Nonlinear Dynamic Positive Feedback Trading and the Complexity of Stock Price

By

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A thesis submitted in partial fulfilment of the University's requirements for the Degree of Doctor of Philosophy

Coventry University ETHICS

Low Risk Research Ethics Approval

Where NO human participants are involved and/or when using secondary data - Undergraduate or Postgraduate or Member of staff evaluating service level quality

Project Title

Nonlinear Dynamic Positive Feedback Trading and the Complexity of Stock Price

Principal Investigator Certification

I believe that this project **does not require research ethics approval**.	X
I confirm that I have answered all relevant questions in the checklist honestly.	X
I confirm that I will carry out the project in the ways described in the checklist. I will immediately suspend research and request a new ethical approval if the project subsequently changes the information I have given in the checklist.	X

Principal Investigator

Name: Bang Xiao..

Date: 28/02/2014 ....

Student's Supervisor (if applicable)

I have read the checklist and confirm that it covers all the ethical issues raised by this project fully and frankly. I confirm that I have discussed this project with the student and agree that it does not require research ethics approval. I will continue to review ethical issues in the course of supervision.

Name: Karl Shutes..

Date: 28/02/2014 ....

Abstract

This thesis has applied the theory from behavioural finance theory and by merging with the concept of chaos theory from natural science, this thesis focuses on the impact of positive feedback trading on the price formation process. By using the Hurst exponent estimation and calculating the correlation dimension value, the market index and individual firms from China have presented the nonlinearity and chaotic characteristics, thus demonstrating the source of complexity.

This thesis proposes a new model that uses the Hurst exponent as the signal for thresholds to indicate changes in market conditions. The result suggested, by combining the threshold and assumptions from the positive feedback model, that the new model offers a better explanation for the complexity of the stock market which presents chaos. The model is found to be statistically significant and superior in all comparative testing.

Keywords and Phrases*:* Positive Feedback Trading, Hurst exponent, Nonlinearity, Complexity, Chaos

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Part I

Introduction

1 The Research Motivation and Its Significance

Inspiration for this research comes from the observation that academic studies and industry are often disconnected. Those masters of traditional finance theories still suffer fatal loss, for example, the failure of Long Term Capital Management.

The most dominant theory is the Efficient Market Hypothesis (EMH) raised by Fama (1970). The EMH suggests that the market is efficient and obeys random walk, which means no one should make abnormal profits. In the real world, however, a lot of participants, institutions and individuals continue to make abnormal profits from the market. This raises the questions:

- "How could this happen?"
- "Is the market trend obeys random walk, so no one could trading based on past information?"
- "If it's not random, what is a better explanation to this phenomenon?"

Previous studies and the real market observation by the author led to the conclusion that behavioural finance offers a better explanation than EMH. Therefore, this thesis follows this interest to further explore the behavioural field. It has two main research streams. One involves doing qualitative research focused on the individual decision making process, the motivation, expectation and bias, mostly using the techniques from the psychology and experimental economics. The other stream uses quantitative research focused on the market anomalies, groups of trader behaviours, and market price formation *etc.*, mostly using the mathematics tool to build a better explanation model.

This thesis followed the later stream, doing quantitative research. The literature review goes back to noise trading, and a specific type of noise trader, the positive feedback trader which becomes the vital component of later research. It described a group of traders just trading on the past price movement but not on the deviation from stock fundamental value. This offers a very good explanation of the market presenting positive feedback feature, but the previous work, the static models assumed the price back to its fundamental value. It is a snapshot but cannot explain why the trend could suddenly change.

The thesis brought the concept from natural science, and believed the Chaos Theory, a branch of mathematics focusing on the behaviour of dynamical systems that are highly sensitive to initial conditions, could offers a better answer for the question above. Chaos theory is an interdisciplinary theory stating that within the apparent randomness of chaotic complex systems, there are underlying patterns, constant feedback loops, repetitions, self-similarities, fractals, as well as self-organization. Complexity is relative to simplicity. It indicates a status between complete disorder and complete order. A complete disordered system on thermal equilibrium state which presents Brownian motion is a simple system. A complete disordered price system which presents random walk is a simple system as well. The complexity of stock price means the change of stock price is neither completely ordered nor completely disordered, but in between of those. It is not perfectly predictive nor is it totally random. It is similar to random motion but it is not random motion. This will be tested in the Chinese market and its individual firms by using the tools introduced in the methodology Chapter.

The main contribution is proposed as a new model based on the work by De Long *et al.* (1990b) that uses the Hurst exponent as the signal for thresholds to indicate the changes of market condition. This is the extension of applied research in behavioural finance on financial markets, and it is also the main novel part of our research. After the evaluation, comparative testing is conducted to demonstrate the improvement that this model gives. Last but not least, the thesis has conducted the empirical test in the Chinese market, for both the index and sample stocks. It opens a new path for further research possibility. Therefore, the work both contributed to current methodology and empirical result up to date.

To sum up, this thesis focuses on the impact of positive feedback trading to the price formation process. There are behavioural finance stream and Chaos theory stream of literature out there, but lacking of model development in between, or fully applied the concept from each other. This thesis stands in the middle and merges the two together to fill the gap in between. Also, this thesis focues more on the emerging market China as the primary target, since it is still insufficient of empirical test due to its short finance market history, this thesis also contributes to the empirical evidence.

2 Thesis Structure

This thesis is organised mainly in three parts. The first part is the literature review. This chapter begins with the concept of "noise" which is the starting point of the following discussion. The pioneer work *i.e.* the "DSSW noise trading model" (De Long *et al.*, 1990a) has been reviewed. Following the path of this research, positive feedback trading has been found a major source of speculation. This type of noise trader is not assumed to trading in a random direction but mostly centralized at one direction. De Long *et al.* (1990b) proposed a new model to describe it and some other theoretical model has emerged. The "DSSW positive feedback trading model" is a static model which assumes the asset prices go back to their fundamental value but cannot explain the reason causing the bubble crashes. The next section brought the concept of natural science revealed in a chaotic system, a tiny disturbance could cause a collapse. Since chaos is a type of nonlinear system, its application in the economics and finance field has been discussed.

The second part is the methodology. This Chapter includes three sections. The Hurst exponent estimation process has been examined closely by comprehensively explaining the technical details and comparing different estimation methods. Also the empirical result of previous works has been reviewed in order to give a comparison for our own estimation in a later Chapter. The correlation dimension calculation has been explained and linked with BDS statistic, also the previous empirical results will be presented. The last part of this Chapter is to introduce two methods that have been used in time series forecasting. The ARIMA model and the exponential smoothing are the two most widely used approaches to time series forecasting, and provide complementary approaches to the problem. In that later Chapter, these two methods will be used to compare with our own models.

The third part of this work is the main contribution. This Chapter is divided into three parts. The first part is introducing the way of data collection, and more importantly, the rationale of the choice. The background information for the research target, the Shanghai Stock Exchange will be introduced as well. The second part to analysing the characteristics of the market as a whole through the SSEC index and individual firms listed by using the Hurst exponent estimation and calculating the correlation dimension value.

It should be noted that the firms have been further sampled into smaller portion in order to conduct more concise and comparable research. The third part raised a new model which is adapted from previous work done of DSSW model. The new model uses the Hurst exponent as the signal for thresholds that indicates the changes of market condition. Then a further test will be conducted to test its statistical significance and there are also comparisons to select the better model.

Part II

Literature Review

In this section, a framework of the literature review chapter will be introduced, illustrating the path of the review to help readers has better understanding of the work.

We begin our literature review from the section of Efficient Market Hypothesis and Behavioural Finance. EMH is the core of the mainstream finance theory, in this section we will introduce the historical background of the EMH and how it has been challenged by the emerging of behavioural finance such as Prospect Theory and BAPM.

The section is noise trading, which is the original point of the topic discussed. The concept of "noise" has been brought to economics and finance and been widely used, although the definition was not exactly the same. We will introduce the main conflict people argued about and explain one of the most important noise trading models, "DSSW noise trading model" (De Long *et al.*, 1990a).

In the following section, we look further at the irrational side of the trading process, especially concentrated on the positive feedback traders. By fully explaining Shiller's "Feedback Model" (Shiller, 1990) and "DSSW noise trading model" De Long *et al.* (1990b), we will also give some theoretical comparison like the contagion model raised by Lux (1995). We also give some evidence for the presence of positive feedback trading in the empirical part. Finally we will discuss the formation mechanism of the positive feedback trading.

In the next Section, the direction of the literatures are getting closer of research done by thesis. We will start to look at the nonlinear part of the trading process and the complexity of the stock price. By introducing the term "chaos" at economics and finance, we illustrated different phenomenon and detective method around it which helps to understand its characteristic and application we used in this thesis. For example, the Hurst exponent estimation, the method we actually using for my work will be further discussed at methodology chapter.

The final Section include some additional literature refereed for our ethology and analysis as well, models we will used later and some further behaviour evidence.

3 Efficient Market Hypothesis and Behavioural Finance

3.1 The Formation of EMH

The fundamental function of the capital market is to reasonably allocate capital, the most valuable resource in the modern economy. The effectiveness of the market determines the efficiency of resource allocation in the capital market. Therefore, the efficiency of the capital market has always been the focus point of the economist.

The early research for the stock market efficiency could be traced back to random walk theory raised by Bachelier (1900). He used the the methodology from gambling analysis into market price, and argued that the random walk process is a Brownian motion.

Cowles 3rd (1933); Cowles 3rd and Jones (1937) discovered that the time series correlation coefficient of stock price changes is zero by studying the trend of US stock prices over the past decades, which makes no investment strategy able to keep making profit in the market for the long run. Therefore, he pointed out that there is no certain pattern of price changes in the US stock market to follow the random walk hypothesis. Working (1934), based on the study of various price time series, proposed that the time series of stock prices can be characterized by random walk model.

Kendall (1953) found that changes in the price of financial assets could not predict future trends by studying their past price and volume data. The difference between the spot price and the price of the next period (previous period) is a set of random numbers, and the change in the price of financial assets follows the pattern of random walk.

Roberts (1959) and Osborne (1959) used stochastic processes to analyze stock market price volatility. They came to a similar conclusion, that is, the process of stock price movement is in line with the "Brown movement", showing the law of random walk. There is no definite rule for price changes for people to explore, and the way in which securities analysts use past stock trading records to predict future stock price movements is a practice that ignores random factors that affect price fluctuations.

Samuelson (1965) believes that the phenomenon of price fluctuations in financial markets is irregular, which does not indicate that financial markets do not run market irrational evidence in accordance with economic laws. Instead, it proves that rational investors continue to use new information arbitrage in financial markets. He believes that if the stock market is effective, all information that can affect the price of the asset will be immediately reflected in the asset price, and the occurrence of various information is random and unpredictable, so the fluctuation of the stock price is also unpredictable.

Fama (1965) studied the stock price volatility from the perspective of information, and on the basis of the previous literature on the analysis of stock market price characteristics. Fama pointed out that changes in information and changes in the price of securities are a serial correlation, and the impact of different types of information on the price of securities is not the same. Information cannot be used to earn excess profits in the market, which makes the stock price consistent with the random walk model. The academic community has gradually begun to use whether the stock market price is in line with the random motion model as a sign of the efficiency of the stock market.

For the definition of an efficient market, Fama *et al.* (1969) first proposed the concept of "efficient market" and regarded information as the core of market efficiency research. In the article, the efficient market is defined as a market that adjusts rapidly to new information. Fama's research on efficient market concepts not only provides a concept of efficiency in the securities market, but also provides a research method to test market efficiency. He also believes that if there is friction in a stock market, that is, when a trader needs to pay a commission to a broker, the market may still become an efficient market. At the same time, he also pointed out that if there is a monopoly in the product market, the profits earned by the manufacturers in the product market will also be effectively reflected in the stock price, that is, the allocation efficiency in the product market will not affect the efficiency of the stock market.

At the same time, based on the summary of Roberts' efficient market form, Fama (1970) believes that the price of securities can reflect the information that affects its fluctuation to varying degrees, and the wider the information that price can reflect, the faster the response, the price will be the closer to the real asset price it represents. Thus, the smaller the chances of a trader gaining risk-free returns, the more stable the price will be, allowing the funds in the securities market to be more efficiently allocated to the production sector. Therefore, Fama proposes three forms of capital market in different information environments: weak form efficient market, semi-strong form efficient market and strong form efficient market.

In weak form efficiency, future prices cannot be predicted by analyzing prices from the past. Excess returns cannot be earned in the long run by using investment strategies based on historical share prices or other historical data. Technical analysis techniques will not be able to consistently produce excess returns, though some forms of fundamental analysis may still provide excess returns.

In semi-strong form efficiency, it is implied that share prices adjust to publicly available new information very rapidly and in an unbiased fashion, such that no excess returns can be earned by trading on that information. Semi-strong-form efficiency implies that neither fundamental analysis nor technical analysis techniques will be able to reliably produce excess returns.

In strong-form efficiency, share prices reflect all information, public and private, and no one can earn excess returns. If there are legal barriers to private information becoming public, as with insider trading laws, strong-form efficiency is impossible, except in the case where the laws are universally ignored.

At the same time as the emergence and development of EMH, Markowitz combines Osborne's expected rate of return distribution, measured by its variance, to measure the portfolio of assets and derive the risk and standard deviation of the investor's choice of effective boundaries. A desirable conclusion of a portfolio of assets with the highest expected rate of return at a given level.

Therefore, the rationality of investors in Markowitz's definition means that they are risk-avoiding; on this basis, Sharpe (1964), Lintner (1965) and Mossin (1966) put EMH and Markowitz's portfolio of assets, named after the capital asset model, establishes an investor behaviour model CAPM based on rational expectations in a general equilibrium framework. Investors in CAPM have homogeneous yield expectations and interpret information in the same way. Under this assumption, CAPM concludes that high-risk assets should be compensated for high yields, and that investors' optimal investment decisions should be made along the capital market line.

If EMH answers the conclusion that the known information has no value for profit, then CAPM indicates that the excess return rate in the market is due to the risk of taking greater risks, so to a certain extent CAPM supplements the theory of EMH's vulnerabilities.

Following the birth of CAPM, research in the 1970s and 1980s was generally focused on applying this model for empirical research and verifying the effectiveness of EMH. However, with the deepening of later research, it is gradually found that the modern financial theory model is inconsistent with the actual investment decision-making behaviour of investors in the securities market.

3.2 The Challenge from Behavioural Finance

Momentum effect refers to stocks that have performed well in the past, and will perform well in the future. Stocks that have not performed well in the past will not perform too much in the future. However, after a long period of time, the stocks that performed well in the past have become poor. The stocks that have not performed well in the past have performed very well. This is called reversal effect.

Jegadeesh and Titman (1993) first discovered that stock prices have momentum effect. They used stock data from 1965 to 1989 in the CRSP database as samples to sell stocks with the lowest yields in 3, 6, 9, and 12 months, buy stocks with high yields in 3, 6, 9, and 12 months and holding these stocks for 3, 6, 9, and 12 months, and found that they could earn excess returns if they sold stocks that had not performed well and bought the stocks have recently performed well.

This shows that in the short term, the trend of stock prices has a great correlation with its previous trend, and the trend of stock price changes in the past can predict the trend of the stock in the future. Chan, Jegadeesh and Lakonishok (1996) explain the excess returns that can be obtained by relying on momentum investment strategies, which they believe are caused by insufficient response to information.

The calendar effect means that stock returns will change regularly as the date changes, for example "January effect" and "weekend effect".

Rozeff and Kinney Jr (1976) found that the NYSE stock price index in January 1904-1974 was significantly higher than the other 11-month yield. Gultekin and Gultekin (1983) studied stock returns in 17 countries from 1959 to 1979 and found that 13 of them had higher stock returns in January than in other months. Chan, Karceski and Lakonishok (1998) found that between 1926 and 1989, in January, the smallest 10% of stock returns outweighed other stock returns.

Cross (1973) and French (1980) studied the S&P 500 index gains and found higher average returns on Friday and lower on Monday. Gibbons and Hess (1981) and Keim and Stambaugh (1984) found that the Dow Jones index had negative returns on Monday. Rogalski (1984) found that the average negative return between all Fridays and Mondays' closings occurred during non-trading hours, and average trading day earnings from opening to closing were consistent across the day. Jaffe and Westerfield (1985) studied the results of four developed markets in Australia, Canada, Japan, and the United Kingdom, indicating a weekend effect in the countries studied.

For the calendar effect that appears in the stock market, behavioural finance believes that the calendar effect that appears in the stock market is caused by people's emotions. When investors are at the beginning of the year, they tend to invest in the new year, causing a January effect in the stock market. At the weekend, people may feel excitement due to income of wages and right before the holiday, the choice of investment strategy may be more radical, the weekend effect in the stock market.

Modern financial theory holds that people's decision-making is based on assumptions such as rational expectations, risk avoidance, and utility function maximization, where behavioural finance challenges on.

Kahneman and Tversky (1973), when investigating investment behaviours, found that investors tend to believe in the future predictions of short-term data when they make future predictions about investment returns, and tend to exaggerate the probability of events and over-confident. The ability of the event to characterize the future rate of return, the result is the short-sightedness of investment behaviour, that is, for some stocks with good performance, it is believed that the yield of these stocks can continue to grow, and ultimately the investor behaviour violates Bayes' law.

Kahneman and Tversky (1979) proposed the Prospect Theory and argued that individuals' attitudes toward risk are different and do not follow the assumptions of the VonNeumann-Morgenstem rational concept. The investor's investment utility function is asymmetric, that is, investors are often accustomed to using the profit and loss level of their past investment as the frame of reference, subjectively judging the income level of the investment strategy, and always over-estimating the loss. Moreover, when investors have losses, they are more willing to hold the stocks that caused their losses, instead of adjusting the investment portfolio in time, and are more willing to hold the losing stocks to avoid losses, therefore the investment is very conservative.

In the Efficient Market Hypothesis, the trading behaviour of irrational investors is random, so the effects can be offset by each other, so that the irrational trading behaviour of investors has no influence on the market price. But according to Kahneman and Riepe (1998), investors' trading behaviour is not random but systematic, and the impact of trading behaviour on market prices cannot be eliminated by statistical average.

Moreover, influenced by various psychological and emotional factors, many investors have a certain sociality when they invest in each other, which causes investors to deviate from rational decision-making in the same way. This situation does not only exist in the middle of personal investment. According to the research of Falkenstein (1996), professional investors such as fund managers are affected by the comparison with other fund managers, and often make decisions that deviate from the maximization of asset value. It can be seen that investor trading behaviour is systematic and group-oriented for deviating from rational decision-making.

The birth and development of behavioural finance is inextricably linked to the constant challenge of the Capital Asset Pricing Model(CAPM), one of the cornerstones of standard finance. Shefrin and Statman (1994) challenged the CAPM and proposed a Behavioural Asset Pricing Model(BAPM). They also challenged the Modern Portfolio Theory(MPT) and proposed the Behavioural Portfolio Theory(BPT).

MPT believes that investors should focus on the entire portfolio, and the optimal combination configuration is on the effective front of the mean variance. BPT believes that real investors can't do this. The asset portfolio they actually build is based on the understanding of the risk level of different assets and the pyramidal behavioural asset portfolio formed by the investment purpose. Assets are located at each level of the pyramid. Both are associated with specific goals and risk attitudes, and the correlation between the layers is ignored.

BAPM is an extension of the Capital Asset Pricing Model (CAPM). Unlike CAPM, investors in BAPM are divided into two categories: information traders and noise traders. The information trader is a rational trader who strictly follows the CAPM, and there is no systematic deviation. The noise trader does not act according to the CAPM, and will make various cognitive deviation errors. The two types of traders influence each other to jointly determine the asset price. In fact, the problem of capital market portfolio still exists in BAPM, because the effective combination of mean variances changes over time.

In the next section, we will look more closely at noise trading.

4 Noise Trading

Kyle (1984) coined the term of "noise trading", but rather than the noise trading as commonly used to represent the liquidity trading that trading at stable price, his paper offered a new method to analyse market information flows (Kyle, 1985). He assumed that there are three types of trader in the financial market: the noise trader, the insider and the market maker. The noise trader he mentioned is essentially from the usage of our discussion later because this term he used to describe the trading is based on liquidity needs. Their demand of trading is based on risk hedge or liquidity, and it is exogenous from his model. The following literature often refers to this type of trader as the liquidity trader.

Black (1986) systematically described noise trading in the financial markets, and pointed out that noise trading is not only the foundation of the existence of financial markets but also brings problems to the financial markets. Black's article focused more on the trading environment including the noise since it is an inaugural speech for president of American Finance Association (AFA), subsequent papers that were inspired by his work have developed the technique details, describing the interaction between noise traders.

According to Black's definition, which he brought from natural science, "noise" is the concept contrasted with "information". Information is a sequence of symbols that can be interpreted as a message and be conveyed. It can be recorded as signs, or transmitted as signals. It is any kind of event that affects the state of a dynamic system that can interpret the information. Noise is a signal that people use to make investment decisions as if it were information. The noise trading is that people are trading based on noise as opposed to information. Black also expands the concept of noise to Econometrics and Macroeconomics, and believes that the common element is the emphasis on a diversified array of unrelated causal elements to explain what happens in the world.

In Black's opinion, most of the time, the noise traders as a group will lose money by trading, while the information traders as a group will make money. The reason is trader who trading maybe due to they assume what they had is information rather than noise, or perhaps they just like to trade. So from an objective point of view they are actually better off not to trade at all, and also their loss on trade goes to the information traders. As the amount of noise trading increases, it will become more profitable for people to trade on information. This idea seems to coincide with Friedman (1953), but Friedman used this point to demonstrate that the financial market is in rational equilibrium. This means that the rational arbitrage behaviour in the market will drive the irrational traders out of the market since they do not have sufficient ability to make a profit. Therefore, later research on noise trading models is not consistent with Black. For example, De Long, Shleifer, Summers and Waldmann (henceforth, DSSW) developed the DSSW noise trading model and it proves that the noise trader creates their own space in the financial market (De Long *et al.*, 1990a). Their work provided the direction for following work, and is also vital for my thesis.

The original DSSW model is an overlapping generation model with two period lived agents which is based on Samuelson (1958). It has two periods only, the first period for work and the second for consumption.

The model has two different assets. One is a safe or risk free asset which is in perfectly elastic supply and pays fixed dividend r per period. It can be created at one unit, and it can be turned backed at one unit. The price is fixed at one unit forever as well, generally it can be treated as short term government bills. The risky asset u pays fixed dividend r the same as the risk free asset. But it is not in elastic supply and generally it can be treated as the equity asset. It is in a fixed and unchangeable quantity, normalized at one unit.

The model contains noise traders (denoted by n) and arbitrageurs (denoted by i). Noise traders have erroneous beliefs, however they believe that they have correct information about future distribution of returns on a risky asset and set their investment portfolio based on it. The arbitrageurs are sophisticated investors and their trading strategies push prices towards its fundamentals.

The total demand of risky asset equals the demand λ came from noise traders and arbitrageurs. The proportion of noise trader is μ, so the proportion of arbitrageurs is $(1-\mu)$. Therefore:

$$(1-\mu)\lambda_t^i + \lambda_t^n \mu = 1$$

A representative sophisticated investor young in period (t) accurately perceives the distribution of returns from holding the risky asset, and so maximizes expected utility given that distribution.

A representative noise trader young in period (t) misperceives the expected price of the risky asset by an independent and identically distributed (IID) normal random variable ρ_t:

$$\rho_t \sim N(\rho^*, \sigma_\rho^2)$$

The ρ^* is a measure of the average "bullishness" of the noise trader, that is the mean of the deviation of pricing risky asset from its true value. σ_ρ^2 is the variance of noise traders' misconception of the expected return per unit of the risky asset.

Both types of agents choose their portfolios when young (t) to maximize perceived expected utility given their own beliefs about *ex ante* mean of the distribution of price of risky asset (u) at old $(t+1)$. When they sell the risky asset they holding at old, and so the demand of the young must sum to one in equilibrium. By further assuming the distribution of P_t same as P_{t+1}, De Long *et al.* (1990a) solved the price function for risky asset recursively:

$$p_t = 1 + \frac{\mu(\rho_t - \rho^*)}{1+r} + \frac{\mu\rho^*}{r} - \frac{(2\gamma\mu^2\sigma_\rho^2)}{r(1+r)^2}$$

Where ρ_t denotes the current optimism, ρ^* denotes the average optimism, γ denotes the coefficient of absolute risk aversion.

The later three terms show the impact of noise traders to the equilibrium price of the risky asset. The equilibrium will converge to its fundamental value "one" when the erroneous beliefs of noise traders converge to zero.

The second term is the discounted value of deviation of the next period's noise traders' misevaluation from their average misconception of value $(\rho_t - \rho^*)$. Even though the risky asset u is not subject to any fundamental uncertainty (all the fundamental uncertainty is in the dividend and cannot be forecast) and so is known by a large class of investors, its price varies substantially as noise traders' opinions shift. So at the time period t prices rise and fall according to whether noise traders are optimistic, when $(\rho_t - \rho^*) > 0$, or pessimistic, *i.e.* $(\rho_t - \rho^*) < 0$. Also we could observe that the more numerous noise traders are relative to sophisticated investors, the more volatile asset prices are.

The third term captures the deviations of p_t from the fundamental value due to the fact that the noise traders are on average bullish. This will generate pressure to push the price up. Optimistic noise traders bear a greater than average share of price risk. Since arbitrageurs bear a smaller share of price risk when ρ^* is higher, they require a lower expected excess return and so are willing to pay a higher price for asset u.

The fourth term is at the heart of the model. The sophisticated investors must be compensated for bearing the risk that the noise traders will become bearish and the price of the risky asset will fall. Both types of trader at period t believed the price of asset u is mispriced but they do not want to bet too much since the future price P_{t+1} is uncertain. The return from enlarging one's position in an asset that all have agreed is mispriced, though in different direction, is offset by the additional price risk that must be run. The noise traders thus "create their own space": the uncertainty over what the next period's noise traders will believe is a factor that makes the otherwise riskless asset u risky and drives its price down and its return up.

This two-staged overlapping generation model is a simplified model. It could simulate the real world if the period between choices becomes smaller. This actually transfers the investment term of each generation from years to a very short interval, and lots of investors assess their result and continue trading.

5 Positive Feedback Trading

In this section, we will first further explain the DSSW positive feedback model in detail. Those the theoretical papers in the next chapter were other behavioural model inspired by DSSW but focus on different aspect. For example, the HS model focuses more on the interaction between different participants rather than cognitive bias. The empirical research part will give some examples across different markets at different times. They witness positive feedback trading at both developed and developing markets and have also shown slightly different characteristics. Last but not least, the final section gives both economical and noneconomical explanations of the positive feedback trading forms.

In the noise trading model of DSSW, the rational trader will not hold a large enough arbitrage position. This is due to the risk aversion feature of arbitrageurs as we explained at the fourth term of the model before. Taking a larger position also means taking more risk, so there is a limit to how large the position the arbitrageurs will take. Therefore the noise trader will be able to essentially influence the price and push it away from its fundamental value. The rational trader is trading in the opposite direction of noise trader, and they push the price back toward its fundamental value in the model. A noise trading model generally assumes the erroneous beliefs of the noise traders are in a random direction. Actually there is a special type of irrational trader that conducts a positive feedback trading strategy, the positive feedback trader.

Their behaviour makes them essentially a form of the noise trader because they are not trading based on the information related to fundamental value, but will buy when price goes up and sell when price goes down. The positive feedback traders magnify the change in price because they are always chasing the trend. The price will deviate from its fundamental value and will show highly speculative if the positive feedback traders dominate the financial market.

The results of natural science research have shown that positive feedback is the original driving force for self-enhancement, self-organization and autocatalysis in various systems such as physics, biology, and chemistry. This force is of great significance to the evolution of the system. Therefore, positive feedback trading in the security market has a special effect on the evolution of securities prices. Since positive feedback trading has a unique effect on the evolution of security price, Shiller (2002) pointed out that feedback models, in the form of difference equations, can produce complicated dynamics. This positive feedback may be an essential source of much of the apparently inexplicable "randomness" that we see in financial market prices.

Note that the "Logistic Model", also known as "Logistic Map", not the "Logistic Choice model", is also represented in the form of a difference equation that generate chaotic process, we will not develop the idea here but instead set a later section on this chaos stream of the literature review.

Shiller (1990) first described his "Feedback Model" in 1990. In this model, the feedback coefficient could be either positive or negative, and he assumed the feedback coefficient must be less than 1, therefore the model has a stationary solution. Shiller defined the feedback model for the financial market as follows:

$$P_t = \pi_t + c \int_{-\infty}^{t} e^{-\gamma(t-\tau)} dp_\tau$$

P_t is the asset price at time t, parameter $c > 0$, $\gamma > 0$, π_t is the valuation people would place on a stock if there were no feedback through price. Generally, $c < 1$ means the stock price is in stationary status, $c \geq 1$ means the stock price will be explosive expanding status. Shiller assumes $c < 1$ implies the evolution system of stock price is a stable system. He did not research the formation process of price but just gave the pricing model empirically.

De Long *et al.* (1990a) developed the noise trading model, where De Long *et al.* (1990b) clarified the concept of positive feedback and explicitly described the behaviour of the positive feedback trader, a specific type of noise trader. This paper researched the impact of rational speculative investors on prices under the condition where there are positive feedback traders existing in the market. This paper demonstrated that some rational traders might join the feedback group which will further destabilize the price while there is positive feedback strategy existing in the market. This makes short term prices become more unstable. This approach extends the traditional understanding to rational speculative investor and shows that the rational trader enhance the power of positive feedback trading at a certain level. This paper set a model which investigates the game mode between the passive investor, the rational speculative investor and the positive feedback trader. It offers clues to explain a lot of empirical observations and becomes an important theory on micro trading mechanism research.

De Long *et al.* (1990a) assumed a model which has 4 periods: 0, 1, 2 and 3. Stock and cash the only two assets involved. Cash pays no net return and in perfectly elastic supply.

Stock is in zero net supply: it should be thought of as side bets that investors make against one another. Stock is liquidated and pays a risky dividend equal to $\Phi + \theta$ in period 3, that is when investors consume all their wealth. θ is distributed normally with a mean of zero and variance σ_θ^2. At any time before period 3, there is no meaningful information about θ released. Φ has a mean zero and can take on three possible values: φ, 0, and $-\varphi$. In period 2, the value of Φ becomes public. In period 1, a signal about Φ is released.

There are three kinds of investors in the model: positive feedback traders "*f*", present in a measure of one; informed rational speculators (arbitrageurs) "*a*", who maximize utility as a function of period 3 consumption, present in a measure of μ; and passive investors "*i*" whose demand in all periods depends only on the price relative to its fundamental value, who are present in a measure of $1 - \mu$. The reason to keep the total of the passive investors and arbitrageurs in constant is that it enables us to derive comparative statics results on the effect of changes in the number of rational speculators holding constant the risk bearing capacity of the market. A pure addition of rational speculators to the market would have the extra effect of raising the market's risk bearing capacity and so dampening price volatility. The amount of positive feedback traders is equal to the rational investors (arbitrageurs plus passive investors) and they both equal 1.

There are no signals received in Period 0 since it is a reference period only. The price kept at zero which its initial fundamental value. Trading is not exist. This period provides a benchmark against which the positive feedback traders can measure the depreciation or appreciation of stock from period 0 to periods 1 and 2.

In period 1, a signal $\varepsilon\,\{\varphi,\ 0,\ -\varphi\}$ is received by arbitrageurs about fundamental news Φ in period 2. The signal can either be noiseless: $\varepsilon = \Phi$ or it could be a noisy signal that satisfies:

$$\begin{aligned}
&Prob \quad (\varepsilon = \varphi,\ \Phi = 0) &&= 25\% \\
&Prob \quad (\varepsilon = \varphi,\ \Phi = \varphi) &&= 25\% \\
&Prob \quad (\varepsilon = -\varphi,\ \Phi = 0) &&= 25\% \\
&Prob \quad (\varepsilon = -\varphi,\ \Phi = -\varphi) &&= 25\%
\end{aligned}$$

If the signal is noisy, when the signal of arbitrageurs ε is φ, the expectation of the subsequent value of Φ is $\varphi/2$; when the signal of arbitrageurs ε is $-\varphi$, the expectation of the subsequent value of Φ is $-\varphi/2$. The arbitrageurs choose their demand D_1^a in period 1 to maximize the same mean variance utility function as in period 2 over the distribution they face as of period 1 of their certain equivalent wealth in period 2. The passive investors sell high and buy low. Their demand is:

$$D_1^i = -\alpha p_1$$

The demand of the positive feedback traders is equal to zero in period 1:

$$D_1^f = 0$$

The reason positive feedback traders do not trade in period 1 is that the form of positive feedback behaviour reacts to past price movements only but not to current price changes.

The value of Φ is revealed to both passive investors and arbitrageurs by period 2. The realized value of Φ is required to be sufficiently small so as not to upset the mean variance approximation used in deriving the demands of arbitrageurs.

In period 1, the demand of the positive feedback traders is:

$$D_2^f = \beta\left(p_1 - p_0\right) = \beta\left(p_1\right)$$

The price in period 1 is p_1, and p_0 which equals to zero is the price in period 0, and β is the positive feedback coefficient. In period 2, the demands of positive feedback traders respond to the price movement from period 0 to period 1. They buy when the price goes up and *vice versa.* Note that positive feedback traders place a market order today in response to price movements in the past. This formulation disallows investors to respond immediately to price changes, *i.e.* they do not react to price movements from period 1 to period 2. An explanation to describe this assumption is that investors react to a past history of capital gains by raising their estimate of the mean rate of return and thus increasing their demand.

The reason arbitrageurs will not follow the positive feedback trading strategy is that the expected period 3 value of the stock is known for them. They will not hold a positive quantity of stocks in period 2 if $p_2 > \Phi$ since such a portfolio is exposed to risk and has a negative expected return. Actually, the purchases of positive feedback traders are not related to the price in period 2 .

In period 2, arbitrageurs choose their demand D_2^a to maximize a mean variance utility function with risk aversion coefficient γ. The aggressiveness of arbitrageurs in betting on reversion to fundamentals in period 2 is limited only by period 3 dividend risk. The demand of arbitrageurs is:

$$D_2^a = \frac{(\Phi - p_2)}{(2\gamma\sigma_\theta^2)} = \alpha\,(\Phi - p_2)$$

For notational convenience, we set $\alpha = \frac{1}{\left(2\gamma\sigma_\theta^2\right)}$. In period 2, the demand of the passive investor is negatively related to price as well:

$$D_2^i = \alpha\left(\Phi - p_2\right)$$

We assume the slope of passive investors' demands is equal to the arbitrageurs' in period 2. We also set the number of passive investors and arbitrageurs equal to $1-\mu$ and μ , respectively. When introducing arbitrageurs, this assumption allows us to examine the consequences of it without changing the market risk bearing capacity due to the changes in μ keeping the risk bearing capacity of the economy in constant.

A rise in the number of arbitrageurs has two opposite effects without passive investors. First, prices become destabilized due to it enhances the stimulus of arbitrageurs' purchases to positive feedback trading. Secondly, prices become stabilized due to it increases the market risk bearing capacity. The second role of arbitrageurs has been emphasized by Friedman (1953) and Stein (1987). However, we abstract from this effect and to this end include passive investors in the model. If we perform the experiment of simply adding arbitrageurs, there are cases in which the risk sharing stabilizing effect is less important than the destabilizing effect of anticipatory purchases.

We set $\alpha > \beta$ in order to obtain the stable solutions, since rational speculation makes prices in period 1 increase one for one with expected period 2 prices. The model will not have a stable equilibrium unless $\alpha > \beta$: the demand will exceed supply for high correctly anticipated values of p_2 .

There is no trading in period 3. Investors pay each other according to the positions they hold in the stock and the publicly known dividend $\Phi+\theta$. In period 3, the arbitrageurs pin the stock price down to its fundamental value of $\Phi+\theta$, because the dividend is known for certain.

There is no trading in periods 0 and 3 so the market clearing conditions are automatically satisfied in those periods. Since there are $1-\mu$ passive investors and μ arbitrageurs, the market clearing conditions for periods 1 and period 2 are, respectively:

$$\begin{aligned} 0 &= D_1^f + \mu D_1^a + (1-\mu) D_1^i \\ 0 &= D_2^f + \mu D_2^a + (1-\mu) D_2^i \end{aligned}$$

To sum up, we are introducing the DSSW positive feedback model explicitly for three reasons. First, it offers a novel insight to analyse the price movement mechanism, *i.e.* the positive feedback trading. Secondly, in contrast to intuition, arbitrageurs make prices become destabilized where positive feedback trading exists. Thirdly, the interaction between positive feedback trading and rational traders could help us better understand the mechanism of bubble and crashes. As our core literature, De Long *et al.* (1990b), the later theoretical papers including our main and empirical papers were inspired and cited by them, such as tools to detect the nonlinearity and empirical result across different regions.

5.1 Theoretical papers

Lux (1995) raised a contagion model that attempts to formalise herd behaviour or mutual mimetic contagion in speculative markets. His work was to construct an elementary model of stock dynamics which explicitly included contagion of opinion and behaviour and to offer a behavioural explanation for the bubble formation and crash in the stock market. Early literature on the noise trading model often assumes that noise traders misperceive expected returns (De Long *et al.*, 1990a, 1991) or describe their behaviour as following a simple feedback rule and study the resulting dynamics of the market (Day and Huang, 1990; Gennotte and Leland, 1990; Chiarella, 1992), where the psychological factors which influence the behaviour of non-sophisticated traders explicitly were modelled. Speculators are not simply blind followers of the crowd but react in readiness simply to avoid missing the opportunity to make a profit.

According to this model, it will be postulated that a fixed number $2N$ of speculative traders exists. These may either be optimistic or pessimistic about the future development of the market, and also assuming there is no neutral subjects. There is an index describing the average opinion of speculative investors $x \in [-1, 1]$. It follows that $x = 0$ corresponds to a situation of balanced dispositions, *i.e.* there exists an equal number of optimistic or pessimistic individuals. Hence, situations with $x > 0$ exhibit predominant optimism, and $x < 0$ exhibit predominant pessimism.

Infection of attitudes means with a big portion of optimistic traders, it would be very probable that the few remaining pessimistic ones would also change their attitude and buy stocks, and *vice versa* for a big portion of pessimistic traders. According to the contagion mechanism:

$$\begin{aligned}\frac{dx}{dt} &= (1-x)\,ve^{ax} - (1+x)\,ve^{-ax} \\ &= 2v\left[\sinh(ax) - x\cosh(ax)\right] \\ &= 2v\left[\tanh(ax) - x\right]\cosh(ax)\end{aligned}$$

Where a gives a measure for the strength of infection or herd behaviour, and v is a variable for the speed of change. For $a \leq 1$, it possesses a unique stable equilibrium at $x = 0$. For $a > 1$, the equilibrium $x = 0$ is unstable and two additional, stable equilibrium, *i.e.* $x > 0$, and $x < 0$ exist. Therefore, if the herd effect is relatively weak, then all deflections into one direction will die out in the course of events and the system will return to a state of balanced dispositions after some disturbance for $a \leq 1$. For $a > 1$, on the other hand, small deviations from the balanced state are sufficient to make a majority of traders bullish or bearish through mutual infection. The model including contagion and price dynamics is as follows:

$$\begin{aligned}\dot{x} &= 2v\left[\tanh\left(\frac{a_1\dot{p}}{v} + a_2x\right) - x\right]\cosh\left(\frac{a_1\dot{p}}{v} + a_2x\right) \\ \dot{p} &= \beta\left[xT_N + T_F\left(p_f - p\right)\right]\end{aligned}$$

The dependence of investment behaviour on price dynamics (which is at the heart of most of the contributions in the noise trader literature like DSSW) enforces the contagion effect. The a_2 is a weight factor describing how much information investors try to draw from the behaviour of others, for $a_2 \leq 1$, there exists a unique equilibrium $E_0 = (0,\ P_f)$. When $a_2 > 1$, two additional equilibrium, optimistic market E_+ and pessimistic market E_- emerge, and E_0 is always unstable. If E_0 is a unique equilibrium, *i.e.* $a_2 < 1$, it may either be stable or unstable. The condition for stability is given by:

$$2\left[a_1 \beta T_N + v\left(a_2 - 1\right)\right] - \beta T_F < 0$$

Where T_N denotes the trading volume of speculative investors, and T_F is a measure for the trading volume of fundamentalists as opposed to T_N. If we consider the variable a_0 captures a general prevailing mood of the market, we have:

$$\begin{aligned} \dot{x} &= 2v\left[\tanh\left(a_0 + a_2 x\right) - x\right]\cosh\left(a_0 + a_2 x\right) \\ \dot{a_0} &= \tau\left\{\left[\frac{r + \tau^{-1}\left(\frac{T_N}{T_F}\right)\dot{x}}{p_f + \left(\frac{T_N}{T_F}\right)x}\right] - R\right\} \end{aligned}$$

The dynamics always possesses the unique equilibrium $E = (0,\ 0)$. The equilibrium is stable (unstable) if $a_2 - 1 + \left(\frac{T_N}{T_F}\right)/p_f < (>)\, 0$. When prices rise due to some random event, a_0 also goes up for an extended period. Once infection has reached the overwhelming majority of speculative traders, a change in basic sentiment occurs because the exhaustion of the pool of potential buyers causes price increases to diminish. There the number of additionally infected speculators still rises for a short time, while there is already some scepticism spreading out (declining a_0) because of the deceleration of the price trend. However, the realisation of declining profit opportunities leads very soon to increasing sales, and prices go down as well reinforcing contagion of fear among traders. This continues until the majority is infected with pessimism. Following this the price decrease weakens and a recovery of returns leads to a similar reversion of the "basic disposition" as described above.

Daniel, Hirshleifer and Subrahmanyam (1998) point out the model of De Long *et al.* (1990b) is delivered with mechanistic positive feedback traders. Their approach differs in explicitly modelling the decisions of quasi-rational individuals. Their research shows "Biased Self-attribution" is one possible psychological foundation for a stochastic tendency for trades to be correlated with past price movements, which can create an appearance of positive feedback trading. This literature provides support to positive feedback trading based on a psychological foundation by applying the DHS model that discussed two situations, the constant confidence and the confidence with variation. Bem (1965, 1967) discovered a cognitive dissonance that individuals too strongly attribute events that confirm the validity of their actions to high ability, and events that disconfirm the action to external noise or sabotage. "Biased Self-attribution" could lead to overconfidence, where individuals observe the outcomes of their actions, and they update their confidence in their own ability in a biased manner.

Hong and Stein (1999) built a unified behavioural model called unified theory model or the HS model. This model shared the same goal as the DHS model but adopted a fundamentally different approach. They emphasized the interaction between heterogeneous agents rather than trying to say a lot about the psychology of the representative agent. That means more of the action in their model comes from the way these traders interact with one another, and less of it comes from particular cognitive biases that they ascribe to individual traders.

They featured two types of agents in the financial market, "news watchers" and "momentum traders". Both types are limited rational, with the bounded rationality being of a simple form: each type of agent is only able to "process" some subset of the available public information. The news watchers make forecasts based on signals that they privately observe about future fundamentals. The limitation of the news watchers is that they do not condition on the past or current prices, but the momentum traders do condition on past price changes. However, the limitation of the momentum traders is that their forecasts must be "simple" functions of the price history. That assumption is different from De Long *et al.* (1990b), where positive feedback traders are extremely irrational and arbitrageurs are fully rational.

Hong and Stein (1999) also assumed that private information diffuses gradually across the news watcher population. Therefore the news watchers are reacting slowly to information, and the momentum traders are trying to arbitrage on this underreaction based on a change of past prices. However, if momentum traders are limited to simple strategies, it turns out that this intuition is incomplete. The momentum traders' attempts to profit from the underreaction caused by news watchers lead to a perverse outcome: the initial reaction of prices in the direction of fundamentals is indeed accelerated, but this comes at the expense of creating an eventual overreaction to any news. The trend-chasing strategy makes money even it is so simple. But it would become apparent that the strategy does better in some circumstances than in others if one could condition on more information. The strategy earns the bulk of its profits early in the "momentum cycle" particularly, by which we mean shortly after substantial news has arrived to the news watchers and loses money late in the cycle, by which time prices have already overshot long run equilibrium values. Thus a crucial insight is that "early" momentum buyers impose a negative externality on "late" momentum buyers. To sum up, the HS model unified the short term underreaction and long term overreaction together into the process when information diffuses gradually.

5.2 The formation mechanism of positive feedback trading

In this section, by reviewing the empirical studies, we will illustrate different possible causes for positive feedback trading, such as herd behaviour, extrapolative expectations and so on.

5.2.1 Herd behaviour

In the formation mechanism of positive feedback trading, the gossip around the general public contributes a lot. Whether it comes from the newspaper, television or an upper class gala, the information circulated has eliminated the rational doubt, push more and more people into the market, i.e. the public demonstrate "Herd Behaviour". Herding refers to the type of conduct involving similarity in behavior following interactive observation of beliefs, actions or action-payoffs (Hirshleifer and Hong Teoh, 2003). This kind of behaviour that concerns following other's actions will cause positive feedback trading at the market.

The first person who raised the concept of herd behaviour was Keynes (1936). He also justified the rationale of the existence of herd behaviour by using the beauty contest as a example.

"It is not a case of choosing those faces that, to the best of one's judgment, are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees. " (Keynes, 1936).

He believed that similar behaviour was at work within the stock market. This would have people pricing shares not based on what they think their fundamental value is, but rather on what they think everyone else thinks their value is, or what everybody else would predict the average assessment of value to be.

Festinger (1957) also pointed out that, when facing conflict, our mind will subconsciously eliminate the view which has the least connection with others and will seek the balance. This phenomenon is called cognitive dissonance reduction. In the stock market, herd behaviour has more than one definition. Lakonishok, Shleifer and Vishny (1992) believed that herd behaviour meant to buy or sell the same stock with other investors at the same time. This concept is much narrower than others, since a group of investors conducting the same trading decision based on the same information they have received is different from a group of investors trying to replicate other's actions. In our research model, the positive feedback trader and the smart arbitrageur could represent both groups.

There is a lot of theoretical analysis on the rationale of herd behaviour. The pay-off externalities herding model is the earliest example. The main concept is that when the market condition is getting worse and the people who take earlier action will damage the benefit of acting later, i.e. the externality exists, the most rational choice for all participants is to act as soon as possible. Bank runs are a good example, and we could also see this when the asset bubble crushes.

Devenow and Welch (1996) and Bikhchandani and Sharma (2000) proposed the information cascade herding model. They analysed market participants who lacked the ability to gather information, therefore they try to mimic other investor that has superior ability to gathering information, such as top ranked fund managers or institutional investors. Trueman (1994) demonstrated that the herding effect is exists between financial analysts. His model shows that analysts with low abilities issue earnings forecasts that are close to those announced by other analysts in order to mimic high ability analysts and get a bigger compensation.

The principal agent herding model has explained another herding behaviour under a specific assessment method. Scharfstein and Stein (1990) demonstrated herd behaviour between fund managers and argued that it could be the consequence of rational attempts by managers to enhance their reputation as decision makers, or it correlated to prediction errors that lead to the "sharing the blame" effect that drives managers to herd. Grinblatt, Titman and Wermers (1995) also proved the herd behaviour between mutual funds investment. This reputation based herding takes place because under a specific assessment method, the fund manager does not need to be the best performer but will try to be the worst among their peers.

The empirical studies about herd behaviour approach the subject from two angles. One is the stock price, Christie and Huang (1995) did some very impactful research by testing the dispersion for the individual return and the market return. Their results for both daily and monthly returns did not support the presence of herding. Their method received criticism due to its lack of sensitivity, that is their method is able to detect sharp herding effects but lacks ability to detect the less dramatic ones. Chang, Cheng and Khorana (2000) proposed a modified method, instead of using cross-sectional standard deviation of returns (CSSD), they proposed using cross-sectional absolute deviation of returns (CSAD) to measure the dispersion, and that allows them to catch the minor herd behaviour. Their result supports Christie and Huang (1995) for developed countries or regions (US, Japan etc.), but found that in the emerging markets like Taiwan or Korea, the dispersion will decline, it means the herding behaviour also takes place in the emerging markets. Our research is targeting mainland China, and we are expecting this stock market behaviour to be like other emerging markets.

Another angle for empirical studies on herding behaviour is looking at the trading subject itself. Graham (1999) divided the analysts into smart and dumb types in order to modelize their reputation. The result implies if an analyst has a high reputation or low ability, or if there is strong public information that is inconsistent with the analyst's private information, he or she is likely to herd.

Jegadeesh and Kim (2010) developed a model that allows them to specifically test for non-information driven herding of sell-side analysts when they make stock recommendations. They found robust results to a variety of controls. Their paper was also the first to directly investigate whether the market recognizes herding behaviour. Their results indicate that the market anticipates analysts' tendencies to herd, and the market price reaction on the revision date accounts for such herding tendencies.

Andrikopoulos, Albin Hoefer and Kallinterakis (2014) studied the impact of exchange mergers over herding in the context of the EURONEXT group. The merger had a mixed impact on different countries, although for all countries except Portugal, their herding structure grew less persitent and noisier post-merger, which could be seen as the result achieved by EURONEXT's improved informational environment. Andrikopoulos *et al.* (2017) further revealed that intraday herding is significant in the EURONEXT by using the tick data. They also found that the herding effect became less strong after the 2007-2009 financial crisis period.

5.2.2 Extrapolative expectations

Beside herd behaviour, there are other possibilities to explain why the positive feedback effect takes places in trading. For example, the extrapolative expectations applied to making decisions within an uncertain environment is a common form of positive feedback. Extrapolative expectations is a prediction method using past change trends of indicator to estimate the future situation of indicator. It is frequently applied in economic analysis and prediction, especially in macroeconomics.

Since extrapolative expectations occurs in decision making, it could often be biased. In behavioural finance, there are several terms to describe the cause of this. Representativeness Heuristic is one, predicting the future by searching for the closest scenario in past experience, without carefully considering the possibility of matching this scenario. When people predict future stock prices, they tend to choose obvious stock price patterns, such as unusual and persistent price trends, which form a positive feedback dynamic.

Andreassen and Kraus (1990) had shown that the investor has a tendency to follow the trend by conducting an experiment. They illustrated some real stock price patterns to experimental subjects and told them they were real. They let them trade on a certain price level by giving them initial capital and some background information on the stock's current price. Then, for every new stock price illustrated, let them change the position they were holding and assume their trading had no impact on the market price. Their experiment result is as expected. After a short period of observation, the experiment subjects started to chase the average price level when the price does not have an obvious trend. They did buy when the price was falling and sell when the price was rising. When the price forms a trend, people will start to chase that trend. That is, buying more when price is rising, and selling when price is falling. At this moment, the experiment subjects are not extrapolated by the price level, but extrapolated by the change of price level. The experiment also found that the changing of trend chasing only take place after a series observation and react on the distinct changing on price level.

In addition to the experimental studies, Case and Shiller (1988) also surveyed house buyers. They found that house buyers anticipate the value of houses continuing to rise when the value of houses are already rising sharply in the city. In contrast, if past house prices do not have a rising trend, the house buyer will not have that anticipation. Shiller (1988) also performed another survey of the investors that sold their stocks after the 1987 stock market crash. He found that almost every investor that sold their own stock put the falling price as the reason for selling . That clearly shows, they are anticipating the price falling even further. Frankel and Froot (1987) analysed the change of exchange rate during the 1980s, showing that even when investors were aware the trend could reverse in the long term, they could still take a trend chasing strategy for the short term.

Gennaioli and Shleifer (2018) argued that business cycles are both predictable and driven by irrationality. They take their cue from a number of recent papers hinting that recessions are actually possible to predict years in advance, if one simply pays attention to the right variables. One of these is by Greenwood and Hanson (2013), showing that when junk bond issuance increases and credit spreads narrow, a credit bust often tends to follow two or three years later. Another one is by Baron and Xiong (2017), showing a similar result for bank lending instead of corporate bonds.

All of these papers have one thing in common, that is they use debt to predict recessions years in advance. Gennaioli and Shleifer explain these patterns by turning to their own preferred theory of human irrationality, the theory of extrapolative expectations (Barberis *et al.*, 2018). Basically, this theory holds that when asset prices rise, home values, stocks and so on, without a break, investors start to believe that this trend represents a new normal.They pile into the asset, pumping up the price even more, and seem to confirm the idea that the trend will never end. But when the extrapolators' money runs out, reality sets in and a crash ensues.

5.2.3 Other behavioural factors indirectly affect the stock price

The research in this section is a bit different, because it focuses on factors which do not directly link to the fundamental value but still cause the price of stock to change. This is important because these factors change trigger the price change, and due to the condition changes in a scale, such as weather, the power of positive feedback trading begins. It gives a idea when the market has no economical related news, but could still form a trend.

