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A modified Teunter-Syntetos-Babai method for intermittent demand forecasting

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ABSTRACT

Intermittent demand refers to the specific demand pattern with frequent periods of zero demand. It occurs in a variety of industries including industrial equipment, automotive and specialty chemicals. In some industries or some sectors of industry, even majority of products are in intermittent demand pattern. Due to the usually small and highly variable demand sizes, accurate forecasting of intermittent demand has always been challenging. However, accurate forecasting of intermittent demand is critical to the effective inventory management. In this study we present a band new method - modified TSB method for the forecasting of intermittent demand. The proposed method is based on TSB method, and adopts similar strategy, which has been used in mSBA method to update demand interval and demand occurrence probability when current demand is zero. To evaluate the proposed method, 16289 daily demand records from the M5 data set that are identified as intermittent demands according to two criteria, and an empirical data set consisting three years' monthly demand history of 1718 medicine products are used. The proposed mTSB method achieves the best results on MASE and RMASE among all comparison methods on the M5 data set. On the empirical data set, the study shows that mTSB attains an ME of 0.07, which is the best among six comparison methods. Additionally, on the MSE measurement, mTSB shows a similar result as SES, both of which outperform other methods.

1. Introduction

Intermittent demand occurs when a commodity experiences compound periods of zero and nonzero demands. This demand pattern is in fact universal. It can be found in stock-keeping units (SKU) in most supply chains. Furthermore, Johnston et al. (2003) claimed that these items could account for up to 60% of the stock's total value. There are also a number of industries where intermittent demand is prevailing, which can include aerospace, information technology and automotive.

Forecasting of intermittent demand has been a difficult yet important task. The first attempt is proposed by Waller (2015) using the Single Exponential Smoothing (SES) method. SES uses a single exponential smoothing for demand size and takes the smoothing value to be the estimation of next time interval. In 1972, Croston proposed the Croston's method (see Croston, 1972; Xu et al., 2012). Croston's methods soon became a standard form of parametric intermittent demand forecasting

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models. It is often used as a base method in the intermittent demand forecasting research field. There are some important advantages underlying this unbiased method as shown in some early studies (see Willemain et al., 1994; Hasni et al., 2018) resulting in its frequent analysis and wide applications. However, there are also studies showing the weaknesses of the method, such as Sani and Kingsman (1997) and Syntetos et al. (2015).

Syntetos and Boylan (2001) pointed out that Croston's method is biased on over-forecast. Therefore, they (Syntetos & Boylan, 2001, 2005) proposed a new method called Sytetos-Boylan Approximation (SBA) which added a new smoothing constant of demand intervals. It has been shown that SBA can outperform Croston's method by means of enhanced forecast accuracy of intermittent demand (see Eaves and Kingsman, 2004; Gutierrez et al., 2008) although the negative bias still remains, it may still cause loss comparing with Croston's method on some special cases.

Intermittent demand forecasting becomes even more challenging for items with high risk of being obsolescence. Discussions (Babai et al., 2014) suggest that Croston's method and SBA method only update demand size and demand intervals during positive demand periods. To address this issue, Teunter & Duncan (2009) proposes a Teunter-Syntetos-Babai (TSB) method in which demand occurrence probabilities are used instead of the demand intervals to estimate the demand sizes. Such estimations are proved to be more effective when there are long zero-demand periods. Syntetos et al. (2015) discussed the difference between SBA method and TSB method, and pointed out that TSB method is unbiased and therefore leads to a lower mean squared error at each time point. Their study shows that, when the demand of an item is linear or the item is in a sudden obsolescence, TSB method outperforms SBA method on a theoretical level, especially for when demands are non-stationary. However, this might not be the case in some empirical researches.

Syntetos and Boylan (2005) (see also Babai, Dallery, Boubaker & Kalai, 2018a; Babai et al., 2018b) proposed a new forecasting method, namely the modified SBA (mSBA) method, based on SBA method which divides the demand interval into positive and zero demands. More specifically, mSBA method categorizes zero demand into two cases. On zero demand intervals, if the estimated demand interval of the previous time period is smaller than the actual demand interval of this time period, the estimation of demand interval remains the same as the estimated demand interval at previous time period. Otherwise, an exponential smoothing will be used as a positive demand interval.

Methods discussed above are a summary of parametric methods, as these methods involve parameters α and β. Non-parametric methods are also used in intermittent demand forecasting (Willemain et al., 2004; Zhou and Viswanathan, 2011; Viswanathan and Zhou, 2008; Teunter and Duncan, 2009). However, Kourentzes (2014) and Wallstrom and Segerstedt (2010) pointed out that although all parametric methods can produce a forecast at a fixed time point, none of the intermittent demand forecasting methods appears to be obviously most suitable.

In this paper, a new intermittent demand forecasting method, the modified TSB (mTSB) method, is proposed. It is based on the TSB method which uses the estimation of occurrence probabilities for both zero and nonzero demands. The mTSB methods adopts a similar strategy as used in mSBA method. For zero demand, mTSB method compares the actual occurrence probabilities of the period with the estimated occurrence probabilities of previous time period. If the estimation is greater than the actual probability, the occurrence probability of the next period would be updated by an exponential smoothing with positive demand; otherwise, it remains the same as it is in the original TSB method. Kwan (1991) showed that occurrence probabilities of intermittent demand is Bernoulli distributed. Therefore, the actual probability can be seen as probability of the Bernoulli distribution. After comparison of actual occurrence probability and estimation probability at previous time period, the estimation probability for next time period will approach to the probability of Bernoulli distribution.

