



Department of Economics

Global Imbalances: Saving and Investment Imbalances

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*Thesis submitted for assessment with a view to obtaining the degree of
Doctor of Economics of the European University Institute*

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To Tommaso, for this and all future challenges we will face together.

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Abstract

The goal of the present thesis is to analyze the diverging savings and investment behavior among countries. The purpose of this work is to suggest possible explanations for the so called “global imbalances”. In particular, the focus is on the negative asset positions of the US and the positive asset positions of emerging economies. The first two chapters study the effects of capital market liberalization among countries with structural differences and in particular different financial market depth. Global imbalances are generated by higher propensity to save as well as lower propensity to invest in financially underdeveloped countries, with respect to countries with better financial institutions. The analysis is able to reproduce medium term net capital flows towards financially advanced economies as a result of financial integration. Moreover, capital liberalization generates welfare losses for emerging economies and a reduction of their capital convergence towards the steady state. The third chapter focuses on one of the most debated aspects of international capital movements, namely sovereign reserve accumulation by emerging countries, as a form of precautionary saving to be employed to face liquidity crises. The analysis investigates the determinants of the opportunity cost of holding reserves, and finds that countries optimally decide to hold a positive amount of reserves. Countries’ lenders set the cost of debt by taking into account countries’ decisions and their economic and financial characteristics.

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Part I

Introduction

Capital flows among countries have dramatically increased in the last 20 years. This intensification has not been evenly distributed across countries and time. There are however some persistent trends in the net foreign assets positions of some countries, and in particular negative asset positions of the US and positive asset positions of emerging economies. In the literature some researchers interpret those trends as the result of virtuous market equilibria; others consider those “global imbalances” a source of further instability that can lead to disruptive reversals. Moreover, during the recent financial crisis the global imbalances of the international financial positions have been judged as one of the possible channels through which the crisis has been exacerbated. Today policy makers in international fora declare the need for joint efforts to reduce those imbalances.

The purpose of this work is to highlight some causes of these diverging asset positions. The underlying hypothesis of the present analysis is that countries behave optimally, given some important constraints, therefore the aim is not to give normative prescriptions on the choice of international capital movements; the goal is to understand how those constraints lead to countries’ optimal, yet very different, choices.

In this thesis I analyze how international capital movements are shaped by different propensity to save and invest in different countries. In the first two chapters I highlight the effects of capital market liberalization among countries with structural differences and in particular different financial market depth. Global imbalances are generated by higher propensity to save as well as lower propensity to invest in financially underdeveloped countries, with respect to countries with better financial institutions. Moreover, I stress the welfare consequences of capital account liberalization as well as the short and long run effects on production and the interest rate for integrated countries with persistent financial depth asymmetries. The third chapter focuses on one of the most debated aspects of international capital movements, namely sovereign reserve accumulation by emerging countries as a form of precautionary saving to be employed to face liquidity crises. Together with my coauthor, Emanuele Tarantino, we study what determines the opportunity cost of holding reserves, and find that countries optimally decide to hold a positive amount of reserves.

In the first chapter I build on the idea of saving glut explanation of global imbalances, proposed by Ben Bernanke, to reproduce the diverging net asset positions of the US with respect to the emerging economies. In particular I explain the materialization of global imbalances as the result of capital account liberalization among countries with structural differences, and in particular different financial market depth. I formalize the financial development as the ability to insure entrepreneurs against idiosyncratic shocks to their production. Therefore the higher the sophistication of the financial market the lower is the impact of the

shocks on agents' wealth. Poor financial institutions bring about low capital accumulation and high levels of savings, with respect to the efficient allocation. When two countries that differ only in their level of financial depth decide to liberalize their capital movements, in the less financially developed economy agents decrease risky investments and increase savings by purchasing foreign risk free bonds, while in the other country agents produce more and accumulate liabilities. This chapter therefore replicates not only large net foreign asset positions of the advanced country as well as of the emerging economy, but also lower risk-free interest rates and lower capital before integration for the latter. Moreover capital account liberalization brings about negative welfare gains for the financially less developed country, and in particular for the poorer inside this country, and positive welfare gains in the financially advanced economy. By making use of this setup, I interpret the recent financial crisis as worsened financial conditions for all, but more so for the poor country, in line with the data. The increased gap in the financial market development pushes the emerging market to further increase savings and decrease investment, which results overall in even larger global imbalances.

In the second chapter I take a different perspective. The goal is to understand the consequences of capital account liberalization for a large emerging economy, in the process of accumulating capital towards its steady state, which decides to open its capital account towards an advanced economy that has already reached its steady state of autarky. In line with the previous chapter, the two economies differ in their level of financial depth. The asymmetry in the levels of financial depth has important effects on the convergence rate. The immediate effect of integration is that capital accumulation slows down in the emerging economy, with respect to the autarky accumulation path. Agents optimally decide to move resources from investment and present consumption to foreign bonds, therefore postponing growth and consumption. Equalization of the marginal productivity of capital, adjusted for risk, with the risk-free interest rate, together with the high precautionary motive drive the present result. If the emerging economy has a low level of accumulated capital at the moment of integration, it experiences an initial capital inflow of modest entity, followed by large capital outflows in search for safe assets. The present analysis can therefore replicate the empirical evidence of emerging economies' negative current account followed by the "uphill" flow of capital, towards advanced countries, as documented among others by Prasad et al. (2007). At the moment of capital account liberalization, agents in the advanced economy experience welfare gains, while agents in the other country undergo welfare losses. Those losses are however attenuated if the country is at an early stage of capital accumulation at the moment of integration, while it is particularly strong for emerging economies that are

close to the steady state.

There is however another important difference between the first and second chapter. The peculiar hypotheses on the functional forms in the first chapter (CARA utility function and normal distribution of the shock) allow insulating the effect of financial inefficiencies on the risk premium for investing in risky activities, which is defined as the difference between the marginal productivity of capital and the risk-free interest rate. In fact, the CARA-Normal specification shuts down any wealth effect on capital choice, therefore it allows to highlight the direct link between financial development and the gap between saving and investment in each country, therefore the global imbalances. In line with the standard (neo-classical) framework, the steady state level of capital depends only on the interest rate. In equilibrium, the amount of installed capital equalizes the marginal productivity of capital adjusted for risk with the interest rate on risk-free bonds. The effect of integration on the steady state levels of capital is higher capital and therefore production in the advanced country and lower in the emerging economy, both with respect to the autarky steady state levels.

In the second chapter, I use a more standard functional specification (CRRA utility function and log-normal distributions of the shocks). In this framework, capital accumulation, beside the effects of the interest-rate and the risk premium, is also affected by each agent's wealth. This seems more in line with empirical evidence and brings about important long run results. Risky capital is positively related to wealth, therefore, the advanced economy, with its increasing negative assets positions, in the long run will converge to a level of capital and production lower than the one of autarky. Instead, the emerging economy, by accumulating positive risk-free assets, converges to a steady state level of capital higher than the autarky one.

The technical differences between the two chapters allow characterizing the mechanisms at work. The short and medium run consequences of capital account liberalization are almost entirely due to the effect of the risk premium for investing in risky activities: this result is common in the two chapters. The long run consequences instead, are determined by the levels of aggregate wealth, which in turn are the result of persistent asymmetries in saving and investment decisions.

The third chapter analyzes one important aspect of the global imbalances, namely sovereign reserve accumulation by emerging economies. In the last years the level of sovereign reserves accumulated by emerging countries has increased dramatically, also in the aftermath of the financial crisis. A large part of researchers as well as policy makers believe the accumulated amounts are too high, in terms of country's opportunity cost of holding them. The

aim of this chapter is to highlight country-specific financial and economic characteristics that affect the opportunity cost of holding reserves. We take into account, among other aspects, the effects of reserves on the sovereign cost of borrowing from foreign markets. We study the contracting game between a sovereign country and its international lenders in order to contemporaneously assess the optimal level of reserves, chosen by the country, and the cost of debt, set by the creditors. Reserves represent, for the country, a safe and liquid investment, that can be employed should a liquidity crisis occur. Moreover, reserves are not pledgeable, in case of sovereign default, and distract resources from a productive but risky and illiquid project. There is perfect information; markets are characterized by two important financial frictions, widely used in the literature: creditors cannot inject resources in case of liquidity crises, the country cannot commit not to default on debt if it finds convenient to do so. In line with the empirical evidence, we find that the cost of the sovereign debt decreases when the level of reserves increases, but only if the probability of a liquidity crisis is high enough. However, the cost of debt could increase with reserves when lenders anticipate that in case of a liquidity crisis the country will prefer to default. We moreover find that other variables play important roles in determining the willing to hold large amounts of reserves: the variability of country's output, the degree of financial market openness and financial institution depth, the productivity and the dimension of the economy.

Part II

Chapters

CHAPTER 1

THE SAVING GLUT EXPLANATION OF GLOBAL IMBALANCES: THE ROLE OF UNDERINVESTMENT

Abstract

According to the ‘Saving Glut hypothesis’, global imbalances are caused by an inefficiently high level of precautionary savings in financially underdeveloped regions, where agents have limited opportunity to diversify idiosyncratic risk. This paper generalizes the approach by modeling idiosyncratic risk in entrepreneurial activities, which can be only partially hedged. As a result, agents save too much and invest too little, relative to the efficient allocation, depressing production activities and the real interest rate. Capital account liberalization towards financially more advanced economies then produces an outflow of capital in search of safer investment, with the effect of further reducing domestic investment in countries with poor financial institutions. The model predicts welfare losses for less financially developed economies, and an increase in wealth inequality for advanced economies. Finally, the present analysis is able to explain the direct link between the financial crisis and global recession and the long run implications of worsening financial conditions on countries’ net external positions.

1.1 Introduction

"Over the past decades a combination of diverse forces has created a significant increase in the global supply of savings - a global saving glut - which helps to explain both the increase in the US current account deficit and the relative low level of long-term real interest rate."

Ben Bernanke, March 10, 2005 speech at the Sandridge Lecture, Richmond, Virginia

Bernanke is credited with the idea that the US current account deficits in recent years are due to a saving glut in the rest of the world. According to this view, financial global imbalances are the equilibrium result of structural differences that emerged among groups of countries. Bernanke's position has recently been developed and formalized in the international portfolio literature. Several contributions point at gaps in financial market development, financial integration or growth potential among countries to generate precautionary savings differences in US with respect to other developed and emerging economies, these in turn cause financial global imbalances, as observed in the data. Only the savings side of the current account is typically analyzed in these studies. The aim of our work is to go one step further and disentangle the two components of the current account: investment and savings. We explicitly model the impact of financial market institutions on investment demand and savings supply in order to show that financial integration among economies with structural differences in their financial markets generates, not only precautionary savings, but also underinvestment in financially less developed countries. The two aspects together combine to generate large and persistent net financial borrowing by the financially more advanced economy.

Finally the present analysis contributes to the discussion on the effects of the financial crisis with two important results: on the one hand worsened global financial conditions have the direct effect of reducing investment and lowering interest rates, bringing about a global reduction in output. On the other hand the widened gap between industrialized and emerging countries in their level of financial development exacerbates the saving glut in its two components of domestic underinvestment and higher precautionary savings, and therefore the negative net external position of the US.

We present a two-country model with heterogeneous agents and idiosyncratic production risks. The development of financial institutions is formalized as the ability of the market to absorb and redistribute idiosyncratic shocks to production, therefore the higher the sophistication of the financial market the lower is the impact of idiosyncratic shocks on agents' wealth. In this framework consumers decide to accumulate more savings the higher is their

uninsurable risk; the new element of our analysis is that entrepreneurs face a risk premium for investing in risky production, instead of risk free bonds, and therefore they invest less the more variable their productive activities are. The development of financial markets has a strong impact on the equilibrium level of interest rate and GDP: the lower the ability to insure entrepreneurs against idiosyncratic shocks, the lower the capital and interest rate in the steady state.¹ When two countries with different financial institutions decide to reciprocally open their capital accounts by exchanging risk free bonds, in the less financially developed economy agents decrease risky investments and increase savings by purchasing foreign risk free bonds, while in the other country agents produce more and accumulate liabilities. This paper therefore replicates not only the large financial imbalances of developed countries, but also lower risk-free interest rates and lower capital accumulation in less financially developed economies: a condition that worsens when the country opens to foreign capital.

Financial integration results in overall negative welfare gains for poorer economies and slightly positive for others. The main channel is the increase in the interest rate with respect to the risk-adjusted return on production investment for emerging economies; this divergence pushes agents to accumulate savings, postpone consumption and shift investment to foreign bonds instead of internal production capital. Wealth conditions worsen for the poorer in developing countries, since they face higher interest on their debt. Only the richest, in the last decile of the wealth distribution, are better off since they receive higher returns on their accumulated assets. In contrast, the financially more developed economies experience a decrease in their interest rates that boosts consumption and investment in entrepreneurial activities. They therefore move resources from safe investment in bonds, which are now less profitable, to risky production capital. This is the main cause of the increased dispersion in the wealth distribution and therefore an increase in wealth inequality.

The immediate consequences of the financial crisis are worsened financial conditions, that in this setup translate into higher uninsurable risks associated with production activities, in all countries; the direct effects are a decrease in the level of invested capital and a lower equilibrium interest rate for all. Poorer economies, however, experience larger drops in the quality of their financial markets compared with financially more advanced economies. This, in an integrated world, results in a further reduction in the level of risky investment and an additional incentive to accumulate precautionary savings in the form of foreign risk free bonds. In countries that are financially more advanced, integration helps to mitigate the negative effects of the financial turmoil since the new and very low level of interest

¹Bandiera et al. (2000) document that the real risk free interest rate, before financial liberalization takes place, is depressed in emerging economies with respect to the advanced ones. Moreover Bonfiglioli (2008), among others, shows that financial development has a positive impact on growth.

rate discourages an excessive drop in production investment, boosts consumption and further reduces the precautionary saving motive. The present analysis therefore predicts an acceleration in the negative and growing net external position of the US.

Welfare consequences are negative for all the countries hit by the financial crisis. However the very poor in financially more advanced economies have slightly positive welfare gains due to lower interest rates paid on their accumulated debt and a boost in consumption (two hypotheses drive this result: no borrowing constraints and no possibilities of default).

In the literature there are several contributions that stress the role of financial institutions in explaining international capital flows. Caballero, Farhi and Gourinchas (2008) stress the importance of the availability of domestic financial instruments for real investments together with growth differentials to generate capital flows toward the US. Prades and Rabitsch (2009) look at differences in financial liberalization processes and aggregate productivity to explain financial global imbalances and in particular the US external deficit. In this context, Fogli and Perri (2006) point to the effects of financial innovation in decreasing output volatility that, in turn, reduces precautionary saving needs and brings about large global imbalances.

In the present work, however, emphasis is put on idiosyncratic risks, since, as documented by Angeledis among others, they explain more than half of total economic variability in the US. We want to show that heterogeneity among agents, by influencing their choices, has an impact on the aggregate equilibrium of the economy, and also on its interactions with other countries. In this sense, the contribution of Mendoza, Quadrini and Rios-Rull (2007) (MQR henceforth), is the closest to our analysis. MQR model economies in which agents are subject only to idiosyncratic shocks to labor productivity. They build a model based on Aiyagari (1994) but extend it to two countries, and formalize financial market institutions by introducing limited liability constraints on net worth. In the first country with less developed financial markets households are subject to tight borrowing constraints, therefore they save more at any level of the interest rate in order to have enough resources to face bad idiosyncratic shocks. When this country frees its capital movements with a financially more developed country (with therefore looser borrowing constraints and higher interest rate in equilibrium) agents of the former country are encouraged to save even more, given the higher interest rate in equilibrium. The opposite happens for households in the developed economy: they prefer to consume more in the present since they can borrow at a cheaper price than before. MQR are able to give an explanation of the large and persistent financial imbalances observed in the data: they are generated by different financial structures of countries that open their capital accounts.

There are two main differences between MQR and the present study. First of all MQR results are driven by the impact of financial depth on consumer's behavior. They find, however, that before opening to global financial markets, a developing country saves and produces more than a developed one, bringing the ambiguous message that better financial institutions are detrimental for countries' capital accumulation. Moreover this is at odds with the data.² In our analysis we focus on the impact of financial markets on entrepreneurial decisions as well as on consumers' choices: in equilibrium poor financial institutions result in lower capital accumulation and a lower interest rate in less financially developed economies. Improvements in developing countries' financial markets generate only positive outcomes since they help to enhance welfare, stimulate investment and dampen large uninsurable financial volatility. The second difference is in the formalization of financial markets: MQR introduce a borrowing limit. In our study, in contrast, agents are not constrained on short sales; this enables the model to fully show the effect of financial incompleteness, formalized as missing insurance markets, on savings and investment decisions. Moreover the investment risk premium, which in the model is given by the wedge between the risk-free interest rate and return on capital and is generated by financial underdevelopment, has its counterpart in financial data. Therefore it creates a clear mapping from the model to our empirical study, as highlighted in the next section.

The rest of the paper is organized as follows. The next section illustrates important stylized facts on financial globalization and financial development. Section 3 presents the closed economy model and focuses, in particular, on the impact of market incompleteness on saving and investment choices for single agents and for the aggregate steady state of the economy. In section 4 we present the steady-state results for the open economy setup, in which risk free bonds can be exchanged across countries. In section 5 we show the results of the quantitative exercise. Section 6 extends the results to account for the financial crisis. In section 7 some sensitivity analysis is conducted. Section 8 concludes with some final remarks.

1.2 Stylized facts

The present section aims at motivating our analysis by explaining the definitions and presenting important figures on financial openness, international capital flows and measures of financial development (before and at the moment of the financial crisis) used in the rest of this study.

²Bonfiglioli (2007) for example finds no direct relation between financial openness and investment while

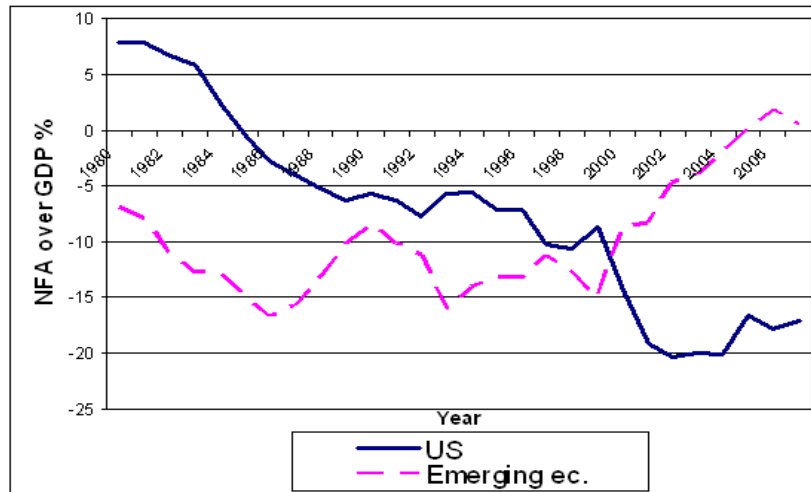


Figure 1.1: Net foreign asset position over GDP. Data source: updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007).

Figure 1.1 shows that the US current account has been negative since the end of the 80's and it has dropped dramatically during the last 10 years reported in the picture. Emerging economies' negative position has instead been recovering in the last 7 years of the sample.

Capital account liberalization is still an ongoing process and there are substantial differences among countries. Chinn and Ito (2007) construct an index of countries' financial integration based on *de jure* and *de facto* measure of financial restrictions. We divide their sample into industrialized and emerging economies³ and plot this information in figures 1.2 and 1.3, respectively. It is clear that emerging countries are far less integrated into international capital markets; therefore it is important for them to understand the possible effects of further liberalization in order to get all positive benefits and avoid negative outcomes.

Figures 1.2 and 1.3 also compare the evolution of gross country liabilities with the one of financial integration. Again there are important differences between the two groups of countries. For emerging economies, increasing financial globalization seems to bring about a slight decrease in gross liabilities, therefore lower capital inflows, while the opposite is true for industrialized economies. This observation is at odds with the neoclassical paradigm which predicts that countries scarce in capital will experience capital inflows once they open their current account.

Figure 1.4 points in the same direction as figure 1.3: it shows that the countries with

she finds a positive relation that links financial depth with capital accumulation.

³This division is taken from Lane and Milesi - Ferretti (2007).

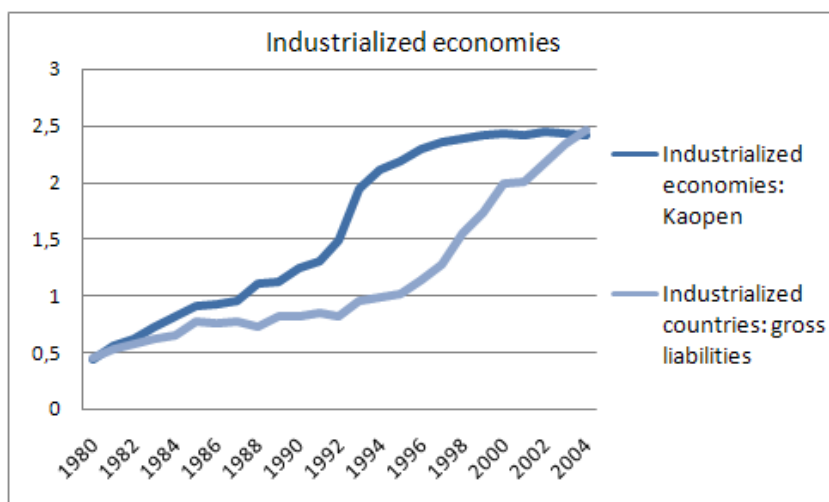


Figure 1.2: Financial integration (Kaopen), source: Chinn and Ito (2007); gross countries' liabilities, source: Lane and Milesi-Ferretti (2007).

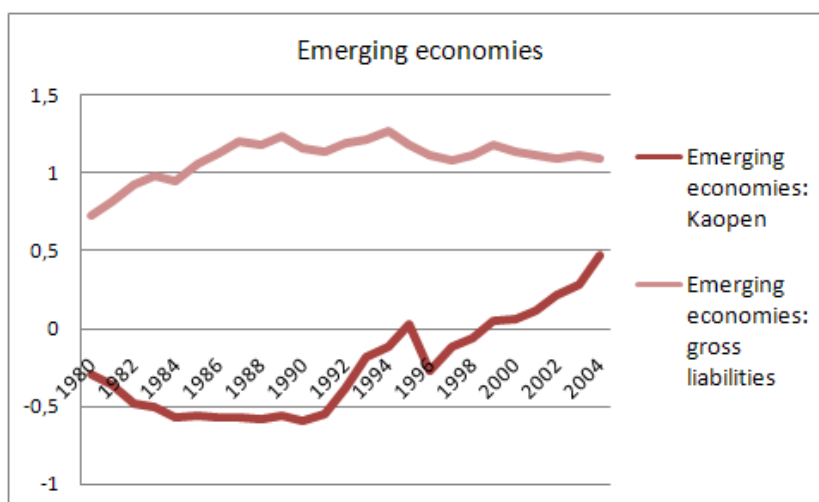


Figure 1.3: Financial integration (Kaopen), source: Chinn and Ito (2007); gross countries' liabilities, source: Lane and Milesi-Ferretti (2007).

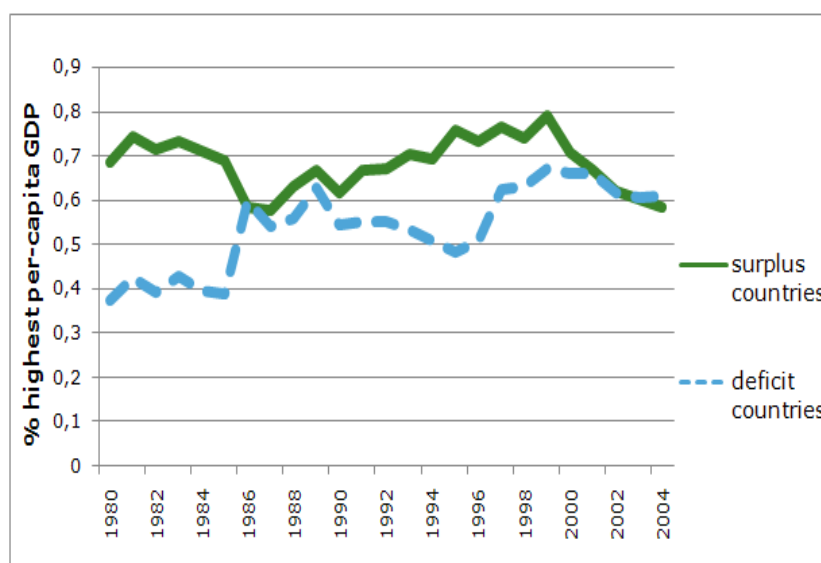


Figure 1.4: Average per-capita GDP, weighted by participation in international financial markets (Assets+Liabilities) over highest per-capita GDP in each year. Source: Lane and Milesi-Ferretti (2007)

negative net asset positions are no longer the emerging economies (with therefore lower per capita GDP). This was first observed by Prasad, Rajan and Subramanian (2007) who talk about the "uphill" flow of capital from developing to developed economies. Moreover they show that nonindustrial countries, that rely less on foreign capital, grow faster. The difference with the figure of Prasad et al. (2007) is that in our estimation countries are weighted by their *de facto* participation in international financial markets, as defined by Lane and Milesi-Ferretti (2007) (sum of foreign assets and liabilities). In contrast, Prasad et al (2007) use net asset positions. As shown in the previous figure, emerging economies experienced an acceleration in the integration process in the last 15 years, but this period does not seem to correspond with an overall capita inflow in those countries.

The analysis of financial depth suggests that there are still large differences among countries. Beck, Demirguc-Kunt and Maksimovic (2002) propose two measures of financial depth (reported in table 1): *PRIVO* is the total credit to the private sector over GDP, which is therefore a *de facto* indicator, while *Laworder* is an index, ranging between 1 and 6, which summarizes the information on the legal system and the protection of citizens.

Figure 1.5 is a scatter plot of data for 2007 on financial development, *PRIVO*, and data on the risk premium estimated by Aswath Damodaran, who elaborates data furnished by

Table 1: *de jure* and *de facto* measures of financial depth. Source: Beck, Demirguc-Kunt and Maksimovic (2002).

	Privo	Laworder		Privo	Laworder
North America and West Europe			East Europe		
Canada	0.83	6	Belarus	0.16	4
Germany	1.06	6	Estonia	0.16	4
Spain	0.79	4	Hungary	0.22	6
France	0.84	5	Poland	0.12	5
Italy	0.57	6	Romania	0.09	4.77
Mexico	0.22	2.41	Slovakia	0.3	5
Sweden	0.82	6	Slovenia	0.26	5
U.K.	1.16	6	Ukraine	0.02	4
U.S.	1.84	6			
Latin America			Asia		
Argentina	0.21	5	China	0.85	5
Bolivia	0.51	3	Indonesia	0.52	2.64
Brazil	0.32	2.05	Malaysia	1.31	4.59
Chile	0.68	5	Pakistan	0.23	3.14
Ecuador	0.3	3.36	Philippines	0.5	4
El Salvador	0.36	3	Singapore	1.11	6
Venezuela	0.1	4			

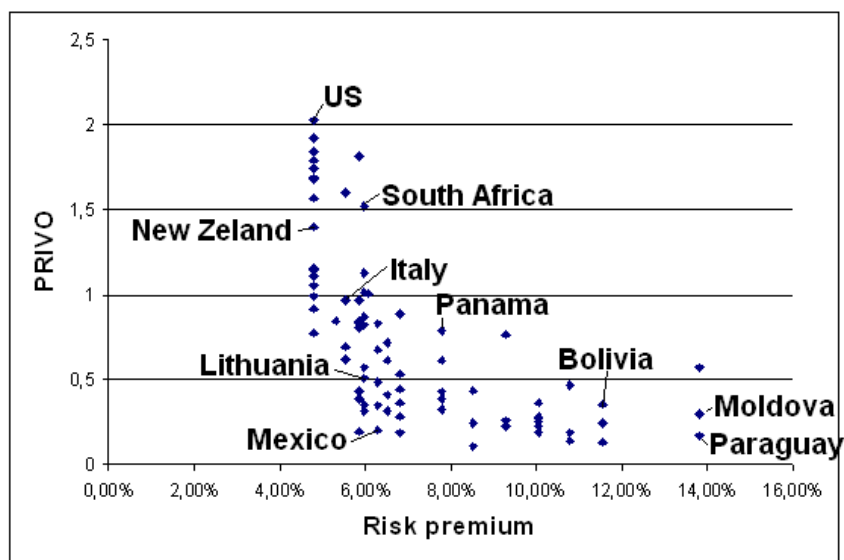


Figure 1.5: Measure of financial market development (Privo), source: Beck, Demirguc-Kunt and Levine (2009); measure of country risk premium, source: Damodaran (2008).

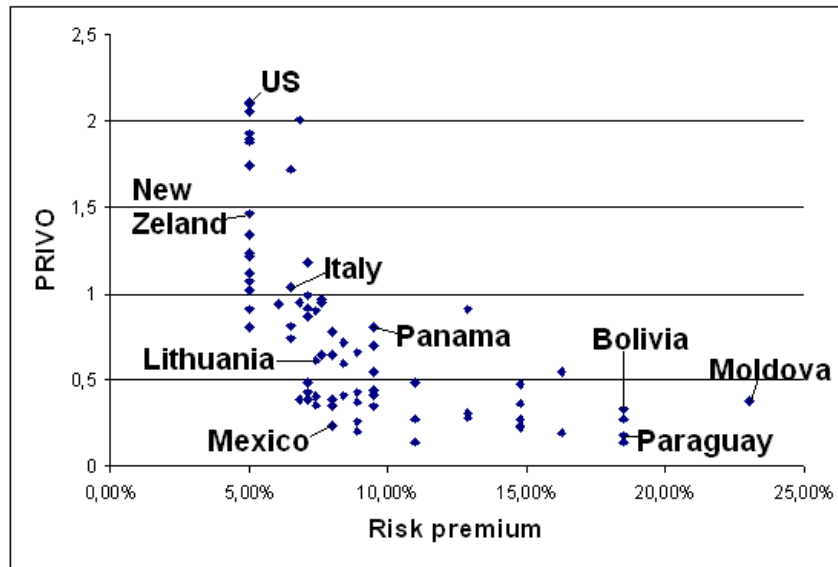


Figure 1.6: Measure of financial market development (Privo), source: Beck, Demirguc-Kunt and Levine (2009); measure of country risk premium, source: Damodaran (2009).

Moody's, Bloomberg and Standard&Poor's.⁴ The measure of financial market development, *PRIVO*, seems to be negatively correlated with the risk premium; therefore countries with large risk premium are also the ones with poor financial market development. The risk premium seems, therefore, to be a good proxy for financial market development. We therefore use this risk premium measure, which represents the depth of financial institutions, to calibrate our parameter on financial development in the quantitative exercise, through the investment risk premium generated by our model.

Figure 1.6 reports the scatter plot of *PRIVO* and the risk premium in 2008. Compared with 2007 data, the risk premium has risen in almost every country, but this increase is particularly strong in emerging markets. Worsened financial conditions have a deeper impact in countries with weaker financial institutions, therefore the gap between industrialized and emerging economies on their level of risk has increased during the current financial crisis.

As mentioned in the previous section, the development of financial markets in the present analysis is defined as the ability of financial institutions to transfer resources among agents and, in particular, to help consumers and entrepreneurs hedge their idiosyncratic shocks. The importance of firm-level shocks is documented by, among others, Angelidis (2008) who

⁴It is the measure of risk premium for a mature equity market adjusted by country rating and default spread for that rating. Data are available at www.pages.stern.nyu.edu/~adamodar/New_Home_Page/data.html

documents that 55% of US market volatility is due to single firms' volatility. Moreover, Jerermann and Quadrini (2006) show that US firms have experienced greater financial flexibility given by their increased ability to issue equities and bonds that, in turn, are used to respond to bad shocks instead of reducing investment and production.

1.3 The model

The model we present is based on Angeletos and Calvet (2006). It is a neoclassical economy with heterogeneous agents, convex technologies and idiosyncratic production risks. Financial markets are incomplete and agents take this into account when deciding how to allocate the production of the final good, either to consumption, investment in risky production or in risk free bonds. Two main assumptions make this model easily tractable: the CARA specification for the utility function and the normal distribution of the shocks. These assumptions allow us to get a closed form solution for the policy functions and therefore to track investments and savings choices and the impact of each parameter on them. There are some drawbacks in this specification. First of all the CARA utility function does not rule out negative consumption, especially in the early stage of capital accumulation or if income is highly variable. The present exercise however looks at the steady state when agents have accumulated capital and the probability of negative wealth is very close to zero (even if still positive). Also the normal distribution of productivity shocks might bring about negative production that could be interpreted as negative profits but still implies that a positive investment brings to destruction of some of the inputs employed in the production. As mentioned above there are no borrowing constraints, however Angeletos and Calvet (2005) prove that, since the optimal decision rules of the infinite setup are the limit of the finite horizon problem, Ponzi schemes are ruled out in the strongest conceivable way, along any possible path. Moreover, Wang (2003) proves that in this setup the transversality condition on bond demand is always satisfied.

Angeletos (2007) shows that a model with more standard assumptions on preferences and technology, CRRA utility function and log-normal support for production risks (and therefore the possibility of defining a natural borrowing limit) produces exactly the same interesting results: lower interest rate and lower capital accumulation with respect to the complete markets case. The CARA-Normal specification is therefore not essential to obtain the big insights of the model in its closed economy version and, in particular, the relationship between financial development and the aggregate dynamics.⁵ The CARA-Normal is indeed

⁵For the open-economy version of Angeletos (2007) see Angeletos and Panousi (2010) and Corneli (2010).

much more tractable.

There are 2 countries 1 and 2. Each country is indexed by i . Time is discrete. There is a continuum of consumer - producers of mass 1 in each country. Each agent has an income of:

$$y_{it} = A_{it}f(k_{it}) \quad (1.1)$$

This is the production of consumption goods each agent produces, by investing the amount of capital k in her own firm (she cannot invest in other production activities). A is a productivity shock that is normally distributed with mean 1 and variance σ_{iA}^2 . A is an iid shock, independent across agents and time. σ_{iA} represents the formalization of financial market underdevelopment: it is the share of the idiosyncratic risk associated with entrepreneurial activities that financial markets are not able to insure. A value of zero for σ_{iA} indicates that financial markets are able to hedge all production risks. Angeletos and Calvet (2006) prove that lowering σ_{iA} , corresponds to introducing financial activities that are able to partially hedge production shocks (Appendix A analyzes this interpretation of the parameter σ_{iA}).

