European University Institute
DEPARTMENT OF ECONOMICS

Optimal Contracts with Non-Bayesian Agents

Sarah Auster

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

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Abstract

This thesis investigates how the theoretical predictions of traditional economic models change when the assumption of Bayesian decision making is relaxed. Bayesian decision theory assumes that decision makers are able to perfectly describe their state space and assign a single prior to every possible event. The theory of unawareness relaxes the first assumption by allowing decision makers to be aware of some contingencies and unaware of others. The theory of ambiguity relaxes the second assumption and allows decision makers to prefer known risks over unknown risks.

The first chapter of this thesis analyzes the effect of ambiguity on bilateral trade in the presence of private information. It demonstrates that in an environment with adverse selection as in Akerlof's (1970) market for lemons, screening the informed party hedges against ambiguity. It further shows that the presence of ambiguity can be both beneficial or harmful for trade. If the adverse selection problem is sufficiently severe, more ambiguity surprisingly leads to more trade and thereby increase surplus. Using these results, a financial market application demonstrates that ambiguity may help to explain why some assets are optimally traded over-the-counter rather than on traditional exchanges, and suggests that opacity may be essential to sustain such trade.

The second chapter of this thesis introduces asymmetric awareness into a classical principal-agent model with moral hazard, and shows how unawareness can give rise to incomplete contracts.[1] The paper investigates the optimal contract between a fully aware principal and an unaware agent, where the principal can enlarge the agent's awareness strategically. When proposing the contract, the principal faces a tradeoff between participation and incentives: leaving the agent unaware allows the principal to exploit the agent's incomplete understanding of the world, relaxing the participation constraint, while making the agent aware enables the principal to use the revealed contingencies as signals about the agent's action choice, relaxing the incentive constraint. The optimal contract reveals contingencies that have low probability but are highly informative about the agent's effort.

[1] This paper was published in *Games and Economic Behavior*, Vol.82, 2013.

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Contents

Chapter 1

Bilateral Trade Under Ambiguity

1.1 Introduction

The optimal design of contracts in the presence of asymmetric information has been the subject of investigation for several decades and its theoretical analysis has generated an array of powerful results. Most of this literature adopts the subjective expected utility model, according to which contracting parties have a single subjective prior belief about the fundamentals. In real-life contracting situations, e.g. when buying a house or when investing in a foreign country, the involved parties rarely have a precise idea about the underlying probability distribution, either because they do not have enough experience or because they do not have sufficient information. It is well established that a lack of knowledge about the probability distribution, typically referred to as ambiguity, has important behavioral implications that are incompatible with the subjective expected utility hypothesis. This gives rise to the question of how the presence of uncertainty over probabilistic scenarios affects the optimal design of contracts and the implemented allocation.

As an illustrative example, consider the exchange of financial assets in over-the-counter (OTC) markets. In contrast to traditional stock exchanges, term of trade in OTC markets are negotiated directly between two parties. OTC markets are frequently referred to as dark or opaque because there is often little or no information about past transaction prices or quantities (Duffie, 2012). It has been argued that this lack of transparency may inhibit traders' ability to understand the odds in such markets, and thereby gives rise to ambiguity (e.g. Cheng and Zhong, 2012). Given the recent debate among academics and policy makers on the economic effects of transparency mandating policies in financial markets, the question of how the presence of ambiguity affects the trading decisions of economic agents in such trading environments has received increasing attention. The paper addresses this question in a general bilateral trade model that allows for common values, an important feature in financial trading, and an ambiguous trading environment.

In the basic model, a risk-neutral buyer (she) makes an offer to a risk-neutral seller (he), who is privately informed about the quality of his good. The buyer has ambiguous beliefs about the distribution of quality and is ambiguity-averse. The preferences of the buyer are represented by the maxmin expected utility model (Gilboa and Schmeidler, 1989), according to which the buyer evaluates her choices with the most pessimistic probability distribution in a set of distributions. The quality of the good can be either high or low, and determines the valuations of both trading parties. The buyer thus faces ambiguity over both her valuation of the good and the seller's willingness to accept her offer. The extent of both types of ambiguity is determined by the buyer's offer: if the buyer offers a price above the valuation of the high-quality seller, she is sure to trade but faces ambiguity over her valuation of the good. If the buyer offers a price below the valuation of the high-quality seller, she is sure to buy low quality but faces ambiguity over her probability of trade. When proposing a contract, the ambiguity-averse buyer faces a tradeoff between these two types of ambiguity.

Samuelson (1984) shows that the optimal contract in the absence of ambiguity is a take-it-or-leave-it price. The first main result of this paper demonstrates that the optimal contract in the presence of ambiguity is a screening menu rather than a posted price: low quality is traded at a low price with certainty, while high quality is traded at a high price probabilistically. The optimal screening menu balances the buyer's ex-ante expected utility across probabilistic scenarios and thereby hedges against ambiguity. To show how optimal screening under ambiguity aversion differs from screening contracts in the existing literature, the properties of the equilibrium contract are contrasted to those of an alternative model in which the buyer is ambiguity-neutral but risk-averse. The paper shows that a key difference between the two models is how the relative gains from trade affect the equilibrium contract and proposes a simple comparative statics exercise along these lines, demonstrating the testable implications of the model.

The second main result of the paper shows that the two types of ambiguity in this contracting problem have opposing effects on the level trade. To identify the two effects, a benchmark setting with ambiguous but independent valuations is considered, allowing for a separate variation of the extent of ambiguity over the buyer's and the seller's valuation. The paper shows that the presence of ambiguity over the buyer's valuation is harmful for trade, while the presence of ambiguity over the seller's valuation is beneficial for trade. The intuition for the first effect is that ambiguity over the buyer's valuation leads the buyer to overweigh the event in which her valuation is low. This reduces her perceived gains from trade and therefore has a negative effect on trade. Ambiguity over the seller's valuation, on the other hand, leads the buyer to overweigh the event in which the seller's valuation is high, which makes pooling more attractive and consequently has a positive effect on trade. When valuations are perfectly correlated, these two cases are mutually exclusive and the net effect of ambiguity on trade may be positive or negative. The analysis shows that if the variation in the seller's valuation is large enough, the effect of ambiguity over the seller's valuation dominates and more ambiguity leads to weakly more trade.

Applying these results to a financial market environment provides some novel insights on off-exchange trading and market transparency. The paper proposes a stylized example which demonstrates that if opacity gives rise to ambiguity, trading financial assets in opaque OTC markets may lead to more trade and a larger surplus compared to traditional exchanges. It is assumed that there is a measure of buyers and sellers that can either trade competitively in a centralized market *à la* Akerlof (1970), or trade bilaterally in a decentralized exchange environment. The analysis shows that if the adverse selection problem is sufficiently severe such that trade of the high-quality asset is precluded in the Akerlof market, decentralization can restore efficiency partially if and only if the trading environment is sufficiently ambiguous. This result stems from the positive effect of ambiguity under bilateral contracting, and highlights a new aspect in the recent debate on 'dark trading' in financial markets. It demonstrates that ambiguity may help to explain why some assets are optimally traded over-the-counter rather than on traditional exchanges, and suggests that opacity may be essential to sustain such trade.

Extensions: For a given set of measures, the maxmin expected utility model may be seen as a limiting case of the smooth ambiguity model with infinite ambiguity aversion. The smooth ambiguity model, developed by Klibanoff et al. (2005), has very robust features and allows for a separation between ambiguity and ambiguity aversion. The paper extends the characterization of the optimal contract to this representation and shows that the optimal contract under smooth ambiguity aversion is a convex combination of the optimal contract under maxmin preferences and ambiguity neutrality. As ambiguity aversion increases, the solution under smooth ambiguity aversion converges to the optimal contract under maxmin expected utility. Next, the assumption on the number of quality types is relaxed and a setting with continuously distributed quality is considered. As in the basic model, the optimal contract is a screening menu if the extent of ambiguity is sufficiently large, whereas it is a posted price if the extent of ambiguity is sufficiently small. In contrast to the binary type setting, there is an intermediate parameter region of ambiguity in which the optimal contract exhibits partial screening. The optimal degree of screening increases in the extent of ambiguity the buyer faces, reflecting the hedging function of screening. The paper derives conditions for these three cases and provides a complete characterization of the optimal contract.

Related Literature: Previous literature on ambiguity and bilateral trade has focussed on private rather than common value environments. Most closely related to this paper is the work by Bergemann and Schlag (2008 and 2011). They analyze a monopoly pricing model in which the principal follows a maxmin decision rule, or alternatively, a minimax regret criterion. The papers' main focus lies on regret preferences because in the private value case, ambiguity aversion does not affect the design of the optimal contract. This can be seen in the context of a special case in this paper: when the buyer's valuation does not depend on quality, the buyer only faces ambiguity over the seller's valuation, and the worst-case probabilistic scenario is generally given by the probability distribution with maximal weight on the high-quality good. This implies that the optimization problem under maxmin preferences is equivalent to the optimization problem under a single pessimistic prior, and the optimal contract is a posted price as in Samuelson (1984). The minimax regret criterion, on the other hand, generates a regret tradeoff that makes posted prices suboptimal. In contrast to this paper, the tradeoff is driven by the principal's aversion to forgone opportunities in a private value setting rather than a hedging motive against the two types of ambiguity that arise in a common value environment.

Furthermore, there is a recent literature on ambiguity and mechanism design in the Myerson-Satterthwaite (1983) environment with private values, e.g. Bodoh-Creed (2012), Bose and Mutuswami (2012), De Castro and Yannelis (2012) and Wolitzky (2014). This literature focusses on the implementability of efficient trade rather than the design of contracts. It explores the link between ambiguity and rent extraction and shows under which conditions ambiguity can change agents' incentives to reveal their type. In contrast to those papers, ambiguity in the proposed model leaves incentives unaffected and enters the principal's payoff function instead, thereby affecting the design of the optimal contract. Other papers that deal with games and mechanisms in ambiguous environments are Lo (1998), Mukerji (1998), Bose, Ozdenoren and Pape (2006), Lopomo et al. (2009), and Carroll (2013). Additionally, recent papers such as Bose and Renou (2013), Di Tillio et al. (2012) and Riedel and Sass (2014) introduce ambiguous mechanisms and strategic ambiguity into games. Finally, there is a considerable body of literature on ambiguity aversion and trade in partial and general equilibrium. Dow and Werlang (1992), Epstein and Wang (1994), Mukerji and Tallon (2001), Rigotti et al. (2008), and De Castro and Chateauneuf (2011) among others provide conditions under which ambiguity aversion precludes trade and leads to inefficient allocations and incomplete markets.

The rest of the paper is organized as follows. Section 1.2 describes the contracting problem, characterizes the optimal contract, and discusses the differences between ambiguity aversion and risk aversion in this setting. Section 1.3 shows the effect of ambiguity on trade and develops the financial market application. Section 1.4 characterizes the optimal contract under smooth ambiguity aversion, and Section 1.5 extends the main results to a framework with a continuum of types. Section 1.6 concludes.

1.2 Optimal Contracting Under Ambiguity

Environment: A risk-neutral buyer makes an offer to a risk-neutral seller. The seller possesses one unit of an indivisible good which can be either of high quality or of low quality. The seller is privately informed about the quality of the object, which determines both his valuation and the buyer's valuation. If the quality of the good is high (low), the seller's valuation is c_h (c_l) and the buyer's valuation is v_h (v_l), where $c_h > c_l$ and $v_h > v_l$. For both types of good, the buyer's valuation exceeds the seller's valuation, implying that there are certain gains from trade: $v_i > c_i, i = l, h$.

Ambiguity: Let σ denote the probability with which the quality of the good is high. In contrast to the standard framework, the buyer does not have a unique subjective prior belief but considers multiple values of σ possible. Ambiguity-sensitive preferences can be captured by a variety of decision-theoretic models (for a recent survey see Gilboa and Marinacci (2013)). The basic set up of this paper employs Gilboa and Schmeidler's (1989) maxmin expected utility model. This representation facilitates the exposition considerably but the basic insights of the model are more general. The analysis will show that the main results are driven by the buyer's desire to hedge against ambiguity, and thus can easily be extended to other models that allow for ambiguity aversion, e.g. the smooth ambiguity model (Klibanoff et al., 2005, see Section 1.4). In the maxmin expected utility model, the decision maker evaluates her choices with the worst probability distribution in a set of distributions. Let $\mathrm{E}_\sigma[\pi_b]$ denote the buyer's expected utility evaluated at σ and let $\Sigma = [\underline{\sigma}, \overline{\sigma}]$ denote a set of probability distributions. The buyer's maxmin expected utility is given by

$$\Pi^{MEU} = \min_{\sigma \in \Sigma} \mathrm{E}_\sigma[\pi_b].$$

Note that if $\underline{\sigma} = \overline{\sigma}$ such that Σ is a singleton, the min-operator has no bite and the buyer is a standard subjective expected utility maximizer. Otherwise ambiguity matters, and ambiguity aversion affects the contracting problem.

Contract: The buyer proposes a menu of contracts, $\{(\alpha_i, p_i)\}_{i=l,h}$, consisting of a trading probability α_i and a price p_i for each type of good.[1] Price p_i is paid independently of whether the good is transferred or not and can be interpreted as an expected price. The buyer's and seller's ex-post expected payoffs are given by $\pi_b = \alpha v_i - p$ and $\pi_s = p - \alpha c_i$, respectively. The buyer maximizes Π^{MEU} subject to the participation and incentive constraints of each type of seller:

$$\begin{aligned} p_l - \alpha_l c_l &\geq 0, && (PC_l), \\ p_h - \alpha_h c_h &\geq 0, && (PC_h), \\ p_l - \alpha_l c_l &\geq p_h - \alpha_h c_l, && (IC_l), \\ p_h - \alpha_h c_h &\geq p_l - \alpha_l c_h, && (IC_h). \end{aligned}$$

Since the seller knows his type, the constraints of the buyer's optimization problem are not affected by the presence of ambiguity in this environment. This, and the fact that the buyer's objective is weakly decreasing in p_l and p_h, implies that the solution to the buyer's optimization problem satisfies the following well established properties (see for example Salanie, 1997, Chapter 2):

1. PC_h is binding: $p_h = \alpha_h c_h$.

[1] The setting corresponds to a classic principal-agent problem with hidden information and an ambiguity-averse principal. The restriction to truthful direct-revelation mechanisms is without loss of generality. To see that the revelation principle applies, note that the seller faces no uncertainty and is a standard subjective expected utility maximizer. The proof of the revelation principle in this setting is thus equivalent to the proof for standard Bayesian Nash Implementation.

2. IC_l is binding: $p_l = \alpha_h c_h + (\alpha_l - \alpha_h)c_l$.
3. PC_l and IC_h are slack.
4. $\alpha_l = 1$.

Properties (1)-(3) state that the high-type seller receives his outside option, while the low-type seller obtains a positive information rent. Property (4) is a standard 'no distortion at the top' result. With these properties, the menu of contracts is completely characterized by the exchange probability of the high-quality good. For notational convenience, let this probability be denoted by α:

$$\{(\alpha_l, p_l), (\alpha_h, p_h)\} = \{(1, \alpha c_h + (1-\alpha)c_l), (\alpha, \alpha c_h)\}.$$

Note that there are two alternative interpretations of the basic model. In the interpretation followed throughout the paper, the good is indivisible and α is a lottery, determining the ex-ante probability of trade. In an alternative interpretation, the seller possesses one unit of a perfectly divisible good, utility functions are multiplicatively linear, and α is a quantity. Under this interpretation, the menu is a non-linear pricing schedule with a quantity discount. Low quality is traded in large quantity at a low price, while high quality is traded in small quantity at a high price.

Optimization Problem: Given the properties stated above, the buyer's objective can be stated as a function of α. The buyer's expected utility under single prior σ is given by

$$\mathrm{E}_\sigma[\pi_b] = \sigma\alpha(v_h - c_h) + (1-\sigma)(v_l - \alpha c_h - (1-\alpha)c_l).$$

The buyer's optimization problem is consequently

$$\max_{\alpha} \quad \min_{\sigma\in\Sigma} \{\sigma\alpha(v_h - c_h) + (1-\sigma)(v_l - \alpha c_h + (1-\alpha)c_l)\}.$$

To derive the solution to this problem, consider first the case in which Σ is a singleton and the buyer is a subjective expected utility maximizer with single prior σ. Samuelson (1984) shows that the buyer's optimal contract is a first-and-final price, which the seller either accepts or rejects. In the two-type setting this result can easily be seen by considering the buyer's objective. Since $\mathrm{E}_\sigma[\pi_b]$ is linear in α, the optimization problem has a corner solution. $\alpha = 0$ corresponds to offering a price equal to c_l and yields expected utility $(1-\sigma)(v_l - c_l)$, while $\alpha = 1$ corresponds to offering a price equal to c_h and yields expected utility $\sigma v_h + (1-\sigma)v_l - c_h$.

Pooling is optimal if and only if the probability that the seller's type is high is large enough, which is the case if

$$\sigma \geq \frac{c_h - c_l}{v_h - c_l}.$$

Under ambiguity and maxmin preferences, Samuelson's result turns out to fail. Proposition 1.2.1 shows that if Σ is not a singleton, the buyer's optimal contract is a posted price if and only if this price is optimal under all probability distributions in Σ. Otherwise, the buyer optimally offers a menu.

Proposition 1.2.1. *Let* $\tilde{\sigma} := \frac{c_h - c_l}{v_h - c_l}$ *and* $\tilde{\alpha} := \frac{v_l - c_l}{v_h - c_l}$. *The optimal menu of contracts is* $\{(1, \alpha^* c_h + (1 - \alpha^*)c_l), (\alpha^*, \alpha^* c_h)\}$ *with*

$$\alpha^* = \begin{cases} 0 & \text{if} \quad \overline{\sigma} \leq \tilde{\sigma}, \\ 1 & \text{if} \quad \underline{\sigma} \geq \tilde{\sigma}, \\ \tilde{\alpha} & \text{otherwise.} \end{cases}$$

Proof See Appendix 1.7.1.

$\tilde{\sigma}$ is the subjective prior under which the buyer is indifferent between all $\alpha \in [0, 1]$. If $\underline{\sigma} \geq \tilde{\sigma}$ ($\overline{\sigma} \leq \tilde{\sigma}$), the contract that maximizes the buyer's expected utility is a posted price equal to c_h (c_l) for all $\sigma \in \Sigma$. Ambiguity aversion thus affects the buyer's utility but not her choice of contract. If $\underline{\sigma} < \tilde{\sigma} < \overline{\sigma}$, the buyer optimally proposes a menu, characterized by $\tilde{\alpha}$. $\tilde{\alpha}$ is the contracting parameter for which the buyer's ex-post expected payoff is constant across quality, i.e.

$$\tilde{\alpha}(v_h - c_h) = v_l - \tilde{\alpha} c_h - (1 - \tilde{\alpha})c_l.$$

The menu characterized by $\tilde{\alpha}$ makes the buyer's payoff independent of the type distribution and yields a "safe payoff" equal to $\frac{(v_h - c_h)(v_l - c_l)}{v_h - c_l}$. Offering a screening menu thus hedges against ambiguity. Hedging is optimal if and only if Σ is large enough, i.e. if and only if the environment is sufficiently ambiguous.

This is illustrated in Figure 1. Under the assumption $\underline{\sigma} < \tilde{\sigma} < \overline{\sigma}$, the expected utility of a buyer with subjective prior $\underline{\sigma}$ is downward sloping in α (solid line), while the expected utility of a buyer with subjective prior $\overline{\sigma}$ is upward sloping in α (dashed line). All expected utility functions $\mathrm{E}_{\sigma}[\pi_b], \sigma \in (\underline{\sigma}, \overline{\sigma})$ lie in between these two benchmarks and intersect at $\tilde{\alpha}$.

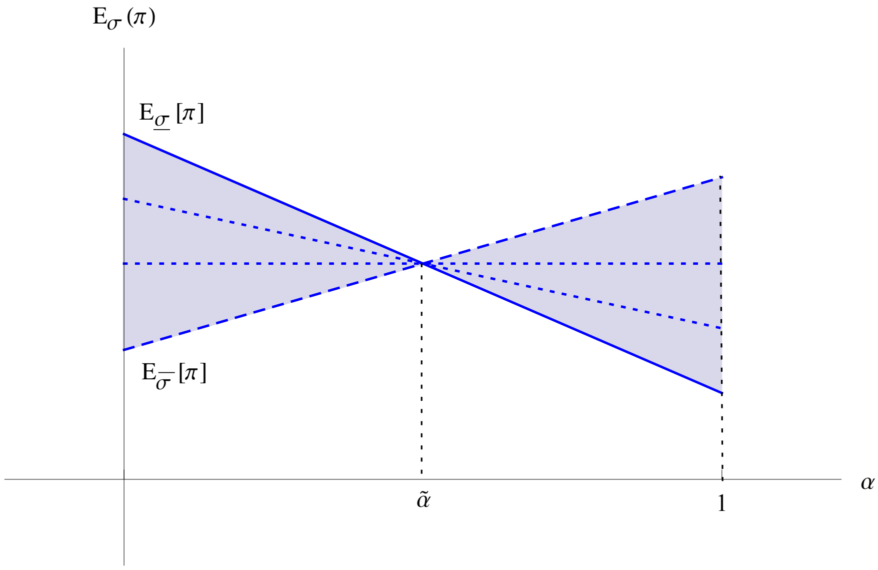


Figure 1.1: $\mathrm{E}_\sigma[\pi_b]$ for $\sigma \in [\underline{\sigma}, \overline{\sigma}]$

For any $\alpha < \tilde{\alpha}$, the worst-case probability distribution in Σ is $\overline{\sigma}$, while for any $\alpha > \tilde{\alpha}$, the worst-case probability distribution is $\underline{\sigma}$. Since $\mathrm{E}_{\overline{\sigma}}[\pi_b]$ is upward sloping in α and $\mathrm{E}_{\underline{\sigma}}[\pi_b]$ is downward sloping in α, the buyer's maxmin payoff is maximized at $\tilde{\alpha}$.

1.2.1 Ambiguity Aversion vs. Risk Aversion

An important question is whether the behavioral implications of ambiguity aversion in this contracting problem differ from those of risk aversion. It is well known that if utility functions are not restricted to be linear, screening can arise in equilibrium, even when there is no ambiguity. Given the long-standing debate on the observability of ambiguity aversion (see for example Bayer et al., 2013, and references therein), the question arises how screening that is driven by an ambiguity hedging motive can be distinguished from screening in the standard model. This section demonstrates that a key difference between the two models is how the relative gains from trade affect the equilibrium contract. Along these lines, a simple comparative statics exercise shows how ambiguity aversion can be distinguished from risk aversion in this contracting problem.

More concretely, this section compares the properties of the equilibrium contract in two alternative settings. The first setting corresponds to the basic model analyzed in the previous section, where the buyer is risk-neutral and ambiguity-averse. The second specification is equivalent to the first, apart from the buyer's objective. The buyer in the alternative model is Bayesian and risk-averse. Her utility from purchasing the good, $u(i,p)$, depends on the good's quality $i \in \{l,h\}$ and the good's price p, and has the following properties:

- Risk aversion: $u_2(i,p) < 0$, $u_{22}(i,p) < 0$, $i = l,h$.
- Certain gains from trade: $u(i,c_i) > 0, i = l,h$
- Common values: $u(h,p) > u(l,p), \forall p$.

Special cases of this general utility function are for example $u(i,p) = v_i - g(p)$, where $g'(\cdot) > 0$ and $g''(\cdot) > 0$ (additive separability), or $u(i,p) = \upsilon(v_i - p)$, where $\upsilon'(\cdot) > 0$ and $\upsilon''(\cdot) < 0$. The outside option of not trading is normalized to zero.

