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SELF-FULFILLING BELIEFS AND BOUNDED BUBBLES
IN THE U.S. HOUSING MARKET

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Abstract

This paper provides an equilibrium framework to organize the following empirical observations on the U.S. housing market from 1975 to 2007: (i) housing tenure and vacancies were approximately constant, (ii) rents were approximately constant, and (iii) in the late 1990s there was a large house price appreciation. Borrowing ideas from search and matching theory, and closing the model with self-fulfilling beliefs about the housing market, the model generates a house price bubble as a consequence of multiple underlying steady state equilibria. To select a deterministic equilibrium, household confidence is assumed to take one of two sunspot-driven values: normal or exuberant. When confidence is normal, both rents and house prices are low. When confidence is exuberant, both rents and house prices are high. Randomization over these two equilibria implies a substantial increase in house prices and constant rents as the probability of the exuberant state increases, although it is not realized. The model can explain a house price bubble as a rational expectations equilibrium driven by self-fulfilling beliefs.

Keywords

search, house prices, sunspots

JEL Classification Codes: E30, E32, R32

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*Masanori Kashiwagi
Max Weber Fellow, 2010-2011*

1 Introduction

This paper explains the recent boom in U.S. house prices as a sunspot phenomenon in a rational expectations equilibrium framework. Empirical data from 1975 to 2007 suggest that the U.S. housing market was stationary in terms of demographics and quantities. Among the total housing units, the shares of rented, owned and vacant units were stable over time. By contrast, house prices grew rapidly from the late 1990s until 2006, while rents were stable. Applying ideas developed in search and matching theory, this paper presents a simple model that accounts for these facts.

In the search and matching model, the meeting of traders creates a surplus which must be divided between them. As Howitt and McAfee (1987) point out, this leads to a situation with fewer equations than unknowns. Consequently, the model displays a steady state indeterminacy. Most of the literature resolves this indeterminacy by assuming Nash bargaining with a fixed bargaining weight. This paper, however, takes a different route. As suggested by Farmer (2009), I close the model by treating confidence as a fundamental.¹ To select a deterministic steady state equilibrium, I assume that household confidence may take one of two values: normal or exuberant. When confidence is normal, both rents and house prices are low. When confidence is exuberant, both rents and house prices are high. The level of household confidence selects one of two underlying deterministic steady state equilibria.

Although the paper focuses on two of the deterministic model equilibria, the set of deterministic equilibria is much larger. There is a continuum of steady state equilibria, each associated with a different division of the surplus between the buyer and the seller of a house. I choose to select just two of these underlying steady state equilibria because I am interested, in this paper, in studying bubbles. I am motivated by the fact that there is very little variation in rents or quantities in the U.S. data, which suggests that most of the time the economy is in what I call a normal steady state equilibrium.

I model a bubble as a non-stationary increase in house prices that is triggered by increasing confidence among buyers and sellers that the economy is likely to move from the normal steady state to the exuberant state. Although the existence of the exuberant state is important to make beliefs rational, I argue that the economy has never reached this state in the observed history of house price bubbles in U.S. data. In effect, I am modeling a non-stationary bubble equilibrium as a sunspot in the sense of

¹Hall (2005) is another example of taking a different approach from Nash bargaining to resolve the indeterminacy, using wage stickiness as an equilibrium selection mechanism.

Cass and Shell (1983).

Sunspots work as a signal that coordinates actions and moves the economy by affecting the likelihood of the exuberant economy occurring. I treat the random arrival of news as a sunspot. While the economy is likely to be in a normal state most of the time, news may signal the possibility that an exuberant state is likely to occur. After receiving such news, people get incremental evidence about the likelihood that it is true. As news arrives randomly, it progressively drives the economy towards the exuberant state. Alternatively, households may receive a signal that the news was incorrect, which triggers a collapse back to the normal state. This randomization by sunspots creates multiple states which are associated with the likelihood of the exuberant state occurring. Note that in the sunspot equilibrium constructed in this paper, the economy does not jump instantly between the two underlying equilibria.

House prices increase as the economy moves closer to the exuberant economy, while rents are stable along the path. Nevertheless, the exuberant state has not been realized in the observed data. Rather, I interpret that the economy experienced a crash before reaching that state. In this framework, news drives prices. This paper approaches the U.S. experience of housing market bubbles within this conceptual framework. Unlike the standard rational bubbles argument, variations in house prices are bounded as traders appropriate positive surpluses. Accordingly, I call the phenomena “bounded bubbles.” A prominent feature of this paper is that it provides an equilibrium framework to support a non-fundamental account of the recent housing boom.

It might be argued, as an alternative explanation, that the large appreciation in house prices was due to an increase in real income associated with economic growth. An increase in household income leads to more spending on housing services. To control for this explanation, I deflate nominal prices by nominal income. The proposed house price series still exhibits the surge from the late 1990s, and the rent series deflated in this way is still stable.

Section 2 of this paper provides a literature review. Section 3 documents three observations regarding the U.S. housing market from 1975 to 2007. Section 4 presents a theoretical model. Section 5 discusses quantitative results and shows that recent housing bubbles can be characterized as a rational expectations equilibrium. Section 6 concludes.

2 Related Literature

The model in this paper applies ideas developed in the labor search literature (for example, see Mortensen and Pissarides (1994)) to the housing market. The pioneering work by Wheaton (1990) studies the homeownership market and presents comparative statics. Subsequent work taking this approach includes Williams (1995) and Krainer (2001).

Among recent literature that studies the empirical implications of search models of the housing market, this paper's focus is related to Piazzesi and Schneider (2009), who use a search model to examine the influence of a small number of optimistic traders on house prices. They model the surge in house prices as a one-time shock to the beliefs of a small fraction of households and present the transition of prices back to the original steady state. Among other recent studies, Ngai and Tenreyro (2009) account for seasonal fluctuations in the housing market through a stochastic job matching model taken from Jovanovic (1979).

My approach to the housing bubble aligns with Shiller (2007), who argues that it does not appear possible to account for the recent house price boom in terms of fundamentals such as rents and construction costs; instead, the boom operates as a speculative bubble driven largely by extravagant expectations of future price appreciations. Unlike Shiller (2007), this paper characterizes the boom in house prices as a rational expectations sunspot equilibrium.²

The notion of sunspot equilibria is taken from the work by Azariadis (1981), Cass and Shell (1983), Azariadis and Guesnerie (1986) and Weil (1987). Sunspot equilibria are constructed by randomizing across a finite set of steady state equilibria.³ Assuming that agents share common beliefs about the sunspot activity and coordinate according to those beliefs, sunspots work as a way of moving from one equilibrium to another. Lastly, the idea of sunspots affecting a search economy is related to Farmer (2009, 2010).

²Peterson (2009) also uses a search model and presents a framework complementary to Shiller's argument. In Peterson (2009), households ignore the effects of search frictions on past prices and think that there has been a permanent change in the value of a house.

³As shown in Cass and Shell (1983) and Azariadis and Guesnerie (1986), a multiplicity of certainty equilibria is not necessary for the existence of a sunspot equilibrium.