The capital stock is chosen at $t-1$ and cannot be reshuffled once agents observe their shocks at time t . The production function f exhibits decreasing returns to scale for capital. We choose a simple specification widely used in the literature: $f(k_{it}) = k_{it}^\alpha$.

At time t , agents can purchase a risk-free bond b_{it+1} , this will yield $(1 + r_{it+1})$ units of the consumption good at time $t+1$. The riskless bond is in zero net supply in the closed economy.⁶

The budget constraint of each agent at time t is therefore:

$$c_{it} + k_{it+1} + b_{it+1} = y_{it} + (1 + r_{it})b_{it} + (1 - \delta)k_{it} \quad (1.2)$$

or

$$c_{it} + k_{it+1} + b_{it+1} = w_{it} \quad (1.3)$$

Where w_{it} represents the total wealth at time t that can be used to consume, invest in the risky production or invest in risk free bonds.

⁶The case of perfect insurance in this model corresponds to the variance σ_{iA}^2 being equal to zero. In our setup this case is computationally equivalent to an economy with contingent bonds.

The utility function is a CARA utility:

$$U_t = -\sum_{l=t}^{\infty} \beta^{-(t-l)} e^{-\gamma c_{i,l}} / \gamma \quad (1.4)$$

Where the parameter γ represents the degree of risk aversion but it also represents the willingness to substitute consumption over time.

1.3.1 Optimization problem

Given a deterministic sequence of prices $\{r_{it+1}\}_{t=0}^{\infty}$, households choose consumption, capital and risk-free bonds $\{c_{it}, k_{it+1}, b_{it+1}\}_{t=0}^{\infty}$ that satisfy their lifetime utility subject to their budget constraints.

The optimization problem for each agent can be written with a value function:

$$V_{it}(w_{it}) = \max_{c_{it}, k_{it+1}, b_{it+1}} \{u(c_{it}) + \beta E_t V_{it+1}(w_{it+1})\} \quad (1.5)$$

Given the properties of the CARA-normal specification, an educated guess for the value function and the consumption rule are:

$$V_{it}(w_{it}) = u(a_{it}w_{it} + d_{it}) \quad (1.6)$$

$$c_{it} = \hat{a}_{it}w_{it} + \hat{d}_{it} \quad (1.7)$$

$a_{it}, \hat{a}_{it}, d_{it}, \hat{d}_{it}$ are four non-random coefficients to be determined. The certainty equivalent for the value function specification of a normal is:

$$E_t V_{it+1}(w_{it+1}) = V_{it+1} \left(E_t w_{it+1} - \frac{\Gamma_{it}}{2} Var_t(w_{it+1}) \right) \quad (1.8)$$

Where Γ_{it} , that represents the effective absolute risk aversion, is equal to:

$$\Gamma_{it} = \gamma a_{it+1} \quad (1.9)$$

Therefore the value function becomes:

$$V_{it}(w_{it}) = \max_{c_{it}, k_{it+1}, b_{it+1}} \left\{ u(c_{it}) + \beta V_{it+1} \left(E_t w_{it+1} - \frac{\Gamma_{it}}{2} Var_t(w_{it+1}) \right) \right\} \quad (1.10)$$

The quantity inside the round brackets can also be expressed as:

$$E_t w_{it+1} - \frac{\Gamma_{it}}{2} \text{Var}_t(w_{it+1}) = (1 + r_{it+1})b_{it+1} + G(k_{it+1}, \Gamma_{it}) \quad (1.11)$$

Where

$$G(k_{it+1}, \Gamma_{it}) = y_{it+1} + (1 - \delta)k_{it+1} - \frac{\Gamma_{it}}{2} (\sigma_{iA}^2 k_{it+1}^\alpha) \quad (1.12)$$

This quantity represents the risk-adjusted level of non-financial wealth.

Combining the first order conditions for capital and bonds we get the relationship for the optimal demand of investment:

$$1 + r_{it+1} = 1 - \delta + \alpha k_{it+1}^{\alpha-1} (1 - \Gamma_{it} \sigma_{iA}^2 k_{it+1}^\alpha) \quad (1.13)$$

The interest rate on the risk free bond is therefore equal to the marginal product of capital minus a risk premium that takes into account the risk of investing in the production, represented by σ_{iA}^2 .

From the envelope condition, and making use of the educated guess, the Euler equation is obtained:

$$u'(c_{it}) = \beta(1 + r_{it+1})u'(E_t c_{it+1} - \frac{\gamma}{2} \text{Var}_t(c_{it+1})) \quad (1.14)$$

The variance of the consumption can be written as:

$$\text{Var}_t(c_{it+1}) = a_{it+1}^2 (k_{it+1}^{2\alpha} \sigma_{iA}^2) \quad (1.15)$$

Therefore the Euler equation becomes:

$$E_t c_{it+1} - c_{it} = \frac{1}{\gamma} \ln(\beta(1 + r_{it+1})) + \frac{\gamma}{2} a_{it+1}^2 (k_{it+1}^{2\alpha} \sigma_{iA}^2) \quad (1.16)$$

The choice between consumption and savings is affected by production variability. It directly pushes agents to postpone consumption. However production risk has also the indirect effect of lowering the level of capital invested in production, so the overall effect is ambiguous.

From the envelope and the Euler equations it is also possible to derive the following relation:

$$\frac{1}{a_{it}} = 1 + \frac{1}{a_{it+1}(1 + r_{it+1})} \quad (1.17)$$

1.3.2 General equilibrium and steady state

Since the agents are characterized by CARA-normal specification, their investment behavior is independent of wealth and also consumption is linear in wealth. The wealth distribution therefore does not affect the aggregate dynamics so it is possible to aggregate agents' choices to solve for the general equilibrium of this economy.

Since the shock to productivity is idiosyncratic, the general equilibrium is deterministic and characterized by the optimal choices of consumption and capital $\{C_{it}, K_{it+1}\}_{t=0}^{\infty}$ and the interest rate $\{r_{it+1}\}_{t=0}^{\infty}$ such that the following relations are satisfied at any $t \geq 0$:

$$C_{it} + K_{it+1} = K_{it}^{\alpha} + (1 - \delta)K_{it} \quad (1.18)$$

$$r_{it+1} + \delta = \alpha K_{it+1}^{\alpha-1} (1 - \Gamma_{it} \sigma_{iA}^2 K_{it+1}^{\alpha}) \quad (1.19)$$

$$C_{it+1} - C_{it} = \frac{1}{\gamma} \ln(\beta(1 + r_{it+1})) + \frac{\gamma}{2} a_{it+1}^2 (K_{it+1}^{2\alpha} \sigma_{iA}^2) \quad (1.20)$$

$$K_{it}^{\alpha} + (1 - \delta)K_{it} \geq 0 \quad (1.21)$$

Where the first equation is the resource constraint, the second represents the aggregate demand for investment and the third is the aggregate supply of savings. The last condition is a limited liability constraint since it implies that countries cannot accumulate negative levels of wealth. In closed economy this condition is always verified and never binding for any level of $K_{it} > 0$.

The steady state relations therefore are:

$$C_{iss} + K_{iss} = K_{iss}^{\alpha} + (1 - \delta)K_{iss} \quad (1.22)$$

$$r_{iss} + \delta = \alpha K_{iss}^{\alpha-1} \left(1 - \frac{\gamma r_{iss}}{1 + r_{iss}} \sigma_{iA}^2 K_{iss}^{\alpha}\right) \quad (1.23)$$

$$\frac{1}{\gamma} \ln(\beta(1 + r_{iss})) = -\frac{\gamma}{2} \left(\frac{r_{iss}}{1 + r_{iss}}\right)^2 (K_{iss}^{2\alpha} \sigma_{iA}^2) \quad (1.24)$$

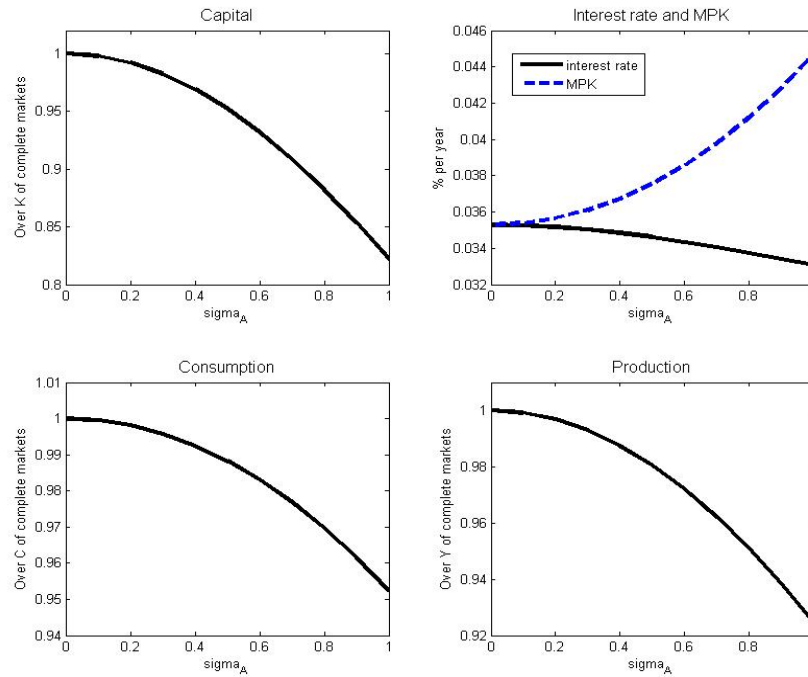


Figure 1.7: The impact of uninsurable risks on the steady state relations.

$$K_{iss}^\alpha + (1 - \delta)K_{iss} \geq 0 \quad (1.25)$$

Figure 1.7 shows how the steady state values for capital, the interest rate, consumption and production vary as the shock to productivity becomes more variable.⁷ In this setting in fact, the variance σ_{iA}^2 (the standard deviation associated to it is the variable on the x-axis, σ_{iA}) represents the coefficient of variation in private consumption and investment returns. In contrast to the Aiyagari (1994) result (which predicts that higher variability on the endowment generates larger precautionary savings, therefore more capital accumulated at any level of the interest rate), a financial market that is not able to fully insure productivity shocks creates the conditions to invest less and save more at any give interest rate. The upper-right panel shows how the spread between marginal productivity of capital and the interest rate increases as the production variability gets larger. It therefore gives a clear picture of the impact of financial development on the investment choice: it represents the risk premium for investing in risky activities. A high risk premium for investing in risky activities discourages capital accumulation and depresses the equilibrium risk free interest

⁷See Section 5 for details on the parameters calibration. The values chosen are standard in the literature and insure locally unique steady states.

rate. GDP is therefore lower in countries with higher uninsurable production variability. The lower-left panel shows that high production risks induce agents to decrease consumption due to lower production and to higher precautionary savings, that push interest rate down even further.

1.4 Two-country integration

Capital account liberalization is formalized as the possibility of exchanging risk free bonds across borders; financial market liberalization is not pre-announced. The integration is not subject to restrictions nor costs for international transactions. The hypotheses are extreme since there are no intermediate steps or constraints.

When the two countries liberalize their capital markets the interest rate is equalized immediately, since it is the interest rate that regulates the exchange of risk free bonds, and equalizes its global demand and supply.

The general equilibrium is again deterministic, since there are no aggregate shocks, and characterized by the optimal choices of consumption, capital and bonds in the two countries $\{C_{1t}, K_{1t+1}, B_{1t+1}, C_{2t}, K_{2t+1}, B_{2t+1}\}_{t=0}^{\infty}$ and the interest rate $\{r_{t+1}\}_{t=0}^{\infty}$ such that the following relations are satisfied at any $t \geq 0$ in both countries:

$$C_{it} + K_{it+1} + B_{it+1} = K_{it}^{\alpha} + (1 - \delta)K_{it} + (1 + r_t)B_{it} \quad (1.26)$$

$$r_{t+1} + \delta = \alpha K_{it+1}^{\alpha-1} (1 - \Gamma_{it} \sigma_{iA}^2 K_{it+1}^{\alpha}) \quad (1.27)$$

$$C_{it+1} - C_{it} = \frac{1}{\gamma} \ln(\beta(1 + r_{t+1})) + \frac{\gamma}{2} a_{it+1}^2 (K_{it+1}^{2\alpha} \sigma_{iA}^2) \quad (1.28)$$

$$B_{1t+1} + B_{2t+1} = 0 \quad (1.29)$$

Where the last equation is the equilibrium condition in the aggregate levels of bonds exchanged between the two countries.

$$K_{it}^{\alpha} + (1 - \delta)K_{it} + (1 + r_t)B_{it} \geq 0 \quad (1.30)$$

The limited liability constraint in this case might be binding since each country can accumulate positive or negative amounts of bonds. This constraint however is not internalized by agents that do not face individual borrowing constraints, therefore the optimal demand and supply of savings are not affected.

Proposition 1 formalizes the characterization of the steady state with financial integration.

Proposition 1: *Suppose that in a two- country world with one country (country 1) financially more developed than the other (country 2), in the autarky steady states it is true that $r_1 > r_2$. The steady state equilibrium with financial integration is characterized by an interest rate r_{ss} , such that $r_2 < r_{ss} < r_1$ and country 1 is net issuer of risk free bonds.*

Appendix B reports the proof of this proposition.

1.4.1 Steady state - intuition

We first of all highlight the main differences in the autarky steady states of two countries that are identical except for their financial depth, measured in terms of completeness of financial markets, and therefore the ability of the financial markets to absorb and redistribute agent's idiosyncratic shocks. Then we explain the result of proposition 1 on the open economy steady state equilibrium.

We analyze the equilibrium without financial integration of the two countries: in the first one financial markets are incomplete, but the variability of uninsurable shock is small ($\sigma_{1A} = 0.9$) and in the second country financial markets are less developed ($\sigma_{2A} = 1.1$).

Figure 1.8 shows that there are important differences in the demand and supply of capital in the two countries before integration. In particular the demand for investment curve represents firms' choice on how much capital to implement in the production. At the steady state the demand is regulated by the equation:

$$r_{iss} + \delta = \alpha K_{iss}^{\alpha-1} \left(1 - \frac{\gamma r_{iss}}{1 + r_{iss}} \sigma_{iA}^2 K_{iss}^\alpha \right) \quad (1.31)$$

Uninsured productivity shocks have a negative impact on the level of steady state capital at any level of the interest rate, r . In fact, at any level of r , in country 1 the capital of steady state is higher than in country 2 since the risk premium for investing in firms located in country 1 is smaller than for the ones in country 2.

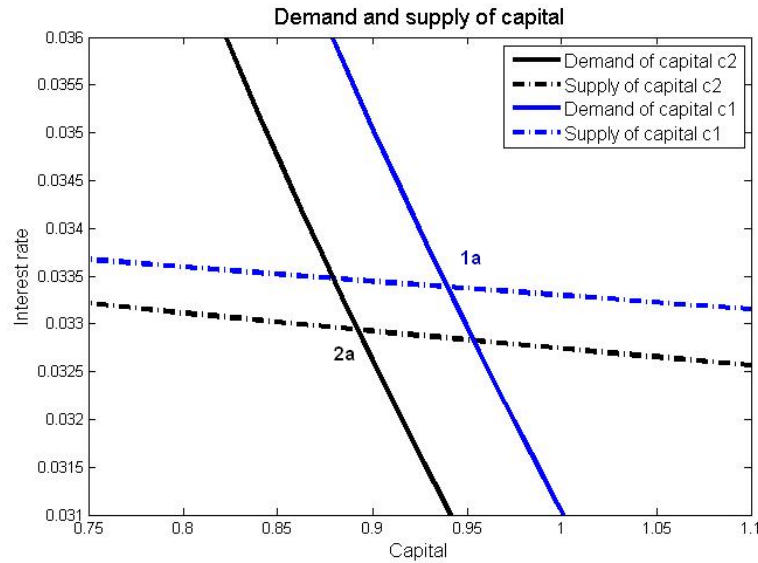


Figure 1.8: Autarky steady state capital and interest rate in two countries with different financial depth.

The supply of capital is the choice made by consumers on how much to save and consume. It is determined by the Euler equation that in the steady state (when $C_{it+1} = C_{it}, \forall i, t$) is:

$$\frac{1}{\gamma} \ln(\beta(1 + r_{iss})) = -\frac{\gamma}{2} \left(\frac{r_{iSS}}{1 + r_{iSS}} \right)^2 (K_{iSS}^{2\alpha} \sigma_{iA}^2) \quad (1.32)$$

Uninsured shocks to production have a direct effect of increasing the precautionary savings at any level of the interest rate. Production risk has however the indirect negative impact due to the lower level of capital chosen by entrepreneurs for higher values of the volatility of the shock. Wealth variability is amplified by the total production, this determines the positive relationship between capital accumulation and precautionary savings, therefore the negative slope of the supply of capital curve in the R-K space.

From figure 1.8 it can be therefore inferred that a less financially developed country at the steady state has a lower level of capital accumulation and lower interest rate.

When the two countries liberalize their capital markets the interest rate is equalized immediately, since it is the interest rate that regulates the exchange of risk free bonds and equalizes its global demand and supply.

Figure 1.9 qualitatively represents what happens after liberalization. Intuitively, when the two countries integrate, the interest rate (called r_{-i} in the figure) gets to an intermediate value between the higher interest rate of country 1, the more financially developed, and the

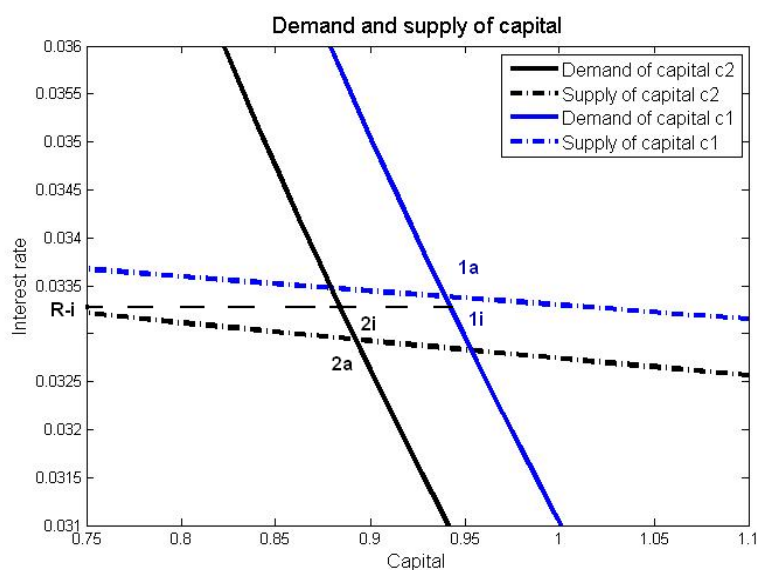


Figure 1.9: Steady state capital and interest rate in two countries with different financial depth that exchange risk free bonds.

lower one of country 2, as shown in the previous section. From the firms' investment decision curves we are able to infer that the new interest rate stimulates investments in country 1, leading to additional capital accumulation; in contrast, it reduces the level of investment in country 2 and therefore its production, since at the new interest rate the risk premium faced by entrepreneurs is too high compared with the risk free bond option. They decrease investment up to the point at which the risk free interest rate and the productivity of capital adjusted for risk are again equalized.

The figure also shows that it is not possible to find an interest rate level for which agents in the two countries contemporaneously have a constant level of consumption - savings. This is because the precautionary saving motive in the two countries is always different due to different wealth variability. In particular, at the lower world interest rate agents in country 1 want to increase immediate consumption and they can do so by issuing bonds. On the other hand, agents in country 2 postpone consumption since at the interest rate of integration they are willing to increase their savings, and are able to do it by purchasing bonds issued by agents in country 1. The only special case of integration in which the two countries reach a steady state with no permanent increase or decrease of savings in equilibrium is when the supply curves coincide (either the two countries have the same level of financial development, or for a combination of the parameters; both cases are however not interesting

from an economic point of view).

1.5 Quantitative results

In this section we provide quantitative simulations of the model presented above. In particular, by calibrating the model with actual data, the simulations presented are able to reproduce long term movements in net capital flows. The assumptions on capital account liberalization are "extreme": the two economies move from autarky to full integration in two subsequent periods, without previous announcements.

1.5.1 Calibration

Each time period is set to 5 years, in order to capture the horizon of an investment project. We calibrate 9 parameters to match some features on the global economy. In particular we consider two blocks in our analysis: the US versus the rest of the world. Since we are interested in aggregate financial flows we calibrate the 2 economies considering that our "country 1", the US, is 29% of the world GDP, while we are not interested in matching differences in productivity or population size.⁸

Parameter values

α	income share of capital	0.4
$1 - \beta$	annual discount rate	0.03
δ	annual capital depreciation rate	0.04
ϕ	annual capital adjustment cost	0.6
γ	absolute risk aversion	1
ψ	elasticity of intertemporal substitution	1
σ_{A1}	uninsured idiosyncratic production shock st. dev., US	0.9
σ_{A2}	uninsured idiosyncratic production shock st. dev., RoW	1.1
ξ	Country 1(US) share of total GDP	0.289

The table above summarizes the calibration. The values of the parameters are the standard values widely used in the literature, and are in line with existing micro-evidence:

⁸Data source: updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007).

the annual discount factor β is set to .97, the share of capital on income α is set to .4, the annual capital depreciation rate δ is .04, the absolute risk aversion γ and elasticity of substitution ψ are equal to 1. In this quantitative simulation quadratic adjustment costs to capital are introduced in order to get smooth capital movements; they however do not alter the main results of the exercise and in particular international capital flows. The standard quadratic form $\phi(k_{it+1}/K_{it} - 1)^2$ is used, where k_{it+1} is the optimal level of capital chosen at period t by each agent, K_{it} is the aggregate capital stock implemented at time t . The annual capital adjustment shock, parameter $\phi = .6$, is taken from Kehoe and Perri (2002). The variability of US production activities that cannot be insured through the financial market is chosen to get an interest rate of 4.79% after integration, while the uninsurable variability of the rest of the world production is assigned to match the average country risk premium over the US interest rate of 1.6%.⁹

1.5.2 Aggregate movements

Figures 1.10 and 1.11 report the transition of the main macroeconomic variables from autarky to the steady state of integration.

In the first period the two economies are in autarky. As documented in the previous sections, country 1 (the US in this exercise) is financially more developed, so in autarky it experiences a higher interest rate, higher ratio of investment in risky activity (capital) over its GDP and therefore higher production and a lower share of consumption in GDP. Once the two economies open their markets for risk free bonds, the interest rate on safe assets is immediately equalized to a value that is between the autarky levels, as proved in proposition 1. In country 1 agents are willing to increase their investment in production activities, since at the margin the return on capital adjusted for risk is too high. They start therefore accumulating capital but due to the costs of adjusting capital stock the process is not immediate, it lasts for 10 periods (or 50 years). The reverse happens in country 2 where the risk premium on production activities is now too high compared with the safe investment in bonds, therefore agents in this economy dismantle installed capital; output is therefore reduced in this country while it increases in country 1. Two forces drive the consumption choice: on the one hand agents face a new interest rate that induces agents in country 1 to consume immediately since they are more impatient, while it induces consumers in 2 to

⁹Data on risk premium is taken from Damodaran, as mentioned in section 2. Countries, in the "rest of the world" group, are weighted by their participation in the international capital flows (IFIGDP from Lane and Milesi-Ferretti (2007)).

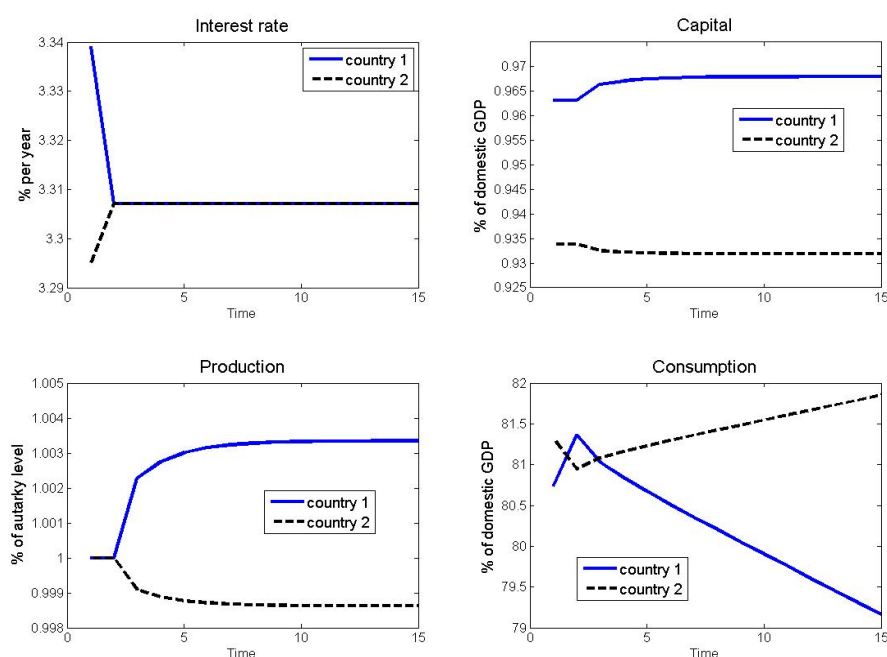


Figure 1.10: Response to capital account liberalization

postpone consumption due to their stronger and increased precautionary savings motive. On the other hand the increase in production in country 1 stimulates precautionary savings since total production amplifies the production risk. The first effect however dominates: it induces a positive jump in country 1 consumption choice for the first periods and a negative one for country 2 (this is preserved also with no capital adjustment costs). In the long run however, due to the high debt issued by country 1, agents in this economy start decreasing consumption in a very long lasting process; on the contrary agents of country 2 enjoy the positive interest on the accumulated bonds.

The net export between the two economies, in figure 1.11, mimics the consumption evolution. In the first 5 periods, first 25 years, after integration, country 1 imports goods from the other country since agents are willing to consume more than what is internally produced, but after period 6 they have to repay the growing interest on accumulated debt and therefore start reducing consumption below their total production. Country 2 accumulates bonds issued by the other economy in a long lasting process, as showed in the net foreign asset position panel, while country 1 has a growing debt and therefore also growing factor payments.

The US current account drops in the first period of liberalization to around -1.5% of GDP then slightly recovers and turns positive 6 periods (or 30 years) after integration. The

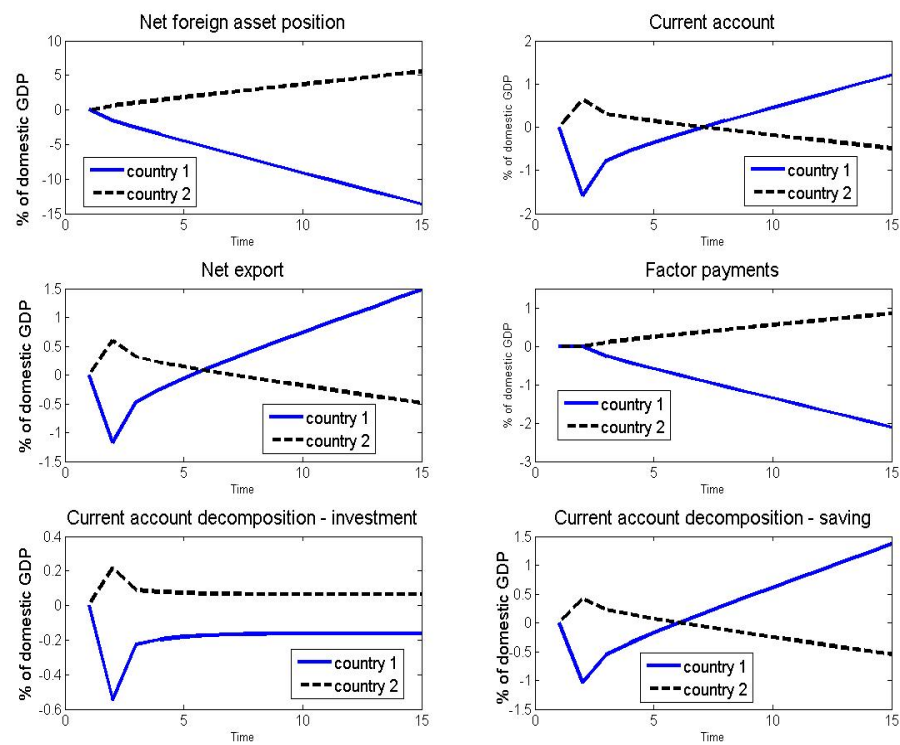


Figure 1.11: Response to capital account liberalization

last two panels represent the decomposition of the current account in variations, with respect to the autarky levels, in investment and saving decisions.¹⁰ Country 1 therefore experiences a negative current account variation because investment increases and saving decreases due to lower precautionary savings at the new interest rate; the stimulus on investment demand from the side of firms generates a drop in the current account of around 0.48% of GDP, while the decrease in the aggregate savings produces a variation in the current account on impact of around 1% of GDP. The contrary happens in country 2: agents want to move their savings from internal risky activities to safe foreign bonds since the risk premium of production investment becomes too high compared with outside options, therefore the variation in investment is negative and of 0.25% of magnitude. Moreover, agents in country 2 increase their precautionary savings with a positive jump of 0.5% in saving variations that subsequently reduces and turns negative when those agents can enjoy the returns on the accumulated assets.

1.5.3 Wealth distribution and welfare

We now turn our attention to the distributional and welfare effects of capital account liberalization in each country. Since the two economies are populated by heterogeneous agents, we can study the consequences of financial globalization for agents with different levels of wealth at the moment of liberalization.

Figure 1.12 shows the wealth distribution in the two countries in the autarky steady states. The uninsurable shock to production risk is higher in country 2, but the aggregate production (that amplifies the impact of the shock) is higher in country 1. The overall result is that wealth is more dispersed in the financially less developed economy. This model generates a variation in wealth dispersion when countries move from autarky to integration. As mentioned before wealth variability depends on the development of the financial markets, as well as the total level of production in the economy.

When moving towards integration agents in country 1 have strong incentives to invest in risky activities, since these activities are now more profitable with respect to safe investments in bonds; they invest more in production capital, since their relative risk aversion decreases. Country 1 therefore experiences an increase in wealth dispersion that, at the new steady state, reaches 0.43% of its autarky wealth standard deviation. In country 2 on the contrary, after

¹⁰ $\Delta CA = \Delta S - \Delta I$, therefore the investment positions in the panel should be interpreted with a minus sign.

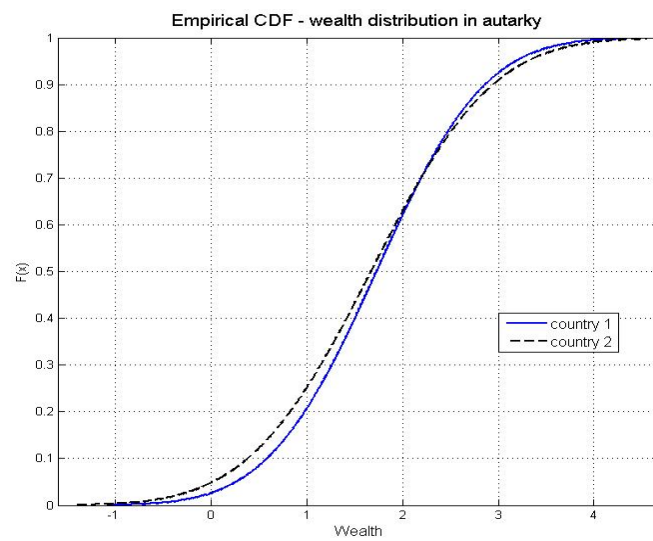


Figure 1.12: Wealth distribution in autarky

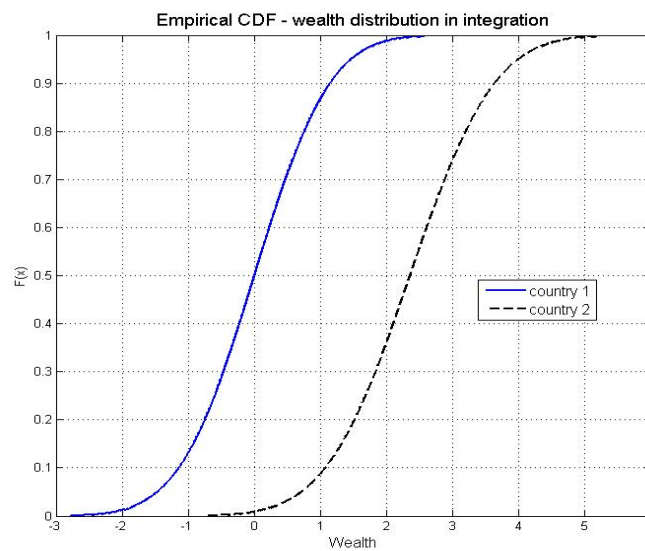


Figure 1.13: Wealth distribution in integration steady state when limited liability becomes binding for country 1.

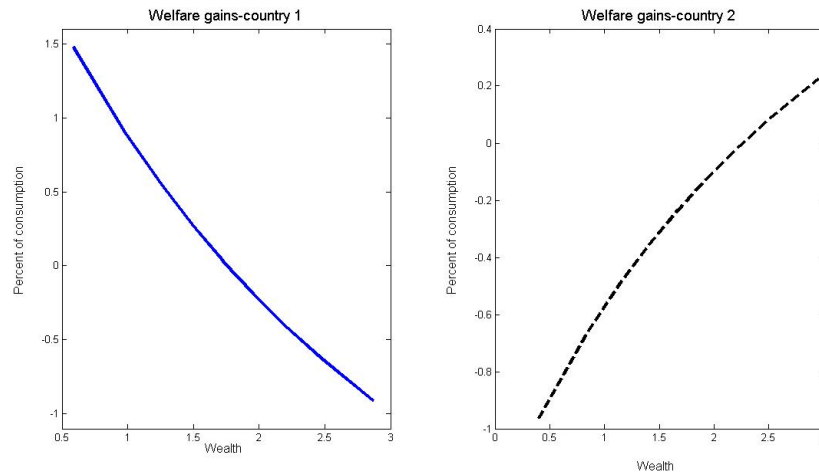


Figure 1.14: Hicksian equivalent variation

integration, agents prefer safer activities therefore they increase their investment in foreign bonds and decrease the exposition to productivity shocks. Wealth dispersion decreases by 0.17% of wealth standard deviation at the steady state of integration. Accumulation of bonds at the country level produces a shift in wealth distribution until the limited liability constraint becomes binding for country 1. Figure 1.13 reports the wealth distribution at the integration steady state when bonds reach their lower bound in country 1 and aggregate wealth is zero (in the picture it coincides with the wealth of agents in the fifth decile). The increasing inequality for country 1 is consistent with the data on wealth distribution registered in the US in recent years.

