



Credit Market Failure and Macroeconomics

Joachim Jungherr

Thesis submitted for assessment with a view to obtaining the degree
of Doctor of Economics of the European University Institute

Florence, November 2013

European University Institute
Department of Economics

Credit Market Failure and Macroeconomics

Joachim Jungherr

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Examining Board

Prof. Árpád Ábrahám, European University Institute (Supervisor)

Prof. Hugo A. Hopenhayn, UCLA

Prof. Ramon Marimon, European University Institute

Prof. Vincenzo Quadrini, University of Southern California

© Joachim Jungherr, 2013

No part of this thesis may be copied, reproduced or transmitted without prior permission of the author

Abstract

This thesis aims to contribute to our understanding of the relationship between market failure on capital markets and macroeconomic outcomes in various forms. The notion of credit markets as a frictionsless veil over real economic activity has proven to be unfruitful with respect to many questions of economic interest. To name only a few examples, in the absence of financial frictions there is no difference between internal and external financing, no trade-off between equity and debt, and there is no reason for banks to exist. In order to correctly identify and address the policy needs which might arise from credit market failure, we need to learn more about the fundamental conditions which give rise to the financial contracts and institutions observed in reality.

The first chapter of this thesis focuses on the phenomenon of the publicly traded firm with its separation of ownership and control. I show how a time-varying misalignment of incentives of firm managers and investors can have important consequences for aggregate business fluctuations. In particular, a rise in idiosyncratic firm-level uncertainty may result in an economy-wide increase in the default rate on corporate bonds together with a drop in measured firm productivity and output.

Bank transparency is the topic of the second chapter. In this model, banks are special because the product they are selling is superior information about investment opportunities. Intransparent balance sheets turn this public good into a marketable private commodity. In the absence of policy intervention, bank competition results in complete bank opacity and a high degree of aggregate uncertainty for households. Mandatory disclosure rules can improve upon the market outcome.

The third chapter is joint work with David Strauss. It focuses on the consequences of credit market failure for development and growth. We show that capital market imperfections may give rise to a poverty trap associated with permanent productivity differences across countries. Key to this phenomenon is a sorting reversal in the matching between human capital and heterogeneous production sectors.

Acknowledgements

This PhD thesis is the product of four rich and exciting years during which I learned more than I had reason to hope for at the outset. Push the envelope. Watch it bend.

The guidance and ever-encouraging support from my adviser Árpád Ábrahám were invaluable for me. Especially at times when both of us were unsure what I was up to. I would like to think that I got contaminated by his enthusiasm for economics. I would also like to thank Vincenzo Quadrini for making my stay at the University of Southern California possible. I benefited a lot from our discussions and I took many memories home with me from this journey to the New World. Ramon Marimon was kind enough to join the team when it was already late in the game. Many thanks for that.

During three years of my PhD studies, I was supported by a scholarship from the German Academic Exchange Service (DAAD), which also contributed to my research visit at the University of Southern California in Los Angeles.

The joint part of this thesis owes much to my co-author David Strauss. Generally, I consider myself as very lucky having spent these four years at the European University Institute. The lively and inspiring atmosphere in Villa San Paolo and the solidarity among that bunch of would-be economists it is packed with are probably unique and certainly a gift which I enjoyed a lot.

Without the superb work of the administrative staff and the many other non-academic members of the EUI, the whole Institute would have fallen apart a long time ago. Therefore, I need to thank Jessica, Lucia, Marcia, Giuseppe, Loredana, Sonia and everybody else who keeps this social engine running day by day.

But I would not have been able to fully appreciate the academic amenities of the Doctoral Programme without the time spent with the IUE Calcio team on rain-soaked mud pitches all over florentine suburbia. I have to express my gratitude to the Squadra Fantastica for uncountable memorable moments and a life beyond my desk in VSP.

Along the journey of these last four years, I had the privilege to share the road with many persons, some of which became very close friends along the way. Every single one of them has made this experience richer. I would not have wanted to undertake this journey alone. My deepest gratitude to all of them.

My family both in Germany as well as in Florence has given me the strength and the confidence which I could not have obtained anywhere else.

Barcelona, September 23rd, 2013

Meinen Eltern

Contents

Preface	vii
Chapter 1 Capital Structure, Uncertainty, and Macroeconomic Fluctuations	1
1.1 Introduction	1
1.2 Model Setup	10
1.2.1 Firms	11
1.2.2 Managers	11
1.2.3 Households	12
1.2.4 Timing	12
1.3 Perfect Information	13
1.3.1 Households	13
1.3.2 Optimal Contract	14
1.3.3 Characterization	16
1.4 Financial Frictions	17
1.4.1 Optimal Contract	18
1.4.2 Capital Structure	27
1.4.3 Households	30
1.4.4 Characterization	31
1.5 Quantitative Analysis	32
1.5.1 Parametrization	33
1.5.2 Optimal Contract	36
1.5.3 Uncertainty and Financial Markets	38
1.5.4 Business Cycle Analysis	40
1.6 Discussion	45
Appendix A Proofs and Derivations	47
Appendix B Model Solution and Simulation	53

Chapter 2 Bank Opacity and Endogenous Uncertainty	61
2.1 Introduction	61
2.2 Model Setup	67
2.2.1 Households	67
2.2.2 Banks	67
2.2.3 Projects and Information	68
2.2.4 Timing	69
2.3 Equilibrium	69
2.3.1 Households	70
2.3.2 Banks: Exogenous Transparency	71
2.3.3 Banks: Endogenous Transparency	74
2.4 Optimal Opacity	76
2.5 Discussion	78
Appendix A Proofs and Derivations	80
Chapter 3 Why Does Misallocation Persist?	91
3.1 Introduction	91
3.2 Model Setup	98
3.2.1 Agents	98
3.2.2 Production	98
3.2.3 Final Good	99
3.2.4 Labor Market	100
3.2.5 Timing	100
3.3 Efficient Allocation	100
3.3.1 Production	101
3.3.2 Sorting	102
3.3.3 Final Good	106
3.3.4 Intermediate Goods	107
3.3.5 Labor Market	107
3.3.6 Dynamics	108
3.4 Misallocation	108
3.4.1 Production	109
3.4.2 Sorting	109
3.4.3 Dynamics	111
3.5 Discussion	114

Preface

This thesis aims to contribute to our understanding of the relationship between market failure on capital markets and macroeconomic outcomes. The formal analysis of financial frictions in its current form has its roots in the literature on private information and limited commitment. These endeavors date back until the 1970s, but the Financial Crisis of 2007-2008 and the Great Recession have certainly added further momentum to the progress of this research agenda. The common understanding underlying the current discussion is that the notion of smoothly functioning credit markets has proven to be unfruitful with respect to many questions of economic interest. To name only a few examples, in the absence of financial frictions there is no difference between internal and external financing, no trade-off between equity and debt, and there is no reason for banks to exist. In order to correctly identify and address the policy needs which might arise from credit market failure, we need to learn more about the fundamental conditions which give rise to the financial contracts and institutions observed in reality. Capital market imperfections can arise for many different reasons and can take on very different forms, each of which potentially yields different policy recommendations. In my view, the literature has only begun to explore these possibilities and their theoretical and practical implications both for the business cycle and economic growth.

One example of a financial institution which cannot be understood without the analysis of financial frictions is the phenomenon of the publicly traded firm with its separation of ownership and control. The recent Financial Crisis did not hit only privately held firms which rely on bank lending as the principal source of financing, but also firms with access to capital markets were considerably affected by adverse credit conditions during the crisis. Credit spreads and default rates soared on corporate bond markets. At the same time, companies were exposed to a particularly sharp rise in sales and growth volatility, while measured total factor productivity (TFP) experienced the sharpest downturn of the post-war era. In the first chapter of this thesis, I employ an optimal contract approach to security design and capital structure to show how an increase in firm-level uncertainty can result in

a rise of the default rate on corporate bonds together with a drop in firm productivity and output. Key to the analysis is a misalignment of the incentives of firm management and investors. Within a dynamic general equilibrium model, I study the impact of exogenous variations in firm-level uncertainty on real and financial aggregates. Uncertainty shocks of plausible size typically cause a recession featuring a rise in default rates and a deleveraging of the corporate sector. An important driver of the business cycle in this model are fluctuations in the Solow residual which are not caused by technology shocks, but by the time-varying severity of agency problems.

Bank transparency is the topic of the second chapter of this thesis. What is special about banks that makes them more opaque than non-financial firms? What exactly are the externalities which give rise to a need for policy intervention? And what is the optimal level of bank transparency? In this model, banks are special because the product they are selling is superior information about investment opportunities. Intransparent balance sheets turn this public good into a marketable private commodity. Voluntary public disclosure of information translates into a competitive disadvantage. Bank competition results in a “race to the bottom” which leads to complete bank opacity and a high degree of aggregate uncertainty for households. Households do value public information as it reduces aggregate uncertainty, but the market does not punish intransparent banks. Policy measures can improve upon the market outcome by imposing minimum disclosure requirements on banks. However, complete disclosure is socially undesirable as this eliminates all private incentives for banks to acquire costly information. The social planner chooses optimal bank transparency by trading off the benefits of reducing aggregate uncertainty for households against banks’ incentives for costly information acquisition.

The third chapter of this thesis is joint work with David Strauss. It focuses on the long term consequences of credit market failure for development and growth. Total factor productivity (TFP) accounts for the major part of cross-country differences in per capita income. Factor misallocation can potentially explain large TFP losses. However, existing models of factor misallocation through credit market frictions fail to robustly generate large effects on TFP in the long run. We propose a new mechanism to show how capital market imperfections may indeed give rise to a permanent misallocation of production factors within a given country and permanent differences in measured TFP across countries. In the presence of binding credit constraints, the assignment of human capital to production sectors is completely reversed with respect to the case of efficient capital markets. Factor misallocation may be permanent because of the possibility of a collective poverty trap which arises for low levels of financial development. Depending on initial conditions, a country converges over

time to one of two different stable steady states characterized by different long-run levels of output, capital, wages, and measured TFP. Manufacturing goods are relatively cheaper in the high-income steady state compared to the low-income equilibrium, while the average firm size and its variance are higher.

Chapter 1

Capital Structure, Uncertainty, and Macroeconomic Fluctuations

1.1 Introduction

Capital market imperfections have been identified as a major determinant of the origin and the severity of the Great Recession which was triggered by the Financial Crisis of 2007-2008. The common narrative attributes a central role to borrowing constraints of privately held firms which rely on bank loans as the principal source of financing.¹ But also firms with access to capital markets were considerably affected by adverse credit conditions during the crisis. The default rate on corporate bonds reached its second highest level of the post-war period in 2009.² Corporate bond spreads almost tripled between 2007 and 2009 (Adrian, Colla and Shin, 2013).³ At the same time, companies were exposed to a particularly sharp rise in sales and growth volatility⁴, while total factor productivity (TFP) experienced the sharpest downturn of the post-war era (Fernald, 2012). This paper employs an optimal contract approach to security design and capital structure to show how an increase in the severity of financial frictions can result in a rise of the default rate on corporate bonds together with a drop in firm profitability, measured TFP, and firm output. Key to the analysis is a misalignment of the incentives of firm management and investors. Time-varying firm-level

¹See Bernanke (2010), Duygan-Bump, Levkov and Montoriol-Garriga (2011), or Shourideh and Zetlin-Jones (2012)

²This data is provided by Giesecke, Longstaff, Schaefer and Strebulaev (2013). During the 2001 U.S. recession, the default rate on corporate bonds was slightly higher than in 2009.

³Likewise, Gilchrist and Zakrajšek (2012) document a dramatic rise in credit spreads on corporate bond markets during the crisis.

⁴See Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry (2012), or Schaal (2012).

uncertainty determines the severity of this agency problem.

This paper explains the emergence of debt and equity securities as efficiently designed instruments to implement an optimal contract between investors and firm managers in an environment subject to asymmetric information. Taking seriously the optimal design and usage of financing instruments at the firm level has two main advantages. First, we can be confident to understand the most important characteristics of the economic environment to the extent that we are able to rationalize the financial contracts observed in reality. Standard macroeconomic models do not perform too well in this respect, as they struggle to rationalize common forms of firm financing such as the prevalent use of a certain combination of equity and bond securities by publicly traded companies. Secondly, the analysis developed below sheds light on the role of firm-level risk in determining the severity of the agency problem between investors and firm managers which gives rise to a non-trivial capital structure choice between equity and debt. In times of high volatility at the firm level, firms need to reduce their leverage in order to avoid a rise in default risk. However, reduced leverage gives firm managers more discretion in pursuing non-profit-maximizing firm policies. The optimal adjustment of the firm's capital structure trades off these two effects, which generally results in some combination of higher default risk, lower firm productivity, and lower firm output.

The financing choice of the individual firm is embedded in a dynamic general equilibrium model subject to exogenous variations in firm-level uncertainty of plausible size. This model features a number of empirical business cycle facts which standard macro models fail to generate. Namely, the model replicates the countercyclical behavior of firm-level uncertainty and corporate bond default rates. In addition, this model economy features fluctuations in the Solow residual which are not caused by technology shocks, but by the time-varying severity of agency conflicts between investors and firm managers. This is an important finding, as Chari, Kehoe and McGrattan (2007) show that variations of total factor productivity (the "efficiency wedge") are a major determinant of the U.S. business cycle.⁵

Preview of the Model

In order to study the financing decision of firms, I build on a standard agency theory of the firm along the lines of Ross (1973) and others. In many situations, the incentives of firm managers and investors are not aligned. Firm managers decide on how hard to work

⁵Chari, Kehoe and McGrattan (2007) conclude their article stating:

"The challenging task is to develop detailed models in which primitive shocks lead to fluctuations in efficiency wedges [...]."

as well as on how hard they try to make their employees work. They choose how much firm resources to spend on the pursuit of managerial benefits such as an overly pleasant work environment or favoring friends in contracting relationships with the firm (Jensen and Meckling, 1976). These incentive problems may even go so far as to affect the selection of large-scale investment projects by choosing firm growth over firm profitability (Jensen, 1986). The underlying reason for this problem is asymmetric information between firm managers and outside investors. Firm managers have the information required to take the right action on behalf of investors but they may not have the appropriate incentives to do so.

This agency problem is modeled in a simple way. Investors want managers to exert costly effort on their behalf. Firm managers observe the productivity state of the firm before they choose the level of effort. Outside investors observe realized firm output but can neither assess the true level of effort provided by the manager nor the stochastic productivity level of the firm. Performance pay is one way for investors to provide incentives for managers. This classical principal-agent setup is augmented with a monitoring technology as introduced by Townsend (1979). Depending on the level of firm productivity announced by the manager, investors can choose to pay for a thorough assessment of the company's true productivity state which allows for a richer set of financial contracts.

Results

The resulting optimal contract lends itself to a straightforward interpretation as a unique combination of equity and debt financing. Optimally, only low realizations of firm productivity are monitored. By identifying the event of monitoring as bankruptcy proceedings, the cash flows to investors can be separated into distinct payment streams to creditors and shareholders. Debt is a fixed claim which triggers monitoring (bankruptcy) in case of default. This notion of bankruptcy as a costly device for outsiders to acquire firm-specific information dates back to Townsend (1979) and Gale and Hellwig (1985). According to this idea, an important feature of bankruptcy proceedings is a transfer of firm-specific information from insiders to outsiders. Creditors of a firm in default pay accountants and trustees to assess the true value of the firm's assets in place in order to recover as much of the face value of debt as possible.⁶ This option to verify the firm manager's announcements reduces agency costs not only in case of actual bankruptcy, but also in all non-bankruptcy states. Conse-

⁶This resembles most closely the process of liquidation of a firm by a trustee according to Chapter 7 of the U.S. Bankruptcy Code. However, also Chapter 11 reorganizations put the debtor under scrutiny by creditors. Bris, Welch and Zhu (2006) estimate that the direct expenses related to Chapter 7 liquidations and Chapter 11 reorganizations are of similar size.

quently, by issuing non-contingent debt securities firms can limit the freedom of managers to deviate from the profit-maximizing production plan. The downside of leverage consists of an elevated risk to incur the costs of bankruptcy. Equity holders are the residual claimants of the firm. Accordingly, they receive a positive dividend after all debt and wage obligations are satisfied.

The result that in this model firms optimally rely on a certain combination of equity and debt instruments to finance investment is important, because it is in line with the design and usage of securities issued by firms in practice. Fama and French (2005) report that 26% of the total asset growth of U.S. listed firms between 1993 and 2002 were financed by net equity issuance, while the growth of total liabilities accounts for 68%.⁷ Stock measures of financing sources convey a similar message. According to Fama and French (2005), the total liabilities of their firm sample account for about two thirds of the aggregate book value of assets, while shareholders' equity sums up to about one third of the aggregate balance sheet.⁸

In the model economy, corporate capital structure is determined as a trade-off between agency costs and the risk of costly bankruptcy. Issuing debt restricts the freedom of firm managers to deviate from profit-maximizing firm policies. This benefit of debt comes at the expense of an increased risk of costly bankruptcy. Up to this point, this model of firm financing is very much in line with the extensive literature on corporate capital structure.^{9,10} What is new in this analysis is the central role of uncertainty about idiosyncratic, firm-specific characteristics. Whenever the business environment of a given firm is particularly volatile, firm performance becomes hard to predict. This gives much room for discretion to the firm's management and exacerbates the agency problem in question. The default risk increases as the optimal monitoring frequency grows in an attempt to put tighter controls on firm management. At the same time, the expected levels of firm output, measured productivity, and return on investment implied by the optimal contract fall relative to their perfect information counterparts.

⁷The remaining 6% of total firm asset growth are financed by retained earnings. This information is based on Table 2 in Fama and French (2005) using dSB as the measure of net equity issuance.

⁸While publicly traded firms are only a trivial fraction of all firms in the U.S., they account for more than 25% of total employment (Davis, Haltiwanger, Jarmin and Miranda, 2007). Shourideh and Zetlin-Jones (2012) calculate that roughly 60% of corporate gross output is produced by publicly traded firms.

⁹Typically, trade-off theories of capital structure also consider interest tax shields as an additional benefit of debt financing. See also the discussion at the end of the paper in Section 1.6.

¹⁰This model of optimal corporate capital structure is also backed up by empirical evidence on the economic significance both of agency and bankruptcy costs. Morellec, Nikolov and Schürhoff (2012) and Nikolov and Schmid (2012) find that agency costs in the manager-shareholder relationship are an important determinant of the capital structure choice of publicly traded firms. Bris, Welch and Zhu (2006) find bankruptcy fees to be increasing in firm size and report an empirical magnitude of about 10% of firm asset value. Depending on firm characteristics, their estimate varies between 0% and 20%.

Various measures of firm-level uncertainty have been documented to move cyclically over time. In particular, Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry (2012) find that for a given firm at time t the volatilities of shocks to total factor productivity, daily stock returns, and sales growth are all positively correlated among each other. Apparently, these measures capture a common underlying state of firm-level uncertainty which varies over time.¹¹ Furthermore, these measures of idiosyncratic firm risk display a robustly countercyclical behavior.¹² The Great Recession 2007-2009 featured a particularly sharp rise in firm-level risk.¹³

The causal relationship between firm-level uncertainty and the business cycle is an open question.¹⁴ In this paper, I study within a dynamic general equilibrium model the effect of exogenous innovations to firm-level uncertainty on the cyclical behavior of financial and real variables.¹⁵ I find that an increase in uncertainty at the firm level aggravates the agency problem between firm managers and investors, which results in a rise of default rates and a drop in measured TFP, aggregate output, consumption, and investment. Firms reduce leverage in times of high uncertainty, as the risk of bankruptcy increases and equity claims gain in value at the expense of bondholders. This is consistent with the extensive empirical literature on corporate finance which regularly finds a negative relationship between firm risk and leverage ratios.¹⁶

Ignoring the effect of time-varying uncertainty at the micro-level results in an overestimation of the significance of other shocks to fundamentals such as aggregate technology shocks.

¹¹Why does idiosyncratic firm risk vary over time? A change of the economic environment can affect different firms in vastly different ways. Some firms might benefit from a given change in economic policy, while others suffer. Accordingly, firm uncertainty could increase whenever important changes of economic policy are implemented or anticipated. Baker, Bloom and Davis (2013) construct an empirical measure of economic policy uncertainty and find it to be correlated with major political events such as elections, wars, the Eurozone crisis, or the U.S. debt-ceiling dispute.

¹²The countercyclical behavior of various measures of firm-level uncertainty has been documented for different countries and firm groups. See for example Campbell, Lettau, Malkiel and Xu (2001), Higson, Holly and Kattuman (2002), Higson, Holly, Kattuman and Platis (2004), Eislefeldt and Rampini (2006), Gourio (2008), Bloom (2009), Gilchrist, Sim and Zakrajšek (2010), or Bloom et al. (2012).

¹³See Bloom et al. (2012) and Schaal (2012).

¹⁴Bloom et al. (2012) find no evidence that this unconditional negative correlation between idiosyncratic uncertainty and aggregate output is merely driven by an endogenous response of uncertainty to the business cycle. But see also Bachmann, Elstner and Sims (2010).

¹⁵Examples of models which feature an endogenous rise of idiosyncratic risk in response to aggregate shocks include Veldkamp (2005), Van Nieuwerburgh and Veldkamp (2006), and Bachmann and Moscarini (2011). A similar feedback channel from aggregate economic activity to idiosyncratic firm risk is absent from my model, but is likely to amplify the quantitative impact of variations in uncertainty.

¹⁶See Bradley, Jarrell and Kim (1984), Friend and Lang (1988), or Korteweg (2010). Campbell and Taksler (2003) use the opposite effects of firm volatility on the value of stock and bond claims to explain the diverging performance of equity and bond markets in the U.S. during the late 1990s.

In this model, the Solow residual fluctuates over time together with firm-level uncertainty even in the absence of technological innovations. This is an important finding, as Chari, Kehoe and McGrattan (2007) show that variations of total factor productivity are an important determinant of the U.S. business cycle. Falls in measured productivity can be caused by an endogenous increase in agency problems, as investors find it harder to incentivize firm managers to pursue efficient business policies in times of high uncertainty. This rationale of declines in the Solow residual is an alternative to the idea of recurring episodes of exogenous technological regress.

Related Literature

The key innovation of this paper consists of embedding an optimal security design approach to firm financing in a general equilibrium macro framework. The principal underlying ideas originate in an earlier literature which rationalizes the design and usage of a certain combination of equity and debt securities focusing on a single firm in partial equilibrium. This research agenda can be divided into three distinct groups. One branch of literature focuses on asymmetric information between firm managers and outside investors. Default on debt payments triggers monitoring by outsiders, which assigns a socially valuable role to costly bankruptcy. This approach is adopted by Chang (1993), Boyd and Smith (1998), Atkeson and Cole (2008), and Cole (2011).¹⁷ A second line of research sees default as a mechanism to withdraw control rights from managers in an environment of incomplete contracts. This idea is explored by Aghion and Bolton (1992), Chang (1992), Dewatripont and Tirole (1994), Zwiebel (1996), and Fluck (1998). A third approach views default as the termination of a long-term financing relationship between firm managers and outside investors, as in DeMarzo and Sannikov (2006), Biais, Mariotti, Plantin and Rochet (2007), and DeMarzo and Fishman (2007).¹⁸

The environment used in the model below is most closely related to the first branch of literature. These models all share one common feature. The agency problem between firm managers and investors does not distort production. Firm output is an exogenous stochastic

¹⁷This literature builds on earlier contributions by Townsend (1979), Diamond (1984), and Gale and Hellwig (1985). In these models, a combination of entrepreneurial (inside) equity and outside debt is optimal. Public equity held by outside investors does not have value in environments in which firm output is private information of firm managers. See Townsend (1979):

“The model as it stands may contribute to our understanding of closely held firms, but it cannot explain the coexistence of publicly held shares and debt.”

¹⁸Two models of the optimal design and usage of equity and debt which do not consider default are Biais and Casamatta (1999) and Koufopoulos (2009).

process and firm managers simply decide on the payout of realized cash flows. In contrast, I study an agency problem in which firm output will generally be inefficiently low. As the severity of the agency problem varies, so does the expected level of firm output. This mechanism will be crucial for the result that agency conflicts at the firm level can affect the Solow residual of an economy.

Also on the macroeconomic level, firms' financing choice between equity and debt has been the subject of inquiry. Examples of models which analyze the interaction between corporate capital structure and the business cycle include Levy and Hennessy (2007), Gomes and Schmid (2010), Covas and Den Haan (2011), and Jermann and Quadrini (2012). These models go a long way in matching empirical facts. However, the set of financial instruments at the disposal of agents is exogenously constrained and not derived from the economic environment. If firms had the option to offer alternative financial contracts to investors in these environments, this would lead to more favorable economic outcomes. Factors which are identified as relevant for the cyclical properties of the model will generally vary with the exogenously imposed contract structure. As long as we do not understand the role of financial contracts in overcoming frictions to economic exchange, we are likely to miss something about these underlying frictions and consequently also about their significance for macroeconomic fluctuations. The role of idiosyncratic firm risk discussed below is one example.

The Financial Accelerator literature, following along the lines of Bernanke and Gertler (1989) and Bernanke, Gertler and Gilchrist (1999), proposes a strictly entrepreneurial model of the firm and does not allow for outside equity financing. Also, Gomes, Yaron and Zhang (2003) show at the example of Carlstrom and Fuerst (1997) that these models tend to generate procyclical default rates which is at odds with empirical evidence. While the Financial Accelerator literature focuses on information frictions, another line of thought follows Kiyotaki and Moore (1997) in putting limits to the enforceability of contracts at the center of their analysis. These models share the exclusively entrepreneurial nature of firms and cannot explain the occurrence of costly default in equilibrium. While Lorenzoni (2008) and others succeed to characterize and explain the problematic nature of excessive borrowing in a similar framework, eventual policy implications for firm financing are put into question by the disregard of equity financing.¹⁹

An important contribution of this paper is the introduction of a novel propagation mechanism of uncertainty (or risk) shocks to the business cycle literature. At the same time, the

¹⁹For other studies of excessive borrowing in a debt-only environment, see also Brunnermeier and Sannikov (2010), or Bianchi (2011).

general idea that idiosyncratic uncertainty may matter for aggregate outcomes is not new at all.²⁰ Bloom et al. (2012) show that non-convex adjustment costs to capital and labor can give rise to a “wait-and-see” effect in response to temporarily elevated levels of firm-level risk. Firms reduce investment in times of high uncertainty if they cannot costlessly reverse their decisions afterwards. As this hampers the optimal reallocation of production factors across plants, this can generate an endogenous decline in the Solow residual. However, Bachmann and Bayer (2013) find this “wait-and-see” effect to be quantitatively small compared to the business cycle impact of a standard aggregate technology shock. Furthermore, Bachmann and Bayer (2011) point out that in this environment large contractionary effects of uncertainty shocks are incompatible with the procyclical behavior of the dispersion of investment levels across firms which they document for German micro data. Also Lang (2012) finds that the “wait-and-see” effect is unlikely to be strong enough such that an increase in the dispersion of productivity shocks at the firm level can trigger an aggregate downturn.

Other studies are closer to the model outlined below in that they examine credit market imperfections as an alternative propagation channel of innovations to the level of firm-level uncertainty. Gilchrist, Sim and Zakrajšek (2010) impose an exogenous contract structure upon firms by restricting their financing choice to equity and debt. Both security types are subject to ad-hoc frictions. They show that uncertainty raises the cost of capital as credit spreads rise in response to a higher risk of bankruptcy. This causes a drop in investment with adverse consequences for optimal factor reallocation across firms and for the Solow residual. The authors depart from the rest of the literature by assuming that firm profits are linear in productivity (instead of being convex). This assumption facilitates to generate countercyclical firm-level risk as shown below.²¹ Both Gilchrist, Sim and Zakrajšek (2010) and Bloom et al. (2012) rely on frictions to the efficient reallocation of production factors across firms to generate endogenous movements of the Solow residual. Using French micro-level data, Osotimehin (2013) finds that the efficiency of factor reallocation is actually higher during recessions than during booms. This result casts a doubt on the important procyclical role of factor reallocation in Gilchrist, Sim and Zakrajšek (2010) and Bloom et al. (2012). In contrast, in the model proposed below the expected marginal product of capital will be equalized across firms at all times.

Christiano, Motto and Rostagno (2013) build on the Financial Accelerator mechanism

²⁰The focus of this paper lies on variations in *idiosyncratic* uncertainty. Recent examples of studies which examine shocks to *aggregate* uncertainty include Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez and Uribe (2011), Basu and Bundick (2012), Leduc and Liu (2013), and Orlik and Veldkamp (2013).

²¹By assuming firm profits to be linear in productivity, the authors switch off the ‘Oi-Hartman-Abel effect’ associated with procyclical idiosyncratic risk. See Oi (1961), Hartman (1972), and Abel (1983).

mentioned above and conclude that shocks to firm-level risk are the most important driver of the business cycle. Importantly, debt is the only source of outside financing within the Financial Accelerator framework. But the effects of increased production risk are very different for the value of equity and bond claims of a given firm. Bondholders participate only in the elevated downside risk of production outcomes, while shareholders benefit from the increase in upside risks. Excluding equity financing from the analysis is problematic, as this prohibits firms to sell their upside risks to investors. The result that the costs of capital rise in uncertainty is somewhat mechanical, if only debt is considered. Furthermore, in Christiano, Motto and Rostagno (2013) nominal rigidities and the endogenous response of monetary policy to uncertainty shocks are crucial elements in generating realistic business cycle co-movements. Dorofeenko, Lee and Salyer (2008) and Chugh (2012) examine idiosyncratic uncertainty within a Financial Accelerator framework without nominal rigidities. They find quantitatively weak results. This is consistent with the finding by Chari, Kehoe and McGrattan (2007), that movements in the “investment wedge” alone are unlikely to generate realistic business cycle properties.

Arellano, Bai and Kehoe (2012) assume exogenously incomplete markets in their study of the role of financial frictions in propagating innovations to firm-level risk. With Dorofeenko, Lee and Salyer (2008), Chugh (2012), and Christiano, Motto and Rostagno (2013) they share the focus on debt as the only source of outside firm financing. The authors are particularly successful in generating variations of labor demand in response to an exogenous shock to idiosyncratic uncertainty. The underlying mechanism is very similar to the effect of a shock to borrowing constraints as in Jermann and Quadrini (2012). Quadrini (2011) points to the similarities between these two types of financial shocks.

Also in Narita (2011), financial frictions give rise to a negative role of uncertainty shocks for the aggregate economy. The author shows that increased firm-level risk causes a rise in the endogenous termination rate of the long-term financial contracts introduced by DeMarzo and Sannikov (2006). Implications for the cyclical properties of real and financial variables are not considered.

The endogenous movements of the Solow residual generated by the model outlined below resemble earlier ideas on variable factor utilization developed by Burnside, Eichenbaum and Rebelo (1993), Basu (1996), or Burnside and Eichenbaum (1996). Keeping the measured units of aggregate capital and labor input fixed, these models allow for variations in the degree of capital utilization and labor effort which are not directly observable to econometricians. This idea can explain movements in the Solow residual which are not caused by technology shocks but, for instance, by innovations to government expenditures. The focus of this model

does not lie on capital or labor, but on the quality of managerial labor as a third production factor which is arguably hard to measure and an important determinant of the productivity of the other two factors. Managerial effort levels do not vary over the business cycle because of aggregate shocks, but because of variations in the severity of agency problems caused by exogenous changes to firm-level uncertainty. The agency problem in question is based on the corporate finance literature on optimal security design and its empirical implications can be tested both on the micro and the macro level.

One key assumption in this model is the central role of managerial effort for the production outcome of the entire firm. This idea is in line with empirical studies on the importance of managerial practices for individual firm performance as documented by Bloom and Van Reenen (2007) and Bloom, Eifert, Mahajan, McKenzie and Roberts (2013). The role of executive managers is special because their decisions affect how efficient the other inputs to production are used. Indirect empirical evidence on this conception of managerial activity is provided by the studies of Baker and Hall (2004), Gabaix and Landier (2008), and Terviö (2008), who estimate the marginal value of the labor input by top executives to increase together with the resources under their control.

Outline

The rest of the paper is organized as follows. The model is set up in Section 1.2. Sections 1.3 and 1.4 characterize the equilibrium allocation for the frictionsless case and the case of asymmetric information, respectively. A quantitative analysis of the model follows in Section 1.5. The paper concludes with a short discussion of future work in Section 1.6.

1.2 Model Setup

Consider a model economy with a continuum of small firms of mass unity. Each firm $j \in [0, 1]$ uses an identical constant returns to scale production technology with capital, labor, and managerial effort as inputs. This technology is subject to both aggregate and idiosyncratic productivity shocks. The economy is inhabited by many small and identical households of unit mass. Households provide labor to companies and allocate their savings across firms in order to smooth consumption over time. Each of the many small companies is run by a single manager who exerts effort.

1.2.1 Firms

All firms produce the same homogeneous final good which can be used either for consumption or for investment. Firm output is given by:

$$y_t(j) = A_t(j) k_t(j)^{\alpha_k} l_t(j)^{\alpha_l} m_t(j)^{\alpha_m},$$

where $k_t(j)$ stands for the amount of capital employed in company j at time t , $l_t(j)$ measures labor input, and $m_t(j)$ indicates managerial effort. The output elasticities of capital, labor, and managerial effort add up to one: $\alpha_k + \alpha_l + \alpha_m = 1$. This functional form is chosen in line with empirical findings by Gabaix and Landier (2008), who estimate CEO compensation to grow linearly in firm size indicating constant returns to scale. Firm-specific productivity $A_t(j)$ is independent and identically distributed across companies according to the cumulative distribution function $F_t(A)$. Its discrete support is completely characterized by the lowest and highest realizations, A^1 and A^n , together with $\Delta_A = A^i - A^{i-1}$ for $i = 2, 3, \dots, n$. The properties of $F_t(A)$ vary stochastically over time with aggregate conditions.

1.2.2 Managers

There is a unit mass of firm managers. Managers have access to the constant returns to scale production technology described above, but they do not own any savings which they could use as capital. In order to produce, they can collect savings from households on a capital market. Each manager can run exactly one firm. Managers provide managerial effort and consume their wages. Their utility is given by the function:

$$u(c_t(j), m_t(j)) = c_t(j) - v(m_t(j)),$$

where $c_t(j)$ and $m_t(j)$ denote consumption and labor effort provided by the manager of firm j at time t , respectively. The function measuring the disutility of effort $v : [0, 1] \rightarrow \mathbb{R}$ is assumed to be increasing and strictly convex. In addition, we assume: $v'(1) = -\infty$. If a firm manager decides not to run a firm, she faces an outside option which generates a utility level of \underline{u} with certainty.

Firm managers live for exactly one period. At the end of each period, the current generation of managers dies and a new generation of managers is born. This assumption implies that firms are modeled as short-term projects which can be studied independently of the particular history of any individual firm manager.²²

²²The assumption of short-term financing contracts is not uncommon in the literature. See for example

1.2.3 Households

The household's preferences regarding an allocation of consumption C_t and labor l_t over time may be described by the function:

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[U(C_{t+i}) - V(l_{t+i}) \right],$$

where \mathbb{E}_t is the expectation operator conditional on date t information, and $\beta \in [0, 1]$ gives the rate of time preference. The function $U : [0, \infty] \rightarrow \mathbb{R}$ is increasing, strictly concave and satisfies the Inada conditions. The function $V : [0, 1] \rightarrow \mathbb{R}$ is increasing and strictly convex. In addition, we assume: $V'(1) = -\infty$.

Households provide labor to firms and try to smooth consumption over time. At the end of each period, they split their wealth W_t between consumption and savings which they allocate across the various investment opportunities offered by a new generation of managers on the capital market. Accordingly, the households' budget constraint reads as:

$$C_{t+i} + \int_0^1 k_{t+i+1}(j) dj \leq w_{t+i} l_{t+i} + \int_0^1 R_{t+i}(j) k_{t+i}(j) dj \equiv W_{t+i},$$

where $k_t(j)$ denotes the individual agent's savings allocated to firm j , $R_t(j)$ indicates the gross return achieved, and w_t gives the wage rate.

1.2.4 Timing

The timing is as follows. A firm j enters period t with a pre-determined capital stock of $k_t(j)$. At the beginning of period t , firm managers learn about the realization of the idiosyncratic shock $A_t(j)$. They choose between staying in the firm and their outside option. Each manager can now make a public announcement $\hat{A}_t(j)$ about the productivity realization of her firm j to investors. This announcement can then be monitored by the investors or not. Through monitoring, investors can learn the true productivity state of the firm. However, this information is not verifiable by the court. Before the labor market opens, next period's productivity distribution $F_{t+1}(A)$ becomes public knowledge. Now, firm manager j contracts labor $l_t(j)$ from households and exerts managerial effort $m_t(j)$. Production takes place and the ex-post value of all firms is distributed among investors, workers, and the current generation of managers which dies after consuming its income $c_t(j)$. A new generation of

Eisfeldt and Rampini (2008) or Greenwood, Sanchez and Wang (2010, 2013).

managers is born and households split their wealth between immediate consumption C_t and savings, which are invested in companies.

1.3 Perfect Information

Before studying optimal financial contracts within an environment of asymmetric information, we take a look at a frictionless world where all variables of interest are public information. In this environment we can abstract from announcements $\hat{A}_t(j)$ and monitoring decisions, as the realization of $A_t(j)$ is costlessly verifiable for the court.

Definition For all histories of aggregate shocks to $F_{t+i+1}(A)$ and given some initial wealth level W_t , a competitive equilibrium in this economy consists of prices w_{t+i+1}^* , $R_{t+i+1}^*(j)$, and quantities C_{t+i} , l_{t+i+1} , and $k_{t+i+1}(j)$, such that (1.) households solve their individual optimization problem, and (2.) labor and capital markets clear.