3 Data from the U.S. Housing Market

This section presents three empirical observations using data from the U.S. housing market between 1975 and 2007: (i) housing tenure and vacancies were approximately constant, (ii) rents deflated by income were approximately constant, and (iii) during the late 1990s, there was a large appreciation in house prices in income units.

3.1 Tenure and Vacancies

Figure 1 presents data on housing broken down by type of occupancy. It shows the shares of rented, owned, and vacant units among the total housing units on the market.⁴ The data are quarterly and taken from the U.S. Census Bureau.⁵

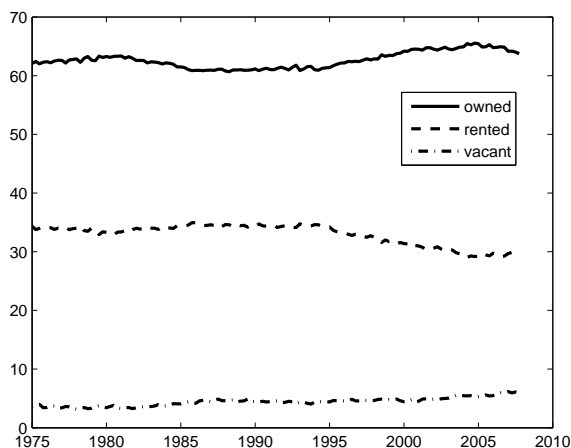


Figure 1: Occupancy of Housing Units (percent)

This figure shows that the proportions of owners and renters remained approximately constant over the sample period. More than 60 percent of all the housing units in the nation were occupied by owners. About 35 percent were rented, and the rest (about four percent) were vacant. Although these series display small variations, I will

⁴I have excluded housing units for occasional use and those occupied by people who usually live elsewhere.

⁵Housing Vacancies and Home Onwership (CPS/HVS), Historical Table 8.

assume in my model that occupancy rates are constant. This is a reasonable assumption because the movements in house prices over this period were huge, relative to the small movements in occupancy rates displayed in Figure 1.

In the census data presented in Figure 1, the count of occupied housing units is the same as the count of households.⁶ This implies that the number of total housing units per household was also constant over time. In fact, over the sample period there were 4-5 percent more housing units than the total number of households. Since the number of houses per household was approximately constant, one might expect that house prices were stable. This was not the case during the sample period.

3.2 House Prices

I use national house price indices published by the Federal Housing Finance Board (FHFB),⁷ which measure changes in single-family house prices.⁸ I deflate the nominal series⁹ in two different ways. The resulting series are plotted in Figure 2, and they are annual from 1975 to 2007 with the first observation normalized to one. The dashed line is nominal house prices deflated by the national consumer price index (CPI),¹⁰ and the solid line is deflated by nominal income.¹¹ The latter series is perhaps a more appropriate analog of the model construct as it abstracts from both inflation and real growth.

⁶See U.S. Census Bureau, Housing Vacancies and Homeownership: Definitions (<http://www.census.gov/hhes/www/housing/hvs/hvs.html>).

⁷Previously, it was the Office of Federal Housing Enterprise Oversight.

⁸These are repeat-sale indices; that is, they measure average price changes of housing properties sold at least twice. Measuring only repeat transactions on the same housing units helps control for changes in housing unit quality. Consequently, the indices show quality-adjusted house prices.

⁹To construct the time series of nominal house prices, I combine the level information about the house prices with FHFB house price indices. I use the nominal median home values reported in Davis *et al.* (2008).

¹⁰I use the CPI for commodities less shelter from the Bureau of Labor Statistics (BLS).

¹¹I have taken the annual median nominal household income series from the U.S. Census Bureau, Income, Table H-6.

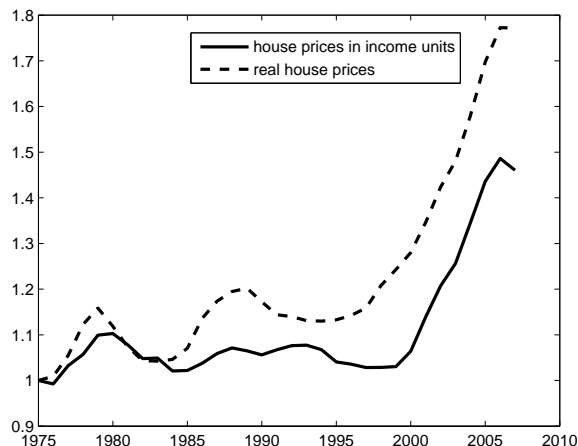


Figure 2: Real House Prices and House Prices in Income Units

Figure 2 shows that deflating by nominal income emphasizes both the price stability that existed until the late 1990s and the subsequent boom. The stability before the boom is consistent with the stationary behavior of quantities noted above. This figure also implies that inflation and growth do not fully account for the recent surge in house prices.¹²

3.3 Rents

While house prices in income units experienced a substantial increase, the rent series deflated by income remained stable. The same experiment was conducted for rents as house prices. Figure 3 presents real rent series (dashed line) and the series of nominal rents¹³ deflated by nominal income (solid line) with the first observations normalized to one. Like the house occupancy series, the rent series in income units was approximately constant over the sample period.

¹²It may be argued that the price increases are due to rises in the average house size because the repeated house price index does not control for trends in the quantity of housing services. To control for an increase in average house size, Van Nieuwerburgh and Weill (2010) deflate the real house price series by the average growth rate of house size between 1975 and 2007. The resulting series still exhibits a substantial increase in house prices.

¹³I construct the nominal rent series by combining the level information of annual rents taken from Davis *et al.* (2008) with the CPI for rents published by BLS.

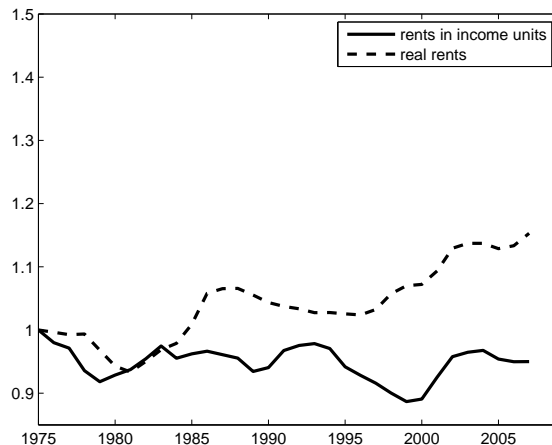


Figure 3: Real Rents and Rents in Income Units

In summary, we observe the following. Quantities in the housing market were stable, in the sense that occupancy rates of housing units were approximately constant. The rent series deflated by income was also stable, with only small fluctuations. Nonetheless, house prices in income units appreciated substantially. This paper provides a search and matching model to explain these data as a rational expectations sunspot equilibrium by allowing a multiplicity of deterministic equilibria.¹⁴

4 Model

Time is discrete and runs forever. There are two groups of people: renters and homeowners. On the supply side, there are competitive real estate agencies that allocate houses for rent or for sale. For simplicity, I assume that there is a fixed supply of houses in this economy. The rental market is assumed to be frictionless. In the ownership market, however, agents must search for a transaction partner.