We analyze the welfare consequences of integration by computing the Hicksian equivalent variation for the two economies overall and inside each country for agents with different levels of wealth, at the moment of financial integration. The Hicksian equivalent variation is defined as the percentage increase in consumption at time zero that makes the agent as well off in autarky as with integration. A positive value then means that agents are better off with integration, a negative value instead says that integration has a negative impact on agents' welfare. This measure is intuitively equal to:¹¹

$$\mu_1 \simeq U^{-1}\left(\frac{U_{1aut}}{U_{1int}}\right) - 1 \quad (1.33)$$

¹¹We use the definition employed in Gourinchas and Jeanne (2006).

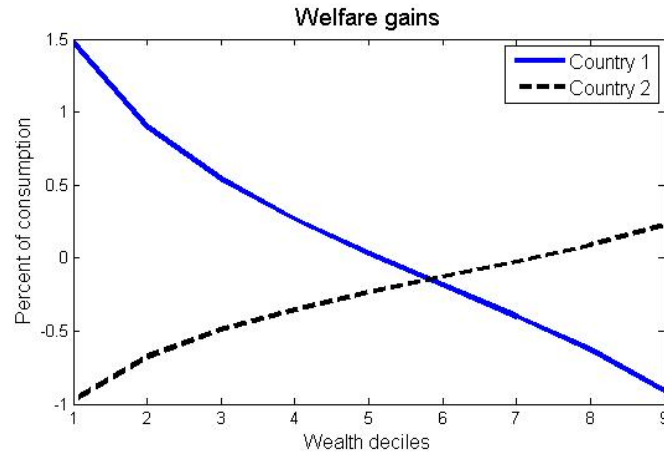


Figure 1.15: Hicksian equivalent variation

$$\mu_2 \simeq U^{-1}\left(\frac{U_{2aut}}{U_{2int}}\right) - 1 \quad (1.34)$$

Figure 1.14 and 1.15 show the Hicksian equivalent variation inside the two countries for agents with different levels of wealth. In figure 1.14 in particular x-axis scales are different because, as mentioned before, country 2 wealth is more dispersed than country 1's. Figure 1.15 reports instead wealth deciles, to show the impact of integration on different income-groups. In country 1 integration has a positive welfare impact for the overall mass of agents, however gains are high for very poor people and decrease as agents wealth increases, until they reach negative values for the upper part of the distribution. The risk free interest rate is the main cause of such different reactions. The poor people experience welfare gains since in integration the interest paid on their accumulated debt drops so they can consume more. The contrary is true for the very rich since the interest they receive on their financial activities falls. The entire population experiences an increase in consumption in the first period, as highlighted above, so on average the population of country 1 is better off with integration.

In country 2 poorer agents have to pay higher interests on their debt after integration, therefore their welfare conditions worsen. Richer people instead receive higher returns on their bonds, therefore are better off. The overall result is however negative, since for the whole population, the new economic conditions push agents to postpone consumption to the future.

Figure 1.16 reports the Hicksian decomposition of integration effects in the two countries

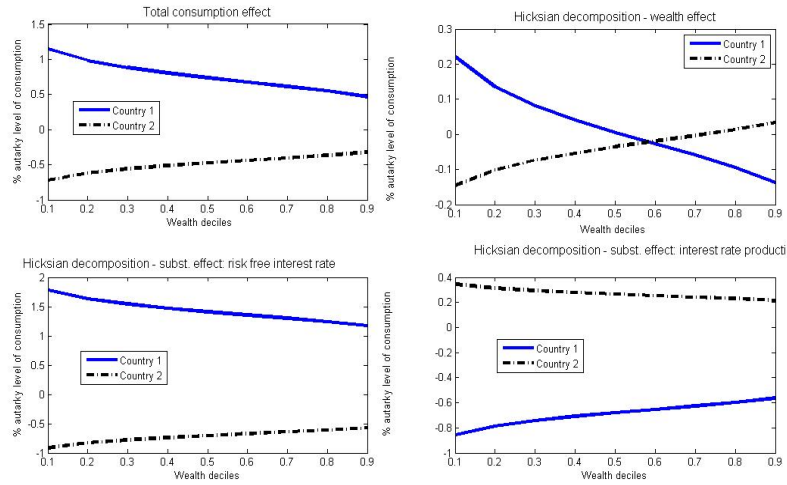


Figure 1.16: Hicksian decomposition - first period

and nine deciles in the first period of integration. The upper-left panel simply shows the percentage change in consumption do to a move from autarky to integration. As expected agents in country 1 increase their consumption, but the effect is stronger for the poor deciles of the population, as explained above. In country 2 the population experiences a generalized drop in consumption, and the effect is again stronger for the poorest. Welfare effects are defined now as the additional consumption in every future period that makes agents as well off in autarky as with integration and are reported in the upper-right panel of figure 1.16. The result is analogous to the one presented in figure 1.15, but is lower in magnitude since the gains/losses are distributed along all periods after integration.

The substitution effect is decomposed in the lower panels. The substitution effect represents the impact of changes in prices on consumption decisions. It can be additionally decomposed into the impact of changes in the interest rate and the impact of variation in the cost of production activities. The lower-left panel shows that in country 1 the drop in the interest rate pushes all agents to consume more due to a lower precautionary motive at the new interest rate. The effect of production costs is instead negative on consumption: the pressure on the marginal return differential with respect to the risk free interest rate pushes agents to invest more in risky activities and therefore consume less.

The opposite is true in country 2. Agents are induced to consume less and accumulate more savings at the new interest rate. They, however, also have incentives to consume more instead of accumulating more capital, since the risk-adjusted capital returns are now too high compared with the returns on foreign bonds.

1.6 Financial crisis

We now turn our attention to the effects of the financial crisis on the main variables of interest examined in the previous section. Before any consequences on the real economy, the immediate effects of the financial crisis are worsened financial conditions and less secure capital markets. As showed in the second section, the risk premium for investing in risky activities increased in all countries between 2007 and 2008: in the US, the interest rate adjusted for the risk premium moved from 4.79% to 5%, while in the rest of the world the increase is stronger, so the gap over the US position moved from 1.6% to 2.6%. Two parameters need therefore to be calibrated to account for the effects of the crisis on the financial market development.

Parameter values - financial crisis

σ_{A1}	uninsured idiosyncratic production shock st. dev., US	0.94
σ_{A2}	uninsured idiosyncratic production shock st. dev., RoW	1.2

In line with estimates of the proxy for financial liberalization, KAOPEN, plotted in section 2, we consider that around 1994, there was a boost in financial integration. In the simulations below we consider that, in the first period, the two economies are in autarky. Then they financially integrate without pre-announcement and for the subsequent three periods, 15 years, they are in the situation illustrated in section 5. In the fifth period, the financial crisis worsens the financial institutions in both economies, bringing about higher risk associated with investment in production activities.

1.6.1 Aggregate movements

Figures 1.17 and 1.18 report the results of the simulations. As illustrated before, after financial integration the two economies reach an interest rate between the values of autarky. The effect of the financial crisis, as illustrated for the closed economy, is to lower the interest rate to a new common level in which both countries optimally decide to reduce their production activities, since now the risk premium associated with risky capital increases in both economies.¹² The overall result is capital accumulation in country 1 for the first three periods, as illustrated above, and smooth decumulation for the subsequent periods up to the new steady state, that is however higher than the autarky initial capital level. In

¹²The new level of interest rate is between the two levels of autarky interest rate but with worsened financial conditions, in line with proposition 1.

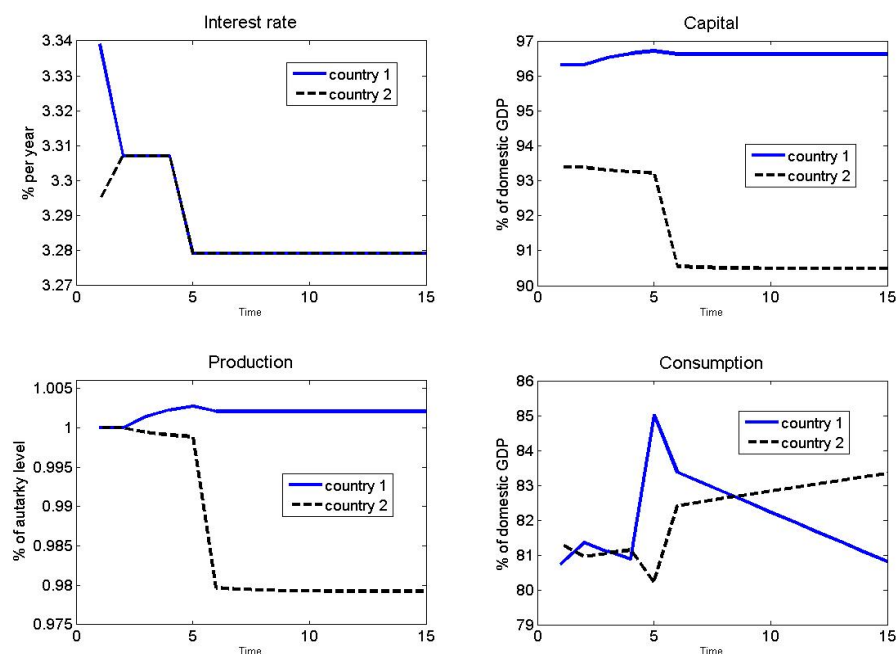


Figure 1.17: Capital account liberalization and financial crisis in period 5.

country 1 capital decumulation is mitigated by the integration with another economy that experiences worse financial turmoil. In country 2 the financial crisis exacerbates the already negative accumulation of capital with a substantial drop due to two forces that point in the same direction of discouraging risky investments: new internal financial conditions and, more importantly, the larger gap with the other economy.

Consumption in country 1 experiences first an increase, as illustrated in the lower-right panel of figure 1.17, due to more impatient agents, the second increase in period 5 is due again to even stronger willingness to anticipate consumption at the new interest rate. Finally consumption of agents in country 1 needs to decrease due to the burden of the interests on the accumulated debt that needs to be paid. Agents in country 2 on the other hand decide to postpone consumption in the first three periods, as illustrated above, due to precautionary savings. This motive is even stronger with the worsened financial conditions, and a larger gap in financial development with respect to the partner economy. Therefore overall agents experience a further and more dramatic reduction in their consumption level. Eventually they will experience higher consumption in the future driven by positive interests earned on financial activities.

Net exports of country 2 towards the US experience first an increase due to integration and then a second one as a consequence of changes in the precautionary savings motive of

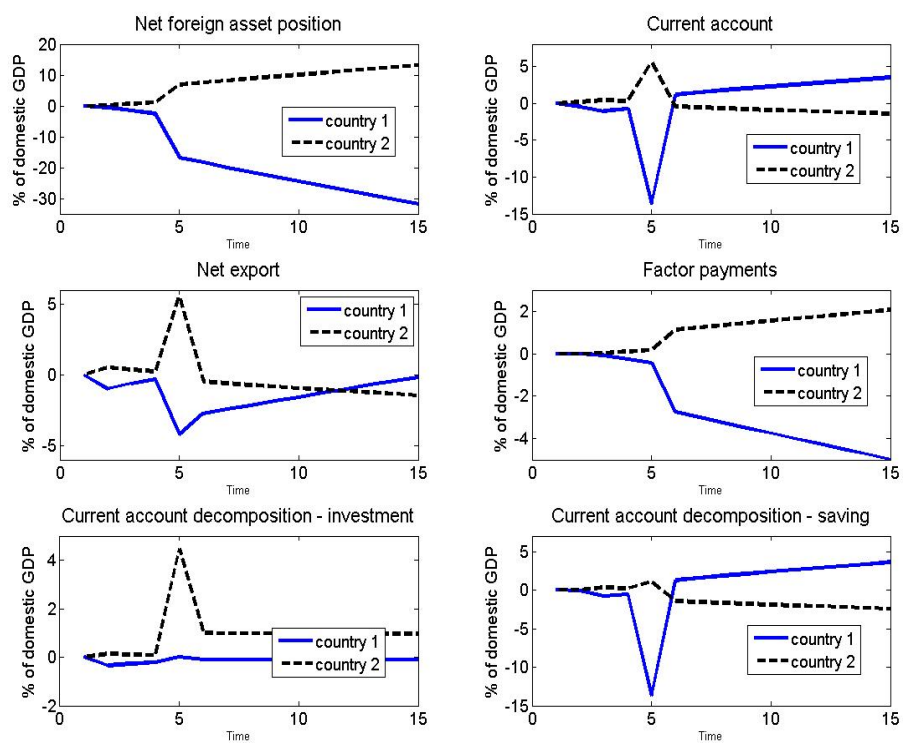


Figure 1.18: Capital account liberalization and financial crisis in period 5.

agents in the two countries, as described above. Country 2 starts immediately acquiring country 1's issued debt, and the financial crisis increases the speed of this accumulation. Factor payments follow the increasing burden of the US debt and experience an acceleration after the financial crisis even if the equilibrium interest rate decreases.

The US current account drops after financial integration and experiences a more dramatic decrease after the financial crisis, reaching a deficit of 14% of production. Two periods after that, in period 6, finally it becomes positive. With financial integration, the investment component of the US current account decreases, so there is a positive variation in risky capital. Three periods after, as a consequence of the financial crisis, capital in country 1 decumulates, so the investment position improves. Country 2 instead experiences two contractions in the investment choice, where the second is more dramatic than the first. When country 2 is hit by the financial crisis the effect on capital accumulation (underinvestment) is so strong that the saving variation experiences a drop, and then a steady decrease. Country 1 instead has negative and growing saving variations after integration and as a consequence of the financial crisis; it then moves to positive positions 35 years after integration.

1.6.2 Welfare analysis

We now investigate the welfare consequences of the financial crisis. In line with section 5.3 we compute the Hicksian equivalent variation, defined as the consumption that makes agents indifferent between the levels of consumption reached three periods after integration and integration with financial crisis.

Figures 1.19 and 1.20 report the welfare gains in the two countries. In particular figure 1.19 shows the gains for agents with different levels of wealth. In country 1 all agents decide to increase current consumption since they become more impatient; they also experience a drop in their production activities due to the crisis. The very poor agents are better off since they experience a drop in the level of interest rates due to the crisis, that makes them enjoy higher consumption; the richers are instead worse off since they gain lower interests on the accumulated activities. In country 2 instead all agents decide to postpone consumption and also produce less. They experience welfare losses with no large differences for different levels of wealth. The positive effects of a lower interest rate are not relevant for agents in country 2 compared with the two strong effects illustrated.

Figure 1.20 highlights different gains at different wealth deciles. Here it is important to notice that the median-mean US citizens experiences welfare losses, as well as the one in the rest of the world, therefore the overall welfare consequences are negative in both countries.

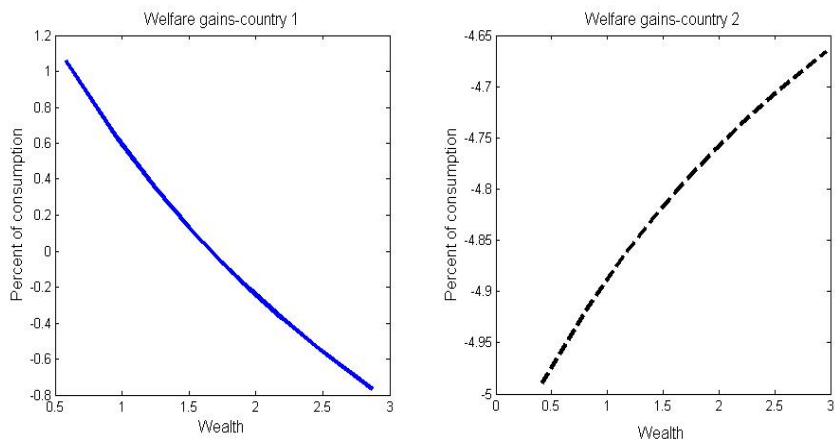


Figure 1.19: Hicksian equivalent variation - financial crisis

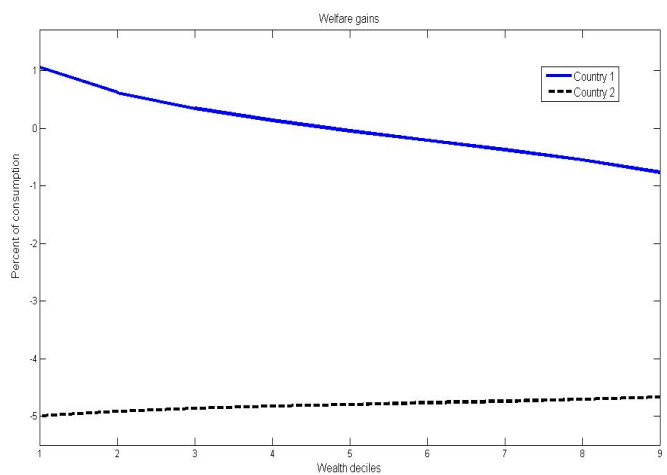


Figure 1.20: Hicksian equivalent variation - financial crisis

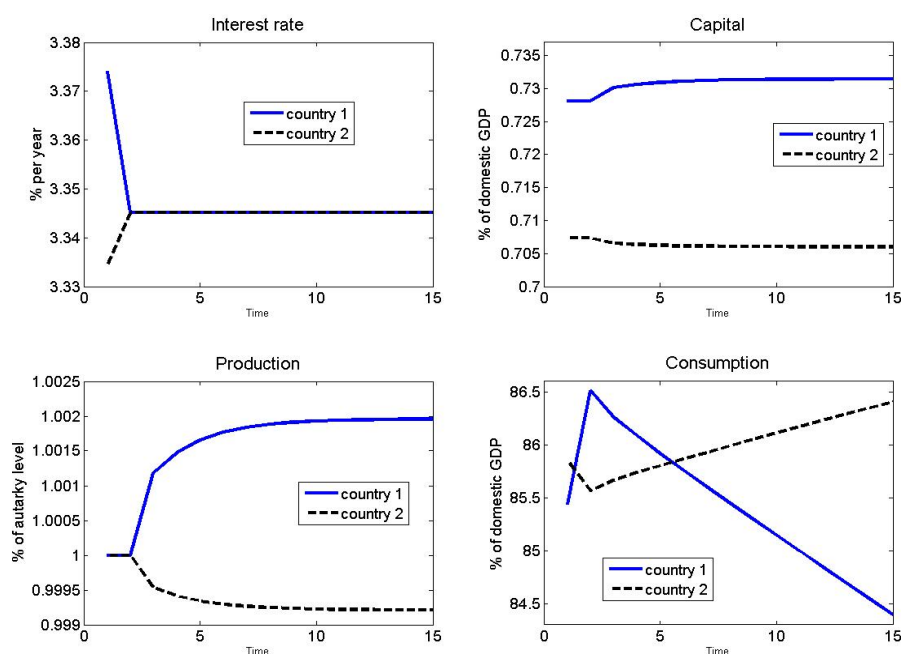


Figure 1.21: Response to capital account liberalization - $\alpha = .3$

1.7 Sensitivity analysis

This section reports the result of the quantitative analysis with different parametrizations of the two economies. In particular in the first part we show that changes in the income share of capital and the annual discount rate do not affect the transition to the integration steady state.

Figure 1.21 and 1.22 reports the simulated transition from the autarky steady state to integration with a different value for the income share of capital: $\alpha = .3$. The dynamics toward the new steady state do not change. The impact of liberalization on the two components of the current account are slightly stronger in both countries.

Figures 1.23 and 1.24 report instead the transition dynamics of the two economies toward the integration steady state when agents are more impatient ($1 - \beta = .05$). Again there are no differences in the response of the main variables with respect to the benchmark specification. The interest rate of equilibrium in autarky and then in integration is higher, and the share of consumption on GDP is always higher. The current account deficit of country 1 is higher in this case, and it is due to higher savings variation while the impact on investment decision remains the same.

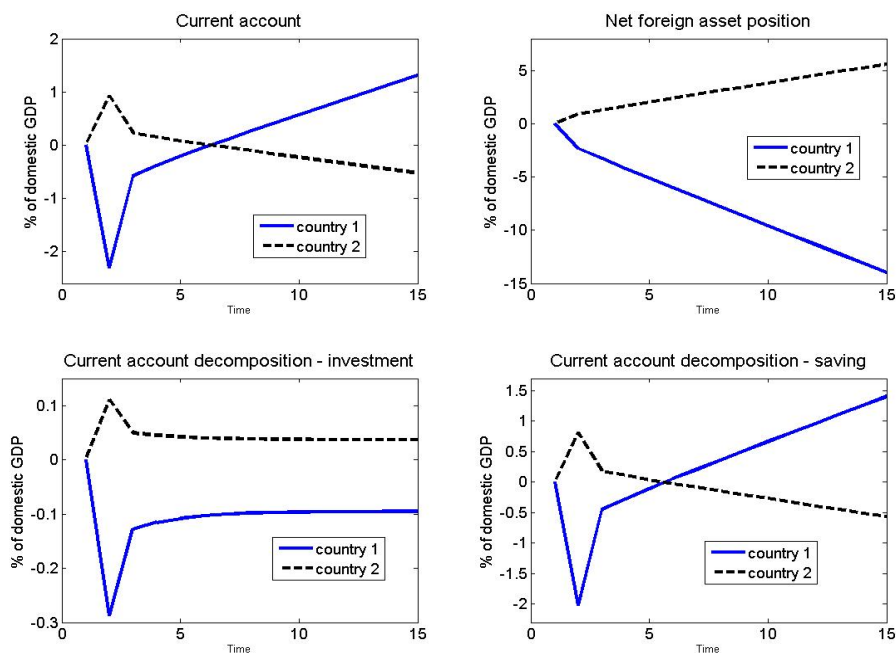


Figure 1.22: Response to capital account liberalization - $\alpha = .3$.

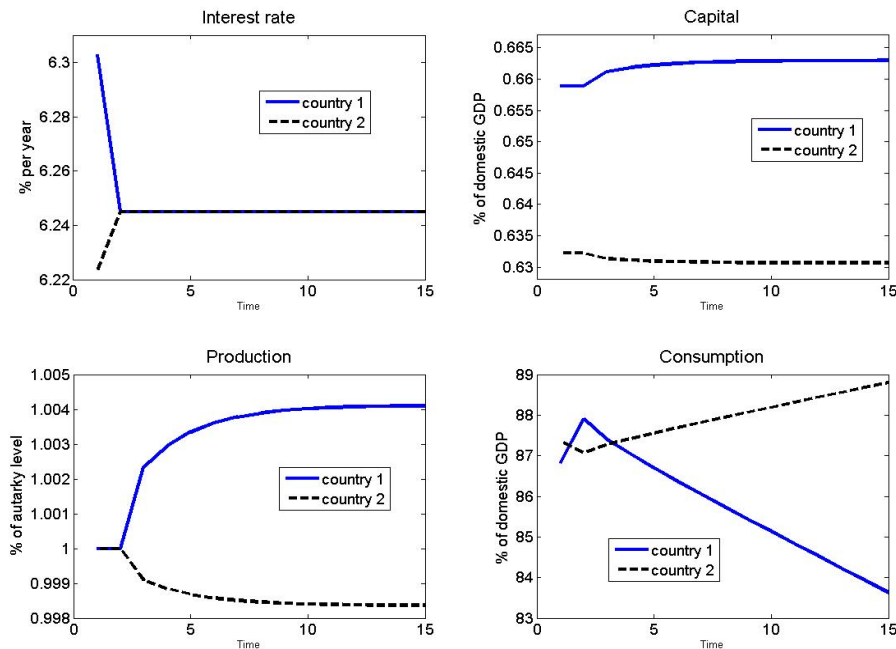
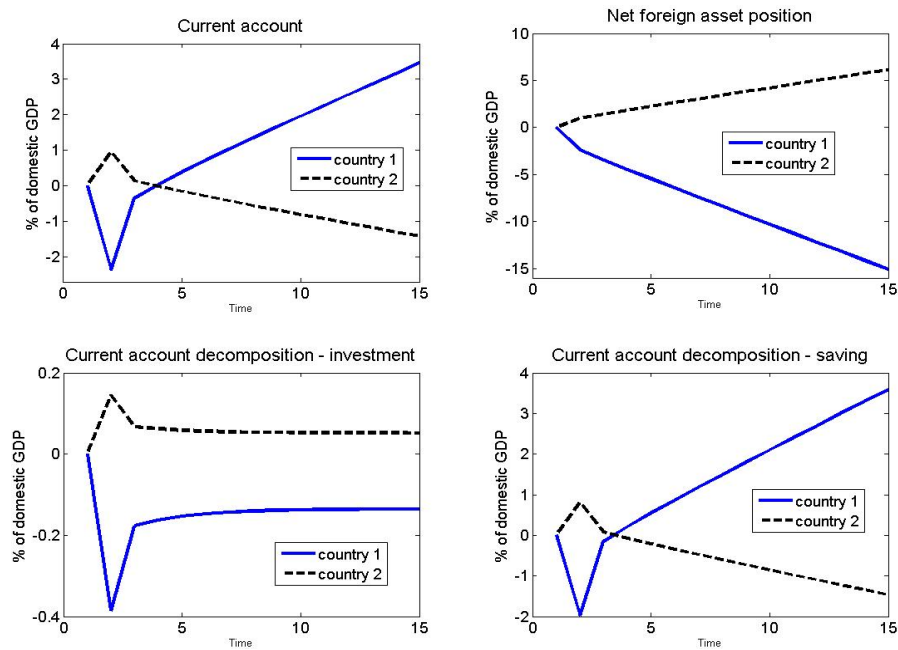


Figure 1.23: Response to capital account liberalization - $1 - \beta = .05$.

Figure 1.24: Response to capital account liberalization - $1 - \beta = .05$.

1.8 Final remarks

The present analysis provides a novel explanation on how financial development differences can shape international portfolio choices. We shed light on the role of financial development in simultaneously affecting consumers' saving preferences as well as entrepreneurial investment decisions. These two effects have important implications for the countries' variables in equilibrium: countries with poor financial institutions have a lower capital accumulation and lower interest rate, and financial liberalization exacerbates the underinvestment condition of those economies. We focus on the long run implications of the impact of financial liberalization on countries' savings and investment decisions; we stress the importance of structural differences among countries in determining, in equilibrium, increasing positive or negative net assets positions. In a two - country model where the two economies differ only in their ability to insure firms against their idiosyncratic shocks, we are able to reproduce financial global imbalances observed in the data: large and rising current account deficit for the financially more advanced economy (in the calibration it is the US), accumulation of the risk-free assets for the rest of the world. Our model provides quantitative evidence that financial integration among economies with different financial development can harm emerging countries since in these economies integration dampens capital accumulation, boosts

precautionary savings and reduces welfare. It also increases wealth inequality in financially more developed economies.

The present work stresses the unambiguously positive effects of improvements in financial market institutions to promote capital accumulation, contrary to the results obtained by Aiyagari (1994) of higher capital levels in equilibrium for less financially advanced economies. Moreover, in line with the recent empirical literature, we question the positive direct effects of financial globalization for countries with weak financial markets.

Finally the present work contributes to the debate on the effects of the financial crisis: it is first of all able to show that worsening financial conditions cause global recession by increasing the risk-adjusted cost of investing in production activities. The welfare consequences of this shock are negative for all economies involved. We then go one step further to explain the long run implications of the financial turmoil. Our model predicts that poorer economies, already integrated with financially more advanced countries, suffer a further reduction in production activities and an even stronger precautionary saving motive. Their capital therefore goes abroad towards foreign risk-free activities. On the contrary financial integration mitigates the real effects of the crisis on richer economies, under-investment is reduced by the large drop in the interest rate, and agents still want to anticipate their consumption. Overall these countries experience negative and growing net foreign asset positions.

This analysis is not able to capture the slight recovery of the current account deficit and savings registered in recent US data. This is due mainly to two features of the model: no borrowing constraints and no possibility of default for single agents. As already mentioned however, ours is an analysis of the long run implications and we believe that while the current data might reflect short run reactions to the recession, there are no signs of changes in agents' preferences with respect to the pre-crisis period.

1.9 Appendix A

Agents' budget constraint (equation 3) can be re-written in the following way:

$$c_{it} + k_{it+1} + b_{it+1} + \boldsymbol{\pi}_t \boldsymbol{\phi}_t = w_{it} \quad (2')$$

Where the term $\boldsymbol{\phi}_t = (\phi_{m,t})_{m=1}^{M_i}$ is a portfolio of M_i risky financial assets and $\boldsymbol{\pi}_t = (\pi_{m,t})_{m=1}^{M_i}$ is the associated price vector. Agents of country i can invest in those assets in order to maximize their utility. In each period wealth is given by:

$$w_{it} = \tilde{A}_{it} k_{it}^\alpha + (1 + r_{it})b_{it} + (1 - \delta)k_{it} + \mathbf{d}_t \boldsymbol{\phi}_{t-1} \quad (3')$$

Where $\mathbf{d}_{t+1} = (d_{m,t})_{m=1}^{M_i}$ is the vector of payoffs of the financial assets that are jointly normal, independent and with expected value $E_t \mathbf{d}_{t+1} = 0$. \tilde{A}_{it} is the productivity shock, distributed as a normal $(1, \sigma_{\tilde{A}}^2)$; $\sigma_{\tilde{A}}^2$ in this representation is the total variance associated with the idiosyncratic production risk.

Agents choose the optimal level of risky activities $\boldsymbol{\phi}_t$ as to minimize the variance of their wealth: the optimal portfolios are able to fully hedge the diversifiable idiosyncratic risk, leaving agents with a residual undiversifiable risk with variance $\sigma_{iA}^2 = \text{Var}(w_{it}) < \sigma_{\tilde{A}}^2$. We can assume that the idiosyncratic production risks are identically distributed in the two countries, while the number of assets available M_i varies; M_i gives the dimension of financial market development in each country i , the higher the number of assets available, the lower is the residual production risk consumers-entrepreneurs have to face.

1.10 Appendix B

Proof of Proposition 1: Given the monotonicity of the R - K relationship in the firm's investment demand at the steady state:

$$r_{SS} + \delta = \alpha K_{iSS}^{\alpha-1} \left(1 - \frac{r_{SS}}{1+r_{SS}} \sigma_{iA}^2 K_{iSS}^\alpha\right)$$

One and only one level of capital choice in each of the two countries corresponds to any level of interest rate r .

We now need to prove that the interest rate, r_{ss} , is $r_2 < r_{ss} < r_1$. If r_{ss} was larger than country 1 steady state interest rate, $r_{ss} \geq r_1$, agents of country 1 would like to have a non-decreasing consumption path, and agents of country 2 an increasing consumption path

(since $r_2 < r_1$), this would bring about total positive consumption growth in the steady state that cannot be an equilibrium. Therefore it has to be that $r_{ss} < r_1$.

If $r_{ss} \leq r_2$ then agents of country 1 would like to have a decreasing consumption path (since $r_2 < r_1$) and agents in country 2 a non-increasing consumption path. The only way to anticipate consumption is by issuing bonds (since capital is constant at the steady state), but the two countries cannot contemporaneously have respectively a negative and a non-positive bond position since the market for bonds has to clear in equilibrium. Therefore it has to be that $r_2 < r_{ss} < r_1$, country 1 holds a negative foreign asset position and country 2 a positive one. Country 1 keeps accumulating debt until its aggregate wealth reaches zero.

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CHAPTER 2

MEDIUM AND LONG RUN IMPLICATIONS OF FINANCIAL INTEGRATION WITHOUT FINANCIAL DEVELOPMENT

Abstract

The textbook neoclassical growth model predicts that countries with lower capital stock benefit from capital account liberalization since integration increases the speed of convergence through the equalization of returns on capital. In the present analysis we show that, in the medium term, financial integration can reduce capital accumulation in developing countries with poor financial institutions, because of a high risk premium in production activities. In the long run, however, integration brings higher capital than in the autarky steady state. The contrary happens for financially advanced economies: they enjoy a period of capital growth in the first years after integration, but in the long run they experience a reduction in capital compared to their autarky steady state. Two forces drive these results: precautionary saving and the propensity to move resources from risky capital to safe assets until the risk-adjusted return on capital equalizes the risk-free interest rate; under the maintained assumption of constant relative risk aversion (CRRA) utility function, those forces are both decreasing in wealth. Overall, financial integration is welfare improving for financially advanced economies and welfare decreasing for developing countries.

2.1 Introduction

Standard neoclassical growth models predict that all economies *conditionally* converge to their steady state and that financial integration - through the equalization of the marginal return of capital across countries - accelerates the convergence rate of those scarce in capital. The role assigned to financial integration has recently been subject to close scrutiny: on one side, a wide range of empirical evidence has not been able to give a univocal answer on the effects of financial integration on growth; on the other hand this literature stresses that, besides transitory effects on accumulation, capital account liberalization could impact countries' long run performance.¹ An important element seems to be able to discriminate the effects of integration on growth: countries' financial market development.² Recent contributions have shown that financial deepness, by shaping the savings and investment behavior of individual agents, has important implications for capital movements among countries and can help explaining the emergence of large external imbalances, as observed in the last decades.

In this paper we argue that financial integration among countries at different stages of financial market development has sizeable transitory as well as permanent effects on their capital levels. In particular, we confront a developing country that is still in the process of accumulating capital and whose financial markets are at an early development stage, and an advanced country that operates at the steady state and has more developed financial markets. We show that when these two economies open up their capital accounts, in the short and medium term the developing country may experience a reduction in the speed of capital accumulation and, therefore, in growth; in the long run, however, this economy achieves steady state levels of capital and production that are higher than the ones under autarky. The opposite is true for the advanced economy: here financial integration boosts capital accumulation in the short and medium term; in the long run however, this result is offset by a decrease in the level of capital and production towards steady state values that are below those of autarky. At the time of liberalization agents in the advanced economy enjoy higher consumption and therefore positive welfare gains, while agents in the poorer country experience a welfare loss. However this negative consequence is partially mitigated if the developing country is at an early stage of capital accumulation. In the first years after integration, high consumption and investment in the advanced economy are

¹See Kose, Prasad, Rogoff and Wei (2009) for a survey on the empirical literature. Aghion et al. (2005) empirically find that countries with poor financial markets converge to a steady state level of capital lower than economies with better institutions; moreover Fung (2009) shows that countries with under-developed financial sectors are more likely to be trapped in poverty.

²For example Bonfiglioli (2008) finds that financial depth boosts capital accumulation.

financed by capital inflows; the accumulation of external debt, however, forces this country to reduce both consumption and the capital stock in the long run. Finally, the short and medium term implications depend on the amount of accumulated capital at the moment of integration, while the long run consequences of financial integration only depend on the financial development gap.

