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TOWARDS A SUSTAINABLE GLOBAL ENERGY SUPPLY  
INFRASTRUCTURE:  
NET ENERGY BALANCE AND DENSITY CONSIDERATIONS

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**ROBERT SCHUMAN CENTRE FOR ADVANCED STUDIES**  
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# Towards a Sustainable Global Energy Supply Infrastructure: Net Energy Balance and Density Considerations

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## Abstract

This paper employs a framework of dynamic energy analysis to model the growth potential of alternative electricity supply infrastructures as constrained by innate physical energy balance and dynamic response limits. Coal-fired generation meets the criteria of longevity (abundance of energy source) and scalability (ability to expand to the multi-terawatt level) which are critical for a sustainable energy supply chain, but carries a very heavy carbon footprint. Renewables and nuclear power, on the other hand, meet both the longevity and environmental friendliness criteria. However, due to their substantially different energy densities and load factors, they vary in terms of their ability to deliver net excess energy and attain the scale needed for meeting the huge global energy demand. The low power density of renewable energy extraction and the intermittency of renewable flows limit their ability to achieve high rates of indigenous infrastructure growth. A significant global nuclear power deployment, on the other hand, could engender serious risks related to proliferation, safety, and waste disposal. Unlike renewable sources of energy, nuclear power is an unforgiving technology because human lapses and errors can have ecological and social impacts that are catastrophic and irreversible. Thus, the transition to a low carbon economy is likely to prove much more challenging than early optimists have claimed.

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The findings, interpretations, and conclusions of this paper are the authors' own and should not be attributed to the World Bank, its Executive Board of Directors, and any of its member countries, or the Argonne National Laboratory.

## 1. INTRODUCTION

Of all the challenges confronting the world today, none is likely to prove as daunting or vital to the global economy and the very future of this planet, as that of energy. Providing sufficient energy to meet the requirements of a growing world population with rising living standards will be a difficult task. Doing it without exacerbating the risks of climate disruption will be an even more challenging undertaking. It will require a significant shift in the historic pattern of fossil-fuel use and a major transformation of the global energy system. The relatively short timescale of the necessary transition to a low carbon economy is likely to prove especially challenging. There are fears that a very rapid transition to a renewable-energy economy could lead to the cannibalization of energy from existing power plants and thus exacerbate the current global energy scarcity (Pearce, 2008; Pearce, 2009; Kenny et al., 2010). Moreover, energy transitions take time, with major innovations in the past having taken decades to diffuse and even longer to have the supporting infrastructures developed (Smil, 2010).

There are high expectations that technological innovation will play a critical role in facilitating the transition to a cleaner and more efficient energy economy, and considerable excitement about the growing importance of renewable technologies in the future energy mix (van der Zwaan, 2006). Already, as part of their efforts to reduce greenhouse gas emissions and improve the security of their energy supply, many governments have made similarly worded pronouncements and set ambitious goals for sourcing a significant portion of electricity generation from renewables. However, the transition to a renewable energy system will be challenging because of the modest energy density of the alternative fuels, the low conversion efficiency and power density of renewable energy extraction, and problems of intermittency (which lead to low load factors). This transition is further complicated by the frequent location of renewable resources away from the major population centers, the uneven geographic distribution of these resources around the globe, and the massive scale of the prospective shift. Indeed, there is still considerable variance in the estimates of the basic performance metrics of renewable technologies. Thus the promises of various substitute technologies in the transition toward increased decarbonization must be subjected to a careful reality check. One should keep in mind that during the 1970s, proponents of nuclear power in the United States were predicting that it would largely replace coal-fired generation by the year 2000. Similarly, during the early 1980s, proponents of small-scale, distributed, green energies (solar, wind, biofuels) were predicting that these technologies would supply between 30 and 50 percent of the country's electricity needs by the first decade of the 21st century (Smil, 2008). Both of these predictions turned out to be wildly optimistic.

In this paper we develop a dynamic energy analysis framework to model the growth potential of alternative electricity supply infrastructures as constrained by innate physical energy balance and dynamic response limits. Both a set of existing and novel figures of merit are utilized for evaluating the technological options for energy growth under such dynamic constraints. We focus in particular on modelling the "doubling time" metric, which measures the time interval required for a given energy facility to produce and accumulate



enough excess energy, after making a contribution to national energy demand, to construct new infrastructure so as to double its power output. In other words, this metric measures the capability of a given energy infrastructure to sustain and reproduce itself from its own output while making sufficient residual energy available for societal use. The doubling time metric for a given energy facility summarizes several fundamental characteristics of its underlying technology, including the: capacity factor; amount of energy required for constructing and emplacing a unit of nameplate capacity; fraction of the facility's gross energy output used for its operation and maintenance; time required for constructing and emplacing a new facility; and the effective lifetime of the facility.

In proposing this metric for self-sustained growth we are mindful of the fact that in the real world, energy manufacturers very rarely, if ever, constrain their factories to self-generated power. Indeed, the future energy supply will likely continue to rely on infrastructures comprised of a mix of technologies in which excess energy can be diverted from high energy surplus/high capacity factor assets (e.g. fossil fuels) to help systems with lower energy surplus/capacity factors and requiring large up-front energy investments for their emplacement (e.g. renewable technologies) grow faster. Moreover, there are a number of different potential pathways characterizing the transition from a fossil-fuel powered economy to a renewable energy base. Our proposed metric is not intended to contribute towards defining the optimal transition path. What we seek instead is to evaluate whether the up-front energy investment in the context of a rapid scale-up of renewable generation is likely to impose a heavy burden on existing energy resources and thus exacerbate the current scarcity and price volatility.<sup>1</sup> If the doubling time of a given low-carbon technology is short, it will be possible to rapidly scale-up its generation by bootstrapping its own energy production to finance in energy terms its own growth. On the other hand, if the doubling time of renewable technologies is very long then the rapid transition to a low carbon, renewable-energy economy could prove more challenging—even if we manage to continuously be on the optimal transition path that minimizes the needed energy subsidy from high carbon fossil-fuel facilities.

Our emphasis on the technical headroom of alternative generating technologies does not seek to supplant the time-honored economic cost-benefit analysis. Nor does it question the power of the incentives provided by market pricing mechanisms for the efficient allocation of scarce energy resources. However, the solutions to the twin challenges of energy and climate change are likely to prove complex, with several important technical (scientific and engineering) and social (economic, political) dimensions to consider. The dynamic energy analysis that we employ in this paper provides a deeper understanding of the powerful physical constraints the alternative generating technologies must respect—constraints that cannot be relaxed through economic policy measures.<sup>2</sup> As such, our dynamic energy balance framework can facilitate a technical reality check on the potential of these technologies to have an impact on the scale required by the global energy problem.

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<sup>1</sup>During the early stages of the transition there will not be self-replication of low-carbon systems but self-destruction of high-carbon ones. After the high-carbon systems are largely replaced, the low-carbon systems will then have to self-replicate. Our proposed analysis and the doubling time metric can be useful in evaluating the ability of alternative generating systems to self-replicate.

<sup>2</sup>As Koonin (2008) astutely observed "...you won't repeal the 2<sup>nd</sup> law of thermodynamics by taxing entropy!"

## **2. TRANSITION TOWARDS A SUSTAINABLE GLOBAL ENERGY SUPPLY INFRASTRUCTURE**

The 21st century is likely to witness a transition to a new energy supply infrastructure that supports the tenets of sustainable development. Key requirements of the enabling energy supply chain will include:

- scalability—ability to expand to the multi-terawatt level;
- environmental friendliness—minimal carbon footprint;
- capacity to deliver net excess energy;
- longevity—abundance of energy source.

According to the International Energy Agency (IEA), assuming no change in government policies, world primary energy demand is projected to rise from 12,013 to 16,790 million tons of oil equivalent (Mtoe) between 2007 and 2030—an increase of 40 percent. Electricity demand is projected to grow from 16,249 to almost 29,000 terawatt-hours (TWh) during the same period—an increase of 78 percent. To meet these needs, the world's electricity generating capacity will have to increase from about 4,509 gigawatts in 2007 to 7,821 gigawatts in 2030, requiring approximately 3,312 gigawatts of capacity additions—over three times the existing US generating capacity (IEA, 2009). Thus the scale of the energy challenge is enormous—it is at the multi-terawatt level.

Climate change is rapidly becoming the defining environmental, economic, and political challenge of our era. With growing concerns about anthropogenic greenhouse warming and climate disruption, the pressures to curtail carbon dioxide emissions from coal-fired electricity generation are likely to escalate sharply. This gives rise to one of the central challenges in global energy policy: in the context of a carbon-constrained world, what energy supply infrastructures will provide the estimated additional 3,312 gigawatts of new electricity generation capacity that it is estimated the world will need by 2030? In view of the projected large absolute increase in global energy demand, such infrastructures will clearly need to display substantial scalability—i.e. ability to expand to the multi-terawatt level. Moreover, to meet the requirements of long-term security of supply and sustainability, these energy sources should be indigenous, abundant, and with a minimal carbon footprint.

