Variability of soil stress-strain non-linearity for use in MSD analyses evaluated using databases of triaxial tests on fine-grained soils

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ABSTRACT: The feasibility of a tunnel, foundation or excavation project is to some extent dependent on limiting the potential ground movements during construction. To make such an assessment it is important to quantify the stress-path dependent behaviour of the soil undergoing undrained lateral and vertical stress relief. At an early stage in the project, site-specific test data is limited and so predictions must necessarily be based upon expected characteristics of the soil deposit. Analysis of a recently compiled database RFG/TXCU-278 has demonstrated a method of quantifying the variability of stress-strain data from stress-path tests that are more frequently encountered in commercial practice (consolidated-undrained triaxial compression and extension tests) for use in design sensitivity analyses. A soil non-linearity parameter (b) is investigated using the database RFG/TXCU-278 and a series of previously reported tests performed on intact Bothkennar Clay. Evidence from both databases suggests that the variability of b is not strongly linked to the effects of shear mode, *OCR*, strain rate, or plasticity. It is shown that reasonable predictions of nonlinear behaviour up to a load factor ($\sigma_{mob}/\sigma_{failure}$) of 0.8 can be achieved using the MSD-MSF method for a rigid pad test.

1 INTRODUCTION

1.1 *MSD for Underground Construction*

The Mobilisable Strength Design (MSD) method has been the subject of considerable research over the past decade. MSD relies on a simple model for soil strength mobilisation i.e. a simple constitutive model for soil behaviour which can also be incorporated into a reliability framework (cf. Vardanega & Bolton 2016a). Early papers on MSD-thinking for retaining structures include (Bolton & Powrie 1988 and Bolton et al. 1990). MSD has been applied to analyses of various construction scenarios such as: retaining structures (Osman & Bolton 2004, Lam & Bolton 2011, Diakoumi and Powrie 2013, Lam et al. 2014, Bolton et al. 2014); shallow foundations (Osman & Bolton 2005, McMahon et al. 2014); deep foundations (Vardanega et al. 2012b, Vardanega 2015, Vardanega et al. 2018, Voyagaki et al. 2019) and tunnels (Klar & Klein 2014).

1.2 *Mobilized Strain Framework*

The mobilized strain framework (MSF) (Vardanega & Bolton 2011a, Vardanega et al. 2012a, and Beesley & Vardanega 2020) uses a simple two-parameter power-law model to describe strength mobilisations in the moderate stress range ($0.2 \leq \tau_{mob}/c_u \leq 0.8$). This has been used as a simple soil model for MSD-style pile design calculations (Vardanega et al. 2012b, Voyagaki et al. 2019, Crispin et al. 2019). The MSF framework can also be used in tunnelling analyses (Klar & Klein 2014 used an exponential function instead of a power function). One advantage of using the MSF framework to characterise soil stress-strain behaviour is that relatively few parameters are needed to capture the key features of soil behaviour, for multiple shear modes, and these parameters can be used in scenario or stochastic analyses to develop envelopes of potential ground movements for various construction scenarios.

2 TRIAXIAL TEST DATABASES

2.1 *Databases on intact soils*

Databases of previous experimental measurements can be used to develop geotechnical correlations (or transformation models) that can serve as design aides (cf. Kulhawy & Mayne 1990, Phoon & Kulhawy 1999a, 1999b). Vardanega and Bolton (2011a) assembled a database of 115 isotropic (CIU) soil tests on 19 natural clays and found that an equation of the form shown as Eq. 1 could describe stress-strain data in the stress range of $0.2 \leq \tau_{mob}/c_u \leq 0.8$ reasonably well, with a mean R^2 of about 0.97 (n=115):

$$\frac{1}{M} = \frac{\tau_{mob}}{c_u} = 0.5\left(\frac{\gamma}{\gamma_{50\,CIU}}\right)^{b_{CIU}} \quad \left[0.2 \leq \frac{\tau_{mob}}{c_u} \leq 0.8\right] \qquad (1)$$

where M = mobilisation factor (cf. BSI 1994); τ_{mob} = mobilised shear strength; c_u = undrained shear strength; γ = shear strain; $\gamma_{50\,CIU}$ = shear strain to 50% of c_u in CIU shear mode and b_{CIU} = a non-linearity parameter for tests with CIU shear mode (if the sample is tested in extension the subscripts on the model parameters are changed to CIUE and similarly CIUC for compression). Power laws for stress-strain response of metals have been proposed by Hollomon (1945) and for soils in p-y calculations (Matlock 1970). In the case of the early p-y work the exponent was generally assigned a single value (Zhang & Anderson 2017).

