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Expressivity of some versions of APAL

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Abstract. Arbitrary public announcement logic (APAL) is a logic of change of knowledge with modalities representing quantification over announcements. We present two rather different versions of APAL wherein this quantification is restricted to formulas only containing a subset of all propositional variables: FSAPAL and SCAPAL; and another version quantifying over all announcements implied by or implying a given formula: IPAL. We then determine the relative expressivity of these logics and APAL. The IPAL quantifier provides a novel perspective on substructural implication as dynamic consequence.

Keywords: Dynamic epistemic logic, expressivity, modal logic

1 Introduction

The modal logic of knowledge was originally proposed to give a relational semantics for the perceived properties of knowledge, such as that what you know is true, and that you know what you know, and to contrast this with the properties of other epistemic notions such as belief [24]. Already in [24] the analysis of paradoxical phenomena that you cannot be informed of factual ignorance while 'losing' that ignorance, so-called Moorean phenomena [26], played an important role. On the heels of the logic of (single agent) knowledge came the multi-agent logics of knowledge, wherein similar phenomena are not so paradoxical: there is no issue with my knowledge of your ignorance. This led on the one hand to the development of group epistemic notions such as common knowledge [5,25] and distributed knowledge [23], topics that we will bypass in this contribution. On the other hand this led to increased interest in the analysis of multiple agents informing each other of their ignorance and knowledge, often inspired by logic puzzles [27,25]. This culminated in Plaza's public announcement logic (PAL) [28], wherein such informative actions became full members of the logical language besides the knowledge modalities; parallel developments of dynamic but not epistemic logics of information change are [19,10].

PAL contains a dynamic operator representing the consequences of information change that is similarly observed by all agents, so-called public (and truthful) announcement. We let $[\psi]\varphi$ stand for 'after truthful public announcement of ψ, φ (is true). Every PAL formula is equivalent to a formula without public announcements, so that PAL is as expressive as epistemic logic EL (a.k.a. the logic S5) [28].

From PAL there were various directions for further generalization. One could consider public announcements in the presence of group epistemic operators such as common knowledge, or non-public information change such as private or secret announcements to some agents while other agents do not or only partially observe that. Both were simultaneously realized in action model logic [8]; parallel, now lesser known, developments are [22].

A different direction of generalizing PAL is to consider quantifying over announcements. Arbitrary public announcement logic APAL was proposed in [6] and contains a construct $[!]\varphi$ standing for 'after any truthful public announcement, φ (is true)', i.e., for all ψ, $[\psi]\varphi$. In order to avoid circularity, the APAL quantifier is only over announcements not containing $[!]$ modalities. There is an infinitary (not RE) axiomatization for the logic [7], where an open question remains whether there is a finitary (RE) axiomatization. APAL is undecidable [20], and the complexity of model checking is PSPACE-complete [1]. There are versions of APAL with finitary axiomatizations or decidable satisfiability problems [14,15,9], or that model aspects of agency [1,2,21]. APAL is more expressive than PAL [6]. The relative expressivity of versions of APAL is rather intricate, and most relevant in view of potential applications. For example, group announcement logic GAL and APAL are incomparable in expressivity [21], and in GAL we can formalize goal reachability in finite two-principal security protocols [1].

In this contribution we investigate some novel versions of APAL. If we quantify over announcements only using atoms in *subsets* $Q \subseteq P$ we obtain the logic SAPAL, and if these subsets are required to be *finite* we get FSAPAL. If we quantify over announcements only using atoms occurring in the formula under the *scope* of the quantifier, we obtain the logic SCAPAL. If we quantify over announcements *implying* a given formula ψ or *implied by* a given formula ψ and if such ψ may also contain quantifiers we obtain logic QIPAL and if they are not allowed to contain quantifiers we obtain IPAL.

In Section 2 we introduce their syntax and semantics, in Section 3 we prove some modal properties of these quantifiers. Section 4 determines the expressivity hierarchy for the reported logics. This section contains our main results. They are depicted in Fig. 1. Let $\prec$ mean '(strictly) less expressive' and $\asymp$ 'incomparable', then the results are that PAL is less expressive than any of the logics with quantifiers, and that SCAPAL $\prec$ FSAPAL, APAL $\asymp$ SCAPAL, APAL $\asymp$ FSAPAL, IPAL $\asymp$ SCAPAL, IPAL $\asymp$ FSAPAL, and APAL $\prec$ IPAL (proof omitted for lack of space and therefore called a conjecture). The complete axiomatizations and the undecidability of satisfiability of our APAL versions all promise to be the same as for APAL. We conclude with Section 5 reinterpreting dynamic consequence in the IPAL setting.

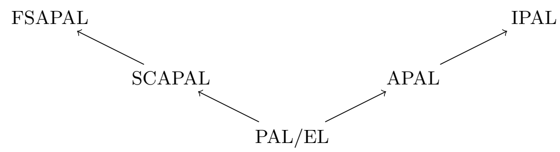

Fig. 1. Expressivity hierarchy of logics presented in this work. An arrow means larger expressivity. Assume transitivity. Absence of an arrow means incomparability.

2 Syntax and semantics: SAPAL, SCAPAL, QIPAL

Throughout this contribution, let a countable set P be of propositional atoms and a finite set A of agents be given.

Definition 1 (Language). *The logical language $\mathcal{L}$ is defined inductively as:*

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\varphi \mid [\varphi]\varphi \mid [!]\varphi \mid [Q]\varphi \mid [\subseteq]\varphi \mid [\varphi^{\downarrow}]\varphi \mid [\varphi^{\uparrow}]\varphi$$

$p \in P$, $a \in A$, and $Q \subseteq P$. The propositional sublanguage is $\mathcal{L}_{PL}$, with additionally modalities K_a we get the epistemic formulas *$\mathcal{L}_{EL}$, with additionally the construct $[\varphi]\varphi$ it is $\mathcal{L}_{PAL}$, and adding one of the* quantifiers *$[!]$, $[Q]$, $[\subseteq]$, $[\varphi^{\downarrow}]\psi$ and $[\varphi^{\uparrow}]\psi$ we obtain, respectively, $\mathcal{L}_{APAL}$, $\mathcal{L}_{SAPAL}$ and $\mathcal{L}_{SCAPAL}$, $\mathcal{L}_{QIPAL^{\downarrow}}$ and $\mathcal{L}_{QIPAL^{\uparrow}}$. Adding both $[\varphi^{\downarrow}]\psi$ and $[\varphi^{\uparrow}]\psi$ we obtain $\mathcal{L}_{QIPAL}$, and if the φ in $[\varphi^{\downarrow}]\psi$ and $[\varphi^{\uparrow}]\psi$ is restricted to $\mathcal{L}_{PAL}$, we get $\mathcal{L}_{IPAL}$. If the Q in $[Q]\varphi$ are (always)* finite *we get $\mathcal{L}_{FSAPAL}$.*