We present a two-country model based on Angeletos (2007). In each economy heterogeneous agents are subject to idiosyncratic production shocks; the presence of idiosyncratic risk generates a wedge between the risk-free interest rate and the marginal return on capital (risk premium). Domestic financial markets provide partial insurance against those shocks.³ Better financial institutions, by lowering the portion of shocks that rests on agents, imply a lower risk premium and lesser need for precautionary savings; in equilibrium, this results in a higher risk-free interest rate and, at the same time, a higher level of capital. The two countries differ not only in terms of financial deepness, but also by the level of accumulated capital: we assume that the advanced economy has already reached the autarky steady state, while the other is still in the process of accumulating capital. When these two countries liberalize their capital accounts (they can therefore exchange risk-free bonds) the interest rate on bonds is immediately equalized at a level that is between the interest rates prevailing before integration. As a consequence of integration, in the medium term agents in the poorer economy tend to postpone consumption and risky investment and buy safe assets issued by the advanced country; agents in the latter, instead, prefer to raise consumption and the capital stock.⁴ The effect is a deceleration of convergence in the developing economy, while for the other country there is a tendency to move resources to risky activities (the exact path for the two levels of capital in the short run depends on the level of accumulated capital at the moment of integration). The developed country keeps accumulating external debt up to the new steady state, where capital, production and consumption end up lower than they were under autarky. The opposite is true for the developing economy: here agents keep accumulating external assets, and in the new steady state reach levels of capital, production and consumption higher than those corresponding to the autarky steady state.

Two forces drive these results: precautionary saving and the propensity to move resources from risky capital to safe assets until the risk-adjusted return on capital is equal to the risk-free interest rate; under the maintained assumption of constant relative risk aversion (CRRA) utility functions, those forces are both decreasing in wealth. Debt accumulation by

³We follow Angeletos and Calvet (2006) and Angeletos (2007) for the specification of the financial markets.

⁴Depending on the level of the risk-free interest rate in the emerging country before integration, agents in this economy may experience a short-lived boost in consumption and capital accumulation.

the agents in the advanced country, by reducing their willingness to take on risk, depresses the steady state aggregate capital stock and production. In the developing country, in contrast, the accumulation of wealth boosts the propensity to take on risk, resulting in higher capital, production and consumption in the long run. Welfare analysis reveals that these long run developments do not compensate for the medium term dynamics: financial integration implies welfare losses for the developing country and gains for the advanced one.⁵

Two contributions are particularly close to the present analysis: Angeletos and Panousi (2010) and Corneli (2009). We share with those papers the idea that financial underdevelopment influences entrepreneurial activities and constrains the ability of the agents to insure against idiosyncratic shocks.⁶ Moreover they also analyze the implications of financial integration between two economies with different levels of financial deepness. However, while they focus on two countries that are at the autarkic steady state at the time of integration, we allow for the economy with less developed financial markets to be still in a transition phase. This way, we are able to assess the effects of integration on the speed of convergence of developing economies at different stages of capital accumulation, therefore providing novel implications for the short and medium run effects of capital account liberalization. In their contemporaneous and independent work, Angeletos and Panousi (2010) establish the same important result for the steady state of integration that we also obtain: lower capital, production and consumption with respect to the autarky levels for a financially developed economy, and higher capital, production and consumption for a financially poorer country with respect to its autarky steady state. The long run effects of financial integration contrast, instead, with Corneli (2009), where capital account liberalization in the long run brings to lower capital accumulation in the developing economy and higher in the other country. The main difference is in the agents' attitude towards risk; due to constant absolute risk aversion, in Corneli (2009) capital choice is independent of the level of agents' wealth; large and widening bond holdings therefore do not affect the level of capital and production, but only the saving-consumption decision. In the present paper, instead, as well as in Angeletos and Panousi (2010), constant relative risk aversion makes optimal capital dependent on wealth.

The welfare consequences of financial integration in the present paper are analogous to Corneli (2009): financial integration brings welfare gains for the agents in the advanced economy, where lower precautionary motive boosts consumption in the first periods after

⁵As mentioned above, welfare losses are larger the closer the developing country is to its autarky steady state, at the moment of the integration.

⁶Analogously to the present work, these two papers adopt the formalization of the financial market development introduced by Angeletos and Calvet (2006).

capital account liberalization, and welfare losses for the developing economy. Mendoza et al. (2007, 2009) reach similar conclusions in terms of welfare but with a different approach: in their analysis, production activities are not risky, and the only source of idiosyncratic risk is labor income, following Aiyagari (1994).

The so far mentioned contributions, in line with the present work, are able to replicate the global imbalances observed in the data, and explain them as a result of differences in savings and investment attitudes. Other studies establish analogous results but focusing on economies hit by aggregate risks (e.g. Caballero et al., 2008, Prades and Rabitsch, 2009, Fogli and Perri, 2006).

Our analysis is also related to the contributions of Carroll and Jeanne (2009) and Sandri (2010): both these papers find that financial integration reduces the speed of capital accumulation for financially underdeveloped economies. However they focus on a small open economy, therefore they are not interested in the two sides of global imbalances; moreover in their analyses the interaction between financial underdevelopment and capital account liberalization has no impact on the steady state level of capital and consumption.

Gourinchas and Jeanne (2006) and Antunes and Cavalcanti (2010) study the welfare consequences of capital account liberalization for a small open economy that is converging towards its steady state.⁷ They find positive welfare gains for the country; those gains are higher for an economy that liberalizes its capital account at an early stage of capital accumulation. The driving force is that, at the moment of integration, the marginal productivity of capital (which coincides with the risk-free interest rate) is higher than the world interest rate; moreover in Antunes and Cavalcanti (2010) the borrowing constraint becomes looser with lower interest rates, therefore the effect of integration is an increase in consumption, capital and net borrowing. We share with these studies the fact that financial integration has better effects for countries at an early stage of capital accumulation than for economies closer to their steady state, however we always obtain welfare losses for the developing country. The main difference is that in our setup, after capital account liberalization, the risk premium for investing in production activities induces agents of the developing country to reduce the speed of capital accumulation. Also in our two-country world, the gap in the precautionary savings (higher in the developing country with respect to the rest of the world) determines an equilibrium interest rate at which the developing economy is a net lender, since its agents tend to postpone consumption and increase savings.

Finally our results are in line with the empirical findings of Reinhart and Rogoff (2010):

⁷In Gourinchas and Jeanne (2006) financial markets are complete, while in Antunes and Cavalcanti (2010) heterogeneous agents are subject to idiosyncratic shocks to their labor income, *à la* Aiyagari.

they estimate that high levels of debt (above 90%) are associated with lower growth. In our simulations we find that the financially advanced economy starts shrinking its level of production for values of debt above 70%. We obtain therefore that widening global imbalances are not neutral for countries' long run performance.

The rest of the paper is organized as follows. In the next section we present the theoretical model and derive the steady state in the two cases of autarky and financial integration. In section 3 we calibrate the model to study the transition path towards the steady state of integration and the implications in terms of convergence rate and welfare for the two economies. We moreover show the results of the simulations with a different hypothesis on the level of capital accumulation in the financially less advanced economy, at the moment of integration. Section 4 concludes with some final remarks and way forward.

2.2 The model

The model we present is based on Angeletos (2007). It is a neoclassical economy with heterogeneous agents, convex technologies and idiosyncratic production risks. Financial markets are incomplete and agents can trade only riskless bonds. Wealth can be allocated either to consumption, risky productive capital or risk free bonds. Time is discrete; there are two countries indexed with i , each populated by a continuum of atomistic agents.⁸ Each household is a consumer - entrepreneur, owns a firm, and supplies inelastically one unit of labor in a competitive labor market; on the other hand she can accumulate capital only in her own firm.⁹

The flows of utility of each agent at time zero can be written as:

$$U_{i0} = \sum_{t=0}^{\infty} \beta^t c_{i,t}^{1-\gamma} / (1 - \gamma) \quad (2.1)$$

We assume a standard CRRA utility function, where $\gamma > 0$ represents the coefficient of relative risk aversion (and the reciprocal of the elasticity of intertemporal substitution) and $c_{i,t}$ is the chosen level of consumption at time t by an individual agent living in country i .

Each period the agent allocates her wealth w_{it} to consumption c_{it} , capital k_{it+1} , to be used in the production of the final good the period after, and risk-free bonds b_{it+1} , according

⁸Small letters represent single agents' variables, while capital letters are aggregate variables.

⁹Alternatively we could think of each "agent's unit" as a couple, one of them owning a firm, the other being a worker.

to the budget constraint:

$$c_{it} + k_{it+1} + b_{it+1} = \pi_{it} + R_{it}b_{it} + \omega_{it} \equiv w_{it} \quad (2.2)$$

Total wealth is given by the profit earned by the household-entrepreneur π_{it} , the gross return R_{it} on the bonds b_{it} purchased the period before and the wage ω_{it} .

Profits are given by total production less labor costs:

$$\pi_{it} = f(a_{it}, k_{it}, n_{it}) - \omega_{it}n_{it} \quad (2.3)$$

Where

$$f(a_{it}, k_{it}, n_{it}) = a_{it}k_{it}^\alpha n_{it}^{1-\alpha} + (1 - \delta)k_{it} \quad (2.4)$$

and

$$a_{i,\min} = 0, E a_{it} = 1, \ln a_{it} \sim N(-\sigma_{ia}^2/2, \sigma_{ia}^2)$$

The production is a Cobb-Douglas aggregator of capital and labor n_{it} . Capital is chosen one period in advance, it depreciates at a rate δ and cannot be reshuffled once the idiosyncratic productivity shock a_{it} realizes. The level of employed labor is instead decided after observing the shock. a_{it} is log-normally distributed with p.d.f. ζ , a_{it} is i.i.d. across agents and time. The parameter σ_{ia}^2 , which is the variance of the associated normal distribution, is the formalization of the financial market development. It represents the portion of the production risk that cannot be insured through the financial market and, therefore, rests on the individual entrepreneurs.¹⁰ Throughout the rest of the paper we maintain the assumption that the variance of the idiosyncratic shock is lower in country 1 (the advanced economy, with developed financial markets) and higher in country 2 (the developing country with less developed financial markets).

Given the assumptions on the distribution of the shock, the model generates an endogenous or "natural" borrowing constraint that must be satisfied in every period:

$$b_{it} \geq -h_{it}, \quad h_{it} = \sum_{j=1}^{\infty} \frac{\omega_{it+j}}{R_{t+1} \dots R_{t+j}} \quad (2.5)$$

h_{it} is the human wealth, computed as the discounted flow of wages; it also represents the wealth of an agent subject to the worse productivity outcome in every period from t

¹⁰ σ is the short-cut for the level of production variability that cannot be insured. We therefore do not make assumption on the "total" variability of the production process, and therefore we do not compare this measure across countries. We are interested in the variability that stays on households, cannot be redistributed, and therefore affects agents' wealth.

onwards (i.e. $a_{i,t}=a_{i,\min} = 0 \forall t$). Since the labor market is competitive, in equilibrium the agents are paid identical wages, therefore human wealth is the same across agents.

2.2.1 Optimization problem

Given a deterministic sequence of prices $\{\omega_{it}, R_{it+1}\}_{t=0}^{\infty}$, agents choose consumption, labor supply, capital and risk-free bonds $\{c_{it}, n_{it}, k_{it+1}, b_{it+1}\}_{t=0}^{\infty}$ in order to maximize their lifetime utility (1), subject to their budget constraint (2) and the non-negativity constraints (see below).

The optimization problem for each agent can be written with a value function:

$$V(k, b, a; t) = \max_{n, c, b', k'} \{U(c) + \beta E_t V(k', b', a'; t + 1)\} \quad (2.6)$$

s.t.

$$c + k' + b' = \pi + Rb + \omega \quad (2.7)$$

$$c \geq 0, n \geq 0, k' \geq 0, b' \geq -h_t$$

Where, again

$$\pi_{it} = f(a_{it}, k_{it}, n_{it}) - \omega_{it}n_{it} \quad (2.8)$$

Angeletos (2007) proves that the policy functions for consumption, capital and bonds can be written as linear functions of financial wealth w_{it} (see appendix A for derivation), in particular:

$$c_{it} = (1 - \psi_{it})(w_{it} + h_{it}) \quad (2.9)$$

$$k_{it+1} = \psi_{it}\phi_{it}(w_{it} + h_{it}) \quad (2.10)$$

$$b_{it} = \psi_{it}(1 - \phi_{it})(w_{it} + h_{it}) - h_{it} \quad (2.11)$$

where $\psi_{it} = \psi(\omega_{it+1}, R_{it+1})$, $\phi_{it} = \phi(\omega_{it+1}, R_{it+1})$
 $\phi_{it} \approx \frac{\ln \bar{r}_{it+1} - \ln R_{it+1}}{\gamma \sigma_{it+1}^2}$

\bar{r}_{it+1} is the mean of the returns to risky capital across the agents, σ_{it+1}^2 their variance that coincides with σ_{ia}^2 , the variability of the idiosyncratic production shock.

The above equations define the equilibrium choices as a linear function of the effective wealth, defined as financial plus the human wealth ($w_{it} + h_{it}$), multiplied by two coefficients

that are deterministic and vary with wage ω_{it+1} and bond return R_{it+1} . The proportion of wealth the agent decides to allocate to savings and investment (ψ_{it}), therefore to future consumption, is a function of the discount rate β and of the return to savings; it in fact comes from the Euler equation combined with the FOC for capital and bonds. The proportion of stored resources that are invested in risky capital (ϕ_{it}) is a measure of the risk premium the agents receive for investing in the production activity instead of risk-free assets, and is decreasing in the riskiness of the financial environment σ_{ia}^2 : the higher is the portion of production risk that cannot be insured through financial markets, the lower the amount of resources invested in risky capital. It is also important to notice that ϕ_{it} is negatively affected by the risk aversion parameter: the more the agent is risk averse (higher γ), the lower the amount of resources employed in risky activities.

2.2.2 General equilibrium and steady state under autarky

Since the policy functions for consumption, capital and bonds are linear in wealth, it is possible to aggregate the equilibrium choices of $c_{it}, k_{it+1}, b_{it+1}$; the wealth distribution does not affect the aggregate dynamics. In addition, since the shock to productivity is idiosyncratic, it cancels out in the aggregate, so that the general equilibrium is deterministic. In the closed economy in every period bonds are in zero net supply since the market has to clear; also, the offer of labor equals its supply:

$$B_{it} = 0 \quad (2.12)$$

$$N_{it} = 1 \quad (2.13)$$

In what follows we assume that $\gamma = 1$, a plausible calibration for the parameter of relative risk aversion that is widely used in the literature and allows to simplify the parameter that defines the propensity to consume, which reduces to $\psi_{it} = \beta$. Aggregating across the agents, the budget and the borrowing constraints (2) and (5) become:

$$C_{it} + K_{it+1} = W_{it} \equiv K_{it}^\alpha N_{it}^{1-\alpha} + (1 - \delta)K_{it} \quad (2.14)$$

$$H_{it} = \frac{\omega_{it+1} + H_{it+1}}{R_{it+1}}, \text{ where } \omega_{it} = (1 - \alpha)K_{it}^\alpha \quad (2.15)$$

The policy functions (9) and (10) become:

$$C_{it} = (1 - \beta)(W_{it} + H_{it}) \quad (2.16)$$

$$K_{it+1} = \beta\phi_{it}(W_{it} + H_{it}) \quad (2.17)$$

The mean of the returns to capital simply becomes equal to the net marginal productivity of capital, taking into account capital depreciation, ($\bar{r}_{it+1} = \alpha K_{it+1}^{\alpha-1} + 1 - \delta$) therefore the propensity to invest in capital is equal to:

$$\phi_{it} \approx \frac{\ln(\alpha K_{it+1}^{\alpha-1} + 1 - \delta) - \ln R_{it+1}}{\gamma \sigma_{ia}^2}$$

The equilibrium path for capital accumulation is positively affected by the financial market development: for any level of the effective wealth the share of resources invested in capital is higher the lower is σ_{ia}^2 .

The steady state versions of (14) - (17) are (given that $N_{it} = 1 \forall t$):

$$C_i + K_i = W_i = K_i^\alpha + (1 - \delta)K_i \quad (2.18)$$

$$H_i = \frac{(1 - \alpha)K_i^\alpha}{R_i - 1} \quad (2.19)$$

$$C_i = (1 - \beta)(K_i^\alpha + (1 - \delta)K_i + H_i) \quad (2.20)$$

$$K_i = \beta\phi_i(K_i^\alpha + (1 - \delta)K_i + H_i) \quad (2.21)$$

Figure 2.1 shows the steady state values of the main variables as function of σ_{ia} , which is varied between zero and 1.1.¹¹ Angeletos (2007) proves that, for plausible values of the model parameters, capital and the risk-free interest rate are decreasing in σ_{ia}^2 , which is also the case in the figure. The risk premium, measured by the gap between the marginal return to capital and the risk-free interest rate, is increasing in the riskiness of production, i.e. in the degree of financial market underdevelopment: this results both from an increase in the risk compensation demanded for investing in the productive capital and from an increase in the demand for precautionary savings, which drives down the risk-free interest rate. Less developed financial markets are associated with a lower steady state capital level and, therefore, with lower production, consumption and wages. It is worth noting that the level of effective wealth decreases less rapidly than production, as it is smoothed by the fall in the risk-free rate; it follows that the fall in consumption is also muted.

¹¹The calibration of the parameters used in this simulation is reported below in section 3.1.

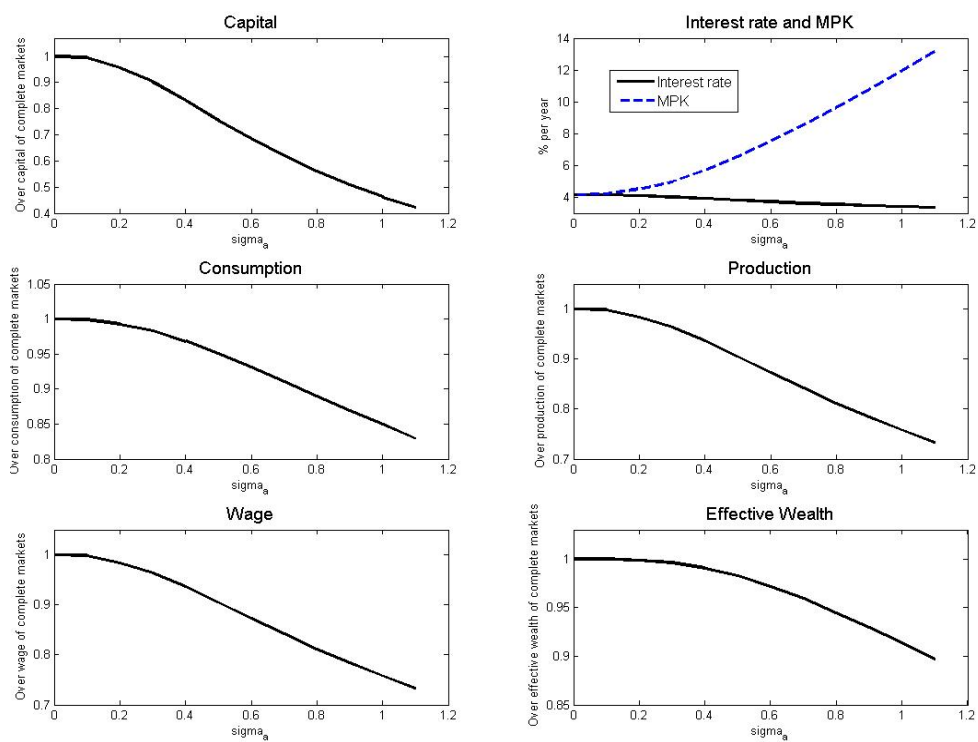


Figure 2.1: The impact of uninsurable risks σ_{ia} on the steady state relations.

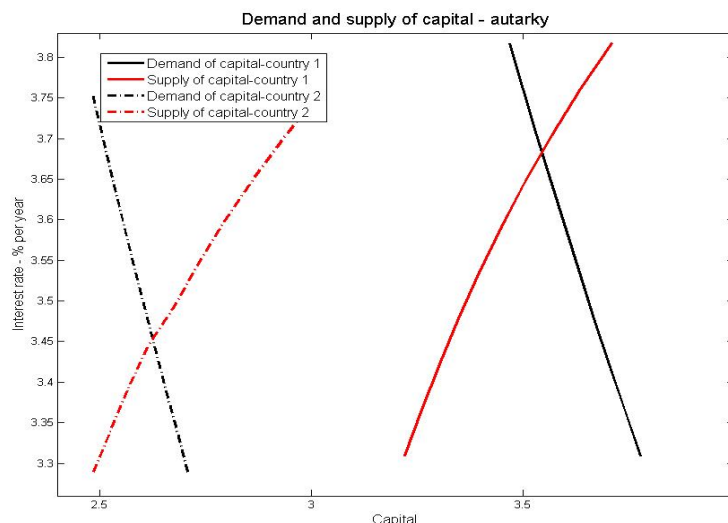


Figure 2.2: Steady state demand and supply of capital in two countries with different degree of financial development.

The equations (18)-(21) can be combined to obtain the following:

$$K_i^{1-\alpha}(\delta + 1 - \beta) = \beta - \frac{(1 - \alpha)(1 - \beta)}{1 - R_i} \quad (2.22)$$

$$\frac{1 - \phi_i}{\phi_i} K_i^{1-\alpha} = \frac{(1 - \alpha)}{(R_i - 1)} \quad (2.23)$$

Equation (22) derives from the Euler condition at the steady state of zero consumption growth; it therefore represents the agents' saving decision or the supply of capital to the firms. This equation implies a positive relationship between the risk-free interest rate and the capital stock: higher interest rates induce agents to postpone consumption and therefore to devote more resources to saving - investment. Equation (23) represents, instead, the demand for capital by the entrepreneurs, or their investment decision. Given decreasing returns to capital, higher interest rates on risk free bonds imply lower levels of capital in order to keep the return on capital adjusted for risk equal to the interest rate.

Equations (22) and (23) are plotted in figure 2 for two countries that differ only in their level of financial development. The figure provides a visual insight of the result that in the autarky steady state deeper financial markets are associated with higher capital and higher risk-free interest rate.

2.2.3 Steady state with integrated financial markets

In this section we solve the model assuming that the two countries open up their capital accounts, i.e. they start exchanging risk-free bonds. Financial integration brings the immediate equalization of the two countries' risk-free interest rates. The individual agents' optimization problem is identical to the closed economy case, therefore conditions (9)-(11) remain valid. The open economy case differs in that, when deriving the general equilibrium and computing the aggregate equilibrium relationships, the condition that bonds are in zero net supply in each country (equation (12)) need not be satisfied, and is replaced by the condition that the world demand and supply of bonds are equal. The general equilibrium is again deterministic, given the absence of aggregate shocks, and is characterized by the sequence of consumption, labor supply, capital and risk-free bonds $\{C_{it}, N_{it}, K_{it+1}, B_{it+1}\}_{t=0}^{\infty}$ and of prices $\{\omega_{it}, R_{t+1}\}_{t=0}^{\infty}$ such that the following aggregate relationships are satisfied in every period:

$$C_{it} + K_{it+1} + B_{it+1} = W_{it} \equiv K_{it}^{\alpha} + (1 - \delta)K_{it} + R_t B_{it} \quad (2.24)$$

$$B_{1t} + B_{2t} = 0 \quad (2.25)$$

$$H_{it} = \frac{(1 - \alpha)K_{it+1}^{\alpha} + H_{it+1}}{R_{t+1}} \quad (2.26)$$

$$C_{it} = (1 - \beta)(W_{it} + H_{it}) \quad (2.27)$$

$$K_{it+1} = \beta\phi_{it}(W_{it} + H_{it}) \quad (2.28)$$

Equation (24) is the analogous of the budget constraint (14) in the closed economy case, except that here aggregate bond holdings can be non-zero. Equation (25) imposes equilibrium in the world bond market. Equations (26) - (28) are identical to the closed economy.

The following proposition establishes the steady state equilibrium result of the model with financial integration among two countries.

Proposition 1: *Suppose that in a two - country world, with country 1 financially more developed than country 2, in the autarky steady states the risk-free interest rates are such that $R_1 > R_2$. The steady state equilibrium with financial integration is characterized by a common risk-free interest rate R_{ss} , such that $R_2 < R_{ss} < R_1$, and country 1 issues a strictly positive level of risk free bonds.*

The steady state versions of (24)-(28) are:

$$C_i = K_i^\alpha - \delta K_i + (R_{ss} - 1)B_i \quad (2.29)$$

$$B_1 + B_2 = 0 \quad (2.30)$$

$$H_i = \frac{(1 - \alpha)K_i^\alpha}{R_{ss} - 1} \quad (2.31)$$

$$C_i = (1 - \beta)(K_i^\alpha + (1 - \delta)K_i + R_{ss}B_i + H_i) \quad (2.32)$$

$$K_i = \beta\phi_i(K_i^\alpha + (1 - \delta)K_i + R_{ss}B_i + H_i) \quad (2.33)$$

Equations (29)-(33) help to understand the novel result of the present analysis, that is in contrast with previous studies (e.g. Mendoza et al, 2007, and Corneli, 2009). The levels of consumption and capital, determined by the aggregation of the policy functions (equations (32) and (33) respectively) are increasing functions of wealth, therefore they are increasing in the steady state level of the bond holdings. In the advanced economy (country 1), capital and consumption are lower now than in the autarky steady state, since this economy is net issuer of bonds. The opposite is true of country 2: the levels of capital and consumption are higher in the integration steady state than in the autarky steady state. The complete dynamics of those variables are simulated in the next section.

Figure 2.3 gives a further visual representation of the movements of capital and the interest rate: it shows the capital supply and demand curves for the two countries at the steady state both in autarky and integration. Equilibria 1_a and 2_a are the autarky interest rate - capital combinations respectively in country 1 and country 2, derived in the previous section, while the equilibria with financial integration are indicated by 1_i and 2_i respectively. As stated in Proposition 1 the world interest rate is between the autarky risk-free interest rates. The most important message from figure 2.3 is that the steady state level of capital in country 1 is smaller than its autarky level while the opposite is true of country 2. By looking at figure 2.3 we also see that the capital accumulated by agents in country 2 always remains below the level in country 1; this happens because the financial development gap generates a higher risk premium and higher precautionary savings in the developing economy, even in the financial integration.

We obtain the surprising result that in the long run, the country with poor financial institutions accumulates more capital than under autarky, while the advanced economy decumulates capital and shrinks production. This novel result is induced by the different behavior of the supply and demand curves. Consider country 1. On the one hand, the supply does not move, since the propensity to consume does not change after integration; the only implication of being able to lend or borrow from abroad is a movement along the

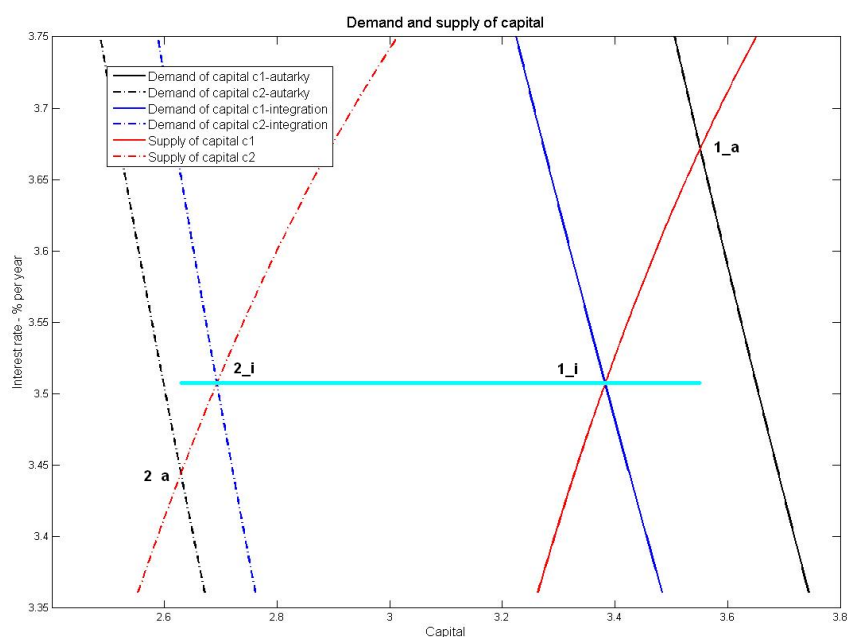


Figure 2.3: Steady state demand and supply of capital in two countries with different degree of financial development. Autarky and integration.

curve: the interest rate of integration, being lower than the one of autarky, induces agents to reduce their savings and increase consumption. On the other hand, two distinct effects influence the demand of capital. The first movement, in line with the supply side, is along the curve: entrepreneurs in country 1 are willing to increase investment at the new interest rate. The second movement is a shift of the demand curve due to changes in the propensity to invest in risky activities: since agents have CRRA utility function, their risk aversion increases with the decrease in wealth therefore they reduce their level of capital further. The positive impact of a smaller risk-free interest rate on the demand of capital is therefore more than offset by the increase in the risk aversion. The opposite is true of the demand curve in country 2: it moves to the right due to the higher propensity to invest in risky activities that derives from the increase in wealth. Moreover, in this economy, agents are willing to save more at the interest rate of integration which is higher than the one of autarkic steady state.

2.3 Quantitative analysis

In this section we calibrate the model and simulate the dynamics during the transition towards the new steady state. In particular, the scope of this exercise is to highlight the implications of financial integration for a developing country that is still in the process of accumulating capital when it opens its capital account towards an economy financially more advanced and which is already at its autarky steady state (country 1).

2.3.1 Calibration

Each period corresponds to one year. We calibrate 8 parameters in order to match some important features of the data. We choose two blocks: on the one side the US, on the other side the rest of the world. This is a conservative measure, since in this way we obtain a financial development gap that is smaller than the one we would obtain considering for the second block only non-OECD countries. Also, this way we can reproduce the path of the US current account deficit and assess the effects that its increasing indebtedness may have on growth. The model is however flexible to different characterizations of the two economies. In order to be consistent with the representation of the two blocks (US - rest of the world), we assume that the first country is smaller, it represents 30% of world GDP; its weight is measured by the parameter ξ .¹² Compared to the analytical representation, we also introduce capital-adjustment costs in order to get smooth transition paths for capital movements and to be consistent with our previous study (Corneli, 2009). We use a standard quadratic form $(\phi(k_{it+1}/K_{it} - 1)^2)$ for capital-adjustment costs as specified in Kehoe and Perri (2002), making use of their calibration of the parameter ϕ .

Parameter values

α	income share of capital	0.36
$1 - \beta$	annual discount rate	0.04
δ	annual capital depreciation rate	0.08
ϕ	annual capital adjustment cost	0.6
γ	risk aversion	1
σ_{1a}	st. dev. of uninsured idiosyncratic prod.shock, US	.6
σ_{2a}	st. dev. of uninsured idiosyncratic prod.shock, RoW	.9
ξ	Country 1(US) share of total GDP	0.289

The income share of capital, the annual discount rate, the annual capital depreciation rate and the parameter of relative risk aversion are taken from the literature. We obtain the

¹²The US represents around 29% of world GDP in 2007(Data source: Lane and Milesi-Ferretti (2007)).

values of the financial market development parameters by matching the risk premium of the two blocks at the final steady state of integration. In particular, we use the data elaborated by Damodaran for 2008 for the interest rate adjusted for risk (our measure of the marginal return to capital).¹³ The estimated interest rate adjusted for risk is of 5% for the US and 7.6% for the rest of the world (therefore a gap over the US of 2.6%).

2.3.2 Simulations

In order to reproduce the dynamic transition from the initial equilibrium in the closed economy to the steady state of integration we adopt the computational procedure proposed by Mendoza et al. (2009): we solve the model backward, updating our guess on one variable until all the policy functions and the resource constraints are satisfied in every period.

We first focus our attention on the transition path towards the autarky steady state for the developing economy (country 2 in our analysis) starting from a given level of capital K_0 . We then make the assumption that the same country, at $K = K_0$, decides to open up its capital account to an economy with more developed financial markets and which is already at its autarky steady state. We compare the speed of convergence towards the two different steady states (autarky and integration) for country 2 and highlight the transition of the main variables of interest for the two economies.

2.3.2.1 Autarky

Figure 2.4 reports the transition to the autarky steady state of the main variables in country 2. The initial level of capital is set equal to 93% of the steady state level. The transition is smooth and takes 35 years to complete.¹⁴ The interest rate declines and capital increases while the risk premium follows a non-monotonic path: first it increases, then it starts tightening as the interest rate gets very close to the steady state level. The risk premium however always moves around its steady state value of 7.95%.

¹³See Corneli (2009) for a discussion on the use of the dataset constructed by Damodaran (http://pages.stern.nyu.edu/~adamodar/New_Home_Page/) to formalize the financial market development. For the second economy (the rest of the world), countries are weighted by their participation in the international capital market (IFIGDP, from Lane and Milesi-Ferretti, 2007).

¹⁴Without capital adjustment costs convergence would take 28 years.

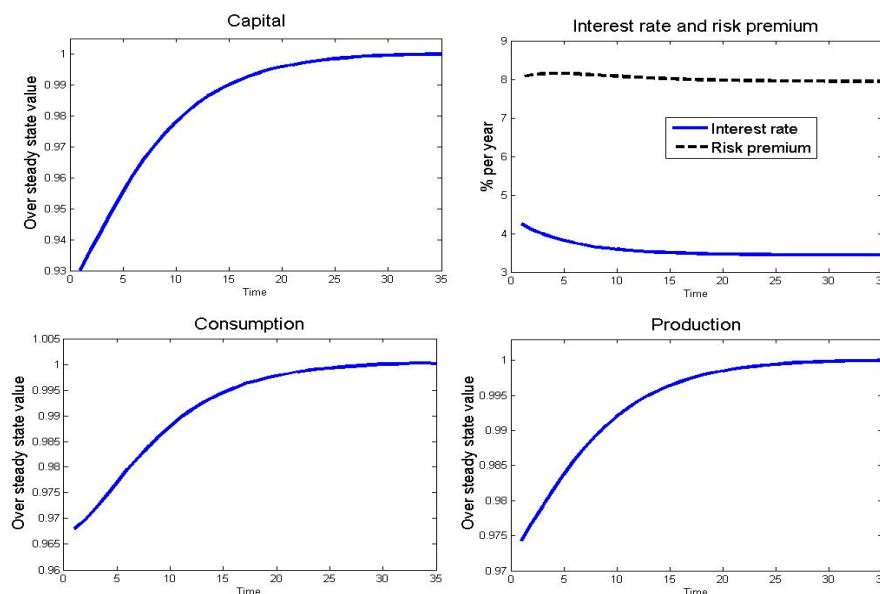


Figure 2.4: Country 2 - transition towards the steady state in autarky.