Later work (Vardanega 2012, Vardanega & Bolton 2016b, Beesley & Vardanega 2020) extended Eq. 1 to account for testing from K_0 conditions (Eq. 2).

$$\frac{\tau_{mob}-\tau_0}{c_u-\tau_0} = 0.5\left(\frac{\gamma}{\gamma_{50\,CKU}}\right)^{b_{CKU}} \quad \left[0.2 \leq \frac{\tau_{mob}-\tau_0}{c_u-\tau_0} \leq 0.8\right] \quad (2)$$

where τ_0 = initial shear stress; $\gamma_{50\,CKU}$ = shear strain to 50% of c_u in CKU shear mode & b_{CKU} = a non-linearity parameter for tests with CKU shear mode (if the sample is tested in extension the subscripts on the model parameters are changed to CKUE and similarly CKUC for compression).

2.2 *RFG/TXCU-278*

Beesley & Vardanega (2020) assembled a large database of published triaxial experiments on reconstituted fine-grained soils tested with a variety of test modes (consolidation type and shear mode): CIUC, CIUE, CKUC and CKUE. The motivation for this work was in part the need to assess the effect of shear mode (as originally suggested by Mayne in Vardanega et al. 2013). The effect of shear mode on undrained shear strength (c_u) has been well documented (e.g., Mayne 1985, Mayne et al. 2009). Beesley & Vardanega (2020) report the following correlations linking the extension and compression modes for CIU tests (Eqs. 3, 5) and CKU tests respectively (Eqs. 4, 6) for the data in RFG/TXCU-278.

Beesley & Vardanega (2020) showed that shear mode does have a significant effect on γ_{50} and c_u/σ'_{vo}, particularly when tested from K_0 conditions. However, the effect of shear mode on b was less obvious. Influence of shear mode, K_0 and isotropic consolidation, overconsolidation ratio (*OCR*), strain rate ($\dot{\varepsilon}_a$), liquid limit (w_L), and plastic limit (w_P), on the MSF parameters (γ_{50}, c_u/σ'_{vo} and b) is discussed in detail in Beesley (2019).

$$\left(\frac{c_u}{\sigma'_{vo}}\right)_{CIUE} = 0.835\left(\frac{c_u}{\sigma'_{vo}}\right)_{CIUC} \quad (3)$$

$[R^2 = 0.94,\ n = 50,\ SE = 0.110,\ p < 0.001]$

$$\left(\frac{c_u}{\sigma'_{vo}}\right)_{CKUE} = 0.649\left(\frac{c_u}{\sigma'_{vo}}\right)_{CKUC} \quad (4)$$

$[R^2 = 0.92,\ n = 29,\ SE = 0.080,\ p < 0.001]$

$$(\gamma_{50\,CIUE}) = 0.749(\gamma_{50\,CIUC}) \quad (5)$$

$[R^2 = 0.71,\ n = 50,\ SE = 0.0031,\ p < 0.001]$

$$(\gamma_{50\,CKUE}) = 3.76(\gamma_{50\,CKUC}) + 0.0054 \quad (6)$$

$[R^2 = 0.46,\ n = 25,\ SE = 0.0099,\ p < 0.001]$

where, R^2 = coefficient of determination; n = number of datapoints used to generate the correlation; SE = standard error and p = p-value of the correlation.

2.3 *Study Aims*

This paper aims to study the effect of various parameters on the soil non-linearity parameter (b) using the database RFG/TXCU-278 (Beesley & Vardanega 2020) and a series of tests performed on Bothkennar clay reported by the Science and Engineering Research Council (SERC 1989). For details of the original data sources see the original publications and Beesley (2019). The variables studied in this paper are: sample depth; sample state; *OCR* & shear mode; test strain rate; plasticity; and sample disturbance. The paper concludes with a simple design example.

3 SOIL NONLINEARITY

3.1 *Sample depth*

When reviewing ground conditions for design, parameter variation with depth is likely to be linked to overburden stress and natural geological variability associated with, for example, stress relief, weathering, and soil composition. Measured properties will also be affected to an uncertain degree by sampling disturbance or a selected testing procedure. Vardanega & Bolton (2011b) showed for a dataset of high-quality tests on London Clay (n=17) from various sites in London that $\gamma_{50\,CIU}$ correlated with sample depth. Figure 1 shows the b_{CIU} parameter plotted with depth for the same database: a weak positive correlation between b_{CIU} and sample depth is observed. Figure 2 shows b_{CKU} for intact Bothkennar Clay samples taken from the Bothkennar Test Site (SERC 1989). Figure 2 shows no positive trend with sample depth but there is a slight upswing of values at shallow depth (for the CKUC tests).

3.2 *Sample state*

Table 1 shows past publications which report ranges of b_{CIU} and b_{CKU} for various soils and soil databases. While there is considerable variation in the mean parameters for each dataset, intact samples tend to have higher b-values (less non-linearity) than reconstituted samples e.g., for the database of intact natural clays from Vardanega & Bolton (2011a) the average b_{CIU} value = 0.608 (n = 92) whereas for the reconstituted soil database RFG/TXCU-278 (Beesley & Vardanega 2020) the average b_{CIUC} value = 0.459 (n=114).