Generation of net excess energy by a given supply infrastructure is a key determinant of its ability to facilitate economic growth. Throughout several centuries of recent history, industrialization and economic growth were facilitated by the emergence of fossil fuels-based energy supply infrastructures capable of delivering increased and highly concentrated net surplus energy. Petroleum, in particular, spawned unprecedented world-economic growth because its excess energy—net of the exploration, extraction, refining, and transport processes—was enormous; and because the energy could be delivered in a highly useful form—a liquid which, due to its high energy density, could effectively power transportation. The rapid

pace of urbanization, especially in developing countries, will render high energy density supply architectures essential. Moreover, economic growth coupled with population growth will almost inevitably lead to an acceleration of energy demand. Keeping up with this energy demand will require supply architectures with both net excess energy output and short energy payback times.

Fossil fuels (oil, coal, and natural gas) supply most of the world's energy. Under business-as-usual scenarios, oil—the most convenient and multipurpose of these fuels—will continue to be the main source of energy, specifically for the transport sector, up to 2030. For electricity generation, coal and natural gas will remain the main primary energy sources, with coal possibly increasing its market share (van der Zwaan, 2005; IEA, 2009).<sup>3</sup> However, the considerable strain experienced by world energy markets in recent years is reviving the debate about the expected lifetime and declining accessibility of conventional fossil fuel resources. Based on current consumption, global oil and gas reserves will last for a few more decades and coal will last for a century and a half. Thus, there are fundamental questions related to the longevity of conventional fossil fuel resources.

There may be abundant unconventional fossil-fuel alternatives to oil and natural gas; and there is a consensus that undiscovered global coal reserves are huge. Oil sands and shale represent very significant potential fossil resources. In addition to being plentiful sources for power generation, these resources can be transformed into gaseous and liquid fuels using known technologies at manageable costs (Bartis et al, 2008). The problem is that all of these options are much more CO<sub>2</sub>-intensive than conventional oil and gas alternatives, and even with carbon capture and sequestration (CCS) their CO<sub>2</sub> intensity is no lower than conventional fossil fuel alternatives. Thus, they carry an even heavier greenhouse gas (GHG) burden than conventional fuels. They also involve a range of local environmental problems from fuel extraction. The availability of these secure, economically feasible, yet environmentally unattractive options lies at the heart of the dilemma facing energy policy makers today.

### **3. FIGURES OF MERIT FOR ENERGY SUPPLY INFRASTRUCTURE**

Assume energy demand increases incrementally due to population growth and/or greater energy use per capita. This rise in demand will be accommodated by expanding the capacity of the supply infrastructure. When the number of deployed converter nodes, the area of extent required to harvest the fuel resource (and thus the associated shipping needed for delivery to the converter), and the incremental delivery infrastructure are increased, then an incremental cost is incurred in the form of incremental energy expended to emplace the new infrastructure and to operate it. This energy cost must be borne by the existing and new

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<sup>3</sup>Bob van der Zwaan (2005) argues that coal is likely to continue to have a high, and perhaps even increasing, share in global electricity generation this century because: " (1) its large resource base; (2) the improving efficiency and competitiveness of conventional and innovative coal technologies; (3) the employability of new coal technologies in conjunction with carbon capture and storage systems; (4) the improving economics of these advanced clean coal technologies."

infrastructure. If this cost gets larger as a fraction of the capacity of the infrastructure to deliver energy, then less energy becomes available for societal use.

Several figures of merit are used to characterize the net (excess) energy output to be derived from emplacing or enlarging an energy supply infrastructure. Two of the most frequently quantified metrics are the Energy Return on (Energy) Investment (*EROI*), which in the case of electricity generation compares the cumulative electricity generated by a given facility to the quantity of primary energy used to emplace, operate, and decommission the facility; and the Energy Payback Time,  $\tau_1$ , which measures the time that it takes a given facility to generate the energy equivalent to that used for emplacing (and decommissioning) it. Both *EROI* and the energy payback time basically depict the amount of energy we expend today in order to obtain energy tomorrow. As such, they are useful in comparing alternative energy systems in terms of their use of society's productive resources for delivering a given amount of energy, and thus ultimately in terms of their efficiency. Clearly, systems characterized by low *EROI* and long  $\tau_1$  present an increased opportunity cost of energy delivery and are limited in terms of their ability to expand rapidly without a subsidy from other sources. We also derive a third metric, the Doubling Time,  $\tau_2$ , which will clarify the way in which certain physical characteristics (e.g. the intermittency and low conversion efficiency and power density of renewable energy flows) limit a given energy infrastructure's capacity for expansion and self-replication.

### 3.1 Energy Return on (Energy) Investment

*EROI* is defined as the ratio of the cumulative lifetime energy delivered by a given facility to the energy used in the manufacture, transport, construction, operation, and decommissioning of the facility (Hall et al, 1986; Hall and Cleveland, 2005). Thus, *EROI* is given by:

$$EROI = \frac{P_{np}\varphi T}{E + P_{np}\varphi hT} \quad (1)$$

where:

$P_{np}$  = is the facility's installed (nominal) nameplate capacity

$\varphi$  = average load or capacity factor

$E$  = is the energy expended to initially emplace and ultimately decommission the facility

$h$  = fraction of gross energy produced that is used for the operation and maintenance of the facility

$T$  = is the facility's effective lifetime.

It should be noted that in this definition: (i) the energy content of the fuel resource harvested and precessed is not included in the denominator—i.e., only the energy needed for the infrastructure per se is accounted for; and (ii) the conversion efficiency from fuel energy carrier to product energy carrier is embedded in the numerator, and the energy unit is joules of heat for both numerator and denominator. It is also easy to see from (1) that *EROI* will be increased by:

- reducing  $\frac{E}{P_{np}}$ , the emplacement energy per unit of nameplate capacity;
- increasing  $\varphi$ , the load factor
- reducing  $h$ , the O&M energy fraction of production
- increasing  $T$ , the facility's lifetime.

An informative way to think about *EROI* is in terms of fractions of lifetime gross energy output that is expended for initial emplacement and for ongoing maintenance and operation:

$$\begin{aligned}
 EROI &= \frac{P_{np}\varphi T}{E + P_{np}\varphi hT} = \frac{1}{\frac{E}{P_{np}\varphi T} + h} \tag{2} \\
 &= \frac{1}{\left[ \begin{array}{c} \text{fraction of gross production} \\ \text{expended for initial emplacement} \end{array} \right] + \left[ \begin{array}{c} \text{fraction of gross production} \\ \text{expended for Operation and Maintenance} \end{array} \right]}
 \end{aligned}$$

The first term represents an initial "capitalization" expenditure of energy; the second term represents an ongoing "variable" expenditure. However, unless the details of the Life Cycle Analysis are available, this breakdown is not evident because when *EROI* values are reported, the two components of the denominator become subsumed and indistinguishable within a single number.

The *EROI* as a figure of merit pertains to the "efficiency" of an infrastructure—its ability to deliver excess energy to society—integrated over its lifetime (given an assumption about the availability of a resource input). However, it does not address a given infrastructure's scalability (because it is a ratio), longevity (because it assumes the availability of a resource), and capacity to achieve required rates of growth.

It should be noted that there are no firm estimates for *EROI*. The reported values vary considerably across studies and are changing over time. This variation is in large part due to the significant methodological issues underlying net energy analysis in general and the calculation of *EROI* in particular. Most vexing among these methodological challenges is the choice of appropriate system boundaries—i.e. the number of stages on each system's vertical chain (upstream and downstream processes) over which the "cradle-to-grave" life cycle analysis is carried out. The unavoidable truncation of the system boundaries can lead to significant systematic errors (Lenzen and Dey, 2000; Lenzen and Treloar, 2003). Moreover, many of the reported system parameters (e.g. power rating, lifetime and load factors) are frequently based on engineering estimates and simulations reflecting ideal operating characteristics rather than actual operating experience.

### 3.2 Energy Payback Time

A second prominent figure of merit is the energy payback time  $\tau_1$  which represents the length of deployment required for an energy system to deliver an amount of net energy equal

to the amount of energy that was expended to emplace the system. Thus,  $\tau_1$  is used to characterize the "capital" component in (2), and is defined by:

$$\tau_1 P_{np}\varphi(1-h) = E \quad (3)$$

or equivalently

$$\tau_1 = \frac{E}{P_{np}\varphi(1-h)} = \frac{q}{\varphi(1-h)} \quad (4)$$

where  $q = \frac{E}{P_{np}}$  is the emplacement energy per unit of nameplate capacity.

It is desirable for the energy payback period  $\tau_1$  of a given infrastructure to be much shorter than its lifetime  $T$ . Writing

$$\frac{\tau_1}{T} = \frac{E}{P_{np}\varphi(1-h)T} = \frac{q}{\varphi(1-h)T} \quad (5)$$

it can be seen that the ratio of  $\frac{\tau_1}{T}$  will be much less than 1.0 if the:

- load factor  $\varphi$  is large
- lifetime  $T$  is long
- O&M plowback fraction  $h$  is much less than 1
- emplacement energy per unit of nameplate capacity  $q$  is small.

