European University Institute
Department of Economics

Three Implications of Learning Behaviour for Price Processes

Matthias Rau-Göhring

Thesis submitted for assessment with a view to obtaining the degree of
Doctor of Economics of the European University Institute

Florence, October 2007

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Meinen Lehrern Wolfgang Schoppet (†) und Eberhard Simon.

Silence invades the breathing wood
Where drowsy limbs a treasure keep,
Now greenly falls the learned shade
Across the sleeping brows
And stirs their secret to a smile

W.H. Auden

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Pro domo

Doctoral studies are not exactly the 'warm and lucky mile' that one might think of initially. Instead, they are a long - and often lonely - walk in the twilight, a continuous struggle with the own personality.

But there is also another dimension, the one of personal and intellectual freedom, which makes this episode of life so valuable and unique. Having been able to go for this venturous 'walk' makes me grateful to those, who have enabled it or accompanied me throughout. In particular I wish to thank

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Part I

Methodological Issues between Epistemology and Ontology

"... we are always searching for something hidden or merely potential or hypothetical, following its traces whenever they appear on the surface." Italo Calvino, Six Memos for the Next Millennium (1986)

0.1 Introductory Remarks

We would certainly support the argument that economic modelling has to conceptualize in a way which is not a mirror image of reality. Especially those bids of an economic model which are unobservable are refutable in an isolated case. And model agents are not real agents at all and so the economists' interpretation of their models is not directed to a proper target. While tractability and parsimony should be guiding principles in (economic) modelling, however, prior to setting up an economic model, economists presumably try to get some sense of how real agents, whose behaviour they are trying to explain, are actually motivated. Model-building in economics has a serious intent if only if it is ultimately directed towards telling us something about the real world and therefore it has to be the role of any social scientist to interpret reality in a meaningful scientific way. This has been brought forward notably by the seminal publication of Samuelson (1947), according to whom a theory is “a set of axioms, postulates, or hypotheses that stipulates something about observable reality”.

However, the underlying paradigms of economic theory are often not helpful in explaining central questions or economic phenomena. One of these conspicuous blemishes is the way expectations formation processes are modelled. With its assumption of rational, self-seeking individuals, economic theory leaves out of account the economic consequences of variations in attitude, value, culture, ideology, collective behaviour or institutional restrictions. While it is impossible to come up with a sensible meta-model for expectations formation, a thorough model should contain both a sub-model of information acquisition as well as one of the learning process. But in the bow wave of the rational expectations revolution, the expectations formation process was washed away and marginalized in the solution process of the intertemporal optimization problem. On the other hand, an accurate model of expectations formation lies beyond the reach of economic science. In his complexity, the economic agent is essentially unpredictable. But this does not imply that patterns of his behaviour are indiscernible, as otherwise there could be no social science at all.

0.2 Friedman (1953) and its Repercussions

The roots of the standard modelling approach of expectations formation in economics can be seen in Milton Friedman's (1953) book on positive economics. Laying down the fundamental differences between normative and positive approaches to economics, it is Friedman's

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idea that assumptions do not need to be realistic in order for theory to be both predictive and relevant. While offering no clear definition of the term "assumptions", he recommends to regard them just as some 'as-if' statement and not as a realistic account.

Friedman's (1953) line of argument basically combines two strands of the literature, namely the inexactness-literature of John Stuart Mill (1836) and Karl Popper's (1935) falsificationist methodology. Mill's idea of economics as a 'separate and inexact science' (see Blaug (1980)) sets the methodological ground for ignoring behavioural aspects, as economics can per se not yield accurate predictions of the behaviour of individuals. Although Friedman (1953) does not make any direct reference to Popper's work, it is the "test of the validity of a hypothesis" which is at the center of his focus. Blaug (1978) elaborates that Friedman's (1953) work is Popperian, not only in character: Prediction is a means of testing the relation of a statement and reality, prediction is the sole criterion of verity and the only instrument of falsification.

But as Redman (1991) states, Popper's philosophy did not see falsification as a demarcation criterion rather than a trial-and-error-learning, where you make progress only if you learn from mistakes. Popper, who can be regarded as having been most influential on mainstream economic methodology, adapted the principle of verifiability of the Logical Positivists and replaced it with his principle of 'falsifiability' serving as the distinction between science and non-science. But Popper (1960) had a much broader research agenda in mind than just testable empirical statements: Following the contribution of Marschak (1943), the methodological aim should first and foremost be scientific and not practical in the sense to produce immediate economic policy responses. The numerical aspect with hypothesis testing is only the means in the endeavour to build a coherent economic theory. Only indirectly, the quantitative aspects will lead to practical consequence. But how well does this apply to economic theory, where probabilistic arguments matter?[1]

Popper (1960, p. 140-1) also nourishes the hypothesis of the rational man. "There are good reasons [...] for the belief that concrete social situations are in general less complicated than concrete physical situations. For in most social situations, if not in all, there is an element of rationality. Admittedly, human beings hardly ever act quite rational (i.e. as they would if they could make the optimal use of available information for the attainment of whatever ends they may have) but they act, none the less, more or less rationally [...]."

Economists have learned to see their own activities through the methodological eyes of Friedman and Popper. Many fundamental economic principles, such as those of expectations

[1] Note that a theory that makes only probabilistic predictions cannot be strictly falsified by any run of its alternative outcomes.

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formation, have been tested empirically with the aim to be falsified by empirical observations. However, by the standards of Popper, there is nothing like "epistemological certainty" (Redman (1991)). Moreover, Popper (1960) claims that verifying a model does not increase its chances of being true. Therefore any empirical foundation of any science is refutable: We can never know for certain.

0.3 Wither Positivism?

The positivist philosophy of science leaves us with important insights about a possible procedure, but it rests upon shaky foundations: If you look at the empirical implications, falsification has to be implementable and operational, which in many cases - especially once dealing with individuals‘ expectations - will prove an impossible undertaking. So what is it that economic methodology might improve upon compared to one of its main pillars?

Already Mises recognized that every method (belonging to the realm of epistemology) presupposes some explicit or implicit conception of reality (ontology), at least to the extent that the nature of reality is such that it can be the object of knowledge: knowledge is necessarily knowledge of something. That is to say that the episteme as well as the methods of the natural sciences entails certain ontological presuppositions that bear on under what kind of conditions their usage is appropriate. But opponents of ontological arguments argue that it is impossible for any human being to derive knowledge about reality which is inaccessible to empirical investigation. Therefore, it is also impossible to derive any ultimate evidence.[2]

In our point of view, a pinch of Thomas Kuhn‘s methodology could prove useful. As stated by Kuhn (1962), each discipline has unexamined postulates, core beliefs, which establish the ontology of the science and set the standards for judging its knowledge claims. Therefore, each discipline is bounded and defined by a particular system of perceptions embodied in a set of theories, hypotheses, techniques and assumptions, which he summarizes as the 'paradigm‘ of a system (Solo (1991)). These paradigms are regarded as the accurate model of scientific behaviour, at least in any positivist aproach. But if observed behaviour does not obey our (core) paradigms, are we then able to state that epistemological assumptions (i.e. everything derived from the conception of knowledge) should be downgraded compared to ontological assumptions (that is, assumptions that are concerned with the known world and the knowing agent), as the former are confined by their respective irrefutable paradigms?

Ontological arguments are made for what we would called "perennial reality", while

[2] One should note that this argument is already contradictory, as it entails knowledge which is inaccessible to empirical analysis.

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epistemological arguments are used in ergodic environments where agents have cognitive limitations that prevent them from using or analyzing (historical) data appropriately. But the cognition of reality remains fuzzy, once uncertainty enters.[3] Uncertainty implies that any action chosen implies a distribution of outcomes and these distributions overlap. This renders any attempt for maximization, which is nicely put forward by Keynes (1936, p. 162-3), who notes that "[w]e are merely reminding ourselves that human decisions affecting the future, whether personal or political or economic, cannot depend on strict mathematical expectations since the basis for making such calculations does not exist [...]."

While epistemic theories are not interested in what we called perennial reality, ontological theories ignore knowledge-related issues. But uncertainty is faced by exactly both features and reducing uncertainty has be mirrored in agents' behavioural assumptions. Only with the interplay of both concepts we can imagine economic agents which not only know or think, but who are able to learn and to create new knowledge.

We therefore truly believe that the reality of economics cannot be that of an never-ending sequence of falsifications or empirical discards. The economic discipline needs to recognize that ontological themes (e.g. rationality principle, maximization principle, focus on equilibrium phenomena) are rarely revealed and important ingredients of economic models are presupposed without being credited appropriately. Let us give three examples. Building an economic theory has become synonymous with mathematical modelling. For Lawson (2003) this proves to be one of the main fallacies of modern economics. He sees a kind of ontological indifference which needs to be reversed. But how should we be able to take this ontological turn to approach (objective) knowledge? We certainly do not propose to give up mathematical modelling in economic theory, which has been *the* major milestone within modern economic history. A second example has to do with the neoclassical focus on equilibrium. While equilibrium is rarely interpreted as a rest point of a dynamic process, considering the dynamics and the real effects of out-of-equilibrium behaviour however, has only been rarely studied (Jean-Michel Grandmont being among the staunchest advocates here). It is especially the discipline of game theory that tries to advance in said respect and the economics discipline should have a close look at results achieved there.

Before proceeding with possible ontological foundations, let me come up with a third example (modified from Brenner (1999)) which is able to summarize our argument. Consider that agents follow an investment decision according to Sharpe, Lintner and Mossin's capital

[3]Knight (1921) distinguishes between risk and uncertainty: The differentiation depends on whether probability estimates are or are not calculable on the basis of objective instances. This difference was later picked up by Keynesian theorists. Also Lucas (1977) states that only in situations of risk the behavioural aspects of Muth's rational expectations hypothesis make sense, while in situations of uncertainty economic reasoning will be of no value.

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asset pricing model (CAPM): Under the assumption that risk averse investors maximize their expected utility, building an efficient portfolio which minimizes the unsystematic risk is they way investors should proceed. Lets assume that traders and investors are trained in this respect and behave accordingly without reference to other tools of investment decisions. We have then transformed an initially normative problem into a positive one, which works if and only if people have confidence in the validity of the investment rule. But this is not the whole story, as the ontological argument is missing: People need to be capable in doing so and need to meet the cognitive abilities. But these are confined within the frame of the CAPM and no ontological discovery can take place here, leaving aside other potentially relevant investment theories.

Although contemporary economic theory tries to overcome the neoclassical foundations by clarifying its ontological assumptions, the problem of ontological opacity remains a real one. It is still difficult to read economists' ontological commitments off their models in any direct or simple way. Economists who routinely assume representative agent economies,[4] perfect competition, common knowledge of rationality and the like, would be surprised if such assumptions were interpreted as a sign of their being committed to the existence of these phenomena as real features of the world. It was only in the last years that a growing interest in modelling agents in a heterodox way started. Bounded rationality features, endogenous preferences, incomplete information etc more or less takes into account that economic agents cannot be modelled while at the same time renouncing the effects of institutions, culture, values and ethical norms.

As mentioned above, the ontological prerequisites of the methods of mathematical modelling used are rarely questioned or even acknowledged, at least not in any systematic or sustained way. As a result, the possibility of a lack of ontological fit (i.e. mismatch of the presuppositions of these modelling methods with those features of social reality being investigated) is not considered.

Ontological analysis reveals human beings to be structured and possessed of capacities that may or may not be exercised. As such, it can sustain the possibility that even if capacities of calculation are possessed, they may remain unexercised in certain contexts. Of course, mainstream economists tend to insist that behaviour is everywhere rational in the calculative sense, i.e. that the relevant capacities are always exercised.

A related example is the presumption that whatever the outcome associated with an action in one situation, the same outcome will follow from this particular action in all cases. Ontological analysis, however, reveals "social reality" (cf. Lawson (1993)) to be open, with

[4] See annex A.

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the likelihood that, in each different context a quite different array of accompanying causal forces and conditions will be in play, affecting the outcome that emerges.

This has fundamental implications with regard to the metaphysical method: it can never be purely deductive in the sense that one is searching for a distinct and clear insight as was done by Descartes at the beginning of modern history. However, the way economists model expectations formation processes is a classic example of deductivism in the aftermath of Friedman‘s (1953) influence. Next to the economic model at hand agents are expected to have a true data generating process, the existence of some true world that agents just approach by subjective means. But any kind of subjective cognition and knowledge is rooted in the existence of some primary reality. There exists only a partial isomorphism between our subjective world and the real world, and any object or matter is only recognizable for an agent, if it fits some category of cognition which is discernible for the agent. According to Kuhn (1962) scientists predict only what is predictable within their "respective frame", ignoring the predictive failures of established theories, which all too often causes researchers to speak of anomalies within their discipline (see e.g. Charles Holt‘s series of anomaly-papers in the Journal of Economic Perspectives).

Economic action depends to a great extent on plans which are itself based on expectations, and it is notably this aspect in which most economic models are unconvincing. While any empirically relevant economic theory needs to have a profound model of expectations formation, only rarely the issue of expectations formation is tackled at all. All too often, expectations in economic models are closed just by including a simple mathematical expected value, leaving aside the three main pillars of expectations formation (see Lewin (1927)), namely recognition of the environment, and choosing appropriate models for the description of the model and prediction. Also matters of uncertainty are usually ignored by either assuming static expectations, that is asserting that people believe that the environment will remain unchanged in the future, or perfect foresight, which claims that agents‘ beliefs will be verified as correct after the decision. These are extreme forms of the "rationality postulate" (cf. Blaug (1980)), meaning that agents choice depends on complete and transitive preference orderings, perfect and costless information, and expected utility maximization with correct probabilistic foundations. The rational expectations hypothesis has been the workhorse of economic theory since more than three decades by now. While serving as a useful normative benchmark, it is inherently related to what can be described as the 'stream of positivist thought' initiated by Friedman (1953). With its assumption of unbounded rationality, it is a direct heir of the 18th century episteme of classical physics but has little to do with an ontologically based approach.

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0.4 A Brief Historical Overview of Rational Expectations

In the considerations of classical economics since Adam Smith, expectations and opinions of economic agents did not have a significant role. Classical economists inquired into “natural laws” as determining factors of the economy, having in mind some pre-stabilized harmony or even a predetermined course of events. Therefore, expectations and uncertainty about future events did not play a (major) role in their considerations.

These methodological presumptions were inherited by the neoclassical school, which was chiefly interested in equilibrium states of the economy, in which optimality of utility and profit maximization ought to be given. For example, Walrasian economics has been predominantly concerned with equilibrium end-states demonstrating under which conditions efficiency in the allocation of economic goods is to be obtained. Neoclassical economists moved the axiom of a homo oeconomicus in the center of their image of man: Having complete market overview, the homo oeconomicus did not need to cope with uncertainty and expectations either. Instead he solely focuses on his self-interest, separates preferences and constraints and behaves rational/optimal.

It was not before the contributions of the Stockholm School (notably Myrdal and Lindhal) and Pigou (1929) that expectations entered into economic research. It was Hayek (1937, p. 41-2) who first treated equilibrium conditions in expectations in terms of an “expectational equilibrium”, in which expectations are correct. He can thus be seen as a precursor of the seminal contributions of Muth (1960, 1961). In the quarter century in between these publications, modelling expectations formation was at the forefront of economic research and tackled by all main economic schools. Tinbergen, Keynes, Hicks, Harrod, Shackle and Morgenstern are just the most prominent economists who studied expectations formation extensively, leaving adaptive processes and cobweb phenomena as the major results from expectations research, also because of a convenient econometric transformation due to Koyck (1954).

Muth (1960, 1961) first introduced the concept of rational expectations in a meaningful and precise way. But it was not until the early 1970s that rational expectations features were widely acknowledged and first included in macroeconomic models (Lucas (1972) and Sargent (1973), independently of each other). Rational expectations implied that expectations about future values of a variable should respond to changes in exogenous variables which itself affects the equilibrium values. Economic policy instruments are meant to be among these exogenous factors, s.t. policy changes should be reflected in changes of agents‘ behaviour.

But rational expectations has not been an unambiguous concept. Many people have criticized the rational expectations hypothesis on the basis that some of the assumptions

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used in rational expectations models are inconsistent with real world observations. However, even Muth (1961, p. 317) himself claimed that the rational expectations hypothesis "does not assert that the scratch work of entrepreneurs resembles the system of equations in any way; nor does it state that predictions of entrepreneurs are perfect [...]." Nevertheless, critics of the rational expectations formation approach almost universally picked on its adequacy with respect to real world expectations formation (being thus an anti-positivist, anti-Friedman approach). But do assumptions in economic theory need to be perfectly realistic, in order to be appropriate?

In our point of view, the basic fallacy of the rational expectations approach is different and relates more to Aristotle's claim that "it is the mark for an educated man to look for precision in each class of things just as far as the nature of subjects admit" (Aristotle (1969, book I,2). Prior to the monetarist and new classical counter-revolutions, the problem of expectations formation was at the forefront of economic research and tackled by all main economic schools. Since the introduction of an equilibrium concept for expectations, the monistic claim for the rational expectations hypothesis has been detrimental to various strands of the economic literature as it marginalized one of the crucial aspects of economic behaviour. In a way the rational expectations hypothesis is not even a theory of expectations formation, as it denies the expectations formation process to matter. According to Benjamin Friedman (1979), rational expectations lack "a clear outline of the way in which economic agents derive the knowledge which they then use to formulate expectations". As mentioned above, a proper model of expectations formation should not be mute about the way information acquisition and learning is handled.

A second strand of criticism focuses on the predictive aspects of rational expectations models. In fact, rational expectations modelling has been only moderately successful in explaining certain empirical regularities such as the volatility, persistence and certain kind of trade-offs of macroeconomic variables. This started a considerable literature on tests of the rational expectations hypothesis, both from an empirical point of view and via survey studies.

0.5 Testing the Rationality Assumption Directly

Between Muth's original contribution and the early nineties, more than 80 publications tackled the issue of the validity of the rational expectations hypothesis (Redman (1991)). The use of survey data to test the validity of the rational expectations hypothesis is widely used. We will therefore emphasize this strand of literature in the following paragraphs.

Testing the rational expectations hypothesis using survey data implies testing for un-

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biasedness and orthogonality. For foreign exchange markets, Dominguez (1986) can reject unbiasedness for the USD for the period 1983-85 using MMS data. Orthogonality test have been carried out by Dominguez (1986), Froot and Frankel (1989), MacDonald and Torrance (1989) and Itô (1990). These studies differ by their definition of the set of information available at time t, but nevertheless their results are similar: The null is rejected for periods longer than three months in all cases.

These results suggest that the expected exchange rates, as reported in the survey data, did not fully incorporate all available information and are thus inconsistent with the rational expectations hypothesis. Jeong and Maddala (1991) translate these results for other survey data and find considerable evidence against the rational expectations hypothesis.

Survey data usually exhibit one main characteristic: Heterogeneity. Any survey data set immediately reveals that expectations have a distribution. There also is a tendency for dispersion to increase for longer term expectations. We will not discuss this inherent contradiction of the rational expectations hypothesis any further, as it will be addressed in annex A.

Conlisk (1996) also surveys data from experimental asset markets, (e.g. Schmalensee (1976), Plott and Sunder (1988), Marimon and Sunder (1993)). Apparently these contributions favour adaptive processes over rational expectations, but under suitable conditions experienced agents may converge towards rational expectations.

Lovell (1986) surveys empirical studies of the rational expectations hypothesis. He concludes that there is only weak support of the validity of rational expectations. He conjectures that this might be due to various reasons, including measurement errors and learning behaviour on behalf of the agents.

The presented evidence of tests of the rational expectations hypothesis is as clear on its rejection as it is unclear on the implications that the economics discipline should bring about. Carroll (2003) forcefully criticizes that despite the availability of historical data on expectations formation, there have been only very view attempts to model the expectations formation process constructively (Roberts (1998), Branch (2004) being two of the view exceptions). Manski (2004) emphasizes the same problem and poses the quest for "basic research on expectations formation". This translates into basic questions of knowledge and belief, where non-basic beliefs ought to be epistemologically justified. We are then back in the epistemological trap of testing over and over again but not venturing in a meaningful ontological way. The problem is that the method or process whereby agents gain their knowledge is unknown. But there is no reason to believe that this process or human behaviour in general is guided solely by some "abstract laws".

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0.6 Is There a Case for Bounded Rationality?

Not all behaviour makes sense, some of it is even unreasonable. But if one is prepared to drop the rational expectations assumption, the door to behavioural wilderness is wide open. In order not to get lost, relaxing the assumption of perfect rationality has to be done in a constructive way. Bounded rationality, as first noted by Herbert Simon in the 1950s, can be regarded as such a way out of our dilemma. In most if not all cases the expression 'bounded rationality' is used to denote the type of rationality that agents resort to when the environment in which they operate is too complex relative to their limited mental abilities. But there is no unique way of modelling these limits in agents' capabilities or their rationality bound.

Simon's notion of bounded rationality is deeply rooted in psychological theories of human behaviour, and one might wonder if economics should not completely dispense with its highly unrealistic homo oeconomicus in favour of the more plausible description provided by behaviourism.[5] As is often the case, this probably depends on the circumstances. Simon's original aim was to analyze complex decision procedures in major businesses, and here it is the motivation of the single employee, or the question of how to simplify every-day procedures with the help of routines that become the focus of attention. In the presence of asymmetric or incomplete information, where humans fail to grasp the complex details, behaviour guided by rules becomes sensible, and should extend the standard economic theory of rationality. What should be done, therefore, is to try to select the constraints most appropriate to the problem at hand, and then to decide whether the homo oeconomicus approach will suffice or whether it should be amended or modified.

We can see that Simon emphasized that economics should not disregard the rules governing human behaviour. But in contrast to neoclassical assumptions, humans were not able to take into account all possible alternatives, to accurately calculate their expected outcomes, and to evaluate these outcomes consistently (see, e.g. Simon (1957)). Agents are unable to optimize, and instead are guided by routines and are content with satisfactory outcomes. Thus, the maximizing agent was replaced by a satisficing agent, who makes use of aspiration levels to find a suitable choice.

Selten's notion of bounded rationality takes a slightly different course. Human capabilities of computation and cogitation impose cognitive limits on rationality. However, cognitive limitations are not the only limits of the human mind. In his 1998 presidential address to the European Economic Association, Selten (1998) basically identifies three relevant aspects of mental processes for human behaviour: motivation, adaptation and cognition, while

[5] See also annex B.

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motivation is the primary source. Selten‘s main procedural advance is the so-called ’learning-direction-theory‘ (see Selten and Stöcker (1986)). Being in its character similar to reinforcement learning, the basic idea of this theory is that agents adapt their behaviour towards the direction in which their payoff improves, according to most recent experiences.

In his work with Gigerenzer (see Gigerenzer and Selten (2002)), the theory of ’probabilistic mental models‘ is highlighted, being basically a subjective probability-based model of choice. It says that inferences about the true state of the world are based on probabilistic hints, combining three strands of literature: Inductive inferences with respect to the environment of the agents, satisficing-type of decision making, and probabilistic assessments of frequencies.