1.3.1 Households

Taking as given wages and the expected return to investment, the representative household solves:

$$\max_{C_{t+i}, k_{t+i+1}(j), l_{t+i} \in \mathbb{R}_{\geq 0}} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[U(C_{t+i}) - V(l_{t+i}) \right] \quad (1.1)$$

$$\text{subject to: } C_{t+i} + \int_0^1 k_{t+i+1}(j) dj \leq w_{t+i} l_{t+i} + \int_0^1 R_{t+i}(j) k_{t+i}(j) dj. \quad (1.2)$$

The first-order condition with respect to labor supply is given by:

$$V'(l_t^h) = w_t U'(C_t^h). \quad (1.3)$$

The marginal disutility of labor must be equalized with the marginal benefit of the associated increase in income. Labor supply is increasing in the wage rate. Inter-temporal optimality is characterized by a standard Euler equation:

$$U'(C_t^h) = \beta \mathbb{E}_t [U'(C_{t+1}^h) R_{t+1}(j)] \quad , \text{ for all } j \in [0, 1]. \quad (1.4)$$

Savings are chosen after capital and labor income is realized. Risk averse households employ their savings in order to achieve a high and steady level of future consumption. The supply of

savings is increasing in the expected rate of return. Facing a continuum of ex-ante identical firms exposed to idiosyncratic risk, the optimal portfolio is perfectly diversified across firms.

1.3.2 Optimal Contract

Managers demand households' savings on a capital market. They offer financial contracts to households which specify payouts to investors contingent on the uncertain realization of firm-specific productivity $A_t(j)$. As the capital market is perfectly competitive, managers design contracts which maximize the expected return to investors subject to a participation constraint for managers. Of course, managers would prefer to offer contracts which grant them more utility than just their outside option. However, with perfectly competitive capital markets no such contract can ever arise in equilibrium (see Lemma 1.3.1 below). Given some amount of capital $k_t(j)$ supplied by households, the return on investment is determined by the aggregate payout to investors:

$$A_t k_t^{\alpha_k} l(A_t)^{\alpha_l} m(A_t)^{\alpha_m} + (1 - \delta) k_t - w_t l(A_t) - c_t(A_t).$$

The firm subscripts have been suppressed for enhanced legibility. The parameter δ gives the rate of capital depreciation. While capital k_t is set before the firm-specific state of productivity is realized, the levels of labor demand $l(A_t)$, managerial effort $m(A_t)$, and manager compensation $c(A_t)$, can all be specified conditional on the respective draw of firm productivity.

Taking as given the competitive wage rate w_t , the optimal contract offered by a manager at the end of period specifies manager compensation, labor demand, and managerial effort as the solution to the following problem:

$$\max_{c(\cdot), l(\cdot), m(\cdot)} \mathbb{E}_{t-1} \left[A_t k_t^{\alpha_k} l(A_t)^{\alpha_l} m(A_t)^{\alpha_m} + (1 - \delta) k_t - w_t l(A_t) - c(A_t) \right] \quad (1.5)$$

$$\text{subject to: } c(A_t) - v(m(A_t)) \geq \underline{u}, \quad \text{for all } A_t. \quad (1.6)$$

Expression (1.6) is the participation constraint for managers. After A_t is realized, managers are free to walk away from their contractual obligations. In this case, they face an outside option which generates a utility level of \underline{u} with certainty. The participation constraint (1.6) makes sure that the firm manager never chooses to leave the firm before production has actually taken place. In the solution to this problem, (1.6) is binding for all realizations of firm productivity.

The first order condition for an optimal choice of $m(A_t)$ is given by:

$$\alpha_m A_t k_t^{\alpha_k} l^*(A_t)^{\alpha_l} m^*(A_t)^{\alpha_m-1} = v'(m^*(A_t)) \quad , \quad \text{for all } A_t. \quad (1.7)$$

In each firm-specific productivity state, the marginal product of effort is equalized with the manager's marginal rate of substitution of leisure for consumption. Managers of high productivity firms will efficiently work harder than others. Also the quantity of labor input $l^*(A_t)$ is strictly increasing in A_t :

$$\alpha_l A_t k_t^{\alpha_k} l^*(A_t)^{\alpha_l-1} m^*(A_t)^{\alpha_m} = w_t. \quad (1.8)$$

Labor demand is falling in the wage rate. Finally, the executive compensation scheme $c(A_t)$ is chosen such that (1.6) holds with equality in each state of firm-specific productivity.²³

The highest expected return which managers can possibly offer to households is accordingly given by:

$$R_t^* = \mathbb{E}_{t-1} \left[\frac{A_t k_t^{\alpha_k} l^*(A_t)^{\alpha_l} m^*(A_t)^{\alpha_m} + (1 - \delta)k_t - w_t l^*(A_t) - c^*(A_t)}{k_t} \right]. \quad (1.9)$$

This expected return is uniform across firms which are all identical ex-ante. It may vary over time together with the characteristics of the distribution of firm productivity $F_t(A)$. It remains to show that R_t^* is indeed the expected return to households' savings in equilibrium.

Lemma 1.3.1. *In equilibrium, the expected rate of return is R_t^* .*

Proof. To see this, assume that contracts trade at an expected return of $R_t < R_t^*$. This implies that all managers can attain an expected level of utility at least as high as:

$$\mathbb{E}_{t-1} \left[c^*(A_t) + [R_t^* - R_t] k_t - v(m^*(A_t)) \right] > \underline{u}.$$

In this case, demanding an additional unit of capital at price R_t increases managerial utility. The capital market does not clear at a price $R_t < R_t^*$ as managers' demand exceeds the supply k_t . \square

Capital owners appropriate the entire surplus of managers. This is because managers are tied to one firm once a financial contract is in place. They sell their managing services as a

²³Throughout the paper, it is assumed that the manager's outside option \underline{u} is low enough such that production is profitable for all possible realizations of firm productivity A_t (and in particular for the lowest possible value A^1).

single package, while the owners of capital and ordinary labor charge the same price for the first marginal unit supplied to the market as for the last unit.

Lemma 1.3.2. *Savers' expected return R_t^* is strictly decreasing in k_t .*

Proof. To see this, rewrite (1.8):

$$\alpha_l y^*(A_t) = w_t l^*(A_t),$$

where $y^*(A_t) = A_t k_t^{\alpha_k} l^*(A_t)^{\alpha_l} m^*(A_t)^{\alpha_m}$. Equation (1.9) becomes then:

$$R_t^* = \mathbb{E}_{t-1} \left[\frac{(1 - \alpha_l) y^*(A_t) - c^*(A_t)}{k_t} + (1 - \delta) \right].$$

Because of constant returns to scale, $y^*(A_t)$ can increase at most proportionally to k_t . However, manager compensation $c^*(A_t)$ must increase overproportionally to k_t , as the marginal rate of substitution of leisure for consumption is strictly increasing in managerial effort. As output $y^*(A_t)$ and managerial effort $m^*(A_t)$ are growing with the firm's capital stock k_t , that part of output, which can be paid out to investors after workers and managers have received their wage payments, is shrinking relative to k_t . \square

1.3.3 Characterization

It follows from Lemma 1.3.2 that all firms are of uniform size. Given that R_t^* is strictly decreasing in k_t , an additional marginal unit of capital will always be allocated to the smaller of two firms. Households hold a perfectly diversified portfolio with a portfolio weight of zero for any given firm. The idiosyncratic risk of each firm due to $A_t(j)$ is perfectly diversified. Households remain exposed to aggregate risk due to variations in $F_t(A)$ over time both through their labor and their capital income.

Proposition 1.3.3. *Firm-level uncertainty is procyclical in a frictionless environment.*

Proof. Rewriting equation (1.8), we derive:

$$l^*(A_t) = \left(\frac{\alpha_l k_t^{\alpha_k}}{w_t} \right)^{\frac{1}{1-\alpha_l}} A_t^{\frac{1}{1-\alpha_l}} m^*(A_t)^{\frac{\alpha_m}{1-\alpha_l}}.$$

Since $A_t^{\frac{1}{1-\alpha_l}}$ is strictly convex in firm productivity and $m(A_t)$ is increasing in A_t , it follows

that labor demand is strictly convex in A_t . So is firm output:

$$y^*(A_t) = \frac{w_t l^*(A_t)}{\alpha_l}.$$

In this model economy, the law of large numbers holds and the population expectation of a variable is identical to its aggregate value. From Jensen's inequality, it follows that a mean-preserving spread in $F_t(A)$ results in a higher value of aggregate output and working hours. \square

Managerial effort is concave or convex in A_t depending on the curvature of managers' disutility of effort:

$$m^*(A_t) = \frac{\alpha_m y^*(A_t)}{v'(m^*(A_t))}.$$

The same is true for the expected return on savings R_t^* . The result that elevated idiosyncratic uncertainty has a positive role in the presence of convex demand curves is known as the 'Oi-Hartman-Abel' effect.²⁴ Note that this result is in stark contrast with empirical findings on the countercyclical behavior of firm-level risk. In the following, we will see that the unambiguously procyclical behavior of firm-level risk crucially depends on the assumption of frictionless capital markets.

1.4 Financial Frictions

In the economy studied so far, all variables of interest are public information and costlessly verifiable by the court. The Modigliani-Miller theorem holds in this frictionless environment. Consequently, firms' capital structure choice is trivial and there is no default in equilibrium. In order to study the effect of idiosyncratic uncertainty on default rates, firm productivity and aggregate output, we introduce informational frictions to the environment. This allows for a non-trivial capital structure choice of firm financing with a positive probability of default on bond claims. We will also see that firm-level uncertainty drives up default rates and may have a negative impact on measured TFP and aggregate output. With respect to the previous section, the economic environment is modified by the following assumptions.

(A1) Asymmetric Information. Assume that productivity $A_t(j)$ and managerial effort $m_t(j)$ are private information of the manager of firm j only. Output $y_t(j)$ and labor input $l_t(j)$ remain public information.

²⁴See Oi (1961), Hartman (1972), and Abel (1983).

(A2) Monitoring. A technology is at the disposal of agents which allows investors to observe the true realization of $A_t(j)$. However, this information is not verifiable by the court. Furthermore, monitoring comes at a cost: the produce of firm j is diminished by $G(y_t(j))$ if the state of $A_t(j)$ is observed by investors. We assume that $G(y_t(j)) < y_t(j)$ and $G'(y_t(j)) < 1$.

We should expect the costs to learn about the true state of firm productivity to be much lower for investors than for a court which is completely extraneous to the business of the firm. This idea is captured by the assumption that the information acquired by investors through monitoring is unverifiable by the court.

1.4.1 Optimal Contract

The structure of the representative household's optimization problem remains unaffected by this change in the economic environment. Firm managers still offer financial contracts on a competitive capital market. As in the previous section, these contracts generally specify a cash flow to investors contingent on the firm-specific realization of productivity. However with $A_t(j)$ and $m_t(j)$ being private information, it becomes more complicated for firm managers to commit themselves to firm policies which maximize households' expected return on capital.

After firm-specific productivity shocks are realized, firm managers make a public announcement $\hat{A}_t(j)$ about the current state of $A_t(j)$. At this point, the state of company j may be monitored or not. As long as monitoring is not used, managers are always free to misreport productivity and choose a different bundle of effort, labor demand, and manager compensation if this is convenient for them. Since there are various combinations of A_t and m_t which result in the same level of output y_t for given amounts of k_t and l_t , the manager can always reduce effort by underreporting the productivity state of the firm. Consider the optimal contract for the case of perfect information as described above. If A_t is the true level of firm-specific productivity, then a manager can obtain more than just her outside option by announcing some level $\hat{A}_t < A_t$, hiring $l^*(\hat{A}_t)$, and providing $m_t < m^*(\hat{A}_t)$ herself:

$$\hat{A}_t k_t^{\alpha_k} l^*(\hat{A}_t)^{\alpha_l} m^*(\hat{A}_t)^{\alpha_m} = A_t k_t^{\alpha_k} l^*(\hat{A}_t)^{\alpha_l} m_t^{\alpha_m} \Leftrightarrow m_t = \left(\frac{\hat{A}_t}{A_t} \right)^{\frac{1}{\alpha_m}} m^*(\hat{A}_t).$$

Due to this incentive to mimic low productivity types, the effort levels described by equation (1.7) are not implementable anymore under private information. A contract between firm managers and investors needs to take this agency problem into account.

Monitoring

Since monitoring does not yield a verifiable signal of the firm-specific state of productivity, the only way in which this tool can mitigate the agency problem is via recontracting after investors have observed A_t . In this case, a new contract between investors and the firm manager can specify managerial effort, firm labor demand, and manager compensation, conditional on the mutually and publicly recognized level of firm productivity. Recontracting after monitoring by investors results in an allocation which is the solution to the following problem:

$$\max_{c(\cdot), l(\cdot), m(\cdot)} A_t k_t^{\alpha_k} l(A_t)^{\alpha_l} m(A_t)^{\alpha_m} - G(y(A_t)) + (1 - \delta)k_t - w_t l(A_t) - c(A_t) \quad (1.10)$$

$$\text{subject to: } c(A_t) - v(m(A_t)) \geq \underline{u}. \quad (1.11)$$

Just as before, managers are free to reject the contract and walk away from the project. The participation constraint (1.11) is binding in the solution to this problem. The first order conditions for an optimal choice of $m(A_t)$ is given by:

$$[1 - G'(y^m(A_t))] \alpha_m A_t k_t^{\alpha_k} l^m(A_t)^{\alpha_l} m^m(A_t)^{\alpha_m - 1} = v'(m^m(A_t)),$$

where $y^m(A_t) = A_t k_t^{\alpha_k} l^m(A_t)^{\alpha_l} m^m(A_t)^{\alpha_m}$. The marginal product of effort is reduced by its impact on the monitoring costs which are increasing in firm output. Otherwise, this optimality condition is identical to condition (1.7) from the frictionless case. Similarly, also the first order condition for $l(A_t)$ is slightly modified relative to (1.8):

$$[1 - G'(y^m(A_t))] \alpha_l A_t k_t^{\alpha_k} l^m(A_t)^{\alpha_l - 1} m^m(A_t)^{\alpha_m} = w_t.$$

Manager compensation $c(A_t)$ is chosen such that (1.11) holds with equality. We see that monitoring results in an allocation which can be fairly close to the frictionless case. However, investors need to pay the associated cost $G(y^m(A_t))$ for its use. If this cost is too high, it will generally not be optimal to monitor all announcements of firm productivity made by the firm manager.

Full Contract

Since the optimal contract between investors and firm managers cannot condition ex-ante on the information acquired by investors during monitoring, recontracting between investors

and managers is optimal in this case. However, the decision whether to monitor the state of the firm or not can be predetermined by the optimal contract conditional on the public announcement by the firm manager \hat{A}_t . Furthermore, also managerial effort, labor demand, and managerial compensation can be specified for all announcements \hat{A}_t which do not trigger monitoring by investors.

Let the function $b(\hat{A}_t)$ assign to each announcement \hat{A}_t a value of 1 if the investors observe the state of the firm and 0 otherwise.²⁵ Lemma 1.4.1 allows us to restrict the set of possible contracts to those which induce firm managers to truthfully reveal their type.

Lemma 1.4.1. *Every allocation implemented by a contract $\{b(\hat{A}_t), c(\hat{A}_t), l(\hat{A}_t), m(\hat{A}_t)\}$ can also be achieved by another contract which has the additional property that the firm manager's announcement is $\hat{A}_t = A_t$.*

Proof. Every contract $\{b(\hat{A}_t), c(\hat{A}_t), l(\hat{A}_t), m(\hat{A}_t)\}$ permits a firm manager of type A_t to choose from this menu any consumption bundle $[c(\hat{A}_t), m(\hat{A}_t)]$, which corresponds to an unmonitored value of firm productivity $\hat{A}_t : b(\hat{A}_t) = 0$. Consider now the consumption bundle chosen by a firm manager of type $A_t = A^j$: $[c(\hat{A}_t), m(\hat{A}_t); A^j] = [c^j, m^j]$. Facing now a modified contract $\{b(\hat{A}_t), c'(\hat{A}_t), l(\hat{A}_t), m'(\hat{A}_t)\}$ with $c'(A^j) = c^j$ and $m'(A^j) = m^j$ for all j , all firm managers will again choose the same consumption bundles as before: $[c'(\hat{A}_t), m'(\hat{A}_t); A^j] = [c^j, m^j]$. This follows from the principle of revealed preference. The new contract achieves truth-telling and implements the same allocation as the original one. \square

Lemma 1.4.1 is a straight-forward application of the well-known revelation principle. It allows us to look for a contract $\{b(A_t), c(A_t), l(A_t), m(A_t)\}$, which is specified directly in terms of the true realization of firm-specific productivity A_t . The optimal contract is then

²⁵In line with most of the literature, we only consider deterministic monitoring schemes here. Cole (2011) shows that stochastic monitoring schemes may result in allocations which imply a random decision between costly bankruptcy and costless settlement in case of default.

given as a solution to the following problem:

$$\max_{b(\cdot), c(\cdot), l(\cdot), m(\cdot)} \mathbb{E}_{t-1} \left[A_t k_t^{\alpha_k} l(A_t)^{\alpha_l} m(A_t)^{\alpha_m} - b(A_t) G(y(A_t)) \right. \\ \left. + (1 - \delta) k_t - w_t l(A_t) - c(A_t) \right] \quad (1.12)$$

$$\text{subject to: } A_t = \arg \max_{\hat{A}_t \in \Omega} c(\hat{A}_t) - v \left(\left(\frac{\hat{A}_t}{A_t} \right)^{\frac{1}{\alpha_m}} m(\hat{A}_t) \right), \quad \text{for all } A_t, \quad (1.13)$$

$$c(A_t) = c^m(A_t), \quad l(A_t) = l^m(A_t), \quad m(A_t) = m^m(A_t), \\ \text{for all } A_t : b(A_t) = 1, \quad \text{and} \quad (1.14)$$

$$c(A_t) - v(m(A_t)) \geq \underline{u}, \quad \text{for all } A_t. \quad (1.15)$$

The solution to this contracting problem maximizes the expected payout to investors subject to the incentive compatibility constraints in equation (1.13), the outcome of recontracting in case of monitoring given by (1.14), and to the participation constraints in (1.15). The set Ω consists of all productivity levels for which $b(A_t) = 0$. This is the set of announcements \hat{A}_t the manager is free to make without being monitored. As is shown by Proposition 1.4.2, incentive compatibility puts tight restrictions on the permissible set of monitoring schemes.

Proposition 1.4.2. *Consider the ordered set of possible realizations of firm productivity: $\{A^1, A^2, \dots, A^{n-1}, A^n\}$, with $A^i < A^j$ if and only if $i < j$. Any functions $b(A_t)$, $c(A_t)$, $l(A_t)$, and $m(A_t)$, solving the optimal contract problem as stated above, must satisfy:*

$$b(A^i) = 1 \Rightarrow b(A^j) = 1 \quad , \text{ for all } A^j \in \{A^1, A^2, \dots, A^{i-1}, A^i\}.$$

Proof. Assume the contrary. Then there must exist some realization A^i such that $b(A^i) = 0$ and $b(A^{i+1}) = 1$. The minimum level of utility which a manager of type A^i has to be granted is given by her outside option \underline{u} . But in this case, a manager of type A^{i+1} can achieve more than \underline{u} by mimicking A^i :

$$c(A^i) - v \left(\left(\frac{A^i}{A^{i+1}} \right)^{\frac{1}{\alpha_m}} m(A^i) \right) > c(A^i) - v(m(A^i)) \geq \underline{u}.$$

But we know from section 1.4.1 that the manager of a monitored firm receives exactly her outside option. This violates the incentive-compatibility constraint (1.13). \square

A similar result is found by Chang (1993). It follows that any solution to the contracting problem above, consisting of the functions $b(A_t)$, $c(A_t)$, $l(A_t)$, and $m(A_t)$, can equivalently be described by the three functions $c(A_t)$, $l(A_t)$, and $m(A_t)$, together with the threshold value A^I , which is defined as: $A^I = \max\{A^i : b(A^i) = 1\}$.

Proposition 1.4.2 is derived directly from incentive compatibility in combination with the unverifiable nature of the information acquired through monitoring by investors. Would a court be able to observe the monitoring outcome on the same terms as investors, then allocations which grant a higher level of managerial utility in case of monitoring could be enforced.

From our analysis above, we know already a lot about the allocation implemented by the optimal contract for the monitored realizations below the threshold value A^I . What can we say about the characteristics of the contract for $A^i > A^I$? In this range, the firm manager is unmonitored. Lemma 1.4.3 shows that this fact allows them to capture some information rents.

Lemma 1.4.3. *The participation constraint is binding for the lowest unmonitored productivity state: $c(A^{I+1}) - v(m(A^{I+1})) = \underline{u}$. Managers of more productive firms are better off: managerial utility is strictly increasing on Ω .*

Proof. First, assume that: $c(A^{I+1}) - v(m(A^{I+1})) > \underline{u}$. In this case, $m(A^{I+1})$ could profitably be increased without violating the participation constraint. Downward incentive compatibility does not apply for A^{I+1} . What about upward incentive compatibility? It can be maintained by profitably increasing $m(A^i)$ somewhat for all $A^i > A^{I+1}$ wherever necessary. Eventually binding participation constraints are not an issue here since upward incentive compatibility for A^{I+1} is satisfied before any participation constraints for $A^i > A^{I+1}$ are binding. This follows from downward incentive compatibility. As long as a manager with a high draw of A_t does not choose to underrepresent firm productivity, she will also not have a reason to prefer leaving the firm to staying inside of the contract. Hence, any solution to the contracting problem in question must feature a binding participation constraint for A^{I+1} .

Second, we characterize managerial utility for the remaining unmonitored realizations. Consider the ordered set $\Omega = \{A^i : A^i > A^I\} = \{A^{I+1}, A^{I+2}, \dots, A^n\}$, with $A^i < A^j$ if and only if $i < j$. Incentive compatibility implies that for any $A^i \in \Omega$:

$$c(A^{i+1}) - v(m(A^{i+1})) \geq c(A^i) - v\left(\left(\frac{A^i}{A^{i+1}}\right)^{\frac{1}{\alpha_m}} m(A^i)\right).$$

Furthermore, it follows from $A^{i+1} > A^i$ that:

$$c(A^i) - v\left(\left(\frac{A^i}{A^{i+1}}\right)^{\frac{1}{\alpha_m}} m(A^i)\right) > c(A^i) - v\left(m(A^i)\right).$$

This concludes the proof. \square

Firms' labor demand is publically observable. It is intuitive that the information frictions which apply to A_t and $m(A_t)$ should not interfere directly with an efficient choice of $l(A_t)$. This intuition is confirmed by Lemma 1.4.4.

Lemma 1.4.4. *The marginal product of labor is equalized with the wage rate in all states of firm productivity.*

Proof. From the first order conditions for $l(A^i)$, it follows directly that:

$$\left[1 - b(A^i) G'(y(A^i))\right] \alpha_l A^i k_t^{\alpha_k} l(A^i)^{\alpha_l - 1} m(A^i)^{\alpha_m} = w_t. \quad (1.16)$$

In all monitored states with $b(A^i) = 1$, the marginal product of labor is diminished by the factor $G'(y(A^i))$, which accounts for the fact that monitoring costs are increasing in firm output. Otherwise, this first order condition is identical to equation (1.8) from the frictionless case. \square

It remains to characterize the allocation of $c(A^i)$ and $m(A^i)$ for $A^i > A^I$. Over this unmonitored range, the manager's preferences over the consumption bundles $[c(\hat{A}_t), m(\hat{A}_t)]$ determine which allocations may be achieved. The marginal rate of substitution between consumption $c(\hat{A}_t)$ and effort $m(\hat{A}_t)$ is given by:

$$\frac{\left(\frac{A_t}{\hat{A}_t}\right)^{\frac{1}{\alpha_m}}}{v'\left(\left(\frac{\hat{A}_t}{A_t}\right)^{\frac{1}{\alpha_m}} m(\hat{A}_t)\right)}.$$

This expression is strictly increasing in A_t . In exchange for a higher compensation payment, managers of high productivity firms are always willing to increase output by a little more, for given values of k_t and $l(\hat{A}_t)$, than those who happen to run a company with a low draw of A_t . Hence, managers' preferences satisfy the single-crossing (or Spence-Mirrlees) property, i.e. the indifference curves of firm managers with different realizations of A_t can cross only once. This property can be used to learn about the state of firm productivity without relying on the monitoring technology. Lemma 1.4.5 describes whether and how the optimal contract

discriminates among different contingencies of firm productivity for realizations above the threshold value A^I .

Lemma 1.4.5. *Consider the ordered set $\Omega = \{A^{I+1}, A^{I+2}, \dots, A^{n-1}, A^n\}$. For each neighbouring pair A^i and A^{i+1} within Ω , there are two possibilities:*

1. A^i and A^{i+1} are 'pooled'. In this case:

$$\begin{aligned} (a) \quad & c(A^i) = c(A^{i+1}) \quad \text{and} \quad A^i m(A^i)^{\alpha_m} = A^{i+1} m(A^{i+1})^{\alpha_m} \quad , \\ (b) \quad & c(A^{i+1}) - v\left(m(A^{i+1})\right) = c(A^i) - v\left(\left(\frac{A^i}{A^{i+1}}\right)^{\frac{1}{\alpha_m}} m(A^i)\right) \quad , \text{ and} \\ (c) \quad & c(A^i) - v\left(m(A^i)\right) = c(A^{i+1}) - v\left(\left(\frac{A^{i+1}}{A^i}\right)^{\frac{1}{\alpha_m}} m(A^{i+1})\right) \quad . \end{aligned}$$

2. A^i and A^{i+1} are 'separated'. In this case:

$$\begin{aligned} (a) \quad & c(A^i) < c(A^{i+1}) \quad \text{and} \quad A^i m(A^i)^{\alpha_m} < A^{i+1} m(A^{i+1})^{\alpha_m} \quad , \\ (b) \quad & c(A^{i+1}) - v\left(m(A^{i+1})\right) = c(A^i) - v\left(\left(\frac{A^i}{A^{i+1}}\right)^{\frac{1}{\alpha_m}} m(A^i)\right) \quad , \text{ and} \\ (c) \quad & c(A^i) - v\left(m(A^i)\right) > c(A^{i+1}) - v\left(\left(\frac{A^{i+1}}{A^i}\right)^{\frac{1}{\alpha_m}} m(A^{i+1})\right) \quad . \end{aligned}$$

A proof of Lemma 1.4.5 is provided in Appendix A. 'Pooling' is a trivial option to deal with the problem of asymmetric information. In this case, the required level of effort for the manager of a firm of given productivity is identical across different announcements of firm productivity, as she always has to provide the same level of output in exchange for a fixed compensation payment. Pooled types are observationally equivalent which means that managers of high productivity firms work less hard than others. Clearly, this constitutes an inefficiency caused by information frictions. By offering a firm manager to compensate her through higher wage payments for choosing a high level of firm output, the optimal contract can avoid uniform output levels across types. If two neighbouring types are 'separated', then the high type will be indifferent between truth-telling and deviating towards the closest state below. At the same time, the low type strictly prefers the announcement of her true state to overreporting.²⁶

Together with Lemma 1.4.4, Lemma 1.4.5 states that within the unmonitored region of firm productivity manager compensation is strictly increasing in firm output. Note, however, that for given levels of managerial effort executive compensation is higher now than in

²⁶In certain variations of the principal-agent problem, it is optimal to 'shut down' production completely for low productivity states in order to limit the amount of information rents granted to higher types. In the contracting problem considered here, monitoring strictly dominates shut-down.

the frictionless case in order to ensure incentive compatibility. In this sense, some part of manager compensation consists in a reward of pure luck, or rather an information rent. This strong relationship between executive compensation and firm performance is consistent with empirical evidence on CEO pay as documented by Clementi and Cooley (2010) and others.

Lemma 1.4.6. *If the hazard rate of $F(A)$ is non-decreasing in A^i , then a complete separation of types is optimal.*

A formal proof of Lemma 1.4.6 can be found in Appendix A. Note that Lemma 1.4.6 merely states a necessary condition for complete separation to be optimal. More generally speaking, as long as the hazard rate is not declining too rapidly, complete separation is optimal. Increasing managerial effort $m(A^i)$ by an infinitesimal amount increases allocative efficiency for the realization of firm productivity $A_t = A^i$. However, this infinitesimal change in $m(A^i)$ also increases the information rent extracted by the firm manager whenever $A_t > A^i$. If the hazard rate drops sharply, it means that an increase in allocative efficiency at A^i is only of minor importance relative to the associated costs of increased information rents for $A_t > A^i$. Pooling is desirable in this case, because a lower value of $m(A^i)$ dampens the increase in information rents over Ω .

An important corollary of Lemma 1.4.6 concerns the efficiency of managerial effort in this environment.

Corollary 1.4.7. *Production is efficient for the highest possible level of firm productivity $A_t = A^n$. Production is inefficiently low for all levels of firm productivity $A^I < A_t < A^n$.*

Again, the proof of this corollary is deferred to Appendix A. Firm production is distorted in the presence of asymmetric information in order to contain the extraction of information rents by firm managers for high realizations of firm productivity. Since this motive is absent for $A_t = A^n$, we encounter in Corollary 1.4.7 the usual ‘no distortion at the top’ result.

The more important managers are for production, the higher are the information rents. But these information rents only arise over the unmonitored range of firm productivity states. A higher threshold value A^I increases the monitoring frequency and reduces agency costs. This trade-off is summarized by Proposition 1.4.8.

Proposition 1.4.8. *Given a sufficiently fine grid $\{A^1, A^2, \dots, A^n\}$ with $\Delta_A = A^i - A^{i-1}$ for $i = 2, 3, \dots, n$, the monitoring frequency is monotonically increasing in the output elasticity of managerial effort α_m and decreasing in monitoring costs $G(y(A_t))$.*

Please refer to Appendix A for a formal proof. The allocations with and without informational frictions coincide, whenever one of the two costs associated with asymmetric

information falls to zero. This is the case if $\alpha_m = 0$, that is, the output elasticity of managerial effort is zero. In this case, firm output is determined entirely by the amount of capital and labor employed together with the firm-specific realization of productivity, which is revealed perfectly by the observable level of firm output. Managerial effort is redundant for production and monitoring is never used. Likewise, asymmetric information is costless if $G(y(A_t)) = 0$. In this case, $A^I = A^n$ and managers are monitored over the whole range of productivity realizations.

The optimal contract described here is incomplete in the sense that it cannot condition on the information gathered by investors through monitoring. This information is not verifiable by the court. If this would be the case, then Lemma 1.4.2 would cease to hold and the monitoring domain would not need to be a convex set. But note that also in this case the benefits of monitoring apply to all higher unmonitored realizations of firm productivity, while the costs accrue only for the monitored state. For a wide class of probability distributions $F(A)$, the solution to the optimal contract problem described in (1.12)-(1.15) is identical with or without the constraints in (1.14).

Uncertainty

Output levels are distorted in this model economy for two reasons: (1.) monitoring costs, and (2.) information rents. Inefficiencies caused by the latter can be measured by the effort wedge $\tau(A^i)$:

$$[1 - \tau(A^i)] [1 - b(A^i) G'(y(A^i))] \alpha_m A^i k_t^{\alpha_k} l(A^i)^{\alpha_l} m(A^i)^{\alpha_m - 1} = v'(m(A^i)).$$

If the degree of information asymmetry between investors and firm managers is growing, then we should also expect that the associated distortions become more severe. A direct measure of asymmetry of information in this model economy is firm-level uncertainty. However, it is generally not true that any mean-reserving spread of $F(A)$ results in more severe distortions to firm production. As a mean-preserving spread can take on vastly different forms, it is also possible to generate various different effects in response. This is why we restrict ourselves in the following to the analysis of a mean-preserving spread in $F(A)$ which largely maintains the properties of the original probability distribution.

Proposition 1.4.9. *Consider the range of all possible productivity levels $\{A^1, A^2, \dots, A^n\}$ with $\Delta_A = A^i - A^{i-1}$ for $i = 2, 3, \dots, n$. Furthermore, consider an increase in Δ_A which leaves all probabilities $p(A^i)$ and the expected value of A_t unchanged. If this spread is of sufficient size, then monitoring is optimally used more frequently in the new contract and the value of $\tau(A^i)$ has to increase for each A^i .*

Proof. First of all, note that information rents are increasing in Δ_A . Consequently, managerial effort levels for $A^{I+1} < A_t < A^n$ are falling in response to an increase in Δ_A as can be seen from equation (1.21). The associated values of $\tau(A^i)$ are growing. This increase in information rents shifts the trade-off which determines the optimal threshold value A^I as described in Proposition 1.4.8. The labor wedge $\tau(A^{I+1})$ is growing towards one in Δ_A , which implies that the benefits of increasing the monitoring range are growing without bounds. Recall that firm technology satisfies the Inada conditions. At some point, the benefits of an additional increase in A^I must outweigh the associated costs. The discrete support of firm productivity allows for a local drop in the value of $\tau(A^i)$ for some A^i in response to an increase in A^I . However, if Δ_A continues to grow, both the monitoring frequency as well as the effort wedge for all unmonitored realizations (except for A^n) need to increase. \square

Now that we have characterized the solution to the contracting problem given in (1.12)-(1.15), and understood the role of agency costs, monitoring costs, and firm-level uncertainty in shaping the production outcomes in this model economy, we go on to examine the implications of these results for the behavior of capital structure, default rates, and firm productivity over the business cycle.

1.4.2 Capital Structure

The previous section examined the problem of a manager who has to decide about the optimal contract to offer on the period $t - 1$ capital market in the presence of informational frictions. Key components of this contract are the optimal monitoring frequency and an executive compensation scheme which incentivizes the firm manager to exert high effort levels even in the presence of asymmetric information. In principle, this contract could be implemented using various types of financial securities as long as they result in the solution to the contracting problem described above. It turns out, however, that one implementation of the optimal contract consists of a certain combination of public equity and corporate bonds. This implementation of the optimal contract corresponds closely to the practice of firm financing of publicly held companies and is therefore particularly attractive in order to

understand the effects of changes in fundamentals on the financing choice and the economic performance of publicly held firms. Equity and bond securities are defined as follows.

Definition Equity holders are the residual claimants of a firm. They are entitled to a pro rata share of the firm's asset value net of wage, manager compensation, and debt payments.

Definition Bond holders are entitled to a fixed payment by the debtor at maturity. In case the debtor fails to comply with this obligation, bankruptcy is declared and bond holders have the right to recover as much of the face value of their bonds as possible.

One way to characterize the key difference between these two financing instruments is: "Equity is soft; debt is hard." A bond represents a precisely defined claim which is senior to equity and highly enforceable with the occasional upshot of bankruptcy of the debtor. Meanwhile, equity generates a highly variable future payoff which is junior to debt and wage obligations.

We follow Townsend (1979) and Gale and Hellwig (1985) in identifying bankruptcy with monitoring. The social value of bankruptcy proceedings consists of a costly transfer of firm-specific information to outsiders, i.e. creditors. This view on bankruptcy and firm default allows us to separate the state-dependent payout to investors implied by the optimal contract into distinct payments to holders of equity securities and bonds, respectively. Whenever a firm fails to pay out the face value of debt:

$$y(A^{I+1}) + (1 - \delta) k_t - w_t l(A^{I+1}) - c(A^{I+1})$$

to bond holders, the firm goes bankrupt and monitoring takes place. This allows for recontracting between the bond holders (or their representatives) and the firm manager without the complications of asymmetric information. However, putting the firm under the scrutiny of creditors comes at a cost as accountants and lawyers need to be paid during the process of bankruptcy. This implies for the aggregate payout to the bond holders of a given company:

$$P^d(A_t) = \begin{cases} y(A^{I+1}) + (1 - \delta) k_t - w_t l(A^{I+1}) - c(A^{I+1}) & , \text{ if } A_t > A^I, \text{ and} \\ y(A_t) - G(y(A_t)) + (1 - \delta) k_t - w_t l(A_t) - c(A_t) & , \text{ otherwise.} \end{cases}$$

Whenever the ex-post value of a firm is high enough to pay out bond holders, they receive a fixed payment equal to the face value of their debt holdings. In case the firm is unable to service these obligations, bankruptcy is declared, monitoring takes place, and the creditors receive whatever is left of the company. Within this bankruptcy region, the fixed debt claim

becomes state-dependent. Holders of equity securities receive the residual asset value after all liabilities have been served. This results in an aggregate payout to holders of equity securities issued by a given company as given by:

$$P^e(A_t) = \begin{cases} y(A_t) - y(A^{I+1}) - w_t[l(A_t) - l(A^{I+1})] - [c(A_t) - c(A^{I+1})] & , \text{if } A_t > A^I, \\ 0 & , \text{otherwise.} \end{cases}$$

Dividends are state-dependent and only paid out as long as the firm is able to service its debt and wage obligations. A look at Propositions 1.4.8 and 1.4.9 through the lens of financial structure links bankruptcy risk to characteristics of the economic environment. Bankruptcy is used in order to contain agency costs. As these costs increase (1.) in the degree of asymmetry of information between investors and managers and (2.) in the importance of managers for production outcomes, bankruptcy is used more frequently in response to an increase in one of these two factors. Obviously, the opposite is true for a rise in bankruptcy costs. The positive relationship between firm-specific uncertainty and bankruptcy risk finds empirical support in Gilchrist, Sim, and Zakrajšek (2010), who estimate a positive effect of uncertainty on corporate bond spreads.