2.3.2.2 Integration

In all the following simulations, at time 1 the two countries are in autarky, at time 2 they financially integrate without pre-announcement. At the moment of financial integration the interest rate on risk free bonds is equalized in the two economies. The transition towards the final steady state is extremely slow (it takes 470 periods). The level of capital accumulated by country 2 at the time of the capital account liberalization crucially determines the transition path in the short and medium run. Consistently with the above exercise we consider K_0 to be 93% of the autarky steady state level (K_0 is moreover equal to 90% of the steady state of integration).

Figure 2.5 compares the capital accumulation path for country 2 in autarky and integration in the first 35 periods. In the first two periods the two curves coincides (recall that capital is set one period in advance). Then there is a jump in the level of capital chosen in integration and capital goes above the level of autarky. In fact, in this scenario, at the time of integration the interest rate moves to a value that is lower than the risk-free interest rate of country 2 (see figure 2.7); in this economy, agents are therefore willing to invest in risky activities, because the return to capital adjusted for risk is above the risk-free interest rate. The boost in capital accumulation is however short-lived; after 5 years the level of capital moves below the one of autarky. Agents in country 2 can now diversify their portfolios by

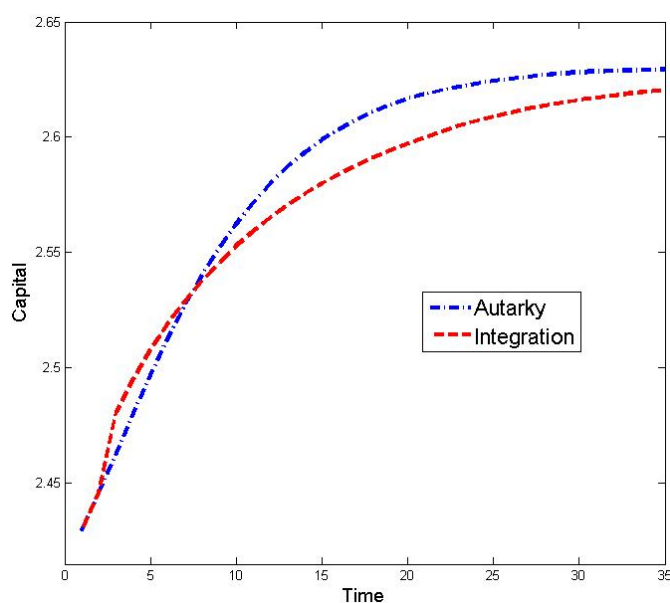


Figure 2.5: Capital accumulation in country 2 for the first 34 years after integration - autarky vs integration.

accumulating (on aggregate) positive levels of bonds, and they do so as they are more risk averse than agents in country 1. The novel result of the present analysis is that financial integration may boost capital accumulation in the very short time because of positive and high differentials of returns on capital between the two countries, even when adjusted for risk; however the implications of different savings and investment behaviors have longer lasting consequences in that they slow down convergence for more than 40 periods.

In fact, from figure 2.6 it emerges that only 50 years after financial integration (instead of 35 in the autarky regime) agents in country 2 reach a level of capital equal to the autarky steady state. From that point on, they keep accumulating capital until they get to the steady state level, which is 2.5% higher than the autarky one.

As mentioned above the very long run result is positive for this economy in terms of final level of capital independently of the variables at the moment of integration: the final steady state depends only on the combination of parameters of the two economies and in particular on the financial development gap.

Figure 2.7 shows the behavior of the variables of interest in the first 34 years after integration.¹⁵ At the moment of integration the interest rate immediately jumps to a common

¹⁵The interest rate is expressed as a percentage rate per year; the capital is reported over the initial level of capital and the same for production, while consumption, current account and bonds are reported as a

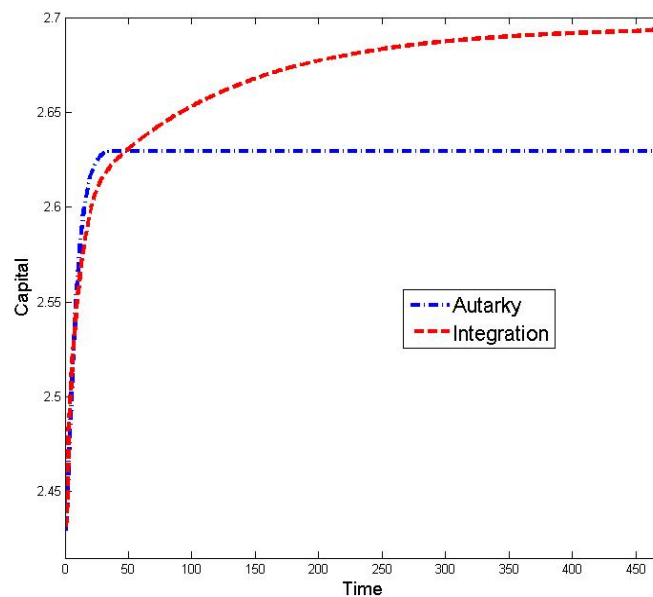


Figure 2.6: Capital accumulation in country 2 in the long run - autarky vs integration.

intermediate level; it then moves to the steady state level in about 28 years¹⁶. The risk-free interest rate differential before integration determines the short time reaction of the other variables: in the present scenario at time 1 country's 2 risk-free interest rate is higher than the interest rate of integration (i.e. higher than country 1's interest rate).¹⁷ At the time of capital account liberalization agents in country 2 are willing to invest more resources in their risky activity, since the return on capital adjusted for risk turns out to be higher than the new interest rate; this boosts capital accumulation as illustrated above. The opposite happens in the other economy: on impact agents in country 1 decreases the level of capital, because the return on capital adjusted for risk is lower than the new risk-free interest rate; they go back to the autarky level in 5 years (as interest rate decreases towards the steady state level which is below country 1 autarky level) and then keep accumulating capital that in 22 years reaches a value that is 1.3% higher than the autarky level. Also at the time of integration agents in country 1 reduces the level of consumption, but only for the first period;

percentage of current domestic production.

¹⁶When the two countries are both at the autarky steady state at the moment of financial integration, the interest rate jumps immediately to the new steady state, since capital liberalization induces only a capital reshuffle, in line with the results obtained in Corneli (2009). The result is very close to the one analysed in section 3.4, where K_0 is 99% of the autarky steady state level.

¹⁷On the contrary if the risk-free interest rate of country 1 is lower than the new common level, agents in this country move resources from risky activities to safe foreign bonds, decumulating capital (this second case, with country's 2 interest rate below the one of integration is presented in section 3.4).

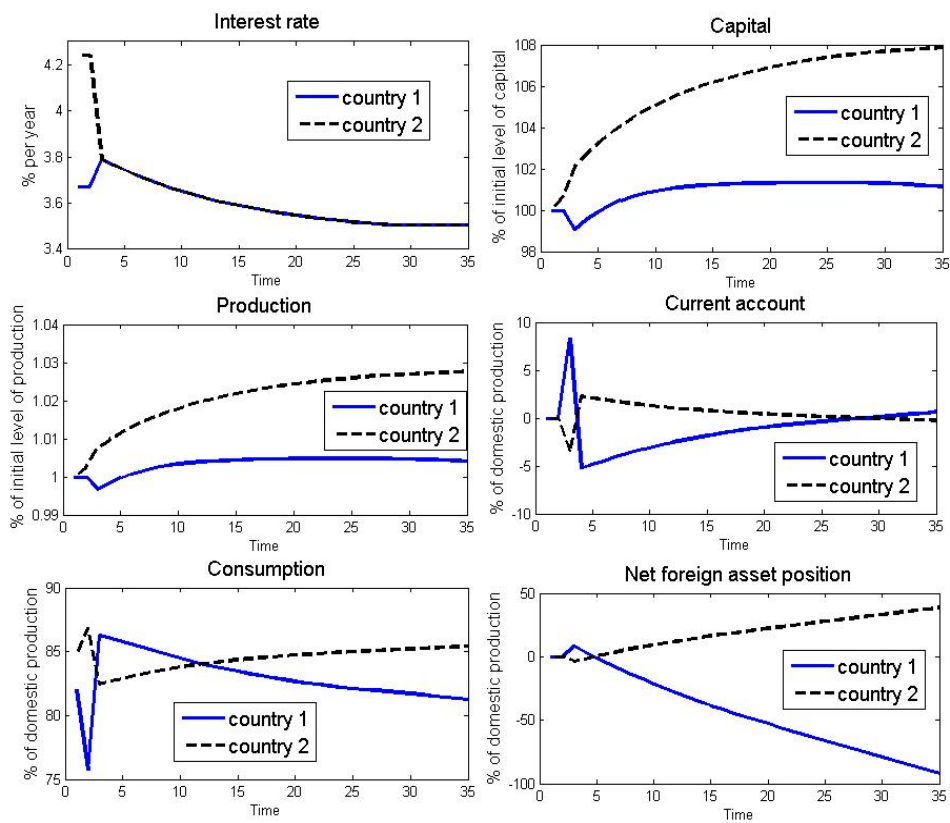


Figure 2.7: Transition dynamics - first year in autarky then first 34 years of integration.

then the lower precautionary motive, with respect to the other economy, induces them to increase the level of consumption and to finance it by issuing debt. The current account therefore first jumps to a surplus of 9% of production for country 1, followed by a jump to a deficit of 6% of its GDP, which subsequently starts shrinking and turns again positive as the debt service rises. Agents in country 1 keep issuing debt in the entire transition until it reaches almost 400% of production at the final steady state. Due to the large increase in the negative asset position, total wealth decreases and therefore consumption and capital, that are a fraction of wealth, decrease as well. On one hand, agents have to repay interests on the accumulated debt; on the other hand, they become more risk-averse as total wealth decreases (given CRRA preferences). These two forces, namely lower wealth and higher risk aversion, push agents to reduce the level of capital, that at the final steady state is 5% lower than the autarky level. The level of consumption falls back to the autarky steady state value in 25 years, it then keeps diminishing down to 17% below the autarky one. The turning point for the level of debt is around 71% of production; having reached this level of debt, country 1 starts decumulating capital, broadly in line with Rehinart and Rogoff (2010) prediction.

On the other hand, agents in country 2 keep accumulating capital: in the first part of the transition this is due mainly to the decrease in the risk-free interest rate, which must be matched by a decline of the risk-adjusted return on capital; in the second very long transition phase, capital accumulation is driven by the increasing share of resources they want to invest in risky activities. In the steady state of integration capital in country 2 is 2.5% higher than its level in autarky steady state. At the moment of integration agents increase their level of consumption given the lower interest rate on savings, they however start consuming less already two years after integration, when they become willing to postpone consumption in order to save and invest more; they get to the autarky steady state level of consumption in 32 years and then they steadily raise it up to a final steady state which is 7.2% higher than the autarky steady state level. The current account of country 2 mirrors the one of the other economy. Therefore a large deficit in the first period is followed by a persistent surplus (for around 30 years); then the current account turns negative again as the level of debt service by the other economy increases. The final level of accumulated foreign assets reaches almost 200% of total production.

Poor countries (in terms of capital level) enjoy a first boost in capital accumulation and consumption, and also capital inflows following capital account liberalization. The sign of the capital account changes however after a few years. This result is in line with the observations of Prasad et al. (2007), who show a tendency for this reversal: until the end of the 1990's capital importing countries were the poorer economies, while in the last decade

richer economies tended to attract net flows of capital.

2.3.3 Welfare analysis

We now turn our attention to the welfare consequences of financial integration. We compute the Hicksian equivalent variation, defined as the amount of consumption agents must receive in order to remain in autarky instead of moving to integration; it therefore represents the amount of immediate consumption agents want at the moment of capital account liberalization in order to be indifferent between autarky and integration. A positive value therefore means that the agent of this economy prefers to move to integration (integration is welfare improving) while a negative value implies a higher utility from remaining in autarky (integration is welfare decreasing).¹⁸

In the advanced country integration implies a positive welfare gain corresponding to a 5.3% increase in immediate consumption. The temporary drop in consumption at the moment of integration is more than offset by consumption levels above the one of autarky for the first 25 periods. Due to the discount factor β , agents in this economy assign low weight to the decrease in consumption in the rest of the transition path.

The opposite is true for the agents in the developing country, where integration has a negative welfare impact of 8% of consumption at time 1. The increase in the consumption level at the time of integration cannot compensate for the decrease in the following years, which is due to the strong precautionary motive.

The welfare implications of the model are sensitive to the initial level of capital in country 2 at the moment of integration. We show below, that the welfare consequences are worse for developing economies with higher levels of capital at the moment of integration, since the precautionary motive dominates over the equalization of the returns to capital adjusted for risk. The contrary is also true: for example, if K_0 is 69% of country 2 autarky steady state (or 67% of the steady state of integration), the overall welfare loss is lower in this economy and equal to 4.41%.

¹⁸We compute the Hicksian equivalent variation for the "average" agent, the one that, at the moment of liberalization, has her wealth and accumulated capital and bonds equal to the average in that economy. This measure is different from the one used by Antunes and Cavalcanti (2010), who assume a benevolent planner that averages across agents' welfare. However given the monotonicity of the utility function the "sign" of the welfare consequence is the same, even if the magnitude of the two measures is not comparable.

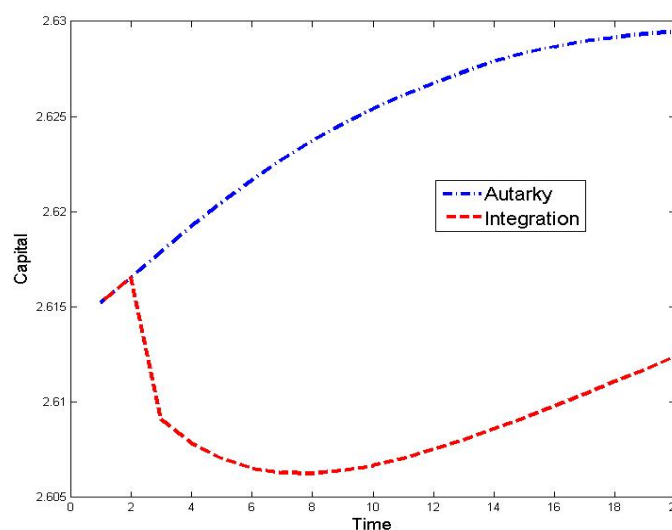


Figure 2.8: Capital accumulation in country 2 for the first 19 years after integration - autarky vs integration.

2.3.4 Alternative hypotheses on the level of initial capital K_0

We now modify the initial value of capital for country 2 at the moment of financial integration. We start from a value of K_0 for country 2 that is 99% of the autarky level and, more importantly, from a level of interest rate in country 2 slightly below the level of integration in steady state (but still above the interest rate of autarkic steady state).

In figure 2.8 we plot together capital accumulation in autarky and integration. The main difference with respect to the scenario analyzed above is that here for country 2, capital account liberalization reduces on impact the level of resources invested in risky activities by almost 3%; the level of capital starts increasing again 5 years after integration and reaches the initial level in about 25 years and the level of autarky steady state in 50 years. The very long run implications are analogous to the ones exposed above (i.e. in country 2, capital increases further and converges to a level higher than the autarkic steady state). The important difference in this case is that at the moment of integration the risk-free interest rate increases for agents in country 2, therefore they are willing to postpone consumption, and to shift resources away from the risky and low profitable investment in capital (taking into account the risk premium) into the now higher yielding risk-free bonds issued by country 1. In this scenario figure 2.9 shows that the interest rate jumps to the steady state level upon

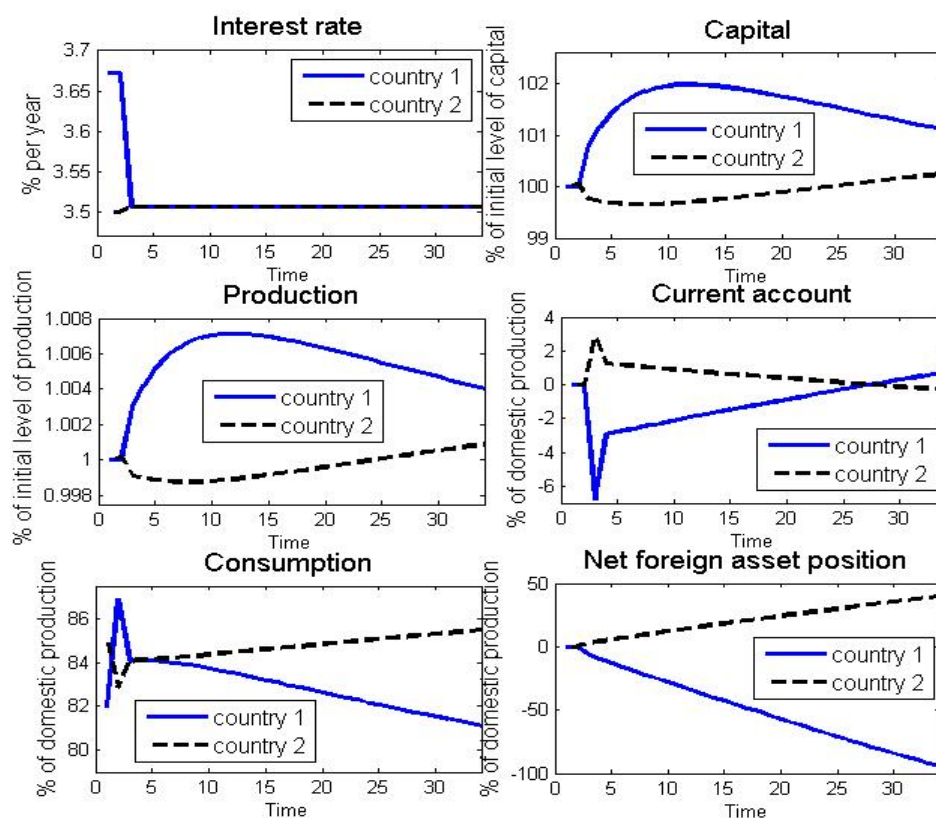


Figure 2.9: Transition dynamics - first period in autarky, then first 33 periods of integration.

integration and capital in country 1 starts increasing immediately, boosted by the lower risk premium. The current account in country 2 jumps to a surplus of 3% of production. On the other side, agents in country 1 are willing to issue debt in order to finance consumption and investment in capital, whose return adjusted for risk turns higher than the new lower risk-free interest rate; in country 1 the current account decreases on impact by 7% of production. At the time of integration, agents in country 2 decrease their level of consumption while agents in the other economy increase consumption on impact; the medium and long run paths for consumption are instead in line with the results of the previous scenario. The welfare implications are therefore stronger than the previous scenario, with a 9% welfare gain in terms of current consumption in country 1, and a 9% welfare loss for the agents of the other economy.

2.4 Final remarks

The goal of the present analysis is to study the effects of integration among countries with different levels of financial market development. We first of all highlight that financial deepness influences agents decisions in terms of investment as well as savings; in this respect our analysis is in line with the empirical findings of Gourinchas and Jeanne (2009):¹⁹ we obtain savings and investment gaps with respect to the complete markets case (that in the present study is equivalent to assuming a production process with zero variance, σ_i^2).

We show that financial integration has negative welfare effects for financially poorer economies, especially if those countries are close to their autarky steady state at the moment of integration; this negative consequence is instead partially mitigated for countries that are at an early stage of capital accumulation. On the other hand, integration is welfare improving for financially advanced economies. In terms of capital accumulation the short term effects of financial integration are very different from the steady state ones. We establish two important results: first, for economies with poor financial institutions, capital account liberalization slows down the speed of capital convergence in the medium term, but in the long run it results in a higher level of capital with respect to the autarkic steady state; second, financially advanced economies enjoy a first period of higher investment in risky activities, but in the very long run they reduce their capital (and, therefore, production) to levels below the autarky steady state. Different behaviors of savings and investment, determined by different appetite for risk, shape the choice of production, consumption and

¹⁹Gourinchas and Jeanne (2009) are able to reproduce international capital movements and growth observed in the data by introducing savings and investment wedges into the neoclassical growth model.

safe investment. Moreover the propensity towards risky production activities decreases with wealth, this mechanism determines the very long run levels of the main variables.

The novel result on the long run consequences of financial integration among economies with different levels of financial development is in stark contrast with previous works: Mendoza et al. (2007) and Corneli (2009) obtain the opposite long run results of steady state in integration. A detailed comparison with them is needed but goes beyond the purpose of the present study; it will be the object of future investigation.

2.5 Appendix A

In what follows, for simplicity we drop the index i for the countries, which is attached to every variable. Following Lemma 1 of Angeletos (2007), we obtain labor demand and capital income as linear functions of k_t , by maximizing $\hat{f}(a, 1, \hat{n})$ with respect to \hat{n} :

$$n_t = n(a_t, \omega_t)k_t \quad (2.34)$$

$$\pi_t = r(a_t, \omega_t)k_t \quad (2.35)$$

Where

$$\hat{f}(a_t, 1, \hat{n}_t) = f(a_t, k_t, n_t)/k_t$$

$$\hat{n}_t = n_t/k_t$$

The maximization problem reduces to:

$$V(w; t) = \max_{c, b', k'} \{U(c) + \beta E_t V(w'; t + 1)\} \quad (2.36)$$

s.t.

$$c + k' + b' = w \quad (2.37)$$

$$w' = r(a', \omega_{t+1})k' + R_{t+1}b' + \omega_{t+1} \quad (2.38)$$

We obtain the solution in equations (9) - (11) by proposing and verifying the following solution:

$$V(w; t) = U(\lambda_t(w + h_t)) \quad (2.39)$$

$$c(w; t) = (1 - \psi_t)(w + h_t) \quad (2.40)$$

$$k(w; t) = \psi_t(w + h_t) \quad (2.41)$$

$$b(w; t) = (1 - \phi_t)\psi_t(w + h_t) - h_t \quad (2.42)$$

Where the last is obtained by substituting (40) and (41) in the budget constraint (37). For simplicity, we call $r(a_{t+1}, \omega_{t+1}) = r_{t+1}$.

The first-order conditions for capital and bonds become:

$$c_t^{-\gamma} = \beta \lambda_{t+1}^{1-\gamma} E_t [r_{t+1}(w_{t+1} + h_{t+1})^{-\gamma}]$$

$$c_t^{-\gamma} = \beta \lambda_{t+1}^{1-\gamma} E_t [R_{t+1}(w_{t+1} + h_{t+1})^{-\gamma}]$$

Combining these two expressions and substituting $(w_{t+1} + h_{t+1})$ from the budget constraint we obtain an expression for $\phi_t = \phi(\omega_{t+1}, R_{t+1})$:

$$E_t[(r_{t+1} - R_{t+1})((r_{t+1} - R_{t+1})\phi_t + R_{t+1})^{-\gamma}] = 0 \quad (2.43)$$

From the envelope condition ($V'(w; t) = U'(c)$) we obtain the following relation:

$$\lambda_t^{1-\gamma} = (1 - \psi_t)^{-\gamma}$$

Moreover summing the FOC above, pre-multiplied respectively by ϕ_t and $(1 - \phi_t)$, we have the recursive structure for the coefficient ψ_t :

$$(1 - \psi_t)^{-1} = 1 + \beta^{1/\gamma} \rho_t^{1/\gamma-1} (1 - \psi_{t+1})^{-1} \quad (2.44)$$

Where

$$\rho_t = E_t[(r_{t+1} - R_{t+1})\phi_t + R_{t+1}]$$

Finally ϕ_t and ρ_t are approximated with a second-order Taylor expansion of $\ln \rho_t$ around $\sigma_t = 0$, by employing the derivation of the log - portfolio return in Campbell and Viceira (2002):

$$\ln \rho_t \approx \phi_t E_t(\ln r_{t+1}) + (1 - \phi_t) \ln R_{t+1} + \frac{1}{2} \phi_t (1 - \phi_t) \sigma_{t+1}^2 + \frac{1-\gamma}{2} \phi_t^2 \sigma_{t+1}^2$$

Where

$$\bar{r}_{it+1} = E_t r_{t+1}$$

Solving for ϕ_t and using the fact that $E_t(\ln r_{t+1}) = \ln E_t(r_{t+1}) - \frac{1}{2} \sigma_{t+1}^2$ we obtain the expressions:

$$\phi_t \approx \frac{\ln \bar{r}_{t+1} - \ln R_{t+1}}{\gamma \sigma_{t+1}^2}$$

$$\rho_t \approx R_{t+1} \exp\left(\frac{(\ln \bar{r}_{t+1} - \ln R_{t+1})^2}{2\gamma \sigma_{t+1}^2}\right).$$

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CHAPTER 3

RESERVE MANAGEMENT AND SOVEREIGN DEBT COST IN A WORLD WITH LIQUIDITY CRISES

This Chapter is joint work with Emanuele Tarantino.

Abstract

The accumulation of large amount of sovereign reserves has fuelled an intense debate on the associated costs. In a world with liquidity crises and strategic default, we model a contracting game between international lenders and a country, which delivers the country's optimal portfolio choice and the cost of sovereign debt: at equilibrium, the sovereign allocates the borrowed resources to either liquid reserves or an illiquid and risky production project. Moreover, we study how the opportunity cost of hoarding reserves is affected by the financial and technological characteristics of the economy. In line with recent empirical evidence, we find two important results: the cost of debt decreases in the level of reserves if the probability of liquidity shocks is high enough; however the cost of debt increases in reserves when the lenders anticipate that the country has an incentive to default after a liquidity shock. Indeed, in the event of such a shock, we show that the country may choose to retain reserves instead of employing them to inject the liquidity needed to bring the production project to maturity.

3.1 Introduction

Reinhart and Rogoff (2010) document that after the Great Depression and consequent banking system crisis, a wave of sovereign debt defaults arose during the 1930s. To establish this domino effect as a stylized fact in contemporary history a piece is missing: the failure of major financial institutions in the recent crisis to trigger a series of sovereign debt crises in the years ahead. This threat calls for a reflection on the management of sovereign liquidity and could become a test for the strategy, mainly employed by emerging economies after the experience of the late '90s Asian crisis, of accumulating large amounts of sovereign reserves as buffer stock to face liquidity shocks.

In early 2000 this accumulation seemed to be justified by the self-insurance motive and to be broadly in line with rules of thumb like the “Guidotti-Greenspan rule”;¹ however, the recent increase in the resources devoted to reserves has made it urgent to understand the costs associated with this build-up. Rodrik (2006) measures the opportunity cost of accumulating large amounts of liquid assets (in particular US treasury bonds) as the spread between external borrowing costs and reserves’ returns. Rodrik (2006) compares this “self-insurance premium” with the expected cost of a financial crisis in terms of reduced output and concludes that in the last decade the amount of reserves accumulated by some countries has grown too much. In order to assess the opportunity cost of holding reserves, we believe that two further elements should be taken into account: the impact of reserves on future domestic output and on the cost of debt.

We analyse the opportunity cost of holding reserves in a world where the sovereign country is subject to liquidity crises and always has the option to default on debt. We develop a model of optimal portfolio choice where country’s resources are determined by a contracting game with international lenders, and characterize the equilibrium level of debt price, sovereign reserves and expected output. Reserves are the country choice variable. Instead, the debt price is set by the lenders and the country’s expected output results from the share of total borrowed resources that is not devoted to reserves.

The strategic interaction between the agents is shaped by two crucial financial frictions: lenders cannot inject the resources needed in the event of a liquidity shock and the country cannot commit not to default on debt.² These frictions introduce asset incompleteness into our model, and this is necessary to disentangle the relationship between the country’s decision to default and the cost of sovereign debt. Indeed, if assets are not contingent, risk-neutral

¹To be consistent with this rule, reserves should be equivalent to the country’s short-term external debt.

²These assumptions are consistent with several papers in the literature and in particular with Holmström and Tirole (1996, 1998), Caballero and Krishnamurty (2002), Caballero and Panageas (2005) and Lorenzoni (2007).

competitive lenders incorporate the probability of default in the premium set on the debt contract.

The optimal choice of reserves made by the country arises from the equilibrium between two forces: on the one hand, reserves are liquid assets that can be injected in the event of liquidity shocks and therefore help to avoid defaults (*precautionary motive*), on the other hand, they distract resources from a more productive but illiquid project. The country also takes into account the impact of reserves on the cost of debt; there are two conflicting effects at work: the positive one is that, by allowing the country to avoid default in the event of liquidity crises, reserves raise the probability for lenders to be repaid; the negative effect is that reserves are not pledgeable in the event of default and distract resources from the productive activity.

The novelty of our approach is that we do not assess whether the level of reserves is optimal in terms of its opportunity cost *per se*. We take a more general perspective, considering that when the country decides the optimal resource allocation it takes into account not only the opportunity cost of holding reserves in terms of the expected return of an alternative (illiquid) project, but also the effects of its choices on the price set by the lenders. In this way, we can address a number of policy relevant questions regarding the impact of reserves on the cost of sovereign debt and on the expected output.

We establish the following results. First of all, *when there is a high probability of liquidity crises, the cost of sovereign debt may decrease if the level of reserves is large enough*; second, *the country can decide to default at equilibrium if it is hit by an adverse shock and finds it optimal to behave this way*. We also analyse the effect of the chosen level of reserves and the cost of debt on the *sovereign expected output*: *we find that if the probability of a liquidity crisis is high enough, then larger reserves, by insuring the country against a shock, have a positive impact on expected output*.

Our framework allows us to analyse how the sovereign investment choices are affected by the financial and technological characteristics of the economy, namely the degree of riskiness of the production project and the level of the country's capital account openness. More specifically, we show that *the opportunity cost of holding reserves increases with the variance of the productive process due to a limited liability effect*: when deciding how to invest its resources the country disregards the lower tail of output realization (since in those cases it would default), therefore the return on the productive project is increasing in the risk associated with it. Finally, we show that, *as the degree of sovereign capital account openness increases, larger capital outflows in the event of sudden stops increases the country's incentives to default in such events*.

We contribute to the literature that studies the optimal decision of sovereign countries to accumulate international reserves in the presence of strategic default on sovereign debt, like Alfaro and Kanczuk (2009). Alfaro and Kanczuk (2009) employ a setup in which the sovereign keeps reserves while losing part of the output in the event of strategic default on debt. In their model, the main upside of reserve accumulation is the possibility of employing them to smooth consumption. They introduce the possibility of sovereign liquidity crises in the form of contagion shocks (an abrupt variation in the interest rate) or sudden stops; however, in their setup reserves do not have a particular role in avoiding such crises, instead they are imperfect substitutes of external debt reduction for smoothing consumption. The main result they obtain is that the optimal policy of the country features nil international reserves and the intuition is that the country prefers to smooth consumption by lowering debt exposure instead of accumulating reserves. In our model, we study the effect of the country's willingness-to-repay concerns on the cost of debt by modelling the funding game between lenders and sovereign and obtain the rather different result that the optimal level of accumulated reserves can be positive at equilibrium. Remarkably, this is independent from the working of the precautionary motive. In our setting the sovereign solves the investment game by comparing the opportunity cost of reserves with respect to the productive activity: provided the former is low enough, then hoarding reserves is rational. On top of this, we perform an analysis of the role that reserves have in providing an insurance device against liquidity shocks and study whether self-insurance is preferable to a strategy that does not feature the injection of the necessary liquidity following a shock.

The level of resources available to the country for the investment game is not exogenous, because we endogenously determine the cost of sovereign debt. Instead, we take the level of outstanding liabilities as given and abstract from considerations about the possibility of reducing short-term debt in order to create liquidity and thus lower the impact of a shock. This assumption is consistent with the evidence in Rodrik (2006), which documents that, in recent years, emerging economies have not reduced their exposure to short-term debt, while accumulating large amounts of foreign liquid assets. Finally, we deliver our result by abstracting from the role that reserves have in stabilizing exchange rates and promoting exports (which instead is the main focus of Dooley, Folkerts-Landau and Garber, 2003).

We also contribute to the literature that studies optimal contractual arrangements in the presence of commitment problems and non-contingent contracts, as in Arellano (2008). In analogy to Arellano (2008), we show that default arises at equilibrium after an adverse shock occurs, consistently with the received empirical evidence. However, while Arellano (2008) studies the relationship between default risk and output, consumption and foreign debt, we

look at the interaction between default risk and reserves, cost of debt and output.

We map the relationship between the cost of sovereign debt and the level of sovereign reserves: the amount of reserves hoarded by a country may reduce the cost of debt. In particular, this happens if the precautionary motive dominates. However, the opposite can occur: if the rationale behind the accumulation of liquid assets is opportunistic (that is, if the country prefers to keep reserves instead of employing them in the event of a liquidity shock), then an increase in their amount raises the cost of debt. This result, namely that the relationship between reserves and the cost of sovereign debt is not monotonic, is supported by two pieces of empirical evidence. Levy Yeyati (2008) finds in the data that holding reserves reduces the spread on the debt issued over the risk-free return on assets to the extent that reserves lower the probability of a run-induced default; moreover, the empirical analysis in Ruiz-Arranz and Zavadjil (2008) shows that the accumulation of reserves reduces the cost of borrowing only up to a threshold and concludes that the level of reserves observed in Asian countries is in line with an optimal insurance model.

The paper is organized as follows: in the next section we present the main features of the model. In Section 3 we solve for the equilibrium of the game. First, we analyse a benchmark framework without reserves, then we introduce the possibility for the country to (diversify its portfolio and) choose between investing in liquid reserves and an illiquid production project. In Section 4 we analyse the model's equilibrium by undertaking numerical simulations, while two possible extensions of the main setting are presented in Section 5: in the first we model the optimal investment choice of a risk-averse country, in the second we introduce a supra-national organization, e.g. the IMF, that can intervene to finance the country in a liquidity crisis. Section 6 concludes.

3.2 The Model

Consider a sovereign country that needs to borrow D from international lenders. At stage $0a$, the lenders set the rate of return on the resources lent to the country and at stage $0b$, the country decides on the allocation of the same resources.³ At stage 1, a liquidity shock may take place: if a shock occurs the country has to decide whether to default at stage 2; in the absence of the shock the game proceeds to stage 3. At stage 3 the productivity shock takes place and at stage 4 the sovereign can again choose whether to default on debt. Figures 3.1 and 3.2 illustrate, respectively, the timing of the game and the game-tree. We solve

³For ease of exposition, we present the borrowing decision and the investment decision in sequence. However, if the two games were solved simultaneously we would obtain the same results.

the model by backward induction and the equilibrium concept we employ is the Sub-game Perfect Nash Equilibrium (SPNE).

[FIGURES 3.1 AND 3.2 ABOUT HERE]

In this Section, we present in detail how we model each relevant node and the main ingredients of the game.