In most if not all cases discussed by Simon or Selten, the expression ’bounded rationality‘ is used in combination with complexity arguments relating to agents‘ limited mental abilities. This observation motivated Albin (1998) who puts complexity at the center of the bounded rationality approach. Building on Gödel‘s theorem[6], Albin (1998) draws a rather pessimistic picture of bounded rationality. According to him, most economic scenarios lie beyond the range of economic calculus as there are no invariant laws that govern economic systems which makes rational decision making futile. Therefore, the economics discipline should abandon its aspiration “to predict the unpredictable”.[7] Instead, measures of complexity of economic systems should be derived in order to derive the factors that contribute or give rise to complexity in the first place. According to Albin (1998), building appropriate taxonomies of complexity-related characteristics of models is therefore vital for any modelling attempt.

Sargent (1993) can be regarded as a milestone in the bounded rationality literature, introducing main concepts of learning behaviour and deriving applications for macroeconomic models for the first time in a concise way. Sargent states that “[t]he elementary object of analysis in rational expectations models and models of bounded rationality is a collection of decision rules, namely, functions mapping people‘s information into decisions. Rational expectations restricts those decision rules by adopting the assumptions of individual optimization and consistency of perceptions. Bounded rationality drops at least the second of these assumptions, and replaces it with heuristic algorithms for representing and updating decision rules.” Sargent, being one of the protagonists of both the rational expectations literature and least-squares learning approaches, proposes to model economic agents as econometricians. This "adaptive learning approach" has nice behavioural aspects and

[6] By Gödel’s theorem, in any sufficiently rich logic some stateable theorems (and indeed, some true theorems) can neither be proved true or false. Moreover, in such logics, no general procedure can be devised that will decide whether any particular question that is presented is decidable.

[7] Note the similarity of Albin’s (1998) argument to John Stuart Mill’s notion of economics as an inexact science.

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typically assumes that agents have to estimate unknown parameters. However, it is also a bounded rationality approach as during the learning process the economic model at hand is usually misspecified. We will discuss this approach in the next section.

Other approaches impose bounded rationality at a more primitive level (Mankiw and Reis (2002), Branch et al. (2004) to mention just two relatively recent contributions) by concentrating on particular rules-of-thumb, a literature disregarded in this chapter. We also disregard an important and considerable theoretical literature, namely that on game theoretical learning concepts such as imitation and evolutionary approaches. For an overview see e.g. Fudenberg and Levine (1998).

But in spite of their neat behavioural interpretations and empirical or psychological backing, are bounded rationality models useful in economic theory? The answer is: it depends. From an empirical point of view, the full reduced-form of a dynamic model may entail a very large number of parameters and estimation of these parameters might be futile given the required degrees of freedom. Adaptive learning with bounded rationality features is therefore a tractable, less complex alternative which guarantees simple but plausible decision rules. On the other hand we have to ask ourselves whether there are any effects of bounded rationality and learning behaviour on market outcomes, that is whether bounded rationality approaches make a difference, and if yes, under which circumstances these differences show up.

0.7 The Quest for a Learning-Based Solution

Whether the epistemic or the ontological perspective is more suitable to economics, remains an open question. However, we have seen that the rational expectations hypothesis, despite being technically neat and convincing from an equilibrium point of view, is not a sensible measure in many markets and can be applied with great care only. According to Krugman (1998), rationality is a modelling device rather than a principle. It should be abandoned in cases in which it is not conducive to solutions of empirical regularities. In this thesis, therefore, we try to identify three aspects in which this traditional neoclassical approach fails to explain empirical regularities or anomalies. We will argue that in the cases under consideration, learning behaviour can be regarded as a worthwhile ontological venture, being a road still very little traveled by.

But how should one best model learning behaviour? Learning behaviour in different economic contexts cannot be modelled in a unique way. If one is interested in learning in specific contexts, a taxonomy of the situations at hand is key. The dominant paradigm is to endow individuals with learning rules which lead to optimizing behaviour in the long

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run/asymptotically. These learning models are generally normative principles rather than attempts to describe how economic agents really learn. For the latter purpose other models such as those of learning based on psychological research or evolutionary theories are more appropriate.[8]

The literature on rational expectations learning is often classified according to whether the agents are fully rational or boundedly rational. Fully rational learning takes place when agents know the model specification well enough to learn by estimating the parameter values consistently. This learning typically takes place via Bayes' Rule. Starting with a prior belief, agent i updates his beliefs once new information becomes available according to

$$P^i(\theta \mid I) = \frac{P^i(I \mid \theta)\, P^i(\theta)}{P^i(I)},$$

where θ are the parameter values to be estimated and I is new information available.

While it is a standard and almost natural way to handle imperfect information situations, Bayes' Rule is also a practical means in cases of asymmetric information in which one agent tries to learn about the beliefs of other agents. Being a normative concept about how to revise prior beliefs, Bayesian learning is the only rational procedure to update these beliefs and to derive consistent posteriors.

While being such a strong normative and theoretical claim, Kahneman et al. (1982) provide evidence for the refutation of Bayesian updating. Already Edwards (1968) showed that individuals react "conservatively" towards new information available, meaning that the actual revision of posterior beliefs lags behind. Kahneman et al. (1982) qualified this result by showing that by Bayes' Rule the more reliable new information is, the larger should be its impact on the posterior beliefs. However, they cannot show that this phenomenon is not given empirically. Also prior probabilities, which should matter in Bayesian updating, do not seem to influence decision making (much). El-Gamal and Grether (1995) derive similar result.

But if Bayesian learning (i.e. rational learning) has only limited empirical support, how should agents choose their learning procedures when they lack the information and capabilities necessary to make a fully rational choice? Bounded rationality can take many different forms and yield many different results. If such a route should be followed, economic theory cannot establish those bounds on rationality on an ad hoc basis. It seems inescapable that boundedly rational learning requires explicit modelling of the institutional and informational

[8] In all our cases learning means 'learning the model'. A different approach to learning are behavioural models in the spirit of Cyert and March (1963). The distinctive mark of this literature is that agents require much less information than is commonly assumed in rational, even boundedly rational learning models.

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environment of economic agents. We can also conject that the optimal procedure will depend on how costly errors are and how easily they can be corrected.

The unattractive prior information assumptions of fully rational models have made the concept of bounded rationality more widely accepted as a starting point for models of rational expectations learning. In recent years, an increasing number of economists has come to use the expression "adaptation" in the context of learning. They understand learning as a process by which individuals try to adapt their behaviour to the (changing) situations they face. The principle motivation of this kind of learning is to find an adequate or satisfactory behaviour under given circumstances. Such a concept is adequate if we assume that individuals have only limited knowledge about the world they live in and try to make the best use of their limited knowledge. From this perspective the concept of adaptive learning has something in common with the concept of bounded rationality (see e.g. Sargent (1993)). The assumption of omniscience, characterizing most of the economic literature, is abandoned. Instead, individuals are assumed to have only partial knowledge about the situation they face.

Note that adaptive learning, like Bayesian learning, excludes any cognitive creativity of agents. They are assumed to choose from a given set of alternatives only, and after each decision they adapt their behaviour to newly provided information. The creation of new alternatives, i.e. the cognitive development of strategies, is not considered by adaptive learning models.

Agents are assumed to have some reasonable initial belief about the model or equilibrium but lack information needed to guarantee consistent estimation. Agents have perceived laws of motion, represented by a parameter vector θ. This perceived law of motion usually nests the rational expectations equilibrium solution $\overline{\theta}$, so the departure from rational expectations modelling is not as great as one might imagine.

Agents are boundedly rational because they do not initially know $\overline{\theta}$ and they attempt to learn this REE solution using an adaptive inference process. Using their perceived law of motion, agents form expectations which enter into the model's reduced-form equations in lieu of rational expectations. The result is an actual law of motion consisting of a map (T-map), from the perceived law of motion to the actual realizations of the variables agents are attempting to forecast. We can briefly introduce this by assuming a reduced-form

$$x_t = \gamma + \alpha x_t^e + \beta' y_{t-1} + \eta_t,$$

where y_{t-1} is the vector of observable shocks.

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Let the rational expectations equilibrium be given by

$$\overline{x}_t = \overline{a} + \overline{b} y_{t-1} + \eta_t,$$

where $\overline{a} = (1-\alpha)^{-1} \gamma$, and $\overline{b} = (1-\alpha)^{-1} \beta$.

Let $\overline{\theta}' = \begin{pmatrix} \overline{a} & \overline{b} \end{pmatrix}$, and $\widetilde{y}'_{t-1} = \begin{pmatrix} 1 & y_{t-1} \end{pmatrix}$. Consider agents having a perceived law of motion according to

$$x_t = \theta' \widetilde{y}_{t-1} + \eta_t,$$

where θ are unknown parameters which need to be estimated. Let the forecast of x_t be given by

$$x_t^e = \theta' \widetilde{y}_{t-1}.$$

Inserting this forecast in the reduced-form equation above and solving for x_t gives us the actual law of motion, namely

$$x_t = (\gamma + \alpha a) + (\beta + \alpha b)' \, y_{t-1} + \eta_t,$$

which entails the T-map from the perceived to the actual law of motion:

$$T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \gamma + \alpha a \\ \beta + \alpha b \end{pmatrix}.$$

Evans and Honkapohja (2001) define the actual law of motion as the "stochastic process followed by the economy if forecasts are made under the fixed rule given by the perceived law of motion".

Referring to this adaptive inference process, it seems that most agents use commonly accepted econometric estimation methods in their learning, typically least-squares regression models. In statistical learning models, in which information arrives in each period independent of the actions of the decision maker and the economic environment is stationary, an agent eventually obtains all available information and his actions are optimal given this information. Therefore, under rather mild regularity conditions these models will converge to REE where the expectations of the agents will be verified by the outcomes, apart from some residual stochastic noise, from which the learning methods of the agents can extract no further information (see Evans and Honkapohja (2001) on a review of expectational stability).

Marcet and Sargent (1989) apply Ljung‘s (1977) theorem to the learning problem of self-referential rational expectations models, in the sense that the actual law of motion depends on the perceived law of motion. To prevent the updating procedure from going outside the verifiable attraction areas a projection facility is often needed that prevents outliers from throwing the estimation out of bounds. Essentially it is an assumption that learning agents throw away certain outlier observations. Grandmont and Laroque (1991) criticize the use of the projection facility, notable in the multiple equilibria context. They deem it contrary to the spirit of enquiry since it presumes that agents have some consensus on which domain of attraction to project estimates into. But how could such consensus arise before any learning has taken place?

The rational expectations equilibrium achieved will be sensitive not only to assumptions about initial information sets, but also towards the length of memory and how learning rules weigh past observations, as well as the confidence agents have in their (initial) beliefs about the appropriate models and learning rules. Boundedly rational learning in the shape of adaptive learning therefore still entails a lot of premises which - at best - only have limited empirical support. As Salmon (1995) states, "the main problem with these learning mechanisms is that they lack a behavioural interpretation."

Two alternatives to imposing "static" learning rules are given by Arthur (1994) and LeBaron (1995), where agents start close to the rationality outcome but are left to decide on their learning rule. Economic agents then have incentives to eliminate forecast errors up to the point where the costs of attaining necessary information become larger. Note that the most readily and least costly available information about the future value of a variable are its past values. In the absence of structural breaks, econometric models can lead under fairly general conditions to univariate conditions between the current and past values of a variable.

What remains important though, is that in a setting, where information is scarce and localized and behaviour is experimental and testing rather than optimizing, the path followed by the economy is likely to depend more on the dynamics of the learning process (let it be rational or boundedly rational) than on any characteristics of a long-run rational expectations equilibrium of the economy. Therefore, learning behaviour can lead to a better understanding of transition and equilibrium dynamics.

0.8 Outline of the Thesis

The following three chapters are concerned with the impact of learning behaviour in different economic environments. In all three cases we were particularly motivated by real world puzzles: The size and persistence of exchange rate volatility, the possibility of financial market crashes without news and progress in disinflation in transition countries despite limited credibility and conflicting monetary and fiscal policies. We introduce learning behaviour in a particular way in all three cases, trying to be close to "sensible real world behaviour".

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However, we have no reason to assume that our models are comprehensive explanations of the three markets under consideration. But - in all modesty - we are able to show that relaxing a standard assumption by factor *mu*, we are able to get interesting dynamics which put the models closer to economic reality.

Chapter 1 shows in a simple two-country model that when adding learning behaviour in which agents discount their knowledge of the past rather heavily, we can end up in a situation where exchange rate volatility does not settle down, just because of our expectational assumption. Adaptive learning behaviour thus adds some additional dynamics to the model, which are not present under full rationality.

Chapter 2 tries to implement adaptive learning behaviour in an empirical model. In the study of disinflation processes in three eastern European accession countries we add expectations derived from a recursive learning process to an otherwise standard VAR model. We can see that differences in the speed of disinflation can - at least partly - be attributed to the presence of expectational effects stemming from learning behaviour. More precisely, in countries in which learning behaviour is stronger, the effects of monetary policy shocks are smaller and less pronounced. This certainly has clear implications for any monetary policy conduct: Credibility and transparency are key mechanisms by which private sector learning behaviour can be facilitated, which results not only in faster disinflation but also in lower sacrifice ratios.

In chapter 3 we consider a simple investment game, in which agents have incomplete and asymmetric information in line with the "agreeing-to-disagree" literature (see e.g. Aumann (1976)). We can show that agents' valuation of an asset will ultimately be the same, even though they start from different information sets and do not interact directly with each other. That is, social learning behaviour (i.e. learning from others) can lead to the convergence of posterior beliefs, which can form the basis for a kind of endogenous market crash which is not accompanied by any exogenous news.

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Part II

Chapters

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CHAPTER 1

ON EXCHANGE RATE VOLATILITY AND ADAPTIVE LEARNING BEHAVIOUR

1.1 Introduction

One of the most puzzling characteristics of international financial markets is the volatility of flexible exchange rates. It is all the more mysterious as with the end of the Bretton Woods System many scholars and professional economists believed that a new era of exchange rate stability would arise. This view was emphasized in the tradition of Friedman's (1953) argument that an international monetary system with flexible exchange rates should cause less volatility due to stabilizing speculation. However, this belief did not prove to hold in reality. Nominal exchange rates have been considerably more volatile, notably in relation to their relevant fundamentals.

Over the last decades, economists have developed an extensive literature to explain the existence of these (persistent) exchange rate fluctuations. However, the conclusion from empirical tests of the principal structural models is that none of them adequately explains the short to medium run dynamics of major exchange rates. The inability to reconcile observed levels of foreign exchange rate volatility with predictions derived from these principal structural models remains to be one of the most persistent challenges in international finance.

One aspect in which most structural models are unconvincing is in their treatment of expectations. As exchange rates should be viewed theoretically as asset prices that reflect - among other things - expectations of a variety of real and monetary factors, the expectations formation process about the future exchange rate is a key determinant of current demand and therefore the current exchange rate. The still dominating paradigm in theories of exchange rate determination is that agents have rational expectations as introduced by Muth (1961) (see e.g. Taylor (1995), and Sarno and Taylor (2002)). Rational expectations models presuppose that agents have complete knowledge of the fundamental structure of the economy, they immediately take all available information into account and maximize expected utility according to an economic model which is common knowledge. As these are rather demanding conditions, it is not surprising that rationality tests of foreign exchange markets

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indicate that full rationality does not seem to be given empirically.[1] Instead, the empirical literature suggests that agents in the foreign exchange (henceforth FX) markets do not use all available information and therefore structural models based on rational expectations are likely to be flawed.

In this paper we drop the full rationality assumption and let agents be boundedly rational. However, we introduce the rationality bound in a rather particular way, namely via adaptive learning. The literature on learning has shown that convergence to rational expectations, even in highly stylized models, occurs only under very special conditions. Thus, the theoretical foundations of rational expectations are weak at best.

Instead of assuming that all past data count equally, we make use of a constant gain learning algorithm that discounts past observations. Constant gain learning is a method of "staying alert" to structural change as it entails a greater sensitivity to more recent observations which may incorporate shifts in the underlying structure. Constant gain learning, which is similar to adaptive expectations in that current expectations depend upon some fixed proportion of the previous period's forecast error, has not been used in a two-country setup so far.

While in deterministic models with a sufficiently small constant gain parameter convergence to rational expectations can be achieved (Evans and Honkapohja (2001)), constant gain learning has been typically proposed as a means to induce persistent dynamics in otherwise converging models (see Sargent (1993), Evans and Honkapohja (1993), Sargent (1999)).[2]

We are interested in the non-convergence case of the constant gain algorithm. We go beyond the existing literature by applying constant gain learning to a non-stochastic two-country overlapping generations (OLG) model. Our model isolates the effects of the adaptive learning algorithm and compares the results to the case of rational expectations. We show that resulting learning equilibria can exhibit volatility that is entirely due to replacing the rational expectations assumption. In particular we show that the parameter region for which a monetary equilibrium exists can be divided into regions of convergence and non-convergence. For the convergence region we derive interesting new stability results: for sufficiently small money growth processes convergence is independent of the size of the gain parameter. This contrasts with most of the existing literature that derives convergence only if the gain parameter is sufficiently small (see e.g. Evans and Honkapohja (2001)). For the non-convergence region we show that simulated exchange rate series exhibit statistical regularities (mean, standard deviation, skewness, kurtosis, normality, stationarity) that come closest to

[1] See also section 1.2

[2] The only exchange rate model with adaptive learning features known to me is Kim (2003).

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weekly and monthly German mark - U.S. dollar data (1979.1-1998.12). Volatility can thus emerge in deterministic models simply because expectations show a greater sensitivity to more recent observations.

The remainder of this paper is organised as follows: Section 1.2 motivates our bounded rationality approach, describes the relevant literature on rationality in FX markets and introduces the adaptive learning approach. In section 1.3 the model setup is presented and the solutions for both the rational expectations and the adaptive learning case are derived. Section 1.4 presents a simulation exercise of our model under non-convergence and discusses its implications for the volatility of exchange rates. Section 1.5 derives concluding remarks.

1.2 Motivation and Related Literature

The economics profession has not reached a consensus about the structure and the appropriate set of fundamental factors, notably macroeconomic variables, to be included in an exchange rate equation. For example, leading structural models such as the monetary approach, portfolio balance and currency substitution models include domestic and foreign money supplies, real incomes, interest rates, price levels and balance of payments. Most models contain also some conditional expectations of the spot exchange rate, based on information available at time t. A general macroeconomic model of the exchange rate can thus be written as

$$e_t = G\left[H_t, E_t(e_{t+1} \mid I_t)\right],$$

where the exchange rate e_t is a function of the set of fundamental variables H_t and the conditional expectation of next period's exchange rate given current information I_t.

The conclusion from the empirical literature on structural models is that none of them adequately explains short to medium run dynamics for all time periods and exchange rates. The most profound negative result is Meese and Rogoff's (1983) finding that no existing structural model could out-predict naïve random walk forecasts at short and medium horizons (see also Meese (1990) and Cheung et al. (2002)).

1.2.1 Stylized Facts of Exchange Rate Volatility

We focus on the log German mark (DEM) to U.S. dollar (USD) bilateral exchange rate for the period 1979-1998, to illustrate the quantative importance of historical exchange rate volatility[3] (table 1 in annex I). We make use of weekly, monthly and quarterly end of period

[3] As it is standard in the FX literature, logarithmic values are used as numeraire conventions are an important factor favouring logarithmic transformations (as problems arising from Jensen's inequality are avoided).

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market rates that are obtained from the Federal Reserve's Statistical Release H.10 (weekly) and IMF's International Financial Statistics (monthly and quarterly). From figure 1 and table 1 we can read two salient features: (i) volatility increased tremendously with the end of the Bretton Woods System, and (ii) volatility increases when we move from weekly to quarterly data.

The most telling bit of evidence is shown in table 2: No fundamental, neither at monthly nor quarterly frequency, has a standard deviation even close to that of the nominal exchange rate, which is not without problems for most structural models using fundamentals.

Many other empirical regularities also remain what they were more than 20 years ago: Table 3a shows that the hypothesis of normality of the log exchange rate returns can be rejected at the 1% level for weekly and quarterly data, and at the 5% level for monthly data. Moreover exchange rate returns exhibit positive skewness at the weekly and monthly frequency, while being marginally negatively skewed at the 3-month horizon. Exchange rate returns are leptokurtic (peaked) relative to the normal distribution at weekly and monthly frequency, while quarterly data are platykurtic (flat). This result mirrors the well-established stylized empirical fact that leptokurtosis becomes less pronounced when proceeding from weekly to quarterly data. These formal tests are mirrored in figure 2, where the graphs of the estimated densities all indicate "fat tails" behaviour.

Moreover, table 3b shows stationarity tests for the DEM-USD exchange rate. While the log spot rates turn out to be non-stationary, the first difference is stationary at all frequencies. This provides support for the use of a constant gain algorithm, as such algorithms are known to work well in non-stationary environments (see e.g. Sargent and Williams (2005)). Moreover, constant gain learning rules lack a sharp prior knowledge of the economic structure and thus reflect well our assumption of boundedly rational behaviour.[4]

1.2.2 Expectations and Rationality in Foreign Exchange Markets

In a survey Dornbusch and Frankel (1987) conclude that "exchange rates are moved by factors other than the obvious, observable, macroeconomic fundamentals". Frankel and Froot (1998) take it even further by stating that "macroeconomics appears to be irrelevant in explaining high- and medium-frequency exchange rate dynamics for low inflation countries", which lets them conclude that "macroeconomics is an inessential piece of the exchange rate volatility puzzle".

We do not share this opinion but rather suspect that macroeconomic models fail to

[4] In addition, constant gain learning rules usually give rise to additional learning dynamics not found in rational expectations equilibria.

identify the correct expectations formation process instead. In particular, the inability to reconcile observed levels of foreign exchange rate volatility with predictions derived from rational expectations models should not mislead us in abolishing macroeconomic approaches in general. Exchange rate models since the 1970s have emphasized that nominal exchange rates are asset prices and are influenced by expectations about the future. It is therefore our objective to take expectations seriously, as the expectations formation process is key to the question of variability.

1.2.2.1 Efficient Market Hypothesis

In testing the validity of the rational expectations hypothesis, tests of the efficient market hypothesis are widely used. The efficient market hypothesis states that asset prices in financial markets should reflect all available information; as a consequence, prices should always be consistent with fundamentals.

In an efficient, risk-neutral foreign exchange market, the current forward exchange rate should be an unbiased predictor of the spot exchange rate at the settlement date of the forward contract. Tests of the efficient market hypothesis are a joint test of rationality of expectations and risk neutrality of agents, which is one assumption for the uncovered interest rate parity to hold. The evidence from the empirical literature (Hodrick (1987)) indicates that there is strong evidence for a time-varying risk premium. This calls into question whether agents in the FX market are actually risk neutral.