¹⁴In Kashiwagi (2010), I consider a change in the interest rate and housing starts as alternative candidates to explain an asset price boom. In view of the time series, they do not seem to be important in generating the surge in prices.

The model has many features of the standard search and matching framework, but the assumption of a fixed bargaining weight is relaxed. By relaxing this assumption I am able to construct a model with multiple underlying steady state equilibria. I exploit this multiplicity to generate a rational expectations equilibrium with a house price bubble in which housing occupancy is always at its steady state.

I focus on two steady state equilibria, one with a high equilibrium rent and the other with a low equilibrium rent. It is important to observe that quantities (by which I mean measures of renters, homeowners, vacant units, houses on the rental market and houses on the ownership market) are the same in both of these equilibria due to the assumption that the housing supply is fixed and exogenous moving shocks govern the evolution of renters and homeowners. These assumptions imply that the determination of quantities in the model is trivial.

In addition to quantities, the model has five objects that are determined in equilibrium: (i) the value of being a renter, (ii) the value of being a homeowner, (iii) house prices, (iv) the present value of a vacant unit, and (v) rents. Below, I present equations that solve for those five objects. The appendix provides a full characterization of the model.

4.1 Households

I assume that the model economy has a fixed measure of households. Households are risk neutral and discount future utility at a constant rate β . Each household either rents or owns one housing unit. I assume that households draw higher utility from owning units than from renting units because of tax benefits and psychological satisfaction (for example, pride and sense of security). Consequently, renters seek houses to purchase. The probability of finding a housing unit to buy is subject to the search and matching friction described below. I assume an endowment economy, and in each period households receive a flow of income. Let R_t be the value of being a renter in period t . This value satisfies the following Bellman equation:

$$R_t = w - q_t + \beta \mathbb{E}_t [\phi_{t+1} (H_{t+1} - p_{t+1}) + (1 - \phi_{t+1}) R_{t+1}], \quad (1)$$

where w is the flow of income.¹⁵ In period t , renters pay rents q_t , and in the following period they search for homeownership. With probability ϕ_{t+1} , they find and purchase a housing unit at price p_{t+1} and then become homeowners. This probability is subject to a search and matching friction described in the appendix. The value of being a homeowner in period t is denoted by H_t . If searching renters do not find a house to buy, with probability $1 - \phi_{t+1}$, they remain renters.

Homeowners receive the same income flow w as renters. I assume that they enjoy extra utility $\alpha_H > 0$ from their owned homes. Unlike renters, homeowners have no rent payment. In the following period, homeowners may have to move out of their housing units due to exogenous events with probability m_H . (Such an event might be, for example, a job reassignment to another location.) In these cases, I assume that homeowners sell their housing units to the real estate sector and change their housing tenure into renters. This assumption simplifies the framework because one does not have to keep track of households' number of housing units. The real estate sector is implicitly owned by households in the model.¹⁶

The present value of a vacant housing unit at time t is denoted by W_t . I assume that the real estate sector is competitive. Hence, when homeowners move out, their housing units are sold to the real estate sector for the value of vacancy W_t , and the homeowners become renters. The value of being a homeowner is described recursively as:

$$H_t = w + \alpha_H + \beta \mathbb{E}_t [m_H (R_{t+1} + W_{t+1}) + (1 - m_H) H_{t+1}]. \quad (2)$$

4.2 Real Estate Sector

I assume a representative real estate sector that supplies housing units to the market. If homeowners move out of their housing units, competition implies that the real estate sector will purchase those units for the present value of a vacant unit W_t . The real estate sector has two options. First, it can rent out houses, in which case they are

¹⁵Due to the assumption of risk neutrality, budget constraints are irrelevant for the determination of rents and prices. They simply determine the timing of consumption, which is a matter of indifference to the households.

¹⁶The real estate sector represents the opportunity cost of holding a vacant unit and trying to sell it in the ownership market at a higher price. Here, I abstract from agents who move from an owned unit to another owned unit. This approach differs from Wheaton's (1990) framework, which focuses on the homeownership market. According to the American Housing Survey data (see footnote 26 for the source), however, about half of homeowners who move become renters.

always occupied by renters and the real estate sector receives rent payment q_t in period t . Second, it can post housing units for sale to potential new homeowners.¹⁷ In such cases, the real estate sector finds a new homeowner with some probability, selling the housing unit at price p_t during period t . This option is subject to a search and matching friction.

Let $W_{FR,t}$ be the value of the first option, renting out a housing unit for period t . Similarly, the value of the second option, posting a housing unit for sale, is denoted by $W_{FS,t}$. The value of renting out today is described as:

$$W_{FR,t} = q_t + \beta \mathbb{E}_t [\max \{W_{FR,t+1}, W_{FS,t+1}\}];$$

that is, the real estate sector receives rent payment for certain today and in the next period it faces the two options again.

If the real estate sector chooses to post a housing unit for sale, it faces a search and matching friction. Let θ_t denote the probability that the real estate sector finds a new homeowner to purchase the housing unit.¹⁸ Then, $W_{FS,t}$, the value of posting a housing unit for sale, satisfies the following Bellman equation:

$$W_{FS,t} = \theta_t p_t + (1 - \theta_t) \beta \mathbb{E}_t [\max \{W_{FR,t+1}, W_{FS,t+1}\}].$$

The real estate sector successfully matches with a renter with probability θ_t , and in this case a transaction occurs at price p_t . With probability $1 - \theta_t$, the real estate sector holds on to the housing unit and can choose between the two options in the following period.

Since the real estate sector is assumed to be competitive, the values of renting out and posting for sale equal the value of a vacant housing unit. This implies that

$$W_{FR,t} = W_{FS,t} = W_t,$$

for all t . Hence, the real estate sector is characterized by zero profit.¹⁹ (Recall that

¹⁷The real estate sector serves as an intermediary in the model economy. Rubinstein and Wolinsky (1987) and Yavas (1994) analyze the activity of intermediaries in bilateral trading. In their model, buyers and sellers can trade directly or indirectly through the intermediaries while here agents trade only through the real estate sector.

¹⁸This probability is characterized in the appendix.

¹⁹This condition attains the interior solution due to the indifference between renting out a house and posting it for sale. If the value of posting housing units for sale were higher than the other option ($W_{FS,t} > W_{FR,t}$), the real estate sector would put every housing unit in the homeownership market; as a result, there would be no housing units for renters. In the reverse case ($W_{FS,t} < W_{FR,t}$), there would be more rental units than renters. There is no friction in the rental market in that renters can find housing units to rent instantly; this pushes rents down until the value of renting out equals that of selling. This indifference condition guarantees that these two corners are avoided in equilibrium.

the real estate sector buys housing units for W_t from homeowners.)