Note that so far we have only separated the firm payout to investors into distinct payments to equity and bond holders. It remains to relate these findings to the optimal capital structure choice of the firm. This can only be done in general equilibrium as is shown below. We denote the household's holdings of debt and equity securities issued by firm j at the end of period $t-1$ with $d_t(j)$ and $e_t(j)$, respectively. The gross return realized on these financial investments is given by $R_t^d(j)$ and $R_t^e(j)$. Our definition of a competitive equilibrium is adapted to the introduction of financial structure in the following way.

Definition For all histories of aggregate shocks to $F_{t+i+1}(A)$ and given some initial wealth level W_t , a competitive equilibrium in this economy consists of prices w_{t+i+1}^* , $R_{t+i+1}^{d*}(j)$, $R_{t+i+1}^{e*}(j)$, and quantities C_{t+i} , l_{t+i+1} , $d_{t+i+1}^*(j)$, and $e_{t+i+1}^*(j)$, such that (1.) households solve their individual optimization problem, and (2.) labor and capital markets clear.

1.4.3 Households

Revisiting the representative household's problem, we examine its portfolio choice among the various equity and debt securities offered by different managers on the capital market:

$$\max_{\substack{C_{t+i}, d_{t+i+1}(j), \\ e_{t+i+1}(j), l_{t+i} \in \mathbb{R}_{\geq 0}}} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[U(C_{t+i}) - V(l_{t+i}) \right] \quad (1.17)$$

$$\begin{aligned} \text{subject to: } C_{t+i} + \int_0^1 [d_{t+i+1}(j) + e_{t+i+1}(j)] dj \\ \leq w_{t+i} l_{t+i} + \int_0^1 [R_{t+i}^d(j) d_{t+i}(j) + R_{t+i}^e(j) e_{t+i}(j)] dj. \end{aligned} \quad (1.18)$$

The first-order condition with respect to labor supply is identical to the frictionless case:

$$V'(l_t^h) = w_t U'(C_t^h). \quad (1.19)$$

Savings are chosen in order to equalize the expected marginal utility from adjusting the portfolio weight of any type of security which is in positive demand:

$$U'(C_t^h) = \beta \mathbb{E}_t [U'(C_{t+1}^h) R_{t+1}^i(j)] \quad , \text{ for } i = d, e, \text{ and for all } j. \quad (1.20)$$

Just as in the frictionless case discussed above, the optimal portfolio of risk-averse households is perfectly diversified across firms which are all subject to idiosyncratic risk and identical ex-ante. Since the portfolio weight of any given security of any given firm is zero, households are perfectly insured against firm-specific risks. Furthermore, households already know at time t next period's distribution of productivity shocks $F_{t+1}(A)$. For this reason, they do not face aggregate uncertainty in their savings decision and only care about the expected return associated with the various securities offered on the capital market.

1.4.4 Characterization

The representative household's portfolio choice has important implications for the optimal capital structure choice of firms.

Proposition 1.4.10. *There is a unique ratio of equity to debt financing which arises in equilibrium. The equilibrium capital structure is given by:*

$$\frac{d_{t+1}^*}{e_{t+1}^*} = \frac{\mathbb{E}_t [P^d(A_{t+1})]}{\mathbb{E}_t [P^e(A_{t+1})]}.$$

Proof. Any type of security which is in positive demand must yield the same expected return: $\mathbb{E}_t [R_{t+1}^{d^*}(j)] = \mathbb{E}_t [R_{t+1}^{e^*}(j)] = R_{t+1}^*$, for all j . Given that all firms are identical ex-ante, they will have identical supply curves of quantities of equity and debt securities offered at given expected rates of return. In this case, adding new firms to the household's portfolio is always profitable as this reduces the variability of the household's financial wealth. This implies a perfectly diversified portfolio. Consequently, only expected rates of return matter. It follows that the solution to the contracting problem given in (1.12)-(1.15) can only be offered by managers who can credibly promise an identical expected return both on the bonds and the shares which they sell. Hence, the optimal capital structure choice is pinned down by:

$$R_{t+1}^* = \frac{\mathbb{E}_t [P^e(A_{t+1})]}{e_{t+1}^*} = \frac{\mathbb{E}_t [P^d(A_{t+1})]}{d_{t+1}^*} \Leftrightarrow \frac{d_{t+1}^*}{e_{t+1}^*} = \frac{\mathbb{E}_t [P^d(A_{t+1})]}{\mathbb{E}_t [P^e(A_{t+1})]}.$$

□

The ratio of debt to equity financing must be equal to the ratio of the expected payouts on the respective securities. The Modigliani-Miller theorem does not hold in this environment. There is a unique capital structure choice which implements the optimal contract and maximizes the return on capital.

The contracting problem described above suggests a theory of optimal capital structure as determined by a trade-off between agency costs and bankruptcy risk. This idea is very much in line with classical contributions to the corporate finance literature dating back until the days of Jensen and Meckling (1976). As changes in fundamentals affect the relative size of the expected payouts to shareholders and bondholders, they also cause variations in the optimal capital structure choice. However, the relationship between the output elasticity α_m , monitoring costs $G(y(A_t))$, or firm-level uncertainty on one hand, and optimal leverage on the other hand is analytically ambiguous. For instance, a rise in firm-level uncertainty

increases the risk of bankruptcy, but the associated change in the optimal face value of debt may be positive or negative. And even if the face value of debt is higher now, the firm is more unlikely to actually meet its credit obligations. If bondholders gain relative to shareholders, then the optimal ratio of debt to equity financing must increase and vice versa. The comparative statics of optimal capital structure generally depend on the functional form of the probability distribution $F(A)$, the monitoring costs $G(y(A_t))$, as well as other parameters.

The theoretical predictions are more clear cut when it comes to examine the impact of a rise in firm-level uncertainty on the distortions to firm production. From Lemma 1.4.9 we know that both the effort wedge and the risk of bankruptcy are increasing in uncertainty. Firm output is therefore falling relative to its perfect information benchmark. This counteracts the procyclical role of uncertainty in the frictionsless benchmark economy (see Proposition 1.3.3). Whether the ‘Oi-Hartman-Abel’ effect or the increase in the costs of asymmetric information prevail in response to a rise in firm-level uncertainty is a question which requires a quantitative answer.

1.5 Quantitative Analysis

In order to get a more precise idea about the role of idiosyncratic uncertainty in shaping the behavior of corporate capital structure and firm production along the business cycle, the theoretical model economy is parameterized using U.S. data and its properties are studied in response to firm-level uncertainty shocks. I choose the following functional forms:

- Firm managers’ disutility of effort is linear: $v(m_t(j)) = \gamma m_t(j)$.
- Households’ disutility of work is zero: $V(l_t) = 0$.

It follows that in equilibrium households always supply their entire endowment of working time $\bar{l} = 1$. Working hours are constant over the business cycle and only the competitive wage rate moves with aggregate conditions. One difference between this simulated economy and the analytical model outlined above concerns the timing.

- Firm managers hire labor before the realization of the firm-specific productivity shock $A_t(j)$.

This implies that all firms hire an identical amount of labor.

1.5.1 Parametrization

The period in the model is a quarter. Most of the parameters can be calibrated using long term averages of stationary target variables. The stochastic process of time-varying uncertainty is specified in order to replicate empirical evidence provided by Bloom et al. (2012) on the properties of the cross-sectional distribution of firm-specific productivity shocks over time.

Parameters with long run average targets

We have seen above that two of the key determinants of capital structure in this model are the output elasticity of managerial effort α_m and bankruptcy costs $G(y_t)$. We set $\alpha_m = 0.05$, which results in a high share of average executive compensation in firm earnings. Bebchuk and Grinstein (2005) estimate the compensation of the top five executives of each firm in their sample of public companies to add up to 6.6% of firm earnings. Arguably, the group of managers which are subject to the agency problem described above can be much bigger than only the top five managers of a given firm. On the other hand, the assumed production function suggests that these managers should be able to influence with their actions the aggregate production outcome of the entire company. The chosen parameter value is a preliminary choice and subject to future adjustment.

Bankruptcy costs are assumed to be proportional to firm output: $G(y_t) = \varphi y_t$. This assumption is motivated by the empirical findings of Bris, Welch and Zhu (2006). The authors document the amount of fees paid to attorneys, accountants and trustees during bankruptcy proceedings to be increasing in firm size.²⁷ The parameter φ is set to 0.24, which implies average bankruptcy costs of 13.65% of firm asset value. This lies well within the range of empirical estimates.²⁸ The model generates yearly default rates of 2.65%, which is slightly higher than the default rate on corporate bonds of 2.20% reported in Covas and Den Haan (2011).

Covas and Den Haan (2011) and Jermann and Quadrini (2012) calculate average leverage of publicly traded firms as the ratio of liabilities to the book value of firm assets. They report values of 0.59 and 0.46, respectively. In order to simultaneously match empirical default rates and the fairly low ratio of debt financing in the data, a high value of capital depreciation is

²⁷Meisenzahl (2011) reports that the assumption of proportional monitoring costs fits the data on US small business credit contracts quite well. Other models which assume proportional monitoring costs include Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), and Christiano, Motto and Rostagno (2013).

²⁸Bris, Welch, and Zhu (2006) report an estimate of 10% with a broad range of fitted values from 0% to 20% depending on firm characteristics.

Table 1.1: Parametrization

Parameters	Values
α_k	0.35
α_l	0.60
α_m	0.05
δ	0.285
φ	0.24
γ	1.00
β	0.9825
\bar{l}	1

necessary. Accordingly, the parameter δ is set to 0.285 which is high in comparison with the literature.²⁹ The average value of leverage in the simulated model is 0.61.

The constant marginal disutility of managerial effort γ turns out to be extremely hard to identify as the significance of managers for production implied by our parametrization is fairly low. Since the model is very robust to various considered values, we normalize $\gamma = 1$.

In line with typical specifications of the business cycle literature, we specify households' preferences as $U(C_t) = \ln(C_t)$, with a discount factor $\beta = 0.9825$. The model generates an average annual rate of return on investment of 7.31%. This is slightly higher than the empirical average stock market return of 5.98% reported by Gomes and Schmid (2010). The time endowment of labor provided by households is normalized at $\bar{l} = 1$. Finally, the labor share of output is chosen to be $\alpha_L = 0.60$, which implies for the output elasticity of capital: $\alpha_K = 0.35$. The full set of parameter values is displayed in Table 1.1.

Uncertainty

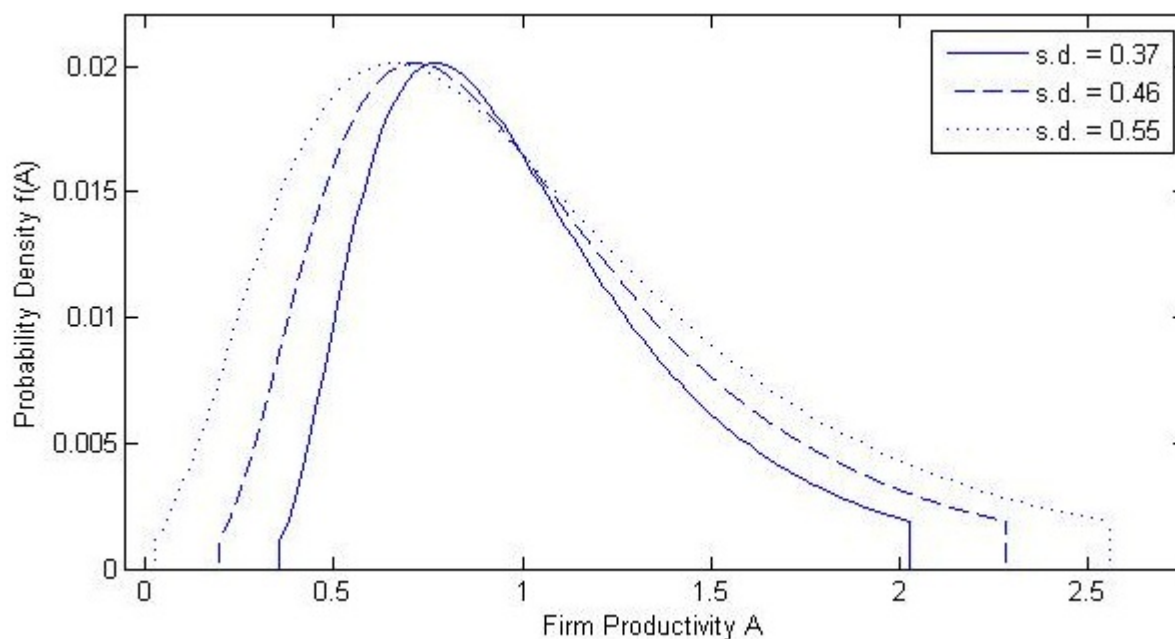
Bloom et al. (2012) estimate establishment-level TFP shocks of U.S. firms at a yearly frequency in the period from 1972-2010. They report that the average interquartile range of shocks to logarithmized TFP lies at a value of 0.39. They use this interquartile range as their measure of firm-level uncertainty and document a variation over time with a standard deviation of 0.05 and a serial correlation of 0.76.

In order to match these features of the uncertainty process, I assume the distribution of TFP shocks across firms in a given time period to be truncated lognormal with a constant mean of 1 and a time-varying standard deviation of firm shocks σ_t . In line with the concept of

²⁹Gilchrist, Sim, and Zakrajšek (2010) argue that an annual depreciation rate of 0.18 is consistent with firm-level Compustat data. The introduction of materials as a fourth input factor of the production function would result in a more realistic choice of δ .

a mean preserving spread used above, the different regimes of uncertainty stretch the support of A_t without changing the associated probability values. The exogenous state of micro-level uncertainty is specified as a discrete Markov chain with nine distinct states. Figure 1.1 shows three different probability densities corresponding to the lowest, the average, and the highest state of firm-level uncertainty. Details of the construction and specification of the uncertainty process are provided in Appendix B.

Figure 1.1: Uncertainty Regimes



The data generated from simulating the model economy at a quarterly frequency is estimated with a procedure which is aimed at replicating the estimation technique used by Bloom et al. (2012). That is, the fictional econometrician estimates a production function which is specified as:

$$y_y(j) = \tilde{A}_y(j) k_y(j)^{\frac{\alpha_k}{1-\alpha_m}} l_y(j)^{\frac{\alpha_l}{1-\alpha_m}},$$

where $y_y(j)$ is yearly output calculated as the sum of output for a given firm j over four consecutive quarters, and $k_y(j)$ and $l_y(j)$ are the corresponding mean values of firm capital and labor input. In correspondence with Bloom et al. (2012), managerial input and agency frictions of any kind are ignored during this procedure. Simulated values of $\log(\tilde{A}_y(j))$ are generated across 10,000 firms over a time period of 250 years. The underlying process of micro uncertainty is chosen such that we match the empirical properties of time-variation in the distribution of $\tilde{A}_y(j)$. The average interquartile range of $\log(\tilde{A}_y(j))$ amounts to 0.39, with a

standard deviation of 0.05 and a persistence of 0.79. By replicating the empirical findings on time-varying firm-level risk, we make sure that our model generates quantitatively plausible movements in uncertainty.

1.5.2 Optimal Contract

Figure 1.2 shows the optimal contract implied by the parametrization described above for long run average values of capital and firm-level uncertainty. This is the optimal contract which achieves a complete separation of productivity types across firms. The dashed line gives the optimal contract for the perfect information benchmark, while the contract under asymmetric information is described by the solid lines in the four panels. In the upper left panel, we see that firm output in the frictionless model is almost linear in productivity. This is due to the relatively small role of managers for production implied by $\alpha_m = 0.05$. Managerial effort is the only input to production which can respond to realizations of firm productivity. This curve becomes more convex for higher values of α_m . Firm output under asymmetric information is strictly lower than the benchmark value except for the highest possible realization of A_t . For all other values of firm productivity, managerial effort is distorted by the agency friction. The threshold value $A^I = 0.24$ is very low as the default rate implied by this contract is 2.65% per year. Firm output over the monitored region is gross of bankruptcy costs. Monitored values are associated with higher effort levels than the unmonitored realizations just above. This is due to the fact that for monitored productivity states no incentive compatibility constraints have to be taken into account.

In the frictionless case, executive compensation is increasing rather linearly in effort and firm output. In contrast to this, managerial pay with asymmetric information is strongly convex and it outpaces its perfect information counterpart even for levels of firm productivity with severely distorted effort levels. This decoupling of effort and compensation of executive managers is a symptom of the agency problem discussed above. However, note that the skewness of the distribution of executive pay across firms lies at 2.67, which is still below empirical values calculated by Clementi and Cooley (2010).

Comparing state dependent payouts to investors, we see that the gap between the case of asymmetric information and the perfect information benchmark derives mainly from the difference in executive compensation and only to a much smaller extent from the costs of monitoring and inefficiently low effort levels. This is due to the relatively small importance of managers implied by $\alpha_m = 0.05$, which reduces the consequences of shirking for output and the need for frequent monitoring.

Figure 1.2: Optimal Contract

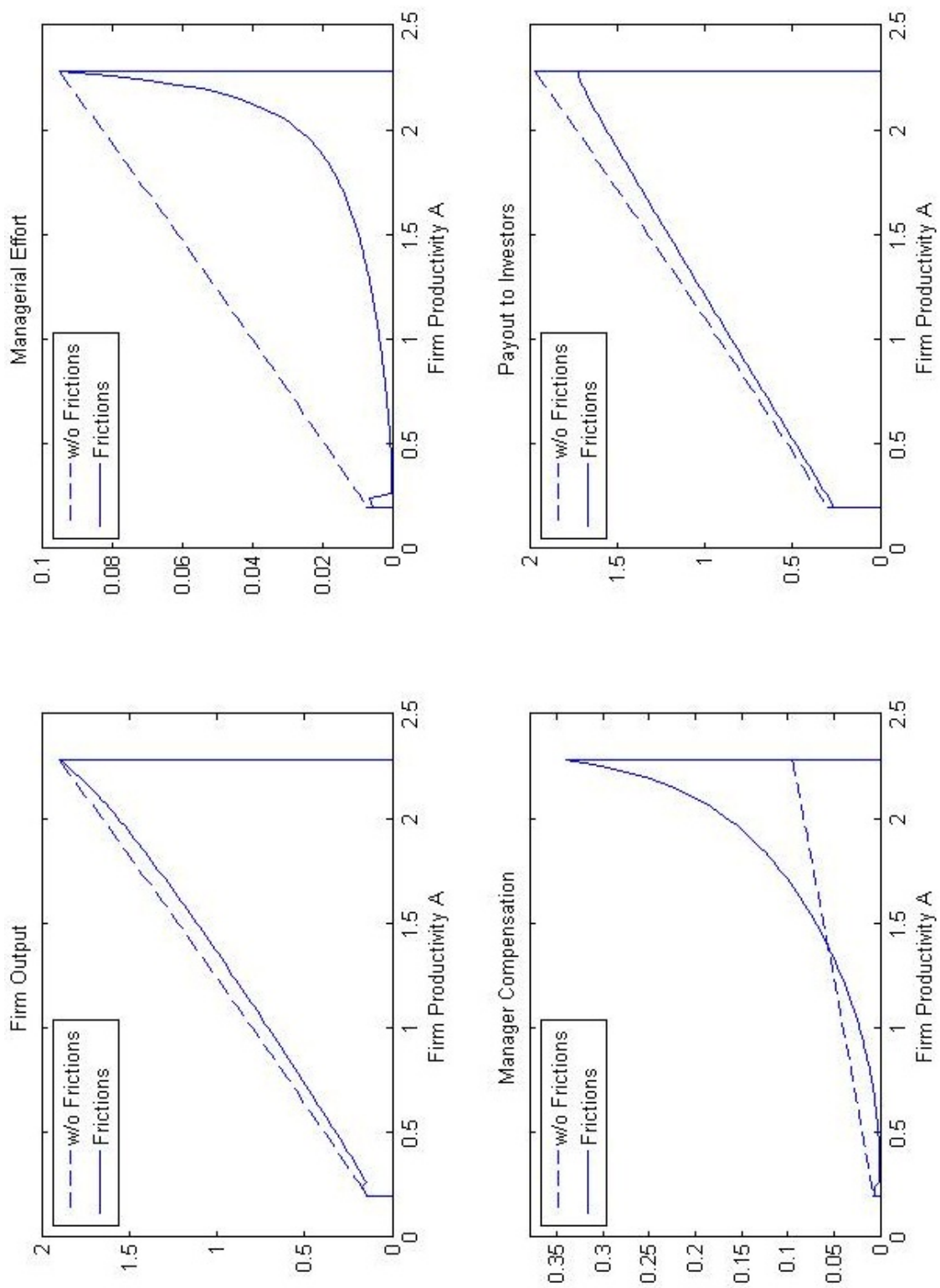
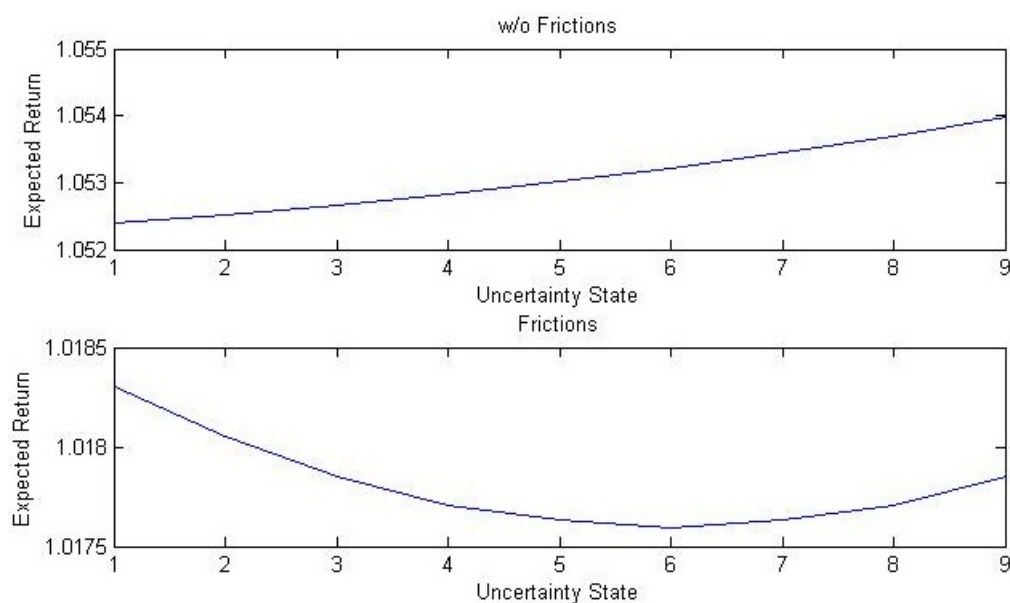


Figure 1.3: Uncertainty and Expected Returns

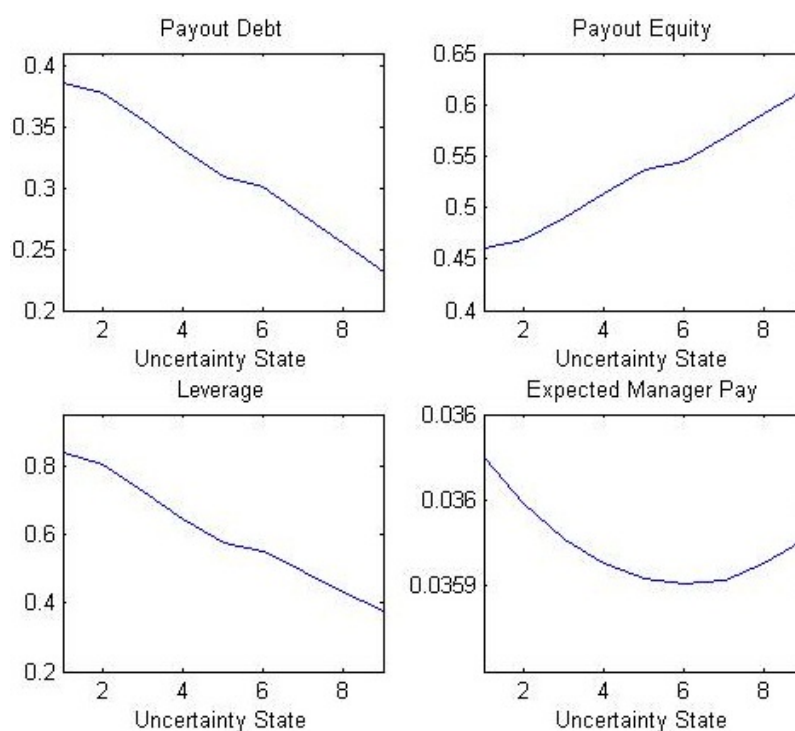


1.5.3 Uncertainty and Financial Markets

Figure 1.3 shows the two competing forces which move the return on investment for differing degrees of firm-level risk. Consider first the case of perfect information in the upper panel. In the version of the model used for the numerical analysis, labor demand does not respond to the firm-specific realization of productivity and the disutility of managerial effort is linear. In this case, efficient effort levels are strictly convex in $A_t(j)$. Consequently, also firm output and the payouts to investors are convex in firm productivity. The ‘Oi-Hartman-Abel’ effect gives rise to the familiar procyclical behavior of firm-level risk.

In the lower panel of Figure 1.3, the expected return to investment is displayed for the case of information frictions. The beneficial impact of rising uncertainty present in the frictionless environment is still at work here. But in addition to this, uncertainty also drives up monitoring and agency costs which depresses investors’ returns. It turns out that uncertainty’s impact on the costs of asymmetric information is getting weaker for higher levels of firm risk. This results in a non-monotonic relationship between the expected return to investment and firm-level uncertainty. What is the effect of rising uncertainty on the optimal financing mix of publicly traded firms? Figure 1.4 shows the aggregate payout to shareholders and bondholders given the long-run average capital stock for the different uncertainty regimes. The expected value of total payouts to debt holders is monotonically decreasing in uncertainty. This negative effect of volatility on the value of debt claims

Figure 1.4: Uncertainty and Financial Markets



is well understood since the classic contribution by Merton (1974) to bond pricing. The expected payout to stockholders, on the other hand, is growing with volatility. Shareholders' downside risk is limited, while dividends in states of high firm productivity are increasing in the variance of A_t . These opposite effects of firm-level volatility on the value of stock and bond claims is used by Campbell and Taksler (2003) to explain the diverging performance of the U.S. equity and corporate bond markets during the late 1990s.

In response to the increased value of equity claims, firms substitute debt financing by the issuance of new shares as can be seen from the lower left panel of Figure 1.4. This is consistent with the empirical findings of a negative relationship between volatility and leverage as reported by Bradley, Jarrell and Kim (1984), Friend and Lang (1988), or Korteweg (2010). The model generates this negative co-movement, because agency costs are increasing only moderately in uncertainty. Would agency costs respond more strongly to high volatility states, then the optimal contract would feature an even higher monitoring frequency and this could potentially result in an increased ratio of debt to equity financing.

The expected value of managerial effort is moving together with the expected levels of firm output and the return on capital as shown in the lower right panel of Figure 1.4. The distortions to effort become monotonically more severe as uncertainty rises, but the expected

first best levels of effort are increasing due to the ‘Oi-Hartman-Abel’ effect. For higher states of firm risk, the latter force prevails.

1.5.4 Business Cycle Analysis

How do innovations to the second moment of firm-specific productivity shocks affect the business cycle in this model economy? In Figure 1.5 and Figure 1.6, impulse response functions of real and financial variables are shown in reaction to an increase in the standard deviation of productivity shocks across firms σ_t of about 9%. This is equivalent to two times of the unconditional standard deviation of σ_t over time.³⁰ The typical effect of a rise in firm-level uncertainty is very different depending on whether the environment is subject to informational frictions or not.

Consider first the response in the frictionless case represented by the dashed lines. The expected return to investment increases in volatility as seen already in Figure 1.3. Households respond by accumulating capital which subsequently lowers the return to investment.³¹ Aggregate firm output, consumption, wages, and manager compensation all jump up with the uncertainty shock and decrease then slowly together with the economy’s stock of capital. Investment actually drops just before the impact, as anticipated higher future wealth levels resulting from high volatility have an income effect on consumption. The initial drop in investment is followed by a large increase, as more investment is required now in order to maintain the elevated capital stock, until it converges back to its long term average value.

The behavior of an economy subject to asymmetric information is quite different. Expected returns fall typically since the effect of rising agency costs is stronger on average than the countervailing force present in the frictionless case. Note that also output falls on impact as the aggregate level of managerial effort drops. This is an important property of the model. Financial frictions do not only manifest themselves in a distortion of savings and investment. They also have a direct effect on the efficiency of currently employed production factors in this model, which is why a rise in firm-level uncertainty can have an instantaneous impact on aggregate output. Households respond to lower future income by initially increasing savings and investment slightly in order to contain the fall in future returns to capital and labor. The capital stock falls subsequently as households decumulate savings in order to smooth consumption over time until the uncertainty state returns to its long-run average level. Aggregate firm output, consumption, and wages fall and rise subsequently together

³⁰The exogenous uncertainty state jumps up two levels. For details, please consult Appendix B.

³¹Firms use a production function with constant returns to scale, but equilibrium labor input is constant over time.

with aggregate capital. Also executive compensation is diminished, because managerial effort is lowered as a result both of the increase in uncertainty and of the lowered stock of firm capital.

While the responses of real variables (aggregate output, consumption, investment) are only of limited magnitude in this specification, financial variables react quite strongly. The increase in the default rate of 20% depresses the value of debt claims, which leads to a fall of the absolute amount of debt financing by 12%. The increase in the value of equity by more than 6% does not fully make up for the loss in total capital. Leverage decreases by about 18%.

Table 1.2: Business Cycle Properties

Correlation with Output	Data	Frictions	w/o Frictions
Firm-level Uncertainty	-0.46	-0.70	0.97
Consumption	0.83	0.98	0.99
Investment	0.87	0.94	0.96
Executive Pay	0.92	0.99	1.00
Default Rate	-0.33	-0.70	

Notes: All variables are logged and measured as deviations from trend. *Firm-level Uncertainty* is the cross-sectional interquartile range of establishment-level shocks to logarithmized TFP. The contemporaneous correlation with GDP is reported by Bloom et al. (2012). The values for *Consumption* and *Investment* can be found in Jermann and Quadrini (2012). The correlation for *Executive Pay* is provided by Eisfeldt and Rampini (2008) based on data by Bebchuk and Grinstein (2005). The correlation of output with default rates is calculated by Gomes and Schmid (2010).

These observations are confirmed by unconditional correlations between aggregate output and selected variables as generated by simulations of the model with and without asymmetric information. Table 1.2 compares this simulated data with stylized facts of the business cycle. In contrast to the frictionless benchmark case, the model with informational frictions performs well in replicating the countercyclical behavior of firm-level uncertainty and the default rate on corporate bonds. Consumption, investment, and executive pay are all highly correlated with aggregate output, both in the data and in the model. Figure 1.7 illustrates how the results above relate to the empirical importance of time-varying TFP in shaping the business cycle. The plot shows three simulated time series generated by the model economy subject to asymmetric information. The first panel shows logged values of σ_t , the standard deviation of productivity shocks across firms, as deviations from the long-run average. The corresponding reaction of aggregate firm output is displayed below. Times of high volatility alternate with periods of low variation in output. Whenever firm-level uncertainty is already

Figure 1.5: Impulse Response Functions - Part I

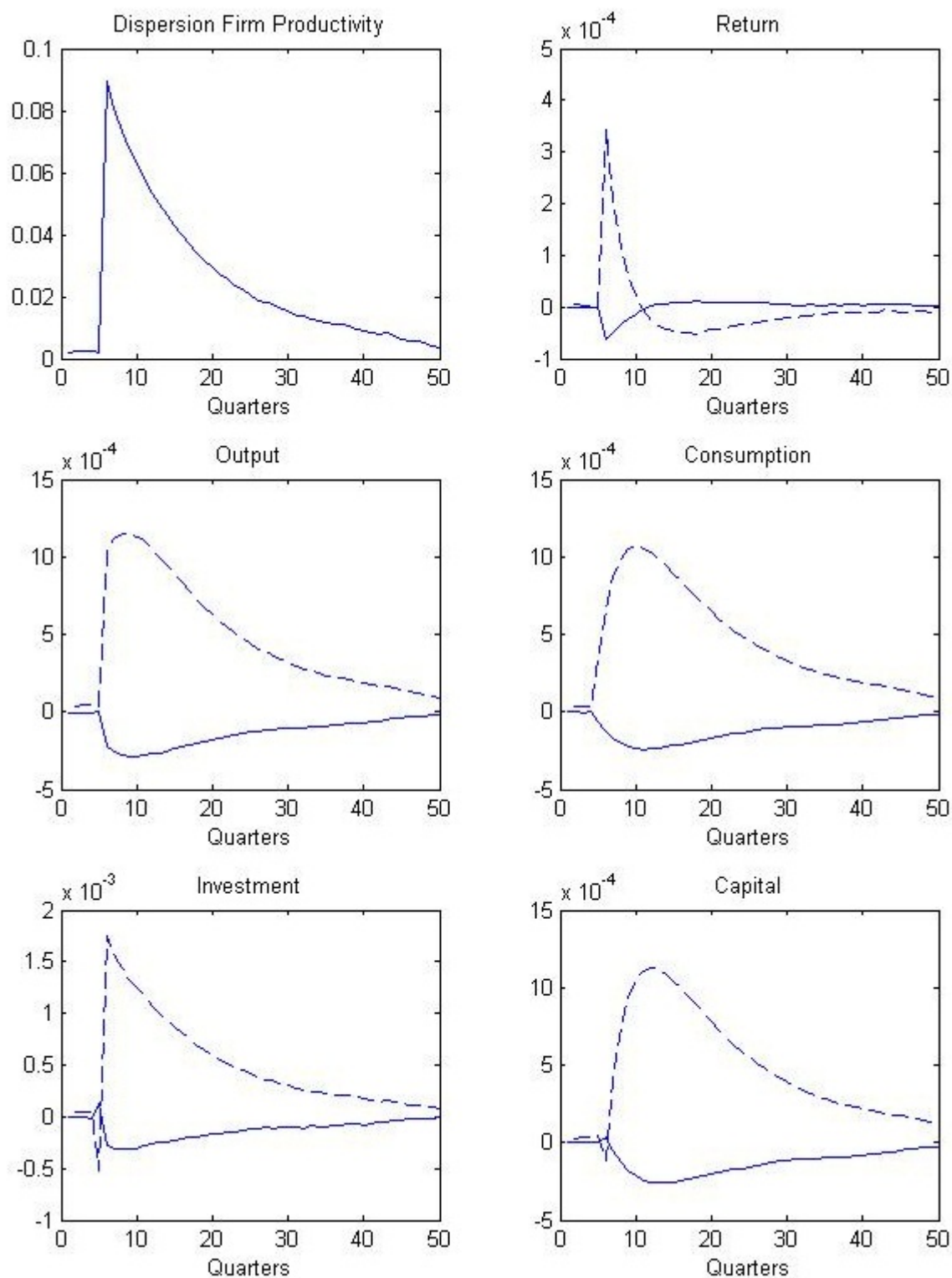


Figure 1.6: Impulse Response Functions - Part II

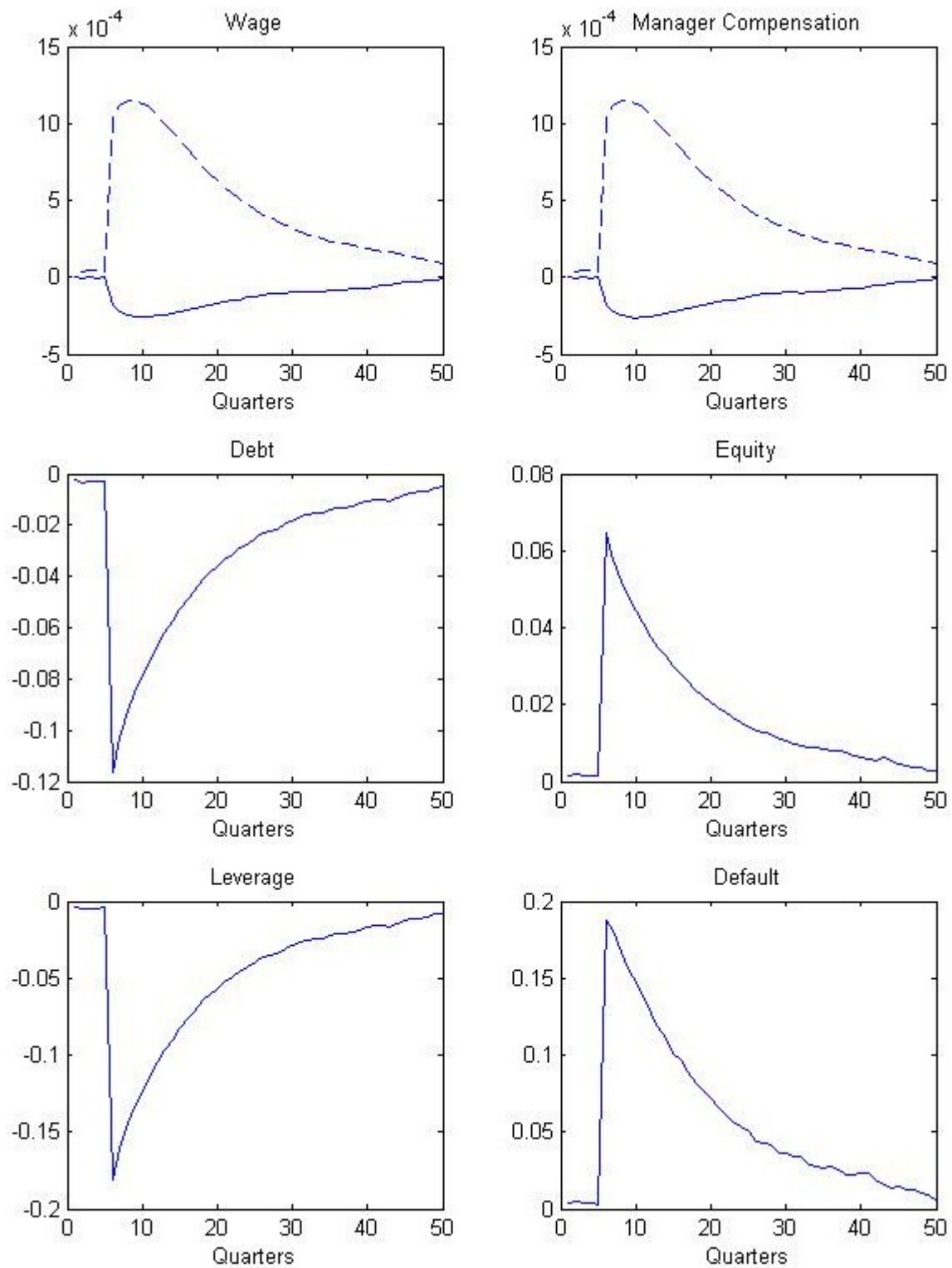
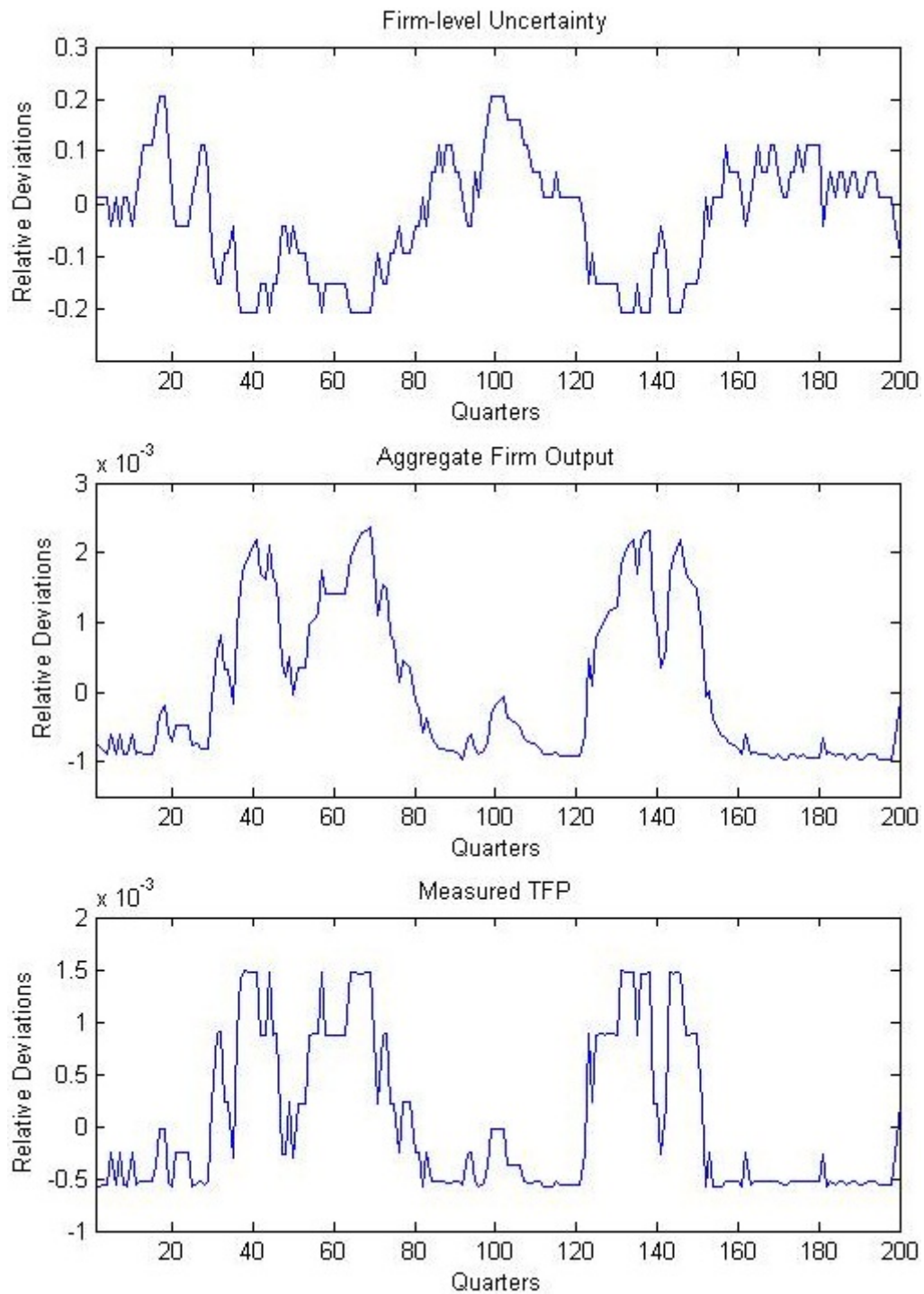


Figure 1.7: Measured TFP



on a high level, then output features a dampened positive co-movement with the uncertainty state. On the other hand, if the dispersion of firm-level shocks is below average, then the relationship between uncertainty and output becomes strongly negative. The strength of this negative correlation gives rise to the overall picture of a robustly countercyclical behavior of firm-level risk.