The paper is arranged as the following. To begin with, the main focus of the paper, the mTSB method, is detailly explained in Section 2 following an overview of other related methods. The experiment is discussed in Section 3 with the corresponding performance analysis. Experiment results and discussions are included in Section 4. Finally, concluding comments are presented in Section 5.

2. Methodology

2.1. Notations

Throughout this paper the following notations will be used.

- D_t is the actual demand of an item at time t
- $\overline{D}_t$ is the estimated average demand per period for time t+1 made at time t
- Z_t is the observed demand size at time t
- Z_t is the estimated average demand size at time t
- T_t is the observed demand interval at time t
- $\overline{T}_t$ is the estimated demand interval at time period t
- p_t is the actual demand occurrence probability at time t
- $\overline{p}_t$ is the estimated demand occurrence probability at time t
- α, β are two smoothing parameters satisfying $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$

Three terms - demand size, demand intervals and demand per period will be frequently used in this paper.

Demand size is the amount of demand if demand occurs at a time point. Demand interval is the interval from one demand occurrence to the next. For instance, if the first demand occurred in the first month, and in the sixth month a second demand occurred, the demand interval would be 5. Demand per period is the mean value of demand size over one period.

2.2. Parametric methods

Single Exponential Smoothing method is one of the earliest methods proposed for intermittent demand forecasting, which can be traced back to 1956. This method only takes into account the demand size of each time period. SES uses the following equation to update the forecast:

$$\overline{D}_t = \overline{D}_{t-1} + \alpha\left(D_t - \overline{D}_{t-1}\right) \tag{1}$$

Croston's method is an improvement of SES method. In CRO method, both demand size and demand interval are updated using exponential smoothing. However, only periods with nonzero demands are updated. The formula of CRO method is as following:

$$\overline{D}_t = \frac{\overline{Z}_t}{\overline{T}_t} \tag{2}$$

where

$$\begin{cases} \overline{Z_t} = \overline{Z}_{t-1} + \alpha\left(Z_t - \overline{Z}_{t-1}\right), \ \overline{T_t} = \overline{T}_{t-1} + \beta\left(T_t - \overline{T}_{t-1}\right) \quad \textit{if } D_t > 0 \qquad \overline{Z}_t = \overline{Z}_{t-1} \quad \textit{and} \quad \overline{T}_t = \overline{T}_{t-1} \quad \textit{otherwise} \end{cases}$$

The original CRO method uses one smoothing parameter α for both demand size and demand interval, whereas the current implementations of CRO method generally use two parameters, α and β, for demand size and demand interval.

SBA also applies exponential smoothing on both demand sizes and demand intervals. The main difference between CRO method and SBA method is a deflating factor $(1 - \frac{\beta}{2})$ applied to the estimated mean demand, in order to correct the bias yielded in CRO method.

$$\overline{D}_t = \left(1 - \frac{\beta}{2}\right)\frac{\overline{Z}_t}{\overline{T}_t} \tag{3}$$

where

$$\begin{cases} \overline{Z_t} = \overline{Z}_{t-1} + \alpha\left(Z_t - \overline{Z}_{t-1}\right), \quad \overline{T_t} = \overline{T}_{t-1} + \beta\left(T_t - \overline{T}_{t-1}\right) \quad \textit{if } D_t > 0 \qquad \overline{Z}_t = \overline{Z}_{t-1} \quad \textit{and} \quad \overline{T}_t = \overline{T}_{t-1} \quad \textit{otherwise} \end{cases}$$

Since only positive demands are updated in CRO method and SBA method, TSB method is proposed which introduced the use of demand occurrence probability to replace the demand interval. The equation of TSB method is as below:

$$\overline{D}_t = \overline{p}_t \overline{Z}_t \tag{4}$$

where

$$\begin{cases} \overline{Z_t} = \overline{Z}_{t-1} + \alpha\left(Z_t - \overline{Z}_{t-1}\right), \quad \overline{p_t} = \overline{p}_{t-1} + \beta\left(p_t - \overline{p}_{t-1}\right) \quad \textit{if } D_t > 0 \overline{Z}_t = \overline{Z}_{t-1} \quad \textit{and} \quad \overline{p}_t \end{cases}$$
$$= \overline{p}_{t-1} + \beta\left(0 - \overline{p}_t\right) \quad \textit{otherwise}$$

In SBA method, zero demands do not contribute to the forecasting process. As a result, Babai et al. (2018a) proposed the mSBA method, in which demand intervals are treated differently according to the actual demand interval and the estimated demand interval. The equation of mSBA method is given as below:

$$\overline{D}_t = \left(1 - \frac{\beta}{2}\right)\frac{\overline{Z}_t}{\overline{T}_t} \tag{5}$$

where

$$\begin{cases} \overline{Z_t} = \overline{Z}_{t-1} + \alpha\left(Z_t - \overline{Z}_{t-1}\right), \quad \overline{T_t} = \overline{T}_{t-1} + \beta\left(T_t - \overline{T}_{t-1}\right) \text{ if } D_t > 0 & \begin{cases} \overline{Z}_t = \overline{Z}_{t-1} \\ \overline{T}_t = \begin{cases} \overline{T}_{t-1} & \text{if } T_t \le \overline{T}_{t-1} \\ \overline{T}_{t-1} + \beta\left(T_t - \overline{T}_{t-1}\right) & \text{if } T_t > \overline{T}_{t-1} \end{cases} \end{cases} \text{otherwise} \end{cases}$$