Goetzmann *et al.* (2015) was the first to examine the impact of weather on institutional investor trading decisions and individual stock returns. Recent research found that the weather patterns of major financial centres has a relationship to the return of stock indices, therefore it provided evidence that investor mood has an impact on asset price. For example, Hirshleifer and Shumway (2003) used data from the international stock exchange demonstrate that the return was higher on sunny days due to investors having a good mood. The founding in finance has approved the theories from psychology, that is the investors falsely take into account the mood induced by the weather into an information source of decision making, and the weather should not have a relationship to the object of decision making, such as stock return.

Their study focused on the economical mechanism of how the weather-induced mood affected the asset price. More specifically, they investigated how mood affected the institutional investor which is the main part of the price decision. Since there is already research which shows that experienced investors will also affect by cognitive bias, they assume the weather will affect their mood and further affect on price.

To test the hypothesis, they used cloud cover as the agent indicator for investor mood, and also data of position hold of institutional investors. They found that relatively cloudier days increase perceived overpricing in both individual stocks and the Dow Jones Industrial Index, and increases selling propensities of institutions. They also used their new indicator show that investor pessimism negatively impacts daily stock returns, most on high cost to arbitrage stocks. Since the existing studies on the weather effect on stock index returns are for individual investors, their founding complementing by showing the weather could also affect the institution as well, which offered an additional channel through which can manifest stock price.

Gao and Lin (2015) demonstrated that there is a substitution effect between stock trading and gambling, at least for individual investors. They found that the trading volume by individual investors decreases between 5.2% and 9.1% among stocks preferred by individual investors and between 6.8% and 8.6% among lottery-like stocks when the jackpot exceeds 500 million Taiwan dollars (about 150 million US dollars).

There are two possible explanations for the decline of trading volume along with the big jackpot appears. One hypothesis is that investors might just be having fun and feeling excited, if there is a substitution effect between stock trading and gambling, the jackpot will make the trading volume of individual investors (both at long and short positions) decline. Another hypothesis is that the individual investor might treat stock trading as having a similar risk exposure as the lottery, so when the jackpot went big, the individual investors would use the lottery to substitute the stocks. So their net position will decrease, that is, they reduce their buy volume more than their sell volume in stocks. The evidence that buying stock is as gratifying as selling stock is consistent with our hypothesis that individual investors trade stocks for fun and excitement.

Guo and Zhang (2016) show that the air quality could affect the stock market participant by different channel thus affect the stock market. Similar to the research done by Hirshleifer and Shumway (2003), air pollution could trigger mood change (Lepori, 2009). Their research used the classical method to demonstrate that the air quality has an impact on the stock market and there is a channel between mood and air quality. Air pollution will raise the volatility of stock mainly due to the mood change stated above, and they used the Copula method to capture the other channel effect besides the mood, such as policy and expectation.

5.3 Empirical Research

Sentana and Wadhwani (1992) present evidence on the links between volatility and returns autocorrelations by using both hourly data around the period of the October 1987 crash and daily data for 1885-1988. They use alternative measures of volatility based on an exponential GARGH model (EGARGH), and on non-parametric methods as well. Their result reported suggests that stock returns at short horizons exhibit positive serial correlation when volatility is low, but returns exhibit negative autocorrelation when volatility is rather high. This time the varying nature of the serial correlation pattern appears to be robust across different periods and different measures of volatility. They also found some evidence which suggests that the extent of positive feedback trading is greater following price declines than it is after price rises. This asymmetry is consistent with both the possibility that the existence of significant distress selling after price declines and with risk aversion declines rapidly with wealth.

Koutmos (1997) has examined the autocorrelation pattern of the returns of the stock market in six major industrialized countries, assuming that some traders follow positive feedback trading strategies. In all six markets, he found that feedback trading is an important factor of short term movements in stock returns. The impact of feedback trading is to produce negative first order autocorrelation in stock returns, which becomes more negative as the level of volatility rises. He also found that the results support Sentana and Wadhwani (1992) that in four out of the six markets, the feedback trading is a little more intense during the market declines.

Koutmos and Saidi (2001) tested for the presence of positive feedback trading in the stock index returns of six emerging stock markets. They also found that feedback trading is an important factor in determining short term movements in stock returns and it produces negative first order autocorrelation stock returns, which become more negative as the level of volatility rises. One contrast from Sentana and Wadhwani (1992) and Koutmos (1997), papers which focus more on the developed market, this time for the majority of the emerging markets that Koutmos and Saidi (2001) examined suggested the positive feedback trading is asymmetric in up and down markets. Unlike the feedback traders from developed markets who are active in up and down, this study suggests that feedback traders at emerging stock market are just intense during market decline but absent on the up side. Such behaviour is also consistent with the notion that the so-called 'leverage effect' may actually be due to intense feedback trading during market declines. Jiang, Shen and Zhao (2008) have the same findings in market characteristics with Koutmos and Saidi (2001), in the Shanghai Stock Exchange Composite (SSEC) Index and ShenZhen Stock Exchange Component (SZEC) Index in the Chinese Stock market from 1996 to 2005.

Watanabe (2002) examined the autocorrelation pattern of the returns in the Tokyo Stock Exchange (TSE) assuming that some traders follow a positive feedback trading strategy. The relation between autocorrelation and volatility is specified in two different ways. One is the linear AR (LAR) model that specifies as a linear function and the other is the exponential AR (EAR) model that specifies as an exponential function.

$$R_t = a + (b_0 + b_1\sigma_t^2)R_{t-1} + b_2|R_{t-1}| + c\sigma_t^2 + \varepsilon_t$$

and

$$\varepsilon_t = \sigma_t z_t,\ z_t \sim i.i.d.,\ E\left[z_t\right] = 0,\ E\left[z_t^2\right] = 1$$

This specification is henceforth called the Linear AR (LAR) model, which is used by Sentana and Wadhwani (1992) and Koutmos (1997). If b_1 is negative, autocorrelation is decreasing in volatility. Because of the presence of the term $b_2|R_{t-1}|$ in the equation, the autocorrelation coefficient varies depending on the sign of R_{t-1}, *i.e.*

$$b_0 + b_1\sigma_t^2 + b2 \text{ if } R_{t-1} \geq 0$$

$$b_0 + b_1\sigma_t^2 - b2 \text{ if } R_{t-1} < 0$$

If b_2 is positive, the autocorrelation coefficient is larger during market advances, indicating that positive feedback trading is more intense during market declines. Using daily data on the US stock returns, Sentana and Wadhwani (1992) document that the estimates of b_1 and b_2 are respectively negative and positive and that both are statistically significant. A similar result is obtained by Koutmos (1997) for other stock markets. To check the robustness of results, the exponential AR (EAR) model is also considered in which the equation is replaced by:

$$R_t = a + [b_0 + b_1 \exp(-\sigma_t^2/\sigma^2)]R_{t-1} + b_2|R_{t-1}| + c\sigma_t^2 + \varepsilon_t$$

This specification is the same as the one employed by LeBaron (1992) and Bollerslev, Engle and Nelson (1994) except that they omit the term $b_2|R_{t-1}|$. σ^2 is an additional parameter whose value is set equal to the unconditional sample variance of R_t following LeBaron (1992) and Bollerslev, Engle and Nelson (1994). In this model, the autocorrelation coefficient is:

$$b_0 + b_1 \exp(-\sigma_t^2/\sigma^2) + b_2 \text{ if } R_{t-1} \geq 0$$

$$b_0 + b_1 \exp(-\sigma_t^2/\sigma^2) - b_2 \text{ if } R_{t-1} < 0$$

Thus, autocorrelation is decreasing in volatility if b_1 is positive. LeBaron (1992) and Bollerslev, Engle and Nelson (1994) obtain positive and statistically significant estimates of b_1 for the US stocks.

It is shown that the EAR model is favoured over the LAR model. However, no matter which model is used, the recent findings of the inverse relation between volatility and autocorrelation, the sign reversal of autocorrelation, and the asymmetry in autocorrelation are confirmed. Evidence is also found that an increase in margin requirements in the TSE makes stock returns more positively autocorrelated, which is consistent with the view that a substantial amount of positive feedback trading is due to margin trading.

6 Nonlinear Positive Feedback Trading and the Complexity of Stock Price

6.1 Introduction

For the word "chaos", the definitions given by most dictionaries are "turmoil", "turbulence", "primordial abyss" or "undesired randomness". But in the field of natural science, chaos is a concept was first discussed in the 1960s. It indicates the internally irregular, non-repetitive and non-periodical movement in a deterministic system. There is an internal nonlinear positive feedback dynamic existing in the system. Its steady state is a complex, disorderly but finite state of motion. Chaos looks like random motion, but it is not, it is driven by its internal deterministic rules.

Chaos Theory is very attractive for economic analysis, for example, it could well explain the endogenous volatility of the economic system, especially for some abnormal fluctuation events, *e.g.*, the stock market crashes in 1929 and 1987. On the one hand, chaos process illustrates the motion of the economic system has internal deterministic rules, and it is good for making short term predictions. On the other hand, a tiny difference at the initial condition can yield widely diverging outcomes, showing that happenstance is vital for the evolution of the economic system and explaining the impossibility of long term forecast for economics. Indeed, Marshall (1890) noticed that, a positive feedback mechanism is very possible in goods market. It generates instability but not equilibrium. Positive feedback could come from large scale fixed start-up costs, learning effect and adaptive expectations. He also noticed that, when investors make investment decisions based on prediction, the instability of goods market might transfer to stock prices.

Complexity is relative to simplicity. It indicates a state between complete disorder and complete order. A complete disordered system on thermal equilibrium state which presents Brownian motion is a simple system. A complete disordered price system which presents random walk is a simple system as well. A complete ordered system, for example, a diamond, is a simple system. Generally, fewer variables are required to describe a simple system, which means more variables are needed to describe a complex system. The complexity of stock price means the change of stock price is neither complete ordered nor complete disordered, but in between the two. It is definitely not perfectly predictive and also not totally random. It is similar to random motion but it is not random motion. For example, it could maintain steady fluctuation and suddenly come up with a drastic fluctuation. As we know, the distribution of return of stock price presenting the characteristic of leptokurtosis and fat tail, that could not describe by normal distribution.

This section will continue to analyse the complexity of stock price in the financial market caused by nonlinear positive feedback. The complexity of stock price comes from the complexity of the stock market itself. The Stock market is a complex and open system. Political, economic and social factors all affect the system at the same time. There are rational investors and irrational investors in the market. The formation and evolution of stock price is the combined action between those factors.

6.2 Complexity

The term "complexity" comes from natural science. The world we live in is an evolutionary system, which is extremely complex. The major reason for the complexity of the real world comes from "nonlinearity". Sadly almost every theory tends to linearisation, in part to deal with the nonlinear problem. To really explorer the complexity, we need to research the nonlinear system as a whole.

The rise of Chaos Theory has really pushed up the research on nonlinear systems. Chaos Theory focuses more on the evolution process. The stock market is a huge open and complex system which contains all kinds of information from economics, culture and politics. Different forces are enhancing or fighting each other, and generate the price continuously through the trading behaviour. So there are different types of investors, such as rational traders trading based on rational strategy, irrational traders trading based on speculative strategy, and positive feedback traders trading according to price movement only. Our following section will demonstrate that the positive feedback trading strategy is a special and vital force in the stock price evolution system. It could lead the stock price to expand explosively in one direction by the way of self enhancing . At the same time, the evolution of stock price has also been limited by other factors, such as investor wealth, government policy, and so on. The evolution of stock market price could appear as a chaotic process under those kinds of conditions, the path of price movement has a fractal structure which means complexity is presented.

6.3 nonlinear Positive Feedback and Chaos

6.3.1 Characteristics of nonlinear systems

A nonlinear system is a relative of a linear system. Generally, a linear system has the following characteristics. First, for every cause, there is an effect. Secondly, the system is looking for the equilibrium position. Thirdly, the system is ordered.

The linear system is simple and perfect which means it has certain and definite solutions, but it has its limitations. The world in which we live is actually a nonlinear system. A linear system is an analytical framework that humans established to simplify the analysis to object only, and it becomes less useful in the analysis of complex systems such as the atmosphere, the financial markets, and social system. For example, in the natural sciences, Newtonian physics, which is based on linear relationships, can explain the interaction between two objects, and can accurately predict their trajectory, but it cannot predict three objects' interacting trajectory. We can say that in the 19th century, most of the time, the three-body problem plagued physicists. Until the 19th century, Henri Poincare demonstrated: the three object interaction involves an inherent nonlinear nature of the system, and therefore cannot provide a single solution, so that confused the three-body problem really be solved.

In general, the nonlinear dynamical system has the following characteristics: first, it is a feedback system, secondly, there is multiple solutions exist, thirdly, the system is away from equilibrium. Due to the presence of nonlinear dynamical system of internal feedback mechanisms may produce chaos and other complex motion, its trajectory has a fractal structure. Therefore, the analysis of nonlinear dynamical systems theory is also called chaos analysis theory.

6.3.2 The Concepts of Chaos

The concept of chaos was first introduced by the American meteorologist Lorenz (1963). The atmosphere is a large area of complex dynamic systems, after the invention of the computer, meteorologists have tried to use mathematical models and numerical calculations to do the weather forecasting, but made little progress.

In the study of weather forecasting model analysis, Lorenz found that the following behaviours can occur in a deterministic system: system oscillating nearby a stable state and is gradually enlarged, when this oscillation expand to a certain extent, it also turned to another unstable state oscillation. Its trajectory is limited to a bounded by area. Small insignificant differences in the initial conditions can produce very different results with the previous results, *i.e.* the system is sensitive to the initial conditions.

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In the study of weather forecasting model analysis, Lorenz found that in a deterministic system can occur following behaviour: system oscillations near the a stable state and is gradually enlarged, when this oscillation expand to a certain extent, it also turned to another unstable state oscillation. Its trajectory confined in a bounded region. Eventually be attracted irregular oscillation in the size of the surface is zero. Small insignificant differences in the initial conditions can produce very different results with the previous results, the system sensitive to initial conditions.

Since Lorenz published his work, scientists have discovered aperiodic irregular movement in different areas. Li and Yorke (1975) mathematically defined the chaotic process. After the 1980s, Chaos Theory has been used in the research of various fields, such as biology, physics, chemistry, economics and finance.

Although academia, lacks a unified chaotic process definition, the author is generally regarded as identifying some of the inherent characteristics of the chaotic process, summarized below:

- The orbit at an exponential rate of divergence;
- Its trajectory presents a fractal structure;
- The system is sensitive to the initial conditions, *i.e.* minor differences in initial conditions will be quickly and dramatically enlarged, and lead to different results;
- There is a threshold value and the bifurcation;
- There is internal feedback motivation;
- It moves away from a static or simple cyclical equilibrium.

Chaos is not random, but seemingly randomness. Randomness is a random process caused by noise disturbance. The chaos is due to the internal deterministic nonlinear positive feedback, so it is also known as deterministic chaos. This will be explained in the following well-known examples.

Logistic Map Logistic mapping function is a one-dimensional nonlinear feedback system and it is one of the most intuitive chaotic processes. Early logistic mapping was developed to give the group dynamics model for natural ecosystems. This system has birth and death rates. On the one hand, the group growth in a non-controlled manner is in accordance with the rate of Ax. On the other hand, the faster the population grew, the less ecosystem resources are available for support. Thus, the system has a mortality rate associated with $Ax^2$and resulting in the equation of system growth :

$$x_t = Ax_{t-1}\left(1 - x_{t-1}\right)$$

Where A is in the range between 0 and 4, and the range of the initial value x_0 is between 0 and 1.

When the value A is small, the iterative system described is a stable system with good characteristics. For example, when $A = 0.50$, the system reaches a stable value after a few iterations. If we increase the value of A, when A = 0. 75, the system is no longer stable at a value, but is oscillating back and forth between two values. Such a split phenomenon from a solution to the two solutions, one called bifurcation. If we continue increasing the value of A, when A = 0 87, the system has four solutions. When A = 0 8911, 16 solutions emerge and the threshold value of$A$ may appear more and more intensive. Until A is approximately equal to 0.90 (actually 0.892486418), the system loses all stability, and the number of solutions is infinite. When A is close to 4, the system becomes a chaotic system. The sequence appears to be random at this time. If we do a statistical analysis of the system, it is also consistent with the randomness of the standard. We can distinguish the difference between this and a stochastic system only in more than two-dimensional systems.

This system has the following features:

- When $t \to \infty$, $\{x_t\}$will filled with interval $[0, 1]$.
- Subtle difference of initial value x_0 will cause the deviation for the forecast of x_t amplification exponentially.
- Where it looks to be random, but it is generated by a deterministic process.
- $x_t$can have a series of small changes in value, and then suddenly produce dramatic changes in scale.

source: Tozzi, Peters and James Iii (2017)

Figure 1: Bifurcation diagram of a simulated logistic map's nonlinear dynamical equation

Logistic mapping function is not a purely hypothetical equation which has nothing connecting to our real life. If we put a microphone connected to a speaker in front of that speaker, and then gradually turn up the microphone volume, it forms a sound amplifying system with positive feedback, since the sound from the speaker has to re-enter the system through the microphone. When the volume is relatively low, we can hear the low hum. When the speaker volume is turned up to a certain extent, the system will suddenly alternate back and forth between the two tones. Continuing to increase the volume will lead to more bifurcation, until at a threshold level, we will find that the system enters a chaotic state, the speaker will make a very shrill scream.

For the Logistic process above, we can also assume that there is a game process in price: the positive feedback traders will pull the securities price away from its fundamental values at a rate of Ax, while the rational traders will push the securities price back to its fundamental value at a rate of Ax^2. The Logistic process shows that the evolution of prices is likely to be unstable in the game between the positive feedback traders and the rational traders.

Lorenz map Lorenz mapping is a three-variable chaotic system, and unlike Logistic mapping, that the system is not represented by a difference equation, but in differential equations:

$$x_t \begin{cases} \frac{dx}{dt} = & a\,(y-x) \quad a = 10 \\ \frac{dy}{dt} = & -y + x\,(b-z) \quad b = 28 \\ \frac{dz}{dt} = & xy - cz \qquad c = 8/3 \end{cases} \tag{1}$$

The above differential equation is the Lorenz (1963) built in the study of weather forecast. The system has two nonlinear part, a change-over switch is controlled by $(b-z)$, the other is the xy to control the z . When $b > z$, is a positive feedback system, and when $b < z$ is the negative feedback system.

By solving equations mathematically, we know that the system has three fixed points, and is unstable. The Trajectory of the system movement will be a spiral that moves around two fixed divergence points, and is limited in a bounded surface with zero volume for continuous irregular oscillations. This irregular oscillating is like a moth seeing two light sources. It flew to a light, felt hot when approached and then flew to another source, so it irregularly fly back and forth, never to repeat its flight trajectory.

The Lorenz process is non-periodic oscillations and seems to never end, it is neither disappearing nor diverging but keeping irregular oscillations.This trajectory of oscillation is a spiral on the three-dimensional phase space, which is very dense and presenting fractal structure. It is infinitely long and sensitive to initial conditions. Insignificant errors in initial conditions can be rapidly amplified by the system, resulting in a very different system evolution path, the so-called "butterfly effect".

6.4 Applications of Chaos Theory on economics

6.4.1 Introduction

In the last thirty years, the research about deterministic chaos has advanced rapidly. This progress comes from both theoretical simulation and empirical tests. Also, Chaos Theory aroused great interest of economics theory researchers. Chaos is attractive to economic analysis. Firstly, in its endogenous fluctuations. In the literature on the economic cycle, in general, there are two ways you can cause fluctuations in output, either an external shock or an internal factor. In the Box-Jenkins time series model, the economic system is in the presence of a stable equilibrium, but constantly getting external shocks such as war, climate change *etc.* The dynamic behaviour of economic systems is caused by those external shocks. But in the chaos model, the economy follow nonlinear dynamics, which can fluctuate spontaneously and never disappear. Therefore, if the chaos model can explain the economic fluctuations, these could spontaneously generated, this is undoubtedly thinking of ingenious and novel. Furthermore, chaos must be a nonlinear system, which is one of the reasons that causes chaos analysis to attract the interest of economists. It is known that the linear model can only generate limited kinds of behaviour patterns. However, a nonlinear system could generate broader behaviours, for example, the system may produce sudden fluctuation, occasional sharp movement. Those features coincided in line with the price volatility characteristics of securities.

Chaos is a bridge connecting the deterministic process and the stochastic process. Deterministic process is completely predictable, and the stochastic processes is completely unpredictable. Chaos is on the boundary between the deterministic process and stochastic process. Since the chaotic process is sensitive to initial conditions, the initial minor differences can be amplified exponentially, so in the long term evolution of the system is unpredictable.

However, if the initial conditions remain stable, applying the chaotic process to predict the short-term evolution of the state of the system means the results we obtained can be much more accurate than using the predicted results by the linear stochastic process. Therefore the chaotic process is vital in the significance of economic analysis and forecasting.

The butterfly effect which the chaotic process has could also explain the global financial market volatility anomalies triggered by accidental events, for example, the Mexican financial crisis in the early 1990s, the South-east Asian financial crisis in the late 1990s, and the subprime mortgage crisis in the late 2000s. On the stock market, due to the irrational behaviour generated by the presence of noise traders and herding, a positive feedback formed and it could cause securities prices to rise, and may even produce an asset price bubble. Subject to various constraints by external factors, when after the asset price bubble reaches a certain level, *i.e.* more than the nonlinear switching threshold, the securities prices may evolve endogenously feedback effect, resulting in securities prices to continue to drop. It could also produce chaotic and strange attractors, the expression of the price movement of the securities oscillating back and forth nearby strange attractors is oscillating back and forth nearby in some prices level.

Thus, the internal positive feedback mechanism and nonlinear switching threshold could generate very complex movements. It could be the cyclical fluctuations, or the aperiodic and never duplicated complex motion. It results in the price of securities return distribution presenting a complex structure *e.g.* fractal structure, showing a high degree of complexity. For example, the sudden price fluctuations have resulted in the distribution of produce fat tail phenomenon. The emergence of chaos and of strange attractors leading to securities prices irregularly oscillating back and forth nearby some prices levels repeatedly, makes the price of securities leptokurtic distribution occurs.

6.4.2 Chaos Theory and Chaos Control in Economics

As mentioned in the introduction, since the late 70s to 80s, economists began to do analysis on the global dynamics by applying Chaos Theory (Medio, 1979; Stutzer, 1980; Grandmont, 1985). Benhabib and Nishimura (1979) find out the way of the discount rate affect properties of an optimal growth model by apply the Hopf bifurcation in their study. Benhabib and Day (1981) and Day (1982) 's work on economic growth attracted a lot of attention and offered a challenge to the macroeconomic theorist. Also, Benhabib and Day (1981), Grandmont (1985) and Boldrin and Montrucchio (1986) derived chaotic business cycle models from utility and profit maximization principles within the general equilibrium paradigm of perfectly competitive markets and rational expectations. Dana and Malgrange (1984) analysed chaos in a multiplier accelerator type model beside a classical growth model and a Solow growth model analysed by Day (1982) . The necessary conditions for chaos has been discussed by Deneckere and Pelikan (1986). With lags in investment and consumption in Hicksian type models, Hommes (1991) showed it is very easy to produce chaos. Bala, Majumdar and Mitra (1998) located sufficient conditions for robust ergodic chaos to appear in growth models. Mitra (2001) shows the existence of chaotic equilibrium growth paths within a model of endogenous growth with externalities.

Grandmont (1986) investigated different government policies' effect, while Grandmont and Laroque (1986) show the significance of the expectations of formation mechanism for the stability of the economy. Farmer (1986) considers production economies and application of Hopf bifurcation that is usually meant to be more robust than the flip bifurcation. His result shows the chaos depends upon the debt policy of the government. Reichlin (1990) replicated the methodology of Farmer (1986) and found an interesting result that if government use fiscal policy to suppress or eliminate chaos, the action could also cure or produce chaos.

Under adaptive expectations, Chiarella (1988) introduced a general nonlinear supply function into the traditional cobweb model. He showed that in its locally unstable region it contains a chaotic regime follows to a period-doubling regime .

By assuming isoelastic demand and constant unit production costs, Puu (1991) studied the nonlinear dynamics of two competing firms in a market in terms of Cournot's duopoly theory. This model shows persistent periodic and chaotic motions. A common feature of the models described above is that nonlinear dynamics tend to arise as the result of relaxing the assumptions underlying the competitive market general equilibrium approach.

Chaos control is another area that attracted interest since it offers a new perspective in system control strategies which attracted the researcher on economic policies. We already know the characteristic of chaos that is able to make a big influence just by a very tiny change in the initial value. Ott, Grebogi and Yorke (1990) demonstrate that a very tiny correction could be made in a parameter in order to adjust the system. It is so useful because it could be applied into government policy decisions. Faggini (2008a,b) shows that a small control could improve the system a lot since the output and the input are made exponential by the sensitivity to initial conditions. Holyst *et al.* (1996), Hołyst and Urbanowicz (2000), and Kaas (1998) applied methods of controlling chaos in economic models. Kaas (1998) stabilizing Chaos in a Dynamic Macroeconomic Model

6.4.3 Chaos Theory and Economic Time Series Analysis

The relevance of addressing chaos in economic models and the potentiality offered by its control techniques is associated to detecting the presence of chaotic motion in economic data. From an empirical point of view, it is hard to distinguish between fluctuations stimulated by endogenous fluctuations determined by the nonlinear nature and random shocks. This would be vital from a policy point of view that if it were possible to clearly separate stochastic and deterministic components of time series. Because the purely stochastic trajectories do not allow forecasting future outcomes, chaotic series are deterministic which do allow. For example, if people are aware of the initial state, the outcome *i.e.* the impact by policy could be predicted accurately.

Trends, noise, and time evolution caused by structural changes are the main difficulties in economic time series analysis (Barnett and Chen, 1988a,b). There are tests to investigate the assumption of random walk and characteristics of chaotic time series.The most used tests for these, applied both in macroeconomic and financial time series, are: the Hurst exponent estimation (with R/S, modified R/S, V/S analysis); the Lyapunov exponent; Correlation Dimension and the BDS test. We will briefly introduce them here and will do an in-depth discussion on technical detail in a later chapter of the methodology.

As we know the stochastic time series could not be affected by historical events, the Hurst exponent raised by Hurst (1951) could be used to identify the non-randomness and non-periodic recycling of series. Therefore it could be used to identify the characteristics of nonlinearity for a time series. It has been widely used outside of the economics field, and has been especially used in financial time series, mostly the results are against the fundamental assumption of EMH, therefore people critique the methodology of estimation and some variant methods created to gives more solid estimation on the Hurst exponent. There is a more detailed explanation in the methodology chapter.

Grassberger and Procaccia (1983a,b) proposed the concept of Correlation Dimension(CD) originally in physics. This method is based on measuring the dimension of a strange attractor. It is applied to identifing the time series has a lower dimensional deterministic process or not. Its major advantage is the simplicity of calculating. However this analysis provides necessary but not sufficient conditions for testing the existence of chaos and requires a rather large and clean data set, also it is a graphical procedure rather than a statistical test. Therefore one usually applied an BDS test proposed by Brock *et al.* (1996) to supplementing their research. It is not a direct test for chaos but tests the much more restrictive null hypothesis that the series is independent and identically distributed (IID). Then with an additional technique by Hsieh (1991), we could rule out the possibility of the tested time series been other than chaotic one. The BDS test is wildly used in economic and financial time series since it is powerful and simple to apply to any kind of structure in a series.

The Lyapunov exponent is another necessary but not sufficient condition for chaos. As chaotic motion is very sensitive to the initial state of the system, the level of sensitivity could be measured by the Lyapunov exponent. The estimate of the Lyapunov exponent has similar requirements to the correlation dimension since they are both brought from natural science, that is it requires a large number of observations. It is not easy to get that many samples in economics therefore the result might not be so reliable, but since few economic series of such a large size are available, at a certain point it still gives a clue of the existence of chaos (Nicolis and Nicolis, 1984).

There are a lot of papers on tests for chaos in economic time series, and the results are controversial. The application of these tests to such data presents problems. Besides the difficulty we stated above, *i.e.* need a rather large data set to test the time series, another problem is that noise of economic time series may render any dimension calculation useless (Brock and Sayers, 1988). The quality and quantity of the samples and data are crucial and vital in applying the test. The noisy and short data sets in empirical economic analysis are where the main obstacles are.

Especially, there are suspicions around the testing on macroeconomic time series since the majority macroeconomic of data sets cannot be shorter than monthly, which indicate this is not sufficient to perform the tests. Another reason is the macroeconomic time series involve mixed effects *i.e.* it is not just the distinction between nonlinearities and noise that is in order, but also the eventual source of nonlinearity.

Although tests in macroeconomic time series are controversial, since relatively weak evidence for chaos has been found, people did find substantial evidence for nonlinearity because most macroeconomic time series have high noise levels and a small set of samples. In contrast to the laboratory experiments where a large amount of data points can easily be obtained, most economic time series consist of monthly, quarterly, or annual data, with the exception of some financial data with daily or weekly time series.

In fact the analysis of the financial time series has much better results that are more reliable than macroeconomic time series in total. Financial time series are a good candidate for analysing chaotic behaviour due to being available in larger quantities and for much more disaggregated time intervals. However the samples might still fall short when applied to the test in the emerging market.

Part III

Methodology

In this section, a framework of the methodology chapter will be introduced, by illustrating the selection of methods commonly used around the research field and applied in the later empirical chapter to help the reader have a better understanding of the work.

The first section will introduce two main methodologies to detect the monlinear characteristics and chaotic characteristics, these are the Hurst exponent and the correlation dimension, including both their estimation processes and previous empirical test results.

The second section will introduce the nonlinear dynamic positive feedback model that we derived on the foundation of De Long *et al.* (1990b), providing further description and explanation. Since we added the threshold factor to presenting the nonlinear and chaotic characteristics, this section will start with the introduction of the simple threshold. The threshold we will be using in the empirical chapter is the value of the Hurst exponent.

The third section will introduce the ARIMA model and the Holt-Winters seasonal method. These will be used for comparative tests in the empirical chapter later on.

7 Nonlinear Characteristics and Chaotic Characteristics

In the first section of this chapter, we will bring in our major tools to detect the nonlinear characteristics and the chaotic characteristics in the market structure. We will introduce their background knowledge first, followed by their theoretical research stream, and report the related empirical results for later comparison.

7.1 The Hurst Exponent

7.1.1 Introduction of the Hurst Exponent

A time series is random only when it has been affected by a lot of probability events. A non-random time series may be fractal. This does not obey a random walk but is a biased random motion. Internal autocorrelation could last in the long run. That means the non-random motion is fractal and shown self-similarity at the time. The "Efficient Market Hypothesis" (EMH) implicitly assumes all investors will react instantly to new information therefore the past, the future and the present have no relationship. Barberis, Shleifer and Vishny (1998) argued that the individuals might display a degree of conservatism, defined as the slow updating of models in the face of new evidence (Edwards, 1968). The underreaction evidence in particular is consistent with conservatism. They will wait until the trend reaches a threshold then react. The unequal reaction to information could lead to a biased random motion. Therefore the rate of return in security price has the fractal structure in time. Hurst comprehensively researched biased random motions in 1940s but it became well-known since Mandelbrot's work called this type of motion as fractional Brownian motion (FBM) (Mandelbrot and Wallis 1969; Mandelbrot 1977, 1983).

Hurst (1951) measured the fluctuation of the reservoirs changes in relation to the time in average. Obviously, the range of the fluctuation is not a constant, and it depends on the length of time used for measurement. To standardize the measurement in time, Hurst used rescaled range analysis (also called rescaled standard deviation analysis, R/S analysis). He built a proportion, without dimension, by using the standard deviation of the observed value divided by the range (Δ) and defined the Hurst exponent.

$$\Delta^2 \propto (\Delta t)^{2H}$$

Where "H" is the Hurst exponent or the Hurst index.

Hurst found that most natural phenomena such as rivers, temperature, rain and so on do not obey random motion, but follow a biased random movement, that is a trend plus a noise, which Mandelbrot called fractional Brownian motion. The relative strength between trend and noise could be measured by rescaled range along with change in time. That means looking at the difference of H value from random motion that $H = 0.5$.

It can be demonstrated that the value of H for Brownian motion equals 0.5. Where the value of H for biased random motion is larger than 0.5, it has been called fractional Brownian motion" since *2H* is not an integer. The fractional dimension of fractional Brownian motion D = 2-H. D = 1.5 if $H = 0.5$, this is the H value and dimension for Brownian motion.

The Hurst exponent could be used to identify the non-randomness and non-periodic recycling of series. Therefore it could be used to identify the characteristics of nonlinearity for a time series.

An observation is not independent if the H value does not equal 0.5. Every observation contains memory of all past events. The memory lasts forever in theory. Although the impact of past events is less than recent events, residual effects still exist. In a wider scale, a system shown Hurst statistical characteristics is the consequence of a series of correlated events.

The effect of current to future could be presented as a relationship C_N that is equal to the correlation over period N, we have:

$$C_N = 2^{(2H-1)} - 1$$

If $H < 0.5$, the system is an anti-persistent time series, also known as a mean-reverting series. This type of system has dramatic volatility and has negative relationships between increments. That means a downward movement in the next term if there was a rise in the previous term and vice versa. This effect is called the anti-persistence effect. The effect becomes stronger if the H value closer to zero.

If $H > 0.5$, the system has positive relationships between increments. The current will affect the future, or we could say that memory exists. This effect is called persistence effect. That means an upward movement in the next term if there is a rise in the previous term and vice versa. The effect becomes stronger if the H value is closer to one.

7.1.2 Theoretical Research Estimating the Hurst exponent

Hurst (1951) proposed the first rescaled range analysis (R/S analysis) for estimating the Hurst exponent when he studied the properties of Nile River. Mandelbrot (1963, 1972) refined this method and brought it to financial economics, since then R/S analysis has become the most canonical way and the most popular way to analyse market efficiency and long term memory properties across different financial markets. The rescaled range statistic is the range of the partial sum of deviations of a time series from its mean, rescaled by its standard deviation. In detail, consider the sub-series from a given time series $X(t) = \{x_1, x_2, \ldots, x_n\}$ with n length, calculate the mean $\bar{x_\tau} = \frac{1}{\tau}\sum_{i=1}^{\tau} x_i$ and $\tau\,(1 \leq \tau \leq n)$ is the time horizon considered. The R/S statistics of the series is shown below:

$$\left(\frac{R}{S}\right)_\tau = \frac{1}{S_\tau}\left[\max_{1\leq t\leq \tau}\sum_{i=1}^{t}(x_i - \bar{x_\tau}) - \min_{1\leq t\leq \tau}\sum_{i=1}^{t}(x_i - \bar{x_\tau})\right]$$

where its standard deviation:

$$S_\tau = \sqrt{\frac{1}{\tau}\sum_{i=1}^{\tau}(x_i - \bar{x_\tau})^2}$$

Hurst (1951) found that many real world phenomena obey the power-law relationship given by the equation above:

$$\left(\frac{R}{S}\right)_\tau \propto \tau^H$$

Therefore the Hurst exponent could be estimated by the linear regression:

$$\ln\left(\frac{R}{S}\right)_\tau = c + H\ln(\tau)$$

Lo (1991) stated that the classical R/S analysis is sensitive to short range dependence, *i.e.* the predicted behaviour does perhaps not arise from long term memory, but may merely be a symptom of short term memory. Therefore he raised a modified version of the R/S analysis in order to correct this short term memory. This involved replacing the standard deviation S_τ by

$\hat{\sigma_n}(q)$, so:

$$Q_n = \frac{1}{\hat{\sigma_n}(q)} \left[\max_{1 \le t \le \tau} \sum_{i=1}^{t} (x_i - \bar{x_\tau}) - \min_{1 \le t \le \tau} \sum_{i=1}^{t} (x_i - \bar{x_\tau}) \right] \propto \tau^H$$

where

$$\begin{aligned} \hat{\sigma_n}(q) &= \frac{1}{\tau} \sum_{i=1}^{\tau} (x_i - \bar{x_\tau})^2 + \frac{2}{\tau} \sum_{i=1}^{q} \omega_i(q) \left[\sum_{j=i+1}^{\tau} (x_j - \bar{x_\tau})(x_{j-i} - \bar{x_\tau}) \right] \\ &= \hat{\sigma_x}^2 + 2 \sum_{i=1}^{q} \omega_i(q) \hat{\gamma_i}, \ \omega_{\mathrm{i}}(\mathrm{q}) = 1 - \frac{i}{q+1} \quad q < n \end{aligned}$$

where $\hat{\sigma_x}^2$ is the usual sample variance estimator and $\hat{\gamma_i}$ is the autocovariance estimator. Also, the parameter q is chosen according to Andrews (1991) and Andrews and Monahan (1992).

$$q = [k_\tau], \qquad k_\tau = \left(\frac{3\tau}{2} \right)^{\frac{1}{3}} \cdot \left(\frac{2\hat{p}}{1 - \hat{p}^2} \right)^{\frac{2}{3}}$$

where $[k_\tau]$ means the greatest integer is less than or equal to k, and $\hat{p}$ is the estimated first order autocorrelation coefficient.

Following this Kwiatkowski *et al.* (1992) introduced the KPSS statistic for trend stationariness against a unit root test. Lee and Schmidt (1996) expanded the KPSS statistic on testing of the long term memory effect for a stationary time series. Lee and Amsler (1997) further expanded the KPSS statistic to testing the long term memory effect for a unstationary time series. The KPSS statistic is:

$$T = \frac{1}{\hat{\sigma_n}(q)\, S_\tau^2} \sum_{k=1}^{\tau} \left(\sum_{i=1}^{k} (x_i - \bar{x_\tau}) \right)^2$$

Cajueiro and Tabak (2005) proposed a new method called rescaled variance (V/S) analysis for the evaluation of the Hurst exponent. Their work is based on Giraitis *et al.* (2003) which centralized the KPSS statistic and raised the V statistic. They claim the V/S method does not have the shortcomings of the R/S method, so more recent empirical work applied the method. The V/S replaced the standard deviation given in the R/S analysis by the sample variance of $\sum_{i=1}^{k} (x_i - \bar{x_\tau})$. That is:

$$\left(\frac{V}{S}\right)_\tau = \frac{1}{\tau S_\tau^2} \left[\sum_{k=1}^{\tau} \left(\sum_{i=1}^{k} (x_i - \bar{x_\tau}) \right)^2 - \frac{1}{\tau} \left(\sum_{k=1}^{\tau} \sum_{i=1}^{k} (x_i - \bar{x_\tau}) \right)^2 \right]$$

Correspondingly, the estimation of the Hurst exponent is different from the previous case since the relationship is different:

$$\left(\frac{V}{S}\right)_\tau \propto \tau^{2H}$$

Therefore:

$$\ln \left(\frac{V}{S}\right)_\tau = c + 2H \ln(\tau)$$

The classical R/S analysis, the modified R/S analysis and the V/S analysis are the three most widely used tools to estimate the Hurst exponent. He and Qian (2012) compared the approaches by using Monte Carlo simulation. They had generated a FBM time series whose theoretical the Hurst exponent was already known. They compared the Hurst exponent generated with theoretical value to investigate which method is better to use to estimate the Hurst exponent.

source: He and Qian (2012)

Figure 2: Comparison of Measurement of R/S, modified R/S and V/S statistics from a fractional Brownian motion

Figure 2 shows the bias which is the mean of the estimated the Hurst exponent by R/S, modified R/S and V/S minus the theoretical value in FBM for the length $L = 10,000$.

It is clear that the R/S and the modified R/S overestimate the Hurst exponent when the theoretical value is smaller than 0.5, but they still give a qualitatively correct conclusion, *i.e.* the theoretical value is 0.4, the R/S is estimated as 0.45, we still know the time series is anti-persistent. Therefore, although the V/S gives the most accurate result in the mean-reverting market, the R/S and the modified R/S is still a reliable tool.

All three methods tend to underestimate the Hurst exponent when the theoretical value is above 0.5. To be more specific, the modified R/S method is slightly more accurate compared with the classical R/S method only when the Hurst exponent is significantly larger than 0.5, around 0.77. Also, both the classical R/S method and the modified R/S method outperformed the V/S method under the persistent time series.

The V/S method is still a better tool to use in the mean-reverting markets *i.e.* when the $H < 0.5$. Many real world financial markets are not mean-reverting but persistent with the long memory. These will be discussed in the next section. Therefore it is better to apply the classical R/S method or the modified R/S method to estimating the Hurst exponent in a persistent time series.

7.1.3 Empirical Research Estimating the Hurst exponent

In this section we will first review some empirical literature on the developed financial markets to get a sense of the development of empirical research on estimating the Hurst exponent. We will review some local empirical literature research related to our research target markets, the Shanghai stock exchange market and the ShenZhen stock exchange market. Finally we will state our method choice and explain why it is worth using and why we chose this method.

After Mandelbrot (1971) emphasized the importance of long term dependence in asset markets and encouraged using the R/S analysis in economics (Mandelbrot, 1972), Greene and Fielitz (1977) applied the R/S analysis to the common stock returns. Some early research including Booth and Kaen (1979) looked at gold price, Booth, Kaen and Koveos (1982) investigated the foreign exchange rates and Helms, Kaen and Rosenman (1984) tested the future contract markets.

This early empirical research has been criticised by Davies and Harte (1987). They said the research did not focus on the R/S statistic itself, but rather on the logarithm on subsample sizes. Also, the proof of statistical significance of their empirical results and the short range dependence are problems as well.

Lo (1991) proposed the modified R/S analysis (for technical details see Section 7.1.2). He also implemented his new method in the empirical test by using the annual Cowles (1938) U.S. stock index extending back to 1872, and he has found little evidence of long term memory in historical U.S. stock exchange market returns. His empirical result has been cited a lot due to its rare and notable outcomes which support EMH from traditional finance, and his new method is believed to be a more correct way for estimating the Hurst exponent.

However, Goetzmann (1993) used Lo's methodology to test the very long stock market series. He found evidence to suggest that long term memory may exist in the London Stock Exchange (LSE) over the period of 1700—1989.

Peters (1989, 1991, 1996) used the R/S analysis in the 1990s, and we summarise the series empirical research and then extract the Hurst exponent results reported into a Table 1. His research reach across different kinds of financial market including the stock index, the individual stocks, the exchange rates market, the treasury bond market and economical indices. He found almost every target series he researched is possessed by long term memory, except he found that Singapore dollars against U.S. dollars had the perfect random walk. We could also see the Hurst exponent value of S&P 500 is higher than any of those individual stocks. This higher Hurst exponent value shows, by eliminating the noise factors and raising the Hurst exponent value, diversifying the asset portfolio could effectively lower the total risk.

Sample Name	Time Span	Hurst exponent
S&P 500 Index	01/1950 – 07/1988	0.78
S&P 500 Index*	02/01/1928 – 31/12/1989	0.60
IBM	01/1963 – 12/1989	0.72
MOBIL	01/1963 – 12/1989	0.72
Coca - Cola	01/1963 – 12/1989	0.70
McDonald's	01/1963 – 12/1989	0.65
Niagara Mohawk	01/1963 – 12/1989	0.69
MSCI U.K.	01/1959 – 02/1990	0.69
MSCI Germany	01/1959 – 02/1990	0.72
MSCI France	01/1959 – 02/1990	0.68
Treasury Bond (30 years)	01/1950 – 02/1989	0.68
Treasury Bill (3,6,12 months)	01/1950 – 02/1989	0.65
USDJPY Exchange rate*	01/1973 – 12/1989	0.64
USDSGD Exchange rate*	01/1981 – 10/1990	0.50
Index of Industrial Production	01/1950 – 01/1990	0.91
Index of New Business Formation	01/1950 – 01/1990	0.81
Index of Housing Starts	01/1950 – 01/1990	0.73
Leading Economic Index	01/1955 – 01/1990	0.83

Source: Peters (1989, 1991, 1996)

Table 1: Comprehensive Empirical research
*Note that * means the Hurst exponent calculated on daily basis data, others used monthly data*

For the Chinese market research side, Xu and Lu (1999) first applied R/S analysis to empirical research in the Shanghai Stock Exchange Composite Index (SSEC) and the ShenZhen Stock Exchange Composite Index (SZEC), the Hurst exponents they obtained are 0.661 and 0.643. These two research are both based on the daily logarithmic rate of return. Shi (2000) used the weekly logarithmic rate of return to do the R/S analysis result and shows the Hurst exponent for SSEC is 0.687 and for SZEC is 0.667. Further research done by Shi and Zhao (2006) reports the Hurst exponent for SSEC (arithmetic weighted) is 0.5828, and for SSEC (market cap weighted) is 0.5831. The result goes higher in the SZEC market, the Hurst exponent for the SZEC (arithmetic weighted) is 0.6913, and for the SZEC (market cap weighted) is 0.6632.

Cao and Li (2003) are focused on the ShenZhen stock market, and they used the daily, weekly, fortnightly, and monthly logarithmic rate of return to calculate the Hurst exponent for SZEC by the R/S analysis. The results they obtained are 0.6507, 0.7000, 0.6906 and 0.7576. Zhu (2004) did R/S analysis on SZEC while using the daily logarithmic rate of return AR (1), and reports that the Hurst exponent is 0.71.

Du (2008) calculated the Hurst exponent by using the daily logarithmic rate of return and he reported the that Hurst exponent for SSEC is 0.6509 and for SZEC is 0.6486. The shortcoming for his R/S analysis is that the time span is only three years(2002 to 2005). Zheng (2013) used longer time span data, nearly ten years of the daily logarithmic rate of return, for both SSEC and SZEC to do R/S analysis. He reported that the Hurst exponent calculated is 0.6367 for SSEC and 0.6728 for SZEC.

The empirical research above shows the Shanghai and ShenZhen stock markets do not present random walk characteristics but present trend persistence effect. Note that the empirical research is not strong enough to assert that the Shanghai stock market and the ShenZhen stock market have long term memory effect due to their data are over dated. The time span of their data samples are not sufficient to justify the result since the time span is vital on investigating the nonlinear characteristics of the sample series. Therefore, in our empirical test, we will double check this long term memory effect by applying a longer time span, from the beginning of the market settle to nowadays.

Also, we will be using the classical R/S analysis to do the estimation for the Hurst exponent rather than using the modified R/S analysis and V/S analysis. One reason is the classical R/S analysis is slightly better than the modified R/S analysis within the range 0.6 to 0.77 for FBM (see Fig.1). That is, all results reported by the empirical research above lies on, so we expect our result will lie on that range as well and the classical R/S analysis will give a better estimation. Another reason is the majority of empirical research done was using the classical R/S analysis, the same methodology is more convenient in the result comparison.

7.2 Correlation Dimension

In this section we will introduce the correlation dimension, another tool commonly used in the detection of nonlinear and chaotic characteristics other than the Hurst exponent mentioned above. Unlike the estimation of the Hurst exponent, the correlation dimension method offers a graphical procedure. The Hurst exponent estimation is more focused on the long-term memory part of the time series, where the correlation dimension is trying to describe the degree of complexity of the internal structure for a time series. The higher the degree of filling, the more complex the internal structure of the time series, *i.e.* the more similar it is to the random process time series.

7.2.1 Introduction of Correlation Dimension

Following Lorenz (1963) introducing chaos theory, Grassberger and Procaccia (1983a,b) proposed the notion of the correlation dimension (CD). The correlation dimension method is applied to identify if the time series has a lower dimensional deterministic process or not. We will illustrate the technical details of the correlation dimension method in 7.2.2.

The basic idea is that if a chaotic process is a $n-$dimensional process, then this process will fill up the $n-$dimensional space. But if we put this $n-$dimensional process into a higher dimensional space, *e.g.* $(n+1)$ dimension, it could only fill up the first $n-$dimensional space, and leaves large "holes" in the $(n+1)$ dimension. This is not the case if the time series is random rather than chaotic. We could make a thought experiment. Thus, imagining a type of gas, its gas molecules doing random motion, if we put the gas into a larger container which has a larger volume, the gas molecules will simply spread to every corner of this new larger space. Solid matter, *e.g.* a rock will keep its original state if we put the rock into a larger container, because its molecules are already bounded together. This is similar for a fractal time series, the correlation brings points together since it has a deterministic process.

For a time series, it is random only when it is affected by a number of different events with equal possibility. In statistical terms, it has a large degree of freedom. A non-random time series reflects the internal correlation of its impact,*i.e.* the time series is fractal. A random time series has no relationships with its original status, there is nothing to attract the points within a closer area to maintain its dimension. But for a fractal dimension or a fractal time series, they depend on how the substance or the time series fills its space. A fractal object fills up its space non-uniformly, because its different parts have correlation or relationships.

Also, we need to emphasize that we only focused on the "lower" dimensional chaotic behaviour when we tried to test the appearance of the chaos. The reason is we might never detect the difference between a higher dimensional chaotic behaviour (*e.g.* a very good pseudo random number generator) from a randomness process using a finite amount of data. This is especially practical since we are focusing on the financial markets which do not have that much data compared to similar research in physics. Given that our research is governed by a not too complex chaotic process, *i.e.* in a lower dimension, it should leave us a window, therefore we could predict its pattern, or at least prove we are able to predict its pattern, in a short period of time.

7.2.2 Theoretical Research Estimating Correlation Dimension

The correlation dimension method proposed by Grassberger and Procaccia (1983a,b) could be performed by four steps.

The first step is to eliminate autocorrelation if it is present. We have to remove it from the data because it could potentially affect some test for chaos. This usually could be done by filtering the raw data by applying an autoregressive model. The lag length is selected based on either the Akaike (1974) or Schwarz *et al.* (1978) information criterion (Hsieh, 1991).

The second step is to construct phase space dimensional vectors. Assuming the data, a time series $\{x_t,\ t = 1,\ 2,\ \cdots,\ n;\ x_t \in \mathbb{R}\}$ which is generated by a nonlinear dynamic system could be embedded in $n-$space by construct $n-$futures. As an $n-$history is a point in $n-$dimensional space, therefore n is called the embedding dimension. We could constructing the $n-$histories of the filtered data in order to obtain the embedding dimension.

The $n-$histories are denoted as follows:

$$
\begin{aligned}
\text{1-history}: \quad x_t^1 = &\quad x_t \\
\text{2-history}: \quad x_t^2 = &\quad (x_{t-1},\ x_t) \\
&\vdots \\
n\text{-history}: \quad x_t^n = &\quad (x_{t-n+1},\ \cdots,\ x_t)
\end{aligned}
$$

The trajectory of a time series process in the phase space is constructed by $n-$dimensional vectors. Note that an attractor is a subset of $n-$dimensional phase space towards which almost all sufficiently close trajectories get "attracted" asymptotically. They tend towards strange attractors on which the motion is chaotic, *i.e.* not (multiply) periodic and unpredictable over long times, being extremely sensitive on the initial conditions (Grassberger and Procaccia, 1983a).

The third step is to calculat the correlation integral $C_n(\gamma)$. It measures the fraction of the total number of pairs $(\vec{x_i},\ \vec{x_j})$ such that the distance between $\vec{x_i}$ and $\vec{x_j}$ is no more than γ, *i.e.* it is a measure of spatial correlation. It is defined according to:

$$C_n(\gamma) = \lim_{T\to\infty} \frac{1}{T^2} \sum_{i,\, j\neq 1}^{T} \Theta\left(\gamma - |\vec{x_i} - \vec{x_j}|\right)$$

where Θ is the Heaviside function *i.e.* an indicator like dummy variable I in our previous model, assumed $x = \gamma - |\vec{x_i} - \vec{x_j}|$, therefore:

$$\Theta\left(x\right) = \begin{cases} 0 & if \quad x \leq 0 \\ \\ 1 & if \quad x > 0 \end{cases}$$

Grassberger and Procaccia (1983a,b) established that for small distance γ , the correlation integral $C_n\left(\gamma\right)$ grows to obey the power law:

$$C_n\left(\gamma\right) \propto \gamma^v$$

Therefore the correlation exponent v_n could be obtained by calculating the slope of the graph of $\ln C\left(\gamma\right)$ versus $\ln\gamma$ for small values of γ . More specifically, we want to calculate the following quantity:

$$v_n = \lim_{\gamma\to\infty} \frac{\ln C\left(\gamma\right)}{\ln\gamma}$$

We want to do this step by increasingly larger values of the embedding dimension and observing the value of the correlation exponent. It will stabilize at the saturation value of the correlation exponent. This value is the value of correlation dimension v (correlation dimension value).

$$v = \lim_{n\to\infty} v_n$$

Note that the phenomenon of stabilization will take place only when the system analysed has lower dimensional chaotic process. In a stochastic system, the correlation exponent will keep raising with increasingly larger embedding dimension, therefore the correlation dimension v will be ∞ . It is also the case for a higher chaotic process since we could not find a stabilized value for the correlation exponent as well.