Now assume a fictional econometrician who observes the data on aggregate output, capital, and labor input generated by the model. This econometrician backs out TFP of a representative firm by estimating the aggregate production function Y_t :

$$Y_t = \tilde{A}_t K_t^{\frac{\alpha_k}{1-\alpha_m}} L_t^{\frac{\alpha_l}{1-\alpha_m}},$$

where Y_t , K_t , and L_t measure aggregate output, capital, and labor input, respectively. Accordingly measured levels of aggregate TFP over time are plotted in the third panel of Figure 1.7. In this model economy, there are no aggregate productivity shocks. Average firm TFP is constant over time at 1. All variation derives from changes in the standard deviation of idiosyncratic productivity shocks. However, an econometric exercise which ignores the effect of time-varying uncertainty will pick up considerable variation in aggregate productivity levels over time. This observation is also applicable to the spurious identification of alternative types of fundamental shocks in the presence of aggregate effects of idiosyncratic uncertainty.

1.6 Discussion

This paper employs an optimal contract approach to capital structure and firm financing in order to study the role of firm-level uncertainty in shaping the business cycle. Debt as a highly enforceable claim with the occasional upshot of bankruptcy serves to contain the agency problem associated with equity financing. An increase in idiosyncratic firm risk has aggregate consequences in this framework. In particular, a rise in uncertainty leads to an increase in the costs of asymmetric information. The default rate on corporate bonds grows optimally in order to mitigate the associated rise in agency costs and the fall of aggregate managerial effort. This drop in effort reduces the productivity of labor and capital. Aggregate output falls. In contrast to the frictionless benchmark, the model is able to replicate the empirically countercyclical behavior of firm level uncertainty and default rates.

The analysis outlined above can be extended in several ways. One important determinant in standard trade-off theories of optimal capital structure is the interest tax shield resulting

from debt financing. An analysis of different tax policies in the current framework could be of interest. Given that firms' financing policy in this model is the solution to an optimal contracting problem, exempting debt payments from taxation has a distortive effect on firms' capital structure choice and should result in inefficiently high default rates. On the other hand, a macroprudential taxation of debt financing as proposed by Bianchi (2011) and others is uncalled for in the current framework in which the Fisherian debt-deflation mechanism is absent.

The introduction of realistic tax rates would certainly benefit the calibration exercise of Section 1.5. Generally, the numerical analysis is far from perfect in many ways. A more careful parametrization is in order. Furthermore, the simulated model should be made completely consistent with the analytical part by introducing heterogeneous labor demand across firms and convex disutility of managerial effort. These adjustments could also amplify the response of the model variables to changes in the uncertainty regime.

In order to compare the quantitative performance of this model to a standard RBC counterpart, a shock to the average level of productivity across firms could be introduced as a second and independent source of aggregate fluctuations. This would also allow for a test of the model predictions with respect to firms' capital structure choice over the business cycle. In addition, the quantitative significance of uncertainty shocks could be assessed relative to the importance of standard technology shocks in driving the business cycle.

Appendix A Proofs and Derivations

Lemma 1.4.5

The proof of Lemma 1.4.5 is partitioned into two distinct parts. First, Lemma 1.6.1 and Lemma 1.6.2 state two properties of the optimal contract which follow directly from the structure of managerial preferences. Building on these statements, we can then go on to verify the validity of Lemma 1.4.5.

Lemma 1.6.1. *Any functions $c(A_t)$, $l(A_t)$, and $m(A_t)$, together with the threshold value A^I , solving the optimal contract problem as stated in section 1.4.1, must satisfy the following monotonicity constraints: $c(A_t)$ and $A_t m(A_t)^{\alpha_m}$ are monotonically increasing on $\Omega = \{A^i : A^i > A^I\}$.*

Proof. Consider the ordered set $\Omega = \{A^i : A^i > A^I\} = \{A^{I+1}, A^{I+2}, \dots, A^{n-1}, A^n\}$, with $A^i < A^j$ if and only if $i < j$. The difference in utility units between the goods bundle $[c(A^i), m(A^i)]$ and $[c(A^{i+1}), m(A^{i+1})]$ for a manager running a company with a productivity level of A_t is given by:

$$\begin{aligned} \Delta u(A^i, A^{i+1}; A_t) \\ \equiv c(A^{i+1}) - v\left(\left(\frac{A^{i+1}}{A_t}\right)^{\frac{1}{\alpha_m}} m(A^{i+1})\right) - c(A^i) + v\left(\left(\frac{A^i}{A_t}\right)^{\frac{1}{\alpha_m}} m(A^i)\right). \end{aligned}$$

Incentive compatibility requires the value of $\Delta u(A^i, A^{i+1}; A_t)$ to be increasing in A_t . This is the case if and only if:

$$A^{i+i} m(A^{i+i})^{\alpha_m} \geq A^i m(A^i)^{\alpha_m}.$$

Again from incentive compatibility it follows then that also $c(A^{i+i}) \geq c(A^i)$. □

Condition (1.13) requires a solution to satisfy *global incentive compatibility*. The contracting problem is simplified if we can focus instead on *local incentive compatibility*.

Definition A contract consisting of the functions $b(A_t)$, $c(A_t)$, $l(A_t)$, and $m(A_t)$, satisfies *local incentive compatibility* if for all i it holds that:

$$c(A^i) - v\left(m(A^i)\right) \geq c(A^j) - v\left(\left(\frac{A^j}{A^i}\right)^{\frac{1}{\alpha_m}} m(A^j)\right),$$

where $j = \max_{x < i} \{x : A^x \in \Omega\}$, $\min_{x > i} \{x : A^x \in \Omega\}$.

A manager should prefer truth-telling to misrepresenting her type by reporting the closest unverified realizations below or above the true level of A_t . As Lemma 1.6.2 states, this is indeed sufficient for overall truth-telling.

Lemma 1.6.2. *Local incentive compatibility implies global incentive compatibility.*

Proof. Consider again the two productivity levels $\{A^i, A^{i+1}\} \in \Omega$. Local incentive compatibility implies that $A^{i+1} m(A^{i+1})^{\alpha_m} \geq A^i m(A^i)^{\alpha_m}$. But in this case, the utility difference $\Delta u(A^i, A^{i+1}; A_t)$ is increasing in A_t . That is, if the goods bundle $[c(A^{i+1}), m(A^{i+1})]$ is preferred to $[c(A^i), m(A^i)]$ by a manager of type A^{i+1} , then *a fortiori* it is also preferred by any manager of a firm with $A_t > A^{i+1}$. On the other hand, if the goods bundle $[c(A^i), m(A^i)]$ is preferred to $[c(A^{i+1}), m(A^{i+1})]$ by a manager of type A^i , then it is also preferred by any manager of a firm with $A_t < A^i$. \square

Using the results of Lemma 1.6.1 and Lemma 1.6.2, we proceed to prove the individual components of Lemma 1.4.5.

Proof. Clearly, the pooling allocation,

$$c(A^i) = c(A^{i+1}) \quad \text{and} \quad A^i m(A^i)^{\alpha_m} = A^{i+1} m(A^{i+1})^{\alpha_m} \quad ,$$

does not violate incentive compatibility. Both, the local downward incentive compatibility constraint for A^{i+1} and the local upward incentive compatibility constraint for A^i , are simultaneously binding in this case.

Now consider the possibility that:

$$c(A^i) < c(A^{i+1}) \quad \text{or} \quad A^i m(A^i)^{\alpha_m} < A^{i+1} m(A^{i+1})^{\alpha_m} \quad .$$

If only one of these two inequalities is strict, incentive compatibility is violated. If both inequalities are strict, incentive compatibility may be satisfied. This is the separating allocation considered in Lemma 1.4.5. In this case, the value of $\Delta u(A^i, A^{i+1}; A_t)$ from Lemma 1.6.1 is strictly increasing in A_t .

Consider now the first order condition for an optimal choice of $c(A^n)$:

$$-p(A^n) + \lambda(A^n) - \mu(A^{n-1}) = 0 \quad ,$$

where $p(A^n)$ is the probability of the event $A_t = A^n$. The Lagrange multipliers $\lambda(A^n)$ and $\mu(A^{n-1})$ correspond to the downward local incentive compatibility constraint of A^n and

upward local incentive compatibility for A^{n-1} , respectively. Obviously, upward incentive compatibility does not apply to A^n and the participation constraint is slack, as follows from Lemma 1.4.3. This optimality condition can only be satisfied if $\lambda(A^n) > 0$.

If A^{n-1} and A^n are pooled, then we know that $\mu(A^{n-1}) > 0$. If, on the other hand, A^{n-1} and A^n are separated, then $\mu(A^{n-1}) = 0$. Why?

From $\lambda(A^n) > 0$, it follows that:

$$\Delta u(A^{n-1}, A^n; A^n) = c(A^n) - v\left(m(A^n)\right) - c(A^{n-1}) + v\left(\left(\frac{A^{n-1}}{A^n}\right)^{\frac{1}{\alpha_m}} m(A^{n-1})\right) = 0.$$

Since $\Delta u(A^{n-1}, A^n; A_t)$ is strictly increasing in A_t for the separating allocation, it follows that $\Delta u(A^{n-1}, A^n; A^{n-1}) < 0$, and therefore $\mu(A^{n-1}) = 0$.

If $A^{n-1} > A^{I+1}$, we can use $\mu(A^{n-1}) = \nu(A^{n-1}) = 0$ to derive again that $\lambda(A^{n-1}) > 0$. Continuing in the manner outlined above, we can verify that $\lambda(A^i) > 0$ and $\nu(A^i) = 0$ for all $A^i > A^{I+1}$, and $\mu(A^i) = 0$ whenever A^i and A^{i+1} are separated. \square

Lemma 1.4.6

Proof. To see this, we consider the range of unmonitored realizations $A^i > A^I$ and define:

$$S(m(A^i), A_t) \equiv A^i k_t^{\alpha_k} l(A^i)^{\alpha_l} m(A^i)^{\alpha_m} + (1 - \delta) k_t - w_t l(A^i) - v\left(\left(\frac{A^i}{A_t}\right)^{\frac{1}{\alpha_m}} m(A^i)\right),$$

where labor demand $l(A^i)$ is chosen as defined by Lemma 1.4.4. The function $S(A^i, A_t)$ gives the social surplus which can be divided between firm managers and investors as specified by the optimal contract for a given announcement $\hat{A}_t = A^i$ and the true level of firm productivity A_t . Incentive compatibility implies that firm investors receive $S(m(A^i), A^i)$ net of the firm manager's information rent:

$$\begin{aligned} A^i k_t^{\alpha_k} l(A^i)^{\alpha_l} m(A^i)^{\alpha_m} + (1 - \delta) k_t - w_t l(A^i) - c(A^i) \\ = S(m(A^i), A^i) - [c(A^i) - v(m(A^i))] . \end{aligned}$$

We know from Lemma 1.4.3 that these information rents are strictly increasing over the unmonitored range $A^i > A^I$. Furthermore, Lemma 1.4.5 states that the local downward incentive compatibility constraint is always binding. This implies for the local increase of

managers' information rents:

$$\begin{aligned}
& c(A^{i+1}) - v(m(A^{i+1})) - [c(A^i) - v(m(A^i))] \\
&= c(A^i) - v\left(\left(\frac{A^i}{A^{i+1}}\right)^{\frac{1}{\alpha_m}} m(A^i)\right) - [c(A^i) - v(m(A^i))] \\
&= S(m(A^i), A^{i+1}) - S(m(A^i), A^i).
\end{aligned}$$

It is precisely the manager's option to misrepresent the firm's level of productivity by announcing A^i instead of A^{i+1} which allows her to participate in the benefit of a high realization of A_t . These information rents add up as productivity levels grow over the unmonitored range of productivity realizations $\Omega = \{A^{I+1}, A^{I+2}, A^{I+3}, \dots, A^n\}$:

$$\begin{aligned}
c(A^{I+1}) - v(m(A^{I+1})) &= \underline{u}, \\
c(A^{I+2}) - v(m(A^{I+2})) &= \underline{u} + S(m(A^{I+1}), A^{I+2}) - S(m(A^{I+1}), A^{I+1}), \\
c(A^{I+3}) - v(m(A^{I+3})) &= \underline{u} + S(m(A^{I+1}), A^{I+2}) - S(m(A^{I+1}), A^{I+1}) \\
&\quad + S(m(A^{I+2}), A^{I+3}) - S(m(A^{I+2}), A^{I+2}), \\
&\dots \\
c(A^n) - v(m(A^n)) &= \underline{u} + \sum_{j=I+1}^{n-1} S(m(A^j), A^{j+1}) - S(m(A^j), A^j).
\end{aligned}$$

This allows us to express the expected payout to investors over the unmonitored range Ω as a function of the surplus term $S(m(A^i), A^i)$ net of the firm manager's information rent:

$$\begin{aligned}
& \sum_{i=I+1}^n p(A^i) [A^i k_t^{\alpha_k} l(A^i)^{\alpha_l} m(A^i)^{\alpha_m} + (1 - \delta)k_t - w_t l(A^i) - c(A^i)] \\
&= \sum_{i=I+1}^n p(A^i) \left[S(m(A^i), A^i) - \left[\underline{u} + \sum_{j=I+1}^{i-1} S(m(A^j), A^{j+1}) - S(m(A^j), A^j) \right] \right].
\end{aligned}$$

Rearranging the terms yields:

$$\begin{aligned}
 & \sum_{i=I+1}^n p(A^i) \left[S(m(A^i), A^i) - \left[\underline{u} + \sum_{j=I+1}^{i-1} S(m(A^j), A^{j+1}) - S(m(A^j), A^j) \right] \right] \\
 = & \left[S(m(A^{I+1}), A^{I+1}) - \underline{u} \right] \sum_{j=I+1}^n p(A^j) \\
 & + \left[S(m(A^{I+2}), A^{I+2}) - S(m(A^{I+1}), A^{I+2}) \right] \sum_{j=I+2}^n p(A^j) \\
 & + \left[S(m(A^{I+3}), A^{I+3}) - S(m(A^{I+2}), A^{I+3}) \right] \sum_{j=I+3}^n p(A^j) \\
 & + \dots \\
 & + \left[S(m(A^n), A^n) - S(m(A^{n-1}), A^n) \right] p(A^n).
 \end{aligned}$$

The first order condition of an optimal choice of $m(A^i)$ now reads as:

$$\begin{aligned}
 & \left[\alpha_m A^i k_t^{\alpha_k} l(A^i)^{\alpha_l} m(A^i)^{\alpha_m-1} - v'(m(A^i)) \right] \sum_{j=i}^n p(A^j) \\
 & - \left[\alpha_m A^i k_t^{\alpha_k} l(A^i)^{\alpha_l} m(A^i)^{\alpha_m-1} - \left(\frac{A^i}{A^{i+1}} \right)^{\frac{1}{\alpha_m}} v' \left(\left(\frac{A^i}{A^{i+1}} \right)^{\frac{1}{\alpha_m}} m(A^i) \right) \right] \sum_{j=i+1}^n p(A^j) \\
 = & \frac{p(A^i)}{[1 - \Pr(A_t \leq A^i)]} \left[\alpha_m A^i k_t^{\alpha_k} l(A^i)^{\alpha_l} m(A^i)^{\alpha_m-1} - v'(m(A^i)) \right] \\
 & - \left[v'(m(A^i)) - \left(\frac{A^i}{A^{i+1}} \right)^{\frac{1}{\alpha_m}} v' \left(\left(\frac{A^i}{A^{i+1}} \right)^{\frac{1}{\alpha_m}} m(A^i) \right) \right] = 0. \quad (1.21)
 \end{aligned}$$

Pooling implies decreasing effort levels as firm productivity grows. The first term in square brackets must be strictly increasing in A^i in this case. At the same time, the second term in square brackets gets smaller as A^i is growing and $m(A^i)$ is falling due to the convexity of $v(m)$. If the hazard rate of $F(A)$ is non-decreasing in A^i , then the first order condition of managerial effort can never be simultaneously satisfied for two pooled types. It follows that in this case a complete separation of all unmonitored productivity types is optimal. \square

Corollary 1.4.7

Proof. Consider the first order condition of $m(A^i)$ in equation (1.21) derived in the proof to Lemma 1.4.6. For $A_t = A^n$, this equation is identical to equation (1.7). For all levels of firm productivity $A^I < A_t < A^n$, managerial effort is inefficiently low. From Lemma 1.4.4 it follows that firm output $y(A^i)$ is distorted whenever managerial effort is inefficiently low. \square

Proposition 1.4.8

Proof. Consider the consequences of increasing the threshold value A^I from A^{i-1} to A^i . The probability of monitoring rises and so does the expected amount of monitoring costs. On the upside, the range of unmonitored productivity states is reduced which lowers the extraction of information rents by the firm manager.

For $A_t = A^{i+1}$, the firm manager gets now a utility level of \underline{u} instead of:

$$c(A^{i+1}) - v(m(A^{i+1})) = \underline{u} + S(m(A^i), A^{i+1}) - S(m(A^i), A^i).$$

Similarly, if $A_t = A^{i+2}$, then the firm manager's utility is now given by:

$$c(A^{i+2}) - v(m(A^{i+2})) = \underline{u} + S(m(A^{i+1}), A^{i+2}) - S(m(A^{i+1}), A^{i+1}),$$

instead of:

$$\begin{aligned} c(A^{i+2}) - v(m(A^{i+2})) = & \underline{u} + S(m(A^i), A^{i+1}) - S(m(A^i), A^i) \\ & + S(m(A^{i+1}), A^{i+2}) - S(m(A^{i+1}), A^{i+1}). \end{aligned}$$

Information rents are reduced over the entire range of unmonitored productivity states. Accordingly, the optimal threshold value A^I is characterized by the following two conditions:

$$p(A^I)G(y(A^I)) \leq [S(m(A^I), A^{I+1}) - S(m(A^I), A^I)] \sum_{j=I+1}^n p(A^j),$$

together with:

$$p(A^{I+1})G(y(A^{I+1})) > [S(m(A^{I+1}), A^{I+2}) - S(m(A^{I+1}), A^{I+1})] \sum_{j=I+2}^n p(A^j).$$

It pays off to monitor A^I , but the additional costs of increasing the monitored range further

exceed the associated reduction in information rents. Generally, we will not always have such an interior solution $A^1 < A^I < A^n$. But if it exists, then it is straightforward to see that a high value of $G(y(A^i))$ lowers the optimal threshold value A^I . What is the role of the output elasticity α_m ? We can rewrite:

$$S(m(A^i), A^{i+1}) - S(m(A^i), A^i) = v(m(A^i)) - v\left(\left(\frac{A^i}{A^{i+1}}\right)^{\frac{1}{\alpha_m}} m(A^i)\right).$$

From equation (1.21), we know that $m(A^i)$ is strictly increasing in α_m which drives up information rents. However, there is one opposing force as:

$$\frac{\partial}{\partial \alpha_m} \left[\left(\frac{A^i}{A^{i+1}}\right)^{\frac{1}{\alpha_m}} \right] > 0.$$

But note that as $\Delta_A = A^i - A^{i-1}$ becomes small enough, this effect loses power. Consequently, for a sufficiently small step size Δ_A firm managers' information rents are increasing in α_m which renders monitoring more attractive. \square

Appendix B Model Solution and Simulation

In this appendix, I first lay out the algorithm for numerically solving the model and then explain how it is simulated to generate artificial data on the behavior of real and financial variables over time.

Solving the Model

Optimal Contract

At the heart of the model lies the optimal contract between investors and managers as a solution to the problem laid out in (1.12)-(1.15). The optimal choice of managerial effort is described by the respective first order conditions given above. Manager compensation is then pinned down by the remaining incentive and participation constraints. The optimal choice of A^I needs to be calculated using simple numerical methods.

In a first step, I solve the optimal contract conditional on some given threshold value A^I . The expected aggregate payout to investors implied by the solution to this constrained maximization problem is then compared across all different threshold values A^I . The highest value gives the solution to the optimal contract problem specified in (1.12)-(1.15). A high value of n is desirable in order to get a fine grid for A_t which allows for a precise identification of A^I . The number n is set to 100.

The optimal contract is solved once for each probability distribution $F(A_t)$. It is not necessary to solve it for different values of firm capital because the optimal contract of the model used in the numerical analysis is scale-independent. Due to constant returns to scale in the production technology and managers' linear disutility of effort, the solution to $m(A^i)$ and $c(A^i)$ is just scaled up or down by changes in capital and labor input, while the optimal threshold value A^I remains unchanged. Given a solution for some arbitrary values of capital k_t and labor l_t , the values for $m(A^i)$ and $c(A^i)$ respond to changes in

$$X = k_t^{\alpha_k} l_t^{\alpha_l}$$

according to:

$$m'(A^i) = \left(\frac{X'}{X}\right)^{\frac{1}{1-\alpha_m}} m(A^i) \quad , \text{ and: } \quad c'(A^i) = \left(\frac{X'}{X}\right)^{\frac{1}{1-\alpha_m}} c(A^i).$$

Equilibrium

The recursive optimization problem of households is given by:

$$V(W, s) = \max_{W'} \log \left(W - \frac{1}{R^{*'}} (W' - w'\bar{l}) \right) + \sum_{s'} \pi(s'|s) \beta V(W', s'),$$

where W is the household's current wealth level:

$$W = w\bar{l} + (e + d) R^* .$$

Note that in equilibrium the portfolio of the representative household is perfectly diversified and she earns the return R^* with certainty. Next period's wealth level W' depends on savings today, $W - c$, and on next period's prices $R^{*'}$ and w' . These prices depend on next period's exogenous state of uncertainty s , which is known at the time of the household's savings decision, and on next period's aggregate stock of capital k' . Taking as given the stock of aggregate capital k' in the economy as a function of the state variables W and s , the households solves the recursive problem outlined above. In equilibrium, the household's savings policy must be equal to the assumed function of aggregate capital:

$$k(W, s) = \frac{1}{R^{*'}} (W'(W, s) - w'\bar{l}).$$

The function $k(W, s)$ is initially specified by an arbitrary guess. The household's savings decision is then calculated and used to update our initial guess for $k(W, s)$. This procedure is iteratively repeated until our guess and the actual solution converge.

Simulation

Uncertainty

The source of all variation in the simulated model economies is time-varying uncertainty about firm productivity A_t . I assume $F(A_t)$ to be truncated lognormal with a constant mean of 1 and a state-dependent standard deviation of σ_t . The exogenous state of firm-level uncertainty is specified as a discrete Markov chain with nine distinct levels.

It is constructed as follows. I calculate the probability values of a lognormal distribution with a mean value of 1 and a standard deviation of 0.84 on a grid of values of firm productivity A_t . This grid is encompassing values one standard deviation below and above the mean: it is limited by the boundaries $A^1 = 0.16$ and $A^n = 1.84$. I proceed to create the corresponding probability values of a truncated lognormal distribution by scaling up the probability mass $p(A^i)$ on this grid such that:

$$\sum_{i=1}^n p(A^i) = 1 .$$

Due to the asymmetry of the lognormal density, the resulting probability distribution will generally not have a mean value of 1 anymore. In order to fix this, I multiply the discrete support of A_t : $\{A^1, A^2, \dots, A^n\}$, by

the factor:

$$f = \left[\sum_{i=1}^n p(A^i) A^i \right]^{-1}.$$

The unchanged values of $p(A^i)$ together with support $\{fA^1, fA^2, \dots, fA^n\}$ gives a truncated lognormal distribution $F(A_t)$ which has a mean value of 1 and a standard deviation of 0.46. This will be the probability distribution for the average state of firm-level uncertainty.

The other states of uncertainty differ from the average level by the variable $D(s)$. This variable increases or diminishes the support and the variance of $F(A_t)$ in the following way. The original grid, encompassing values from $A^1 = 0.16$ until $A^n = 1.84$, is expanded or truncated across different states of uncertainty according to: $A^1(s) = 0.16 - D(s)$ and $A^n(s) = 1.84 + D(s)$. The probability values $p(A^i)$ for this grid are the same as for the average state of uncertainty. In order to normalize the mean value of this distribution to 1, the discrete support of A_t : $\{A^1, A^2, \dots, A^n\}$ is multiplied by the factor $f(s)$. The unchanged values of $p(A^i)$ together with support $\{f(s)A^1(s), f(s)A^2(s), \dots, f(s)A^n(s)\}$ gives again a truncated lognormal distribution with mean 1 and a standard deviation σ_t which is higher (smaller) than 0.46 if $D(s)$ is positive (negative).

In order to define the variation of firm-level risk over time, I specify $D(s)$ as a discrete-valued Markov chain. The method of Tauchen (1986) is used to approximate an AR(1) process with an unconditional mean 0 and a standard deviation of the white noise process of 0.034. The persistence parameter is chosen as 0.97, and the grid encompasses one times the unconditional standard deviation below and above the unconditional mean 0. The number of grid points is set to 9. The nine resulting exogenous states of uncertainty feature the following levels of standard deviation: $\{0.37, 0.39, 0.41, 0.43, 0.46, 0.48, 0.50, 0.53, 0.55\}$.

Impulse Response Functions

In order to measure the typical response of real and financial variables to an increase in firm-level uncertainty, the model is simulated for 10,000 different model economies over a time period of 180 quarters each. During the first 50 quarters, each simulated model economy is following an individual stochastic time path. In the 50th quarter, agents across all 10,000 simulated economies learn that the exogenous uncertainty state will jump up by two levels in the next period (or will be equal to the highest value on the grid in case the current state of uncertainty is already on the 8th or 9th level). Investment and consumption respond on impact, while the other endogenous variables react to the shock starting from quarter 51 onwards. For all remaining time periods, the exogenous state is following an individual time path again for each model economy. Impulse responses are generated by taking the average of the variables of interest at a given point in time across all 10,000 economies.

Bibliography

- Abel, Andrew B. (1983). 'Optimal Investment Under Uncertainty,' *The American Economic Review*, Vol. 73(1), pp. 228-233.
- Adrian, Tobias, Paolo Colla, and Hyun Song Shin (2013). 'Which Financial Frictions? Parsing the Evidence from the Financial Crisis of 2007-9.' in: D. Acemoglu, J. Parker & M. Woodford (ed.), *NBER Macroeconomics Annual 2012*, Vol. 27, pp. 159-214, University of Chicago Press.
- Aghion, Philippe and Patrick Bolton (1992). 'An Incomplete Contracts Approach to Financial Contracting,' *The Review of Economic Studies*, Vol. 59(3), pp. 473-494.
- Arellano, Cristina, Yan Bai, and Patrick Kehoe (2012). 'Financial Frictions and Fluctuations in Volatility,' Federal Reserve Bank of Minneapolis Research Department Staff Report no. 466, July 2012.
- Atkeson, Andrew and Harold Cole (2008). 'A Dynamic Theory of Optimal Capital Structure and Executive Compensation,' mimeo, January 2008.
- Bachmann, Rüdiger, Steffen Elstner, and Eric R. Sims (2010). 'Uncertainty and Economic Activity: Evidence from Business Survey Data,' NBER Working Paper no. 16143, June 2010.
- Bachmann, Rüdiger and Christian Bayer (2011). 'Investment Dispersion and the Business Cycle,' NBER Working Paper no. 16861, February 2011.
- Bachmann, Rüdiger and Giuseppe Moscarini (2011). 'Business Cycles and Endogenous Uncertainty,' mimeo, July 2011.
- Bachmann, Rüdiger and Christian Bayer (2013). '“Wait-and-See” Business Cycles?' *Journal of Monetary Economics*, Vol. 60(6), pp. 704-719.
- Baker, George P. and Brian J. Hall (2004). 'CEO Incentives and Firm Size,' *Journal of Labor Economics*, Vol. 22(4), pp. 767-798.
- Baker, Scott, Nicholas Bloom, and Steven Davis (2013). 'Measuring Economic Policy Uncertainty,' Chicago Booth Research Paper no. 13-02, January 2013.
- Basu, Susanto (1996). 'Procyclical Productivity: Increasing Returns or Cyclical Utilization?' *The Quarterly Journal of Economics*, Vol. 111(3), pp. 719-751.
- Basu, Susanto and Brent Bundick (2012). 'Uncertainty Shocks in a Model of Effective Demand,' NBER Working Paper no. 18420, September 2012.
- Bebchuk, Lucian and Yaniv Grinstein (2005). 'The Growth of Executive Pay,' *Oxford Review of Economic Policy*, Vol. 21(2), pp. 283-303.
- Bernanke, Ben S. and Mark Gertler (1989). 'Agency Costs, Net Worth, and Business Fluctuations,' *The American Economic Review*, Vol. 79(1), pp. 14-31.
- Bernanke, Ben S., Mark Gertler, and Simon Gilchrist (1999). 'The Financial Accelerator in a Quantitative Business Cycle Framework,' in: J. B. Taylor & M. Woodford (ed.), *Handbook of Macroeconomics*, Ed. 1, Vol. 1, Chapter 21, pp. 1341-1393, Elsevier.
- Bernanke, Ben S. (2010). 'Restoring the Flow of Credit to Small Businesses,' Remarks at the Federal Reserve Meeting Series: 'Addressing the Financing Needs of Small Businesses', Washington, D.C., U.S., July 12, 2010.
- Biais, Bruno and Catherine Casamatta (1999). 'Optimal Leverage and Aggregate Investment,' *The Journal of Finance*, Vol. 54(4), pp. 1291-1323.

- Biais, Bruno, Thomas Mariotti, Guillaume Plantin, and Jean-Charles Rochet (2007). 'Dynamic Security Design: Convergence to Continuous Time and Asset Pricing Implications,' *The Review of Economic Studies*, Vol. 74(2), pp. 345-390.
- Bianchi, Javier (2011). 'Overborrowing and Systemic Externalities in the Business Cycle,' *The American Economic Review*, Vol. 101(7), pp. 3400-3426.
- Bloom, Nicholas and John Van Reenen (2007). 'Measuring and Explaining Management Practices across Firms and Countries,' *The Quarterly Journal of Economics*, Vol. 122(4), pp. 1351-1408.
- Bloom, Nicholas (2009). 'The Impact of Uncertainty Shocks,' *Econometrica*, Vol. 77(3), pp. 623-685.
- Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J. Terry (2012). 'Really Uncertain Business Cycles,' NBER Working Paper no. 18245, July 2012.
- Bloom, Nicholas, Benn Eifert, Aprajit Mahajan, David McKenzie, and John Roberts (2013). 'Does Management Matter? Evidence from India,' *The Quarterly Journal of Economics*, Vol. 128(1), pp. 1-51.
- Boyd, John H. and Bruce D. Smith (1998). 'The Evolution of Debt and Equity Markets in Economic Development,' *Economic Theory*, Vol. 12(3), pp. 519-560.
- Bradley, Michael, Gregg A. Jarrell, and E. Han Kim (1984). 'On the Existence of an Optimal Capital Structure: Theory and Evidence,' *The Journal of Finance*, Vol. 39(3), pp. 857-878.
- Bris, Arturo, Ivo Welch, and Ning Zhu (2006). 'The Costs of Bankruptcy: Chapter 7 Liquidation versus Chapter 11 Reorganization,' *The Journal of Finance*, Vol. 61(3), pp. 1253-1303.
- Brunnermeier, Markus K. and Yuliy Sannikov (2010). 'A Macroeconomic Model with a Financial Sector,' mimeo, May 2010.
- Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo (1993). 'Labor Hoarding and the Business Cycle,' *The Journal of Political Economy*, Vol. 101(2), pp. 245-273.
- Burnside, Craig and Martin Eichenbaum (1996). 'Factor-Hoarding and the Propagation of Business-Cycle Shocks,' *The American Economic Review*, Vol. 86(5), pp. 1154-1174.
- Campbell, John Y., Martin Lettau, Burton G. Malkiel, and Yexiao Xu (2001). 'Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk,' *The Journal of Finance*, Vol. 56(1), pp. 1-43.
- Campbell, John Y. and Glen B. Taksler (2003). 'Equity Volatility and Corporate Bond Yields,' *The Journal of Finance*, Vol. 58(6), pp. 2321-2350.
- Carlstrom, Charles T. and Timothy S. Fuerst (1997). 'Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis,' *The American Economic Review*, Vol. 87(5), pp. 893-910.
- Chang, Chun (1992). 'Capital Structure as an Optimal Contract between Employees and Investors,' *The Journal of Finance*, Vol. 47(3), pp. 1141-1158.
- Chang, Chun (1993). 'Payout Policy, Capital Structure, and Compensation Contracts when Managers Value Control,' *The Review of Financial Studies*, Vol. 6(4), pp. 911-933.
- Chari, Varadarajan V., Patrick J. Kehoe, and Ellen R. McGrattan (2007). 'Business Cycle Accounting,' *Econometrica*, Vol. 75(3), pp. 781-836.
- Christiano, Lawrence, Roberto Motto, and Massimo Rostagno (2013). 'Risk Shocks,' NBER Working Paper no. 18682, January 2013.
- Chugh, Sanjay K. (2012). 'Firm Risk and Leverage Based Business Cycles,' mimeo, July 2012.