The meaning of all constructs will be explained after defining the semantics. The dual modalities for $[!]$, $[Q]$, $[\subseteq]$, $[\varphi^{\downarrow}]$, and $[\varphi^{\uparrow}]$ are, respectively, $\langle ! \rangle$, $\langle Q \rangle$, $\langle \subseteq \rangle$, $\langle \varphi^{\downarrow} \rangle$, and $\langle \varphi^{\uparrow} \rangle$. Instead of $\varphi \in \mathcal{L}_X$ we also say that φ *is an X formula*. For any language $\mathcal{L}$, $\mathcal{L}|Q$ is the sublanguage only containing atoms in $Q \subseteq P$. Given $\varphi \in \mathcal{L}$, $P(\varphi) \subseteq$ denotes the set of atoms occurring in φ. For $[\{p_1, \dots, p_n\}]\varphi$ we may write $[p_1 \dots p_n]\varphi$. The *modal depth* $d(\varphi)$ of a formula is the maximum stack of epistemic modalities; it is defined as: $d(\bot) = d(p) = 0$, $d(\varphi \wedge \psi) = \max\{d(\varphi, d(\psi)\}$, $d(K_a\varphi) = d(\varphi) + 1$, $d([\varphi]\psi) = d([\varphi^{\downarrow}]\psi) = d([\varphi^{\uparrow}]\psi) = d(\varphi) + d(\psi)$, and $d([!]\varphi) = d([\subseteq]\varphi) = d([Q]\varphi) = d(\neg\varphi) = d(\varphi)$.

Definition 2 (Structures). *An* epistemic model *(or* model*) is a triple $M = (S, \sim, V)$ where S is a domain of states, $\sim$ is a set of binary relations $\sim_a \subseteq S \times S$ that are all equivalence relations, and $V : P \to \mathcal{P}(S)$ maps each atom $p \in P$ to its denotation $V(p)$.*

Given a model M, we may refer to its domain, relations, and valuation as S^M, $\sim_a^M$, and V^M respectively, and we also refer to the domain of M as $\mathcal{D}(M)$. Bisimulation to compare models will be defined later. A model N is a *submodel* of M, notation $N \subseteq M$, if $S^N \subseteq S^M$, for all $a \in A$, $\sim_a^N = \sim_a^M \cap (S^N \times S^N)$, and for all $p \in P$, $V^N(p) = V^M(p) \cap S^N$.

Definition 3 (Semantics). *Given model $M = (S, \sim, V)$, $s \in S$ and $\varphi \in \mathcal{L}$ we inductively define $M, s \models \varphi$ (φ is true in state s of model M) as:*

$$
\begin{array}{lll}
M, s \models p & \textit{iff} & s \in V(p) \\
M, s \models \neg\varphi & \textit{iff} & M, s \models \varphi \\
M, s \models \varphi \wedge \psi & \textit{iff} & M, s \models \varphi \textit{ and } M, s \models \psi \\
M, s \models K_a\varphi & \textit{iff} & \textit{for all } t \in S,\ s \sim_a t \textit{ implies } M, t \models \varphi \\
M, s \models [\psi]\varphi & \textit{iff} & M, s \models \psi \textit{ implies } M|\psi, s \models \varphi \\
M, s \models [!]\varphi & \textit{iff} & \textit{for any } \psi \in \mathcal{L}_{PAL} : M, s \models [\psi]\varphi \\
M, s \models [Q]\varphi & \textit{iff} & \textit{for any } \psi \in \mathcal{L}_{PAL}|Q \ : M, s \models [\psi]\varphi \\
M, s \models [\subseteq]\varphi & \textit{iff} & \textit{for any } \psi \in \mathcal{L}_{PAL}|P(\varphi) : M, s \models [\psi]\varphi \\
M, s \models [\chi^{\downarrow}]\varphi & \textit{iff} & \textit{for any } \psi \in \mathcal{L}_{PAL} \textit{ implying } \chi : M, s \models [\psi]\varphi \\
M, s \models [\chi^{\uparrow}]\varphi & \textit{iff} & \textit{for any } \psi \in \mathcal{L}_{PAL} \textit{ implied by } \chi : M, s \models [\psi]\varphi
\end{array}
$$

where $M|\varphi = (S', \sim', V')$ is such that $S' = [\![\varphi]\!]_M = \{s \in S \mid M, s \models \varphi\}$, $\sim'_a = \sim_a \cap ([\![\varphi]\!]_M \times [\![\varphi]\!]_M)$, and $V'(p) = V(p) \cap [\![\varphi]\!]_M$.

A formula φ is valid on model *M, notation $M \models \varphi$, iff for all $s \in S$, $M, s \models \varphi$, and φ* is valid*, notation $\models \varphi$, iff φ is valid on all models M. A formula φ is a* distinguishing formula *of a subset $S' \subseteq S$ of M (or S' is* definable *by φ) if for all $t \in S$, $M, t \models \varphi$ and for all $t \notin S$, $M, t \not\models \varphi$.*

In the dual existential reading of the semantics of the quantifiers, the ψ in 'there is a $\psi \in \mathcal{L}_{PAL}$' is the *witness* of the quantifier. In the semantics of the last two, 'ψ implies χ' means $\models \psi \to \chi$ and 'ψ is implied by χ' means $\models \chi \to \psi$.

PAL and APAL *Public announcement logic* PAL and *arbitrary public announcement logic* APAL were already introduced.

SAPAL and FSAPAL The logic with construct $[Q]\varphi$, for 'after any announcement only containing atoms in $Q \subseteq P$', is called SAPAL, for APAL with quantification over formulas restricted to *subsets* of variables. If those subsets are required to be *finite* we get FSAPAL.

SCAPAL The logic with construct $[\subseteq]\varphi$, for 'after any announcement only containing atoms ocurring in φ', is called SCAPAL (where φ is the formula under the *scope* of the quantifier $[\subseteq]$).

QIPAL The logic with constructs $[\psi^{\downarrow}]\varphi$ and $[\psi^{\uparrow}]\varphi$ is called QIPAL; where $[\psi^{\downarrow}]\varphi$ stands for 'after every announcement implying ψ, φ is true', and $[\psi^{\uparrow}]\varphi$ stands for 'after every announcement implied by ψ, φ is true'. In QIPAL we can reason over restrictions of a given model M that are submodels of $M|\psi$, or over restrictions that contain $M|\psi$ as a submodel.

Bisimulation We define several notions of bisimulation between models and obtain some elementary invariance results for our logics. They will be used much in the expressivity Section 4.