3.2.1 The Lending Game

Our economy is populated by a continuum of atomistic and *risk-neutral lenders* (indexed by $i \in I$), from which the country can borrow. Each of these lenders is able to lend D , so that the sovereign must borrow from a subset of mass 1 of them. The total mass of lenders is large, ensuring that perfect competition prevails and lenders do not extract any rent; moreover, the lenders are risk-neutral and have unlimited access to fund at the risk-less interest (that we assume to be nil).

The lending game can be viewed as a general (common agency) contracting game between a principal (the sovereign) and multiple agents (the lenders):⁴ as in Bolton and Jeanne (2007), investors participate in a bidding game following the sovereign's announcement of a fund raising goal of D . Lenders move first by each making a bid simultaneously. The sovereign then decides which bids to accept.

The lenders' utility is equal to the value of the repayment D discounted by the probability that the sovereign repays in full. At the bidding stage of the game, then, each lender i makes an offer on the rate of return, $r(i)$, that is required to break even in expectation. A lender i solves a problem of the following sort:

$$\begin{aligned} D &= D(1 + r(i))\text{Prob}\{\text{The Country is Solvent}\} \iff \\ 1/(1 + r(i)) &= \text{Prob}\{\text{The Country is Solvent}\} \iff \\ \delta(i) &\equiv \text{Prob}\{\text{The Country is Solvent}\}. \end{aligned}$$

Consequently, in the model the contract specifies the discount factor $\delta(i) = 1/(1 + r(i))$ that a lender i asks in exchange for a loan D . A Nash equilibrium of the lending game is defined by a set of bids $(\delta(i))_{i \in I}$ such that, for all i , bid $\delta(i)$ maximizes lender i 's utility taking all the other bids $\delta(j)$, with $j \neq i$, as given. Clearly, at equilibrium the sovereign squeezes all the surplus from the lending relationship and (randomly) selects among a set of identical bids, $\delta(j) = \delta(i) = \delta$, so *we can focus on a representative sovereign-lender pair*.

⁴See, for example, Bernheim and Whinston (1986a,b), Hart and Tirole (1990).

3.2.2 The Investment Game

The *country is risk-neutral* and can invest its funds in public expenditure, g , and/or reserves, R . Therefore, the sovereign feasibility constraint is given by

$$\delta D = g + R \iff g = \delta D - R. \quad (3.1)$$

When deciding resource allocation, the country maximizes its expected utility, denoted by $E(U(\cdot))$: this is determined by the amount of liquid resources gathered and by the expected value of output. In the model, reserves (R) yield the risk-less interest rate and are a storage and liquid technology that can be carried over from the stage in which they are accumulated to the final stage of the game.

The production activity is the illiquid technology: it requires public expenditure (g) as sole input and is subject to a productivity shock, z . The assumption we make is that the output materializes at stage 4, after the country has decided on the allocation of its resources and uncertainty over the shocks occurs. More specifically, the output is generated by a production function, $Y(z, g)$, that is linearly affected by the productivity shock and that, using (1), can be rewritten as:

$$Y(z, g) = zY(g) = zY(\delta, R).$$

The shock z is such that $z \sim F(z)$, where $F(z)$, the cumulative distribution function, is twice differentiable and continuous and $f(z)$, the probability distribution function, is given by $dF(z)/dz = f(z)$. For the sake of simplicity, we assume that z follows a continuous uniform distribution over $[1 - c, 1 + c]$; however, our results hold under alternative assumptions on the distribution of z .⁵

Finally, the function $Y(\delta, R)$ is twice differentiable,⁶ with $Y'(\cdot) \geq 0$, $Y''(\cdot) < 0$, it is increasing in δ , decreasing in R and satisfies standard Inada Conditions ($\lim_{g \rightarrow 0} Y(\cdot) = 0$, $\lim_{g \rightarrow 0} Y'(\cdot) = \infty$ and $\lim_{g \rightarrow +\infty} Y'(\cdot) = 0$).

3.2.3 The Liquidity Shock

We assume that the country incurs in the risk of a liquidity shock at an intermediate stage, before the outcome of the production activity.

Following Chang and Velasco (2000) we assume that, with probability $\eta \in (0, 1)$, the illiquid project needs a further infusion of capital ϵ at stage two in order to be completed.

⁵In particular, Appendix D shows that the results of the model carry over under continuous distributions with exponential density functions, like the normal and log-normal distributions.

⁶We choose this compact notation instead of $Y(\delta D - R)$ to stress that the endogenous variables that affect $Y(\cdot, \cdot)$ are δ and R .

If a liquidity shock occurs the country can only use the accumulated reserves to inject ϵ , while it cannot dismantle the capital invested in the production process. If it decides not to tackle the shock with the infusion of capital, the country defaults on the process and retains reserves.

3.2.4 The Default Decisions

There are two stages in the model at which the country may choose to default. The first is after the realization of the liquidity shock at stage two. The second is after the realization of the productivity shock at the final stage (when output is realized). In this way, we introduce the two main frictions of the model: the first is that the lender commits not to inject the resources needed by the country if a liquidity shock occurs. The second is that the country cannot undertake not to default when the realized output is lower than the face value of debt.⁷

In analogy to what is typically assumed in the literature (e.g. Bolton and Jeanne, 2007), the cost of default for the country consists of losing the entire output.⁸ However, in the event of default the country keeps the reserves that it has accumulated, while the lender gets nothing.⁹

For simplicity, we distinguish between two plans (that we denote \mathcal{F} and \mathcal{N}) depending on the country's choice at stage two to inject the needed liquidity should the shock occur. We find it a useful the distinction as it allows us to easily characterize all possible branches of the game-tree. Indeed, because of the financial friction of no commitment on the country side, at each stage the sovereign can decide to default if it is convenient to do so, i.e. if the expected utility from defaulting is greater than the one from continuation.¹⁰

[FIGURES 3.3 ABOUT HERE]

Figure 3.3 illustrates the branches of the game-tree corresponding to the two plans. Under plan \mathcal{N} the country chooses to default if hit by the *liquidity shock*. Instead, under plan \mathcal{F} the country chooses to face the shock, but for this decision to be credible it must also be feasible and time-consistent. Indeed, if at the investment game the country accumulates a level of reserves lower than ϵ then it cannot inject the needed capital to face the shock

⁷Or, equivalently, that the lender cannot act as residual claimant if realized output is lower than the face value of debt.

⁸See Alfaro and Kanczuk (2005) for a discussion on output losses in the event of default.

⁹This assumption is consistent with Alfaro and Kanczuk (2009).

¹⁰Notice, however, that the crucial decision at time zero is how to allocate resources between R and g , and the final equilibrium is always at most one.

(resource constraint). Moreover, a problem of limited commitment may arise: if the country's payoff from the continuation of the game following the shock is less than ϵ then the country will prefer to default after the shock and retain reserves (no-default constraint).

Finally, the country may default after the realization of the *productivity shock*, in which case the lender is not repaid.

3.2.5 A Discussion of Our Assumptions

To keep our model tractable and study the interaction between the effects that crucially determine the country's choices, we make a number of important assumptions.

First of all, we treat the level of the face value of debt D as an exogenous variable. Alfaro and Kanczuk (2009) show that debt and reserves can both be employed to smooth a country's utility, and therefore their levels are connected. However, Rodrik (2006) finds that emerging economies seem to be more active in deciding how to allocate resources and, in particular, in increasing the level of accumulated reserves; instead, the level of short-term debt has not changed much in time. In this sense, ours can be considered a short-term perspective theory, in which creditors set the debt price, either in the principal or secondary market, by looking at the economic fundamentals of each country and taking the amount of outstanding debt as given. Moreover, the fact that in the model the lender decides the debt discount factor (δ) implies that she influences the total amount of resources available for sovereign investment, and in this sense we develop a model with endogenous country resources.

In the setting presented above there are two shocks; at time two a liquidity shock may hit the country and at time three the productivity process z is realized. We assume that these two shocks are orthogonal to each other and to the main variables of the model (especially reserves). An alternative approach would be to assume that, if a liquidity shock occurs, output would be adversely affected, by the shock itself, but also by a common underlying process that guides the two uncertainties. Moreover, the liquidity and productive shock processes may be linked to the variables set at time zero: for example, empirical studies show that countries with higher reserves performed better during the recent financial crisis, both in terms of output and of lower exchange rate depreciation and capital flight. The analysis of the correlation among the shocks could lead to new dynamics and add interesting results to the model; however, we leave it for future research.

The interest rate on the safe assets bought to build up reserves is set to zero. Also, in the event of default the country loses the entire output. These two assumptions may look somewhat extreme; however, (i) they are not uncommon in the literature and (ii)

we do believe that they can be introduced without loss of generality, as a more realistic parameterization would not change the nature of the results we obtain.

3.3 Solution of the Model

The analysis is conducted under the following simplifying technical restriction:

$$\mathbf{A1} : |\epsilon_{Y',R}| > |\epsilon_{\bar{z},R}|.$$

Assumption (A1) is needed to make sure that the solution to the country's maximization problem is unique and implies that the output is more sensitive to changes in reserves than the expected productivity level in the case of no default. More specifically, $\epsilon_{Y',R}$ represents the elasticity of the derivative of the production function to changes in the reserve level, while $\epsilon_{\bar{z},R}$ is the elasticity to changes in reserves of the default threshold \bar{z} , which will be defined in what follows.¹¹

In order to better disentangle the impact of reserves in this setting, in Section 3.1 we solve the model assuming that reserves cannot be accumulated: clearly, in this benchmark the country can never cover the liquidity shock. In Section 3.2, we allow the country to decide how to allocate δD between g and R both in plan \mathcal{F} and \mathcal{N} .

Our objective is to compare the equilibrium levels of δ , R and the expected output in the two plans with the values resulting from the benchmark without reserves. Then, we assess which plan maximizes the country's expected utility and is implemented at equilibrium by performing numerical simulations.

3.3.1 Benchmark without Reserves

If the country is hit by the shock in $t = 1$, then it is not able to repay ϵ and is already in default at $t = 1$. Conversely, if the country is not hit by the shock, then it defaults at the final stage if:

$$0 \geq zY(\delta) - D \iff z \leq \bar{z}(\delta) \equiv \frac{D}{Y(\delta)}.$$

\bar{z} represents the threshold level of the productive process: if the realized productivity is higher, then the country repays its creditors and keeps the residual, otherwise the country defaults on debt and loses the entire output. This introduces a problem of limited liability on the country side, which influences the outcome of the investment game.

¹¹In Section 4, we perform a set of numerical simulations of the model and show that (A1) is naturally satisfied by a standard Cobb-Douglas production function.

At the investment stage, resources are entirely invested in g . At $t = 0$, the lender sets the optimal δ as to break-even in expectation:

$$\delta D = D(1 - \eta) \int_{\bar{z}(\delta)}^{\infty} dF(z) \iff \delta = (1 - \eta)[1 - F(\bar{z}(\delta))] \equiv (1 - \eta)G(\delta). \quad (3.2)$$

In the absence of reserves, the lender takes into account that the break-even is attained only if the liquidity shock does not take place, that is with probability $(1 - \eta)$. For convenience, when we solve for the lender's problem we denote by $G(\cdot)$ the survival function and interpret it as the probability that the country is fully solvent after the productivity shock.

$$\exists! \delta^* \in (0, 1) \quad \text{s.t.} \quad \delta^* = (1 - \eta)G(\delta^*).$$

Proof. See Appendix A.

[FIGURE 3.4(a) ABOUT HERE]

Figure 3.4(a) illustrates the result of Lemma 3.3.1. The deal signed at the initial stage between the lender and the sovereign can be implemented by a debt contract in which the lender earns $(D - \delta^*D)$ (or, equivalently, $Dr^*/(1 + r^*)$, with $r/(1 + r) = 1 - \delta$) provided the country repays in full, zero otherwise.

3.3.2 Framework with Reserves

In what follows, we allow the country to accumulate reserves at the investment game stage.

If the liquidity shock does not occur at $t = 1$, then the model proceeds as in the framework without reserves and at $t = 4$ the country defaults after the realization of the productivity shock if the following condition holds:

$$zY(\delta, R) - D + R - \epsilon \leq R - \epsilon \iff z \leq \bar{z}(\delta, R) \equiv \frac{D}{Y(\delta, R)}.$$

Conversely, if the liquidity shock occurs at $t = 1$, we distinguish between two cases, depending on whether the country decides to face the liquidity shock (plan \mathcal{F}) or not (plan \mathcal{N}).

Plan \mathcal{F}

At $t = 4$ the country defaults after the realization of the productivity shock if the following condition holds:

$$zY(\delta, R) - D + R - \epsilon \leq R - \epsilon \iff z \leq \bar{z}(\delta, R) \equiv \frac{D}{Y(\delta, R)}.$$

At $t = 2$, the country would inject the new capital and go on with the project. Clearly, a necessary condition for the country to do so is that it accumulates enough liquidity. In other words, the outcome of the investment stage must be such that $R \geq \epsilon$ (*resource constraint*).

Moreover, we analyse whether the decision to inject liquidity after the shock is sub-game perfect. Given the choice on R by the country under plan \mathcal{F} and the consequent contract offered at $t = 0$ by the lender, it must be that the sovereign has incentive to continue with the liquid project instead of defaulting strategically at $t = 2$, otherwise the same plan would not be time-consistent. The *no-default constraint* that has to hold follows:

$$\begin{aligned} R - \epsilon + \int_{\bar{z}(\delta, R)}^{\infty} [zY(\delta, R) - D]dF(z) &\geq R \iff \\ \int_{\bar{z}(\delta, R)}^{\infty} [zY(\delta, R) - D]dF(z) &\geq \epsilon. \end{aligned}$$

At $t = 0a$, the lender solves the following zero-profit condition:

$$\delta D = D \int_{\bar{z}(\delta, R)}^{\infty} dF(z) \iff \delta = 1 - F(\bar{z}(\delta, R)) \equiv G(\delta, R).$$

In this case, the formulation of the investor's problem takes into account that the sovereign does not default after a liquidity shock.

At $t = 0b$, the country decides to invest δD in g and/or R by solving the following problem:

$$\max_R E(U_F(\delta, R)) = R + \int_{\bar{z}(\delta, R)}^{\infty} [zY(\delta, R) - D]dF(z) - \eta\epsilon, \quad (3.3)$$

The country is certain to R , because reserves are not lost in the event of default. The second term in (3.3) is the expected value of the production project net of D : this second term is positive by construction, because the expected value of output is truncated downwards by \bar{z} . The third term in expression (3.3) is the expected value of the liquidity shock.

The maximization problem in (3.3) is solved under the aforementioned constraints:

$$\int_{\bar{z}(\delta, R)}^{\infty} [zY(\delta, R) - D]dF(z) \geq \epsilon, \quad (3.4)$$

$$R \geq \epsilon, \quad (3.5)$$

which make sure that the implementation of plan \mathcal{F} is sub-game perfect.

Plan \mathcal{N}

At $t = 2$, if the sovereign chooses to default after the liquidity shock, it loses output but keeps the accumulated liquidity (R).

At $t = 0a$, the lender replies by setting δ as to break-even in expectation:

$$\delta D = D(1 - \eta) \int_{\bar{z}(\delta, R)}^{\infty} dF(z) \iff \delta = (1 - \eta)[1 - F(\bar{z}(\delta, R))] \equiv (1 - \eta)G(\delta, R).$$

In this case, the lender anticipates that the country defaults on the liquidity shock in $t = 2$.

At $t = 0b$, the country's choice of R is derived by solving the following problem:

$$\max_{R \in [0, \delta D]} E(U_N(\delta, R)) = R + (1 - \eta) \int_{\bar{z}(\delta, R)}^{\infty} [zY(\delta, R) - D]dF(z).$$

The decision to default on the liquidity shock implies two things: the first is that the sovereign obtains the net expected payoff from the production project only if exempted from the liquidity shock (with probability $1 - \eta$); the second is that the sovereign does not need to inject ϵ to bring the illiquid project to the end. Finally, the country gets R in all states of the world.

3.3.3 Equilibrium Definition

The equilibrium is defined by the vector $\{\delta, R\}$ such that the country's behaviour is optimal and the lender breaks even in expectation. Moreover, with liquidity shock coverage (that is, when plan \mathcal{F} is chosen), the country's actions must be sub-game perfect, insofar as both the no-default constraint and the resource constraint must be satisfied.

Definition 3.1

Define $\mathcal{F} \equiv \{\delta_F^{**}, R_F^{**}\}$ as the pair that characterizes the plan in which the country decides to face the liquidity shock. At $\mathcal{F} \equiv \{\delta_F^{**}, R_F^{**}\}$, the resource constraint is satisfied

$$R_F^{**} \geq \epsilon,$$

and the no-default constraint is satisfied

$$\int_{\bar{z}(\delta_F^{**}, R_F^{**})}^{\infty} [zY(\delta_F^{**}, R_F^{**}) - D]dF(z) \geq \epsilon.$$

Define $\mathcal{N} \equiv \{\delta_N^{**}, R_N^{**}\}$ as the pair that characterizes the plan in which the country decides not to face the liquidity shock.

The SPNE of the game is given by plan \mathcal{F} if $E(U(\delta_F^{**}, R_F^{**})) \geq E(U(\delta_N^{**}, R_N^{**}))$ and the relevant constraints are satisfied, otherwise plan \mathcal{N} is chosen by the country.

In the following, we first determine $\{\delta_N^{**}, R_N^{**}\}$ and $\{\delta_F^{**}, R_F^{**}\}$. Then, we analyse the country's choice between plan \mathcal{N} and plan \mathcal{F} , and the features of the corresponding equilibrium.

We find that when the country accumulates reserves, the discount factor set by the lender is higher than in the benchmark without reserves if the country has an incentive to inject liquidity after the liquidity shock (that is, under plan \mathcal{F}) and the probability of the liquidity shock (η) is big enough. At the same time, a high enough value of η implies that the country's expected output is greater under plan \mathcal{F} than under plan \mathcal{N} and in the no-reserves case. Finally, we show that the lower the incidence of the liquidity shock (ϵ), the more likely it is that the country will have an incentive to face the shock (and that plan \mathcal{F} will arise at equilibrium).

3.3.4 Equilibrium Analysis

First of all, we analyse country's decision to accumulate reserves and the consequent choice of the discount factor δ taken by the lender under plan \mathcal{N} .

Assume (A1) holds. If

$$\lim_{R \rightarrow 0} 1 + (1 - \eta) \frac{\partial Y(\tilde{\delta}_N, R)}{\partial R} \int_{\bar{z}(\tilde{\delta}_N, R)}^{\infty} z dF(z) > 0 \quad (3.6)$$

is satisfied, then

$$\begin{aligned} \exists! \quad & \{\delta_N^{**}, R_N^{**}\} \quad \text{s.t.} \\ R_N^{**} &= \arg \max_R E(U_N(\delta_N^{**}, R)), \\ \delta_N^{**} &= (1 - \eta)G(\delta_N^{**}, R_N^{**}). \end{aligned}$$

Proof. See Appendix B.

Lemma 3.3.4 shows that there exists a unique equilibrium in pure strategies under plan \mathcal{N} , provided condition (3.6) is satisfied. We now turn to the determination of the value of reserves and discount factor under plan \mathcal{F} , that is when the country chooses to inject the needed liquidity after the shock ϵ occurs at stage $t = 1$.

Assume (A1) holds. If

$$\lim_{R \rightarrow \epsilon} 1 + \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z dF(z) > 0 \quad (3.7)$$

then

$$\exists! \{ \delta_F^{**}, R_F^{**} \} \quad \text{with} \quad R_F^{**} \in [\epsilon, \delta_F^{**} D), \quad (3.8)$$

$$\int_{\bar{z}(\delta_F^{**}, R_F^{**})}^{\infty} [zY(\delta_F^{**}, R_F^{**}) - D] dF(z) > \epsilon, \quad (3.9)$$

$$\delta_F^{**} = G(\delta_F^{**}, R_F^{**}). \quad (3.10)$$

Moreover, if, together with (7), the following condition holds

$$\left(1 + \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} dF(z) \right) \Big|_{R=R_F^{**}} > 0,$$

then

$$\exists! \{ \delta_F^{**}, R_F^{**} \} \quad \text{such that}$$

$$\int_{\bar{z}(\delta_F^{**}, R_F^{**})}^{\infty} [zY(\delta_F^{**}, R_F^{**}) - D] dF(z) = \epsilon, \quad (3.11)$$

$$R_F^{**} \in (\epsilon, \delta_F^{**} D), \quad (3.12)$$

$$\delta_F^{**} = G(\delta_F^{**}, R_F^{**}), \quad (3.13)$$

Proof. See Appendix C.

Lemma 3.3.4 shows that, under the relevant conditions outlined above, there exists a unique equilibrium in pure strategies under plan \mathcal{F} if and only if either the resource constraint

or the no-default constraint binds. Instead, there is no Nash equilibrium in pure strategies if both the resource and the no-default constraints are binding.¹²

When deciding the optimal value of R , the country trades-off the marginal return of reserves, equal to 1, with the marginal return of the illiquid project, which is equal to $-\int_{\bar{z}(\delta,R)}^{\infty} z \frac{\partial Y(\delta,R)}{\partial R} dF(z)$ under plan \mathcal{F} and $-(1-\eta) \int_{\bar{z}(\delta,R)}^{\infty} z \frac{\partial Y(\delta,R)}{\partial R} dF(z)$ under plan \mathcal{N} . Overall, then, the sovereign accumulates a positive level of reserves provided the marginal return of the illiquid project is low enough. However, the fact that under plan \mathcal{N} the country reaches the final stage only if it is exempted from the liquidity shock (with probability $1-\eta$) implies that, *ceteris paribus*, the level of accumulated liquidity is higher there.

The country defaults when the value of the productivity shock z falls below \bar{z} , that is, it is *limitedly liable in the obligation to repay the lender*. This affects the sovereign decisions at the investment game, because, as will be further explored in the section on comparative statics, the expected return of the output is boosted by a higher volatility of the shock z , at the expense of the investment in the liquid asset (R).

3.3.4.1 The Impact of Reserves on the Discount Factor set by the Lender

The sovereign choice to inject liquidity after the shock occurs at $t = 1$ affects the discount factor set by the lender. If the country decides to default at $t = 2$ (as it does under plan \mathcal{N}) then the problem solved by the lender is analogous to the one in Lemma 1, with the difference that the value of reserves was nil there. The following lemma compares the value of the discount factor in the benchmark with the one that the country obtains when choosing plan \mathcal{N} .

The optimal value of δ set by the lender in response to plan \mathcal{N} , δ_N^{**} , is lower than the one set in the benchmark without reserves, δ^* .

Without the reserves' insurance role, from the point of view of the lender the accumulation of liquid assets diverts resources from the production activity and increases the

¹²The intuition for this outcome is given in the following. If the resource constraint is binding, then the country would like to accumulate a lower level of reserves than the necessary injection, but the constraint satisfaction fixes R exactly at ϵ . In other words, the resource constraint sets a higher R than in an unconstrained optimum. The role of the no-default constraint is the opposite. Since the expected residual output after repaying the creditors (in the event of no default, that is the left-hand-side of the no-default constraint) is decreasing in R , a binding no-default constraint implies that although the country would like to accumulate a higher level of reserves, it has to reduce R in order to satisfy the no-default constraint with an equality. In other words, the impact of the level of reserves in the two constraints goes in two opposite directions: R should increase with respect to the unconstrained optimum in order to satisfy the resource constraint; R should decrease with respect to the unconstrained optimum to satisfy the no-default constraint. Overall, this implies that an equilibrium in which both constraints are contemporaneously binding cannot exist.

likelihood of sovereign default after the realization of the productivity shock. Consequently, the amount of resources available to the country to sustain sovereign investment production in plan \mathcal{N} shrinks.¹³

If the country chooses to face the shock at $t = 1$ (plan \mathcal{F}), then the impact on the problem solved by the lender is ambiguous, and eventually depends on η . With respect to the framework without reserves, on the one hand the accumulation of liquid assets reduces expected output, on the other hand it allows the country to face the liquidity shock successfully.

Below, we compare the discount factor set by the lender in plan \mathcal{F} with the discount factor in the benchmark case without reserves.

$$\text{At } \delta = \delta^*, R = R_F^{**}, \quad G(\delta^*, R_F^{**}) \geq (1 - \eta)G(\delta^*) \iff$$

$$\eta \geq \eta^{**} \equiv \frac{F(\bar{z}(\delta^*, R_F^{**})) - F(\bar{z}(\delta^*))}{[1 - F(\bar{z}(\delta^*))]} \in (0, 1).$$

Proof. The proof follows by comparing $G(\delta^*, R_F^{**})$ with $G(\delta^*)$.

Therefore, if the probability of a shock occurring is greater than η^{**} , the discount factor of debt with reserves is higher than in the case without reserves. Remember that, in the model, there is an inverse relationship between the discount factor δ and the rate of return r , since $\delta = 1/(1 + r)$; consequently, if δ increases the rate of return decreases.

[FIGURES 3.4(b) and 3.4(c) ABOUT HERE]

Lemma 3.3.4.1 is illustrated in Figure 3.4(b) and 3.4(c). More specifically, we show that when the probability attached to the occurrence of the liquidity shock (η) is high enough, then the discount factor is higher when the country accumulates reserves in excess of ϵ and injects liquidity (plan \mathcal{F}) than in the benchmark case without reserves (Figure 3.4c). The lender takes into account that reserves help the country to be solvent when the liquidity shock occurs and “prices” D accordingly. In the complementary case, the lender anticipates that the accumulated reserves are “too high” relative to the real danger of the shock occurring

¹³Remember that there is an inverse relationship between the discount factor and the rate of return asked by the lender: a higher discount factor is equivalent to a lower rate of return.

($\eta < \eta^{**}$) and sets a lower discount factor (δ) under plan \mathcal{F} with respect to the benchmark without reserves (see Figure 3.4b).

Proposition 3.3.4.1 summarizes the resulting impact of reserves on the lender's decisions under plan \mathcal{F} and plan \mathcal{N} with respect to the case without reserves.

- If the country does not face the liquidity shock, then $\delta^* > \delta_N^{**}$.
- If the country faces the liquidity shock, then the impact of the accumulation of reserves on the value of the discount factor δ depends on the probability of the liquidity shock, η :
 - If $\eta < \eta^{**}$ then $\delta^* > \delta_F^{**}$.
 - If $\eta \geq \eta^{**}$ then $\delta^* \leq \delta_F^{**}$.

Proof. See the discussion above.

In the following section, we study the impact of the accumulation of reserves on the country's expected output.

3.3.4.2 The Impact of Reserves on the Country's Expected Output

The decisions to accumulate reserves and face the liquidity shock affect the country's expected output in a way that is even clearer than in the case of the discount factor.

At equilibrium, the expected value of the production project in the benchmark without reserves is

$$(1 - \eta) \int_{\bar{z}(\delta^*)}^{\infty} zY(\delta^*)dF(z).$$

In turn, the expected value of output in plan \mathcal{N} is equal to

$$(1 - \eta) \int_{\bar{z}(\delta_N^{**}, R_N^{**})}^{\infty} zY(\delta_N^{**}, R_N^{**})dF(z).$$

Finally, in the case of plan \mathcal{F} it is given by

$$\int_{\bar{z}(\delta_F^{**}, R_F^{**})}^{\infty} zY(\delta_F^{**}, R_F^{**})dF(z).$$

First of all, we compare the outcomes of plan \mathcal{N} with the benchmark, in terms of expected output.

The expected value of output in plan \mathcal{N} is lower than in the benchmark without reserves.

The result in Lemma 3.3.4.2 is a consequence of the result in Lemma 3.3.4.1, in which it is shown that when the country does not inject the capital needed to offset the liquidity shock (plan \mathcal{N}), the lender sets a discount factor that is lower than in the benchmark, implying that $\delta^*D > \delta_N^{**}D$. Moreover, in plan \mathcal{N} the country invests in reserves, further reducing the resources allotted to the illiquid project.

In plan \mathcal{F} the country is fully insured against the liquidity shock; instead, in the benchmark the country is certain to default if it is hit by the same shock. This consideration leads us to the following result.

If $\eta > \hat{\eta}$, the expected value of output in plan \mathcal{F} is larger than in the benchmark without reserves.

Summarizing the results in Lemmata 3.3.4.2 and 3.3.4.2, as η grows to 1 the expected value of production in the case where the country injects liquidity after the shock (plan \mathcal{F}) dominates the other two cases. This leads directly to Proposition 3.3.4.2 below.

- If the country does not face the liquidity shock, then

$$(1 - \eta) \int_{\bar{z}(\delta^*)}^{\infty} zY(\delta^*)dF(z) > (1 - \eta) \int_{\bar{z}(\delta_N^{**}, R_N^{**})}^{\infty} zY(\delta_N^{**}, R_N^{**})dF(z).$$

- If the country faces the liquidity shock, then the impact of the accumulation of reserves on the value of the expected output depends on the probability of the liquidity shock, η :

- If $\eta < \hat{\eta}$ then $(1 - \eta) \int_{\bar{z}(\delta^*)}^{\infty} zY(\delta^*)dF(z) > \int_{\bar{z}(\delta_F^{**}, R_F^{**})}^{\infty} zY(\delta_F^{**}, R_F^{**})dF(z)$.
- If $\eta \geq \hat{\eta}$ then $(1 - \eta) \int_{\bar{z}(\delta^*)}^{\infty} zY(\delta^*)dF(z) \leq \int_{\bar{z}(\delta_F^{**}, R_F^{**})}^{\infty} zY(\delta_F^{**}, R_F^{**})dF(z)$.

Proof. See the discussion above.

In Lemmata 2 and 3, we determined the pairs $\{\delta_N^{**}, R_N^{**}\}$ and $\{\delta_F^{**}, R_F^{**}\}$ that solve the country's maximization problem and the lender's zero-profit condition under plan \mathcal{N} and plan \mathcal{F} , respectively. In Proposition 1 and Proposition 2 we analyse how the discount factor and the expected output are affected by the probability of a liquidity shock.

In the next section, we continue studying the country's choice at equilibrium by undertaking some comparative statics on the exogenous variables of the model.

3.3.4.3 Comparative Statics

The goal of this section is to analyse the impact of other characteristics of the economy that have so far been kept fixed. We investigate the effects on the resource allocation – in reserves or public expenditure – of an increase in the variability of the investment project, and therefore its risk, and of an increase in the dimension of capital outflows in the event of a liquidity crisis, ϵ .

The level of reserves is optimally chosen by looking at the opportunity cost of holding them in terms of the alternative investment. The expected return of the investment in output is given by, for plan \mathcal{F} :

$$-\frac{\partial Y(\delta, R)}{\partial R} \int_{\bar{z}(\delta, R)}^{\infty} z dF(z).$$

And for plan \mathcal{N} :

$$-(1 - \eta) \frac{\partial Y(\delta, R)}{\partial R} \int_{\bar{z}(\delta, R)}^{\infty} z dF(z).$$

In both cases, the opportunity cost has to equal the return on reserves (recall that, for simplicity, we fixed the interest rate on the liquid and safe asset at zero, and therefore the return on reserves is equal to 1).

To assess the impact of the distribution of the shock, and in particular of its variance, on the opportunity cost of holding reserves, we begin by using the assumption that z is distributed as a continuous uniform with support $[1 - c, 1 + c]$; this implies that:

$$\int_{\bar{z}(\delta, R)}^{\infty} z dF(z) = [(1 + c)^2 - (\bar{z}(\delta, R))^2]/4c,$$

and

$$\frac{\partial}{\partial c} \left(\frac{(1 + c)^2 - (\bar{z}(\delta, R))^2}{4c} \right) = \frac{(\bar{z}(\delta, R))^2 - (1 + c)(1 - c)}{16c^2}.$$

Therefore, in the case of a continuous uniform distribution, as c increases above unity the incentive that a country has to invest an additional unit of resources in reserves decreases in both plan \mathcal{F} and plan \mathcal{N} .¹⁴ An increase of c also triggers an increase in the variance of the uniform distribution, which is equal to $c^2/3$. Consequently, we can conclude that in the case of the uniform distribution, an increase in the variance can boost the incentive to de-cumulate reserves. This result is even more robust in the case of exponential probability density functions, for which a jump in the variance of the distribution increases the value of $\int_{\bar{z}(\delta, R)}^{\infty} z dF(z)$ and makes hoarding reserves relatively less profitable.¹⁵

An increase in the variance of the productivity shock z increases the opportunity cost of reserves.

In both the distributions considered above, an increase in the variance does not change the expected value of the productivity shock; however, it increases the expected value of z given the possibility of the country defaulting. In other words, Corollary 1 is linked to a *limited liability effect*: the country enjoys the upper tail of the distribution of the realizations of the productivity shock while it disregards the lower tail. Therefore, an increase in the riskiness of the illiquid project raises the opportunity cost of holding reserves and inflates sovereign incentive to invest in g instead of R .¹⁶ We further investigate the impact of the limited liability effect in the section where the optimal portfolio allocation of a risk-averse country is studied in the extension presented below.

We now turn our attention to ϵ , the amount of liquid resources the country needs to inject in the event of a liquidity shock at $t = 1$. This parameter could be interpreted as a proxy for the capital account openness of the country and it therefore represents capital flight in a sudden stop. In line with Obstfeld et al. (2009), ϵ could more generally represent the fraction of the M2 aggregate (a proxy for the size of the banking system) the country needs to inject should a double drain occur.

In our model, ϵ does not affect the opportunity cost directly; however, it adversely affects country's expected utility of plan \mathcal{F} . Moreover, ϵ has an impact on the *no-default constraint*

¹⁴Notice that $\partial Y(\delta, R)/\partial R$ is independent from the shape of the distribution of z .

¹⁵To see this, note that, in the case of exponential probability density functions, were the variance to rise, the value of $\int_{-\infty}^{\infty} z dF(z)$ would stay the same. Instead, as long as $\bar{z}(\delta, R) < \infty$, an increase in the variance implies that the truncated expected value of z weighted by its density $f(z)$ rises.

¹⁶Another implication of this is that if the country could choose among different projects, it would prefer a riskier one (other things being equal).

the country has to satisfy in order to credibly adhere to plan \mathcal{F} :

$$\int_{\bar{z}(\delta, R)}^{\infty} [zY(\delta, R) - D]dF(z) \geq \epsilon.$$

Intuitively, as the liquidity needs increase, the constraint becomes more binding, and therefore it is more likely that the equilibrium choice of reserves under plan \mathcal{F} is constrained. All these reasons lead us to the conclusion in the following corollary.

An increase in the needed liquidity injection (ϵ) makes the emergence of plan \mathcal{F} less likely at equilibrium.

In the next Section, to gain further insights on the value of the model's key variables (reserves, debt discount factor and public expenditures) at equilibrium, and see under which conditions a country decides to default should the liquidity shock occur, we undertake a numerical analysis.

