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Quantitative scientific theories have been crucial to science for four-hundred years. Yet a random sample of the voluminous metaphysics literature on laws is still likely to pay mere lip-service to this fact—the schema '*All Fs are Gs'* standing in for much of the ongoing debate. Wolff therefore undersells her new book-length treatment, *The Metaphysics of Quantities* (2020, OUP), when she remarks that 'we do not have a good metaphysical understanding of quantities' (1). Indeed, the debate has entirely failed to have the prominence it should have.

Nevertheless, there is a rich background on which Wolff draws to develop her 'substantival structuralism' about quantities. Within metaphysics-proper this includes Field's science without numbers, Mundy's Platonism about quantities, and Arntzenius and Dorr's view of calculus as geometry. However, the central inspiration behind Wolff's considered view is the Representationalist Theory of Measurement (RTM), developed by Krantz, et al. (1971-90), Narens and others. As initially conceived, RTM is more concerned with the mathematical nature of quantitative representation than any metaphysical project (although there are deflationary undertones). But Wolff puts it to work informing a genuinely novel account of quantities which is both realist and rigorously defined.

One thing which could have been made more explicit early on in the book is that Wolff is concerned only with *classical* quantities. Hence, the peculiar discreteness of certain quantum scales is not within her purview (fn.9, p.20). Moreover, Wolff tends to treat the quantities with which she does engage as homogenous, with complications of zero-values considered as an afterthought (e.g. 154). One might wonder therefore about the scope of her metaphysical view when moving into these other domains. For instance, homogeneity is crucial for her rejection of absolutism (ch.8), but should we be absolutist about zero-values? But given the caveats, the path Wolff leads us down is generally well-supported and plausible.

The book is divided into three parts. In the first (chs 2-4) Wolff, argues for a 'restricted realism' about quantities. She begins (ch.2) by rejecting treatments of quantities in terms of the determinate-determinable model (e.g. Funkhouser 2006). Clearly the ordering and distance relations among a quantity's magnitudes are not implied by it. But Wolff observes further that 'paradigmatic' determinables exhibit a high degree of nesting whereas quantities do not. This is perhaps overplayed. Her own example of determinate isotopes of iron doesn't exhibit significant nesting, and quantities like charge might be conceived as 'nested' within positive and negative magnitudes. More persuasively, Wolff notes that unlike the determinate-determinable model, the weak ordering of quantities' magnitudes provides an *explanation* of the *single value principle* (that different magnitudes of the same quantity cannot be co-instantiated).

The rejection of the determinate-determinable model motivates Wolff's anti-permissivism (ch.3), that quantitativeness makes for a significant difference among attributes rather than being a feature of our choice of representation. Quantities, she points out, are prized in science, and if we could get them by changing our representation of mere qualitative attributes (including kinds) we would surely be doing so (35). Having said that, Wolff also makes clear that quantities do not themselves have the richness of the numerical structure with which they are represented, since multiple scales can represent the same quantity. In sum, quantitative representation is hard, but not *that* hard.

Wolff also aims to undermine operationalist views about quantities (ch.4), which ground quantitativeness in the operations of measurement. In making her case she asks why the temperature scale can be strengthened to admit equal intervals but the Mohs hardness scale cannot. It seems there is something in the empirical phenomena themselves which makes for the difference. Only temperature, for instance, is able to be related by empirical law to other

measurable quantities. By comparison, there are many ways of measuring hardness, none of which supply a practically relevant interval scale.

In the second part (chs 5-6) Wolff spells out RTM's formal details. The Alper-Narens theorem shows that there are no homogeneous scales (i.e. with no structurally distinguished elements) requiring more than two but fewer than infinite elements to be mapped onto themselves for an order-preserving automorphism to be an identity. The theorem therefore indicates a sweet spot of '*super-ratio scales'* between those structures which are mere orderings and require infinite self-mappings to maintain identity under transformation (like the Mohs scale), and structures which aren't homogeneous (like the reals inclusive of 0). These super-ratio scales are, so Wolff claims, the quantitative scales; crucially, they include both interval scales (e.g. temperature) and ratio scales (e.g. mass).