However, the presence of risk aversion in preferences does not seem to be the whole story. MacDonald and Torrance (1990) and Lewis (1994) show that the rejection of the EMH is due to both a risk premium *and* expectational failures. One therefore has to question, in which way such expectational failures may be explained.

1.2.2.2 Survey Studies

The use of survey data as a proxy for exchange rate expectations enables us to unravel the joint hypothesis of rational expectations and risk neutrality. There are two kinds of tests of the rational expectations hypothesis using survey data. The first kind, surveyed by Takagi (1991), tests for unbiasedness and orthogonality in the survey data. Prominent contributions in that respect are Dominguez (1986), MacDonald (1990), MacDonald and Torrance (1990), Frankel and Froot (1987, 1989), and Itô (1990). A useful starting point for their analysis is equation $e_{t+k} - e_t = \alpha + \beta X_t + u_{t+k}$, where X_t is a set of known information at time t and u is a zero mean error. Empirically the β coefficient should equal zero for rational expectations.

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This null is rejected strongly by the authors mentioned above. The results suggest that the expected exchange rate, as reported in the survey data, did not fully incorporate all available information and thus is inconsistent with the rational expectations hypothesis.

A second type of rationality test is based on Liu and Maddala (1992) who use the data set of Frankel and Froot (1989). They extend the above mentioned analysis by using cointegration techniques as both spot and forward exchange rates and the expected spot rate are $I(1)$. Testing monthly expectations and spot exchange rate data they are able to show that the rejection of the null hypothesis of no cointegration is due to both expectational failures and some degree of risk aversion. This results holds for a wide range of currencies, including the Swiss Franc, the British Pound and the German Mark.

Sarno and Taylor (2002) conclude "that the true, unknown expectations formation process used by agents operating in the foreign exchange market is likely to be more complex than typically assumed [...]."

1.2.2.3 On Learning and Exchange Rates

The first one to introduce learning behaviour into a model of exchange rates was Tabellini (1988). He assumes a simple flexible price monetary model where agents face uncertainty about a parameter of the model that they try to learn by means of Bayes' rule. Tabellini shows that uncertainty about the stochastic process governing the market fundamentals leads to systematic overestimations of the permanent components of random shocks. Although the magnifying effect decreases exponentially as agents' prior precision increases with increasing horizon, short run variability of exchange rates is reasonably well explained. Tabellini shows that the rejection of standard econometric tests on the non-existence of bubbles may be explained by the dynamics caused by learning behaviour. However, in absence of continuous structural change, learning effects and FX market volatility fade out quickly

Lewis (1989) applies a similar framework to the performance of the U.S. dollar in the early 1980s. She shows that the tightening of the U.S. money supply at that time was not immediately recognized by the market as a persistent change in monetary policy. Market participants instead "learned" this shift gradually. Her main result is that although exchange rate forecasts have been systematically wrong *ex post,* this need not be an argument for irrationality of agents. It is in line with the assumption of Bayesian learning about an unknown parameter shift in the fundamental process.

Gourinchas and Tornell (2000) try to explain some delayed overshooting features of the exchange rate by introducing Bayesian learning. In their two-country infinite horizon model, interest rate expectations are the essential factor in exchange rate determination.

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The interest rate differential between two countries is supposed to consist of the sum of a persistent and a transitory component, $di_t = \mu + di_t^p + \upsilon_t$, where μ represents a constant, di_t^p the unobservable persistent and υ_t the transitory component. The persistent component is assumed to follow an AR(1) process, that is i_t^p is assumed to consist of two components: past persistent shocks and current persistent shocks. The uncertainty agents face is that they do not know whether a change in the interest rate is caused by a shock in the transitory or the permanent component. Gourinchas and Tornell (2000) assume that agents perceive a relative variance of the two components which is different from the true variance. Agents use Bayesian inference to update their priors. The results of this analysis are that the model is capable of explaining the delayed overshooting pattern as a reaction to monetary policy shocks that was first mentioned by Eichenbaum and Evans (1995).

Bayesian learning, as it is applied by Tabellini, Lewis, Gourinchas and Tornell, is applied in contexts where the presence of fully rational but not fully informed agents is assumed. This results in learning processes that vanish quickly and - without the presumption of continuous parameter shifts - are not able to depict real world exchange rate patterns.

Arifovic (1996) and Arifovic and Gencay (2000) use artificially intelligent agents that "learn" by means of a genetic algorithm. Their analysis is embedded in the two-country overlapping generations (OLG) model of Kareken and Wallace (1981). Agents face cash-in-advance constraints and use genetic algorithms to update their decisions regarding their portfolio composition. The resulting equilibria exhibit large and persistent volatility of exchange rates.

The genetic algorithm is a simple model of natural evolution. The problem with the genetic algorithm is that it introduces uncertainty in a rather extrinsic way via mutation. Moreover the applied fitness criterion is an adaptation rather than a learning process.

A more plausible assumption is that agents act like econometricians when doing forecasts about the future state of the economy. Expectational rules, or *perceived laws of motion,* of future variables are conditioned on current and past information as with rational expectations, but the relationship between the expectations variable and the information set evolves recursively as a result of expectational errors which are incorporated into econometric procedures which update coefficients.

Timmermann (1993, 1996), Bullard (1992) and Bullard and Duffy (1998, 2001) apply adaptive learning algorithms to stock market models. Timmermann explains the excess volatility of stock prices with (self-referential) adaptive learning behaviour, while Bullard and Duffy show the existence of complicated limit dynamics under learning, notably non-convergence to rational expectations equilibria. All in all, these authors argue that the observed volatility in asset markets can be explained by changes in investors' expectations via adaptive learning against a background of relatively small changes in economic fundamentals.

Our conjecture is that similar arguments hold also for the FX markets, where one has a similar discrepancy between sample moments of exchanges rates and economic fundamentals.

1.2.3 Adaptive Learning Behaviour

Models with rational expectations are mute about behaviour off an equilibrium path. Likewise, since rational expectations is an equilibrium concept, the process of expectations formation is left undefined since expectations are assumed to be fulfilled.

This paper suggests that agents form expectations about future variables in a way that their beliefs are consistent with the observed realizations in a linear statistical sense. In other words, it is supposed that agents act like econometricians using linear statistical techniques and, in doing so, they do not make systematic forecast errors. For example, agents might compute their expectations from actual time series observations in the past by using least squares regressions (see Bray (1982), Marcet and Sargent (1989), Bullard (1994) and Evans and Honkapohja (2001)). We refer to these relatively simple forecasting rules as *adaptive learning* rules (see also annex C. on the relationship of adaptive learning and least-squares estimation).[5, 6] As Sargent (1993, p. 22) puts it, "[W]e can interpret the idea of bounded rationality broadly as a research program to build models populated by agents who behave like working economists or econometricians".

Learning rules are called adaptive, if agents adjust their forecast rule as new data points become available over time. The rules usually depend on a single parameter, which describes how quickly expectations react to present conditions. The parameter then reflects the relative weight of present observations and of an aggregate index of past observations.

An agent's expectational formation process is completely described by such a forecasting rule. At each date t, agents fit their forecasting rule to the set of past price data. Forecasts of future price levels are then generated and converted into forecasts of rates of return. Given these forecasts, agents determine their consumption and savings decisions. Those are optimal for each agent given the forecasts generated by the forecasting rule at period t.

Let us now have a look at the methodology employed in models of adaptive learning by considering a simplified example of the model employed later in this paper. We analyze

[5] Gans (1995) shows that the paths generated under best reply dynamics are a subset of what is called adaptive learning.

[6] R*ational learning*, instead, is demanding in terms of its informational requirements as it assumes knowledge of the true structure of the economy. Rational learning thus retains the rational expectations equilibrium assumption at each point in time.

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a standard overlapping generations (OLG) model with non-stochastic money supply. The economy consists of N identical agents, each living for two periods.[7] Agents maximize their lifetime utility. They receive (perishable) endowments in each period and can decide on the amount of savings in the first period. Real money balances are the only means to hold these savings. There is no production in the economy and money supply is given by a non-constant, non-stochastic money supply rule.

Consider the agents' utility function to be defined in the following way:

$$u\left(c_t, c_{t+1}\right) = \ln c_t + \ln c_{t+1}.$$

Agents face cash-in-advance (CIA) constraints according to

$$\begin{aligned} c_t &\leq \omega^1 - \frac{M_t}{p_t} \\ c_{t+1} &\leq \omega^2 + \frac{M_t}{p^e_{t+1}}. \end{aligned}$$

ω^i denotes the endowments in period i, while M_t is money demand at time t. Agents' intertemporal budget constraint becomes

$$c_{t+1} \leq \omega^2 + R^e_{t+1}\left(\omega^1 - c_t\right),$$

where $R^e_{t+1} = p_t / p^e_{t+1}$ is the expected rate of return on money holdings.

Deriving the first order conditions for that maximization problem, we can write individual savings of each agent as

$$s_t = \left(\omega^1 - c_t\right) = \frac{1}{2}\left[\omega^1 - \frac{\omega^2}{R^e_{t+1}}\right].$$

Let the money supply be given by

$$M_t = \theta M_{t-1},$$

where θ is a constant money growth factor. The money market equilibrium[8] is then given by

$$\frac{M_t}{p_t} = N s_t,$$

which gives us a solution for the price level,

$$p_t = \left(M_t + \omega^2 p^e_{t+1}\right)\frac{2}{N\omega^1}. \tag{1.1}$$

[7] It is assumed that generations pass their information about the PLM parameters to the next generation. Note that under learning, if the recursive system is of order one, then only the information about PLM parameters in the immediately preceding period needs to be passed along.

[8] Note that due to Walras' Law the equilibrium for the good market follows.

Today's price level is thus a function of the expected price level,

$$p_t = F\left(p_{t+1}^e\right). \tag{1.2}$$

Eliminating the expectations operator from equation (1.1) by applying rational expectations would leave us with the perfect foresight solution of the economy. Note that the perfect foresight steady state in that case is given by

$$\overline{p} = \frac{2M}{N\omega^1 - 2\omega^2}. \tag{1.3}$$

We now posit an alternative forecasting rule for p_{t+1}^e, the agents' "perceived law of motion". This is the equation that they believe is governing the economy. We assume agents to be reasonably rational in that they understand the basic linear structure of the model. Suppose they form their expectations adaptively according to

$$p_{t+1}^e = p_t^e + \alpha_t\left(p_{t-1} - p_t^e\right), \tag{1.4}$$

where α_t is known as the gain parameter. Note that time-t prices are not entailed in the learning rule: We assume that agents form their expectations at the beginning of period t, where only $t-1$ prices are known.

Generally one can consider different types of gain parameters. However, in the learning literature two types are most prominent: A decreasing gain, s.t. $\alpha_t = t^{-1}$ and a constant gain, $\alpha_t = \alpha$. The decreasing gain function is only practical if p_t follows a stationary process, agents start reasonably close to the rational expectations equilibrium and believe that the structure of the economy never changes. Marcet and Sargent (1993) and Evans and Honkapohja (2000) show that under the assumption of a decreasing gain sequence, expectations derived according to equation (1.4) converge to RE asymptotically (see annex D.).

By contrast, the constant gain learning with $\alpha_t = \alpha$ for all t implies a model of continuous learning. It involves a trade-off between bias and variance (or "tracking" and "precision") when used in adapting to an exogenous time-varying process. In its recursive formulation it places a greater weight on more recent observations. Compared to OLS estimation, the algorithm is equivalent to estimating with weighted least squares where the weights decline geometrically with the distance in time between the observation being weighted and the most recent observation.[9] It is also equivalent to the rolling-window regression of Friedman (1979), where his rolling window of length ℓ is equivalent to a constant gain α of $2/\ell$. The advantage of the constant gain algorithm over rolling regressions and weighted least squares

[9] By contrast, in the decreasing gain algorithm with $\alpha_t = 1/t$ all observations have equal weights as in ordinary least squares procedures.

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is that the evolution of the former system is fully described by a small set of variables and is thus both operationally tractable and in line with our behavioural assumption of bounded rationality.

As we make use of the constant gain learning algorithm in our model, let us now see how it applies to the simple one-country case above.[10] Using the perceived law of motion (1.4), one is left with the *actual law of motion* of the economy, given by

$$p^e_{t+1} = p^e_t + \alpha \left(F \left(p^e_t \right) - p^e_t \right), \tag{1.5}$$

where

$$F \left(p^e_t \right) \overset{(1.2)}{=} \frac{2M_{t-1}}{N\omega^1} + \frac{2\omega^2}{N\omega^1} p^e_t .$$

In order to derive the stability conditions, we need to linearize equation (1.5) around the steady state. We employ a first order Taylor approximation around the steady state $p_t = \overline{p}$, which leaves us with

$$\begin{aligned} p^e_{t+1} &= \left(1 + \alpha \left(F' \left(\cdot \right) - 1 \right) \right) p^e_t \\ &= \left(1 + \alpha \left(\frac{2\omega^2}{N\omega^1} - 1 \right) \right) p^e_t . \end{aligned} \tag{1.6}$$

This implies that the steady state is stable under constant gain learning, iff

$$\left| 1 + \alpha \left(\frac{2\omega^2}{N\omega^1} - 1 \right) \right| < 1, \tag{1.7}$$

which is given for

$$1 - \frac{2}{\alpha} < \frac{2\omega^2}{N\omega^1} < 1.$$

This condition, which is simple to derive in the scalar case, is known as the *E-stability condition*, a key concept of the adaptive learning literature, which governs whether the system converges under adaptive learning.

To define *E-stability* properly, let us first introduce the notion of a *T-map*. For this, rewrite (1.5) as

$$p^e_{t+1} = \lambda + \phi p^e_t ,$$

where $\lambda = \left(\frac{2\alpha M_{t-1}}{N\omega^1} \right)$ and $\phi = \left[1 + \alpha \left(\frac{2\omega^2}{N\omega^1} - 1 \right) \right]$.

In matrix notation we can now write the map from the perceived, represented by $\sigma' = (\lambda, \phi)$, to the actual law of motion as

$$T \left(\sigma \right) = \begin{bmatrix} \frac{2\alpha M_{t-1}}{N\omega^1} \\ \\ 1 + \alpha \left(\frac{2\omega^2}{N\omega^1} - 1 \right) \end{bmatrix} . \tag{1.8}$$

[10] The analysis for the decreasing gain algorithm can be found in appendix D.

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Agents begin with an initial set of beliefs for the values of σ and revise those beliefs through the learning process. The latter is modeled by means of the differential equation

$$\frac{d\sigma}{d\tau} = T\left(\sigma\left(\tau\right)\right) - \sigma\left(\tau\right), \tag{1.9}$$

where τ is virtual time. Equation (1.9) represents a revision process in which the learning rule σ is adapted towards the actual law of motion. This revision takes place at a fixed rate per unit of virtual time τ and proportional to the discrepancy $T(\sigma) - \sigma$. Rational expectations equilibria correspond to fixed points of $T(\cdot)$.

Definition 1.1 (E-stability) *Consider a finitely parametrized family Σ of possible models of the endogenous data. Define a map $T : \Sigma \rightarrow \Sigma$ that indicates the pattern $T(\sigma)$ that the data will exhibit, if agents act on the basis of beliefs $\sigma \in \Sigma$. The fixed points of the T-map are the rational expectations equilibria. The equilibrium $\sigma^* \in \Sigma$ is E-stable if it is a stable rest point of the discrete-time dynamics $\sigma_{t+1} = T(\sigma_t)$.*

E-stability thus depends on the properties of the T-map from the PLM to the ALM near the respective fixed points (i.e. the rational expectations equilibria). We say that a rational expectations equilibrium (REE) is locally learnable, or E-stable, if the REE is locally asymptotically stable under equation (1.9), that is if and only if all eigenvalues of the Jacobian have real parts less than one.

1.3 The Model

The benchmark economy we are considering throughout the paper is a version of the two-country OLG model with fiat money of Kareken and Wallace (1981) in which we explicitly introduce adaptive learning features. We choose to analyze a version of the OLG model because it is one of the simplest fully specified dynamic general equilibrium models in which expectations matter.

There are two countries, A and B. At each date t, $t \geq 1$, there is a constant population of N young agents born in each country, said to be of generation t. They are young in period t and old in period $t+1$. Each agent of generation t is endowed with ω^1 units of a single perishable consumption good in his first period and ω^2 of the good in the second period. There is no production. In order to have a non-trivial decision problem, we assume that the endowment in period 1 is sufficiently large compared to the endowment in period 2, that is $0 < \omega^2 \lll \omega^1$.[11]

[11] The way we specify endowments and preferences in our setup gives agents an incentive to save a positive amount of the period t endowments. Precautionary savings are encouraged by an isoelastic utility specification.

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The world we are considering is one with free-trade and a flexible exchange rate regime without central bank intervention. Agents in both countries are permitted to borrow from and lend freely to each other and to hold each other's currency without any legal restrictions.

Agents in the two economies derive utility from consumption only. The consumption good is identical in both countries. Agents consume c_t of the good when young and c_{t+1} when old. We assume that consumption is a normal good in both periods. As there is no production or storage technology and fiat money is the only financial asset, agents can save by acquiring currency 1 or currency 2 only. Moreover there is no bequest.

Agents in both countries are characterized by the separable utility function

$$u\left(c_t, c_{t+1}\right) = \ln c_t + \ln c_{t+1}. \tag{1.10}$$

Moreover we assume that agents face cash-in-advance constraints, which will enable us to get tractable money demand functions.

Fiat currency is introduced into this economy by a government that endures for ever. The government in country i has control over a constant level of real deficit, which it finances entirely through seigniorage. Let real money supply in country i be given by

$$m_{i,t} = \theta m_{i,t-1} R_{i,t-1}, \tag{1.11}$$

where $\theta > 1$ is a constant money growth factor that cannot be observed by the agents. $R_{i,t-1} \equiv p_{i,t-1}/p_{i,t}$ is defined as the gross rate of return on currency i. We assume that all government revenue leaves the economy and does not affect agents' optimizing decisions.

We define the exchange rate in this setup to be the relative price of the two monies, that is

$$e_t = \frac{p_{1,t}}{p_{2,t}}. \tag{1.12}$$

Before analyzing the agents maximization problem, let us briefly look at the sequencing of actions within a period. An agent lives for two periods, t and $t+1$. In the first period he decides on how much of his endowment to consume in the first period and how much to save with a portfolio of two currencies. In the second period, the agent consumes his endowment plus whatever is being saved in the first period. Consumption takes place at current market prices which are determined in the money markets.

It is noteworthy that the update of the agent's expectations takes place *before* the prices of the current period are known. That is, while forming expectations p^e_{t+1}, the current price p_t does not belong to the information set of the perceived law of motion.

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1.3.1 Households' Maximization Problem

An agent of generation t has to solve the following maximization problem:

$$\max_{c_t, M_{1,t}} \ln c_t + \ln c_{t+1}, \quad (1.13)$$

$$\text{s.t.} \quad c_t \leq \omega^1 - \frac{M_{1,t}}{p_{1,t}} - \frac{M_{2,t}}{p_{2,t}}, \quad (1.14)$$

$$c_{t+1} \leq \omega^2 + \frac{M_{1,t}}{p^e_{1,t+1}} + \frac{M_{2,t}}{p^e_{2,t+1}}, \quad (1.15)$$

where $M_{i,t}$ is the agent's nominal holding of currency i acquired at time t. $p_{1,t}$ is the nominal price of the good in terms of currency 1 at time t, while $p_{2,t}$ is the nominal price of the good in terms of currency 2 at time t. $p^e_{i,t+1}$ denotes the time t expectation of the respective price levels in period $t+1$. Agents thus have to decide on how much to save of their period t endowment and secondly, how to allocate these savings to currency 1 and currency 2. To avoid corner solutions, an arbitrage-condition requires equal rates of return on both assets, as otherwise one currency would be dominated and money holdings of that currency would be zero. Thus, expected rates of return on both currencies have to be equal, namely

$$R^e_{1,t+1} = R^e_{2,t+1}. \quad (1.16)$$

For the solution of the individual household's maximization problem it is convenient to collapse the two one-period budget constraints into a single intertemporal constraint. This is given by[12]

$$c_{t+1} = \omega^2 + \omega^1 R^e_{t+1} - c_t R^e_{t+1}. \quad (1.17)$$

The first order condition for the maximization problem is then

$$\frac{1}{c_t} = \frac{R^e_{t+1}}{\omega^2 + \omega^1 R^e_{t+1} - c_t R^e_{t+1}} \Leftrightarrow$$

$$c_t = 1/2 \left[\omega^1 + \frac{1}{R^e_{t+1}} \omega^2 \right].$$

From this we can derive the agent's savings function,

$$s_t = \omega^1 - c_t = 1/2 \left[\omega^1 - \frac{1}{R^e_{t+1}} \omega^2 \right].$$

[12] As there is no uncertainty in the model, an equilibrium condition (1.16) requires equal rates of return for both currencies to be valued.

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Aggregate savings in country i are then given by

$$S_{i,t} = Ns_t, \ \forall \ i = 1, 2.$$

This determines world aggregate savings at time t as

$$S_t = N \left[\omega^1 - \frac{1}{R^e_{t+1}} \omega^2 \right].$$

Given the money supply in country i at time t, $M_{i,t}$, the equilibrium for the money market requires that aggregate savings equal money supplies, that is

$$S_t = N \left[\omega^1 - \frac{1}{R^e_{t+1}} \omega^2 \right] = \frac{M_{1,t}}{p_{1,t}} + \frac{M_{2,t}}{p_{2,t}} = \frac{M_{1,t}}{p_{1,t}} + \frac{M_{2,t}}{p_{1,t}} e_t. \tag{1.18}$$

Without loss of generality we can set $R^e_{t+1} = R^e_{1,t+1}$ and derive the solutions for the price level[13],

$$\begin{aligned} p_{1,t} &= \frac{\omega^2}{\omega^1} p^e_{1,t+1} + \left[M_{1,t-1} + M_{2,t-1} \frac{p^e_{1t+1}}{p^e_{2,t+1}} \right] \frac{\theta}{N\omega^1}, \\ p_{2,t} &= \frac{\omega^2}{\omega^1} p^e_{2,t+1} + \left[M_{1,t-1} \frac{p^e_{2t+1}}{p^e_{1,t+1}} + M_{2,t-1} \right] \frac{\theta}{N\omega^1}. \end{aligned} \tag{1.19}$$

Using definition (1.12), we can derive the solution for the current exchange rate which is a function of money supplies and the expected price levels in both countries.[14]

Due to Walras' Law, the equilibria for the goods markets follow.

1.3.2 Perfect Foresight Solution

To derive the households' maximization problem under perfect foresight, we rewrite (1.17) according to

$$c_{t+1} = \omega^2 + \omega^1 R_{t+1} - c_t R_{t+1}.$$

The individual agent's savings function then becomes

$$s_t = \omega^1 - c_t = 1/2 \left[\omega^1 - \frac{1}{R_{t+1}} \omega^2 \right]. \tag{1.20}$$

The equilibrium condition for both currencies determines world aggregate savings at time t as

$$S_t = N \left[\omega^1 - \frac{1}{R_{t+1}} \omega^2 \right]. \tag{1.21}$$

[13] Grandmont and Laroque (1991) refer to this as the *temporary equilibrium relation*.