Definition 4 (Bisimulation). *Let M and N be epistemic models. A non-empty relation $Z \subseteq S^M \times S^N$ is a* bisimulation *between M and N if for all Zst, $p \in P$ and $a \in A$:*
*— **atoms**: $s \in V^M(p)$ iff $t \in V^N(p)$.*
*— **forth**: if $s \sim_a^M s'$, then there is a $t' \in S^N$ such that $t \sim_a^N t'$ and $Zs't'$.*
*— **back**: if $t \sim_a^N t'$, then there is a $s' \in S^M$ such that $s \sim_a^M s'$ and $Zs't'$.*
If there exists a bisimulation Z between M and N we write $M \leftrightarroweq N$ (or $Z : M \leftrightarroweq N$, to indicate the relation), and if it contains pair (s,t), we write $(M,s) \leftrightarroweq (N,t)$. If the **atoms** *clause is only satisfied for atoms $Q \subseteq P$, we write $M \leftrightarroweq^Q N$ and Z is called a* Q-bisimulation *or a* (Q-)restricted bisimulation.

Definition 5 (Bounded bisimulation). *Let M and N be epistemic models. For $n \in \mathbb{N}$ we define a sequence $Z^0 \supseteq \cdots \supseteq Z^n$ of relations on $S_M \times S_N$.*
A non-empty relation Z^0 is a 0-bisimulation if for all Z^0st and $p \in P$:
*— **atoms**: $s \in V^M(p)$ iff $t \in V^N(p)$.*
A non-empty relation Z^{n+1} is an $(n+1)$-bisimulation if for all $Z^{n+1}st$, $a \in A$:
*— $(n+1)$-**forth**: if $s \sim_a^M s'$, then there is a $t' \in S^N$ s.t. $t \sim_a^N t'$ and $Z^n s't'$.*
*— $(n+1)$-**back**: if $t \sim_a^N t'$, then there is a $s' \in S^M$ s.t. $s \sim_a^M s'$ and $Z^n s't'$.*
If there exists a n-bisimulation Z^n between M and N we write $M \leftrightarroweq^n N$. (We also combine the notations $\leftrightarroweq^Q$ and $\leftrightarroweq^n$ in the obvious way, writing $\leftrightarroweq^{Q,n}$.)

Given pointed models (M,s) and (N,t) and a logic L with language $\mathcal{L}_L$, $(M,s) \equiv_L (N,t)$ denotes: for all $\varphi \in \mathcal{L}_L$, $M,s \models \varphi$ iff $N,t \models \varphi$. Given $Q \subseteq P$ and $n \in \mathbb{N}$, annotations $\equiv_L^n$ and $\equiv_L^Q$ restrict the evaluated formulas $\varphi \in \mathcal{L}_L$ to those of modal depth $d(\varphi) \leq n$ and (resp.) to $\varphi \in \mathcal{L}_L|Q$. APAL is invariant for bisimilarity, but not for restricted bisimilarity or bounded bisimilarity: $(M,s) \leftrightarroweq (N,t)$ implies $(M,s) \equiv_{APAL} (N,t)$, whereas $(M,s) \leftrightarroweq^n (N,t)$ may not imply $(M,s) \equiv_{APAL}^n (N,t)$, and $(M,s) \leftrightarroweq^Q (N,t)$ may not imply $(M,s) \equiv_{APAL}^Q (N,t)$ [6,16]. This is because the APAL modality [!] implicitly quantifies over formulas of arbitrarily large modal depth and over infinitely many atoms. All logics we consider in this paper are invariant for bisimilarity.

Lemma 1. *For any L considered, $(M,s) \leftrightarroweq (N,t)$ implies $(M,s) \equiv_L (N,t)$.*

Proof. For example, case quantifier for FAPAL: $M,s \models [Q]\psi$, iff $M,s \models [\varphi]\psi$ for all $\varphi \in \mathcal{L}_{PAL}|Q$, iff $M,s \models \varphi$ implies $M|\varphi,s \models \psi$ for all $\varphi \in \mathcal{L}_{PAL}|Q$, iff (by induction) $N,t \models \varphi$ implies $N|\varphi,t \models \psi$ for all $\varphi \in \mathcal{L}_{PAL}|Q$, (…) iff $N,t \models [Q]\psi$.

Corollary 1. *Let $M,s \models \varphi$. Then $(M,s) \leftrightarroweq (N,t)$ implies $(M|\varphi,s) \leftrightarroweq (N|\varphi,t)$.*

For bounded bisimilarity this only holds for L = EL, PAL (a special case of [13], and given that PAL is as expressive as EL). As we use this result virtually identically for the inductive case announcement in subsequent proofs in the expressivity section, we give its full proof.

Lemma 2. *Let $n \in \mathbb{N}$ and $\varphi \in \mathcal{L}_{PAL}$ with $d(\varphi) = k \leq n$, models (M,s) and (N,t), and $M,s \models \varphi$ be given. If $(M,s) \leftrightarroweq^n (N,t)$, then $(M|\varphi,s) \leftrightarroweq^{n-k} (N|\varphi,t)$.*

Proof. Let $Z^0 \supseteq \cdots \supseteq Z^n$ be such that $Z^0 : (M,s) \leftrightarroweq^0 (N,t)$, ..., $Z^n : (M,s) \leftrightarroweq^n (N,t)$. For all $i = 0,\dots,n-k$, let $Z^i_\varphi : \mathcal{D}(M) \to \mathcal{D}(N)$ be defined as: $Z^i_\varphi st$ iff $Z^{i+k}st$ and $M,s \models \varphi$. As $d(\varphi) \leq n$, from n-bisimulation invariance for PAL and $M,s \models \varphi$ also follows that $N,t \models \varphi$.

By natural induction on $n-k$ we show that $Z^n : (M,s) \leftrightarroweq^n (N,t)$ implies $Z^{n-k}_\varphi : (M|\varphi,s) \leftrightarroweq^{n-k} (N|\varphi,t)$, from which the required follows.

Case $n-k=0$. We show **atoms**. We have that $Z^0_\varphi st$ iff $Z^k st$, where the latter follows from $Z^k \supseteq Z^n$ and $Z^n st$. Therefore, $Z^0_\varphi : (M|\varphi,s) \leftrightarroweq^0 (N|\varphi,t)$.

Case $n-k>0$. We show $(n-k)$-**forth**. Let $s \sim_a s'$ and $M,s' \models \varphi$, i.e., $s \sim_a s'$ in $M|\varphi$. From $Z^n : (M,s) \leftrightarroweq^n (N,t)$ and $s \sim_a s'$ follows that there is a $t' \sim_a t$ such that $Z^{n-1} : (M,s') \leftrightarroweq^{n-1} (N,t')$. As $n-k = n-d(\varphi) > 0$, $d(\varphi) < n$, so $d(\varphi) \leq n-1$. From $Z^{n-1} : (M,s') \leftrightarroweq^{n-1} (N,t')$, $M,s' \models \varphi$ and $d(\varphi) \leq n-1$ it follows by bisimulation invariance that $N,t' \models \varphi$. Therefore t' is in the domain of $N|\varphi$. By induction, from $Z^{n-1} : (M,s') \leftrightarroweq^{n-1} (N,t')$ it follows that $Z^{n-k-1}_\varphi : (M|\varphi,s') \leftrightarroweq^{n-k-1} (N|\varphi,t')$. Therefore, t' satisfies the requirement for $(n-k)$-**forth** for relation Z^{n-k}_φ.