Early researchers could not fully convince themselves that a time series had lower dimensional chaotic process when they found a stable correlation dimension value. There is a shortcoming in the correlation dimension method, *i.e.* the correlation dimension method is a graphical procedure rather than a statistical test. Therefore a different but related test the "BDS" test (Brock *et al.*, 1996) was introduced by Brock, Dechert, and Scheinkman originally in 1987. It was motivated by the special problems raised by designing a test on time series data to detect whether such data came from a (possibly noisy) chaotic data generation process.

The null hypothesis that is tested for is that a time series sample comes from a data generating process that is "Independent and Identically Distributed" (IID). A time series has nonlinearity if the null of IID has been rejected. The BDS tests are often conducted simultaneously when calculating the correlation dimension value, since BDS statistics are very sensitive to any deviation from IID for different sorts of models. If $\{x_t : t = 1, \cdots, T\}$ is a random sample of IID observations, then:

$$C_n(\gamma) = C_1(\gamma)^n$$

One can estimate $C_n(\gamma)$ and $C_1(\gamma)$ by the usual sample versions $C_{n,T}(\gamma)$ and $C_{1,T}(\gamma)$. The BDS statistic $W_{n,T}(\gamma)$ has a standard normal limiting distribution and is calculated by:

$$W_{n,T}(\gamma) = \frac{\sqrt{T}\left[C_{n,T}(\gamma) - C_{1,T}(\gamma)^n\right]}{\sigma_{n,T}(\gamma)}, \quad as\ T \to \infty$$

Here $\sigma_{n,T}(\gamma)$ is an estimate of the asymptotic standard error of $\left[C_{n,T}(\gamma) - C_{1,T}(\gamma)^n\right]$. The BDS statistic shows that it should be asymptotically $N(0,1)$ as $T \to \infty$ if the residuals from the estimated model are actually IID whether it is a linear or nonlinear model. The larger the value of the BDS statistic, the stronger the evidence of nonlinearity in the data. Note that Dechert (1988) has given several counter examples for the statement that $C_n(\gamma) = C_1(\gamma)^n$ does imply IID.

Both Scheinkman and Lebaron (1989) and Brock and Baek (1991) (their empirical result to be discussed in Section 7.2.3) reported the null of IID was rejected for returns (including dividends) on the value weighted portfolio of the US stock market. However, rejection of the null of IID by the BDS statistic is not direct evidence that the time series exhibits a low complexity chaotic behaviour. Hsieh (1991) concluded the rejection of IID could be consistent with any of the following four types of non-IID behaviour: linear dependence, nonstationarity, nonlinear stochastic processes and chaos (*i.e.* nonlinear deterministic processes). The BDS has good power to detect those four types of behaviour. Therefore to safely draw the conclusion that the data is chaotic, we need to eliminate the other three possibilities.

The linear dependence could be easily ruled out due to the fact there is little of it in the raw data. Also we could remove whatever correlation there is by filtering the data.

The nonstationarity could also be ruled out because it can be eliminated by differencing the data. Hsieh (1991) also reported it is unlikely that infrequent structural changes are causing the rejection of IID.

To rule out the nonlinear stochastic process is a much more complicated task. Scheinkman and Lebaron (1989) pointed out that some nonlinear stochastic models, such as autoregressive conditional heteroskedasticity (ARCH) model (Engle, 1982), exhibit dependence similar to that of chaotic maps. Urrutia *et al.* (2002) and Urrutia and Vu (2006) did a further investigation by following the technique from Hsieh (1991). We proceed to consider whether stock returns are nonlinear in variance: $x_t = g\left(x_{t-1,\,\ldots}\right)\epsilon_t$. The ARCH type models as the special cases included from this general model of conditional heteroskedasticity. The object is to find the evidence of the conditional heteroskedasticity. We could observe that if we take the absolute value:

$$|x_t| = |g\,(x_{t-1}\,,\,...)|\,|\epsilon_t|$$

If $g\,()$ is differentiable, a Taylor series expansion would yield the result that $|x_t|$ depends on $|x_{t-i}|$. Thus, if we compute the autocorrelation of the absolute valued data, the finding of the correlation of $|x_t|$ with $|x_{t-i}|$ is the evidence of the conditional heteroskedasticity.

The second objective is to determine whether the conditional heteroskedasticity captured by ARCH type models account for all the nonlinearity in the stock return. To achieve that objective we need to fit an EGARCH model to the data:

$$x_t \sim N\left(0\,,\ \sigma_t^2\right)\ ,$$

$$\log\sigma_t^2 = \phi_0 + \phi\left|\frac{x_{t-1}}{\sigma_{t-1}}\right| + \psi\log\sigma_{t-1}^2 + \frac{\gamma x_{t-1}}{\sigma_{t-1}}$$

One reason to choose the EGARCH model is that the simple ARCH or the GARCH models impose restrictions on the signs of the parameters to guarantee that estimated variances are positive. Since EGARCH does not, numerical problems associated with constrained optimization are therefore avoided.. Another reason is that the EGARCH model could accommodate conditional skewness. If the EGARCH model is correctly specified, the standardized residuals:

$$z_t = \frac{x_t}{\hat{\sigma}_t}\,,$$

should be IID in large samples. Note that $\hat{\sigma_t}$ is the fitted value of the standard deviation from the variance equation. The correlation dimension method and the BDS statistic could be applied to the standardized residuals to test if the EGARCH captures all nonlinearity present in stock returns. If the correlation dimension value does not explode with increments in the embedding dimension, and the BDS statistic rejects the null of IID. That is, the correlation dimension and the BDS detect the presence of nonlinearity in the data, even after controlling for heteroskedasticity. Then we could conclud that the conditional heteroskedasticity could not account for the presence of nonlinear structures in the stock returns (Urrutia *et al.*, 2002; Urrutia and Vu, 2006).

7.2.3 Empirical Research for Estimating Correlation Dimension

Scheinkman and Lebaron (1989) constructed a weekly returns series by using the data set consisting of more than 5200 daily returns (including dividends) on the value weighted portfolio of the Centre for Research in Security Prices at the University of Chicago (CRSP). The correlation dimension value they obtained is approximately around 6, therefore they argued the weekly returns series of US stock presents nonlinear dependence. This nonlinear dependence could be used to explain phenomena such as the leptokurtic, the fat tails and so on.

Brock and Baek (1991) expanded the work from Scheinkman and Lebaron

(1989). The results of the correlation dimension value they obtained are between 7 and 9. The null hypothesis of IID is rejected as well, and more in favour of a lower dimensional alternative hypothesis. Hsieh (1991) pointed out the reason of rejection in the null of IID might not be chaotic process but another nonlinear stochastic process such as ARCH proposed by Engle (1982).

Urrutia *et al.* (2002) used insurance stocks trade on the New York Stock Exchange, American Stock Exchange and NASDAQ to investigate this. They constructed the weighted portfolios for both Life - Health insurance and Property - Casualty insurance. The correlation dimension value they reported is around 6.3 for both types of insurance portfolio. Urrutia and Vu (2006) conducted similar research by using the returns of American Depository Receipts (ADRs) traded on the NYSE, AMEX, and NASDAQ. The correlation dimension value they reported is approximately 5.5. Most importantly, Urrutia *et al.* (2002) and Urrutia and Vu (2006) did the further investigation and proved that the lower dimensional chaotic process is account for the rejection in the null of IID, but not conditional heteroskedasticity.

For the Chinese market research side, Gao, Pan and Chen (2000a) first obtained the correlation dimension value by using the daily logarithmic rate of return and he reported the correlation dimension value for SSEC is 2.65, for SEEC is 3.8(Gao, Pan and Chen, 2000b). Sun and Zhang (2001) reported the correlation dimension value for SSEC is 1.58 by using the daily logarithmic rate of return. The empirical results from China's stock market are much smaller than the results reported from developed countries which have a mature market, so we expect our empirical results will give a relatively smaller correlation dimension value.

8 Nonlinear Dynamic Positive Feedback Model

After detecting the nonlinear characteristics and the chaotic characteristics in the market structure, the next step is to build a model in order to make a better in description and explanation. The model here is required to have nonlinear characteristics and the chaotic characteristics, which we believe our proposed model satisfies. The model we derived here is a nonlinear dynamic positive feedback model, the work is based on the foundation of the DSSW positive feedback trading model (De Long *et al.*, 1990b), which offers solid financial economical sense.

8.1 Threshold

The threshold autoregressive (TAR) family proposed by Tong (1978) and further explained by Tong (1983) are contained within the state-dependent (regime-switching) model family, along with the bilinear and exponential autoregressive (EAR) models.

The simplest class of TAR models is the self exciting threshold autoregressive (SETAR) models of order p introduced by Tong (1983) and specified by the following equation:

$$Y_t = \begin{cases} a_0 + \sum_{i=1}^{p} a_j Y_{t-j} + \varepsilon_{t-d} & if \quad Y_{t-d} \leq r \\ (a_0 + b_0) + \sum_{i=1}^{p} (a_j + b_j) Y_{t-j} + \varepsilon_{t-d} & if \quad Y_{t-d} > r \end{cases}$$

TAR models are piecewise linear. The threshold process divides one dimensional Euclidean space into k regimes, with a linear autoregressive model in each regime. Such a process makes the model nonlinear for at least two regimes, but remains locally linear (Tsay, 1989). One of the simplest of TAR models equates the state determining variable with the lagged response, producing what is known as a self-exciting threshold autoregressive (SETAR) model.

A comparatively recent development is the smooth transition autoregressive (STAR) model, developed by (Terasvirta and Anderson, 1992). The STAR model of order p model is defined by:

$$Y_t = a_0 + a_1 Y_{t-1} + \ldots + a_p Y_{t-p} + (b_0 + b_1 Y_{t-1} + \ldots + b_p Y_{t-p}) G(\frac{Y_{t-d} - r}{z}) + \varepsilon_t$$

where d; p; r; $\{\varepsilon_t\}$ are as defined above, z is a smoothing parameter $z \in \mathbb{R}^+$

and G is a known distribution function which is assumed to be continuous. Transitions are now possible along a continuous scale, making the regime-switching process 'smooth'. This helps overcome the abrupt switch in parameter values characteristic of simpler TAR models.

8.2 Nonlinear Dynamic Positive Feedback Model

The reason we begin this section by introducing the threshold is because our model will use two thresholds as the regime switcher. From the previous section on chaos, we already knew this could generate a chaos process. This part is the theoretical work of the empirical estimation in a later chapter. We will use the Hurst exponent as the indicator of the threshold (so called H_TAR), when the Hurst exponent changes reached a threshold and entering a new regime, that indicating the market condition changes. Therefore trader behaviour changes accordingly, which will make the market present nonlinear and chaotic characteristics.

8.2.1 Model Assumptions

There are three types of investors: rational informed speculators (arbitrageurs), positive feedback traders and passive investors. There is a security asset pays no dividend, net supply equals zero and fixed fundamental value for a certain period.

8.2.2 Type I: Arbitrageurs

The first type of market participant is arbitrageurs, present in a measure of u. Their demand for an asset is based on deviation of the asset price from its fundamental value. If the logarithmic asset price is p_t and logarithmic fundamental value is f_t, the changing in demand for arbitrageurs is:

$$S_{1,t} = \begin{cases} u \cdot (-\beta_1) \cdot (p_t - f_t) & if \ (p_t - f_t) < k \\ u \cdot (-\beta_2) \cdot (p_t - f_t) & if \ (p_t - f_t) \geq k \end{cases}$$

$$\beta_1 < \beta_2$$

Where β represents the coefficient of sensitivity for arbitrageurs, and k is the threshold value of price deviation from its fundamental value that arbitrageurs settle on. If the price deviates far enough from its fundamental value, the arbitrageurs will become more aggressive ($\beta_1 \to \beta_2$). More of them will enter the market, and they will take larger positions. They may even initiate merges, leveraged buyouts, and other forms of restructuring. Thus the dummy variable I_k indicates if the threshold k has been reached or not. It will switch to 1 if $(p_t - f_t) \geq k$, otherwise $I_k = 0$. Thus:

$$
\begin{aligned}
S_{1,t} &= u\left(-\beta_1\right)\left(p_t - f_t\right) + I_k\left[u\left(-\beta_2\right)\left(p_t - f_t\right) - u\left(-\beta_1\right)\left(p_t - f_t\right)\right] \quad (2)\\
&= -u\beta_1\left(p_t - f_t\right) + I_k\left[-u\left(p_t - f_t\right)\left(\beta_2 - \beta_1\right)\right]\\
&= -u\beta_1\left(p_t - f_t\right) - \left[I_k u\left(p_t - f_t\right)\left(\beta_2 - \beta_1\right)\right]\\
&= -u\beta_1\left(p_t - f_t\right) - \left[I_k u\beta_2\left(p_t - f_t\right) - I_k u\beta_1\left(p_t - f_t\right)\right]\\
&= -u\beta_1\left(p_t - f_t\right) - I_k u\beta_2\left(p_t - f_t\right) + I_k u\beta_1\left(p_t - f_t\right)\\
\therefore S_{1,t} &= u\beta_1\left(p_t - f_t\right)\left(I_k - 1\right) - I_k u\beta_2\left(p_t - f_t\right) \qquad (3)
\end{aligned}
$$

The logarithmic fundamental value f_t is affected by all kinds of factors and is present in random walk. That is:

$$f_t = f_0 + \epsilon_t \qquad \epsilon_t \sim N(0, \sigma^2)$$

8.2.3 Type II: Positive Feedback Traders

The second type of the market participant is positive feedback traders, present in a measure of v. They are defined as same the in DSSW (De Long *et al.*, 1990b), chasing the trend. Thus the changing in demand for positive feedback traders is:

$$S_{2,t} = \begin{cases} v \cdot \theta_1 \cdot (p_t - p_{t-1}) & if \ \ (p_t - p_{t-1}) < q \\ v \cdot \theta_2 \cdot (p_t - p_{t-1}) & if \ \ (p_t - p_{t-1}) \geq q \end{cases}$$

$$\theta_1 < \theta_2$$

Where θ represents the coefficient of sensitivity for positive feedback traders, and q is the threshold value for the amount of the past price change that positive feedback traders settle on. For example, if the price has increased so much that it breaks the threshold, the demand from the positive feedback traders will change. There is increased uncertainty as to whether the current trend will continue or not. The larger the increase in price, the more passive the positive feedback traders will become ($\theta_1 \rightarrow \theta_2$). Thus:

$$\begin{aligned}
S_{2,t} &= v\theta_1 \left(p_t - p_{t-1}\right) + I_q \left[v\theta_2 \left(p_t - p_{t-1}\right) - v\theta_1 \left(p_t - p_{t-1}\right)\right] &\quad (4) \\
&= v\theta_1 \left(p_t - p_{t-1}\right) + I_q \left[v \left(p_t - p_{t-1}\right)\left(\theta_2 - \theta_1\right)\right] \\
&= v\theta_1 \left(p_t - p_{t-1}\right) + I_q v \left(p_t - p_{t-1}\right)\left(\theta_2 - \theta_1\right) \\
&= v\theta_1 \left(p_t - p_{t-1}\right) + I_q v\theta_2 \left(p_t - p_{t-1}\right) - I_q v\theta_1 \left(p_t - p_{t-1}\right) \\
\therefore S_{2,t} &= v\theta_1 \left(p_t - p_{t-1}\right)\left(1 - I_q\right) + I_q v\theta_2 \left(p_t - p_{t-1}\right) &\quad (5)
\end{aligned}$$

The dummy variable I_q indicates if the threshold q has been reached or not. It will switch to *1* if$(p_t - p_{t-1}) \geq q$, otherwise $I_q = 0$.

8.2.4 Type III: Passive Investors

The third type of market participant is passive investors that just take a long hold position, present in a measure of $(1 - u - v)$. Their changing in demand of the asset is based on the changing of the asset's fundamental value but not price. So as long as the fundamental value remains unchanged, their changing in demand will be zero:

$$S_{3,t} = 0 \tag{6}$$

8.3 Model Equilibrium

8.3.1 Solution

Under market clearing condition, the market has equilibrium below:

$$S_{1,t} + S_{2,t} + S_{3,t} = 0 \tag{7}$$

Thus:

$$u\beta_1 (p_t - f_t)(I_k - 1) - I_k u\beta_2 (p_t - f_t) + v\theta_1 (p_t - p_{t-1})(1 - I_q) + I_q v\theta_2 (p_t - p_{t-1}) = 0 \tag{8}$$

$$\begin{aligned} & p_t u\beta_1 (I_k - 1) - f_t u\beta_1 (I_k - 1) - p_t I_k u\beta_2 - f_t I_k u\beta_2 \\ & + p_t v\theta_1 (1 - I_q) - p_{t-1} v\theta_1 (1 - I_q) + p_t I_q v\theta_2 - p_{t-1} I_q v\theta_2 = 0 \end{aligned}$$

$$\begin{aligned} & p_t u\beta_1 (I_k - 1) - p_t I_k u\beta_2 + p_t v\theta_1 (1 - I_q) + p_t I_q v\theta_2 \\ = \; & f_t u\beta_1 (I_k - 1) + f_t I_k u\beta_2 + p_{t-1} v\theta_1 (1 - I_q) + p_{t-1} I_q v\theta_2 \end{aligned}$$

$$
\begin{aligned}
& p_t\left[u\beta_1\left(I_k-1\right)-I_k u\beta_2+v\theta_1\left(1-I_q\right)+I_q v\theta_2\right] \\
= \; & f_t\left[u\beta_1\left(I_k-1\right)+I_k u\beta_2\right]+p_{t-1}\left[v\theta_1\left(1-I_q\right)+I_q v\theta_2\right]
\end{aligned}
$$

Finally:

$$
\begin{aligned}
p_t \; = \; & \frac{u\beta_1\left(I_k-1\right)+I_k u\beta_2}{u\beta_1\left(I_k-1\right)-I_k u\beta_2+v\theta_1\left(1-I_q\right)+I_q v\theta_2} \; f_t \\
+ \; & \frac{v\theta_1\left(1-I_q\right)+I_q v\theta_2}{u\beta_1\left(I_k-1\right)-I_k u\beta_2+v\theta_1\left(1-I_q\right)+I_q v\theta_2} \; p_{t-1}
\end{aligned} \tag{9}
$$

Let:

$$
\varphi=\frac{v\theta_1\left(1-I_q\right)+I_q v\theta_2}{u\beta_1\left(I_k-1\right)-I_k u\beta_2+v\theta_1\left(1-I_q\right)+I_q v\theta_2} \tag{10}
$$

Therefore:

$$
p_t=(1-\varphi) f_t+\varphi p_{t-1} \tag{11}
$$

8.3.2 Analysis

So from the result above we could see that the condition for p_t is close to stable is$|\varphi|<1$, that is:

Case I:

$$
|\varphi|= \frac{v\theta_1\left(1-I_q\right)+I_q v\theta_2}{u\beta_1\left(I_k-1\right)-I_k u\beta_2+v\theta_1\left(1-I_q\right)+I_q v\theta_2} <1
$$

The asset price is stable. When the asset price gets any shock, the arbitrageurs will take the up-wind. That means the price will converge back to its fundamental value.

Case II:

$$|\varphi| = \frac{v\theta_1 (1 - I_q) + I_q v\theta_2}{u\beta_1 (I_k - 1) - I_k u\beta_2 + v\theta_1 (1 - I_q) + I_q v\theta_2} = 1$$

In this case, the price will neither back to the equilibrium position nor towards infinity after been shocked, the asset price might be a constant or swing at equal amplitude.

Case III:

$$|\varphi| = \frac{v\theta_1 (1 - I_q) + I_q v\theta_2}{u\beta_1 (I_k - 1) - I_k u\beta_2 + v\theta_1 (1 - I_q) + I_q v\theta_2} > 1$$

The asset price is unstable, any shock to the asset price will leads it to infinity.

Stage 0 At this time, the price trend is flattened, we can say the price equal to its fundamental values. This is similar to Case II above where $|\varphi| = 1$:

$$p_t = (1 - \varphi) f_t + \varphi p_{t-1}$$

$$p_t = p_{t-1}$$

At this stage, neither the positive feedback trader nor the rational trader take part. Then, there is a trigger, for example the type III investors could be treated as institutional investors with a passive index following strategy. They are buying (and have to do this) stocks when that company has been added to the market index, therefore their return is a replicated return of the market index. But actually adding a company into the index did not change its fundamental value, but it results in its price going up, and others will follow to buy. Or maybe the price fluctuates simply by luck. Either way, the price goes up and we move on to the next Stage.

Stage 1 At this time, the price trend is upward, so we can say the that price is higher than its fundamental values. This is similar to Case III above, where $|\varphi| > 1$.

At this stage, the buy power is larger than the sell power so the price goes up, the reason is the positive feedback trader is buying, and some arbitrageurs might also join to buy. The threshold q (for the positive feedback trader) and k (for arbitrageurs) has not been reached yet. (see Section 8.2.2 and 8.2.3 for details).

Stage 2 At this stage, the price is flattened or swings at equal amplitude like Case II. Because the force of buying and selling are equal now due to the threshold having been reached. the demand from the positive feedback traders decreases since they fear it's too high, and also due to the trend not going up as rapidly as before because more and more arbitrageurs are starting to sell. That is due to the price deviating too far from its fundamental value, and they may even initiate merges, leveraged buyouts, and other restructuring in order to sell. At this stage, the positive feedback traders will become ($\theta_1 \rightarrow \theta_2$), and the arbitrageurs will become ($\beta_1 \rightarrow \beta_2$).

Stage 3 At this time, the price trend is upward, so we can say that the price is higher than its fundamental value. This is similar to Case I above, where $|\varphi| < 1$.

At this stage, the sell power is larger than the buy power so the price goes down, the reason is the positive feedback trader closing out or even join to short. The price will converge back to its fundamental value.

This is a long-term description of bubbles and crashes. We could extend this to at smaller intervals by setting multiple threshold values. Similar to technical analysis reports published daily, different institutions give similar resistance and support levels, say if the price breaks through level 1, then it will aim to level 2 *etc.* This kind of price generate bubbles and crashes is a dynamic process and relies heavily by its initial setting and coefficient, therefore a random event could simply affect the path of the price evolution. This may therefore potentially lead to a chaos phenomenon in price evolution.

9 The ARIMA model and the Holt-Winters seasonal method

In this Section, We will introduce two method streams that has been used in time series forecasting. The ARIMA model and the exponential smoothing are the two most widely used approaches to time series forecasting, and provide complementary approaches to the problem.

While exponential smoothing models were based on a description of trend and seasonality in the data, ARIMA models aim to describe the autocorrelations in the data.

9.1 ARIMA model

In statistics and econometrics, and in particular in time series analysis, an autoregressive integrated moving average (ARIMA) model is a generalization of an autoregressive moving average (ARMA) model. Both of these models are fitted to time series data either to better understand the data or to predict future points in the series.

The ARMA model is mixed by two parts, an autoregressive (AR) model and a moving average (MA) model. To put it in another way, both the AR model and the MA model are the special form of the ARMA model. The ARMA(p,q) model contains p autoregressive terms and q moving average terms. So the ARMA(0,q) is MA(q), the ARMA(p,0) is AR(p), and the ARMA defining as follows:

$$X_t = c + \varepsilon_t + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}$$

The AR part of ARMA indicates the return of the past. More specifically, it shows that the evolving variable of interest is regressed on its own lagged values. The MA part indicates the predication error. More specifically, it shows that the regression error is actually a linear combination of error terms whose values occurred contemporaneously and at various times in the past.

The ARMA model is used when the time series is stationary, while the autoregressive integrated moving average (ARIMA) models are applied in some cases where data shows evidence of nonstationarity, where an initial differencing step can be applied one or multiple times to eliminate the non-stationarity to make the model fit the data better. This (for "integrated") indicates that the data values have been replaced with the difference between their values and the previous values. Note that the ARIMA model used deal the first order difference, and the with GARCH model deals the second order difference.

Therefore, in ARIMA(p,d,q), p is the autoregressive terms and q is the moving average terms, where the d expresses the integer order of differencing to be applied to the series before estimation to render it stationary. If a non-stationary time series could transform into a stationary time series by apply differencing d times, then we could have model:

$$\phi(L)(1-L)^d X_t = \theta(L)\varepsilon_t$$

where $\phi(L)$ is the stationary autoregressive lag operator polynomial, $\theta(L)$ is the inverted autoregressive lag operator polynomial, the integer order of differencing d:

$$(1-L)^d = 1 - C_d^1 L + C_d^2 L^2 + \ldots + (-1)^{d-1} C_d^{d-1} L^{d-1} + (-1)^d L^d$$

where C_d^r is the combination number r picked from d.

ARIMA(p,d,q) makes the non-stationary time series become stationary by differencing, and is often focused on the non-stationary time series has a trend. There is another possibility for a time series which has seasonal volatility. For example, monthly or seasonal data usually has this type of non-stationary times series. For monthly data, the non-stationary could be eliminated by differencing:

$$(1-L^{12})x_t = y_t, \qquad [(1-L^{12})x_t = x_t - x_{t-12}$$

For seasonal data, the non-stationary from the original time series could be eliminated by differencing:

$$(1-L^4)x_t = y_t, \qquad [(1-L^4)x_t = x_t - x_{t-4}$$

9.1.1 The ARFIMA model

Traditional time analysis models include the AR model, MA model, ARMA model and ARIMA models, but those models are more focused on the short-term memory. Since the importance of long-term memory gets the spotlight due to its characteristics of nonlinearity, the ARFIMA model proposed by Granger and Joyeux (1980) and Hosking (1981) is getting popular in different areas, especially financial economics. Liu, Liu and Zhang (2002) did use the ARFIMA model in the Chinese stock market and found the prediction failed, they argued that the reason is that the long-term memory effect on the Chinese stock market is not strong enough.

The ARFIMA model definition and estimation process is shown below:

If $\{x_t\}$is a stationary process and satisfes the difference equation:

$$\phi(L)(1-L)^d x_t = \theta(L)\alpha_t$$

Then the $\{x_t\}$ is ARFIMA(p,d,q) process, and:

$$\phi(L) = 1 - \phi_1 L - \phi_2 L - \ldots \phi_p L^p$$

$$\theta(L) = 1 - \theta_1 L - \theta_2 L - \ldots \theta_q L^q$$

1. Analyse the long-term memory effect, that is get the value of d.

2. Determining the order, that is get the value of p and q.

3. To make a prediction, we need to get the value of $\phi_1, \ldots \phi_p$and $\theta_1, \ldots \theta_p$.

Since the difference order d has a relationship with the Hurst exponent,

$$H = 0.5 + d$$

Therefore we are usually using the R/S method to calculate the value of the Hurst exponent to then get the value of d.

9.2 Introduction of the Holt-Winters seasonal method

The simple exponential smoothing method is suitable for forecasting data with no trend or seasonal pattern, such as our stock market data which does not display any seasonality or obviously trending behaviour. Around the late 1950s, Holt (1957) and Winters (1960) developed an advance method to capture the seasonality, the Holt-Winters method.

The Holt-Winters method was originally developed as a better prediction tool, so it has been applied to different research domains, and finance is one of them. Valakevicius and Brazenas (2015) used the seasonal Holt-Winters model to study the hourly exchange rate of the Euro (EUR) and US dollar (USD). They used two Holt-Winters methods, the addictive one and the multiplicative one, which we are going to cover in the next section; to analyse and predict on hourly EUR/USD exchange rate volatility. They found that the volatility is best predicted by a simplified version of the multiplicative Holt-Winters model.

The predictive effectiveness of the Holt-Winters model, no matter which variation type (addictive or multiplicative), could be compared with the method we raised before or a similar model. Omane-Adjepong, Oduro and Oduro (2013) applied both the ARIMA model and the Holt-Winters model to forecast Ghana's short-term inflation. They used monthly inflation data and four selected seasonal ARIMA models and made comparison with two Holt-Winters variation models, they found that the seasonal ARIMA models are the most appropriate method for obtaining result at short-term.

9.2.1 Theoretical Research

The Holt-Winters seasonal method contains the forecast equation and three smoothing ones. This is the core part of the Holt-Winters seasonal method, and there are two variations of equations, the additive method and the multiplicative method. For the additive method, the forecast equation is:

$$\hat{y}_{t+h|t} = \ell_t + hb_t + s_{t-m+h_m^+}$$

The three smoothing equations are the three components in the forecast equation. ℓ_t is the level equation is:

$$\ell_t = \alpha(y_t - s_{t-m}) + (1 - \alpha)(\ell_{t-1} + b_{t-1})$$

The ℓ_t itself means an estimate of the level of the series at time t. More specifically, it is the weighted average between the seasonal index of the past and the current seasonal index $(\ell_{t-1} + b_{t-1})$. Where α is the smoothing parameter of the level equation. The trend equation is:

$$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}$$

The b_t itself means an estimate of the trend for the timer series t. Where β^* is the smoothing parameter of the trend equation. The third equation is the seasonal equation.

$$s_t = \gamma(y_t - \ell_{t-1} - b_{t-1}) + (1 - \gamma)s_{t-m}$$

The seasonal equation is usually expressed as:

$$s_t = \gamma^*(y_t - \ell_t) + (1 - \gamma^*)s_{t-m}$$

Then we are able to substitute the level equation into the seasonal equation:

$$s_t = \gamma^*(1 - \alpha)(y_t - \ell_{t-1} - b_{t-1}) + [1 - \gamma^*(1 - \alpha)]s_{t-m}$$

which is identical to the smoothing equation for the seasonal component we specify here with $\gamma = \gamma^*(1-\alpha)$. The usual parameter restriction is $0 \leq \gamma^* \leq 1$, which translates to $0 \leq \gamma^* \leq 1-\alpha$.The error correction form of the smoothing equations is:

$$\ell_t = \ell_{t-1} + b_{t-1} + \alpha e_t$$

$$b_t = b_{t-1} + \alpha\beta^* e_t$$

$$s_t = s_{t-m} + \gamma e_t$$

where $e_t = y_t - (\ell_{t-1} + b_{t-1} + s_{t-m} = y_t - \hat{y}_{t|t-1}$are the one step training forecast errors.

Holt-Winters multiplicative method The component form for the multiplicative method is:

$$\hat{y}_{t+h|t} = (\ell_t + hb_t)s_{t-m+h_m^+}$$

$$\ell_t = \alpha \frac{y_t}{s_{t-m}} + (1-\alpha)(\ell_{t-1} + b_{t-1})$$

$$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1}$$

$$s_t = \gamma \frac{y_t}{\ell_{t-1} + b_{t-1}} + (1-\gamma)s_{t-m}$$

and the error correction representation is:

$$\ell_t = \ell_{t-1} + b_{t-1} + \alpha \tfrac{e_t}{s_{t-m}}$$

$$b_t = b_{t-1} + \alpha\beta^* \tfrac{e_t}{s_{t-m}}$$

$$s_t = s_t + \gamma \frac{e_t}{\ell_{t-1} + b_{t-1}}$$

where $e_t = y_t - (\ell_{t-1} + b_{t-1})s_{t-m}$

9.2.2 Applicability to the Financial Time Series

As the previous section illustrated, there are two variations of the Holt-Winters method, the additive method and the multiplicative method. The analysis conducted in the next chapter will be using the additive method.

The reason is that, first, the multiplicative Holt-Winters requires strictly positive data points, *i.e.* > 0. Because in the multiplicative model for the Holt Winter trend is calculated as division of two data points rather than subtraction (in case of additive model).

Due to our data contains some negative values, which should be expected in terms of returns. The Holt-Winters method we run in the next chapter will be the additive method and that method only.

Another reason for our analysis using the additive method is that, the value of return that was used in our data has been already logged. Therefore the additive method would be equivalent to the multiplicative method for logged return anyway, so we apply only one method to avoid the redundancy.

Part IV

Data Collection, Analysis and Evaluation

In this section, the framework of the data chapter will be introduced. This chapter is divided into three parts. The first part will introduce the method of data collection, and more importantly, the rationale of the choice. The background information for the research target, the Shanghai Stock Exchange will be introduced as well.

The second part will analyse the characteristics of the market as a whole through the SSEC index and individual firms listed by using the Hurst exponent estimation and calculating the correlation dimension value. It should be noted that the firms has been further sampled into smaller portions in order to conduct more concise and comparable research.

The third part raises a new model which is adapted from previous work done of DSSW model. The new model uses the Hurst exponent as the signal for thresholds that indicates the changes of market condition. The further test has approved its statistical significance and comparison shows the new model is superior among them.

10 Data Collection

The data that we will be evaluating in the next section is the Shanghai Stock Exchange Composite Index (SSEC), from when the stock market started trading in December 1990 to January 2015 (5897 trading days as variables). Also the data of daily logarithm returns will be used for all firms listed in the Shanghai Stock Exchange which is over 1000 companies. The time span for those companies varies since they listed at different times, but this thesis traces back the data to its Initial Public Offerings (IPO) to the year 2015 in best efforts.

China is a very interesting research target to choose, not just due to the fact it repsents an emering market that the author is most familiar with, but it has unique charateristics that other emerging market lack. Besides the well-known growth rate and the trading business over the world, China's stock market is not as open as other areas. It is relatively young, born after the 1990s, and forigner could not access it until the 21st century, which means the market is dominated by local investment. In the view of behavioural research, the tradition of Chinese culture tends to ignore the individual personality and encourage people to follow the mass. We would like to know if this could have an impact on the trader's behaviour, thus increasing the degree of positive feedback effect in the market. Also, the emipirical test on the Shanghai stock market, nonliearity-wise, is not sufficient. Therefore this thesis chosen China as the research target.

The data source is downloaded from the Thomson Reuters Datastream database for the stock market, and has been compared with the CSMAR Chinese financial database, Whist very tiny differences exist for a few firms, these are trivial differences that have no impact on the results *i.e.* the research conclusion remains the same. Note that in a later chapter the data for all firms has been narrowed down to samples in order to conduct a better estimation, details of the sampling procedure will be explained at section 12.2.

11 Background of the Exchange

Shanghai was the first city in China where stocks including stock exchanges and stock trading appeared. Stock trading started in Shanghai as early as the 1860s. In the primitive form of stock bourses, the Shanghai Sharebrokers Association was established in 1891. Later in 1920 and 1921, the Shanghai Security Goods Exchange and the Shanghai Chinese Security Exchange started operations. By the 1930s, Shanghai had emerged as the financial centre of the Far East, where both Chinese and foreign investors could trade stocks, debentures, government bonds and futures. In 1946, Shanghai Securities Exchange Co., Ltd. was created on the basis of the Chinese Security Exchange, but ceased operations three years later in 1949.

After Chinese economic reform and a series of policies that intended to open up the domestic market were introduced from 1978, the securities market started to resume. The first indication is the treasury bonds started trading again in 1981. Then in 1984, stocks and corporate bonds emerged in Shanghai and a few other cities. On 26th November 1990, the Shanghai Stock Exchange (SSE) came into existence, and it started formal operations on 19th December of the same year. There are three types of securities listed at the exchange, bond, fund and the one we mainly focus on, stock.

Table 2: SSE Market Overview

	2014	*2013*	*2012*
No.of Trading Days	245	238	243
No.of Listed Companies	995	953	954
No.of New Listed Companies	43	1	26
No.of Listed Securities	3758	2786	2098
Shares	1039	997	998
A Shares	986	944	944
B Shares	53	53	54
Bond	2646	1731	1059
Government Bond	267	218	191
Corporate Bond	2336	1468	830
Repo	43	45	38
Fund	68	58	41
Close end	3	9	12
ETF	61	47	29
Transaction currency fund	4	2	-
Preferred Share	5	-	-
Issued Vol (100 M)			
Share	27085.17	25751.69	24617.62
Preferred Share	10.3	-	-
Capital Raised (100 M)			
Share	3962.59	2515.72	2890.31

Table 2: SSE Market Overview

	2014	*2013*	*2012*
Preferred Share	1030	-	-
Negotiable Share(100 M)	24914.59	23731.13	19521.33
Market Capitalization (100 M)	243974.02	151165.27	158698.44
Negotiable Capitalization (100 M)	220495.87	136526.38	134294.45
WFE Rank of Market Cap	4	6	7
WFE Rank of Total Capital Raised	4	5	3
WFE Rank of Total Trading Value	3	5	6
Trading Value(100 M)	1281497.98	865098.34	547535.22
Share	377162.12	230266.03	164545.01
A-Share	375149.95	228918.82	164047.38
B-Share	484.45	689.94	413.48
Stock Repurchase	1527.72	657.27	84.14
Bond	866848.59	625839.41	379818.85
Goverment Bond	1247.47	771.6	905.56
Corporate Bond	24198.95	14540.88	7537.43
Repo	841402.16	610526.93	371375.86
Fund	37479.25	8989.48	3171.36
Close end	193	231.86	144.53
ETF	10142.68	6706.52	3026.59
Transaction currency fund	27141.81	2050.41	-
Preferred Share	4.27	-	-
P/E Ratio	15.99	10.99	12.3
A Shares	15.99	10.99	12.29
B Shares	15.77	11.62	13.18
SSE Composite Index	3234.68	2115.98	2269.13
SSE 50 Index	2581.57	1574.78	1857.68

Table 2: SSE Market Overview

Source: Exchange (2016)

Table 2 gives some general information on the Shanghai Stock Exchange, dated to 2015. China, as one of the key markets, shown rapid growth on different sorts of indicators, such as the number of seats and newly listed companies. The growth is also clearly reflected in the World Federation of Exchanges(WFE) ranking on market capital.

Source: Exchange (2015)

Table 3: Historical Number of Accounts Opened

Table 3 shows the participants of the market increase over time. Note that there is a leap in the number of accounts opened in 2007, that is due to the global financial boom at that time.

The Shanghai Stock Exchange Composite index (SSEC index) is the main index describing the stock market since it contains every listed firm at the exchange. Table 4 and Table 5 illustrate some factors about The SSEC index in the first quarter of 2015. Notice the high turnover rate, actually the highest in the world, reflecting that trading in the Chinese stock market is very active.

Total Market Capitalization (100 M Yuan)	29.22
Index total market capitalization coverage	100%
Negotiable Capitalization (100 M Yuan)	26.16
Index market capitalization coverage	100%
Sample stocks average daily turnover (100 M Yuan)	3.81
Turnover Rate	79.71%
Correlation coefficient of daily Index returns	98.16%

Table 4: SSEC Index Basic Market Indicator (at 2015 Q1)

Profit after Tax (100 M Yuan)	15673.23
Shareholders' equity (100 M Yuan)	121081.7
Earnings Per Share (1 Yuan)	0.56
Per Share Net Asset (1 Yuan)	4.36
Rate of Return	12.94%
Equity Ratio	12.66 %
Asset liability ratio	87.34%
Return on Total Assets Ratio	1.64%

Table 5: SSEC Index Basic Financial Indicator (at 2015 Q1)

The SSEC index was first published on 15th July 1991 and calculated on the basis of 100 of the day on 19th December 1990. Figure 3 demonstrates the historical trend of this index.

Source: Exchange (2015)

Figure 3: Historical Trends of SSEC Index

To sum up, the trend remains approximately flat with little upward movement before 2007. There are a few noticeable peaks worth mentioning. The first peak around mid-1992 is due to the fact that SSE removed the daily fluctuation limit, it was very strictly at 0.5%. There are numbers of passive index funds begins to trading as well. There is also a little boom and crush around 2001. That is due to the high-tech boom around the world, where firms with web features get speculated. The later huge up and down since 2007 is due to the financial crisis.

The downward around 1994 and flat trend for quite long time is due to the exchange has implemented the board limitation on trading stocks and funds. Since then, trading in A-shares, B-shares and securities investment funds is subject to a maximum daily price fluctuation limit of plus or minus 10%, except for the first trading day of its IPO(Initial Public Offering). Stocks subject to a risk warning are subject to a maximum daily price fluctuation limit of plus or minus 5%.

Moreover, all the firms have been characterised by an industrial category. Note that the total number added (995) is little below the observation we find in the next section (1028), that is due to the small amount of firms listed both in A shares and B shares. A shares are priced in the local renminbi yuan currency, while B shares are quoted in U.S. dollars. Initially, trading in A shares are restricted to domestic investors only while B shares are available to both domestic (since 2001) and foreign investors.

More than half of the listed firms are in the manufacturing industry from table 6, and the firms in the service sector are few. The majority of listed firms in the table are related to infrastructure. "Infrastructure" refers to long lived and costly capital assets often with complex design architectures that are required for economic growth and development in the public and private sectors. In the stock market, it also means their fundamental value should change in a rather slow pace, compared to those firms in the retail or high technology industry. This also implied a characteristic the Chinese stock market should have, but it turns other way round.

Industry	Number of firms
Agriculture, forestry, animal husbandry and fishery	15
Mining industry	47
Manufacturing	552
Electricity, heat, gas and water production and supply	52
Building industry	31
Wholesale and retail trade	89
Transportation, storage and postal services	57
Accommodation and catering industry	2
Transmission of information, software and IT services industry	26
Financial sector	32
Real estate	71
Leasing and Business Services	9
Scientific research and technical services	5
Water conservancy, environment and public facilities management industry	8
Education	1
Health and social work	1
Culture, Sports and Entertainment	12
Other	18
Total	1028

Table 6: Sector Distribution and Number of Companies

12 Nonlinear Characteristics and Chaotic Characteristics Detection and Estimation

In this section, we are going to test and estimate the nonlinear characteristics and chaotic characteristics by useing the Hurst exponent and correlation dimensions. Then we will analyse our result. In addition, we will make comparisons to other literature.

In the methodology chapter, we have already illustrated the development history of the Hurst exponent estimation, and we have compared the three methods of estimating the Hurst exponent including the rescaled range analysis (Classical R/S analysis) refined by Mandelbrot (1963, 1972), the modified rescaled range analysis (modified R/S analysis) proposed by Lo (1991) and the rescaled bariance analysis (V/S analysis) introduced by Cajueiro and Tabak (2005). The comparison at the end of the section shows that we are better using the classical R/S analysis or modified R/S analysis to get the most accurate result. We will apply the classical R/S analysis to the data, both the Shanghai Stock Exchange Composite Index (SSEC) and all listed firms individually.

The rescaled range statistic is the range of the partial sum of deviations of a time series from its mean, rescaled by its standard deviation. In detail, consider the sub-series from a given time series such as the SSEC index or each individual firm $X(t) = \{x_1, x_2, \ldots, x_n\}$ with n length, calculate the mean $\bar{x_\tau} = \frac{1}{\tau} \sum_{i=1}^{\tau} x_i$ and $\tau \, (1 \leq \tau \leq n)$ is the time horizon considered.

The R/S statistic of the series is shown below:

$$\left(\frac{R}{S}\right)_\tau = \frac{1}{S_\tau} \left[\max_{1 \leq t \leq \tau} \sum_{i=1}^{t} (x_i - \bar{x_\tau}) - \min_{1 \leq t \leq \tau} \sum_{i=1}^{t} (x_i - \bar{x_\tau}) \right]$$

where its standard deviation:

$$S_\tau = \sqrt{\frac{1}{\tau} \sum_{i=1}^{\tau} (x_i - \bar{x_\tau})^2}$$

Hurst (1951) found that many real world phenomena obey the power law relationship given by the equation above:

$$\left(\frac{R}{S}\right)_\tau \propto \tau^H$$

Therefore the Hurst exponent could be estimated by linear regression:

$$\ln \left(\frac{R}{S}\right)_\tau = c + H \ln (\tau)$$

12.1 The Hurst exponent for the SSEC Index and All listed firms

So the value of the Hurst exponent is the slope of the linear regression. We choose the SSEC index from 20th December 1990 to 23rd January 2015 to calculate its Hurst exponent and found an estimate that fitted the data well (see Figure 4).

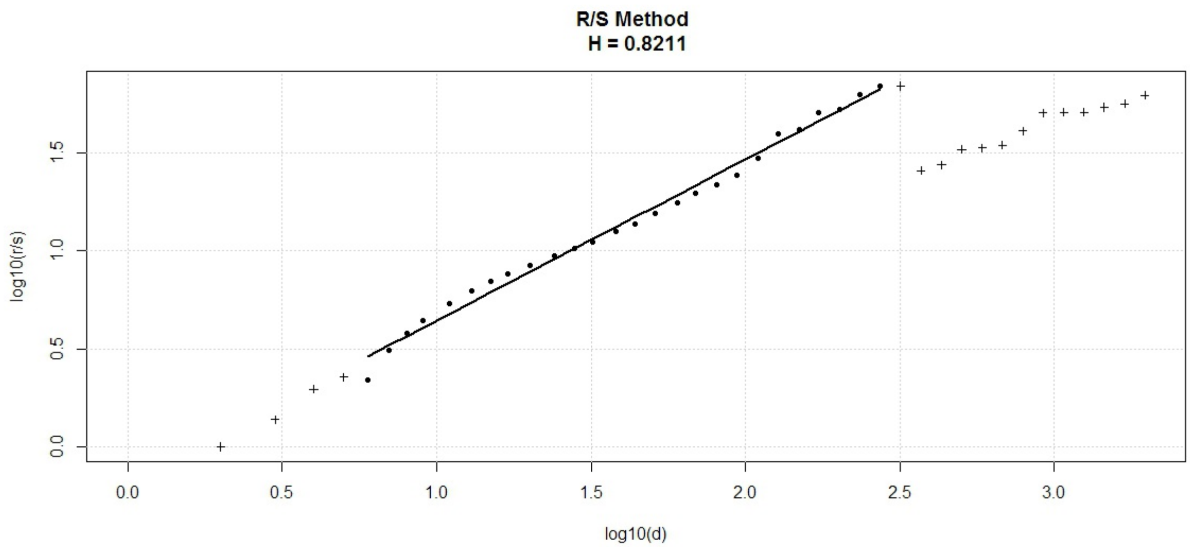

Figure 4: Hurst exponent estimated by R/S analysis for SSEC Daily Logarithm Return

Note that the τ(levels) is artificial, so we tried to apply different values of τ and found the value of the Hurst exponent converges around 0.82 since the value of standard error narrows after we increase the number of τ. The result is very stable after τ larger than 50. In Table 7 we present the result of the Hurst exponent estimation along with the t value for $H > 0.5$ to show its significance for different levels. The critical value is 1.96 at 5% level, and the results are statistically significant across all levels except at the lowest level which is still on the margin of significance.

n(levels)	Hurst exponent estimate	Std. error	t value for $H > 0.5$	Pr(>|t|)
20	0.827753	0.1934831	1.693962	0.04518816
40	0.8181216	0.1360529	2.33822	0.009720608
60	0.8166608	0.1126635	2.810678	0.002487901
80	0.8236014	0.09505867	3.404228	0.000336112
100	0.8204199	0.08643336	3.707132	0.000106705
120	0.8163717	0.0761169	4.156392	1.66178E-05
140	0.8157547	0.07111182	4.440256	4.6548E-06
160	0.8227396	0.06619549	4.875553	5.70751E-07
180	0.8218027	0.06291657	5.114753	1.66953E-07
200	0.8192747	0.0595812	5.358648	4.50972E-08

Table 7: Hurst exponent value for different levels

The H value being larger than 0.5 means the events of today indeed affect the events tomorrow. That is saying the information received today is still taking account by the market after its been received. This also reflect the price of SSEC is not obey random walk. There is a correlation between return series, and it is not a simple series correlation. Because the impact of information will decay quickly in a series correlation, and here is a long-term memory.

If we compare the value of the Hurst exponent of the SSEC index with other market indices from the developed countries, the Chinese one is higher than those developed markets (Table 1) such as S&P 500. This is reasonable and coherent with our expectation, since as an emerging market, the Chinese stock market shows more chaotic characteristics therefore the persistence effect is expected to last longer.

We also proposed a new way to see the whole pattern of the development of the Hurst exponent.The Hurst exponent could be estimating in a rolling way rather than processing the time series as a whole. The way to rolling it is to cut the original time series into different segments, and only move one step for each rolling.

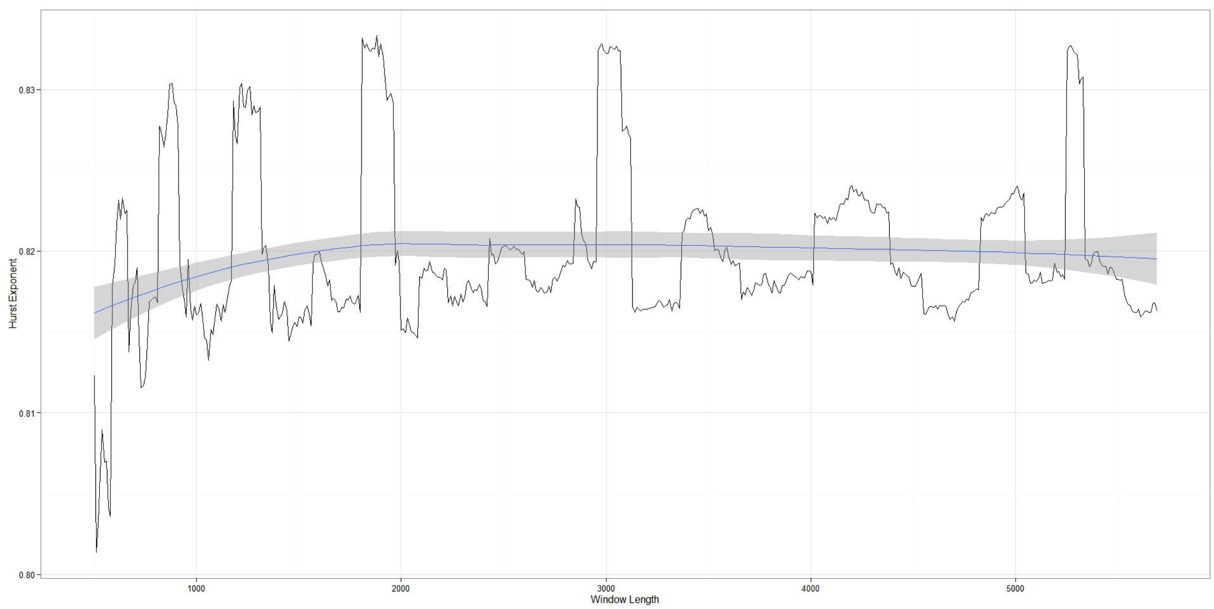

Figure 5: Hurst exponent Estimation by Starting Rolling Methods

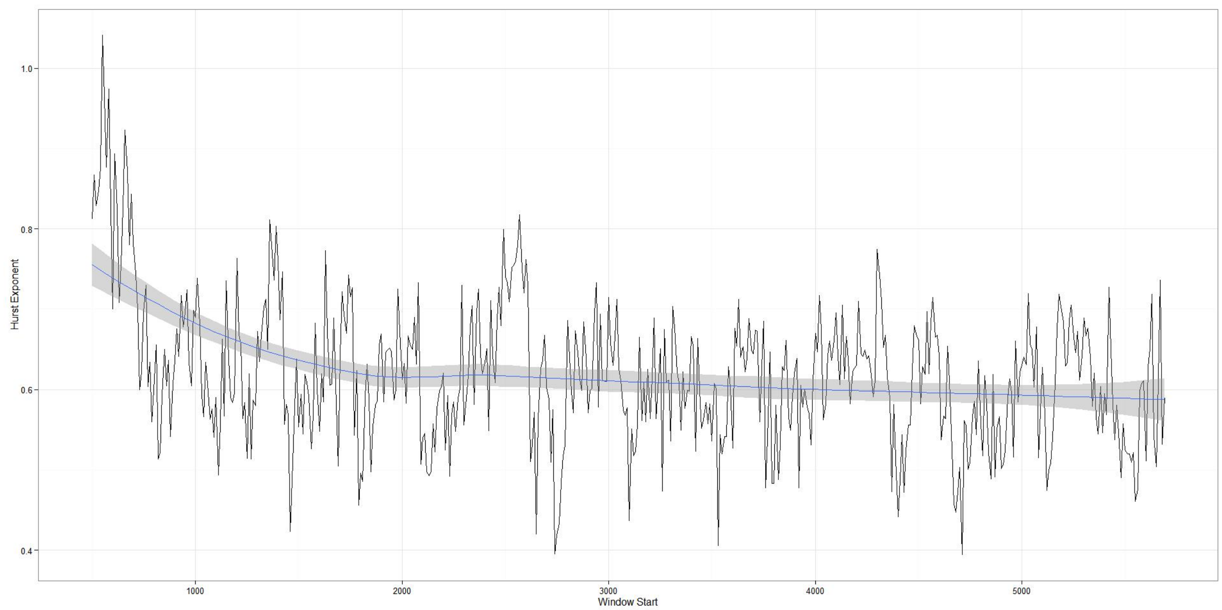

Figure 6: Hurst exponent Estimation by Simple Rolling Methods

Figure 5 and Figure 6 are Hurst exponent estimations presented using two different methods. If we use a sub-sample of the data from the SSEC Index (Figure 4), we see a similar estimate to the whole series, the value of the Hurst exponent is approximately 0.82 again. However, if we do the rolling method by a small step each time, taking account of the previous information, we will obtain a smaller value of the Hurst exponent. We believe this new method which takes into account the previous information could reflect the development of the market. The recent market is becoming more mature since the value of the Hurst exponent is slowly going down.