Taken together, the equilibrium equations that characterize the real estate sector's behavior are

$$W_t = q_t + \beta \mathbb{E}_t [W_{t+1}], \quad (3)$$

and

$$W_t = \theta_t p_t + (1 - \theta_t) \beta \mathbb{E}_t [W_{t+1}]. \quad (4)$$

Equation (3) implies that the value of a vacant housing unit is the present discounted sum of rent flows. According to Equation (4), there is a “liquidity premium” in that the transaction price in the ownership market is higher than the value of a vacant unit that can be rented out for certain. This premium is characterized by the search friction and the discount rate. In the steady state, the transaction price p and the value of a vacant unit W satisfy $p = \frac{1-(1-\theta)\beta}{\theta} W$ and the coefficient is greater than one if $0 < \theta < 1$. The premium compensates for the risk that the real estate sector might fail to find a new homeowner and holds on to a housing unit as vacancy until the next period.

4.3 Rational Expectations Sunspot Equilibrium

While most of the literature assumes Nash bargaining between traders with a fixed bargaining weight, this paper's model is closed with self-fulfilling beliefs about the housing market. This approach exploits the view that the model exhibits multiple underlying steady state equilibria, as Howitt and McAfee (1987) point out. Consequently, there can be a house price bubble characterized as a rational expectations sunspot equilibrium.

To select a deterministic steady state equilibrium, I assume that household confidence may take one of two values: “normal” or “exuberant”. When confidence is normal, both rents and house prices are low. When confidence is exuberant, both rents and house prices are high. The normal rents are denoted by q^0 , and the exuberant rents are denoted by q^+ . The level of household confidence selects one of two underlying deterministic steady state equilibria.

I construct a rational expectations sunspot equilibrium by randomizing over the normal and exuberant steady state equilibria. In the construction of the equilibrium, I allow exogenous sunspots to affect the likelihood of the exuberant economy occurring.

In the model, states are associated with this likelihood. I treat the arrival of “news” as a sunspot that works as a randomization device.²⁰ This randomization creates *news states* in addition to the normal state and the exuberant state that correspond to the two underlying steady state equilibria.

Note that in the rational expectations equilibrium examined in this paper, the economy does not jump instantly between the two underlying equilibria. Rather, the economy makes incremental steps as the likelihood of the exuberant state changes. Each of these steps is itself a state. News acts as a sunspot that drives the economy from one state to another. The sunspot-driven states are denoted by s_t . Hence a variable X_t is a function of s_t and is denoted by $X_t(s_t)$. As can be seen in the quantitative analysis, house prices can increase substantially as the probability of the exuberant economy increases, without the realization of the exuberant state.

In the model, quantities are the same in both of the deterministic steady states and accordingly quantities are constant in a rational expectations equilibrium obtained by randomization.²¹ I assume that there are N states and I define the first and N th states respectively as the exuberant state and the normal state. The other states are “news” states that represent new information about the likelihood of the exuberant state happening. I construct a sunspot equilibrium in which rents are normal irrespective of sunspot realizations as long as the economy is not in the exuberant state. Although this is only one of the possible sunspot equilibria, it is one that is consistent with the data.

To explain why house prices appreciate but rents are constant, households must believe that eventually rents will be higher. This assumption is rational since the exuberant state, by assumption, is one in which rents, and house prices, are much higher than in the normal state. Rents in the rational expectations equilibrium are described by the following equations:

$$q_t(s_t) = q^+ \text{ if } s_t = 1 \text{ and } q_t(s_t) = q^0 \text{ if } s_t = 2, \dots, N. \quad (5)$$

I assume that the evolution of sunspots is Markovian with transition probability matrix Π , and I determine house prices endogenously for each state. The prices and rents in

²⁰The terminology here is different from the recent literature, including Beaudry and Portier (2004) and Jaimovich and Rebelo (2009). While news is about future productivity in their works, here news is about the likelihood of the exuberant economy occurring.

²¹State variables (which appear in the appendix) $\mu_{R,t+1}$, $\mu_{H,t+1}$ and $\mu_{V,t+1}$ are respectively denoted by $\mu_{R,t+1}(s_t)$, $\mu_{H,t+1}(s_t)$ and $\mu_{V,t+1}(s_t)$. Since this paper focuses on the steady state, they are constant.

each state are:

$$\underbrace{(p(1), q^+)}_{\text{exuberant state}}, \underbrace{(p(2), q^0), \dots, (p(N-1), q^0)}_{\text{news states}}, \underbrace{(p(N), q^0)}_{\text{normal state}}.$$

The self-fulfilling property of the rational expectations equilibrium implies that prices change because people believe that they will change in response to news. This exploits the idea of market psychology affecting house prices.

I assume that the transition probability matrix Π has the following structure:

$$\Pi = \begin{bmatrix} 1 - \pi_{1,N} & 0 & \cdots & 0 & \pi_{1,N} \\ 1 - \pi & 0 & \cdots & 0 & \pi \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & 1 - \pi & 0 & \pi \\ 0 & \cdots & 0 & 1 - \pi_{N,N} & \pi_{N,N} \end{bmatrix},$$

where the rows and columns represent current and subsequent states, respectively. I define the N th state as the normal state and assume that it is very stable in that the probability of remaining there ($\pi_{N,N}$) is very high. In the normal state, however, there is a small probability of moving to the $(N-1)$ th state. Once this shock occurs, there is a fixed probability of $1 - \pi$ of moving up by one state. With probability π , the economy goes back to the normal state. This structure allows easy computation of the expected duration of booms conditional on the economy being in the $(N-1)$ th state. In the later part, I parameterize the probabilities by targeting the expected duration of a boom. The constant probability assumption is not crucial for the computation of duration. One can assume different probabilities of moving up by one state for different states, but the assumption that in news states the economy will move up by one state or return to the normal state is important for simplifying the algorithm.²²

Due to the structure of the transition probability matrix, when there is a crash in house prices, prices crash back to the normal state. It would be relatively simple to modify this matrix to allow for a more gentle decline. I have not done this in the

²²Suppose more generally, the probability of returning to the normal state is different for different states. Let the probability of returning to the normal state if the economy is in state s be $\pi_{s,N}$. As I assume that in news states, the economy either moves up by one state or collapses to the normal state, the expected duration conditional on the economy being in the $(N-1)$ th state is

$$D = \pi_{N-1,N} + \sum_{i=2}^{N-2} \left[i \times \pi_{N-i,N} \left(\prod_{j=N-i+1}^{N-1} (1 - \pi_{j,N}) \right) \right] + \left(N - 1 + \frac{1 - \pi_{1,N}}{\pi_{1,N}} \right) \prod_{j=2}^{N-1} (1 - \pi_{j,N}).$$

present paper because I want to provide a stylized model of a bubble and a crash, and although declines are not instantaneous, they are much faster in the data, than price appreciations.

In this model, shifts in states generate a house price appreciation. On the other hand, these shifts have no feedback on the measures of rented, owned or vacant units. This is due to the assumption that housing units are fixed in the economy and that exogenous moving shocks govern the evolution of renters and homeowners. I focus on a balanced flow of renters and homeowners in order to match the observation discussed in Section 3. For each state, quantities are the same.