2.3. mTSB method

As has been shown in the literature, the TSB method is unbiased and can lead to a lower mean squared error at each time point. The TSB method also shows good performance in dealing with linear and sudden obsolescence cases as it always provides up-to-date forecasts, even after long zero demand intervals. However, TSB can be empirically outperformed by SBA method in some situation, due to updated demand probability in every period, which may affect the performance estimated by MSE. mSBA is an extension of SBA method, which updates the demand interval on zero demand periods according to the actual demand interval and the estimated demand interval. mSBA has shown better performance compared to the SBA method. In this study, we have proposed a new method named modified TSB (mTSB) method. Our proposed method is an extension of TSB method, which aims to combine the advantages of both the TSB and the mSBA method. The formula of mTSB method is as following.

$$\overline{D}_t = \overline{p}_t \overline{Z}_t \tag{6}$$

where

$$\begin{cases} \overline{Z_t} = \overline{Z}_{t-1} + \alpha\left(Z_t - \overline{Z}_{t-1}\right), \quad \overline{p_t} = \overline{p}_{t-1} + \beta\left(p_t - \overline{p}_{t-1}\right) \quad \text{if } D_t > 0 & \begin{cases} \overline{Z}_t = \overline{Z}_{t-1} \\ \overline{p}_t = \begin{cases} \overline{p}_{t-1} + \beta\left(0 - \overline{p}_t\right) & \text{if } p_t \le \overline{p}_{t-1} \\ \overline{p}_{t-1} + \beta\left(1 - \overline{p}_t\right) & \text{if } p_t > \overline{p}_{t-1} \end{cases} \end{cases} \text{otherwise} \end{cases}$$

In mTSB method, when the demand size is positive, the update to the demand size and demand occurrence probability is treated the same as the update in TSB method. When the current demand size is zero, the update to the demand is determined based on the comparison between the observed occurrence probability at period t and the estimated occurrence probability for period t at period t-1. If the former one is less than the latter one, the estimated occurrence probability for period t+1 at period t will be updated in the same way as it is in TSB method, otherwise, it will be updated as a positive demand.

Kwan (1991) has shown that intermittent demand size and occurrence probability follow logarithmic distribution and Bernoulli distribution respectively. Hence, in mTSB method, the observation of occurrence probability can be seen as the probability of actual occurrence in each time period. The estimated occurrence probability is based on the comparison of previous estimated occurrence probability and the observed occurrence probability in the same period. It ensures the accuracy of the estimated probability. Even after many continuous zero demand periods, mTSB method can still produce an estimated occurrence probability which is close to the actual value.

3. Experiment

3.1. Accuracy measure and smoothing parameter setting up

In this study, mean error (ME) and mean squared error (MSE) are used as the main metric to assess the performance of the forecasting. ME is used to reflect the bias because it is a popularly used metric in performance assessments while it should be noted that its negative and positive error can counteract each other. MSE is another popularly used accuracy measurement metric. It can show the variance and the bias of the forecast errors. For the definition of these two measures, readers can refer to Babai et al. (2014). Meanwhile, two additional measurements, mean absolute scaled error (MASE) introduced by Hyndman and Koehler (2006) and revised mean absolute scaled error (RMASE) proposed by Li and Lim (2018), will be used in this study. These two methods do not take into account the order of magnitude, therefore performance is assessed directly.

To find the best smoothing parameters α and β, empirically we set a range [0.05, 0.3] for both parameters and tried both parameters at every 0.05 within the range. In total there are 36 different combinations of α and β. For each combination of the two parameters, we calculate the ME and MSE. We can then locate the minimum ME and MSE and find the best combination of α and β. The setting of the parameters can be reproduced as described in Boylan (2015). In the same way, we can find the best performance of α and β combination by measuring the MASE and RMASE.

3.2. M5 forecasting competition data set

Organized by the M Open Forecasting Center (MOFC), the M5 Competition is a popular competition in the field of forecast which is to advance the theory of forecasting and improve its utilization by business and non-profit organizations (see Makridakis & Spiliotis, 2021; Koning et al., 2005). The M5 dataset, provided by Walmart, comprises the unit sales of 3049 individual products sold in 10 stores in USA. Totally there are 30490 time series of sales in 1913 days from January 29, 2011 to June 19, 2016. The reason that this study chooses the M5 dataset, rather than the other M series dataset, is because the M5 dataset contains many intermittent demands.

A criterion for demand pattern classification was mentioned by Ghobbar and Friend (2002) based on two indicators, that is, Average inter-demand interval (ADI) and Square coefficient of variation (CV^2). The formula of ADI and CV^2 are shown as below:

$$\mathrm{ADI} = \frac{Total\ number\ of\ observation}{Number\ of\ non-zero\ demand} \tag{7}$$

$$CV^2 = \frac{standard\ deviation}{mean\ value} \tag{8}$$

The same criteria is used in this study, that is, only the accuracy with ADI $\geq$ 1.32 and $CV^2 \leq 0.49$, are considered as “intermittent but not very erratic” demand. After data screening, there are 16289 items satisfy the requirement of ADI and CV^2 and will be used in this study (it will be named M5 data set in the rest of the paper).

Table 1 lists the statistical properties of the selected M5 competition data set including the mean and standard deviation (std) of the minimum, 25th percentile, median, 75th percentile and the maximum of the demand size, demand interval and mean demand per period across all items. In Table 1, the degree of lumpiness is high. The demand size, demand interval and demand per period has a sharp increasing for the 75th percentile to the maximum value both in mean and standard deviation. For demand size, the range of standard deviation is from 2.3792 to 108.612 with a median 1.4353. The standard deviation value of demand interval is from 112.8 to 457.4 with the median value 31.56. At the same time, the standard deviation of demand per period is from 2.2730 to 107.889 with a median as 1.2832.