There is a large decrease in the first 1500 days of the sample, which lessened after this point. This is believed to be at least in part due to the introduction of the trading board limitation system on the SSEC in December 1996. Before that time, there was no limit to the rise or fall for an individual firm stock. Therefore the positive feedback effect is very powerful, and it creates a much bigger impact on the future. The scale of the changing in price was limited by 10% in whatever direction after that time. Then the market became less one-sided before.

Qin and Ying (2011) used the moving blocks bootstrap method which is similar to our rolling window approach to calculate the development of the Hurst exponent from the SSEC. The data they used is from 1990 to 2010, and their findings is very similar to ours. The Hurst exponent was large than 0.8 at the beginning, then it started to drop to a lower but more stationary value for recent years. Though the stationary value which they get is slightly higher, about 0.65 to 0.7.

The research in the next section will be changed from market index to individual firms. The R/S method will be applied in order to estimate the Hurst exponent for all listed firms in the SSEC. Figure 7 is the Hurst exponent estimation for all listed firms stock return in SSEC, and Figure 8 is the distribution of it.

12.2 The Hurst exponent for the Individual Firms

12.2.1 The Hurst exponent for All Listed Firms

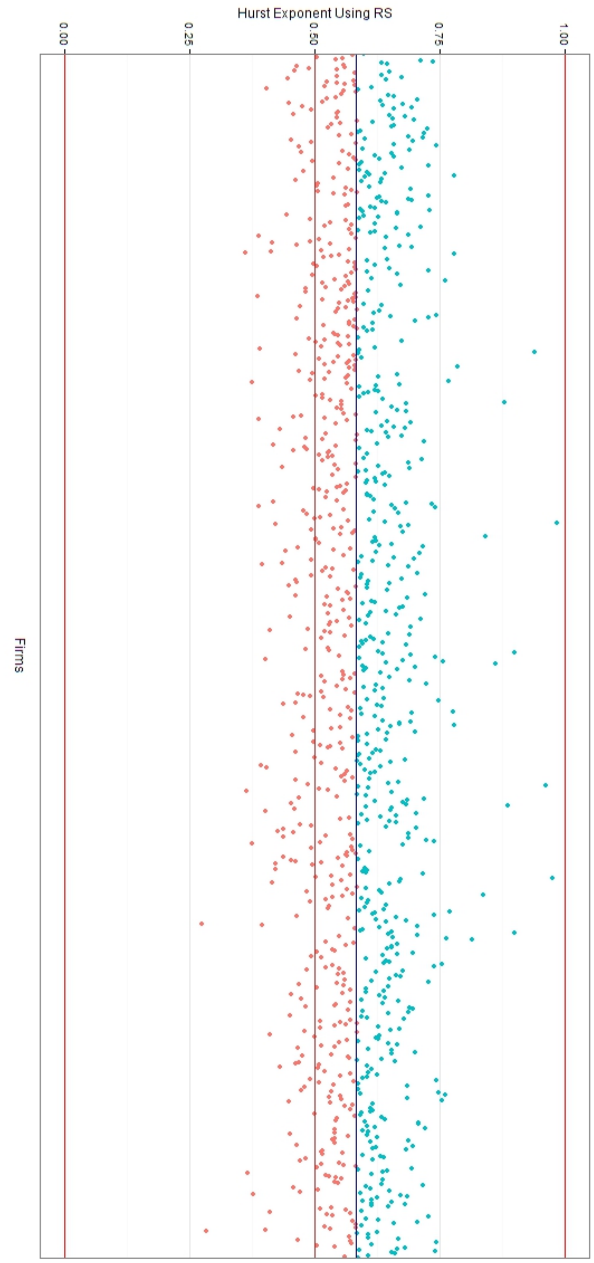

Figure 7: Hurst exponent Estimation for All Listed Firms Stock Return in SSEC

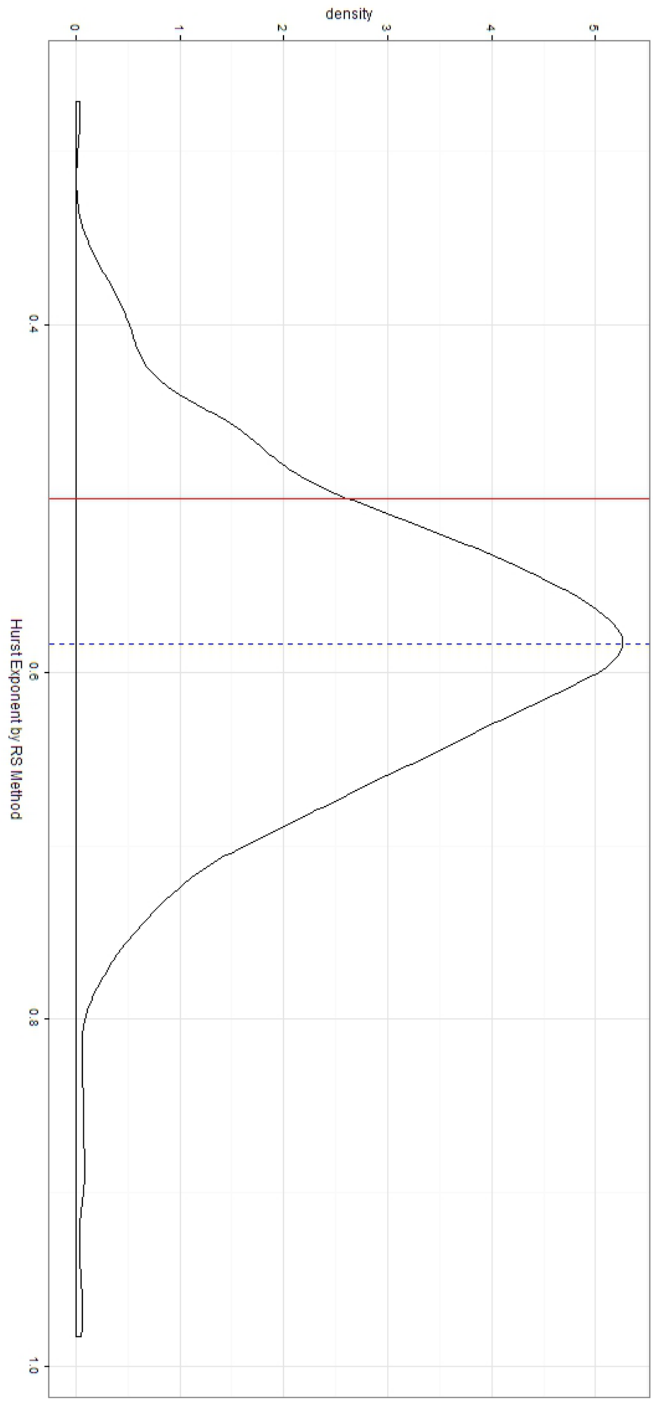

Figure 8: Hurst exponent Estimation Distribution for All Listed Firms Stock Return in SSEC

Each dot in Figure 7 represents a firm. We can see the line that indicates the median lays about 0.59. In Figure 8 it is represents from another angle, the peak of the distribution is significantly deviated from 0.5 which is the point the traditional finance would expect. The importance can be illustrated by an example.

For example, suppose a group of people picked a group of dots, *i.e.* a group of firms, by careful research. In traditional finance, they cannot get the abnormal return since the efficiency market hypothesis (EMH) says that past information cannot affect the future, *i.e.* the historical stock price cannot be used to predicted the future price, so the stock price is independent. That is saying that the market obeys random walk. Selecting those firms carefully should be no different to picking them randomly, in this case, all dots should be at the 0.5 line.

However, as our result suggested, now those group of people are highly likely to find that for some of the firms they picked the value of the Hurst exponent significantly differs from 0.5, which indicates random. For those firms that the Hurst exponent differs from 0.5, their time series is not independent *i.e.* random but biased. That is saying that the past information could indeed affect the future, as from the diagram, the majority of the firms for which the value of the Hurst exponent is different from 0.5 is larger than 0.5. For example, those firms that raised in the past are more likely to go up in the future.

That leaves an opportunity for those investors using historical stock prices to forecast the future trend. There is a difference that exist between randomly picked and careful selection. Once they are able to earn the abnormal return from that, the efficiency market hypothesis does not hold anymore. To conclude, this is vital evidence to chanllenge the EMH and stock price obey random walk status.

12.2.2 The Hurst exponent for the Sample Firms

We have picked a number of firms for our sample, and using the closing prices as the basis for logging first differences, we generated a series for estimation. From this, we estimated the Hurst exponent(Taqqu *et al.*, 2013). The selection method is as follows: the firm is required to have at least 15 years trading history (427 firms available); the firm also needs to be currently alive so the price is able to be traced (for example, firms that only traded during 1993 to 2008 and delisted are not suitable); then we want our samples to have almost a full record of trading, that means their trading continuously without too much gap. This narrowed the sample down to 50 firms to fulfil our filter requirements. The name and code of our sample firms will appear in the Appendix A at the end. Note that these 50 sample firms are coming from different industries, and should be distinguish from the 50 firms of the SSE 50 index. Those 50 firms from SSE 50 index are representing the top 50 listed companies by "float adjusted" capitalization, which means they are all blue chip stocks.

The prices were transformed into returns using the logarithm of first differences and the Hurst exponent was estimated using a rolling window. Note that Chinese stocks could only fluctuate 10% maximum per day except on the day of a special event, such as IPO or restructuring. The BDS test was also used to test the statistical significance. The tests were found to show a rejection of the null hypothesis of Independent Identity Distribution (I.I.D.) in all cases.

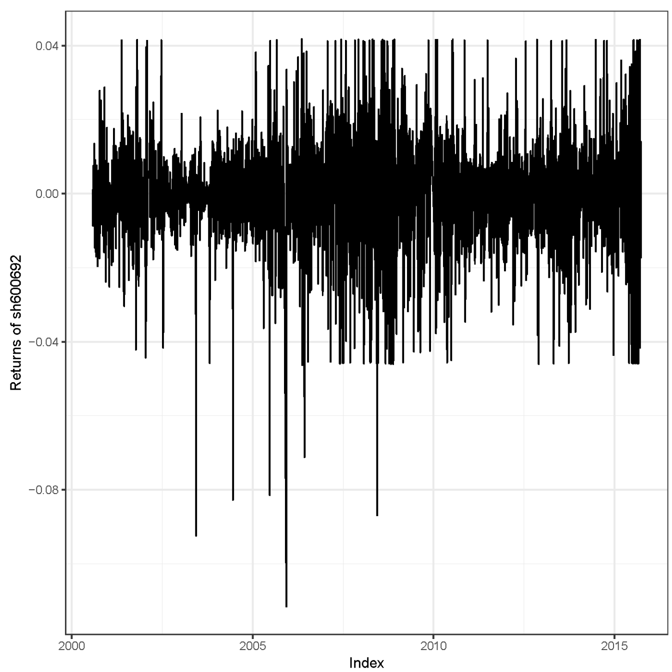

Figure 9: Return plot of sh600692

The return against time plot is drawn before the Hurst exponent discussion, an interesting find being that, most firms have spikes. With the trading board fluctuation limitation, that is equivalent to the price hitting the limit in one direction for consequent days, which immediately reflects the concept of positive feedback trading. Figure 9 gives one example, and the full plots can be found in Appendix C. It can be seen from those figures that the daily returns have obvious characteristics of volatility clustering, indicating that investors' response to information is not linear, but cumulative and lagging.

The value of the Hurst exponent being larger than 0.5 means the events of today indeed affect the future. That means the information received today is still taking account of the market after it has been received. We found that the mean of the Hurst exponent from all sample firms is larger than 0.5, which indicates the historical price has some impact on the consequence. Table 8 gives the Hurst exponent range of sh600645 (Zhongyuan Union Cell & Gene Engineering). The firm which has the highest mean Hurst exponent is sh600645 as well. Note that the larger the window, the bigger the range of Hurst exponent. This pattern is the same for all sample firms.

Range	HS_100	HS_250	HS_500
Min.	0.3787	0.4335	0.4751
1st.	0.4969	0.5159	0.5320
Median	0.5363	0.5523	0.5594
Mean	0.5375	0.5509	0.5554
3rd.	0.576	0.5830	0.5759
Max.	0.6911	0.6559	0.6381

Table 8: Hurst exponent Rang of sh600645

Compared with the Hurst exponent result for Western firms, such as 0.7

for Coca Cola and 0.72 for IBM found by Peters, our sample firms seem to have a lower value and closer to 0.5 which indicates randomness. It could be speculated that the sample firms have been invested in an untransparent way and did not really shown a trend of growth in the long term.

Next we will looking further at some individual cases, since we used a rolling method to estimate the individual firms across the 15 years, we could see the density of its Hurst exponent. The distribution of our sample firms could be seen in Figure 10. Results of every individual firm has the mean of Hurst exponent larger than 0.5, and the majority of samples presented as bi-modal. The full plots can be found in Appendix D. It is informative to consider a number of individual firms. We are able to show the density of the Hurst exponents from the estimates series. Qualitatively the results are very similar, *i.e.* above 0.5 for every part of the data. The main difference between histograms is the spread of the data.

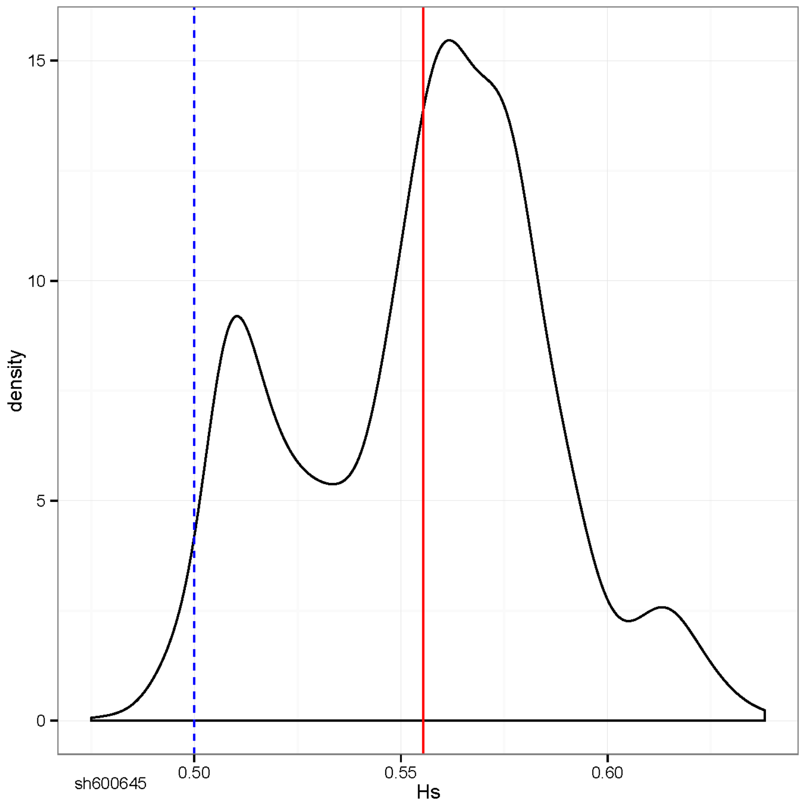

Figure 10: Hurst exponent Distribution of sh600645

12.3 The Correlation Dimension Value for both the SSEC Index and the Sample Firms

The correlation dimension value could be estimated through a graphic procedure illustrated in the Methodology chapter, and the core ideas follow here, (full technical details can be found at section 7.2.2).

It is assumed that the data, in this case, is the time series we used in the last section. The time series $\{x_t,\ t = 1,\ 2,\ \cdots,\ n;\ x_t \in \mathbb{R}\}$ which is generated by a nonlinear dynamic system could be embedded in $n-$space by constructing $n-$futures. As an $n-$history is a point in $n-$dimensional space, therefore n is called the embedding dimension. We could construct the $n-$histories of the filtered data in order to obtain the embedding dimension. The $n-$histories are denoted as follows:

$$\begin{aligned}
\text{1-history}: \quad x_t^1 &= \quad x_t \\
\text{2-history}: \quad x_t^2 &= \quad (x_{t-1},\ x_t) \\
&\vdots \\
n\text{-history}: \quad x_t^n &= \quad (x_{t-n+1},\ \cdots,\ x_t)
\end{aligned}$$

The trajectory of a time series process in the phase space is constructed by $n-$dimensional vectors. Note that an attractor is a subset of $n-$dimensional phase space towards which almost all sufficiently close trajectories get "attracted" asymptotically. They tend towards strange attractors on which the motion is chaotic, *i.e.* not (multiply) periodic and unpredictable over long times, being extremely sensitive on the initial conditions (Grassberger and Procaccia, 1983a).

The next step is to calculate the correlation integral $C_n(\gamma)$. It measures the fraction of the total number of pairs $(\vec{x_i},\ \vec{x_j})$ such that the distance between $\vec{x_i}$ and $\vec{x_j}$ is no more than γ, *i.e.* it is a measure of spatial correlation. It is defined according to:

$$C_n\left(\gamma\right) = \lim_{T\to\infty} \frac{1}{T^2} \overset{T}{\underset{i,\,j\neq 1}{}} \Theta\left(\gamma - \left|\vec{x_i} - \vec{x_j}\right|\right)$$

where Θ is the Heaviside function i.e. an indicator like dummy variable I in our previous model, assumed $x = \gamma - \left|\vec{x_i} - \vec{x_j}\right|$, therefore:

$$\Theta\left(x\right) = \begin{cases} 0 \quad if \quad x \leq 0 \\ 1 \quad if \quad x > 0 \end{cases}$$

Grassberger and Procaccia (1983a,b) established that for small distance γ , the correlation integral $C_n\left(\gamma\right)$ grows to obey the power law:

$$C_n\left(\gamma\right) \propto \gamma^v$$

Therefore the correlation exponent v_n could be obtained by calculating the slope of the graph of $\ln C\left(\gamma\right)$ versus $\ln \gamma$ for small values of γ . More specifically, we want to calculate the following quantity:

$$v_n = \lim_{\gamma\to\infty} \frac{\ln C\left(\gamma\right)}{\ln \gamma}$$

We want to do this step by increasingly larger values of the embedding dimension and observing the value of the correlation exponent. It will stabilize at the saturation value of the correlation exponent. This value is the value of correlation dimension v (correlation dimension value):

$$v = \lim_{n \to \infty} v_n$$

We applied this estimation process by using the SSEC index data first, and generated Figure 11. This graphic procedure illustrates the correlation of integral changes with the distance (radius). When the radius is between 0.0001 and 0.0005, the correlation integral grows to obey the power law. After increasing the embedding dimension, the correlation exponent stabilized at approximately 4.43.

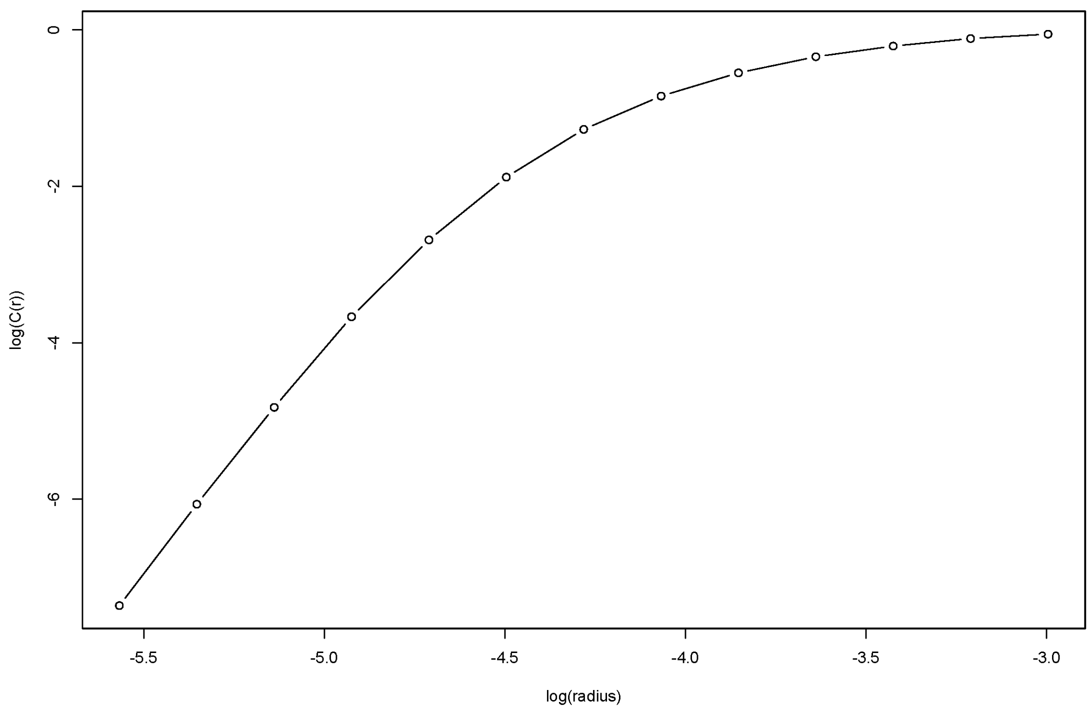

Figure 11: Correlation Dimension Graphic Procedure for SSEC index

Compared with the empirical research reviewed in section 7.2.3, such as Urrutia and Vu (2006), Scheinkman and Lebaron (1989), and other calculated correlations of the mature capital market, the correlation dimension of the Chinese stock market we estimated is significantly lower. This result is what we expected. Also, comparing with previous empirical estimations on correlation dimension for SSEC, Gao, Pan and Chen (2000a) reported 2.65, Sun and Zhang (2001) reported 1.58, Li *et al.* (2003) reported 1.32, while the result we obtained is slightly higher. It can still be treated as a lower dimensional deterministic chaotic series, but it seems improved from the previous result.

In general, the correlation dimension measures the degree of the phase space, which is filled by a set of time series. The larger the correlation dimension, *i.e.* the higher the degree of filling, indicates the more complex the internal structure of the time series, *i.e.* the more similar it is to the random process time series. When the correlation dimension is high, it is difficult for us to identify its complex structure with limited sample data. At this point it looks similar to a good pseudo-random generator, therefore the higher dimensional deterministic processes and random processes will have no practical significance.

However, if the time series is a lower dimensional deterministic process, it means that the time series is predictable in the short term. In this sense, according to our result, we believe that the Chinese stock market is less random and relatively predictable in the short term compared to Western mature capital markets. Note that even our result is lower than the mature capital market in the West, the correlation dimension value estimated is still larger than the previous empirical research for the Chinese stock market at early 21st century. Those empirical results are around 2 which is extremely low, and about 10 to 15 years later, the correlation dimension value has increased to 4.4, which implied the similarity to randomness of Chinese stock market has raised and its structure becoming more complex.

Table 9 summarized the result of the correlation dimension value when we repeated the graphic procedure into sample firms.

	Correlation	Dimension	
SSEC Index	4.43		
sh600000	4.40	sh600640	5.89
sh600009	4.02	sh600642	4.49
sh600054	5.19	sh600643	5.10
sh600064	5.37	sh600644	4.20
sh600067	4.81	sh600645	4.42
sh600082	5.10	sh600650	5.01
sh600097	5.26	sh600651	4.57
sh600159	5.44	sh600658	6.13
sh600162	5.26	sh600663	5.26
sh600256	4.65	sh600668	5.61
sh600601	4.72	sh600674	5.41
sh600609	5.87	sh600683	5.11
sh600611	5.44	sh600684	5.19
sh600612	5.43	sh600692	4.73
sh600618	4.83	sh600802	5.59
sh600620	5.30	sh600812	4.72
sh600621	5.49	sh600821	5.09
sh600622	5.33	sh600824	5.01
sh600624	5.46	sh600826	5.74
sh600626	4.89	sh600830	5.18
sh600628	4.60	sh600831	5.50
sh600630	5.68	sh600834	4.98
sh600636	4.90	sh600835	4.76
sh600638	5.06	sh600846	5.42
sh600639	4.75	sh600859	4.91

Table 9: Correlation Dimension for SSE Composite Index and Sampled Firms

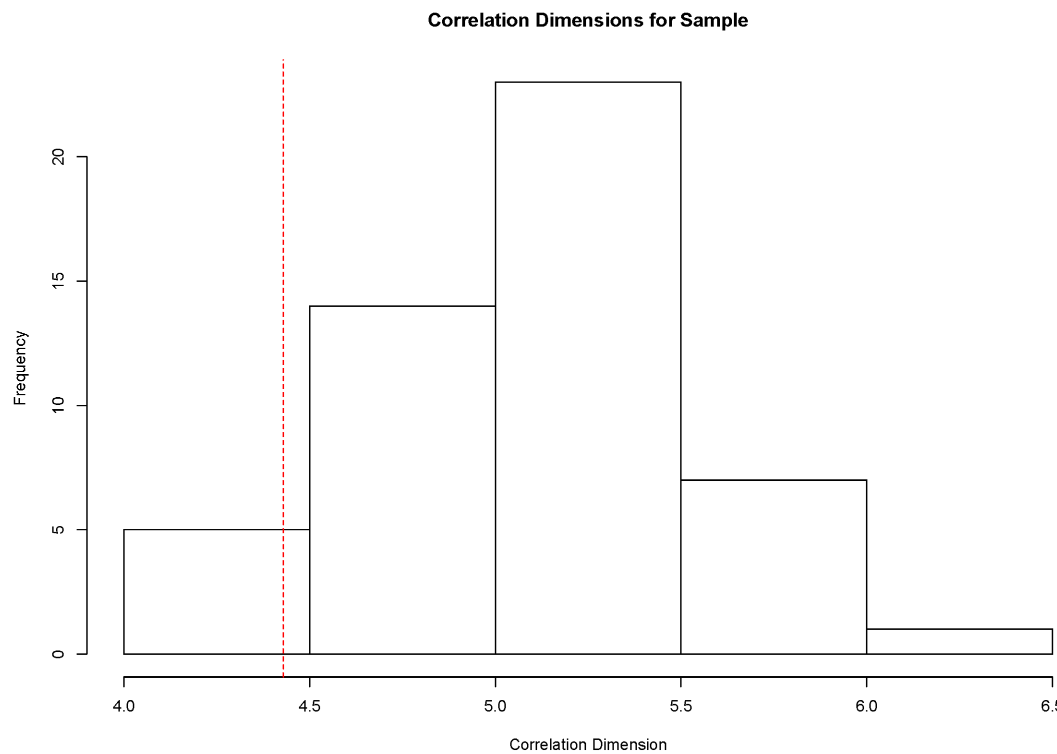

Figure 12: Histogram of Correlation Dimension for SSE Composite Index and Sampled Firms

Figure 12 illustrates the histogram of correlation dimension for SSEC index and sampled firms. It can be seen the correlation dimension for most sampled firms is higher than the SSEC index (red line at 4.4). This result of correlation dimension is coherent with the Hurst exponent estimation previously. This means most firms have higher correlation dimension with a more complex internal structure of the time series, and are more similar to the random process time series than SSEC index which represent the market. In the previous Hurst exponent estimation, most of the firms have a lower Hurst exponents value that is closer to 0.5 compared to the SSEC index, indicating the firms are closer to random walk than the market.

These two results are coherent with each other, and implies that the market tends to be more predictable than a individual stocks, which meets with the general observation. It is easier to forecast the general market trend in a month than to pick a few individual stocks and predict their price. In the real world, the example is earning stable at a 15% with a passive index fund, or either earning or losing 85% with active stock selection strategy. It sounds very reasonable but if thinking deeply, the market is the one that has more variables affected than a single firm. Investors should know a single firm much more easily than they know the market, information-wise. Therefore the market should be more random than a firm in theory, but both real world experience and our estimation denied this hypothesis. The reason we offered is that the market is affected by more variables and has more participants itself is exactly why they are less random. Because the market has a large amount of participants, a positive feedback effect is much easier to create and harder to turn around than a single stock. Then the market has more a clearer trend than an individual stock, *i.e.* less randomness.

13 Nonlinear Dynamic Positive Feedback Model

This section will be divided into four parts. The first part is the model raised from the methodology chapter, where the empirical research will be conducted for sample firms. Then the model selection will be performed to be compared with the ARIMA, effectively the AR(1) by using AIC. Then further tests including the LR test and the BDS test will be used to test the statistical significance. Later the model will be compared with the sub-period and the Holt-Winters additive technique.

13.1 Model Evaluation

Our main contribution is a new model that brought concepts from natural science to interpret financial markets. The theoretical work has been developed from section 8.2. We will use the Hurst exponent as the indicator of the threshold, when the Hurst exponent changes reached a threshold and entered a new regime, this indicated the market condition changes.

Therefore the traders' behaviour changes accordingly, which makes the market present nonlinear and chaotic characteristics. By combining the threshold model and assumptions from DSSW which offers economic intuition, we believe the new model could offers a better explanation on the complexity of the stock market which present in chaos. The model is as follows:

$$R_t \;= \alpha + \quad \delta_1 I + \delta_2 J + \beta R_{t-1} + \gamma_1 R_{t-1} I + \gamma_2 R_{t-1} J + \varepsilon_t \tag{12}$$

$$I = \begin{cases} 1 & if\ H < T_A \\ 0 & if\ H \geq T_A \end{cases} \quad ; J = \begin{cases} 1 & if\ H < T_B \\ 0 & if\ H \geq T_B \end{cases} \quad T_A < T_B$$

Where the value of the Hurst exponent is the signal, in this model, there are three possibilities.

1. $H < T_A$ which means $H < T_B$ as well, since $T_A < T_B$.
2. $T_A \leq H < T_B$.
3. $T_B \leq H$.

Then:

$$\begin{cases} R_t = & \alpha + \delta_1 I + \delta_2 J + \beta R_{t-1} + \gamma_1 R_{t-1} I + \varepsilon_t \quad if\ I = J = 1 \\ R_t = & \alpha + \delta_2 J + \beta R_{t-1} + \gamma_2 R_{t-1} J + \varepsilon_t \quad if\ J = 1\ and\ I = 0 \\ R_t = & \alpha + \beta R_{t-1} + \varepsilon_t \quad if\ I = J = 0 \end{cases} \tag{13}$$

$$R_t = \alpha + \delta_1 T_A R_{t-1} + \delta_2 T_B R_{t-1} + \beta R_{t-1} + \gamma_1 T_A + \gamma_2 T_B + \varepsilon_t$$

	sh600000 Estimate	sh600000 t value	sh600009 Estimate	sh600009 t value	sh600054 Estimate	sh600054 t value	sh600064 Estimate	sh600064 t value	sh600067 Estimate	sh600067 t value
(Intercept)	0.0011	1.4686	2e-04	0.6006	5e-04	1.6616	0.0027	2.5615	-4e-04	-0.3688
(Intercept) SE	(7e-04)		(4e-04)		(3e-04)		(0.001)		(0.0011)	
lag(x, 1)	-0.203	-3.0486	-0.1066	-2.5006	-0.0453	-1.6583	-0.1531	-2.0005	0.4484	5.0633
lag(x, 1) SE	(0.0666)		(0.0426)		(0.0273)		(0.0766)		(0.0886)	
T_A	-0.0014	-1.6699	-2e-04	-0.2204	-8e-04	-1.4557	-0.0031	-2.7258	4e-04	0.309
T_A SE	(8e-04)		(7e-04)		(6e-04)		(0.0011)		(0.0012)	
lag(x, 1):T_A	0.3035	4.116	0.298	3.4398	0.2071	4.0566	0.2895	3.3403	-0.4046	-4.4404
lag(x, 1):T_A SE	(0.0737)		(0.0866)		(0.051)		(0.0867)		(0.0911)	
T_B	-0.001	-1.3407	-1e-04	-0.2425	-7e-04	-1.7779	-0.0027	-2.5123	2e-04	0.1874
T_B SE	(8e-04)		(4e-04)		(4e-04)		(0.0011)		(0.0012)	
lag(x, 1):T_B	0.1689	2.4215	0.1401	3.0037	0.0875	2.3759	0.1678	2.1266	-0.3959	-4.2531
lag(x, 1):T_B SE	(0.0698)		(0.0467)		(0.0368)		(0.0789)		(0.0931)	
F Stat	4.98012	(5,3494)	3.16908	(5,3494)	4.68037	(5,3494)	4.72084	(5,3494)	6.65787	(5,3494)
R Sq	0.00708		0.00451		0.00665		0.00671		0.00944	
R Sq Adj	0.00566		0.00309		0.00523		0.00529		0.00802	

Table 10: H_TAR result for sh600000, sh600009, sh600054, sh600064 and sh600067

Table 10 is an example of the result. The test statistic for each sample firm, with window 100, 250, and 500 respectively can be found in Appendix E. We have found that the majority of sample firms are statistically significant($|t| > 1.96$) for the threshold coefficient estimation. Also, the standard error of our results is very small across the board, so we could safely say that the coefficient estimated are quite stable.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.0002	0.0004	0.60	0.5481
lag(x, 1)	-0.1066	0.0426	-2.50	0.0124
T_A	-0.0002	0.0007	-0.22	0.8256
T_B	-0.0001	0.0004	-0.24	0.8084
lag(x, 1):T_A	0.2980	0.0866	3.44	0.0006
lag(x, 1):T_B	0.1401	0.0467	3.00	0.0027
F Stat	3.16908			
R Sq	0.00451			
R Sq Adj	0.00309			

Table 11: At window 100, H_TAR result for firm sh600009

This is the detail coefficient for one estimation if our scope continues been narrow down. In Table 11, the difference between two thresholds is quite big: lag T_A = (0.30) and lag T_B = (0.14). The difference between the thresholds is quite large while in some cases the difference between two threshold is very small.

The result shows our proposed model gives a better explanation to the market. At the beginning, the magnitude of the stock price fluctuation from the underlying value of the securities is still within a certain small range. Then assuming that the stock price is subject to a positive disturbance, at this time, it reaches a threshold where the demand from the positive feedback trader exceeds the arbitrageurs, and if so, the positive feedback trader's purchase will push the stock price to rise rapidly in a self-reinforcing manner, thus causing a certain degree of bubble in prices.

However, after the stock price deviates from the underlying value far enough to reach another threshold, at this point, the selling by arbitrageurs will take the upper hand. In this situation, the stock may fall in a self-reinforcing manner since those positive feedback trader will join this process and speed it up, prompting the price to return to the underlying value of stocks. This can be seen as the asset price bubble burst.

Notice the above process could take place on a rather small scale like day or even hour, and it will continue to recycle and make the stock price form a dynamic equilibrium. In this dynamic equilibrium, the price of securities may be driven by the internal positive feedback mechanism, and the asset price bubble will continue to be generated. The development of the bubble may be shattered to a certain extent and return to its underlying value in a rapid collapse. It does not need to be a huge bubble, this could be the process of a trend development in a smaller scale.

In general, the above dynamic process is greatly influenced by the initial values and the equation parameters. For example, when the price deviates from the base value of the asset to a certain extent, the system may become unstable. At this point, the occurrence of accidental events is likely to change the path of the evolution of the price, such as the exposure of a political scandal, the stock being selected into the composition of an index, or even baseless rumours, could drive to the evolution of stock prices to different paths. Moreover, this evolution of the stock price is a chaos process, which means the distribution of stock price changes and presents complex structures such as fractals, showing a high degree of complexity.

Our model integrated the nonlinear and chaotic factors and empirically found success in China's stock. Next we want to explore the possible reasons causing China's stock market to present this way, in simple words, not mature. Apart from the reason for the positive feedback trading formation, which has been discussed in previous chapters already, there are some unique reasons that might explain why China's stock market is presenting less maturely compared to other developed countries.

The first possible reason is that China has a large portion of non-circulated shares. The circulated shares can be freely traded on the stock exchange, whereas non-circulated shares cannot. Non-circulated shares come from three sources: the state-owned shares converted from state-owned companies, the shares of company founders and the shares obtained by other organizations through private placements. As non-circulated shares constitute as much as two-thirds of the total shares in the market, every time converting non-circulated into circulated ones will lead to an over-supply of shares, thus forming a clear trend.

Beginning mid-2005, the administrator started the circulation reform, mostly by starting to allow non-circulating shareholders to compensate circulating shareholders with shares or other types of distributions. Also, before this reform, venture capitalists could not sell their shares even when the firm they financed went public, because their stocks are categorized as non-circulated shares. The reform essentially removed important roadblock in the venture capitalists' exit mechanism and will gradually make the stock market become more mature.

The second possible reason is that China limited access to the stock market from foreign investors for a long time. Basically, there are two ways to invest in China's stock market from overseas, starting from 2004. The first option is becoming a Qualified Foreign Institutional Investors (QFII), and there is a quota of how much you could invest. The administrator kept the approval limit in a very low level before 2011, therefore China's stock market has been dominated by local investors.

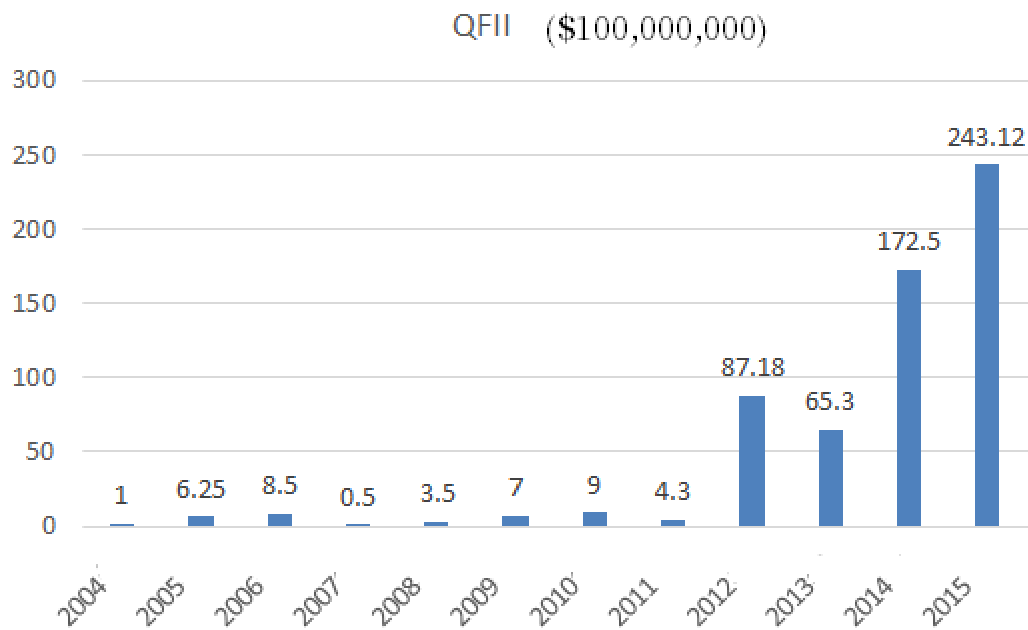

Figure 13: QFII Approval Limit(2004-2015)

The second way is becoming a RenMinBi Qualified Foreign Institutional Investors (RQFII), this is very similar to QFII, the difference is the funds raised for investment are off-shore RMB instead of American dollars. But this option was started after 2011, the same as QFII dramatically increased the approval limit to invest in China's stock market.

China now is starting the third way, called the connect program, to let the foreigner investor invest the China's stock market directly. The connect program links two exchange markets and allows investors of the two sides to trade eligible shares listed on the other's market, for example, the Shanghai-Hong

Kong Stock Connect program or Shanghai-London Stock Connect program. This is mainly aimed at attracting individual investors in the hope that the mature investor could bring some changes to the stock market. Since these connect program have only just started, we expect to see the stock market becoming more mature in the future.

The third possible reason is from investor education. Chinese traditional culture ignores individual personality and encourages people to follow the masses. But education, especially higher education, encourages students to think independently.

Figure 14: College Graduates (1990-2017)

From the figure of annual college graduates, we can see the number of graduates start to increase rapidly from 2003. This is due to the policy of expanding admission in 1999. More people receiving higher education means there are more people thinking independently instead of following others. Receiving higher education usually leads to a higher income, and higher income could create more eligible investors and who are more capable of doing sophisticated activities like arbitrage. All these things could make the stock market more mature.

13.2 Comparative Testing

This section first compare the ARIMA model, effectively AR(1) model with our models by using AIC. Also the justification of using AIC rather than other criteria will be discussed. Then we will conduct two common tests to test the significance of our model. Both the LR test and BDS test reported a very positive result.

13.2.1 Model Selection by Using AIC

The AIC is a tool that quantified the goodness of fit for the statistical model. Akaike (1974) raised the concept of the Akaike information criterion (AIC), which is an estimator of the relative quality of statistical models for a given set of data. Given a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models. Thus, the AIC provides a means for model selection.

In this section the AIC will be used to make a comparison between our model and the ARIMA model. The technical details of the autoregressive integrated moving average model (ARIMA model) were previously shown at section 9.1. In short, an ARIMA(1,0,0) model with zero degree of differencing and zero order of the moving-average is essentially a AR(1) model.

For a statistical model, let k be the number of estimated parameters in the model, and let$\hat{L}$ be the maximum value of the likelihood function for the model. Note that there is further testing in the next section using an LR test with the null hypothesis that the thresholds and threshold AR interaction coefficients were all jointly equal to zero. The results are presented in Table 14, and that test is distributed as χ_4^2.

The AIC value of the model is the following:

$$AIC = 2k - 2\ln \quad \hat{L}$$

Table 12 used Shanghai International Airport (sh600009) as an example here in order to show the comparison.

Firms	ARIMA	Res100	Res250	Res500
sh600009	-22688.07	**-32116.74**	-30637.64	-28211.50

Table 12: AIC for firm sh600009

This table has shown the AIC result for the ARIMA and our model with different window sized variation, 100, 250, and 500 respectively. The criteria for comparison is that the one with the minimum AIC is better when comparing with the other model. We have highlighted the minimum AIC result and it indicate our model gives the better fit since our result has a lower AIC result. This table is just for one firm only, the full list will be available in Table 13.

Table 13: AIC for All Firms in Samples for ARIMA(1,0,0) and H_TAR Models

Firms	ARIMA(1,0,0)	Res100	Res250	Res500
sh600000	-21366.88	-31095.91	-29628.15	-27335.04
sh600009	-22688.07	-32116.74	-30637.64	-28211.50
sh600054	-22654.45	-31327.94	-29902.67	-27617.51
sh600064	-21012.16	-30268.14	-28850.72	-26625.19
sh600067	-20525.81	-29536.74	-28155.31	-25902.38
sh600082	-20534.21	-29334.81	-27970.79	-25797.16
sh600097	-22245.01	-30672.98	-29338.83	-27055.95
sh600159	-20573.22	-29439.53	-28152.83	-25983.20
sh600162	-19735.43	-29132.01	-27772.40	-25628.97
sh600256	-19354.22	-29396.59	-28027.50	-26054.40
sh600601	-20245.65	-29483.72	-28605.48	-26372.32
sh600609	-21373.74	-30773.71	-29387.09	-27081.50
sh600611	-20999.87	-30240.90	-28819.55	-26583.57

Table 13: AIC for All Firms in Samples for ARIMA(1,0,0) and H_TAR Models

Firms	ARIMA(1,0,0)	Res100	Res250	Res500
sh600612	-21057.02	-30269.63	-28911.36	-26727.78
sh600618	-20998.12	-29747.13	-28450.58	-26229.33
sh600620	-20124.21	-29797.03	-28418.34	-26165.20
sh600621	-20022.43	-30199.74	-28905.46	-26664.89
sh600622	-20918.53	-30116.96	-28721.38	-26594.24
sh600624	-21050.11	-29810.15	-28438.98	-26220.31
sh600626	-20520.83	-29959.14	-28596.82	-26345.18
sh600628	-21875.64	-30975.84	-29566.32	-27387.32
sh600630	-23543.33	-29636.71	-28294.38	-26061.53
sh600636	-20953.99	-29778.18	-28433.37	-26182.18
sh600638	-21967.13	-30415.94	-29025.91	-26785.94
sh600639	-21053.64	-30077.72	-28712.33	-26529.64
sh600640	-19783.41	-29017.58	-27683.66	-25512.74
sh600642	-21029.27	-31324.27	-29874.80	-27553.87
sh600643	-20005.46	-29863.89	-28487.73	-26232.92
sh600644	-21788.67	-30537.35	-29117.26	-26800.61
sh600645	-21737.63	-30602.98	-29242.15	-27193.66
sh600650	-21078.99	-30432.82	-29081.39	-26897.39

Table 13: AIC for All Firms in Samples for ARIMA(1,0,0) and H_TAR Models

Firms	ARIMA(1,0,0)	Res100	Res250	Res500
sh600651	-21002.67	-29628.28	-28351.94	-26355.00
sh600658	-20496.13	-29705.73	-28342.42	-26122.23
sh600663	-21073.83	-30478.52	-29105.61	-26840.56
sh600668	-21045.65	-30193.19	-28799.01	-26588.32
sh600674	-20124.37	-29264.46	-28055.06	-25918.55
sh600683	-21057.87	-29610.61	-28260.22	-26027.48
sh600684	-20072.38	-29604.50	-28242.61	-26011.31
sh600692	-21001.38	-29744.08	-28393.52	-26185.83
sh600802	-20387.35	-29730.87	-28356.33	-26159.80
sh600812	-21015.34	-30413.31	-28994.42	-26722.60
sh600821	-20035.65	-29639.74	-28306.64	-26117.21
sh600824	-19998.12	-30301.24	-28931.55	-26678.13
sh600826	-20053.37	-29758.21	-28371.80	-26158.06
sh600830	-20357.42	-29666.22	-28282.65	-26017.11
sh600831	-20237.38	-29618.01	-28297.65	-26024.58
sh600834	-21237.53	-30831.87	-29469.17	-27235.34
sh600835	-20067.10	-30409.61	-29070.49	-26778.99
sh600846	-20758.30	-29668.39	-28281.85	-26072.65

Table 13: AIC for All Firms in Samples for ARIMA(1,0,0) and H_TAR Models

Firms	ARIMA(1,0,0)	Res100	Res250	Res500
sh600859	-21983.35	-31473.31	-30037.86	-27715.93

To sum up, all our model variants were compared directly to an ARIMA(1,0,0) by using the AIC metric for all sample firms, and our model variants were found to be superior for all cases. Note that the smaller window sized models are better quality than than bigger window sized.

There are some solid reasons for AIC being used for our model selection criteria but not others. The corrected version of the AIC (AICc) raised by Burnham and Anderson (2004) is used to deal with the over-drifting problem when the sample size is small. Assuming that the model is univariate, is linear in its parameters, and has normally-distributed residuals (conditional upon regressors), then the formula for AICc is as follows:

$$AICc = AIC + \frac{2k^2 + 2k}{n - k - 1}$$

Where n denotes the sample size and k denotes the number of parameters. Thus, AICc is essentially AIC with an extra penalty term for the number of parameters. Note that as $n \rightarrow \infty$, the extra penalty term converges to 0, and thus AICc converges to AIC. However, using the AICc is unnecessary in our case, because first, if n is many times larger than k^2, then the extra penalty term will be negligible *i.e.* the disadvantage in using AIC, instead of AICc, will be negligible. Secondly, if the candidate models have the same k and the same formula for AICc such as our models just different in window size, then AICc and AIC will give identical (relative) valuations *i.e.* there will be no disadvantage in using AIC, instead of AICc. Further asymptotically this is equivalent to cross validation.

The Bayesian information criterion (BIC) raised by Schwarz *et al.* (1978) is similar to AIC but with stronger penalty:

$$BIC = \ln(n)\, k - 2 \ln \quad \hat{L}$$

The BIC tended to select the "true model" from the set of candidate models where the AIC is not suitable, as $n \rightarrow \infty$, although the "true model" is unlikely there. Vrieze (2012) ran a simulation and found that, even though the "true model" is allowed to be in the candidate set, the AIC occasionally selects a much better model than BIC due to the fact that BIC can have a substantial risk of selecting a very bad model from the candidate set when n is finite.This reason can arise even when n is much larger than k^2. With AIC, the risk of selecting a very bad model is minimized. If the "true model" is not in the candidate set, then the most that we can hope to do is select the model that best approximates the “true model”. AIC is appropriate for finding the best approximating model.

Yang (2005) made a comparison between AIC and BIC in the context of regression. In regression, AIC is asymptotically optimal for selecting the model with the least Mean Squared Error (MSE), under the assumption that the "true model" is not in the candidate set. BIC is not asymptotically optimal under the assumption. Yang (2005) additionally shows that the rate at which AIC converges to the optimum is the best possible in a certain sense.

To sum up, the AIC is useful in finding the best approximate model with the least MSE. Using OLS with Gaussian errors (as assumed in most cases), this is also equivalent to the Colin Lingwood Mallows (Mallow’s C_p) in the case of Gaussian linear regression.

13.2.2 LR test

A likelihood ratio test (LR test) is a statistical test used for comparing the goodness of fit of two statistical models, one of which (the null model) is a special case of another more complex model (the alternative model, *i.e.* “nested”). The test is based on the likelihood ratio, which expresses how many times more likely the data are under one model than the other. The LR test can be presented as a difference in the log likelihoods, and this can be expressed in terms of deviance. Then:

$$LRT = -2\ln \quad \frac{L_0}{L_a} \tag{14}$$

$$LRT = 2\ln \quad \frac{L_a}{L_0} \tag{15}$$

$$LRT = 2\left(\ln L_a - \ln L_0\right) \sim \chi^2_{df_a - df_0} \tag{16}$$

Thus, the LRT can be computed as a difference in the deviance for the two models. This is convenient as the deviance is a value of interest in other respects.

$$LRT = deviance_0 - deviance_a \tag{17}$$

The model with more parameters (the alternative) should fit as good as, *i.e.* have the same or greater log-likelihood, than the model with fewer parameters (the null). Whether the fit is significantly better and should thus be preferred is determined by deriving the probability or p value of the difference LRT.

$$H_0 : \delta_1 = \delta_2 \equiv \delta$$

$$H_a : \delta_1 \neq \delta_2$$

Where the null hypothesis represents a special case of the alternative hypothesis, the probability distribution of the test statistic is approximately a χ^2 distribution with degrees of freedom equal to $df_a - df_0$ respectively the number of free parameters of models alternative and null, in our case the calculated degrees of freedom is 4 due to the number of restrictions.

Firm	χ^2_4	Firm	χ^2_4	Firm	χ^2_4
sh600000	**8.864361**	sh600601	**7.231459**	sh600628	14.329748
sh600009	**9.441634**	sh600609	29.748056	sh600630	10.54023
sh600054	17.904507	sh600611	19.373857	sh600636	13.764872
sh600064	10.175221	sh600612	12.184229	sh600638	19.192787
sh600067	16.991199	sh600618	19.702961	sh600639	12.690464
sh600082	12.002481	sh600620	17.306614	sh600640	20.471534
sh600097	45.667326	sh600621	15.08517	sh600642	21.627012
sh600159	27.393757	sh600622	26.390293	sh600643	29.567801
sh600162	16.289212	sh600624	11.470028	sh600644	74.48897
sh600256	117.07765	sh600626	16.138162	sh600645	22.126249
sh600650	32.027213	sh600674	34.290767	sh600812	19.989034
sh600651	15.317647	sh600683	17.05813	sh600821	13.945831
sh600658	25.508461	sh600684	18.422093	sh600824	22.432162
sh600663	15.898048	sh600692	18.087239	sh600826	24.339815
sh600668	18.036898	sh600802	18.16007	sh600830	22.293522
sh600831	16.429318	sh600834	34.010263	sh600835	23.623658
sh600846	12.204339	sh600859	16.579469	$df = 4$	

Table 14: The LR test statistic for all samples in LR test

At 5% (0.95) level with calculated degrees of freedom of 4, the critical value of χ_4^2 is 9.488. In our result, 47 out of 50 reported χ_4^2 larger than the critical value, therefore reject the null, which means our model has better fit for 94% of samples.

If we loosen the probability from 0.95 to 0.9, at 10% level, the critical value of χ_4^2 is 7.779. In our result, 49 out of 50 reported χ_4^2 value larger than the critical value therefore reject the null, which means our model has better fit for 98% of samples.

13.2.3 BDS test

Chaos theory is based on the assumption that the underlying system is a nonlinear process, and the underlying system is a deterministic system. The BDS test is a powerful tool for detecting serial dependence in time series and it was first developed by Brock *et al.* (1996).The technical details have been shown in section 7.2.2.