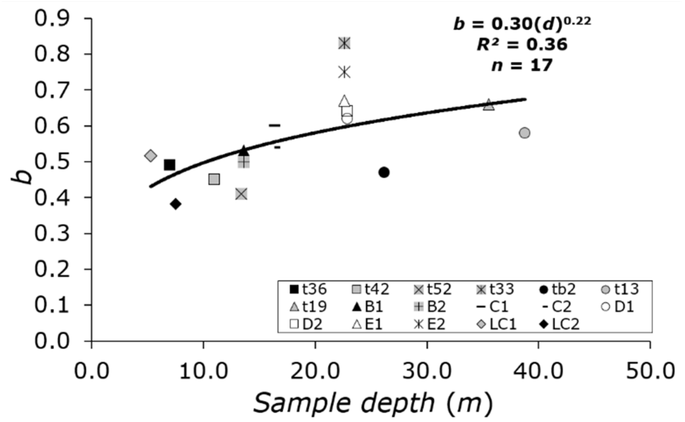

Figure 1. Variation of b_{CIU} with sample depth for the database of intact London Clay specimens sheared from isotropic conditions reported in Vardanega & Bolton (2011b) (original data from Gasparre 2005, Yimsiri 2001 and Jardine et al. 1984)

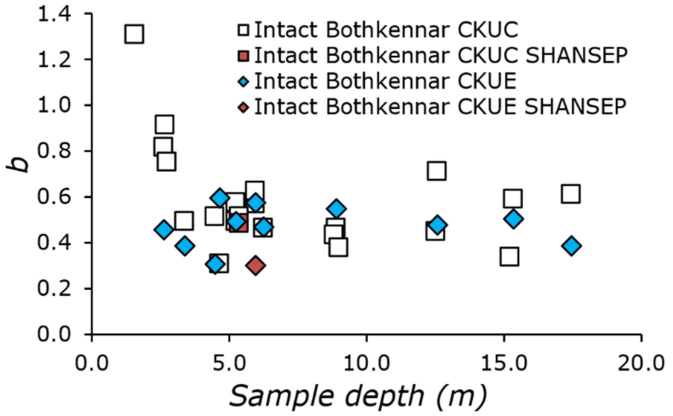

Figure 2. Variation of b_{CKU} with sample depth for the database of intact Bothkennar Clay specimens sheared from K_0 conditions reported in Beesley (2019) (original data from SERC 1989)

Table 1. Values of b_{CIU} and b_{CKU} for various data-sets

Reference/Material	Sample Condi-tion*	*OCR* range ** (-)	*b*-value						Notes
			Test mode	Max (-)	Mean (-)	Min (-)	*SD* (-)	*n* (-)	
Vardanega & Bolton (2011a) /various soils	*I*	*n/k*	CIU	1.21	**0.608**	0.32	0.158	92	
	I	*n/k*	DSS	0.92	**0.610**	0.36	0.163	12	
	I	*n/k*	Cyclic	0.64	**0.548**	0.39	0.083	11	
Vardanega & Bolton (2011b) /London Clay	*I*	*n/k*	CIU	0.83	**0.57**	0.38	0.12	17	Some data used in this paper were also included in the data-base of Vardanega & Bolton (2011a)
Bolton et al. (2014) /Shanghai Clay	*I*	*n/k*	CIUC	0.630		0.365		3	
	I	*n/k*	CIUE	0.363		0.308		3	
Vardanega et al. 2012a/kaolin	*R*	1-20	CIUC	0.602	**0.464**	0.291	0.096	18	Original test data also used in Beesley & Vardanega (2020)
Beesley et al. 2019/kaolin	*R*	1-8	CIUC	0.416	**0.342**	0.263	0.060	6	Data using right cylinder assumption (excludes data from a CIUE test)
Beesley & Vardanega (2020) /various soils	*R*	1-32	CIUC	1.131	**0.459**	0.232	0.143	114	RFG/TXCU-278
	R	1-12	CIUE	0.589	**0.399**	0.220	0.082	55	RFG/TXCU-278
	R	1-10	CKUC	1.126	**0.581**	0.123	0.167	68	RFG/TXCU-278
	R	1-17	CKUE	0.745	**0.350**	0.177	0.100	34	RFG/TXCU-278
Beesley (2019) data from SERC (1989)/Bothkennar	*I*	3.33-	CKUC	1.304	**0.586**	0.311	0.223	21	Excludes SHANSEP (*OCR*=1)
	I	1.33	CKUE	0.598	**0.472**	0.307	0.088	11	tests

* *R* = Reconstituted; *I* = Intact; ** *n/k* = not known

3.3 *OCR & Shear Mode*

Later work revealed that increasing $\gamma_{50\,CIU}$ and $\gamma_{50\,CKU}$ was reasonably well linked to increasing *OCR* (Vardanega et al. 2012a; Casey 2016; Beesley & Vardanega 2020) but the weaker correlation of b_{CIU} (the non-linearity parameter) with *OCR* reported in Vardanega et al. (2012a) was not found for the databases analysed in Casey (2016) and Beesley & Vardanega (2020).