3.3. Empirical data set

In this study, a data set, provided by a local medicine wholesaler which contains monthly demand history of 1718 medicine products over a period of three years (36 months), is used to evaluate the proposed intermittent demand forecasting method. Each month is treated as a time period, with the total quantity of demand size for each item being over 400 and its average being over 10. The data set is logarithmically distributed according to Kwan (1991).

Table 2 lists the statistical property of the data set, including the mean and standard deviation (std) of the minimum, 25th percentile, median, 75th percentile and the maximum of the demand size, demand interval and mean demand per period across all items.

From Table 2, the data set has a high degree of lumpiness. The standard deviations of demand size, demand interval and demand per period all witness a sharp increase from 75th percentile to the maximum. For demand size, standard deviation range is from 2.984 to 155.614 with a median of 16.899. The standard deviation value of demand interval ranges from 0.167 to

Table 1
Statistics of M5 data set.

	Demand Size		Demand Interval		Demand per Period	
	mean	std	mean	std	mean	std
minimum	0.2838	0.5358	1.0037	0.060	0.1328	0.3132
25%ile	0.5938	1.0045	4.3837	9.390	0.4251	0.8220
50%ile	0.9733	1.4353	13.728	31.56	0.7880	1.2832
75%ile	1.8756	2.3792	59.698	112.8	1.6958	2.2730
maximum	130.95	108.612	496.32	457.4	129.10	107.889

Table 2
Statistics of empirical data set.

	Demand Size		Demand Interval		Demand per Period	
	mean	std	mean	std	mean	std
minimum	11.111	2.984	1.028	0.167	2.955	3.052
25%ile	14.500	11.985	1.111	0.398	13.708	11.985
50%ile	18.972	16.899	1.444	0.921	17.861	16.899
75%ile	24.889	24.639	1.861	1.833	23.756	24.639
maximum	34.611	155.614	9.444	8.175	33.833	155.614

8.175 with the median value being 0.921. At the same time, the standard deviation of demand per period goes from 3.052 to 155.614 with a median of 16.899.

To determine the best parameters of each parametric intermittent demand forecasting methods, the data set is divided into two parts. The first part contains the first one third of demand histories and is used as the initial set. Specifically, the mean of demand size and demand interval of the initial set is used as the initial value.

4. Results and discussions

4.1. Results of M5 competition data set

The proposed mTSB method and other methods mentioned in this study applied on the M5 data set. Table 3 and Table 4 show the details of ME, MSE, MASE and RMASE results of all methods with different combinations of parameter α and β. The minimum results of ME and MSE are picked in Tables 4 and 5 with corresponding α and β. Tables 6 and 7 take the minimum results of MASE and RMASE with corresponding α and β as well.

From Tables 3, 5 and 6, mTSB gets the minimum ME and MSE results with smoothing parameter $\alpha = 0.3$ and $\beta = 0.3$. The minimum ME result of mTSB method is 0.145, and minimum MSE result of mTSB is 4.074. However, the minimum ME result which equals 0.001was taken by CRO method with $\alpha = 0.25$, $\beta = 0.3$. The minimum MSE result of all methods which equals 3.7398 was taken by SES method with $\alpha = 0.3$. In Tables 4, 7 and 8, mTSB gets the minimum MASE and RMASE results with

Table 3
Results of ME and MSE on M5 Data set.

	β/α	ME						MSE					
		0.05	0.10	0.15	0.20	0.25	0.30	0.05	0.10	0.15	0.20	0.25	0.30
SES		0.030	0.015	0.010	0.007	0.006	0.005	7.023	6.048	5.352	4.761	4.230	3.740
CRO	0.05	0.703	0.335	0.195	0.119	0.069	0.031	10.824	9.349	8.892	8.658	8.527	8.444
	0.10	0.686	0.306	0.156	0.074	0.018	−0.023	10.422	8.880	8.412	8.174	8.042	7.961
	0.15	0.682	0.300	0.148	0.064	0.006	−0.037	10.115	8.499	8.009	7.761	7.625	7.544
	0.20	0.681	0.298	0.146	0.060	0.002	−0.042	9.843	8.153	7.641	7.384	7.242	7.158
	0.25	0.681	0.298	0.145	0.060	0.001	−0.044	9.591	7.833	7.300	7.033	6.885	6.797
	0.30	0.681	0.298	0.146	0.060	0.001	−0.044	9.354	7.533	6.981	6.705	6.552	6.460
SBA	0.05	0.735	0.416	0.328	0.304	0.306	0.322	10.888	9.400	8.936	8.712	8.604	8.560
	0.10	0.718	0.389	0.292	0.263	0.262	0.276	10.491	8.936	8.457	8.224	8.110	8.062
	0.15	0.714	0.383	0.285	0.254	0.252	0.264	10.190	8.567	8.070	7.832	7.717	7.671
	0.20	0.713	0.382	0.283	0.251	0.248	0.260	9.922	8.232	7.720	7.478	7.363	7.320
	0.25	0.713	0.382	0.282	0.251	0.247	0.258	9.675	7.922	7.396	7.149	7.035	6.994
	0.30	0.713	0.382	0.282	0.251	0.247	0.258	9.441	7.633	7.092	6.842	6.727	6.688
TSB	0.05	0.148	0.128	0.122	0.120	0.119	0.118	7.296	6.859	6.605	6.420	6.270	6.142
	0.10	0.096	0.072	0.065	0.061	0.059	0.058	6.725	6.298	6.044	5.855	5.699	5.565
	0.15	0.080	0.054	0.046	0.042	0.040	0.038	6.286	5.862	5.606	5.415	5.256	5.118
	0.20	0.073	0.046	0.037	0.033	0.030	0.029	5.900	5.474	5.217	5.023	4.862	4.721
	0.25	0.069	0.041	0.032	0.027	0.025	0.023	5.546	5.118	4.858	4.661	4.496	4.352
	0.30	0.067	0.039	0.029	0.024	0.021	0.020	5.217	4.784	4.521	4.321	4.153	4.006
mSBA	0.05	0.871	0.584	0.497	0.470	0.468	0.480	11.373	9.353	8.603	8.240	8.051	7.960
	0.10	0.856	0.560	0.466	0.434	0.429	0.439	11.009	8.912	8.135	7.755	7.554	7.454
	0.15	0.853	0.554	0.459	0.425	0.419	0.428	10.737	8.579	7.781	7.392	7.184	7.082
	0.20	0.851	0.552	0.456	0.422	0.415	0.424	10.498	8.285	7.470	7.072	6.861	6.757
	0.25	0.851	0.552	0.455	0.421	0.414	0.422	10.278	8.014	7.183	6.778	6.563	6.458
	0.30	0.850	0.551	0.454	0.420	0.413	0.421	10.070	7.760	6.914	6.502	6.284	6.178
mTSB	0.05	0.635	0.469	0.373	0.311	0.269	0.236	8.158	7.231	6.795	6.515	6.320	6.170
	0.10	0.598	0.425	0.323	0.259	0.215	0.181	7.652	6.710	6.262	5.971	5.765	5.606
	0.15	0.587	0.411	0.307	0.242	0.197	0.162	7.271	6.306	5.847	5.547	5.333	5.168
	0.20	0.582	0.404	0.300	0.234	0.188	0.153	6.939	5.948	5.477	5.168	4.948	4.777
	0.25	0.580	0.401	0.295	0.229	0.183	0.148	6.637	5.619	5.135	4.818	4.591	4.415
	0.30	0.578	0.399	0.293	0.226	0.180	0.145	6.356	5.311	4.814	4.488	4.255	4.074