The BDS tests are often conducted simultaneously when calculating the correlation dimension value like we did in section 12.3. Since BDS statistics are very sensitive to any deviation from independent and identically distributed (I.I.D) for different sorts of models. If $\{x_t : t = 1, \cdots, T\}$ is a random sample of I.I.D observations, then:

$$C_n(\gamma) = C_1(\gamma)^n$$

One can estimate $C_n(\gamma)$ and $C_1(\gamma)$ by the usual sample versions $C_{n,T}(\gamma)$ and $C_{1,T}(\gamma)$. The BDS statistic $W_{n,T}(\gamma)$ has a standard normal limiting distribution and is calculated by:

$$W_{n,T}(\gamma) = \frac{\sqrt{T}\left[C_{n,T}(\gamma) - C_{1,T}(\gamma)^n\right]}{\sigma_{n,T}(\gamma)}, \quad as\ T \to \infty$$

Here $\sigma_{n,T}(\gamma)$ is an estimate of the asymptotic standard error of $\left[C_{n,T}(\gamma) - C_{1,T}(\gamma)^n\right]$. The BDS statistic shows that it should be asymptotically $N(0,1)$ as $T \to \infty$, if the residuals from the estimated model are actually IID whether it is a linear or nonlinear model. The larger the value of the BDS statistic, the stronger the evidence of nonlinearity in the data.

To sum up, BDS tests the null hypothesis of independent and identically distributed (IID) against an unspecified alternative.

$$H_0 : \{x_t : t = 1, \cdots, T\} \in i.i.d$$

$$H_a : \{x_t : t = 1, \cdots, T\} \notin i.i.d$$

The null hypothesis that is tested for is that a time series sample comes from a data generating process that is IID. A time series has nonlinearity if the null of IID has been rejected, *i.e.* implies that the time series is nonlinearly dependent if first differences of the natural logarithm have been taken.

Radius	0.005034	0.010068	0.015102	0.0201361
Dim=2	11.91	12.59	11.92	11.43
3	17.80	17.99	16.57	15.06
4	22.02	21.66	19.64	17.47
5	27.22	25.48	22.21	19.22

Table 15: BDS Test Result for sh600009

Table 15 gives the BDS test statistics from one of our samples, sh600009. The full BDS results for all sample firms are available in Appendix D.

Levels of significance (α) of 10%, 5% and 1% are taken in this hypothesis testing. The critical values are 1.645, 1.96 and 2.575 for each level of significance respectively. We can see the BDS test result for sh600009 is larger than the most strict level of significance (1%).

To sum up, the BDS test for all sample firms has shown the results are greater than the critical value by a large margin. Therefore the results strongly suggest that the time series in Chinese stock markets are nonlinearly dependent, which is one of the strong indicators of chaotic behaviour.

13.3 Comparison with Sub-period

In this section we will conduct an empirical test to make a comparison between the key sub-period to double check the performance of our model in different market conditions.

Figure 15: SSEC(1990-2015)

The selection criteria for sub-period is based on the historical trend of the SSEC index. The tranquil period we picked is 1997 to 1999, and the volatile period picked is 2007 to 2009. The result of the comparison is as follows:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.0004	0.0005	0.51	0.6102
lag(x, 1)	-0.0980	0.0405	-2.70	0.0071
T_A	-0.0003	0.0006	-0.23	0.8181
T_B	-0.0001	0.0003	-0.25	0.8026
lag(x, 1):T_A	0.3002	0.0851	3.49	0.0005
lag(x, 1):T_B	0.1412	0.0468	2.98	0.0030
F Stat	3.29002			
R Sq	0.00354			
R Sq Adj	0.00377			

Table 16: 1997-1999 Results

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.0003	0.0005	0.63	0.5288
lag(x, 1)	-0.1024	0.0397	-2.65	0.0081
T_A	-0.0002	0.0006	-0.23	0.8181
T_B	-0.0001	0.0004	-0.26	0.7949
lag(x, 1):T_A	0.2996	0.0798	3.46	0.0006
lag(x, 1):T_B	0.1408	0.0455	3.04	0.0024
F Stat	3.73106			
R Sq	0.00475			
R Sq Adj	0.00341			

Table 17: 2007-2009 Results

From the two tables for the tranquil and volatile periods, the result obtained is very similar. The value of estimates has a slight difference, but these similar ones are still statistically significant.

The result obtained here suggests that different market conditions will not affect the robustness of our model, which is expected due to the fact that our model integrated the Hurst exponent as thresholds that specialize in dealing with market condition changes.

13.4 Comparison with Holt-Winters

The Holt-Winters model uses a modified form of exponential smoothing. It applies three exponential smoothing formulae to the series. The Holt-Winters seasonal method has two different approaches, the multiplicative technique and the additive technique. The core idea presented follows, with full technical details are available in section 9.2.1.

The exponential smoothing formulae applied to a series with a trend and constant seasonal component using the Holt-Winters additive technique are:

$$\ell_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(\ell_{t-1} + b_{t-1}) \tag{18}$$

$$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1} \tag{19}$$

$$s_t = \gamma^*(y_t - \ell_t) + (1-\gamma^*)s_{t-m} \tag{20}$$

$$\hat{y}_{t+h|t} = \ell_t + hb_t + s_{t-m+h_m^+} \tag{21}$$

where:

The ℓ_t itself means an estimate of the level of the series at time t.

The b_t itself means an estimate of the trend for the time series t.

The s_t is the smoothed estimate of the appropriate seasonal component at t.

Table 18 presented the comparison result between H_TAR and Holt-Winters for the sample of firms. In this comparison, the additive approach is the only approach we ran, because the multiplicative technique cannot be used due to the data containing some negative values, which should be expected in terms of returns.

Firms	RSS(HW)	RSS(H_TAR)	Firms	RSS(HW)	RSS(H_TAR)
sh600000	0.5170263	0.483137053	sh600620	0.7511346	0.700229731
sh600009	0.3864761	0.360911955	sh600621	0.6734282	0.624122525
sh600054	0.4917362	0.4521458	sh600622	0.6989071	0.639060664
sh600064	0.6489051	0.612044425	sh600624	0.7529254	0.697609874
sh600067	0.7984525	0.754290063	sh600626	0.7034446	0.668537144
sh600082	0.8591887	0.799087897	sh600650	0.640152	0.583914369
sh600097	0.6017734	0.545190394	sh600651	0.7743442	0.734816513
sh600159	0.8243984	0.775533349	sh600658	0.7754309	0.71873602
sh600162	0.9091785	0.846755864	sh600663	0.6241408	0.57633851
sh600256	0.9333768	0.785106088	sh600668	0.6754053	0.625291894
sh600628	0.5367244	0.499998429	sh600674	0.8939446	0.815311072
sh600630	0.7870646	0.733049741	sh600683	0.7963787	0.738537364
sh600636	0.7486895	0.704010106	sh600684	0.7921019	0.739827419
sh600638	0.6267133	0.586736802	sh600692	0.7568699	0.710903843
sh600639	0.6991002	0.646265403	sh600802	0.7692829	0.713591165
sh600640	0.9449527	0.87489966	sh600812	0.614193	0.587178701
sh600642	0.4855508	0.452620177	sh600821	0.7849319	0.732415689
sh600643	0.7537666	0.686980303	sh600824	0.6549466	0.606283882
sh600644	0.6505414	0.566732777	sh600826	0.7622252	0.70803998
sh600645	0.5991913	0.556205153	sh600830	0.7801519	0.726924306
sh600601	0.817638	0.765803104	sh600831	0.7926543	0.736976461
sh600609	0.5819514	0.529724341	sh600834	0.5725191	0.520994223
sh600611	0.6688973	0.616826218	sh600835	0.6367546	0.587798512
sh600612	0.6550884	0.611784023	sh600846	0.7843323	0.726445194
sh600618	0.7677631	0.710283499	sh600859	0.4691576	0.433751286

Table 18: Holt-Winters Comparison for Sample Firms

Residual sum of squares (RSS) is a measure of the discrepancy between the data and an estimation model. It is used as an optimality criterion in parameter selection and model selection. A small RSS indicates a tight fit of the model to the data. We can see the result of the Holt-Winters additive method has a bigger RSS for all sample firm, which means the our model produces a better prediction for all sample firms. Note that there are a few of them getting very close such as for sh600009, the difference in RSS between two models is as small as 0.02. In total, the Holt-Winters method still cannot produce a better result for the samples firms than our model.

The comparison result is not surprising to us because the Holt-Winters model assumes that the seasonal pattern is relatively constant over the time period. Therefore changes in the seasonal pattern, sometimes dramatic changes, will causing a potential problem with the model, particularly if long-term predictions are made. Where our model has better strength to deal with the changes when the series break a threshold to a new regime. Note that the Holt-Winters multiplicative model is considered to deal the with inconsistent seasonal patterns better than the additive model, but we cannot apply it in our estimation due to the returns containing some negative values.

Part V

Conclusion

14 Research Summary

Traditional finance theory is based on the assumption of the "rational economic man". It is believed that the economic man's decision making is rational choice according to the expected utility theory, and the pursuit of the expected utility maximization, thus proposing the efficient market hypothesis (EMH) that the price has already reflected all current information. Behavioural finance theory denies the assumption of rational economic man, and believes that investors have limited rationale. The prospect theory is proposed based on the research results of psychology on human decision-making behaviour in the risk environment. It is believed that investors have cognitive biases such as representativeness heuristic, anchoring, mental accounts, and loss aversion in the process of making decisions in an uncertain environment. Therefore, it does not follow the expected utility theory, but follows the prospect theory.

The volatility of stock price is far more random than that predicted by traditional financial theory. It exhibits some random characteristics, but it cannot withstand strict random fluctuation tests. The distribution of price return rate is not normally distributed as predicted by the random walk model. It has the characteristics of leptokurtic and fat tails. Large and sudden fluctuations often occur, and financial asset price bubbles often appear and are eventually crushed. The stock price series shows a certain sequence correlation, but it does not seem to be persistent. The part of a series is highly like with the whole series. In short, the stock price shows a high degree of complexity, which cannot be fully explained by traditional finance theory.

Behavioural finance theory believes that due to the cognitive bias of some investors, there are irrational noise traders in the financial market. Their understanding of the market is wrong, and it is in a random status which is unpredictable, thus creating risks for asset pricing. Due to the existence of noise trading, stock prices will also generate irrational fluctuations without information. At the same time, market arbitrage mechanisms have its limitations. Therefore, in the process of rational traders playing with irrational traders, the irrational traders may survive for a long time.

The author believes that the behaviour of positive feedback trading in the financial market is significant to the evolution of securities prices. Due to irrational factors such as cognitive bias, extrapolation expectations and herding effects, there is a special irrational trading behaviour in financial markets, the positive feedback trading. This kind of trading behaviour leads to the existence of an internal positive feedback dynamic mechanism for the evolution of stock prices, which may lead to overreaction in the stock market and the emergence and destruction of asset price bubbles. In the process of mutual games between rational traders and positive feedback traders, risk preference and decision mode all determine that this feedback is a kind of nonlinear feedback. This kind of nonlinear positive feedback system may cause the chaotic process of the evolution of the stock price, resulting in a complex structure like fractals in the distribution of returns, showing a high degree of complexity. Therefore, this thesis aimed to find a model to better represent this system, and used data from China's stock market to conduct empirical research.

15 Main Contribution and Findings

Based on the basic assumptions of investor's limited rationality, this thesis studied the noise trading behaviour in financial markets, focusing on a special type of noise trading, positive feedback trading. Based on this, a dynamic positive feedback trading model is established to analyse the evolution process of securities prices and further analyse the nonlinear characteristics of the model.

This is the key contribution of this thesis, that proposed the nonlinear dynamic positive feedback model using the Hurst exponent as the signal for thresholds to indicate the changes of market condition. It is a extension of applied research about behavioural finance on the financial market. Exploring the chaotic process in the evolution of stock prices and analysing the complexity of stock price is the novel part of our research. This thesis has also conducted an empirical test that estimated the Hurst exponent and correlation dimension values on China's stock market.

The author's empirical research shows that the time series of index returns of the Shanghai market shows nonlinear characteristics, and there are lower dimensional deterministic chaotic processes, and the degree of randomness is significantly lower than that of mature capital markets. It has stronger short-term predictability and also shows a lower degree of market efficiency. The empirical research results are consistent with behavioural finance theory, thus providing empirical evidence for behavioural finance theory.

The result of the Hurst exponent estimation to the SSEC index is 0.82, which not only shows the non-randomness and persistence effect, but is also relatively stronger than the mature market reviewed. The Hurst exponent estimation to all individual firms is conducted to see their distribution. It shows that their mean is around 0.59, which means the individual stock looks closer to random than the market as a whole.

Then the individual firms were sampled into a smaller portion to meet a criteria of feasible comparison in order to do further empirical tests. With a rather longer term and stable trading history and no gaps, the Hurst exponent of all sample firms was found to be greater than 0.5. Also the result suggests that the window size will affect the range of the estimation result;

the smaller the window size, the smaller the estimation range *i.e.* higher degree of precision.

The calculation of the correlation dimension value for the SSEC Index found that the result is approximately 4.4, which is lower than the mature market, indicating the market index is to farther away to randomness than the mature market. But the result is relatively larger than studies 10 to 15 years ago, which shows that the Chinese market has better efficiency now. The calculation of correlation dimension value for the sample firms shows that the majority are above the result of the index. This result is coherent with the Hurst exponent estimation before, and it implies that the market tends to be more predictable than a individual stocks, which meets with the general observation. It is easier to forecast the general market trend in a month than to pick a few individual stocks and predict their price.

The result of the model evaluation suggests the new model offers a better explanation for the complexity of the stock market which presenting chaos. By using the AIC as the model selection criteria and comparing with ARIMA, the new model is found to be superior, and the small window size variant is the best. The LR test shows that 94% of the sample firms' results are statistically significant at 5% level. The BDS test for all sample firms has shown the results are greater than the critical value by a large margin. Therefore the results strongly suggest that the time series in Chinese stock markets are nonlinearly dependent, which is one of the strong indications of the chaotic behaviour. Finally, the result shows our model has a small RSS for all sample firms which means it produces a better prediction than the Holt-Winters additive method.

16 Policy and Investment Implications

According to traditional financial economics, the market is efficient, and the stock price reflects its intrinsic value. Therefore, the capital market can achieve the rational allocation of resources. However, the author believes that due to various cognitive factors of investors, the capital market cannot reach an efficient state. The following aspects can be instructive for investment practice and financial supervision policies.

Stock prices are not always based on their underlying value, but may be the result of a combination of investor perceptions, feelings, and social factors. Therefore, these factors should be considered when analysing changes in securities prices.

There a lot of noise trading in the financial market, which leads to limitations in the market arbitrage mechanism. In investment practice, the risks caused by noise trading should be considered when conducting arbitrages.

The positive feedback mechanism caused by the irrational factors of the financial market may form an asset price bubble, and the bubble will eventually collapse. From the perspective of investment practice, investors should be aware of the existence of irrational market factors, maintain rationality when the market generates a high level of bubbles, and overcome irrational overreaction behaviour when investing in specific asset classes. From the perspective of financial supervision, because the irrational market factors may lead to asset price bubbles, the irrational prosperity of the capital market may eventually have a serious impact on the real economy.

Therefore, financial market regulators should make effective financial regulatory policies, vigorously develop institutional investors, and improve the structure of capital market investors. If necessary, they should adopt macro-control measures to curb excessive speculation in the market, improve the efficiency of the capital market, maintain the stable development of the capital market, and prevent irrational prosperity. This is crucial to the function of the capital market to allocate resources.

nonlinear positive feedback in financial markets may lead to complex motion processes like chaos in stock prices. Changes in initial conditions caused by external accidental factors may lead to very different paths for the evolution of securities prices. Therefore, in theory, it has certain predictability for the short-term evolution of securities prices, but long-term prediction of the evolution of securities prices is extremely difficult, if not impossible.

The implications to investment practice are, first, the predictability of short-term behaviour of stock prices provides a profitable space for short-term transactions, but it is necessary to prevent the risk of significant reverse changes in the price.

Secondly, investors should strengthen risk management and diversify their investments to avoid huge losses due to large fluctuations in individual securities.

Finally, long-term predictions of the evolution of market prices are difficult. Therefore, from the perspective of long-term investment, the effect of market timing strategy is limited. A reasonable investor strategy is to focus on strategic asset allocation, while focusing on selection based on the underlying value of stocks.

17 Further Questions

Based on the limitations of the thesis, this section will point out a possible expansion plan that could be used for further research.

The first expansion would be extending the model to a number of different Hurst exponents for the thresholds. Second, the research target could be another emerging market, or even another type of asset to make the comparison. Third, the Chinese market has a relatively short history, so our model might perform differently with a longer data history.

Fourth, the time span is another important parameter, this work has used the daily one which is easy to make comparisons with, but as the "high frequency trading" is trending, shorterening the time span to seconds might reveal a different structure. Fifth, Auto Regressive Fractionally Integrated Moving Average (ARFIMA) model could be used in future research. Sixth, conducting the work in a in-sample estimation and evaluating in an out-of-sample estimation is a possible expansion.

Last but not least, a Locally Weighted Regression (LWR) could be conducted after the BDS test, to further rule out the nonlinear stochastic process which could also cause the rejection of IID as well. There are lots of test options and its further extension could be implemented to justify the statistical significance of the estimation.

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Appendices

- A Company code reference
- B Histograms of Hurst exponent estimation
- C The Chart of return for each sample firms.
- D BDS Test
- E Main Result of H_Tar model estimation

A Appendix A

There are 50 sample firms data has been proceed in the model, those 50 firms represent different industry, the table below gives the name of the firm related to the their stock code.

Table 19: Company Code Reference

No.	Code	Name
1	sh600000	Shanghai Pudong Development Bank
2	sh600009	Shanghai International Airport
3	sh600054	Huangshan Tourism Development
4	sh600064	Nanjing Gaoke
5	sh600067	Citychamp Dartong
6	sh600082	Tianjin Hi-Tech Development
7	sh600097	Shanghai Kaichuang Marine
8	sh600159	Beijing Dalong Weiye Real Estate
9	sh600162	Shenzhen Heungkong Holding
10	sh600256	Guanghui Energy
11	sh600601	Founder Technology
12	sh600609	Shenyang Jinbei Automotive
13	sh600611	Dazhong Transportation (Group)
14	sh600612	Lao Feng Xiang
15	sh600618	Shanghai Chlor-Alkali Chemical
16	sh600620	Shanghai Tianchen
17	sh600621	Shanghai Chinafortune
18	sh600622	Shanghai Jiabao Industry & Commerce
19	sh600624	Shanghai Fudan Forward S&T

Table 19: Company Code Reference

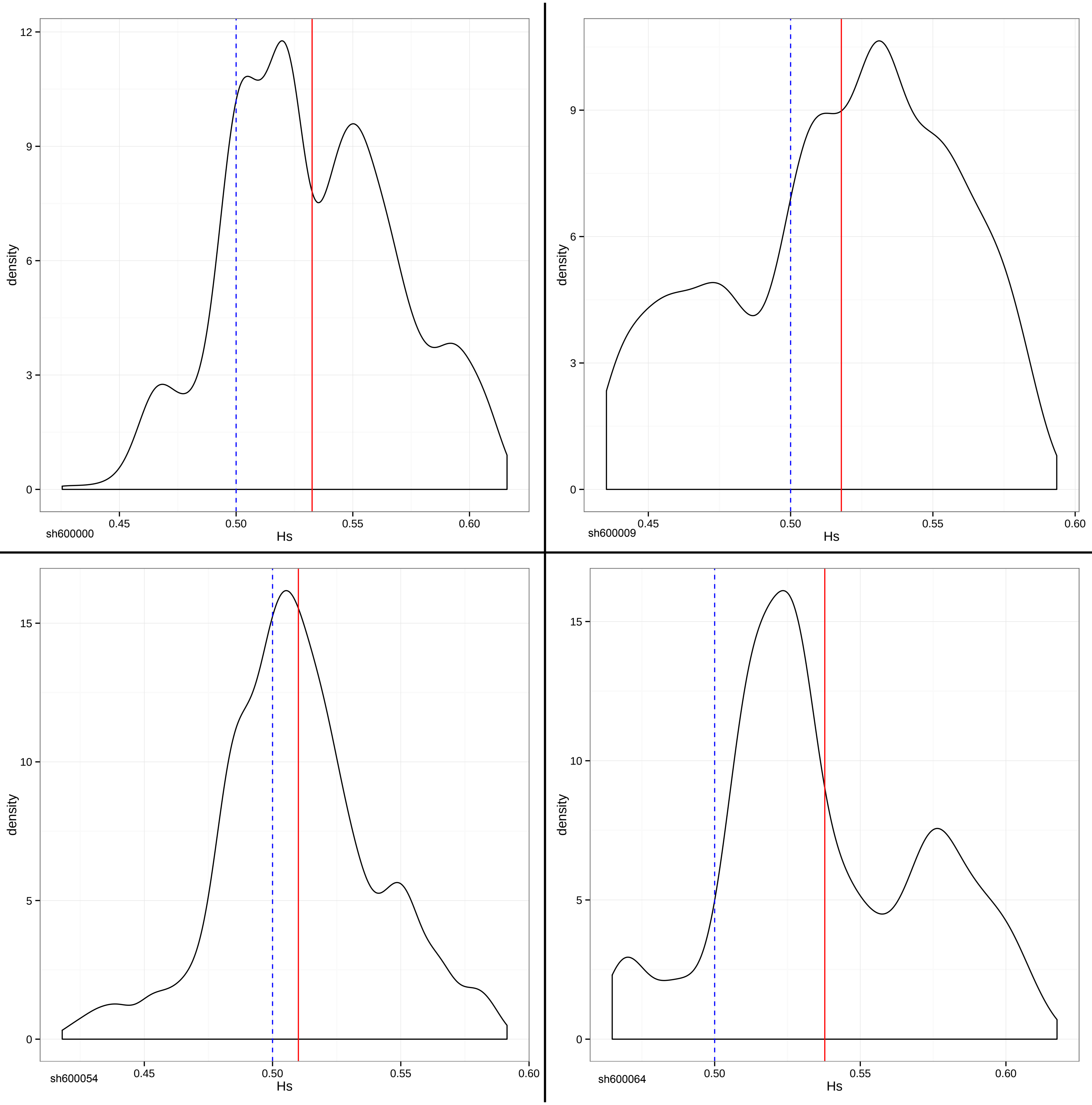
No.	Code	Name
20	sh600626	Shanghai Shenda
21	sh600628	Shanghai New World
22	sh600630	Shanghai Dragon
23	sh600636	Shanghai 3F New Materials
24	sh600638	Shanghai New Huang Pu Real Estate
25	sh600639	Shanghai Jinqiao Export Processing Zone
26	sh600640	Besttone Holding
27	sh600642	Shenergy
28	sh600643	Shanghai AJ
29	sh600644	Leshan Electric Power
30	sh600645	Zhongyuan Union Cell & Gene Engineering
31	sh600650	Shanghai Jin Jiang International Industrial Investment
32	sh600651	Shanghai Feilo Acoustics
33	sh600658	Beijing Electronic Zone Investment and Development
34	sh600663	Shanghai Lujiazui Finance & Trade Zone
35	sh600668	Zhejiang Jianfeng Group
36	sh600674	Sichuan Chuantou Energy Stock
37	sh600683	Metro Land
38	sh600684	Guangzhou Pearl River Industrial Development

Table 19: Company Code Reference

No.	Code	Name
39	sh600692	Shanghai Ya Tong
40	sh600802	Fujian Cement
41	sh600812	North China Pharmaceutical
42	sh600821	Tianjin Quanyechang (Group)
43	sh600824	Shanghai Yimin Commercial Group
44	sh600826	Shanghai Lansheng
45	sh600830	Sunny Loan Top
46	sh600831	Shaanxi Broadcast & TV network intermediary (Group)
47	sh600834	Shanghai Shentong Metro
48	sh600835	Shanghai Mechanical & Electrical Industry
49	sh600846	Shanghai Tongji Science & Technology
50	sh600859	Beijing Wangfujing Department Store

B Appendix B

The Histograms of hurst estimation for each sample firms.



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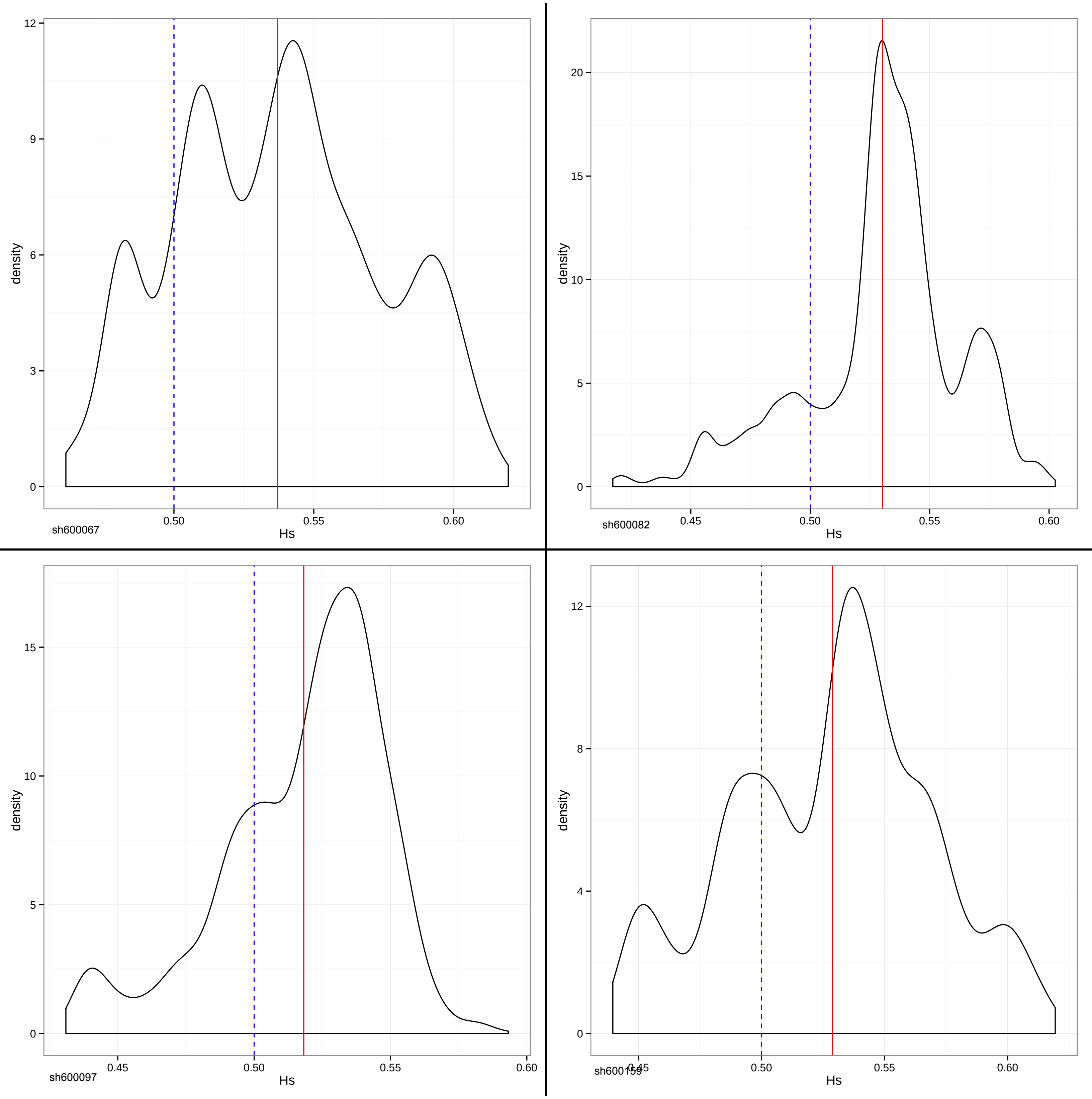
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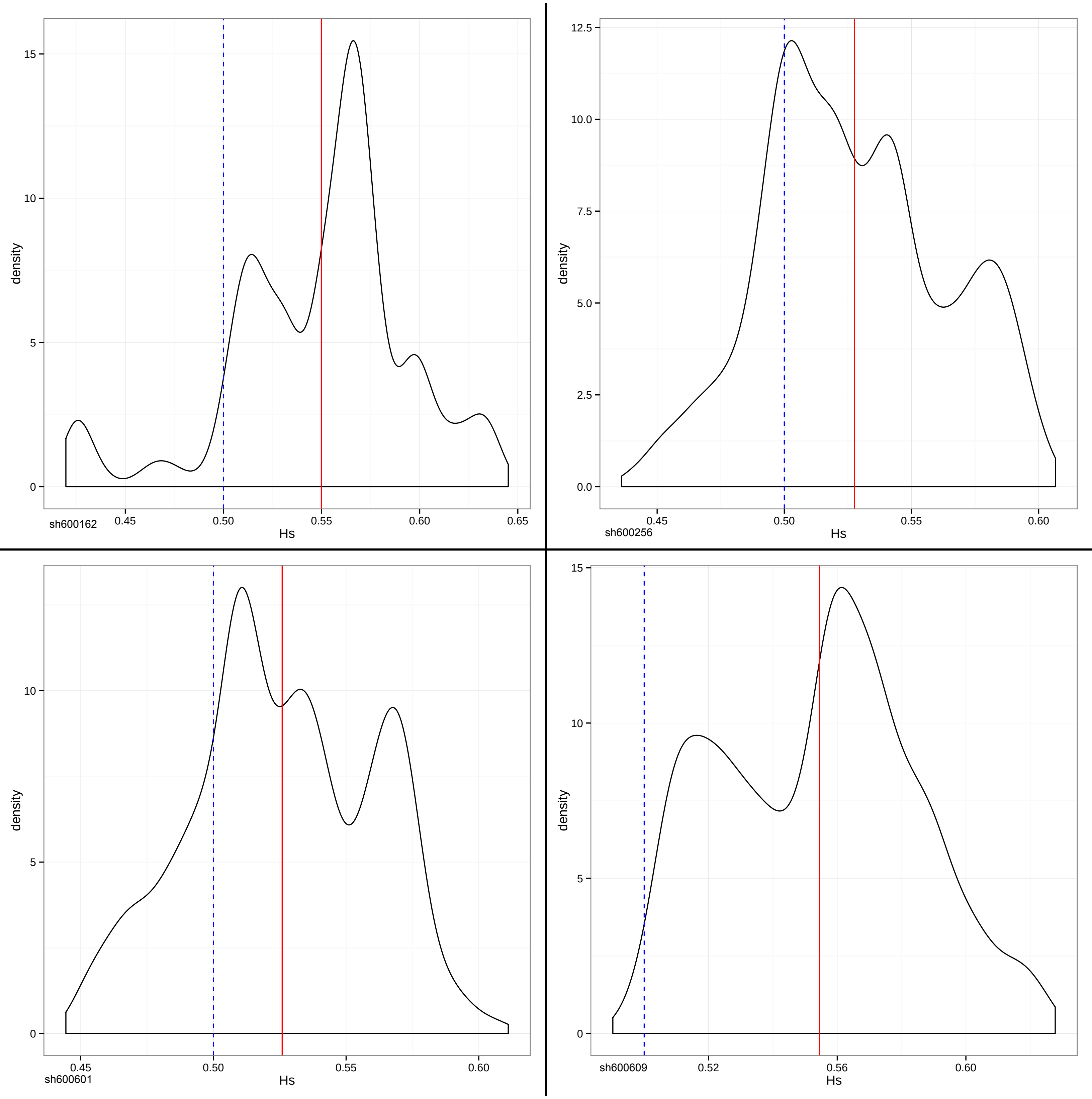

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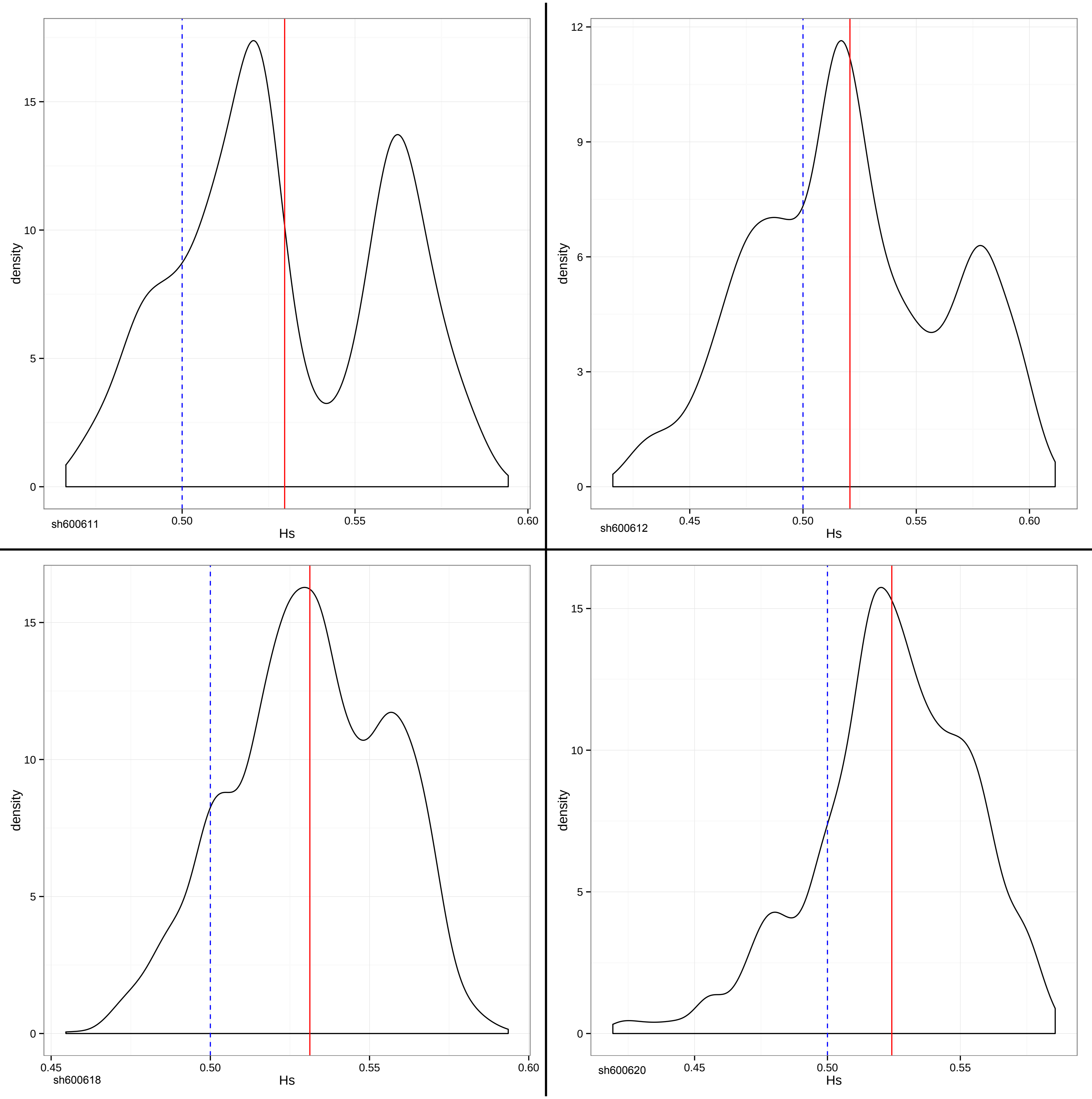
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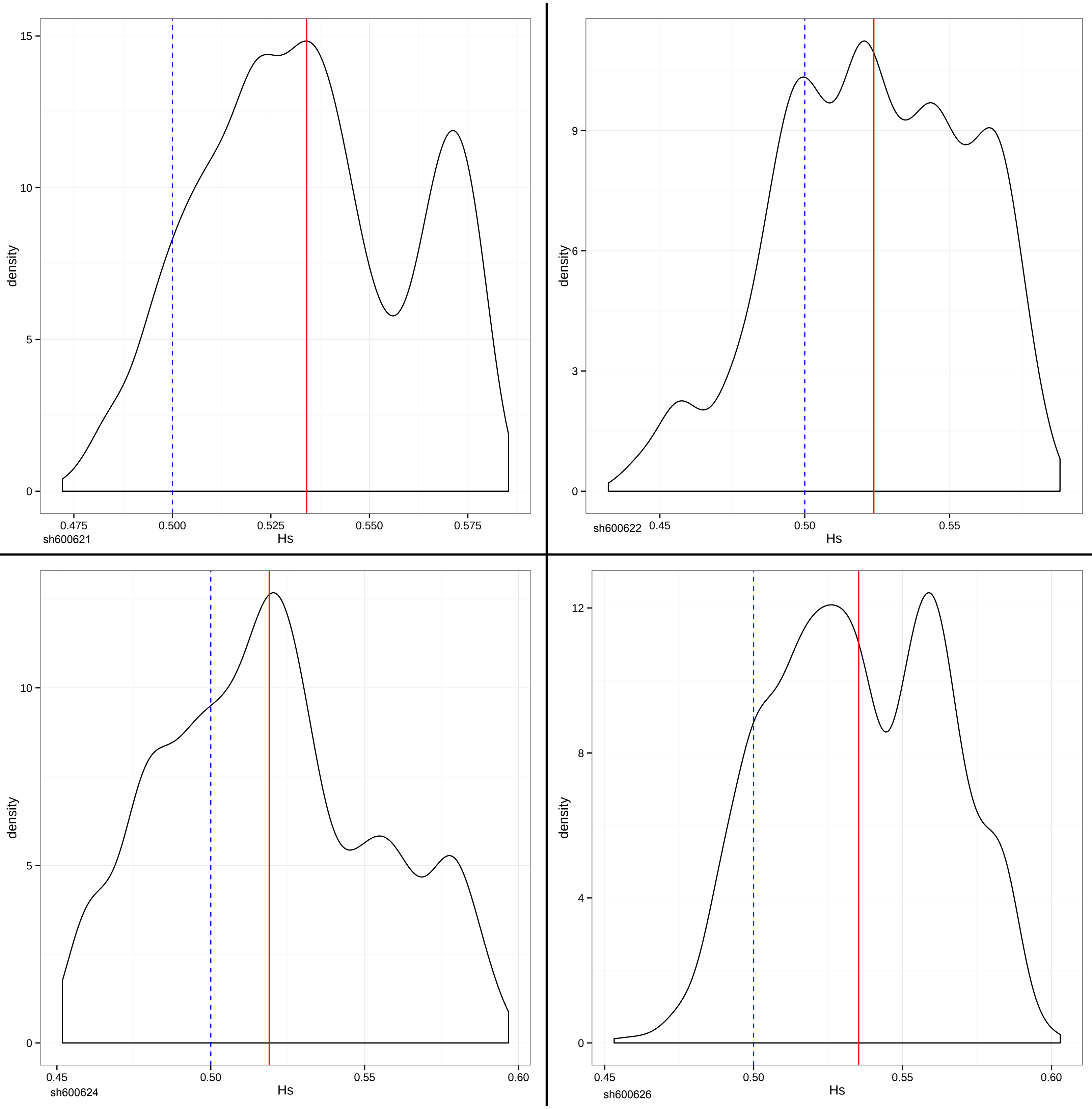
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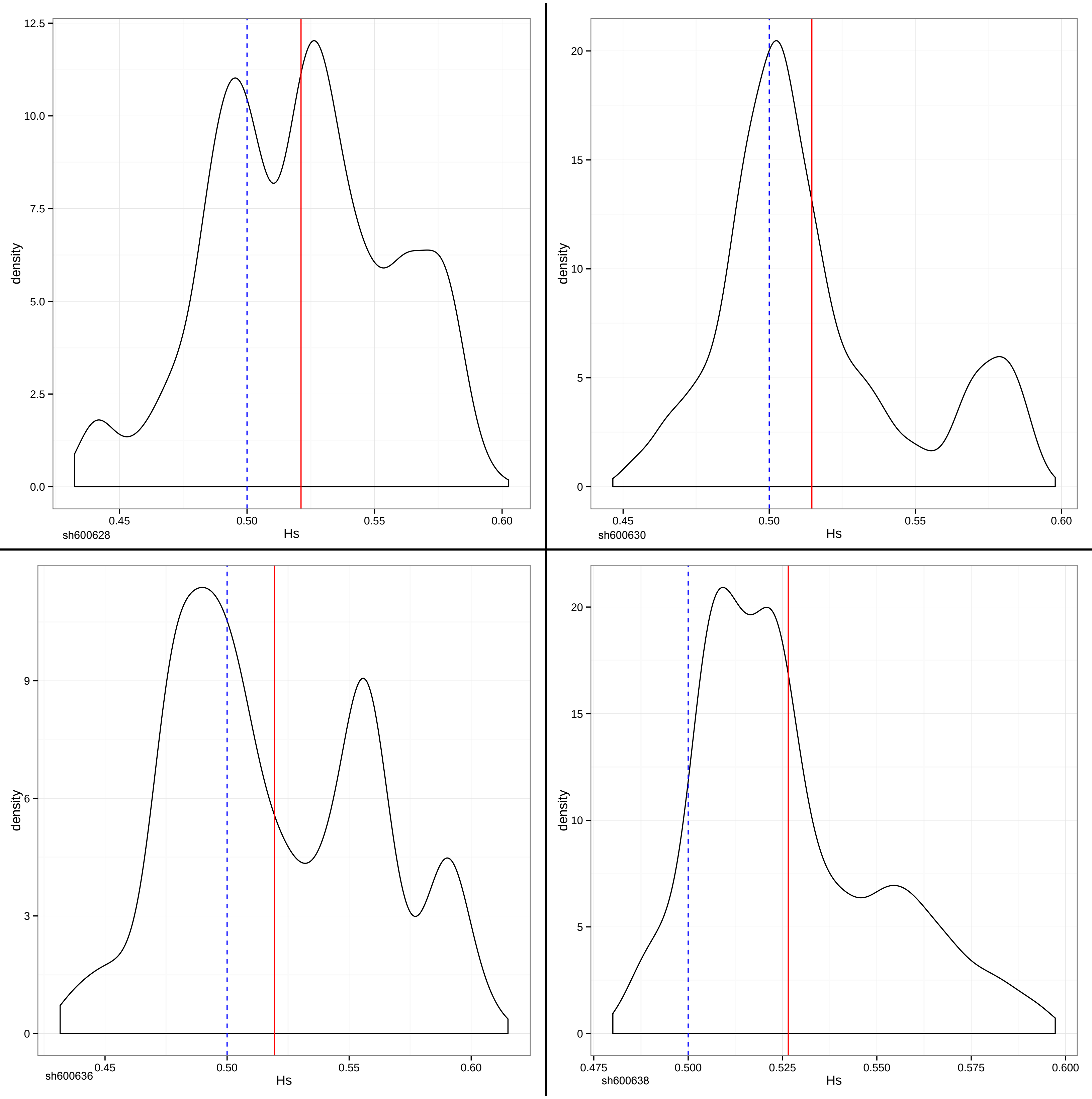