A rational expectations equilibrium must satisfy the individual rationality condition that each trader in the ownership market appropriates positive surplus from a match. For each period t , the surplus from a match is the difference in values of being a homeowner and a renter less the value of a vacant unit, $H_t(s_t) - R_t(s_t) - W_t(s_t)$. Then, a house buyer receives the surplus $H_t(s_t) - R_t(s_t) - p_t(s_t)$ from a transaction with price $p_t(s_t)$ and a seller appropriates the rest. Hence, individual rationality implies that prices are subject to $H_t(s_t) - R_t(s_t) > p_t(s_t) > W_t(s_t)$.²³

Since I focus on an equilibrium in which quantities²⁴ are constant and equal to their steady state values,²⁵ a rational expectations equilibrium is characterized by the values of being a renter and a homeowner, $\{R_t(s_t), H_t(s_t)\}$, house prices and rents $\{p_t(s_t), q_t(s_t)\}$, and the present value of a vacant housing unit $\{W_t(s_t)\}$. These five objects are subject to the Bellman equations (1) and (2), pricing equations (3) and (4), the stability property of rents outside the exuberant state (5), sunspot events $\{s_t\}$ evolving according to the transition probability matrix Π and the individual rationality condition $H_t(s_t) - R_t(s_t) > p_t(s_t) > W_t(s_t)$.

²³If the model is closed with a Nash bargain, prices are solved given the Nash bargaining weight. Let λ_t be the bargaining weight of a renter. In this approach, prices are formed through maximizing the Nash product $(H_t - R_t - p_t)^{\lambda_t} (p_t - W_t)^{1-\lambda_t}$ and the resulting surplus sharing rule $\frac{\lambda_t}{H_t - R_t - p_t} = \frac{1-\lambda_t}{p_t - W_t}$ works as a condition to solve for prices. Hence, using the model closed with confidence, the implied surplus sharing rate can be computed for state s_t by $\lambda_t(s_t) = \frac{H_t(s_t) - R_t(s_t) - p_t(s_t)}{H_t(s_t) - R_t(s_t) - W_t(s_t)}$. With the implied sharing rate for each state and the transition probability matrix, the model closed with Nash bargaining implies the same rents and prices. In this sense, the model closed with beliefs is isomorphic to the bargaining framework. Moreover, the individual rationality condition is equivalent to the notion that the implied surplus sharing rate λ_t is in an open interval $(0, 1)$.

²⁴Recall that quantities in this model are measures of renters, homeowners, vacant units, houses on the rental market and houses on the ownership market.

²⁵See the appendix for a full description of a rational expectations sunspot equilibrium.

5 Quantitative Analysis

This section discusses some quantitative implications of the model and shows that a house price bubble can be characterized as a rational expectations equilibrium. With the calibrated parameters presented below, the model generates a house price bubble as a consequence of multiple underlying steady state equilibria. I conduct a sensitivity analysis to show that the existence of house price bubbles is a robust feature of the model.

5.1 Parameterization

I calibrate the fixed parameters for a deterministic steady state that corresponds to the period of stability in U.S. house prices from 1975 to 1998. First, the probability of moving of homeowners m_H is calibrated at 0.04, which is the mean ratio of homeowners who become renters to total homeowners, according to the American Housing Survey.²⁶ I normalize the total measure of households to one, $\bar{N} = 1$. The total housing units relative to the number of households \bar{H} is taken from the U.S. Census Bureau data used to produce Figure 1. From the total housing units in the data, I subtract vacant units occupied by people who usually live elsewhere, units for temporary use and seasonally vacant units. In other words, the vacant units in the model correspond to vacant units on the market. The measure of total housing units is 1.045 per household.

As described in the appendix, the measure of matches M_t depends on the measure of renters $\mu_{R,t}$ and the measure of houses posted on the ownership market $\mu_{FS,t}$. I assume that the matching technology takes a Cobb-Douglas form:²⁷

$$M(\mu_{R,t}, \mu_{FS,t}) = \kappa (\mu_{R,t})^\gamma (\mu_{FS,t})^{1-\gamma}.$$

To calibrate the parameters of the matching function, I target the homeownership rate, which was stable at 65 percent. The curvature parameter γ is set at 0.5.²⁸ Given this curvature parameter, targeting a steady state homeownership rate of 65 percent implies that κ is 0.144. Note that the choice of γ by itself is not important for the main

²⁶I use the “Introductory Characteristics–All Housing Units” table and the “Previous Unit of Recent Movers” table.

²⁷This specification potentially leads to a situation in which $M_t > \min\{\mu_{R,t}, \mu_{FS,t}\}$, while this is not the case with the parameterization in the quantitative experiment.

²⁸The choice of the curvature parameter does not have welfare consequences because the model assumes fixed housing units.

analysis, as long as the scale parameter κ is properly specified to match the observed homeownership rate; this is because the relevant probabilities ϕ and θ in the ownership market are determined by the steady state homeownership rate. Those probabilities are computed as $\phi = 0.074$ and $\theta = 0.37$. The rate that renters become homeowners is consistent with the data on tenure change. According to the American Housing Survey data used to parameterize the probability of moving of homeowners m_H , the rate was about seven percent.

Data on the rent-price ratio are used to calibrate the time discount rate. The series was very stable around five percent before the housing price boom. Assuming that this is the steady state value of the rent-price ratio, the discount rate β can be calibrated at 0.945.

To calibrate the parameter of utility flow from a housing unit, I use observed rents. Assuming that the surplus is equally split in the targeted steady state, then rents are proportional to the extra utility flow from owning a housing unit, α_H . Nominal rents deflated by nominal income were stable at around 0.196, which is the average value between 1975 and 1998. The extra utility flow from homeownership α_H is calibrated at 0.0866. Note that to be compatible with the prices in income units presented in Section 3, I normalize the income flow w to one. Hence, homeowners enjoy extra utility equivalent to 8.66 percent of income from their own housing units relative to renters. This number is reasonable considering that there are tax benefits for homeownership²⁹ and that owning a house gives people psychological satisfaction in the form of pride and a sense of safety.

To parameterize the transition probabilities, I assume that the exuberant state is very stable once the economy gets there. For the probability of staying in the exuberant state, I assume a value of 0.99.³⁰ Also, the probability of moving to the boom path from the normal state ($1 - \pi_{N,N}$) is set at 0.03, which is based on the frequency of booms in asset prices observed in other asset markets.³¹ According to the price-earnings ratio data used in Shiller (2005),³² there were two rapid booms in stock prices over the sample period of 1881 to 2008. These two booms lasted from 1922 to 1929 and from

²⁹One could imagine a situation in which one third of an agent's income goes on mortgage interest payments and the tax rate is 25 percent.

³⁰This is also hypothetical, like the value of the exuberant rents. Sensitivity analysis for this parameterization is conducted below.

³¹As long as the probability of moving to the boom path $1 - \pi_{N,N}$ is a small number, the parameterization is insignificant for the result.

³²The most recent data can be found at <http://www.econ.yale.edu/~shiller/data.htm>

1991 to 2000, respectively. Excluding the boom and crash periods, there were about 100 periods.

Given that the economy is on the boom path, it remains on this path for an average of eight periods. Thus, I assume twelve states in the model, so that, on average, the boom path reaches the two-thirds of the way to the exuberant state.³³ Given this assumption, the structure of the transition probability matrix and the probability of staying in the exuberant state once reached, one can calibrate the probability of a crash for news states by targeting expected boom durations of eight periods. Then, the probability of a crash for news states is calibrated at $\pi = 0.268$.