[14] For later reference let me introduce the elasticity of the current price with respect to agents price expectations, $\varepsilon_{i,t} = \left(\partial p_{i,t} / \partial p^e_{i,t+1}\right) \left(p^e_{i,t+1} / p_{i,t}\right) = \left[\frac{\omega^2}{\omega^1} + \frac{M_{j,t-1}}{P^e_{j,t+1}} \frac{\theta}{N\omega^1} \right] \frac{P^e_{i,t+1}}{P_{i,t}}$, that follows from (1.19).

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Given money supply in country i at time t, $M_{i,t}$, the equilibrium for the money market requires that aggregate savings equal money supplies, that is

$$S_t = N\left[\omega^1 - \frac{1}{R_{t+1}}\omega^2\right] = \frac{M_{1,t}}{p_{1,t}} + \frac{M_{2,t}}{p_{2,t}}. \tag{1.22}$$

Using equation (1.21), we can derive the price level in country A:

$$\begin{aligned} N\left[\omega^1 - \frac{1}{R_{t+1}}\omega^2\right] &= \frac{M_{1,t}}{p_{1,t}} + \frac{M_{2,t}}{p_{2,t}} \overset{(1.12)}{=} \frac{M_{1,t}}{p_{1,t}} + \frac{M_{2,t}}{p_{1,t}}e_t \Leftrightarrow \\ p_{1,t} &= \frac{\omega^2}{\omega^1}p_{1,t+1} + \frac{M_{1,t} + M_{2,t}e_t}{N\omega^1}. \end{aligned} \tag{1.23}$$

Similarly we can derive the price level of country B.

Using equations (1.11), we can rewrite the solutions for the price levels in both countries as

$$\begin{aligned} p_{1,t} &= \frac{\omega^2}{\omega^1}p_{1,t+1} + \left(M_{1,t-1} + M_{2,t-1}\frac{p_{1,t+1}}{p_{2,t+1}}\right)\frac{\theta}{N\omega^1} \\ p_{2,t} &= \frac{\omega^2}{\omega^1}p_{2,t+1} + \left(M_{1,t-1}\frac{p_{2,t+1}}{p_{1,t+1}} + M_{2,t-1}\right)\frac{\theta}{N\omega^1}. \end{aligned} \tag{1.24}$$

From these price sequences we can derive the exchange rate solution via (1.12).

As the system (1.24) is rather complex to work with, let us rewrite the solutions for the prices according to

$$\begin{aligned} p_{1,t} &= F\left(p_{1,t+1}, p_{2,t+1}, M_{1,t-1}, M_{2,t-1}\right) \\ p_{2,t} &= G\left(p_{1,t+1}, p_{2,t+1}, M_{1,t-1}, M_{2,t-1}\right). \end{aligned} \tag{1.25}$$

We can now define a perfect foresight equilibrium:

Definition 1.2 (Perfect Foresight Equilibrium) *A perfect foresight equilibrium is a sequence of prices* $\{p_{1,t}, p_{2,t}\}$*, s.t.*

$$p_{1,t+1} = p^e_{1,t+1},$$

$$p_{2,t+1} = p^e_{2,t+1},$$

and the sequence of prices satisfies the equilibrium conditions (1.24).

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Steady state price processes are then given by

$$\overline{p}_1 = F(\overline{p}_1, \overline{p}_2, M_1, M_2)$$

$$\overline{p}_2 = G(\overline{p}_1, \overline{p}_2, M_1, M_2). \quad (1.26)$$

The requirement for a monetary equilibrium in which both currencies are valued is that the expected rate of return on currency 1, i.e. $R^e_{1,t+1}$, and the expected rate of return on currency 2, i.e. $R^e_{2,t+1}$, are equal. Under perfect foresight this reduces to

$$R_{1,t+1} = \frac{p_{1,t}}{p_{1,t+1}} = \frac{p_{2,t}}{p_{2,t+1}} = R_{2,t+1}. \quad (1.27)$$

Rearranging (1.27) and using the definition of the exchange rate according to (1.12), we end up with

$$e^e_{t+1} = e_t = e \qquad t \geqslant 1. \quad (1.28)$$

The rational expectations solution thus implies that the exchange rate is constant over time.

Proposition 1.3 (Kareken and Wallace, 1981) *Let e denote an exchange rate that sustains a monetary equilibrium, where both currencies are valued. Then for any exchange rate $e \in (0, \infty)$ there exists a monetary equilibrium.*

Proof. See annex E. ■

This indeterminacy result stems from the fact that there is no legal restriction on money holdings and only one equation for world money demand, namely equation (1.22). Therefore individual real demands for each currency are not well defined.

We consider these results as our benchmark case.

1.3.3 Adaptive Learning Solution

The learning behaviour we use in this paper can be described by the perceived law of motion[15]

$$p^e_{t+1} = p^e_t + \alpha_t (p_{t-1} - p^e_t). \quad (1.29)$$

The choice of the appropriate functional form for the gain sequence depends on the properties of p_t. A decreasing gain (s.t. $\alpha_t = 1/t$) would be appropriate if agents confidently believe that they are in an economy in which the forecasted variable has no unit root. This does not seem to be given empirically for the exchange rate.[16] Thus, the proposed procedure

[15] As already mentioned in section 1.2. We assume that generations pass their information about the PLM parameters to the next generation.

[16] See also section 1.2

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is a constant gain learning algorithm that is able to track changes in the level of p_t more easily.

The dynamic system - our actual law of motion - is given by

$$\begin{aligned} p^e_{1,t+1} &= p^e_{1,t} + \alpha\left[p_{1,t-1} - p^e_{1,t}\right] \\ p^e_{2,t+1} &= p^e_{2,t} + \alpha\left[p_{2,t-1} - p^e_{2,t}\right] \end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned} p^e_{1,t+1} &= p^e_{1,t} + \alpha\left[F\left(p^e_{1,t}, p^e_{2,t}\right) - p^e_{1,t}\right] \\ p^e_{2,t+1} &= p^e_{2,t} + \alpha\left[G\left(p^e_{1,t}, p^e_{2,t}\right) - p^e_{2,t}\right]. \end{aligned} \tag{1.30}$$

We thus have a fully recursive system in the variables $p^e_{i,t}$.

1.3.4 Local stability of the steady state under adaptive learning

Let us now analyze whether the system under adaptive learning converges, that is conditions for the two price processes to be locally stable under adaptive learning. Applying learning rule (1.29), we get the following system of equations,

$$\begin{aligned} p^e_{1,t+1} &= p^e_{1,t} + \alpha\left[\frac{\omega^2}{\omega^1}p^e_{1,t+1} + \left[M_{1,t-1} + M_{2,t-1}\frac{p^e_{1,t+1}}{p^e_{2,t+1}}\right]\frac{\theta}{N\omega^1} - p^e_{1,t}\right] \\ p^e_{2,t+1} &= p^e_{2,t} + \alpha\left[\frac{\omega^2}{\omega^1}p^e_{2,t+1} + \left[M_{1,t-1}\frac{p^e_{2,t+1}}{p^e_{1,t+1}} + M_{2,t-1}\right]\frac{\theta}{N\omega^1} - p^e_{2,t}\right] \end{aligned} \tag{1.31}$$

These equations can be solved for $p^e_{i,t+1}$which gives us a fully recursive system,

$$\begin{aligned} p^e_{1,t+1} &= \frac{(1-\alpha)\,p^e_{1,t} + \alpha\left(M_{1,t-1}\frac{\theta}{N\omega^1}\right)}{1-\alpha\left(\frac{\omega^2}{\omega^1} + M_{2,t-1}\frac{\theta}{N\omega^1 p^e_{2,t}}\right)} \\ p^e_{2,t+1} &= \frac{(1-\alpha)\,p^e_{2,t} + \alpha\left(M_{2,t-1}\frac{\theta}{N\omega^1}\right)}{1-\alpha\left(\frac{\omega^2}{\omega^1} + M_{1,t-1}\frac{\theta}{N\omega^1 p^e_{1,t}}\right)} \end{aligned} \tag{1.32}$$

To ensure that an equilibrium exists in which fiat money is valued, we need one further assumption:[17] We require that $\theta \in (0, \overline{\theta})$,where $\overline{\theta}$ is the maximum value for money growth,

[17] A monetary equilibrium, i.e. an equilibrium with money being valued, exists if price levels are finite. In this case, since goods are perishable and money is the only store of value, money has a strictly positive value.

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i.e. the amount of money collected by the government through seigniorage. Thus, the public deficit cannot take on any value as fiat money becomes not valued. We can show that θ is bounded by the inequality $\frac{\theta}{\upsilon} \leq 1$, where $\upsilon = \frac{\left[\omega^2-\omega^1-\sqrt{10\omega^1\omega^2+(\omega^1)^2+9(\omega^2)^2}\right]}{2\omega^2}$ is the marginal rate of substitution (see annex F.).

To analyze convergence of this system, one has to linearize the system (1.31) and find the eigenvalues of the Jacobian evaluated at the steady state. Only if all eigenvalues have real parts less than one, local asymptotic stability is achieved.

For our model we can state the following result:

Proposition 1.4 (Convergence under adaptive learning) *There exists $\overline{\theta} > 0$, s.t. $\frac{\overline{\theta}}{\upsilon} \leq 1$ for $i = 1, 2$ and for all t. Let $\widetilde{\theta} = \min\left[(\omega^1 - \omega^2)\frac{\overline{p}_1}{M_1}\frac{N}{\theta}, (\omega^1 - \omega^2)\frac{\overline{p}_2}{M_2}\frac{N}{\theta}\right]$. Then $\widetilde{\theta} < \overline{\theta}$. If $\theta < \widetilde{\theta}$, the dynamic system (1.31) is stable for all possible values of $\alpha \in (0,1)$. If $\widetilde{\theta} < \theta < \overline{\theta}$, convergence depends on the $\theta - \alpha -$ relationship. The system (1.31) is instable if and only if*

$$\theta > \frac{1}{\overline{S}} \frac{\alpha\left(\frac{2}{\omega^1}\left(\omega^2 - 2\right) + a^2\left(\left(\frac{\omega^2}{\omega^1}\right)^2 + 1\right)\right)}{\frac{\alpha}{N\omega^1} - \frac{\alpha^2}{N\omega^1} + \frac{\alpha^2\omega^2}{N\omega^1}}.$$

Proof. See annex G. ■

To illustrate the statement of this proposition, let us focus on figure 3 which displays a parameter space for which the equilibrium under learning exists and fiat money is valued. There are regions in which the equilibrium is locally stable, and parameter regions for which the equilibrium is unstable.

For the first part of proposition 4, we basically have qualifications on θ: $\theta < \widetilde{\theta}$, gives us convergence for any gain parameter in the assumed range. For $\theta > \widetilde{\theta}$, however, stability of the system under adaptive learning depends on the parameter values of both θ *and* α. Notably a higher gain parameter (i.e. agents that learn faster, that are "more impatient") and a higher money growth factor drive the system towards instability. Applying the implicit function theorem one can show that $\partial\theta/\partial\alpha < 0$. For the parameter values employed in our simulation analysis of section 4 the stability frontier, that is a well defined border between convergence and non-convergence regions, is given in figure 3.

The results indicate that for a given θ, a higher gain parameter implies that the local stability criteria are not necessarily satisfied. Similarly, for a given value of the gain parameter, a higher level of money creation leaves the system locally unstable. This instability results in an exchange rate that does not converge to its steady state value even in the limit leaving us with perpetual learning dynamics. This supports our hypothesis, that learning

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behaviour can be regarded as an independent source of volatility in the foreign exchange market. Agents are uncertain about the process determining the exchange rate. It is this 'model uncertainty' that can partly explain the exchange rate volatility puzzle.

1.4 Simulations and Discussion of Results

Our theoretical analysis lets us conclude that learning can be an independent contribution to exchange rate volatility. Under adaptive learning behaviour fluctuations arise because expectations are continually revised and these expectations feed back into actual exchange rate processes. This allows for systematic forecast errors in succession. In this section we try to quantify these effects by analyzing simulations of the model at hand. As canonical monetary models with rational expectations fail to match the observed pattern of exchange rate data, it is the persistence of exchange rate volatility is the primary scope of our analysis.

Given the use of a highly stylized model, simulations discussed in this section are meant to be suggestive rather than an actual mirror image of real world exchange rate behaviour. Nevertheless, from a heuristic point of view, the comparison of our results to actual DEM-USD exchange rate data produces interesting insights in the analysis of common statistical characteristics.

All simulations start with appropriate initial conditions that are in the immediate neighbourhood of the steady state. We perturb the system under learning with a small one-time shock. Throughout this section, we assume a constant population with $N = 30$ and initial endowments and money supplies to be $\omega^1 = 200$, $\omega^2 = 50$, $M_{1,0} = 100$, and $M_{2,0} = 110$ respectively.[18] We run simulations for $T = 500$ periods and focus on four endogenous variables, namely the nominal exchange rate, price expectations in country 1 and 2 and the average utility of country 1. Respective steady state values are $\overline{e} = 0.94$, $\overline{p}_1 = 3$, $\overline{p}_2 = 3.2$ and $2,897$ for the average utility of country 1.

Though we also describe the behaviour of price expectations and average utility levels, the most notable result of our analysis concerns the volatility of the nominal exchange rate. Under rational expectations, the exchange rate is a constant in our model. Under adaptive learning however, there exist parameter regions for which the exchange rate does not converge.

Figure 4 exhibits a non-convergence case for which $\theta = 3$ and $\alpha = 0.5$. We make use of a high gain parameter, i.e. agents are pronouncedly impatient. Both expected prices and the

[18] Note that increasing endowment in period t relative to endowment in period $t + 1$ makes agents in our model willing to save more. The quantitative aspects of learning on price and exchange rate behaviour thus becomes more pronounced.

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exchange rate exhibit a relatively large volatility which is confined within a certain band. Moreover figure 4 exhibits volatility clustering.

For figure 5 we decrease the gain parameter to 0.1 and increase the money growth factor to 7. It displays a markedly different outcome; effects of volatility clustering are clearly less pronounced, while the exchange rate shows larger swings. It seems that a smaller gain parameter lets the exchange rate fluctuate in a wider "band", while a larger money growth factor induces larger swings. To gain more insight, let us now focus on figure 6 that shows an example for the parametrization $\alpha = 0.5$ and $\theta = 7$. We can see that the larger gain parameter contributes in a particular way: While it decreases the amplitude of the swings in the exchange rate, it increases the volatility around these swings, showing signs of volatility clustering. This difference between figures 5 and 6 highlights an important property of the adaptive learning algorithm: the higher the gain parameter, the more pronounced is the trade off between "tracking" ($\alpha = 0.1$) and "precision" ($\alpha = 0.5$).

We now turn to the discussion of the descriptive statistics of our simulated learning equilibria. We focus again on the instability case. We make use of the stationary distribution to derive results that are comparable to those of tables 1-3 which displays actual German mark-U.S. dollar exchange rate data. Table 4 indicates that the volatility of the simulated series is somewhere in between that of the weekly and the monthly German mark-U.S. dollar data. The same resemblance holds once we look at the normality tests: While excess kurtosis of the simulated exchange rate returns is close to the monthly German mark-U.S. dollar data, skewness and chi^2 tests are closer to that of weekly data. Last but not least stationarity tests indicate that the simulated series are non-stationary. From these results we can tell that our simulated data face characteristics that are closest to weekly and monthly German mark-U.S. dollar data.

However, having shown this similarity to actual German mark-U.S. dollar data, one should also be concerned about the appropriateness of our adaptive learning assumption. A useful criterion could be that forecast errors are sufficiently random (i.e. they can not be detected too easily). We test that by analyzing the autocorrelation and partial autocorrelation functions of the simulated exchange rate forecast errors. We restrict our analysis to a lag of 2 periods which covers the whole lifetime of our OLG agents. Figure 7 indicates that although forecast errors do not vanish asymptotically, agents do not tend to produce forecast errors that can be detected by standard econometric tests.[19] While expectational

[19] Values within the upper and lower bound indicate that the autocorrelation of errors is not significant.- Our results indicate that agents *do* make systematic forecast errors, which hints for at least some degree of bounded rationality.

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errors are the driving force behind our results, we thus have an indication that learning and adaptation are anything but irrational in our context. In this sense agents in our setup seem to be "procedurally rational" while learning the true exchange rate process.

A more formal test of said results is the regression of the forecast error against the lagged forecast error as in

$$e^e_{t+1} - e_{t+1} = \alpha + \gamma \left(e^e_t - e_t\right).$$

The result is given in table 7: We exhibit a small but significant amount of positive serial correlation that indicates that adaptive expectations are mildly insufficient and agents should put even more weight on the current spot rate.

1.5 Conclusion

We considered a standard two-country OLG model similar to Kareken and Wallace (1981) and highlighted the difference between rational expectations and adaptive learning solutions. With rational expectations, the exchange rate process is constant. We can show that with adaptive learning behaviour, local stability criteria are not satisfied for the entire parameter space for which the monetary equilibrium exists. More precisely we get parameter regions in which the adaptive learning solution is unstable, leaving the exchange rate to fluctuate even in the limit. This exchange rate volatility is entirely due to the learning assumptions made in the paper.

We simulate our model to get a clearer idea of the quantitative relevance of our setup. As it turns out, the model can generate time series which closely mimic statistical characteristics of empirical weekly and monthly German mark- U.S. dollar data (1979-1998). Statistical tests of the empirical distributions confirm this closeness.

The results provide support for the view that persistent volatility dynamics can be generated even without introducing complex preference specifications, nominal rigidities or market imperfections, if the rational expectations assumption is relaxed. The introduction of mildly boundedly rational beliefs may introduce a permanent effect of a one-time shock such that the system does not settle down again asymptotically. We are thus able to give an alternative interpretation of the large and persistent exchange rate volatilities beyond conventional macroeconomic explanations.

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CHAPTER 2

LEANING AGAINST TOMORROW'S WIND - THE EXPECTATIONAL CONTEXT OF DISINFLATION IN TRANSITION

2.1 Introduction

The transformation from a socially planned to a market economy was accompanied by rapidly increasing inflation rates in many central and eastern European countries. The reasons for that were manifold, such as swift measures of price liberalization, large monetary overhangs, increased seigniorage to finance public deficits that could not be financed otherwise, and a lack of central bank independence and credibility.

Over several decades most prices in the centrally planned economies were set administratively. Therefore, when the transition started there was very little economic experience to rely upon and most economic agents, whether from the private or public sector, knew next to nothing about basic macroeconomic mechanisms, including monetary policy. Even for central and eastern European central banks, modelling inflation is an inherently difficult task that is further complicated in transition economies due to the short time-span during which free-market prices have existed. Furthermore, as a result of the major structural changes that have been taken place over the last fifteen years, the relation between inflation, money, and other variables have been susceptible to instability. Central banks therefore possessed only scant monetary policy instruments, and the private sector was not well enough informed about changes in market conditions, at least at the time that these changes occurred, to be able immediately to react in the way that would most fully serve their own interests. Not surprisingly, many central and eastern European countries started out with massive inflation, in some cases close to or even above Cagan's standard hyperinflation threshold of 50 per cent per month.

However, while the run-up to hyperinflationary kind of scenarios is relatively well understood, the miracle of escaping from these hyperinflationary paths - that many central and eastern European countries experienced in the 1990s - deserves further analysis. Tight monetary and fiscal policies, as favoured by leading economists and supported by IMF programmes, cannot be regarded as the full story of this success (see e.g. EBRD (1999)). King

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(1996) and Svensson (2003), besides others, give theoretical reasons why learning behaviour in disinflation processes could be important and show how disinflation depends crucially on whether the private sector immediately believes in a new inflation regime or not. We try to tackle these questions in the context of disinflation processes in three eastern European countries and try to show empirically, if private sector learning was important for successful disinflation. We introduce an empirical setup to tackle these questions by introducing expectations derived from learning behaviour in an otherwise standard VAR.

While there is only one kind of full rationality, modelling learning behaviour can take various expressions. We start from a relatively simple form of adaptive learning, which has proven to match survey inflation forecasts of the Survey of Professional Forecasters quite well. While the theoretical literature on private sector learning and inflation has gained a considerable momentum recently (Tetlow and Muehlen (1999), Bullard and Mitra (2002), Orphanides and Williams (2003, 2004), Preston (2003), Molnar and Santoro (2005), Gaspar et al. (2006) besides others), studies that empirically analyze the macroeconomic implications of private sector learning are rare (see e.g. Timmermann (1993, 1996), Marcet and Nicolini (2003)). This can be partly attributed to the fact that most of the models employed are not able to address the issues of changing expectations because their dynamic lag structure does not distinguish between sources of intrinsic and expectational dynamics. This paper therefore considers an alternative approach for identifying the effects of monetary policy shocks. It is our aim to provide a simple and tractable econometric model that enables us to evaluate the potential for learning as a mechanism capable of improving the empirical fit compared to the canonical model. As any empirical learning model is confronted with unobservable price expectations, we make use of Kalman filter estimates of the perceived inflation rate. The Kalman filter is an elegant way of deriving optimal model-based forecasts.[1] The paper contributes to the literature on empirical adaptive learning models by applying a method introduced by Hamilton (1985) to uncover unobservable price expectations and using expectational variables in an otherwise standard VAR setup.

We examine the effects of monetary policy and private sector learning in three transition countries (Lithuania, Poland, and Slovenia) from the early 1990s until end 2000.[2] Using monthly data, we will show that private sector learning of the inflation rate played a role in the transition from high to low inflation in the countries under consideration. Moreover, the speed of disinflation seems to depend on how strong the private sector's learning behaviour

[1] Based on linear models and normal errors, the Kalman filter is an optimal procedure that minimizes the estimated error covariance. It is still an optimal linear predictor when errors are non-normal.

[2] We use different sample periods for each country, thus taking into account that stabilization policies started at different dates.

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is. The "management" of private sector expectations thus becomes an important task of monetary policy, making a strong call for instance for central bank transparency to facilitate the learning process (see e.g. Blanchard (1998)).

A few caveats and qualifications should immediately be emphasized: First and foremost we do not intend to model the whole process of transition from a centrally-planned to a market economy. Most notably we disregard the initial liberalization phase (which gave rise to hyperinflation) and consider only the subsequent stabilization period starting around 1991/3. We acknowledge also that our analysis contributes only partly to the understanding of the complex patterns of interaction of monetary policy and inflation, as we are especially interested in the importance of monetary policy and private sector learning. Moreover, our analysis has a specific reference to central and eastern European economies. While this term applies to a highly heterogeneous array of institutional and economic realities, we deliberately abstract from some considerations that would appear germane in setting monetary policy, such as central bank independence, fiscal constraints, and time-inconsistency problems.