The clause $(n-k)$-**back** is shown similarly.

Proposition 1. *$(M,s) \leftrightarroweq^Q (N,t)$ implies $(M,s) \equiv^Q_{SAPAL} (N,t)$ and implies $(M,s) \equiv^Q_{SCAPAL} (N,t)$.*

Proof. The proof is by induction on formulas true in (M,s). The crucial case quantifier is satisfied because (let $R \subseteq Q$): $M,s \models [R]\varphi$, iff $M,s \models [\psi]\varphi$ for all $\psi \in \mathcal{L}_{PAL}|R$, iff for all $\psi \in \mathcal{L}_{PAL}|R$, $M,s \models \psi$ implies $M|\psi,s \models \varphi$, iff (induction, Cor. 1) for all $\psi \in \mathcal{L}_{PAL}|R$, $N,s \models \psi$ implies $N|\psi,s \models \varphi$, iff $(\dots)$ $N,s \models [R]\varphi$.

The proof for SCAPAL is similar.

3 Modal properties of the quantifiers

We continue by discussing some peculiarities of the semantics, where we focus on modal properties of the quantifiers. We recall that APAL satisfies: $[!]\varphi \to \varphi$ (T), $[!]\varphi \to [!][!]\varphi$ (4), $\langle ! \rangle [!]\varphi \to [!]\langle ! \rangle\varphi$ (CR), and $[!]\langle ! \rangle\varphi \to \langle ! \rangle[!]\varphi$ (MK) [6,16].

3.1 SAPAL and FSAPAL

The logic SAPAL generalizes APAL, as $[P]\varphi$ is equivalent to $[!]\varphi$. We also considered FSAPAL where $Q \subseteq P$ in $[Q]\varphi$ is required to be finite.

Proposition 2. *SAPAL-valid are $[Q]\varphi \to \varphi$ (T) and $[Q \cup R]\varphi \to [Q][R]\varphi$ (4)*

Proof. The validity of $[Q]\varphi \to \varphi$ follows from the validity of $[\top]\varphi \leftrightarrow \varphi$. Just as for APAL, $[Q \cup R]\varphi \to [Q][R]\varphi$ is valid because two announcements can be made into one announcement, as in the PAL validity $[\psi][\chi]\varphi \leftrightarrow [\psi \wedge [\psi]\chi]\varphi$, and because $P(\psi \wedge [\psi]\chi) \subseteq Q \cup R$ if $P(\psi) \subseteq Q$ and $P(\chi) \subseteq R$.

Concerning $\langle Q\rangle[R]\varphi \to [Q]\langle R\rangle\varphi$ (CR) and $[Q]\langle R\rangle\varphi \to \langle Q\rangle[R]\varphi$ (MK) the first is invalid in SAPAL (counterexample omitted) and we do not know whether the second one is valid, but this seems unlikely. The proof of their APAL validity consists in first announcing the value of all variables occurring in the formula φ, and then using that φ is true in the subsequent model restriction iff it is valid on that restriction. This announcement cannot be made if $Q \cup R \subset P(\varphi)$.

3.2 SCAPAL

The SCAPAL quantifier does not distribute over conjunction: $[\subseteq]\varphi \wedge [\subseteq]\psi$ is not equivalent to $[\subseteq](\varphi \wedge \psi)$. This is easily demonstrated by an example.

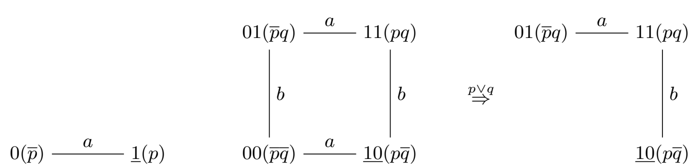

Fig. 2. Model $(N,1)$ on the left, $(M,10)$ in the middle, $(M|(p \vee q),10)$ on the right.

Example 1. Consider model $(M,10)$ in Fig. 2 ($p\overline{q}$: p is true and q is false). Then:

$$
\begin{array}{l}
M,10 \not\models [\subseteq]((K_a p \to K_b K_a p) \wedge \neg q) \\
M,10 \models [\subseteq](K_a p \to K_b K_a p) \\
M,10 \models [\subseteq]\neg q
\end{array}
$$

The first is false because, as depicted:

$$
\begin{array}{l}
M,10 \models \langle p \vee q\rangle(K_a p \wedge \neg K_b K_a p), \text{ so} \\
M,10 \models \langle p \vee q\rangle((K_a p \wedge \neg K_b K_a p) \vee q), \text{ and therefore} \\
M,10 \models \langle\subseteq\rangle((K_a p \wedge \neg K_b K_a p) \vee q), \text{ which is equivalent to} \\
M,10 \not\models [\subseteq]((K_a p \to K_b K_a p) \wedge \neg q).
\end{array}
$$

The second is true because the only model restrictions containing 10 that we can obtain with formulas involving p are $\{10,11\}$ and $\{10,11,00,01\}$. The third is true because q is false in state 10.

Therefore, $[\subseteq]\varphi \wedge [\subseteq]\psi$ is not equivalent to $[\subseteq](\varphi \wedge \psi)$.

Proposition 3. *Valid in SCAPAL are:* $[\subseteq]\varphi \to \varphi$ *(T),* $[\subseteq]\varphi \to [\subseteq][\subseteq]\varphi$ *(F),* $[\subseteq]\langle\subseteq\rangle\varphi \to \langle\subseteq\rangle[\subseteq]\varphi$ *(MK) and* $\langle\subseteq\rangle[\subseteq]\varphi \to [\subseteq]\langle\subseteq\rangle\varphi$ *(CR).*

Proof. T and 4 are valid for the same reason as in SAPAL. For CR and MK we can now (unlike for SAPAL) use the same method as in APAL, as in any state of a model we can announce the value of all variables occurring in φ. A proof of CR is found in [16, Prop. 3.10] (for the similar logic APAL$^+$), which corrects the incorrect proof of CR for APAL in [6]). A proof of MK it is found in [6].

3.3 QIPAL and IPAL

We recall that in APAL the quantification is over $\varphi \in \mathcal{L}_{PAL}$. Fairly complex counterexamples demonstrate that $[!]\varphi \rightarrow [\psi]\varphi$ is invalid for certain $\psi \in \mathcal{L}_{APAL}$ containing quantifiers. Now in $[\psi^{\downarrow}]\varphi$, $\psi \in \mathcal{L}_{QIPAL}$ may also contain quantifiers. This makes the relation to $[!]$ unclear. In $\mathcal{L}_{IPAL}$, that ψ must be in $\mathcal{L}_{PAL}$ and the relation is clearer.