Figure 3a shows the data for RFG/TXCU-278 plotted against *OCR* for the CIUC and CIUE tests. No obvious correlation between b_{CKUC} and *OCR* can be seen (perhaps a slight elevation in values can be observed at high *OCR* values – although less data is present in this range). For RFG/TXCU-278, mean values of b_{CIU} for the CIUC (0.46) and CIUE (0.40) tests are similar; however, Figure 3b shows that the mean values are clearly different for the CKUC and CKUE test series.

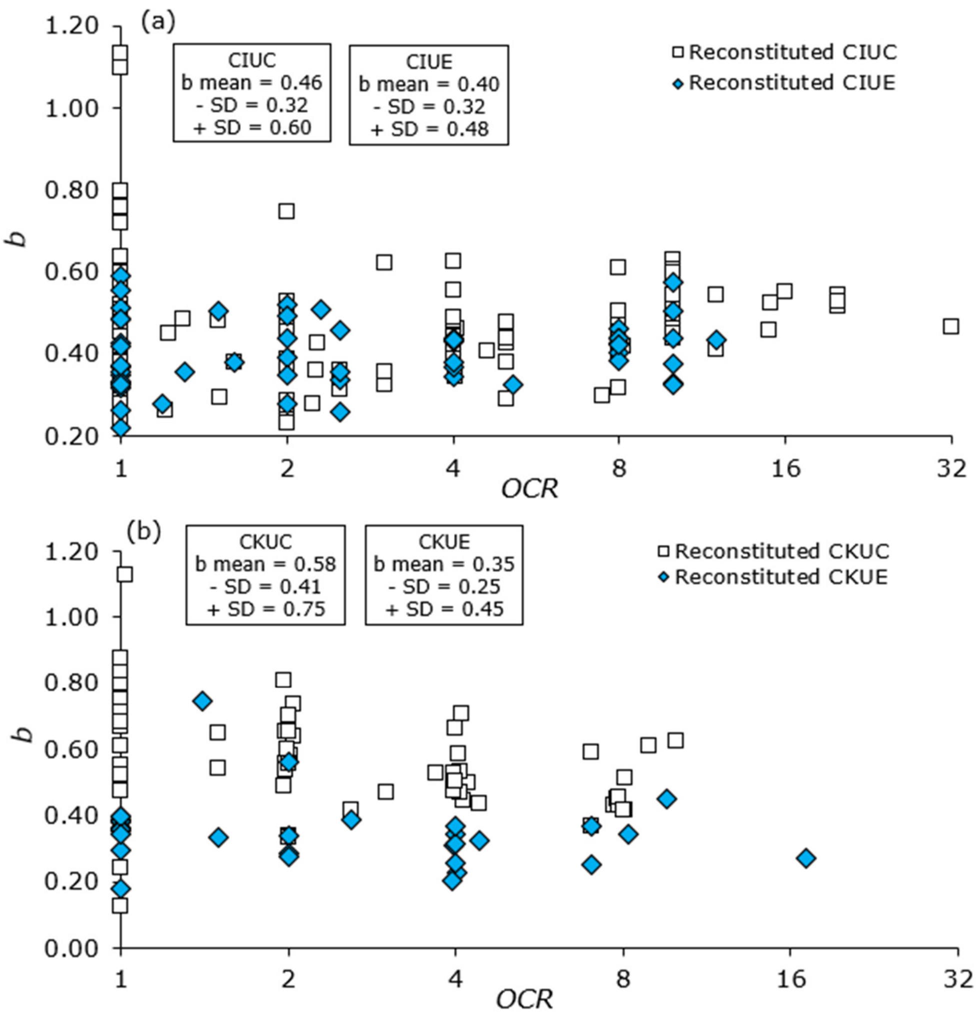

Figure 3. Variation of *b* with *OCR* for reconstituted specimens sheared from (a) isotropic consolidation stresses (b) K_0 consolidation stresses (data from RFG/TXCU-278).