- Clementi, Gian Luca and Thomas Cooley (2010). 'Executive Compensation: Facts,' mimeo, July 2010.
- Cole, Harold (2011). 'Self-Enforcing Stochastic Monitoring and the Separation of Debt and Equity Claims,' mimeo, November 2011.
- Covas, Francisco and Wouter J. Den Haan (2011). 'The Role of Debt and Equity Finance Over the Business Cycle,' mimeo, January 2011.
- Davis, Steven J., John Haltiwanger, Ron Jarmin, and Javier Miranda (2007). 'Volatility and Dispersion in Business Growth Rates: Publicly Traded versus Privately Held Firms,' in: D. Acemoglu, K. Rogoff & M. Woodford (ed.), *NBER Macroeconomics Annual 2006*, Vol. 21, pp. 107-180, The MIT Press.
- DeMarzo, Peter M. and Yuliy Sannikov (2006). 'Optimal Security Design and Dynamic Capital Structure in a Continuous-Time Agency Model,' *The Journal of Finance*, Vol. 61(6), pp. 2681-2724.
- DeMarzo, Peter M. and Michael J. Fishman (2007). 'Optimal Long-Term Financial Contracting,' *The Review of Financial Studies*, Vol. 20(6), pp. 2079-2128.
- Dewatripont, Mathias and Jean Tirole (1994). 'A Theory of Debt and Equity: Diversity of Securities and Manager-Shareholder Congruence,' *The Quarterly Journal of Economics*, Vol. 109(4), pp. 1027-1054.
- Diamond, Douglas W. (1984). 'Financial Intermediation and Delegated Monitoring,' *The Review of Economic Studies*, Vol. 51(3), pp. 393-414.
- Dorofeenko, Victor, Gabriel S. Lee and Kevin D. Salyer (2008). 'Time-varying Uncertainty and the Credit Channel,' *Bulletin of Economic Research*, Vol. 60(4), pp. 375-403.
- Duygan-Bump, Burcu, Alexey Levkov, and Judit Montoriol-Garriga (2011). 'Financing Constraints and Unemployment: Evidence from the Great Recession,' Federal Reserve Bank of Boston Working Paper no. QAU10-6, December 2011.
- Eisfeldt, Andrea L. and Adriano A. Rampini (2006). 'Capital Reallocation and Liquidity,' *Journal of Monetary Economics*, Vol. 53(3), pp. 369-399.
- Eisfeldt, Andrea L. and Adriano A. Rampini (2008). 'Managerial Incentives, Capital Reallocation, and the Business Cycle,' *Journal of Financial Economics*, Vol. 87(1), pp. 177-199.
- Fama, Eugene F. and Kenneth R. French (2005). 'Financing Decisions: Who Issues Stock?' *Journal of Financial Economics*, Vol. 76(3), pp. 549-582.
- Fernald, John (2012). 'Productivity and Potential Output before, during, and after the Great Recession,' Federal Reserve Bank of San Francisco Working Paper no. 2012-18, September 2012.
- Fernández-Villaverde, Jesús, Pablo Guerrón-Quintana, Juan F. Rubio-Ramírez, and Martin Uribe (2011). 'Risk Matters: The Real Effects of Volatility Shocks,' *The American Economic Review*, Vol 101(6), pp. 2530-2561.
- Fluck, Zsuzsanna (1998). 'Optimal Financial Contracting: Debt versus Outside Equity,' *The Review of Financial Studies*, Vol. 11(2), pp. 383-418.
- Friend, Irwin and Larry H.P. Lang (1988). 'An Empirical Test of the Impact of Managerial Self-interest on Corporate Capital Structure,' *The Journal of Finance*, Vol. 43(2), pp. 271-281.
- Gabaix, Xavier and Augustin Landier (2008). 'Why Has CEO Pay Increased So Much?' *The Quarterly Journal of Economics*, Vol. 123(1), pp. 49-100.
- Gale, Douglas and Martin Hellwig (1985). 'Incentive-Compatible Debt Contracts. The One-Period Problem,' *The Review of Economic Studies*, Vol. 52(4), pp. 647-663.

- Giesecke, Kay, Francis A. Longstaff, Stephen Schaefer, and Ilya Strebulaev (2013). 'Macroeconomic Effects of Corporate Default Crises: A Long-Term Perspective,' NBER Working Paper no. 17854, July 2013.
- Gilchrist, Simon, Jae W. Sim, and Egon Zakrajšek (2010). 'Uncertainty, Financial Frictions, and Investment Dynamics,' mimeo, September 2010.
- Gilchrist, Simon and Egon Zakrajšek (2012). 'Credit Spreads and Business Cycle Fluctuations,' *American Economic Review*, Vol. 102(4), pp. 1692-1720.
- Gomes, João F., Amir Yaron, and Lu Zhang (2003). 'Asset Prices and Business Cycles with Costly External Finance,' *Review of Economic Dynamics*, Vol. 6, pp. 767-788.
- Gomes, João F. and Lukas Schmid (2010). 'Equilibrium Credit Spreads and the Macroeconomy,' mimeo, July 2010.
- Gourio, François (2008). 'Estimating Firm-level Risk,' mimeo, October 2008.
- Greenwood, Jeremy, Juan M. Sanchez, and Cheng Wang (2010). 'Financing Development: The Role of Information Costs,' *American Economic Review*, Vol. 100(4), pp. 1875-1891.
- Greenwood, Jeremy, Juan M. Sanchez, and Cheng Wang (2013). 'Quantifying the Impact of Financial Development on Economic Development,' *Review of Economic Dynamics*, Vol. 16(1), pp. 194-215.
- Hartman, Richard (1972). 'The Effects of Price and Cost Uncertainty on Investment,' *Journal of Economic Theory*, Vol. 5, pp. 258-266.
- Higson, Chris, Sean Holly, and Paul Kattuman (2002). 'The Cross-sectional Dynamics of the US Business Cycle: 1950-1999,' *Journal of Economic Dynamics and Control*, Vol. 26(9), pp. 1539-1555.
- Higson, Chris, Sean Holly, Paul Kattuman, and Stelios Platis (2004) 'The Business Cycle, Macroeconomic Shocks and the Cross-Section: The Growth of UK Quoted Companies,' *Economica*, Vol. 71(282) pp. 299-318.
- Jensen, Michael C. and William H. Meckling (1976), 'Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure,' *Journal of Financial Economics*, Vol. 3(4), pp. 305-360.
- Jensen, Michael C. (1986). 'Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers,' *The American Economic Review*, Volume 76(2), pp. 323-329.
- Jermann, Urban and Vincenzo Quadrini (2012). 'Macroeconomic Effects of Financial Shocks,' *The American Economic Review*, Volume 102(1), pp. 238-271.
- Kiyotaki, Nobuhiro and John Moore (1997). 'Credit Cycles,' *Journal of Political Economy*, Vol. 105(2), pp. 211-248.
- Korteweg, Artur (2010). 'The Net Benefits to Leverage,' *Journal of Finance*, Vol. 65(6), pp. 2137-2170.
- Koufopoulos, Kostas (2009). 'Optimal Securities under Adverse Selection and Moral Hazard,' *Journal of Mathematical Economics*, Vol. 45(5), pp. 341-360.
- Lang, Jan Hannes (2012). 'The Role of Uncertainty Shocks in US Business Cycles,' mimeo, August 2012.
- Leduc, Sylvain and Zheng Liu (2013). 'Uncertainty Shocks are Aggregate Demand Shocks,' Federal Reserve Bank of San Francisco Working Paper no. 2012-10, January 2013.
- Levy, Amnon and Christopher Hennessy (2007). 'Why Does Capital Structure Choice Vary with Macroeconomic Conditions?' *Journal of Monetary Economics*, Vol. 54(6), pp. 1545-1564.

- Lorenzoni, Guido (2008). 'Inefficient Credit Booms,' *The Review of Economic Studies*, Vol. 75(3), pp. 809-833.
- Meisenzahl, Ralf R. (2011). 'Verifying the State of Financing Constraints: Evidence from US Business Credit Contracts,' Federal Reserve Board Finance and Economics Discussion Series Working Paper no. 2011-04, January 2011.
- Merton, Robert C. (1974). 'On The Pricing of Corporate Debt: The Risk Structure of Interest Rates,' *The Journal of Finance*, Vol. 29(2), pp. 449-470.
- Morellec, Erwan, Boris Nikolov, and Norman Schürhoff (2012). 'Corporate Governance and Capital Structure Dynamics,' *The Journal of Finance*, Vol. 67(3), pp. 803-848.
- Narita, Futoshi (2011). 'Hidden Actions, Risk-Taking, and Uncertainty Shocks,' mimeo, February 2011.
- Nikolov, Boris and Norman Schürhoff (2012). 'Testing Dynamic Agency Theory via Structural Estimation,' mimeo, March 2012.
- Oi, Walter Y. (1961). 'The Desirability of Price Instability under Perfect Competition,' *Econometrica*, Vol. 29(1), pp. 58-64.
- Orlik, Anna and Laura Veldkamp (2013). 'Understanding Uncertainty Shocks,' mimeo, February 2013.
- Osotimehin, Sophie (2013). 'Aggregate Productivity and the Allocation of Resources over the Business Cycle,' mimeo, August 2013.
- Quadrini, Vincenzo (2011). 'Financial Frictions in Macroeconomic Fluctuations,' *Federal Reserve Bank of Richmond Economic Quarterly*, Vol. 97(3), pp. 209-254.
- Ross, Stephen A. (1973). 'The Economic Theory of Agency: The Principal's Problem,' *American Economic Review*, Vol. 63(2), pp. 134-139.
- Schaal, Edouard (2012). 'Uncertainty, Productivity and Unemployment in the Great Recession,' mimeo, January 2012.
- Shourideh, Ali and Ariel Zetlin-Jones (2012). 'External Financing and the Role of Financial Frictions over the Business Cycle: Measurement and Theory,' mimeo, June 2012.
- Tauchen, George (1986). 'Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions,' *Economics Letters*, Vol. 20, pp. 177-181.
- Terviö, Marko (2008). 'The Difference that CEOs Make: An Assignment Model Approach,' *The American Economic Review*, Vol. 98(3), pp. 642-668.
- Townsend, Robert M. (1979). 'Optimal Contracts and Competitive Markets with Costly State Verification,' *Journal of Economic Theory*, Vol. 21, pp. 265-293.
- Van Nieuwerburgh, Stijn and Laura Veldkamp (2006). 'Learning Asymmetries in Real Business Cycles,' *Journal of Monetary Economics*, Vol. 53(4), pp. 753-772.
- Veldkamp, Laura (2005). 'Slow Boom, Sudden Crash,' *Journal of Economic Theory*, Vol. 124(2), pp. 230-257.
- Zwiebel, Jeffrey (1996). 'Dynamic Capital Structure under Managerial Entrenchment,' *The American Economic Review*, Vol. 86(5), pp. 1197-1215.

Chapter 2

Bank Opacity and Endogenous Uncertainty

2.1 Introduction

It seems to be a widely held view that the financial system is particularly opaque. Indeed, empirical evidence suggests that the profitability of banks is harder to predict for outsiders than the performance of non-financial firms.¹ Also in the aftermath of the 2008 financial crisis, insufficient transparency has frequently been put forward as an important factor to understand the origins and severity of the crisis.² Consequently, policy has implemented measures which are meant to improve public disclosure and bank transparency. The Dodd-Frank Act of 2010 requires the Federal Reserve to publish a summary of the results of its annual supervisory stress test of large bank holding companies. The published stress test results include company-specific measures of risk exposure to selected scenarios. Pillar 3 of the Basel Accords specifies public disclosure requirements for banks including information on asset holdings and risk exposure. However, these policy measures explicitly did not constitute an attempt to achieve a maximum level of transparency. The Squam Lake Report (2010) acknowledges that “it is important to protect proprietary business models and incentives to innovate. Public disclosure of a firm’s positions also raises concerns about predatory or

¹See Morgan (2002), Hirtle (2006), Iannotta (2006), or Jones, Lee and Yeager (2012). Flannery, Kwan and Nimalendran (2004) find banks not to be more opaque than non-financial firms.

²The Squam Lake Report (2010) identifies a critical lack of information about the risk exposure of financial firms. The authors argue in favor of access for regulators to more information about banks’ asset positions and risk sensitivities. They also advocate the release of this information to the public with a suitable time lag. Bernanke (2010) agrees with the perception of opaqueness as one of “the structural weaknesses in the shadow banking system”.

copycat trading by competitors". Implemented disclosure requirements reflect this warning by specifying deliberate time lags and a suitable degree of aggregation.

What does economic theory have to say about this question? In order to justify mandatory disclosure rules for banks, one needs to show that in the absence of regulation banks choose to transmit too little information to the public. So what is special about banks that makes them more opaque than non-financial firms? What exactly are the externalities which give rise to a need for policy intervention? And what is the optimal level of bank transparency? Existing contributions have focused on the particularly fragile liabilities structure of banks and the role of transparency in preventing (or triggering) socially costly banking crises. This paper adopts a different perspective and points to the asset side of banks' balance sheets. In this model, banks are special because the product they are selling to households is superior information about investment opportunities. Intransparent balance sheets turn this public good into a marketable private commodity. Unilateral disclosure of this information translates into a competitive disadvantage. Complete bank opacity is the only equilibrium in the absence of policy intervention. Households do value public information as it reduces aggregate uncertainty, but the market does not punish intransparent banks. Mandatory disclosure rules can improve upon the market allocation because of the public good character of banks' information.

Preview of the Model and Results

The banking sector is modelled as a simple duopoly with Bertrand competition. Two banks compete for households' savings. Banks use these funds to invest in some combination of riskless and risky investment projects. The more information a bank has about the future profitability of risky investment projects, the better and safer its portfolio choice will be. This information is not only valuable for banks, which choose the composition of households' investment portfolio, but also for households, who have to decide on how much to save and thereby on the aggregate size of their investment portfolio.

If households can choose between investing in a fully transparent bank, which shares all its information with the public, and a competitor bank, which remains opaque, then each single household will paradoxically choose the latter. Why is it that the socially harmful behavior of opaque banks gets rewarded by the market? The opaque bank knows more about future investment opportunities than its transparent competitor and therefore its portfolio choice will be better. The opaque bank can rely on its private information as well as on the information shared by its transparent competitor, while the transparent bank only knows its own information and does not participate in its rival's information set. In this situation,

households face a prisoner's dilemma: if they could coordinate to invest only in transparent banks, each one of them would be better off, as in this case there would be no incentive for banks to hide information from the public. But once a bank reduces disclosure, it earns a competitive advantage over the transparent rival because of its superior information and it becomes profitable for households to invest in the opaque bank. Competition between the two banks results in a "race to the bottom" which leads to complete bank opacity and a high degree of aggregate uncertainty for households.

Policy measures can improve upon this market outcome by imposing minimum disclosure requirements on banks. Complete disclosure is socially undesirable as this eliminates all private incentives to acquire costly information. The social planner chooses optimal bank transparency by trading off the benefits of reducing aggregate uncertainty for households (Blackwell effect³) against the incentives for costly information acquisition (Grossman-Stiglitz effect⁴).

Related Literature

The main argument used in support of mandatory public disclosure is improved market discipline. Public information about the expected profitability of individual banks helps financial markets to allocate resources efficiently across financial firms. Allegedly, it also prevents bank managers from excessive risk taking and thereby contributes to financial stability. These points seem to be very much in line with plain common sense and this might be the reason why economic research has tended to focus on the potential costs of financial transparency rather than on its social benefits.⁵ There are only a few examples of formal models which explain why market forces on their own are not capable of creating a sufficient level of bank transparency.

These models are generally of recent vintage. Chen and Hasan (2006) show that bank managers may want to delay disclosure in order to avoid efficient bank runs. Mandatory disclosure rules can restore market discipline in this case. An important assumption here is that bank managers cannot commit to a pre-selected timing of disclosure. This would remove the need for policy intervention.

The experience of the Financial Crisis 2007-2008 is reflected in an increased interest in

³Blackwell (1951) shows that for a single decision maker more information about fundamentals is always desirable.

⁴Grossman and Stiglitz (1976, 1980) famously demonstrate that full transparency eliminates all incentives for costly information acquisition.

⁵This is true also for two recent review articles on the trade-offs involved in financial transparency. See Landier and Thesmar (2011) and Goldstein and Sapra (2012).

the topic. In Bouvard, Chaigneau, and de Motta (2012), depositors know the health of the average bank in the economy but only a regulator knows the asset quality of each individual bank. During normal times, informational opacity prevents inefficient bank runs. However, if investors observe that the financial sector is hit by a crisis, public information about individual banks is desirable in order to prevent a run on the whole financial system. Only the regulator can provide this information, as banks' announcements are not verifiable. A similar result is found by Spargoli (2012). During normal times, there is no policy need as banks with high quality assets can separate themselves from low quality banks. However, during a financial crisis separation becomes too costly and financial markets are unable to discriminate between banks of different quality.

Also Alvarez and Barlevy (2012) study an endogenous lack of information about the location and size of bank losses. Banks form a financial network in this model which exposes them to the credit risk of their counterparties. This gives rise to an information externality as information about the financial health of one bank is also valuable with respect to the risk exposure of its counterparties. Crucial for the authors' results is an exogenous fixed cost of public disclosure.

The contributions cited above do not model banks' portfolio choice and there is no feedback effect from public disclosure to a bank's market share and the quality of its assets. In contrast, this paper introduces the problem of costly information acquisition to the analysis which endogenizes the costs of public disclosure.

As mentioned above, the social costs of bank transparency have been studied at least as extensively as the potential benefits. For instance, Moreno and Takalo (2012) find that negative spillovers of bank failures result in an oversupply of voluntary disclosure. If anything, policy should induce banks to disclose less information to the public than they would like to. Also Dang, Gorton, Holmström and Ordoñez (2013) warn of the perils of bank transparency. In their model, it is precisely the role of banks to collect socially valuable information about asset quality without disclosing it to the public. The negative role of public information in this model is related to Hirshleifer (1971).⁶ Consumers are exposed to liquidity shocks. This makes them unwilling to invest in risky projects if information about project losses become public. A bank which can hide these project losses from the public is able to shut down the Hirshleifer effect and to channel households' savings to investment projects. Banks allow households to share both the risks of production and of stochastic liquidity needs. More opacity is better in this environment.

⁶Hirshleifer (1971) shows that disclosure is socially harmful whenever its primary effect is to redistribute wealth among agents.

Pagano and Volpin (2012) address the phenomenon of intransparent securities traded on secondary markets rather than intransparent bank balance sheets. Also here banks can increase liquidity through opacity. But in contrast to the findings of Dang, Gorton, Holmström and Ordoñez (2013), imposing mandatory disclosure rules can be welfare increasing in Pagano and Volpin (2012). The authors study the problem of a bank which offers asset-backed securities of heterogeneous quality. The quality of these securities is unknown to the bank. The fact that sophisticated investors can learn the quality of these securities renders them unattractive for unsophisticated potential buyers. The bank can increase the liquidity of its securities in this case by rendering them intransparent and hard to assess even for sophisticated investors. But this might create a problem of adverse selection on a secondary market triggering social costs which the issuing bank does not fully internalize.

In Kurlat and Veldkamp (2012), a risky asset in fixed supply is sold on a market consisting of rational investors and noise traders. The price-insensitive noise traders systematically lose money as they move the asset return against themselves. The sensitivity of the asset return to noise demand is increasing in uncertainty. This is because uncertainty about asset quality increases the price of arbitrage performed by rational investors. The option of rational investors to respond with their demand to the actions of noise traders introduces a convexity to their objective function which makes them effectively risk-loving. Public disclosure reduces uncertainty and therefore also the opportunity of investors to benefit from noise traders' erratic actions. This result is overcome in case of an equilibrium with asymmetric information among rational investors. Noise traders always benefit from public disclosure.

The setup used by Kurlat and Veldkamp (2012) relates to earlier contributions by Admati and Pfleiderer (1988, 1990). These authors consider the problem of a single agent with an exogenous endowment of socially valuable information. They show that under certain conditions the information monopolist may find it profitable to act as a financial intermediary for uninformed investors. Admati and Pfleiderer (1988, 1990) and Kurlat and Veldkamp (2012) differ from our model in the assumption that assets are not in perfectly elastic supply and therefore asset prices partially reveal information. Furthermore, these contributions do not consider the endogenous production of information nor the role of competition in determining its supply to the public.

This paper is also related to the more general role of public information in shaping market outcomes. Morris and Shin (2002) study the social value of public information in an environment prone to coordination failures. Whenever public information is sufficiently imprecise, this impedes social coordination and can be welfare decreasing. In the model studied

below, coordination failures play no role for the analysis of bank transparency. Vives (2012) examines a general setting in which agent's actions are partly reflected by a public signal. He finds that the precision of public information always improves the market allocation.

While the formal analysis of public disclosure is a fairly recent phenomenon in the banking context, it can build on an extensive tradition in the literature on corporate finance and accounting. This literature has generally acknowledged that even in the absence of policy intervention, there are good reasons to expect a considerable degree of voluntary disclosure by firms which compete for funds on capital markets (see for instance Grossman and Hart, 1981). Diamond (1985) shows that public disclosure is preferred by shareholders because it prevents investors from wasting resources on private information acquisition.

We have seen above that existing models of bank transparency abstract from the costs of releasing proprietary information. This is at odds with the central role which is generally attributed to confidential information in banking services. In the context of non-financial firms, proprietary information has been considered by the accounting literature from very early on. Verrecchia (1983) studies the trade-off between transparency and an exogenous fixed proprietary cost of information disclosure. A similar trade-off is examined by Dye (1986). Darrough and Stoughton (1990) endogenize the private costs of proprietary disclosure in an entry game. However, these models do not allow for a formal welfare analysis of eventual policy interventions.

Information externalities as a justification of mandatory disclosure rules are considered in an early contribution by Foster (1980). In a formal model, Dye (1990) demonstrates that in the presence of externalities (e.g. due to proprietary information) mandatory and voluntary disclosure tend to diverge. Likewise, Admati and Pfleiderer (2000) study information externalities. Since there are private costs to increasing the precision of public signals, the supply of public information is inefficiently low in their model. These models are tailored primarily to non-financial firms and do not capture the peculiarities of the financial sector which are examined below.

Outline

The rest of the paper is organized as follows. The model is set up in section 2.2. Section 2.3 characterizes the equilibrium allocation on the market for financial intermediation in the absence of mandatory disclosure rules. Optimal bank transparency is studied in section 2.4. Section 2.5 concludes the paper with a short discussion of potential enhancements of the model.

2.2 Model Setup

Consider a simple model economy inhabited by many small and identical households of unit mass. Households aim at smoothing consumption over time by investing in two different banks. These two banks have access to risky investment opportunities.

2.2.1 Households

In period t , the representative household owns a certain amount w_t of the the numéraire good. She decides how to allocate consumption over time. Her preferences regarding any consumption path $\{c_{t+i}\}_{i=0}^{\infty}$ may be described by the function:

$$\mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}) \middle| \mathbb{Q}_t^H \right\}, \quad (2.1)$$

where \mathbb{E} is the expectation operator conditional on the date t information of the representative household \mathbb{Q}_t^H , and $\beta \in [0, 1]$ gives the rate of time preference. The function $u : [0, \infty] \rightarrow \mathbb{R}$ is increasing, strictly concave and satisfies the Inada conditions. In addition, we assume non-increasing absolute risk aversion. This implies: $u'''(c) > 0$.

Households have no direct access to investment projects. They can invest in the two banks which are active in this model economy. Accordingly, the household's budget constraint is given by:

$$c_t + b_{t+1}^A + b_{t+1}^B \leq b_t^A r_t^A + b_t^B r_t^B + T_t^A + T_t^B \equiv w_t, \quad (2.2)$$

where b_t^A and b_t^B indicate the amount of securities bought from bank A and bank B , respectively. The associated gross returns are indicated by r_t^A and r_t^B . The households are the joint owners of the two banks. Accordingly, eventual bank profits T_t^A and T_t^B are uniformly distributed among households.

2.2.2 Banks

In contrast to households, banks have access to risky investment projects in addition to common storage. These projects are completely homogeneous and in perfectly elastic supply. The return to risky investment projects is perfectly correlated across projects. Return risk is therefore systematic and not insurable.⁷ Banks spend resources in order to learn about the future performance of these risky projects. They maximize the expected utility of their

⁷Alternatively, one could think of a single risky project with a linear return.

owners as given by (2.1) subject to the budget constraint:

$$T_{t+1}^j = (b_{t+1}^j - k_{t+1}^j) + k_{t+1}^j R_{t+1} - b_{t+1}^j r_{t+1}^j - g_{t+1}^j, \quad \text{for } j = A, B. \quad (2.3)$$

Bank j manages an amount b_{t+1}^j of the numéraire good lent to it by households. The amount of funds invested in risky projects by bank j is indicated by k_{t+1}^j . These funds yield an uncertain return of R_{t+1} . The remainder $(b_{t+1}^j - k_{t+1}^j)$ is put into riskless storage. Resources spent on learning about R_{t+1} are given by g_{t+1}^j .

2.2.3 Projects and Information

The gross return on risky projects is persistent over time:

$$R_{t+1} = \zeta_0 + \zeta_1 R_t + \varepsilon_{t+1},$$

where $\zeta_0 > 0$, $0 < \zeta_1 < 1$, and $\varepsilon_{t+1} \sim \mathcal{N}(0, \sigma^2)$. All agents in the economy, households and bankers, publicly observe R_t after it is realized. In addition to R_t , each bank observes at time t also a second signal \hat{R}_{t+1}^j which likewise contains information about R_{t+1} :

$$\hat{R}_{t+1}^j \sim \mathcal{N}(R_{t+1}, \Sigma_{t+1}^j), \quad \text{for } j = A, B.$$

The precision of this additional signal can be improved at a cost:

$$\Sigma_{t+1}^j = \frac{1}{f(g_{t+1}^j)}, \quad \text{for } j = A, B,$$

where $f(g)$ is increasing, strictly concave and satisfies the Inada conditions. That is, $f(0) = 0$ and zero expenditures on signal precision result in a bank signal which does not contain any information about R_{t+1} . If a bank acquires a lot of information, this informational advantage might result in a certain degree of market power. In order to protect it, a bank may choose to hide its current portfolio choice from its competitor. We assume that a bank is able to costlessly hide its current investment policy from outsiders. This is important, because a bank's portfolio choice could reveal its private information about future project returns.

Informational opacity of bank balance sheets may protect a bank's market power but it also creates additional uncertainty for households. If banks wish to reveal some part of their superior information, they can use a costless signal which is transmitted to the public:

$$Q_{t+1}^j \sim \mathcal{N}(\hat{R}_{t+1}^j, \hat{\Sigma}_{t+1}^j), \quad \text{for } j = A, B.$$

In this manner, banks are free to give away any part of their informational advantage to the public. A perfect correlation between Q_{t+1}^j and \hat{R}_{t+1}^j ($\hat{\Sigma}_{t+1}^j = 0$) corresponds to complete transparency and consequently also zero uncertainty for outsiders about bank j 's current portfolio choice. On the other hand, zero correlation ($\hat{\Sigma}_{t+1}^j = \infty$) is equivalent to complete opacity of bank j 's balance sheet and a maximum level of information asymmetry.

2.2.4 Timing

The timing is as follows. Bank A and bank B enter period t with a predetermined portfolio of riskless storage and risky investment projects. At the beginning of period t , the gross return R_t is realized and publicly observed by all agents in the economy. Households who invested in bank j last period receive a cash flow of $b_t^j r_t^j$ in return. Eventual bank profits are distributed among households. Banks choose how much resources g_{t+1}^j to spend on information acquisition and they choose how much of this information to share with others. Private and public signals of the future return are realized. Households divide their wealth between consumption and bank investment. The two banks A and B choose a portfolio of investment projects and storage.

2.3 Equilibrium

Definition Given some initial wealth level of households w_t , a competitive equilibrium in this economy consists of values for $\hat{\Sigma}_{t+i+1}^A$ and $\hat{\Sigma}_{t+i+1}^B$, of prices r_{t+i+1}^A and r_{t+i+1}^B , and quantities c_{t+i} , b_{t+i+1}^j , k_{t+i+1}^j , g_{t+i+1}^j , T_{t+i+1}^j , for $j = A, B$ and $i = 0, 1, 2, \dots$, such that for all histories: (1.) households solve their individual optimization problem, (2.) bank A and bank B maximize the expected utility of households subject to price competition in the market for financial intermediation, and (3.) the market for financial intermediation clears.

Households observe the performance of the bank's chosen portfolio of intermediated funds:

$$\Pi_{t+1}^j = \frac{(b_{t+1}^j - k_{t+1}^j) + k_{t+1}^j R_{t+1}}{b_{t+1}^j}, \quad \text{for } j = A, B.$$

Hence, the return on bank securities r_{t+1}^j can condition on this information. In principle, the remuneration of banks for providing financial services could take on many forms. In the following, we consider contracts of financial intermediation of the following class:

Assumption Households' return on bank funding is given by $r_{t+1}^j = \Pi_{t+1}^j - \delta_{t+1}^j$, where δ_{t+1}^j is a non-negative scalar which is known at time t with certainty.

This assumption is without loss of generality. Since households are both the holders of bank securities as well as the owners of the two banks, no insurance contract between households and banks can be profitable. Ultimately, households bear all the risk associated with R_{t+1} , no matter how it is divided between banks and households. It follows that a non-stochastic price of banking services δ_{t+1}^j is optimal.

The model economy described above may be understood as a team decision problem as defined by Marschak (1955) and Radner (1962). Households and banks pursue a common objective function by maximizing expected lifetime utility of households. To this end, households choose a consumption and savings policy, while banks invest in information acquisition, decide on how much of this information to share with other agents, set a price of financial intermediation, and select an investment portfolio on the basis of the information available to them. In principle, it would be desirable in this environment of costless communication that every agent knows all information available in the model economy at any given point in time. In the following, we will see that bank competition in combination with the public good character of information puts severe restrictions on the information allocations which are compatible with a competitive equilibrium.

2.3.1 Households

In period t , the representative household divides her wealth w_t between consumption and risky bank securities:

$$\begin{aligned} \max_{c_t, b_{t+1}^A, b_{t+1}^B \in \mathbb{R}_{\geq 0}} \quad & \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}) \mid Q_{t+1}^A, Q_{t+1}^B, R_t \right\} \\ \text{subject to:} \quad & c_{t+i} + b_{t+i+1}^A + b_{t+i+1}^B \leq b_{t+i}^A r_{t+i}^A + b_{t+i}^B r_{t+i}^B + T_{t+i}^A + T_{t+i}^B \equiv w_{t+i}. \end{aligned}$$

The information $\mathbb{Q}_t^H = \{Q_{t+1}^A, Q_{t+1}^B, R_t\}$ on which her decision at time t is based depends on the quality of information collected by banks as well as on the precision of Q_{t+1}^A and Q_{t+1}^B , i.e. bank transparency. More precise information reduces the exposure of households' consumption plans to aggregate uncertainty. The intertemporal Euler equation is given by:

$$u'(c_t) = \beta \mathbb{E} \left\{ u'(c_{t+1}) r_{t+1}^j \mid Q_{t+1}^A, Q_{t+1}^B, R_t \right\}, \quad \text{for } j = A, B.$$

Households demand bank securities with high and safe returns.

2.3.2 Banks: Exogenous Transparency

Consider first bank behavior for the special case that exogenously $\hat{\Sigma}_{t+1}^A = \hat{\Sigma}_{t+1}^B = 0$. There is no asymmetry of information in this economy, as Q_{t+1}^A and Q_{t+1}^B are perfect signals of \hat{R}_{t+1}^A and \hat{R}_{t+1}^B . Consequently, all agents share identical expectations about the distribution of future project returns. Bayesian inference yields as the updated probability distribution of future project returns:

$$R_{t+1} \sim \mathcal{N}\left(\mathbb{E}\{R_{t+1} \mid \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t\}, \frac{\sigma^2 \Sigma_{t+1}^A \Sigma_{t+1}^B}{\sigma^2 \Sigma_{t+1}^A + \sigma^2 \Sigma_{t+1}^B + \Sigma_{t+1}^A \Sigma_{t+1}^B}\right).$$

The optimal portfolio choice by banks is perfectly inferable for everyone. In this sense, banks' balance sheets are completely transparent.

The two bankers play a Bertrand game. Their intermediation services are perfect substitutes, as both banks have access to the same information set. Hence, also their portfolio choice and the distribution of future bank returns are identical. If the two banks charge the same price $\delta_{t+1}^A = \delta_{t+1}^B$, we assume that the households' demand is split evenly between them. The resulting equilibrium allocation shows a number of characteristics which are familiar from the literature on Bertrand competition games.

Lemma 2.3.1. *In equilibrium, both banks make exactly zero profits: $T_{t+1}^j = 0$, which implies: $b_{t+1}^j \delta_{t+1}^j = g_{t+1}^j$, for $j = A, B$. They both choose a portfolio of intermediated funds which maximizes the expected utility of households subject to the available information.*

Proof. The proof works by contradiction. Consider first equilibrium bank profits.

1. *Banks make exactly zero profits:* Assume that bank A makes positive profits. In this case, bank B can capture the whole demand for financial intermediation by choosing the same portfolio as bank A and charging $\delta_{t+1}^B = \delta_{t+1}^A - \eta$ (with $\eta > 0$). If η is sufficiently small, this increases bank B 's profit. This excludes the possibility that banks make positive profits in equilibrium.
2. *Banks choose a portfolio which maximizes the expected utility of households:* Assume that bank A chooses a portfolio which does not maximize the expected utility of households, taking as given banks' expenditures g_{t+1}^A and g_{t+1}^B . In this case, bank B can choose a portfolio which caters more to the needs of households. Note that there is an $\eta > 0$, such that bank B charges a spread $\delta_{t+1}^B = \delta_{t+1}^A + \eta$ and still captures the complete demand for financial intermediation. Bank B makes positive profits in this case.

This excludes the possibility that banks choose a portfolio which does not maximize the expected utility of households. □

What does this imply for banks' investment policy? Bank j chooses its portfolio of investment projects according to:

$$\begin{aligned} & \max_{k_{t+1}^j \in \mathbb{R}_{\geq 0}} \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}) \mid \mathbb{Q}_t \right\} \\ \text{subject to: } & c_{t+1} + b_{t+2}^A + b_{t+2}^B \leq b_{t+1}^A (\Pi_{t+1}^A - \delta_{t+1}^A) + b_{t+1}^B (\Pi_{t+1}^B - \delta_{t+1}^B), \\ & b_{t+1}^j (\Pi_{t+1}^j - \delta_{t+1}^j) = (b_{t+1}^j - k_{t+1}^j) + k_{t+1}^j R_{t+1} - g_{t+1}^j, \quad \text{for } j = A, B, \\ & \mathbb{Q}_t = \{ \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t \}, \text{ and} \\ & \hat{R}_{t+1}^j \sim \mathcal{N}(R_{t+1}, \Sigma_{t+1}^j), \quad \text{with: } \Sigma_{t+1}^j = \frac{1}{f(g_{t+1}^j)}, \quad \text{for } j = A, B. \end{aligned}$$

The chosen portfolio is characterized by the following first order condition:

$$\mathbb{E} \left\{ \frac{\partial u(c_{t+1})}{\partial c_{t+1}} (R_{t+1} - 1) \mid \mathbb{Q}_t \right\} = 0. \quad (2.4)$$

Some part of the risk associated with investment projects is endogenous, as banks can spend resources to reduce uncertainty. Proposition 2.3.2 describes the market allocation of information expenditures.

Proposition 2.3.2. *In equilibrium, uncertainty about future project returns is maximum: $g_{t+1}^A = g_{t+1}^B = 0$. This implies: $\Sigma_{t+1}^A = \Sigma_{t+1}^B = \infty$.*

Proof. Assume that bank A spends $g_{t+1}^A > 0$ on reducing public uncertainty about future project returns. In this case, also bank B must spend $g_{t+1}^B = g_{t+1}^A$ on information acquisition in equilibrium. Otherwise, one bank could charge a lower spread than the other bank and make positive profits.

Consider now an equilibrium with $g_{t+1}^A = g_{t+1}^B > 0$. In this case, bank A can reduce g_{t+1}^A somewhat and charge $\delta_{t+1}^A = \delta_{t+1}^B - \eta$ (for $\eta > 0$). Uncertainty is higher now and total demand for bank securities lower. But bank B 's forecast is hurt by this in the same way as bank A 's prediction of future returns. Hence, bank A captures the whole demand for

bank securities. If η is sufficiently small, this increases bank A 's profit. This excludes the possibility that $g_{t+1}^A > 0$ or $g_{t+1}^B > 0$ in equilibrium. \square

The public signals \hat{R}_{t+1}^A and \hat{R}_{t+1}^B provided by banks contain no information at all: $\mathbb{E}\{R_{t+1} | \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t\} = \mathbb{E}\{R_{t+1} | R_t\}$. Precision of the public signal \hat{R}_{t+1}^j is a public good. If bank A spends resources on improving its signal, this increases the information set for bank A in the same way as for bank B (as well as for all the households). Bertrand competition between the two banks does not permit bank A to incur these extra costs, which reduce the return on its securities but which do not translate into a competitive advantage with respect to bank B . Atomistic bank investors do not internalize that their investment behavior influences the quality of public information in this economy.

