, 0.0358) & (0.41, 0.0551) \\ (0.134, 0.024) & (0.1774, 0.0656) & (0.3735, 0.0965) \end{pmatrix}$$

Step 2.2. Based on Eq. (23) and Eq. (24), we can calculate the combined probability mass $\tilde{n}_{q,t_k}(x_i)$ and $\tilde{n}_{H,t_k}(x_i)$ w.r.t. alternative x_i at the period t_k as follows:

$$n_{1,3}(x_1) = 0.635, n_{2,3}(x_1) = 0.0465, n_{1,3}(x_2) = 0.5171, n_{2,3}(x_2) = 0.0959, n_{1,3}(x_3) = 0.5051,$$
$$n_{2,3}(x_3) = 0.1372, n_{1,3}(x_4) = 0.6169, n_{2,3}(x_4) = 0.0634, n_{1,3}(x_5) = 0.5214, n_{2,3}(x_5) = 0.1163,$$
$$n_{H,3}(x_1) = 0.3185, n_{H,3}(x_2) = 0.3871, n_{H,3}(x_3) = 0.3576, n_{H,3}(x_4) = 0.3197, n_{H,3}(x_5) = 0.3622.$$

Step 2.3. Aggregate the evaluating values of decision makers with respect to alternative x_i to obtain the belief distributions based on Eq. (25)

$$\beta_1(x_1) = 0.8842, \beta_2(x_1) = 0.0648, \beta_1(x_2) = 0.7053, \beta_2(x_2) = 0.1307, \beta_1(x_3) = 0.7163,$$
$$\beta_2(x_3) = 0.1947, \beta_1(x_4) = 0.8679, \beta_2(x_4) = 0.0891, \beta_1(x_5) = 0.7121, \beta_2(x_5) = 0.1589.$$

Let the aggregated value obtained by the belief distributions $\beta_1(x_i), \beta_2(x_i)$ form the intuitionistic fuzzy values $(\beta_1(x_i), \beta_2(x_i))$ as follows:

$$(\beta_1(x_1), \beta_2(x_1)) = (0.8842, 0.0648), (\beta_1(x_2), \beta_2(x_2)) = (0.7053, 0.1307),$$
$$(\beta_1(x_3), \beta_2(x_3)) = (0.7163, 0.1947), (\beta_1(x_4), \beta_2(x_4)) = (0.8679, 0.0891),$$
$$(\beta_1(x_5), \beta_2(x_5)) = (0.7121, 0.1589).$$

Step 3 Calculate the scores of IFN $(\beta_1(x_i), \beta_2(x_i))$ obtained by the aggregation result of Step 3. Let $\alpha_i = (\beta_1(x_i), \beta_2(x_i))(i = 1, 2, 3, 4, 5)$

Step 3.1 According to Eq. (2), calculate $H_M(\alpha_i)(i = 1, 2, 3, 4, 5)$ as follows:
$H_M(\alpha_1) = 0.5665, H_M(\alpha_2) = 0.2634, H_M(\alpha_3) = 0.2085, H_M(\alpha_4) = 0.5, H_M(\alpha_5) = 0.2391.$

Step 3.2 Calculate the score of α_i and denote as $S(\alpha_i)(i = 1, 2, 3)$ according to Eq. (3):
$S(\alpha_1) = 0.8484, S(\alpha_2) = 0.6177, S(\alpha_3) = 0.5402, S(\alpha_4) = 0.8, S(\alpha_5) = 0.584$

Step 4 Determine the ranking of alternatives according to Step 3. We can obtained the preference order of alternatives as $x_1 > x_4 > x_2 > x_5 > x_3$, that is, x_1 is the desirable one.

(2) **Method II:** We utilize the MDIFWG which is presented in Section 4.2.2.