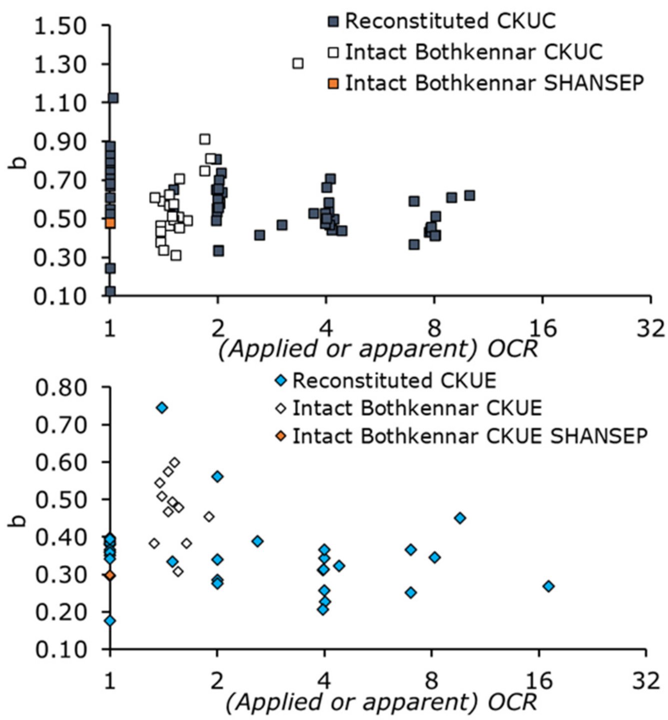

Figure 4. Variation of b_{CKU} with *OCR* from RFG/TXCU-278 compared with the CKU data from SERC (1989) (a) CKUC tests (b) CKUE tests

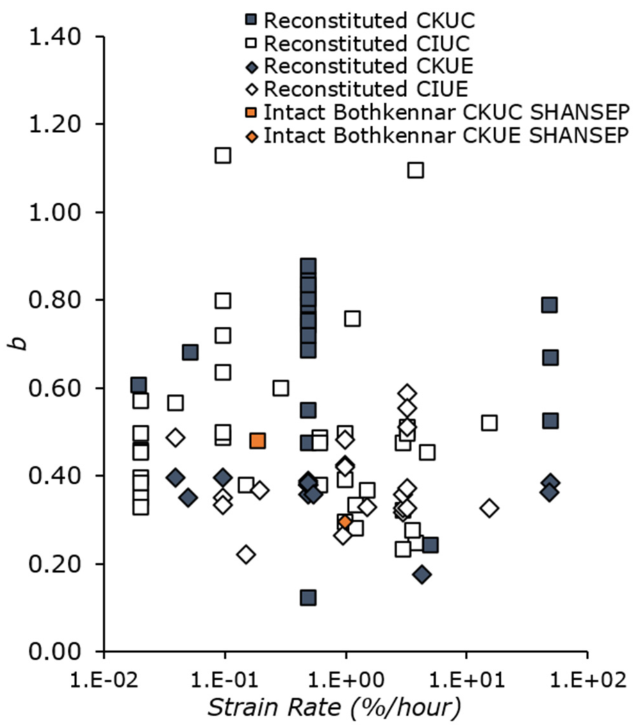

Figure 5. Variation of b_{CIU} & b_{CKU} with axial strain rate for data from RFG/TXCU-278 and Bothkennar (SERC 1989) all tests *OCR*=1

As for the data in Figure 3a, no obvious correlation between b_{CKUE} and OCR is seen in Figure 3b. Tentatively it can be concluded that CKUE tests do produce rather different b-values on average compared to CKUC tests (and γ_{50} values, see Eq. 6).

Figure 4 shows b_{CKU} values from Figure 3b compared with values determined from CKU tests on intact Bothkennar Clay (SERC 1989). Values of b_{CIU} and b_{CKU} for RFG/TXCU-278 are plotted against the OCR 'applied' to the sample during the reconstitution and consolidation process. Test parameters of intact Bothkennar Clay have been plotted using the 'apparent' OCR (yield stress ratio, YSR) recommended by Hight et al. (1992a) for the sample depth. (The yield stress ratio profile was based on experimental measurements reported by Nash et al. 1992 using incremental load oedometer tests). The range of b_{CKUC} for Bothkennar Clay appears to conform well to the general range of the larger database RFG/TXCU-278. However, Figure 4b shows that the CKUE data from the Bothkennar site plots on average higher than the data from RFG/TXCU-278, which is possibly linked with the more intact structure of the Bothkennar specimens (see Beesley 2019 for further details).

3.4 *Effect of strain rate*

Sheahan et al. (1996) demonstrated with an experimental programme of CKUC tests on reconstituted Boston Blue Clay that for normally and overconsolidated samples (OCR varying from 1 to 8) an increase in undrained shear strength normalised by preconsolidation stress (c_u/σ'_{vo}) with strain rate ($\dot{\varepsilon}_a$) was associated with suppressed shear-induced pore pressure. The tests published by Sheahan et al. (1996) represent about 40% of the CKUC tests in RFG/TXCU-278. The normally consolidated specimens showed the greatest rate-dependency of 7 to 11.5% increase in $c_u/\sigma'_{vo\ NC}$ per log cycle $\dot{\varepsilon}_a$ (%/hour) (Sheahan et al. 1996). However, the same tests showed stress-strain curves that were rate-independent and unique to OCR when plotted using $(\tau-\tau_0)/(c_u-\tau_0)$.