Table 4
Results of MASE and RMASE on M5 data set.

	β/α	MASE						RMASE					
		0.05	0.1	0.15	0.2	0.25	0.3	0.05	0.1	0.15	0.2	0.25	0.3
SES		0.968	0.906	0.856	0.809	0.763	0.717	0.260	0.244	0.230	0.218	0.206	0.193
CRO	0.05	1.078	1.046	1.042	1.042	1.043	1.046	0.287	0.274	0.271	0.270	0.270	0.270
	0.1	1.073	1.039	1.034	1.035	1.038	1.042	0.285	0.271	0.268	0.267	0.267	0.267
	0.15	1.067	1.029	1.024	1.024	1.027	1.031	0.283	0.268	0.264	0.263	0.263	0.264
	0.2	1.062	1.019	1.013	1.012	1.014	1.018	0.281	0.265	0.261	0.260	0.260	0.260
	0.25	1.056	1.010	1.001	1.000	1.002	1.005	0.279	0.262	0.257	0.256	0.256	0.256
	0.3	1.050	1.000	0.990	0.988	0.989	0.992	0.277	0.259	0.254	0.252	0.252	0.252
SBA	0.05	1.077	1.041	1.032	1.027	1.024	1.022	0.287	0.274	0.270	0.268	0.267	0.266
	0.1	1.072	1.034	1.024	1.020	1.017	1.016	0.285	0.270	0.266	0.265	0.264	0.263
	0.15	1.067	1.025	1.015	1.010	1.008	1.007	0.283	0.268	0.263	0.261	0.261	0.260
	0.2	1.062	1.016	1.005	1.000	0.998	0.998	0.281	0.265	0.260	0.258	0.257	0.257
	0.25	1.056	1.007	0.995	0.990	0.988	0.988	0.279	0.262	0.257	0.255	0.254	0.254
	0.3	1.051	0.998	0.985	0.980	0.978	0.978	0.277	0.259	0.254	0.252	0.251	0.251
TSB	0.05	0.967	0.929	0.900	0.874	0.849	0.825	0.256	0.248	0.241	0.234	0.229	0.223
	0.1	0.954	0.915	0.885	0.857	0.831	0.807	0.252	0.243	0.236	0.229	0.223	0.218
	0.15	0.941	0.901	0.870	0.841	0.815	0.789	0.248	0.239	0.231	0.225	0.218	0.212
	0.2	0.928	0.887	0.855	0.826	0.798	0.772	0.244	0.234	0.227	0.220	0.213	0.207
	0.25	0.915	0.873	0.840	0.810	0.782	0.755	0.240	0.230	0.222	0.215	0.208	0.202
	0.3	0.902	0.858	0.825	0.794	0.765	0.738	0.236	0.225	0.217	0.210	0.203	0.197
mSBA	0.05	1.058	1.000	0.977	0.965	0.956	0.950	0.285	0.269	0.263	0.260	0.257	0.256
	0.1	1.053	0.991	0.968	0.955	0.946	0.940	0.283	0.266	0.260	0.256	0.254	0.252
	0.15	1.048	0.983	0.958	0.945	0.936	0.930	0.281	0.263	0.257	0.253	0.251	0.249
	0.2	1.043	0.975	0.949	0.935	0.927	0.920	0.279	0.261	0.254	0.250	0.247	0.246
	0.25	1.038	0.967	0.940	0.926	0.917	0.911	0.277	0.258	0.251	0.247	0.244	0.243
	0.3	1.033	0.959	0.931	0.917	0.908	0.902	0.276	0.255	0.248	0.244	0.241	0.240
mTSB	0.05	0.919	0.867	0.837	0.814	0.793	0.775	0.253	0.239	0.230	0.224	0.218	0.214
	0.1	0.903	0.850	0.819	0.795	0.773	0.754	0.249	0.234	0.225	0.218	0.212	0.207
	0.15	0.890	0.835	0.803	0.779	0.756	0.736	0.245	0.229	0.220	0.213	0.207	0.202
	0.2	0.879	0.822	0.789	0.763	0.740	0.720	0.241	0.225	0.215	0.208	0.202	0.197
	0.25	0.868	0.809	0.775	0.748	0.724	0.703	0.238	0.221	0.211	0.203	0.197	0.192
	0.3	0.857	0.796	0.761	0.733	0.708	0.687	0.234	0.217	0.207	0.199	0.192	0.186