If the detailed reporting of a Life Cycle Analysis includes values for both  $EROI$  and  $\tau_1$ , then it is possible to "back calculate" the two subcomponents of  $EROI$  in (2). The energy payback time is the energy analog to financial payback. It can be a useful metric for comparing alternative energy systems in terms of how quickly they can yield net excess energy for societal use.

### 3.3 Doubling Time

We derive now another metric that is important in the context of dynamic energy analysis: the doubling time  $\tau_2$  which represents the time interval required for a given energy facility to produce and accumulate enough excess energy to deploy new infrastructure that is sufficient to double the facility's power output. This figure of merit provides the basis for assessing the capability of a given energy infrastructure to sustain and reproduce itself from its own output (i.e. without an energy subsidy from other sources) while still making sufficient residual energy available to support ongoing economic growth.

Consider an energy facility with  $P_{np}(t)$  installed nameplate (nominal) capacity at time  $t$ . Let  $P(t)$  denote the power generated by the facility that is available for societal use. Then

$$P(t) = (1 - h)(1 - \beta)\varphi P_{np}(t) \quad (6)$$

where  $\beta$  is the fraction of net energy produced that is plowed back into the construction of new plants and their associated resource supply and delivery infrastructures.

Assume further that  $C(t)$  is the nameplate plant capacity that is under construction at time  $t$ . Then, at the end of time interval  $\delta t$

$$C(t + \delta t) = C(t) + \frac{\beta(1 - h)\varphi P_{np}(t)\delta t}{q} - \frac{\delta t}{\tilde{t}}C(t) \quad (7)$$

where  $\tilde{t}$  is the average construction time for a new plant.

The term  $\beta(1-h)\varphi P_{np}(t)\delta t$  represents the amount of energy produced during time interval  $\delta t$  that is plowed back into the construction of new plants, while  $\frac{\delta t}{\tilde{t}}C(t)$  is the fraction of capacity  $C(t)$  under construction at time  $t$  which is completed by the end of the time interval  $\delta t$ .

Similarly,

$$P_{np}(t + \delta t) = P_{np}(t) - \frac{\delta t}{T}P_{np}(t) + \frac{\delta t}{\tilde{t}}C(t) \quad (8)$$

where  $T$  is the lifetime of the plant before decommissioning. The term  $\frac{\delta t}{T}P_{np}(t)$  represents the portion of the nameplate capacity at  $t$  that is decommissioned during the time interval  $\delta t$ .

Equations (7) and (8), expressed in differential form, constitute a source-sink system that defines the dynamics of growth for the energy supply under an energy plowback constraint ( $0 \leq \beta \leq 1$ ):

$$\dot{P}_{np}(t) = -\frac{1}{T}P_{np}(t) + \frac{1}{\tilde{t}}C(t) \quad (9)$$

$$\dot{C}(t) = \frac{\beta(1 - h)\varphi P_{np}(t)}{q}P_{np}(t) - \frac{1}{\tilde{t}}C(t) \quad (10)$$

The solution to this system of differential equations describing the infrastructure's dynamic behavior is given in the Annex. It is shown there that

$$\tau_2 = \frac{\ln 2}{\alpha} \quad (11)$$

where  $\alpha$  is the infrastructure's expansion growth rate which in turn is given by

$$\alpha = \frac{-\left(\frac{1}{\tilde{t}} + \frac{1}{T}\right) + \sqrt{\left(\frac{1}{\tilde{t}} + \frac{1}{T}\right)^2 + \frac{4}{\tilde{t}} \left[\frac{\beta\varphi(1-h)}{q} - \frac{1}{T}\right]}}{2}. \quad (12)$$

Thus, the growth potential of a specific infrastructure depends on several innate characteristics of its underlying technology:  $\varphi$ , the capacity factor;  $q$ , the amount of energy expended to construct a unit of nameplate capacity;  $h$ , the O&M plowback fraction;  $\tilde{t}$ , the required construction time for a new plant; and  $T$ , the effective lifetime of the plant. It also depends on  $\beta$ , the fraction of produced energy that is plowed back into the construction of new plants—which is fundamentally an economic decision.

Clearly, to meet rapid growth in energy demand a high value of  $\alpha$ , or equivalently a short doubling time  $\tau_2$ , is desirable. Simple comparative static analysis indicates that  $\tau_2$  will be shorter: the larger is  $\varphi$ ; the smaller is  $q$ ; the smaller is  $h$ ; the longer is  $T$ ; the shorter is  $\tilde{t}$ ; and the larger is  $\beta$ . The parameters  $\varphi$ ,  $q$ , and  $h$  determine how rapidly surplus energy becomes available from newly installed capacity to support the system's expansion;  $\tilde{t}$  defines how rapidly such surplus energy, once made available, can be transformed to new capacity; and  $T$  defines how rapidly existing capacity is depleted because of aging. Finally,  $\beta$  defines the fraction of net energy produced that is plowed back into the construction of new plants and their associated resource supply and delivery infrastructures.

The term  $\frac{4}{\tilde{t}} \left[\frac{\beta\varphi(1-h)}{q} - \frac{1}{T}\right]$  in (12) clarifies the critical role of the three time scales in determining the dynamic expansion and consequently the attainable expansion rate of a given infrastructure under a plowback constraint:  $\tau_1$ , defining how rapidly surplus energy will become available from newly installed capacity to support the system's expansion;  $\tilde{t}$ , defining how rapidly such surplus energy, once made available, can be transformed to new capacity; and  $T$ , defining how rapidly existing capacity is depleted because of ageing. Noting that  $\frac{\varphi(1-h)}{q} = \frac{1}{\tau_1}$ , then according to (12) it is the relationship between  $\frac{\beta}{\tau_1}$  and  $\frac{1}{T}$ , augmented by  $\tilde{t}$ , that is the key determinant of the attainable rate of growth. Technologies with low capacity factors and/or large up-front investment of energy relative to their lifetime outputs, and thus relative long payback periods ( $\tau_1$  large in comparison to  $T$ ), require large plowback ratios (large  $\beta$ ) to achieve any reasonable rate of indigenous infrastructure growth. Also, the potential for expansion will be limited if the surplus energy generated by a given infrastructure can be transformed only very slowly to new capacity—i.e. if  $\tilde{t}$  is long. This implies that such technologies could not be relied upon to make any substantial power available for societal use during the early stages of their expansion—which would be especially problematic in developing countries experiencing high rates of growth in energy demand.

#### 4. GROWTH POTENTIAL WITH ALTERNATIVE INFRASTRUCTURES UNDER AN ENERGY PLOWBACK CONSTRAINT

In this section we examine the dynamics of growth of electricity supply under the constraint of energy plowback for several representative technologies: coal, oil, gas, nuclear,



hydro, wind, and PV. Our analysis is limited to a few representative technologies for which the data needed for our calculations are available. CCS is a promising technology for reducing the heavy carbon footprint of coal-fired generation (Stephens and van der Zwaan, 2005). However, it is clouded by large uncertainties regarding the speed of implementation and the ultimate feasibility of large-scale application. Moreover, CCS is doubly capital intensive, both in terms of the extra equipment to handle the carbon dioxide and the lowered efficiency of the overall plant. Thus its performance metrics ( $EROI$  and  $\tau_1$ ) are likely to be inferior in comparison to traditional coal-fired plants. There is also considerable excitement surrounding the development of concentrated solar power (CSP). The parameter values of both CCS and CSP remain speculative.

Using the idealized model developed above and literature values of the relevant parameters— $EROI$ ,  $T$ ,  $\varphi$ , and  $\tau_1$ —it is possible to identify the essential constraints on feasible rates of growth of alternative technologies. For illustrative purposes, we first employ the parameter values reported in one specific study (IEA, 2002). Subsequently, we utilize values reported in more recent studies that take into account the impacts of technological innovation on plant performance and the energy parameters of alternative infrastructures.

It is important to acknowledge up-front the limitations of the data reported in the IEA study. First, these data reflect the operating experience of existing plants as well as engineering estimates for hypothetical installations of electricity generation systems in Japan. As such, they might not be representative of global options. In particular, the single source of data does not adequately take into account the site-dependent nature of renewable energy performance. Capacity factors and consequently net energy returns for wind and solar can vary dramatically across different locations. Clearly, the wind and solar resources of Japan are not necessarily comparable to those found in the best sites around the globe. Second, we must recognize that some of the values of the parameter estimates reported in the IEA study might be considerably outdated. Wind and solar power have experienced significant technological advances in recent years. On the positive side, since the estimates come from a single study they are likely to suffer less from problems of methodological inconsistency than the updated estimates we employ that are based on different studies. Moreover, the estimates reported in the IEA study permit us to back-calculate the two subcomponents of  $EROI$ : the initial capitalization expenditure of energy  $\frac{E}{P_{np}\varphi T}$  and the ongoing variable expenditure  $h$ . To do so with the updated parameter values requires some additional assumptions.

The goal of our illustrative analysis is not to define the most accurate point estimates or representative values of net energy parameters. Instead, we seek to: (i) evaluate the efficacy of postulated symbiotic mixes of infrastructures for attaining sustainability goals; (ii) determine the bounds on the self-sustained growth rate for each category of infrastructure; and (iii) assess the impacts of technological change on the relevant parameters.