The buyer maximizes her expected utility subject to the participation and incentive constraints of the seller. The set of constraints is equivalent to the one in the basic model, implying that $p_h = \alpha c_h$ and $p_l = \alpha c_h + (1-\alpha)c_l$. As noted before, p_h can be seen as an expected price: the buyer pays price c_h with probability α and zero otherwise. The buyer thus solves

$$\max_{\alpha} \quad \sigma\alpha u(h,c_h) + (1-\sigma)u(l,\alpha c_h + (1-\alpha)c_l).$$

The first-order condition to this optimization problem is given by

$$\sigma u(h,c_h) = -(1-\sigma)u_2(l,\alpha c_h + (1-\alpha)c_l)(c_h - c_l).$$

The first-order condition shows the classic tradeoff between efficiency and rent extraction. The left-hand side corresponds to the marginal benefit from increasing α, the additional gains from trade of the high quality good. The right-hand side corresponds to the marginal cost from increasing α, the additional information rent paid to the low-type seller. The marginal benefit is constant in α, while the marginal cost is increasing in α. Thus, if $\sigma u(h,c_h) > -(1-\sigma)u_2(l,c_l)(c_h - c_l)$ and $\sigma u(h,c_h) < -(1-\sigma)u_2(l,c_h)(c_h - c_l)$, the buyer's problem has an interior solution and the optimal contract is a menu rather than a posted price.

Screening under risk aversion can be distinguished from screening under ambiguity aversion through a simple comparative statics exercise. Consider an increase in the buyer's valuation of the high-quality good. Under risk aversion, this increase leads to an increase in the optimal contracting parameter α^*: an increase in the buyer's valuation of the high-quality good corresponds to an increase in $u(h, c_h)$, raising the marginal benefit of increasing α and thereby distorting the tradeoff between efficiency and rent extraction. Trading the high-quality good becomes more attractive, leading to an increase in the optimal contracting parameter α^*. This can easily be seen from the first-order condition above.

In the case of ambiguity aversion, the analysis of the optimal contract showed that the optimal contracting parameter is given by $\tilde{\alpha} = \frac{v_l - c_l}{v_h - c_l}$, provided that $\underline{\sigma} < \tilde{\sigma} < \overline{\sigma}$. An increase in the buyer's valuation of the high-quality good corresponds to an increase in v_h. Since $\tilde{\alpha}$ is decreasing in v_h, it follows immediately that an increase in the buyer's valuation of the high-quality good leads to a decrease in the optimal contracting parameter α^*. The intuition is as follows. The contracting parameter $\tilde{\alpha}$ balances the buyer's ex-post expected payoff across quality and thereby hedges against ambiguity over the quality distribution. As v_h increases, the buyer's expected payoff conditional on quality being high, $\alpha(v_h - c_h)$, increases, while the buyer's expected payoff conditional on quality being low, $v_l - \alpha c_h - (1 - \alpha)c_l$, remains unchanged. In order to re-balance these expected payoffs, the buyer reduces the probability of trade with the high-quality seller which in turn reduces the information rent paid to the low type seller. Observation 1.2.2 summarizes this result.[2]

Observation 1.2.2. *Let α^* denote the buyer's optimal contracting parameter.*

- *If the buyer is risk-averse and ambiguity-neutral, α^* is weakly increases in the buyer's valuation of the high-quality good.*
- *If the buyer is risk-neutral and ambiguity-averse, α^* is weakly decreases in the buyer's valuation of the high-quality good.*

Observation 1.2.2 thus demonstrates that even if ambiguity aversion is not directly observable through the equilibrium contract, a simple comparative statics exercise can resolve the question of whether screening is driven by the well-known tradeoff between efficiency and rent extraction or by the buyer's desire to hedge against ambiguity.

[2]The comparative statics effect under ambiguity aversion illustrates a more general property of choice under maxmin expected utility. In recent work, Ghili and Klibanoff (2014) propose a monotonicity axiom, and show that Ellsberg behavior under maxmin expected utility preferences is incompatible with this axiom. The axiom, called 'Monotonicity in Mixtures', roughly states that if an act is replaced by a dominating act, it is optimal to mix more towards the dominant act. The trading probability α in this framework can be re-interpreted as a randomizing probability between offering price c_h and offering price c_l. The comparative statics exercise considered increases the payoff associated to offering c_h but leads to a decrease in the optimal probability with which c_h is offered, a violation of 'Monotonicity in Mixtures'.

1.3 Ambiguity and Trade

This section addresses the question of how the presence of ambiguity affects the level of trade. In the maxmin expected utility model, the extent of ambiguity the buyer perceives is captured by the size of the interval Σ. Since valuations in the proposed contracting problem are perfectly correlated, an expansion of this interval corresponds to both an increase in ambiguity over the seller's valuation and an increase in ambiguity over the buyer's valuation. In order to separate the two effects, this section first considers a benchmark model in which the buyer faces ambiguity over both the seller's valuation and her own valuation, but where these valuations are independent. This model shows that more ambiguity over the seller's valuation leads to weakly more trade, whereas more ambiguity over the buyer's valuation leads to weakly less trade. We will then return to the basic model with perfect correlation and show under which conditions one effect outweighs the other.

In the model with independent valuations, the buyer's set of prior probabilities is given by $\Sigma = [\underline{\sigma}_s, \overline{\sigma}_s] \times [\underline{\sigma}_b, \overline{\sigma}_b]$, where σ_s (σ_b) denotes the probability with which the seller's (buyer's) valuation is high. The extent of ambiguity over the seller's valuation is captured by the interval $[\underline{\sigma}_s, \overline{\sigma}_s]$, while the extent of ambiguity over the buyer's valuation is captured by the interval $[\underline{\sigma}_b, \overline{\sigma}_b]$. Due to the independence of valuations, the worst-case probability distribution in Σ does not depend on the buyer's contract choice. To see this, consider the buyer's single prior payoff

$$E_{\sigma_s,\sigma_b}[\pi_b] = \sigma_s \alpha(\sigma_b v_h + (1-\sigma_b)v_l - c_h) + (1-\sigma_s)(\sigma_b v_h + (1-\sigma_b)v_l - \alpha c_h - (1-\alpha)c_l),$$

with

$$\begin{aligned} \frac{\partial \mathrm{E}_{\sigma_s,\sigma_b}[\pi_b]}{\partial \sigma_s} &= -(1-\alpha)(\sigma_b v_h + (1-\sigma_b)v_l - c_l) \leq 0, \\ \frac{\partial \mathrm{E}_{\sigma_s,\sigma_b}[\pi_b]}{\partial \sigma_b} &= (1-\sigma_s+\sigma_s\alpha)(v_h - v_l) \geq 0. \end{aligned}$$

$E_{\sigma_s,\sigma_b}[\pi_b]$ is weakly decreasing in σ_s and weakly increasing in σ_b, for any $\alpha \in [0,1]$. This implies that the minimizing probability in Σ is generally given by $\{\overline{\sigma}_s, \underline{\sigma}_b\}$, the probability distribution with maximal weight on the high-type seller and low-type buyer. The buyer's optimization problem under ambiguity thus corresponds to a standard optimization problem under a single subjective prior $\{\overline{\sigma}_s, \underline{\sigma}_b\}$ and is given by

$$\max_{\alpha} \quad \Pi^{MEU} = \overline{\sigma}_s \alpha(\underline{\sigma}_b v_h + (1-\underline{\sigma}_b)v_l - c_h) + (1-\overline{\sigma}_s)(\underline{\sigma}_b v_h + (1-\underline{\sigma}_b)v_l - \alpha c_h - (1-\alpha)c_l).$$

Samuelson's (1984) result applies and the problem has a corner solution. The optimal contract is characterized by

$$\alpha^* = \begin{cases} 0 & \text{if} \quad \overline{\sigma}_s < \frac{c_h - c_l}{\underline{\sigma}_b v_h + (1-\underline{\sigma}_b)v_l - c_l}, \\ 1 & \text{if} \quad \overline{\sigma}_s \geq \frac{c_h - c_l}{\underline{\sigma}_b v_h + (1-\underline{\sigma}_b)v_l - c_l}. \end{cases}$$

Ambiguity over the buyer's valuation: Consider first an increase in the extent of ambiguity the buyer faces over her own valuation by considering an expansion of the interval $[\underline{\sigma}_b, \overline{\sigma}_b]$. Let $[\underline{\sigma}_b, \overline{\sigma}_b] \subset [\underline{\sigma}'_b, \overline{\sigma}'_b]$, or equivalently $\underline{\sigma}'_b \leq \underline{\sigma}_b$ and $\overline{\sigma}'_b \geq \overline{\sigma}_b$. Since

$$\frac{c_h - c_l}{\underline{\sigma}_b v_h + (1-\underline{\sigma}_b)v_l - c_l} < \frac{c_h - c_l}{\underline{\sigma}'_b v_h + (1-\underline{\sigma}'_b)v_l - c_l},$$

an increase in ambiguity over the buyer's valuation reduces the range of parameter values under which pooling is optimal. This implies that whenever the buyer does not offer the pooling price under $[\underline{\sigma}_b, \overline{\sigma}_b]$, she does not either under $[\underline{\sigma}'_b, \overline{\sigma}'_b]$. More ambiguity over the buyer's valuation thus leads to weakly less trade. The intuition is that the buyer's aversion to ambiguity over her own valuation implies that the buyer overweighs the probability with which her valuation is low. This diminishes the buyer's perceived gains from trade and thus has a negative effect on trade.

Ambiguity over the seller's valuation: Similarly, consider an increase in the degree of ambiguity over the seller's valuation by considering an expansion of the interval $[\underline{\sigma}_s, \overline{\sigma}_s]$. Let $[\underline{\sigma}_s, \overline{\sigma}_s] \subset [\underline{\sigma}'_s, \overline{\sigma}'_s]$. $\overline{\sigma}'_s \geq \overline{\sigma}_s$ implies

$$\overline{\sigma}_s > \frac{c_h - c_l}{\underline{\sigma}_b v_h + (1-\underline{\sigma}_b)v_l - c_l} \quad \Rightarrow \quad \overline{\sigma}'_s > \frac{c_h - c_l}{\underline{\sigma}'_b v_h + (1-\underline{\sigma}'_b)v_l - c_l}.$$

Whenever the buyer offers the pooling price under $[\underline{\sigma}_s, \overline{\sigma}_s]$, she does so under $[\underline{\sigma}'_s, \overline{\sigma}'_s]$. More ambiguity over the seller's valuation thus leads to weakly more trade. The intuition is that the buyer's aversion to ambiguity over the seller's valuation leads the buyer to overweigh the probability with which the seller's valuation is high. Since under such beliefs pooling is optimal, this type of ambiguity has a positive effect on trade.

Net effect: When types are perfectly correlated as in the basic model, the seller's and buyer's valuation cannot be simultaneously high and low. This implies that the worst-case probability distribution depends on the contract the buyer offers. The previous section showed that if the buyer offers a menu characterized by $\alpha < \tilde{\alpha}$, the minimizing probability distribution in $\Sigma = [\underline{\sigma}, \overline{\sigma}]$ is given by $\overline{\sigma}$, corresponding to the distribution with maximal weight on the high-quality good. On the other hand, if the buyer offers a menu characterized by $\alpha > \tilde{\alpha}$, the minimizing probability distribution in Σ is given by $\underline{\sigma}$, corresponding to the distribution with maximal weight on the low-quality good. This implies that for low values of α, the buyer's aversion to ambiguity over the seller's valuation outweighs her aversion to ambiguity over her own valuation, whereas for large values of α, the reverse implication holds.

To see the net effect of ambiguity on trade, let α^*_Σ denote the buyer's optimal contracting parameter under $\Sigma = [\underline{\sigma}, \overline{\sigma}]$. Section 1.2 showed that $\alpha^*_\Sigma = 0$ if $\underline{\sigma} < \overline{\sigma} < \tilde{\sigma}$, $\alpha^*_\Sigma = \tilde{\alpha}$ if $\underline{\sigma} < \tilde{\sigma} < \overline{\sigma}$, and $\alpha^*_\Sigma = 1$ if $\tilde{\sigma} < \underline{\sigma} < \overline{\sigma}$. An increase in the extent of ambiguity implies that $\underline{\sigma}$ weakly decreases, while $\overline{\sigma}$ weakly increases. If the decrease in $\underline{\sigma}$ and the increase in $\overline{\sigma}$ is sufficiently large, the condition $\underline{\sigma} < \tilde{\sigma} < \overline{\sigma}$ eventually becomes satisfied and the buyer's optimal contract is characterized by $\tilde{\alpha}$. Whether an increase in ambiguity has a positive or negative effect on trade thus depends on the initial equilibrium contract: if $\alpha^*_\Sigma = 0$, more ambiguity leads to weakly more trade, whereas if $\alpha^*_\Sigma = 1$, more ambiguity leads to weakly less trade. If $\alpha^*_\Sigma = \tilde{\alpha}$, an increase in ambiguity has no effect. This is summarized in Proposition 1.3.1.

Proposition 1.3.1. *Let $\Sigma \subset \Sigma'$. If $\underline{\sigma} < \tilde{\sigma}$, then $\alpha^*_\Sigma \leq \alpha^*_{\Sigma'}$. If $\tilde{\sigma} < \overline{\sigma}$, then $\alpha^*_\Sigma \geq \alpha^*_{\Sigma'}$.*

Proof See Appendix 1.7.2.

Note that $\tilde{\sigma} = \frac{c_h - c_l}{v_h - c_l}$ corresponds to the relative variation in the seller's valuation across quality, while $\tilde{\alpha} = 1 - \frac{v_h - v_l}{v_h - c_l}$ corresponds to the inverse of the relative variation in the buyer's valuation across quality. The variation of the seller's valuation thus determines the sign of the effect of ambiguity on trade, while the variation of the buyer's valuation determines the extent of this effect. The larger $\frac{c_h - c_l}{v_h - c_l}$ is, the larger is the set of parameters for which more ambiguity leads to more trade, reflecting the positive effect of ambiguity over the seller's valuation. The larger $\frac{v_h - v_l}{v_h - c_l}$ is, the smaller is the contracting parameter $\tilde{\alpha}$ to which the solution under increasing ambiguity jumps, reflecting the negative effect of ambiguity over the buyer's valuation. The net effect is positive and sizable if $\frac{c_h - c_l}{v_h - c_l}$ is large and $\frac{v_h - v_l}{v_h - c_l}$ is small, while the reverse implication holds if $\frac{c_h - c_l}{v_h - c_l}$ is small and $\frac{v_h - v_l}{v_h - c_l}$ is large.

Remark: *Note that if $v_l = v_h$, the buyer's type does not depend on the seller's type. Under this parameter restriction, the buyer only faces ambiguity over the seller's valuation and ambiguity always has a positive effect on trade. To see this, note that $\tilde{\alpha} = \frac{v_l - c_l}{v_h - c_l} = 1$ and the worst-case probability distribution is generally given by the distribution with maximal weight on the high-type seller, $\overline{\sigma}$. The buyer's optimization problem under maxmin preferences thus corresponds to the optimization problem of a subjective expected utility maximizer with single prior $\overline{\sigma}$. Samuelson's result applies and the optimal contract is a posted price. The buyer offers c_l if $\overline{\sigma} < \tilde{\sigma}$, while she offers the pooling price c_h if $\overline{\sigma} \geq \tilde{\sigma}$. As the extent of ambiguity increases, the latter condition eventually becomes satisfied, implying that more ambiguity leads to weakly more trade. The intuition is that offering the pooling contract yields a safe payoff equal to the difference between the buyer's valuation and the pooling price. The more ambiguity the buyer faces, the more attractive the safe pooling option becomes. Thus, in the private value case, ambiguity generally has a positive effect on trade.*

1.3.1 Ambiguity in Financial Markets

The lack of transparency in financial markets has received increasing attention by regulators and academics. Recent papers have argued that missing transparency may give rise to ambiguity, often perceived to be harmful for trade, e.g. Easley and O'Hara (2010) and Cheng and Zhong (2012). The previous analysis demonstrates that the effect of ambiguity on trade may itself be ambiguous and provides conditions under which ambiguity improves the trading outcome. In the context of financial markets this suggests that if trade is bilateral, as in OTC markets, opacity may be beneficial for trade and may increase surplus. This section shows that if the positive effect of ambiguity is large enough, the presence of ambiguity not only improves the bilateral trading outcome but this outcome may even outperform a competitive centralized market allocation. Specifically, a stylized example is constructed to demonstrate that if the adverse selection problem is sufficiently severe, bilateral exchange implements a more efficient allocation than centralized markets if and only if there is sufficient ambiguity. This result highlights a new aspect in the recent debate on why a large class of assets is predominantly traded over the counter rather than on traditional exchanges (see for example Malamud and Rostek (2013) and the references therein) and why opacity plays a critical role.

Suppose there is an equal measure of buyers and sellers that can either be matched bilaterally or trade in a centralized market *à la* Akerlof. In the first setting, we abstract from possible matching frictions and assume that each buyer meets exactly one seller.[3] In the second setting, assets are exchanged competitively at a market clearing price.[4] As in the basic model, sellers are endowed with one unit of an asset and have private information about its quality. At the market clearing price, buyers face no ambiguity over their probability of trade but only over their valuation of the asset. Ambiguity in a competitive market with adverse selection thus generally has a negative effect on trade. The worst-case probabilistic scenario for a buyer is given by the distribution under which the probability of buying low quality is maximal, $\underline{\sigma}$. The behavior of ambiguity-averse buyers in this market is consequently equivalent to that of subjective expected utility maximizers with a single prior $\underline{\sigma}$. Trade of the high-quality asset is feasible if and only if the high-type seller's valuation does not exceed the buyer expected valuation evaluated at $\underline{\sigma}$, i.e.

$$c_h \leq \underline{\sigma} v_h + (1 - \underline{\sigma}) v_l.$$

If this inequality is not satisfied, trade of the high-quality asset is precluded. Note that the larger the extent of ambiguity is, the smaller is $\underline{\sigma}$ and the more likely it is that only low quality can be traded. Ambiguity thus exacerbates the adverse selection problem in a competitive market à la Akerlof.

Now consider the case in which buyers and seller are matched bilaterally. Assume that the buyer makes a take-it-or-leave-it offer as in Section 1.2, which the seller accepts or rejects, and consider the following condition:

$$\underline{\sigma} < \frac{c_h - v_l}{v_h - v_l} < \underbrace{\frac{c_h - c_l}{v_h - c_l}}_{=\tilde{\sigma}} < \overline{\sigma}.$$

The first inequality is equivalent to $c_h > \underline{\sigma} v_h + (1 - \underline{\sigma}) v_l$, which precludes trade of the high-quality asset in the centralized market. The second inequality follows from $c_l < v_l$, and the third inequality together with the first implies that the optimal contract under bilateral trade is characterized by $\tilde{\alpha}$.[5] Since $\tilde{\alpha} > 0$, the above condition implies that the high-quality asset

[3] In reality, matching frictions are relevant of course. The following results and statements hold as long as these frictions are sufficiently small.

[4] This implicitly assumes that trade is non-exclusive, a natural assumption in the context of financial markets. Non-exclusivity implies that competitive screening *à la* Rothschild and Stiglitz (1976) cannot be sustained in equilibrium.

[5] The basic model assumes that the good is indivisible and that $\tilde{\alpha}$ is a probability. As noted before, there is an alternative interpretation in which the good is perfectly divisible and $\tilde{\alpha}$ is a quantity. The optimal contract can then be interpreted as a non-linear pricing schedule with quantity discounts. This type of

is traded with positive probability under bilateral contracting but not in the competitive market. Since the condition requires $\underline{\sigma} < \overline{\sigma}$, the argument crucially relies on the presence of ambiguity, and suggests that if traders have sufficiently ambiguous beliefs about the fundamentals, decentralized trade may yield strictly more surplus than traditional exchanges.

Suppose now that regulators had means to reduce the extent of ambiguity traders face, e.g. through transparency mandating policies. In particular, assume that there is an objective probability that quality is high, $\hat{\sigma} \in \Sigma$, that can be credibly revealed to traders. If $c_h > \hat{\sigma} v_h + (1 - \hat{\sigma}) v_l$ and $\hat{\sigma}$ is disclosed, trade of the high-quality asset is precluded in the centralized market and strictly sub-optimal under bilateral contracting. If, on the other hand, $\hat{\sigma}$ is not disclosed and traders have ambiguous beliefs, optimal contracting in bilateral meetings implements trade of the high-quality asset with positive probability as long as $\overline{\sigma} > \tilde{\sigma}$. This suggests that if the adverse-selection problem is sufficiently severe, the presence of ambiguity may be essential for high-quality assets to be traded. Presuming that transparency mandating policies reduce the extent of ambiguity traders face, this demonstrates a new effect of opacity on financial market outcomes, adding to a recent literature on the welfare effects of opaque trading, e.g. Di Maggio and Pagano (2014) and Pancs (2014).[6]

1.4 Smooth Ambiguity Aversion

For a given set of measures, the maxmin expected utility model may be seen as a special case of the smooth ambiguity model with infinite ambiguity aversion. The smooth ambiguity model, developed by Klibanoff et al. (2005), not only has very robust features but also allows for a separation between ambiguity and ambiguity attitude. This section extends the characterization of the optimal contract to the case of smooth ambiguity aversion and shows how the properties of the solution are connected to the optimal contract under maxmin expected utility. The buyer's utility function is given by

$$\Pi^{SM} = \mathrm{E}_{\mu} \left[\Phi \left(\mathrm{E}_{\sigma}[\pi_b] \right) \right],$$

non-linear pricing is indeed commonly observed in OTC markets, for example in the form of dealers posting quotes which only hold for small quantities. The exchange of larger quantities is negotiated directly between dealers and traders and typically executed at a lower price (Jankowitsch et al., 2011).

[6] In these papers, decision makers are subjective expected utility maximizers and more transparency provides more information rather than less ambiguity. Both papers show that opaque trading may be efficient.

where $\mu : [0,1] \to [0,1]$ is a subjective prior on a set of probability measures and $\Phi : \mathbb{R} \to \mathbb{R}$ is a function that weighs different realizations of the decision maker's expected utility $E_\sigma[\pi_b]$. Ambiguity is captured by the second-order belief μ, which measures the buyer's belief about a particular σ being the "correct" probability distribution. Ambiguity attitude is captured by the function Φ. If Φ is linear, the buyer is ambiguity neutral and her preferences are observationally equivalent to those of a subjective expected utility maximizer. If, on the other hand, Φ is concave, the buyer is ambiguity-averse and prefers known risks over unknown risks. The degree of ambiguity aversion is measured by the coefficient of absolute ambiguity aversion $-\frac{\Phi''(x)}{\Phi'(x)}$.[7]

As in the basic model, the buyer proposes a menu of the form $\{(1, \alpha c_h + (1-\alpha)c_l), (\alpha, \alpha c_h)\}$. The optimization problem is given by

$$\max_\alpha \mathrm{E}_\mu \left[\Phi\left(\sigma\alpha(v_h - c_h) + (1-\sigma)(v_l - \alpha c_h - (1-\alpha)c_l)\right)\right].$$

To make ambiguity matter assume that μ has positive mass on both $[0, \tilde{\sigma}]$ and $[\tilde{\sigma}, 1]$ and assume that $\Phi'(\cdot) > 0, \Phi''(\cdot) < 0$. The first order condition to the buyer's optimization problem is given by

$$\begin{aligned} &\underbrace{\mathrm{E}_\mu \left[\Phi'\left(\mathrm{E}_\sigma[\pi_b(\alpha)]\right)(\tilde{\sigma} - \sigma) | \sigma < \tilde{\sigma}\right]}_{\equiv MC(\alpha)} \\ = &\underbrace{\mathrm{E}_\mu \left[\Phi'\left(\mathrm{E}_\sigma[\pi_b(\alpha)]\right)(\sigma - \tilde{\sigma}) | \sigma > \tilde{\sigma}\right]}_{\equiv MG(\alpha)}. \end{aligned}$$

The marginal cost of increasing α, $MC(\alpha)$, is the marginal decrease in expected utility in the probabilistic scenario that pooling is not optimal ($\sigma < \tilde{\sigma}$), while the marginal gain of increasing α, $MG(\alpha)$, is the marginal increase in expected utility in the probabilistic scenario that pooling is optimal ($\sigma > \tilde{\sigma}$). Concavity of Φ implies that the marginal cost is increasing in α, whereas the marginal gain is decreasing in α. This implies that there is a unique α that maximizes the buyer's expected payoff. The conditions for an interior solution are

$$MC(0) < MG(0) \quad \text{and} \quad MC(1) > MG(1).$$

[7] $-\frac{\Phi''(x)}{\Phi'(x)}$ measures aversion to subjective uncertainty about ex-ante payoffs. Analogous to risk aversion, Klibanoff et al. (2005) define ambiguity aversion as an aversion to mean preserving spreads in the subjective probability distribution over the set of expected utility values.