I construct a rational expectations sunspot equilibrium in which rents are normal irrespective of sunspot realizations as long as the economy is not in the exuberant state. Considering the observation that rents in income units were stable over time, the normal rents are taken from the data on nominal rents deflated by nominal income, whose average value over the stable period (1975-1998) was 0.196. Rents in the exuberant state are assumed to be 2.5 times higher than normal rents. Among the states of the economy, only the one I call the exuberant state is associated with high rents; the others are associated with normal rents. I argue that rents and prices observed in the data are associated with those on the path towards the exuberant state. The existence of the exuberant state is the key to account for the surge in house prices and stable rents during the boom.

In the data presented in this paper, exuberant rents are not observed. But that is not unexpected given that the probability of observing that state in 100 years of data is only 0.1 given my parameterization of the transition matrix, and thus it is hard to empirically justify a parameterization of 2.5 times higher than normal rents. I interpret the recent house price appreciation as a move towards the exuberant state that crashed before the state was reached. Although, in the model, the crash is instantaneous, it should be clear for the structure of my argument that a more complex specification of transition probabilities would allow for a gentler decline. Table 1 summarizes the parameterization.

³³While the target of two-thirds of the way towards the exuberant state may still be arbitrary, I view that this is not too extreme.

Parameter	Description	Value
β	Discount rate	0.945
α_H	Extra utility from owning a house	0.0866
γ	Curvature parameter on matching function	0.5
\bar{H}	Measure of housing units in a location	1.045
\bar{N}	Total measure of households	1
κ	Scaling parameter on matching function	0.144
w	Flow income	1
m_H	Probability of homeowners moving	0.04
q^+	Exuberant rents	0.49
q^0	Normal rents	0.196
N	Number of states	12
$\pi_{1,N}$	Crash probability in the exuberant state	0.01
$\pi_{N,N}$	Staying probability in the normal state	0.97
π	Crash probability in a news state	0.268

Table 1: Parameter Values

5.2 Bounded Bubbles as a Rational Expectations Equilibrium

Assuming that the measures of renters and homeowners are constant, the price profile is computed as

$$\mathbf{p} = [9.02 \quad 7.11 \quad 6.14 \quad 5.47 \quad 5.01 \quad 4.69 \quad 4.46 \quad 4.31 \quad 4.20 \quad 4.13 \quad 4.08 \quad 3.97]',$$

and house prices substantially increase as the economy moves to higher states. As long as the economy is not in the exuberant state, rents are constant. As I show below, this house price profile satisfies individual rationality. Thus, the recent U.S. housing market can be characterized as a rational expectations sunspot equilibrium.

House prices in the exuberant state are high because of the stability and high rents in that state. To support the constant rent profile for the other states, house prices need to be expected to appreciate moderately. If house prices were expected to decrease, rents would have to be very high to compensate the expected depreciation. Also, if the expected appreciation were too high, rents would be very small (or even negative) because the value of a vacant house is the discounted sum of future rent payments. Note that the expected value of a house in a news state s is characterized by house prices in the $(s - 1)$ th state and in the normal state due to the structure of

the transition probability matrix. For states with high house values, moving further towards the exuberant state implies a substantial house price increase for moderate expected appreciation; this is because there could be a crash and prices could plummet. By modeling the exuberant state where rents are high, the framework implies that a surge in house prices could occur while rents are stable. This framework generates a boom in house prices with stable rents.

I compare prices in the bottom nine states with data from the recent boom observed from 1998 through 2006.³⁴ As Figure 4 shows, the model closely matches the empirical surge in house prices. This addresses the main theme of the paper: due to a steady state indeterminacy in the search and matching framework, self-fulfilling beliefs can drive a boom in house prices. If people believe that there is a small chance of reaching the exuberant state, house prices will appreciate substantially while rents remain constant.

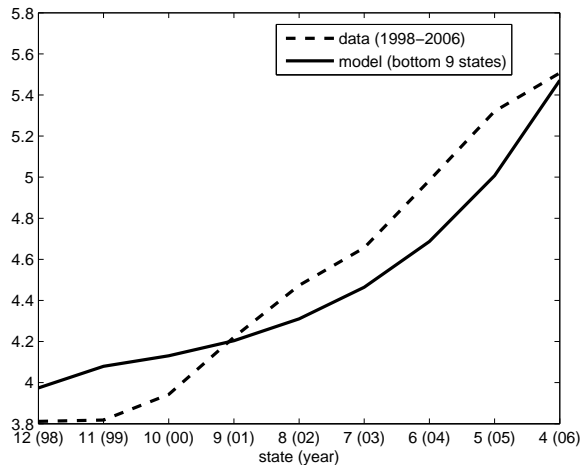


Figure 4: House Prices in Income Units Implied by the Model and Data 1998-2006

Figure 5 illustrates the fact that the division of the surplus in each state is individually rational. Accordingly, the model characterizes a house price bubble as a rational expectations equilibrium. A match surplus in period t is the gain in values from changing housing tenure $H_t - R_t$ minus the intrinsic value of a housing unit W_t .

³⁴The series are nominal house prices (including level information) deflated by nominal income. Note that in Figure 2, I normalize the first observation to one.

The transaction price p_t must stay inside the set $[W_t, H_t - R_t]$ so that each side of the match appropriates a nonnegative surplus. Even in the exuberant state with very high prices, agents are willing to trade if they match.

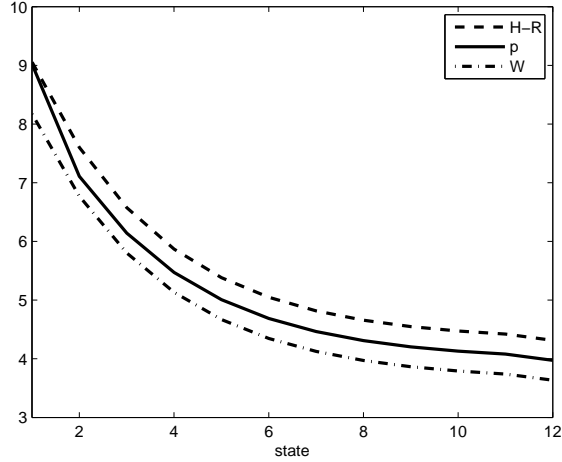


Figure 5: Individual Rationality in Each State

The figure shows how the surplus is split between renters and the real estate sector. In the normal state (12th state) the total surplus is equally split.³⁵ For most states, the surplus is shared approximately equally. Even in the state in which the price appreciates almost 40 percent (4th state), the sharing rate on the renter's side is 0.54. An obvious exception is the exuberant state, where the real estate sector appropriates most surplus. Such an extreme state is still supported as a rational expectations equilibrium. Table 2 summarizes rents, house prices and the fraction of surplus that renters appropriate in each state.