The paper is organized as follows: Chapter 2.2 gives an overview of the related literature and introduces the econometric methodology employed in the paper. We also discuss descriptively the economic (stabilization) policie in the three countries under concern. Chapter 2.3 introduces the model. Estimates for monthly data for Poland, Lithuania, and Slovenia are derived in chapter 2.4. We then analyze the impulse response functions generated by our model, followed by concluding remarks in chapter 2.5.

2.2 Related Literature and Methodological Issues

While the long-run relationship between monetary variables and inflation is relatively well understood, a stable positive relationship between the price level and the nominal quantity of money often fails once looking at high- or hyperinflationary environments. On the one hand, hyperinflationary paths need not be generated by excessive seigniorage (Marcet and Nicolini (2003), Woodford (2003)), but also disinflation periods do not necessarily come along with increasingly tight monetary and fiscal policies.

According to Svensson (2003), "central banks control [...] inflation mostly through the private-sector expectations they give rise to." This implies that disinflationary strategies by the central bank involve changing prevailing expectations of private agents. While it is well established among economists that expectations play a prominent role in economic decision making,[3] the disagreement about the basis on which agents form expectations and thus

[3] Classic articles include Phelps (1967), Friedman (1968), Lucas (1972), Sargent (1973), Sargent and Wallace (1975), Taylor (1975) and Barro (1976).

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about the way to model them is strong. This debate continues partly because obtaining data on expectations is difficult. Therefore, although inflation expectations play an increasingly important role in models of monetary policy, there has been little research done since the seminal paper of Carlson and Parkin (1975) who showed that when inflation is rapid, expectations approximate roughly a second-order error-learning process. Figlewski and Wachtel (1981) show that post-war inflation expectations as collected by the Livingston Survey are inconsistent with rational expectations and best described by an adaptive expectations model. Pesaran (1985) analyzes the inflation expectations of the British manufacturing industry with the result that learning behaviour seems to be important in the expectations formation process. Madsen (1996) analyzes producer's inflation expectations for several EC countries and shows that rationality of expectations is likely not to be satisfied. Instead, an N-order error-learning specifications seem to fit the survey data much better. More recently, Caroll (2003) and Branch (2004), using survey expectations of both households and professional forecasters, show that deviations from rational expectations yield macroeconomic dynamics that are more plausible for a variety of dimensions. Purely adaptive expectations perform poorly too, while expectations derived from adaptive learning algorithms are very similar to survey-based forecasts. Thomas (1999) summarizes the results for U.S. survey measures.

Although there is no indicator for a unique model of expectations formation, according to Mankiw et al. (2003) survey data on inflation expectations show substantial lack of full rationality, thus pointing into the direction of a learning-based setup. A burgeoning literature examining learning behaviour, both of central banks and the private sector, has therefore started recently. On the theoretical side, learning behaviour of central banks has been identified as one of the reasons for escaping from high inflation regimes.[4] On the other hand, it has been shown that activist monetary policy in presence of learning is likely to reduce the likelihood of a sustained inflationary bias due to incorrect beliefs, but is prone to induce nonstationary behaviour in economic observables which puts additional uncertainty in private sector expectations that might offset the benefits of an activist policy.[5]

While there have been only few empirical contributions that look at private sector learning,[6] it is noteworthy that even the most recent Federal Reserve macromodel ("FRB-World") uses some form of non-fully-rational adaptive learning (see Brayton et al. (1997)). The underlying justification for using learning behaviour is that although agents understand the main features of the macroeconomy, their expectations are derived just from a small model with time-unvarying parameters. One of the major results from this strand of literature is

[4] See Sargent (1999), Carlstrom and Fuerst (2001), Gerali and Lippi (2002).
[5] See e.g. Wieland (2000), Ellison and Valla (2001), Schaling (2003), Lim and McNelis (2003).
[6] E.g. Bullard and Eusepi (2003), Orphanides and Williams (2003).

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that in dynamic economies with continuously learning agents, an activist disinflation policy[7] may cause inflation expectations to "unmoor", which again might lead to inflation persistence that would otherwise not be given. The "management" of private sector inflation expectations therefore becomes a crucial part of any stabilization policy.

2.2.1 Disinflation in Central and Eastern Europe

Over several decades, most prices in the centrally planned economies of central and eastern Europe (CEE) were set administratively, with little regard towards costs or supply and demand. Prices on the whole were subject to strict controls and measured inflation was most of the time low or at least repressed. Already in 1989, most CEE countries liberalized a large proportion of producer and consumer prices, leading into lasting, volatile, high and open inflation. The peak of average inflation among the 15 transition countries was in 1990/2, although one has to note that inflationary processes varied considerably throughout the region (see table 8 for a selection of countries).

The literature on the determinants of inflation has traditionally identified various demand-pull and cost-push factors that have successfully explained the temporal behaviour of inflationary processes. Classic demand-pull factors are money and credit growth as well as the monetization of fiscal deficits. Cost-push factors have centered on wage growth in excess of productivity gains. Concerning the CEE countries, all factors seem to be more or less relevant, at least part of the time. Conducting monetary policy in such circumstances was a difficult (many observers said futile) task. Moreover, there was a control problem for monetary policy: Available time series were often short, highly non-stationary and even unreliable. Furthermore, ongoing structural adjustment in the real sectors of the economy or changes in the financial sector often seriously interfered with any credible attempt to bring down inflation considerably. Needless to say that there were no official nor reliable measures for inflation expectations.

However, despite all these difficulties, the median inflation rate in CEE was brought down from nearly 100% in 1992 to approximately 11% in 1997. Many economists believed that this impressive record of disinflation was mainly due to strict monetary policy and fiscal restraints.[8] However, by using an eyeball-inspection of the data on our three countries under consideration (see figures 8-10 for illustration), this argument is not as clear cut as it seems. So why did the transition countries do so well in bringing inflation under control? Certainly,

[7] For instance a more than one-for-one response in the monetary policy instrument due to changing inflation.

[8] The latter argument being based on Sargent's (1982) argument that in hyperinflationary environments inflation only comes to a halt when the government credibly changes fiscal policy.

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the interplay of monetary, fiscal and exchange rate policies did have an important impact in the initial stabilization phase. Given the wide variety of potential sources for inflation, any effective disinflation strategy has to rely on a broad set of measures from different policy fields. However, empirical studies were only partially successful in explaining the disinflation process by macroeconomic policies (see e.g. Elbourne and de Haan (2005) for a recent study). Thus, while the initial disinflation was a remarkable achievement, the explanation for sound macroeconomic stability were weak in many countries. It is therefore not surprising that since the beginning of the post-communist transition period, an enormous amount of research on inflation and disinflation in transition has been published.[9]

Recent studies (e.g. Lyziak (2001) or Dabrowsky (2003)) indicate that anchoring inflationary expectations can be regarded as one of the pillars of a successful disinflation policy by breaking down accumulated inflationary inertia. This view is highlighted for instance by the monetary policy council of the National Bank of Poland, stating that the monetary regime needs to break through "inflationary expectations, which constitute one of the main obstacles in the process of steadily reducing inflation" (NBP (1998)). While private sector expectations are important, most empirical contributions assume that the private sector catches on immediately to a new policy. The low interest in inflation expectations in transition economies can perhaps be explained by the lack of experience with open inflation in the majority of these economies, the lack of survey data on expectations of inflation and initially the unavailability of reliable time series.

Following the pioneering work by Sargent (1993) and Evans and Honkapohja (2001), agents' forecasting rules ("perceived laws of motion") are often specified as some (infinite memory) ordinary least-square estimate. In the presence of potential structural changes, however, a learning specification with finite memory is more appropriate. The benefit from such a characterization is that learning remains tractable and particularly alert to shifts in the underlying economic structure, which is appropriate in the transition perspective relevant to our model.

We assume that agents, when forming their expectations about period $t+1$, have access only to information up to period t. In particular, agents' updating mechanism evolve according to

$$\pi_{t+1}^{e} = \pi_{t}^{e} + \lambda \left(\pi_{t} - \pi_{t}^{e} \right), \tag{2.1}$$

where λ is called the gain parameter, π_t is the period t inflation rate, and π_t^e is the expected inflation rate of period t where expectations are derived in period $t-1$.[10] From this charac-

[9] For comprehensive reviews of the first decade of transition see e.g. Cottarelli and Doyle (1999), EBRD (1999), and Wyplosz (2000).

[10] Note that the gain parameter is not an arbitrary learning parameter, but rather a specific value deter-

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terization it becomes clear that agents' behaviour shows bounded rationality features: While agents use equation (2.1) to update their expectations, they do not take into account their subsequent updating behaviour in periods $T > t$. Agents thus take their beliefs (and also the model) as a constant, even though they evolve recursively.

One can show easily that the updating formula (2.1) gives a higher weight to more recent observations. This "fading memory" has a convenient Bayesian interpretation: If there had never been a regime change in the past, agents would optimally put equal weight on all past observations. By contrast, if there has been a history of occasional regime shifts or if agents believe that such a regime shift might occur in the future, discounting past observations mirrors exactly these beliefs. The constant gain λ implies that agents somehow overreact to a measure of their past forecast error. This will be of utmost importance for our analysis as it allows for the possibility that agents learn a new policy more quickly, which is the channel through which learning behaviour has an impact in our empirical analysis.

2.2.2 Economic Policies in Poland, Slovenia and Lithuania

2.2.2.1 Poland

Poland became known for its comprehensive and speedy reforms (so-called "big-bang" reform, or "shock-therapy") which started with the Balcerowicz Plan released on October 12, 1989. Interest rates increased sharply, prices were liberalized on a large scale, fiscal policy was restrained, the exchange rate sharply devalued and wage control was introduced. Just after his inauguration, Polish finance minister Balcerowicz stated that "[w]e see monetary and price stabilization as an immediate task and a precondition for structural adjustment". It was thus diagnosed that microeconomic liberalization and macroeconomic stabilization were mutually conditional, but macroeconomic stability was key for a successful reform.[11] With Poland's IMF supported programme the economy started to undergo a massive restructuring on its adjustment path to the postcommunist era. This laid the ground for an enormous expansion of private business activity which cushioned the otherwise collapsing industrial and agricultural activity, both in terms of absolute GDP and in terms of employment. However, these structural changes in the economy did not occur without initial sacrifice: GDP declined sharply in 1989-91 by approximately 17% p.a., but then experienced a lasting U-turn in 1992 with positive growth rates ever since.

Besides the official goal to bring down inflation quickly, the National Bank of Poland

mined by the behaviour of the process in question.

[11] Note that this kind of reform is still not without criticism. Many economists plead instead for a more gradual kind of reform, which is supposed to put less burden on economic agents.

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(NBP) faced considerable difficulties in conducting monetary policy in the early stabilization period. In fact there was not even a consensus, whether monetary policy mechanisms were an important factor at all for bringing down inflation. It is therefore not surprising that Polish monetary policy in the 1990s is a patchwork of continuous revisions of strategies and targets. While the initial fixed exchange-rate regime (against the US-$) was an anti-inflationary anchor in the beginning of economic transition in 1990, it was abandoned already in 1991 after several one-off devaluations and converted into a crawling-peg regime. Already at that time other channels of monetary policy gained importance (see e.g. Golinelli and Rovelli (2002)): Open market operations and from 1993 onwards a shift towards reserve and money market rates became the dominant monetary policy tool, while the exchange rate band was subsequently widened and abolished. In 1995 the currency was reformed with the new zloty worth 10,000 times the old unit.

The increasingly flexible exchange rate regime was introduced to pre-empt currency crises and to introduce inflation targeting via managing the domestic interest rate (in 1998). However, the official inflation target was overshot for the first three years and undershot in 2001-2003.

Already since 1994, the NBP experienced a considerable independence, which was formalized in the 1997 constitutional act. While the NBP is supposed to control exchange and interest rates, the finance ministry has responsibility for the state budget which implies an inherent political conflict. Moreover, the central bank law defines price stability as the ultimate goal of monetary policy, but also explicitly allows for a secondary objective, which is loosely defined by providing support for the overall economic policies. Besides this apparent policy mix, the NBP spares no effort in improving its credibility and transparency. Not only does the NBP publish monthly monetary aggregates, but also statements by the monetary policy committee are released timely on its website.

2.2.2.2 Slovenia

Slovenia has adopted a gradualist approach to economic reform, maintaining that the country's advanced level of economic development made certain types of reform unnecessary which were pursued in other CEE countries. Slovenia was also the only accession country which never adopted an IMF programme, which gave considerable leeway to government and central bank policies. In fact this resulted in a number of unorthodox policy measures. Prices were only liberalized partly,[12] and capital controls were used extensively in the 1990s to discourage portfolio investment. These controls were abolished only in 2001, and the

[12] In 2003 still 16% of all prices were administered.

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capital account was not liberalized before 2002. But also in the real sector restructuring was particularly slow, especially in the enterprise sector. As Arvai and Vincze (2002) note, "Slovenia is a case of avoiding all sorts of crises at the cost of maintaining relatively strong capital controls, and pursuing extremely cautious macroeconomic policies".

Immediately after its formal independence in late 1991, Slovenia has made remarkable progress in reducing inflation through a money-based stabilization programme. While initial liberalization lead to hyperinflationary kind of behaviour, the annual inflation rate fell below 100% already in 1992. Monetary policy, for which the Bank of Slovenia (BS) is primarily responsible, set the rediscount rate at 25% in 1992, and introduced base money as its chief operating target. Moreover, M3 was set as an intermediate target for monetary policy, as well as an active exchange rate policy within a managed float regime. The latter is owed to the fact that the central bank law explicitly spells out the importance of national currency stability. Officially, however, Slovenia did not make any exchange rate announcement.

The government was keeping its spending broadly within the limits of revenue and small budget deficits started only in the late 1990s. Serious budget deficits were however never an issue of concern, which is also contributed to the fact that Slovenia started with a relatively low level of outstanding public debt when it separated from rest-Yugoslavia.

While bringing down inflation to moderate levels was successful and materialized quickly, Slovenia exhibited a stubbornly persistent inflation with high single-digit rates since the mid 1990s. Therefore, on it's way towards EU-membership, the BS increased interest rates by 300 basis points in early 2001 and declared a formal inflation target in 2002 in line with the Maastricht criterion. However, this target was missed by a significant margin in subsequent years and met only very recently. Main reason for the strong persistence in inflation was seen in the ineffective coordination of fiscal and monetary policies together with the mentioned protracted reforms of the real and financial sectors.

2.2.2.3 Lithuania

Shortly after its independence in 1990, Lithuania experienced a sharp decline in its GDP (7% in 1990, 14% in 1991 and 38% in 1992). The highly industrialized country was hit particularly by the rise of energy prices to world market levels on the one hand and a large drop of demand for agricultural products of its eastern neighbours. Despite these unfavorable preconditions, the government embarked on rapid, market-oriented reforms. Prices were liberalized comprehensively by 1993 which resulted in sharp increases of CPI rates to hyperinflationary levels. The government subsequently introduced a law to compensate depositors whose savings were destroyed by the hyperinflation, which led to severe budget deficits that got more or less out of control by end 1993. The IMF-supported programme introduced in 1994 therefore targeted notably at macroeconomic stabilization. But it took until the end of the 1990s until a sound austerity programme was started by the government which reduced the public deficit towards the level of the Maastricht criterion.

Monetary policy in Lithuania is characterized by its imposition of a currency board system in 1994. Initially the litas was pegged to the US-$, due to the importance of energy imports. This peg was changed in 2002, when the litas was set in reference to the Euro. In general, in a currency board the monetary authority will no longer issue fiat money but instead will only issue a set number of units of the local currency for each unit of foreign currency it possesses. However, Lithuania did not opt for such a "pure" currency board which resulted in a more active role for the Bank of Lithuania (BoL) and less stringent requirements in terms of budgetary discipline for the government. Moreover, as in Poland, the central bank law entails a reference to a secondary objective of central bank policy next to the primary goal of price stability. This policy mismatch resulted in considerable foreign-exchange outflows which automatically resulted in a contraction of the monetary base and higher interest rates and domestic credit squeeze. Nevertheless inflation was kept under control after 1995 with the help of the currency board arrangement and favourite external factors such as a drop in oil prices and a collapse of the Russian export markets after the crisis in 1998.

2.2.3 VARs and Adaptive Learning

Sims' (1980) seminal paper on vector autoregressions (VAR) prepared the ground for replacing the 1970s approach of simultaneous equations modelling, advocated by the Cowles Commission and until then the dominating paradigm in econometrics.[13] However, VARs have the status of a reduced-form and are thus merely vehicles to summarize the dynamic properties of the data.[14] Without reference to a specific economic structure such reduced-form VARs are difficult to understand (see e.g. Bernanke (1986), Cooley and LeRoy (1985)). As long as the parameters of a VAR are not related to structural ones, they do not have an economic meaning and are subject to the Lucas critique.[15] According to Sims (2000), the original meaning of a structural model in econometrics is explained in an article by Hurwicz (1962): A model is structural if it allows us to predict the effect of interventions. In order to

[13] The Cowles Commission approach tries to evaluate the effects of changes in exogenous variables on the endogenous variables of a system.

[14] One has to admit, however, that the basic reason for introducing VARs was economic forecasting rather than macroeconometric policy analysis.

[15] Lucas (1976) argues that the parameters of a macroeconomic models depend on agents' expectations of the policy conduct and are unlikely to remain stable as economic policy changes.

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be able to make these predictions, the model must tell us how the intervention corresponds to changes in some of the variables or parameters of the model. Coming from this point, VAR models have become a major tool in analyzing macroeconometric time series, notably in monetary economics. In a VAR, a group of economic variables is interpreted as being driven exclusively by unobservable economic shocks. In order to recover these shocks from the data, one has to estimate a VAR by maximum likelihood estimation and decompose the residuals into economically meaningful structural shocks. Let us look briefly at a simple example which substantiates our introduction.

A VAR model is the generalization of a univariate autoregression, i.e.

$$A(L)x_t = \varepsilon_t, \tag{2.2}$$

where $A(L)$ denotes a lag polynomial of order p. For ε_t being normally distributed ($\varepsilon_t \sim iid\,(0, \Sigma_\varepsilon)$), one can estimate matrix A with least-squares which is in this case equivalent with maximum-likelihood estimation. Note that all right-hand variables can interact with x_t. A priori it is only the shocks which are exogenous, all other influences are endogenous. Note that in the Cowles Commission approach, constraints on the matrix A would be used to identify the system (see e.g. Favero (2001) for a discussion). In contrast, the VAR approach restricts the covariance matrix Σ_ε, that is the influence of the structural shocks on the variables.

In a VAR, vector ε_t is regarded as a vector of independent structural disturbances. To recover them econometrically, one has to transform the structural model (2.2) into an estimable reduced-form, i.e.

$$x_t = B(L)\,x_t + u_t, \tag{2.3}$$

where $u \sim N(0, \Sigma_u)$.[16] Note that the reduced-form errors u_t are linear combinations of the structural errors ε_t, where

$$\begin{aligned} E[u_t u_t'] &= BE[\varepsilon_t \varepsilon_t']\,B' \\ &= \Sigma. \end{aligned}$$

Interpretation of the reduced-form is difficult because matrix B involves many parameters. Without some restrictions, the parameters in the VAR would not be identified. That is, given the values of the reduced-form parameters, it would not be possible to uniquely solve for the structural parameters. Whereas Sims (1980) has argued that economic theory is not rich enough to suggest proper identifying restrictions, Bernanke (1986), Sims (1986), and others

[16]Note that this transformation hinges on the assumption that the inverse of $A(L)$ exists, meaning that A has full rank.

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have argued forcefully that VAR analysis without identification of independent shocks will leave the researcher without meaningful theoretical implications. However, although VARs have become a popular tool for evaluating economic models, especially in monetary policy issues, the debate about what constitutes appropriate identifying restrictions is still vivid. The only apparent consensus is that structural shocks should be mutually uncorrelated ("orthogonal"), which implies that we can analyze the impact of just one structural shock on the VAR system. However, the assumption of orthogonal shocks is mostly not sufficient for an exact identification. In particular, for an n-dimensional system, a total of $n(n-1)/2$ additional restrictions are needed.[17] Thus, there remains to be a wide range of possible identifying restrictions which come along with the same reduced-form representation. Typical identifying restrictions are zero restrictions and linear restrictions on the elements of B.

Note that we only tackle identifying assumptions on the contemporaneous movements of the variables and do not perform an explicit long-run analysis for the countries under concern.[18]

In the applied VAR literature economists implemented model specifications that induced "reasonable" behaviour. Such reasonableness is usually evaluated by means of an impulse response analysis. Structural impulse responses are plots of the structural dynamic multipliers of an identified system and thus summarize how unit impulses of a particular structural shock at time t affects the level of the economic variables of interest at time $t+s$. In order to find these impulse responses, the VAR needs to be transformed in its moving average representation (also called Wold representation),

$$x_t = \sum_{\tau=0}^{\infty} C_\tau \varepsilon_{t-\tau}, \tag{2.4}$$

where the elements of the matrices C_τ give the impulse responses of x_t to changes in ε_t. In many cases, identification of the parameters in the VAR is given via restricting parameters in the Wold representation. Any such restriction is equivalent to restrictions in the VAR representation.

Note that even VARs are susceptible to the Lucas critique, that is policy changes are in some cases not appropriately reflected in changes of private sector forecasting. However, empirical macroeconomic models for monetary policy analysis usually abstract from mod-

[17]That is a total number of n^2 restrictions are needed for a $n-$ dimensional system.

[18]For the monetary transmission process considered in our paper, long-run behaviour is per se not pertinent. Moreover, long-run restrictions as proposed by Blanchard and Quah (1989) have been criticized e.g. by Faust and Leeper (1997) for statistical reasons. Moreover, informal short-run restrictions remain necessary to identify the structural shocks completely.

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elling expectations explicitly,[19] although expectational effects, caused by policy changes, are likely to feed back in the reduced-form dynamics of the model. But how could one possibly tackle expectations in a VAR approach? Several ways of proceeding in that respect were considered, such as introducing available Greenbook expectations for U.S. data (e.g. Barth and Ramey (2000), introducing control variables such as the federal funds futures rate (e.g. Brissimis and Magginas (2006) or commodity prices (e.g. Sims (1992)). Another alternative is the so-called "expectations theory" of the interest rate term structure (e.g. Fung et al. (1999)). However, all approaches had it's limitations and are moreover not directly applicable to accession countries.

In our model, we therefore proceed differently. By introducing expectations exogenously as the outcome of an unobservable learning process and using the Kalman filter method of Hamilton (1985), we are able to reveal optimal model-based private sector inflation expectations which are subsequently introduced into our VAR analysis.

2.3 Model Setup

Already Friedman (1979) argued that the assumption of rational expectations in this context may be unrealistic, especially in periods which follow a major policy change because agents have not had sufficient time to fully comprehend the implications of the new policy. In our view this directly applies to the experience in CEE and we therefore explicitly model private sector expectations by assuming learning about the conduct of the monetary policy by means of the inflation rate.[20] As it is common, we compare this result to a case where learning behaviour is not present.