Proposition 1.4.1 summarizes this result.

Proposition 1.4.1. *The optimal menu of contracts for a buyer with smooth ambiguity aversion is* $\{(1, \alpha^* c_h + (1-\alpha^*)c_l), (\alpha^*, \alpha^* c_h)\}$ *with*

$$\alpha^* = \begin{cases} 0 & \text{if} \quad MC(0) \geq MG(0), \\ 1 & \text{if} \quad MC(1) \leq MG(1), \\ s.th. \quad MC(\alpha^*) = MG(\alpha^*) & \text{otherwise.} \end{cases}$$

Proposition 1.4.1 is the smooth counterpart of the equilibrium characterization under maxmin expected utility in Section 1.2. To see the connection between the two models, assume that absolute ambiguity aversion is constant, i.e. $-\frac{\Phi''(x)}{\Phi'(x)} = \gamma$.[8] Assume further that the support of μ is Σ and that $\underline{\sigma} < \tilde{\sigma} < \overline{\sigma}$.[9] Under maxmin preferences, the buyer's optimal contract, characterized by $\tilde{\alpha}$, makes her payoff unambiguous. Under smooth ambiguity aversion, the buyer compromises between maximizing the second-order expectation of her expected utility, $\mathrm{E}_\mu \mathrm{E}_\sigma[\pi_b]$, and limiting her exposure to ambiguity. $\mathrm{E}_\mu \mathrm{E}_\sigma[\pi_b]$ is maximized by offering a posted price (either equal to c_l or c_h), whereas ambiguity is eliminated by offering the menu characterized by $\tilde{\alpha}$. The optimal contract under smooth ambiguity aversion is a convex combination of the two. If offering the pooling price maximizes $\mathrm{E}_\mu \mathrm{E}_\sigma[\pi_b]$, then the optimal contracting parameter α^* lies in the interval $[\tilde{\alpha}, 1]$, otherwise α^* lies in the interval $[0, \tilde{\alpha}]$. The more ambiguity-averse the buyer is, the closer is α^* to $\tilde{\alpha}$. This is summarized in Proposition 1.4.2.

Proposition 1.4.2. *Assume* $-\frac{\Phi''(x)}{\Phi'(x)} = \gamma$.

- *If* $\mathrm{E}_\mu[\sigma] < \tilde{\sigma}$, *then* $\alpha^* \in [0, \tilde{\alpha}]$ *and* $\frac{d\alpha^*}{d\gamma} \geq 0$.
- *If* $\mathrm{E}_\mu[\sigma] > \tilde{\sigma}$, *then* $\alpha^* \in [\tilde{\alpha}, 1]$ *and* $\frac{d\alpha^*}{d\gamma} \leq 0$.

Proof See Appendix 1.7.3.

Proposition 1.4.2 is illustrated in Figure 2. The figure shows the optimal contracting parameter α^* as a function of $\tilde{\sigma}$ for different degrees of ambiguity aversion, parameterized by γ. The solid curve represents the optimal contract under ambiguity neutrality ($\gamma = 0$), while the dashed curve represents the optimal contract under maxmin expected utility ($\gamma \to \infty$). The curves in between these two benchmarks show the optimal contracting parameter under

[8] Klibanoff et al. (2005) show that under this assumption the maxmin expected utility model is a limiting case of the smooth ambiguity model.

[9] Otherwise the solution is the same posted price under both representations.

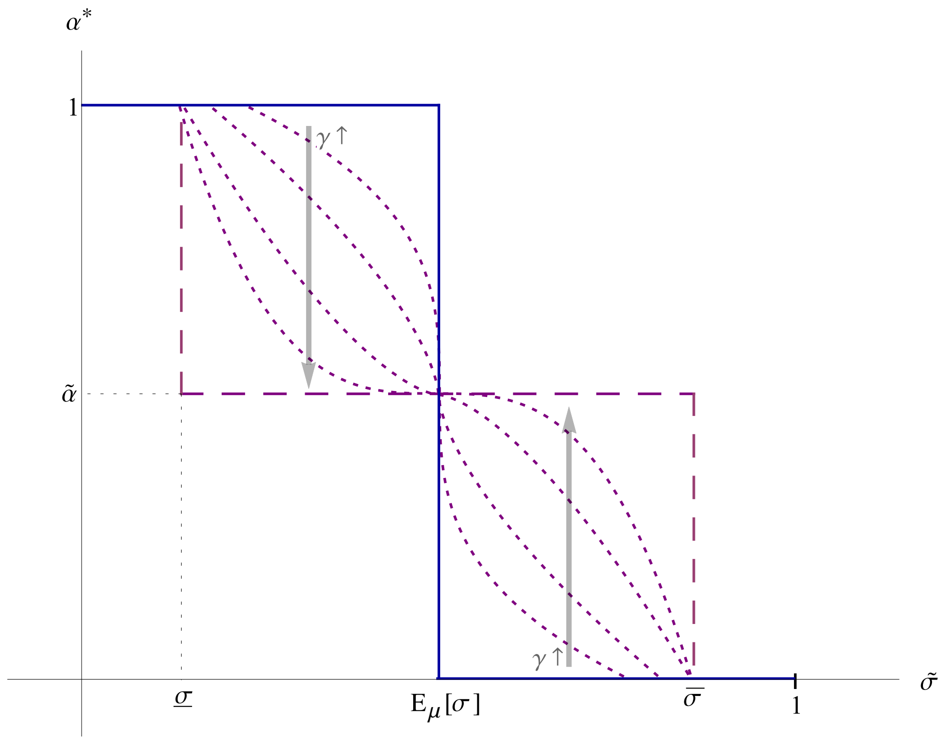


Figure 1.2: α^* for different levels of γ.

smooth ambiguity aversion with intermediate values of γ. If $\tilde{\sigma} < \underline{\sigma}$ ($\tilde{\sigma} > \overline{\sigma}$), ambiguity is irrelevant and the optimal contract is characterized by $\alpha^* = 1$ ($\alpha^* = 0$). If, on the other hand, $\tilde{\sigma} \in [\underline{\sigma}, \mathrm{E}_\mu[\sigma]]$ ($\tilde{\sigma} \in [\mathrm{E}_\mu[\sigma], \overline{\sigma}]$), the optimal contract under ambiguity neutrality is a price equal to c_h (c_l), whereas the optimal contract under maxmin expected utility is a menu characterized by $\tilde{\alpha}$. For intermediate values of γ, the solution of the buyer's optimization problem lies in between these two benchmarks. As γ increases, the optimal contracting parameter under smooth ambiguity aversion approaches the maxmin solution $\tilde{\alpha}$.

1.5 Continuous Types

This section extends the results derived in the two-type setting to a model with a continuum of types. The seller's valuation c is distributed on the interval $[0, 1]$ and the buyer's valuation v is a function of the seller's valuation. To facilitate the exposition, assume that gains from trade are constant across types: $v(c) = c + \Delta, \Delta > 0$.[10]

[10] An online appendix available on my homepage, https://sites.google.com/site/austersarah/, extends the analysis to general linear valuation functions.

Ambiguity: The buyer considers multiple probability distribution functions on $[0,1]$ possible. The set of probability measures, denoted by $\mathcal{F}$, is defined by the set of all distribution functions that have some minimal mass g_c on each type $c \in [0,1]$. Working with this general set of priors is complicated, because the extent of ambiguity is determined by an infinite set of parameters $g_c, c \in [0,1]$. To make the analysis tractable, I assume that the extent of ambiguity is symmetric across types, i.e. $g_c = g, \forall c$:

$$\mathcal{F} = \left\{ P \in \mathbb{P} : P[B] \geq \int_B g \mathrm{d}t, \forall B \in \mathcal{B}[0,1] \right\}, \quad g \in [0,1],$$

where $\mathbb{P}$ is the set of all probability distribution functions on $[0,1]$ and $\mathcal{B}[0,1]$ is the Borel σ-algebra of the set $[0,1]$. Note that the symmetric structure on the set of measures does not imply that the distribution functions in the set are symmetric: within bound g, $f \in \mathcal{F}^g$ can be arbitrarily skewed to the right or to the left. The advantage of this structure is that the extent of ambiguity is captured by a single parameter g, where a larger g corresponds to a smaller set of probability distributions: $\mathcal{F}^{g'} \subset \mathcal{F}^g, g' > g$. If $g = 0$, $\mathcal{F}^g$ contains every possible distribution function on $[0,1]$, while if $g = 1$, there is no ambiguity and the unique probability distribution in $\mathcal{F}^g$ is the uniform distribution on $[0,1]$.

Contract: The optimal contract is characterized by a trading probability $\alpha(c)$ and a price $p(c)$ for each $c \in [0,1]$. The participation and incentive constraints in this setting are

$$\begin{aligned} p(c) - \alpha(c)c &\geq 0, & \forall c, \\ p(c) - \alpha(c)c &\geq p(\tilde{c}) - \alpha(\tilde{c})c, & \forall c, \tilde{c}, \end{aligned}$$

which corresponds to the set of constraints in Samuelson (1984). Given that the seller truthfully reveals, the buyer's and seller's ex-post expected payoffs are given by

$$\pi_b(c) = \alpha(c)v(c) - p(c) \quad \text{and} \quad \pi_s(c) = p(c) - \alpha(c)c,$$

respectively. Samuelson (1984, p. 997) shows that the set of constraints implies $\alpha'(c) \leq 0$ and $\pi_s'(c) = -\alpha(c)$. With these properties, the buyer's and seller's payoffs can be derived as a function of $\alpha(c)$ only. $\pi_s'(c) = -\alpha(c)$ together with $\pi_s(1) = 0$ yields

$$\pi_s(c) = \int_c^1 \alpha(u)\mathrm{d}u.$$

$\int_c^1 \alpha(u)\mathrm{d}u$ corresponds to the information rent paid to type c. The buyer's ex-post expected payoff is the difference between the expected gains from trade and the information rent paid to the seller, i.e.

$$\pi_b(c) = \alpha(c)\Delta - \int_c^1 \alpha(u)\mathrm{d}u.$$

Optimization Problem: The buyer solves

$$\max_{\alpha(c)} \quad \Pi^{MEU} = \min_{f \in \mathcal{F}^g} \int_0^1 \left(\alpha(c)\Delta - \int_c^1 \alpha(u)\mathrm{d}u \right) f(c)\mathrm{d}c, \quad \text{s.t.} \quad \alpha'(c) \leq 0.$$

The main challenge in finding the solution to this problem is to identify the minimizing probability distribution in $\mathcal{F}^g$ for each function $\alpha(c)$ that satisfies the monotonicity constraint. The structure imposed on $\mathcal{F}^g$ makes this task tractable: The set of minimizing distributions, given $\alpha(c)$, is the set of distribution functions with maximal mass on any type that yields the lowest expected payoff for the buyer. This set contains every distribution function with a mass point $1-g$ on some minimizing type and density g on the rest of the type space. The buyer's payoff is consequently

$$\Pi^{MEU} = g \int_0^1 \pi_b(c)\mathrm{d}c + (1-g) \min_{c \in [0,1]} \pi_b(c).$$

This payoff is the weighted sum of the buyer's expected utility with a single uniform prior and the buyer's maxmin payoff under complete ambiguity.[11] Before presenting the full characterization of the contract that maximizes this objective, some intuition can be gained by considering the two extreme cases $g = 1$ (no ambiguity) and $g = 0$ (maximal ambiguity).

If $g = 1$, the buyer is a standard subjective expected utility maximizer and the optimal contract is a posted price (Samuelson, 1984). To see this, consider the buyer's payoff under

[11]Note that this payoff function represents a well-established parametric ambiguity model, the ε-contamination model axiomatized by Kopylov (2009). In this model, the decision maker holds a subjective probabilistic opinion, here the uniform distribution, but likes to hedge against the case that it turns out to be false. $\varepsilon(= 1-g)$ parameterizes the decision maker's subjective willingness to hedge.

a single uniform prior:[12]

$$\int_0^1 \pi_b(c)\mathrm{d}c \quad = \quad \int_0^1 \left(\alpha(c)\Delta\mathrm{d}c - \int_c^1 \alpha(u)\mathrm{d}u\right)\mathrm{d}c = \int_0^1 \alpha(c)(\Delta - c)\mathrm{d}c.$$

This payoff is maximized by the contract

$$\alpha^{NA}(c) = \begin{cases} 1 & \text{if} \quad c \leq \Delta, \\ 0 & \text{if} \quad c > \Delta, \end{cases}$$

which corresponds to a posted price $p^* = \Delta$.

If $g = 0$, $\mathcal{F}^g$ contains every possible probability distribution on $[0, 1]$ and the buyer's payoff is given by $\min_{c\in[0,1]} \pi_b(c)$. The contract that maximizes this payoff is a contract under which $\pi_b(c)$ is constant across types (see Proposition 1.5.1):

$$\pi_b'(c) = 0 \qquad \Longleftrightarrow \qquad \alpha'(c)\Delta = \alpha(c).$$

The solution to this differential equation, together with $\alpha(0) = 1$, is

$$\alpha^{MA}(c) = e^{-\frac{c}{\Delta}},$$

a decreasing function of c that perfectly separates all types. The contract characterized by α^{MA} is a screening menu that makes the buyer indifferent between all types and thereby indifferent between all distributions over types. A decreasing function $\alpha(c)$ allows the buyer to balances her rent across types by exactly offsetting the increased information rent paid to lower types by the increased trading probability with these types. α^{MA} therefore hedges perfectly against the ambiguity in this contracting problem and can be seen as the continuous analogue of $\tilde{\alpha}$ in the binary type environment of Section 1.2.

For intermediate levels of ambiguity, the optimal contract compromises between maximizing the buyer's expected utility under a single uniform prior, $\int_0^1 \alpha(c)(\Delta - c)\mathrm{d}c$, and limiting her worst-case payoff, $\min_c \pi_b(c)$. The ambiguity parameter g weighs these two objectives.

[12]The second equality can be derived by changing the order of integration:

$$\int_0^1 \int_c^1 \alpha(u)\mathrm{d}u\mathrm{d}c = \int_0^1 \int_0^u \alpha(u)\mathrm{d}c\mathrm{d}u = \int_0^1 \alpha(u)u\mathrm{d}u.$$

Proposition 1.5.1 characterizes the buyer's optimal contract for all $g \in [0,1]$. It shows that for intermediate levels of ambiguity the optimal contract exhibits partial bunching and partial separation, where the extent of separation increases in the extent of ambiguity the buyer faces.

Proposition 1.5.1. *Assume* $v(c) = c + \Delta$ *and* $\mathcal{F} = \mathcal{F}^g$ *for some* $g \in [0,1]$.

- ***Perfect Separation:*** *if* $g \leq \frac{1}{\Delta}e^{-\frac{1}{\Delta}}$,

$$\alpha^*(c) = e^{-\frac{c}{\Delta}}, \forall c.$$

- ***Partial Separation:*** *if* $g \in \left(\frac{1}{\Delta}e^{-\frac{1}{\Delta}}, \frac{1}{\Delta}e^{-\frac{1-\Delta}{\Delta}}\right)$,

$$\alpha^*(c) = \begin{cases} 1 & \text{if } c \leq \hat{c}, \\ \frac{\Delta-\hat{c}}{\Delta}e^{-\frac{c-\hat{c}}{\Delta}} & \text{if } c > \hat{c}, \end{cases}$$

where $\hat{c} = 1 + \Delta \ln(g\Delta)$.

- ***Posted Price:*** *if* $g \geq \frac{1}{\Delta}e^{-\frac{1-\Delta}{\Delta}}$,

$$\alpha^*(c) = \begin{cases} 1 & \text{if} \quad c \leq \Delta, \\ 0 & \text{if} \quad c > \Delta. \end{cases}$$

Proof See Appendix 1.7.4.

There are three regions to be distinguished. First, if the extent of ambiguity is sufficiently small $\left(g \geq \frac{1}{\Delta}e^{-\frac{1-\Delta}{\Delta}}\right)$, the buyer proposes α^{NA}, the optimal contract in the absence of ambiguity ($g = 1$). Under α^{NA}, the buyer's expected utility under a single uniform prior is maximized, while the buyer's worst-case payoff is equal to zero. Second, if the extent of ambiguity is sufficiently large $\left(g \leq \frac{1}{\Delta}e^{-\frac{1}{\Delta}}\right)$, the buyer proposes α^{MA}, the optimal contract under maximal ambiguity ($g = 0$). By definition of α^{MA}, all types yield the same expected ex-post payoff, equal to

$$\overline{\Pi} = \pi_b(1) = \Delta\alpha^{MA}(1) = \Delta e^{-\frac{1}{\Delta}}.$$

Finally, for intermediate levels of ambiguity, the buyer faces a tradeoff between maximizing her expected utility under a single uniform prior and limiting her minimal payoff. She solves this tradeoff by partially separating types. In particular, there is a separating region, $(\hat{c}, 1]$,

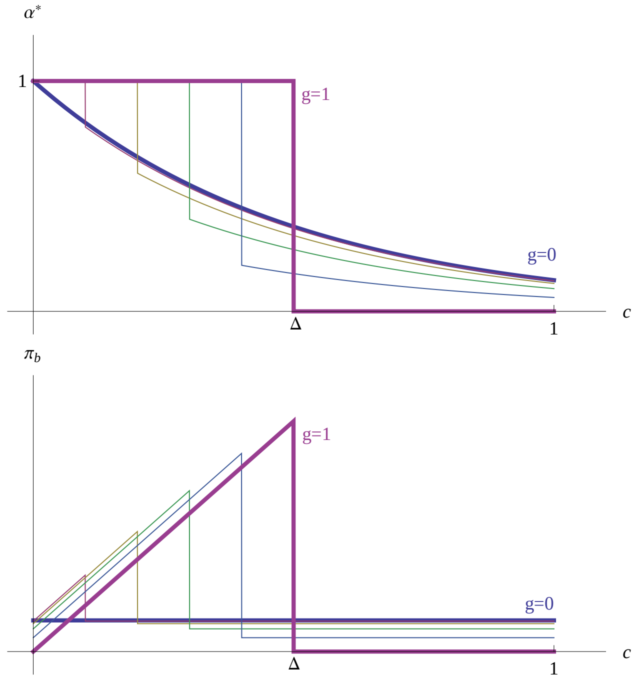


Figure 1.3: $\alpha^*(c)$ and $\pi_b(c)$ for different values of $g \in [0, 1]$

in which $\alpha(c)$ is strictly decreasing in c and $\pi_b(c)$ is constant across types. The rest of the types, $[0, \hat{c}]$, are bunched and trade with probability one. The larger the buyer's desired level of $\min_c \pi_b(c)$ is, the smaller is the bunching region $[0, \hat{c}]$.

To see this, consider the optimal contract in the absence of ambiguity, α^{NA}. If the buyer wishes to raise her minimal payoff, she needs to raise $\alpha(c)$ for all $c \in (\Delta, 1]$. This implies that the information rent paid to types $c \in [0, \Delta]$ increases, which consequently implies that the buyer's ex-post expected payoff when trading with the lowest type is negative:

$$\pi_b(0) = \Delta - \int_0^\Delta 1\text{du} - \int_\Delta^1 \alpha(c)\text{d}u < 0.$$

To increase $\pi_b(0)$, the buyer needs to decrease the information rent paid to the lowest type. This is optimally done by shrinking the bunching region $[0, \hat{c}]$ and by keeping $\pi_b(c)$ constant across the remaining types, $c \in (\hat{c}, 1]$. The buyer's ex-post expected payoff is minimal either when trading with the lowest type or when trading with any type in $(\hat{c}, 1]$. The more the buyer wishes to raise her minimal payoff, the smaller the bunching region $[0, \hat{c}]$ becomes, illustrated in Figure 3. The optimal size of $[0, \hat{c}]$ is determined by the ambiguity parameter

g (see Proposition 1.5.1):

$$\hat{c} = 1 + \Delta \ln(g\Delta).$$

The marginal type $\hat{c}$ is increasing in g. Since g is the inverse measure of the extent of ambiguity in this environment, this implies that the degree of separation in equilibrium increases in the extent of ambiguity the buyer faces. The intuition is that bunching maximizes the buyer's subjective expected utility while separation allows the buyer to hedge against ambiguity. The more ambiguity the buyer faces, the more profitable hedging becomes.

1.6 Conclusion

This paper demonstrates the implications of ambiguity aversion on optimal contracting with asymmetric information and common values. It shows how non-linear pricing hedges against ambiguity over the valuations of the contracting parties and provides conditions under which the optimal contract under ambiguity differs from the optimal contract under any single prior. Moreover, the analysis demonstrates that ambiguity does not necessarily inhibit trade, a rather surprising result given the previous literature on ambiguity and trade, e.g. Dow and Werlang (1992), De Castro and Chateauneuf (2011), etc. The paper provides conditions under which ambiguity has a positive effect on trade and shows that under further restrictions, the trading outcome under bilateral negotiations and ambiguity may be strictly more efficient than in competitive markets. In the context of financial markets, this suggests that if opacity gives rise to ambiguity, opaque OTC markets may implement strictly better trading outcomes than traditional exchange markets, and thereby highlights a new aspect in the debate on why opaque off-exchange trading may emerge.

An interesting question for future research is how the results derived in this paper extend to trading environments with more than two contracting parties. One promising approach is to consider ambiguity in auction settings with interdependent values. In interdependent value auctions, bidders compete for a good of uncertain value and receive private signals, determining each other's valuations. If bidders have ambiguous beliefs about the signal distribution, they face ambiguity over both their valuation of the good and their probability to win, and are thus confronted with a decision problem akin to the one presented in this paper. The results of this work suggest that if the bidders' aversion to ambiguity over the probability to win outweighs their aversion to ambiguity over their valuation of the object, the presence of ambiguity may lead to strictly higher bids, thereby benefiting the auctioneer.