smoothing parameters $\alpha = 0.3$ and $\beta = 0.3$. The minimum MASE result of mTSB which is also the minimum MASE result of all methods equals to 0.145. Meanwhile, the minimum RMASE of mTSB equals to 0.186 becoming the minimum RMASE results among the performance of all methods.

From Tables 3 and 4, it shows that with the increasing of smoothing parameter α and β, the better performance of mTSB method and TSB method take. We can also find that with the increasing of α, the ME, MSE, MASE and RMASE results of SES method decreasing. AS for ME result, the performances of CRO method, SBA method and mSBA method become better with β increasing. When α is greater than 0.25, the performance of these methods become worse.

4.2. Results of empirical data set

The proposed mTSB, together with SES, CRO, SBA, TSB and mSBA, are applied on the empirical data set. Table 9 shows all ME and MSE results and Table 10 shows all MASE and RMASE results of all methods under different combinations of parameters α and β. Tables 11 and 12 pick the minimum ME and MSE of each method with the corresponding α and β. Table 13 takes the minimum values of MASE and RMASE for each method with their corresponding α and β.

From these three tables, mTSB method achieves the minimum ME result 0.06 with smoothing constants $\alpha = 0.2$ and $\beta = 0.3$. SES method also works well with minimum ME result 0.07 when $\alpha = 0.3$. Regarding the minimum MSE result, both SES method and mTSB method are shown to be better than other methods, with the minimum MSE result of SES method being 196.24, and the minimum MSE result of mTSB method being 198.21. As for the MASE results, the proposed method mTSB attains better results for all methods mentioned with minimum MASE result 0.45 with $\alpha = 0.3$ and $\beta = 0.3$. With $\alpha = 0.3$ and $\beta = 0.3$, mTSB also performs better than other methods with a RMASE result of 0.40. SBA method achieves the minimum RMASE result 0.37 with $\alpha = 0.3$ and $\beta = 0.3$. However, the MASE result of SBA method is the second largest among all methods.

From Table 9, we can find that as α increases, both ME and MSE of SES method decrease. For CRO method, SBA method and mSBA method, smaller α and bigger β values contribute to better ME results. TSB method arrives at a smaller ME result when α and β value are greater. There is no clear relation between smoothing constants and ME result for mTSB method. However, considering MSE result, all methods achieve smaller MSE results with bigger smoothing constants. From the three tables, SES method and mTSB method outperform other methods. In terms of ME, mTSB method obtains the best result.

From Table 13, both MASE and RMASE obtain their minimum value for the same α and β.

Table 5
Minimum ME result.

Forecasting method	α	β	ME	MSE
SES	0.3		0.004	3.7398
CRO	0.25	0.3	0.001	6.5515
SBA	0.25	0.3	0.247	6.7272
TSB	0.3	0.3	0.020	4.0063
mSBA	0.25	0.3	0.412	6.2838
mTSB	0.3	0.3	0.145	4.074

Table 6
Minimum MSE result.

Forecasting method	α	β	ME	MSE
SES	0.3		0.004	3.7398
CRO	0.3	0.3	−0.04	6.7050
SBA	0.3	0.3	0.258	6.6880
TSB	0.3	0.3	0.020	4.0063
mSBA	0.3	0.3	0.421	6.178
mTSB	0.3	0.3	0.145	4.074

Table 7
Minimum MASE result.

Forecasting method	α	β	MASE	RMASE
SES	0.3		0.717	0.193
CRO	0.2	0.3	0.985	0.252
SBA	0.3	0.3	0.975	0.251
TSB	0.3	0.3	0.738	0.197
mSBA	0.3	0.3	0.902	0.240
mTSB	0.3	0.3	0.696	0.186

Table 8
Minimum RMASE result.

Forecasting method	α	β	MASE	RMASE
SES	0.3		0.717	0.193
CRO	0.3	0.3	0.989	0.252
SBA	0.3	0.3	0.978	0.251
TSB	0.3	0.3	0.738	0.197
mSBA	0.3	0.3	0.902	0.240
mTSB	0.3	0.3	0.696	0.186

5. Conclusion

Accurate forecasting of intermittent demand has always been challenging due to the existence of time periods with zero demand size. On the other hand, this type of demand tends to be ubiquitous, especially in industries such as heavy machinery and electronics. The paper aims to investigate how effective a new method, the modified TSB method, is in forecasting intermittent demand.