For the normally consolidated specimen data included in RFG/TXCU-278 a slight decrease in γ_{50} is observed at strain rates greater than 1%/hour but strain rate does not appear to influence the non-linearity parameter (Fig. 5).

3.5 *Effect of plasticity index*

Using multiple linear regression analysis, Vardanega & Bolton (2011a) identified that a higher plasticity index (I_p) would predict greater γ_{50} values from intact specimens. The normally consolidated reconstituted soils included in RFG/TXCU-278 show a weak positive trend between I_p and γ_{50} (see Beesley 2019) and a weak negative correlation exists with b (Fig. 6). No obvious difference in trends are found whether plotting the parameters with liquid limit or I_P. The database results suggest that b_{CKUC} of 0.24 to 0.88 can occur in similar unstructured materials (I_p=0.23).

3.6 *Effect of sampling disturbance*

The effect of sampling disturbance on Bothkennar Clay was studied in detail by Clayton et al. (1992) by reconsolidating samples under various stress paths and applying controlled cycles of strain. In Figure 7, only five CKUC tests have magnitudes of $\gamma_{50\ CKUC}$ less than $\gamma = 1.5\varepsilon_a = 0.0015$. From the few available tests there is no evidence to suggest that larger strain parameters were measured from 'Laval' samples than from 'Sherbrooke' samples. Differences in peak stress and stress-strain data in triaxial tests undertaken by City University and Imperial College were found to be explained by different procedures of sampling and extrusion (Hight et al. 1992b). Lower values of b_{CKU} are associated with more disturbed specimens (City University data from SERC 1989). SHANSEP test procedures (OCR=1), which cause "incomplete" destructuration (Smith et al. 1992), also noticeably reduce b_{CKU} compared to intact tests from similar depths (Fig. 7).

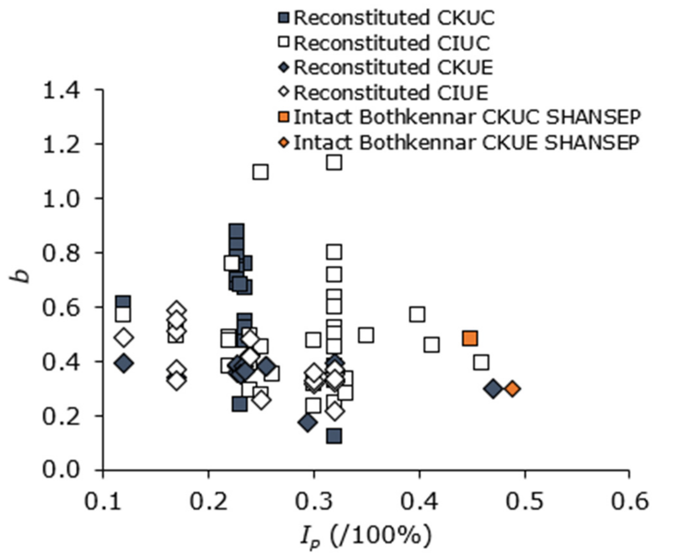

Figure 6. Variation of b with plasticity index for data from RFG/TXCU-278 and Bothkennar (SERC 1989) all tests OCR=1

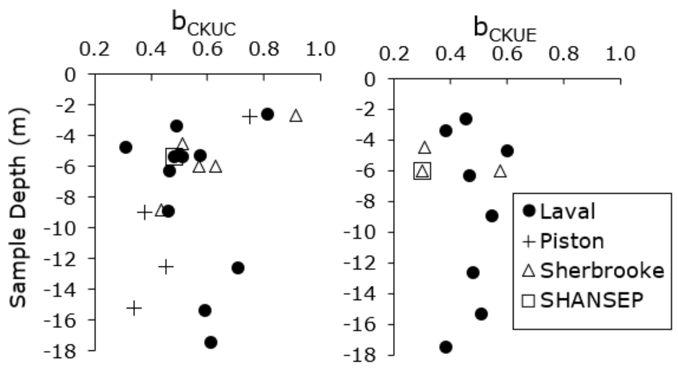

Figure 7. Variation of b with sample depth and sampling procedure using test data from Bothkennar (SERC 1989).

4 DESIGN EXAMPLE

Taking the example from Osman and Bolton (2005) that was discussed in Vardanega & Bolton (2011a) for a rough shallow footing with a variable *b*-factor it can be shown that:

$$\left(\frac{\sigma_{mob}}{6.05\Delta\tau_{peak}}\right) = \frac{1}{2}\left(\frac{1.35w}{\gamma_{50}D}\right)^{b} \quad (7)$$

where: w = undrained footing settlement; D = footing diameter; $\Delta\tau_{peak} = (\tau - \tau_0)/(c_u - \tau_0)$ = maximum change in average mobilised shear stress; σ_{mob} = vertical bearing pressure; and using a bearing capacity factor of 6.05 (Eason & Shield 1960).