A social planner would choose the precision of the public signals \hat{R}_{t+1}^A and \hat{R}_{t+1}^B by solving the following optimization program:

$$\begin{aligned} \max_{g_{t+1}^A, g_{t+1}^B \in \mathbb{R}_{\geq 0}} \quad & \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^*) \mid R_t \right\} \\ \text{subject to:} \quad & c_{t+i}^* = (b_{t+i}^* - k_{t+i}^*) + k_{t+i}^* R_{t+i} - b_{t+i+1}^* - g_{t+i}^A - g_{t+i}^B, \\ & b_{t+i+1}^* = b(\hat{R}_{t+i+1}^A, \hat{R}_{t+i+1}^B, R_{t+i}), \quad k_{t+i+1}^* = k(\hat{R}_{t+i+1}^A, \hat{R}_{t+i+1}^B, R_{t+i}), \\ & \text{and } \hat{R}_{t+i+1}^j \sim \mathcal{N}(R_{t+i+1}, \Sigma_{t+i+1}^j), \quad \text{with: } \Sigma_{t+i+1}^j = \frac{1}{f(g_{t+i+1}^j)}, \text{ for } j = A, B. \end{aligned}$$

At that point in time when the planner chooses g_{t+1}^A and g_{t+1}^B , she anticipates the benefits of observing more reliable signals \hat{R}_{t+1}^A and \hat{R}_{t+1}^B . The amount of savings b_{t+1}^* and the investment policy k_{t+1}^* can both be set more precisely when information is better. Proposition 2.3.3 describes the solution to this problem.

Proposition 2.3.3. *The first best level of information expenditures is positive:*

$$0 < g_{t+1}^{A*} = g_{t+1}^{B*} < \infty.$$

A proof of this proposition can be found in Appendix A. But the result is quite intuitive. The first marginal unit of resources spent on information acquisition has a very high marginal impact on the precision of the respective signal. This reduction in aggregate uncertainty is valuable as it allows for a more precise savings decision by the planner. Only an interior choice can be optimal as the marginal impact of an additional increase in information expenditures

is falling towards zero. The concavity of $f(g)$ implies that the planner will optimally invest equal amounts in the precision of both signals.

A high degree of risk aversion increases the marginal benefit of an additional unit of the numéraire good spent on information acquisition. Likewise, high uncertainty, e.g. because of a high value of σ^2 , and the efficiency of learning, as measured by the steepness of $f(g)$, contribute to a high optimal level of information expenditures.

We have seen how the planner chooses the optimal level of signal precision. The market allocation under full transparency of bank balance sheets falls short of this first best level of public information. Expectations about the future are perfectly homogeneous across all agents in the model economy, but these expectations are based on a minimum amount of information.

2.3.3 Banks: Endogenous Transparency

So far we have assumed that banks' balance sheets are completely transparent and everybody can infer banks' expectations about future returns. Now, we consider the more general case which allows banks to choose the precision of their public signals Q_{t+1}^A and Q_{t+1}^B themselves. Proposition 2.3.4 states that banks will always choose a maximum level of informational opacity if they are free to do so.

Proposition 2.3.4. *In equilibrium, households' uncertainty about future project returns is maximum: $\hat{\Sigma}_{t+1}^A = \hat{\Sigma}_{t+1}^B = \infty$.*

Proof. Assume that bank A and bank B spend any amount g_{t+1}^A and g_{t+1}^B on improving the precision of the signals \hat{R}_{t+1}^A and \hat{R}_{t+1}^B . By reducing the precision of Q_{t+1}^A , bank A can costlessly reduce the precision of bank B 's forecast $\mathbb{E}\{R_{t+1} - 1 | \mathbb{Q}_t^B\}$. Bank A 's forecast remains unaffected by the precision of Q_{t+1}^A . Since households observe the precision of the banks' signals Q_{t+1}^A and Q_{t+1}^B and information expenditures g_{t+1}^A and g_{t+1}^B , they know the forecast accuracy of banks. Ceteris paribus, households buy securities of the bank with more information about future project returns. This gives bank A a strong incentive to marginally decrease the precision of its signal Q_{t+1}^A . Bank B in turn can regain competitiveness by reducing the precision of Q_{t+1}^B . The only equilibrium allocation is given by $Q_{t+1}^A = Q_{t+1}^B = \infty$. \square

Transparency implies that bank j 's signal \hat{R}_{t+1}^j is public information. By keeping an opaque balance sheet, the information of bank j 's private signal becomes private. This does not change bank j 's information set, but it creates more uncertainty for the competitor bank.

Households like transparency, but the market does not punish a bank for being opaque if this bank has more information about future returns than the competitor bank. As information is private now, does this provide incentives for banks to invest in information about future returns? On the one hand, bank A securities lose in value as information expenditures are costs which depress the return. On the other hand, bank A 's portfolio choice $k(\hat{R}_{t+1}^A, R_t)$ benefits from the higher precision of \hat{R}_{t+1}^A :

$$b_{t+1}^A r_{t+1}^A = b_{t+1}^A [\Pi_{t+1}^A - \delta_{t+1}^A] = b_{t+1}^A + k(\hat{R}_{t+1}^A, R_t) [R_{t+1} - 1] - g_{t+1}^A.$$

Under Bertrand competition, bank A 's market share is extremely sensitive to the attractiveness of its intermediation services in comparison with the rival bank. Therefore, bank A invests in information acquisition in order to increase households' valuation of bank A securities relative to bank B securities:

$$\begin{aligned} & \max_{g_{t+1}^A \in \mathbb{R}_{\geq 0}} \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \middle| R_t \right\} - \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^B) \middle| R_t \right\} \\ \text{subject to: } & c_{t+i}^j = b(R_{t+i-1}) + k(\hat{R}_{t+i}^j, R_{t+i-1}) [R_{t+i} - 1] - b(R_{t+i}) - g_{t+i}^j, \\ & \text{and } \hat{R}_{t+1}^j \sim \mathcal{N}(R_{t+1}, \Sigma_{t+1}^j), \quad \text{with: } \Sigma_{t+1}^j = \frac{1}{f(g_{t+1}^j)}, \text{ for } j = A, B. \end{aligned}$$

Proposition 2.3.5. *Under full opacity, banks' investment in information about future returns is higher than the first best allocation under full transparency:*

$$0 < g_{t+1}^{A*} = g_{t+1}^{B*} < g_{t+1}^{A**} = g_{t+1}^{B**} < \infty.$$

The formal proof of Proposition 2.3.5 is deferred to the appendix. Bank opacity provides an environment in which it is profitable for banks to invest in the precision of their private signals. They even spend more resources on information acquisition than a planner would choose to in a world of complete transparency. Under transparency, one unit of the numéraire good spent on informational precision improves the portfolio choice of both banks as well as households' savings decision. In the opacity case, each bank observes only its own signal and households do not learn anything about R_{t+1} in addition to the observation of R_t . Therefore, a given level of information expenditures results in a much higher level of uncertainty under opacity than in the case of complete transparency. As the marginal value of information expenditures is increasing in uncertainty, this leads to the result of overproduction of information in combination with an undersupply of communication.

2.4 Optimal Opacity

From the previous analysis, it has become clear that there is a trade-off between information production and information transmission. Maximum transmission induces minimum production and vice versa. The problem of bank regulation is to find an intermediate level of bank opacity which sacrifices some degree of information production by banks in favor of a reduced level of uncertainty for households. Consider bank A 's optimal choice of g_{t+1}^A for some intermediate level of opacity $\hat{\Sigma}_{t+1}$:

$$\begin{aligned} & \max_{g_{t+1}^A \in \mathbb{R}_{\geq 0}} \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \middle| R_t \right\} - \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^B) \middle| R_t \right\} \\ \text{subject to: } & c_{t+i}^j = b(Q_{t+i-1}^H) + k(Q_{t+i-1}^j) [R_{t+i} - 1] - b(Q_{t+i}^H) - g_{t+i}^j, \\ & Q_{t+i}^H = \{Q_{t+i+1}^A, Q_{t+i+1}^B, R_{t+i}\}, \quad Q_{t+i}^A = \{\hat{R}_{t+i+1}^A, Q_{t+i+1}^B, R_{t+i}\}, \\ & Q_{t+i}^B = \{Q_{t+i+1}^A, \hat{R}_{t+i+1}^B, R_{t+i}\}, \quad Q_{t+1}^j \sim \mathcal{N}(\hat{R}_{t+1}^j, \hat{\Sigma}_{t+1}), \\ & \text{and } \hat{R}_{t+1}^j \sim \mathcal{N}(R_{t+1}, \Sigma_{t+1}^j), \quad \text{with: } \Sigma_{t+1}^j = \frac{1}{f(g_{t+1}^j)}, \text{ for } j = A, B. \end{aligned}$$

Each bank optimally invests more in information as the informational spillovers to its rival get reduced through increased opacity.

Lemma 2.4.1. *Banks' investment in information about future returns is strictly increasing in $\hat{\Sigma}_{t+1}$. Information expenditures become less sensitive as opacity tends towards infinity:*

$$\lim_{\hat{\Sigma}_{t+1} \rightarrow \infty} g'(\hat{\Sigma}_{t+1}) = 0.$$

A proof of this lemma can be found in Appendix A. Under complete transparency (i.e. $\hat{\Sigma}_{t+1} = 0$), investments in information benefit the rival bank just as much as the bank which actually pays for the improvements in public information. This public good character of information renders its costly acquisition unprofitable. As $\hat{\Sigma}_{t+1}$ is growing, the rival bank participates less and less in improvements to bank A 's information set. The optimal choice of information expenditures increases in value until it converges to the solution to the bank's problem under complete opacity as derived above. For high values of opacity, the signal-to-noise ratio of banks' public signals becomes less and less responsive to additional changes in opacity. This is reflected by the vanishing sensitivity of $g(\hat{\Sigma}_{t+1})$.

Bank competition in combination with the public good character of information puts severe restrictions on the feasible allocations in this economy. The problem of the social planner is now to set an optimal level of bank transparency. By changing the information structure, the planner can indirectly influence the equilibrium outcome. The optimal choice trades off two effects: more transparency (1.) reduces the information asymmetry between households and banks, but it also (2.) results in less production of socially valuable information by banks.

$$\begin{aligned} & \max_{\hat{\Sigma}_{t+1} \in \mathbb{R}_{\geq 0}} \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \mid \mathbb{Q}_t^H \right\} \\ \text{subject to: } & c_{t+i}^A = b(\mathbb{Q}_{t+i-1}^H) + k(\mathbb{Q}_{t+i-1}^A) [R_{t+i} - 1] - b(\mathbb{Q}_{t+i}^H) - g(\hat{\Sigma}_{t+1}), \\ & \mathbb{Q}_{t+i}^H = \{Q_{t+i+1}^A, Q_{t+i+1}^B, R_{t+i}\}, \quad \mathbb{Q}_{t+i}^A = \{\hat{R}_{t+i+1}^A, Q_{t+i+1}^B, R_{t+i}\}, \\ & Q_{t+1}^j \sim \mathcal{N}(\hat{R}_{t+1}^j, \hat{\Sigma}_{t+1}) \quad , \text{ and} \\ & \hat{R}_{t+1}^j \sim \mathcal{N}(R_{t+1}, \Sigma_{t+1}^j), \quad \text{with: } \Sigma_{t+1}^j = \frac{1}{f(g(\hat{\Sigma}_{t+1}))} \quad , \text{ for } j = A, B. \end{aligned}$$

Recall that households regard both banks as equally well informed in equilibrium. Therefore, the two types of bank securities are perfect substitutes. Without loss of generality, here we will consider the impact of changes in opacity on the expected value of bank *A* securities. The same reasoning holds for the case of bank *B* securities. Proposition 2.4.2 states that the optimal degree of bank transparency has no corner solution. It must therefore differ from the market allocation.

Proposition 2.4.2. *The socially optimal choice of bank opacity is $0 < \hat{\Sigma}_{t+1}^* < \infty$.*

A formal proof of Proposition 2.4.2 is deferred to Appendix A. In the neighborhood of complete transparency, a local increase in opacity actually decreases uncertainty. This is because the positive effect of opacity on information production outweighs the increase in noise of banks' public signals. The opposite is true for high levels of opacity. Here, a marginal reduction of opacity reduces aggregate uncertainty for households without affecting information production by banks in any significant way.

Proposition 2.4.2 demonstrates the potential gains from policy intervention. Imposing minimum transparency requirements on banks leads to a Pareto improvement in this environment. The optimal degree of bank transparency generally depends on the functional

form of $f(g)$ which determines the social value of costly information acquisition by banks. If this function is very steep, then society has a lot to lose from reductions in information expenditures by banks and the optimal level of bank transparency will be relatively low. The same is true for high levels of fundamental uncertainty (σ^2) and risk aversion, as these two factors likewise increase the social benefit of costly information acquisition. Note however one interesting aspect of bank opacity in general: an increase in the degree of asymmetry of information between households and banks may result in a welfare gain.

2.5 Discussion

We have seen that the private costs of public disclosure of banks' asset positions and risk exposure are particularly high if proprietary information becomes public. The mechanism described above applies to a wide range of credit decisions and asset classes. The problem of opacity becomes particularly severe whenever (1.) bank competition is fierce, and whenever (2.) investment in information acquisition by banks can result in a considerable information advantage.

Note that bank competition is part of the problem in this model and not part of the solution. The equilibrium allocation does not change qualitatively whether two banks compete for households' savings or a large number of N banks. On the other hand, a monopolist banker in a non-contestable market for financial intermediation would be in a position to reveal all available information to the public without the threat of adverse consequences for her market share. The inefficiencies with respect to the supply of public information described above would cease to exist. However, other well-known inefficiencies are bound to arise in the presence of market power.

In the analysis above it is assumed that the information which banks choose to transmit to the markets are verifiable. In practice, banks report summary statistics of aggregated asset positions and risk sensitivities estimated for selected scenarios. These reporting instruments still leave some room for financial window dressing. This may even be intended by regulation as complete transparency is not desirable. On the other hand, information about asset positions is socially valuable to the extent that the risk characteristics of the products held by banks are understood by the public. If opacity results in a competitive advantage, then we should expect banks to invest resources in the development of assets which are hard to understand and to value for competitor banks. Cheng, Dhaliwal and Neamtiu (2008) find that empirically banks that engage in securitization transactions are more opaque than banks with no asset securitizations.

The review of the related literature has demonstrated that existing approaches to bank transparency have found that not only the asset side of banks' balance sheet but also the particularities of their liability structure yields interesting implications for the problem of optimal bank transparency. The model outlined above is sufficiently general to encompass a wide range of financial intermediaries (e.g. mutual funds, hedge funds). Arguably, maturity transformation is a central characteristic of banks and should be incorporated in the analysis in order to study the impact of public disclosure on the stability of banks. After all, the renewed interest in the topic of bank transparency has started with the recent crisis.

The introduction of a fragile liability structure could also shed new light on the related topic of bank contagion. Jones, Lee and Yeager (2012) have demonstrated the tight empirical link between informational opacity and bank contagion. Slovin, Sushka and Polonchek (1999) show that informational contagion occurs more frequently among money center banks which process large financial flows through global networks, and less often among regional banks which service a domestic-based clientele through branches and subsidiaries. This finding is consistent with the notion that fierce competition and information-intensive and complex investment activities in the market of money center banks result in increased informational opacity relative to less competitive regional banking markets, where eventual information advantages are limited by the size of the market and the characteristics of available assets.

Appendix A Proofs and Derivations

Proposition 2.3.3

Proof. The costs of a marginal investment in information acquisition must be equal to the marginal benefits in terms of a more profitable and safer portfolio. The first order condition for a socially efficient choice of g_{t+1}^A reads as:

$$\begin{aligned} & \frac{\partial}{\partial g_{t+1}^A} \left[\mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^*) \mid R_t \right\} \right] \\ &= \frac{\partial}{\partial g_{t+1}^A} \left[\mathbb{E} \left\{ \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^*) \mid \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t \right\} \mid R_t \right\} \right] \\ &= \mathbb{E} \left\{ \frac{\partial}{\partial g_{t+1}^A} \left[\mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^*) \mid \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t \right\} \mid R_t \right] \right\} = 0. \end{aligned}$$

For a given sample of observations $\mathbb{Q}_t = \{\hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t\}$, information expenditures must pay off by making this information more reliable and thereby increasing its social value. Let $\varphi(R_{t+1} | \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t)$ denote the density of R_{t+1} for a given sample of observations. Then we can rewrite:

$$\begin{aligned} & \frac{\partial}{\partial g_{t+1}^A} \left[\mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^*) \mid \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t \right\} \right] \\ &= \frac{\partial}{\partial g_{t+1}^A} \left[\int \left(\sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^*) \right) \varphi(R_{t+1} | \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t) dR_{t+1} \right] \\ &= \int \left[\frac{\partial}{\partial g_{t+1}^A} \left(\sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^*) \right) \varphi(R_{t+1} | \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t) \right. \\ & \quad \left. + \left(\sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^*) \right) \frac{\partial \varphi(R_{t+1} | \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t)}{\partial g_{t+1}^A} \right] dR_{t+1} \\ &= \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} \frac{\partial u(c_{t+i}^*)}{\partial g_{t+1}^A} \mid \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t \right\} \\ & \quad + \int \left(\sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^*) \right) \frac{\partial \varphi(R_{t+1} | \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t)}{\partial g_{t+1}^A} dR_{t+1}. \end{aligned}$$

The first term of this sum captures the consequences of information expenditures in terms of a reallocation of resources, while the second term measures the implied changes in the

uncertainty regime which the planner has to face.

The envelope theorem allows us to abstract from indirect effects of changes in g_{t+1}^A which are transmitted through its impact on other choice variables. To see this, consider the welfare effect of a change in b_{t+1}^* induced by the variation in g_{t+1}^A :

$$\begin{aligned} & \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} \frac{\partial u(c_{t+i}^*)}{\partial b_{t+1}^*} \frac{\partial b_{t+1}^*}{\partial g_{t+1}^A} \middle| \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t \right\} \\ &= \frac{\partial b_{t+1}^*}{\partial g_{t+1}^A} \left[- \frac{\partial u(c_t^*)}{\partial c_t^*} + \beta \mathbb{E} \left\{ \frac{\partial u(c_{t+1}^*)}{\partial c_{t+1}^*} \middle| \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t \right\} \right]. \end{aligned}$$

Note that the optimal choice of $b_{t+1}^* = (\hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t)$ is defined by:

$$\frac{\partial u(c_t^*)}{\partial c_t^*} = \beta \mathbb{E} \left\{ \frac{\partial u(c_{t+1}^*)}{\partial c_{t+1}^*} \middle| \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t \right\}.$$

The same reasoning applies to indirect effects of changes in g_{t+1}^A transmitted through other choice variables, e.g. k_{t+1}^* . It follows that:

$$\mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} \frac{\partial u(c_{t+i}^*)}{\partial g_{t+1}^A} \middle| \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t \right\} = - \beta \mathbb{E} \left\{ \frac{\partial u(c_{t+1}^*)}{\partial c_{t+1}^*} \middle| \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t \right\}.$$

Increasing investment in information acquisition reduces the resources available at date $t+1$ for consumption. Now, the first order condition for a socially efficient choice of g_t^A boils down to:

$$\beta \mathbb{E} \left\{ \frac{\partial u(c_{t+1}^*)}{\partial c_{t+1}^*} \middle| R_t \right\} = \mathbb{E} \left\{ \int \left(\sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^*) \right) \frac{\partial \varphi(R_{t+1} | \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t)}{\partial g_{t+1}^A} dR_{t+1} \middle| R_t \right\}.$$

The density of R_{t+1} depends on g_{t+1}^A through its conditional variance as given by:

$$\text{Var} \{ R_{t+1} | \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t \} = \frac{\sigma^2 \Sigma_{t+1}^A \Sigma_{t+1}^B}{\sigma^2 \Sigma_{t+1}^A + \sigma^2 \Sigma_{t+1}^B + \Sigma_{t+1}^A \Sigma_{t+1}^B}.$$

Since we know that:

$$\frac{\partial \text{Var} \{ R_{t+1} | \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t \}}{\partial g_{t+1}^A} = - \text{Var} \{ R_{t+1} | \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t \}^2 f'(g_{t+1}^A),$$

we can rewrite the first order condition for g_t^A according to:

$$\begin{aligned} & \beta \mathbb{E} \left\{ \frac{\partial u(c_{t+1}^*)}{\partial c_{t+1}^*} \middle| R_t \right\} \\ &= \mathbb{E} \left\{ \int \left(\sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^*) \right) \frac{\partial \varphi(R_{t+1} | \mathbb{Q}_t)}{\partial \text{Var}\{R_{t+1} | \mathbb{Q}_t\}} \frac{\partial \text{Var}\{R_{t+1} | \mathbb{Q}_t\}}{\partial g_{t+1}^A} dR_{t+1} \middle| R_t \right\} \\ &= -\text{Var}\{R_{t+1} | \mathbb{Q}_t\}^2 f'(g_{t+1}^A) \mathbb{E} \left\{ \int \left(\sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^*) \right) \frac{\partial \varphi(R_{t+1} | \mathbb{Q}_t)}{\partial \text{Var}\{R_{t+1} | \mathbb{Q}_t\}} dR_{t+1} \middle| R_t \right\}, \end{aligned}$$

where $\mathbb{Q}_t = \{\hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t\}$. The function $f(g)$ is increasing, strictly concave and satisfies the Inada conditions, e.g. $f'(0) = \infty$ and $f'(\infty) = 0$. Furthermore, $u(c)$ is strictly concave. By Jensen's inequality, a mean preserving spread in the distribution of R_{t+1} lowers expected welfare. It follows that the right hand side of the equation above is strictly positive.

For $g_{t+1}^A = 0$, the marginal benefit of increasing information expenses on the right hand side of the equation above exceeds the associated costs on the left hand side. As g_{t+1}^A goes towards infinity, its marginal benefits shrink while the marginal costs in terms of expected welfare are growing without bounds. Only an interior choice of g_{t+1}^{A*} can satisfy the first order condition. The analogue reasoning holds for the entirely symmetric problem of selecting g_{t+1}^{B*} . Concavity of $f(g)$ implies that the planner will optimally invest equal amounts in the precision of both signals. \square

Proposition 2.3.5

Proof. In the neighborhood of the optimal level of g_{t+1}^A , a marginal adjustment must increase households' valuation of bank A securities just as much as it increases the value of bank B securities.

$$\begin{aligned} & \mathbb{E} \left\{ \frac{\partial}{\partial g_{t+1}^A} \left[\mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \middle| \hat{R}_{t+1}^A, R_t \right\} \right] \middle| R_t \right\} \\ &= \mathbb{E} \left\{ \frac{\partial}{\partial g_{t+1}^A} \left[\mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^B) \middle| \hat{R}_{t+1}^B, R_t \right\} \right] \middle| R_t \right\}. \end{aligned}$$

Under complete opacity, the latter term is zero. The first order condition for an optimal choice of g_{t+1}^A becomes:

$$\mathbb{E} \left\{ \frac{\partial}{\partial g_{t+1}^A} \left[\mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \middle| \hat{R}_{t+1}^A, R_t \right\} \right] \middle| R_t \right\} = 0.$$

Applying the same reasoning as in the proof to Proposition 2.3.3 above, we can rewrite this first order condition according to:

$$\beta \mathbb{E} \left\{ \frac{\partial u(c_{t+1})}{\partial c_{t+1}} \middle| R_t \right\} = \mathbb{E} \left\{ \int \left(\sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \right) \frac{\partial \varphi(R_{t+1} | \hat{R}_{t+1}^A, R_t)}{\partial g_{t+1}^A} dR_{t+1} \middle| R_t \right\}.$$

This is equivalent to:

$$\begin{aligned} & \beta \mathbb{E} \left\{ \frac{\partial u(c_{t+1})}{\partial c_{t+1}} \middle| R_t \right\} \\ &= -\text{Var}\{R_{t+1} | \mathbb{Q}_t^A\}^2 f'(g_{t+1}^A) \mathbb{E} \left\{ \int \left(\sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \right) \frac{\partial \varphi(R_{t+1} | \mathbb{Q}_t^A)}{\partial \text{Var}\{R_{t+1} | \mathbb{Q}_t^A\}} dR_{t+1} \middle| R_t \right\}, \end{aligned}$$

where $\mathbb{Q}_t^A = \{\hat{R}_{t+1}^A, R_t\}$. Under opacity, bankers face a higher degree of uncertainty than under transparency for given levels of information expenses:

$$\begin{aligned} \text{Var}\{R_{t+1} | \hat{R}_{t+1}^A, R_t\} &= \frac{\sigma^2 \Sigma_{t+1}^A}{\sigma^2 + \Sigma_{t+1}^A} > \\ & \frac{\sigma^2 \Sigma_{t+1}^A \Sigma_{t+1}^B}{\sigma^2 \Sigma_{t+1}^A + \sigma^2 \Sigma_{t+1}^B + \Sigma_{t+1}^A \Sigma_{t+1}^B} = \text{Var}\{R_{t+1} | \hat{R}_{t+1}^A, \hat{R}_{t+1}^B, R_t\}. \end{aligned}$$

Also, household welfare is reduced with respect to the case of exogenous transparency for a given level of g_{t+1}^A . This is because both households' savings decisions as well as banks' portfolio choice are based on less information now. Non-increasing absolute risk aversion implies that given increases in uncertainty become more costly as expected consumption levels fall. It follows that:

$$0 < g_{t+1}^{A*} = g_{t+1}^{B*} < g_{t+1}^{A**} = g_{t+1}^{B**} < \infty.$$

□

Lemma 2.4.1

Proof. The first order condition of g_{t+1}^A is given by:

$$\begin{aligned} \mathbb{E} \left\{ \frac{\partial}{\partial g_{t+1}^A} \left[\mathbb{E} \left\{ \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \mid \mathbb{Q}_t^A \right\} \mid \mathbb{Q}_t^H \right\} \right] \mid R_t \right\} \\ = \mathbb{E} \left\{ \frac{\partial}{\partial g_{t+1}^A} \left[\mathbb{E} \left\{ \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^B) \mid \mathbb{Q}_t^B \right\} \mid \mathbb{Q}_t^H \right\} \right] \mid R_t \right\}. \end{aligned}$$

In the absence of complete opacity, the term on the right hand side of the equation above is generally not zero. The quality of bank B 's portfolio choice benefits to some degree from the increased precision of bank A 's private signal. The uncertainty which banker B faces when she chooses her investment portfolio depends on the precision of bank A 's signal:

$$\text{Var}\{R_{t+1} \mid Q_{t+1}^A, \hat{R}_{t+1}^B, R_t\} = \frac{\sigma^2 (\Sigma_{t+1}^A + \hat{\Sigma}_{t+1}) \Sigma_{t+1}^B}{\sigma^2 (\Sigma_{t+1}^A + \hat{\Sigma}_{t+1}) + \sigma^2 \Sigma_{t+1}^B + (\Sigma_{t+1}^A + \hat{\Sigma}_{t+1}) \Sigma_{t+1}^B}.$$

Information expenditures by bank A reduce this uncertainty:

$$\frac{\partial \text{Var}\{R_{t+1} \mid Q_{t+1}^A, \hat{R}_{t+1}^B, R_t\}}{\partial g_{t+1}^A} = -\text{Var}\{R_{t+1} \mid Q_{t+1}^A, \hat{R}_{t+1}^B, R_t\}^2 \left(\frac{\Sigma_{t+1}^A}{\Sigma_{t+1}^A + \hat{\Sigma}_{t+1}} \right)^2 f'(g_{t+1}^A).$$

Note that banker B 's level of uncertainty is independent of g_{t+1}^A under complete opacity ($\hat{\Sigma}_{t+1} = \infty$). The dependence on g_{t+1}^A becomes stronger for higher degrees of transparency and information spillovers. Applying the same reasoning as in the proof to Proposition 2.3.3 above, we can rewrite the first order condition of g_{t+1}^A according to:

$$\begin{aligned} \beta \mathbb{E} \left\{ \frac{\partial u(c_{t+1})}{\partial c_{t+1}} \mid R_t \right\} \\ = -\text{Var}\{R_{t+1} \mid \mathbb{Q}_t^A\}^2 f'(g_{t+1}^A) \mathbb{E} \left\{ \int \left(\sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \right) \frac{\partial \varphi(R_{t+1} \mid \mathbb{Q}_t^A)}{\partial \text{Var}\{R_{t+1} \mid \mathbb{Q}_t^A\}} dR_{t+1} \mid R_t \right\} \\ + \text{Var}\{R_{t+1} \mid \mathbb{Q}_t^B\}^2 f'(g_{t+1}^A) \left(\frac{\Sigma_{t+1}^A}{\Sigma_{t+1}^A + \hat{\Sigma}_{t+1}} \right)^2 \\ \mathbb{E} \left\{ \int \left(\sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^B) \right) \frac{\partial \varphi(R_{t+1} \mid \mathbb{Q}_t^B)}{\partial \text{Var}\{R_{t+1} \mid \mathbb{Q}_t^B\}} dR_{t+1} \mid R_t \right\}. \end{aligned}$$

In equilibrium, both banks spend identical amounts on information acquisition. Hence, ex-ante their expected beliefs are identical:

$$\mathbb{E}\left\{\varphi(R_{t+1} | \mathbb{Q}_t^A) \middle| R_t\right\} = \mathbb{E}\left\{\varphi(R_{t+1} | \mathbb{Q}_t^B) \middle| R_t\right\}.$$

Likewise, the symmetry of equilibrium implies:

$$\text{Var}\{R_{t+1} | \mathbb{Q}_t^A\} = \text{Var}\{R_{t+1} | \mathbb{Q}_t^B\}.$$

Hence, we can rewrite the first order condition of g_{t+1}^A :

$$\begin{aligned} \beta \mathbb{E}\left\{\frac{\partial u(c_{t+1})}{\partial c_{t+1}} \middle| R_t\right\} &= -\text{Var}\{R_{t+1} | \mathbb{Q}_t^A\}^2 f'(g_{t+1}^A) \left[1 - \left(\frac{\Sigma_{t+1}^A}{\Sigma_{t+1}^A + \hat{\Sigma}_{t+1}}\right)^2\right] \\ &\quad \mathbb{E}\left\{\int \left(\sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A)\right) \frac{\partial \varphi(R_{t+1} | \mathbb{Q}_t^A)}{\partial \text{Var}\{R_{t+1} | \mathbb{Q}_t^A\}} dR_{t+1} \middle| R_t\right\}. \end{aligned}$$

Under complete transparency ($\hat{\Sigma}_{t+1} = 0$), the right hand side of this equation is always zero and so is the optimal choice of g_{t+1}^A . As $\hat{\Sigma}_{t+1}$ is growing, the rival bank participates less and less in improvements to bank A 's information set.

Note that:

$$\lim_{\hat{\Sigma}_{t+1} \rightarrow \infty} \frac{\partial}{\partial \hat{\Sigma}_{t+1}} \left[\left(\frac{\Sigma_{t+1}^A}{\Sigma_{t+1}^A + \hat{\Sigma}_{t+1}} \right)^2 \right] = \lim_{\hat{\Sigma}_{t+1} \rightarrow \infty} \left[-2 \frac{\Sigma_{t+1}^A{}^2}{(\Sigma_{t+1}^A + \hat{\Sigma}_{t+1})^3} \right] = 0,$$

and also:

$$\lim_{\hat{\Sigma}_{t+1} \rightarrow \infty} \frac{\partial}{\partial \hat{\Sigma}_{t+1}} \left[\text{Var}\{R_{t+1} | \mathbb{Q}_t^A\} \right] = \lim_{\hat{\Sigma}_{t+1} \rightarrow \infty} \left[\left(\frac{\text{Var}\{R_{t+1} | \mathbb{Q}_t^A\}}{\Sigma_{t+1}^A + \hat{\Sigma}_{t+1}} \right)^2 \right] = 0.$$

For high values of opacity, the signal-to-noise ratio of banks' public signals becomes less and less responsive to additional changes in opacity. This is reflected by the vanishing dependence of the optimal choice of g_{t+1}^A on the degree of information spillovers:

$$\lim_{\hat{\Sigma}_{t+1} \rightarrow \infty} g'(\hat{\Sigma}_{t+1}) = 0.$$

□

Proposition 2.4.2

Proof. The first order condition for a socially optimal choice of $\hat{\Sigma}_{t+1}$ is given by:

$$\frac{\partial}{\partial \hat{\Sigma}_{t+1}} \left[\mathbb{E} \left\{ \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \middle| \mathbb{Q}_t^A \right\} \middle| \mathbb{Q}_t^H \right\} \right] = 0.$$

A change in opacity has two effects on welfare: (1.) uncertainty for households varies with the informational content of $\mathbb{Q}_t^H = \{Q_{t+1}^A, Q_{t+1}^B, R_t\}$, and (2.) the uncertainty for banks is affected through $\mathbb{Q}_t^A = \{\hat{R}_{t+1}^A, Q_{t+1}^B, R_t\}$. This can be seen from rewriting:

$$\begin{aligned} & \frac{\partial}{\partial \hat{\Sigma}_{t+1}} \left[\mathbb{E} \left\{ \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \middle| \mathbb{Q}_t^A \right\} \middle| \mathbb{Q}_t^H \right\} \right] \\ &= \frac{\partial}{\partial \hat{\Sigma}_{t+1}} \left[\int \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \middle| \mathbb{Q}_t^A \right\} \varphi(R_{t+1} | \mathbb{Q}_t^H) dR_{t+1} \right]. \end{aligned}$$

From the product rule, it follows that:

$$\begin{aligned} & \frac{\partial}{\partial \hat{\Sigma}_{t+1}} \left[\int \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \middle| \mathbb{Q}_t^A \right\} \varphi(R_{t+1} | \mathbb{Q}_t^H) dR_{t+1} \right] \\ &= \int \frac{\partial}{\partial \hat{\Sigma}_{t+1}} \left[\mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \middle| \mathbb{Q}_t^A \right\} \right] \varphi(R_{t+1} | \mathbb{Q}_t^H) dR_{t+1} \\ & \quad + \int \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \middle| \mathbb{Q}_t^A \right\} \frac{\partial \varphi(R_{t+1} | \mathbb{Q}_t^H)}{\partial \hat{\Sigma}_{t+1}} dR_{t+1} \\ &= \mathbb{E} \left\{ \frac{\partial}{\partial \hat{\Sigma}_{t+1}} \left[\mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \middle| \mathbb{Q}_t^A \right\} \right] \middle| \mathbb{Q}_t^H \right\} \\ & \quad + \int \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \middle| \mathbb{Q}_t^A \right\} \frac{\partial \varphi(R_{t+1} | \mathbb{Q}_t^H)}{\partial \hat{\Sigma}_{t+1}} dR_{t+1}. \end{aligned}$$

The first term of this sum captures households' expectations about how the change in opacity will affect the precision of bank A's forecast of R_{t+1} :

$$\begin{aligned} & \mathbb{E} \left\{ \frac{\partial}{\partial \hat{\Sigma}_{t+1}} \left[\mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \middle| \mathbb{Q}_t^A \right\} \right] \middle| \mathbb{Q}_t^H \right\} \\ &= \mathbb{E} \left\{ \int \left(\sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \right) \frac{\partial \varphi(R_{t+1} | \mathbb{Q}_t^A)}{\partial \hat{\Sigma}_{t+1}} dR_{t+1} \middle| \mathbb{Q}_t^H \right\}. \end{aligned}$$

Hence, the first order condition for $\hat{\Sigma}_{t+1}$ becomes:

$$\mathbb{E} \left\{ \int \left(\sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \right) \frac{\partial \varphi(R_{t+1} | \mathbb{Q}_t^A)}{\partial \hat{\Sigma}_{t+1}} dR_{t+1} \middle| \mathbb{Q}_t^H \right\} + \int \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \middle| \mathbb{Q}_t^A \right\} \frac{\partial \varphi(R_{t+1} | \mathbb{Q}_t^H)}{\partial \hat{\Sigma}_{t+1}} dR_{t+1} = 0.$$

Opacity affects the beliefs of banker A (first term of the sum above) as well as the expectations of households (second term). This effect works through the induced variations in uncertainty. Households' uncertainty is given by:

$$\text{Var}\{R_{t+1} | \mathbb{Q}_t^H\} = \frac{\sigma^2 (\Sigma_{t+1}^A + \hat{\Sigma}_{t+1}) (\Sigma_{t+1}^B + \hat{\Sigma}_{t+1})}{\sigma^2 (\Sigma_{t+1}^A + \hat{\Sigma}_{t+1}) + \sigma^2 (\Sigma_{t+1}^B + \hat{\Sigma}_{t+1}) + (\Sigma_{t+1}^A + \hat{\Sigma}_{t+1})(\Sigma_{t+1}^B + \hat{\Sigma}_{t+1})}.$$

This uncertainty responds to changes in opacity in the following way:

$$\frac{\partial \text{Var}\{R_{t+1} | \mathbb{Q}_t^H\}}{\partial \hat{\Sigma}_{t+1}} = \text{Var}\{R_{t+1} | \mathbb{Q}_t^H\}^2 \frac{\left(\frac{\partial \Sigma_{t+1}^A}{\partial \hat{\Sigma}_{t+1}} + 1 \right) (\Sigma_{t+1}^B + \hat{\Sigma}_{t+1})^2 + \left(\frac{\partial \Sigma_{t+1}^B}{\partial \hat{\Sigma}_{t+1}} + 1 \right) (\Sigma_{t+1}^A + \hat{\Sigma}_{t+1})^2}{(\Sigma_{t+1}^A + \hat{\Sigma}_{t+1})^2 (\Sigma_{t+1}^B + \hat{\Sigma}_{t+1})^2}.$$

In the absence of positive effects on information production, households' uncertainty would always increase in opacity. What about banks' uncertainty?

$$\text{Var}\{R_{t+1} | \mathbb{Q}_t^A\} = \frac{\sigma^2 \Sigma_{t+1}^A (\Sigma_{t+1}^B + \hat{\Sigma}_{t+1})}{\sigma^2 \Sigma_{t+1}^A + \sigma^2 (\Sigma_{t+1}^B + \hat{\Sigma}_{t+1}) + \Sigma_{t+1}^A (\Sigma_{t+1}^B + \hat{\Sigma}_{t+1})}.$$

Also banks' uncertainty varies with opacity:

$$\frac{\partial \text{Var}\{R_{t+1} | \mathbb{Q}_t^A\}}{\partial \hat{\Sigma}_{t+1}} = \text{Var}\{R_{t+1} | \mathbb{Q}_t^A\}^2 \frac{\frac{\partial \Sigma_{t+1}^A}{\partial \hat{\Sigma}_{t+1}} (\Sigma_{t+1}^B + \hat{\Sigma}_{t+1})^2 + \left(\frac{\partial \Sigma_{t+1}^B}{\partial \hat{\Sigma}_{t+1}} + 1 \right) \Sigma_{t+1}^A{}^2}{\Sigma_{t+1}^A{}^2 (\Sigma_{t+1}^B + \hat{\Sigma}_{t+1})^2}.$$

Again, in the absence of positive effects on information production, banks' uncertainty would always increase in opacity. But note that:

$$\frac{\partial \Sigma_{t+1}^j}{\partial \hat{\Sigma}_{t+1}} = - \Sigma_{t+1}^j{}^2 f'(g_{t+1}^j) g'(\hat{\Sigma}_{t+1}),$$

which is always negative. By Lemma 2.4.1, this term converges to zero as opacity tends towards infinity. At $\hat{\Sigma}_{t+1} = 0$, its value is $-\infty$. Information production is highly responsive for low levels of opacity. This sensitivity falls as opacity is increased.