The modified TSB (mTSB) method is based on the TSB method while adopting a slightly different strategy. The proposed mTSB method was applied to 16289 daily demand records, selected out from M5 data set as intermittent demand records with two criteria. The proposed mTSB method achieves the best results in terms of MASE and RMASE among all comparison methods. An empirical data set consisting three years' monthly demand history of 1718 medicine products from a local medicine wholesaler is also used to evaluate the proposed method. The experimental result has shown that mTSB performs the most effectively among all other methods in terms of ME, MASE and RMASE measurement. On MSE, mTSB showed similar result as SES, both outperformed other comparison methods. Considering the MASE and RMASE, mTSB both performed better than other methods. It only performed worse than SBA method on RMASE result. However, the MASE result of SBA method is too large to be considered as a suitable method.

It should be noted that, though mTSB showed good performance with the data set used in this study, it was pointed out in Watson (1987); Porras and Dekker (2008) that intermittent demand forecasting depends heavily on the characteristic of intermittent demand data set. Performance of our proposed mTSB method on intermittent demands of different

Table 9
Results of ME and MSE on empirical experiment.

	β/α	ME						MSE					
		0.05	0.10	0.15	0.20	0.25	0.30	0.05	0.10	0.15	0.20	0.25	0.30
SES		8.50	4.41	2.47	1.50	0.98	0.07	480.33	382.63	318.41	269.22	229.49	196.24
CRO	0.05	−2.66	−3.38	−3.92	−4.30	−4.56	−4.73	489.45	492.51	488.47	485.19	482.25	479.73
	0.10	−1.98	−2.76	−3.38	−3.81	−4.11	−4.32	4337.55	430.63	426.44	423.32	420.49	417.94
	0.15	−1.47	−2.28	−2.92	−3.38	−3.70	−3.93	400.54	392.84	399.26	385.11	382.35	379.88
	0.20	−1.08	−1.89	−2.55	−3.02	−3.35	−3.59	373.87	365.4	360.26	356.85	354.03	351.58
	0.25	−0.77	−1.59	−2.24	−2.72	−3.06	−3.31	352.64	343.46	337.75	334	331.04	328.55
	0.30	−0.52	−1.33	−1.99	−2.47	−2.82	−3.07	334.73	324.93	318.71	314.63	311.51	308.94
SBA	0.05	−2.16	−2.33	−2.31	−2.11	−1.78	−1.38	491.7	478.81	467.81	457.97	449.18	441.61
	0.10	−1.49	−1.74	−1.80	−1.67	−1.39	−1.03	433.27	421.71	412.61	404.77	397.83	391.93
	0.15	−0.99	−1.28	−1.38	−1.28	−1.04	−0.70	397.63	386.6	378.3	371.48	365.59	360.69
	0.20	−0.61	−0.92	−1.03	−0.96	−0.73	−0.41	371.87	360.95	352.94	346.63	341.37	337.15
	0.25	−0.31	−0.63	−0.75	−0.69	−0.48	−0.17	351.28	340.32	332.39	326.36	321.52	317.77
	0.30	−0.07	−0.38	−0.52	−0.46	−0.26	0.03	333.87	322.81	314.87	309.02	304.46	301.06
TSB	0.05	15.59	14.73	13.97	13.30	12.70	12.16	674.03	639.77	610.2	584.36	561.6	541.46
	0.10	15.61	14.75	14.00	13.34	12.75	12.21	669.58	632.94	601.14	573.2	548.47	526.47
	0.15	15.62	14.78	14.04	13.38	12.80	12.26	665.88	627.46	594.01	564.54	539.39	515.06
	0.20	15.63	14.80	14.07	13.42	12.84	12.31	662.65	622.75	587.93	557.21	529.91	505.51
	0.25	15.64	14.82	14.10	13.45	12.88	12.35	659.75	618.53	582.52	550.7	522.39	497.05
	0.30	15.65	14.84	14.12	13.48	12.91	12.39	657.1	614.68	577.56	544.74	515.51	489.33
mSBA	0.05	−1.27	−1.09	−0.87	−0.55	−0.16	0.27	457.21	434.96	419.27	407.35	398.22	391.35
	0.10	−0.66	−0.58	−0.44	−0.18	0.16	0.55	404.18	383.94	369.89	359.52	354.8	346.24
	0.15	−0.21	−0.18	−0.08	0.14	0.45	0.82	371.61	352.25	338.92	329.32	322.4	317.6
	0.20	0.14	0.14	0.22	0.41	0.70	1.05	347.88	328.9	315.83	306.61	300.15	295.86
	0.25	0.41	0.40	0.46	0.64	0.91	1.25	328.79	309.96	296.95	287.91	381.75	277.8
	0.30	0.63	0.60	0.66	0.83	1.09	1.41	312.54	293.75	280.73	271.76	265.77	262.06
mTSB	0.05	0.74	−0.09	−0.46	−0.69	−0.86	−0.99	406.95	384.45	373.02	364.59	358.22	353.64
	0.10	1.07	0.11	−0.3	−0.55	−0.73	−0.73	358.44	335.53	322.45	321.74	305.18	299.52
	0.15	1.34	0.32	−0.11	−0.38	−0.57	−0.71	327.98	304.34	290.20	279376	271.52	365.25
	0.20	1.55	0.50	0.5	0.06	−0.21	−0.41	305.36	280.69	265.61	254.58	245.83	239.15
	0.25	1.72	0.66	0.21	−0.07	−0.27	0.42	286.86	261.13	245.13	233.54	224.34	217.34
	0.30	1.86	0.80	0.34	0.06	−0.15	−0.3	270.87	244.18	227.26	215.14	205.50	198.21

Table 10
Results of MASE and RMASE on empirical experiment.