The stability of rents in all the states before the exuberant state is supported by small changes in the surplus in favor of renters. To see this, consider first the exuberant state, in which most of the surplus is appropriated by the real estate sector. Suppose that the surplus were to be equally appropriated by renters and the real estate sector in States 2 to N . As the economy approaches the exuberant state, house prices and

³⁵Precisely speaking, the implied sharing rate differs slightly from 0.5 (0.5002) due to randomization. The parameters are calibrated so that the surplus is split equally at the normal deterministic steady state.

the value of vacancy would increase since it becomes more likely in the future that the real estate sector will appropriate most of the surplus. With a constant surplus sharing rule, rents increase with house prices and the value of a vacancy. It follows that a small adjustment to the surplus sharing rule in favor of renters is required to maintain flat rents as the economy approaches the exuberant state.

State	Rents	House prices	Sharing rate of renter
1 (exuberant)	0.49	9.02	0.04
2	0.196	7.11	0.59
3	0.196	6.14	0.56
4	0.196	5.47	0.54
5	0.196	5.01	0.52
6	0.196	4.69	0.52
7	0.196	4.46	0.51
8	0.196	4.31	0.51
9	0.196	4.20	0.50
10	0.196	4.13	0.50
11	0.196	4.08	0.50
12 (normal)	0.196	3.97	0.50

Table 2: Prices and Implied Sharing Rate in Each State

5.3 House Price Distribution

Since the model implies a substantial house price appreciation similar to the data, it can address the probability distribution of house prices. The model is simulated for 30 periods starting from the normal state, which corresponds to the period from 1975 to 2006. This is iterated 10^5 times. The simulation is of the house price distribution in 2006 with the initial period (1975) in the normal state. Figure 6 presents the result. The vertical axis is the probability and the horizontal axis is the state indexed as in Table 2.

It is highly likely that the economy is in the normal state with probability 0.88. With probability 0.1, the economy is in one of the news states. Since the economy moves towards the exuberant state one state at a time and potentially collapses to the normal state from each state, the probability monotonically decreases as the econ-

omy approaches the exuberant state. Among the news states, the 4th state, which corresponds to the price level observed in 2006, occurs with probability 0.003. The simulation also suggests that the economy is in the exuberant state with probability 0.02. This probability is low but higher than the probabilities of relatively high news states being observed, due to the assumption that the exuberant state is stable once the economy gets there.

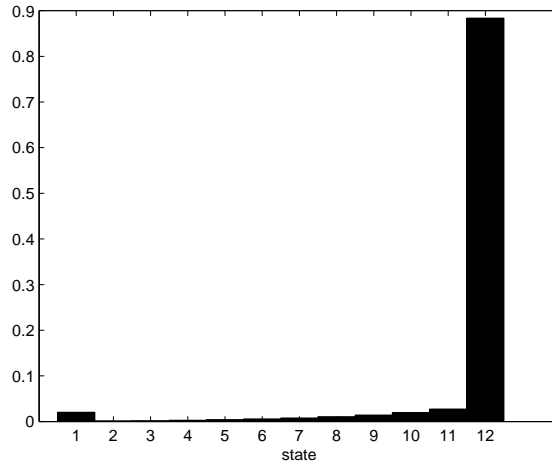


Figure 6: Probability Distribution of House Prices

5.4 Sensitivity

This subsection examines the implications of different values of rents and sustainability in the exuberant state. First, I consider variation in rents in the exuberant state, keeping the transition probability matrix fixed. I use rents in the exuberant state (q^+) of 2.5 times (as used in the analysis above), 2.25 times and two times as much as the normal rents (q^0). In Figure 7, I plot the appreciation of house prices as moving from the normal state upward by eight states for different values of rents in the exuberant state. The graph with rents 2.5 times higher than normal corresponds to Figure 4. With rents of 2.25 times and two times as high as the normal rents, house prices appreciate more than 30 percent and about 25 percent, respectively.

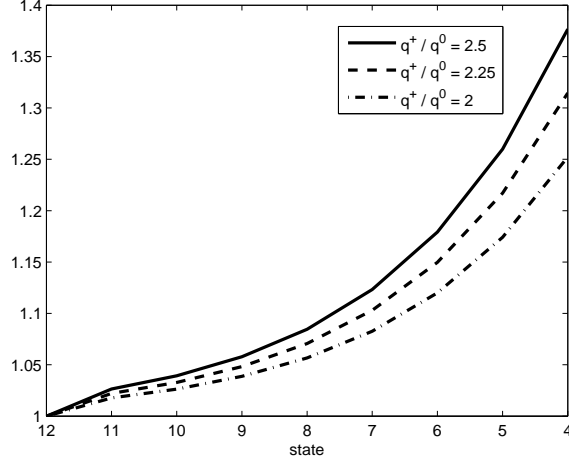


Figure 7: Price Appreciation for Variations in Rents in the Exuberant State

In the second experiment, I consider a case in which the probability of collapsing from the exuberant state to the normal state is the same as the probabilities of crashing from the news states, $\pi_{1,N} = \pi$. This probability is set at 0.125 by targeting the same expected duration of a boom. The same experiment, summarized in Figure 8, shows that there is still a considerable increase in house prices moving towards the exuberant state, although the magnitude is not as large as the case with the original crash probability in the exuberant state. With the selected values of rents, prices appreciate more than 15 percent in eight periods.

With the transition probabilities used in the second experiment, exuberant rents 2.5 times higher than normal violate the individual rationality condition. Changes in the transition probability matrix affect not only prices but also households' values. The smaller sustaining probability of the exuberant state lowers the upper bound of house prices more than it lowers house prices in the exuberant state.

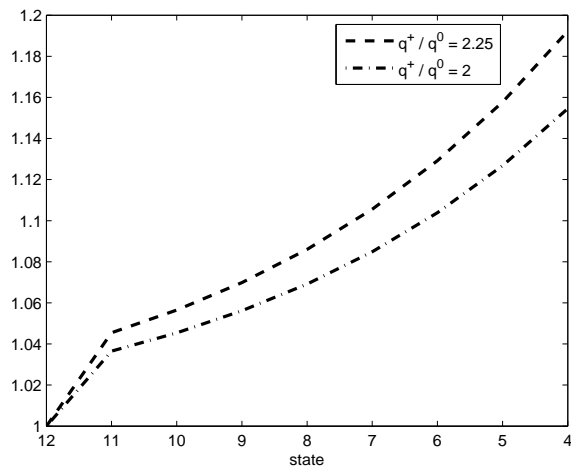


Figure 8: Price Appreciation for the Same Crash Probabilities over the States

6 Conclusion

This paper has focused on three observations about the U.S. housing market from 1975 to 2007. First, housing tenure and vacancy were approximately constant. Second, rents in income units were approximately constant. Third, house price data deflated by income nonetheless showed a large appreciation after the late 1990s. It has then provided a model that characterizes those observations as an equilibrium.

The model takes ideas developed in search and matching theory, and is closed with self-fulfilling beliefs. In particular, it focuses on two levels of confidence (normal and exuberant) regarding the housing market. The level of household confidence selects one of two underlying deterministic steady state equilibria.