#### 4.1 Individual Infrastructures

IEA (2002) provides estimates of  $EROI$ ,  $\varphi$ , and  $\tau_1$  in table 8.2, figure 8.8, and table 8.4, respectively. Moreover, it assumes that  $T = 30$  years for all infrastructures. From these

values we can back-calculate  $\frac{E}{P_{np}\varphi T}$  and  $h$  (and hence  $q$ ) using (2) and (5). We subsequently employ reasonable estimates of the required construction time  $\tilde{t}$  for new generating plants, set the plowback ratio  $\beta$  to .2, and use (11) and (12) to estimate  $\alpha$  and  $\tau_2$ . All of these values are presented in table 1.

With a 20% plowback (i.e.  $\beta = .2$ ), oil and gas-fired plants can attain 84% annual expansion growth rate, coal-fired 73%, nuclear 58%, hydro 23%, wind 2% and PV 1%. Thus, conventional fossil fuels (oil, natural gas, coal), nuclear power, and to a lesser extent hydro, show potential to support rapid growth. Wind and especially PV, on the other hand, seem quite constrained. This is surprising in light of wind's *EROI* being as large as coal's and its O&M plowback fraction  $h$  being smaller than that of coal. To understand this outcome is it necessary to examine the fixed vs variable components of *EROI* in (2). The fixed component for wind, representing the fraction of gross production expended for initial emplacement, is over 25 times larger than that of coal's. This also is reflected in the comparison of their energy payback period  $\tau_1$ , whose computed value is 3.39 years for wind and only 0.15 years for coal.

The doubling time  $\tau_2$ , again with a 20% plowback, is 1.1 years for oil and gas plants, 1.3 years for coal, 1.5 years for nuclear, 3.3 years for hydro, 28.5 years for wind, and 82.9 years for PV systems. Thus, the ability of wind and PV systems to rapidly scale up their production by bootstrapping their own energy appears to be limited. Even if all the energy produced by these systems was plowed back ( $\beta = 1.0$ ) to support their scale up, their expansion rates would still be well-below those achieved by fossil fuel and nuclear plants with a plowback of just 20%. For example, if we set  $\beta = 1.0$ , then we obtain:  $\alpha = .21, \tau_2 = 3.29$  for wind; and  $\alpha = .14, \tau_2 = 4.88$  for PV. According to these estimates, which are based on pre-2002 technologies, under a 100% plowback, wind power could attain an annual growth rate of 23% and double its capacity every 3.3 years. However, global wind power achieved 29.2% annual growth rate in 1998, 41.7% in 1999, 31.7% in 2000, 34.8% in 2001, and 28.2% in 2002 (WWEA, 2009). Clearly, that type of expansion could not have been achieved without an energy subsidy from conventional fossil fuels and nuclear plants.

It is also important to note and interpret the significant difference between the estimated values of the energy payback period  $\tau_1$  and doubling time  $\tau_2$ , especially for wind and PV. According to Table 1, the energy payback period  $\tau_1$  for a PV system is 4.76 years—although, as it will be noted below, in the last few years this number has been shortened considerably due to technological change. Under a 20% plowback constraint, it will take it another 5-4.76 or 23.80 years to produce enough energy to double its emplaced infrastructure. Thus, it will take a PV system 28.56 years in total to replicate itself. However, given that the system's expected lifetime is 30 years, by the time it produces the required amount of energy to double its capacity, the system becomes largely obsolete and requires replacement. The system's growth will be very slow—indeed in (12),  $\frac{\beta\varphi(1-h)}{q} \sim \frac{1}{T}$  and thus  $\alpha$  is small.

	$EROI$	$T$	$\varphi$	$\tau_1$	$\frac{E}{\varphi P_{np} T}$	$h$	$q = \frac{E}{P_{np}}$	$\tilde{t}$	$\beta$	$\alpha$	$\tau_2$
Oil	17	30	.75	0.09	.003	.056	.063	4	.2	.61	1.1
LNG	21	30	.75	0.09	.003	.045	.065	4	.2	.61	1.1
Coal	6	30	.75	0.15	.004	.163	.094	4	.2	.55	1.3
Nuclear	24	30	.75	0.11	.004	.038	.079	6	.2	.46	1.5
Hydro	50	30	.45	0.45	.005	.015	.069	4	.2	.21	3.3
Wind	6	30	.20	3.39	.106	.061	.637	1	.2	.02	28.5
PV	5	30	.15	4.76	.151	.049	.680	1	.2	.01	82.9
60% nuclear, 40% wind	11	30	.53	1.98	.047	.071	.106	4	.2	.36	1.95

Table 1: Breakout of Components of EROI and Growth Potential Under Energy Plowback Constraint

## 4.2 Mixed Infrastructures

The analysis described above can be applied to a mixed infrastructure by using parameters that are power-fraction weighted average of all the technologies present in the supply infrastructure:

$$\left\langle \frac{\beta\varphi(1-h)}{q} \right\rangle = \sum_i \left( \frac{\beta(1-h)\varphi}{q} \right)_i f_i \quad (13)$$

$$\left\langle \frac{1}{\tilde{t}} \right\rangle = \sum_i \left( \frac{1}{\tilde{t}} \right)_i f_i \quad (14)$$

$$\left\langle \frac{1}{T} \right\rangle = \sum_i \left( \frac{1}{T} \right)_i f_i \quad (15)$$

where  $f_i$  is the fraction of the total supply provided by technology  $i$ .

Consider, for example an electricity supply infrastructure comprised of 60% nuclear and 40% wind. Assuming again a 20% energy plowback and the assumed and estimated parameter values of Table 1, the mixed infrastructure would have an  $EROI$  of 10.91 and its growth parameter  $\alpha$  and doubling time  $\tau_2$  are estimated to be .36 and 1.95 years respectively.

Such a mixed infrastructure might be considered in the future for deep water offshore wind farms located near the population centers. Large wind farms of ~500 MW nameplate rating might be deployed in tandem with onshore 100 to 200 MW small reactors having a load following capability (in place of using conventional combustion gas turbine plants for load following). Such a mixed-system deployment would eliminate greenhouse gas emissions, mitigate the intermittency problem, and would be capable of rapid growth under an energy plowback constraint.

## 4.3 Structure of Net Excess Energy and its Effect on Doubling Time

As seen in the examples above, the single numerical values of life cycle energy balances  $EROI$  and  $\tau_1$  are not sufficient for determining the capacity of a given infrastructure to support rapid growth rates. It is important to understand the "structure" and time dependencies of the energy investments. As it was noted, the following features of an infrastructure favor a fast growth rate potential ( $\alpha$ ) and short doubling time ( $\tau_2$ ): high capacity factor  $\varphi$ ; low hotel load fraction  $h$ ; and low energy input per nameplate rating  $q$ . Table 1 reveals substantially different structures across the various infrastructures.

In the case of coal, 0.4% of gross lifetime energy is required for initial emplacement, and 16.3% of gross lifetime energy is consumed in mining and shipping. For wind, the structure reverses to 10.6% for initial emplacement and 6.1% for O&M. For nuclear, only 0.35% of gross lifetime energy is expended for initial emplacement, and 3.8% for O&M. The corresponding figures for oil, gas, and hydro are: .3%, .3%, and .5% for initial emplacement; and 5.6%, 4.5%, and 1.5% for O&M. For PV, on the other hand, the corresponding structure is 15.1% for initial emplacement and 4.9% for O&M. Thus, with the exception of hydro, renewables require much greater up-front energy investment as a fraction of their lifetime output relative to fossil fuels and nuclear.

The inverse of the sum of the two components  $\frac{E}{\varphi P_{np} T}$  and  $h$ , the fraction of gross production expended for initial emplacement and for O&M respectively, is equal to  $EROI$ . For hydro both  $\frac{E}{\varphi P_{np} T}$  and  $h$  are very low. As a result, hydro has by far the highest  $EROI$  and excess energy delivery. It is followed by nuclear, gas, oil, coal and wind. Coal and wind are lower and nearly equal because they each require significant energy plowback just to maintain the steady state. Standalone PV has somewhat lower  $EROI$  because of its high initial emplacement energy requirements.

Our analysis of infrastructure's dynamic response indicates that although wind and PV have short construction periods, none-the-less they are characterized by long doubling times because of their low capacity factors and large up-front emplacement energy requirements. Even with much longer construction periods, conventional fossil fuels and nuclear benefit from their high capacity factors and relatively low initial emplacement energies and thereby attain short doubling times.

#### 4.4 Role of Technological Innovation for Future Improvements

The parameters determining net excess energy and the doubling time of energy systems are not static. As research and development gives rise to technical change and process innovation, the values of these parameters, and consequently  $EROI$ , will likely change.

The fossil era witnessed an ever improving value for  $EROI$  as technological innovation increased the efficiency of energy converters and as economies of scale lowered energy consumption of shipping resource energy carriers and distributing end use energy carriers. That trend persisted for 200 years, but reached an apogee in the mid 1980s. Since then, fossil-based  $EROI$ s have tended to decrease as the rich fossil reserves have become more

scarce—forcing a move to poorer grade or more difficult to reach resources; and to longer shipping distances as local resources became depleted or uneconomic. Additionally, societal focus on reducing the environmental impact of energy supply has led to energy-consuming additions to O&M operations which have generally reduced *EROI*.