	β/α	MASE						RMASE					
		0.05	0.10	0.15	0.20	0.25	0.30	0.05	0.10	0.15	0.20	0.25	0.30
SES		1.54	1.33	1.14	0.99	0.85	0.74	0.75	0.67	0.61	0.55	0.50	0.45
CRO	0.05	1.50	1.50	1.49	1.49	1.49	1.49	0.63	0.62	0.62	0.63	0.63	0.63
	0.1	1.45	1.44	1.44	1.44	1.44	1.44	0.58	0.58	0.58	0.58	0.58	0.58
	0.15	1.40	1.40	1.39	1.39	1.39	1.39	0.54	0.54	0.53	0.53	0.53	0.53
	0.2	1.36	1.35	1.35	1.35	1.35	1.35	0.50	0.50	0.50	0.50	0.49	0.49
	0.25	1.32	1.31	1.31	1.31	1.31	1.31	0.47	0.46	0.46	0.46	0.46	0.46
	0.3	1.28	1.28	1.27	1.27	1.27	1.27	0.43	0.43	0.43	0.43	0.43	0.43
SBA	0.05	1.47	1.43	1.40	1.36	1.33	1.30	0.61	0.59	0.58	0.56	0.55	0.53
	0.1	1.42	1.38	1.35	1.32	1.29	1.26	0.56	0.55	0.53	0.52	0.51	0.49
	0.15	1.37	1.34	1.31	1.28	1.25	1.22	0.52	0.51	0.50	0.48	0.47	0.46
	0.2	1.33	1.30	1.27	1.24	1.21	1.18	0.49	0.47	0.46	0.45	0.44	0.43
	0.25	1.29	1.26	1.23	1.20	1.18	1.15	0.46	0.44	0.43	0.42	0.41	0.40
	0.3	1.25	1.22	1.20	1.17	1.15	1.12	0.43	0.41	0.40	0.39	0.38	0.37
TSB	0.05	1.41	1.28	1.16	1.06	0.97	0.90	0.66	0.65	0.64	0.64	0.63	0.63
	0.1	1.36	1.22	1.11	1.01	0.92	0.85	0.60	0.60	0.59	0.59	0.58	0.58
	0.15	1.31	1.18	1.06	0.96	0.88	0.81	0.56	0.55	0.55	0.54	0.53	0.53
	0.2	1.26	1.13	1.02	0.92	0.84	0.77	0.52	0.51	0.50	0.50	0.49	0.49
	0.25	1.22	1.09	0.98	0.88	0.80	0.73	0.48	0.47	0.47	0.46	0.45	0.45
	0.3	1.19	1.06	0.95	0.85	0.77	0.70	0.45	0.44	0.43	0.42	0.42	0.41
mSBA	0.05	1.28	1.16	1.08	1.02	0.96	0.92	0.60	0.58	0.57	0.55	0.53	0.52
	0.1	1.23	1.11	1.03	0.97	0.92	0.88	0.56	0.54	0.52	0.51	0.49	0.48
	0.15	1.18	1.07	0.99	0.93	0.88	0.84	0.52	0.50	0.48	0.47	0.45	0.44
	0.2	1.14	1.03	0.95	0.90	0.85	0.81	0.48	0.47	0.45	0.44	0.42	0.41
	0.25	1.11	1.00	0.92	0.86	0.82	0.78	0.45	0.43	0.42	0.41	0.39	0.38
	0.3	1.07	0.96	0.89	0.84	0.79	0.76	0.42	0.40	0.39	0.38	0.37	0.36
mTSB	0.05	0.68	0.67	0.66	0.64	0.63	0.62	0.63	0.62	0.62	0.62	0.62	0.61
	0.1	0.64	0.63	0.61	0.60	0.59	0.58	0.58	0.57	0.57	0.57	0.56	0.56
	0.15	0.60	0.59	0.58	0.56	0.55	0.54	0.53	0.53	0.52	0.52	0.52	0.51
	0.2	0.57	0.56	0.54	0.53	0.52	0.50	0.49	0.49	0.48	0.48	0.47	0.47
	0.25	0.54	0.52	0.51	0.50	0.49	0.47	0.45	0.45	0.44	0.44	0.44	0.43
	0.3	0.51	0.50	0.48	0.47	0.46	0.45	0.42	0.42	0.41	0.41	0.40	0.40

Table 11
Minimum ME result.

Forecasting method	α	β	ME	MSE
SES	0.3		0.07	196.24
CRO	0.05	0.3	−0.52	334.73
SBA	0.3	0.3	0.03	301.06
TSB	0.3	0.05	12.16	541.46
mSBA	0.1	0.15	−0.18	352.25
mTSB	0.2	0.3	0.06	215.14

Table 12
Minimum MSE result.

Forecasting method	α	β	ME	MSE
SES	0.3		0.07	196.24
CRO	0.3	0.3	−3.07	308.94
SBA	0.3	0.3	0.03	301.06
TSB	0.3	0.3	12.39	489.33
mSBA	0.3	0.3	1.41	262.06
mTSB	0.3	0.3	−0.3	198.21

Table 13
Minimum MASE and RMASE result.

Forecasting method	α	β	MASE	RMASE
SES	0.3		0.74	0.45
CRO	0.3	0.3	1.27	0.43
SBA	0.3	0.3	1.12	0.37
TSB	0.3	0.3	0.70	0.41
mSBA	0.3	0.3	0.76	0.36
mTSB	0.2	0.3	0.45	0.40

characteristics, such as sudden obsolescence demand, decreasing demand, seasonal demand and so on, has not been explored in this study. The performance of the method can be better revealed if more experiments with intermittent demands of different characteristics can be used. Unfortunately, such data is not available for this study.

Declaration of competing interest

The authors declare no conflict of interest.

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