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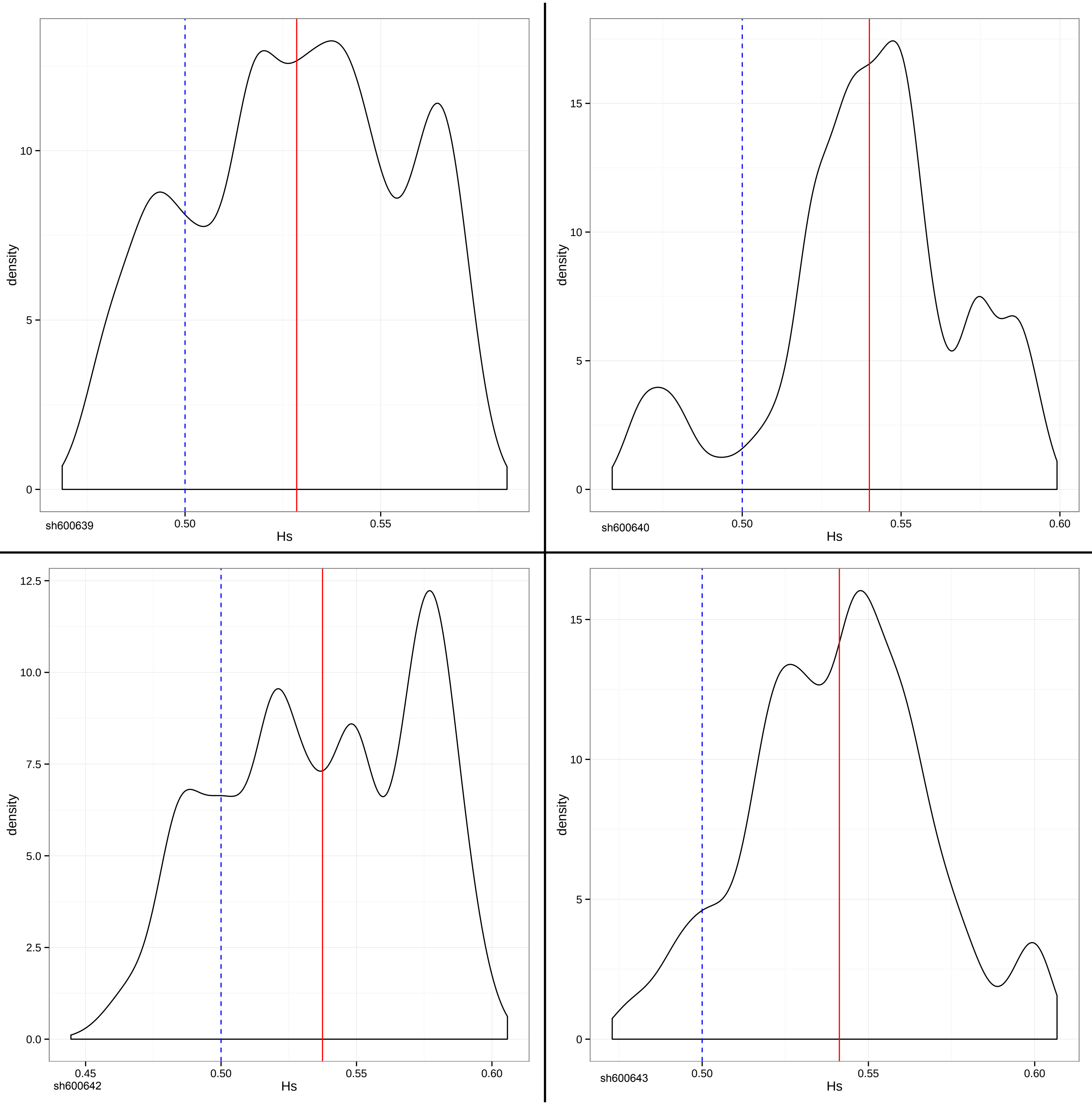
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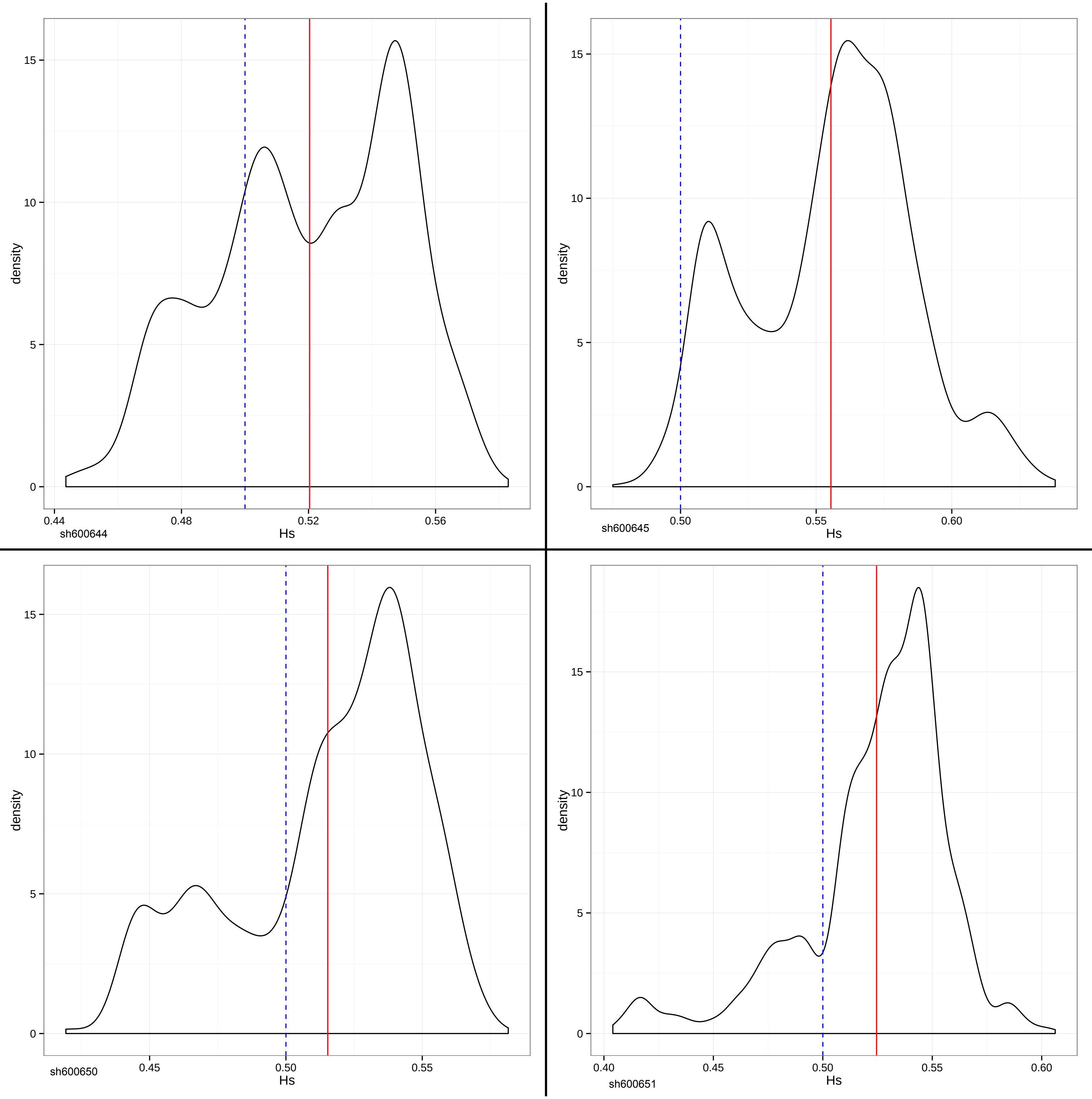
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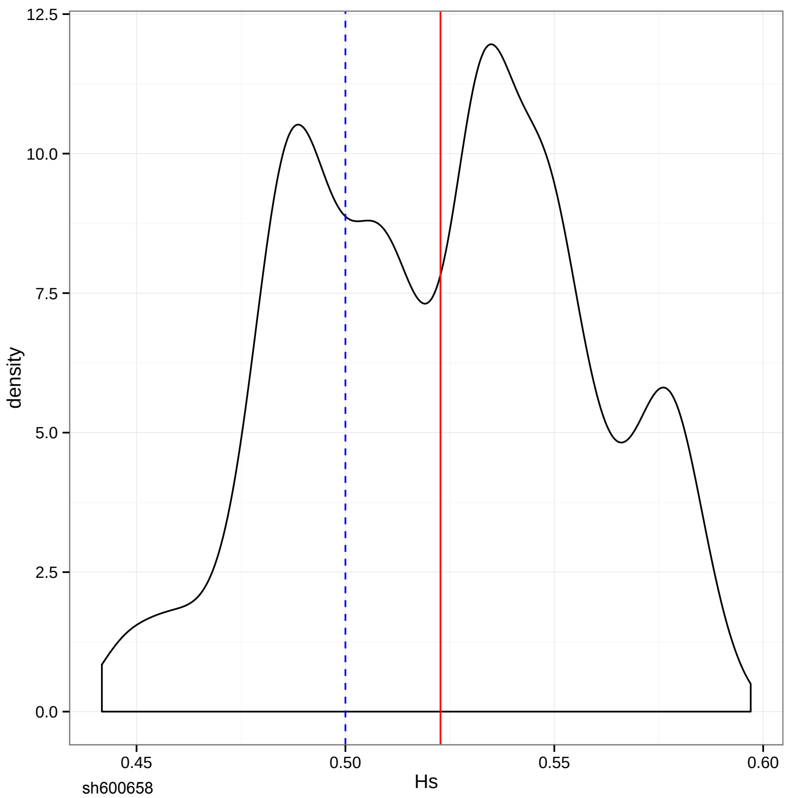
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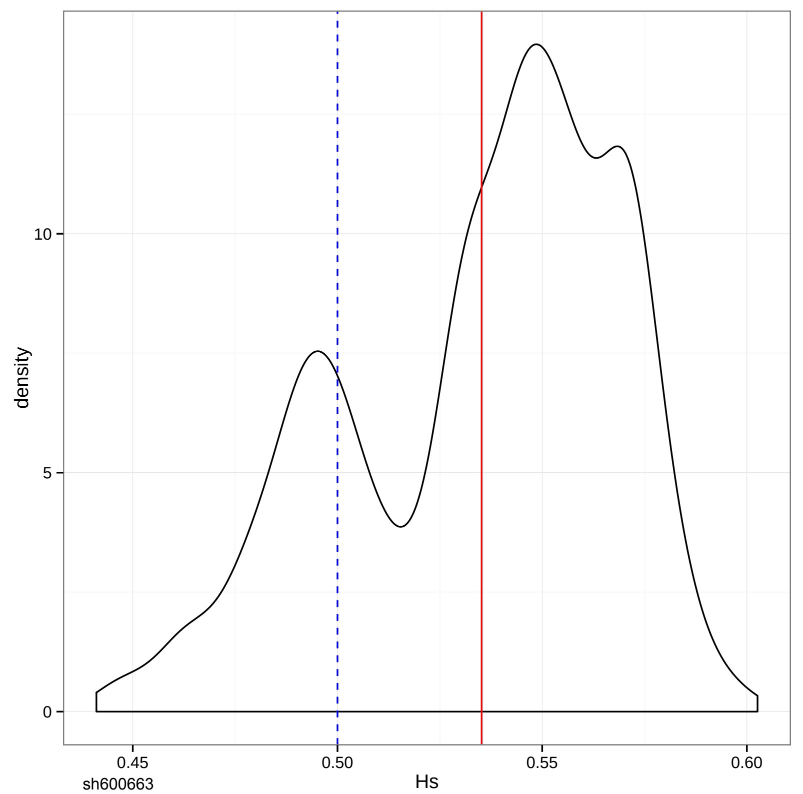
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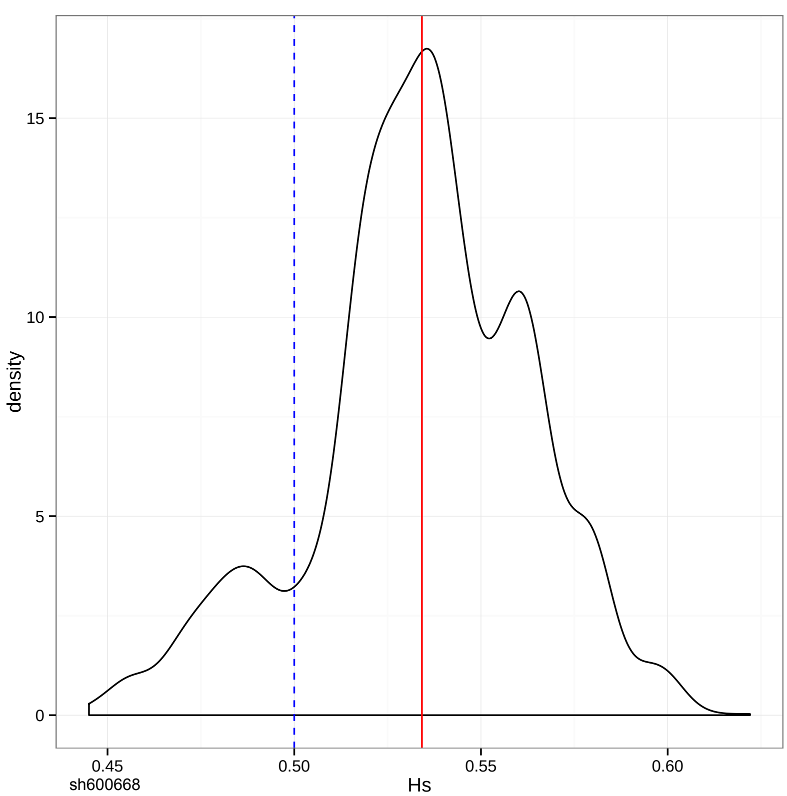
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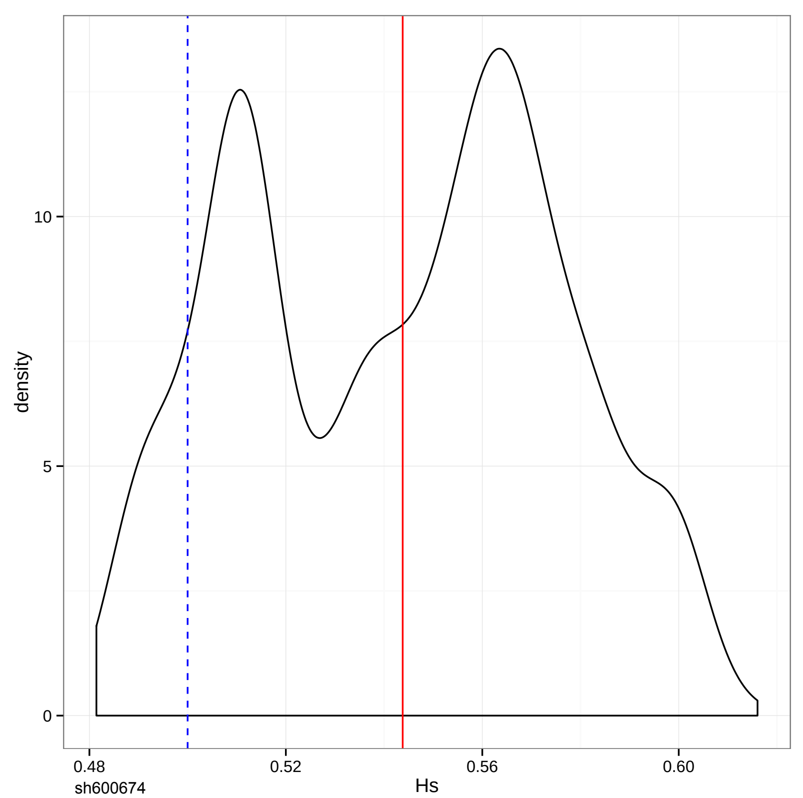
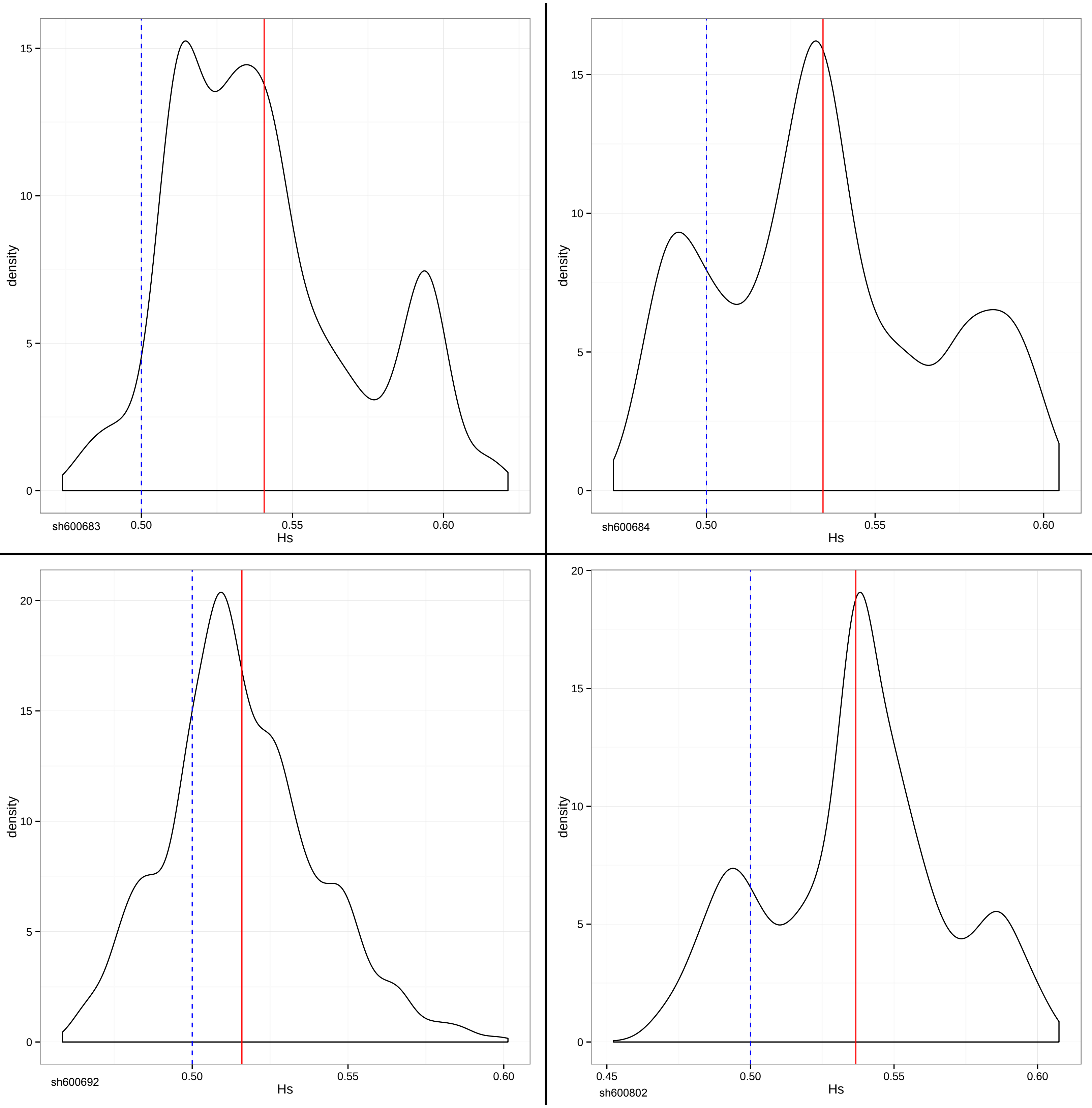
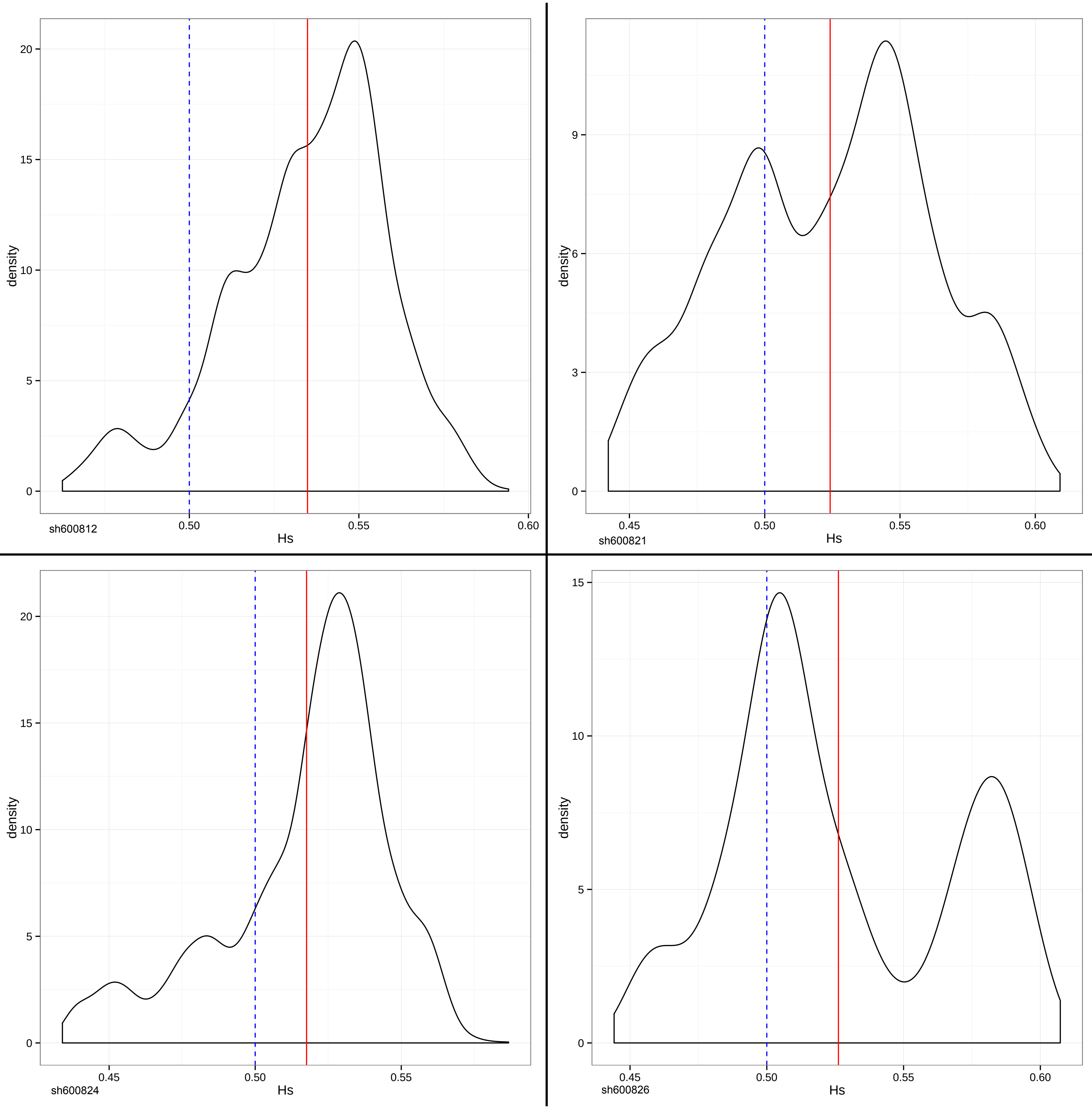
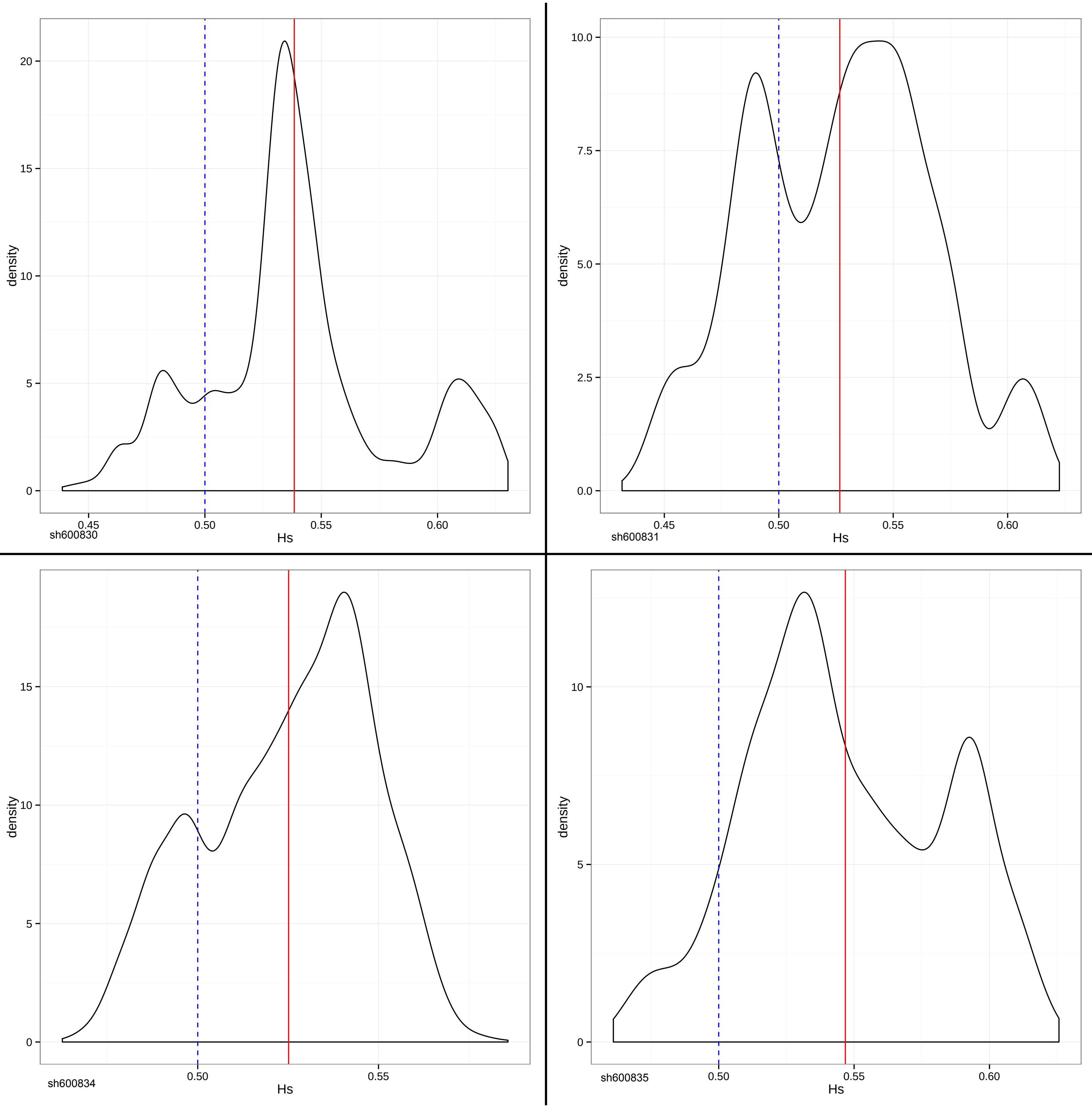
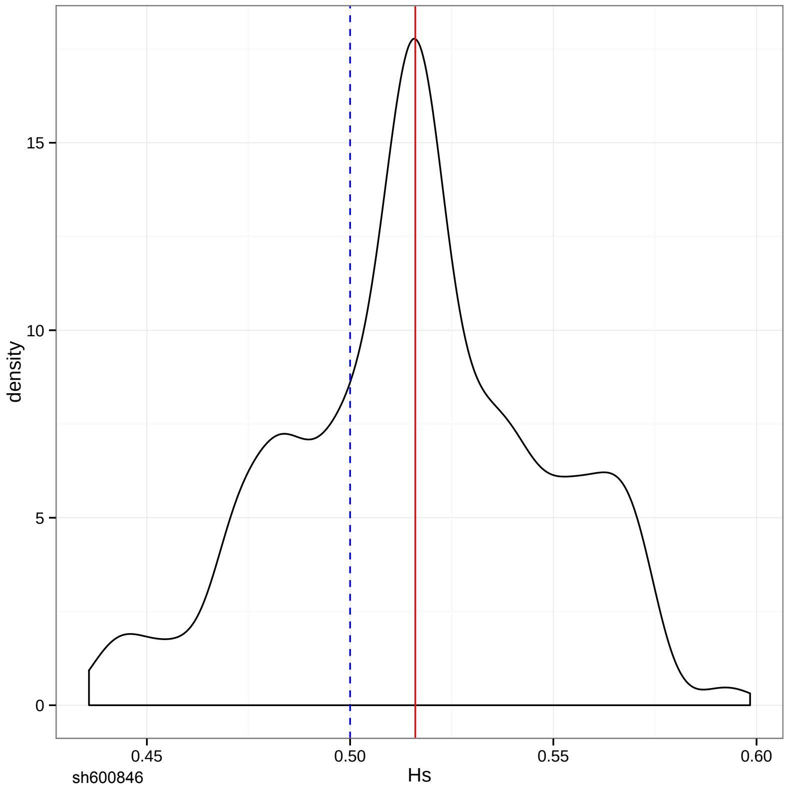

10
5
0
density
0.48
0.52
0.56
0.60
Hs
sh600674



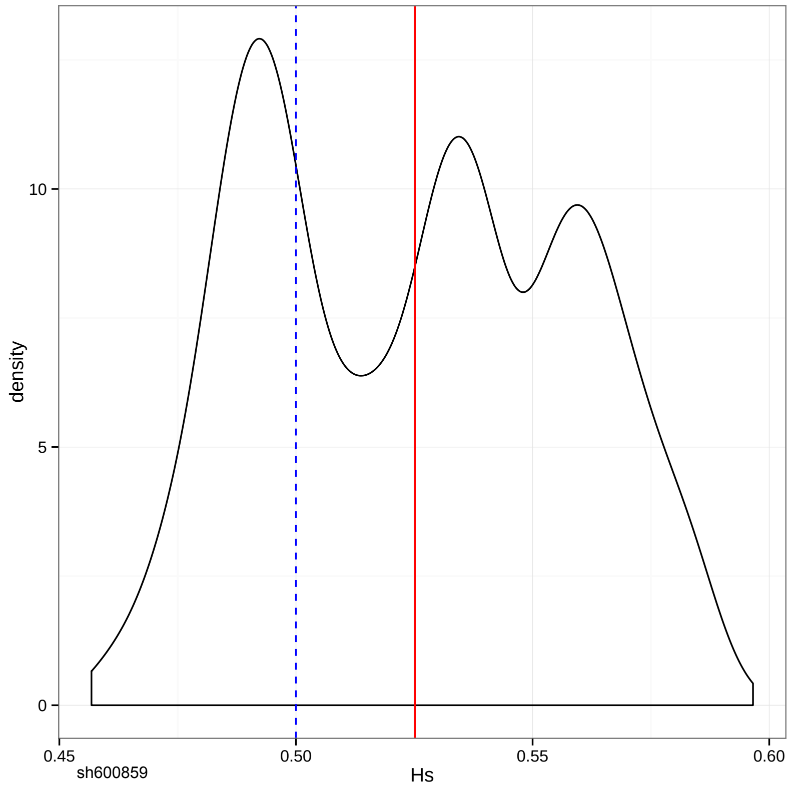
density
Hs
sh600683
sh600684
sh600692
sh600802
0
5
10
15
20
0.45
0.50
0.55
0.60


density
Hs
sh600812
sh600821
sh600824
sh600826


density
Hs
sh600830
sh600831
sh600834
sh600835
0
5
10
15
20
0.0
2.5
5.0
7.5
10.0
0.45
0.50
0.55
0.60



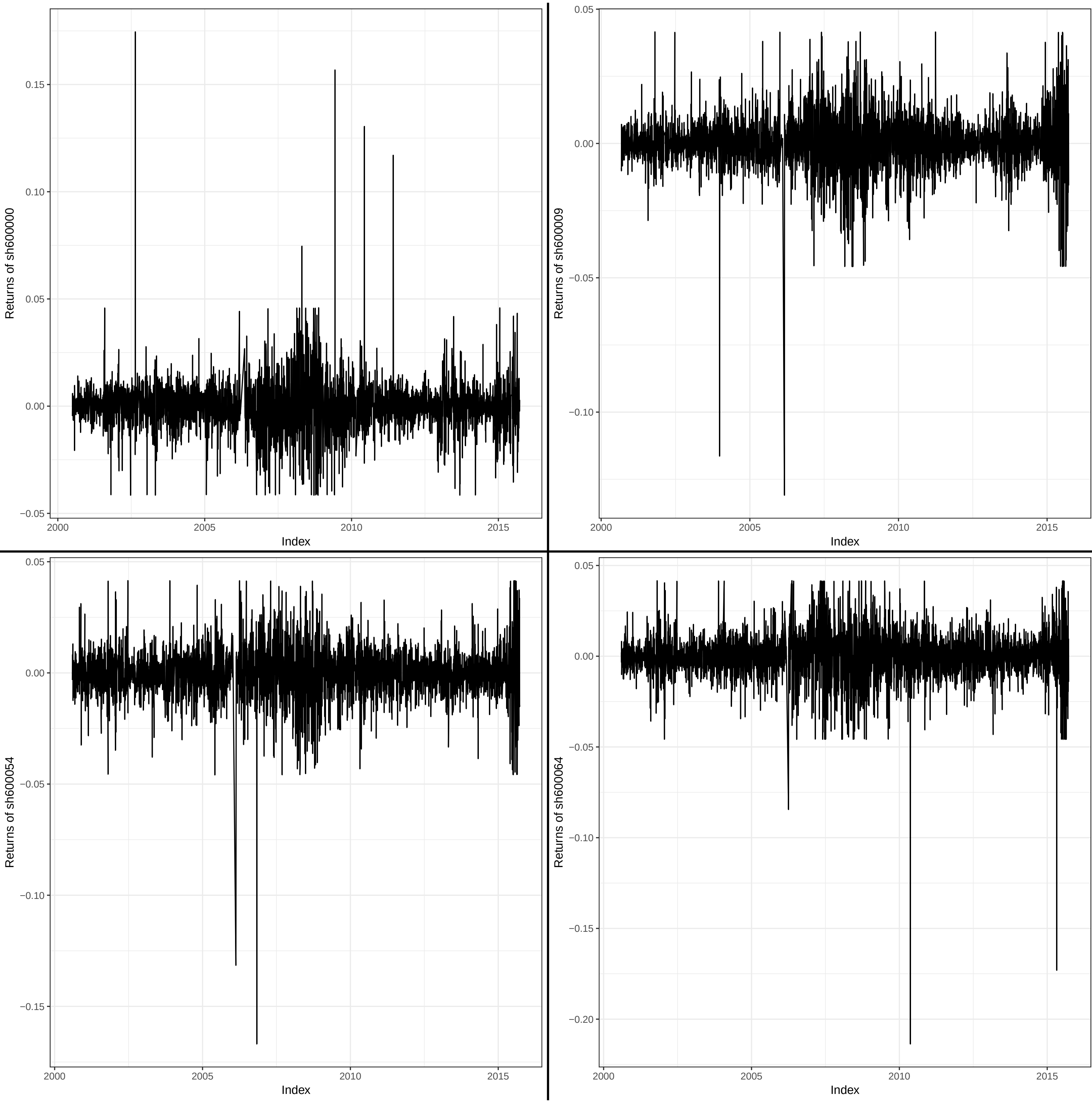
density
15
10
5
0
0.45
0.50
0.55
0.60
Hs
sh600846



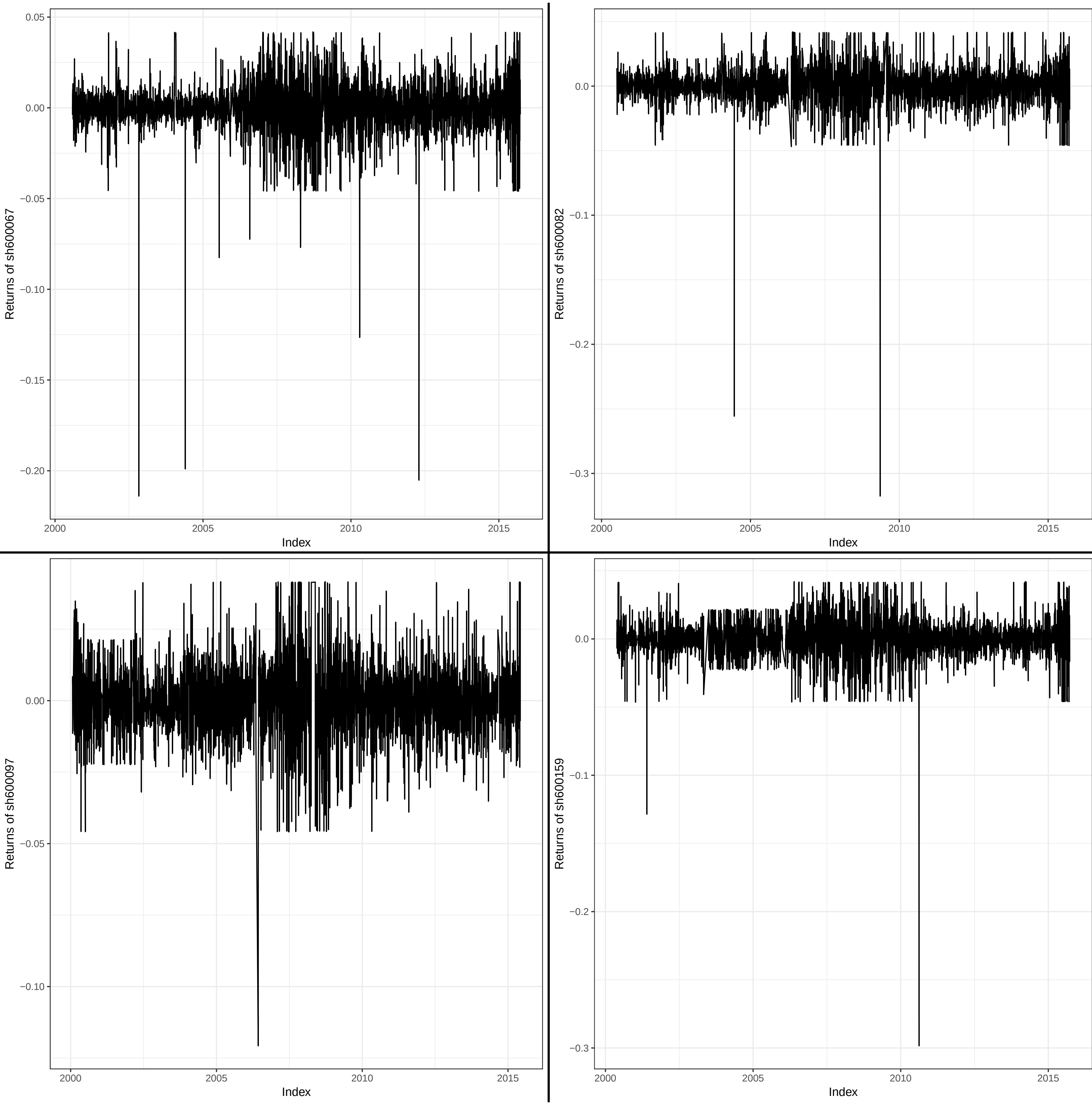
density
10
5
0
0.45
0.50
0.55
0.60
Hs
sh600859

C Appendix C

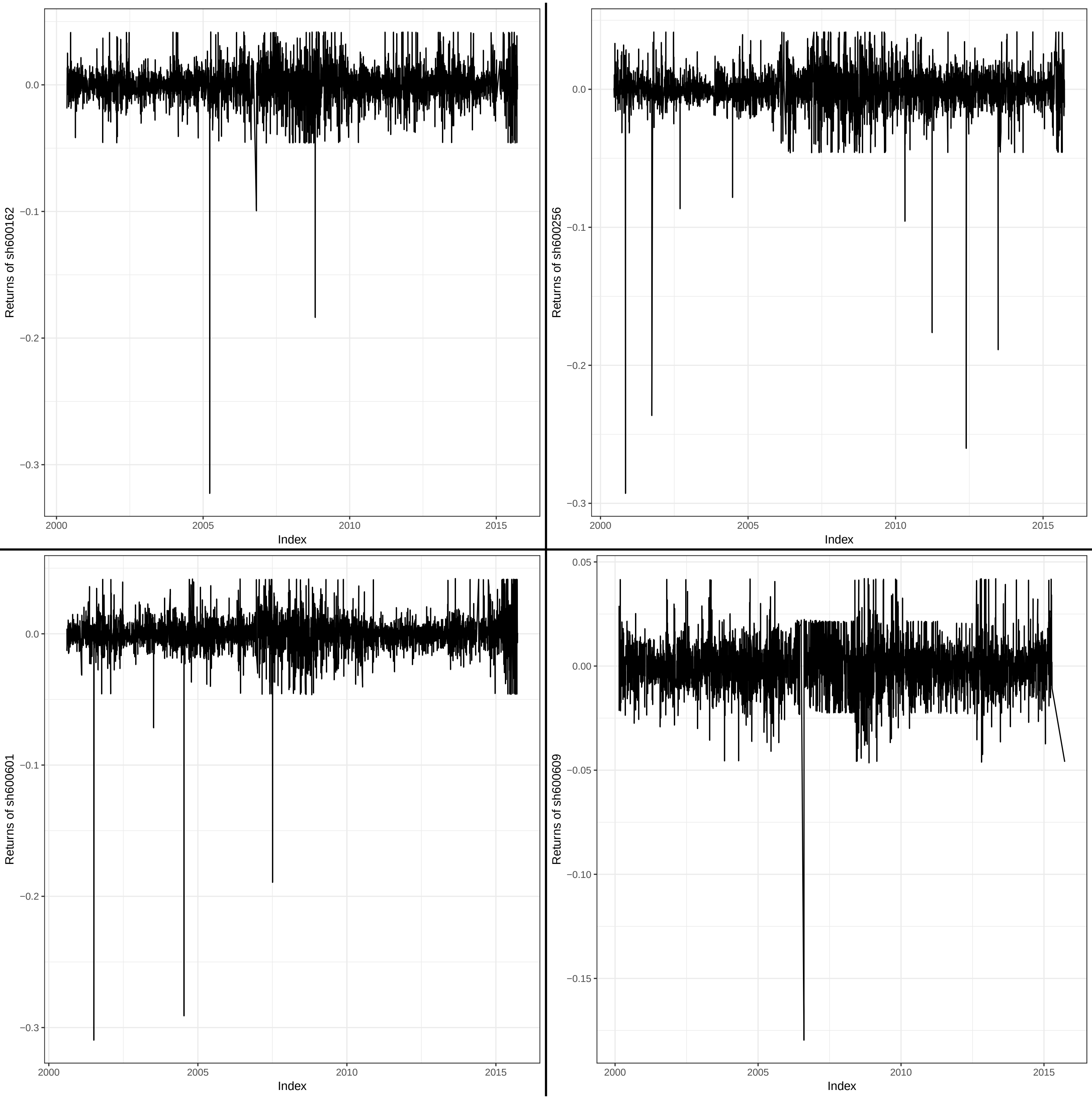
The Chart of return for each sample firms.

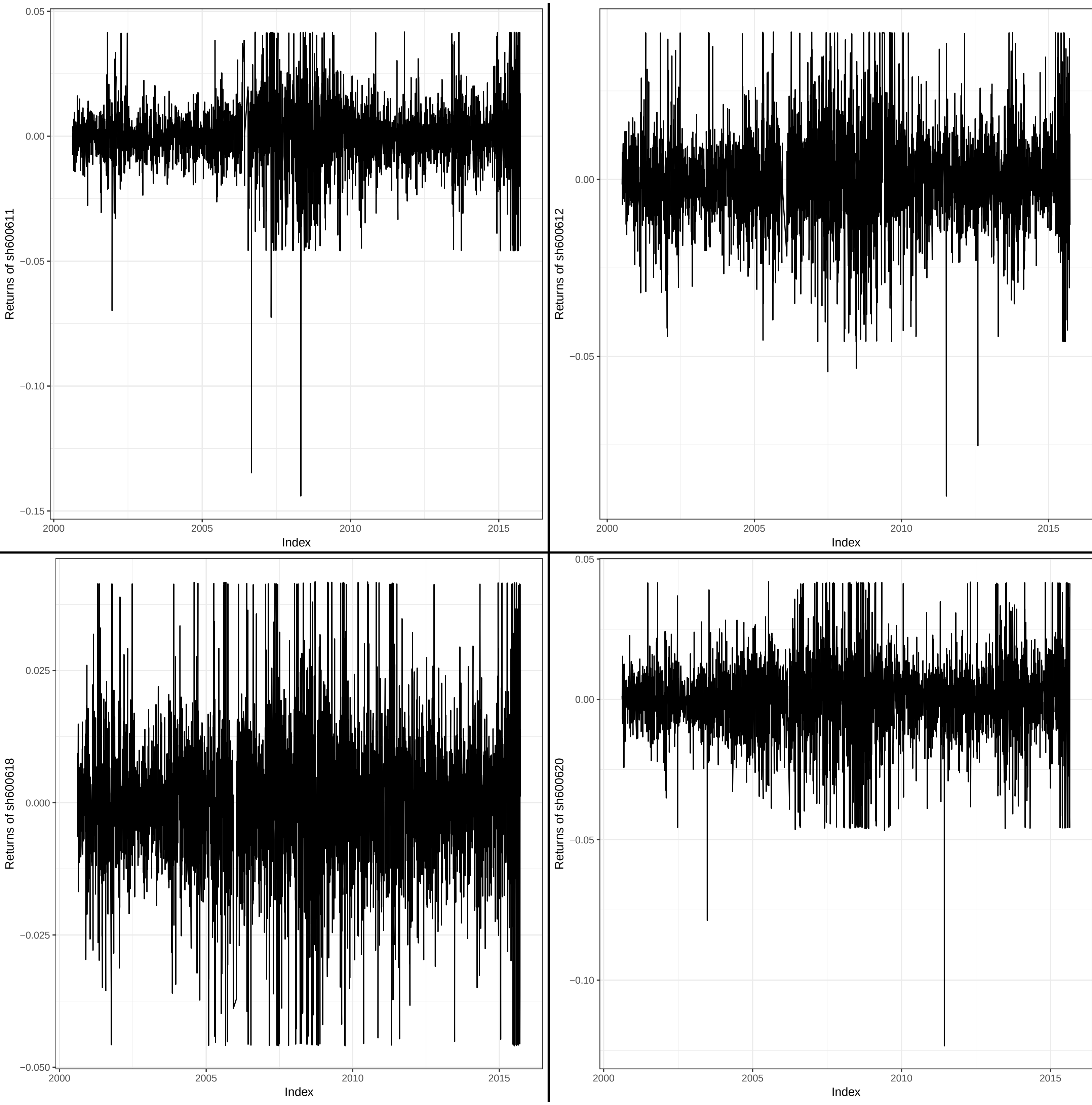


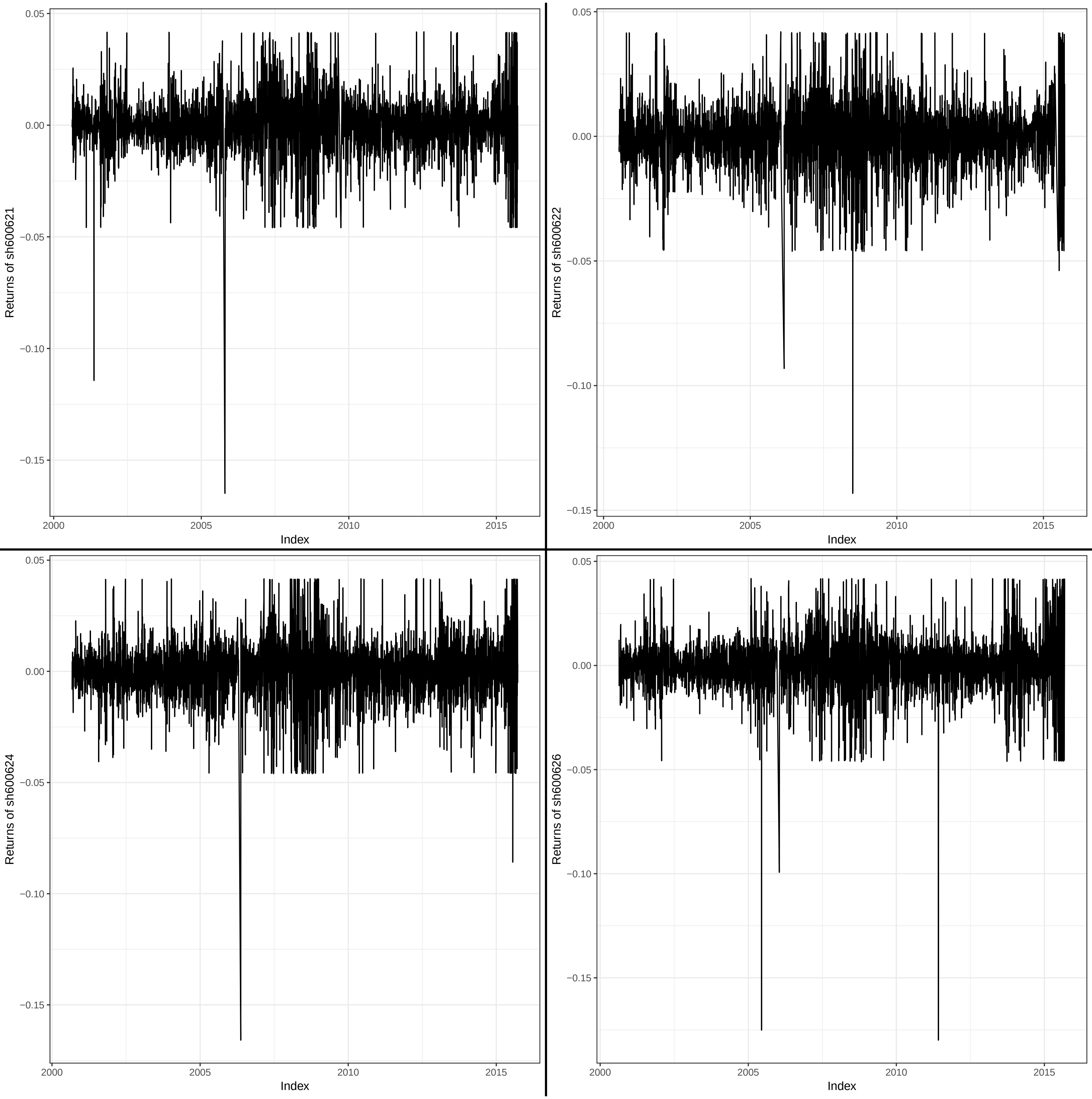
Returns of sh600000
Returns of sh600009
Returns of sh600054
Returns of sh600064
Index
Index
Index
Index
2000
2005
2010
2015



Returns of sh600067
0.05
0.00
−0.05
−0.10
−0.15
−0.20
2000
2005
2010
2015
Index
Returns of sh600082
0.0
−0.1
−0.2
−0.3
2000
2005
2010
2015
Index
Returns of sh600097
0.00
−0.05
−0.10
2000
2005
2010
2015
Index
Returns of sh600159
0.0
−0.1
−0.2
−0.3
2000
2005
2010
2015
Index


Returns of sh600162
0.0
−0.1
−0.2
−0.3
2000
2005
2010
2015
Index
Returns of sh600256
0.0
−0.1
−0.2
−0.3
2000
2005
2010
2015
Index
Returns of sh600601
0.0
−0.1
−0.2
−0.3
2000
2005
2010
2015
Index
Returns of sh600609
0.05
0.00
−0.05
−0.10
−0.15
2000
2005
2010
2015
Index


Returns of sh600611
Returns of sh600612
Returns of sh600618
Returns of sh600620
Index
0.05
0.00
−0.05
−0.10
−0.15
−0.050
−0.025
0.000
0.025
2000
2005
2010
2015


Returns of sh600621
Returns of sh600622
Returns of sh600624
Returns of sh600626
Index
0.05
0.00
−0.05
−0.10
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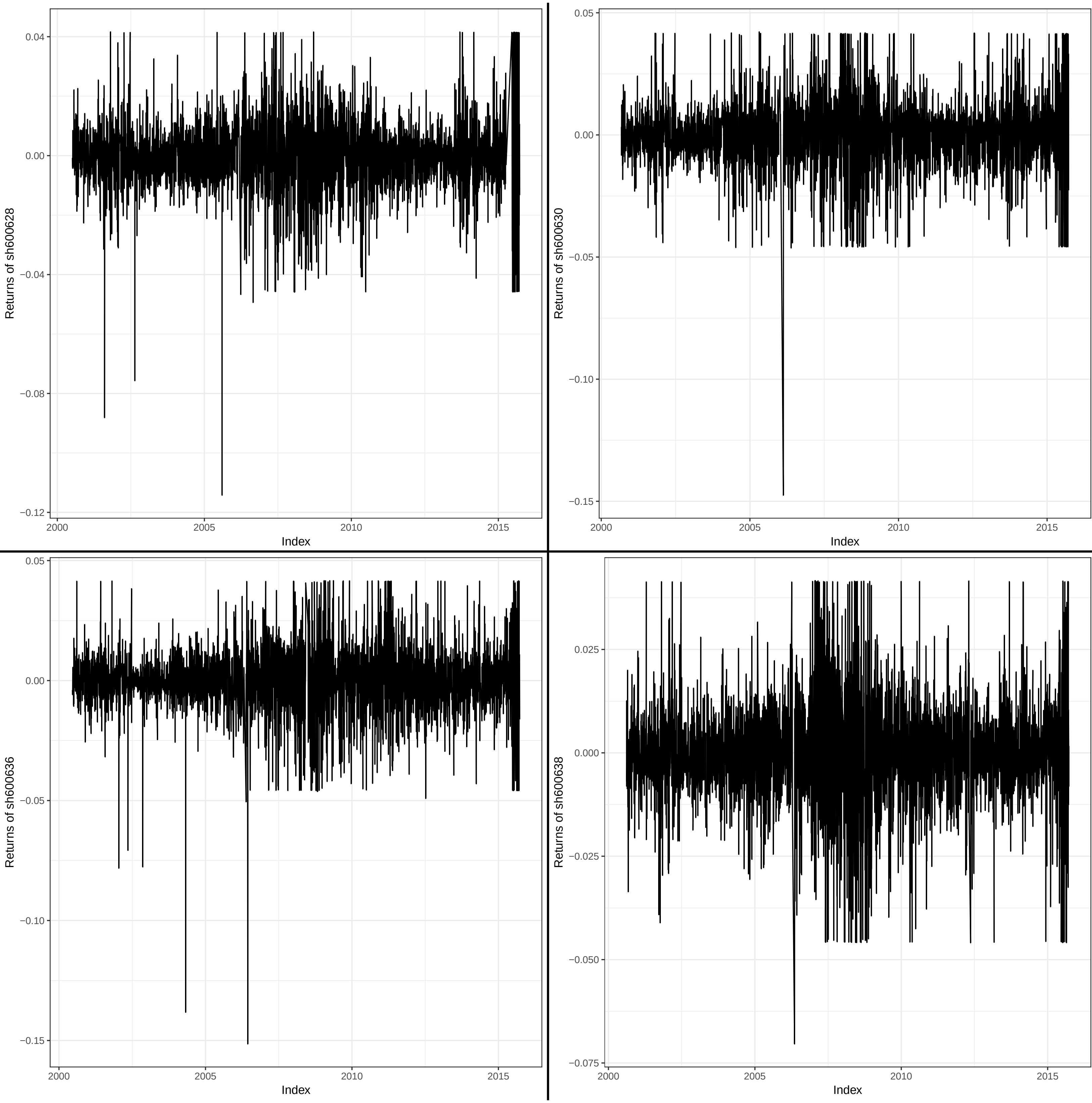
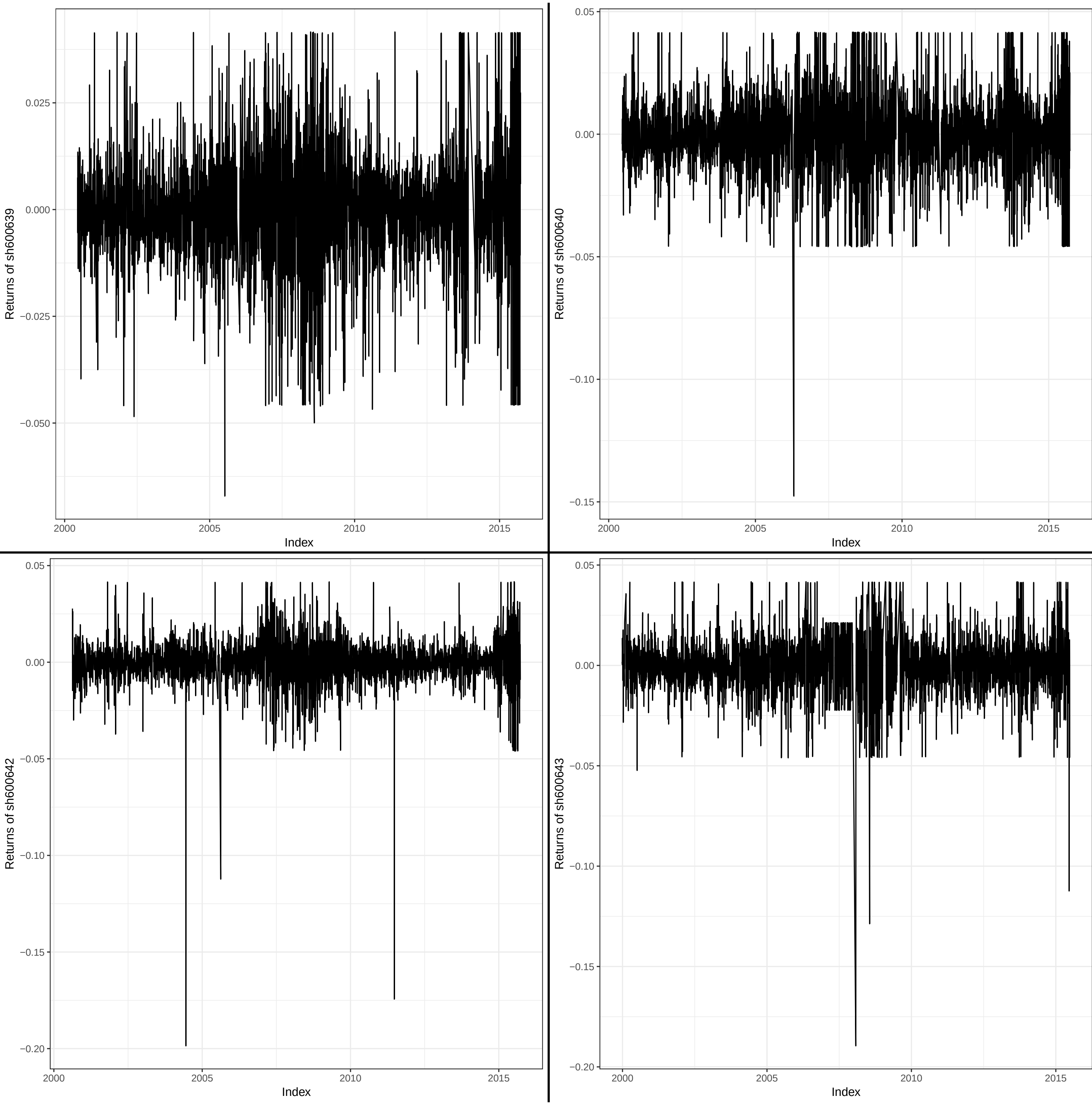
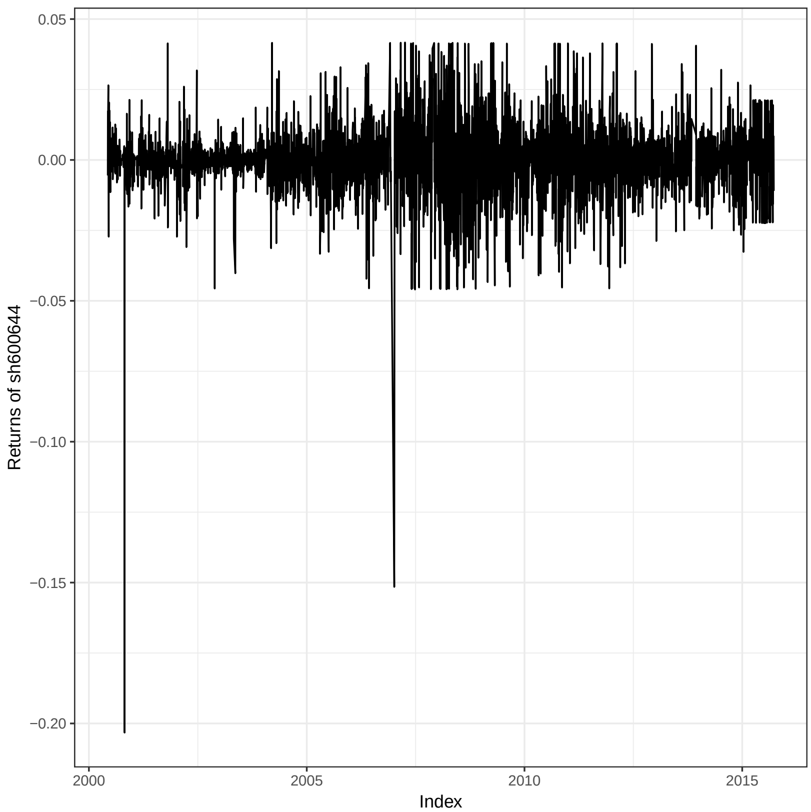
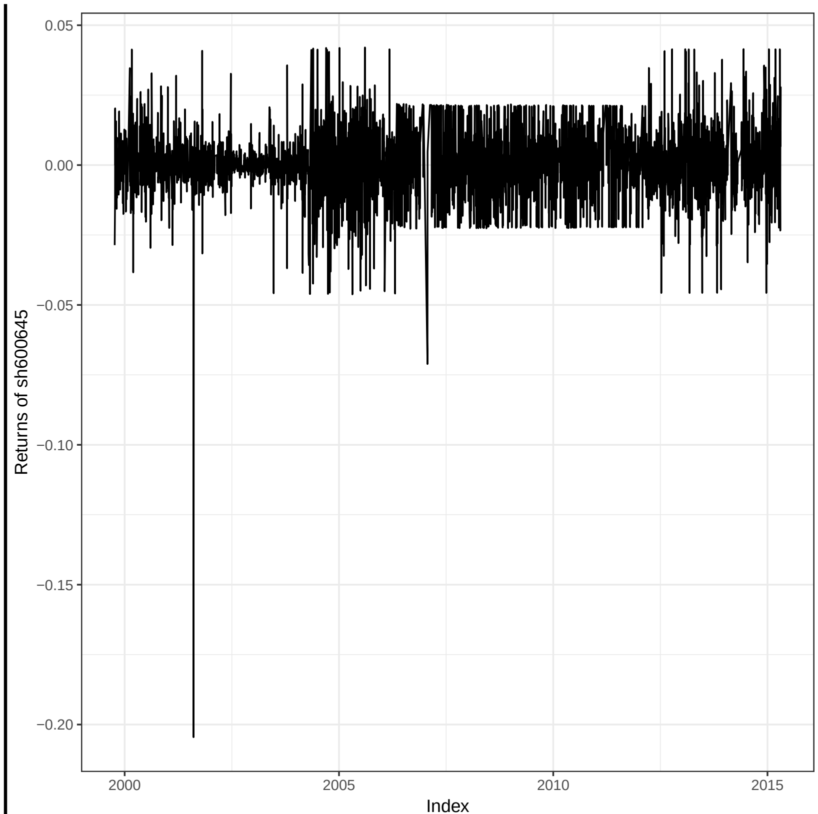
2000
2005
2010
2015