3.4 Simulations

Here we present a numerical example that illustrates the results above.¹⁷ More specifically, it is assumed that the production function is a Cobb-Douglas of the following type:

$$Y(z, g) = zY(g) = zY(\delta, R) = z(\delta D - R)^\alpha.$$

Also, we maintain the assumption for which the productivity shock is distributed as a uniform random variable:

$$z \sim U(1, c^2/3), z \in [1 - c; 1 + c].$$

There are five parameters to be set:

α	<i>income share of capital</i>	0.3
D	<i>amount of debt as % of GDP</i>	40%
ϵ	<i>capital infusion as % of GDP</i>	10%
c	<i>support of the productivity process</i>	2
η	<i>probability of the liquidity shock</i>	15%

¹⁷For the sake of the exposition, we focus on cases in which an equilibrium in pure strategies exists and is well-defined.

The income share of capital is taken from the literature. The face value of debt D is chosen in order to obtain on average an amount of gross debt over GDP of around 40%, in line with the data for 2009 reported by the IMF in *World Economic Outlook* for emerging and developing countries. The needed capital infusion in the event of a liquidity shock is fixed at 10% of the expected GDP, as proposed in Rodrik (2006) and Obstfeld et al. (2009). The variance of the productivity process and the probability of the liquidity shock are arbitrary. We then make them vary to check how those parameters affect the choice variables, δ and R , and to check the robustness of our results. With the proposed parameters we obtain estimates of the value of reserves over expected output of around 15%, slightly below the 20% estimated by Jeanne (2007). Moreover, in our simulations the discount factor generates a rate of return r that oscillates around 1: this is the same rate of return that would be generated by a 10-year bond capitalized at an annual interest rate of about 7%, which is a value of the annual rate in line with the empirical evidence in Borri and Verdelhan (2009).¹⁸

[FIGURE 5 ABOUT HERE]

Figure 5 reports the main results of the simulation when the probability of the liquidity shock moves from zero to 50%. It shows how the cost of debt, the expected output, the reserve level and the country's expected welfare vary when the probability of the liquidity shock increases.

The upper-left panel shows that the cost of debt increases with the increase in the risk of the liquidity shock in two cases, namely in the framework without reserves and when the country chooses not to face the liquidity shock when it occurs (plan \mathcal{N}).¹⁹ Instead, if the country decides to face the liquidity shock (plan \mathcal{F}) the cost of debt is constant for any value of η , because the country is perfectly insured independently of the probability of the adverse event occurring.

The upper-left panel of Figure 5 graphically presents the result of Proposition 1. More specifically, it shows the threshold η^{**} (which here corresponds to a probability of the liquidity shock around 4%): if the probability of the liquidity shock is higher than η^{**} , then the discount factor in the framework without reserves (δ^*) rises above the discount factor with

¹⁸Recall that the discount factor δ is equal to $1/(1+r)$. In our model, δ is inversely correlated to the cost of debt: an increase in the value of δ set by the lender reflects an increase in the probability that the lender expects the country to be solvent and stands for a decrease in the cost of debt.

¹⁹Notice that in the framework in which the country can accumulate a positive amount of reserves, the case "no reserves" cannot be a credible choice since, in the absence of commitment, the country will always prefer a positive amount of reserves at any discount factor offered by the lender as represented by plan \mathcal{F} and plan \mathcal{N} . We simulate the equilibrium variables in the framework with no reserves in order to have means of comparison.

reserves when the lender anticipates that the country will face the liquidity shock (δ_F^{**}). From this simulation, we also infer that when the country decides not to face the liquidity shock the cost of debt is always higher (δ lower) because this choice makes the lender worse off.

The upper-right panel of Figure 5 displays the path of sovereign expected output. It presents the results of Proposition 2. The expected output of the benchmark case without reserves is always higher than the expected output in plan \mathcal{N} . Moreover, the expected output in the benchmark is above the one of plan \mathcal{F} if η is below a certain threshold $\hat{\eta}$ defined in Lemma 7 (which here corresponds to a probability of the liquidity shock of around 30%).

The amount of accumulated reserves, reported in the lower-left panel, decreases when η increases with no capital infusion, because the resources available to the country (equal to $\delta_N^{**}D$) decrease with η . If, instead, the country decides to face the liquidity shock, should it occur, the level of reserves does not depend on the probability of the shock and the two constraints (resource constraint and no-default constraint) are slack at any level of η .

Finally, the lower-right panel plots the country's welfare in the two plans which the country can choose between: when the country accumulates reserves and faces the liquidity shock (plan \mathcal{F}), the welfare is always higher than if it decides not to inject capital (plan \mathcal{N}). Moreover, the resource constraint (lower-left panel) and the no-default constraint (not reported in the figure) are always satisfied under plan \mathcal{F} , so the country chooses to face the liquidity shock if it occurs.

From this first simulation of the model we can conclude that, under the chosen parameters, the country accumulates reserves and injects capital if the liquidity shock occurs at any value of η . The lender anticipates this choice when setting the discount factor of debt and sets δ_F^{**} to satisfy its zero-profit condition.

[FIGURE 3.6 ABOUT HERE]

The simulations in Figure 3.6 show how the variables of interest and the final equilibrium of the model vary when the variance of the underlying productivity process (equal to $c^2/3$) moves from 1 to $5\frac{2}{5}$. As reported in Corollary 1, when the variability of the productivity shock increases, the opportunity cost of holding reserves increases, because of a limited liability effect. Consequently, the country chooses a lower level of reserves and the discount factor on debt decided by the lender in response is lower, because more resources are invested in the productive activity.

Therefore, as the variance rises, the trade-off between accumulating enough reserves to face the shock and putting more resources in the productive project leads to a smaller pile of liquid and safe assets. There is a threshold of the variance for which the country decides

to gather $R = \epsilon$ and at which the resource constraint becomes binding under plan \mathcal{F} .²⁰ Moreover, as the variance increases, the level of reserves chosen under plan \mathcal{N} decreases to zero.

The overall impact on expected output and welfare is that they both increase as the sovereign decides to de-cumulate reserves. This is due to the role of residual claimant of the country if it decides not to default. Finally, when the country decides to face the liquidity shock (plan \mathcal{F}), the expected output is higher than when the country chooses not to inject the necessary liquidity (plan \mathcal{N}), but for the case of a very high level of variability of the shock. This is due to the fact that under plan \mathcal{F} the level of R cannot get smaller than the needed liquidity infusion.

The equilibrium chosen is plan \mathcal{N} for very low levels of the variance, due to the high cost of facing the shock compared with the opportunity cost of investing in output (weighted by the probability of the liquidity shock not occurring).

For higher levels of the variability, however, the country prefers to accumulate more reserves and face the liquidity shock since the expected output increases. The welfare under plan \mathcal{F} increases more than under plan \mathcal{N} because the underlying cost is fixed in the first case ($\eta\epsilon$) while it increases with the variance in the second case (it is the expected output weighted by the probability of the liquidity shock).

[FIGURE 3.7 ABOUT HERE]

The last simulation performed shows how the country's choice of equilibrium plan is affected by the importance of the liquidity shock (ϵ). In Figure 3.7, we let the value of ϵ vary to illustrate the result in Corollary 2. To obtain this figure, we change the values of parameters D and c as follows:

α	<i>income share of capital</i>	0.3
D	<i>amount of debt as % of GDP</i>	60%
c	<i>support of the productivity process</i>	$\sqrt{3}$
η	<i>probability of a liquidity shock</i>	15%

We increase the value of D , the available liquidity, and decrease the value of c , the variability of the productivity process. In this way, besides the results of Corollary 2, we are able to show the influence of the no-default constraint, reported in the lower-left panel. Under plan \mathcal{F} , when the no-default constraint becomes binding, the country has to choose

²⁰In this simulation, the no-default constraint is always verified for plan \mathcal{F} and is not reported in the figure.

a level of reserves lower than the unconstrained optimum; this in turn induces the lender to offer a better discount for the debt (higher δ). The two effects contribute to an increase in the expected output and welfare. As ϵ further increases the level of reserves approaches the resource constraint and therefore, as proved in Appendix C, an equilibrium no longer exists under plan \mathcal{F} . The only equilibrium of the game features the choice of plan \mathcal{N} .

3.5 Extensions

In what follows we study two possible extensions of the present analysis. In the first, we introduce risk-aversion in order to show that the optimal level of reserves can be significantly higher if we change the shape of the country's utility function. In the second extension, a third player is introduced: we assume that a supranational organization can provide the liquidity needed in the event of a liquidity crisis. In this way we relax the friction on the financial markets' commitment not to intervene in the event of a liquidity shock.

3.5.1 Risk-averse utility function

In the main model, we assume that the country is risk-neutral, therefore it takes into account only the expected value of alternative investments either in safe and liquid assets (reserves) or in a risky project (output). In line with Arellano (2008), in the following we assume that the country is risk-averse, that is, it also looks at the second moment of alternative investment opportunities. We expect to find that, all else being equal, the country prefers to invest more in the safe asset, so that the relative level of reserves chosen should be higher than in the previous analysis with linear utility. Notice that nothing changes for the lender, who is assumed to be risk-neutral and price the debt taking into account the same trade-off induced by the reserves as in the main model: reserves divert resources from the productive investment, but on the other hand they can be employed to face the liquidity crisis, should it occur, and therefore avoid default at time two.

We make use of a standard CRRA utility function with a parameter of risk aversion equal to 1. Following Section 3.2, we re-write the expected output and the related constraints for the two plans (\mathcal{F} and \mathcal{N}) by making use of a logarithmic utility function.

Plan \mathcal{F}

At $t = 0b$, the country maximizes the following expression:

$$\max_R E(U_F(\delta, R)) = (1 - \eta) \left[\int_{\bar{z}(\delta, R)}^{\infty} \log[zY(\delta, R) - D + R]dF(z) + \int_{-\infty}^{\bar{z}(\delta, R)} \log(R)dF(z) \right] + \eta \left[\int_{\bar{z}(\delta, R)}^{\infty} \log[zY(\delta, R) - D + R - \epsilon]dF(z) + \int_{-\infty}^{\bar{z}(\delta, R)} \log(R - \epsilon)dF(z) \right],$$

subject to the no-default and resource constraints:

$$\int_{-\infty}^{\bar{z}(\delta, R)} \log(R - \epsilon)dF(z) + \int_{\bar{z}(\delta, R)}^{\infty} \log[zY(\delta, R) - D + R - \epsilon]dF(z) \geq \log(R),$$

$$R \geq \epsilon.$$

The resource constraint does not change in the new setup, while the *no-default constraint* incorporates the disutility generated by the uncertainty over the output realization.

Plan \mathcal{N}

In this case, at $t = 0b$ the country maximizes the following expression:

$$\max_R E(U_N(\delta, R)) = (1 - \eta) \left[\int_{\bar{z}(\delta, R)}^{\infty} \log[zY(\delta, R) - D + R]dF(z) + \int_{-\infty}^{\bar{z}(\delta, R)} \log(R)dF(z) \right] + \eta \log(R).$$

We conduct a numerical simulation to find the equilibrium pair $(\tilde{\delta}, \tilde{R})$. In line with the analysis conducted for the probability of liquidity shock (η) that varies, we set the four parameters as reported in the table below:

α	<i>income share of capital</i>	0.3
D	<i>amount of debt as % of GDP</i>	40%
ϵ	<i>capital infusion as % of GDP</i>	10%
c	<i>support of the productivity process</i>	2

[FIGURE 3.8 ABOUT HERE]

The simulations are reported in Figure 3.8. In line with the previous analysis, plan \mathcal{F} is the one implemented at any value of η between 0 and 50%. Notice that in this case, even under plan \mathcal{F} , the probability of a liquidity crisis affects all the equilibrium variables. In particular, it reduces the expected welfare, induces the country to invest more in reserves, which

in turn makes the investor fix a lower debt discount factor (or a higher rate of return). Plan \mathcal{N} , with this parametrization, coincides with the case of no-reserves, since the country would like to accumulate a negative amount of reserves, and therefore the constrained equilibrium is at $R = 0$. As a consequence, in this case the country's expected welfare collapses, because of the shape of the utility function, and this plan is never implemented at equilibrium.

In the present simulations the threshold η^{**} - defined in Proposition 1 - is higher than with linear utility and equal to about 10%. Instead, the threshold $\hat{\eta}$ - defined in Proposition 2 - is not reported in the second panel. For η between 0 and 50%, the output with no reserves is always higher than under plan \mathcal{F} . This happens for two reasons: the first is that a larger fraction of the available resources is put in reserves instead of in the risky investment, the second is that in the case of a risk-averse country the expected output decreases with η under plan \mathcal{F} .

[FIGURE 3.9 ABOUT HERE]

In Figure 3.9, we compare the amount of resources invested in reserves in terms of expected output in the two cases, namely with linear and logarithmic utility. With logarithmic utility, the amount of resources saved in reserves is about 70% of the expected output. It is a very high level, exceeding by far that of reserves in the case of linear utility.

With our analysis, we do not pretend to predict the optimal level of reserves; however, *the results that emerge from this extension allow us to conclude that, by making the hypothesis of linear utility, researchers can obtain only a lower bound for the optimal level of reserves. This lower bound needs to be corrected upwards in the case of perceived risk aversion by the economic authorities.*

3.5.2 IMF intervention

In this section, we introduce a supranational organization, e.g. the IMF, which can intervene in the interim period and help the country by injecting ϵ should the liquidity shock occur.²¹ We assume that the application for IMF intervention is decided by the country before the contracting stage; therefore, at time zero the lender and the country already know whether the IMF will intervene in a liquidity shock. Moreover, the intervention takes place by means of an instrument that covers the entire amount of liquidity needed.

At $t = 4$, if not hit by the liquidity shock, the country defaults if $z \leq \bar{z}(\delta, R) \equiv D/Y(\delta, R)$.

²¹This way, we propose a simplified setting for the analysis of Flexible Credit Lines (FCL), a new IMF instrument available without ex-post conditionality for member countries that satisfy pre-determined criteria (for details see: <http://www.imf.org/external/np/pdr/fac/2009/032409.htm>)

Instead, if hit by the shock ϵ , at $t = 4$ the country has to repay the IMF at the market price (recall the risk-free interest rate is nil); however, as in the case of the international lender, the IMF cannot act as a residual claimant and is not repaid in the event of default. More specifically, at the final stage the country decides to default strategically if the following is true:

$$zY(\delta, R) - D + R - \epsilon \leq R \iff z \leq \hat{z}(\delta, R) \equiv \frac{D + \epsilon}{Y(\delta, R)} > \bar{z}(\delta, R) \equiv \frac{D}{Y(\delta, R)}.$$

At $t = 2$, if it is hit by a liquidity shock in $t = 1$, the country's no-default constraint is in what follows:

$$R + \int_{\hat{z}(\delta, R)}^{\infty} [zY(\delta, R) - D - \epsilon] dF(z) \geq R.$$

The condition above is satisfied by construction, implying that the IMF intervention clears the incentive to default on the liquidity shock at $t = 2$. At $t = 0$ the country chooses how to allocate resources between public expenditure and reserves by maximizing the following expected utility:

$$\max_{R \in [0, \delta D]} E(U_{IMF}(\delta, R)) = R + (1 - \eta) \int_{\bar{z}(p, R)}^{\infty} [zY(\delta, R) - D] dF(z) + \eta \int_{\hat{z}(p, R)}^{\infty} [zY(\delta, R) - D - \epsilon] dF(z).$$

Instead, the lender fixes δ at the level that solves the zero-profit condition:

$$\begin{aligned} \delta D &= D\eta \int_{\hat{z}(\delta, R)}^{\infty} dF(z) + (1 - \eta)D \int_{\bar{z}(\delta, R)}^{\infty} dF(z) \iff \\ \delta &= \eta(1 - F(\hat{z}(\delta, R))) + (1 - \eta)(1 - F(\bar{z}(\delta, R))). \end{aligned}$$

The intervention of the IMF introduces a trade-off into the model. *On the one hand, the problem of insurance against liquidity shock disappears. On the other hand, the probability of the country's default on debt at $t = 4$ is higher than in the case of no liquidity shock, because the outstanding debt is higher in that case. Consequently, the associated threshold below which the country defaults increases in the event of a liquidity shock (with probability η).*

The trade-off above implies that the intervention of the IMF impacts on the lender's break-even problem solution in an interesting fashion, especially when compared to the solution of the lender problem in the cases of plan \mathcal{F} and plan \mathcal{N} . More specifically, since, with respect to plan \mathcal{F} (see equation 3.4), the country will default more often on its debt under IMF intervention, the lender will set a lower discount factor δ when the IMF intervenes. However, if we compare the optimal value of δ with the IMF against the one under plan \mathcal{N} , the contrary is true, so that, everything else being equal, the value of δ under IMF intervention should be higher than in the no injection case (\mathcal{N}).

[FIGURE 3.10 ABOUT HERE]

The simulations are reported in Figure 3.10. By comparing the levels of expected welfare, it is clear that IMF intervention is preferred to the other plans. Under the IMF scenario, the rate of return (respectively, discount factor) set by the lender is always higher (lower) than the one set under plan \mathcal{F} due to the trade-off discussed above. However, the rate of return under IMF intervention is lower than under plan \mathcal{N} , since when the IMF intervenes the lender expects the country not to default should a liquidity crisis occur. Reserves are also slightly lower with the IMF than under plan \mathcal{F} , although this difference is not very large.

Overall, this extension predicts that the intervention by an international organization to solve the liquidity crisis does not reduce significantly the will to accumulate reserves. In fact, even though reserves are no longer needed to face the liquidity shock, they are still a form of saving in which the country can invest and retain in the event of default. Consequently, the optimal portfolio allocation still prescribes a positive amount of safe and liquid assets.

Finally, the combined effect of the accumulated level of reserves and the higher probability of default in a liquidity crisis implies that the lender fixes a cost of debt with the IMF that is larger than under plan \mathcal{F} . This shows that, per-se, *when liquidity crises can occur IMF intervention does not necessarily ease the pressure on the sovereign debt rate of return. To accomplish this result, the financial instrument proposed by a supranational organization, or simply bilateral agreements, should be designed to limit country's incentive to default on debt.*

Summarizing, on the one hand the way the IMF instrument is designed in this analysis decreases the probability of default, since it eliminates the need for the country to default in case of liquidity crises, on the other hand it exacerbates the solvency problem by increasing the probability of default at the last stage.

3.6 Concluding Remarks

This paper contributes to the discussion about the management of liquidity and the optimal level of sovereign reserves for countries that are subject to liquidity crises. We are able to model simultaneously endogenous debt prices, optimal reserve accumulation and strategic default decisions. In our setup, the country optimally decides the level of international reserves that maximizes its expected welfare taking into account the probability of a liquidity crisis and the possibility of defaulting (with the cost in terms of output of this action). Competitive international lenders anticipate the country choice and set a rate of

return over the lent resources that satisfies the zero-profit condition. Therefore, we deliver a model that abstracts from the role of reserves in managing the exchange rate and that instead draws on the opportunity cost of holding reserves in terms of reduced expected output.

Our results rationalize the self-insurance motive for accumulating reserves and facing liquidity shocks, as well as the limited liability effect due to the impossibility for the country to commit not to default on its debt. The combination of these two forces drives our results: we show that both an equilibrium featuring the country self-insuring against the shock and one with the country not facing the shock can emerge depending on the productivity process, the amount of resources available in the country and the probability and dimension of the liquidity crisis. Finally, we are able to reproduce the empirical evidence of Levy Yeyati (2008) and Ruiz-Arranz and Zavadjil (2008): the level of international reserves reduces the costs of external debt for sovereign countries that face a high probability of being hit by a liquidity crisis.

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APPENDICES

A. Proof of Lemma 1

The equilibrium value of δ , denoted δ^* , exists and is unique if and only if $G(\delta)$ satisfies the following five conditions.

Condition (i): $\exists \lim_{\delta \rightarrow 0} G(\cdot) < \infty$.

Taking limits, it can be shown that this condition holds:

$$\delta \rightarrow 0 \Rightarrow \bar{z}(\delta) \rightarrow \infty \Rightarrow F(\bar{z}(\delta)) \rightarrow 1 \Rightarrow G(\delta) \rightarrow 0.$$

Condition (ii): $\lim_{\delta \rightarrow 1} G(\cdot) < 1$.

Again, taking limits, one has that also condition (ii) holds:

$$\delta \rightarrow 1 \Rightarrow \bar{z}(\delta) \rightarrow \frac{D}{Y(1)} \Rightarrow F(\bar{z}(1)) \rightarrow k \in (0, 1) \Rightarrow G(\delta) \rightarrow 1 - k < 1.$$

Condition (iii): $dG(\delta)/d\delta > 0$.

Computing the first derivative of $G(\delta)$, one finds that condition (iii) is satisfied:

$$\begin{aligned} \frac{dG(\delta)}{d\delta} &= - \frac{\partial F(\bar{z}(\delta))}{\partial z} \frac{d\bar{z}(\delta)}{d\delta} = \\ &= f(\bar{z}(\delta)) \frac{D^2 Y'(\delta)}{(Y(\delta))^2} > 0. \end{aligned}$$

Condition (iv): $d^2G(\delta)/d\delta^2 < 0$.

Condition (iv) is necessary (but not sufficient) for uniqueness and imposes a restriction on the curvature of $G(\delta)$. In particular, it is satisfied if $G(\delta)$ is concave:

$$\frac{d^2G(\delta)}{d\delta^2} = \frac{df(\bar{z}(\delta))}{d\delta} \frac{D^2 Y'(\delta)}{(Y(\delta))^2} + D^3 f(\bar{z}(\delta)) \left\{ \frac{Y''(\delta)}{(Y(\delta))^2} - \frac{2(Y'(\delta))^2}{(Y(\delta))^4} \right\} < 0. \quad (\text{A.14})$$

The second term of (A.14) is strictly negative, thus the sign of the inequality depends on the sign of the first term: we then use the assumption of uniform distribution of z , which implies that $df(\bar{z}(\delta))/d\delta = 0$, to get that (A.14) holds true.

Condition (v): $\lim_{\delta \rightarrow 0} dG(\delta)/d\delta > 1$.

This condition is necessary to make sure that an interior solution exists that is different from the trivial one, i.e. $\delta = 0$, to the fixed point problem that we are analysing. Taking limits, one has that in the case of the uniform distribution the following is true:

$$\lim_{\delta \rightarrow 0} \frac{dG(\delta)}{d\delta} = \lim_{\delta \rightarrow 0} \frac{D^2 Y'(\delta)}{(Y(\delta))^2} = \infty. \quad (\text{A.15})$$

This result uses the assumption of uniform distribution of z , for which $f(\bar{z}(\delta))$ does not depend on δ ,²² and that

$$Y'(\delta) \rightarrow_{\delta \rightarrow 0} \infty,$$

$$\frac{1}{(Y(\delta))^2} \rightarrow_{\delta \rightarrow 0} \infty.$$

■

B. Proof of Lemma 2

The problem solved by the lender under plan \mathcal{N} and the one we solved in Lemma 3.3.1 are isomorphic. Therefore, following the steps of the proof in Appendix A with the use of the Envelope Theorem allows us to pin down the unique reaction function $\tilde{\delta}_N = \delta_N(R)$ that solves the lender's zero-profit condition.²³

Under plan \mathcal{N} , the sovereign faces the following problem:

$$\max_{R \in [0, \tilde{\delta}_N D)} E(U_N(\tilde{\delta}_N, R)) = R + (1 - \eta) \int_{\bar{z}(\tilde{\delta}_N, R)}^{\infty} [zY(\tilde{\delta}_N, R) - D] dF(z). \quad (\text{B.16})$$

²²Here, we are implicitly restricting our attention to the cases where $\bar{z}(\delta) \in [1 - c, 1 + c]$, since otherwise the lender would never serve the country and one would have that the solution to the problem is given by $\delta = 0$.

²³In particular, notice that when evaluating the limit of $G(.,.)$ as δ approaches zero also R tends to zero, because a nil discount factor implies that the country has no resources available at the investment stage.

We now compute the first order condition of the expression above (and employ the Envelope Theorem when disregarding the effect of a change of R on $\tilde{\delta}_N$). After setting the resulting derivative with respect to R to zero, we have:

$$H_N(R) = 1 + (1 - \eta) \frac{\partial Y(\tilde{\delta}_N, R)}{\partial R} \int_{\bar{z}(\tilde{\delta}_N, R)}^{\infty} z dF(z) = 0. \quad (\text{B.17})$$

In order to determine the existence of a solution to our problem we need to show two preliminary results. First, we need to prove that the country does not have an incentive to employ all resources in reserves (i.e. $R_N^{**} \neq \tilde{\delta}_N D$). To see that this is the case, we compute the value of $H_N(R)$ as R approaches $\tilde{\delta}_N D$:

$$\begin{aligned} \lim_{R \rightarrow \tilde{\delta}_N D} H_N(R) &= 1 + (1 - \eta) \frac{\partial Y(\tilde{\delta}_N, R)}{\partial R} \Big|_{R \rightarrow \tilde{\delta}_N D} \int_{\bar{z}(\tilde{\delta}_N, R)|_{R \rightarrow \tilde{\delta}_N D}}^{\infty} z dF(z) = \\ &= 1 + (-\infty)(+\infty) = -\infty. \end{aligned}$$

Indeed, from the Inada Conditions we have that:

$$\frac{\partial Y(\tilde{\delta}_N, R)}{\partial R} \Big|_{R \rightarrow \tilde{\delta}_N D} = -Y'(\tilde{\delta}_N, R)|_{R \rightarrow \tilde{\delta}_N D} \rightarrow -\infty.$$

Moreover,

$$\int_{\bar{z}(\tilde{\delta}_N, R)|_{R \rightarrow \tilde{\delta}_N D}}^{\infty} z dF(z) = \frac{E(z|z \geq \bar{z}(\tilde{\delta}_N, R))}{1 - F(\bar{z}(\tilde{\delta}_N, R))} \rightarrow \frac{\infty}{0} = \infty.$$

This is because:

$$\lim_{R \rightarrow \tilde{\delta}_N D} \bar{z}(\tilde{\delta}_N, R) = \lim_{R \rightarrow \tilde{\delta}_N D} \frac{D}{Y(\tilde{\delta}_N, R)} = \infty \Rightarrow \lim_{R \rightarrow \tilde{\delta}_N D} E(z|z \geq \bar{z}(\tilde{\delta}_N, R)) = \infty$$

and

$$\lim_{R \rightarrow \tilde{\delta}_N D} 1 - F(\bar{z}(\tilde{\delta}_N, R)) = 0.$$

Clearly, investing all resources into liquidity is not an optimum for the sovereign.

Now, we need to check that the country does not want to invest the entire allowance in public expenditure (i.e. $R_N^{**} \neq 0$). In particular, the value of $H_N(R)$ as R approaches 0 is equal to:

$$\lim_{R \rightarrow 0} H_N(R) = 1 + (1 - \eta) \frac{\partial Y(\tilde{\delta}_N, R)}{\partial R} \Big|_{R=0} \int_{\bar{z}(\tilde{\delta}_N, R)|_{R=0}}^{\infty} z dF(z) = H_N(R)|_{R=0}.$$

So, if $H_N(R)|_{R=0} > 0$ there exists a solution to the problem under consideration. For $H_N(R)|_{R=0}$ to be positive it must be that, when all resources are invested in productive inputs, the production function is close to exhausting its returns to scale and its derivative with respect to R is small enough to bring the second term of $H_N(R)|_{R=0}$ to a value smaller than 1 (recall that $\partial Y(.,.)/\partial R = -Y'(.,.) < 0$ and $\partial^2 Y/\partial R^2 = Y'' < 0$).²⁴

Finally, the uniqueness of the solution to the problem in (B.16) is granted if the maximand is concave. More specifically, the derivative of $H_N(R)$ must be negative.

$$\frac{dH_N(R)}{dR} = (1 - \eta) \frac{\partial^2 Y(\tilde{\delta}_N, R)}{\partial R^2} \int_{\bar{z}(\tilde{\delta}_N, R)}^{\infty} z dF(z) + (1 - \eta) \frac{\partial Y(\tilde{\delta}_N, R)}{\partial R} \left[- \frac{\partial \bar{z}(\tilde{\delta}_N, R)}{\partial R} \bar{z}(\tilde{\delta}_N, R) \right] < \text{(B.18)}$$

Where the inequality holds by (A1). Indeed, (B.18) can be rewritten as:²⁵

$$\frac{\partial Y}{\partial R} \left(- \bar{z} \frac{\partial \bar{z}}{\partial R} \right) < - \frac{\partial^2 Y}{\partial R^2} \int_{\bar{z}}^{\infty} z dF(z) \iff \text{(B.19)}$$

$$\frac{\frac{\partial Y}{\partial R}}{\frac{\partial^2 Y}{\partial R^2}} < \frac{- \int_{\bar{z}}^{\infty} z dF(z)}{- \bar{z} \frac{\partial \bar{z}}{\partial R}} \text{(B.20)}$$

and

$$- \bar{z} \frac{\partial \bar{z}}{\partial R} = \frac{\partial}{\partial R} \left(\int_{\bar{z}}^{\infty} z dF(z) \right) < 0. \text{(B.21)}$$

²⁴For example, take the production function $Y(\tilde{\delta}D - R) = (\delta D - R)^\alpha$, with $\alpha < 1$; its first order derivative with respect to R is equal to:

$$\frac{\partial}{\partial R} \left\{ (\delta D - R)^\alpha \right\} \Big|_{R=0} = -\alpha \frac{1}{(\delta D)^{1-\alpha}} = -\alpha \frac{1}{g^{1-\alpha}}.$$

For given $\delta \in (0, 1)$ and g , such expression goes to zero if α is small enough.

²⁵Notice that, for ease of exposition, in the following we are dropping functional forms.

Where the sign of (B.21) uses the fact that $\partial\bar{z}/\partial R > 0$. Therefore, (B.20) can be formulated as:

$$\frac{\frac{\partial Y}{\partial R}}{\frac{\partial^2 Y}{\partial R^2}} < \frac{-\int_{\bar{z}}^{\infty} z dF(z)}{\frac{\partial}{\partial R} \left(\int_{\bar{z}}^{\infty} z dF(z) \right)}. \quad (\text{B.22})$$

Now, by using the definition of elasticity, one can re-write (B.22) as

$$\frac{1}{\epsilon_{Y',R}} < \frac{1}{\epsilon_{\bar{z},R}} \iff |\epsilon_{\bar{z},R}| < |\epsilon_{Y',R}|. \quad (\text{B.23})$$

Which is equivalent to (A1).

To close the proof, the value of R_N^{**} that solves (B.17) needs to be plugged into $\tilde{\delta}_N$ to compute the optimal discount factor set by creditors under plan \mathcal{N} . ■

C. Proof of Lemma 3

Also in this case, the problem solved by the lender under plan \mathcal{F} and the one we solved in Lemma 3.3.1 are isomorphic, so we follow the steps of the proof in Appendix A with the use of the Envelope Theorem to pin down the reaction function $\tilde{\delta}_F = \delta_F(R)$ that solves the lender's participation constraint.

Under plan \mathcal{F} , the sovereign faces the following maximization problem:

$$\max_R E(U_F(\tilde{\delta}_F, R)) = R + \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} [zY(\tilde{\delta}_F, R) - D] dF(z) - \eta\epsilon. \quad (\text{C.24})$$

Under the resource and the no-default constraints:

$$\begin{aligned} \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} [zY(\tilde{\delta}_F, R) - D] dF(z) &\geq \epsilon, \\ R &\geq \epsilon. \end{aligned}$$

We solve the problem by maximizing the following expression, where we denote the Lagrange multipliers by λ and μ :²⁶

$$\mathcal{L}(R, \lambda, \mu) = R + \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} [zY(\tilde{\delta}_F, R) - D] dF(z) - \eta\epsilon + \lambda \left\{ \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} [zY(\tilde{\delta}_F, R) - D] dF(z) - \epsilon \right\} - \mu(R - \epsilon).$$

²⁶Note that, as shown in the proof of Lemma 3.3.4, the maximand is concave under (A1).

The relevant constraints are given below:

$$\frac{\partial \mathcal{L}(R, \lambda, \mu)}{\partial R} = 1 + (1 + \lambda) \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} dF(z) - \mu = 0 \quad (\text{C.25})$$

$$\frac{\partial \mathcal{L}(R, \lambda, \mu)}{\partial \lambda} = \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} [zY(\tilde{\delta}_F, R) - D] dF(z) - \epsilon = 0 \quad (\text{C.26})$$

$$\frac{\partial \mathcal{L}(R, \lambda, \mu)}{\partial \mu} = R - \epsilon = 0 \quad (\text{C.27})$$

$$\lambda \left\{ \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} [zY(\tilde{\delta}_F, R) - D] dF(z) - \epsilon \right\} = 0 \quad (\text{C.28})$$

$$\mu (R - \epsilon) = 0 \quad (\text{C.29})$$

$$\mu \geq 0, \lambda \geq 0 \quad (\text{C.30})$$

There are four cases to be discussed.

Case I: $\lambda = \mu = 0$.

In this case, the two constraints are both slack. Then, the optimal level of reserves is determined by the following condition:

$$1 + \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} dF(z) = 0. \quad (\text{C.31})$$

We know from Lemma 3.3.4 that the left-hand-side of the equality above tends to $(-\infty)$ as R tends to $\tilde{\delta}_F D$. Thus, one has that if the following holds,

$$\lim_{R \rightarrow \epsilon} H_F(R)|_{R=\epsilon} \equiv 1 + \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} \Big|_{R=\epsilon} \int_{\bar{z}(\tilde{\delta}_F, R)|_{R=\epsilon}}^{\infty} z dF(z) > 0, \quad (\text{C.32})$$

the solution to the problem is given by $R_F^{**} \in (\epsilon, \tilde{\delta}_F D)$.

Case II: $\lambda > 0, \mu = 0$.

In this case, the no-default constraint is binding, while the resource constraint is slack (meaning that $R > \epsilon$ at this candidate equilibrium). The optimal level of reserves and the Lagrange multiplier λ are determined by the following conditions:

$$1 + (1 + \lambda) \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} dF(z) = 0,$$

$$\int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} [zY(\tilde{\delta}_F, R) - D] dF(z) = \epsilon.$$

Therefore, the optimal value of reserves R_F^{**} is computed by solving the no-default constraint for R and the value of the multiplier is equal to:

$$\lambda^{**} = \frac{1 + \int_{\bar{z}(\tilde{\delta}_F, R_F^{**})}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R_F^{**})}{\partial R} dF(z)}{- \int_{\bar{z}(\tilde{\delta}_F, R_F^{**})}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R_F^{**})}{\partial R} dF(z)}.$$

Note that for λ^{**} to be strictly positive it has to be that at $R_F^{**} > \epsilon$ the following condition is satisfied:

$$\left(1 + \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} dF(z) \right) \Big|_{R=R_F^{**}} > 0. \quad (\text{C.33})$$

This condition holds if the value of R that solves the no-default condition in this sub-case is lower than the one given in *case II*, because there the no-default constraint is slack at the equilibrium and the same no-default constraint is decreasing in R .²⁷ Therefore, for $R_F^{**} > \epsilon$ to be an equilibrium in this case two conditions must hold; namely, (C.31) and (C.33).