1.7 Appendix

1.7.1 Proof of Proposition 1.2.1

The buyer maximizes $\min_{\sigma\in\Sigma} \mathrm{E}_\sigma[\pi_b]$. To identify the minimizing prior, consider

$$\frac{\partial \mathrm{E}_\sigma[\pi_b]}{\partial \sigma} = \alpha(v_h - c_l) - (v_l - c_l) \gtreqless 0 \text{ if } \alpha \gtreqless \tilde{\alpha}.$$

This implies $\overline{\sigma} \in \arg\min_{\sigma\in\Sigma} \{\mathrm{E}_\sigma[\pi_b(\alpha)]\}$ if $\alpha \leq \tilde{\alpha}$ and $\underline{\sigma} \in \arg\min_{\sigma\in\Sigma} \{\mathrm{E}_\sigma[\pi_b(\alpha)]\}$ if $\alpha \geq \tilde{\alpha}$. The buyer consequently maximizes the step function

$$\Pi^{MEU} = \begin{cases} \overline{\sigma}\alpha(v_h - c_h) + (1-\overline{\sigma})(v_l - \alpha c_h + (1-\alpha)c_l) & \text{if} \quad \alpha \leq \tilde{\alpha}, \\ \underline{\sigma}\alpha(v_h - c_h) + (1-\underline{\sigma})(v_l - \alpha c_h + (1-\alpha)c_l) & \text{if} \quad \alpha > \tilde{\alpha}. \end{cases}$$

Note further:

$$\frac{\partial \mathrm{E}_\sigma[\pi_b(\alpha)]}{\partial \alpha} = \sigma(v_h - c_l) - (c_h - c_l) \gtreqless 0 \text{ if } \sigma \gtreqless \tilde{\sigma}.$$

If $\overline{\sigma} \leq \tilde{\sigma}$, both parts of the step function are weakly decreasing in α and Π^{MEU} is maximized at $\alpha = 0$. Similarly, if $\underline{\sigma} \geq \tilde{\sigma}$, both parts of the step function are weakly increasing in α and Π^{MEU} is maximized at $\alpha = 1$. If $\underline{\sigma} < \tilde{\sigma} < \overline{\sigma}$, Π^{MEU} is increasing in α on the interval $[0, \tilde{\alpha}]$ and decreasing in α on the interval $[\tilde{\alpha}, 1]$, and therefore maximized at $\tilde{\alpha}$. □

1.7.2 Proof of Proposition 1.3.1

Suppose $\underline{\sigma} < \tilde{\sigma}$.
If $\tilde{\sigma} < \overline{\sigma}$, then $\alpha^*_\Sigma = \tilde{\alpha}$. Since $\underline{\sigma} < \tilde{\sigma} < \overline{\sigma}$ implies $\underline{\sigma}' < \tilde{\sigma} < \overline{\sigma}'$, $\alpha^*_{\Sigma'} = \tilde{\alpha}$. An increase in ambiguity has no effect.
If $\overline{\sigma} < \tilde{\sigma}$, then $\alpha^*_\Sigma = 0$. This implies either $\underline{\sigma}' < \overline{\sigma}' < \tilde{\sigma}$ or $\underline{\sigma}' < \tilde{\sigma} < \overline{\sigma}'$, in which case $\alpha^*_{\Sigma'} = 0$ or $\alpha^*_{\Sigma'} = \tilde{\alpha}$, respectively. Hence, $\alpha^*_\Sigma \leq \alpha^*_{\Sigma'}$.
The proof for the case $\tilde{\sigma} < \overline{\sigma}$ is analogous.

1.7.3 Proof of Proposition 1.4.2

Under constant absolute ambiguity aversion, we have $\Phi(x) = -\frac{1}{\gamma}e^{-\gamma x}$.

Suppose first $\mathrm{E}_\mu[\sigma] > \tilde{\sigma}$. Note that $\mathrm{E}_\mu \mathrm{E}_\sigma[\pi_b(\alpha)]$ increases in α and that $\mathrm{Var}_\mu\left[\mathrm{E}_\sigma[\pi_b(\alpha)]\right] > 0$ for all $\alpha < \tilde{\alpha}$. This implies that the expected utility distribution induced by any $\alpha < \tilde{\alpha}$ is second-order stochastically dominated by the (degenerate) distribution induced by $\tilde{\alpha}$. Since Φ is strictly concave, this implies $\alpha^* \in [\tilde{\alpha}, 1]$.

The optimal value of α^* is characterized by

$$\mathrm{E}_\mu\left[e^{-\gamma \mathrm{E}_\sigma[\pi(\alpha^*)]}(\sigma - \tilde{\sigma})\right] = 0,$$

if the solution is interior (otherwise a marginal change in γ has no effect). The implicit function theorem implies

$$\frac{\mathrm{d}\alpha^*}{\mathrm{d}\gamma} = -\frac{\mathrm{E}_\mu\left[e^{-\gamma \mathrm{E}_\sigma[\pi(\alpha^*)]}(\sigma - \tilde{\sigma})\mathrm{E}_\sigma[\pi(\alpha^*)]\right]}{\gamma \mathrm{E}_\mu\left[e^{-\gamma \mathrm{E}_\sigma[\pi(\alpha^*)]}(\sigma - \tilde{\sigma})^2\right]}.$$

$\frac{\mathrm{d}\alpha^*}{\mathrm{d}\gamma} \leq 0$, if the numerator is positive on the interval $[\tilde{\alpha}, 1]$. Suppose not. Then

$$\mathrm{E}_\mu\left[e^{-\gamma \mathrm{E}_\sigma[\pi(\alpha^*)]}(\tilde{\sigma} - \sigma)\mathrm{E}_\sigma[\pi(\alpha^*)] \mid \sigma \leq \tilde{\sigma}\right] > \mathrm{E}_\mu\left[e^{-\gamma \mathrm{E}_\sigma[\pi(\alpha^*)]}(\sigma - \tilde{\sigma})\mathrm{E}_\sigma[\pi(\alpha^*)] \mid \sigma \geq \tilde{\sigma}\right].$$

But

$$\begin{aligned}
\mathrm{E}_\mu\left[e^{-\gamma \mathrm{E}_\sigma[\pi(\alpha^*)]}(\tilde{\sigma} - \sigma)\mathrm{E}_\sigma[\pi(\alpha^*)] \mid \sigma \leq \tilde{\sigma}\right] &\leq \mathrm{E}_{\tilde{\sigma}}[\pi(\alpha^*)]\mathrm{E}_\mu\left[e^{-\gamma \mathrm{E}_\sigma[\pi(\alpha^*)]}(\tilde{\sigma} - \sigma) \mid \sigma \leq \tilde{\sigma}\right], \\
&= \mathrm{E}_{\tilde{\sigma}}[\pi(\alpha^*)]\mathrm{E}_\mu\left[e^{-\gamma \mathrm{E}_\sigma[\pi(\alpha^*)]}(\sigma - \tilde{\sigma}) \mid \sigma \geq \tilde{\sigma}\right], \\
&\leq \mathrm{E}_\mu\left[e^{-\gamma \mathrm{E}_\sigma[\pi(\alpha^*)]}(\sigma - \tilde{\sigma})\mathrm{E}_\sigma[\pi(\alpha^*)] \mid \sigma \geq \tilde{\sigma}\right],
\end{aligned}$$

which follows from $\frac{\partial \mathrm{E}_\sigma[\pi_b(\alpha)]}{\partial \sigma} > 0, \forall \alpha \in [\tilde{\alpha}, 1]$. A contradiction. Hence, $\frac{\mathrm{d}\alpha^*}{\mathrm{d}\gamma} \leq 0$.

The proof for the case $\mathrm{E}_\mu[\sigma] > \tilde{\sigma}$ is analogous, where the above inequalities are reversed. Just note that second-order stochastic dominance implies $\alpha^* \in [0, \tilde{\alpha}]$ and that $\frac{\partial \mathrm{E}_\sigma[\pi_b(\alpha)]}{\partial \sigma} < 0, \forall \alpha \in [0, \tilde{\alpha}]$. □

1.7.4 Proof of Proposition 1.5.1

The optimal contract α^* will be derived in a series of lemmas. Let $\Pi^{NA} := \int_0^1 \alpha(c)[\Delta - c]\mathrm{d}c$ and $\Pi^{MA} := \min_c \left\{\alpha(c)\Delta - \int_c^1 \alpha(u)\mathrm{d}u\right\}$ denote the buyer's maxmin payoff for no ambiguity and maximal ambiguity, respectively. Further, let the set of types that yield the buyer's lowest expected ex-post payoff given contract α be denoted by

$$\mathcal{C}^{min}(\alpha) = \left\{c : c \in \min_{\tilde{c}} \left\{\alpha(\tilde{c})\Delta - \int_{\tilde{c}}^1 \alpha(u)\mathrm{d}u\right\}\right\}.$$

The buyer solves

$$\max_{\alpha(c)} \quad g\int_0^1 \alpha(c)(\Delta - c)\mathrm{d}c + (1-g)\min_{c\in[0,1]}\left\{\alpha(c)\Delta - \int_c^1 \alpha(u)\mathrm{d}u\right\} \quad \text{s.t.} \quad \alpha'(c) \leq 0.$$

The first lemma shows that in the optimal contract, the minimal ex-post payoff is non-negative.

Lemma 1.7.1. *Assume $v(c) = c + \Delta$ and $\mathcal{F} = \mathcal{F}^g$ for some $g \in [0,1]$. $\pi_b(c) \geq 0, c \in \mathcal{C}^{min}(\alpha^*)$.*

Proof Π^{NA} is maximized by the contract

$$\alpha(c) = \begin{cases} 1 & \text{if} \quad c \leq \Delta, \\ 0 & \text{if} \quad c > \Delta. \end{cases}$$

The set of minimizing types given this contract is $\mathcal{C}^{min}(\alpha^{NA}) = \{0 \cup (\Delta, 1]\}$, where $\pi_b(c) = 0, \forall c \in \mathcal{C}^{min}(\alpha^{NA})$. This implies that any contract $\tilde{\alpha}$ with $\pi_b(c) < 0, c \in \mathcal{C}^{min}(\tilde{\alpha})$ is strictly dominated by α^{NA}. Hence, a minimizing type in the optimal contract α^* yields a payoff that is weakly greater than zero. □

Lemma 1.7.2. *Assume $v(c) = c + \Delta$ and $\mathcal{F} = \mathcal{F}^g$ for some $g \in [0,1]$. For any $\bar{\pi} \in \left[0, \Delta e^{-\frac{1}{\Delta}}\right]$, there exists a strictly decreasing function $\alpha^{\bar{\pi}} : [0,1] \to [0,1]$ such that $\alpha^{\bar{\pi}}(c)\Delta - \int_c^1 \alpha^{\bar{\pi}}(u)du = \bar{\pi}, \forall c$.*

Proof $\alpha(c)\Delta - \int_c^1 \alpha(u)\mathrm{d}u$ is constant in c if

$$\pi_b'(c) = \alpha'(c)\Delta + \alpha(c) = 0.$$

The solution to the differential equation $-\alpha'(c)\Delta = \alpha(c)$ is $\alpha(c) = De^{-\frac{c}{\Delta}}$. The level of constant ex-post payoff is equal to $\bar{\pi}$ if $\pi_b(1) = \bar{\pi}$, i.e. if

$$\alpha(1)\Delta = \bar{\pi} \quad \Rightarrow \quad D = \frac{\bar{\pi}}{\Delta}e^{\frac{1-c}{\Delta}}.$$

Consequently

$$\alpha^{\bar{\pi}}(c) = \frac{\bar{\pi}}{\Delta}e^{\frac{1-c}{\Delta}}.$$

Note that $\alpha^{\bar{\pi}}(c)$ is decreasing in c and thus, satisfies the monotonicity constraint. Finally, $\alpha(c)$ may not exceed one for any c, which is satisfied if

$$\alpha^{\bar{\pi}}(0) \leq 1 \quad \Rightarrow \quad \bar{\pi} \leq \Delta e^{-\frac{1}{\Delta}}.$$

Lemma 1.7.3. *Assume $v(c) = c + \Delta$ and $\mathcal{F} = \mathcal{F}^g$ for some $g \in [0,1]$. Let $\pi^* = \pi_b(c), c \in \mathcal{C}^{min}(\alpha^*)$. Then $\alpha^*(c) \geq \alpha^{\pi^*}(c)$ for all c.*

Proof Suppose not. Let $\tilde{c}$ be the largest type such that $\alpha^*(c) < \alpha^{\pi^*}(c)$. Then $\alpha^*(\tilde{c}) - \int_{\tilde{c}}^1 \alpha(u)\mathrm{d}u \geq \pi^*$ implies

$$\int_{\tilde{c}}^1 \alpha^*(u)\mathrm{d}u < \int_{\tilde{c}}^1 \alpha^{\pi^*}(u)\mathrm{d}u,$$

a contradiction. □

Lemma 1.7.4. *Assume $v(c) = c+\Delta$ and $\mathcal{F} = \mathcal{F}^g$ for some $g \in [0,1]$. There exists some $\hat{c} \in [0,\Delta]$ and some $\bar{\pi} \in \left[0, \Delta e^{-\frac{1}{\Delta}}\right]$ such that the optimal contract α^* satisfies $\alpha^*(c) = 1, \forall c \leq \hat{c}$ and $\alpha^*(c) = \alpha^{\bar{\pi}}(c), \forall c < \hat{c}$.*

Proof For given minimal ex-post payoff $\bar{\pi}$, the buyer's optimization problem is

$$\max_{\alpha(c)} \quad \Pi^{NA} \quad \text{s.t.} \quad \alpha(c)\Delta - \int_c^1 \alpha(u)\mathrm{d}u \geq \bar{\pi}, \forall c \quad \text{and} \quad \alpha'(c) \leq 0.$$

If $\bar{\pi} = 0$, the optimal contract is α^{NA} (see the proof of Lemma 1.7.1), i.e. $\hat{c} = \Delta$. If $\bar{\pi} > 0$, the optimal contract given $\bar{\pi}$ can be derived by considering a less restricted optimization problem and then showing that the solution to this problem satisfies all constraints of the original problem:

$$\max_{\alpha(c)} \quad \Pi^{NA} \quad \text{s.t.} \quad \alpha(0)\Delta - \int_0^1 \alpha(u)\mathrm{d}u \geq \bar{\pi} \quad \text{and} \quad \alpha(c) \geq \alpha^{\bar{\pi}}(c), \forall c.$$

Let λ denote the Lagrange-multiplier of the constraint $\pi_b(0) \geq \bar{\pi}$ and let $\mu_c, c \in [0,1]$ denote the Lagrange-multipliers of the constraints $\alpha(c) \geq \alpha^{\bar{\pi}}(c)$. First, we can show that the constraint $\pi_b(0) \geq \bar{\pi}$ is binding. Suppose not.

$$\frac{\partial \mathcal{L}}{\partial \alpha(c)} = \Delta - c + \mu_c > 0 \quad \text{for all} \quad c < \Delta,$$

implying that $\alpha^*(c) = 1, \forall c < \Delta$. Then

$$\pi_b(0) = \Delta - \int_0^\Delta 1 \mathrm{d}c - \int_\Delta^1 \alpha(c) \mathrm{d}c = 0 - \int_\Delta^1 \alpha(c) \mathrm{d}c \leq 0,$$

a contradiction. The binding constraint implies that $\lambda > 0$. Let $\hat{c}$ denote the type such that

$$\Delta - \hat{c} - \lambda = 0,$$

Since $\frac{\partial \mathcal{L}}{\partial \alpha(c)} = \Delta - c - \lambda + \mu_c > 0$ for all $c < \hat{c}$, we have $\alpha^*(c) = 1, \forall c \leq \hat{c}$. For the remaining types, the constraint $\alpha(c) \geq \alpha^{\bar{\pi}}(c)$ is binding as

$$\frac{\partial \mathcal{L}}{\partial \alpha(c)} = \Delta - c - \lambda + \mu_c = 0,$$

requires $\mu_c > 0$ for all $c > \hat{c}$. Finally, we need to check whether this solution satisfies the original constraints of the principal's optimization problem, i.e. monotonicity and $\pi_b(c) \geq \bar{\pi}, \forall c$. Given that $\alpha^{\bar{\pi}}(c)$ is strictly decreasing, monotonicity is not violated. To see that $\pi_b(c) \geq \bar{\pi}, \forall c$, note that for $c \leq \hat{c}$

$$\pi_b(c) = \Delta - \int_c^{\hat{c}} 1 \mathrm{d}c - \int_{\hat{c}}^1 \alpha^{\bar{\pi}}(c) \mathrm{d}c \geq \Delta - \int_0^{\hat{c}} 1 \mathrm{d}c - \int_{\hat{c}}^1 \alpha(c) \mathrm{d}c = \pi_b(0) = \bar{\pi},$$

while for $c > \hat{c}$, $\pi_b(c) \geq \bar{\pi}$ is satisfied by definition of $\alpha^{\bar{\pi}}(c)$. □

Lemmas 1.7.1-1.7.4 allow us to derive the optimal contract α^*. First, the minimal ex-post payoff $\bar{\pi}$ can be expressed as a function of $\hat{c}$:

$$\begin{aligned}
\bar{\pi} &= \pi_b(0), \\
\Leftrightarrow \quad \bar{\pi} &= \Delta - \hat{c} - \int_{\hat{c}}^1 \frac{\bar{\pi}}{\Delta} e^{\frac{1-u}{\Delta}} \mathrm{d}u, \\
\Leftrightarrow \quad \bar{\pi} &= \Delta - \hat{c} + \bar{\pi} - \bar{\pi} e^{\frac{1-\hat{c}}{\Delta}}, \\
\Leftrightarrow \quad \bar{\pi} &= (\Delta - \hat{c}) e^{-\frac{1-\hat{c}}{\Delta}}.
\end{aligned}$$

This implies

$$\alpha(c) = \begin{cases} 1 & \text{if } c \leq \hat{c}, \\ \frac{\Delta - \hat{c}}{\Delta} e^{-\frac{c-\hat{c}}{\Delta}} & \text{if } c > \hat{c}. \end{cases}$$

To derive the buyer's objective as a function of $\hat{c}$ note that

$$p(\hat{c}) = \hat{c} + \int_{\hat{c}}^{1} \frac{\Delta - \hat{c}}{\Delta} e^{-\frac{c-\hat{c}}{\Delta}} = \Delta - \bar{\pi}(\hat{c}),$$

where $\bar{\pi}(\hat{c}) = (\Delta - \hat{c}) e^{-\frac{1-\hat{c}}{\Delta}}$. The buyer's optimization problem is then given by

$$\max_{\hat{c}} \quad g \left[\hat{c} \left(\frac{1}{2} \hat{c} + \Delta - p(\hat{c}) \right) + (1 - \hat{c}) \bar{\pi}(\hat{c}) \right] + (1 - g) \bar{\pi}(\hat{c}),$$

or simply

$$\max_{\hat{c}} \quad \frac{1}{2} g \hat{c}^2 + \bar{\pi}(\hat{c}).$$

The first order condition to the problem is

$$g\hat{c} = -\bar{\pi}'(\hat{c}),$$

where $\bar{\pi}'(\hat{c}) = -\frac{\hat{c}}{\Delta} e^{-\frac{1-\hat{c}}{\Delta}}$. To derive the bounds for an interior solution, note that $g\hat{c}$ is linearly increasing in $\hat{c}$ and $-\bar{\pi}'(\hat{c})$ is convexly increasing in $\hat{c}$:

$$-\bar{\pi}''(\hat{c}) = e^{-\frac{1-\hat{c}}{\Delta}} \left(\frac{1}{\Delta} + \frac{\hat{c}}{\Delta^2} \right) > 0, \quad -\bar{\pi}'''(\hat{c}) = e^{-\frac{1-\hat{c}}{\Delta}} \left(\frac{2}{\Delta^2} + \frac{\hat{c}}{\Delta^3} \right) > 0,$$

both of them intersecting at the origin. This implies that $g\hat{c} \geq -\bar{\pi}'(\hat{c})$ on the interval $[0, \Delta]$ if

$$-\bar{\pi}''(0) \geq g \quad \Rightarrow \quad g \leq \frac{1}{\Delta} e^{-\frac{1}{\Delta}},$$

whereas $g\hat{c} \leq -\bar{\pi}'(\hat{c})$ on the interval $[0, \Delta]$ if

$$g\Delta \geq -\bar{\pi}'(\Delta) \quad \Rightarrow \quad g \geq \frac{1}{\Delta} e^{-\frac{1-\Delta}{\Delta}}.$$

Finally, if $g \in \left(\frac{1}{\Delta}e^{-\frac{1}{\Delta}}, \frac{1}{\Delta}e^{-\frac{1-\Delta}{\Delta}}\right)$, the problem has an interior solution. Solving the first order condition for $\hat{c}$ yields

$$\hat{c} = 1 + \ln(g\Delta).$$

To see that $\hat{c}$ is a maximum, note that

$$\frac{\partial^2 \Pi}{\partial \hat{c}^2}(1 + \ln(g\Delta)) = -\left(\frac{1}{\Delta} + \ln(g\Delta)\right) < 0 \quad \text{for all} \quad g \in \left(\frac{1}{\Delta}e^{-\frac{1}{\Delta}}, \frac{1}{\Delta}e^{-\frac{1-\Delta}{\Delta}}\right),$$

making the characterization complete. □

1.8 Bibliography

Akerlof, G.A., 1970. The Market for 'Lemons': Quality Uncertainty and the Market Mechanism. Quarterly Journal of Economics, 84(3), 488500.

Bayer, R.C., Bose, S., Polisson, M., Renou, L., 2013. Ambiguity Revealed. Working Paper.

Bergemann, D., Schlag, K., 2008. Pricing without Priors. Journal of the European Economic Association, 6(2-3), 560569.

Bergemann, D., Schlag, K., 2011. Robust Monopoly Pricing. Journal of Economic Theory, 146(6), 2527-2543.

Bodoh-Creed, A., 2012. Ambiguous Beliefs and Mechanism Design. Games and Economic Behavior, 75(2), 518-537.

Bose, S., Mutuswami, S., 2012. Bilateral Bargaining in an Ambiguous Environment. Working Paper.

Bose, S., Ozdenoren, E., Pape, A., 2006. Optimal Auctions with Ambiguity. Theoretical Economics, 1(4), 411-438.

Bose, S., Renou, L., 2014. Mechanism Design with Ambiguous Communication Devices. Econometrica, *forthcoming*.

Carroll, G., 2013. Robustness and Linear Contracts. Working Paper.

Chen, F., Zhong, Z., 2012. Pre-trade Transparency in Over-the-Counter Markets. Working Papper.

De Castro, L., Chateauneuf, A., 2011. Ambiguity Aversion and Trade. Economic Theory, 48(2-3), 243-273.

De Castro, L., Yannelis, N.C.,2012. Uncertainty, Efficiency and Incentive Compatibility. Working Paper.

Di Tillio, A., Kos, N., Messner, M., 2012. The Design of Ambiguous Mechanisms. Working Paper.

Dow, J., Werlang, S., 1992. Uncertainty Aversion, Risk Aversion, and the Optimal Choice of Portfolio. Econometrica, 60(1), 197-204.

Duffie, D., 2012. Dark Markets: Asset Pricing and Information Transmission in Over-the-Counter Markets. Princeton University Press.

Easley, D., O'Hara, M., 2010. Microstructure and Ambiguity. Journal of Finance, 65(5), 1817-1846.

Epstein, L., Wang, T., 1994. Intertemporal Asset Pricing under Knightian Uncertainty. Econometrica, 62(3), 283-322.

Gilboa, I., Marinacci, M., 2013. Ambiguity and the Bayesian Paradigm, in: D. Acemoglu, M. Arellano, E. Dekel (Eds.), Advances in Economics and Econometrics, Tenth World Congress, Vol. 1, Cambridge University Press.