Figure 8 shows field measurements from a pad loading test at Bothkennar reported by Jardine et al. (1995). Shown for comparison are predicted load-settlement curves using Eq. 7 from 16 of the reported CKUC and CKUE triaxial tests performed on Bothkennar Clay (SERC 1989).

Table 2 lists parameter values (Tests 1, 2 and 4) and mean values (by test mode) derived from the digitised test data (SERC 1989). An additional curve (Test 3) using mean b_{CKUE} demonstrates the use of Eqs. 4 and 6 to estimate variation of strain from a single triaxial compression test (Test 1) due to shear mode effects.

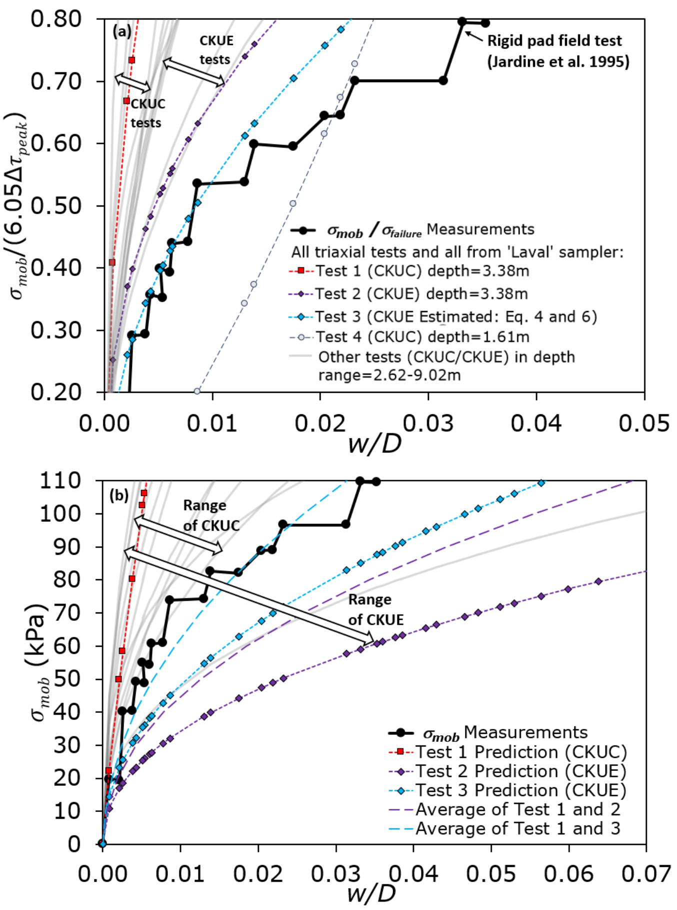

Figure 8. Load-settlement predictions, using MSD-MSF and test data reported by SERC (1989), compared with field measurements: (a) predicted stress ratio (b) predicted bearing pressure

Table 2. Mobilized Strain Framework parameters for triaxial tests shown in Figure 8 (mean values shown for depth range 2.62-9.02m, specimens from 'Laval' samplers only).

Parameter	$c_{u\,CKU}/\sigma'_{v0}$ (-)	$\gamma_{50\,CKU}$ (-)	b_{CKU} (-)
Test 1	0.624	0.0036	0.814
Test 2	0.326	0.0070	0.453
Test 3	0.405	0.0189	0.490
(Test 4)*	(1.060)	(0.0235)	(1.304)
Mean CKUC n=8	0.506	0.0024	0.516
Mean CKUE n=6	0.242	0.0051	0.490

* Test 4 data not included in the mean values

The curves shown in Figure 8a indicate a range in non-linearity of b = 0.307 to 1.304 for all test modes (for values plotted with sample depth, see Fig. 2). This parameter range includes the effect of sample depth, material variability, different sampling procedures, two strain rates, and measurement accuracy of two laboratories. Lowest b (0.298) was measured in CKUE mode after SHANSEP consolidation (to OCR=1, not shown in Fig. 8). One CKUC test was performed for a sample depth of 1.61m – close to the characteristic depth (1.5m) used in the MSD case study of Osman and Bolton (2005). However, the value of b_{CKUC} is very high (1.304) and Eq. 7 predicts load-settlement that does not match field behaviour.

Figure 8a shows that, using test results from sample depths of 2.62 to 9.02m below ground level and without accounting for the maximum change in shear stress, the MSD-MSF method (Eq. 7) underpredicts w/D needed to mobilise stress ratio beneath the test pad. $\Delta\tau_{peak}$ has a noticeable effect on the prediction accuracy of Eq. 7: predicted bearing pressures (Fig. 8b) for 0.8m embedment (Brinch Hansen 1970; following Osman & Bolton 2005) are a closer approximation of field measurements. A better prediction of nonlinear behaviour is achieved using the average strains of Tests 1 and 3 compared with Tests 1 and 2, with the result governed by $\Delta\tau_{peak}$. Note that the power-law model adopted in MSF has been calibrated to a moderate bearing pressure range equal to $27 \leq \sigma_{mob} \leq 110$ kPa for the 2.2m square pad.