Technologies for wind, solar, and nuclear infrastructures are relatively immature on the time scales of energy infrastructure evolution, and as such they entail significant remaining opportunities for innovation and improvement. Wind and the photovoltaics in particular have experienced extensive technological progress during the past decade. Thus the parameters we employed for the illustrative calculations of Table 1 are based on data that can be updated. Table 2 presents estimates based as far as possible on current assumptions and data for these technologies.

Let us consider first the case of wind. Kubiszewski *et al* (2010) present a meta-analysis of *EROI* for 119 wind turbines from 50 studies between 1977 and 2007. Their analysis notes that there is a positive relationship between *EROI* and the power rating of the new generation of windmills which they attribute to: (i) utilization of more advanced technologies (e.g. improved aerodynamic designs) which permit the newer, larger windmills to achieve higher levels of energy conversion efficiency; (ii) greater hub height and rotor diameters on new windmills—wind speed generally increases with height, and the power delivered by the turbine is proportional to the cube of wind-speed and the square of its diameter; and (iii) economies of scale. As a result, the significant increase in the power rating of wind turbines in recent years has been accompanied by a substantial increase in their *EROI*.

The average *EROI* for all the studies (operational and conceptual) surveyed by Kubiszewski *et al* is 25.2 ( $n = 114$ ,  $std.dev = 22.3$ ) while the corresponding average for just the operational studies is 19.8 ( $n = 60$ ,  $std.dev = 13.7$ ). If we use the average *EROI* from the operational studies, then from (2) and (4)—assuming  $T = 30$  and a constant capitalization-to-variable expenditure ratio—the implied value for the energy payback period  $\tau_1$  is approximately .98 years. From (11) and (12), the corresponding growth parameter  $\alpha$  and doubling time  $\tau_2$  of wind infrastructure are estimated to be .15 and 4.80 years respectively. The large variance in the published *EROI* estimates raises doubts about the meaning of representative (e.g. average) values. Moreover, some existing studies do not properly account for the backup capacity that is needed to compensate for wind fluctuations and thus some reported *EROI* ratios are overestimated. As wind penetration rises above certain threshold levels, the required investment in backup capacity and the supporting grid could potentially increase nonlinearly. While the surplus energy that can be realistically delivered by wind needs to be studied very carefully, the trend is unmistakable. The performance metrics of wind power, especially those related to sustainability, have experienced substantial improvements in recent years and now compare favorably to those of baseload electricity mainstays coal and nuclear fission.

The figures of merit for PV systems have been a matter of considerable controversy during the past decade. Some studies have asserted that PV systems will never achieve an *EROI* greater than 1—i.e., their cumulative electricity generated will be less than their embodied primary energy. The energy costs of refining and purifying crystalline silicon and

the complex process of turning silicon wafers into PV cells are mainly responsible for the high fraction of gross production expended for the PVs' initial emplacement. Such studies have raised doubts about the sustainability of PV systems (Odum, 1996). More recent studies, however, reflecting the significant technological change of the last few years, claim that PV systems are able to reimburse their initial energy requirements within a few years of operation (Bankier and Gale, 2006). Thus, there is a huge variance across studies in their estimates of *EROI* and the payback time  $\tau_1$  for PV systems—reflecting both methodological differences (choice of system boundaries) as well as differences in cell manufacturing technology (silicon purification and crystallization process), insolation, type of encapsulation, frame and array of support, and the type of system application (autonomous vs grid-based).

A recent report based on data from 26 OECD countries presented the following estimates: (i) for roof-mounted PV systems (taking into account not only the panels but also the wires and electronic devices) the payback time  $\tau_1$  is in the range of 19 to 40 months and for PV-facades the corresponding range is 32 to 56 months; (ii) *EROI* is between 8 to 18 for roof-mounted systems and between 5.4 to 10 for facades (IEA, 2006). Based on these ranges, the implied values for the growth parameter  $\alpha$  are between .03 and .08 for roof-mounted systems and between .01 and .04 for facades; the doubling time  $\tau_2$  is: between 8.3 and 27.5 years for roof-mounted systems and between 17.8 and 75.7 years for facades. A study by Fthenakis and Alsema (2006), based on more recent Life Cycle Inventory data from 12 European and US photovoltaic companies, estimated the payback times for ribbon, multi-, and mono-Si technologies to be 1.7, 2.2, and 2.7 years respectively. The corresponding doubling times then are estimated to be 9.11, 13.07, and 18.20 years.

The *EROI* estimates for nuclear power have been equally disparate. As with the other generating technologies, one of the main issues has been defining the appropriate boundaries of analysis. For nuclear lifecycles employing the gaseous diffusion technology, fuel enrichment is the key energy input—more than half the lifetime total. Centrifuge technology reduces enrichment energy consumption to 1/12 of its diffusion-based value and correspondingly reduces  $h$ , the O&M component of *EROI*, down to a value closer to  $\frac{E}{P_{np}\varphi T}$ , the initial emplacement component. During the past two decades, nuclear power plants have also achieved increasingly higher capacity factors. In the United States, for example, the average capacity factor  $\varphi$  for plants in operation in 1980 was 56.3 percent. By 1990, it had risen to 66 percent, and in 2007 to 91.8 percent. Similar developments occurred in other countries. In Russia, capacity factors increased from 56% in 1998 to 75% in 2007 (NEI, 2008). Most of today's nuclear plants were originally designed for 30 or 40-year operating lives. However, improvements in system components and upgrading of structures have extended effective plant lifetimes. Reflecting these technical improvements, many of the currently operating reactors in the United States are expected to be granted licence extensions raising  $T$  from 40 to 60 years (Nuclear News, May 2009).

The following set of calculations is based on the most recent data reported in WNA (2009) on the total energy required to build and run a nuclear power plant and the lifecycle data provided by Vattenfall for its 3090 MW Forsmark power plant in Sweden. These

	$EROI$	$T$	$\varphi$	$\tau_1$	$\frac{E}{\varphi P_{np} T}$	$h$	$q = \frac{E}{P_{np}}$	$\tilde{t}$	$\beta$	$\alpha$	$\tau_2$
Nuclear	74	40	.85	.05	.001	.012	.043	6	.2	.90	.8
Wind	20	30	.20	.98	.032	.018	.193	1	.2	.15	4.8
PV	5.4-10	30	.15	4.67-2.67	.150-.088	.035-.012	.676-.395	1	.2	.01-.04	75.7-17.8

Table 2: Technology-induced Improvements in the Energy Balance and Growth Potential of Renewables and Nuclear Power

data permit us to compute the fraction  $\frac{E}{\varphi P_{np} T}$  of gross production expended for the initial emplacement of the plant and the fraction  $h$  of O&M plowback. From (2), (11) and (12) the implied values for  $EROI$ ,  $\alpha$ , and  $\tau_2$  are 74, .90, and .8 years respectively.

A comparison of tables 1 and 2 reveals significant improvements in the performance metrics of nuclear power, wind, and solar. The huge increase in nuclear power's  $EROI$  comes about from converting to centrifuge technology and the improvements in capacity factors and expected lifetime of nuclear plants. Significant increases in the power rating of wind turbines and concomitant reductions in the fraction of gross production expended for initial emplacement and for O&M led to an equally impressive increase in the  $EROI$  and dramatic reductions in the payback and doubling times of wind power. Similarly, improvements in the manufacturing technology, refining and purification processes of silicon, led to significant increases in the  $EROI$  of PV systems.

## 5. Summary of Underlying Technological Characteristics

The historical evolution of energy infrastructures has led to very suitable energy carriers (electricity and liquid chemical fuels) for the distribution links of the supply chain. These carriers exhibit high energy density and exceptional versatility and convenience. They are unlikely to experience significant substitution in the next several decades.

Table 3 provides a summary of the innate differences among the alternative electricity generation technologies.

### Longevity, Environmental Friendliness, and Carbon Intensity

Based on current consumption, proven economic global reserves of coal will last another 137 years, natural gas 63 years, and oil 43 years (EIA, 2009). Probable reserves for oil and natural gas may be substantially higher. Extracting these reserves will eventually become more difficult and costly, putting upward pressure on prices and the required energy plowback—though how quickly remains a matter of debate. Regardless of how rapidly prices rise, however, they are likely to remain volatile. Thus there are fundamental issues

related to the longevity (timescale of availability) of the primary energy source in the current fossil-based global energy pathway (Hall, 2004).