We consider a small-open economy case similar to Peersman and Smets (2001), where we can write the VAR as

$$Y_t = A(L)\,Y_{t-1} + B(L)\,X_t + \eta_t, \tag{2.5}$$

where Y_t is a (5x1) vector of endogenous variables, namely x_t as output, p_t as CPI, m_t as money, s_t as the short–term interest rate and f_t as the real effective exchange rate. η_t is a (5x1) vector of reduced-form shocks, $A(L)$ and $B(L)$ are lag polynomials and X_t a (3x1) vector of exogenous variables. The inclusion of exogenous variables can be justified as we

[19] Exceptions are the publications by Brunner (1996) and Rudebusch (2005).

[20] Note that abstracting from deeper accounts here can be justified by the poor standards of official statistics in CEE, the lag in providing adequate data and the general obscurity of the monetary transmission mechanism in these countries. The burgeoning literature on DSGE-VARs (see e.g. Caputo (2003), Del Negro et al. (2006), Smets and Wouters (2004)) can be seen as a useful workhorse in applying learning behaviour which stems from deeper accounts. According to Del Negro et al. (2006), a "VAR can be interpreted as an approximation to the moving-average representation of the DSGE model." However, in the analysis of CEE countries, this approach seems to be less useful, as the use of log-linearized approximations of steady-state relationships is likely to produce implausible results for accession countries.

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model small-open CEE economies which are likely to be affected from shocks abroad but the (big) foreign country is not affected by the small-open economy. In our first setup we consider three types of possible exogenous factors, namely change in EU countries' demand and inflation. We define $X_t' = \begin{bmatrix} cp_t^{EU} & x_t^{EU} & s_t^{EU} \end{bmatrix}$, where x_t^{EU} is the average GDP of EU countries, cp_t^{EU} denotes the EU's average commodity price index, and s_t^{EU} is the average short-term interest rate. The inclusion of cp_t^{EU} instead of average EU-CPI can be justified because of the so-called price puzzle.[21]

All data used are monthly, seasonally adjusted series in levels, measured in natural logs (except for the interest rate) and taken from the IMF's International Financial Statistics (IFS) database, EBRD transition reports as well as data from the European Central Bank. Output is measured in terms of real GDP which is not available at a monthly frequency. Instead of using a proxy for output such as measures of industrial production,[22] we make of use of interpolated quarterly GDP series constructed along the lines of Bernanke et al. (1997). Money is measured by M1 unless otherwise noted. The short-term interest rate is the discount or money market rate, depending on the availability. f_t is the real effective exchange rate. Note that - according to standard ADF tests - most variables in our model are integrated of order one. However, Sims et al. (1990), Christiano, Eichenbaum and Evans (1999) and Kim and Roubini (2000) suggest to estimate the VAR in levels instead of first differences in this case which allows for implicit cointegrating relationships asymptotically.[23] This is particularly useful when using monthly series, as the power of the cointegration tests are likely to be low. Moreover, the considered data samples are not long enough to perform more detailed long-run analyses.

A key component in any monetary VAR is the definition of the monetary policy reaction function. We follow the literature by using the interest rate as the main policy tool, which is a prominent and widely used monetary policy tool in CEE countries. We do that by modelling one of the VAR equations as a variant of Taylor rule, which is an interest rate rule that responds to output and inflation. As McCallum (1999) argues, a policy reaction function should not include current variables such as current prices as those are only observable with a lag. This feature was also highlighted by Sims and Zha (1998) and Kim and Roubini (2000) and will be used in our identification procedure as well be seen below.

[21] Originally documented by Sims (1992), a positive shock to the federal funds rate (short-term interest rate) often leads to persistent increases in consumer prices, the so-called "price-puzzle". Solutions were proposed inter alia by Sims and Zha (1998) by including a commodity price index. The reason behind this is that it is assumed that the central bank has information about future inflation beyond the data available in the endogenous variables of the VAR, which are capture by the commodity price.

[22] Note that this proxy would have just been given for two of our three countries anyway.

[23] See also Hamilton (1994, ch. 18) on this issue.

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2.3.1 Identification Issues

A crucial feature of VAR models is the imposition of identifying restrictions on the dynamic behaviour of structural shocks. As mentioned before, identification is key if we want to arrive at interpretable results for the structural shocks. We begin with recognizing that the reduced-form VAR (2.5) can be represented in a structural form according to

$$A_0 Y_t = A(L)\, Y_{t-1} + B(L)\, X_t + \varepsilon_t, \tag{2.6}$$

where $\varepsilon_t \sim (0, \Sigma_\varepsilon)$. Note that representations (2.5) and (2.6) are related to each other via the following rules:

$$A_0 \eta_t = \varepsilon_t, \text{ and}$$

$$A_0^{-1} \Sigma_\varepsilon A_0'^{-1} = \Sigma_\eta. \tag{2.7}$$

The structural identification usually begins with restrictions on the variance covariance matrix Σ_ε by making it diagonal and normalizing the elements to one. Economically this guarantees that the structural errors are mutually uncorrelated. However, this leaves us with $n(n-1)/2$ additional restrictions needed for an exact identification of the VAR. One of way of achieving this, is a recursive identification scheme, usually referred to as the Choleski decomposition. By this we isolate the underlying structural errors by recursive orthogonalization. The key assumption in any Choleski decomposition is related to the order in which the variables are stacked in the vector of endogenous variables Y_t. The ordering of the variables in Y_t determines the level of exogeneity of the variables. A widely used application of the Choleski decomposition is an ordering of the vector of endogenous variables according to

$$Y_t' = \begin{bmatrix} x_t & p_t & m_t & s_t & f_t \end{bmatrix}. \tag{2.8}$$

This ordering assumes that policy shocks have no contemporaneous effect on output, prices or money.[24] Note that if the exchange rate is ordered last in the VAR, as in equation (2.8), this basically means that neither the home nor the foreign country reacts contemporaneously to exchange rate fluctuations. That means that the exchange rate is not part of the central bank's information set when the level for policy variable is being chosen. Note that the Choleski decomposition imposes a just-identifying scheme and is uniquely determined

[24] See McCallum (1999). Note also that this definition entails a time series perspective: If one variable is causal for another, it has to be realized before.

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up to an orthogonal transformation. This implies that each structural shock is identified up to its sign. Our VAR would then be

$$\begin{bmatrix} x_t \\ p_t \\ m_t \\ s_t \\ f_t \end{bmatrix} = A(L) \begin{bmatrix} x_{t-1} \\ p_{t-1} \\ m_{t-1} \\ s_{t-1} \\ f_{t-1} \end{bmatrix} + B(L) X_t + \eta_t. \quad (2.9)$$

However, in a small open economy setup this identification scheme is not appealing, as small open economies are particularly concerned about exchange rate shocks and it is not plausible that the monetary policy rule does not interact with the exchange rate. This was the major concern of Kim and Roubini (2000) which let them derive an alternative identification scheme for the study of small-open economies. We consider the following (modified) version of these restrictions which have already been used by Peersman and Smets (2001) with some success:

$$\begin{aligned} A_0 \eta_t &= \varepsilon_t \\ &\Longleftrightarrow \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 & 0 \\ a_{31} & a_{32} & 1 & a_{34} & 0 \\ 0 & 0 & a_{43} & 1 & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & 1 \end{bmatrix} \begin{bmatrix} \eta_t^x \\ \eta_t^p \\ \eta_t^m \\ \eta_t^s \\ \eta_t^f \end{bmatrix} &= \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^p \\ \varepsilon_t^m \\ \varepsilon_t^s \\ \varepsilon_t^f \end{bmatrix}. \end{aligned} \quad (2.10)$$

Note that in our 5-dimensional VAR, the imposition of 10 additional restrictions as above leads to a just-identified system. Moreover, the interpretation of (2.10) is handy: Following Sims and Zha (1998), it is assumed that monetary policy does not respond contemporaneously to output or prices, an assumption of information delay which seems plausible when using CEE data. Even more this assumption seems appropriate as we use monthly data. Besides this, money, the short-term interest rate (as the policy tool) and the exchange rate are allowed to interact contemporaneously and are just mildly restricted.[25] This is an important feature of our setup, as monetary policy in many CEE countries has been a hybrid construct of interest rate rules combined with elements of money supply and exchange rate policies. Restricting these interactions contemporaneously with zero restrictions would therefore likely to be flawed.

[25] As Peersman and Smets (2001) argue, the exchange rate is supposed to react like an asset price and is therefore contemporaneously influenced.

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The money demand equation is fairly standard and depends on income and the opportunity cost of holding money. Moreover, we have a Phillips-curve relationship, where the change in inflation depends positively on lagged output.

2.3.1.1 The Fixed-Exchange Rate Model for Lithuania

For the case of Lithuania, which has a currency board arrangement, a fixed exchange rate without fluctuations is given. This means that the exchange rate is not an endogenously determined variable in Y_t but exogenous to the model. For our data sample under consideration, the litas was pegged to the U.S. dollar from April 1994 onwards. Note that the monetary policy authority loses its policy tool in a fixed exchange rate system as its primary goal is to keep the parity to the foreign exchange rate anchor. Similar to the spring 2004 report of the European Forecasting Network (EFN (2004)), we assume that the exchange rate becomes an exogenous variable, while the foreign short-term interest rate is included as the most exogenous variable in Y_t. However, instead of using the average EU interest rate, we take the U.S. Federal Funds Rate, s_t^{FF}, as the Lithuanian Litas was pegged to the US-$ until 2002.

$$\begin{bmatrix} s_t^{FF} \\ x_t \\ p_t \\ m_t \\ s_t \end{bmatrix} = A(L) \begin{bmatrix} s_{t-1}^{FF} \\ x_{t-1} \\ p_{t-1} \\ m_{t-1} \\ s_{t-1} \end{bmatrix} + B(L) \begin{bmatrix} f_t \\ cp_t^{EU} \\ x_t^{EU} \end{bmatrix} + \mu_t. \tag{2.11}$$

We make use of a Choleski identification scheme as in EFN (2004), which is sensible as it endogenizes domestic interest rates and leaves the foreign interest rate as a completely exogenous variable which affects all other endogenous variables within the system.

2.3.2 Including Expectations in VAR Models

Sometimes endogenous variables are presumed to depend not only on endogenous and exogenous variables as in equation (2.5), but additionally on expectations about endogenous variables. A standard VAR model is based on the information summarized in the contemporaneous and lagged values of the endogenous variables and does not permit the inclusion of any learning-based forward-looking expectations. When considering expectations, assuming rational expectations is still the standard workhorse. However, for CEE this assumption seems to be too strong and there are certain indicators that learning behaviour mattered (see e.g. Lyziak (2003)). We therefore make use of adaptive learning according to (2.1), which is the so-called statistical approach to learning. We do that by applying the Kalman filter to derive inflation forecasts from lagged inflation and inflation expectations. This is a variant of a time-varying recursive least squares learning algorithm, which produces the "best" estimates of inflation expectations.[26] These learning-based expectations data are then used in our VAR analysis.

Let us now have a closed look at the way the inflation expectations are generated. According to standard ADF tests, inflation rates in Lithuania, Poland and Slovenia are best described as a variable which is integrated of order one.[27] In other words, inflation can be described as a random walk process,

$$\pi_t = \mu_t + v_t, \tag{2.12}$$

where v_t is iid with zero mean. As Ljung and Söderström (1983) show, equation (2.12) can be seen as a special case of the general state-space representation

$$\begin{aligned} \pi_t &= \mu_t + v_t \\ \mu_t &= \mu_{t-1} + w_t, \end{aligned} \tag{2.13}$$

where μ_t is the state variable. We consider $\text{var}(v_t) = \sigma_v^2$, and $\text{var}(w_t) = \sigma_w^2 I$ for technical reasons. Denote $\widehat{\mu}_{t+1}$ as the estimator of the state μ_{t+1}. We can then write the Kalman filter (see e.g. Harvey (1989)) as

$$\widehat{\mu}_{t+1} = \widehat{\mu}_t + \kappa_t \left(\pi_t - \widehat{\mu}_t\right), \tag{2.14}$$

The Kalman gain, κ_t, is

$$\kappa_t = P_{t-1} \left[\sigma_v^2 + P_{t-1}\right]^{-1}. \tag{2.15}$$

where P_t is the mean square error of the estimator $\widehat{\mu}_t$, which can be represented as

$$P_t = P_{t-1} - \kappa_t P_{t-1} + var\left(w_t\right). \tag{2.16}$$

We are then able to use the Kalman filter for a one-step ahead prediction of the inflation rate. Note that this is a learning model which shows a considerable rationality on the side of the private sector, as it produces optimal forecasts in an unstable economic environment. It is a useful concept for our transition countries, however, as it entails implicitly a time-varying framework which is able to capture changes in the inflation process.[28]

[26] "Best" here means that the Kalman filter is the best linear forecast of the unobserved variable based on available information. However, in the relatively small samples under consideration, the Kalman filter might not be optimal and there might exist non-linear filters which would do better.

[27] To save space, ADF tests are not reported here but available on request.

[28] To implement the Kalman filter, one has to specify the starting values for P_0 and μ_0. We make use of educated guesses, which is the best we can do given that we have a random walk process.

We have seen how we derive our inflation expectations from real world data recursively. In the next step we can advance on our initial VAR setup (2.9) and include inflation expectations. We do that in the following way

$$\begin{bmatrix} x_t \\ p_t \\ m_t \\ s_t \\ f_t \end{bmatrix} = A(L) \begin{bmatrix} x_{t-1} \\ p_{t-1} \\ m_{t-1} \\ s_{t-1} \\ f_{t-1} \end{bmatrix} + B(L) \begin{bmatrix} cp_t^{EU} \\ x_t^{EU} \\ s_t^{EU} \\ \pi_t^e \end{bmatrix} + \eta_t. \tag{2.17}$$

Our new model tries to improve on standard VAR setups by including additional inflation expectations as an exogenous variable. This adds some additional informational content to the VAR but does not condition the expectations on other endogenous variables of the VAR. Assuming this kind of exogeneity for inflation expectations implies also that they are independent of the residual process (see also Engle, Hendry and Richard (1983)). Let me briefly explain, why this assumption seems to be reasonable: Firstly, data availability of monetary policy aggregates was generally poor in CEE, at least in the early stabilization phase. Moreover, macroeconomic variables such as output were available only with considerable lags which made them less useful for expectations formation. On the other hand, the monetary authorities had only scant ideas about the expectations formation process of the private sector, due to a general lack of understanding of the monetary policy transmission mechanism and also because surveys such as the ones conducted in industrialized countries were not known. From this perspective it seems to be reasonable to assume that expectations were exogenous to the rest of the VAR variables.[29]

2.4 Results of the Identified VAR

Before presenting the VAR results, let us briefly note some important prerequisites for our analysis. The first one concerns the differing sample periods under consideration. As noted before, we do not intend to model the whole process of economic liberalization and stabilization in transition countries, but try to focus on the stabilization period only. As economic reform did not start at the same time in all countries, we use Polish data from 01/1991, Lithuanian data from 01/1993 and Slovenian data from 01/1992. We only consider data until 12/2000, as afterwards the European Commission's DG ECFIN started to publish consumer surveys for price trends, which would have a considerable impact on our model.

[29] We also checked our assumption technically. Huh (2003) has shown that a variables is weakly exogenous to the structural model if there is no Granger causality from the variable of interest to the other variables in the reduced form. We therefore perform a Granger causality test which confirms our assumption.

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For the time period under consideration, potential parameter instability due to policy shifts has to be dealt with in order to secure that identifying restrictions on parameters are meaningful. We therefore run break-point Chow tests[30] for all series, which can be rejected at the 1 % level for Slovenia and Lithuania; Polish inflation data exhibit some instability, which vanish once looking at the 5 % confidence level (see figures 11-13). In fact we were quite surprised by these results ourself, as there were changes taken place in the monetary policy conduct in all three countries. However, as already mentioned above, monetary policy was never as clear-cut as officials tried to explain to the public. There was always a policy mix and a hybrid construct of using various instruments at the same time. In this light, officially declared changes in monetary policy (if abstracting from fixed exchange rate rules) gave only an indication that the balance of instruments used might have changed but not that the country was facing a well-defined regime change.

The determination of the lag length is another important issue, as VAR results are very sensitive to the lag lengths used. We employ the Akaike Information Criterion (AIC) (see table 2). We decided to use a lag length of 2 for all countries, which is a parsimonious choice given our relatively short sample sizes.[31] Note that we additionally ran Portmanteau statistics as a goodness-of-fit test for each equation in the systems that do not indicate any misspecification.[32] For the lag polynomial $B(L)$ we only consider $B(0)$ which enables us to interpret the estimated parameters as elasticities.

Note that we consider two types of VARs. The first one (2.9) being the standard formulation and the second one including a measure of inflation expectations (2.17). We will discuss the results country-by-country first and then compare the two setups to each other.

The empirical analysis is conducted with two software programmes, *GiveWin* (PcGive Version 2.10) and *Structural VAR* (Version 0.31). Impulse responses are shown in figures 14-19, where we also report the 95% confidence interval.

2.4.1 Results of the VAR without Expectations

Let us first consider the results for Poland. The responses for all variables seem all reasonable from a theoretical point of view, although it is somehow surprising that the negative impact on inflation gets mildly reversed after 10 months and fades out completely after 2.5 years. Especially the so-called price puzzle does not seem to be present, due to the

[30] The idea of the breakpoint Chow test is to test for any sizable differences in the estimated equations. A significant difference indicates a structural change in the relationship.

[31] Note as well, that the AIC is known to favour higher VAR orders in comparison to other criteria, such as the Schwarz Criterion (SC).

[32] Results are available on request.

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inclusion of the foreign commodity price in the vector of exogenous variables. The Slovenian responses are less stringent. Most notably the monetary aggregate shows an unusual increase. The 95% confidence intervals do not seem to narrow down after time, so there seems to be a lot of "noise" in the model. Besides this, especially the initial responses of CPI are satisfactory, although also here a positive effect starts to enter after 15 months.

For Lithuania things are different, not only because of the differing exchange rate regime. CPI responds sharply with no indication of a price puzzle However, the error bands are quite large. There seems to be an effect of imported inflation, which only slowly fades out after about 4 years. Output seems to be only little effected, which is why we can conject that exchange rate related stabilization policy entails a larger credibility and benign output losses.

2.4.2 Results of the VAR with Expectations

Let us now discuss the results of the VAR with expectations added to the vector of exogenous variables. Remember that these variables are derived from Kalman filter estimates given historical inflation data. The Kalman filter, as mentioned before, is a special recursive updating mechanism which can be compared to recursive least squares estimation. We report the different Kalman gains in table 11. As can be seen, the Kalman gain is largest in Lithuania and smallest in Slovenia. Given that we are discussing the updating mechanism of a time-varying estimation, this result can be interpreted that learning was faster in Lithuania as it was in Poland and even faster than it turned out to be in Slovenia. The estimated coefficients point into the same direction as the estimated "learning speed". In economies which managed to facilitate faster learning, the impact of inflation expectations is more pronounced.

In the case of Poland the introduction of expected inflation changed the qualitative results only for CPI, which does not exhibit the reversal to positive price effects of a monetary policy shock anymore. Moreover, the confidence intervals of all variables narrowed down, which is why we speculate that the precision of this model is much stronger. For Slovenia, the initial price puzzle vanishes too, and also the precision of our estimates increased by large. Only the impulse response of the monetary variable remains puzzling, which is strongly positive initially, turns negative only after six months and reverses after just two years. Maybe this indicates the inherent contradictions in Slovenian monetary policy mentioned earlier, where heterodox policy measures were chosen until recently. The Lithuanian results also improve in terms of their precision and the effects of the shock die out faster than under the basic VAR setup. This somehow reflects again that the fast learning behaviour in this country was an important factor behind the successful disinflation process, fueled of course by bought-in

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credibility via the currency board arrangement.

2.4.3 Summary of Results

All countries faced more or less hybrid disinflationary strategies with sometimes conflicting policy goals as was discussed earlier. Whereas Poland's and Slovenia's main emphasis was on interest rate measures, Lithuania embarked on a currency peg. The disinflation achieved under these monetary regimes are characterized by rapid elimination of hyperinflation but subsequent differences in the persistence of moderate inflation. Especially the persistence of inflation in Slovenia is seen as puzzle due to enormous efforts in fiscal and monetary policies.

The dynamic responses that we yield from our VAR analyses are broadly consistent with theoretical presumptions. This supports our identifying assumptions in the small-open economy VAR. Most notably, there is almost no price puzzle present, which favours our assumption of including exogenous variables such as foreign commodity prices in the VAR setup. However, wide confidence bands indicate that we could do better.

In our second model, by introducing expected inflation rates as an exogenous variable, we can show that confidence bands tend to be narrower than before and most remaining inconsistencies in impulse responses seem to vanish. This points to the fact that the informativeness of an extra exogenous variable somehow improved the efficiency of our estimates. Moreover, in all three countries the inclusion of inflation expectations leads to faster adjustments in the output variable. That implies that the effects of monetary policy are small and transitory in those countries, in which private sector learning matters much, and are bigger and more persistent in those countries, in which learning behaviour is less pronounced. This has important implications not only for the monetary policy conduct but also for achievable sacrifice ratios, where potential output losses are smaller in countries with faster private sector learning.

2.4.4 Checking Robustness

Although impulse responses give a clear sign that our model setup is quite successful in explaining real world phenomena, we performed a couple of robustness checks.[33] We checked for differences within sub-samples of our benchmark results, but find out that qualitative effects of our baseline study are robust across sub-samples. However, one has to note that these additional tests entail difficulties concerning the degree of freedom.

A second kind of check is due to Giordani (2004) who shows that adding a time trend

[33] Results are not reported here but are available on request.

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can lead to greater efficiency of the results, as one accommodates for non-stationarity in the data. We find out that the results are mixed and we do not see the clear improvement as in Giordani (2004).

Last but not least we consider a larger number of lags in the VAR specification. As mentioned earlier, there is always a trade-off for the degrees of freedom; we find out that increasing the lag length does not change our results qualitatively and reduces the validity of our estimates of the implied model by far.

2.5 Conclusions

It is unquestionable that inflation expectations have a major impact on actual inflation. Thus, the management of private sector expectations has to be at the focus of any monetary authority which tries to conduct a disinflationary policy. This is particularly true for CEE countries in which economic reform and transition were intense for the first decade after liberalization, and forming inflation expectations was an inherently difficult task.

Our analysis was particularly motivated by a puzzle in the disinflation processes of CEE countries: While domestic monetary and fiscal policies did not necessarily exhibit signs of austerity, inflation rates were decreasing at a rising speed in many countries. In other cases inflation remained above a certain threshold even though the macroeconomic environment was sound. That is, the textbook-style link between monetary and fiscal policies on the one hand and the behaviour of the inflation rate on the other hand was fuzzy. At least there seems to be an indication that other factors mattered too. We believe that the expectations formation process is such a factor and pose the hypothesis that learning behaviour matters in the disinflation process of CEE countries.