Step 1. Utilize the MDIFWG operator to aggregate all the intuitionistic fuzzy decision matrices $D_{t_k} = (\alpha_{ij,t_k})_{5\times3}(k = 1, 2, 3)$ into a complex intuitionistic fuzzy decision matrix $D = (\alpha_{ij})_{5\times3}$ as follows, where $\alpha_{ij} = (\mu_{ij}, \nu_{ij})$ is an IFN obtained by Eq. (30).

$$D = \begin{pmatrix} (0.801, 0.199) & (0.8769, 0.1231) & (0.8466, 0.1534) \\ (0.8466, 0.1534) & (0.5723, 0.1815) & (0.5898, 0.237) \\ (0.4523, 0.5477) & (0.7551, 0.2449) & (0.7186, 0.2814) \\ (0.8586, 0.1414) & (0.7842, 0.2158) & (0.8403, 0.1597) \\ (0.5723, 0.2545) & (0.7538, 0.2462), & (0.6378, 0.1998) \end{pmatrix}$$

.

Step 2-3. Calculate the distance between the alternative x_i and the IFPIS α^+ and the distance between the alternative x_i and the IFNIS α^-, respectively:

$$d(x_1, \alpha^+) = 0.1549, d(x_2, \alpha^+) = 0.3402, d(x_3, \alpha^+) = 0.3467, d(x_4, \alpha^+) = 0.1766, d(x_5, \alpha^+) = 0.3355;$$
$$d(x_1, \alpha^-) = 0.8451, d(x_2, \alpha^-) = 0.8103, d(x_3, \alpha^-) = 0.6533, d(x_4, \alpha^-) = 0.8234, d(x_5, \alpha^-) = 0.7652.$$

Step 4. Calculate the closeness coefficient of each alternative

$$c(x_1) = 0.8451, c(x_2) = 0.7043, c(x_3) = 0.6533, c(x_4) = 0.8234, c(x_5) = 0.6952.$$

Step 5. According to Step 4, we can obtain the preference order as $x_1 > x_4 > x_2 > x_5 > x_3$, which coincides with the order obtained by using Method I.

Table 4 shows a comparison of the preference order of the alternatives for different methods for Example 3.

Table 4: A comparison of preference order for different methos

methods	preference order
DIFWA[39]	$x_1 > x_4 > x_2 > x_5 > x_3$
DIFWG[32]	$x_1 > x_4 > x_2 > x_5 > x_3$
DIFWG$^{\epsilon}$[15]	$x_1 > x_4 > x_2 > x_5 > x_3$
Extended VIKOR based on DIFWG[26]	$x_1 > x_4 > x_5 > x_2 > x_3$
The proposed method based MDIFWG	$x_1 > x_4 > x_2 > x_5 > x_3$
The proposed method based ER algorithm	$x_1 > x_4 > x_2 > x_5 > x_3$

It follows from the Table 4 that the preference order of alternatives obtained by our proposed method are the same with the preference order obtained by Xu's [39], Gumus's [15] and Wei's [32] methods. It is also shown that our proposed methods based on ER algorithm and MDIFWG operators are valid.

Now, the following two examples will be used to show the our proposed methods can overcome effectively the Drawbacks A, B and C listed in Section 3.

Example 4. *Considering Example 2, we illustrate how this example can be solved by using the proposed two methods in Section 4.*

(1) **Method I.** Utilizing the ER algorithm. The specific steps are detailed as follows:

Step 1. Based on the decision matrices D_{t_1}, D_{t_2} and D_{t_3}, as well as the weights of attributes and Eq. (15), Eq. (16), we can obtain the basic probability mass P_{t_1}, P_{t_2} and P_{t_3}:

$$P_{t_1} = \begin{pmatrix} (0.0833, 0.2333) & (0.1667, 0.0667) & (0.2333, 0.0667) \\ (0.1667, 0.0667) & (0.1333, 0.16665) & (0.1333, 0.0333) \\ (0.1333, 0.1) & (0.1667, 0.1000) & (0.2, 0.1) \end{pmatrix}$$

$$P_{t_2} = \begin{pmatrix} (0.2, 0.1) & (0.1333, 0.0333) & (0.20.0333) \\ (0.2333, 0.0333) & (0.0833, 0.2333) & (0.1667, 0.1) \\ (0.1667, 0.0667) & (0.2333, 0.0667) & (0.1333, 0.1667) \end{pmatrix}$$

$$P_{t_3} = \begin{pmatrix} (0.1333, 0.1667) & (0.2333, 0.1) & (0.1661, 0.1) \\ (0.2, 0.1) & (0.2, 0.1) & (0.2333, 0.0667) \\ (0.2333, 0.0333) & (0.2, 0.0333) & (0.08333, 0.2333) \end{pmatrix}$$