Returns of sh600628
Index
Returns of sh600630
Index
Returns of sh600636
Index
Returns of sh600638
Index

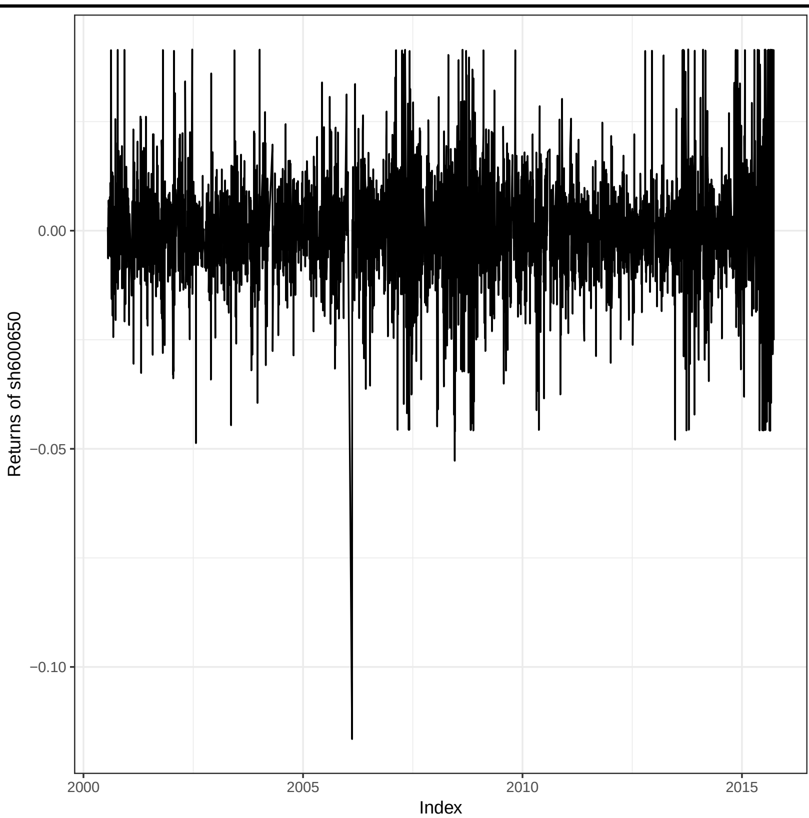

Returns of sh600639
Index
Returns of sh600640
Index
Returns of sh600642
Index
Returns of sh600643
Index



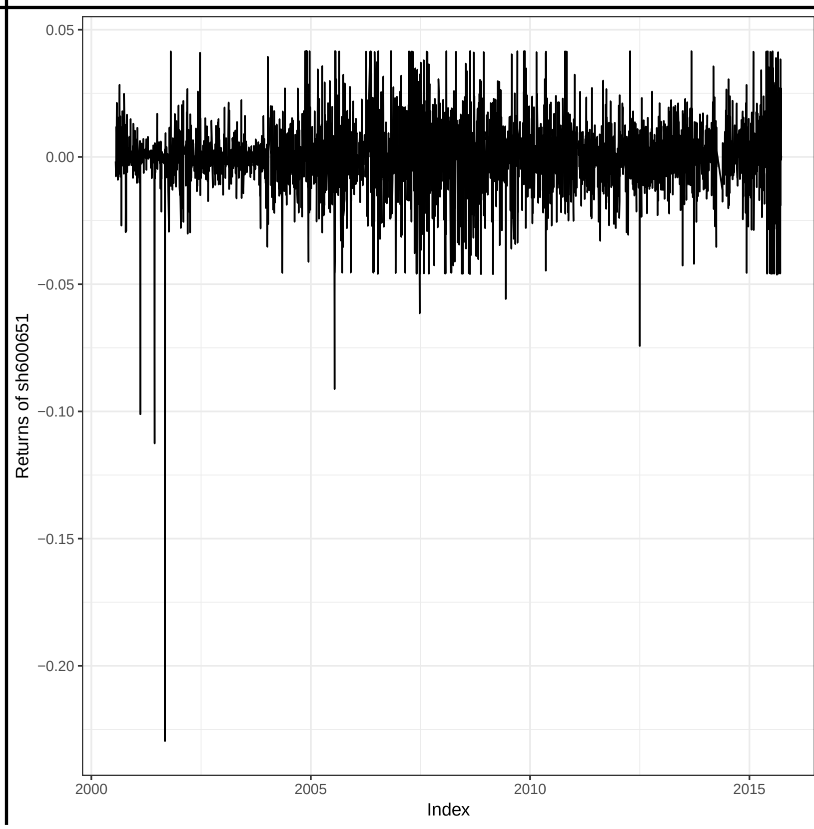
0.05
0.00
−0.05
−0.10
−0.15
−0.20
Returns of sh600644
2000
2005
2010
2015
Index



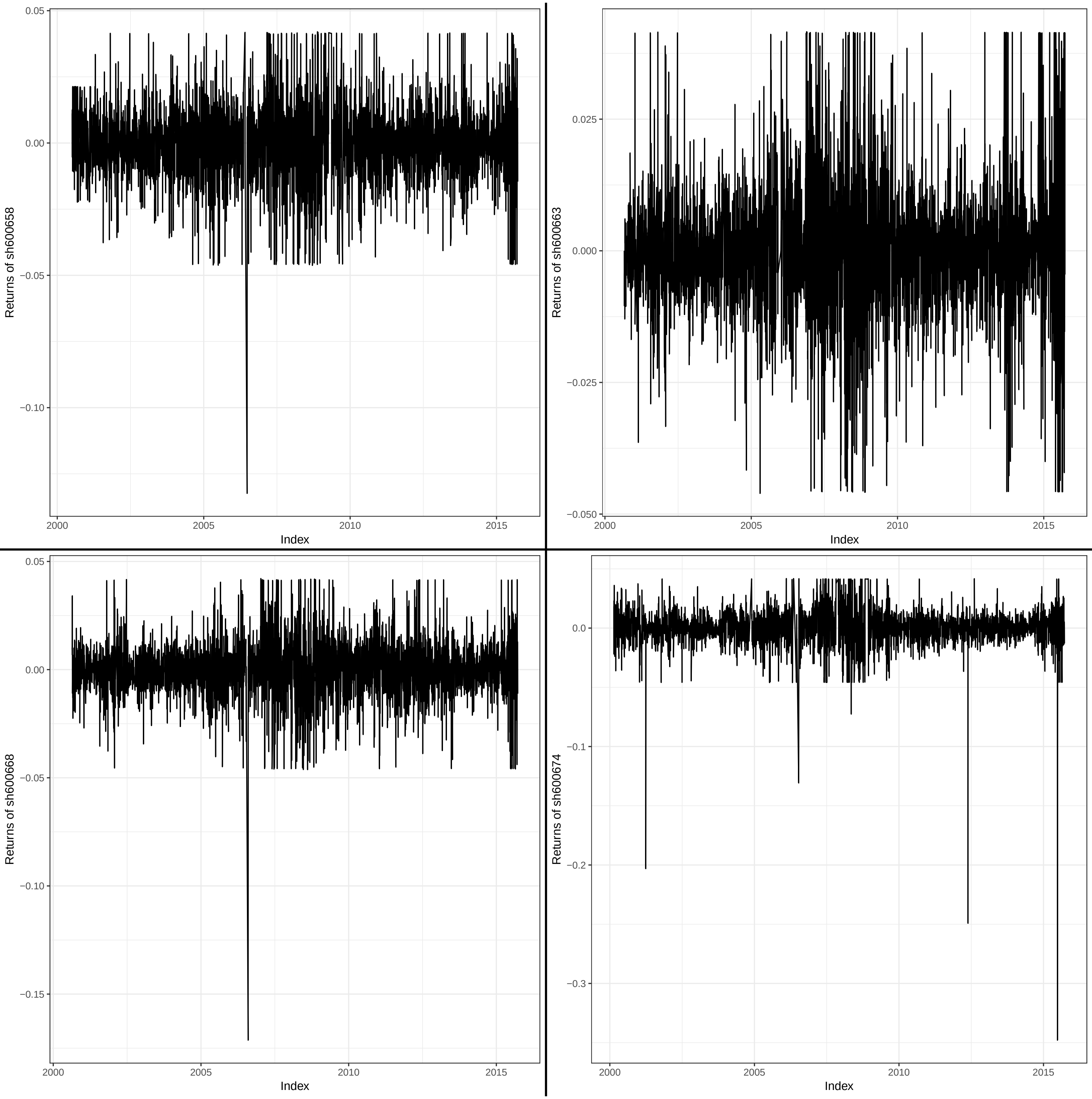
0.05
0.00
−0.05
−0.10
−0.15
−0.20
Returns of sh600645
2000
2005
2010
2015
Index

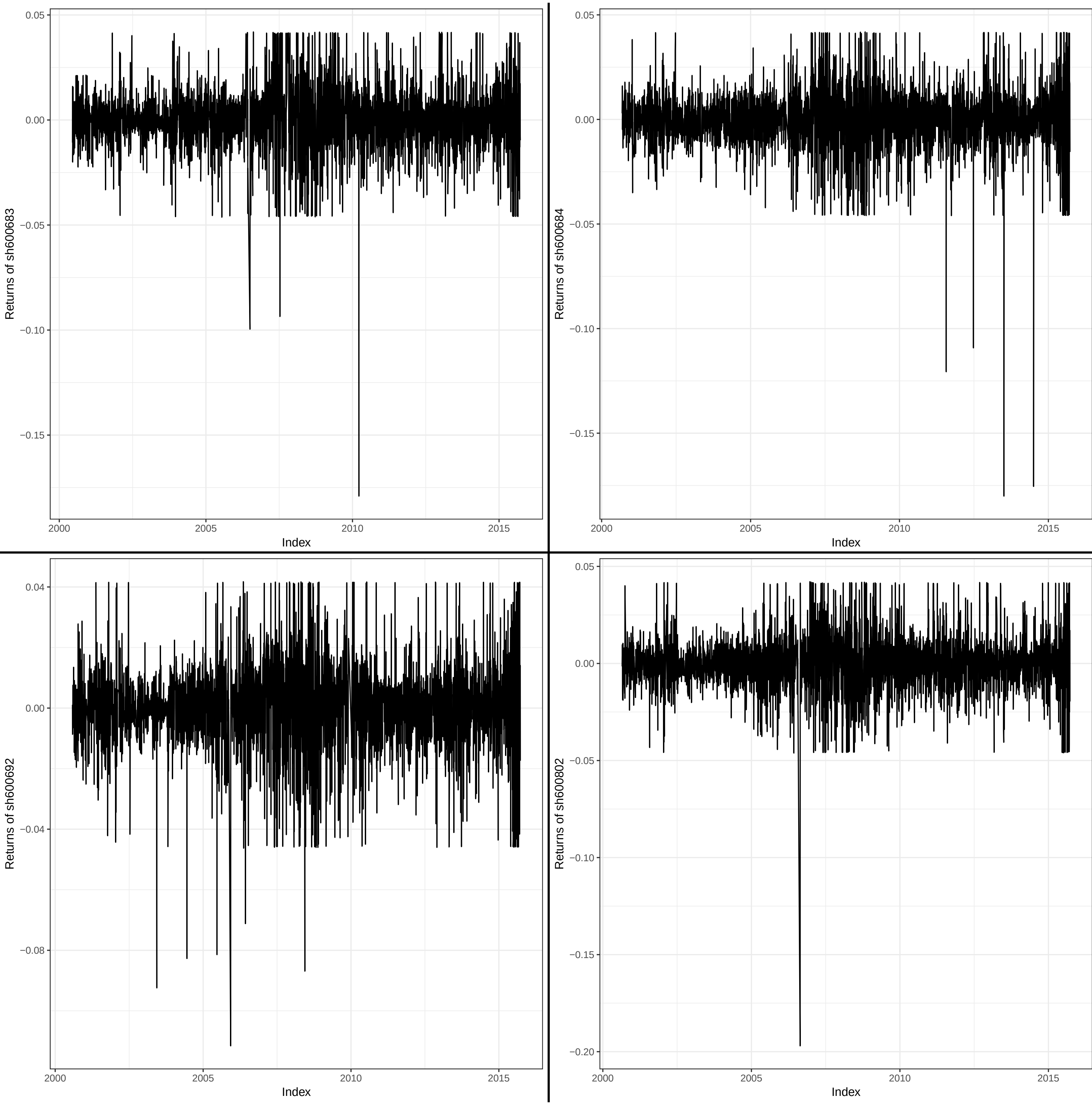


0.00
−0.05
−0.10
Returns of sh600650
2000
2005
2010
2015
Index

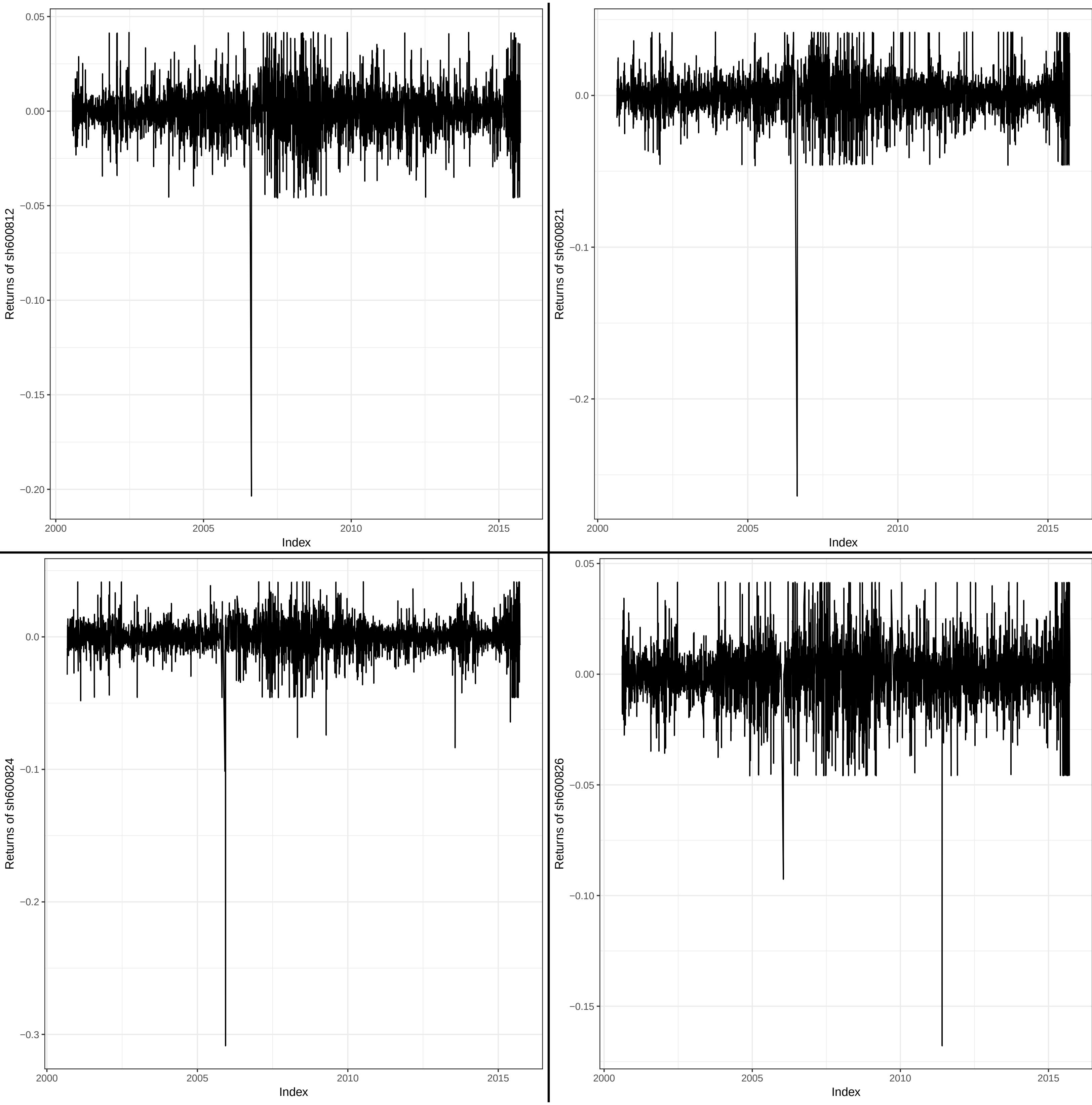


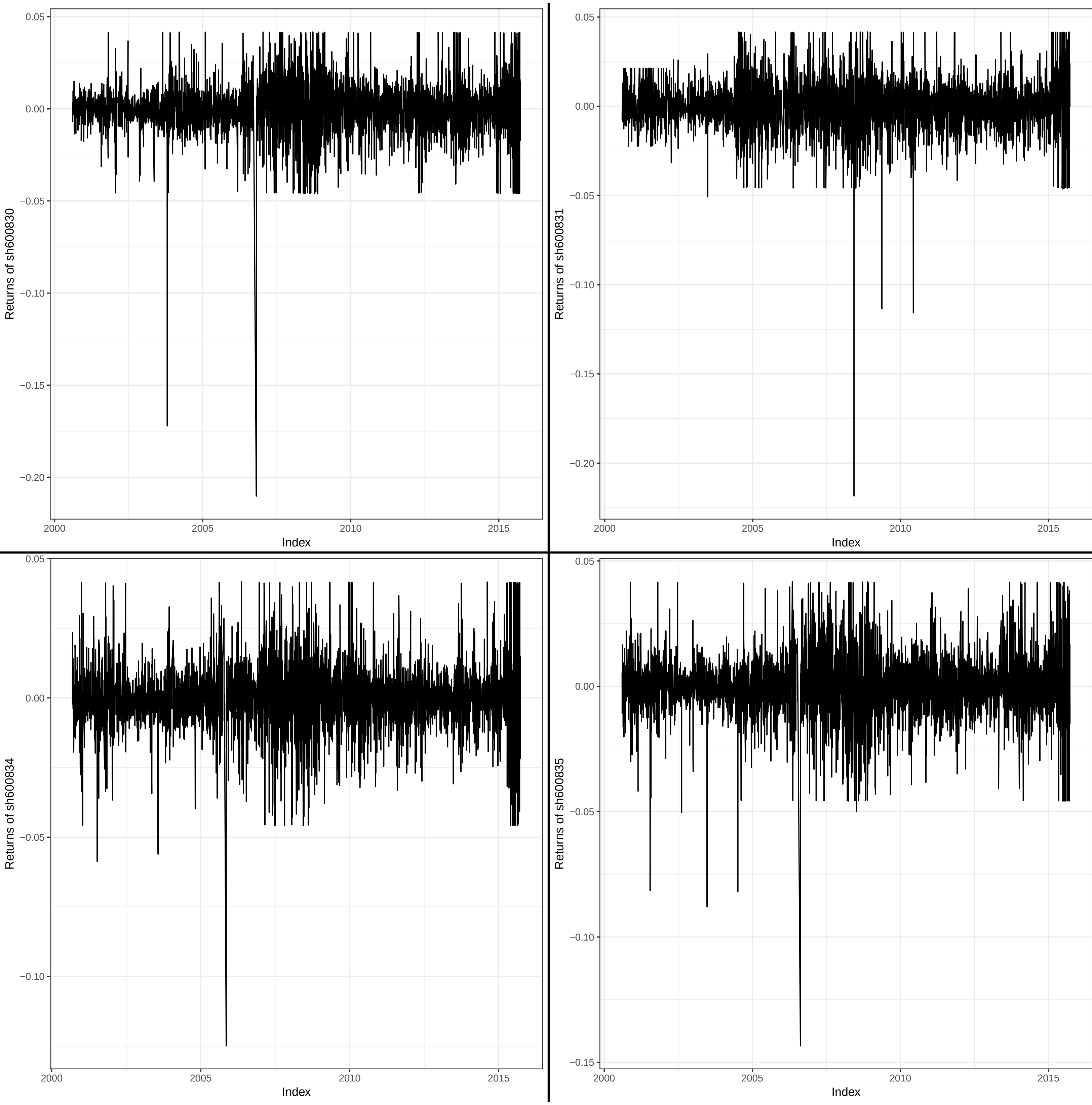
0.05
0.00
−0.05
−0.10
−0.15
−0.20
Returns of sh600651
2000
2005
2010
2015
Index


Returns of sh600658
0.05
0.00
−0.05
−0.10
2000
2005
2010
2015
Index
Returns of sh600663
0.025
0.000
−0.025
−0.050
2000
2005
2010
2015
Index
Returns of sh600668
0.05
0.00
−0.05
−0.10
−0.15
2000
2005
2010
2015
Index
Returns of sh600674
0.0
−0.1
−0.2
−0.3
2000
2005
2010
2015
Index

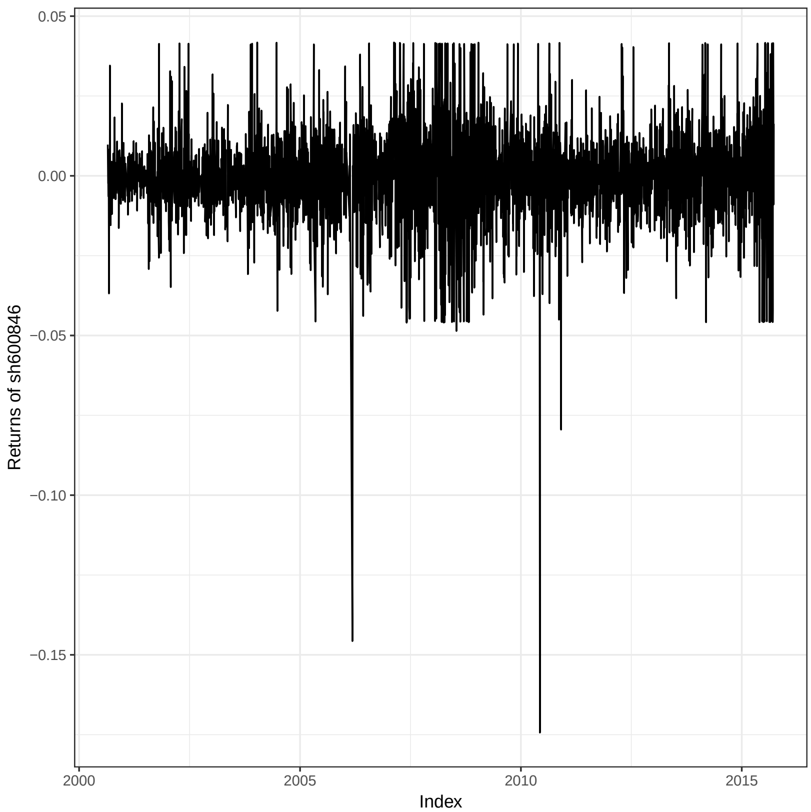


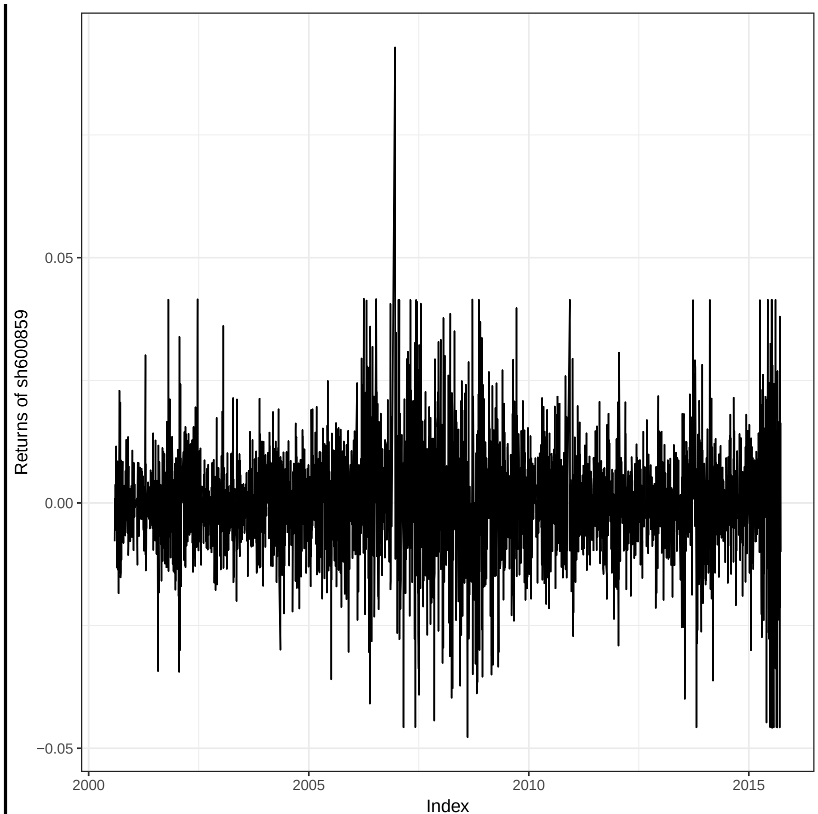
Returns of sh600683
Returns of sh600684
Returns of sh600692
Returns of sh600802
Index
0.05
0.00
−0.05
−0.10
−0.15
−0.20
0.04
−0.04
−0.08
2000
2005
2010
2015


Returns of sh600812
Returns of sh600821
Returns of sh600824
Returns of sh600826
Index
0.05
0.00
−0.05
−0.10
−0.15
−0.20
0.0
−0.1
−0.2
−0.3
2000
2005
2010
2015



Returns of sh600830
0.05
0.00
−0.05
−0.10
−0.15
−0.20
2000
2005
2010
2015
Index
Returns of sh600831
0.05
0.00
−0.05
−0.10
−0.15
−0.20
2000
2005
2010
2015
Index
Returns of sh600834
0.05
0.00
−0.05
−0.10
2000
2005
2010
2015
Index
Returns of sh600835
0.05
0.00
−0.05
−0.10
−0.15
2000
2005
2010
2015
Index


0.05
0.00
−0.05
−0.10
−0.15
Returns of sh600846
2000
2005
2010
2015
Index


0.05
0.00
−0.05
Returns of sh600859
2000
2005
2010
2015
Index

D Appendix D

The BDS test result for each sample firms.

Radius	0.00582645601447355	0.0116529120289471	0.0174793680434206	0.023305824057894
Dim=2	10.74	10.42	9.29	7.20
3	15.19	14.60	12.99	10.24
4	18.37	17.21	15.36	12.24
5	22.50	19.90	17.37	13.89

Table 20: BDS test result for firm sh600000

Radius	0.00503401393770168	0.0100680278754034	0.0151020418131051	0.020136055750806
Dim=2	11.91	12.59	11.92	11.43
3	17.80	17.99	16.57	15.06
4	22.02	21.66	19.64	17.47
5	27.22	25.48	22.21	19.22

Table 21: BDS test result for firm sh600009

Radius	0.00566151920759271	0.0113230384151854	0.0169845576227781	0.022646076830370
Dim=2	9.26	11.95	13.76	13.95
3	12.10	14.46	15.82	15.61
4	14.66	16.64	17.35	16.84
5	16.92	18.61	18.70	17.73

Table 22: BDS test result for firm sh600054

Radius	0.0065667519493443	0.0131335038986886	0.0197002558480329	0.0262670077973772
Dim=2	9.59	11.35	12.64	13.32
3	13.53	14.75	15.26	15.36
4	16.88	17.29	17.18	16.85
5	20.66	19.89	19.01	18.14

Table 23: BDS test result for firm sh600064

Radius	0.00731244388236091	0.0146248877647218	0.0219373316470827	0.0292497755294436
Dim=2	14.78	14.41	14.61	15.16
3	20.70	19.45	18.39	18.15
4	27.28	23.32	20.76	19.71
5	35.86	26.76	22.46	20.59

Table 24: BDS test result for firm sh600067

Radius	0.00750956018324116	0.0150191203664823	0.0225286805497235	0.0300382407329646
Dim=2	12.78	13.93	14.41	14.57
3	15.99	16.83	17.07	17.20
4	19.56	19.17	18.84	18.26
5	23.87	21.47	20.42	19.25

Table 25: BDS test result for firm sh600082

Radius	0.00631358083341951	0.012627161666839	0.0189407425002585	0.025254323333678
Dim=2	7.72	9.23	10.57	11.05
3	9.83	11.26	12.56	13.17
4	12.19	13.01	13.71	14.13
5	14.32	14.41	14.52	14.61

Table 26: BDS test result for firm sh600097

Radius	0.00745716053980172	0.0149143210796034	0.0223714816194052	0.0298286421592069
Dim=2	13.69	14.20	13.94	13.50
3	18.18	18.26	17.66	16.39
4	23.27	21.52	19.91	18.09
5	29.37	24.87	21.83	19.34

Table 27: BDS test result for firm sh600159

Radius	0.00774653406033291	0.0154930681206658	0.0232396021809987	0.030986136241331
Dim=2	12.41	13.85	12.97	11.58
3	16.75	17.96	16.62	14.47
4	19.71	20.27	18.26	15.44
5	23.46	22.80	19.81	16.36

Table 28: BDS test result for firm sh600162

Radius	0.00786777997038638	0.0157355599407728	0.0236033399111592	0.031471119881545
Dim=2	12.05	11.96	11.23	8.92
3	16.54	15.57	14.15	11.37
4	20.22	17.81	15.91	12.77
5	25.33	20.18	17.58	14.04

Table 29: BDS test result for firm sh600256

Radius	0.00732447668665853	0.0146489533733171	0.0219734300599756	0.029297906746634
Dim=2	15.18	16.78	16.40	14.97
3	18.51	19.91	19.21	17.34
4	21.57	22.41	21.38	19.15
5	24.41	24.66	22.95	20.15

Table 30: BDS test result for firm sh600601

Radius	0.00617241480080595	0.0123448296016119	0.0185172444024179	0.024689659203223
Dim=2	9.93	12.07	12.65	12.72
3	11.92	14.18	14.43	14.12
4	13.54	15.69	15.61	14.85
5	14.79	16.91	16.48	15.24

Table 31: BDS test result for firm sh600609

Radius	0.0066279013922828	0.0132558027845656	0.0198837041768484	0.0265116055691312
Dim=2	14.88	17.29	18.57	19.02
3	19.32	21.06	21.50	21.19
4	22.96	23.78	23.62	22.92
5	27.32	26.66	25.57	24.15

Table 32: BDS test result for firm sh600611

Radius	0.00656426574094101	0.013128531481882	0.019692797222823	0.026257062963764
Dim=2	10.15	11.36	11.99	12.39
3	13.04	13.79	13.58	13.41
4	15.58	16.15	15.59	14.93
5	18.10	18.18	17.11	16.07

Table 33: BDS test result for firm sh600612

Radius	0.00711484525360679	0.0142296905072136	0.0213445357608204	0.0284593810144272
Dim=2	10.61	12.75	14.27	15.27
3	14.63	16.89	17.96	17.99
4	18.24	19.67	20.09	19.42
5	21.67	22.07	21.63	20.41

Table 34: BDS test result for firm sh600618

Radius	0.00704735752717124	0.0140947150543425	0.0211420725815137	0.0281894301086850
Dim=2	11.96	13.51	14.89	15.57
3	15.23	16.76	17.85	18.05
4	19.09	19.53	19.95	19.73
5	23.06	21.93	21.31	20.57

Table 35: BDS test result for firm sh600620

Radius	0.00664605216026426	0.0132921043205285	0.0199381564807928	0.026584208641057
Dim=2	13.78	15.80	17.12	17.44
3	17.44	19.25	20.40	20.59
4	21.76	22.27	22.48	22.16
5	27.13	25.41	24.45	23.45

Table 36: BDS test result for firm sh600621

Radius	0.00676434287814983	0.0135286857562997	0.0202930286344495	0.027057371512599
Dim=2	12.59	14.16	14.93	14.67
3	16.09	17.65	18.37	18.02
4	18.77	19.84	20.13	19.63
5	21.65	22.00	21.59	20.79

Table 37: BDS test result for firm sh600622

Radius	0.00700720304094066	0.0140144060818813	0.021021609122822	0.0280288121637627
Dim=2	12.34	14.59	16.05	16.03
3	15.47	18.16	19.79	19.49
4	17.25	20.09	21.69	21.23
5	20.04	22.37	23.28	22.41

Table 38: BDS test result for firm sh600624

Radius	0.00687895768304932	0.0137579153660986	0.020636873049148	0.0275158307321973
Dim=2	11.01	13.43	14.71	15.34
3	14.52	17.11	18.21	18.22
4	16.54	19.22	19.98	19.42
5	19.40	21.46	21.71	20.60

Table 39: BDS test result for firm sh600626

Radius	0.00594284250689915	0.0118856850137983	0.0178285275206975	0.023771370027596
Dim=2	8.38	11.90	14.97	17.75
3	12.27	15.32	18.06	20.68
4	15.22	17.77	19.81	21.70
5	18.20	19.98	21.07	22.23

Table 40: BDS test result for firm sh600628

Radius	0.00717878008957325	0.0143575601791465	0.0215363402687198	0.028715120358293
Dim=2	13.51	15.94	17.01	16.59
3	17.13	19.27	19.80	19.14
4	20.30	22.01	21.85	20.88
5	24.00	24.57	23.21	21.71

Table 41: BDS test result for firm sh600630

Radius	0.00704909668627298	0.014098193372546	0.0211472900588189	0.0281963867450919
Dim=2	13.51	15.10	15.91	15.78
3	19.59	19.49	19.39	18.87
4	25.75	22.70	21.39	20.17
5	33.42	25.90	23.18	21.07

Table 42: BDS test result for firm sh600636

Radius	0.00645581828488345	0.0129116365697669	0.0193674548546503	0.025823273139533
Dim=2	12.51	14.44	15.28	15.27
3	15.64	17.90	19.20	19.31
4	18.97	21.09	21.96	21.45
5	22.38	24.10	24.14	22.89

Table 43: BDS test result for firm sh600638

Radius	0.00675234063871852	0.013504681277437	0.0202570219161556	0.0270093625548741
Dim=2	15.84	16.87	16.60	15.66
3	20.48	20.97	19.94	18.48
4	24.92	24.36	22.39	20.33
5	30.56	27.82	24.48	21.63

Table 44: BDS test result for firm sh600639

Radius	0.0078986251236455	0.015797250247291	0.0236958753709365	0.031594500494582
Dim=2	10.24	11.66	13.58	14.56
3	14.14	14.63	16.21	17.23
4	17.00	16.97	18.24	18.97
5	20.30	19.23	19.90	20.18

Table 45: BDS test result for firm sh600640

Radius	0.00568389497638818	0.0113677899527764	0.0170516849291646	0.0227355799055528
Dim=2	12.43	15.76	17.65	18.00
3	16.08	18.94	20.07	19.92
4	19.23	21.42	21.54	20.57
5	22.43	23.72	22.83	20.92

Table 46: BDS test result for firm sh600642

Radius	0.0070375019831925	0.014075003966385	0.0211125059495775	0.02815000793277
Dim=2	11.06	12.78	13.99	14.29
3	15.03	16.43	17.10	16.96
4	17.95	18.90	19.24	18.88
5	21.06	21.11	20.76	19.91

Table 47: BDS test result for firm sh600643

Radius	0.00653858176245908	0.0130771635249182	0.0196157452873772	0.0261543270498364
Dim=2	17.60	14.93	13.68	13.02
3	26.58	20.63	17.81	15.93
4	37.47	25.68	20.71	17.62
5	52.63	30.45	23.11	18.83

Table 48: BDS test result for firm sh600644

Radius	0.00632108825740602	0.012642176514812	0.0189632647722181	0.0252843530296241
Dim=2	14.10	12.70	11.19	10.82
3	24.37	17.68	14.54	13.63
4	36.35	21.90	16.77	15.12
5	53.45	25.82	18.31	16.12

Table 49: BDS test result for firm sh600645

Radius	0.0064862161348664	0.0129724322697328	0.0194586484045992	0.0259448645394656
Dim=2	12.56	15.58	17.55	18.61
3	14.79	17.87	19.61	20.18
4	16.91	19.71	20.81	20.85
5	19.17	21.48	21.82	21.42

Table 50: BDS test result for firm sh600650

Radius	0.00720860514967893	0.0144172102993579	0.0216258154490368	0.0288344205987157
Dim=2	11.86	12.75	14.38	16.01
3	17.67	15.95	16.67	17.89
4	24.69	18.64	18.21	18.56
5	35.26	21.71	19.74	19.29

Table 51: BDS test result for firm sh600651

Radius	0.00716903286470049	0.014338065729401	0.0215070985941015	0.028676131458802
Dim=2	7.21	9.09	11.38	13.05
3	9.34	11.05	13.18	14.62
4	11.39	12.58	14.14	15.19
5	13.69	14.17	15.03	15.62

Table 52: BDS test result for firm sh600658

Radius	0.00638844029565609	0.0127768805913122	0.0191653208869683	0.0255537611826244
Dim=2	13.64	15.33	15.81	15.56
3	18.56	19.65	19.28	18.64
4	22.43	22.73	21.66	20.55
5	26.56	25.59	23.52	21.79

Table 53: BDS test result for firm sh600663

Radius	0.00666277613544237	0.0133255522708847	0.0199883284063271	0.0266511045417695
Dim=2	10.32	12.03	13.36	13.96
3	13.24	14.85	15.64	15.73
4	17.04	17.57	17.71	17.50
5	20.54	19.98	19.32	18.75

Table 54: BDS test result for firm sh600668

Radius	0.00767755876876913	0.0153551175375383	0.0230326763063074	0.0307102350750765
Dim=2	15.04	16.17	18.83	20.15
3	19.34	19.81	21.62	21.86
4	22.56	22.26	23.39	22.86
5	25.93	24.28	24.64	23.48

Table 55: BDS test result for firm sh600674

Radius	0.00725498090398285	0.0145099618079657	0.0217649427119485	0.029019923615931
Dim=2	9.97	12.10	13.52	13.65
3	12.53	14.16	14.82	14.54
4	15.66	16.05	16.03	15.54
5	18.21	17.64	17.12	16.49

Table 56: BDS test result for firm sh600683

Radius	0.00724815102327576	0.0144963020465515	0.0217444530698273	0.028992604093103
Dim=2	11.15	12.52	13.69	14.47
3	15.39	16.67	17.28	17.41
4	18.43	19.24	19.28	18.96
5	21.69	21.80	21.04	20.14

Table 57: BDS test result for firm sh600684

Radius	0.00711269526500004	0.0142253905300001	0.0213380857950001	0.028450781060000
Dim=2	11.42	12.53	13.24	13.81
3	15.89	16.18	16.49	16.74
4	20.52	19.28	18.55	18.11
5	25.98	22.16	19.99	18.91

Table 58: BDS test result for firm sh600692

Radius	0.00712322429750197	0.0142464485950039	0.0213696728925059	0.028492897190007
Dim=2	12.42	13.93	14.05	13.69
3	16.75	17.92	17.35	16.44
4	20.97	21.25	20.04	18.57
5	25.42	24.10	21.85	19.85

Table 59: BDS test result for firm sh600802

Radius	0.00646143592479034	0.0129228718495807	0.019384307774371	0.0258457436991614
Dim=2	11.29	12.71	13.94	15.28
3	15.34	16.44	17.23	18.18
4	18.36	18.67	18.95	19.29
5	22.00	20.63	20.10	19.98

Table 60: BDS test result for firm sh600812

Radius	0.00719070605516185	0.0143814121103237	0.0215721181654856	0.028762824220647
Dim=2	11.51	13.39	14.89	15.08
3	15.54	16.77	17.14	16.79
4	19.30	19.67	19.17	18.47
5	23.10	22.52	21.06	19.78

Table 61: BDS test result for firm sh600821

Radius	0.00657274491803434	0.0131454898360687	0.019718234754103	0.0262909796721374
Dim=2	12.76	13.16	12.78	11.88
3	17.22	17.08	16.19	14.80
4	21.38	20.28	18.43	16.69
5	26.34	23.47	20.38	17.94

Table 62: BDS test result for firm sh600824

Radius	0.00711362333375592	0.0142272466675118	0.0213408700012678	0.028454493335023
Dim=2	10.14	11.83	12.98	14.44
3	12.67	14.15	15.14	16.42
4	15.01	16.08	16.57	17.37
5	17.95	17.99	17.91	18.29

Table 63: BDS test result for firm sh600826

Radius	0.00715973905008686	0.0143194781001737	0.0214792171502606	0.028638956200347
Dim=2	13.58	15.01	15.83	16.45
3	19.13	19.51	19.08	18.72
4	23.58	22.25	20.84	20.06
5	29.38	25.29	22.38	20.89

Table 64: BDS test result for firm sh600830

Radius	0.00722234141412636	0.0144446828282527	0.0216670242423791	0.028889365656505
Dim=2	9.70	11.17	12.84	14.59
3	13.05	13.95	14.91	16.19
4	16.41	16.53	16.86	17.56
5	19.97	18.82	18.45	18.79

Table 65: BDS test result for firm sh600831

Radius	0.00613382087232855	0.0122676417446571	0.0184014626169857	0.024535283489314
Dim=2	10.91	13.22	14.64	15.27
3	15.12	16.78	17.71	17.84
4	18.74	19.56	19.80	19.37
5	22.75	22.05	21.19	20.31

Table 66: BDS test result for firm sh600834

Radius	0.00647677080424589	0.0129535416084918	0.0194303124127377	0.025907083216983
Dim=2	9.20	10.34	11.15	11.03
3	13.03	13.22	13.32	12.86
4	16.48	15.42	14.72	13.85
5	20.43	17.95	16.40	14.93

Table 67: BDS test result for firm sh600835

Radius	0.0071581449341135	0.014316289868227	0.0214744348023405	0.028632579736454
Dim=2	14.70	16.55	17.71	17.49
3	18.42	19.83	20.41	19.88
4	22.45	22.31	21.99	21.21
5	27.91	25.08	23.28	21.92

Table 68: BDS test result for firm sh600846

Radius	0.00554117926045896	0.0110823585209179	0.0166235377813769	0.022164717041835
Dim=2	11.03	12.50	13.68	14.20
3	14.95	16.08	16.81	16.70
4	18.23	18.98	19.10	18.35
5	22.58	21.91	21.16	19.77

Table 69: BDS test result for firm sh600859

E Appendix E

The test statistic for each sample firms, with window 100, 250, and 500 respectively.

	sh600000 Estimate	sh600000 t value	sh600009 Estimate	sh600009 t value	sh600054 Estimate	sh600054 t value	sh600064 Estimate	sh600064 t value	sh600067 Estimate	sh600067 t value
(Intercept)	0.0011	1.4686	2e-04	0.6006	5e-04	1.6616	0.0027	2.5615	-4e-04	-0.3688
(Intercept) SE	(7e-04)		(4e-04)		(3e-04)		(0.001)		(0.0011)	
lag(x, 1)	-0.203	-3.0486	-0.1066	-2.5006	-0.0453	-1.6583	-0.1531	-2.0005	0.4484	5.0633
lag(x, 1) SE	(0.0666)		(0.0426)		(0.0273)		(0.0766)		(0.0886)	
T_A	-0.0014	-1.6699	-2e-04	-0.2204	-8e-04	-1.4557	-0.0031	-2.7258	4e-04	0.309
T_A SE	(8e-04)		(7e-04)		(6e-04)		(0.0011)		(0.0012)	
lag(x, 1):T_A	0.3035	4.116	0.298	3.4398	0.2071	4.0566	0.2895	3.3403	-0.4046	-4.4404
lag(x, 1):T_A SE	(0.0737)		(0.0866)		(0.051)		(0.0867)		(0.0911)	
T_B	-0.001	-1.3407	-1e-04	-0.2425	-7e-04	-1.7779	-0.0027	-2.5123	2e-04	0.1874
T_B SE	(8e-04)		(4e-04)		(4e-04)		(0.0011)		(0.0012)	
lag(x, 1):T_B	0.1689	2.4215	0.1401	3.0037	0.0875	2.3759	0.1678	2.1266	-0.3959	-4.2531
lag(x, 1):T_B SE	(0.0698)		(0.0467)		(0.0368)		(0.0789)		(0.0931)	
F Stat	4.98012	(5,3494)	3.16908	(5,3494)	4.68037	(5,3494)	4.72084	(5,3494)	6.65787	(5,3494)
R Sq	0.00708		0.00451		0.00665		0.00671		0.00944	
R Sq Adj	0.00566		0.00309		0.00523		0.00529		0.00802	

	sh600082 Estimate	sh600082 t value	sh600097 Estimate	sh600097 t value	sh600159 Estimate	sh600159 t value	sh600162 Estimate	sh600162 t value	sh600256 Estimate	sh600256 t value
(Intercept)	-3e-04	-0.9381	0.0011	1.5752	-0.002	-2.873	-5e-04	-1.3065	-7e-04	-0.8659
(Intercept) SE	(3e-04)		(7e-04)		(7e-04)		(4e-04)		(9e-04)	
lag(x, 1)	0.0373	1.8463	0.2873	5.7576	0.0962	2.3517	0.1086	3.6569	0.3946	5.4883
lag(x, 1) SE	(0.0202)		(0.0499)		(0.0409)		(0.0297)		(0.0719)	
T_A	9e-04	1.1568	-0.0013	-1.7532	0.0021	2.7538	8e-04	1.4379	6e-04	0.6245
T_A SE	(8e-04)		(7e-04)		(7e-04)		(6e-04)		(0.001)	
lag(x, 1):T_A	0.1203	2.2373	-0.2466	-4.3795	-0.0643	-1.4113	-0.0795	-2.1704	-0.3518	-4.3878
lag(x, 1):T_A SE	(0.0538)		(0.0563)		(0.0456)		(0.0366)		(0.0802)	
T_B	5e-04	0.6418	-8e-04	-0.9932	0.0026	2.4797	8e-04	0.7299	7e-04	0.7191
T_B SE	(8e-04)		(8e-04)		(0.001)		(0.0011)		(9e-04)	
lag(x, 1):T_B	0.0956	2.1904	-0.2796	-5.0256	0.0456	0.7335	-0.2049	-2.6568	-0.378	-5.0689
lag(x, 1):T_B SE	(0.0436)		(0.0556)		(0.0621)		(0.0771)		(0.0746)	
F Stat	5.4122	(5,3494)	7.70652	(5,3494)	5.13128	(5,3494)	3.80572	(5,3494)	6.49717	(5,3494)
R Sq	0.00769		0.01091		0.00729		0.00542		0.00921	
R Sq Adj	0.00627		0.00949		0.00587		0.00399		0.00779	

	sh600601 Estimate	sh600601 t value	sh600609 Estimate	sh600609 t value	sh600611 Estimate	sh600611 t value	sh600612 Estimate	sh600612 t value	sh600618 Estimate	sh600618 t value
(Intercept)	-9e-04	-0.9186	-0.0024	-2.471	-0.0012	-0.8002	0.0032	2.4337	-0.0033	-1.5014
(Intercept) SE	(0.001)		(0.001)		(0.0015)		(0.0013)		(0.0022)	
lag(x, 1)	-0.19	-2.6335	0.0028	0.0332	-0.5021	-4.48	0.4316	4.5241	0.6471	4.4919
lag(x, 1) SE	(0.0722)		(0.084)		(0.1121)		(0.0954)		(0.1441)	
T_A	8e-04	0.8071	0.0028	2.688	0.0013	0.8233	-0.0034	-2.4279	0.0034	1.5026
T_A SE	(0.0011)		(0.001)		(0.0015)		(0.0014)		(0.0022)	
lag(x, 1):T_A	0.2168	2.8632	0.0647	0.7278	0.5795	5.1024	-0.4109	-3.9885	-0.57	-3.9251
lag(x, 1):T_A SE	(0.0757)		(0.0888)		(0.1136)		(0.103)		(0.1452)	
T_B	4e-04	0.3577	0.0024	2.3004	7e-04	0.4368	-0.003	-2.2738	0.0032	1.3718
T_B SE	(0.0012)		(0.001)		(0.0017)		(0.0013)		(0.0024)	
lag(x, 1):T_B	0.233	3.0313	0.0853	0.9834	0.6741	5.6183	-0.4082	-4.1954	-0.5146	-3.4163
lag(x, 1):T_B SE	(0.0769)		(0.0867)		(0.12)		(0.0973)		(0.1506)	
F Stat	2.3176	(5,3494)	6.02476	(5,3494)	10.89793	(5,3494)	4.93976	(5,3494)	9.70188	(5,3494)
R Sq	0.00331		0.00855		0.01536		0.00702		0.01369	
R Sq Adj	0.00188		0.00713		0.01395		0.0056		0.01228	

	sh600620 Estimate	sh600620 t value	sh600621 Estimate	sh600621 t value	sh600622 Estimate	sh600622 t value	sh600624 Estimate	sh600624 t value	sh600626 Estimate	sh600626 t value
(Intercept)	-2e-04	-0.5355	0.0019	1.0594	0.0019	1.412	-0.0012	-1.6536	5e-04	1.064
(Intercept) SE	(3e-04)		(0.0018)		(0.0014)		(7e-04)		(5e-04)	
lag(x, 1)	-0.0241	-0.9777	0.3526	3.3379	-0.3491	-3.5122	-0.1283	-2.1433	-0.0212	-0.5791
lag(x, 1) SE	(0.0247)		(0.1056)		(0.0994)		(0.0598)		(0.0366)	
T_A	9e-04	1.7799	-0.0018	-1.0268	-0.0016	-1.0672	0.0016	1.8488	-0.001	-1.6312
T_A SE	(5e-04)		(0.0018)		(0.0015)		(9e-04)		(6e-04)	
lag(x, 1):T_A	0.1113	2.8952	-0.3121	-2.9032	0.4557	4.274	0.2074	2.8675	0.0805	1.7698
lag(x, 1):T_A SE	(0.0384)		(0.1075)		(0.1066)		(0.0723)		(0.0455)	
T_B	-0.0012	-1.6099	-0.0025	-1.3307	-0.0021	-1.4904	0.0012	1.5263	-3e-04	-0.4987
T_B SE	(7e-04)		(0.0019)		(0.0014)		(8e-04)		(6e-04)	
lag(x, 1):T_B	0.1964	4.4525	-0.2445	-2.2109	0.4252	4.2018	0.1849	2.9385	0.1448	3.195
lag(x, 1):T_B SE	(0.0441)		(0.1106)		(0.1012)		(0.0629)		(0.0453)	
F Stat	8.15255	(5,3494)	5.81424	(5,3494)	7.76567	(5,3494)	4.03803	(5,3494)	6.01702	(5,3494)
R Sq	0.01153		0.00825		0.01099		0.00575		0.00854	
R Sq Adj	0.01012		0.00683		0.00958		0.00432		0.00712	

	sh600628 Estimate	sh600628 t value	sh600630 Estimate	sh600630 t value	sh600636 Estimate	sh600636 t value	sh600638 Estimate	sh600638 t value	sh600639 Estimate	sh600639 t value
(Intercept)	-9e-04	-0.8831	-0.0028	-1.5343	6e-04	1.7154	-6e-04	-0.529	0.0031	2.6489
(Intercept) SE	(0.001)		(0.0018)		(4e-04)		(0.0011)		(0.0012)	
lag(x, 1)	0.3684	4.1955	0.3164	1.7782	0.0011	0.0415	-0.2568	-2.8174	-0.1691	-1.7071
lag(x, 1) SE	(0.0878)		(0.1779)		(0.0275)		(0.0911)		(0.099)	
T_A	0.001	0.9232	0.0027	1.4692	-0.0011	-2.0825	6e-04	0.5572	-0.003	-2.4672
T_A SE	(0.0011)		(0.0018)		(5e-04)		(0.0011)		(0.0012)	
lag(x, 1):T_A	-0.3451	-3.8109	-0.2873	-1.6054	0.0589	1.4526	0.3301	3.544	0.2263	2.2267
lag(x, 1):T_A SE	(0.0906)		(0.179)		(0.0405)		(0.0931)		(0.1016)	
T_B	9e-04	0.8393	0.0044	2.2427	-0.0011	-1.6593	2e-04	0.1742	-0.0034	-2.7641
T_B SE	(0.0011)		(0.002)		(7e-04)		(0.0012)		(0.0012)	
lag(x, 1):T_B	-0.3267	-3.5535	-0.2415	-1.3319	0.1263	3.0706	0.386	3.9077	0.2329	2.2752
lag(x, 1):T_B SE	(0.0919)		(0.1813)		(0.0411)		(0.0988)		(0.1024)	
F Stat	4.26179	(5,3494)	3.60889	(5,3494)	5.40754	(5,3494)	6.91971	(5,3494)	4.35643	(5,3494)
R Sq	0.00606		0.00514		0.00768		0.00981		0.0062	
R Sq Adj	0.00464		0.00371		0.00626		0.00839		0.00477	