Both renewables and nuclear can meet the longevity and environmental friendliness criteria for sustainable development. Renewable energy sources are indigenous and abundant. Solar and wind power are inexhaustible as long as the sun shines and it does so unevenly. Hydroelectric power is the world's largest source of renewable electricity, with considerable unexploited potential. Less than a fifth of technically exploitable hydropower has been used to date (WEC, 2007). Much of the potential for new development is in Asia, Latin America, Africa, and the former Soviet Union. Uranium is present at a concentration of 2 - 3 parts per million in the Earth's crust which is about 600 times greater than that of gold. The total known global uranium reserves which can be mined at less than \$US 130 are estimated to be around 4.7 million tonnes. At current rates of usage, in a once-through cycle these uranium resources would be adequate to supply the world's nuclear power stations for at least 100 years. Deployment of fast breeder reactors and recycling of transuranic from recycled spent fuel could lengthen this period by 50-fold or more and thus increase the long-term availability of nuclear energy to several thousand years. It should also be noted that in recent years, growing demand and higher prices have spurred increased investment in exploration activities and led to substantially larger identified uranium resources. Indeed, the identified world reserves of uranium have risen by over 50% since the end of 2003. In Russia alone, proven uranium reserves have reached 545,000 metric tons, a 275 percent increase from 2006. Speculative uranium resources, i.e. deposits that are likely to be discovered on the basis of the geological characteristics of already identified resources, have risen to over 10.5 million tonnes (NEA, 2008).

Renewable technologies and nuclear power have significant environmental benefits compared to fossil fuel based electricity generation. Under normal operating conditions, wind, solar, and nuclear plants produce almost none of the residual fossil fuel pollutants which include particulate matter, sulfur dioxide, nitrogen oxides, and a variety of heavy metals—mercury being the most prominent. In the case of nuclear power, small quantities of radioactive gases are emitted under controlled conditions imposed and supervised by regulatory authorities and pose no significant threat to plant workers or surrounding populations. Moreover, numerous LCA cradle-to-grave studies point to a very substantial gap between coal, oil and gas at the high end and wind, hydro, solar, and nuclear at the low end, in terms of their CO<sub>2</sub> emissions per unit of electricity generated—the average estimates being 1.2 kg CO<sub>2</sub>/kWh for coal, .4 kg CO<sub>2</sub>/kWh for gas, .06 kg CO<sub>2</sub>/kWh for solar PV, .011 kg CO<sub>2</sub>/kWh for wind (onshore), .008 kg CO<sub>2</sub>/kWh for nuclear, and .005 kg CO<sub>2</sub>/kWh for hydro (OECD/NEA, 2008).

### Capacity to Deliver Net Excess Energy

There is substantial variance across the different electricity generation options in terms of their lifetime capacity to deliver net excess energy. With the exception of hydro, renewables



Resource	Energy Density	Longevity	Carbon Footprint	Scalability	Growth Rate	Surplus Energy
Oil	High	Decades	Heavy	High	High	High
Gas	High	Decades	Heavy	High	High	High
Coal	High	Centuries	Heavy	High	High	High
Nuclear	High	Centuries	Negligible	High	High	High
Hydro	Low	Unlimited	Negligible	Moderate	High	High
Wind	Low	Unlimited	Negligible	Moderate	Moderate	High
Solar	Low	Unlimited	Negligible	Moderate	Limited	Low

Table 3: Characteristics of Alternative Electricity Supply Infrastructures

are constrained because of their low energy density and low capacity factors. Hydropower has the highest performance with *EROI* ratios between 50 and 260—exclusive of required backup to deal with precipitation variations and seasonal intermittency. It is followed by nuclear power whose high *EROI* is driven by the extraordinary energy density of uranium. Coal-fired generation is constrained by the high energy costs of mining and transporting its resource fuel. Carbon capture and sequestration will further depress coal’s *EROI*. Wind exhibits very good and steadily improving performance, although its intermittency is a constraining factor. Moreover, as wind penetration exceeds certain threshold levels, the required investment in backup capacity and the supporting grid will increase significantly.

#### Capacity to Support Rapid Growth

All generating technologies have an *EROI* exceeding 1.0. Thus, it is a mythconception that the manufacturing of solar panels consumes more energy than PV systems will ever deliver during their lifetime (MacKay, 2008). Still, the optimism driven by a positive *EROI* could be misplaced because the comparative assessment of the energy surplus delivered by alternative generating technologies is useful only in a steady state. It is less so under conditions of rapid growth of demand and during periods of major energy transitions—like the one the world is facing today. Given that a significant portion of renewables’ lifetime energy production is expended for their initial emplacement, too rapid a growth of these infrastructures could potentially lead to the cannibalization of the energy of existing power plants. Thus the front-loading of energy investment that is characteristic of renewables might indeed lead to a renewables hump.

Our dynamic energy analysis framework facilitates a careful comparison among the alternative generating technologies in terms of their capacity to support rapid growth under conditions of self-plowback. The estimated values of the growth parameter  $\alpha$  and doubling time  $\tau_2$  from tables 1 and 2 reveal that conventional fossil fuels, nuclear power, and hydro, exhibit the greatest capacity to support rapid growth. On this measure, renewables are not performing as well mainly because of the intermittency and low conversion efficiency and power density of their energy flows. However, as it was noted above, the performance of wind power and PVs has been improving steadily.

#### Scalability Potential

There are two components to consider for scalability potential: total availability (joules); intensity (joules/m<sup>2</sup>). Already, the majority of the world's population lives in cities and a high population density demands a high energy density of supply—i.e. intensity. To support sustainable development for a world population centered in cities, the infrastructure must be capable of intense supply.

The renewables infrastructures rely on harvesting and converting a dilute but widespread resource. As the harvesting area expands, shipping distances and the associated energy consumption devoted to resource shipping rise. Thus, as scale increases it drives excess energy availability downwards.

Hydro and wind benefit from relying on natural processes for delivering the energy resource to the converter node—rain water collecting and running downhill, and wind generated by atmospheric pressure inhomogeneities. Transportation costs are nearly zero. However, since the energy density of the carrier is relatively small, the wind and water conversion process requires the harvesting of a large flux of resource to generate any significant levels of power. This limits the scalability potential of hydro and wind. Deployment of these technologies at the terawatt level is also impeded by their intermittancy.

In the case of hydro, river systems collect and concentrate rainwater over their drainage area which may comprise thousands of square miles and significant fractions of a continent. The Three Gorges Dam (the largest hydroelectric power station in the world), for example, controls a drainage area of 1 million km<sup>2</sup> with an average annual runoff of 451 billion m<sup>3</sup>. To illustrate the large land area that is needed to obtain a given amount of hydroelectric energy from rain water, we consider a region with an average rainfall of .5 m per year and an altitude of 100 m above the sea level. Given water's density of 1000 kg/m<sup>3</sup> and the gravitational force of 10 m/s<sup>2</sup>, then the total gravitational potential energy of the water that falls during the year on an area of 1 m<sup>2</sup> is approximately 500,000 joules.<sup>4</sup> Thus, the power per unit of land derived from rainfall is approximately .02 W/m<sup>2</sup>. To obtain 1 kWh per day (which would light a 40 W lightbulb switched on all the time) would require a catchment area of approximately 2,000 m<sup>2</sup>. We also consider a geographic region with a higher rainfall of 2 m per year and elevation of 300 m. The power per unit of highland is approximately .24 W/m<sup>2</sup>. However these calculations do not take into account evaporation and assume that every drop of rain water is fully exploited. Under the very optimistic assumption that 30% of this potential is exploited, we obtain a limit of .07 W/m<sup>2</sup> for this more favorable geographic region (MacKay, 2008). Clearly, hillsides, valleys, aqueducts and tunnels act as power concentrators. However, to the extent that the sites with the most effective concentrators have already been tapped, the low energy density of rain as it runs down to sea level limits the scalability of hydro.

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<sup>4</sup>The gravitational potential energy  $U_g$  of an object with mass  $m$  that is lifted to a height  $h$  is given by  $U_g = mgh$  where  $g$  is the gravitational acceleration constant. Thus,  $U_g = \text{rainfall (m per year)} \times \text{area (m}^2\text{)} \times \text{density of water (kg/m}^3\text{)} \times \text{gravitational acceleration (m/s}^2\text{)} \times \text{elevation above sea level (m)} = .5 \text{ (m)} \times 1 \text{ (m}^2\text{)} \times 1000 \text{ (kg/m}^3\text{)} \times 10 \text{ (m/s}^2\text{)} \times 100 \text{ (m)} = 500,000 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$ .

MacKay (2008) estimates that the power per unit of land area of a wind farm with an average wind speed of 6 m/s and where windmills are spaced 5 times their diameter, is approximately  $2 \text{ W/m}^2$ .<sup>5</sup> According to the IEA's *World Energy Outlook 2009*, electricity generation in the United States will reach 5,300 TWh by 2030. A "20% Wind Energy by 2030" scenario would require approximately 60,000 km<sup>2</sup> of land for the siting of wind farms where over 100,000 turbines would be installed (USDOE, 2008). Under the ambitious scenario of the Global Wind Energy Council, world wind energy capacity would increase from 121 GW at the end of 2008 to 2,400 GW by 2030 (GWEC, 2009). Similarly, under the IEA's 450 ppm Scenario, wind onshore generation would increase from 173 TWh in 2007 to just over 2,000 TWh by 2030 (IEA, 2009). The vast amount of land that must be covered will be a significant but not insurmountable barrier to the expanded deployment of wind power. More importantly, however, deployment of wind power at the terawatt level would impose substantial requirements for several key materials (e.g. about 9 tons of fiberglass are required per megawatt of wind turbine capacity) and manufacturing capacity.