Proposition 4. *Let $\psi \in \mathcal{L}_{PAL}$, $\chi \in \mathcal{L}_{IPAL}$ and pointed model (M, s) be given. The following are equivalent:*

1. $M, s \models \langle \psi^{\downarrow} \rangle \chi$
2. *there is a $\varphi \in \mathcal{L}_{PAL}$ such that $\models \varphi \rightarrow \psi$ and $M, s \models \langle \varphi \rangle \chi$,*
3. *there is a $\varphi \in \mathcal{L}_{PAL}$ such that $M \models \varphi \rightarrow \psi$ and $M, s \models \langle \varphi \rangle \chi$,*
4. *there is a $\varphi \in \mathcal{L}_{PAL}$ such that $M, s \models \langle \varphi \wedge \psi \rangle \chi$.*

Proof.
$1 \Leftrightarrow 2$ This is the semantics of the $\langle \psi^{\downarrow} \rangle$ quantifier (in dual form).
$2 \Rightarrow 3$ From $\models \varphi \rightarrow \psi$ it trivially follows that $M \models \varphi \rightarrow \psi$.
$3 \Rightarrow 4$ Suppose that there is a $\varphi \in \mathcal{L}_{PAL}$ such that $M \models \varphi \rightarrow \psi$ and $M, s \models \langle \varphi \rangle \chi$. Because $M \models \varphi \rightarrow \psi$, we have $M \models \varphi \leftrightarrow (\varphi \wedge \psi)$, and therefore $M|\varphi = M|(\varphi \wedge \psi)$. From $M, s \models \langle \varphi \rangle \chi$ then follows that $M, s \models \langle \varphi \wedge \psi \rangle \chi$.
$4 \Rightarrow 2$ Suppose that there is a $\varphi \in \mathcal{L}_{PAL}$ such that $M, s \models \langle \varphi \wedge \psi \rangle \chi$. Let $\varphi' = \varphi \wedge \psi$, and note that $\varphi' \in \mathcal{L}_{PAL}$. We have $\models \varphi' \rightarrow \psi$ and $M, s \models \langle \varphi' \rangle \chi$.

The *positive formulas* $\mathcal{L}^{+}_{PAL}$ are the PAL-fragment $p \mid \neg p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid K_a \varphi \mid [\neg \varphi]\varphi$. The truth of positive formulas (corresponding to the universal fragment in first-order logic) is preserved after update [17].

Corollary 2. *Let $\psi \in \mathcal{L}^{+}_{PAL}$. Then $\langle \psi^{\downarrow} \rangle \chi$ is equivalent to $\langle ! \rangle \langle \psi \rangle \chi$.*

Proof. Let $M, s \models \langle \psi^{\downarrow} \rangle \chi$. From Prop. 4.4 we obtain that there is $\varphi \in \mathcal{L}_{PAL}$ such that $M, s \models \langle \varphi \wedge \psi \rangle \chi$. As ψ is positive, from that we obtain $M, s \models \langle \varphi \rangle \langle \psi \rangle \chi$. By the definition of the APAL quantifier, it follows that $M, s \models \langle ! \rangle \langle \psi \rangle \chi$.

Proposition 5. *Let $\varphi \in \mathcal{L}_{IPAL}$. Then $[\top^{\downarrow}]\varphi$ is equivalent to $[!]\varphi$ and $[\bot^{\uparrow}]\varphi$ is equivalent to $[!]\varphi$.*

Proof. Let model (M, s) and $\varphi \in \mathcal{L}_{QIPAL}$ be given. Then: $M, s \models [\top^{\downarrow}]\varphi$, iff $M, s \models [\psi]\varphi$ for all $\psi \in \mathcal{L}_{PAL}$ with $\models \psi \rightarrow \top$, iff $M, s \models [\psi]\varphi$ for all $\psi \in \mathcal{L}_{PAL}$, iff $M, s \models [!]\varphi$.

Similarly, $M, s \models [\bot^{\uparrow}]\varphi$, iff $M, s \models [\psi]\varphi$ for all $\psi \in \mathcal{L}_{PAL}$ with $\models \bot \rightarrow \psi$, iff $M, s \models [\psi]\varphi$ for all $\psi \in \mathcal{L}_{PAL}$, iff $M, s \models [!]\varphi$.

Proposition 6. *Valid in QIPAL are $[\psi^{\uparrow}]\varphi \rightarrow \varphi$ (T) and also $[\psi^{\uparrow}]\varphi \rightarrow [\psi^{\uparrow}][\chi^{\uparrow}]\varphi$ and $[\psi^{\downarrow}]\varphi \rightarrow [\psi^{\downarrow}][\chi^{\downarrow}]\varphi$ (4)*

Proof. All proofs are as in Prop. 2 and 3.

However, $[\psi^{\downarrow}]\varphi \rightarrow \varphi$ (T) is invalid. Whenever $M|\psi$ is a proper submodel of a given model M, the trivial announcement is not allowed. Also, $[\psi^{\uparrow}]\varphi \rightarrow [\chi^{\uparrow}][\psi^{\uparrow}]\varphi$ and $[\psi^{\downarrow}]\varphi \rightarrow [\chi^{\downarrow}][\psi^{\downarrow}]\varphi$ are invalid because of Moorean phenomena.

We envisage similar results for the more general QIPAL quantifier in future.

4 Expressivity

We now address the relative expressivity of APAL, FSAPAL and SCAPAL and IPAL. Given logics L and L' with languages $\mathcal{L}_L$ and $\mathcal{L}_{L'}$, L is at least as expressive as L', notation $L_1 \preceq L_2$, iff for $\varphi \in \mathcal{L}_L$ there is a $\varphi' \in \mathcal{L}_{L'}$ such that φ is equivalent to φ'. Logics L and L' are equally expressive iff $L \preceq L'$ and $L' \preceq L$, L is less expressive than L', notation $L \prec L'$, iff $L \preceq L'$ but $L' \not\preceq L$; L and L' are incomparable (in expressivity), notation $L \asymp L'$, iff $L \not\preceq L'$ and $L' \not\preceq L$.

4.1 APAL $\not\preceq$ FSAPAL and APAL $\not\preceq$ SCAPAL

We show that there is an APAL-formula that can distinguish two pointed models that cannot be distinguished by any FSAPAL-formula. We use that APAL, unlike FSAPAL, quantifies over arbitrarily many atoms. The proof is similar to the proof that APAL $\not\preceq$ PAL in [6].