This implies for the uncertainty of households and bankers, respectively:

$$\frac{\partial \text{Var}\{R_{t+1} | \mathbb{Q}_t^H\}}{\partial \hat{\Sigma}_{t+1}} < 0 \quad , \text{ and } \quad \frac{\partial \text{Var}\{R_{t+1} | \mathbb{Q}_t^A\}}{\partial \hat{\Sigma}_{t+1}} < 0 \quad , \text{ for } \hat{\Sigma}_{t+1} = 0 .$$

In the neighborhood of complete transparency, a local increase in opacity actually *decreases* uncertainty. This is because the positive effect of opacity on information production outweighs the increase in noise of banks' public signals. The opposite is true for high levels of opacity:

$$\frac{\partial \text{Var}\{R_{t+1} | \mathbb{Q}_t^H\}}{\partial \hat{\Sigma}_{t+1}} > 0 \quad , \text{ and } \quad \frac{\partial \text{Var}\{R_{t+1} | \mathbb{Q}_t^A\}}{\partial \hat{\Sigma}_{t+1}} > 0 \quad , \text{ for } \hat{\Sigma}_{t+1} = \infty .$$

In the neighborhood of complete opacity, a marginal reduction of opacity reduces aggregate uncertainty for households without affecting information production by banks in any significant way. Reconsider now the first order condition of $\hat{\Sigma}_{t+1}$:

$$\begin{aligned} & \frac{\partial \text{Var}\{R_{t+1} | \mathbb{Q}_t^A\}}{\partial \hat{\Sigma}_{t+1}} \mathbb{E} \left\{ \int \left(\sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \right) \frac{\partial \varphi(R_{t+1} | \mathbb{Q}_t^A)}{\partial \text{Var}\{R_{t+1} | \mathbb{Q}_t^A\}} dR_{t+1} \middle| \mathbb{Q}_t^H \right\} \\ & + \frac{\partial \text{Var}\{R_{t+1} | \mathbb{Q}_t^H\}}{\partial \hat{\Sigma}_{t+1}} \int \mathbb{E} \left\{ \sum_{i=0}^{\infty} \beta^{t+i} u(c_{t+i}^A) \middle| \mathbb{Q}_t^A \right\} \frac{\partial \varphi(R_{t+1} | \mathbb{Q}_t^H)}{\partial \text{Var}\{R_{t+1} | \mathbb{Q}_t^H\}} dR_{t+1} = 0 . \end{aligned}$$

It becomes clear that the social benefits of increasing opacity are positive at $\hat{\Sigma}_{t+1} = 0$. The opposite is true for $\hat{\Sigma}_{t+1} = \infty$. \square

Bibliography

- Admati, Anat R. and Paul Pfleiderer (1988). 'Selling and Trading on Information in Financial Markets,' *The American Economic Review*, Vol. 78(2), pp. 96-103.
- Admati, Anat R. and Paul Pfleiderer (1990). 'Direct and Indirect Sale of Information,' *Econometrica*, Vol. 58(4), pp. 901-928.
- Admati, Anat R. and Paul Pfleiderer (2000). 'Forcing Firms to Talk: Financial Disclosure Regulation and Externalities,' *The Review of Financial Studies*, Vol. 13(3), pp. 479-519.
- Alvarez, Fernando and Gadi Barlevy (2013). 'Mandatory Disclosure and Financial Contagion,' mimeo, July 2013.
- Bernanke, Ben S. (2010). 'Remarks on "The Squam Lake Report: Fixing the Financial System",' Remarks at the Squam Lake Conference, New York, New York, U.S., June 16, 2010.
- Blackwell, David (1951). 'Comparison of Experiments,' in: Jerzy Neyman (ed.), *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability. July 31-August 12, 1950. Statistical Laboratory of the University of California, Berkeley*, pp. 93-102, University of California Press, Berkeley.
- Bouvard, Matthieu, Pierre Chaigneau, and Adolfo de Motta (2012). 'Transparency in the Financial System: Rollover Risk and Crises,' mimeo, August 2012.
- Chen, Yehning and Iftexhar Hasan (2006). 'The Transparency of the Banking System and the Efficiency of Information-Based Bank Runs,' *Journal of Financial Intermediation*, Vol. 15, pp. 307-331.
- Cheng, Mei, Dan Dhaliwal and Monica Neamtiu (2008). 'Banks' Asset Securitization and Information Opacity,' mimeo, April 2008.
- Dang, Tri Vi, Gary Gorton, Bengt Holmström, and Guillermo Ordoñez (2013). 'Banks as Secret Keepers,' mimeo, May 2013.
- Darrough, Masako N. and Neal M. Stoughton (1990). 'Financial Disclosure Policy in an Entry Game,' *Journal of Accounting and Economics*, Vol. 12, pp. 219-243.
- Diamond, Douglas D. (1985). 'Optimal Release of Information of Firms,' *The Journal of Finance*, Vol. 40(4), pp. 1071-1094.
- Dye, Ronald A. (1986). 'Proprietary and Nonproprietary Disclosures,' *The Journal of Business*, Vol. 59(2), pp. 331-366.
- Dye, Ronald A. (1990). 'Mandatory versus Voluntary Disclosures: The Cases of Financial and Real Externalities,' *The Accounting Review*, Vol. 65(1), pp. 1-24.
- Flannery, Mark J., Simon H. Kwan, and M. Nimalendran (2004). 'Market Evidence on the Opaqueness of Banking Firms' Assets,' *Journal of Financial Economics*, Vol. 71, pp. 419-460.
- Foster, George (1980). 'Externalities and Financial Reporting,' *The Journal of Finance*, Vol. 35(2), pp. 521-533.
- French, K., M. Baily, J. Campbell, J. Cochrane, D. Diamond, D. Duffie, A. Kashyap, F. Mishkin, R. Rajan, D. Scharfstein, R. Shiller, H. Shin, M. Slaughter, J. Stein, and R. Stulz (2010). *The Squam Lake Report: Fixing the Financial System*, Princeton University Press, Princeton and Oxford.
- Goldstein, Itay and Haresh Saprà (2012). 'Should Banks' Stress Test Results be Disclosed? An Analysis of the Costs and Benefits,' mimeo, April 2012.

- Grossman, Sanford J. and Oliver D. Hart (1980). 'Disclosure Laws and Takeover Bids,' *The Journal of Finance*, Vol. 35(2), pp. 323-334.
- Grossman, Sanford J. and Joseph E. Stiglitz (1976). 'Information and Competitive Price Systems,' *The American Economic Review*, Vol. 66(2), pp. 246-253.
- Grossman, Sanford J. and Joseph E. Stiglitz (1980). 'On the Impossibility of Informationally Efficient Markets,' *The American Economic Review*, Vol. 70(3), pp. 393-408.
- Hirshleifer, Jack (1971). 'The Private and Social Value of Information and the Reward to Inventive Activity,' *The American Economic Review*, Vol. 61(4), pp. 561-574.
- Hirtle, Beverly (2006). 'Stock Market Reaction to Financial Statement Certification by Bank Holding Company CEOs,' *Journal of Money, Credit and Banking*, Vol. 38(5), pp. 1263-1291.
- Iannotta, Giuliano (2006). 'Testing for Opaqueness in the European Banking Industry: Evidence from Bond Credit Ratings,' *Journal of Financial Services Research*, Vol. 30, pp. 287-309.
- Jones, Jeffrey S., Wayne Y. Lee, and Timothy J. Yeager (2012). 'Opaque Banks, Price Discovery, and Financial Instability,' *Journal of Financial Intermediation*, Vol. 21, pp. 383-408.
- Kurlat, Pablo and Laura Veldkamp (2012). 'Should We Regulate Financial Information?' mimeo, May 2012.
- Landier, Augustin and David Thesmar (2011). 'Regulating Systemic Risk through Transparency: Tradeoffs in Making Data Public,' in: Markus K. Brunnermeier and Arvind Krishnamurthy (ed.), *Risk Topography: Systemic Risk and Macro Modeling*, forthcoming from University of Chicago Press, Chicago.
- Marschak, Jacob (1955). 'Elements for a Theory of Teams,' *Management Science*, Vol. 1(2), pp. 127-137.
- Moreno, Diego and Tuomas Takalo (2012). 'Optimal Bank Transparency,' mimeo, November 2012.
- Morgan, Donald P. (2002). 'Rating Banks: Risk and Uncertainty in an Opaque Industry,' *The American Economic Review*, Vol. 92(4), pp. 874-888.
- Morris, Stephen and Hyun Song Shin (2002). 'Social Value of Public Information,' *The American Economic Review*, Vol. 92(5), pp. 1521-1534.
- Pagano, Marco and Paolo Volpin (2012). 'Securitization, Transparency, and Liquidity,' *The Review of Financial Studies*, Vol. 25(8), pp. 2417-2453.
- Radner, Roy (1962). 'Team Decision Problems,' *The Annals of Mathematical Statistics*, Vol. 33(3), pp. 857-881.
- Slovin, Myron B., Marie E. Sushka and John A. Polonchek (1999). 'An Analysis of Contagion and Competitive Effects at Commercial Banks,' *Journal of Financial Economics*, Vol. 54(2), pp. 197-225.
- Spargoli, Fabrizio (2012). 'Bank's Optimal Information Disclosure and Banks' Opacity,' mimeo, October 2012.
- Verrecchia, Robert E. (1983). 'Discretionary Disclosure,' *Journal of Accounting and Economics*, Vol. 5, pp. 179-194.
- Vives, Xavier (2012), 'Endogenous Public Information and Welfare,' IESE Working Paper No. 925, April 2012.

Chapter 3

Why Does Misallocation Persist?

with David Strauss (European University Institute)

3.1 Introduction

Why are some countries so rich while others are poor? Hsieh and Klenow (2010) calculate that 10-30 percent of cross-country income differences can be explained by different levels of human capital, while about 20 percent are attributable to differences in physical capital. The most important part of 50-70 percent is accounted for by differences in total factor productivity (TFP) across countries. But why do TFP levels differ so vastly across countries? The recent growth literature has identified the misallocation of production factors within countries as a potentially powerful explanation device for large TFP differences.¹ In search for a reason for severe factor misallocation, capital market imperfections have been a popular subject of inquiry.² However, the quantitative effect on TFP of factor misallocation caused by credit market frictions can be quite disappointing.³ One important reason for this negative result is that borrowing constraints are only a temporary obstacle for capital accumulation if self-financing is possible.⁴

¹See Banerjee and Duflo (2005), Guner, Ventura and Xu (2008), Restuccia and Rogerson (2008), Hsieh and Klenow (2009), or Bartelsman, Haltiwanger and Scarpetta (2013). An early study of the role of factor misallocation for TFP is Hopenhayn and Rogerson (1993).

²The role of factor misallocation caused by credit market frictions for growth and development has recently been explored by Giné and Townsend (2004), Jeong and Townsend (2007), Greenwood, Sanchez and Wang (2010), Buera, Kaboski and Shin (2011), Moll (2012), Buera and Shin (2013), Caselli and Gennaioli (2013), Greenwood, Sanchez and Wang (2013), and others.

³See Hosono and Takizawa (2012), Gilchrist, Sim and Zakrajšek (2013), and Midrigan and Xu (2013).

⁴This point has been made by Banerjee and Moll (2010) and Moll (2012).

We contribute to this literature by proposing a model in which factor misallocation caused by capital market imperfections may indeed give rise to permanent income and TFP differences across countries. In the presence of binding credit constraints, the assignment of human capital to heterogeneous production sectors is completely reversed with respect to the case of efficient capital markets. This misallocation of skill across sectors may be permanent because the model features a collective poverty trap which arises for low levels of financial development. This poverty trap is the result of a pecuniary externality which firm owners inflict on workers by selecting the sector of production. Depending on initial conditions, a country converges over time to one of two different stable steady states characterized by different long-run levels of output, capital, wages, and measured TFP.

The comparison of these two stable long-run equilibria yields additional interesting results. First, manufactured goods are relatively cheaper in a country which has reached the high-income steady state compared to a country which has converged to the low-income equilibrium. This corresponds well with the empirical results of Parente and Prescott (2002) and Hsieh and Klenow (2007) who find that manufactured goods are relatively more expensive in financially underdeveloped economies. Furthermore, both the average firm size in terms of capital as well as its variance are higher in the high-income steady state. Hopenhayn (2012) reports that the firm-size distributions of India and Mexico, as measured by the number of employees, are compressed with respect to the U.S. (but also with respect to China) once average firm size is controlled for.

Preview of the Model

At the core of the model lies a static assignment problem. There are two production sectors: manufacturing and services. Production takes place in many firms. Each of them uses a single worker and some quantity of capital as input factors. The manufacturing sector is more capital-intensive than services, as the input productivity of capital is higher in manufacturing. Workers differ according to their human capital (or talent). Workers of high ability reduce the failure rate of production and they also produce more than low ability workers in case production succeeds. Manufactured goods and service goods are aggregated to the numéraire good which is a simple Cobb-Douglas composite of the two intermediate goods. Firm owners choose to produce one of the two intermediate goods depending on the ability of their randomly drawn worker. This assignment problem of workers of different quality to production sectors of different capital efficiency is embedded in a highly stylized overlapping generations model. Young agents work and save in order to become a firm owner once they are too old to work themselves. In addition to their own savings, firm owners have

access to the international capital market at an exogenous interest rate.

Results

The key result of the paper is the central importance of financial development for the growth trajectory and the convergence properties of the model economy. A low level of financial development is characterized by tight credit constraints for domestic firm owners who are seeking to obtain capital on the international credit market.⁵ First, we show that the equilibrium assignment of human capital to production sectors of different capital input productivity is completely reversed in the presence of binding borrowing constraints with respect to the case of efficient capital markets. This formal result confirms a conjecture by Sampson (2011).

Assume that worker talent is a close substitute for capital productivity in the production function. In this case, labor quality and capital productivity are also gross substitutes in firm earnings as long as capital is exogenously fixed. However, under increasing returns to worker talent, capital productivity and labor quality become gross complements in firm earnings once capital is set optimally. As the matching of human capital to production sectors depends on the degree of complementarity between the attributes of workers and sectors, an assignment reversal occurs.

We go on to show that this assignment reversal gives rise to the possibility of multiple stable steady states. Depending on the initial domestic stock of capital, the model economy converges to one of two different long-run equilibria which are characterized by different sorting patterns and consequently also different levels of measured TFP, wages, capital, and per capita income. The reason for the possibility of multiple steady states is that an assignment reversal causes a discrete upward jump in a country's level of measured TFP. But firm owners need to be able to produce on a sufficiently large scale before it becomes profitable for firms with workers of different quality to switch production sectors. Financially underdeveloped countries may reach a low-income and low-capital steady state before the sorting reversal occurs. In this case, the misallocation of human capital to production sectors is permanent.

Self-financing provides an imperfect substitute for functioning credit markets. If a country starts with a sufficiently high level of domestic capital, it may converge to a second steady state which is characterized by an efficient sorting pattern of workers into production sectors, and consequently also high levels of measured TFP, wages, capital, and per capita

⁵Borrowing constraints may arise in equilibrium in the presence of limited contract enforceability. In this case, the upper limit for credit is falling in the share of borrowed funds which can be diverted by debtors, while it rises with the pledgeable fraction of firm owners' private wealth.

income. This high-income steady state may be reached even under financial autarky, if initial conditions are sufficiently favorable.

The allocation of human capital across production sectors is efficient in the high-income steady state. If worker ability and capital enter the production function as gross substitutes, then it is efficient for high talent agents to produce capital-intensive manufacturing goods since they are the ones who benefit most from large scales of production. A given quantity of manufactured goods can be produced at a lower cost by high talent workers than by low ability agents. Consequently, manufacturing goods are relatively cheaper in the high-income equilibrium.

Also the firm size distribution differs across steady states depending on the respective sorting pattern. Even if not all firm owners are credit constrained in the low-income equilibrium, the firm size distribution will still be compressed as high talent workers are employed in the sector with low capital productivity and vice versa. This reduces the benefits of leveraging workers' talent through capital. In contrast, in the high-income steady state it is the high ability workers which produce in the sector in which capital is used most efficiently. Consequently, the incentives to combine high talent with a large amount of capital are much higher now. The opposite is true for low ability workers who produce using a technology characterized by low capital productivity. Therefore, both the average firm size and its variance are higher in the high-income steady state than in the low-income equilibrium.

Related Literature

Our paper starts from the observation that quantitative models of credit market imperfections have difficulties to generate permanent and sizeable TFP differences through pure factor misallocation. Using U.S. data on firm-specific borrowing costs, Gilchrist, Sim and Zakrajšek (2013) find large and persistent differences in borrowing costs across firms on the corporate bond market. However, in their model these differences in financing conditions do not translate into large TFP losses due to resource misallocation. Hosono and Takizawa (2012) estimate only a slightly higher impact of borrowing constraints on factor misallocation and aggregate TFP in Japanese plant-level data.

While these two studies analyze data from highly developed economies, Midrigan and Xu (2013) compare South Korea as a country with a well developed financial sector with China and Colombia as examples for financially underdeveloped economies. The authors find a strong effect of borrowing constraints on entry and technology adoption, but only a weak impact on pure factor misallocation across establishments. The reason for this negative result is that credit constraints are only a temporary obstacle for capital accumulation if

self-financing is possible. In the data, firm productivity is sufficiently persistent in order to allow productive establishments to simply grow out of their financial constraints.

These findings are in contrast to the large TFP losses attributed to credit market frictions by other studies. These contributions tend to rely on the extensive margin of factor misallocation. Giné and Townsend (2004) and Jeong and Townsend (2007) study the high growth period in Thailand 1976-1996. They find that exogenous financial deepening allowed productive agents to leave the subsistence sector and become entrepreneurs. Similarly, also in Erosa and Cabrillana (2008), Amaral and Quintin (2010), and Greenwood, Sanchez and Wang (2013), it is an exogenous improvement in the efficiency of the financial intermediation sector which results in the usage of more efficient production technologies. Caselli and Gennaioli (2013) focus on dynastic management of family-owned firms, which can be a source of the mismatch between talent and wealth. The demographic structure of these studies (or the modeling of firms as short-term projects) excludes or severely restricts the possibility of self-financing. This is a key reason for the important role of financial development in these models. If agents (or firms) were allowed to accumulate wealth over time, then this would provide a substitute for credit markets and could result in a minor role of financial frictions for long-run factor misallocation.⁶

Note that this criticism does not apply to our model. In the studies cited above, differences in financial development are a sufficient condition for differences in TFP across countries. In our OLG framework, binding borrowing constraints are merely a necessary condition for permanent TFP differences between countries. It is possible that in both steady states at least some firm owners are credit constrained. Still, in one of these steady states the allocation of human capital across plants is efficient while in the other steady state it is not. In the absence of a poverty trap, the model economy would grow over time until the allocation of worker talent is fully efficient. It is in this sense that we contribute to the literature by introducing a new mechanism that allows for permanent TFP losses through factor misallocation caused by credit market frictions.⁷

Pursuing a similar research question to ours, Buera, Kaboski and Shin (2011) examine the effect of credit constraints on the allocation of entrepreneurial talent and capital across manufacturing and services. The authors explicitly show the role of the self-financing channel in mitigating the adverse consequences of credit constraints by comparing the quantitative results of a model featuring long-lived agents with an OLG setup. But even allowing for

⁶For a formal discussion of the self-financing channel, see also Banerjee and Moll (2010) and Moll (2012).

⁷Of course, within an OLG framework it is impossible to fully address the concerns arising from the self-financing channel. It is an important open question whether the described poverty trap can exist also in a model populated by long-lived agents. See also Section 3.5 for a short discussion of future work.

the possibility of self-financing, they still find that financial frictions can account for TFP losses of up to 40 percent. Non-convexities in production play a key role. Also, the authors use a less persistent productivity process for individual entrepreneurs than Midrigan and Xu (2013). Moll (2012) argues that the magnitude of TFP losses is highly sensitive to the exact value of persistence in productivity processes. Large and permanent TFP losses do not seem to be a robust result in existing models of factor misallocation through credit market frictions. Buera and Shin (2013) focus on the role of financial frictions in delaying the speed of convergence towards a stationary equilibrium. They find that self-financing is only an imperfect substitute for functioning credit markets as transition takes roughly twice as long as in the neoclassical growth model. In contrast to our paper, both Buera, Kaboski and Shin (2011) and Buera and Shin (2013) feature a unique stationary equilibrium.

Obviously, we are not the first to show how credit market imperfections may give rise to multiple stable equilibria along the growth path of an economy. Banerjee and Newman (1993) discuss the central importance of the income distribution for the development process of a country. If a lot of agents are rich enough to become entrepreneurs, this results in high wages for poor workers in their model which in turn allows a lot of agents to start their own business in the future. A more skewed initial income distribution results in less entrepreneurial activity and lower wages in the long run. Buera (2008) shows that this multiplicity result survives in an environment where self-financing is possible. Also in Galor and Zeira (1993), credit market frictions result in a pecuniary externality which current generations inflict on their offspring. Similarly to Banerjee and Newman (1993), the income distribution is a state variable in their model which features multiple steady states. Both in Banerjee and Newman (1993) as well as in Galor and Zeira (1993), agents are homogeneous and TFP in a given production sector is always constant across different equilibria. In our model, TFP within a given sector differs across equilibria. This is an important result in light of the empirical evidence on sector-specific productivity differences across countries provided by Erosa and Cabrillana (2008) or Buera, Kaboski and Shin (2011).⁸

Our work also relates to the literature on equilibrium matching. We build on Sampson (2011) who describes the conditions under which log-submodularity in production translates into log-supermodularity in earnings and vice versa.⁹ He also provides empirical evidence on sorting reversals across countries. In the data, the assignment of human capital to production

⁸Examples of poverty traps which do not affect technological choice or factor misallocation can be found in Piketty (1997), Ghatak, Morelli and Sjöström (2001), or Mookerjee and Ray (2002).

⁹Complementarity has been identified as the decisive factor in driving efficient matching since the classic contribution by Becker (1973). See also Costinot (2009) for a definition and discussion of log-supermodularity and log-submodularity.

sectors seems to depend crucially on the respective state of economic development. The model by Sampson (2011) features assignment reversals across countries because of exogenous differences in the ranking of sectors according to capital input productivity. In our setting, the sorting pattern of worker talent into heterogeneous production sectors depends on a country's domestic stock of capital. If firms are able to set capital optimally, the matching pattern reverses with respect to the case of binding borrowing constraints.

Also Legros and Newman (2002) point out that imperfect credit markets may distort the equilibrium sorting pattern associated with a frictionless environment. In some cases, this may even result in an assignment reversal. In contrast to our model, Legros and Newman (2002) assume non-convexities in production. In their model, assignment reversals may or may not arise in the presence of financial frictions depending on the underlying distribution of types, while in our model sorting reversals are a necessary consequence of binding borrowing constraints.

Poverty traps arise in our model only for low levels of financial development. This central role of financial development for growth has been established by numerous empirical studies. King and Levine (1993) and Levine and Zervos (1998) estimate a close link between aggregate measures of credit and financial development across countries on the one hand and output per capita on the other hand. Further evidence is provided using sector-level data (Rajan and Zingales, 1998) and firm-level data (Beck, Demirgüç-Kunt and Maksimovic, 2005) across countries. Importantly, Beck, Levine and Loayza (2000) find that the positive impact of financial development on economic growth is transmitted through TFP growth. Also the results by La Porta, Lopez-de-Silanes, Shleifer and Vishny (1998) on institutional differences such as contract enforcement and creditor protection support the idea that financial development is tightly linked to economic growth. Banerjee and Duflo (2005) review micro-level evidence for credit constraints in poor countries and the resulting misallocation of capital.

Outline

The rest of the paper is organized as follows. The model is set up in Section 3.2. Section 3.3 characterizes the efficient allocation of production factors across sectors. In Section 3.4, the implications of credit constraints for static and dynamic inefficiencies are discussed. The paper concludes with a short discussion of potential enhancements of the model in Section 3.5.

3.2 Model Setup

Consider an overlapping generations model. Each agent lives for two periods and each generation has unit mass. When agents are young, they are endowed with their ability as workers and their working time. Workers are heterogeneous with respect to their talent. During their youth, they can save their wages in order to become firm owners once they are too old to work. A firm owner can choose to produce one of two intermediate goods: manufacturing or services. The manufacturing sector is more capital-intensive than the service sector. These intermediate goods are used in the production of the numéraire good which can be consumed or sold on the international goods market.

3.2.1 Agents

All agents in this model economy are risk averse and they consume only when they are old. There is no disutility of work and therefore each young agent supplies in equilibrium her full working time endowment $\bar{l} = 1$ on the labor market. A young agent's preferences at time t can be described by the function:

$$\mathbb{E}_t u(c_{t+1}),$$

where \mathbb{E}_t is the expectation operator conditional on date t information. The function $u : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ satisfies the Inada conditions. Young agents use their wage w_t to buy capital at the end of their youth:

$$w_t = k_{t+1}.$$

Old agents employ this capital k_{t+1} in a firm. Production is risky and all agents are risk averse. Hence, old agents will optimally insure each other against their idiosyncratic risks. Old agents consume all of their wealth. Accordingly, the budget constraint of an old agent $z \in [0, 1]$ reads as:

$$c_{t+1}(z) \leq \Pi_{t+1}(z) + T_{t+1}(z),$$

where $\Pi_{t+1}(z)$ gives the firm profits of agent z 's firm and $T_{t+1}(z)$ denotes the insurance payment which is conditional on the production outcome. This insurance transfer can be positive or negative.

3.2.2 Production

A firm owner has access to two different production technologies: manufacturing or services. In both cases, output is produced using capital and a single worker as inputs. Workers differ

with respect to their talent. A fraction φ of workers has a high ability $\theta = \theta^H$ and a mass $1 - \varphi$ of young agents has a low level of talent: $\theta = \theta^L$, with $\theta^H > \theta^L$. The output $y_{t+1}^j(z)$ of an old agent z 's firm in sector $j = m, s$ is given by:

$$y_{t+1}^j(z) = \begin{cases} F(\theta(z), Q^j K_{t+1}(z)) & , \text{ with probability } \pi(\theta(z)), \text{ and} \\ 0 & , \text{ otherwise.} \end{cases}$$

The production function $F : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ has constant returns to scale and is defined over the hired worker's talent θ and capital K_{t+1} . The function is increasing and strictly concave in both arguments with: $F_2(\theta, 0) = \infty$ and $F_2(\theta, \infty) = 0$. The manufacturing sector is more efficient in using capital: $Q^m > Q^s$. Production is risky as a firm owner faces each period a probability of $1 - \pi(\theta)$ that output is zero. The success probability $\pi(\theta)$ is strictly increasing in the worker's talent. Note that this implies that expected output features increasing returns to scale.¹⁰

Firm owner $z \in [0, 1]$ decides conditional on the talent of her worker $\theta(z)$ whether to produce in sector m or in sector s . She also has the option to rent capital $d_{t+1}(z)$ on an international capital market at a rate of r in order to lever up firm profits. These profits are given by:

$$\Pi_{t+1}(z) = p_{t+1}^j y_{t+1}^j(z) - w_{t+1} - (1 + r) K_{t+1}(z),$$

where: $K_{t+1}(z) = k_{t+1}(z) + d_{t+1}(z)$.

3.2.3 Final Good

Manufacturing and service goods are used in the production of the numéraire good which can be consumed or sold on the international goods market. This final good is produced using a Cobb-Douglas technology according to:

$$y_{t+1} = (y_{t+1}^m)^{\alpha_m} (y_{t+1}^s)^{\alpha_s},$$

¹⁰The function of expected output is similar to the output of a manager-worker pair in Rosen (1982) who studies a single sector model of firm hierarchies. In Rosen (1982), increasing returns to managerial skill arise due to the scale-independent importance of managerial decisions for all other factors of production. In our model, it is the talent-dependent success probability which introduces increasing returns to worker ability.

where $\alpha_m + \alpha_s = 1$. There are no additional costs in the production of this final good and the final goods sector is perfectly competitive:

$$y_{t+1} = p_{t+1}^m y_{t+1}^m + p_{t+1}^s y_{t+1}^s.$$

The final goods producers make exactly zero profits in each period.

3.2.4 Labor Market

A young agent supplies her labor force on the period t labor market. An agent can only work for a single firm and every firm owner can hire only one worker. For the sake of tractability, we assume that both firm owners and workers learn the ability of a worker only after the labor contract is signed. Therefore, all workers on the labor market are ex-ante identical and there is a homogeneous wage rate w_t for all worker-firm owner pairs. As workers sell their labor in one piece and not at the margin, the division of the surplus between workers and firm owners depends on the bargaining weight $\gamma \in (0, 1)$ of workers. The outside option of workers is zero, while the outside option of firm owners is the international capital market.

3.2.5 Timing

At time t , young agents are born with an endowment of working time and some level of talent θ . Old agents enter the period with a predetermined stock of savings k_t . They insure themselves against their idiosyncratic risks of production. The labor market opens and old agents hire workers at a wage rate w_t . After they learn the talent of their worker, each firm owner now decides in which sector to produce and how much capital d_t to rent on the international capital market. Production takes place and the intermediate goods are bought by the final goods producers which subsequently offer the final good on the consumption goods market. Old agents consume their wealth and young agents use their wages to buy capital goods for period $t + 1$.

3.3 Efficient Allocation

Before studying the implications of credit constraints for the misallocation of production factors in this model economy, we take a look at a frictionless world where capital markets work smoothly and the allocation of resources is perfectly efficient.

Definition Given some initial capital stock of old agents k_t and the rental rate r on the international capital market, a competitive equilibrium in this economy consists of an assignment function $j^*(\theta, k_{t+i})$, a production policy $K^*(\theta, j, k_{t+i})$, prices w_{t+i}^* , p_{t+i}^m , p_{t+i}^s , and a capital stock k_{t+i+1}^* , such that (1.) young agents save their wages, (2.) old agents solve their individual optimization problem by choosing $j^*(\theta, k_{t+i})$ and $K^*(\theta, j, k_{t+i})$, (3.) the labor market clears, and (4.) the intermediate goods market clears.

3.3.1 Production

Consider first the problem of firm owner $z \in [0, 1]$ at time t who has drawn a worker of talent θ on the labor market and who has chosen to produce in sector j . Firm owners optimally insure each other against the idiosyncratic risk of production failure as well as the risk of ending up with a worker of low talent. The law of large numbers holds and the optimal insurance transfer makes sure that each firm owner in this model economy receives the ex-ante expected level of firm output with certainty. Firm owners have to decide on the optimal scale of production. Expected profits conditional on the worker's talent θ and the production sector j are maximized according to:

$$\max_{K_t \in [0, +\infty)} \pi(\theta) p_t^j F(\theta, Q^j K_t) - w_t - (1+r)[K_t - k_t]. \quad (3.1)$$

Constant returns to scale allow us to express production in terms of a single variable:

$$f\left(\frac{Q^j K_t}{\theta}\right) \equiv \frac{1}{\theta} F(\theta, Q^j K_t).$$

The optimal scale of production satisfies the following first order condition:

$$\pi(\theta) p_t^j Q^j f'\left(\frac{Q^j K^*(\theta, j, k_t)}{\theta}\right) = (1+r).$$

The expected marginal product of capital must be equal to its costs. This can be rewritten according to:

$$f'\left(\frac{Q^j K^*(\theta, j, k_t)}{\theta}\right) = \frac{1+r}{\pi(\theta) p_t^j Q^j}. \quad (3.2)$$

We have expressed the optimal scale of production in terms of a single variable. This variable is the span of control of a worker:

$$\sigma_t^j(\theta) \equiv \frac{Q^j K_t}{\theta}.$$

The span of control measures the efficiency units of capital used at time t per unit of talent and captures the extent to which a firm chooses to leverage the ability of its worker by contracting capital. As the function $f : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ is strictly concave, it follows from the first order condition in (3.2) that $\sigma_t^j(\theta)$ is optimally increasing in the worker's talent θ , the intermediate good's price p_t^j , and input productivity Q^j .¹¹

3.3.2 Sorting

After firm owners learn the talent of their randomly assigned worker, they select their preferred production sector anticipating the respective optimal capital policy $K^*(\theta, j, k_t)$ as described by (3.2). Both intermediate goods are necessary in order to produce a positive quantity of the final good. Therefore, some firm owners should choose to produce the manufactured good while others must sort into the services industry. It is not possible that intermediate goods markets clear if all firm owners strictly prefer one sector of production over the other.

Firm owners differ according to their worker's ability. The two sectors of production differ according to their efficiency in using capital as an input factor. The manufacturing sector's technology strictly dominates services in terms of efficiency. If firm owners with high talent workers strictly prefer the manufacturing sector and/or if firm owners with low talent workers strictly prefer the service sector over manufacturing, then we will have positive assortative matching (PAM). In this case, it holds that:

$$\begin{aligned} \pi(\theta^H) p_t^m \theta^H f\left(\frac{Q^m K^*(\theta^H, m, k_t)}{\theta^H}\right) - w_t - (1+r) [K^*(\theta^H, m, k_t) - k_t] \\ \geq \pi(\theta^H) p_t^s \theta^H f\left(\frac{Q^s K^*(\theta^H, s, k_t)}{\theta^H}\right) - w_t - (1+r) [K^*(\theta^H, s, k_t) - k_t], \end{aligned}$$

¹¹Production per unit of worker talent features diminishing returns to the span of control as in Lucas (1978). If a worker is using more machines, she has less time for each single unit and marginal returns are falling in scale.

and:

$$\begin{aligned} \pi(\theta^L) p_t^m \theta^L f\left(\frac{Q^m K^*(\theta^L, m, k_t)}{\theta^L}\right) - w_t - (1+r) [K^*(\theta^L, m, k_t) - k_t] \\ \leq \pi(\theta^L) p_t^s \theta^L f\left(\frac{Q^s K^*(\theta^L, s, k_t)}{\theta^L}\right) - w_t - (1+r) [K^*(\theta^L, s, k_t) - k_t], \end{aligned}$$

with at least one of the two inequalities above being strict. The opposite case is negative assortative matching (NAM). In any case, intermediate good prices adjust in a way such that the optimal span of control of a worker of given talent is higher in manufacturing than in services as shown by Lemma 3.3.1.

Lemma 3.3.1. *In equilibrium, it must hold that: $\sigma_t^{m*}(\theta) > \sigma_t^{s*}(\theta)$.*

Proof. Taking as given the intermediate goods prices and using:

$$K^*(\theta, j, k_t) = \frac{\theta \sigma_t^{j*}(\theta)}{Q^j},$$

as well as the first order condition in (3.2), we can rewrite firm profits according to:

$$\Pi_t^{j*}(\theta) = p_t^j \pi(\theta) \theta \left[f(\sigma_t^{j*}(\theta)) - \sigma_t^{j*}(\theta) f'(\sigma_t^{j*}(\theta)) \right] - w_t + (1+r) k_t.$$

The term in square brackets is strictly increasing in the optimal span of control:

$$\frac{\partial}{\partial \sigma_t^{j*}(\theta)} \left[f(\sigma_t^{j*}(\theta)) - \sigma_t^{j*}(\theta) f'(\sigma_t^{j*}(\theta)) \right] = -\sigma_t^{j*}(\theta) f''(\sigma_t^{j*}(\theta)).$$

Since by equation (3.2) the optimal span of control is strictly increasing in $p_t^j Q_t^j$, it follows that also the term above is strictly increasing in $p_t^j Q_t^j$.