The amount of energy provided by sunlight is staggering. Indeed, the total amount of energy humans use annually (around 12,000 Mtoe or  $5 \times 10^{20}$  Joules in 2007) is radiated by the Sun to Earth in just over one hour (Crabtree and Lewis, 2007). Yet, in 2007, with a total capacity of just 53 GW, solar's share in world electricity production was less than .03% (IEA, 2009). Thus there is a huge gap between our present use of solar energy and its enormous potential. The barriers to greater use of the huge solar resource are the modest power density of solar radiation and its low conversion efficiency.

Though its potential is huge, solar radiation has a modest energy density and is intermittent. The average power density of the sun's radiation at Earth's distance is approximately  $1.4 \text{ kW/m}^2$ . However, some of this energy is deflected and absorbed in Earth's atmosphere. On a sunny day at ground level we receive about  $1000 \text{ W/m}^2$  per square meter of area perpendicular to the Sun. To estimate the power per square meter of land area in a specific location we need to: (i) compensate for the tilt between the sun and the land in that location; (ii) take into account the effect of cloud cover; (iii) compute the average intensity which is lower than the midday intensity; and (iv) adjust for the wobble of the seasons. Once these factors taken into account, the average raw power of sunshine per square meter of typical rooftop of a northern country (e.g. England) is around  $110 \text{ W/m}^2$ . The corresponding average raw power for flat ground is approximately  $100 \text{ W/m}^2$ . Some of the more expensive photovoltaic panes that convert sunlight into electricity have an efficiency of

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<sup>5</sup>The kinetic energy per unit of time of a piece of air going through a hoop of cross-sectional area  $A$  is equal to  $\frac{1}{2}\rho Av^3$  where  $\rho$  is the density of air and  $v$  its speed. Thus the power of a windmill with diameter  $d$  will be given by  $\zeta \cdot \frac{1}{2}\rho v^3 \pi \frac{d^2}{4}$ , where  $\zeta$  is the windmill's efficiency factor (i.e. the maximum fraction of the incoming energy that can be extracted by the windmill). If we assume that the windmills are 50% efficient and are spaced from each other at a distance of 5 times their diameter (closer spacing could lead to a significant loss of power), then the power per windmill divided by the land area per windmill will equal  $\frac{(\frac{1}{2}) \cdot \frac{1}{2}\rho v^3 \pi \frac{d^2}{4}}{(5d)^2} = \frac{\pi}{200} \frac{1}{2}\rho v^3$ . For air density of  $1.3 \text{ kg/m}^3$  and wind speed of 6 m/s, this lead to a power density of  $2.2 \text{ W/m}^2$ .

around 20%.(MacKay, 2008). Because of their low conversion efficiency, the average power delivered by such panels would be just around 22 W/m<sup>2</sup>.

In summary, the low energy density of renewable resources places an innate bound on their scalability. Nuclear power, exhibits an enormous energy density of the resource energy carrier (uranium). Shipping costs are correspondingly small. The resource flux through the converter is small and the energy density of the converter (reactor core) is quite high (100 to 800 MW/m<sup>3</sup>). There is no innate scalability constraint on nuclear power.

## 6. Matching Innate Characteristics to Diverse Missions

As summarized above, energy infrastructures are distinguished by innate limits that derive from the energy density of the resource energy carriers and the converters. It is important to identify categories of end use application that are suited to these innate properties of the several energy infrastructure types.

Scalability and capacity for rapid growth are not important where population density and economic growth rates are low. This is a characteristic of rural areas, especially in developing countries where population redistribution to cities is the future trend. This may also be the case for suburban areas of developed countries where economic growth rates are moderate and stabilized. Call these "Category 1 missions".

Scalability and capacity for rapid growth are important where population density is high and rising and where the rate of economic growth rate is high. Such is the case in the cities of many developing countries. Call these "Category 2 missions".

### Where Renewables Meet Requirements

The low energy density of renewables presents a disadvantage for Category 2 missions by increasing the transportation components of *EROI* and lowering its value. Additionally their low capacity factor lengthens their attainable doubling time which is a second disadvantage for Category 2 missions.

On the other hand, the properties of renewables suit them well for Category 1 missions. This is because growth rates are not high; and, in a distributed architecture of local collection, conversion, and use, the shipping component of *EROI* plays no role. For example, when deploying rooftop solar to provide hot water or electricity for a building, the full energy cost of emplacing the infrastructure will have a small transportation component in *EROI* because the panel is the collector and converter and the delivery is confined to a single building. Also, for small additions to an established local grid (e.g. for a small wind farm), the incremental addition may fit within the excess capacity of the grid and simply displace alternative existing energy sources on the delivery side.

These situations can be expected to be found in rural areas. Even in urban areas there are instances among the Category 1 missions where the transportation infrastructure to deliver fuel to the converter and to deliver product to the end user is already in place; therefore the energy required to expand those transportation infrastructures need not be included in

the denominator of *EROI*. This is a good assumption in instances where decommissioned assets are being replaced in the steady-state. It could be the situation in developed countries where legacy fossil is coming offline, growth is slow, and renewables are socially preferred over nuclear.

### Where Nuclear Meets Requirements

Nuclear has a high *EROI* is scalable and can support rapid growth. These are the requirements for Category 2 missions—missions found primarily in the cities of the currently transitioning and developing countries.<sup>6</sup>

Even in developed countries where economic growth rate is modest at 1.5 - 2% per year, the decommissioning of legacy infrastructure with lifetimes  $T \sim 40$  years requires an overall emplacement rate of new infrastructure in the range of

$$\alpha \simeq 1.5 \text{ to } 2.0 + \frac{1}{40} = 4.0\% \text{ to } 4.5\% \text{ per year}$$

which exceeds the capability of some renewables (see Table 2). However, in slow growing developed countries renewables may find use in situations where the supply and delivery infrastructure is already in place or where shipping is not required (e.g., rooftop solar). Additionally, where the existing resource shipping infrastructure is saturated, a major payoff from converting to a higher energy density carrier can be attained. For example, if an existing transportation infrastructure can be switched from delivering a lower energy density carrier to deliver a higher energy density fuel to the conversion node, then the transportation infrastructure need not be expanded, and no additional term need be added in the denominator of *EROI*.

### Where Symbiosis Meets Requirements

Mixed renewable/nuclear infrastructures operating in symbiosis may find future application. For example, new designs of nuclear power plants downsized for local grids and capable of load follow operation may find deployment in tandem with wind farms to solve intermittency issues. In fast-growing cities of developing countries, the nuclear characteristics may best meet the needs. In depopulating rural areas of developing countries, on the other hand, renewables may represent the most appropriate choice. In remote small towns having difficult or intermittent transportation links, either renewables or small nuclear reactors of long refueling interval may best meet the needs.

## **7. Summary**

Market systems exhibit remarkable adaptability and flexibility, and market forces can be expected to play a crucial role in addressing the global energy challenge. With appropriate

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<sup>6</sup>However, as van der Zwaan (2010) rightly points out, for nuclear power to be successfully deployed on a large global scale the international community must tackle three important issues: nuclear safety, waste disposal, and proliferation. Unless these issues are sufficiently addressed, nuclear power's share in the global energy mix will not grow substantially or, if it does, there could be calamitous externalities.

public policies, powerful price signals will help promote new technologies for energy supply, distribution, and use, and ultimately facilitate the transition to a sustainable energy supply infrastructure. However, rising concerns about climate disruption have compressed the timescale of the requisite transition to an energy-efficient low carbon global economy. Due to over-enthusiastic predictions in the past related to the potential contributions of alternative technologies (both nuclear and renewables) to the global energy mix, there is an urgent need for a hard nosed assessment of the low carbon supply options. A reality check is in order.

The paper's dynamic energy analysis framework focuses on the innate physical energy balance and dynamic response limits of alternative electricity supply technologies. Our analysis clarifies the impacts of such fundamental physical attributes as the power density of energy production and the intermittency of energy flows on the growth potential of these supply options. Technologies with significant underlying cumulative energy demand and concomitant payback periods that are long relative to their effective lifetime are constrained in terms of their capacity to achieve any reasonable rates of self-sustained growth.<sup>7</sup> This is the case for most renewables. Our physical assessment indicates that, especially in view of the massive scale of the prospective shift, the transition to a non-fossil world is likely to prove much more challenging than optimists have been claiming. The low power density of renewable energy extraction and the intermittency of renewable flows limit considerably their ability to achieve high rates of indigenous infrastructure growth.

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<sup>7</sup>Cumulative energy demand of a given generating facility is the total quantity of primary energy needed to emplace, use, and decommission the facility. Thus it represents the facility's lifecycle load on primary energy resources (Mathur et al, 2004).