Proposition 7. *APAL $\not\preceq$ FSAPAL and APAL $\not\preceq$ SCAPAL.*

Proof. Consider APAL formula $\langle ! \rangle(K_a p \land \neg K_b K_a p)$, and assume towards a contradiction that ψ is an equivalent FSAPAL formula. Let $q \notin P(\psi)$. Now consider models $(M, 10)$ and $(N, 1)$ in Fig. 2 (where the value of q in states 0 and 1 of N is irrelevant). These models are p-bisimilar. We now have that:

1. $M, 10 \models \langle ! \rangle(K_a p \land \neg K_b K_a p)$ (observe $M|(p \lor q)$ in Fig. 2)
2. $N, 1 \not\models \langle ! \rangle(K_a p \land \neg K_b K_a p)$
3. $M, 10 \models \psi$ iff $N, 1 \models \psi$ ($(M, 10) \leftrightarroweq^p (N, 1)$ implies $(M, 10) \equiv^p_{FSAPAL} (N, 1)$ by Prop. 1)

This is a contradiction. Therefore APAL $\not\preceq$ FSAPAL.

As Prop. 1 also applies to SCAPAL, this also proves that APAL $\not\preceq$ SCAPAL.

4.2 SCAPAL $\not\preceq$ APAL and FSAPAL $\not\preceq$ APAL

The proof is similar to that of the previous section, but more involved. We now show that the assumption that there is an APAL formula ψ equivalent to SCAPAL formula $\langle \subseteq \rangle(\neg q \land K_a p \land \neg K_b K_a p)$ leads to a contradiction. Prior to that we present models and lemmas used in the proof.

Consider models M_n and N_n as follows, where $n \in \mathbb{N}$ is odd. Model $M_n = (S, \sim, V)$ is such that (i) $S = [0, 2n-1]$, (ii) for any $i < n$, $2i \sim_b (2i+1)$ and, except for $i = 0$, $(2i-1) \sim_a 2i$ and also $(2n-1) \sim_a 0$, and (iii) for any $i < n$, variable p is true in states $2i$, variable q is only true in state n and variable r is always false. Model N_n is like model M_n except that variable r is only true in n and variable q is always false. Fig. 3 depicts M_3 and N_3.

Lemma 3. *Let $M \subseteq M_n$, $N \subseteq N_n$, $i, j, k \in \mathbb{N}$, with $i \in \mathcal{D}(M)$ and $j \in \mathcal{D}(N)$. If $(M, i) \simeq^k (N, j)$, then for all $\chi \in \mathcal{L}_{PAL}$ such that $M, i \models \chi$ there is a $\chi' \in \mathcal{L}_{PAL}$ such that $N, j \models \chi'$ and $M|\chi \simeq^k N|\chi'$.*

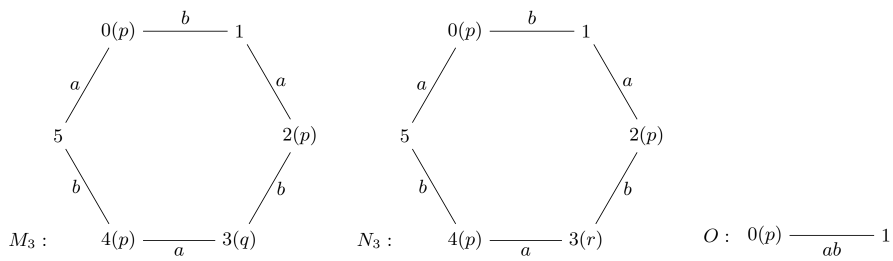

Fig. 3. The models M_3 and N_3

Proof. All subsets of M_n and all subsets of N_n are distinguishable in $\mathcal{L}_{EL}$ (where we use that PAL is as expressive as EL), using the distance from the q-state respectively r-state.[4] Also, any proper submodel of M_n or N_n, where without loss of generality we only consider connected submodels containing the evaluation point, is a finite chain of alternating a-links and b-links of which all subsets are distinguishable, using that both edges of the chain are distinguishable: either a or b knows either p or $\neg p$ in one edge but not in the other edge, except for the singleton model that however is a trivial case.

Lemma 4. *Let $M \subseteq M_n$, $N \subseteq N_n$ and $i, j, k \in \mathbb{N}$, with $i \in \mathcal{D}(M)$ and $j \in \mathcal{D}(N)$. If $(M, i) \simeq^k (N, j)$, then $(M, i) \equiv^k_{APAL} (N, j)$.*

Proof. We show the equivalent formulation:

> For all $\varphi \in \mathcal{L}_{APAL}$, $M \subseteq M_n$, $N \subseteq N_n$ and $i, j, k \in \mathbb{N}$ with $i \in \mathcal{D}(M)$ and $j \in \mathcal{D}(N)$: if $(M, i) \simeq^k (N, j)$ and $d(\varphi) \leq k$, then $M, i \models \varphi$ iff $N, j \models \varphi$.

The proof is by induction on the structure of φ. The cases of interest are $K_b\varphi$, $[\psi]\varphi$, and $[!]\varphi$. As k-bisimilarity is a symmetric relation, it suffices to show only one direction of the equivalence.

Case $K_a\varphi$**:** Suppose $d(K_a\varphi) \leq k$. We have $M, i \models K_a\varphi$ iff for all $i' \sim_a i$, $M, i' \models \varphi$. As $(M, i) \simeq^n (N, j)$, for all $j' \sim_a j$ there is some $i' \sim_a i$ such that $(M, i') \simeq^{k-1} (N, j')$. As $d(K_a\varphi) \leq n$, $d(\varphi) \leq k - 1$. Therefore, by induction, $N, j' \models \varphi$. And therefore $N, j \models K_a\varphi$.

Case $[\psi]\varphi$**:** Suppose $d([\psi]\varphi) \leq k$, and $M, i \models [\psi]\varphi$. Let $d(\psi) = x$ and $d(\varphi) = y$, then $x + y = d(\psi) + d(\varphi) = d([\psi]\varphi) \leq k$. By definition, $M, i \models [\psi]\varphi$ iff $M, i \models \psi$ implies $M|\psi, i \models \varphi$. From $M, i \models \psi$, $(M, i) \simeq^k (N, j)$ and $d(\psi) = x \leq k$ and induction we obtain $N, j \models \psi$. From $(M, i) \simeq^k (N, j)$, $M, i \models \psi$, $d(\psi) = x \leq k - y$, a part identical to that of Lemma 2 except that where bisimulation invariance for PAL is used on $\psi \in \mathcal{L}_{PAL}$ we now use induction on $\psi \in \mathcal{L}_{APAL}$, we obtain that $(M|\psi, i) \simeq^y (N|\psi, j)$. From that, $M|\psi, i \models \varphi$, $d(\varphi) = y$ and

[4] For example, state 0 is M_3 is distinguished by $\hat{K}_b\hat{K}_a\hat{K}_bq \wedge \neg\hat{K}_a\hat{K}_bq$; 0 is the unique state where we can get with three steps but not with two.

induction we obtain $N|\psi, j \models \varphi$. Then, $N, j \models \psi$ implies $N|\psi, j \models \varphi$ is by definition $N, j \models [\psi]\varphi$.