Step 1.2. We can obtain the combined probability based on Eq. (17):

$$\begin{aligned}
&\tilde{n}_{13,t_1}(x_1) = 0.3396, \tilde{n}_{23,t_1}(x_1) = 0.2467, \tilde{n}_{13,t_1}(x_2) = 0.3273, \tilde{n}_{23,t_1}(x_2) = 0.1920, \\
&\tilde{n}_{13,t_1}(x_2) = 0.3673, \tilde{n}_{23,t_1}(x_3) = 0.2017, \tilde{n}_{13,t_2}(x_1) = 0.4152, \tilde{n}_{13,t_2}(x_1) = 0.1133, \\
&\tilde{n}_{13,t_2}(x_2) = 0.3384, \tilde{n}_{13,t_2}(x_2) = 0.2465, \tilde{n}_{13,t_2}(x_3) = 0.3874, \tilde{n}_{13,t_2}(x_3) = 0.1969, \\
&\tilde{n}_{13,t_3}(x_1) = 0.3754, \tilde{n}_{13,t_3}(x_1) = 0.2385, \tilde{n}_{13,t_3}(x_2) = 0.4570, \tilde{n}_{13,t_3}(x_2) = 0.1634, \\
&\tilde{n}_{13,t_3}(x_3) = 0.3714, \tilde{n}_{13,t_3}(x_3) = 0.1990.
\end{aligned}$$

We can then obtain the remaining combined probability based on Eq. (18):

$$\begin{aligned}
&\tilde{n}_{H3,t_1}(x_1) = 0.4136, \tilde{n}_{H3,t_1}(x_2) = 0.4806, \tilde{n}_{H3,t_1}(x_3) = 0.431; \\
&\tilde{n}_{H3,t_2}(x_1) = 0.4716, \tilde{n}_{H3,t_2}(x_2) = 0.4152, \tilde{n}_{H3,t_2}(x_3) = 0.4156; \\
&\tilde{n}_{H3,t_3}(x_1) = 0.3861, \tilde{n}_{H3,t_3}(x_2) = 0.3796, \tilde{n}_{H3,t_3}(x_3) = 0.4296.
\end{aligned}$$

Step 1.3. Aggregate the evaluating values with respect to attribute a_1, a_2, a_3 of alternative x_1, x_2, x_3 at the periods t_1, t_2, t_3 to obtain the belief distributions based on Eq.(19):

$$\begin{aligned}
&\beta_{1,t_1}(x_1) = 0.4922, \beta_{2,t_1}(x_1) = 0.3578, \beta_{1,t_1}(x_2) = 0.4412, \beta_{2,t_1}(x_2) = 0.2588, \\
&\beta_{1,t_1}(x_3) = 0.5164, \beta_{2,t_1}(x_3) = 0.2836, \beta_{1,t_2}(x_1) = 0.55, \beta_{1,t_2}(x_1) = 0.1501, \\
&\beta_{1,t_2}(x_2) = 0.4918, \beta_{1,t_2}(x_2) = 0.3582, \beta_{1,t_2}(x_3) = 0.5525, \beta_{1,t_2}(x_3) = 0.2808, \\
&\beta_{1,t_3}(x_1) = 0.5503, \beta_{1,t_3}(x_1) = 0.3497, \beta_{1,t_3}(x_2) = 0.663, \beta_{1,t_3}(x_2) = 0.237, \\
&\beta_{1,t_3}(x_3) = 0.5317, \beta_{1,t_3}(x_3) = 0.285.
\end{aligned}$$

Let the aggregated values be obtained by the above belief distributions $\beta_{1,t_k}(x_i), \beta_{2,t_k}(x_i)$ form the intuitionistic fuzzy values $(\beta_{1,t_k}(x_i), \beta_{2,t_k}(x_i))$ as follows:

$$\begin{aligned}
&(\beta_{1,t_1}(x_1), \beta_{2,t_1}(x_1)) = (0.4922, 0.3578), (\beta_{1,t_2}(x_1), \beta_{1,t_2}(x_1)) = (0.55, 0.1501), \\
&(\beta_{1,t_3}(x_1), \beta_{1,t_3}(x_1)) = (0.5503, 0.3497), (\beta_{1,t_1}(x_2), \beta_{2,t_1}(x_2)) = (0.4412, 0.2588), \\
&(\beta_{1,t_2}(x_2), \beta_{1,t_2}(x_2)) = (0.4918, 0.3582), (\beta_{1,t_3}(x_2), \beta_{1,t_3}(x_2)) = (0.663, 0.237), \\
&(\beta_{1,t_1}(x_3), \beta_{2,t_1}(x_3)) = (0.5164, 0.2836), (\beta_{1,t_2}(x_3), \beta_{1,t_2}(x_3)) = (0.5525, 0.2808), \\
&(\beta_{1,t_3}(x_3), \beta_{1,t_3}(x_3)) = (0.5317, 0.285).
\end{aligned}$$

Step 2. Based on Step 1, construct the aggregation decision making matrix Q as follows:

$$
\begin{aligned}
Q &= (\beta_{1,t_k}(x_i), \beta_{2,t_k}(x_i))_{3\times 3} \\
&= \begin{pmatrix} (\beta_{1,t_1}(x_1), \beta_{2,t_1}(x_1)) & (\beta_{1,t_2}(x_1), \beta_{2,t_2}(x_1)) & (\beta_{1,t_3}(x_1), \beta_{2,t_3}(x_1)) \\ (\beta_{1,t_1}(x_2), \beta_{2,t_1}(x_2)) & (\beta_{1,t_2}(x_2), \beta_{2,t_2}(x_2)) & (\beta_{1,t_3}(x_2), \beta_{2,t_3}(x_2)) \\ (\beta_{1,t_1}(x_3), \beta_{2,t_1}(x_3)) & (\beta_{1,t_2}(x_3), \beta_{2,t_2}(x_3)) & (\beta_{1,t_3}(x_3), \beta_{2,t_3}(x_3)) \end{pmatrix} \\
&= \begin{pmatrix} (0.4922, 0.3578) & (0.55, 0.1501) & (0.5503, 0.3497) \\ (0.4412, 0.2588) & (0.4918, 0.3582) & (0.663, 0.237) \\ (0.5164, 0.2836) & (0.5525, 0.2808) & (0.5317, 0.285) \end{pmatrix}
\end{aligned}
$$

Step 2.1. Based on Step 2 and Eq. (21), we can obtain the basic probability mass matrix:

$$
Q = \begin{pmatrix} (0.164, 0.119) & (0.1833, 0.05) & (0.1834, 0.1166) \\ (0.1471, 0.0863) & (0.1639, 0.1194) & (0.221, 0.07901) \\ (0.1721, 0.09453) & (0.18415, 0.0936) & (0.17723, 0.095) \end{pmatrix}
$$

Step 2.2. Based on Eq. (23) and Eq. (24), we can calculate the combined probability mass $\tilde{n}_{q,t_k}(x_i)$ and $\tilde{n}_{C,t_k}(x_i)$ w.r.t. alternative x_i at period t_k as follows:

$$
\begin{aligned}
& n_{1,3}(x_1) = 0.3887, n_{2,3}(x_1) = 0.1895, n_{1,3}(x_2) = 0.3908, n_{2,3}(x_2) = 0.1874, \\
& n_{1,3}(x_3) = 0.3913, n_{2,3}(x_3) = 0.1865; \\
& n_{H,3}(x_1) = 0.4218, n_{H,3}(x_2) = 0.4218, n_{H,3}(x_3) = 0.4222
\end{aligned}
$$

Step 2.3. Aggregate the evaluating values of decision makers with respect to alternative x_i to obtain the following belief distributions based on Eq. (25):

$$
\begin{aligned}
\beta_1(x_1) = 0.549, \beta_2(x_1) = 0.2677; \\
\beta_1(x_2) = 0.552, \beta_2(x_2) = 0.2647; \\
\beta_1(x_3) = 0.5531, \beta_2(x_3) = 0.2636.
\end{aligned}
$$

Let the aggregated values obtained by the above belief distributions $\beta_1(x_i), \beta_2(x_i)$ form the intuitionistic fuzzy values $(\beta_1(x_i), \beta_2(x_i))$ as follows:

$$
\begin{aligned}
(\beta_1(x_1), \beta_2(x_1)) = (0.549, 0.2677); \\
(\beta_1(x_2), \beta_2(x_2)) = (0.552, 0.2647); \\
\beta_1(x_3), \beta_2(x_3)) = (0.5531, 0.2636).
\end{aligned}
$$

Step 3. Calculate the scores of IFN $(\beta_1(x_i), \beta_2(x_i))$ obtained by the aggregation result of Step 3.