Figure 9 shows Eq. 7 with varying values of b based on the data in RFG/TXCU-278 and γ_{50} values expected for OCR=1 and 2 (Beesley & Vardanega 2020). Lower b values are associated with CIU and CKUE triaxial tests and higher γ_{50} values are expected at a greater degree of overconsolidation. Using average $\gamma_{50\,CKUC}$, the envelope of normalised nonlinear behaviour is relatively narrow for CKUC test data on reconstituted soils compared with the other test modes. Larger envelopes of strain due to varying b, particularly where stress ratios exceed 0.5, are associated with higher γ_{50} values.

Comparing Figures 8a and 9 and referring to Tables 1 and 2, the average values of $\gamma_{50\,CKUC}$ and b_{CKUC} for reconstituted soils are smaller than those of the reported tests on intact Bothkennar Clay for similar

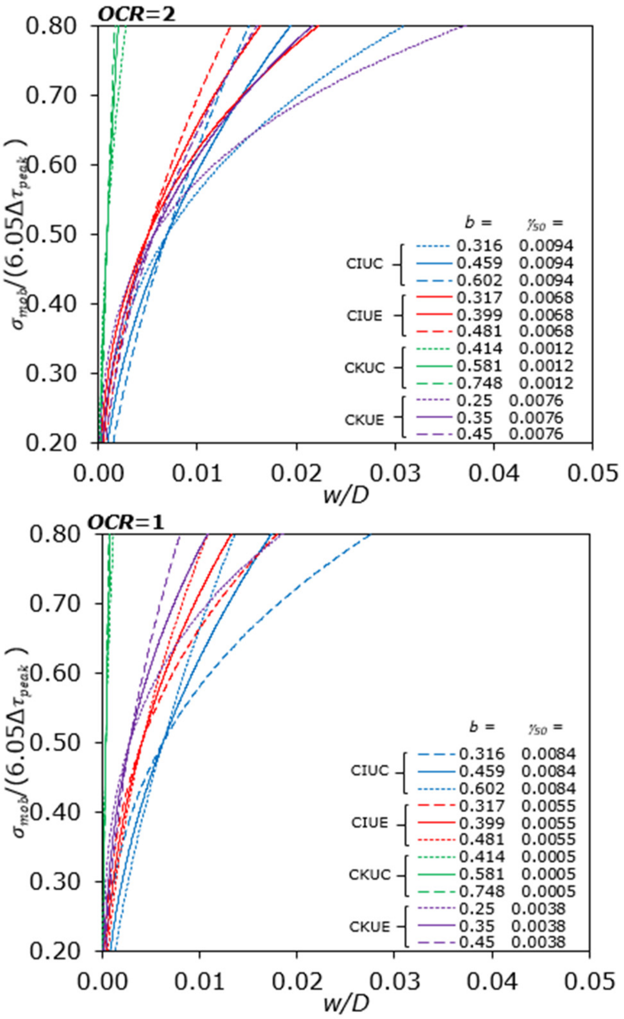

Figure 9. Load-settlement predictions in terms of stress ratio using MSD-MSF and data from RFG/TXCU-278: average γ_{50} for *OCR*=1 and 2 and varying *b*-values from Table 1

values of *OCR* (or *YSR*). However, in CKUE tests, MSF parameters are similar; slightly higher values of b_{CKUE} define the narrower envelope shown in Figure 8a. The Bothkennar Clay was reconsolidated to in-situ stresses (σ'_{v0}=33-75kPa) (SERC 1989) lower than the consolidation stresses needed to achieve *OCR*=1 and 2 (minimum σ'_{v0}=142kPa) for the CKU tests in RFG/TXCU-278. Therefore, void ratio of the reconstituted soils would have been lower than the intact Bothkennar Clay. Notwithstanding this, shear mode has a greater effect on the variation of stress-strain in reconstituted soils.

5 SUMMARY

Two databases of high-quality tests on fine-grained soils have been used to analyse the effects of: sample depth; sample state; *YSR* and *OCR*; shear mode; test strain rate; plasticity; and sample disturbance on the non-linearity parameter (*b*) from the MSF framework. Comparisons with field measurements from a pad loading test at Bothkennar (Jardine et al. 1995) were made with MSD-MSF predictions.

In view of the potential effects of sample disturbance and sample state on the MSF parameters, a data-driven framework for assessing parameter variability is likely to be valuable when selecting values for design. The work shown in this paper offers a framework to incorporate parameter variability from CU triaxial test data into design sensitivity analyses. The framework could also be adopted with other test modes where the soil has been sheared to peak strength. Although the development of larger intact soil test databases is warranted, this study demonstrates the potential value of using a simple nonlinear constitutive model to investigate ranges of behaviour with different triaxial test modes.

6 ACKNOWLEDGEMENTS

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