Case $[!]\varphi$**:**
$M, i \models [!]\varphi$, iff
$M, i \models [\psi]\varphi$ for all $\psi \in \mathcal{L}_{PAL}$, iff
$M, i \models \psi$ implies $M|\psi, i \models \varphi$ for all $\psi \in \mathcal{L}_{PAL}$, iff (Lemma 3)
$N, j \models \psi'$ implies $N|\psi', i \models \varphi$ for all $\psi' \in \mathcal{L}_{PAL}$, iff
$N, j \models [\psi']\varphi$ for all $\psi' \in \mathcal{L}_{PAL}$, iff
$N, j \models [!]\varphi$.

Proposition 8. *SCAPAL* $\not\preceq$ *APAL.*

Proof. Consider $\mathcal{L}_{SCAPAL}$ formula $\varphi = \langle\subseteq\rangle(\neg q \wedge K_a p \wedge \neg K_b K_a p)$. Let ψ be the supposedly equivalent $\mathcal{L}_{APAL}$ formula. Take $n > d(\psi)$. We now show that:

1. $M_n, 0 \models \langle\subseteq\rangle(\neg q \wedge K_a p \wedge \neg K_b K_a p)$
2. $N_n, 0 \not\models \langle\subseteq\rangle(\neg q \wedge K_a p \wedge \neg K_b K_a p)$
3. $M_n, 0 \models \psi$ iff $N_n, 0 \models \psi$

These items are proved by the following arguments:

1. The state n is distinguished by formula q. This allows us to distinguish each finite subset of the domain, in the usual way, in $\mathcal{L}_{EL}$ (note that there is no mirror symmetry along the 0—n 'diameter' of the circular models M_n and N_n). Thus there is a formula $\eta \in \mathcal{L}_{EL}|q$ that distinguishes the set of states $\{0, 1\}$. We now have that:

$$\begin{array}{l} M_n, 0 \models \eta \\ M_n|\eta, 0 \models \neg q \wedge K_a p \wedge \neg K_b K_a p \\ M_n, 0 \models \langle\eta\rangle(\neg q \wedge K_a p \wedge \neg K_b K_a p) \\ M_n, 0 \models \langle\subseteq\rangle(\neg q \wedge K_a p \wedge \neg K_b K_a p) \end{array}$$

2. On the other hand, $N_n, 0 \not\models \langle\subseteq\rangle(\neg q \wedge K_a p \wedge \neg K_b K_a p)$. This is because we cannot use that r is only true in n, as $r \notin P(\neg q \wedge K_a p \wedge \neg K_b K_a p)$, and because $(N_n, 0) \simeq^{pq} (O, 0)$. Clearly $O, 0 \not\models \langle\subseteq\rangle(\neg q \wedge K_a p \wedge \neg K_b K_a p)$.
3. However, $M_n, 0 \models \psi$ iff $N_n, 0 \models \psi$. This follows from Lemma 4, as $n > d(\psi)$ and $(M_n, 0) \leftrightarroweq^{d(\psi)} (N_n, 0)$.

Proposition 9. *FSAPAL* $\not\preceq$ *APAL.*

Proof. As Prop. 8, but we now take FSAPAL formula $\langle q\rangle(\neg q \wedge K_a p \wedge \neg K_b K_a p)$ instead of SCAPAL formula $\langle\subseteq\rangle(\neg q \wedge K_a p \wedge \neg K_b K_a p)$.

As $[!]\varphi$ is equivalent to $[P]\varphi$ we rather trivially have that APAL $\preceq$ SAPAL, so that with Prop. 9 and its consequence SAPAL $\not\preceq$ APAL we immediately obtain:

Corollary 3. *APAL* $\prec$ *SAPAL.*

4.3 SCAPAL $\prec$ FSAPAL

Proposition 10. *SCAPAL $\preceq$ FSAPAL.*

Proof. It is trivial that SCAPAL $\preceq$ FSAPAL, since $\models [\subseteq]\varphi \leftrightarrow [P(\varphi)]\varphi$. Formally, we inductively define a translation function f from SCAPAL to FSAPAL by

$$\begin{array}{ccc} f(p) = p & f(\varphi \vee \psi) = f(\varphi) \vee f(\psi) & f([\varphi]\psi) = [f(\varphi)]f(\psi) \\ f(\neg\varphi) = \neg f(\varphi) & f(K_a\varphi) = K_a f(\varphi) & f([\subseteq]\varphi) = [P(\varphi)]f(\varphi) \end{array}$$

In the final line we could equivalently have written $f([\subseteq]\varphi) = [P(f(\varphi))]f(\varphi)$, as f does not affect the set of atoms that occur in a formula. We then have $\models \varphi \leftrightarrow f(\varphi)$ (which is shown by induction), and therefore SCAPAL $\preceq$ FSAPAL.

We now show SCAPAL $\prec$ FSAPAL. In the proof we use models $M_{-n,n}$ and $N_{-n,n}$ similar to M_n and N_n used in the previous subsection. They are depicted in Fig. 4 for $n = 3$, compare to Fig. 3. (Imagine 'cutting open' M_3 and N_3 at the q resp. r state, and remove r as we can now use the distinguishing power of p on the edges of the chain.) Similarly to Lemma 4, we first show a Lemma 5.

$$M_{-3,3}: \quad -3(q) \overset{a}{\rule{2em}{0.4pt}} -2(p) \overset{b}{\rule{2em}{0.4pt}} -1 \overset{a}{\rule{2em}{0.4pt}} 0(p) \overset{b}{\rule{2em}{0.4pt}} 1 \overset{a}{\rule{2em}{0.4pt}} 2(p) \overset{b}{\rule{2em}{0.4pt}} 3(q)$$

$$N_{-3,3}: \quad -3 \overset{a}{\rule{2em}{0.4pt}} -2(p) \overset{b}{\rule{2em}{0.4pt}} -1 \overset{a}{\rule{2em}{0.4pt}} 0(p) \overset{b}{\rule{2em}{0.4pt}} 1 \overset{a}{\rule{2em}{0.4pt}} 2(p) \overset{b}{\rule{2em}{0.4pt}} 3$$

Fig. 4. The models $M_{-3,3}$ and $N_{-3,3}$

Lemma 5. *Let $M \subseteq M_{-n,n}$, $N \subseteq N_{-n,n}$ and $i, j, k \in \mathbb{N}$, with $i \in \mathcal{D}(M)$ and $j \in \mathcal{D}(N)$. If $(M, i) \simeq^k (N, j)$, then $(M, i) \equiv^k_{SCAPAL} (N, j)$.*

Proof. We show by formula induction that $M, i \models \varphi$ iff $N, j \models \varphi$ for any $\varphi \in \mathcal{L}_{SCAPAL}$ with $d(\varphi) \leq k$. Cases $K_a\psi$ and $[\chi]\psi$ are the same. The case quantifier $[\subseteq]\psi$ is different and shown as follows.