Assume now for a moment that manufacturing goods are more expensive than service goods: $p_t^m > p_t^s$. Then it also holds that: $p_t^m Q_t^m > p_t^s Q_t^s$. But in this case $\Pi_t^{m*}(\theta) > \Pi_t^{s*}(\theta)$ for all firm owners and nobody chooses to produce service goods. This is incompatible with intermediate goods markets clearing. It follows that in equilibrium: $p_t^m < p_t^s$.

Since $p_t^m < p_t^s$, it follows in turn that $p_t^m Q_t^m > p_t^s Q_t^s$ in equilibrium. Otherwise, nobody chooses to produce manufactured goods. From (3.2), it follows then that for a worker of given talent the optimal span of control is strictly higher in manufacturing than in the service sector. \square

No matter which level of talent the workers in manufacturing possess, firm owners will

optimally lever up their abilities using a higher span of control than firm owners in the service sector do. Firm output per unit of worker ability responds to an increase in the span of control according to the point elasticity of $f(\sigma_t(\theta))$:

$$\varepsilon_f(\sigma_t(\theta)) \equiv \frac{f'(\sigma_t(\theta)) \sigma_t(\theta)}{f(\sigma_t(\theta))}.$$

This elasticity differs across sectors together with the optimal span of control. In any given sector, the optimal span of control is higher if high talent workers are employed. Proposition 3.3.2 shows that the efficient assignment of human capital to production sectors crucially depends on the output elasticity defined above.

Proposition 3.3.2. *Given well functioning credit markets, the equilibrium is characterized by positive assortative matching (PAM) if the output elasticity $\varepsilon_f(\sigma_t(\theta))$ is strictly increasing in the span of control. Negative assortative matching (NAM) arises if $\varepsilon_f(\sigma_t(\theta))$ is strictly decreasing. Sorting is underdetermined if $\varepsilon_f(\sigma_t(\theta))$ is constant.*

Proof. Consider again the conditions for PAM as defined above. Taking as given intermediate goods prices, we can rewrite this condition applying the same reasoning as in the proof of Lemma 3.3.1:

$$\frac{f(\sigma_t^{m*}(\theta^H)) - \sigma_t^{m*}(\theta^H) f'(\sigma_t^{m*}(\theta^H))}{f(\sigma_t^{s*}(\theta^H)) - \sigma_t^{s*}(\theta^H) f'(\sigma_t^{s*}(\theta^H))} > \frac{f(\sigma_t^{m*}(\theta^L)) - \sigma_t^{m*}(\theta^L) f'(\sigma_t^{m*}(\theta^L))}{f(\sigma_t^{s*}(\theta^L)) - \sigma_t^{s*}(\theta^L) f'(\sigma_t^{s*}(\theta^L))}.$$

Rewriting this ratio across sectors using (3.2):

$$\frac{f(\sigma_t^{m*}(\theta)) - \sigma_t^{m*}(\theta) f'(\sigma_t^{m*}(\theta))}{f(\sigma_t^{s*}(\theta)) - \sigma_t^{s*}(\theta) f'(\sigma_t^{s*}(\theta))} = \frac{f(\sigma_t^{m*}(\theta)) - \frac{\sigma_t^{m*}(\theta) (1+r)}{\pi(\theta) p_t^m Q^m}}{f(\sigma_t^{s*}(\theta)) - \frac{\sigma_t^{s*}(\theta) (1+r)}{\pi(\theta) p_t^s Q^s}},$$

and taking its derivative with respect to θ yields:

$$\begin{aligned} \frac{\partial}{\partial \theta} & \left[\frac{f(\sigma_t^{m*}(\theta)) - \frac{\sigma_t^{m*}(\theta) (1+r)}{\pi(\theta) p_t^m Q^m}}{f(\sigma_t^{s*}(\theta)) - \frac{\sigma_t^{s*}(\theta) (1+r)}{\pi(\theta) p_t^s Q^s}} \right] \\ & \propto f(\sigma_t^{m*}(\theta)) \left[\pi(\theta) f(\sigma_t^{s*}(\theta)) - \frac{\sigma_t^{s*}(\theta) (1+r)}{p_t^s Q^s} \right] \\ & \quad - f(\sigma_t^{s*}(\theta)) \left[\pi(\theta) f(\sigma_t^{m*}(\theta)) - \frac{\sigma_t^{m*}(\theta) (1+r)}{p_t^m Q^m} \right]. \end{aligned}$$

In turn, this last expression can be rewritten using optimality:

$$\begin{aligned}
& f(\sigma_t^{m*}(\theta)) \left[\pi(\theta) f(\sigma_t^{s*}(\theta)) - \frac{\sigma_t^{s*}(\theta) (1+r)}{p_t^s Q^s} \right] \\
& \quad - f(\sigma_t^{s*}(\theta)) \left[\pi(\theta) f(\sigma_t^{m*}(\theta)) - \frac{\sigma_t^{m*}(\theta) (1+r)}{p_t^m Q^m} \right] \\
= & f(\sigma_t^{m*}(\theta)) \left[f(\sigma_t^{s*}(\theta)) - \sigma_t^{s*}(\theta) f'(\sigma_t^{s*}(\theta)) \right] \\
& \quad - f(\sigma_t^{s*}(\theta)) \left[f(\sigma_t^{m*}(\theta)) - \sigma_t^{m*}(\theta) f'(\sigma_t^{m*}(\theta)) \right] \\
\propto & \varepsilon_f(\sigma_t^{m*}(\theta)) - \varepsilon_f(\sigma_t^{s*}(\theta)).
\end{aligned}$$

We know from Lemma 3.3.1 that $\sigma_t^{m*}(\theta) > \sigma_t^{s*}(\theta)$. It follows that the equilibrium is characterized by PAM if the output elasticity $\varepsilon_f(\sigma_t(\theta))$ is strictly increasing in the span of control. NAM arises for the opposite case of $\varepsilon_f(\sigma_t(\theta))$ being a strictly decreasing function. If $\varepsilon_f(\sigma_t(\theta))$ is constant for any span of control, then sorting is underdetermined. \square

This result is due to Sampson (2011). Its intuition is the following. High talent workers are valuable for two reasons: (1.) the marginal product of labor is high, and (2.) production failures are unlikely. It is precisely for this second reason that the optimal span of control is increasing in worker ability. If the success probability of production was identical for both talent levels, then the optimal span of control would not respond to skill levels as can be seen from equation (3.2). In this case, the output elasticity $\varepsilon_f(\sigma_t(\theta))$ would be constant across sectors and sorting would be underdetermined. We see that the assumption of non-constant returns to scale of expected output is necessary for firm owners to select into different sectors.

High talent workers allow for a high optimal span of control, i.e. high leverage through capital. Depending on the properties of the production function $f(\sigma_t(\theta))$, it will be efficient to use this high leverage in manufacturing or services, i.e. in a sector of high capital efficiency or in a sector in which input productivity is low. If the output elasticity increases in the span of control, then it is optimal to have very unequal levels of leverage across the two sectors which is achieved through PAM. By employing large quantities of capital per unit of talent in some firms it is gained more than is lost in the firms with a low span of control. On the other hand, if the output elasticity is falling in the span of control, then it is efficient to match talent and input productivity in an attempt to equalize the span of control across production sectors. NAM tends to level out the differences in leverage across sectors as high talent is matched with an inefficient production technology and vice versa.

Two separately equivalent conditions for PAM are (1.) the elasticity of substitution

between worker talent and capital in $F(\theta, QK_t)$ is bigger than one, and (2.) $F(\theta, QK_t)$ is log-submodular.¹² Only a high degree of substitutability between capital and labor skill allows for an increasing output elasticity $\varepsilon_f(\sigma_t(\theta))$. If capital and worker talent interact in a strongly complementary fashion, then increasing the scale of production through an increase in capital alone is not very effective and output becomes less and less sensitive to additional changes in the span of control. But if capital and skill are close substitutes, then production becomes more responsive to capital if worker talent is low and vice versa.

This is why high talent agents benefit relatively more from an increase in capital productivity if and only if talent and capital are close substitutes. Increasing returns to talent cannot be readily exploited through high leverage if labor input is fixed and very complementary to capital. In this case, high talent is matched with unproductive capital where skill is more valuable at the margin. But if worker ability and capital are easily substitutable in production, then increasing returns to talent imply that capital productivity and worker talent interact very complementary in driving up the optimal span of control and therefore also firm earnings. Having less production failures in firms with high capital productivity is then more important because these are the establishments which benefit most from an additional increase in the scale of production. Gross substitutes in production turn into gross complements in earnings and vice versa.

3.3.3 Final Good

Final goods producers demand intermediate goods in order to maximize:

$$\max_{y_t^m, y_t^s \in [0, +\infty)} (y_t^m)^{\alpha_m} (y_t^s)^{\alpha_s} - p_t^m y_t^m - p_t^s y_t^s. \quad (3.3)$$

The optimal quantities demanded are given by:

$$p_t^m y_t^m = \alpha^m y_t, \text{ and: } p_t^s y_t^s = \alpha^s y_t. \quad (3.4)$$

As already argued above, both intermediate goods are in strictly positive demand for any given vector of goods prices. Demand is falling in the intermediate goods prices. The total expenditures on intermediate goods will vary with the quantity of the numéraire good which final good producers are able to supply on the international goods market at the world price $p_t = 1$.

¹²See Sampson (2011) for a formal proof.

3.3.4 Intermediate Goods

Market clearing for manufactured goods is accordingly given by:

$$\frac{\alpha^m y_t}{p_t^m} = \varphi \psi_t^H \pi(\theta^H) \theta^H f(\sigma_t^{m*}(\theta^H)) + (1 - \varphi) \psi_t^L \pi(\theta^L) \theta^L f(\sigma_t^{m*}(\theta^L)),$$

where ψ_t^H and ψ_t^L denote the fraction of high talent and low talent workers employed in manufacturing. The demand for manufactured goods comes from the final goods producers who sell the numéraire good to the world and the domestic consumption goods market, as well as to young agents who purchase capital. A similar market clearing condition applies to the service sector:

$$\frac{\alpha^s y_t}{p_t^s} = \varphi (1 - \psi_t^H) \pi(\theta^H) \theta^H f(\sigma_t^{s*}(\theta^H)) + (1 - \varphi) (1 - \psi_t^L) \pi(\theta^L) \theta^L f(\sigma_t^{s*}(\theta^L)).$$

Just as manufacturing goods, service goods are demanded by final goods producers.

Note that generally there is no reason to assume that workers of a given production sector are all homogeneous. Consider for instance the case of PAM. If the percentage of high talent agents in the population is high, then not all of them are needed to satisfy the demand for manufactured goods. In this case, intermediate goods prices adjust such that firm owners with high talent agents are indifferent between both production sectors. All low talent agents will work in the service sector in this case, while high ability workers are employed in both sectors. The opposite holds if the ratio of low talent agents is relatively high.

3.3.5 Labor Market

The division of the production surplus between workers and firm owners is determined by workers' relative bargaining weight γ together with the respective outside options of zero and the international capital market, respectively. Accordingly, the homogeneous wage rate is given by:

$$w_t = \gamma \mathbb{E}_t \left\{ p_t^{j^*(\theta)} y_t^{j^*(\theta)} - (1 + r) K^*(\theta, j^*(\theta)) \right\}.$$

The law of large numbers holds and allows us to rewrite:

$$\begin{aligned}
 w_t = \gamma & \left[\varphi \psi_t^H \left[\pi(\theta^H) p_t^m \theta^H f(\sigma_t^{m*}(\theta^H)) - (1+r) K^*(\theta^H, m) \right] \right. \\
 & + (1-\varphi) \psi_t^L \left[\pi(\theta^L) p_t^m \theta^L f(\sigma_t^{m*}(\theta^L)) - (1+r) K^*(\theta^L, m) \right] \\
 & + \varphi (1-\psi_t^H) \left[\pi(\theta^H) p_t^s \theta^H f(\sigma_t^{s*}(\theta^H)) - (1+r) K^*(\theta^H, s) \right] \\
 & \left. + (1-\varphi) (1-\psi_t^L) \left[\pi(\theta^L) p_t^s \theta^L f(\sigma_t^{s*}(\theta^L)) - (1+r) K^*(\theta^L, s) \right] \right].
 \end{aligned}$$

Recall that firm owners are insured against idiosyncratic production risks and therefore always able to pay the fixed wage rate w_t .

3.3.6 Dynamics

The equilibrium assignment $j^*(\theta)$ and the optimal scale of production $K^*(\theta, j^*(\theta))$ are both independent of the firm owners' own stock of capital k_t . This is a characteristic property of an efficient allocation of resources under well functioning credit markets. Note that this implies that also the wage rate w_t is independent of initial conditions. It follows that the dynamics of this competitive equilibrium are very simple. Regardless of the initial capital stock k_t , this model economy jumps immediately to the time-invariant steady state level of the equilibrium allocation described above. There is no role for history in the presence of well functioning capital markets.

3.4 Misallocation

Up until now, credit markets have been working perfectly and both capital and workers' abilities were allocated efficiently across production sectors. Now we will consider the implications of potentially binding credit constraints for firm owners. These credit market frictions will generally result in an inefficient allocation of capital in this model economy. Importantly, the presence of credit constraints implies that two otherwise identical countries may or may not converge to the same steady state equilibrium depending on their initial domestic stock of capital. The corresponding steady states feature different allocations of human capital to production sectors and consequently also different levels of measured TFP. In this sense, credit market frictions can give rise to a permanent misallocation of resources.

With respect to the previous section, the economic environment is modified by the following assumption:

(A1) Credit Constraint. There is a maximum amount which a firm owner can borrow on the international capital market: $d_t \leq \bar{d}$.

The borrowing constraint \bar{d} may be a function of firm owner's private wealth, the world interest rate r or other variables. In the following, we will merely assume that \bar{d} is always identical for all firm owners in both production sectors and that the constraint is binding given the initial domestic stock of capital k_t .¹³

3.4.1 Production

Revisit the problem of a firm owner who has drawn a worker of talent θ on the labor market and who has chosen to produce in sector j . Firm owners face a binding borrowing constraint on the international capital market: $d_t \leq \bar{d}$. This implies:

$$\pi(\theta) p_t^j Q^j f' \left(\frac{Q^j [k_t + \bar{d}]}{\theta} \right) > (1 + r).$$

The optimal span of control associated with the efficient allocation is bigger for high ability workers. Once capital is not set optimally anymore, this is no longer the case. The span of control is still increasing in the capital productivity parameter Q^j , but for a given production sector the span of control is now higher for low talent workers than for high ability agents!

3.4.2 Sorting

How does this change in the environment affect the sorting pattern in this model economy? The amount of borrowed capital is now identical across production sectors. The necessary and sufficient condition for PAM can be simplified to:

$$\frac{f \left(\frac{Q^m [k_t + \bar{d}]}{\theta^H} \right)}{f \left(\frac{Q^s [k_t + \bar{d}]}{\theta^H} \right)} > \frac{f \left(\frac{Q^m [k_t + \bar{d}]}{\theta^L} \right)}{f \left(\frac{Q^s [k_t + \bar{d}]}{\theta^L} \right)}.$$

¹³We do not explicitly model the particular market failure which gives to this credit constraint in equilibrium. Limited contract enforceability on the capital market is one possibility. Note that we maintain our assumption of perfect insurance among firm owners.

The opposite condition holds for negative assortative matching (NAM). Proposition 3.4.1 shows that the equilibrium assignment of worker talent to production sectors is completely reversed with respect to the efficient allocation described above.

Proposition 3.4.1. *In the presence of binding credit constraints, the equilibrium is characterized by negative assortative matching (NAM) if the output elasticity $\varepsilon_f(\sigma_t(\theta))$ is strictly increasing in the span of control. Positive assortative matching (PAM) arises if $\varepsilon_f(\sigma_t(\theta))$ is strictly decreasing. Sorting is underdetermined if $\varepsilon_f(\sigma_t(\theta))$ is constant.*

Proof. Taking the derivative of the output ratio considered above with respect to θ yields:

$$\begin{aligned} \frac{\partial}{\partial \theta} \left[\frac{f\left(\frac{Q^m [k_t + \bar{d}]}{\theta}\right)}{f\left(\frac{Q^s [k_t + \bar{d}]}{\theta}\right)} \right] &\propto f(\sigma_t^m(\theta)) f'(\sigma_t^s(\theta)) \frac{\sigma_t^s(\theta)}{\theta} - f(\sigma_t^s(\theta)) f'(\sigma_t^m(\theta)) \frac{\sigma_t^m(\theta)}{\theta} \\ &\propto \varepsilon_f(\sigma_t^s(\theta)) - \varepsilon_f(\sigma_t^m(\theta)). \end{aligned}$$

We know that $\sigma_t^s(\theta) > \sigma_t^m(\theta)$. It follows that the equilibrium is characterized by NAM if the output elasticity $\varepsilon_f(\sigma_t(\theta))$ is strictly increasing in the span of control. PAM arises for the opposite case of $\varepsilon_f(\sigma_t(\theta))$ being a strictly decreasing function. If $\varepsilon_f(\sigma_t(\theta))$ is constant for any span of control, then sorting is underdetermined. \square

Compare this result to Proposition 3.3.2. The key difference is that in the presence of binding borrowing constraints all sectors employ an identical quantity of capital. The span of control in a given production sector is now falling in workers' ability. This is true irrespective of whether the success probability of production varies with θ or not. The assumption of increasing returns to scale of the expected level of output is not important for the sorting pattern under binding credit constraints.

Now it is the low talent workers which allow for a high span of control, i.e. a relatively high leverage ratio of talent through capital. If the output elasticity $\varepsilon_f(\sigma_t(\theta))$ increases in the span of control, then it is optimal to have very unequal levels of leverage across the two sectors. This is true with or without binding borrowing constraints. But now unequal levels of the span of control across sectors arise under NAM instead of PAM. On the other hand, if the output elasticity is falling in the span of control, then it is efficient to match talent and input productivity in an attempt to equalize the span of control across production sectors. This is achieved best under PAM, as high talent and a low span of control is matched with high input productivity and vice versa. Once capital is fixed, gross substitutes in production are also gross substitutes in earnings. This is why now high talent agents benefit relatively

less from an increase in capital productivity than low ability workers. High talent is relatively more productive at the margin in firms with low capital productivity.

3.4.3 Dynamics

How does this model economy evolve over time? Consider the equilibrium path for a given initial capital stock k_t . The borrowing constraint \bar{d} is identical for all firm owners in both production sectors. The equilibrium scale of production is now given by:

$$\hat{K}(\theta, j, k_t) = \min \left\{ k_t + \bar{d}, K^*(\theta, j, k_t) \right\},$$

where the efficient capital input $K^*(\theta, j, k_t)$ is characterized by (3.2). The law of motion for the domestic capital stock reads as:

$$\begin{aligned} k_{t+1} &= \gamma \left[\varphi \psi_t^H \left[\pi(\theta^H) p_t^m \theta^H f \left(\frac{Q^m \hat{K}(\theta^H, m, k_t)}{\theta^H} \right) - (1+r) \hat{K}(\theta^H, m, k_t) \right] \right. \\ &\quad + (1-\varphi) \psi_t^L \left[\pi(\theta^L) p_t^m \theta^L f \left(\frac{Q^m \hat{K}(\theta^L, m, k_t)}{\theta^L} \right) - (1+r) \hat{K}(\theta^L, m, k_t) \right] \\ &\quad + \varphi (1-\psi_t^H) \left[\pi(\theta^H) p_t^s \theta^H f \left(\frac{Q^s \hat{K}(\theta^H, s, k_t)}{\theta^H} \right) - (1+r) \hat{K}(\theta^H, s, k_t) \right] \\ &\quad \left. + (1-\varphi)(1-\psi_t^L) \left[\pi(\theta^L) p_t^s \theta^L f \left(\frac{Q^s \hat{K}(\theta^L, s, k_t)}{\theta^L} \right) - (1+r) \hat{K}(\theta^L, s, k_t) \right] \right] \\ &= \gamma \left[\Phi(\bar{K}(k_t)) - (1+r) \bar{K}(k_t) \right], \end{aligned}$$

where $\Phi(\bar{K}(k_t))$ denotes the total revenue generated by the intermediate goods sector at time t and $\bar{K}(k_t)$ gives the aggregate stock of capital employed in the economy. Note that $\bar{K}(k_t)$ depends on the domestic stock of capital k_t due to potentially binding borrowing constraints. In the worst case of financial autarky, we have: $\bar{K}(k_t) = k_t$. In any steady state it must hold that:

$$k = \gamma \left[\Phi(\bar{K}(k)) - (1+r) \bar{K}(k) \right]. \quad (3.5)$$

The wages earned by young agents must be exactly enough to replace the existing stock of domestic capital. But how does this dynamic system behave outside of a steady state? The equilibrium path of k_t crucially depends on the function $\Phi(\bar{K}(k_t))$. The assumed production technology satisfies the Inada conditions $F_2(\theta, 0) = \infty$ and $F_2(\theta, \infty) = 0$. It follows that

also $\Phi'(0) = \infty$ and $\Phi'(\infty) = 0$. Hence, there is at least one stable steady state. In the following, we will argue that there may be more than one stable steady state in this dynamic system. The reason for this is the assignment reversal described above. We put the following additional restriction on the available technology:

(A2) Increasing Output Elasticity. The output elasticity $\varepsilon_f(\sigma_t(\theta))$ is strictly increasing in the span of control.

This assumption is equivalent to an elasticity of substitution between worker talent and capital in $F(\theta, QK_t)$ which is bigger than one: capital and worker talent are gross substitutes.¹⁴

Consider a model economy with an initial stock of capital k_t . This economy is subject to binding borrowing constraints for all firm owners in both sectors. The span of control is strictly higher for low ability workers than for high talent agents. Hence, the sorting pattern of human capital across production sectors is characterized by NAM, that is, low ability agents are primarily employed in manufacturing, while firm owners with high talent workers select into the service industry. As k_t is growing, also the span of control is increasing for all firms in both sectors. We know that eventually k_t is high enough that irrespective of credit market frictions no firm owner has to operate on an inefficiently low scale anymore because self-financing is sufficient for an optimal allocation of capital. At this point, also the sorting pattern of human capital to production sectors has reversed. Depending on the point along the equilibrium path of k_t at which this assignment reversal takes place, this dynamic system may have more than one stable steady state.

Proposition 3.4.2. *There is an upward discontinuity in $\Phi(\bar{K}(k_t))$. This upward discontinuity may give rise to multiple stable steady states.*

Proof. First of all note that it is impossible that firm owners with a high talent worker and firm owners with a low ability agent are both at the same time indifferent between the two production sectors. This could only be the case if the output elasticity $\varepsilon_f(\sigma_t(\theta))$ is constant. It follows that at each point along the equilibrium path the sorting pattern is either PAM or NAM.

Second, there must be a level of capital \tilde{k} such that the sorting pattern is PAM at $\tilde{k} + \varepsilon$ and NAM at $\tilde{k} - \varepsilon$ for $\varepsilon \rightarrow 0$. At \tilde{k} , an assignment reversal occurs. In the immediate

¹⁴There is no strong empirical guidance for choosing the elasticity of substitution between capital and worker talent in our model. Krusell, Ohanian, Ríos-Rull and Violante (2000) estimate an elasticity of substitution of 1.67 between unskilled labor and equipment capital, while they find a value of 0.67 for the elasticity of substitution between skilled labor and equipment capital. See also Hamermesh (1993) for a survey of the various empirical results on the topic.

neighborhood of \tilde{k} , some firm owners are credit constrained and some are not. This is a necessary condition for a sorting reversal to occur. As the change in the allocation of talent to sectors is discrete, also the aggregate stock of capital $\bar{K}(k_t)$ makes a discrete upward jump at \tilde{k} . This follows from the fact that unconstrained firm owners will generally employ different amounts of capital depending on the talent of their worker and the production sector. At least one of these two factors is bound to change at the switching point \tilde{k} . Since $\bar{K}(k_t)$ features an upward discontinuity at \tilde{k} , so does $\Phi(\bar{K}(k_t))$ which is an increasing function of $\bar{K}(k_t)$.

Consider now some steady state value of capital k' satisfying condition (3.5). If $\tilde{k} > k'$, then it is possible that there is a second value k'' which also satisfies (3.5) as the equilibrium path of k_t cuts the 45-degree line a second time from above. \square

If there are two steady states on the equilibrium path of k_t , then the allocations corresponding to the two stable equilibria are very different. Most importantly, the assignment of worker talent to production sectors is completely reversed across the two steady states. The stable capital stock is not only higher in the high-income steady state, but the total stock of capital is also used more productively. The sorting reversal has also an impact on the relative prices of the two intermediate goods.

Proposition 3.4.3. *If there are two stable steady states, then the price of manufacturing goods in relation to service goods is lower in the higher steady state with PAM than in the lower steady state with NAM.*

Proof. It follows from intermediate goods market clearing that:

$$\frac{p^s}{p^m} = \frac{\alpha^s}{\alpha^m} \frac{\varphi \psi_t^H \pi(\theta^H) \theta^H f(\sigma_t^m(\theta^H)) + (1 - \varphi) \psi_t^L \pi(\theta^L) \theta^L f(\sigma_t^m(\theta^L))}{\varphi (1 - \psi_t^H) \pi(\theta^H) \theta^H f(\sigma_t^s(\theta^H)) + (1 - \varphi) (1 - \psi_t^L) \pi(\theta^L) \theta^L f(\sigma_t^s(\theta^L))}.$$

High talent agents are always more productive in any given sector than low ability workers. The ratio of high talent workers in manufacturing is higher in a PAM equilibrium than under NAM. It follows that in the PAM steady state, a given quantity of manufactured goods can be produced using less capital input than under NAM. The opposite is true for the service sector which is less efficient under PAM than with NAM. Intermediate goods market clearing implies that the price ratio of p_t^s over p_t^m needs to be higher in the PAM steady state than under NAM. \square

This result is in line with empirical findings by Parente and Prescott (2002) and Hsieh and Klenow (2007). The authors report that manufactured goods are relatively more expensive in poorer economies. Sorting reversals provide an explanation for the particularly low

manufacturing productivity in countries with a low capital stock and low aggregate TFP. Also the firm size distribution of a given country is affected by assignment reversals.

Proposition 3.4.4. *If there are two stable steady states, then the average firm size, measured in the amount of capital employed, is bigger and the distribution of the firm size is more spread out in the higher steady state with PAM compared to the lower steady state with NAM.*

Proof. The fact that the average firm size is higher is a direct consequence of the higher stock of capital in the higher steady state.

As the output elasticity $\varepsilon_f(\sigma_t(\theta))$ is a strictly increasing function, PAM will arise if and only if this results in a higher dispersion of the span of control across firms than under NAM. It follows that the dispersion of the firm size distribution is also higher under PAM than with NAM. \square

The result that credit market frictions compress the efficient plant-size distribution is common to many models (e.g. Hosono and Takizawa, 2012). Hopenhayn (2012) finds that the firm-size distributions of India and Mexico, as measured by the number of employees, are compressed with respect to the U.S. once average firm size is controlled for. China, on the other hand, does not seem to suffer from a similar compression. Alfaro, Charlton and Kanczuk (2009) find that the average firm size and its variance are negatively correlated with per-capita income across countries. This result may be driven by an underrepresentation of small firms in poor countries in their data set.

3.5 Discussion

We propose a new mechanism to show how credit market failure may give rise to a permanent misallocation of production factors across sectors and permanent differences in measured TFP across countries. At this point, the biggest challenge in this literature is the role of self-financing in providing an imperfect substitute for functioning capital markets and thereby mitigating any long-run effects of financial frictions for factor allocation and TFP.¹⁵ We have derived our theoretical results within a highly stylized OLG framework. The advantages of this model with respect to analytical tractability go along with limitations in its capability to fully address the concerns arising from the mentioned self-financing channel. It is an important theoretical question whether sorting reversals of the kind described above may also give rise to a poverty trap in a model populated by long-lived agents.

¹⁵See Banerjee and Moll (2010), Buera, Kaboski and Shin (2011), Moll (2012), Buera and Shin (2013), and Midrigan and Xu (2013).

To answer this question, we have to study the assignment of human capital to production sectors allowing for heterogeneous wealth levels across agents. The matching problem between firms and workers becomes three-dimensional as some firms own more equity capital than others. Only a quantitative analysis of sorting reversals in an environment of long-lived agents can clarify if the existence of poverty traps is robust with respect to the self-financing channel. Furthermore, this work would also give an idea about the magnitude of TFP losses generated by this model along the transition path as well as in a stationary equilibrium.

One implication of sorting reversals is that manufacturing goods may be relatively more expensive in poor countries than in rich economies. Arguably, capital goods consist to a higher degree of manufactured goods than consumption goods do. In our model, we have assumed that capital and consumption goods are produced using identical input shares of the two intermediate goods. This assumption could be modified to test the robustness of the results derived above. If capital consists primarily of manufactured goods and if the manufacturing sector is particularly unproductive at initial stages of economic development, this would also have an impact on the accumulation of capital and the associated growth path of the model economy.

Poverty traps may exist in the model described above whenever the constant elasticity of substitution between capital and labor skill is not exactly unity. However, the model predictions on sector-specific TFP gaps and on the firm-size distribution across countries depend on the exact value or at least on the sign of this substitution elasticity. Further empirical evidence is desirable to inform our parameter choice in this question.

Bibliography

- Alfaro, Laura, Andrew Charlton, and Fabio Kanczuk (2009). ‘Plant-Size Distribution and Cross-Country Income Differences,’ in: *NBER International Seminar on Macroeconomics 2008*, pp. 243-272, University of Chicago Press, Chicago.
- Amaral, Pedro S. and Erwan Quintin (2010). ‘Limited Enforcement, Financial Intermediation, and Economic Development: A Quantitative Assessment,’ *International Economic Review*, Vol. 51(3), pp. 785-811.
- Banerjee, Abhijit V. and Andrew F. Newman (1993). ‘Occupational Choice and the Process of Development,’ *Journal of Political Economy*, Vol. 101(2), pp. 274-298.
- Banerjee, Abhijit V. and Esther Dufo (2005). ‘Growth Theory through the Lens of Development Economics,’ *Handbook of Economic Growth*, Vol. 1, pp. 473-552.
- Banerjee, Abhijit V. and Benjamin Moll (2010). ‘Why Does Misallocation Persist?’ *American Economic Journal: Macroeconomics*, Vol. 2(1), pp. 189-206.
- Bartelsman, Eric, John Haltiwanger, and Stefano Scarpetta (2013). ‘Cross-Country Differences in Productivity: The Role of Allocation and Selection,’ *The American Economic Review*, Vol. 103(1), pp. 305-334.
- Beck, Thorsten, Ross Levine, and Norman Loayza (2000). ‘Finance and the Sources of Growth,’ *Journal of Financial Economics*, Vol. 58(1), pp. 261-300.
- Beck, Thorsten, Asli Demirgüç-Kunt, and Vojislav Maksimovic (2005). ‘Financial and Legal Constraints to Growth: Does Firm Size Matter?’ *The Journal of Finance*, Vol. 60(1), pp. 137-177.
- Becker, Gary S. (1973). ‘A Theory of Marriage: Part I,’ *Journal of Political Economy*, Vol. 81(4), pp. 813-846.
- Buera, Francisco J. (2008). ‘Persistency of Poverty, Financial Frictions, and Entrepreneurship,’ mimeo, January 2008.
- Buera, Francisco J., Joseph P. Kaboski, and Yongseok Shin (2011). ‘Finance and Development: A Tale of Two Sectors,’ *The American Economic Review*, Vol. 101(5), pp. 1964-2002.
- Buera, Francisco J. and Yongseok Shin (2013). ‘Financial Frictions and the Persistence of History: A Quantitative Exploration,’ *Journal of Political Economy*, Vol. 121(2), pp. 221-272.
- Caselli, Francesco and Nicola Gennaioli (2013). ‘Dynastic Management,’ *Economic Inquiry*, Vol. 51(1), pp. 971-996.
- Costinot, Arnaud (2009). ‘An Elementary Theory of Comparative Advantage,’ *Econometrica*, Vol. 77(4), pp. 1165-1192.
- Erosa, Andrés and Ana Hidalgo Cabrillana (2008). ‘On Finance as a Theory of TFP, Cross-Industry Productivity Differences, and Economic Rents,’ *International Economic Review*, Vol. 49(2), pp. 437-473.
- Galor, Oded and Joseph Zeira (1993). ‘Income Distribution and Macroeconomics,’ *Review of Economic Studies*, Vol. 60(1), pp. 35-52.
- Ghatak, Maitreesh, Massimo Morelli, and Thomas Sjöström (2001). ‘Occupational Choice and Dynamic Incentives,’ *Review of Economic Studies*, Vol. 68(4), pp. 781-810.
- Gilchrist, Simon, Jae W. Sim, and Egon Zakrajšek (2013). ‘Misallocation and Financial Market Frictions: Some Direct Evidence from the Dispersion in Borrowing Costs,’ *Review of Economic Dynamics*, Vol. 16, pp. 159-176.

- Giné, Xavier and Robert M. Townsend (2004). 'Evaluation of Financial Liberalization: A General Equilibrium Model with Constrained Occupation Choice,' *Journal of Development Economics*, Vol. 74, pp. 269-307.
- Greenwood, Jeremy, Juan M. Sanchez, and Cheng Wang (2010). 'Financing Development: The Role of Information Costs,' *The American Economic Review*, Vol. 100(4), pp. 1875-1891.
- Greenwood, Jeremy, Juan M. Sanchez, and Cheng Wang (2013). 'Quantifying the Impact of Financial Development on Economic Development,' *Review of Economic Dynamics*, Vol. 16, pp. 194-215.
- Guner, Nezih, Gustavo Ventura, and Yi Xu (2008). 'Macroeconomic Implications of Size-Dependent Policies,' *Review of Economic Dynamics*, Vol. 11, pp. 721-744.
- Hamermesh, Daniel S. (1993). *Labor Demand*, Princeton University Press, Princeton.
- Hopenhayn, Hugo A. and Richard Rogerson (1993). 'Job Turnover and Policy Evaluation: A General Equilibrium Analysis,' *Journal of Political Economy*, Vol. 101(5), pp. 915-938.
- Hopenhayn, Hugo A. (2012). 'On the Measure of Distortions,' mimeo, May 2012.
- Hosono, Kaoru and Miho Takizawa (2012). 'Do Financial Frictions Matter as a Source of Misallocation? Evidence from Japan,' mimeo, December 2012.
- Hsieh, Chang-Tai and Peter J. Klenow (2007). 'Relative Prices and Relative Prosperity,' *The American Economic Review*, Vol. 97(3), pp. 562-585.
- Hsieh, Chang-Tai and Peter J. Klenow (2009). 'Misallocation and Manufacturing TFP in China and India,' *The Quarterly Journal of Economics*, Vol. 124(4), pp. 1403-1448.
- Hsieh, Chang-Tai and Peter J. Klenow (2010). 'Development Accounting,' *American Economic Journal: Macroeconomics*, Vol. 2(1), pp. 207-223.
- Jeong, Hyeok and Robert M. Townsend (2007). 'Sources of TFP Growth: Occupational Choice and Financial Deepening,' *Economic Theory*, Vol. 32(1), pp. 179-221.
- King, Robert G. and Ross Levine (1993). 'Finance and Growth: Schumpeter Might be Right,' *The Quarterly Journal of Economics*, Vol. 108(3), pp. 717-737.
- Krusell, Per, Lee E. Ohanian, José-Victor Ríos-Rull, and Giovanni L. Violante (2000). 'Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis,' *Econometrica*, Vol. 68(5), pp. 1029-1053.
- La Porta, Rafael, Florencio Lopez-de-Silanes, Andrei Shleifer, and Robert W. Vishny (1998). 'Law and Finance,' *Journal of Political Economy*, Vol. 106(6), pp. 1113-1155.
- Legros, Patrick and Andrew F. Newman (2002). 'Monotone Matching in Perfect and Imperfect Worlds,' *Review of Economic Studies*, Vol. 69(4), pp. 925-942.
- Levine, Ross and Sara Zervos (1998). 'Stock Markets, Banks, and Economic Growth,' *The American Economic Review*, Vol. 88(3), pp. 537-558.
- Lucas, Robert E. (1978). 'On the Size Distribution of Business Firms,' *The Bell Journal of Economics*, Vol. 9(2), pp. 508-523.
- Midrigan, Virgiliu and Daniel Yi Xu (2013). 'Finance and Misallocation: Evidence from Plant-Level Data,' *The American Economic Review*, forthcoming.
- Moll, Benjamin (2012). 'Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?' mimeo, August 2012.

Mookherjee, Dilip and Debraj Ray (2002). ‘Contractual Structure and Wealth Accumulation,’ *The American Economic Review*, Vol. 92(4), pp. 818-849.

Parente, Stephen L. and Edward C. Prescott (2002). *Barriers to Riches*, MIT Press, Cambridge.

Piketty, Thomas (1997). ‘The Dynamics of the Wealth Distribution and the Interest Rate with Credit Rationing,’ *Review of Economic Studies*, Vol. 64(2), pp. 173-189.

Rajan, Raghuram G. and Luigi Zingales (1998). ‘Financial Dependence and Growth,’ *The American Economic Review*, Vol. 88(3), pp. 559-586.

Restuccia, Diego and Richard Rogerson (2008). ‘Policy Distortions and Aggregate Productivity with Heterogeneous Establishments,’ *Review of Economic Dynamics*, Vol. 11, pp. 707-720.

Rosen, Sherwin (1982). ‘Authority, Control, and the Distribution of Earnings,’ *The Bell Journal of Economics*, Vol. 13(2), pp. 311-323.

Sampson, Thomas (2011). ‘Assignment Reversals: Trade, Skill Allocation and Wage Inequality,’ mimeo, June 2011.