A further result from Luce (2001) is that super-ratio scales' translations (automorphisms which preserve order by mapping some element to another) form a higher-order group which themselves satisfy the axioms of a the ratio scale. It is *this* characteristic which Wolff settles on as the distinctive and unifying mark of quantitativeness. The argumentation at this stage goes by fairly swiftly and if one is not already familiar with the measurement-theoretic results a degree of trust must be placed in Wolff's deployment of them. Luce's theorem especially deserved further elaboration. Nevertheless, the analysis of quantitativeness struck this reviewer as plausible. As Wolff stresses, it distinguishes quantitative attributes from attributes for which numbered representations seem superfluous without demanding a preference for any particular numerical scale. Moreover, the key criterion has a realist flavour, being in terms of (automorphisms among) the scales themselves rather than our numerical representations of them.

The final part of the book (chs 7-10) turns to more established metaphysical questions. Wolff suggests (ch.7) that an appropriate ontology for quantities must be substantivalist, in the sense of having an infinite range of relata over which to formulate the structural relations essential to quantities. Although one might wonder whether Wolff exhausts the possible alternatives, this leaves her deciding between Mundy's (1987) Platonism, and Arntzenius and Dorr's (2012) development of Field's nominalist deployment of spacetime points. Since Mundy's approach would require second-order relations (to structure the first-order magnitudes) Wolff tentatively opts for Arntzenius and Dorr's approach, according to which particulars occupy a 'location' in a space of points associated with each of the base quantities (e.g. those specified in the ISU). Wolff calls this view 'locationism' (cf. Cowling 2014).

Locationism is in many ways an elegant position, but it has a glaring problem. So long as it is the *occupants* of spacetime which get located in other quantity-spaces, a composed object may either fail to be located where its parts are or it will not be located where its own magnitudes are. This is presumably why Cowling restricts locationism to mereologically simple objects. Wolff suggests a different solution: *supersubstantivalism* (140), the view that spacetime regions themselves occupy locations in quality-space. But the dilemma might seem to carry over to regions too. Larger regions will either fail to be located on some dimension of quality-space where their sub-regions are, or they will not be located where their own magnitudes are. A possible way round this might be to restrict quantities to densities, but regardless, this issue was not dealt with convincingly in the book.

The remaining chapters steer the substantivalist view towards structuralism. In chapter 8 the motivation is to find an agreeable middle-ground between comparativism and absolutism about the relata of quantitative relations (such as *more-massive-than*, *twice-as-massive*). In another densely argued section, Wolff argues that absolutism's commitment to intrinsic identities

outstrips the homogeneity of quantitative scales and is hence empirically otiose (§8.2). However, Wolff also references a thought-experiment from Baker (2020) showing that comparativism (as typically conceived) is insufficient to make sense of empirical differences between magnitudes, such as the relevance of a planet's mass to whether or not a projectile launched from it achieves escape velocity.

Wolff claims that a 'sophisticated' substantivalism about quantity-spaces (expanding on Pooley's (2006) space-time sunstantivalism) will avoid both issues. First, its elements are not intrinsically individuated so there is no otiose structure imposed. Second, there is a clear way to contrast sparse worlds containing (e.g.) planets of differing escape velocities: the planets simply occupy different locations in mass space (161). I have doubts about this response to Baker's problem-case. Spacetime is typically taken to be world-bound (even necessarily so). Yet for meaningful comparisons of mass across worlds (as in Baker's problem case) mass space must be *trans-world*. This seems to lead to an inconsistent form of locationism. Moreover, if cross-world spaces are permitted then why not simply opt for a comparativism in which the mass scale is defined by all possible worlds?

Wolff is more persuasive when critiquing comparativism's pull towards fundamentalism about quantities' ordering relations (ch.9). There are, she points out, numerous ways of interpreting the relationship between a quantitative structure and a representing structure, as well as numerous equally valid relations to define the representing structure itself. So it seems unlikely that representations can be sought which are free of redundancy or equivalent competitors thereby casting doubt on the defence of any particular representation as fundamental (176). By contrast, a radical sophisticated substantivalist view can eschew the need for an explanation of the invariance of quantitative structures. Wolff draws on Dewar (2019) to motivate an external analysis of quantitative structures according to which certain transformations among

representing structures are stipulated to be homomorphisms and any further structure is deemed redundant.

As well as being thoroughly structuralist (ch.10), Wolff admits the radical sophistication of her view is an instance of what Sider (2020) has recently dubbed ‘quotienting’. Sider demures, but the approach is quickly becoming a competitor for addressing problems of invariance. Wolff is one of the vanguard taking quotienting up in the case of quantities. Though I suspect her view will come under serious scrutiny, it should surely count as the new standard by which to judge any alternative.

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