Gilboa, I., Schmeidler, D., 1989. Maxmin Expected Utility with a Non-Unique Prior. Journal of Mathematical Economics, 18, 141-153.

Jankowitsch, R., Nashikkar, A., Subrahmanyam, M.G., 2011. Price dispersion in OTC markets: A new measure of liquidity, Journal of Banking and Finance, 35(2), 343357.

Klibanoff, P., Marinacci, M., Mukerji, S., 2005. A Smooth Model of Decision Making under Ambiguity. Econometrica, 73 (6), 18491892.

Lo, K., 1998. Sealed Bid Auctions with Uncertainty Averse Bidders. Economic Theory, 12(1), 1-20.

Lopomo, G., Rigotti, L., Shannon, C., 2009. Uncertainty in Mechanism Design. Working Paper.

Di Maggio, M., Pagano, M., 2014. Financial Disclosure and Market Transparency with Costly Information Processing. Working Paper.

Malamud, S., Rostek, M., 2013. Decentralized Exchange. Working Paper.

Mukerji, S., 1998. Ambiguity Aversion and Incompleteness of Contractual Form. American Economic Re-

view, 88(5), 1207-1231.
Mukerji, S., Tallon, J.M., 2001. Ambiguity Aversion and Incompleteness of Financial Markets. Review of Economic Studies, 68(4), 883904.
Myerson, R.B., Satterthwaite, M.A., 1983. Efficient Mechanisms for Bilateral Trading. Journal of Economic Theory, 29(2), 265-281.
Pancs, R., 2013. Efficient Dark Markets. Working Paper.
Riedel, F., Sass, L., 2011. The Strategic Use of Ambiguity. Working Paper.
Rigotti, L., Shannon, C., Strzalecki, T., 2008. Subjective Beliefs and Ex Ante Trade. Econometrica, 76(5), 1167-1190.
Rothschild, M., Stiglitz, J., 1976. Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. The Quarterly Journal of Economics, 90(4), 629-649.
Samuelson, W., 1984. Bargaining under Asymmetric Information. Econometrica, 52(4), 995-1005.
Salanie, B., 1997. The Economics of Contracts. MIT Press.
Wolitzky, A., 2013. Mechanism Design with Maxmin Agents: Theory and an Application to Bilateral Trade. Working Paper.

Chapter 2

Asymmetric Awareness and Moral Hazard

2.1 Introduction

The canonical moral hazard model analyzes the optimal contract between a principal and an agent in the presence of privately observable effort. As in standard economic models, the underlying assumption is that all decision makers are fully aware. That is, both the principal and the agent know every possible outcome realization and its distribution conditional on the agent's effort. However, in reality there are contracting situations where one party has a better understanding of the underlying uncertainties than the other. The question I want to address in this paper is whether the party with superior awareness can use his better understanding of the world strategically in the presence of moral hazard.

To illustrate this, consider the owner of a firm who wants to hire a manager. It is possible that the firm owner is aware of more opportunities and liabilities concerning his firm than the manager. Suppose, for example, that there is the possibility that one of the firm's products has adverse effects on the health of consumers. As a consequence, it is possible that the firm has to recall the product and faces severe legal liabilities. Whether the product's potential health threat becomes public or not is uncertain and depends on the manager's effort. Suppose further that the possibility of this event never crossed the manager's mind. The question is then, under which conditions it is profitable for the firm owner to disclose a possible recall and its legal consequences to the manager when offering the contract. The model shows that there is a trade off between participation and incentives. First, if the firm owner does not reveal the potential threat to the manager, he designs the contract such that the manager receives the minimal payment if the recall is realized. Since the manager does not take into account such an event, the participation constraint is less costly to satisfy. The size of the participation effect depends on the probability of the event. If the potential recall and its legal consequences are highly probable, it is easier to hire the manager without disclosing the possibility of the product's health threat. Second, since the probability of the health threat becoming public depends on the effort of the manager, its realization is a signal about the manager's effort. Disclosing the potential threat makes the incentive constraint less costly to satisfy. The size of the incentive effect depends on how much the manager's effort affects the probability of the event. If the manager can reduce the probability of the potential recall significantly, it is optimal to make his payment contingent on its realization.

This paper proposes a theoretical model which introduces asymmetric awareness in the canonical moral hazard model. The model analyzes the optimal contract between a fully aware principal and an unaware agent. A decision maker is called unaware when there exist contingencies that he does not know, and he does not know that he does not know, and so on *ad infinitum* (Modica and Rustichini, 1994, 1999). In the proposed model, the agent is assumed to be unaware of some relevant events, meaning that there are contingencies that affect the agent's payoff but that have never crossed his mind. Further, the agent is assumed to be unaware of his unawareness, so he believes that his description of the world is correct and complete. This implies that the agent is oblivious to the possibility that the principal is aware of contingencies that he is unaware of. The principal, on the other hand, is assumed to be fully aware. Moreover, the principal knows that the agent is unaware and he knows what the agent is unaware of. When writing the contract the principal can make the agent aware of some or all relevant contingencies. Note that a contract which transmits awareness is distinct from a contract that transmits information. A contract carrying information generally narrows the state space of the agent, whereas a contract carrying awareness expands the agent's state space by adding new dimensions. Thus, the analysis of the optimization problem of a principal with superior awareness complements the literature on moral hazard with an informed principal (e.g. Benabou and Tirole (2003)).

The risk-neutral principal proposes a contract to the risk-averse agent. The principal is the owner of a risky project, whose outcome is a function of the realization of a finite number of elementary contingencies of which the agent only knows a subset. These contingencies can be thought of as elementary propositions that can be either true or false. The probability of a contingency to be realized depends on the agent's privately taken effort. The agent's effort can be high or low and it is assumed that implementing high effort is always optimal. Since the principal cannot observe the agent's action, the terms of the contract have to be such that it is in the agent's best interest to exert the level of effort the principal wishes to implement. The compensation scheme is made contingent on the observable and verifiable outcome, rewarding the agent for outcome realizations that are relatively likely under high effort. The agent is assumed to have limited liability, thus transfers have to be non-negative in each state of the world. If the principal leaves the agent unaware of some contingencies, there is a non-empty set of possible outcomes that the agent does not take into account. It is optimal for the principal to construct the contract such that the agent receives the minimal payment whenever an unforeseen outcome is realized.

The main question this paper addresses is whether and under which conditions the principal enlarges the agent's awareness. The rationale for leaving the agent unaware is what I refer to as the participation effect. If the agent is unaware, his beliefs are systematically biased, which is exploited by the principal. The principal pays in expectation less than the agent's reservation utility, because there is positive probability that he pays zero and because the agent does not take this into account. The rationale for making the agent aware is what I refer to as the incentive effect. Since the probability of a contingency to be realized depends on the effort of the agent, including it in the contract allows the principal to use its realization as a signal about the agent's action choice. This implies that the information structure is richer and providing incentives is less costly.

The principal includes contingencies in the contract for which the incentive gains outweighs the participation loss, determined by the distributional properties of these contingencies. The participation cost of announcing a contingency is the payment to the agent in the states where the contingency is realized. This cost increases with the probability that the unforeseen contingency is realized. The gain of including the contingency is the richer information structure, where this gain increases with the informativeness of the signal. Roughly speaking, contingencies for which the incentive effect dominates the participation effect are contingencies that are very unlikely but highly informative. The characterization of the tradeoff between participation and incentive effect is the key contribution of this paper.

If the agent is unaware after reading the contract, his perception of the world differs from the perception of the principal. The question arises whether the agent can rationalize the proposed contract given his beliefs or whether he should get suspicious. To answer this question I analyze the principal's optimization problem from the viewpoint of the agent. The solution to this problem coincides with the proposed contract whenever the principal's expected profit evaluated at the agent's beliefs is non-negative and the optimal effort choice is the same for both beliefs. Given these conditions, the proposed contract is fully rationalizable for the agent, i.e. the agent has no reason to become suspicious upon reading the contract. The reason for this is that the principal uses the signals within the agent's awareness optimally. Since the agent is unaware of the existence of other relevant contingencies, the proposed contract maximizes the principal's expected payoff evaluated at the agent's beliefs.

Next, I allow for competition among principals. In the benchmark model without unawareness, principals engage in a Bertrand competition over the compensation scheme. In equilibrium, principals make zero profits and the second-best surplus goes to the agent. If the agent is unaware, the symmetric awareness equilibrium exists but there may be other equilibria in which the agent stays unaware, even when competition is tight. As these equilibria are generally inefficient, this result is rather surprising. The reason why unawareness may persist in equilibrium is that if the agent's perception of the world is sufficiently distorted, principals can make "generous" offers for outcomes within the agent's awareness. Revealing the unforeseen states allows the agent to adjust his action choice, exploiting the equilibrium offer in his favor at the cost of the principals. If the agent can reduce the probability to receive the minimal payment sufficiently, accepting the equilibrium contract may yield a higher payoff than the second-best surplus, making any deviating contract unprofitable.

Finally, some generalizations are discussed. Throughout the main part of the analysis it is assumed that output is a discrete one-to-one mapping from contingencies to real numbers. I discuss how results change when more general forms of output functions are considered. Next, the analysis abstracts from the optimal action choice. I show that whenever the principal wishes to implement low effort, it is optimal to not reveal any contingencies, because there is no incentive effect. Further, I extend the analysis to an environment with heterogeneous awareness of agents and show that unawareness is preserved in equilibrium only if the extent of initial unawareness is large enough. Next, I discuss the optimal contract when the agent is the residual claimant. There is an additional effect on the participation constraint because the agent's evaluation of the project generally depends on his level of awareness. It is favorable to the principal to disclose negative outcome shocks, because their revelation lowers the agent's outside option. Lastly, a frequently raised concern is whether unawareness is observationally equivalent to full awareness with zero probability beliefs. I discuss in what sense my model can be interpreted as a standard principal-agent model with heterogeneous priors.

Section 2.2 gives an overview of the related literature. Section 2.3 introduces the theoretical model. In Section 2.4 the optimal contract with observable effort is characterized as a benchmark. The main part of the paper, Section 2.5, is devoted to the analysis of the optimal contract with unobservable effort. Section 2.6 introduces competition among principals and Section 2.7 discusses generalizations of the basic model. Section 2.8 offers some concluding remarks. All proofs can be found in the appendix.

2.2 Related Literature

It is not possible to incorporate non-trivial unawareness in the standard state space model. This has been shown in the seminal paper by Dekel et al. (1998). In response, Heifetz et al. (2006), Li (2009), Board and Chung (2011) and Galanis (2013) have proposed generalized state space models that allow for non-trivial unawareness. My model is based on the generalized state space model introduced by Heifetz et al. (2006). Their unawareness structure consists of a lattice of state spaces, ordered according to their expressive power, where each state space captures a particular horizon of propositions. In a companion work Heifetz et al. (2013) introduce probabilistic beliefs to the model.

Filiz-Ozbay (2012) was one of the first to incorporate unawareness into contracting problems. She considers a contracting situation between a fully aware insurer and an unaware insuree. The key difference between my work and her paper is the presence of moral hazard and the assumption on beliefs. In Filiz-Ozbay (2012) there is no hidden action and consequently no incentive effect. Her set up restricts my framework to the case where the agent is the residual claimant and the revelation of new states involves a participation effect only. However, Filiz-Ozbay (2012) allows for a wider range of equilibrium beliefs. She assumes that the agent assigns arbitrary probability beliefs to newly revealed states with the restriction that the principal's payoff evaluated at the agent's beliefs is non-negative and that relative probability beliefs previously held are unchanged. Given this assumption, it is possible that the agent's beliefs deviate stronger from objective probabilities when becoming aware than before. Due to the insurance motive and the wider range of equilibrium beliefs, the effect of revelation on the participation constraint in her environment is ambiguous. Depending on the effect on the participation constraint, disclosure can be profitable or not. Also Ozbay (2008) analyzes a setting where the decision maker is unaware of some events and a fully aware announcer strategically mentions contingencies before the decision maker takes an action. Both Filiz-Ozbay (2012) and Ozbay (2008) explore the possibility that the unaware agent is able to reason why the other agent proposed the observed contract.

A second strand of literature analyzes contracting problems with unawareness of actions. In these models agents are aware of all of nature's moves but are unaware of their own action space. Von Thadden and Zhao (2012a, 2012b) propose a moral hazard model with a fully aware principal and an unaware agent. At first glance this set up may seem similar to mine, however the underlying intuition and the results are very distinct. In contrast to my model, the agent in their model understands all relevant contingencies but is unaware of his action space. Von Thadden and Zhao (2012a) assume that if the principal leaves the agent unaware, the agent chooses a default action unconsciously, but assesses his expected utility with respect to such default action correctly. The principal decides whether to make the agent aware of his action space or whether to leave him unaware. In a standard moral hazard framework this can be interpreted as the decision whether to restrict the agent's action choice to some sub-optimal level ex-ante or whether to leave the action choice to the agent's discretion. Making the agent aware enlarges the agent's action space and consequently relaxes the participation constraint. However, enlarging the agent's action space adds further incentive constraints to the principal's optimization problem. Consequently, the principal faces a trade off between participation and incentives, but the effects are reversed compared to my model. Their main result is, that it is optimal to leave the agent unaware whenever the default action is close enough to the first best effort level. They extend their analysis to the case where agents differ in their level of awareness and derive the optimal menu of contracts. Also Zhao (2008) considers a moral hazard problem with unawareness of actions and default actions. In his setup both the principal and the agent can be unaware of their action space.

Finally, this work is related to the literature on moral hazard and heterogeneous priors. Santos-Pinto (2008) analyzes a principal-agent model with an agent that holds wrong beliefs about the impact of his effort and calls such biased beliefs self-image. He shows that if positive self-image and effort are complements, the impact of positive self-image is favorable to the principal.[1] If unawareness in my model is interpreted as assigning probability zero to certain outcomes, the resulting distribution does not satisfy the imposed restrictions in Santos-Pinto (2008). Consequently his results do not apply in my framework.[2] Also De la Rosa (2011) analyzes a moral hazard problem with overconfidence.

2.3 The Model

There is a risk-neutral principal and a risk-averse agent. The agent receives utility from monetary transfers C and disutility from effort e. I assume that the utility function is separable in money and effort: $U(C, e) = \upsilon(C) - e$, where υ satisfies the Inada conditions. Effort can take two possible values $e \in \{e^L, e^H\}$, where $e^L < e^H$.

The uncertainty of the environment is captured by a finite set of elementary contingencies, denoted by Θ. A contingency $\theta \in \Theta$ is a random variable with realizations 0 and 1. It can be thought of as an elementary proposition that can be either true or false. The probability of $\theta = 1$ depends on the effort of the agent. Throughout the main part of the analysis it will be assumed that, given e, the contingencies in Θ are conditionally independent of each other.

Assumption 1. *The random variables θ and θ' are conditionally independent given e, for any $\theta, \theta' \in \Theta$.*

The assumption of conditional independence may not always be satisfied in reality. For example, the potential recall in the introductory example may very well be correlated with the sales volume of other products of the firm. However, the tradeoff between participation and incentive effect does not hinge on Assumption 1 and the analysis is made tractable. In contrast, the result on justifiability of the optimal contract depends crucially on this assumption. The implications of relaxing Assumption 1 on the optimal contract and its justifiability

[1] The agent is said to have a positive self-image, whenever there is first-order stochastic dominance of the agent's perceived distribution over the actual distribution for any action. Santos-Pinto (2008) defines effort and self-image as complements if first-order stochastic dominance is stronger for high effort than for low effort.

[2] In my set up the agent implicitly has a positive self-image if he is unaware of outcome decreasing contingencies. If I assume that the true distribution conditional on high effort first-order stochastically dominates the true distribution conditional on low effort and that the agent is unaware of negative outcome shocks only, unawareness implies that effort and positive self-image are substitutes instead of complements. This is because first-order stochastic dominance implies that the probability of low outcomes is more likely under low than high effort. Unawareness implicitly implies that the agent assigns probability zero to some of these outcomes given high and low effort. Thus, the agent's beliefs deviate stronger for low than high effort.

are discussed in Section 2.5.1 and Section 2.5.4 respectively.

Awareness Structure: Unlike in the standard moral hazard problem, the agent is unaware of some contingencies. The subset the agent is aware of is denoted by $\Theta_A \subset \Theta$. The principal is aware of the entire set Θ. Further, he knows that the agent is unaware and he knows which contingencies the agent is unaware of. The agent is unaware of his unawareness and is unaware of the principal's superior awareness. When the principal writes the contract he can enlarge the agent's awareness by mentioning contingencies in the contract, denoted by $X \subseteq \Theta \backslash \Theta_A$. The agent updates his awareness and considers henceforth all contingencies in the set $\widehat{\Theta} = \Theta_A \cup X$.

State Spaces: A state of the world in this environment can be thought of as a sequence of 0's and 1's of length $|\Theta|$ that specifies the realization of each $\theta \in \Theta$. Let S denote the collection of these sequences. Since the agent is unaware of some contingencies, he does not perceive the actual state space but a less expressive one. A state in the agent's subjective state space can be thought of as a sequence of 0's and 1's of length $|\widehat{\Theta}| < |\Theta|$ that specifies the realization of each $\theta \in \widehat{\Theta}$. Let $\widehat{S}$ denote the collection of these sequences. For example, let $\Theta = \{\theta_1, \theta_2\}$ and $\widehat{\Theta} = \{\theta_1\}$. Objectively there are four states of the world $S = \{(0,0),(0,1),(1,0),(1,1)\}$, but the agent only perceives two $\widehat{S} = \{(0),(1)\}$. In terms of the introductory example, suppose there are two relevant contingencies: the marketing strategy being a success and the product having adverse effects on consumers' health. If the manager is unaware of the latter, his subjective state space distinguishes between the marketing strategy being a success or a failure but misses the dimension about the adverse health effects. Thus, the set of contingencies the agent is aware of determines the dimension of his subjective state space. Disclosing a contingency in the contract implies adding another dimension to his subjective state space.

Outcomes: There is a project with stochastic outcome Y, which is observable and verifiable. Outcome is a function of the contingencies in Θ. Since state $s \in S$ specifies the realization of each contingency $\theta \in \Theta$, we can define outcome realization y directly as a function of the state $s \in S$:

$$y = f(s), \quad s \in S.$$

Let $\mathcal{Y}$ denote the range of function f. Since the agent does not know the objective state space S, he cannot know $\mathcal{Y}$. Instead he perceives outcome as a function of the contingencies he is aware of. I assume that the agent's perceived outcome is equivalent to the objective outcome when the contingencies the agent is unaware of are not realized, denoted by $\theta = 0$, implying that the agent considers a subset of possible worlds. Note that $\theta = 0$ can refer to an elementary proposition being true or false.[3] A way to think about this assumption is that there are events that the agent has never observed and that have never crossed his mind. Instead, the agent has some implicit assumptions about the underlying state of the world, but is unaware of these implicit assumptions (Li, 2008). Consequently, he cannot imagine a world in which a proposition implicitly assumed to be true (false) turns out to be false (true). This assumption is prevalent in the literature of unawareness.[4]

The agent's outcome function is

$$y = \widehat{f}(s), \quad s \in \widehat{S}.$$

where $\widehat{f}(s) = f(s, 0, 0, 0, ...)$, $s \in \widehat{S}$.[5] Let $\widehat{\mathcal{Y}}$ denote the range of function $\widehat{f}$.

Assumption 2. $|\mathcal{Y}| = 2^{|\Theta|}$.

A2 imposes that outcome differs across every state of the world, which implies that the agent knows a subset of possible outcomes whenever he is not fully aware, $\widehat{\mathcal{Y}} \subset \mathcal{Y}$. This implies that the agent is not only unaware of some contingencies but also of their consequences. Coming back to the introductory example, the manager is not only unaware of a possible recall but also of its effect on the firm's performance. If the recall is realized, the manager is surprised and receives the minimal payment. The moral hazard problem to have in mind is one, where unforeseen contingencies lead to unforeseen consequences and the agent may be surprised ex-post. The assumption that f is a one-to-one function is important for tractability of the characterization of the tradeoff between participation and incentives.[6]

[3] For example, if a decision maker is unaware of the concept of gravity and θ is the elementary statement that there is gravity, then $\theta = 0$ means that the elementary statement is true. Similarly, if a decision maker is unaware of global warming and θ is the elementary statement that there is global warming, then $\theta = 0$ means that the elementary statement is false.

[4] See for example Modica et al. (1998) and Heifetz et al. (2013). Furthermore, the concept of a default dimension is closely related to the assumption of a default action in the literature on unawareness of actions (e.g. von Thadden and Zhao (2012a)).

[5] Assume that any sequence $s \in S$ is ordered such that $s = \{s', s''\}$, with $s' \in \widehat{S}$.

[6] A2 implies that disclosing contingencies allows the principal to use a more informative outcome distribution, which implies that the effect of disclosure on incentives is always positive. If this assumption is given up, the incentive effect is ambiguous.

More general outcome functions are discussed in Section 2.7.1.

Probability Measures: Let $\pi(y|e)$ denote the probability of $y \in \mathcal{Y}$ given effort e and assume $\pi(y|e) > 0, \forall y \in \mathcal{Y}$. The distribution over $\mathcal{Y}$ is known to the principal. The agent is assumed to have correct beliefs over the distribution of contingencies within his awareness. So whenever the principal expands the agent's state space by making him aware of a new contingency, the agent understands the conditional probability distribution of this contingency. This belief updating rule implies that the likelihood ratios of events in the original state space remain unchanged and that the relative probability mass assigned to the newly revealed outcomes is correct.[7]

Under the assumption of independence the probability that $\theta = 0$ for all $\theta \notin \widehat{\Theta}$ is constant across all $y \in \widehat{\mathcal{Y}}$. Let $\Pi(\widehat{\Theta}|e) := \prod_{\theta \notin \widehat{\Theta}} \Pr[\theta = 0|e]$ denote the probability that $y \in \widehat{\mathcal{Y}}$. Then the agent assigns probability

$$\widehat{\pi}(y|e) := \frac{\pi(y|e)}{\Pi(\widehat{\Theta}|e)}, \quad y \in \widehat{\mathcal{Y}},$$

to outcome $y \in \widehat{\mathcal{Y}}$ conditional on effort e, which is simply the conditional probability given that none of the unforeseen contingencies are realized.

The Contract: As in the standard principal-agent problem, effort is assumed to be non-observable; thus, the principal offers a contract based on the observable and verifiable outcome Y. The distribution of Y depends on the effort of the agent. It is assumed that $E[Y|e^H] > E[Y|e^L]$ and that $E[Y|e^H] - E[Y|e^L]$ is large enough such that it is always optimal to induce high effort. This allows me to abstract the analysis from the choice of effort. In Section 2.7.2 the optimal action choice will be discussed. The agent is assumed to have limited liability; thus, the outcome contingent compensation C is non-negative for all $y \in \mathcal{Y}$. Due to the Inada conditions imposed on $v(\cdot)$, the limited liability constraint will not be

[7] For a detailed study of belief updating under growing awareness see Karni and Vierø (2012). They derive a belief updating rule that requires that the likelihood ratios of events in the original state space remain unchanged, but their model stays silent about the absolute levels of these probabilities. The belief updating rule in my framework is a special case of the belief updating rule derived by Karni and Vierø (2012). The additional assumption that the relative probability mass assigned to newly revealed states is correct facilitates the exposition of the tradeoff between participation and incentive effect. If this assumption is given up, principal and agent may hold different beliefs about the likelihood of newly revealed contingencies, making the principal want to bet with the agent on the realization of these contingencies. With the assumption that the agent understands the likelihood of contingencies once aware, I abstract from such side bets.

binding for outcomes within the agent's awareness. Instead, it implements a lower bound on payments for outcomes the agent is unaware of.