Let $\alpha_i = (\beta_1(x_i), \beta_2(x_i))(i = 1, 2, m)$.

Step 3.1. According to Eq. (2), calculate $H_M(\alpha_i)(i = 1, 2, 3)$ and shown as follows: $H_M(\alpha_1) = 0.06, H_M(\alpha_1) = 0.06266, H_M(\alpha_1) = 0.06367$.

Step 3.2. Calculate the score of α_i and denote $S(\alpha_i)(i = 1, 2, 3)$ according to Eq. (3): $S(\alpha_1) = 0.2923, S(\alpha_2) = 0.2988, S(\alpha_3) = 0.3013$.

Step 4. Determine the ranking of alternatives according to Step 3. We can obtain the preference order of alternatives is $x_3 > x_2 > x_1$, that is, x_3 is the desirable one.

We can see from Exa. 2 that the DIF-MADM methods proposed by Xu [39], Gumus [15] and Wei [32] can not distinguish the preference order of alternatives x_1, x_2, x_3. However, we can see from above Method I that our DIF-MADM method based on ER algorithm can distinguish the preference order of alternatives x_1, x_2, x_3. It is also shown that our method based on ER algorithm can overcome the Drawback C. That is, Drawback C is not the drawback anymore in this new method based on ER algorithm.

(2) **Method II.** We utilized the MDIFWG which is presented in Section 4.2.2. for Example 2.

Step 1. Utilize the MDIFWG operator to aggregate all the intuitionistic fuzzy decision matrices $D_{t_k} = (\alpha_{ij,t_k})_{3\times3}(k = 1, 2, 3)$ into a complex intuitionistic fuzzy decision matrix $D = (\alpha_{ij})_{3\times3}$ as follows, where $\alpha_{ij} = (\mu_{ij}, \nu_{ij})$ is an IFN obtained by Eq. (30).

$$D = \begin{pmatrix} (0.3926, 0.5279) & (0.796, 0.204) & (0.614, 0.204) \\ (0.614, 0.204) & (0.3926, 0.5279) & (0.5802, 0.204) \\ (0.5336, 0.204) & (0.614, 0.204) & (0.3926, 0.5279) \end{pmatrix}.$$

Step 2-3. Calculate the distance between the alternative x_i and the IFPIS α^+ and the distance between the alternative x_i and the IFNIS α^-, respectively:

$$d(x_1, \alpha^+) = 0.5768, d(x_2, \alpha^+) = 0.5191, d(x_3, \alpha^+) = 0.5048;$$
$$d(x_1, \alpha^-) = d(x_2, \alpha^-) = d(x_3, \alpha^-) = 0.3309.$$

Step 4. Calculate the closeness coefficient of each alternative

$$c(x_1) = 0.3646, c(x_2) = 0.3893, c(x_3) = 0.396.$$

Step 5. According to Step 4, we can obtain the preference order is $x_3 > x_2 > x_1$, which coincided with the order obtained by using Method I as detailed above.

Table 5 shows a comparison of the preference order of the alternatives for different methods for Example 4.

Table 5: A comparison of preference order for different methods for Example 3

Methods	Preference order
DIFWA[39]	$x_1 = x_2 = x_3$
DIFWG[32]	$x_1 = x_2 = x_3$
DIFWG$^\epsilon$[15]	$x_1 = x_2 = x_3$
Extended VIKOR based on DIFWG[26]	$x_1 = x_2 = x_3$
The proposed method based on MDIFWG	$x_1 < x_2 < x_3$
The proposed method based on the ER algorithm	$x_1 < x_2 < x_3$

We can see from Table 4 that the DIF-MADM methods proposed by Xu [39], Gumus [15]and Wei [32] can not distinguish the preference order of alternatives x_1, x_2, x_3. The same problem is also obtained by extend VIKOR method based on DIFWG, the root of this problem is the related aggregation operators (or the definition of operation of dynamic intuitionistic fuzzy numbers). However, we can see from above Table 5 that our DIF-MADM methods based on ER algorithm and MDIFWG operator can distinguish the preference order of alternatives x_1, x_2, x_3. It is also shown that our methods based on ER algorithm and MDIFWG operator can overcome effectively the Drawback C.