First, suppose that $q \notin P(\psi)$. Then from $(M, i) \leftrightarrows^{P(\psi)} (N, j)$ and Lemma 1 it directly follows that $M, i \models [\subseteq]\psi$ iff $N, j \models [\subseteq]\psi$.

Next, suppose that $q \in P(\psi)$; w.l.o.g. we may also assume that $p \in P(\psi)$. By assumption, $(M, i) \leftrightarrows^k (N, j)$. Just as for Lemma 3, every $M' \subseteq M$ is definable in M by a formula in $\mathcal{L}_{PAL}|pq$, and every $N' \subseteq N$ is definable in N by a formula in $\mathcal{L}_{PAL}|pq$. It follows that for every $\chi \in \mathcal{L}_{PAL}|pq$ with $M, i \models \chi$ there is a $\xi \in \mathcal{L}_{PAL}|pq$ such that $(M|\chi, i) \leftrightarrows^k (N|\xi, j)$, and vice versa. Therefore, $M, i \models [\subseteq]\psi$ iff $N, j \models [\subseteq]\psi$.

Proposition 11. *SCAPAL $\prec$ FSAPAL.*

Proof. We proceed as usual, however, with distinguishing FSAPAL formula $\langle q \rangle (K_a p \wedge \neg K_b K_a p)$. Let ψ be the supposedly equivalent $\mathcal{L}_{SCAPAL}$ formula. Take $n > d(\psi)$. Then:

1. $M_{-n,n}, 0 \models \langle q \rangle (K_a p \wedge \neg K_b K_a p)$
2. $N_{-n,n}, 0 \not\models \langle q \rangle (K_a p \wedge \neg K_b K_a p)$ (obvious)
3. $M_{-n,n}, 0 \models \psi$ iff $N_{-n,n}, 0 \models \psi$ (use $(M_{-n,n}, 0) \leftrightarroweq^{d(\psi)} (N_{-n,n}, 0)$ & Lemma 5)

4.4 IPAL

Proposition 12. *APAL $\preceq$ IPAL.*

Proof. This follows from Prop. 5 that $[\top^{\downarrow}]\varphi$ is equivalent to $[!]\varphi$.

We also obtained strictness, by a rather involved proof that is omitted from the submission for space constraints and therefore called a conjecture.

Conjecture 1. *APAL $\prec$ IPAL.*

The relative expressivity between IPAL and FSAPAL/SCAPAL mirrors the results already obtained between APAL and FSAPAL/SCAPAL.

Proposition 13. *IPAL $\asymp$ FSAPAL and IPAL $\asymp$ SCAPAL.*

Proof. IPAL $\not\preceq$ FSAPAL and IPAL $\not\preceq$ SCAPAL are shown as APAL $\not\preceq$ FSAPAL (Prop. 9) and APAL $\not\preceq$ SCAPAL (Prop. 8), except that in the inductive case for the quantifier of the proof of Lemma 4 we do not consider all witnesses ψ for the quantifier $\langle ! \rangle$ but only those that imply the given χ in $\langle \chi^{\downarrow} \rangle$ or that are implied by the given χ in $\langle \chi^{\downarrow} \rangle$.

From APAL $\preceq$ IPAL, FSAPAL $\not\preceq$ APAL and SCAPAL $\not\preceq$ APAL (Prop. 7), we immediately obtain FSAPAL $\not\preceq$ IPAL and SCAPAL $\not\preceq$ IPAL.

5 Substructural implication, PAL and IPAL

The satisfaction clause for IPAL announcements $[\varphi^{\downarrow}]\psi$ is loosely inspired by the satisfaction clause for substructural implication $\varphi \Rightarrow \psi$ in the relational semantics for substructural logics [30,31] and the informational interpretation of the semantics. Relational models for substructural logics comprise a set of states S and a *ternary* accessibility relation R on S. The substructural implication is a box-like binary modal operator with the following satisfaction clause:

$$x \Vdash \varphi \Rightarrow \psi \iff (\forall y, z \in S)(Rxyz \;\;\&\;\; y \Vdash \varphi \implies z \Vdash \psi)$$

Dunn and Restall point out that "perhaps the best reading [of $Rxyz$] is to say that the combination of the pieces of information x and y (not necessarily the union) is a piece of information in z" [18, p. 67]. Restall adds that "a body of information warrants $\varphi \Rightarrow \psi$ if and only if whenever you *update* that information with new information which warrants φ, the resulting (perhaps new) body of information warrants ψ" [29, p. 362] (notation adjusted).

On the informational reading, substructural implication clearly resembles an information update operator. It is therefore natural to inquire into the similarities and differences between substructural implication and epistemic update operators such as public announcements. Similarities between substructural logic and information update have been noted before. Van Benthem [11,12] observes that various *dynamic consequence relations*, arising from defining consequence in terms of the effects of successive updates (such that $\varphi \Rightarrow \psi$ roughly corresponds to $[\varphi]\psi$), lack most of the standard structural properties. Aucher [3,4] observes that dynamic epistemic logic can be seen as a two-sorted substructural logic and that the product update is a special case of the ternary accessibility relation.

Differences between substructural logics and PAL are plentiful. For example, the former are closed under substitution and the latter contains Boolean negation and conservatively extends classical propositional logic. The aspect of substructural implication that directly influenced our formulation of the IPAL$^{\downarrow}$ announcement is that *multiple bodies of information y supporting* φ are taken into account when evaluating $\varphi \Rightarrow \psi$. This aspect is easily incorporated in PAL by requiring that, in evaluating "after announcing φ, ψ is the case" in (M, s), we have to look at multiple submodels N of M containing s such that φ is satisfied in *all* states in N. This is precisely what the satisfaction clause for $[\varphi^{\downarrow}]\psi$ requires. This new $[\varphi^{\downarrow}]$ operator has interesting properties that we wish to explore later. For example, Prop. 6 established $[\psi^{\downarrow}]\varphi \to [\psi^{\downarrow}][\chi^{\downarrow}]\varphi$ which translates into the substructural property of *right weakening* 'from $\psi \Rightarrow \varphi$ infer $\psi, \chi \Rightarrow \varphi$'; whereas 'van Benthem' dynamic consequence does not satisfy right weakening.

6 Conclusions and further research

We investigated the expressivity of the logics FSAPAL, SCAPAL and IPAL. Let us finally also observe that their axiomatizations promise to be very similar to that of APAL, and the we also expect these logics still to have undecidable satisfiability problems. The use of the IPAL quantification $[\varphi^{\downarrow}]\psi$ to model substructural implication $\varphi \Rightarrow \psi$ clearly needs further research.

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