Definition 1. *A contract is a pair* $(\widehat{\Theta}, C)$ *with* $\Theta_A \subseteq \widehat{\Theta} \subseteq \Theta$ *and* $C : \mathcal{Y} \longrightarrow \mathbb{R}_0^+$.

Let $(\widehat{\Theta}^*, \widehat{C}^*)$ denote the contract that maximizes the principal's expected payoff. Following Filiz-Ozbay (2012), Definition 2 introduces a notion of incompleteness.

Definition 2. *A contract* $(\widehat{\Theta}, C)$ *is incomplete if* $\widehat{\Theta} \neq \Theta$. *Otherwise it is complete.*

Suppose the proposed contract is incomplete such that $\mathcal{Y}\backslash\widehat{\mathcal{Y}}$ is non-empty. The principal can construct the contract such that he pays zero to the agent when an unforeseen level of outcome is realized, e.g. by finding a functional form of the compensation scheme on $\mathcal{Y}$ satisfying zero payments for all $y \in \mathcal{Y}\backslash\widehat{\mathcal{Y}}$ or by including a "zero payment otherwise" clause in the contract. I will abstract from the question of how the principal can implement zero payments in the unforeseen states, but analyze a reduced form of this model. Zero payments at $y \in \mathcal{Y}\backslash\widehat{\mathcal{Y}}$ are optimal because any positive payment in these states will leave the agent's expected utility unaffected, but make the principal strictly worse off. This implies that whenever the contract is incomplete, the agent's expected utility evaluated at objective beliefs is strictly lower than his reservation utility. It is important to note that zero payments facilitate notation considerably, but that the results hold for any other minimal payment as long as it is low enough.[8] Thus, the optimal contract in this environment can be interpreted as a contract that promises a fixed payment and that rewards the agent with boni for certain outcomes. Whenever a contingency is realized that is not anticipated by the agent, the bonus is not paid.

Expected Payoffs: The principal's outside option in the case of rejection is assumed to be zero. His expected payoff is given by

$$EU_P = \begin{cases} \sum_{y\in\mathcal{Y}} \pi(y|e)\,[y - C(y)] & \text{if the agent accepts,} \\ 0 & \text{if the agent rejects.} \end{cases}$$

The agent assesses his expected utility with respect to his restricted state space. The outside option of rejecting the contract is $\bar{U}$:

$$EU_A = \begin{cases} \sum_{y\in\widehat{\mathcal{Y}}} \widehat{\pi}(y|e) v(C(y)) - e & \text{if the agent accepts,} \\ \overline{U} & \text{if the agent rejects.} \end{cases}$$

[8]Low enough means that the minimal payment constraint is not binding for outcomes within the agent's awareness. Otherwise the tradeoff for the respective outcomes changes.

2.4 The Optimal Contract with Observable Effort

In order to have a benchmark it is useful to first characterize the contract when effort is observable. If effort is observable and verifiable the contract can be made directly contingent on the action of the agent. The principal solves the problem:

$$\max_{\widehat{\Theta},C(\cdot)} \sum_{y\in\mathcal{Y}} \pi(y|e^H)\left[y - C(y)\right]$$

subject to

$$\sum_{y\in\widehat{\mathcal{Y}}} \widehat{\pi}(y|e^H)\upsilon\left(C(y)\right) - e^H \geq \overline{U}$$

$$C(y) \geq 0, \quad \forall y \in \mathcal{Y}.$$

When the contract is complete, it is optimal to give the agent full insurance. This can be seen from the first order conditions

$$\frac{1}{\upsilon'(C(y))} = \lambda, \qquad \forall y \in \mathcal{Y}.$$

The agent receives $C^{FB} = \upsilon^{-1}\left(\bar{U} + e^H\right)$ independent of the realization of Y. The first best is achieved. Now suppose the principal leaves the agent unaware of some contingencies. The first order conditions for $C(y), y \in \widehat{\mathcal{Y}}$ are

$$\frac{1}{\upsilon'(C(y))} = \lambda\frac{1}{\Pi(\widehat{\Theta}|e^H)}.$$

The first-order conditions imply that the transfer across $y \in \widehat{\mathcal{Y}}$ is constant. The optimal compensation scheme is simply $\widehat{C}^*(y) = C^{FB}, \forall y \in \widehat{\mathcal{Y}}$ and $\widehat{C}^*(y) = 0, \forall y \in \mathcal{Y}\backslash\widehat{\mathcal{Y}}$. The expected payment to the agent is $\Pi(\widehat{\Theta}|e^H)C^{FB}$. It is minimized when the probability of paying, $\Pi(\widehat{\Theta}|e^H)$, is minimized, which is achieved when the agent's awareness level is lowest. Consequently, if effort is observable, it is optimal to reveal nothing to the agent.

Proposition 2.4.1. *Under A1, A2 and observable effort,* $\widehat{\Theta}^* = \Theta_A$.

The reason for result 2.4.1 is that disclosing contingencies to the agent makes the participation constraint more costly to satisfy. Since effort is observable there is no incentive effect and only the participation effect matters.

2.5 The Optimal Contract with Unobservable Effort

If effort is unobservable, the principal maximizes his expected profit subject to the participation constraint, the incentive constraint and the limited liability constraints. The participation constraint assures that the agent accepts the contract. The incentive constraint leads the agent to exert high effort e^H. The principal solves:

$$\max_{\widehat{\Theta}, C(\cdot)} \sum_{y \in \mathcal{Y}} \pi(y|e^H) \left[y - C(y)\right], \tag{2.1}$$

subject to

$$\sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^H) v\left(C\left(y\right)\right) - e^H \geq \overline{U}, \tag{2.2}$$

$$e^H \in \arg\max_e \left\{ \sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e) v\left(C\left(y\right)\right) - e \right\}, \tag{2.3}$$

$$C(y) \geq 0, \qquad \forall y \in \mathcal{Y}, \tag{2.4}$$

where (2.2) is the participation constraint, (2.3) is the incentive constraint and (2.4) is the limited liability constraint. Assume that a solution to the maximization problem exists.[9] The analysis of the optimal contract can be divided into two steps. In step one the principal chooses the optimal compensation scheme $\widehat{C}$ given announcement $\widehat{\Theta}$. In step two he chooses the optimal level of awareness $\widehat{\Theta}$.

2.5.1 Step 1: Optimal Compensation Scheme given Awareness $\widehat{\Theta}$

The optimal compensation scheme given awareness $\widehat{\Theta}$ is characterized by the necessary condition

$$\frac{1}{v'(C(y))} = \frac{1}{\Pi(\widehat{\Theta}|e^H)} \left(\lambda + \gamma \left[1 - \frac{\widehat{\pi}\left(y|e^L\right)}{\widehat{\pi}\left(y|e^H\right)} \right] \right), \quad \forall y \in \widehat{\mathcal{Y}}, \tag{2.5}$$

as well as $C(y) = 0, \forall y \in \mathcal{Y} \backslash \widehat{\mathcal{Y}}$.[10] Note that the optimal compensation scheme varies with the likelihood ratio $\frac{\widehat{\pi}(y|e^L)}{\widehat{\pi}(y|e^H)}$ of the restricted information structure $\widehat{\Theta}$ instead of Θ. Since the agent is unaware of the contingencies in $\Theta \backslash \widehat{\Theta}$, these signals cannot be used to induce e^H. As

[9] For details see Grossman and Hart (1983).

[10] Inada conditions assure that (2.4) is not binding for $C(y), y \in \widehat{\mathcal{Y}}$.

in the standard moral hazard problem both participation and incentive constraint hold with equality. Let $\widehat{C}$ denote the solution to this system of equations.

Lemma 2.5.1. *Assume A1 and A2. Under $\widehat{C}$, both $\lambda > 0$ and $\gamma > 0$.*

Proof. See Appendix 2.9.2. □

Remark: If the assumption of conditional independence is relaxed, the optimal compensation scheme additionally varies with the ratio $\frac{\widehat{\pi}(y|e^H)}{\pi(y|e^H)}, y \in \widehat{\mathcal{Y}}$.[11] *Under Assumption 1, $\frac{\widehat{\pi}(y|e^H)}{\pi(y|e^H)}$ is constant across all $y \in \widehat{\mathcal{Y}}$, but if conditional dependence is not ruled out, this ratio generally varies across outcomes. In such a case, it is optimal to promise relative large payments for outcomes that are positively correlated with contingencies the agent is left unaware of, because these payments are less likely to be realized.*

The optimal compensation scheme $\widehat{C}(\cdot)$ across $y \in \widehat{\mathcal{Y}}$ coincides with the the optimal compensation scheme of the standard principal-agent model with symmetric awareness and restricted information structure $\widehat{\Theta}$. To see this, suppose that $\widehat{S}$ is the objective state space and that both, the principal and the agent, are symmetrically aware of $\widehat{S}$. Then the principal solves

$$\min_{C(\cdot)} \quad \sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^H) C(y) \tag{2.6}$$

subject to (2.2) and (2.3). Let $C^C_{\widehat{\Theta}}$ denote the solution to this problem. $C^C_{\widehat{\Theta}}$ is the optimal complete contract under symmetric awareness and information structure $\widehat{\Theta}$. Under asymmetric awareness, the expected payment to the agent is

$$\sum_{y \in \mathcal{Y}} \pi(y|e^H) C(y) = \Pi(\widehat{\Theta}|e^H) \sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^H) C(y) - (1 - \Pi(\widehat{\Theta})) \cdot 0,$$

which is equivalent to (2.6) except for the scaling factor $\Pi(\widehat{\Theta}|e^H)$. Since $\Pi(\widehat{\Theta}|e^H)$ is nothing but a constant for a given $\widehat{\Theta}$, the two optimization problems are equivalent and $\widehat{C}(y) = C^C_{\widehat{\Theta}}(y)$ for all $y \in \widehat{\mathcal{Y}}$. The expected profit of the optimal incomplete contract is simply the expected

[11] If Assumption 1 is relaxed, the first-order condition is

$$\frac{1}{v'(C(y))} = \frac{\widehat{\pi}(y|e^H)}{\pi(y|e^H)} \left\{ \lambda + \gamma \left[1 - \frac{\widehat{\pi}(y|e^L)}{\widehat{\pi}(y|e^H)} \right] \right\}, \quad \forall y \in \widehat{\mathcal{Y}}.$$

payment of $C^C_{\widehat{\Theta}}$ weighted by the probability that none of the unforeseen contingencies are realized:

$$E[\widehat{C}(Y)|e^H] = \Pi(\widehat{\Theta}|e^H)E[C^C_{\widehat{\Theta}}(Y)|e^H].$$

2.5.2 Step 2: Optimal Awareness $\widehat{\Theta}^*$

The optimal level of disclosure is characterized by identifying the basic trade off between participation and incentives of disclosing contingencies to the agent. To separate the effect on incentives from the effect on participation, it is useful to compare the expected payment of complete contracts under different information structures.

Lemma 2.5.2. *Let Z be a non-empty subset of $\Theta \backslash \widehat{\Theta}$. Then*

$$\Delta C^Z_{\widehat{\Theta}} := E\left[C^C_{\widehat{\Theta}}(Y)|e^H\right] - E\left[C^C_{\widehat{\Theta}\cup Z}(Y)|e^H\right] \geq 0,$$

with strict inequality if and only if $\exists \theta \in Z$ such that $\Pr[\theta = 1|e^H] \neq \Pr[\theta = 1|e^L]$.

Proof. See Appendix 2.9.3. □

This result is in line with Holmström's *Sufficient Statistic Theorem* (1979), which states that a signal θ is valuable if and only if it is informative.[12] Valuable means that both, principal and agent, can be made better off by including θ because agency costs are reduced. Under independence, θ is informative if and only if $\Pr[\theta = 1|e^H] \neq \Pr[\theta = 1|e^L]$.

2.5.3 The Basic Tradeoff

To understand the effect of disclosing a subset of $\Theta \backslash \Theta_A$ on participation and incentives, compare the expected payoff of the principal when revealing nothing ($\widehat{\Theta} = \Theta_A$) and revealing set X ($\widehat{\Theta} = \Theta_A \cup X$):

$$\Pi(\Theta_A|e^H)E\left[C^C_{\Theta_A}(Y)|e^H\right] \gtrless \Pi(\Theta_A \cup X|e^H)E\left[C^C_{\Theta_A \cup X}(Y)|e^H\right], \tag{2.7}$$

or simply

$$\prod_{\theta \in X} \Pr\left[\theta = 0 \mid e^H\right] E\left[C^C_{\Theta_A}(Y)|e^H\right] \gtrless E\left[C^C_{\Theta_A \cup X}(Y)|e^H\right].$$

[12] Holmström (1979) shows this for continuous outcome and continuous effort.

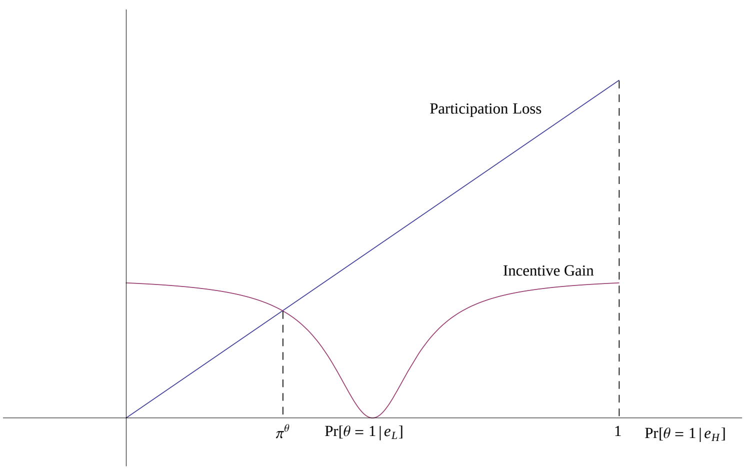
Using $\Delta C^X_{\Theta_A} = E\left[C^C_{\Theta_A}(Y)|e^H\right] - E\left[C^C_{\Theta_A \cup X}(Y)|e^H\right]$ we can restate (2.7) in terms of gains and losses of revealing set X:

$$\Delta C^X_{\Theta_A} \gtrless \left(1 - \prod_{\theta \in X} \Pr\left[\theta = 0|e^H\right]\right) E\left[C^C_{\Theta_A}(Y)|e^H\right]. \tag{2.8}$$

$\Delta C^X_{\Theta_A}$ captures the incentive effect of disclosing set X. The announcement of a set of informative contingencies allows the principal to use these contingencies as a signal about the agent's effort. This information gain is $\Delta C^X_{\Theta_A}$. $(1 - \prod_{\theta \in X} \Pr[\theta = 0|e^H])E[C^C_{\Theta_A}(Y)|e^H]$ captures the participation effect of disclosing X. With probability $1 - \prod_{\theta \in X} \Pr\left[\theta = 0|e^H\right]$ one of the contingencies in X is realized. When announcing $\Theta_A \cup X$, the principal has to pay the agent a positive wage, while when announcing Θ_A, he pays zero. Disclosing a contingency to the agent consequently tightens the participation constraint.

When effort is observable, there is no incentive effect, which is why it is optimal to keep the agent unaware. When effort is unobservable, the principal chooses $\widehat{\Theta}$ such that the net gain of revelation is maximized. Since Θ is finite, he compares a finite number of announcement strategies and their respective expected payoffs. Whenever there exists a $\widehat{\Theta}$ such that the incentive effect outweighs the participation effect, the principal enlarges the agent's awareness.

Figure 2.1: Tradeoff



To illustrate the basic tradeoff consider the announcement of a single contingency $\theta \notin \Theta_A$:

$$\Delta C^{\theta}_{\Theta_A} \quad \text{vs.} \quad \Pr\left[\theta = 1|e^H\right] E\left[C^C_{\Theta_A}(Y)|e^H\right].$$

Figure 1 shows the incentive gain and the participation loss as a function of $\Pr[\theta = 1|e^H]$.[13]

Lemma 2.5.3. *Assume A1 and A2. Let* $\Delta^{\theta} := \left|\Pr[\theta = 1|e^H] - \Pr[\theta = 1|e^L]\right|$. $\Delta C^{\theta}_{\Theta_A}$ *is monotonically increasing in* Δ^{θ}.

Proof. See Appendix 2.9.4 □

Δ^{θ} is a measure of informativeness of signal θ. Lemma 2.5.3 states that the incentive gain $\Delta C^{\theta}_{\Theta_A}$ is increasing in the informativeness of θ. Consequently, the incentive gain is decreasing on the interval $[0, \Pr[\theta = 1|e^L])$ and increasing on the interval $(\Pr[\theta = 1, e^L], 1]$. When $\Pr[\theta = 1|e^L] = \Pr[\theta = 1|e^H]$, the signal is uninformative and the incentive gain is zero. The participation loss $\Pr\left[\theta = 1|e^H\right] E\left[C^C_{\Theta_A}(Y)|e^H\right]$ is linearly increasing in $\Pr[\theta = 1|e^H]$. Figure 1 shows that it is optimal to reveal any informative contingency θ if the probability that $\theta = 1$ conditional on high effort is small enough. This result is summarized in Observation 2.5.4.

Observation 2.5.4. *Assume A1 and A2. For every* $\theta \in \Theta \backslash \Theta_A$ *there exists a threshold* $\pi^{\theta} \in (0, \Pr[\theta = 1|e^L])$ *such that if* $\Pr[\theta = 1|e^H] < \pi^{\theta}$, *the incentive effect outweighs the participation effect.*

Example 5.1 The tradeoff associated to the announcement of a single contingency generally depends on the announcement of other contingencies. Going back to the introductory example, suppose the manager is not only unaware of a possible lawsuit but also of a possible merger. Whether it is profitable to reveal the merger not only depends on its likelihood conditional on the manager's effort, but also on whether the firm reveals the lawsuit or not. If the lawsuit is revealed and its realization is highly informative about the manager's effort, the incentive effect of revealing the possibility of a merger is negligible. Thus, the tradeoff between incentives and participation associated to the announcement of a single contingency generally depends on the announcement of other contingencies. To see how the single contingency tradeoff changes as the underlying awareness extends, consider the example of N symmetric contingencies, $\Theta \backslash \Theta_A = \{\theta_1, ..., \theta_N\}$, where symmetric means that every contingency has the same distribution. Let $\alpha := \Pr[\theta_i = 0|e^H], i = 1, ...N$ and $\widehat{\Theta}_n := \Theta_A \cup \theta_1 \cup ... \cup \theta_n$.[14] The net

[13] This figure shows $\Delta C^{\theta}_{\Theta_A}$ and $\Pr[\theta = 1|e^H] E[C^C_{\Theta_A}(Y)|e^H]$ for the following specification: $v(C) = \frac{C^{1-\sigma}}{1-\sigma}, \sigma = 0.5, e^H = 1, e^L = 0, \bar{U} = 5$. There are two contingencies. Contingency $\theta \in \Theta_A$ with $\Pr[\theta = 1|e^H] = 0.65$ and $\Pr[\theta = 1|e^L] = 0.5$ and contingency $\theta' \notin \Theta_A$ with $\Pr[\theta' = 1|e^L] = 0.5$.

[14] For notational convenience let $\widehat{\Theta}_0 = \Theta_A$.

gain of revealing the nth contingency is:

$$NG(\theta_n) := \alpha^{N-n}\left(\Delta C^{\theta_n}_{\widehat{\Theta}_{n-1}} - (1-\alpha)E[C^C_{\widehat{\Theta}_{n-1}}(Y)|e^H]\right).$$

The value of $NG(\theta_n)$ is determined by two factors. First, there is the difference between incentive gain and participation loss as illustrated above. This difference is weighted by the probability that the gains and losses are realized, α^{N-n}. Generally, $NG(\theta_n)$ can decrease or increase in n. A sufficient condition for $NG(\theta_n)$ to be decreasing in n is:

$$E[C^C_{\widehat{\Theta}_n}(Y)|e^H]^2 < E[C^C_{\widehat{\Theta}_{n-1}}(Y)|e^H]E[C^C_{\widehat{\Theta}_{n+1}}(Y)|e^H], \quad \text{for all} \quad n \in \{1, ..., N-1\}. \tag{2.9}$$

Condition (2.9) implies that the expected payment of the complete contract $C^C_{\widehat{\Theta}_n}$ as a function of n is sufficiently convex, i.e. the inclusion of signal θ_n in the information structure reduces agency costs strongly when n is small but only marginally when n is large.[15] If this condition holds, the net gain of revealing a contingency is decreasing in the number of other contingencies that are revealed and the optimal level of disclosure is $\widehat{\Theta}^* = \{\Theta_A, \theta_1, ..., \theta_{n^*}\}$ such that:

$$NG(\theta_{n^*}) \geq 0 \quad \text{and} \quad NG(\theta_{n^*+1}) < 0.$$

2.5.4 Justifiability of the Contract

If the contract is incomplete, the agent's perception of the world differs from the perception of the principal. An important question is whether the proposed contract can elicit suspicion on the side of the agent. Upon receiving contract $(\widehat{\Theta}, C)$ the agent may ask herself whether announcement $\widehat{\Theta}$ and compensation scheme C are optimal for the principal. Filiz-Ozbay (2012) introduces an equilibrium refinement which requires that the equilibrium contract maximizes the principal's expected payoff from the viewpoint of the agent. The agent can only contemplate contracts within his awareness. The set of contracts the agent is aware of is given by the set of all contracts $(\tilde{\Theta}, \tilde{C})$ such that $\Theta_A \subseteq \tilde{\Theta} \subseteq \widehat{\Theta}$. Let $E_{\tilde{\Theta}}$ and $\tilde{Y}$ denote the expectation operator and the set of outcomes associated to awareness $\tilde{\Theta}$. A contract $(\widehat{\Theta}, C)$ is called justifiable if is optimal for the principal from the viewpoint of the agent, both in terms of compensation and in terms of the level of disclosure.

[15] Condition (2.9) is satisfied, for example when $v(C) = \frac{C^{1-\sigma}}{1-\sigma}$, $\sigma = 0.5$, $e^H = 1$, $e^L = 0$, $\bar{U} = 9$, $N = 5$, $\alpha = 0.6$ and $\Pr[\theta = 0|e^L] = \Pr[\theta = 1|e^H] = 0.5$.