The following example can show the proposed methods can overcome the drawbacks A and B of existing methods analysed in Section 3.

Example 5. *A company wants to invest in one of renewable energy sources, Geothermal, solar, biomass among renewable energy sources have been determined as alternatives. The company has determined three criteria for the evaluation of renewable energy resources: risk factor, the growth rate in the sector, payback reliability. The company thinks that these evaluations need to be done in a dynamic process due to the increasing energy demand, environmental awareness and government support for energy projects in recent three years. The three alternative $x_i(i = 1, 2, 3)$: (1) x_1 is Geothermal; (2) x_2 is solar; (3) x_3 is biomass. The investment company must take a decision according to the following three attributes: (1) A_1 is risk factor; (2) A_2 is the growth rate in the sector; (3) A_3 is the payback reliability. The three possible alternatives $x_i(i = 1, 2, 3)$ are to be evaluated using the intuitionistic fuzzy information by the decision maker under the above four attributes at the periods $t_k(k = 1, 2, 3)$, as listed in the following matrix, shown as Table 6. Let $\lambda(t) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ be weight vector of the periods $t_k(k = 1, 2, 3)$, and $w = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ be weight vector of the attributes $A_j(j = 1, 2, 3)$.*

Table 6: Individual IF decision matrix $D_{t_k}(k = 1, 2, 3)$

years		A_1	A_2	A_3
t_1	x_1	(0.6, 0.1)	(0.5, 0.2)	(0.7, 0.2)
	x_2	(0, 0.1)	(0.4, 0.5)	(0.4, 0.1)
	x_3	(0.4, 0.3)	(0.5, 0.1)	(0.6, 0.3)
t_2	x_1	(0.6, 0.3)	(0.7, 0.1)	(0.6, 0.1)
	x_2	(0.7, 0.2)	(0.1, 0.3)	(0.5, 0.3)
	x_3	(0.5, 0.2)	(0.7, 0.2)	(0, 0.1)
t_3	x_1	(0.4, 0.5)	(0.6, 0.3)	(0.5, 0.4)
	x_2	(0.5, 0.4)	(0.6, 0.1)	(0.7, 0.2)
	x_3	(0.7, 0.1)	(0.5, 0.4)	(0.1, 0.5)

The results obtained by the proposed methods based on Method I and Method II are listed in Table 7. The details of process are the same as the ones in Example 3 and Example 4, so skipped. Whilst, Table 7 also shows the comparisons with some existing methods.

We can see from Table 7 that the DIF-MADM methods Gumus [15] and Wei [32] can not distinguish the preference order of alternatives x_2, x_3. The reason is that there is only one membership degree of IFNs is equal to 0, the aggregation membership degree of IFNs is 0 even if the membership degrees of $n - 1$ IFNs are not 0, which leads to inappropriate preference order of alternatives in this situation. However, we can see from above Table 7 that our DIF-MADM methods based on ER algorithm and MDIFWG operator can distinguish the preference order of alternatives x_1, x_2, x_3. It is also shown that our methods based on ER algorithm and MDIFWG operator can overcome effectively the Drawback B.

In Exa 5., if $\mu_{\alpha(t_1)}(x_2) = 1$ at the period t_1, $\mu_{\alpha(t_2)}(x_2) = \mu_{\alpha(t_3)}(x_2) = 0$, the modified decision matrix is shown as follows:

Table 7: A comparison of preference order for different methods for Example 5

Methods	Preference order
DIFWA[39]	$x_1 > x_3 > x_2$
DIFWG[32]	$x_2 = x_3 > x_1$
DIFWG$^{\epsilon}$[15]	$x_2 = x_3 > x_1$
The proposed method based MDIFWG	$x_1 > x_3 > x_2$
The proposed method based ER algorithm	$x_1 > x_3 > x_2$