We introduce the expectational process by means of a recursive least-squares learning rule, mirrored by Kalman filter forecasts. That is, agents are perceived to behave like econometricians: They do not know the relevant parameters of the model but use available data and derive statistical inferences. These learning processes are incomplete and can only be boundedly rational, which emphasizes the importance for the monetary authorities to give clear signals to anchor inflation expectations. This can substantially increase the speed of learning by the private sector, similar to the description of Flemming's (1976) "changing gears".[34] The Kalman gain, as a measure of the speed of adjustment of inflation expectations,

[34] Flemming (1976) showed that the senstivity of the expected prices to current prices rises as the monetary authority moves up through the "gears". That is, monetary policy should be more aggressive, the more private agents discount past dara.

is therefore not an arbitrary learning parameter, but rather determined by the behaviour of the process under consideration.

We make use of a small-open economy VAR setup with two specifications. Following Peersman and Smets (2001), our first specification allows for contemporaneous interactions of the exchange rate, money and the interest rate. This VAR specification yields surprisingly stringent impulse responses for the three countries under concern. Most notable, the almost standard price puzzle is almost absent.

Our second specification adds inflation expectations as an exogenous variable to the initial VAR setup. Expectations are derived from a time-varying recursive learning process mirrored by using the Kalman filter to generate "optimal" - in terms of estimated error covariance - forecasts. We are able to show that when we relax the standard assumption of rational expectations by adding expectations derived from learning processes, we are able to improve our results in two aspects: While the qualitative results are not much different to the initial specification, the inclusion of inflation expectations enhances the precision of our estimates and produces responses which point to small and transitory effects of monetary policy shocks. Moreover we are able to capture a key issue of cross-country differences in disinflation by showing that countries with a higher speed of learning experience faster and less burdensome disinflation processes in terms of smaller sacrifice ratios.

But how can central banks try to pursue the "management of expectations" if it is of such big importance for monetary policy? As Cottarelli and Doyle (1999) note, "[t]he absence of publicly announced monetary targets [...] reflects structural change and uncertainty" which gives only little orientation for private sector inflation forecasts. Gaspar et al. (2005) show, that central banks behave optimally, if they try to "anchor inflation expectations" by decreasing the limiting variance of the private sector's inflation expectations. To put it differently: if a central bank gives rise to unstable inflation (expectations), it will be harder to disinflate due to slower adjustments of inflation expectations.

Once can even go one step further and say that only central banks which conduct a credible monetary policy (with a transparent monetary target) facilitate private sector learning. In that respect, a faster learning behaviour by the private sector can be seen as an indicator for the quality of the policy conducted by the central bank.[35,36]

[35] Note that with rational expectations, which represents one end of the possible range of expectational mechanisms, one would get an immediate response and adjustment to monetary policy changes.

[36] Looking at measures of central bank independence (such as the ones conducted by Dvorsky (2000) and Cukierman et al. (2001)) for CEE countries shows that independence has an unsurprising correspondence with the size of our estimated learning parameters. According to their studies, central bank independence is higher in Lithuania and Poland as in Slovenia, where the first two countries cannot be ranked unambiguously.

This has several implications for the specific policy conduct of monetary authorities. As Issing and Gaspar (2005) note, the presence of forward-looking variables in the economy implies additional advantages apart from the credibility issues. The monetary authority can influence economic outcomes by credibly influencing private sector expectations, which can be perceived as an additional monetary policy channel. For that transparency in terms of publishing monetary policy targets and relevant economic data should be given. Moreover, codified central bank independence can be an important factor in facilitating private sector learning, as it substantiates perceived credibility of policy targets, even if the monetary authority has preferences for surprise inflation.

Besides these interesting results, our analysis gives rise to two more questions, which cannot be tackled in this paper but should be subject to future research. The first one concerns the direct impact of the monetary policy reaction function on expected inflation. As inflation is not part of our vector of endogenous variables, we have no impulse responses at hand which can tell us something about this relationship. Related to this is the issue that if learning depends on the actual inflation experience, then any discretionary monetary policy interacts with the speed of disinflation. But in which way and how strong is this effect? To give answers to this question, one would need to model the central bank behaviour in more realistic way, which we have started working on with only very preliminary results.

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CHAPTER 3

INFORMATION DISCLOSURE THROUGH MARKET ACTIVITY WITH AN APPLICATION TO MARKET CRASHES

"We must look at the price system as a mechanism for communicating information if we want to understand its real function." (Hayek (1945))

3.1 Introduction

If one considers a market in which agents' information about the true state of nature is incomplete, in planning his actions, how does an agent take into account what other agents know about this true state? Is there a tendency that agents in this economy *learn* all the collective information over time? And what role does the price system play in that context?

In standard (rational expectations based) financial market models, economic agents are assumed to possess private, idiosyncratic information and to incorporate additional, publicly available information. In determining whether to give a sell or buy order of an asset, individual investors combine these two sources of information to derive rational decisions. Due to this, the asset price is informative also about the private information of investors. But how is private and public information being exactly aggregated in the market price? Hayek's (1945) critique of central planning suggests that the market *process* itself is able to accomplish full aggregation. That is, through collective action of individually rational agents, aggregation of dispersed information occurs, which results in prices that coincide with the asset's fundamentals. This line of argument was formalized at the end of the seventies, when an influential literature has developed focusing attention on the transmission of information through prices (see also box 1). According to Radner (1979), when the state space is finite, a rational expectations equilibrium exists generically, is finite in number and fully revealing in the sense that every agent can infer from prices alone the combined knowledge of all other agents. Some economists therefore argue that theorems on the existence of fully revealing equilibria support Hayek's views on the virtues of the market as an information processor.

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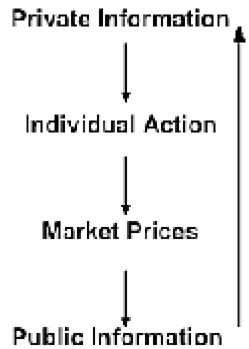

Box 1 - Flow of Information

Rational expectations (hereafter RE) models suggest that markets achieve this result, even though traders are unable to communicate information directly. Grossman (1976) was the first one to analyze a market with n-types of informed agents, where each agent holds only a part of the relevant information. In his model, the only source of uncertainty is the liquidation value of the asset. Grossman (1976) shows that in such a case the price system perfectly aggregates information.[1] But if this hypothesis would prove to be true, how would we deal with episodes of extreme market reversals such as the October 1987 crash, which is difficult to be justified by real news? Obviously there is a need to look more carefully at individuals' beliefs.

There are many possible ways to analyze how and why asset prices might deviate from fundamentals in financial markets. One may model such deviations as some kind of irrationality[2] or as an equilibrium outcome of the interaction of rational agents. However, whatever one's approach is, one of the central questions in terms of market crashes is, how agents coordinate their beliefs. One might think about anecdotal evidences (e.g. Kindleberger (1989)), sunspots (e.g. Greenspan (1996)), news effects etc. We argue that it is the mere fact that information among agents is incomplete and asymmetric and that it takes time to make inferences from initially given private information. This might result in a lumpy and "hidden" adjustment process that causes what we perceive to be runs or panics in asset markets. In this sense, we believe Kindleberger (1989) to be right that market reversals are "foreshadowed by a gradual erosion of confidence in the market". However, this erosion is not always visible to market participants.

The formal methodology which we employ is not new and goes back to Aumann (1976), who considers common priors and common knowledge of posterior beliefs. Aumann (1976)

[1] The theoretical literature requires that a fully revealing price has to be a *sufficient statistic* of the market for the unknown variable (i.e. the liquidiation value).

[2] See Shleifer (2000), Shiller (2000), or Thaler (1993) on issues of behavioural finance.

shows that under such circumstances, one cannot "agree to disagree", so posteriors need to be equal. In a finance context this could be interpreted as a case of initially asymmetric information, in which ultimately all agents agree on the same valuation of an asset.

But the value of an asset is not only influenced by existing information. It also depends on tomorrow's price, so that investors naturally need to form expectations of future market expectations. Therefore, in forming his decisions, an agent clearly faces two types of uncertainty, *fundamental uncertainty* and *strategic uncertainty*. Fundamental uncertainty refers to uncertainty concerning the payoff-relevant state of nature. Strategic uncertainty refers to the uncertainty concerning the beliefs of other investors. Even if fundamental uncertainty becomes smaller and smaller, strategic uncertainty may remain as large as ever. However, when the informational environment of the economy changes for some reason, both fundamental uncertainty and strategic uncertainty will undergo changes. The net effect on the outcomes will depend on the interplay of the two. This is especially true when the new information received is public. As well as providing information on the underlying state, public information plays a role in conveying information on what others believe and know, and may work as a coordinating device.

We will focus on a stylized investment model with incomplete and asymmetric information, which employs the notion of common knowledge. Note that investment decisions lead to a positive net payoff only if the state is good and all other agents also choose to invest. For payoff externalities to matter, agents would therefore need to exhibit both backward-looking and forward-looking behaviour. In our model, we relax the rational expectations assumption by assuming learning behaviour: agents observe the market price and use it together with their private information to derive their investment decision. Agents are assumed to be perfect Bayesians and to that extent they are substantively rational. However, they do not take into account the full hierarchy of higher-order beliefs ("beliefs about the beliefs ... about the beliefs of others"). Our model builds directly on the seminal papers by Geanakoplos and Polemarchakis (1982) who show that, employing the common prior assumption, convergence of posterior beliefs takes place in finite time. While they have to assume that agents directly communicate their posterior beliefs with each other, we are able to show in chapter (3.4), that it is sufficient if they can observe each others' actions. This is already a big step towards an adequate description of real world markets.

A second strand of this chapter is an extension of McKelvey and Talbot (1986), who have shown that if an aggregate market statistic is made public, this can ultimately lead to convergence of posterior beliefs. These two authors say that this aggregate statistic could be the price system. However, they abstract from strategic complementarity which is an

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important ingredient of any financial market model. Once added to the model, we arrive at our main result: Market crashes, defined as a consensus on the agents' willingness to sell the asset at the same time, can take place without changes in external conditions provided that the informativeness of the public signal (e.g. the market price) is not too high. We will specify later what we mean by "not too high". Compared to the results of McKelvey and Talbot (1986) this means that if the market price can be considered to be a sufficient statistic, convergence of posterior beliefs will take place only under specific circumstances.

The structure of the paper is as follows. Section 3.2 reviews the related literature. In Section 3.3 we introduce Aumann's (1976) famous "agreeing to disagree" result and describe the Geanakoplos and Polemarchakis (1982) model. We also solve a numerical example for the latter model. In section 3.4 we introduce our model and expand on Geanakoplos and Polemarchakis' (1982) work by showing that direct communication of posterior beliefs can be substituted by observing others' actions only. We sketch how convergence of posteriors is obtained by employing Geanakoplos and Polemarchakis' (1982) numerical example. In section 3.5, only market prices are observable and we analyze how this influences the basic result. We conclude by stating the main results and pointing at potential policy recommendations.

3.2 Related Literature

Our work is related to different branches in finance theory and information economics.

The assumption that prices reveal information has been extensively studied in the literature. Radner (1979) proves the generic existence of a fully revealing RE equilibrium under the restriction that the mapping from signals to prices is invertible. Given that this assumption does in general not hold, Jordan (1983) was able to show that there can be equilibria that are not fully revealing. The assumption needed is that the dimension of the signal space is larger than the dimension of the space of relative prices.

In models developed by Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1982), prices are prevented from fully revealing the fundamentals of the economy by assuming the existence of liquidity traders.

Radner (1979) resumes that "an economic agent who has a good understanding of the market is in a position to use market prices to make inferences about the non-price information received by other agents". But trading strategies that rely on interpretations of other investors' actions as revealed in prices can create and amplify noise.[3] The results may be

[3] Note that the market price, besides its allocative function can fulfill an additional informational function which is both bounded from below and above: It is bounded from below by the information contained in the information held privately. It is bounded from above by the fully-revealing market price.

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markets characterized by price overreactions and instability, with intermittent price run-ups followed by sudden price reversals.

While the role of information in price formation has also been a concern of finance researchers, there have been only few models of speculative behaviour with heterogeneously informed agents. Based on Kyle (1985), a sizeable literature on models with identically informed agents has been developed, being unable to address the issues of information aggregation. It was only due to the 1987 stock market crash that a sudden interest in information-based explanations recurred, as the drop in asset prices at that time was not accompanied by any major news and could not be explained by standard models. Subsequently, most models of market crashes tried to explain market crashes by introducing explanations that come from outside the model. In sunspot examples, such as Allen et al. (1993) and Sandroni (1998), there are multiple equilibria and agents observe the realization of a uninformative public signal, a sunspot. This signal changes agents' beliefs and serves as a coordination device to move from one equilibrium to another. This shift might cause drastic price changes that deviate from fundamental values. However, the idea that an intrinsically useless signal can have an influence on the real economy is somehow puzzling. Even if it is only supposed to work as a coordination device, it seems to undermine the rationality assumption of agents strongly. Other examples of models of market crashes include models of liquidity shortage and news models (see the excellent review by Brunnermeier (2001)). Models with news effects (e.g. Abreu and Brunnermeier (2003)) are somewhat similar in character to sunspot explanations. The difference is that public signals/news are supposed to be informative. However, their main impact is again that of a coordination device, which causes a disproportionate effect relative to their intrinsic information content. The shortcoming here is that real world market crashes often do not behave like that (cf. Kindleberger (1989)), the most notable example of a market crash without sizeable news being the crash of the Dow Jones in October 1987. Also experimental work (e.g. Plott and Sunder (1988)) has shown that market crashes do not need to be accompanied by any public signal or news.

The papers closest to ours are those of Gavious and Mizrahi (2000) and Hart and Tauman (2004), which in fact uses almost the same principle idea as we do in section (3.4). Their model, however, has two shortcomings. Firstly they give an example of market crashes in cases in which the true state of the world is good, where this result strongly hinges on their decision rule which is not derived from first principles. Moreover, they do not really look at a finance context, as the inclusion of prices adds an additional dimension to the model,

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namely strategic complementarities, which is not given in their setup.[4]

As mentioned in the introduction, strategic uncertainty is at the core of any finance application. The literature on coordination and higher-order beliefs commences with Halpern (1986) and Rubinstein (1989). Subsequently, in the 1990s, various contributions of Stephen Morris and his coauthors have emphasized the importance of common knowledge and higher-order beliefs and focused on various finance applications. Morris et al. (1995) provide conditions under which qualitative results of models without higher-order beliefs can be reversed when those beliefs are added to models of asymmetric information. More precisely they can identify necessary conditions for the existence of bubbles in financial models, i.e. the depth of knowledge[5] must not be too big.

What these authors disregard is that investment can lead to a positive payoff only if other agents do not choose to disinvest at the same time. Thus, there is some kind of payoff externality (called strategic complementarity (cf. Milgrom and Roberts (1990)), in which each player's marginal utility of increasing his strategy rises with increases in other agents' strategies. Only if this additional aspect is taken into account, a financial market model entails not only backward-looking, but also forward-looking elements.

The papers closest to ours, are Geanakoplos and Polemarchakis (1982) and McKelvey and Talbot (1986). Geanakoplos and Polemarchakis' (1982) main result is that one can construct an *indirect communication equilibrium*[6] in which revisions of individual actions only take place in the last period. McKelvey and Talbot (1986) translate this result into a model, in which some aggregate public statistic is sufficient for convergence of posterior beliefs.[7] However, this result implicitly assumes a one-to-one relationship between posterior beliefs and actions, which in terms of a financial market model with binary action is not valid. If the set of actions is discrete, a particular decision can be based on a large amount of different beliefs.

3.3 Learning through Direct Communication

An information structure for agent i is a set of possible states of nature and a function that assigns to each state a non-empty subset of states. More formally, we are talking about

[4]The idea of pushing the direct communication case of Geanakoplos and Polemarchakis (1982) one step forward and looking at prices instead of posterior beliefs was given by these authors in their concluding remarks to above paper.

[5]A state space Ω whose states specify first-order knowledge is said to have depth equal to one (cf. Morris et al. (1995)). Depth of knowledge of order n is defined correspondingly.

[6]Which implies convergence of posteriors to some equilibrium posterior, simply by communicating posteriors between agents back and forth.

[7]Nielsen et al. (1990) generalize these results to conditional expectations.

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a measurable space $(\Omega, \mathcal{F})$, where Ω is the state space, that is the set of possible states of the world and $\mathcal{F}$ is a function that assigns to each state ω a nonempty subset of states $\mathcal{F}(\omega)$. We assume Ω to be finite and any nonempty subset of Ω is called an *event, A*.

Aumann's (1976) seminal paper showed that if two rational agents (we call them Alice and Bob, or simply α and β from now onwards) start from a common prior and posterior beliefs are common knowledge between them, they cannot "agree to disagree" about the probability of an event A: That is, updating the probability of event A on the basis of private information cannot lead to an ultimately different assessment of the probability of A.

Proposition 3.1 (cf. Aumann (1976)) *Consider a set of possible states Ω. Posterior beliefs of α and β about an event $A \subset \Omega$ are common knowledge between them and both have the same prior distribution over Ω. Then posterior beliefs must be equal.*

Proof. *See Aumann (1976).* ■

Intuitively, Aumann' s result says that differences in information alone are not a valid source of belief heterogeneity. Bacharach (1985) extends the results to an n-person setup, which does not alter the basic result. It is not surprising that Aumann' result formed the basis for a large literature, both in game theory ("interactive epistemology"), economics, finance and beyond.[8] However, the assumption that posterior beliefs are common knowledge is quite strong. We will now see whether relaxing this assumption will change the basic result.[9]

3.3.1 Geanakoplos and Polemarchakis (1982) - Revisited

The convergence result of Aumann (1976) holds also if posterior beliefs are not common knowledge but are communicated back and forth between α and β and both revise their posteriors after each round of communication. This is the famous result of Geanakoplos and Polemarchakis (1982), which we will briefly reformulate now.

Consider the finite set Ω of possible states of the world, where

$$\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

We say that a state of the world provides a full description of all important aspects. A subset $A \subset \Omega$ is called an event. We say that this event is realized if the true state of the world

[8] A special application of Aumann's (1976) result is Milgrom and Stokey's (1982) no-trade theorem and Tirole's (1982) impossibility result of speculation in rational expectations contexts.

[9] Note that the assumption of common priors is crucial for Aumann 's (1976) result. This assumption is among the most disputed ones, see e.g. Morris (1995) and Bonanno and Nehring (1998) for an exhaustive discussion.

$\omega \in \Omega$ is in A. There are two agents, α and β, and their private information is modelled in terms of partitions of the state space, denoted as P_α and P_β for α and β respectively. A partition is a collection of subsets of Ω, which need to be disjoint and have Ω as the union. Being better informed means in that context that you have a "finer" partition, i.e. you are more sure about the true state of the world. Geanakoplos and Polemarchakis (1982) consider the following partitions, which are supposed to be common knowledge:

$$\begin{aligned} P_\alpha &= \{\{1,2,3\},\{4,5,6\},\{7,8,9\}\} \\ P_\beta &= \{\{1,2,3,4\},\{5,6,7,8\},\{9\}\}. \end{aligned} \tag{3.1}$$

The smaller a particular partition cell, the better is the information of the agent. For example, if the true state of the world would be $\omega^* = 9$, β would know it for certain, whereas α would know it only with probability $1/3$.

Both agents have common priors of the set of states of the world, $p_\alpha(\omega) = p_\beta(\omega) = 1/9$ for all $\omega \in \Omega$. Geanakoplos and Polemarchakis (1982) consider that α and β are able to communicate directly with each other. More precisely they announce posterior beliefs to each other and each agent revises his or her own beliefs by using the additionally acquainted information. The revision process itself is a simple Bayesian updating rule.[10]

To see how the updating and learning process works, consider an example. Denote p_i^j as the cell j of agent i's partition, e.g. for agent α

$$P_\alpha = \left\{ \underbrace{\{1,2,3\}}_{p_\alpha^1}, \underbrace{\{4,5,6\}}_{p_\alpha^2}, \underbrace{\{7,8,9\}}_{p_\alpha^3} \right\}.$$

Let $\omega^* = 3$ be the true state of the world and let us consider the event $A = \{3,6,9\}$ to be the bad outcome. This setup implies that α is informed that the true state of the world lies in p_α^1, while β is informed that the true state of the world is in p_β^1. α's posterior beliefs for

[10] Bayes' Rule is a natural way to derive the conditional probability $prob(A_i \mid B)$, if $prob(A_i)$ and $prob(B \mid A_i)$ are known $(i = 1,2,...,n)$. In this context, B is an event with $prob(B) \neq 0$, and $A_1, A_2, ..., A_n$ are pairwise disjunct events with $P(A_i) \neq 0\ \forall i$ and $A_1 \cup A_2 \cup ... \cup A_n = \Omega$. Bayes' Rule then says that

$$prob(A_i \mid B) = \frac{prob(B \mid A_i)\, prob(A_i)}{\sum_{j=1}^{n} prob(B \mid A_j)\, prob(A_j)}, \quad \text{for } i = 1,2,...,n.$$

Using the total probability formula, this expression is identical to

$$prob(B)\, prob(A_i \mid B) = prob(A_i)\, prob(B \mid A_i), \text{ for } i = 1,2,...,n.$$

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event A are

$$PB_{\alpha}(A) = \begin{cases} prob\left(A \mid p_{\alpha}^{1}(\omega)\right) = \frac{1}{3} \\ prob\left(A \mid p_{\alpha}^{2}(\omega)\right) = \frac{1}{3} \\ prob\left(A \mid p_{\alpha}^{3}(\omega)\right) = \frac{1}{3} \end{cases}, \text{ and}$$

$$PB_{\beta}(A) = \begin{cases} prob\left(A \mid p_{\beta}^{1}(\omega)\right) = \frac{1}{4} \\ prob\left(A \mid p_{\beta}^{2}(\omega)\right) = \frac{1}{4} \\ prob\left(A \mid p_{\beta}^{3}(\omega)\right) = 1 \end{cases}$$

Why do we consider all partition cells here, although we know that the true state of the world is $\omega^{*} = 3$? The reason is that this information is *mutual* but not *common* knowledge! As Rubinstein (1989) illustrates, a game in which a particular piece of information is common knowledge can behave profoundly different to a game in which this information is mutually known up to some finite order, no matter how many orders one takes into account. We therefore have to consider the whole state space Ω. In our particular example, α would communicate a posterior of $1/3$ and β a posterior of $1/4$. To β this entails hardly any news, but α can infer from β's posterior that the true state can not be $\omega = 9$. The "common knowledge state space" has thus reduced to $\Omega' = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Step by step this revision process leads us to posterior beliefs which are $1/3$, where no further revision takes place as both α and β have exactly the same opinion.