Definition 3. *For any $\tilde{\Theta}$ such that $\Theta_A \subseteq \tilde{\Theta} \subseteq \widehat{\Theta}$, let $C_{\tilde{\Theta}} : \mathcal{Y} \to \mathbb{R}_0^+$ denote the solution to*

$$\begin{aligned} \max_C \quad & E_{\tilde{\Theta}}[Y - C(Y)|e], \\ s.t. \quad & E_{\tilde{\Theta}}[\upsilon(C(Y))|e] - e \geq \overline{U}, \\ & e \in \arg\max_{\tilde{e}} \left\{ E_{\tilde{\Theta}}[\upsilon(C(Y))|\tilde{e}] - \tilde{e} \right\}, \end{aligned}$$

and $C(y) = 0, \forall y \in \mathcal{Y} \backslash \tilde{\mathcal{Y}}$. A contract $(\widehat{\Theta}, C)$ is called justifiable if

$$\widehat{\Theta} \in \arg\max_{\tilde{\Theta}} E_{\widehat{\Theta}}[Y - C_{\tilde{\Theta}}(Y)|e] \quad and \quad C(y) = C_{\widehat{\Theta}}(y), \forall y \in \widehat{\mathcal{Y}}.$$

$\{C_{\tilde{\Theta}}\}_{\Theta_A \subseteq \tilde{\Theta} \subseteq \widehat{\Theta}}$ is the set of justifiable compensation schemes for each level of disclosure the agent can contemplate. A contract $(\widehat{\Theta}, C)$ is called justifiable if it satisfies two requirements. First, compensation scheme C has to be justifiable for awareness $\widehat{\Theta}$ and second, form the viewpoint of the agent, compensation scheme C is the best of all justifiable compensation schemes in the set $\{C_{\tilde{\Theta}}\}_{\Theta_A \subseteq \tilde{\Theta} \subseteq \widehat{\Theta}}$. Note that a justifiable contract generally exists, because if none of the incomplete contracts is justifiable, the principal can always resort to the full awareness contract (Θ, C_{Θ}^C). Since full disclosure implies that the principal and the agent share the same beliefs, (Θ, C_{Θ}^C) is necessarily justifiable. Thus, provided that there is a solution to the principal's optimization problem under symmetric awareness Θ, a justifiable contract exists. Since the optimal contract $(\widehat{\Theta}^*, \widehat{C}^*)$ is not necessarily complete, the question arises under which conditions the optimal contract is justifiable for the agent.

Proposition 2.5.5. *Assume A1 and A2. $(\widehat{\Theta}^*, \widehat{C}^*)$ is justifiable according to Definition 3 if and only if $E_{\widehat{\Theta}^*}[Y - C_{\widehat{\Theta}^*}(Y)|e^H] \geq 0$ and $E_{\widehat{\Theta}^*}[Y - C_{\widehat{\Theta}^*}(Y)|e^H] \geq E_{\widehat{\Theta}^*}[Y|e^L] - \bar{C}$, where $\bar{C} = \upsilon^{-1}(\overline{U} + e^L)$.*

Proof. See Appendix 2.9.5 □

A necessary and sufficient condition for the optimal contract to be justifiable is that the principal's expected utility is non-negative and that e^H is the optimal action choice from the viewpoint of the agent. Note that in the characterization of the optimal contract, actual outcome levels in $\mathcal{Y}$ play no role because only the likelihood ratio associated with each outcome level is relevant for providing of incentives. If we require the contract to be justifiable, this is no longer necessarily the case. Whenever the optimal contract is incomplete, the agent perceives only a subset of possible outcomes. It is possible that the optimal contract leaves the agent unaware of high outcomes such that $E_{\widehat{\Theta}^*}[Y - C_{\widehat{\Theta}^*}(Y)|e^H] < 0$ whereas $E[Y - C_{\widehat{\Theta}^*}(Y)|e^H] \geq 0$. Similarly, it is possible that the optimal contract leaves the agent unaware of outcomes that are strongly correlated with effort such that $E_{\widehat{\Theta}^*}[Y - C_{\widehat{\Theta}^*}(Y)|e^H] < E_{\widehat{\Theta}^*}[Y|e^L] - \bar{C}$ whereas $E[Y - C_{\widehat{\Theta}^*}(Y)|e^H] \geq E[Y|e^L] - \bar{C}$.[16] If any of the two conditions in Proposition 2.5.5 is violated, the refinement introduces another dimension in the tradeoff. The principal is no longer only concerned with the distributional properties of the contingencies in $\Theta \backslash \Theta_A$, but also with the outcomes an announcement reveals.

Given that the expected profit evaluated at the agent's beliefs is non-negative and higher at e^H than at e^L, the optimal contract $(\widehat{\Theta}^*, \widehat{C}^*)$ is justifiable. Justifiability of $\widehat{C}^*$ is straight forward. If the optimal contract is complete, principal and agent share the same beliefs. Hence, $\widehat{C}^*$ maximizes the principal's expected payoff from the agent's perspective. If the optimal contract is incomplete, we know that the transfer rule for outcomes within the agent's awareness coincides with the optimal compensation scheme of the complete contract given information structure $\widehat{\Theta}$. Since the agent thinks that the contract is complete, $\widehat{C}^*$ solves the principal's optimization problem given the agent's beliefs.

To see justifiability of $\widehat{\Theta}^*$, remember that the agent can only consider the announcement of contingencies within his awareness. Suppose $\Theta_A \subset \widehat{\Theta}^*$. The agent evaluates the principal's expected payoff for every announcement $\widehat{\Theta}^* \backslash Z$, where $Z \in \widehat{\Theta}^* \backslash \Theta_A$. The agent knows that given announcement $\widehat{\Theta}^* \backslash Z$ the optimal compensation scheme is $C(y) = C^C_{\widehat{\Theta}^* \backslash Z}(y)$ for all y within the agent's hypothetical awareness $\widehat{\Theta}^* \backslash Z$ and zero otherwise. Thus, after reading the contract, the agent understands the principal's optimal contract for any level of awareness lower or equal than his actual awareness, but he does not understand that there may remain contingencies that he is unaware of. For $\widehat{\Theta}^*$ to be justifiable, the following condition has to be satisfied

$$E_{\widehat{\Theta}}\left[C^C_{\widehat{\Theta}^*}(Y)|e^H\right] \leq \prod_{\theta \in \widehat{\Theta}^* \backslash Z} \Pr[\theta = 0|e^H] E_{\widehat{\Theta}}\left[C^C_{\widehat{\Theta}^* \backslash Z}(Y)|e^H\right], \tag{2.10}$$

for any $Z \subseteq \widehat{\Theta}^* \backslash \Theta_A$. This coincides with the optimality condition of the principal. Consequently, (2.10) is fulfilled and $\widehat{\Theta}^*$ can be rationalized by the agent.

[16] An example for the first violation is a potential innovation that the manager is unaware of without which the firm is not profitable. It seems more realistic that firms stay silent about bad news rather than good news, which may have reasons not captured in this model (e.g. restriction to monotone compensation schemes). The second violation requires that the agent is unaware of an event that is sufficiently correlated with his effort and that is sufficiently likely, e.g. the success of an advertisement campaign. It seems more realistic that decision makers are unaware of low probability events rather than events that occur frequently, so the economic relevance of both violations may be restricted.

Note that the assumption of conditional independence is crucial for justifiability of the optimal contract. If this assumption is not satisfied the optimal compensation scheme varies with the ratio $\frac{\widehat{\pi}(y)}{\pi(y)}, y \in \widehat{\mathcal{Y}}$, which cannot be rationalized by the agent. If we assume that the agent rejects any non-justifiable contract, this implies that the principal proposes the contract that is optimal under conditional independence, $\{\widehat{C}^*, \widehat{\Theta}^*\}$, in order to ensure acceptance by the agent.

Chen and Zhao (2009) propose two additional equilibrium concepts: the trap-filtered equilibrium and the trap-filtered equilibrium with cognition. In the trap-filtered equilibrium the agent assigns some probability ρ to the event that a non-justifiable contract is a trap and probability $1 - \rho$ to the event that the contract is a consequence of the principal's mistake. If the contract is a trap the agent is better of rejecting it, if the contract is proposed by mistake, the agent prefers to accept it. Applying this equilibrium concept to my framework implies that, given $E_{\widehat{\Theta}^*}[Y - C_{\widehat{\Theta}^*}(Y)|e^H] < 0$ or $E_{\widehat{\Theta}^*}[Y - C_{\widehat{\Theta}^*}(Y)|e^H] < E_{\widehat{\Theta}^*}[Y|e^L] - \bar{C}$, the principal finds it optimal to propose a non-justifiable contract if and only if the agent's belief that the contract is a trap is sufficiently low. In the trap-filtered equilibrium with cognition the agent can exert cognitive effort to learn about the probability that the contract is a trap. Cognitive effort is costly, so the agent chooses a level such that the marginal cost of cognitive effort equals its marginal information gain. When designing the contract the principal takes the agent's cognitive effort choice into account. Depending on the agent's prior and the cognitive cost function, the principal offers either a justifiable contract that is accepted with probability one or he offers a non-justifiable contract that is only accepted when the agent does not learn that the contract is a trap.

2.6 Competing Principals

In the basic setup I analyze the optimization problem of a monopolistic principal. This section addresses the question of how these results change when principals compete against each other. Suppose there are N principals that are aware of Θ. They make simultaneous offers, denoted by $(\widehat{\Theta}_i, C_i), i = 1, ..., N$. The agent updates his awareness after hearing all the offers and accepts at most one. He considers henceforth every contingency in $\widehat{\Theta}_1 \cup ... \cup \widehat{\Theta}_N$. If the agent is indifferent between two or more contracts he accepts each contract with equal probability. I focus on symmetric equilibria in pure strategies, $(\widehat{\Theta}_i, C_i) = (\widehat{\Theta}, C), i = 1, ..., N$.

In the absence of asymmetric awareness, principals engage in a Bertrand competition over transfer rule C. In equilibrium they make zero profits and the surplus goes to the agent. Assume that under full awareness the agent's expected payoff is maximized when high effort is implemented and let C^* denote the equilibrium compensation scheme.

Assumption 3. *Let* $E[\upsilon(C^*(y))] - e^H \geq \overline{U}$ *and* $E[\upsilon(C^*(y))] - e^H \geq \upsilon(E[Y|e^L]) - e^L$*, where* C^* *maximizes the aware agent's payoff subject to his incentive constraint and the zero profit constraint.*

Proposition 2.6.1. *Assume A1, A2 and A3. There is a symmetric Nash equilibrium in which the agent is fully aware and each principal offers the complete zero profit contract* (Θ, C^*).

Proof. See Appendix 2.9.6. □

Proposition 2.6.1 states that the full awareness equilibrium exists.[17] To see this, note that whenever the announcements of the other principals promote full awareness the own announcement is not payoff relevant. Further, any change in the compensation scheme yields either negative or zero expected profits for the standard Bertrand competition argument. Thus, the full awareness equilibrium exists. In general there can be other equilibria in which the agent is not fully aware. To see this, consider the following example.

Example 6.1 Suppose there are two contingencies, $\Theta = \{\theta_1, \theta_2\}$, of which the agent is aware of one, $\Theta_A = \{\theta_1\}$. Assume that θ_1 is not informative about the agent's action, i.e. $\Pr[\theta_1 = 0|e^L] = \Pr[\theta_1 = 0|e^H]$. There are two possible equilibria, one in which all principals reveal θ_2 and one in which the principals leave the agent unaware. In the latter there is no incentive compatible compensation scheme, so the equilibrium action is e^L. The principals propose a fixed payment $\bar{C}$ for all outcomes within the agent's awareness and pay zero otherwise, Whether the unawareness equilibrium exists or not depends on the distribution of θ_2.

If $\Pr[\theta_2 = 0|e^H] \leq \Pr[\theta_2 = 0|e^L]$, θ_2 is always revealed and the full awareness equilibrium is unique. To see this, suppose $(\theta_1, \bar{C})$ is the equilibrium contract and consider a deviation by principal i, making the agent aware of contingency θ_2 and splitting the second-best surplus between them. The agent updates his awareness and chooses between the equilibrium contract and the deviating contract. If he chooses the equilibrium contract, the agent exerts low

[17] Filiz-Ozbay (2012) derives a similar result in her framework.

effort and obtains payoff $\Pr[\theta_2 = 0|e^L]\upsilon(\bar{C}) - e^L$. Since $\bar{C}$ is feasible but not optimal under symmetric awareness, the deviating principal can find a compensation scheme that is strictly preferred by the agent and yields a positive profit. Thus, the full awareness equilibrium is unique.

On the other hand, if $\Pr[\theta_2 = 0|e^H] > \Pr[\theta_2 = 0|e^L]$, the equilibrium in which all principals offer $(\theta_1, \bar{C})$ and the agent stays unaware may exist. $\bar{C}$ is determined by the zero profit condition of the principals:

$$E[Y|e^L] - \Pr[\theta_2 = 0|e^L]\bar{C} = 0.$$

Note that the zero profit condition implies that $\bar{C}$ is large if the probability that the principals have to pay, $\Pr[\theta_2 = 0|e^L]$, is small. Now suppose principal i deviates by offering a complete contract $(\Theta, \tilde{C})$. The agent chooses between the deviating contract and the equilibrium contract. In contrast to the previous case the agent may find it optimal to adjust his action choice when accepting the incomplete equilibrium contract, because the probability of receiving $\bar{C}$ is increasing in effort. If the agent accepts the equilibrium contract $(\theta_1, \bar{C})$ and exerts high effort, principals offering the equilibrium contract make a loss and the agent obtains payoff

$$\Pr[\theta_2 = 0|e^H]\upsilon(\bar{C}) - e_H.$$

If $\Pr[\theta_2 = 0|e^L]$ is sufficiently small such that $\bar{C}$ is sufficiently large, this payoff exceeds the surplus of the incentive compatible complete contract (Θ, C^*).[18] In this case there is no contract that attracts the agent and yields a positive expected payoff, making the deviation unprofitable. Thus, the unawareness equilibrium exist.[19]

□

The intuition why competition may not necessarily lead to full revelation is that if the

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[18]This is the case if

$$\Pr[\theta_2 = 0|e^H]\upsilon\left(\frac{E[Y|e^L]}{\Pr[\theta_2 = 0|e^L]}\right) \geq E[\upsilon(C^*)|e^H].$$

[19]Going back to the introductory example, suppose there is more than one firm and suppose all firms are aware of their product's effect on consumer health. In the unawareness equilibrium, firms do not reveal possible adverse health effects to the manager. The manager, not taking this possibility into account, exerts too little effort and may be surprised ex-post.

agent's perception of the world is sufficiently distorted, principals can make "generous" offers for outcomes the agent is aware of. Making the agent aware allows the agent to adjust his action choice, exploiting the equilibrium offer in his favor at the cost of the principals. Since this argument does not depend on the numbers of principals, even under intense competition there may exist equilibria in which the agent is unaware and the constrained-efficient action is not implemented.[20]

The possibility of unawareness despite competition in this setting is closely related to the markets with shrouded attributes as described in Gabaix and Laibson (2006). The authors analyze a competitive market setting in which firms offer a product that has hidden add-on prices of which some consumers are unaware.[21] Consumers can avoid paying for the add-on by exerting substitution effort before purchasing the basic good, assumed to be inefficient. Firms choose a price for the basic good and a price for the add-on. There exists an efficient equilibrium in which firms set the price for the add-on low enough such that all consumers, aware or unaware, find it optimal to purchase the add-on instead of exerting substitution effort. However, if the share of unaware types is large enough, there exists another inefficient equilibrium in which the add-on is shrouded and unaware consumers are exploited by the firms. In this equilibrium firms offer a low price for the basic good and charge a high price for the add-on. A consumer that is aware of the add-on optimally exerts substitution effort prior to purchasing the good while an unaware consumer is forced to pay the high price for the add-on. If the unaware consumer is made aware, he can profit from the low price for the basic good and if this price is low enough, the efficient contract cannot attract any consumer. Just as in the competition environment in my setting, there may exist an inefficient equilibrium with unawareness in which the competitive effect is overturned by a "curse of debiasing" (Gabaix and Laibson, 2006): disclosing the features of the inefficient equilibrium contract makes this contract more attractive. Such a curse arises when the contract that takes advantage of the unaware type may be taken advantage of by the aware type.

[20]Note that there may also be equilibria in which the agent is unaware and exerts high effort. In such an equilibrium, upon becoming aware, the agent exploits the principals' equilibrium offer in his favor by exerting low effort.

[21]Gabaix and Laibson (2006) call their consumers myopic and non-myopic instead of aware and unaware, but the spirit is the same.

2.7 Discussion

2.7.1 The Output Function

In the basic model, output is discrete and differs across every state of the world. If y is not one-to-one and contingencies are not observable, both participation loss and incentive gain of revelation are affected. The participation loss is generally diminished, because the revelation of an unforeseen contingency does not necessarily imply the revelation of an unforeseen outcome. The incentive gain is no longer unambiguous, because the agent's perceived distribution of outcome can be more informative about the agent's effort than the actual distribution of outcome.

To see this, suppose there are only two possible realizations of outcome. The project can be either a success or a failure, $\mathcal{Y} = \{s, f\}$ with $s > f$. Whether the project is a success or a failure depends on the realization of Θ. If $\widehat{\Theta} \subset \Theta$ the agent is aware of outcomes $\{s, f\}$ but believes probability distribution $\widehat{\pi}(.|e)$. Now consider the tradeoff the principal faces when disclosing $\theta \notin \Theta_A$. Since the revelation of θ does not reveal any new outcomes, the participation loss of the announcement is zero. The effect on incentives depends on how the revealed contingency affects the perceived distribution of the agent. To see that the incentive effect can be negative, suppose $\Theta = \{\theta_1, \theta_2\}$, $\Theta_A = \{\theta_1\}$ and assume θ_2 is not informative, i.e. $\Pr[\theta_2 = 1|e^H] = \Pr[\theta_2 = 1|e^L]$. In the basic model $\Pr[\theta = 1|e^H] = \Pr[\theta = 1|e^L]$ implies $\Delta C^{\theta}_{\Theta_A} = 0$, because the principal has the choice to ignore the realization of θ_2. Under $\mathcal{Y} = \{s, f\}$ this is no longer the case. Suppose that $y = s$ whenever $\theta_1 = \theta_2 = 0$ and $y = f$ otherwise. Solving for the optimal compensation scheme, the expected payment under $\widehat{\Theta}^* = \Theta_A$ is smaller than the expected payment under $\widehat{\Theta}^* = \Theta$.[22] There is an incentive loss

[22] If the principal leaves the agent unaware, the optimal compensation scheme is

$$C^u(s) = v^{-1}\left(\bar{U} + \frac{\Pr[\theta_1 = 1|e^L]e^H - \Pr[\theta_1 = 1|e^H]e^L}{\Pr[\theta_1 = 1|e^L] - \Pr[\theta_1 = 1|e^H]}\right), C^u(f) = v^{-1}\left(\bar{U} - \frac{\Pr[\theta_1 = 0|e^L]e^H - \Pr[\theta_1 = 0|e^H]e^L}{\Pr[\theta_1 = 1|e^L] - \Pr[\theta_1 = 1|e^H]}\right).$$

If the principal reveals contingency θ_2, the optimal compensation scheme is

$$\begin{aligned} C^a(s) &= v^{-1}\left(\bar{U} + \frac{\left(1 - \Pr[\theta_1 = 0|e^L]\Pr[\theta_2 = 0|e^L]\right)e^H - \left(1 - \Pr[\theta_1 = 0|e^H]\Pr[\theta_2 = 0|e^H]\right)e^L}{\Pr[\theta_1 = 0|e^H]\Pr[\theta_2 = 0|e^H] - \Pr[\theta_1 = 0|e^L]\Pr[\theta_2 = 0|e^L]}\right) \\ C^a(f) &= v^{-1}\left(\bar{U} - \frac{\Pr[\theta_1 = 0|e^L]e^H - \Pr[\theta_1 = 0|e^H]e^L}{\Pr[\theta_1 = 1|e^L] - \Pr[\theta_1 = 1|e^H]}\right). \end{aligned}$$

So $C^u(f) = C^a(f)$ and $v(C^u(s)) - v(C^a(s)) = -\frac{\Pr[\theta_2=1|e^H]}{\Pr[\theta_2=0|e^H]}\frac{e^H - e^L}{\Pr[\theta_1=0|e^H] - \Pr[\theta_1=0|e^L]} < 0$, which implies that $E[C^u(Y)|e^H] < E[C^a(Y)|e^H]$.

of revealing θ_2 because the perceived distribution of the unaware agent is more informative about the action choice than the true distribution.[23]

2.7.2 Optimal Action Choice

Throughout the analysis I assumed that $E[Y|e^H] - E[Y|e^L]$ is large enough such that the principal always finds it optimal to induce e^H. Unawareness makes incentives more costly, hence it is generally possible that for different levels of awareness different levels of effort are optimal. Since effort can only be high or low, the analysis of the optimal contract under low effort is straightforward.

Proposition 2.7.1. *Assume A1 and A2. If* e^L *is the action choice, the optimal contract is* (Θ_A, C^L) *with* $C^L(y) = v^{-1}(\bar{U} - e^L), \forall y \in \widehat{\mathcal{Y}}$ *and* $C^L(y) = 0, \forall y \in \mathcal{Y} \backslash \widehat{\mathcal{Y}}$.

Proof. See Appendix 2.9.7. □

The optimal contract inducing low effort leaves the agent unaware because there is no incentive effect. The principal induces low effort in equilibrium if

$$E[Y|e^L] - \prod_{\theta \in \Theta \backslash \Theta_A} \Pr[\theta = 0|e^L] v^{-1}(\bar{U} - e^L) > E[Y|e^H] - \prod_{\theta \in \Theta \backslash \widehat{\Theta}^*} \Pr[\theta = 0|e^L] E[C^C_{\widehat{\Theta}^*}(Y)|e^H].$$

Whether this is the case or not depends on the distributional properties of the random variable Y, but it is easy to find examples where e^H is the optimal action choice under full awareness and e^L is the optimal action choice under asymmetric awareness.

2.7.3 Heterogeneous Agents

In the basic model it is assumed that the principal knows the agent's awareness Θ_A. Suppose now that awareness is private information and let β denote the principal's prior on the event that the agent is fully aware. Consider first the case of a monopolistic principal. To make things interesting assume that the optimal contract for the unaware type is incomplete. The principal cannot screen the agent's type, because this would require specifying non-zero payments for outcomes the unaware type is unaware of, which consequently makes

[23] When the agent is aware of all outcomes as in the example, the problem is closely related to a framework without unawareness but with heterogeneous priors. A further discussion on heterogeneous priors and moral hazard can be found in Santos-Pinto (2008) and De la Rosa (2011).

him aware.[24] Hence, the principal either offers the optimal incomplete contract, which is rejected by the aware type, or the optimal complete contract, which is accepted by all types. The first option is optimal if the probability that the agent is unaware is large enough. This is in line with von Thadden and Zhao (2012a), who show that in populations with a large extent of unawareness, contracts are incomplete.[25]

Next, suppose that there are N principals competing with each other. The results derived in Section 2.6 extend to this environment. There exists a full awareness equilibrium in which principals reveal all contingencies and make zero profits in expectation. Furthermore, there may exist equilibria in which the unaware type stays unaware, even under intense competition. In such equilibria the aware type is cross-subsidized by the unaware type. To see this consider Example 6.1 but assume that the agent is aware of contingency θ_2 with probability β. Consider an equilibrium in which principals offer an incomplete contract which promises a fixed transfer $\bar{C}$ for all outcomes the unaware type is aware of and zero otherwise. Suppose that $\bar{C}$ is large enough such that the contract is accepted by both types and such that the aware type finds it optimal to exert high effort. The unaware type does not take the zero payments into account and exerts low effort.

Principals do not know whether the agent is aware or unaware and their payoff depends the agent's action choice. Ex-ante, the equilibrium contract yields payoff:

$$\beta\left(E[Y|e^H] - \Pr[\theta_2 = 0|e^H]\bar{C}\right) + (1-\beta)\left(E[Y|e^L] - \Pr[\theta_2 = 0|e^L]\bar{C}\right).$$

The zero-profit condition implies

$$\bar{C} = \frac{\beta E[Y|e^H] + (1-\beta)E[Y|e^L]}{\beta \Pr[\theta_2 = 0|e^H] + (1-\beta)\Pr[\theta_2 = 0|e^L]}.$$

If the share of unaware types is large enough and $\Pr[\theta_2 = 0|e^L]$ is small enough such that $\bar{C}$ is large enough, there is no complete contract that makes the aware type better of and

[24]This assumes that the principal wants to induce high effort for both types. However, it is possible that the principal offers a (menu of) incomplete contract(s) and the two types choose different actions. This can only be optimal if β is small enough.