Table 8: Modified individual IF decision matrix $D_{t_k}(k = 1, 2, 3)$

years		A_1	A_2	A_3
t_1	x_1	(0.2, 0.6)	(0.5, 0.2)	(0.7, 0.2)
	x_2	(1, 0)	(0.4, 0.5)	(0.4, 0.1)
	x_3	(0.4, 0.3)	(0.5, 0.1)	(0.6, 0.3)
t_2	x_1	(0.4, 0.3)	(0.7, 0.1)	(0.6, 0.1)
	x_2	(0, 0.2)	(0.1, 0.3)	(0.5, 0.3)
	x_3	(0.5, 0.2)	(0.7, 0.2)	(0, 0.1)
t_3	x_1	(0.2, 0.5)	(0.6, 0.3)	(0.5, 0.4)
	x_2	(0, 0.4)	(0.6, 0.1)	(0.7, 0.2)
	x_3	(0.5, 0.4)	(0.5, 0.4)	(0.1, 0.5)

The results obtained by the proposed methods based on Method I is $x_2 > x_1 > x_3$. The preference order of alternatives is the same with order obtained by Xu's method based on DIFWA [39]. However, the preference order of alternatives obtained by Xu's method [39] will be the same no matter how the non-membership degrees of A_1 regarding on x_2 change at period t_2, t_3, this situation is shown in Fig .1. Obviously, it is unreasonable.

As far as our proposed method I is concerned, the preference order will be changed with the non-membership degree of A_1 regarding on x_2 change. For example, when the non-membership degrees of A_1 regarding on x_2 change at period t_2, t_3 , the preference order of alternatives obtained by our proposed method I is shown in Fig. 2.

We can see from the above analysis, Fig. 1 and Fig. 2 that our proposed method I can overcome the Drawback A.

5.2. Sensitivity analysis

Baird [6] pointed out that sensitivity analysis (SA) is the investigation of some potential changes and errors of rating values and their impact on the final ranking order. In this subsection, we conduct some sensitivity analyses to analyze the impact of changing the membership and non-membership degrees of the rating values on the alternatives ranking order based on Method I (DIF-MADM based on the ER algorithm).

For the original membership and non-membership degrees $\alpha_{t_k} = (\mu_{ij,t_k}, \nu_{ij,t_k})$, because the sum of membership degree and the non-membership degree of a intuitionistic number is not more than 1, so we can assume it is updated as $(\mu_{ij,t_k} + \Delta_{ij,t_k}, \nu_{ij,t_k} - \Delta_{ij,t_k})$, where $\mu_{ij,t_k} + \Delta_{ij,t_k}, \nu_{ij,t_k} - \Delta_{ij,t_k} \in$

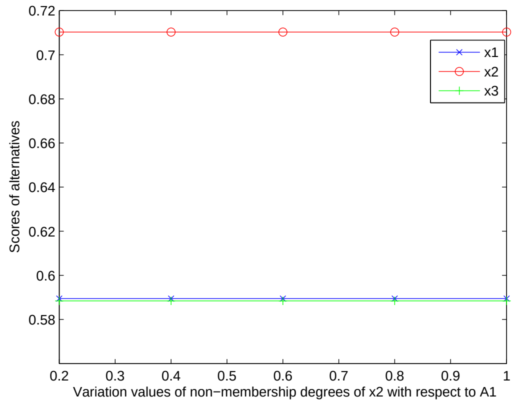

Figure 1: Ranking order sensitivity to the non-membership degrees of x_2 with respect to the first attribute A_1 by Xu's Method [39]

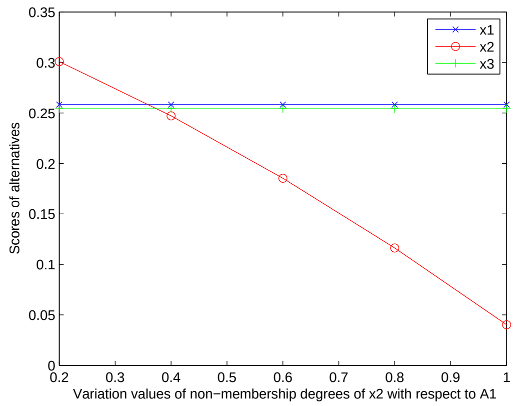

Figure 2: Ranking order sensitivity to the non-membership degrees of x_2 with respect to the first attribute A_1 by our Method I