	sh600640 Estimate	sh600640 t value	sh600642 Estimate	sh600642 t value	sh600643 Estimate	sh600643 t value	sh600644 Estimate	sh600644 t value	sh600645 Estimate	sh600645 t value
(Intercept)	-0.0022	-1.7762	0.0022	2.6058	-3e-04	-0.8872	-1e-04	-0.3167	0.0026	2.4223
(Intercept) SE	(0.0013)		(8e-04)		(4e-04)		(3e-04)		(0.0011)	
lag(x, 1)	-0.1004	-1.2109	-0.1999	-2.3752	0.1021	3.5619	-0.0101	-0.5117	0.2864	3.3916
lag(x, 1) SE	(0.0829)		(0.0841)		(0.0287)		(0.0198)		(0.0844)	
T_A	0.002	1.5363	-0.0018	-1.7998	6e-04	0.7049	-1e-04	-0.1465	-0.0024	-2.109
T_A SE	(0.0013)		(0.001)		(8e-04)		(6e-04)		(0.0011)	
lag(x, 1):T_A	0.1945	2.2201	0.1951	2.0347	0.2379	3.6245	0.1609	3.0487	-0.2404	-2.7225
lag(x, 1):T_A SE	(0.0876)		(0.0959)		(0.0656)		(0.0528)		(0.0883)	
T_B	0.0024	1.8798	-0.0025	-2.9118	7e-04	1.2942	0	-0.0681	-0.0027	-2.3693
T_B SE	(0.0013)		(9e-04)		(5e-04)		(7e-04)		(0.0011)	
lag(x, 1):T_B	0.1761	2.0551	0.2176	2.5255	-0.0746	-2.061	0.1212	2.5616	-0.1778	-2.0319
lag(x, 1):T_B SE	(0.0857)		(0.0862)		(0.0362)		(0.0473)		(0.0875)	
F Stat	5.69383	(5,3494)	3.02365	(5,3494)	9.90944	(5,3494)	3.29177	(5,3494)	8.31617	(5,3494)
R Sq	0.00808		0.00431		0.01398		0.00469		0.01176	
R Sq Adj	0.00666		0.00288		0.01257		0.00326		0.01035	

	sh600650 Estimate	sh600650 t value	sh600651 Estimate	sh600651 t value	sh600658 Estimate	sh600658 t value	sh600663 Estimate	sh600663 t value	sh600668 Estimate	sh600668 t value
(Intercept)	-1e-04	-0.3411	2e-04	0.5606	0.0016	1.5896	0.0013	1.2653	-0.0043	-2.502
(Intercept) SE	(3e-04)		(4e-04)		(0.001)		(0.001)		(0.0017)	
lag(x, 1)	0.0271	1.1802	0.1325	4.6343	0.2824	4.4316	0.3465	4.8846	-0.0719	-0.617
lag(x, 1) SE	(0.023)		(0.0286)		(0.0637)		(0.0709)		(0.1166)	
T_A	6e-04	1.0247	-4e-04	-0.8328	-0.0019	-1.8021	-0.0013	-1.2585	0.0043	2.4986
T_A SE	(5e-04)		(5e-04)		(0.001)		(0.0011)		(0.0017)	
lag(x, 1):T_A	0.0995	2.2405	-0.1302	-3.5076	-0.242	-3.6308	-0.3083	-4.1846	0.1114	0.9429
lag(x, 1):T_A SE	(0.0444)		(0.0371)		(0.0667)		(0.0737)		(0.1182)	
T_B	5e-04	0.6348	-0.0013	-1.4751	-0.001	-0.8392	-5e-04	-0.4278	0.0045	2.5021
T_B SE	(7e-04)		(9e-04)		(0.0012)		(0.0012)		(0.0018)	
lag(x, 1):T_B	0.0975	2.4417	-0.1484	-2.8102	-0.2899	-3.8819	-0.2863	-3.6163	0.2038	1.6665
lag(x, 1):T_B SE	(0.0399)		(0.0528)		(0.0747)		(0.0792)		(0.1223)	
F Stat	5.75301	(5,3494)	4.74912	(5,3494)	5.74128	(5,3494)	6.65163	(5,3494)	4.76573	(5,3494)
R Sq	0.00817		0.00675		0.00815		0.00943		0.00677	
R Sq Adj	0.00675		0.00533		0.00673		0.00801		0.00535	

	sh600674 Estimate	sh600674 t value	sh600683 Estimate	sh600683 t value	sh600684 Estimate	sh600684 t value	sh600692 Estimate	sh600692 t value	sh600802 Estimate	sh600802 t value
(Intercept)	0.0068	4.4094	0.0013	1.3376	-6e-04	-0.9574	-0.0013	-0.877	-0.0012	-1.457
(Intercept) SE	(0.0015)		(0.001)		(6e-04)		(0.0015)		(8e-04)	
lag(x, 1)	0.2639	2.8857	0.2619	3.7563	-0.065	-1.4315	0.463	4.31	-0.059	-0.99
lag(x, 1) SE	(0.0914)		(0.0697)		(0.0454)		(0.1074)		(0.0596)	
T_A	-0.0071	-4.498	-0.0016	-1.4042	5e-04	0.7019	0.0014	0.8841	0.0012	1.4028
T_A SE	(0.0016)		(0.0011)		(7e-04)		(0.0016)		(9e-04)	
lag(x, 1):T_A	-0.178	-1.9072	-0.1274	-1.61	0.151	2.9955	-0.3503	-3.0461	0.1374	2.179
lag(x, 1):T_A SE	(0.0934)		(0.0791)		(0.0504)		(0.115)		(0.063)	
T_B	-0.0067	-4.0454	-0.0014	-1.3784	0.0012	1.5393	0.0013	0.8517	0.0014	1.4847
T_B SE	(0.0017)		(0.001)		(8e-04)		(0.0015)		(0.001)	
lag(x, 1):T_B	-0.2482	-2.4802	-0.1783	-2.4623	0.1868	3.3484	-0.3922	-3.5967	0.2092	3.0494
lag(x, 1):T_B SE	(0.1001)		(0.0724)		(0.0558)		(0.109)		(0.0686)	
F Stat	10.33842	(5,3494)	9.53037	(5,3494)	6.87594	(5,3494)	8.1839	(5,3494)	7.52414	(5,3494)
R Sq	0.01458		0.01345		0.00974		0.01158		0.01065	
R Sq Adj	0.01317		0.01204		0.00833		0.01016		0.00924	

	sh600812 Estimate	sh600812 t value	sh600821 Estimate	sh600821 t value	sh600824 Estimate	sh600824 t value	sh600826 Estimate	sh600826 t value	sh600830 Estimate	sh600830 t value
(Intercept)	-0.0028	-3.7066	-9e-04	-0.5286	-0.0025	-2.4085	-0.0013	-1.0979	1e-04	0.2629
(Intercept) SE	(8e-04)		(0.0016)		(0.001)		(0.0012)		(4e-04)	
lag(x, 1)	0.0939	1.7352	-0.2708	-2.1902	-0.2994	-3.2098	-0.2228	-2.7318	0.1385	5.3289
lag(x, 1) SE	(0.0541)		(0.1236)		(0.0933)		(0.0815)		(0.026)	
T_A	0.0032	3.8695	0.001	0.596	0.0027	2.5215	0.0017	1.3581	-2e-04	-0.4695
T_A SE	(8e-04)		(0.0016)		(0.0011)		(0.0013)		(5e-04)	
lag(x, 1):T_A	0.0027	0.0457	0.2773	2.2144	0.3527	3.6798	0.3128	3.656	-0.1206	-3.5355
lag(x, 1):T_A SE	(0.0595)		(0.1252)		(0.0959)		(0.0855)		(0.0341)	
T_B	0.003	3.5106	1e-04	0.0861	0.0023	2.0004	0.0011	0.8621	0.6548	4.8434
T_B SE	(8e-04)		(0.0017)		(0.0011)		(0.0013)		(0.1438)	
lag(x, 1):T_B	-0.0639	-1.0701	0.4135	3.2259	0.3449	3.5495	0.3151	3.7193	-0.0513	-0.2654
lag(x, 1):T_B SE	(0.0597)		(0.1282)		(0.0972)		(0.0847)		(0.0035)	
F Stat	6.93902	(5,3494)	4.93376	(5,3494)	5.50306	(5,3494)	7.82256	(5,3494)	9.79099	(3,3496)
R Sq	0.00983		0.00701		0.00781		0.01107		0.00833	
R Sq Adj	0.00842		0.00559		0.00639		0.00966		0.00748	

	sh600831 Estimate	sh600831 t value	sh600834 Estimate	sh600834 t value	sh600835 Estimate	sh600835 t value	sh600846 Estimate	sh600846 t value	sh600859 Estimate	sh600859 t value
(Intercept)	0.0019	1.4605	5e-04	1.4364	-7e-04	-2.4802	-6e-04	-0.6335	-0.0013	-1.2165
(Intercept) SE	(0.0013)		(3e-04)		(3e-04)		(0.001)		(0.0011)	
lag(x, 1)	-0.1452	-2.0726	0.0967	3.3931	0.0362	1.5899	-0.1467	-2.2934	0.3573	4.0827
lag(x, 1) SE	(0.0701)		(0.0285)		(0.0227)		(0.064)		(0.0875)	
T_A	-0.0023	-1.7401	-6e-04	-1.1731	0.0014	2.7982	6e-04	0.581	0.0016	1.4342
T_A SE	(0.0013)		(5e-04)		(5e-04)		(0.001)		(0.0011)	
lag(x, 1):T_A	0.1349	1.8051	-0.0862	-2.0688	0.0363	0.9249	0.1961	2.9278	-0.333	-3.6954
lag(x, 1):T_A SE	(0.0747)		(0.0417)		(0.0392)		(0.067)		(0.0901)	
T_B	-0.0015	-1.1217	-0.0012	-2.2524	0.0022	3.4933	7e-04	0.5582	0.0012	1.023
T_B SE	(0.0014)		(5e-04)		(6e-04)		(0.0012)		(0.0011)	
lag(x, 1):T_B	0.2343	3.1722	-0.0455	-1.1207	-0.0065	-0.1387	0.2617	3.5564	-0.3194	-3.4668
lag(x, 1):T_B SE	(0.0739)		(0.0406)		(0.0466)		(0.0736)		(0.0921)	
F Stat	4.9401	(5,3494)	4.18789	(5,3494)	4.95052	(5,3494)	4.34655	(5,3494)	4.56786	(5,3494)
R Sq	0.00702		0.00596		0.00703		0.00618		0.00649	
R Sq Adj	0.0056		0.00453		0.00561		0.00476		0.00507	

	sh600000 Estimate	sh600000 t value	sh600009 Estimate	sh600009 t value	sh600054 Estimate	sh600054 t value	sh600064 Estimate	sh600064 t value	sh600067 Estimate	sh600067 t value
(Intercept)	0.0017	2.0073	-5e-04	-0.9865	-8e-04	-0.9195	-3e-04	-0.7568	0	0.0421
(Intercept) SE	(9e-04)		(5e-04)		(9e-04)		(4e-04)		(7e-04)	
lag(x, 1)	-0.2571	-2.9076	0.1796	3.9804	-0.265	-2.745	-0.0379	-1.1082	0.2689	5.4006
lag(x, 1) SE	(0.0884)		(0.0451)		(0.0965)		(0.0342)		(0.0498)	
T_A	-0.0017	-1.8146	7e-04	1.2448	9e-04	0.8207	2e-04	0.3673	-2e-04	-0.205
T_A SE	(9e-04)		(6e-04)		(0.001)		(6e-04)		(8e-04)	
lag(x, 1):T_A	0.2497	2.6911	-0.1831	-3.659	0.4437	3.9589	0.087	1.9734	-0.2084	-3.5871
lag(x, 1):T_A SE	(0.0928)		(0.05)		(0.1121)		(0.0441)		(0.0581)	
T_B	-0.0018	-2.0489	9e-04	1.4015	9e-04	0.9893	0.001	1.6377	-1e-04	-0.0972
T_B SE	(9e-04)		(6e-04)		(9e-04)		(6e-04)		(8e-04)	
lag(x, 1):T_B	0.2665	2.9199	-0.1913	-3.2813	0.2895	2.9461	0.0902	2.0178	-0.2508	-4.5653
lag(x, 1):T_B SE	(0.0913)		(0.0583)		(0.0983)		(0.0447)		(0.0549)	
F Stat	2.56685	(5,3344)	3.58195	(5,3344)	4.11836	(5,3344)	2.12756	(5,3344)	6.78072	(5,3344)
R Sq	0.00382		0.00533		0.00612		0.00317		0.01004	
R Sq Adj	0.00233		0.00384		0.00463		0.00168		0.00856	

	sh600082 Estimate	sh600082 t value	sh600097 Estimate	sh600097 t value	sh600159 Estimate	sh600159 t value	sh600162 Estimate	sh600162 t value	sh600256 Estimate	sh600256 t value
(Intercept)	-0.0012	-0.9039	0.0024	2.2499	-6e-04	-0.8947	1e-04	0.4779	0.0018	1.9952
(Intercept) SE	(0.0013)		(0.0011)		(7e-04)		(3e-04)		(9e-04)	
lag(x, 1)	0.4979	5.0534	0.3618	4.2488	0.2172	4.4678	0.0661	3.4821	0.0757	1.3821
lag(x, 1) SE	(0.0985)		(0.0852)		(0.0486)		(0.019)		(0.0548)	
T_A	0.0012	0.8415	-0.0024	-2.2124	4e-04	0.5224	-0.0013	-1.6246	-0.0022	-2.291
T_A SE	(0.0014)		(0.0011)		(8e-04)		(8e-04)		(9e-04)	
lag(x, 1):T_A	-0.4699	-4.6068	-0.3308	-3.7813	-0.1735	-3.2854	-0.1337	-2.7069	-0.0382	-0.6614
lag(x, 1):T_A SE	(0.102)		(0.0875)		(0.0528)		(0.0494)		(0.0577)	
T_B	0.0011	0.8069	-0.0021	-1.7996	0.0011	1.1701	-0.0046	-2.4177	9e-04	0.9453
T_B SE	(0.0014)		(0.0012)		(0.001)		(0.0019)		(6e-04)	
lag(x, 1):T_B	-0.4199	-4.1473	-0.2761	-2.9864	-0.1882	-2.9603	0.0242	0.2434	-0.5572	-6.4511
lag(x, 1):T_B SE	(0.1013)		(0.0925)		(0.0636)		(0.0995)		(0.0454)	
F Stat	7.87541	(5,3344)	5.82911	(5,3344)	5.29115	(5,3344)	4.75435	(5,3344)	3.8547	(3,3346)
R Sq	0.01164		0.00864		0.00785		0.00706		0.00344	
R Sq Adj	0.01016		0.00716		0.00637		0.00557		0.00255	

	sh600601 Estimate	sh600601 t value	sh600609 Estimate	sh600609 t value	sh600611 Estimate	sh600611 t value	sh600612 Estimate	sh600612 t value	sh600618 Estimate	sh600618 t value
(Intercept)	-0.0031	-2.8473	-0.0027	-3.6249	-0.0044	-3.0233	0.0022	1.0252	-3e-04	-0.8196
(Intercept) SE	(0.0011)		(8e-04)		(0.0015)		(0.0022)		(4e-04)	
lag(x, 1)	-0.0832	-1.1832	0.1502	2.5447	-0.1794	-2.0742	0.4434	3.4278	0.06	2.1949
lag(x, 1) SE	(0.0703)		(0.059)		(0.0865)		(0.1293)		(0.0273)	
T_A	0.003	2.7286	0.0031	3.8288	0.0045	3.0327	-0.0022	-0.9888	0.0016	0.7721
T_A SE	(0.0011)		(8e-04)		(0.0015)		(0.0022)		(0.002)	
lag(x, 1):T_A	0.0764	1.0131	-0.0927	-1.4297	0.2664	3.012	-0.4168	-3.1907	-0.3947	-3.0573
lag(x, 1):T_A SE	(0.0754)		(0.0649)		(0.0884)		(0.1306)		(0.1291)	
T_B	0.0033	2.8191	0.0028	3.4411	0.0056	3.0199	-0.0013	-0.5273	5e-04	0.9081
T_B SE	(0.0012)		(8e-04)		(0.0019)		(0.0024)		(5e-04)	
lag(x, 1):T_B	0.1264	1.705	-0.0692	-1.0848	0.2825	2.7518	-0.3997	-2.8043	0.069	1.9522
lag(x, 1):T_B SE	(0.0741)		(0.0638)		(0.1027)		(0.1425)		(0.0354)	
F Stat	2.66323	(5,3344)	8.06214	(5,3344)	7.79651	(5,3344)	3.08394	(5,3344)	9.26809	(5,3344)
R Sq	0.00397		0.01191		0.01152		0.00459		0.01367	
R Sq Adj	0.00248		0.01043		0.01005		0.0031		0.01219	

	sh600620 Estimate	sh600620 t value	sh600621 Estimate	sh600621 t value	sh600622 Estimate	sh600622 t value	sh600624 Estimate	sh600624 t value	sh600626 Estimate	sh600626 t value
(Intercept)	-1e-04	-0.2477	-2e-04	-0.2925	0.0017	1.1283	1e-04	0.2546	0.0013	2.9165
(Intercept) SE	(3e-04)		(7e-04)		(0.0015)		(6e-04)		(4e-04)	
lag(x, 1)	-0.0112	-0.4981	-0.1572	-2.5077	0.3595	2.8311	0.1423	4.8313	0.0485	1.6398
lag(x, 1) SE	(0.0225)		(0.0627)		(0.127)		(0.0295)		(0.0296)	
T_A	2e-04	0.3312	2e-04	0.265	-0.0018	-1.154	-2e-04	-0.3175	-0.0019	-3.435
T_A SE	(6e-04)		(8e-04)		(0.0015)		(6e-04)		(5e-04)	
lag(x, 1):T_A	0.1	2.4031	0.1969	2.9268	-0.2861	-2.2298	-0.1436	-3.9543	-0.0184	-0.4674
lag(x, 1):T_A SE	(0.0416)		(0.0673)		(0.1283)		(0.0363)		(0.0393)	
T_B	5e-04	0.6745	5e-04	0.5906	-0.0014	-0.8443	0.0419	2.6553	-0.0019	-2.3977
T_B SE	(8e-04)		(8e-04)		(0.0017)		(7e-04)		(8e-04)	
lag(x, 1):T_B	0.2399	5.1243	0.2935	4.3196	-0.3625	-2.6117	0.2511	4.223	0.1305	2.7934
lag(x, 1):T_B SE	(0.0468)		(0.0679)		(0.1388)		(0.0591)		(0.0467)	
F Stat	7.64286	(5,3344)	7.28162	(5,3344)	4.97171	(5,3344)	7.82676	(3,3346)	8.23627	(5,3344)
R Sq	0.0113		0.01077		0.00738		0.00697		0.01217	
R Sq Adj	0.00982		0.00929		0.00589		0.00608		0.01069	

	sh600628 Estimate	sh600628 t value	sh600630 Estimate	sh600630 t value	sh600636 Estimate	sh600636 t value	sh600638 Estimate	sh600638 t value	sh600639 Estimate	sh600639 t value
(Intercept)	0.0022	2.3816	0.0015	0.6656	-5e-04	-0.7082	3e-04	0.7605	0	0.0841
(Intercept) SE	(9e-04)		(0.0022)		(7e-04)		(3e-04)		(3e-04)	
lag(x, 1)	0.3214	4.2473	-0.598	-4.4749	0.2642	6.9142	0.0036	0.1301	0.0994	4.2911
lag(x, 1) SE	(0.0757)		(0.1336)		(0.0382)		(0.0275)		(0.0232)	
T_A	-0.0025	-2.5893	-0.0016	-0.7308	3e-04	0.4632	-6e-04	-1.2986	-2e-04	-0.4376
T_A SE	(0.001)		(0.0022)		(7e-04)		(5e-04)		(5e-04)	
lag(x, 1):T_A	-0.325	-4.0635	0.608	4.4934	-0.2489	-5.7257	0.0774	2.0156	-0.0979	-2.7416
lag(x, 1):T_A SE	(0.08)		(0.1353)		(0.0435)		(0.0384)		(0.0357)	
T_B	-0.0021	-2.2133	-5e-04	-0.2287	0.0014	1.2978	-2e-04	-0.2837	0.0047	2.9136
T_B SE	(0.001)		(0.0023)		(0.0011)		(7e-04)		(0.0016)	
lag(x, 1):T_B	-0.2683	-3.378	0.7446	5.4359	-0.2749	-4.2943	0.2022	4.2637	-0.1355	-1.6127
lag(x, 1):T_B SE	(0.0794)		(0.137)		(0.064)		(0.0474)		(0.084)	
F Stat	7.09753	(5,3344)	9.52371	(5,3344)	9.97374	(5,3344)	7.88035	(5,3344)	5.53592	(5,3344)
R Sq	0.0105		0.01404		0.01469		0.01165		0.00821	
R Sq Adj	0.00902		0.01257		0.01322		0.01017		0.00673	

	sh600640 Estimate	sh600640 t value	sh600642 Estimate	sh600642 t value	sh600643 Estimate	sh600643 t value	sh600644 Estimate	sh600644 t value	sh600645 Estimate	sh600645 t value
(Intercept)	0.0044	3.4582	0.0014	1.0999	-4e-04	-0.3006	1e-04	0.2239	-4e-04	-1.3521
(Intercept) SE	(0.0013)		(0.0013)		(0.0013)		(4e-04)		(3e-04)	
lag(x, 1)	-0.0205	-0.2535	-0.4691	-3.2294	-0.108	-1.2304	-0.0476	-1.5118	0.0379	1.4035
lag(x, 1) SE	(0.0807)		(0.1453)		(0.0878)		(0.0315)		(0.027)	
T_A	-0.0048	-3.6205	-0.0016	-1.2474	-2e-04	-0.1048	-6e-04	-1.1442	0.0023	2.2817
T_A SE	(0.0013)		(0.0013)		(0.0015)		(5e-04)		(0.001)	
lag(x, 1):T_A	0.1195	1.4327	0.474	3.2369	0.3227	3.1888	0.0836	1.8139	0.1502	1.5644
lag(x, 1):T_A SE	(0.0834)		(0.1464)		(0.1012)		(0.0461)		(0.096)	
T_B	-0.0044	-3.19	-8e-04	-0.5762	6e-04	0.4169	2e-04	0.2739	9e-04	2.0605
T_B SE	(0.0014)		(0.0015)		(0.0013)		(6e-04)		(5e-04)	
lag(x, 1):T_B	0.0519	0.5967	0.5582	3.6303	0.1804	2.01	0.1245	3.0457	0.0784	2.2106
lag(x, 1):T_B SE	(0.087)		(0.1538)		(0.0897)		(0.0409)		(0.0354)	
F Stat	7.31094	(5,3344)	3.51248	(5,3344)	7.20887	(5,3344)	2.92716	(5,3344)	8.25303	(5,3344)
R Sq	0.01081		0.00522		0.01066		0.00436		0.01219	
R Sq Adj	0.00933		0.00374		0.00918		0.00287		0.01071	

	sh600650 Estimate	sh600650 t value	sh600651 Estimate	sh600651 t value	sh600658 Estimate	sh600658 t value	sh600663 Estimate	sh600663 t value	sh600668 Estimate	sh600668 t value
(Intercept)	-1e-04	-0.1489	-0.0013	-1.5567	-1e-04	-0.4095	2e-04	0.6687	9e-04	1.5643
(Intercept) SE	(8e-04)		(9e-04)		(3e-04)		(2e-04)		(6e-04)	
lag(x, 1)	0.2959	5.4972	0.2613	6.3289	0.0623	2.9952	0.0528	3.0471	-0.0305	-0.6971
lag(x, 1) SE	(0.0538)		(0.0413)		(0.0208)		(0.0173)		(0.0438)	
T_A	2e-04	0.2168	0.0014	1.5621	1e-04	0.2051	-3e-04	-0.1185	-0.001	-1.5823
T_A SE	(8e-04)		(9e-04)		(6e-04)		(0.0025)		(6e-04)	
lag(x, 1):T_A	-0.2377	-4.1417	-0.2555	-5.534	-0.1335	-3.0642	0.6735	4.6637	0.0716	1.4689
lag(x, 1):T_A SE	(0.0574)		(0.0462)		(0.0436)		(0.1444)		(0.0488)	
T_B	6e-04	0.5668	0.001	0.8246	0	-0.0047	0.0048	2.1583	-0.0013	-1.4315
T_B SE	(0.0011)		(0.0012)		(9e-04)		(0.0013)		(9e-04)	
lag(x, 1):T_B	-0.2767	-3.9783	-0.2803	-4.4828	0.1191	2.1401	-0.2291	-3.9713	0.2198	3.7579
lag(x, 1):T_B SE	(0.0695)		(0.0625)		(0.0556)		(0.0825)		(0.0585)	
F Stat	7.8609	(5,3344)	8.81636	(5,3344)	4.97464	(5,3344)	11.84658	(3,3346)	6.19821	(5,3344)
R Sq	0.01162		0.01301		0.00738		0.01051		0.00918	
R Sq Adj	0.01014		0.01154		0.0059		0.00962		0.0077	

	sh600674 Estimate	sh600674 t value	sh600683 Estimate	sh600683 t value	sh600684 Estimate	sh600684 t value	sh600692 Estimate	sh600692 t value	sh600802 Estimate	sh600802 t value
(Intercept)	-0.0046	-2.8403	-0.002	-3.117	-0.0027	-2.1132	-0.0017	-1.8994	0.001	1.603
(Intercept) SE	(0.0016)		(7e-04)		(0.0013)		(9e-04)		(6e-04)	
lag(x, 1)	1.7522	11.5859	0.1441	3.1026	0.4811	5.7264	-0.0035	-0.06	0.2526	5.1978
lag(x, 1) SE	(0.1512)		(0.0464)		(0.084)		(0.0591)		(0.0486)	
T_A	0.0047	2.7348	0.002	2.7304	0.0027	2.0268	0.0031	2.944	-0.0018	-1.6578
T_A SE	(0.0017)		(7e-04)		(0.0013)		(0.001)		(0.0011)	
lag(x, 1):T_A	-1.707	-11.0904	-0.0495	-0.9701	-0.4205	-4.8732	0.0595	0.8363	-0.1996	-2.928
lag(x, 1):T_A SE	(0.1539)		(0.0511)		(0.0863)		(0.0711)		(0.0682)	
T_B	0.0045	2.785	0.0044	4.535	0.0037	2.4538	0.0014	1.5556	-0.0012	-1.675
T_B SE	(0.0016)		(0.001)		(0.0015)		(9e-04)		(7e-04)	
lag(x, 1):T_B	-1.6784	-10.9907	-0.0422	-0.7078	-0.4247	-4.587	0.1089	1.7445	-0.1891	-3.6008
lag(x, 1):T_B SE	(0.1527)		(0.0596)		(0.0926)		(0.0624)		(0.0525)	
F Stat	30.64091	(5,3344)	11.68145	(5,3344)	9.73783	(5,3344)	8.08201	(5,3344)	8.50787	(5,3344)
R Sq	0.04381		0.01717		0.01435		0.01194		0.01256	
R Sq Adj	0.04238		0.0157		0.01288		0.01046		0.01108	

	sh600812 Estimate	sh600812 t value	sh600821 Estimate	sh600821 t value	sh600824 Estimate	sh600824 t value	sh600826 Estimate	sh600826 t value	sh600830 Estimate	sh600830 t value
(Intercept)	1e-04	0.2867	0.0015	1.9759	0.0018	2.0229	-4e-04	-1.4659	-9e-04	-0.5379
(Intercept) SE	(5e-04)		(8e-04)		(9e-04)		(3e-04)		(0.0017)	
lag(x, 1)	0.2108	5.2456	-0.0616	-1.1921	0.1353	2.1557	0.0916	4.2162	-0.2719	-2.537
lag(x, 1) SE	(0.0402)		(0.0517)		(0.0628)		(0.0217)		(0.1072)	
T_A	-6e-04	-0.8772	-0.0016	-1.9773	-0.002	-2.1167	0.0015	2.4033	0.001	0.5733
T_A SE	(7e-04)		(8e-04)		(9e-04)		(6e-04)		(0.0017)	
lag(x, 1):T_A	-0.1491	-3.0411	0.0723	1.3076	-0.1369	-2.0793	0.019	0.4509	0.3356	3.0878
lag(x, 1):T_A SE	(0.049)		(0.0553)		(0.0659)		(0.0421)		(0.1087)	
T_B	1e-04	0.1893	-0.0031	-2.3021	-0.0021	-1.9602	0.002	2.5296	-0.0012	-0.5621
T_B SE	(6e-04)		(0.0013)		(0.0011)		(8e-04)		(0.0021)	
lag(x, 1):T_B	-0.2	-4.1836	0.3349	4.7168	0.0383	0.5101	-0.1258	-2.5084	0.5353	4.3061
lag(x, 1):T_B SE	(0.0478)		(0.071)		(0.0751)		(0.0502)		(0.1243)	
F Stat	6.8899	(5,3344)	8.26519	(5,3344)	5.57182	(5,3344)	7.59387	(5,3344)	8.19734	(5,3344)
R Sq	0.0102		0.01221		0.00826		0.01123		0.01211	
R Sq Adj	0.00872		0.01073		0.00678		0.00975		0.01063	

	sh600831 Estimate	sh600831 t value	sh600834 Estimate	sh600834 t value	sh600835 Estimate	sh600835 t value	sh600846 Estimate	sh600846 t value	sh600859 Estimate	sh600859 t value
(Intercept)	-0.0034	-3.488	0.0028	2.7268	-0.0018	-1.6335	0.0027	2.2761	0.0014	1.4629
(Intercept) SE	(0.001)		(0.001)		(0.0011)		(0.0012)		(0.001)	
lag(x, 1)	0.0966	2.0097	0.0777	1.4005	0.1631	2.1391	0.0485	0.6744	0.3677	5.3502
lag(x, 1) SE	(0.0481)		(0.0555)		(0.0762)		(0.0718)		(0.0687)	
T_A	0.0036	3.5003	-0.0028	-2.6324	0.0019	1.6442	-0.0028	-2.3189	-0.0013	-1.3174
T_A SE	(0.001)		(0.0011)		(0.0011)		(0.0012)		(0.001)	
lag(x, 1):T_A	-0.1228	-2.2791	-0.0357	-0.6081	-0.1236	-1.5746	-0.0246	-0.3307	-0.3503	-4.9199
lag(x, 1):T_A SE	(0.0539)		(0.0587)		(0.0785)		(0.0745)		(0.0712)	
T_B	0.0035	3.2692	-0.0043	-3.3158	0.0032	2.3189	-0.0032	-2.3136	-0.002	-1.5809
T_B SE	(0.0011)		(0.0013)		(0.0014)		(0.0014)		(0.0013)	
lag(x, 1):T_B	-0.0175	-0.3134	0.0771	0.9765	-0.0939	-0.9686	0.1161	1.4137	-0.317	-3.4894
lag(x, 1):T_B SE	(0.0558)		(0.079)		(0.0969)		(0.0821)		(0.0908)	
F Stat	5.06378	(5,3344)	5.07996	(5,3344)	3.28944	(5,3344)	4.90305	(5,3344)	6.55085	(5,3344)
R Sq	0.00751		0.00754		0.00489		0.00728		0.0097	
R Sq Adj	0.00603		0.00605		0.00341		0.00579		0.00822	

	sh600000 Estimate	sh600000 t value	sh600009 Estimate	sh600009 t value	sh600054 Estimate	sh600054 t value	sh600064 Estimate	sh600064 t value	sh600067 Estimate	sh600067 t value
(Intercept)	0.0041	2.9193	0.0024	2.914	-0.0024	-2.5445	8e-04	0.4648	-0.0072	-3.6683
(Intercept) SE	(0.0014)		(8e-04)		(9e-04)		(0.0018)		(0.002)	
lag(x, 1)	-1.0653	-4.7778	-0.105	-1.366	-0.0215	-0.3447	-0.3036	-2.7156	0.1913	1.8868
lag(x, 1) SE	(0.223)		(0.0768)		(0.0623)		(0.1118)		(0.1014)	
T_A	-0.0032	-2.0472	-0.0021	-2.4527	0.0026	2.6708	-6e-04	-0.3273	0.0072	3.6231
T_A SE	(0.0016)		(8e-04)		(0.001)		(0.0018)		(0.002)	
lag(x, 1):T_A	1.1045	4.6696	0.0846	1.0304	0.0702	1.0766	0.346	3.0071	-0.1378	-1.3329
lag(x, 1):T_A SE	(0.2365)		(0.0821)		(0.0652)		(0.1151)		(0.1034)	
T_B	-0.0042	-2.997	-0.0028	-3.1755	0.0039	2.2686	-0.0013	-0.6993	0.0075	3.5614
T_B SE	(0.0014)		(9e-04)		(0.0017)		(0.0019)		(0.0021)	
lag(x, 1):T_B	1.0595	4.7353	0.1714	2.1305	-0.2517	-2.1577	0.3335	2.9143	-0.1064	-0.9744
lag(x, 1):T_B SE	(0.2237)		(0.0805)		(0.1166)		(0.1144)		(0.1092)	
F Stat	6.27909	(5,3094)	4.15901	(5,3094)	4.33824	(5,3094)	2.66821	(5,3094)	5.30496	(5,3094)
R Sq	0.01005		0.00668		0.00696		0.00429		0.0085	
R Sq Adj	0.00845		0.00507		0.00536		0.00268		0.0069	

	sh600082 Estimate	sh600082 t value	sh600097 Estimate	sh600097 t value	sh600159 Estimate	sh600159 t value	sh600162 Estimate	sh600162 t value	sh600256 Estimate	sh600256 t value
(Intercept)	-0.0016	-1.7626	-0.0024	-2.4804	-0.0048	-2.8279	0.0014	2.6746	-0.0036	-1.7055
(Intercept) SE	(9e-04)		(0.001)		(0.0017)		(5e-04)		(0.0021)	
lag(x, 1)	0.2932	4.5343	0.2399	3.2758	0.4162	4.0023	0.0687	2.0569	1.002	5.2185
lag(x, 1) SE	(0.0647)		(0.0732)		(0.104)		(0.0334)		(0.192)	
T_A	0.0019	1.7914	0.0028	2.728	0.0049	2.8369	-0.002	-2.9584	0.0031	1.3315
T_A SE	(0.0011)		(0.001)		(0.0017)		(7e-04)		(0.0023)	
lag(x, 1):T_A	-0.2771	-3.6923	-0.192	-2.5266	-0.3599	-3.3755	-0.0313	-0.7198	-0.9569	-4.7509
lag(x, 1):T_A SE	(0.075)		(0.076)		(0.1066)		(0.0435)		(0.2014)	
T_B	0.0016	1.6186	0.0022	1.8998	0.0046	2.6125	-0.0024	-2.9524	0.0035	1.6699
T_B SE	(0.001)		(0.0012)		(0.0018)		(8e-04)		(0.0021)	
lag(x, 1):T_B	-0.2284	-3.355	-0.229	-2.6779	-0.3634	-3.3717	-0.015	-0.3204	-0.9698	-5.0267
lag(x, 1):T_B SE	(0.0681)		(0.0855)		(0.1078)		(0.0469)		(0.1929)	
F Stat	6.61345	(5,3094)	4.91941	(5,3094)	6.87529	(5,3094)	4.22522	(5,3094)	6.69568	(5,3094)
R Sq	0.01057		0.00789		0.01099		0.00678		0.0107	
R Sq Adj	0.00898		0.00628		0.00939		0.00518		0.00911	

	sh600601 Estimate	sh600601 t value	sh600609 Estimate	sh600609 t value	sh600611 Estimate	sh600611 t value	sh600612 Estimate	sh600612 t value	sh600618 Estimate	sh600618 t value
(Intercept)	6e-04	0.7657	0	0.0942	0.0016	2.0199	0	0.1705	0.0012	1.4857
(Intercept) SE	(7e-04)		(2e-04)		(8e-04)		(3e-04)		(8e-04)	
lag(x, 1)	-0.122	-1.7697	0.0831	4.6535	0.1699	3.2087	0.0346	1.6934	0.2936	5.574
lag(x, 1) SE	(0.0689)		(0.0179)		(0.053)		(0.0204)		(0.0527)	
T_A	1e-04	0.0709	0.0029	2.9284	-0.0013	-1.5463	-0.001	-1.1858	-0.0013	-1.5508
T_A SE	(0.0011)		(6e-04)		(8e-04)		(9e-04)		(9e-04)	
lag(x, 1):T_A	0.2742	2.4645	-0.0842	-1.9892	-0.1755	-2.8164	-0.1659	-2.3198	-0.2169	-3.8531
lag(x, 1):T_A SE	(0.1112)		(0.0748)		(0.0623)		(0.0715)		(0.0563)	
T_B	-8e-04	-1.0361	0.0024	3.4486	-0.0024	-2.6626	0.0018	2.3269	-0.0015	-1.1279
T_B SE	(8e-04)		(6e-04)		(9e-04)		(8e-04)		(0.0013)	
lag(x, 1):T_B	0.144	2.0145	-0.0551	-1.7818	-0.068	-1.1754	0.0932	1.9076	-0.2437	-2.9451
lag(x, 1):T_B SE	(0.0715)		(0.0734)		(0.0578)		(0.0489)		(0.0827)	
F Stat	1.92883	(5,3094)	21.65464	(5,3094)	7.69418	(5,3094)	4.56259	(5,3094)	9.63494	(5,3094)
R Sq	0.00311		0.00694		0.01228		0.00732		0.01533	
R Sq Adj	0.0015		0.00662		0.01069		0.00572		0.01374	

	sh600620 Estimate	sh600620 t value	sh600621 Estimate	sh600621 t value	sh600622 Estimate	sh600622 t value	sh600624 Estimate	sh600624 t value	sh600626 Estimate	sh600626 t value
(Intercept)	0.001	0.8741	-9e-04	-1.7331	0.0019	1.8013	0	0.047	6e-04	0.7626
(Intercept) SE	(0.0011)		(5e-04)		(0.0011)		(3e-04)		(7e-04)	
lag(x, 1)	0.3617	5.5515	0.1392	4.5859	0.3065	3.8721	-0.0055	-0.2403	-0.0236	-0.5255
lag(x, 1) SE	(0.0651)		(0.0304)		(0.0791)		(0.0227)		(0.0449)	
T_A	-0.0012	-1.0216	0.0013	2.1189	-0.0013	-1.0069	1e-04	0.1163	-6e-04	-0.8007
T_A SE	(0.0012)		(6e-04)		(0.0013)		(8e-04)		(8e-04)	
lag(x, 1):T_A	-0.3328	-4.8617	-0.0761	-1.9854	-0.3419	-3.4385	0.1753	2.6258	0.119	2.4308
lag(x, 1):T_A SE	(0.0684)		(0.0383)		(0.0994)		(0.0667)		(0.049)	
T_B	-4e-04	-0.3419	0.001	0.9355	-0.0021	-1.8953	-1e-04	-0.2179	-0.0021	-2.7714
T_B SE	(0.0013)		(0.001)		(0.0011)		(7e-04)		(7e-04)	
lag(x, 1):T_B	-0.3086	-4.0435	-0.3369	-4.6244	-0.2377	-2.9183	0.1395	3.491	0.1545	3.783
lag(x, 1):T_B SE	(0.0763)		(0.0729)		(0.0815)		(0.04)		(0.0436)	
F Stat	7.52327	(5,3094)	8.38706	(5,3094)	6.50647	(5,3094)	4.81695	(5,3094)	8.29992	(3,3096)
R Sq	0.01201		0.01337		0.01041		0.00772		0.00798	
R Sq Adj	0.01042		0.01178		0.00881		0.00612		0.00702	

	sh600628 Estimate	sh600628 t value	sh600630 Estimate	sh600630 t value	sh600636 Estimate	sh600636 t value	sh600638 Estimate	sh600638 t value	sh600639 Estimate	sh600639 t value
(Intercept)	0.0011	1.1265	2e-04	0.4832	-2e-04	-0.6059	0.0029	3.2421	2e-04	0.1267
(Intercept) SE	(9e-04)		(4e-04)		(3e-04)		(9e-04)		(0.0013)	
lag(x, 1)	0.3688	5.2539	-0.0318	-1.0833	0.0882	4.387	0.0452	0.8702	0.5771	6.0774
lag(x, 1) SE	(0.0702)		(0.0293)		(0.0201)		(0.0519)		(0.095)	
T_A	0	0.0332	5e-04	0.4857	6e-04	1.0294	-0.003	-3.2312	-2e-04	-0.1508
T_A SE	(0.0011)		(0.0011)		(6e-04)		(9e-04)		(0.0014)	
lag(x, 1):T_A	-0.3756	-4.2437	0.2716	3.7805	-0.1179	-2.6622	0.0464	0.8113	-0.5489	-5.3622
lag(x, 1):T_A SE	(0.0885)		(0.0718)		(0.0443)		(0.0572)		(0.1024)	
T_B	-0.0013	-1.2985	-2e-04	-0.3266	-0.0013	-2.4519	-0.0036	-3.2439	0	0.0106
T_B SE	(0.001)		(6e-04)		(5e-04)		(0.0011)		(0.0014)	
lag(x, 1):T_B	-0.3413	-4.6813	0.1081	2.8477	-0.3253	-4.7244	0.0147	0.2417	-0.5471	-5.6293
lag(x, 1):T_B SE	(0.0729)		(0.038)		(0.0572)		(0.0608)		(0.0972)	
F Stat	7.3073	(5,3094)	4.98942	(5,3094)	6.96434	(3,3096)	6.02867	(5,3094)	8.20449	(5,3094)
R Sq	0.01167		0.008		0.0067		0.00965		0.01309	
R Sq Adj	0.01007		0.0064		0.00574		0.00805		0.01149	

	sh600640 Estimate	sh600640 t value	sh600642 Estimate	sh600642 t value	sh600643 Estimate	sh600643 t value	sh600644 Estimate	sh600644 t value	sh600645 Estimate	sh600645 t value
(Intercept)	0.0013	1.9658	-4e-04	-1.0436	-0.0019	-1.1102	-9e-04	-1.6172	-9e-04	-2.1127
(Intercept) SE	(7e-04)		(4e-04)		(0.0017)		(6e-04)		(4e-04)	
lag(x, 1)	0.1318	3.8531	0.0758	2.8342	0.4534	4.0524	0.1075	2.9637	0.1342	3.9903
lag(x, 1) SE	(0.0342)		(0.0267)		(0.1119)		(0.0363)		(0.0336)	
T_A	-0.0016	-2.2033	6e-04	1.2372	0.0024	1.2684	0.0011	1.8156	0.0014	2.8273
T_A SE	(7e-04)		(5e-04)		(0.0019)		(6e-04)		(5e-04)	
lag(x, 1):T_A	-0.0627	-1.5553	-0.1107	-2.9176	-0.4044	-3.1867	-0.0905	-2.1591	-0.0866	-2.1077
lag(x, 1):T_A SE	(0.0403)		(0.0379)		(0.1269)		(0.0419)		(0.0411)	
T_B	0	-8e-04	2e-04	0.2744	0.002	1.1597	-0.0013	-0.7013	0.0023	2.5461
T_B SE	(0.0022)		(8e-04)		(0.0017)		(0.0018)		(9e-04)	
lag(x, 1):T_B	-0.4122	-3.3252	-0.151	-2.4636	-0.3733	-3.2895	-0.2727	-2.398	0.0968	1.6948
lag(x, 1):T_B SE	(0.124)		(0.0613)		(0.1135)		(0.1137)		(0.0571)	
F Stat	7.24828	(5,3094)	2.65861	(5,3094)	7.12046	(5,3094)	3.30631	(5,3094)	11.19449	(5,3094)
R Sq	0.01158		0.00428		0.01138		0.00531		0.01777	
R Sq Adj	0.00998		0.00267		0.00978		0.00371		0.01618	

	sh600650 Estimate	sh600650 t value	sh600651 Estimate	sh600651 t value	sh600658 Estimate	sh600658 t value	sh600663 Estimate	sh600663 t value	sh600668 Estimate	sh600668 t value
(Intercept)	0.0042	1.9622	-0.0018	-1.8476	0.0022	2.6913	0.0014	1.1318	-0.0014	-1.95
(Intercept) SE	(0.0022)		(0.001)		(8e-04)		(0.0012)		(7e-04)	
lag(x, 1)	-0.3347	-3.132	0.24	5.1356	0.0842	1.837	0.2265	3.2994	0.1836	4.9436
lag(x, 1) SE	(0.1069)		(0.0467)		(0.0459)		(0.0686)		(0.0371)	
T_A	-0.0042	-1.9473	0.0019	1.9012	-0.0025	-2.9077	-0.0015	-1.1669	0.0017	2.1706
T_A SE	(0.0022)		(0.001)		(9e-04)		(0.0013)		(8e-04)	
lag(x, 1):T_A	0.4353	4.0164	-0.2048	-4.0216	-0.069	-1.342	-0.1825	-2.558	-0.156	-3.6817
lag(x, 1):T_A SE	(0.1084)		(0.0509)		(0.0514)		(0.0713)		(0.0424)	
T_B	0.0114	2.6312	0.003	2.0998	-0.0024	-2.3715	0.0032	1.8482	0.0093	2.9742
T_B SE	(0.0044)		(0.0014)		(0.001)		(0.0017)		(8e-04)	
lag(x, 1):T_B	0.1064	0.5949	-0.4508	-5.5926	0.0235	0.404	-0.1351	-1.4781	0.2205	4.1697
lag(x, 1):T_B SE	(0.1789)		(0.0806)		(0.0582)		(0.0914)		(0.0681)	
F Stat	14.37981	(5,3094)	9.235	(5,3094)	4.50957	(5,3094)	6.56757	(5,3094)	10.62566	(3,3096)
R Sq	0.02271		0.0147		0.00723		0.0105		0.01019	
R Sq Adj	0.02113		0.01311		0.00563		0.0089		0.00923	

	sh600674 Estimate	sh600674 t value	sh600683 Estimate	sh600683 t value	sh600684 Estimate	sh600684 t value	sh600692 Estimate	sh600692 t value	sh600802 Estimate	sh600802 t value
(Intercept)	-0.0015	-1.4758	-4e-04	-0.7369	-0.004	-2.6154	-1e-04	-0.3754	-3e-04	-1.0126
(Intercept) SE	(0.001)		(5e-04)		(0.0015)		(3e-04)		(3e-04)	
lag(x, 1)	0.6186	6.494	0.2087	6.4822	0.4469	5.0192	0.0984	5.2123	0.1085	5.2425
lag(x, 1) SE	(0.0953)		(0.0322)		(0.089)		(0.0189)		(0.0207)	
T_A	0.0014	1.1779	6e-04	0.9056	0.0041	2.6068	0.0011	1.419	0.001	1.7856
T_A SE	(0.0012)		(6e-04)		(0.0016)		(8e-04)		(5e-04)	
lag(x, 1):T_A	-0.6024	-5.6933	-0.1431	-3.4447	-0.4057	-4.4185	-0.1423	-2.3746	-0.0856	-2.0837
lag(x, 1):T_A SE	(0.1058)		(0.0416)		(0.0918)		(0.0599)		(0.0411)	
T_B	0.0016	1.5465	2e-04	0.2102	0.0043	2.6331	-0.0028	-1.1226	-0.0016	-2.112
T_B SE	(0.001)		(8e-04)		(0.0016)		(0.0025)		(4e-04)	
lag(x, 1):T_B	-0.5426	-5.5782	-0.1486	-3.032	-0.3337	-3.5366	0.4062	1.6729	-0.1764	-3.7804
lag(x, 1):T_B SE	(0.0973)		(0.049)		(0.0944)		(0.2428)		(0.0421)	
F Stat	11.59686	(5,3094)	10.3919	(5,3094)	9.42693	(5,3094)	7.16547	(5,3094)	10.38802	(3,3096)
R Sq	0.0184		0.01652		0.01501		0.01145		0.00997	
R Sq Adj	0.01681		0.01493		0.01341		0.00985		0.00901	

	sh600812 Estimate	sh600812 t value	sh600821 Estimate	sh600821 t value	sh600824 Estimate	sh600824 t value	sh600826 Estimate	sh600826 t value	sh600830 Estimate	sh600830 t value
(Intercept)	-0.0014	-1.6008	0.0016	1.1098	-0.0024	-1.5943	0.0023	0.7605	-0.0024	-1.838
(Intercept) SE	(9e-04)		(0.0015)		(0.0015)		(0.003)		(0.0013)	
lag(x, 1)	-0.0701	-1.0984	0.3527	4.4048	-0.3969	-3.1161	0.6923	4.5231	0.3321	5.0824
lag(x, 1) SE	(0.0638)		(0.0801)		(0.1274)		(0.1531)		(0.0653)	
T_A	0.0016	1.6773	-0.0017	-1.1095	0.0025	1.6288	-0.0023	-0.7707	0.0024	1.8422
T_A SE	(9e-04)		(0.0015)		(0.0015)		(0.003)		(0.0013)	
lag(x, 1):T_A	0.1571	2.3293	-0.3233	-3.9241	0.4464	3.469	-0.603	-3.8959	-0.2697	-3.9362
lag(x, 1):T_A SE	(0.0675)		(0.0824)		(0.1287)		(0.1548)		(0.0685)	
T_B	0.0014	1.2594	-0.002	-1.2096	-3e-04	-0.1162	-0.0019	-0.6358	0.0025	1.7119
T_B SE	(0.0011)		(0.0017)		(0.0025)		(0.0031)		(0.0015)	
lag(x, 1):T_B	0.1554	2.1347	-0.4011	-4.1174	0.5734	3.0714	-0.6342	-4.071	-0.3037	-3.8949
lag(x, 1):T_B SE	(0.0728)		(0.0974)		(0.1867)		(0.1558)		(0.078)	
F Stat	5.1149	(5,3094)	4.9092	(5,3094)	4.77868	(5,3094)	8.66309	(5,3094)	7.84287	(5,3094)
R Sq	0.0082		0.00787		0.00766		0.01381		0.01252	
R Sq Adj	0.0066		0.00627		0.00606		0.01221		0.01092	

	sh600831 Estimate	sh600831 t value	sh600834 Estimate	sh600834 t value	sh600835 Estimate	sh600835 t value	sh600846 Estimate	sh600846 t value	sh600859 Estimate	sh600859 t value
(Intercept)	-0.0012	-2.2282	-2e-04	-0.9093	-1e-04	-0.0587	-0.0011	-1.2342	3e-04	0.469
(Intercept) SE	(5e-04)		(3e-04)		(0.0017)		(9e-04)		(6e-04)	
lag(x, 1)	0.0775	2.8338	0.0604	2.9563	0.3629	3.4617	0.2828	6.1495	0.1681	4.7201
lag(x, 1) SE	(0.0274)		(0.0204)		(0.1048)		(0.046)		(0.0356)	
T_A	0.0015	2.321	0.0011	2.3833	-0.0013	-0.6954	0.0011	1.2377	-3e-04	-0.412
T_A SE	(6e-04)		(5e-04)		(0.0019)		(9e-04)		(6e-04)	
lag(x, 1):T_A	-0.0912	-2.3491	-0.0375	-0.8642	-0.3223	-2.377	-0.2656	-5.2228	-0.1394	-3.2314
lag(x, 1):T_A SE	(0.0388)		(0.0434)		(0.1356)		(0.0508)		(0.0431)	
T_B	0.0022	2.4333	-0.0214	-5.1627	3e-04	0.177	0.0014	1.2352	3e-04	0.4363
T_B SE	(9e-04)		(0.0041)		(0.0017)		(0.0012)		(8e-04)	
lag(x, 1):T_B	-0.1233	-2.2594	0.4527	3.594	-0.321	-3.0145	-0.2939	-4.6618	-0.2284	-4.3026
lag(x, 1):T_B SE	(0.0546)		(0.126)		(0.1065)		(0.063)		(0.0531)	
F Stat	3.53657	(5,3094)	11.29158	(5,3094)	4.03312	(5,3094)	8.05287	(5,3094)	5.44882	(5,3094)
R Sq	0.00568		0.01792		0.00648		0.01285		0.00873	
R Sq Adj	0.00408		0.01633		0.00487		0.01125		0.00713	