The paper formalizes the idea that sunspots drove the recent boom in the U.S. housing market using an equilibrium framework. I have shown that because of multiple underlying deterministic equilibria, a house price bubble driven by self-fulfilling beliefs can exist in a rational expectations equilibrium. The bubble occurs as the economy moves towards the exuberant equilibrium and is driven by the arrival of news.

Due to the simplicity of the model, it is not capable of explaining changes in housing occupancy or house price depreciation, although a straightforward extension can accommodate the latter phenomenon. Also, we do not observe the exuberant steady

state equilibrium in the national data of the U.S. housing market. While the model is certainly not the sole explanation for the recent episode in the housing market, it does provide one formal explanation using the concept of a rational expectations equilibrium intersected with self-fulfilling beliefs.

Appendix

This appendix presents the determination of quantities in the model and provides the full characterization of the model.

Evolution of Housing Inventory

This subsection describes how the measures of renters and homeowners evolve in the model over time. Let $\mu_{R,t}$ and $\mu_{H,t}$ be the measures of renters and homeowners at the beginning of period t , respectively. As I assume that each household either rents or owns one housing unit, these variables also indicate the respective measures of rented and owned units. I consider a stationary model by abstracting from population growth and increases in housing units. The total number of households is denoted by \bar{N} , which implies that

$$\mu_{R,t} + \mu_{H,t} = \bar{N},$$

for all t . I also assume that the total number of housing units is exogenous and constant. Let $\mu_{V,t}$ denote the stock of vacant housing units at the beginning of period t . As all the housing units are rented, owned or vacant, we have

$$\mu_{R,t} + \mu_{H,t} + \mu_{V,t} = \bar{H},$$

for all t , where \bar{H} denotes the total measure of housing units and is constant. In this framework, the measure of vacant units is simply given by $\mu_{V,t} = \bar{H} - \bar{N}$. The model abstracts from fluctuations in vacant units, based on the observation that the numbers of households and housing units grew at approximately the same rate. Carefully examining this margin is interesting.³⁶ I, however, take the opposite approach of fixed housing units in the model economy.

Renters seek housing units to purchase but face a search and matching friction. Let M_t denote the measure of new matches between renters and housing units posted for sale. Among the stock of renters, measure M_t of them purchase houses and become homeowners. At the same time, there is an inflow of renters, as homeowners who move sell their units to the real estate sector and become renters. The probability of

³⁶For example, Kiyotaki *et al.* (2011) present a model in which housing units are produced from capital and fixed land.

drawing a moving shock is m_H . Hence, the stock of renters in the following period is characterized by the law of motion of renters:

$$\mu_{R,t+1} = \mu_{R,t} - M_t + m_H \mu_{H,t}, \quad (6)$$

for all t . Accordingly, the new flow of homeowners is given by the new match, M_t . The outflow is those who are hit by the moving shock. The law of motion of homeowners is given by:

$$\mu_{H,t+1} = \mu_{H,t} + M_t - m_H \mu_{H,t}, \quad (7)$$

for all t . This follows from the law of motion of renters (Equation (6)) and the fixed measure of households.

Matching

The measure of new matches, M_t , is a function of the measure of households searching for housing units to purchase and the measure of housing units that the real estate sector posts for sale. As every renter searches for a housing unit in the model, the first element is the measure of renters, $\mu_{R,t}$. Let $\mu_{FS,t}$ be the measure of housing units posted for sale. As in the bulk of the labor search literature, the matching function is assumed to be constant returns to scale and increasing in both arguments. The measure of new matches is given by

$$M_t = M(\mu_{R,t}, \mu_{FS,t}).$$

After the moving shock is realized, the real estate sector adds the new flow of houses from homeowners who move out to its stock of houses, rental and vacant units. In period t , the new flow is $m_H \mu_{H,t}$ and the existing stock of houses in the real estate sector is the rental and vacant units, $\mu_{R,t} + \mu_{V,t}$. The real estate sector allocates these housing units between those marketed for rent and those marketed for sale. Let $\mu_{FR,t}$ be the measure of housing units to be rented out. The measure of houses posted for sale is denoted by $\mu_{FS,t}$. Accordingly, the resource constraint on housing stock is

$$\mu_{FS,t} + \mu_{FR,t} = \mu_{R,t} + \mu_{V,t} + m_H \mu_{H,t}. \quad (8)$$

As I assume no friction in the rental market, the real estate sector should supply the same rental units as renters to clear the market. The measure of renters at the

beginning of period $t + 1$ is denoted by $\mu_{R,t+1}$. Recall that this is a state variable in period $t + 1$ and determined within period t . Hence the real estate sector rents out the measure $\mu_{R,t+1}$ of housing units to clear the rental market. Therefore, the measure of housing units to be rented out $\mu_{FR,t}$ satisfies:

$$\mu_{FR,t} = \mu_{R,t+1}. \quad (9)$$

The law of motion of renters, equation (6), implies that

$$\mu_{FS,t} - M_t = \mu_{V,t+1}.$$

The stock of vacant units at the beginning of each period equals the measure of housing units that are posted for sale but have failed to match with renters.³⁷

The measure of matches depends on the measure of renters $\mu_{R,t}$ and the measure of housing units posted for sale $\mu_{FS,t}$. The measure of matches divided by the measure of renters is the probability that a renter succeeds in finding a house, ϕ . Similarly, the measure of matches divided by the measure of houses posted for sale is the probability that a housing unit on the homeownership market is occupied, θ . Hence, these probabilities are determined through the evolution of the measure of renters and houses posted for sale:

$$\phi_t = \frac{M_t}{\mu_{R,t}}, \quad \theta_t = \frac{M_t}{\mu_{FS,t}},$$

for all t .

Full Description of a Rational Expectations Sunspot Equilibrium

Having described the determination of quantities in the model, a rational expectations sunspot equilibrium is defined as follows.

Definition. A rational expectations sunspot equilibrium is a sequence of measures of renters, homeowners and vacant units $\{\mu_{R,t+1}(s_t), \mu_{H,t+1}(s_t), \mu_{V,t+1}(s_t)\}$ given that their initial values $(\mu_{R,1}(s_0), \mu_{H,1}(s_0), \mu_{V,1}(s_0))$ are at their steady state, the measure of housing units rented out and posted for sale $\{\mu_{FR,t}(s_t), \mu_{FS,t}(s_t)\}$, the values of being a renter and a homeowner, $\{R_t(s_t), H_t(s_t)\}$, house prices and rents $\{p_t(s_t), q_t(s_t)\}$,

³⁷The time subscript is for purposes of correct interpretation. For derivation, the time subscript on the measure of vacant units can be ignored because it is constant.

and the present value of a vacant housing unit $\{W_t(s_t)\}$. These variables are subject to the Bellman equations (1) and (2), pricing equations (3) and (4), the stability property of rents except the exuberant state (5), laws of motion (6) and (7), the resource constraint on housing units (8), the rent market clearing condition (9), vacancy equation $\mu_{V,t+1}(s_t) = \bar{H} - \bar{N}$, sunspot events $\{s_t\}$ evolving according to the transition probability matrix Π and the individual rationality condition $H_t(s_t) - R_t(s_t) > p_t(s_t) > W_t(s_t)$.

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