[25]In von Thadden and Zhao (2012a) the principal can screen the types, which comes at a cost of not incentivizing the aware type optimally, because otherwise he pretends to be unaware. If the share of aware types is small, this cost is small and it is optimal to offer the screening menu that leaves the unaware agent unaware.

yields a non-negative expected payoff.[26] Thus, no principal has incentives to deviate and the unawareness equilibrium exists. Note that condition (2.11) implies that principals make losses on the aware type while they make positive profits on the unaware type. In a market with a population of agents this implies that aware types are cross-subsidized by unaware types, just as in Gabaix and Laibson (2006). Such cross-subsidization between types can occur in equilibrium only if there are sufficiently many unaware agents. Thus, the inefficient equilibrium exists only if the extent of unawareness in the population is large enough.

2.7.4 The Agent as the Residual Claimant

The basic model assumes that the principal is the residual claimant. In the presence of asymmetric awareness, ownership of the project matters, because the agent's valuation of the project depends on his level of awareness. If the agent is the residual claimant, unawareness generally affects his perceived outside option $\sum_{y\in\widehat{\mathcal{Y}}}\widehat{\pi}(y|e^H)\upsilon(y)$. The principal solves:

$$\max_{\widehat{\Theta},C(\cdot)} \quad \sum_{y\in\mathcal{Y}}\pi(y|e^H)[P-C(y)]$$

subject to

$$\sum_{y\in\widehat{\mathcal{Y}}}\widehat{\pi}(y|e^H)\upsilon\left(y+C(y)-P\right) \geq \sum_{y\in\widehat{\mathcal{Y}}}\widehat{\pi}(y|e^H)\upsilon(y)$$

$$e^H \in \arg\max_e \left\{\sum_{y\in\widehat{\mathcal{Y}}}\widehat{\pi}(y|e)\upsilon(y+C(y)-P)-e\right\}$$

$$C(y)\geq 0, \qquad \forall y\in\mathcal{Y},$$

where P is the premium paid by the agent and $C(y)$ is the outcome contingent transfer.[27] As in in the basic model, revealing a contingency $\theta\notin\Theta_A$ involves a tradeoff between participation and incentives. In addition, enlarging the agent's awareness affects his perceived

[26]This is the case if

$$\Pr[\theta_2=0|e^H]\upsilon\left(\frac{\beta E[Y|e^H]+(1-\beta)E[Y|e^L]}{\beta\Pr[\theta_2=0|e^H]+(1-\beta)\Pr[\theta_2=0|e^L]}\right)\geq E[\upsilon(C^*)|e^H]. \tag{2.11}$$

[27]Due to the limited liability constraint the principal would like to scale up both P and C. In order to have a solution, one has to assume that such a contract elicits suspicion on the side of the agent.

outside option. This effect is favorable to the principal if

$$\sum_{y \in \tilde{\mathcal{Y}}} \tilde{\pi}(y|e^H)v(y) < \sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^H)v(y),$$

where $\tilde{\Theta} = \widehat{\Theta} \cup \theta, \theta \in \Theta \backslash \widehat{\Theta}$. This is the case if $E[Y|\theta = 1] < E[Y|\theta = 0]$, i.e. if $\theta = 1$ is a negative outcome shock. Consequently, if the agent is the residual claimant, not only do the distributional properties of $\theta \notin \Theta_A$ matter but also the outcome its announcement reveals. Roughly speaking, when the agent is the residual claimant the principal includes contingencies in the contract that are very unlikely, highly informative and that reveal "bad" outcomes. We can think of this setting as a contract between an insurer and an insuree, where the insuree is partly unaware and effort affects the probability of incurring a loss. The additional participation effect gives the insurer incentives to reveal severe calamities to the insuree, such that the insuree is willing to buy insurance at a higher price.

2.7.5 Unawareness and Zero Probability Beliefs

A frequently raised concern is whether unawareness is observationally equivalent to full awareness with zero probability beliefs (Li, 2008). Epistemically, unawareness has very different properties from zero probability beliefs. An agent is unaware if and only if he assigns probability zero to an event and to its negation (Heifetz et al., 2013). Schipper (2013) shows how this feature also implies behavioral differences between unawareness and zero probability beliefs. However, the results derived in my model can be generated in a framework with full awareness and zero probability beliefs. Under the interpretation of heterogeneous priors, there are some caveats to be taken into account.

In order to derive my results in a framework with full awareness and zero probability beliefs, the agent needs to update a zero probability prior to a non-zero posterior. Note that such updating cannot be interpreted as a consequence of the arrival of new information, since information in the standard state space model expands the set of null states instead of narrowing it. Generating my results in the standard state space framework requires a model that allows for manipulation of beliefs rather than revelation of information. Allowing the principal to manipulate the agent's beliefs without the presence of hard information is rather difficult to motivate. Further, it is important to note that reporting the true distribution is not incentive compatible for the principal if lying is possible. Consequently, there are strong assumptions on the message set available to the principal necessary.[28] Given these caveats, asymmetric awareness seems to be a more natural way to think about this environment and the arising tradeoff.

2.8 Conclusion

This paper incorporates asymmetric awareness in the classical principal-agent model. It shows that the principal makes the agent strategically aware and that the optimal contract can be incomplete. Enlarging the agent's awareness involves a tradeoff between participation and incentives. The cost of disclosing contingencies to the agent is the payment in the states that the agent is initially unaware of. The gain of disclosing contingencies to the agent is the richer information structure that is used to induce incentives. Hence, it is profitable to announce contingencies that have a low probability but are highly correlated with the effort of the agent. If we allow for competition among principals, there exists a symmetric Nash equilibrium in which the agent is fully aware and principals make zero profits. Remarkably, there may exist other equilibria in which the agent stays unaware, even when competition is tight. The existence of incomplete contracts in equilibrium may have important implications for welfare because whenever the agent is left unaware, the principal uses an inefficient information structure to induce incentives.

In the proposed model, the principal is able to implement zero payments whenever there is an event the agent is initially unaware of. This may not be feasible in real-life contracting situations. If, for example, the compensation scheme is restricted to be monotone in outcome, it is most costly to disclose low outcomes to the contracting partner. Hence, the magnitude of the participation effect depends on the revealed outcome. Restricting the set of feasible contracts adds interesting features to the optimal compensation scheme and revelation strategy, but as long as the contracting partner with superior awareness is able to profit from the other's limited understanding of the underlying uncertainties the basic tradeoff prevails.

An open question is how the results of the basic model change in a repeated game setting. Whenever a contingency outside the agent's awareness is realized, the agent observes an

[28] One may argue that also in the framework with unawareness an implicit assumption is that the principal can only reveal "true" contingencies. One could imagine a case, where the principal can include virtual events, but since I assume that the agent, once aware, completely understands all consequences and the probability distribution of a contingency, the restriction to "true" contingencies seems to be natural.

outcome considered impossible and consequently becomes aware of his initial unawareness. This raises the question on how such a discovery affects the agent's updated understanding of the world, i.e whether the agent passively updates his state space or whether he understands that there may be other events he is unaware of. To answer these questions, it is necessary to enter the debate on decision making under awareness of unawareness, a largely unexplored field in the literature. It seems plausible that also in repeated games asymmetric awareness has interesting implications, for example on turnover rates and its indirect effects on incentives, and thus poses interesting challenges for future research.

2.9 Appendix

2.9.1 Proof of Proposition 2.4.1

Suppose $\Theta_A \subset \widehat{\Theta}^*$. The optimal compensation scheme is $\widehat{C}^*(y) = \upsilon^{-1}(\bar{U} + e^H)$ for all $y \in \widehat{\mathcal{Y}}$ and $\widehat{C}^*(y) = 0$ for all $y \in \mathcal{Y} \backslash \widehat{\mathcal{Y}}$. Then the expected payment is

$$\prod_{\theta \in \Theta \backslash \widehat{\Theta}^*} \Pr[\theta = 0 | e^H] \upsilon^{-1}(\bar{U} + e^H),$$

which is clearly greater than $\prod_{\theta \in \Theta \backslash \Theta_A} \Pr[\theta = 0|e^H]\upsilon^{-1}(\bar{U} + e^H)$, the expected payment under $\widehat{\Theta}^* = \Theta_A$, due to the assumption $\pi(y|e) > 0, \forall y \in \mathcal{Y}$. Hence $\widehat{\Theta}^*$ cannot be optimal. □

2.9.2 Proof of Lemma 2.5.1

Suppose $\lambda = 0$. Since $\sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^H) = \sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^L) = 1$ and $\widehat{\pi}(.|e^H) \neq \widehat{\pi}(.|e^L)$ there must exist some $y \in \widehat{\mathcal{Y}}$ such that $\widehat{\pi}(y|e^H) - \widehat{\pi}(y|e^L) < 0$. But since $\gamma \geq 0$, $\lambda = 0$ would imply that $\frac{1}{\upsilon'(C(y))} \leq 0$ for some $y \in \widehat{\mathcal{Y}}$, which violates the assumption $\upsilon'(\cdot) > 0$. Hence $\lambda > 0$.

Now suppose $\gamma = 0$. Then, the first-order conditions of the optimization problem imply that compensation is fixed across outcomes within the agent's awareness. But this implies that the incentive constraint is no longer satisfied. Hence, $\gamma > 0$. □

2.9.3 Proof of Lemma 2.5.2

Let $\tilde{S}$ denote the state space, let $\tilde{y}$ denote the output function with range $\tilde{\mathcal{Y}}$ and let $\tilde{\pi}(.|e)$ denote the probability belief given awareness $\widehat{\Theta} \cup Z$. Further, let $\rho : \tilde{\mathcal{Y}} \longrightarrow \widehat{\mathcal{Y}}$ be the mapping from set $\tilde{\mathcal{Y}}$ to set $\widehat{\mathcal{Y}}$, where $\rho(\tilde{y}(\tilde{s})) = \widehat{y}(\widehat{s})$ and $\widehat{s} \in \widehat{S}$ is a subsequence of $\tilde{s} \in \tilde{S}$. Now consider the compensation scheme $\tilde{C}$ with $\tilde{C}(y) = C^C_{\widehat{\Theta}}(\rho(y))$ for all $y \in \tilde{\mathcal{Y}}$. Note that $\tilde{C}$ satisfies both participation and incentive constraint with equality.

Suppose $\Pr[\theta = 1|e^H] = \Pr[\theta = 1|e^L], \forall \theta \in Z$. Then, for any $y \in \tilde{\mathcal{Y}}$ we have

$$\frac{\tilde{\pi}(y|e^L)}{\tilde{\pi}(y|e^H)} = \frac{\widehat{\pi}(y|e^L)}{\widehat{\pi}(y|e^H)}.$$

Hence, $C^C_{\widehat{\Theta}}$ satisfies the first order conditions and consequently solves the optimization problem. $\Delta C^Z_{\widehat{\Theta}} = 0$.

Now, suppose $\Pr[\theta = 1|e^H] \neq \Pr[\theta = 1|e^L]$ for some $\theta \in Z$. Then, there must exist some $y, y' \in \rho^{-1}(y), y \in \widehat{\mathcal{Y}}$ such that

$$\frac{\tilde{\pi}(y|e^L)}{\tilde{\pi}(y|e^H)} \neq \frac{\tilde{\pi}(y'|e^L)}{\tilde{\pi}(y'|e^H)}.$$

Consequently, $C^C_{\widehat{\Theta}}$ does not satisfy the first-order conditions. Hence, $C^C_{\widehat{\Theta}}$ is feasible but not optimal, which implies that $E[C^C_{\widehat{\Theta}\cup Z}(Y|e^H)] < E[C^C_{\widehat{\Theta}}(Y|e^H)]$ and $\Delta C^Z_{\widehat{\Theta}} > 0$. □

2.9.4 Proof of Lemma 2.5.3

To show that $\Delta C^\theta_{\Theta_A} = E\left[C^C_{\Theta_A}(Y)|e^H\right] - E\left[C^C_{\Theta_A\cup\theta}(Y)|e^H\right]$ is monotonically increasing in Δ^θ it is sufficient to show that $E\left[C^C_{\Theta_A\cup\theta}(Y)|e^H\right]$ is monotonically decreasing in Δ^θ. W.l.o.g. we can assume $\Pr[\theta = 1|e^H] > \Pr[\theta = 1|e^L]$. Let $E_{\Theta_A\cup\theta}$ denote the expectation operator with respect to awareness $\Theta_A \cup \theta$. Then we have:

$$E_{\Theta_A\cup\theta}\left[v(C^C_{\Theta_A\cup\theta}(Y))|e, \theta = 1\right] > E_{\Theta_A\cup\theta}\left[v(C^C_{\Theta_A\cup\theta}(Y))|e, \theta = 0\right], \quad e = e^L, e^H,$$

which follows directly from the first-order conditions. The incentive constraint can be rewritten as

$$\begin{aligned} & \Pr[\theta = 1|e^H]E_{\Theta_A\cup\theta}\left[v(C^C_{\Theta_A\cup\theta}(Y))|e^H, \theta = 1\right] + \Pr[\theta = 0|e^H]E_{\Theta_A\cup\theta}\left[v(C^C_{\Theta_A\cup\theta}(Y))|e^H, \theta = 0\right] - e^H \\ = \; & \Pr[\theta = 1|e^L]E_{\Theta_A\cup\theta}\left[v(C^C_{\Theta_A\cup\theta}(Y))|e^L, \theta = 1\right] + \Pr[\theta = 0|e^L]E_{\Theta_A\cup\theta}\left[v(C^C_{\Theta_A\cup\theta}(Y))|e^L, \theta = 0\right] - e^L. \end{aligned}$$

Now, consider probability $\Pr[\theta = 1|e^L] - \varepsilon$ with $\varepsilon > 0$. Under $C^C_{\Theta_A\cup\theta}$ and $\Pr[\theta = 1|e^L] - \varepsilon$ the participation constraint is clearly satisfied with equality. Looking at the incentive constraint it is easy to see that

$$\begin{aligned} & \Pr[\theta=1|e^H]E_{\Theta_A\cup\theta}\left[\upsilon(C^C_{\Theta_A\cup\theta}(Y))|e^H,\theta=1\right]+\Pr[\theta=0|e^H]E_{\Theta_A\cup\theta}\left[\upsilon(C^C_{\Theta_A\cup\theta}(Y))|e^H,\theta=0\right]-e^H \\ > \quad & \Pr[\theta=1|e^L]E_{\Theta_A\cup\theta}\left[\upsilon(C^C_{\Theta_A\cup\theta}(Y))|e^L,\theta=1\right]+\Pr[\theta=0|e^L]E_{\Theta_A\cup\theta}\left[\upsilon(C^C_{\Theta_A\cup\theta}(Y))|e^L,\theta=0\right]-e^L \\ & -\varepsilon\left(E_{\Theta_A\cup\theta}\left[\upsilon(C^C_{\Theta_A\cup\theta}(Y))|e^L,\theta=1\right]-E_{\Theta_A\cup\theta}\left[\upsilon(C^C_{\Theta_A\cup\theta}(Y))|e^L,\theta=0\right]\right). \end{aligned}$$

We know that under the optimal transfer rule both constraints are satisfied with equality. Consequently, given $\Pr[\theta=1|e^L]-\varepsilon$, $C^C_{\Theta_A\cup\theta}$ is feasible but not optimal. The same line of reasoning applies to $\Pr[\theta=1|e^H]$, in which case both constraints are slack. Thus, the expected payment given information structure $\Theta_A\cup\theta$ is decreasing in Δ^θ. □

2.9.5 Proof of Proposition 2.5.5

It is clear that whenever $E_{\widehat{\Theta}^*}[Y-C_{\widehat{\Theta}^*}(Y)|e^H]<0$, the agent thinks that the principal would be strictly better off by not offering the contract. Similarly if $E_{\widehat{\Theta}^*}[Y-C_{\widehat{\Theta}^*}(Y)|e^H]<E_{\widehat{\Theta}^*}[Y|e^L]-\bar{C}$ the agent cannot rationalize why the principal proposes an incentive compatible contract. Hence, $(\widehat{\Theta}^*,\widehat{C}^*)$ cannot be justifiable.

Now suppose $E_{\widehat{\Theta}^*}[Y-C_{\widehat{\Theta}^*}(Y)|e^H]\geq 0$ and $E_{\widehat{\Theta}^*}[Y-C_{\widehat{\Theta}^*}(Y)|e^H]\geq E_{\widehat{\Theta}^*}[Y|e^L]-\bar{C}$.

Justifiability of $\widehat{C}^*$: If $\widehat{\Theta}^*=\Theta$, principal and agent share the same beliefs. Hence, $\widehat{C}^*$ maximizes the principal's expected payoff from the agent's perspective. If $\widehat{\Theta}^*\neq\Theta$, $\widehat{C}^*(y)=C^C_{\widehat{\Theta}^*}(y)$ for all $y\in\widehat{\mathcal{Y}}$. Since the agent thinks that the contract is complete, $\widehat{C}^*$ maximizes the principal's payoff according to the beliefs of the agent.

Justifiability of $\widehat{\Theta}^*$: $\widehat{\Theta}^*$ is optimal for the principal given the agent's beliefs if

$$E\left[C^C_{\widehat{\Theta}^*}(Y|e^H)\right]\leq\prod_{\theta\in\widehat{\Theta}^*\backslash Z}\Pr[\theta=0|e^H]E\left[C^C_{\widehat{\Theta}^*\backslash Z}(Y)|e^H\right],$$

for any $Z\subseteq\widehat{\Theta}^*\backslash\Theta_A$. This coincides with the optimality condition of the principal. Hence, whenever $E_{\widehat{\Theta}^*}[Y-C_{\widehat{\Theta}^*}(Y)|e^H]\geq 0$ and $E_{\widehat{\Theta}^*}[Y-C_{\widehat{\Theta}^*}(Y)|e^H]\geq E_{\widehat{\Theta}^*}[Y|e^L]-\bar{C}$, $(\widehat{\Theta}^*,\widehat{C}^*)$ is justifiable. □

2.9.6 Proof of Proposition 2.6.1

In the proposed equilibrium principals make zero profits and the agent obtains the positive second-best surplus. Consider a deviation of principal i. The strategies are $\widehat{\Theta}_j = \Theta$ and $C_j = C^*$ for all $j = 1, ..., i-1, i+1, ..., N$. A deviation in $\widehat{\Theta}_i$ leaves the expected payoff unaffected because the agent is aware of $\widehat{\Theta}_1 \cup ... \cup \widehat{\Theta}_N = \Theta$ for all $\widehat{\Theta}_i \subseteq \Theta$. A deviation in C_i is not profitable for the standard Bertrand argument. C^* maximizes the agent's expected utility subject to the zero profit constraint and the incentive constraint. A deviation $C_i \neq C^*$ such that the incentive constraint is satisfied must make either the agent or the principal worse off. If the agent is worse off, he rejects the contract and the expected payoff is zero. If the principal is worse off, he has a negative expected payoff. A deviation in $C_i \neq C^*$ such that the incentive constraint is not satisfied makes either the agent worse off or the principal worse off or both by Assumption 3. Hence, there is no profitable deviation and (Θ, C^*) is an equilibrium. □

2.9.7 Proof of Proposition 2.7.1

When e^L is optimal, the principal solves

$$\max_{\widehat{\Theta}, C(\cdot)} \quad \sum_{y \in \mathcal{Y}} \pi(y|e^L)\,[y - C(y)]$$

subject to

$$\sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^L) v\left(C\left(y\right)\right) - e^L \geq \overline{U}$$

$$C(y) \geq 0, \quad \forall y \in \mathcal{Y}.$$

The optimal compensation scheme for a given $\widehat{\Theta}$ is $C^L(y) = v^{-1}(\bar{U} - e^L), \forall y \in \widehat{\mathcal{Y}}$ and $C^L(y) = 0, \forall y \in \mathcal{Y} \backslash \widehat{\mathcal{Y}}$. The expected payment is

$$E[C^L(Y)] = \prod_{\theta \in \Theta \backslash \widehat{\Theta}} \Pr[\theta = 0|e^L] v^{-1}(\bar{U} - e^L),$$

which is clearly minimized for $\widehat{\Theta} = \Theta_A$. □

2.10 Bibliography

Benabou, R., Tirole, J., 2003. Intrinsic and Extrinsic Motivation. Review of Economic Studies, 70 (3), 489-520.

Board, O.J., Chung, K.S., 2011. Object-Based Unawareness: Theory and Applications. Mimeo.

Chen, Y.J., Zhao, X., 2009. Contractual traps. Proceedings of the 12th Conference on Theoretical Aspects of Rationality and Knowledge, 51-60.

Dekel, E., Lipman, B., Rustichini, A., 1998. Standard State-Space Models Preclude Unawareness. Econometrica, 66(1), 159-173.

Filiz-Ozbay, E., 2012. Incorporating Unawareness into Contract Theory. Games and Economic Behavior, 76(1), 181-194.

Gabaix, X., Laibson, D., 2006. Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets. Quarterly Journal of Economics, 121(2), 505-540.

Galanis, S., 2013. Unawareness of Theorems. Economic Theory, 52(1), 41-73.

Grossman, S., Hart, O., 1983. An Analysis of the Principal-Agent Problem. Econometrica, 51(1), 7-45.

Heifetz, A., Meier, M., Schipper, B.C., 2006. Interactive Unawareness. Journal of Economic Theory, 130(1), 78-94.

Heifetz, A., Meier, M., Schipper, B.C., 2008. A Canonical Model for Interactive Unawareness. Games and Economic Behavior, 62(1), 304-324

Heifetz, A., Meier, M., Schipper, B.C., 2013. Unawareness, Beliefs and Speculative Trade. Games and Economic Behavior , 77(1), 100-121.

Holmström, B., 1979. Moral Hazard and Observability. Bell Journal of Economics, 10(1), 74-91.

Karni, E., Vierø, M.-L., 2012. "Reverse Bayesianism": A Choice-Based Theory of Growing Awareness. Mimeo.

Li, J., 2008. A Note on Unawareness and Zero Probability. Mimeo.

Li, J., 2009. Information Structures with Unawareness. Journal of Economic Theory, 144(3), 977-993.

Modica, S., Rustichini, A., 1994. Awareness and Partitional Information Structures, Theory and Decision, 37(1), 107-124.

Modica, S., Rustichini, A., 1999. Unawareness and Partitional Information Structures, Games and Economic Behavior, 27(2), 265-298.

Modica, S., Rustichini, A., Tallon, J.M., 1998. Unawareness and Bankruptcy: A General Equilibrium Model. Economic Theory, 12(2), 259-292.

Ozbay, E., 2008. Unawareness and Strategic Announcements in Games with Uncertainty. Mimeo.

De la Rosa, L.E., 2011. Overconfidence and Moral Hazard. Games and Economic Behavior, 73(2), 429-451.

Santos-Pinto, L., 2008. Positive Self-Image and Incentives in Organizations. Economic Journal, 118(531), 1315-1332.

Schipper, B.C., 2013. Awareness-Dependent Subjective Expected Utility. International Journal of Game Theory, 42(3), 725753.

Von Thadden, E.L., Zhao, X.J., 2012a. Incentives for Unaware Agents. Review of Economic Studies, 79(3), 1151-1174.

Von Thadden, E.L., Zhao, X.J., 2012b. Multi-Task Agency with Unawareness. Mimeo.

Zhao, X.J., 2008. Moral Hazard with Unawareness. Rationality and Society, 20(4), 471-496.