Case III: $\lambda = 0$, $\mu > 0$.

In this case, the no-default constraint is slack. Instead, the resource constraint is binding. Then, the optimal level of reserves and the Lagrange multiplier μ are determined by the following conditions:

$$1 + \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} dF(z) = \mu,$$

$$R = \epsilon.$$

²⁷For completeness, notice that

$$(-\partial Y(\tilde{\delta}_F, R)/\partial R) \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z dF(z) \Big|_{R=R_F^{**}} > 0 \quad (\text{C.34})$$

at any finite value of R in $(\epsilon, \tilde{\delta}_F D)$.

More specifically, $R_F^{**} = \epsilon$ is an equilibrium if

$$\lim_{R \rightarrow \epsilon} H_F(R)|_{R=\epsilon} \equiv 1 + \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} \Big|_{R=\epsilon} \int_{\bar{z}(\tilde{\delta}_F, R)|_{R=\epsilon}}^{\infty} z dF(z) > 0,$$

which is equivalent to condition (C.32) above and implies that $\mu^{**} > 0$.

Case IV: $\lambda > 0, \mu > 0$.

In this case, both the no-default constraint and the resource constraint are binding. The equilibrium would be determined by:

$$\begin{aligned} 1 + (1 + \lambda) \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} dF(z) &= \mu, \\ \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} [zY(\tilde{\delta}_F, R) - D] dF(z) &= \epsilon, \\ R &= \epsilon. \end{aligned}$$

However, notice that

$$\lim_{R \rightarrow \epsilon} \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} [zY(\tilde{\delta}_F, R) - D] dF(z) > \epsilon \quad \forall \epsilon \in [0, \tilde{\delta}_F D).$$

Indeed, if $\epsilon = 0$, then, by construction, $\int_{\bar{z}(\tilde{\delta}_F, \epsilon)}^{\infty} [zY(\tilde{\delta}_F, \epsilon) - D] dF(z) > 0$. Moreover, given that $\partial Y(.,.)/\partial R < 0$, if ϵ increases to $\tilde{\delta}_F D$ then $\int_{\bar{z}(\tilde{\delta}_F, \epsilon)}^{\infty} [zY(\tilde{\delta}_F, \epsilon) - D] dF(z)$ goes to 0. However, the country cannot deplete available resources in reserves, because the lender would reply by setting a nil value of the discount factor. Hence, this case cannot be an equilibrium of the maximization problem.

Finally, to get the equilibrium value of δ one has to use the optimal level of R obtained in each case above and plug it into $\tilde{\delta}_F$. ■

D. Extension with Distributions with Exponential Density Functions

In this extension, we relax the assumption of uniform distribution to show under which conditions the results of the paper carry over in a setting with exponential density functions.

We use the assumption of uniform distribution to derive the fixed point that solves the lender's break-even condition. More specifically, we employ the assumption on the shape of z to show that condition (iv) and condition (v) hold true in the proof of Lemma 3.3.1.

More specifically, condition (iv) is satisfied if $G(\delta)$ is concave, that is if:

$$\frac{d^2G(\delta)}{d\delta^2} = \frac{df(\bar{z}(\delta))}{d\delta} \frac{D^2Y'(\delta)}{(Y(\delta))^2} + D^3f(\bar{z}(\delta)) \left\{ \frac{Y''(\delta)}{(Y(\delta))^2} - \frac{2(Y'(\delta))^2}{(Y(\delta))^4} \right\} < 0. \quad (D.35)$$

In the case of uniform distributions (D.35) is satisfied, because $f(\bar{z}(\cdot))$ is constant in δ (the second term is negative independently of the type of distribution assumed). In the case of exponential distributions, a sufficient condition for (D.35) to hold is that $df(\bar{z}(\delta))/d\delta \geq 0$ is negative. More specifically, it has to be that:

$$\begin{aligned} \frac{df(\bar{z}(\delta))}{d\delta} &= \frac{\partial f(\bar{z}(\delta))}{\partial z} \frac{d\bar{z}(\delta)}{d\delta} = \\ &= -\frac{\partial f(\bar{z}(\delta))}{\partial z} \left(\frac{Y'D^2}{(Y(\delta))^2} \right) \leq 0. \end{aligned} \quad (D.36)$$

(D.36) is negative if $\partial f(\bar{z}(\delta))/\partial z \geq 0$ and this is the case if the following assumption holds:

- A2 :**
- i. $z \sim F(z)$ s.t. $\arg \max_z f(z) = E(z) = 1$.
 - ii. $E(z)Y(\delta, R) - D = Y(\delta, R) - D > 0$.

The implications of (A2) are two: the first is that $f(z)$ reaches its maximum at $E(z) = 1$ and the second is that the expected value of the production plan is ex-ante profitable, so that $\bar{z}(\delta) < 1$. Consequently, $f(z)$ is increasing in z when it is evaluated in $\bar{z}(\delta)$.

Condition (v) is satisfied if $\lim_{\delta \rightarrow 0} dG(\delta)/d\delta > 1$. Taking limits, one has that:

$$\lim_{\delta \rightarrow 0} \frac{dG(\delta)}{d\delta} = \lim_{\delta \rightarrow 0} \frac{f(\bar{z}(\delta))}{(Y(\delta))^2} Y'(\delta) D^2 = \infty. \quad (D.37)$$

In the proof of Lemma 3.3.1, the result above follows from

$$Y'(\delta) \rightarrow_{\delta \rightarrow 0} \infty$$

and from the fact that for a uniform distribution the value of $f(\bar{z}(\delta))$ is a constant in δ , hence

$$\frac{1}{(Y(\delta))^2} \xrightarrow{\delta \rightarrow 0} \infty.$$

When analysing the case of exponential distributions, one needs to show that

$$\frac{f(\bar{z}(\delta))}{(Y(\delta))^2} \xrightarrow{\delta \rightarrow 0} \infty.$$

We can prove it by making use of the following condition:

A3 : $\lim_{\delta \rightarrow 0} Y^2(\delta) < \lim_{\delta \rightarrow 0} f(\bar{z}(\delta))$.

The inequality above uses a specific infinitesimal order that is satisfied by a production function that meets Inada Conditions and exponential density functions. An example may provide a better understanding of why this is the case. Assume $z \sim N(0,1)$ and define $(1/Y(\delta)) = x^\alpha$, then

$$f(\bar{z}(\delta)) = f\left(\frac{D}{Y(\delta)}\right) \approx f(x) \xrightarrow{\delta \rightarrow 0} e^x.$$

Indeed, as δ goes to zero, $1/x^\alpha$ goes to infinity, like e^x does. Therefore,

$$\lim_{\delta \rightarrow 0} \frac{f(x)}{x^{-2\alpha}} \approx \lim_{\delta \rightarrow 0} \frac{e^x}{x^{-2\alpha}} = \infty,$$

And the last result follows from:

$$ord_\infty e^x \geq ord_\infty x^a.$$

■

E. Figures

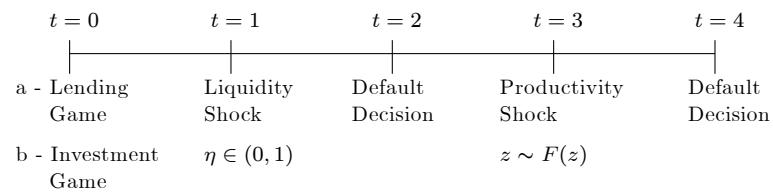


Figure 3.1: Timeline

t=0b - Investment game

t=1 - Liquidity shock

t=2 - Default decision

t=3 - Productivity shock

t=4 - Default decision

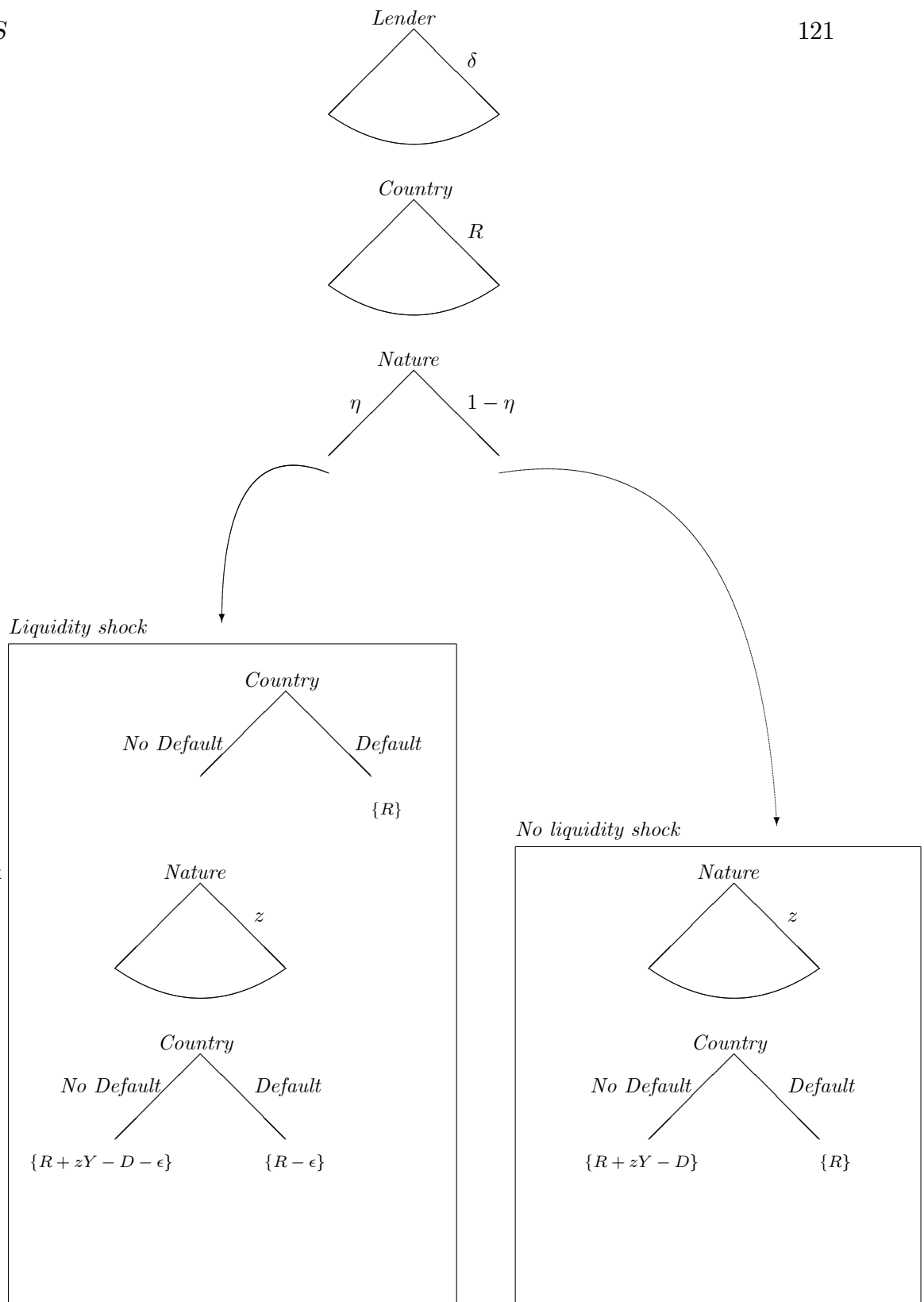


Figure 3.2: Game-tree with the country's payoffs

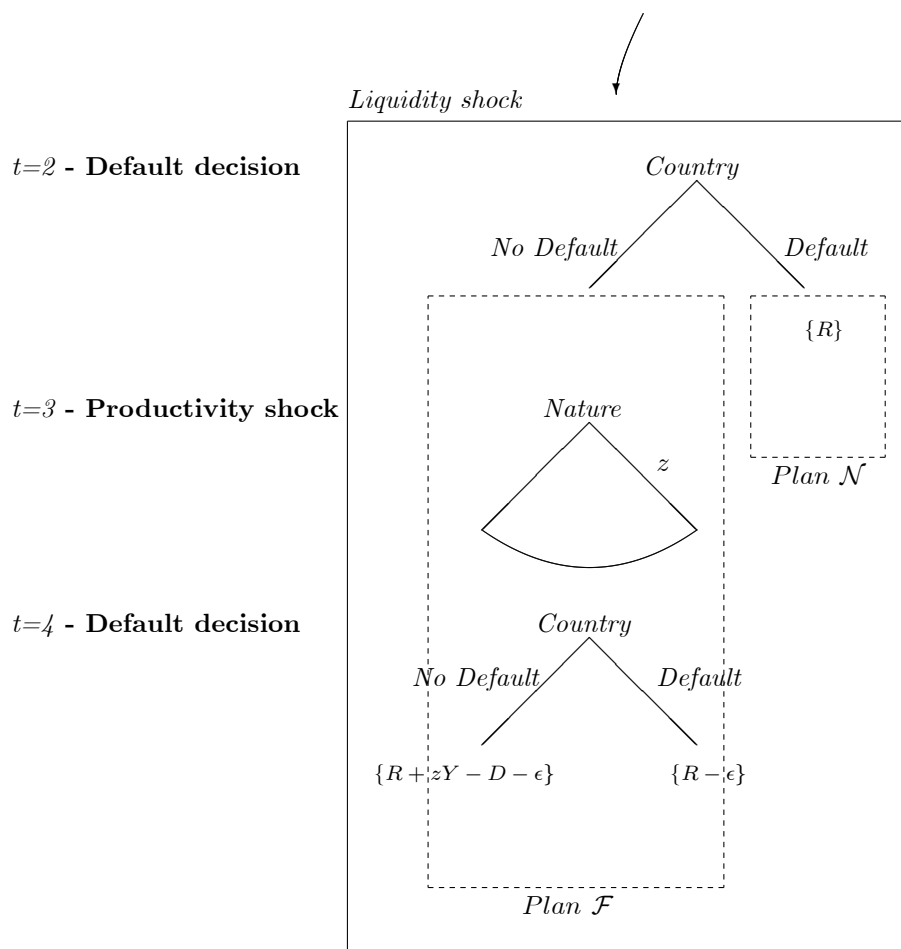


Figure 3.3: Liquidity shock case, plan \mathcal{F} and plan \mathcal{N}

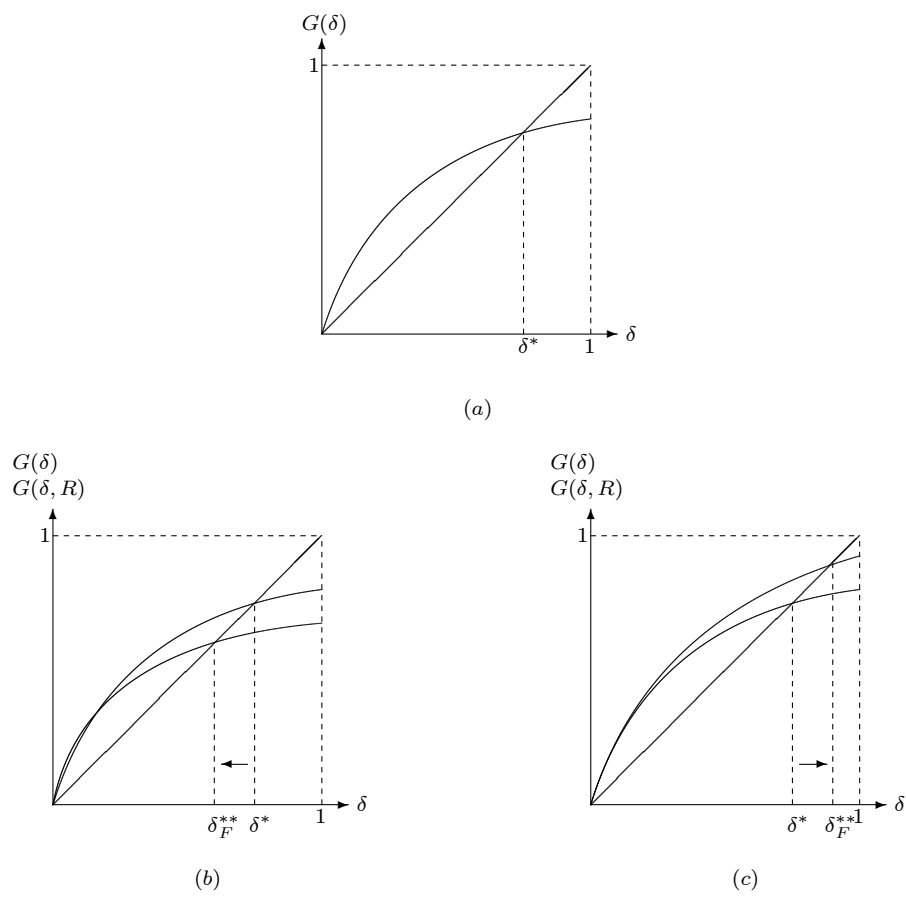


Figure 3.4: Equilibrium discount factors

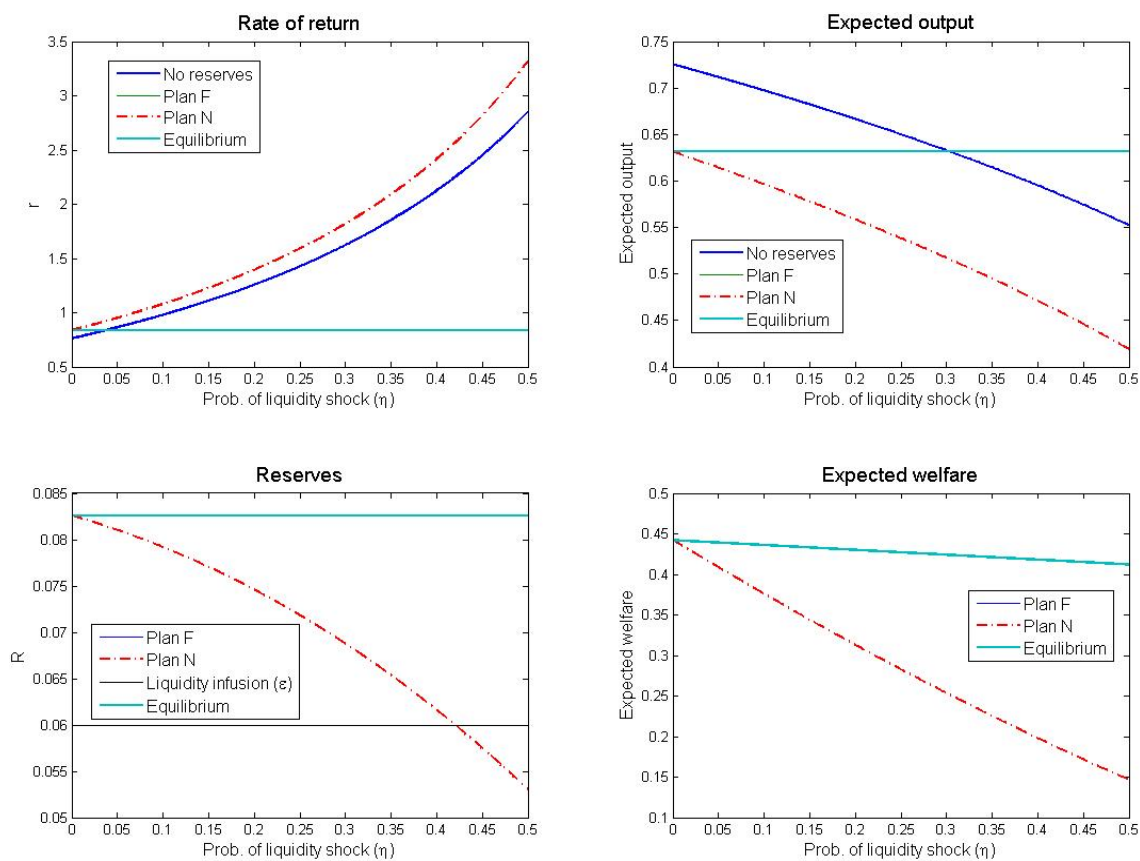


Figure 3.5: Numerical Example - Liquidity Shock Likelihood (η)

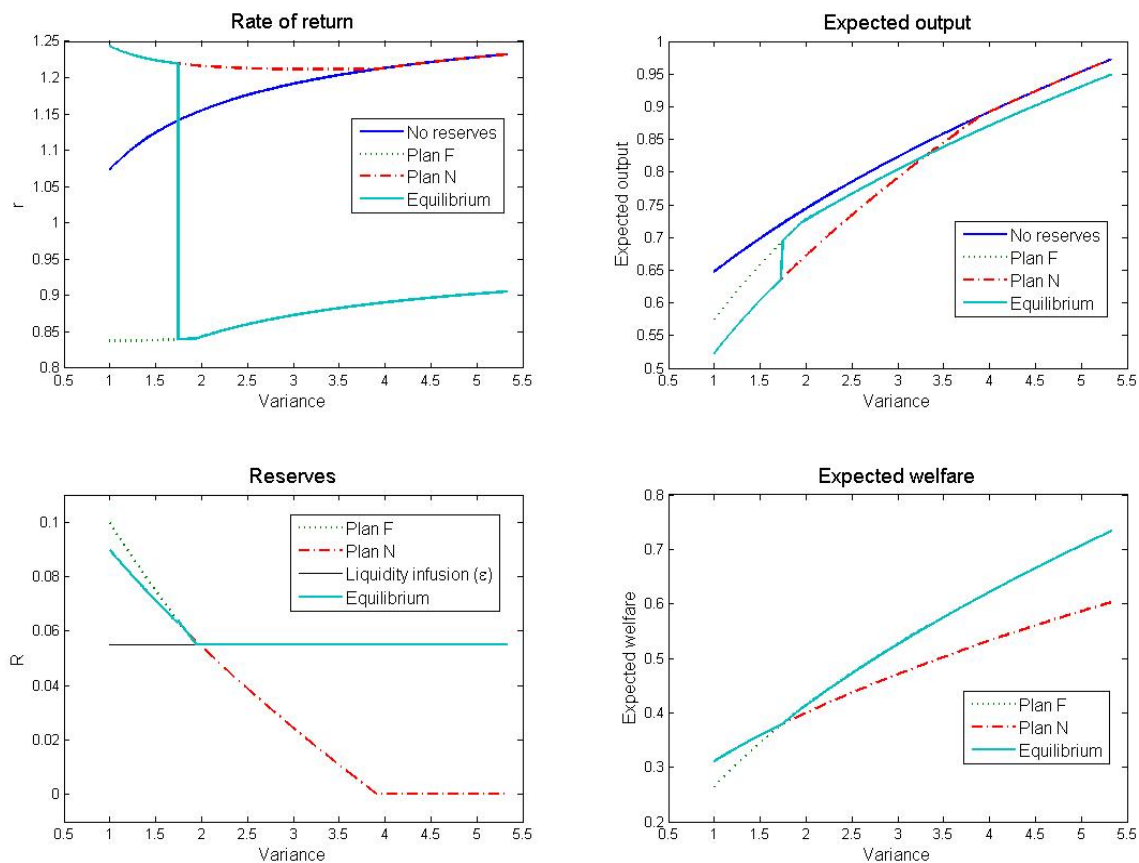


Figure 3.6: Numerical Example - Productivity Shock Variance ($c^2/3$)

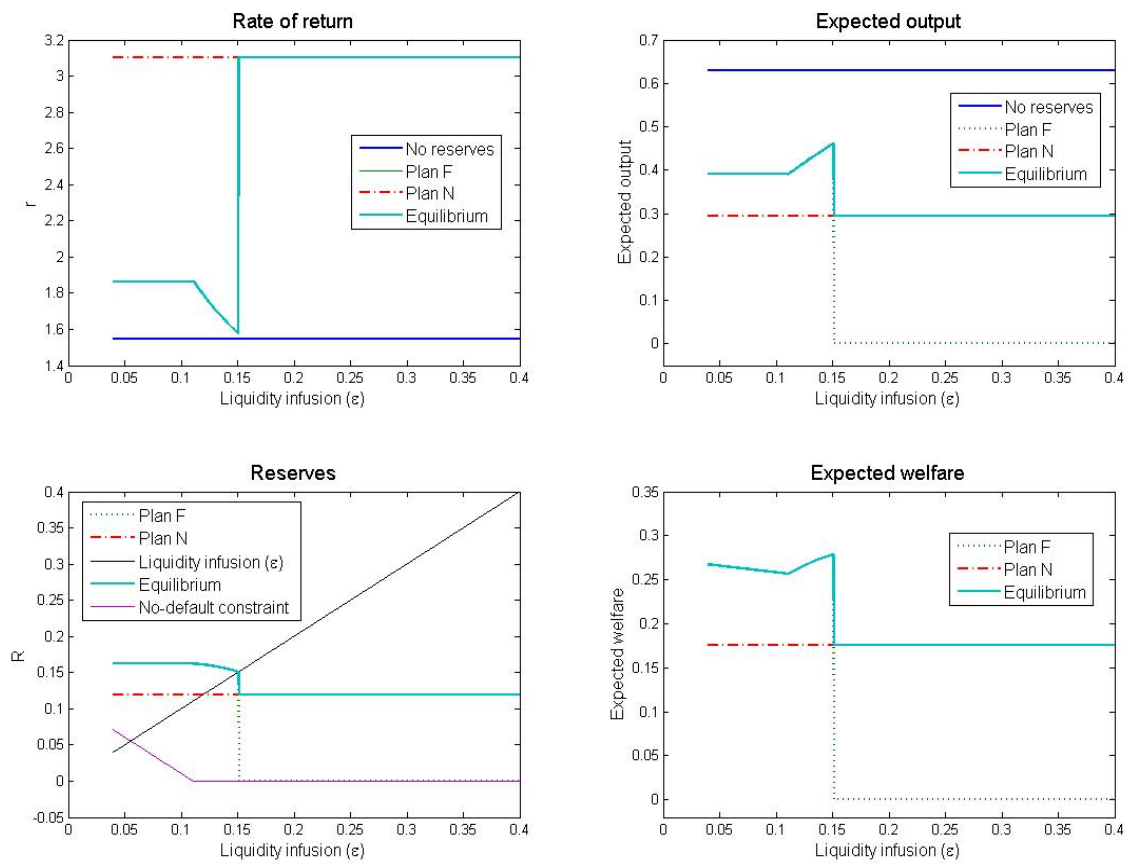


Figure 3.7: Numerical Example - Liquidity Injection (ϵ)

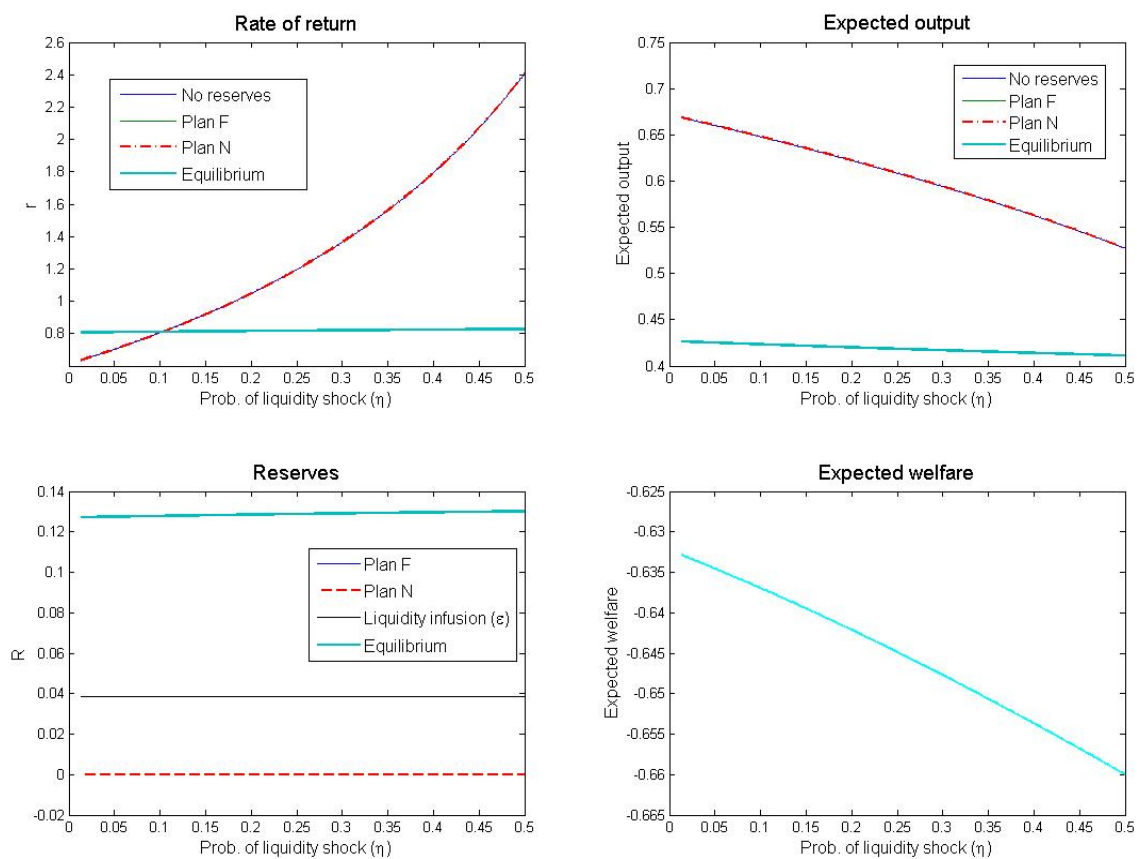


Figure 3.8: Risk-averse country

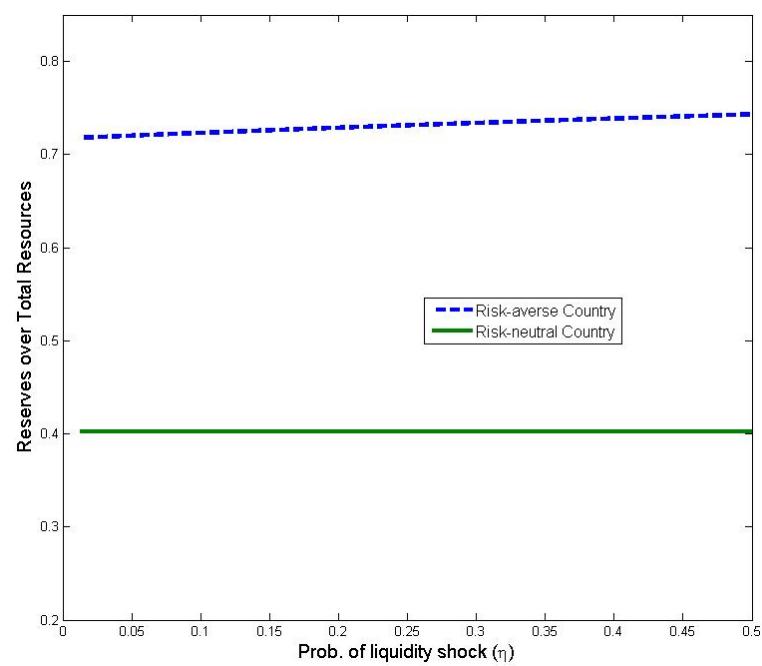


Figure 3.9: Comparison, risk-neutral v. risk-averse country

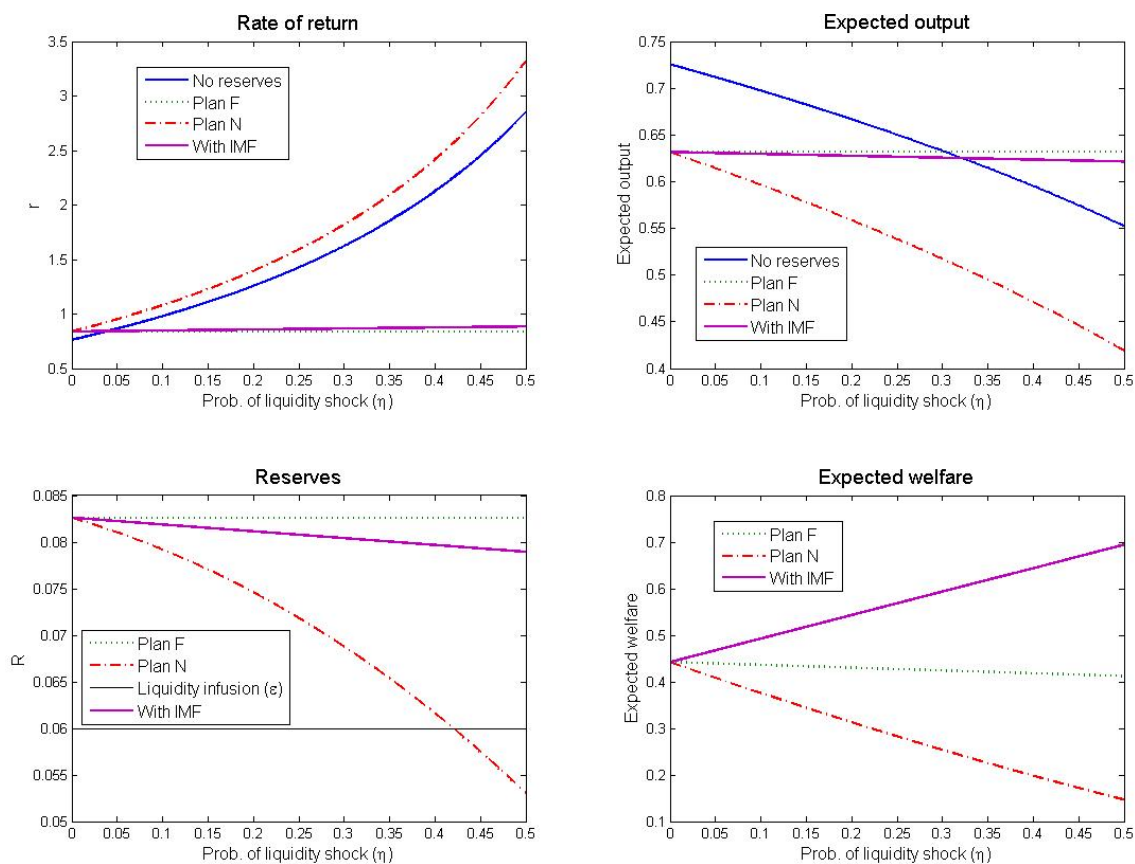


Figure 3.10: IMF intervention