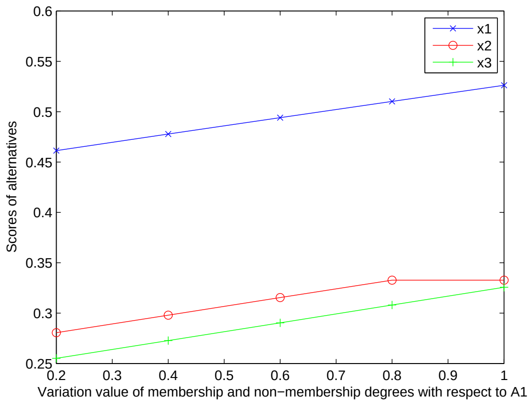

Figure 3: Ranking order sensitivity to the membership and non-membership degrees with respect to the first attribute A_1

$[0, 1]$. Therefore, we can determine the step size Δ_{ij,t_k} according to the condition $\mu_{ij,t_k}+\Delta_{ij,t_k}, \nu_{ij,t_k}-\Delta_{ij,t_k} \in [0, 1]$.

Now, we take Example 5 (Section 5.1) as an example, we can obtain the preference order of the alternatives by changing the membership and non-membership degrees of three attributes, the details are shown in Figures 3-5, which also show the desirable alternatives will remain constant when the variation values of the membership and non-membership degrees with respect to the three attributes vary in the range from 0.1 to 1. But regarding the range of membership degree and non-membership degree, the ranking order of the two alternatives A_2 and A_3 will change with the membership and non-membership degrees. It demonstrates that the alternatives A_2 and A_3 are more sensitive to membership and non-membership degrees than A_1.

6. Conclusions

In this paper, we have proposed two kinds of dynamic fuzzy multi-attribute decision making (DIF-MADM) methods in order to overcome the drawback of the existing DIF-MADM methods: the first one is using the ER methodology; the other one is based on the modified dynamic intuitionistic fuzzy weighted geometric aggregation (MDIFWG) operator. From the experimental results of several examples shown in Tables 3, 5, 7 and the comparative analysis, we can concluded that the proposed methods can overcome the drawbacks of some existing DIF-MADM methods, the details are shown in Table 9, so have shown the good potential in handling DIF-MADM problem.

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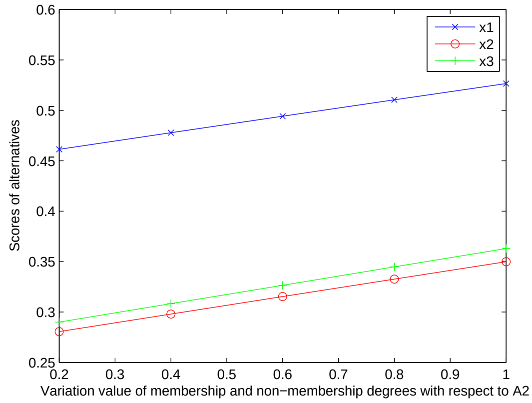

Figure 4: Ranking order sensitivity to the membership and non-membership degrees with respect to the second attribute A_2

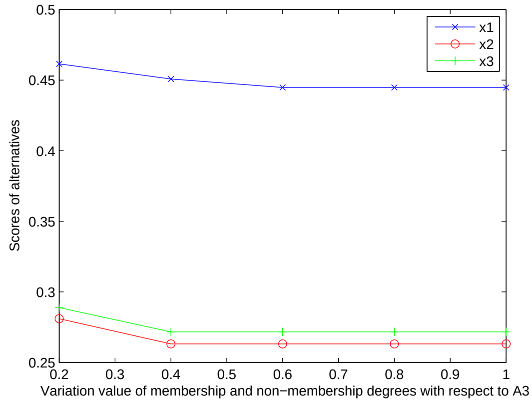

Figure 5: Ranking order sensitivity to the membership and non-membership degrees with respect to the third attribute A_3

Table 9: Corresponding Drawbacks and solutions by proposed methods

	Method I	Method II
Drawback A	Y	N/A
Drawback B	Y	Y
Drawback C	Y	Y

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