## References

- [1] Bankier, C., and S. Gale. 2006. "Energy Payback of Roof Mounted Photovoltaic Cells." *The Environmental Engineer* 7(4): 11-14.
- [2] Bartis, J., Camm, F., and D. Ortiz. 2008. *Producing Liquid Fuels from Coal: Prospects and Policy Issues*. Santa Monica, CA, Rand Corporation.
- [3] Crabtree, G., and N. Lewis. 2007. "Solar Energy Conversion." *Physics Today*, March.
- [4] Fthenakis, V., and E. Alsema. 2006. Photovoltaics Energy Payback Times, Greenhouse Gas Emissions and External Costs: 2004-Early 2005 Status. *Progress in Photovoltaics: Research and Applications* 14: 275-280.
- [5] GWEC (Global Wind Energy Council). 2009. *Wind Power is Crucial for Combatting Climate Change*. [www.gwec.net/uploads/media/Wind\\_\\_\\_climate\\_fact\\_sheet\\_low\\_res.pdf](http://www.gwec.net/uploads/media/Wind___climate_fact_sheet_low_res.pdf)
- [6] EIA (Energy Information Administration). 2009. *International Energy Outlook*. Washington, D.C.
- [7] Hall, C. 2004. "The Myth of Sustainable Development: Personal Reflections on Energy, its Relation to Neoclassical Economics, and Stanley Jevons." *Journal of Energy Resources Technology*. Vol.126: 85-89.
- [8] Hall, C., and C. Cleveland. 2005. "EROI: Definition, History and Future Implications." Presented at ASPO-US Conference, Denver, November 10, 2005.
- [9] Hall, C., Cleveland, C., and R. Kaufmann. 1986. *Energy and Resource Quality: The ecology of the economic process*. Wiley Interscience, NY. (Second Edition. University Press of Colorado).
- [10] IEA (International Energy Agency). 2002. *Environmental and Health Impacts of Electricity Generation*.
- [11] IEA (International Energy Agency). 2006. *Compared Assessment of Selected Environmental Indicators of Photovoltaic Electricity in OECD Cities*. Report IEA-PVPS-T10-01:2006
- [12] IEA (International Energy Agency). 2009. *World Energy Outlook*. Paris: OECD.
- [13] Kenny, R., Law, C., and J. Pearce. 2010. "Towards real energy economics: Energy policy driven by life-cycle carbon emission." *Energy Policy* 38 (2010): 1969-1978.
- [14] Koonin, S. 2008. "Energy, Environment, Security: Can We Have It all?" CISAC Drell Lecture, Stanford University.

- [15] Kubiszewski, I., Cleveland, C., and P. Endres. 2010. "Meta-analysis of Net Energy Return for Wind Power Systems." *Renewable Energy* 35(1): 218-225.
- [16] Lenzen, M., and C. Dey. 2000. "Truncation Error in Embodied Energy Analyses of Basic Iron and Steel Products." *Energy* 25(6): 577-85.
- [17] Lenzen, M., and G. Treloar. 2003. "Differential Convergence of Life-Cycle Inventories Towards Upstream Production Layers." *Journal of Industrial Ecology* 6(3-4): 137-60.
- [18] MacKay, D. 2008. *Sustainable Energy—without the hot air*. UIT, Cambridge, United Kingdom.
- [19] Meier, P. 2002. *Life-Cycle Assessment of Electricity Generation Systems and Applications for climate Change Policy Analysis*. Ph.D. thesis, University of Wisconsin.
- [20] Mathur, J., Bnsal, N., and H-J. Wagner. 2004. Dynamic Energy Analysis to Assess Maximum Growth Rates in Developing Power Generation Capacity: Cade Study of India. *Energy Policy* 32(2004): 281-87.
- [21] NEI (Nuclear Energy Institute), 2008c. Nuclear Statistics.
- [22] NEA (Nuclear Energy Agency).2008. Uranium Resources Sufficient to Meet Projected Nuclear Energy Requirements Long Into the Future." [www.nea.fr/html/general/press/2008/2008-02.html](http://www.nea.fr/html/general/press/2008/2008-02.html)
- [23] Odum, H. 1996. *Environmental Accounting: Energy and Environmental Decision Making*. New York: John Wiley & Sons.
- [24] OECD/NEA (Organisation for Economic Co-operation and Development / Nuclear Energy Agency). 2008. *Nuclear Energy Outlook 2008*. OECD: NEA No. 6348.
- [25] Pearce, J. 2008. "Limitations of Greenhouse Gas Mitigation Technologies Set by Rapid Growth and Energy Cannibalism" *Climate 2008/Klima 2008*. [www.climate2008.net/?a1=pap&cat=1&e=61](http://www.climate2008.net/?a1=pap&cat=1&e=61)
- [26] Pearce, J. 2009. "Optimizing Greenhouse Gas Mitigation Strategies to Suppress Energy Cannibalism." 2nd Climate Change Technology Conference, May 12-15, Hamilton, Ontario, Canada.
- [27] Smil, V. 2008. "Moore's Curse and the Great Energy Delusion." *The American*, November-December.
- [28] Smil, V. 2010. *Energy Transitions: History, Requirements, Prospects*. Westport, CT: Praeger.
- [29] Stephens, J. and B. van der Zwaan. 2005. "The Case for Carbon Capture and Storage." *Issues in Science and Technology* XXII (1): 69-76.



- [30] USDOE (United States Department of Energy). 2008. *20% Wind Energy by 2030: Increasing Wind Energy's Contribution to U.S. Electricity Supply*. DOE/GO-102008-2567.
- [31] van der Zwaan, B. 2005. "Will Coal Depart or Will It Continue to Dominate Global Power Production during the 21st Century?." *Climate Policy* 5 (4): 445-453.
- [32] van der Zwaan, B. 2006. "Technological Innovation in the Energy Sector: R&D, Deployment, and Learning-by-Doing." *Energy Policy* 34(17): 2601-2608.
- [33] van der Zwaan, B. 2010. The Nuclear Wedge. ECN Policy Studies, Report Number ECN-V-10-015. [http://sipa.columbia.edu/students/sea/envision/documents/Envision\\_Spring\\_2010\\_p44-47\\_Nuclear\\_Wedge.pdf](http://sipa.columbia.edu/students/sea/envision/documents/Envision_Spring_2010_p44-47_Nuclear_Wedge.pdf)
- [34] Vattenfall. 1999. *Vattenfall's Life Cycle Studies of Electricity*. Vattenfall AB.
- [35] WEC (World Energy Council). 2007. *Survey of Energy Resources*. [www.worldenergy.org/documents/ser2007\\_final\\_online\\_version\\_1.pdf](http://www.worldenergy.org/documents/ser2007_final_online_version_1.pdf)
- [36] WWEA (World Wind Energy Association). 2010. *World Wind Energy Report 2009*. WWEA, Bonn, Germany.

## Annex

Equations (9) and (10) comprise a homogeneous system of first order differential equations which can be solved with the eigenvalue method. We can write this system as

$$\begin{bmatrix} \dot{P}_{np}(t) \\ \dot{C}(t) \end{bmatrix} = A \begin{bmatrix} P_{np}(t) \\ C(t) \end{bmatrix} \quad (16)$$

$$\text{where } A = \begin{bmatrix} -\frac{1}{T} & \frac{1}{\tilde{t}} \\ \frac{\beta}{\tau_1} & -\frac{1}{\tilde{t}} \end{bmatrix}.$$

Let  $p(\alpha)$  denote the characteristic polynomial of  $A$ . The eigenvalues  $\alpha$  of  $A$  are the solutions of  $p(\alpha) = |A - \alpha I| = \alpha^2 + (\frac{1}{\tilde{t}} + \frac{1}{T})\alpha - \frac{1}{\tilde{t}}(\frac{\beta}{\tau_1} - \frac{1}{T}) = 0$  which are given by

$$\alpha_{1,2} = \frac{-\left(\frac{1}{\tilde{t}} + \frac{1}{T}\right) \pm \sqrt{\left(\frac{1}{\tilde{t}} + \frac{1}{T}\right)^2 + \frac{4}{\tilde{t}}\left(\frac{\beta}{\tau_1} - \frac{1}{T}\right)}}{2}. \quad (17)$$

If  $\beta \neq 0$  and  $\tilde{t} \neq T$ , then there are two distinct real solutions. Moreover, if  $\frac{\tau_1}{\beta} < T$  or equivalently  $\beta(1-h)\varphi P_{np}T > E$  (i.e. the amount of energy that is plowed back into the system during its lifetime exceeds the amount of energy that was expended to emplace the infrastructure), then one of the eigenvalues ( $\alpha_1$ ) is positive. This eigenvalue sets the upper bound on the rate of energy supply growth under a self plowback constraint. The second eigenvalue ( $\alpha_2$ ) is negative.

If the initial boundary conditions are

$$P_{np}(t_0) = P_0 \quad (18)$$

$$C(t_0) = \tilde{t} \frac{\beta(1-h)\varphi P_0}{q} = \frac{\tilde{t}}{\tau_1} \beta P_0 \quad (19)$$

then, the persisting solution is

$$P_{np}(t) = P_0 e^{\alpha_1 t} \quad (20)$$

$$C(t) = \frac{\tilde{t}}{\frac{\beta}{\tau_1} - \frac{1}{T}} \beta P_0 e^{\alpha_1 t}. \quad (21)$$

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