Proposition 3.2 (cf. Geanakoplos and Polemarchakis (1982)) *Under the assumption that the two information partitions are finite, given any event A, the revision process converges in finitely many steps.*

Proof. *See Geanakoplos and Polemarchakis (1982).* ■

This result is quite strong. Not only does direct communication lead to a convergence of posterior beliefs, which relaxes Aumann's assumption of common knowledge, the result also shows that this convergence takes place in finite time if both partitions are finite.

Geanakoplos and Polemarchakis (1982) are able to show another interesting fact. It may be that the revision process is "hidden", in the sense that posterior beliefs are revised after each period, but no apparent revision in actions occurs until the final round. That is, a small change in information of agents is not necessarily reflected in a (small) change in action. We see this result as a valid starting point for analyses of financial markets, in which crashes are not apparently accompanied by bad news hitting the market or other obvious reasons for an abrupt price reversal. The "hidden" process of gradual convergence of posterior beliefs of market participants might be a key here, where information changes

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gradually while individual action changes abrupt between buy and sell orders. We will set up a stylized model in the next section, which nests the setup of Geanakoplos and Polemarchakis (1982).

3.4 Learning from Observing Others' Actions

In many economic situations, agents possess private information that is imperfectly correlated with the true state of nature. If information of agents is asymmetric, they might be able to infer some of the privately held information from other agents. One way in which this might happen is through direct communication. Geanakoplos and Polemarchakis (1982) have shown that if two players have common priors and finite information partitions, repeated communication of posteriors will induce a common posterior in finite time, that is agents' posterior beliefs become identical (see chapter (3.3.1)) Thus, direct communication of posterior beliefs ultimately leads to a state in which no further revisions occur.

The question is what happens, if direct communication is not feasible, as is true for many real world markets, especially financial ones? We introduce a model of social learning, in which agents take decisions that can be observed and agents might be able to infer some of the privately held information from these actions. Agents can not, however, directly communicate with each other, so it is only the channel of observing others's actions through which the level of common knowledge about the true state improves. Or to put it differently: Trading activity itself plays an essential role in both the release and the dissemination of privately held information.

Let us consider a two-player investment game, where the finite state space Ω consists of nine states $\Omega = \{1, 2, 3, ..., 9\}$ with equal prior probabilities. There are two agents, α and β, also called investors, who will decide in each period whether they want to invest or disinvest. If both investors disinvest at the same time, we define this reaction as a market crash.

Consider that we can divide the outcomes of each state of nature into two groups: bad outcomes and good outcomes. Let $\{1, 3, 6, 7, 8, 9\}$ denote the states in which one yields the bad liquidation value and $\{2, 4, 5\}$ to be the states with a good liquidation value. We assign a zero liquidation value for the asset in a bad state and a liquidation value of one in a good state. For the decision rule this implies that "invest if the expected return is strictly higher than the initial posterior belief ", and "disinvest if the expected return is not strictly higher than the initial posterior belief" is the action profile for both agents. Note that both investors initially face an expected payoff of $1/3$.

In period t, agents' private information is described by means of information partitions. A partition has to be a collection of subsets (called *cells*) that are mutually disjoint and have

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a union Ω. In our example, consider that agents initially have the following partitions:[11]

$$P_\alpha = \{\{1,2,3\},\{4,5,6\},\{7,8,9\}\}$$

for agent α and

$$P_\beta = \{\{1,2,3,4\},\{5,6,7,8\},\{9\}\}$$

for agent β. Let us define the cells of α's partition in period t as $p^1_{\alpha,t}$, $p^2_{\alpha,t}$, $p^3_{\alpha,t}$. β's cells are defined accordingly. We assume that agents know each others' partition.

Let A be a bad event and assume that $A = \{3,6,9\}$, thus representing the bad outcome. We assume that both agents know that they are facing a bad market outcome (i.e. event A occurred).[12] Assume further that the true state of nature is $\omega^* = 3$. As in Geanakoplos and Polemarchakis (1982), in the case of the true state ω^* the agents are informed to which cell $p^j_{i,t}(\cdot)$ the true state belongs. If this cell is not a singleton, we can imagine that everybody knows the true state only with some noise.

In the subsequent paragraphs we will show that a crash will occur, which will be caused by the information transmission between agents only. While we assume that direct communication of posterior beliefs does not take place but only actions are observable, we nevertheless will get a result similar to the one in Geanakoplos and Polemarchakis (1982) in that no obvious revision occurs until the last period. In particular, agent α will disinvest and agent β will invest in the asset and after several trading periods both investors want to disinvest at the same time. As mentioned before, we interpret this action profile as a crash.

Period 1 We begin by defining the set of states which is common knowledge in $t = 0$. As shown above, it turns out to be the whole state space, that is Ω. Moreover, we again assume a common prior that is uniformly distributed. We are thus able to calculate agents' posterior beliefs,

$$\alpha : \begin{cases} prob\left(A \mid p^1_{\alpha,1}(\omega)\right) = \frac{prob\left(A \cap p^1_{\alpha,1}\right)}{prob\left(p^1_{\alpha,1}(\omega)\right)} = \frac{1/9}{3/9} = 1/3 \\ prob\left(A \mid p^2_{\alpha,1}(\omega)\right) = \frac{prob\left(A \cap p^2_{\alpha,1}\right)}{prob\left(p^2_{\alpha,1}(\omega)\right)} = \frac{1/9}{3/9} = 1/3 \\ prob\left(A \mid p^3_{\alpha,1}(\omega)\right) = \frac{prob\left(A \cap p^3_{\alpha,1}\right)}{prob\left(p^3_{\alpha,1}(\omega)\right)} = \frac{1/9}{3/9} = 1/3 \end{cases}$$

[11] Our description of the state space is taken from Geanakoplos and Polemarchakis (1982).

[12] As mentioned before, if you consider the pair $\langle \Omega, \mathcal{F} \rangle$ we define $\mathcal{F}$ to be the information function for the set Ω. Given this definition, an agent who knows $\mathcal{F}(\omega) \subseteq A$, knows that some state in the event A has occurred. We say that the agent knows A. Or in the words of Geanakoplos and Polemarchakis (1982): "Given $\omega \in \Omega$, an event A is called common knowledge at ω if A includes that member of the meet of the two partitions that contains ω." In our case the meet of the two partitions is Ω itself, so the assumption is satisfied.

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and

$$\beta : \begin{cases} prob\left(A \mid p^1_{\beta,1}(\omega)\right) = \frac{prob\left(A \cap p^1_{\beta,1}(\omega)\right)}{prob\left(p^1_{\beta,1}(\omega)\right)} = \frac{1/9}{4/9} = 1/4 \\ prob\left(A \mid p^2_{\beta,1}(\omega)\right) = \frac{prob\left(A \cap p^2_{\beta,1}(\omega)\right)}{prob\left(p^2_{b,1}(\omega)\right)} = \frac{1/9}{4/9} = 1/4 \\ prob\left(A \mid p^3_{\beta,1}(\omega)\right) = \frac{prob\left(A \cap p^3_{\beta,1}(\omega)\right)}{prob\left(p^3_{\beta,1}(\omega)\right)} = 1 \end{cases}$$

How can we interpret these results and what implications does this have for the actions of agents? Take the case of agent α. He holds prior beliefs and has incomplete private information about the true state. His initial posterior beliefs are 1/3 for the bad state which yields and initial expected payoff of 2/3. Recall that exactly three states of the world yield a positive outcome while six states of the world yield a good one. Thus posterior beliefs of 1/3 just leave him indifferent. We proposed the decision rule that only in cases of strictly positive expected payoffs, agents invest. Thus, agent α disinvests. In addition he has beliefs about the other agent's beliefs. Since α knows agent β's partition, α knows that if the true state is in $p^1_{\beta,1}$ or $p^2_{\beta,1}$, then agent β's conditional probability for the event A is 1/4. Given that three out of nine states are bad states, a posterior probability for the bad outcome of 1/4 leads agent β to invest. If, however, the true state would be $\{9\}$, agent β would know that for sure (i.e. with probability 1) and thus would disinvest, as his expected payoff in that case would be zero.

Agent β, as mentioned above, knows for himself in which cell of the partition the true state is and this is $p^1_{\beta,1}$, but agent α does not know that agent β knows that.[13] We therefore have to compute the probabilities for all cells. However, the behaviour of the agents takes into consideration that they know for themselves in which cell the true state is. Thus, for agent α only $prob\left(A \mid p^1_{\alpha,1}(\omega)\right)$ enters the decision rule, while it is $prob\left(A \mid p^1_{\beta,1}(\omega)\right)$ for agent β. As one can see, this leads to an investment decision of agent β and a disinvestment decision of agent α in the first period.

Besides the decision process of the investors, there is an informational context that we need to discuss. As agents observe each other's actions, α knows that β invested. He can thus conclude, that the true state of nature cannot be $\{9\}$. In fact agent β also knows that $\{9\}$ is not true state and moreover he knows that agent α knows that he knows that etc. As in Geanakoplos and Polemarchakis (1982) $\{9\}$ can be deleted from the "common knowledge state space" and the new meet encloses $\{1,2,3,4,5,6,7,8\}$.

[13] Note that A knows that the true state is in p^1_A, so what he can infer is that agent B also thinks that the true state is in his first cell, namely p^1_B. However, B could think that A thinks that the true state is in p^2_A, as for instance $\{4\}$ is part of the second cell of A's partition.

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Period 2 Agents can now derive their new posterior according to

$$\alpha : \begin{cases} prob\left(A \mid p^1_{\alpha,2}(\omega)\right) = \frac{prob\left(A \cap p^1_{\alpha,2}(\omega)\right)}{prob\left(p^1_{\alpha,2}(\omega)\right)} = \frac{1/9}{3/9} = 1/3 \\ prob\left(A \mid p^2_{\alpha,2}(\omega)\right) = \frac{prob\left(A \cap p^2_{\alpha,2}(\omega)\right)}{prob\left(p^2_{\alpha,2}(\omega)\right)} = \frac{1/9}{3/9} = 1/3 \\ prob\left(A \mid p^3_{\alpha,2}(\omega)\right) = \frac{prob\left(A \cap p^3_{\alpha,2}(\omega)\right)}{prob\left(p^3_{\alpha,2}(\omega)\right)} = 0 \end{cases},$$

and

$$\beta : \begin{cases} prob\left(A \mid p^1_{\beta,2}(\omega)\right) = \frac{prob\left(A \cap p^1_{\beta,2}(\omega)\right)}{prob\left(p^1_{\beta,2}(\omega)\right)} = \frac{1/9}{4/9} = 1/4 \\ prob\left(A \mid p^2_{\beta,2}(\omega)\right) = \frac{prob\left(A \cap p^2_{\beta,2}(\omega)\right)}{prob\left(p^2_{\beta,2}(\omega)\right)} = \frac{1/9}{4/9} = 1/4 \end{cases}.$$

The actions that are implied by these posteriors are the same ones as in period 1: agent α disinvests and agent β invests in the asset. When looking at the market, nothing new has taken place. Neither has there been an increase in exogenous information or news, nor has there been a revision of actions. However, also in this case agents can learn from each other's actions. Most notably it is now common knowledge that the remaining elements in p^3_α can be ruled out. To see this, remember that states $\{7, 8\}$ are states with a positive payoff and agent α would have been irrational if he would have been disinvesting in such a state. The meet thus becomes further refined and is now $\{1, 2, 3, 4, 5, 6\}$.

Period 3 Again we derive the agents posteriors, which are

$$\alpha : \begin{cases} prob\left(A \mid p^1_{\alpha,3}(\omega)\right) = \frac{prob\left(A \cap p^1_{\alpha,3}(\omega)\right)}{prob\left(p^1_{\alpha,3}(\omega)\right)} = \frac{1/9}{3/9} = 1/3 \\ prob\left(A \mid p^2_{\alpha,3}(\omega)\right) = \frac{prob\left(A \cap p^2_{\alpha,3}(\omega)\right)}{P\left(p^2_{\alpha,3}(\omega)\right)} = \frac{1/9}{3/9} = 1/3 \end{cases},$$

and

$$\beta : \begin{cases} prob\left(A \mid p^1_{\beta,3}(\omega)\right) = \frac{prob\left(A \cap p^1_{\beta,3}(\omega)\right)}{prob\left(p^1_{\beta,3}(\omega)\right)} = \frac{1/9}{4/9} = 1/4 \\ prob\left(A \mid p^2_{\beta,3}(\omega)\right) = \frac{prob\left(A \cap p^2_{\beta,3}(\omega)\right)}{prob\left(p^2_{\beta,3}(\omega)\right)} = \frac{1/9}{2/9} = 1/2 \end{cases},$$

respectively. These results imply again that it is optimal for agent α to disinvest in the asset and for agent β to buy it. Again, this generates further information for the market, as p^2_β can be ruled out, as agent β did not disinvest. This further reduces elements $\{5, 6\}$ from our set of possible true states of nature, the meet therefore becomes $\{1, 2, 3, 4\}$.

Period 4 In the next round the posterior beliefs are given by

$$\alpha : \begin{cases} prob\left(A \mid p^1_{\alpha,4}(\omega)\right) = \frac{prob\left(A \cap p^1_{\alpha,4}(\omega)\right)}{prob\left(p^1_{\alpha,4}(\omega)\right)} = \frac{1/9}{3/9} = 1/3 \\ prob\left(A \mid p^2_{\alpha,4}(\omega)\right) = \frac{prob\left(A \cap p^2_{\alpha,4}(\omega)\right)}{prob\left(p^2_{\alpha,4}(\omega)\right)} = 0 \end{cases},$$

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and

$$\beta : \left\{ prob\left(A \mid p^1_{\beta,4}(\omega)\right) = \frac{prob\left(A \cap p^1_{\beta,4}(\omega)\right)}{prob\left(p^1_{\beta,4}(\omega)\right)} = \frac{1/9}{4/9} = 1/4 \right. .$$

Action profiles are still the same, namely α disinvests and β invests. However, state $\{4\}$ can be commonly ruled out as agent α disinvested. Note that the meet becomes $\{1,2,3\}$.

Period 5 The posteriors in this period are

$$\alpha : \left\{ prob\left(A \mid p^1_{\alpha,5}(\omega)\right) = \frac{prob\left(A \cap p^1_{\alpha,5}(\omega)\right)}{prob\left(p^1_{\alpha,5}(\omega)\right)} = \frac{1/9}{3/9} = 1/3 \right. , \text{ and}$$

$$\beta : \left\{ prob\left(A \mid p^1_{\beta,5}(\omega)\right) = \frac{prob\left(A \cap p^1_{\beta,5}(\omega)\right)}{prob\left(p^1_{\beta,5}(\omega)\right)} = \frac{1/9}{3/9} = 1/3 \right. .$$

That is, agent α disinvests, and agent β decides to sell the asset in period 5 as well after investing in the asset in all initial rounds. Note that, although β was learning continuously from period one to five, the additional information in the first four rounds did not make any difference to him in terms of his decision making process. It is only in period 5 that the "amount of common knowledge" has surpassed a certain threshold that makes him change his action. To say it differently, the meet has reached a certain level, beyond which no further revision will take place.

The fact that both agents want to sell the asset is a phenomenon that we defined as a "market crash". As the liquidation value of the asset is zero, agent β, who was buying the asset in all preceding periods, did wrong and finally recognizes that he should sell the asset as well.

3.4.0.1 Remarks

The most remarkable distinction in this example is the difference between the concepts of knowledge and belief. When knowledge is formalized as here, an agent cannot know something that is false, he can just believe it. That is, if the player *knows* and event A, then A is true and the true state lies in A.

Note moreover that no agent has superior information. A finer information partition is the formal definition for better information. Not all information partitions are refinements or coarsenings of each other, so as in our case, we cannot rank them. Whether a partition is better than any other depends on the true state and the event A. However, note that having superior information with respect to the true state of nature, which would apply for agent

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α in our case, does not necessarily provide a strategic tangible advantage. Thus, improving the privately held information does not induce an inevitable increase in equilibrium payoffs.

However, the assumption that agents can observe each others' actions is not very realistic, especially in a deep and decentralized financial market. It is much more realistic to assume agents to observe the market price instead of others' actions only. We will now proceed with a formal model that incorporates this idea.

3.5 Learning from Market Prices

We analyze the dynamics of a discrete multiperiod financial market with (public) securities. Those assets are completely characterized by their payoffs at a terminal date T. We assume that the terminal payoff equals one in the good state and zero in the bad state. Trade of the securities occurs at each $t = 1, ..., T$. The security is a claim on the fundamental state of the economy which is not directly observable. For simplifying and comparative purposes we follow Aumann (1976) and concentrate on finite state space of nine possible states of nature, that is $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Each state occurs with probability $prob\,(\omega) = \frac{1}{9}$ $\forall\ \omega \in \Omega$. The states are to be thought of as alternative possible descriptions of whatever is relevant to the agents. Thus, only one state is the true state, but no agent knows which one it is. The true state can be either good (G) or bad (B).

We consider a two-player model with agents $i = \{\alpha, \beta\}$ being risk neutral[14] with a zero discount rate. Each agent i receives an endowment of a perishable consumption good at times $0, 1, ..., T$ according to

$$e_i = \{e_{i,0}, ..., e_{i,T}\}$$

and is assumed to be an expected utility maximizers. In this two-player setup, the agents neglect their possible market power and do not consider the informational externalities that their actions cause.

Note that in our case agents' information is both asymmetric[15] and incomplete. Note moreover that although one agent has an informational lead with respect to certain possible states of nature, the entire information structures are such that no agent has superior information.[16] This implies that neither of the two agents has an ex-ante comparative advantage by being better informed than his opponent.

[14] If agents would be risk averse, there would typically be a deviation between the expected value of the asset and the price that agents are willing to pay. Or to put it differently: The more risk averse agents are, the less sensitive is their reaction towards prices and prices would therefore be badly-revealing or in the limit non-revealing.

[15] The asymmetry of information among agents is an essential ingredient for prices to have an informational role.

[16] A partition I_i is finer than partition I_{-i} if and only if every cell of I_i is a subset of some cell in I_{-i}.

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We focus on a simultaneous-move investment game, in which both agents can decide to invest (I) or to disinvest (D) the asset, that is

$$s_i \in S_i = (I, D), \tag{3.2}$$

where $S = \prod_i S_i$.

The utility of agent i is affected not only by his own action but also by the true state of nature and the action of the other agent, that is agent i's utility entails both fundamental and strategic uncertainty,

$$u_i : \{G, B\} \times S \to \mathbb{R}. \tag{3.3}$$

Agents investment or disinvestment decision is such that they announce a bid λ at which they are willing to buy/sell and communicate this bid λ to a competitive market maker. The bid is interpreted as an investment decision if λ is lower than the market price and vice versa. The market maker sets the price at the highest market clearing level, i.e.

$$p_{t+1} = \inf \{p : D_t(p) \geq I_t(p)\}. \tag{3.4}$$

Consider also the graphical representation of the timing of decisions in figure 2.

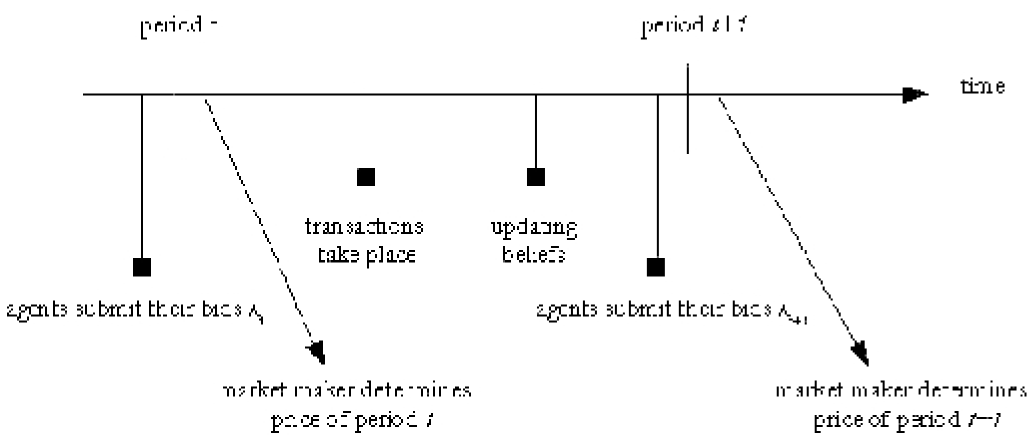

Figure 2 - Sequencing of Decisions

As in chapter (3.4), the event A is the bad outcome and $A = \{3, 6, 9\}$. For illustrative purposes assume again that the true state of the world is $\omega^* = 3$. In contrast to chapter 3, we abandon the assumption of common knowledge of the partitional information structure. That is, agents know and observe only their own information structure and the market price announced by the market maker. The true state of nature is located in the first cell $\wp_i^1(\omega)$ of the agents' partitional structure. By Bayes' Rule we get that

$$\begin{aligned} P\left(A \mid \wp_{\alpha,1}^1(\omega)\right) &= \frac{P\left(A \cap \wp_{\alpha,1}^1(\omega)\right)}{P\left(\wp_{\alpha,1}^1(\omega)\right)} = 1/3, \text{ and} \\ P\left(A \mid \wp_{\beta,1}^1(\omega)\right) &= \frac{P\left(A \cap \wp_{\beta,1}^1(\omega)\right)}{P\left(\wp_{\beta,1}^1(\omega)\right)} = 1/4. \end{aligned}$$

Remember that the event A is the bad event, therefore $P\left(A \mid \wp_{\alpha,1}^1(\omega)\right) = 1/3$ implies that $\alpha's$ (risk neutral) bid equals

$$\lambda_{\alpha,1} = \frac{1}{3} \times 0 + \frac{2}{3} \times 1 = \frac{2}{3}, \tag{3.5}$$

as the security's payoff is either zero or one.

Likewise we can derive $\beta's$ valuation:

$$\lambda_{\beta,1} = \frac{1}{4} \times 0 + \frac{3}{4} \times 1 = \frac{3}{4}. \tag{3.6}$$

Note that the two bids are different, which is the prerequisite for trade to take place. The market maker now uses his pricing formula (3.4), which yields a first

market clearing price at $p_1 = \frac{2}{3}$. At this market price, α's bid will be executed as a sell-order and $\beta's$ bid will be interpreted as a buy-order. Clearly, this price is informative to both agents. Agent α learns that β obviously holds a higher reservation price et vice versa. How do agents incorporate this additional information in the analysis for next period's bid $\lambda_{\alpha,2}$?

We assume for simplicity that agents follow a linear rule while adjusting their private information: They move towards other agents' reservation price at a fixed speed c. But the previous bid of the other agent could not be detected, as only the market price and the own information partition are known to each agent. If λ_{-i} would be known to agent i, we could derive

$$\lambda_{i,2} = (1 - c)\,\lambda_{i,1} + c\lambda_{-i,1}. \tag{3.7}$$

Note that if you look at the informational efficiency of the market price, it is clearly bounded from below by the information contained in the information held privately. It is moreover bounded from above by the fully-revealing market price. This implies that instead of using the unobservable λ_{-i}, agents take a proxy. Note that agent α just forms a belief

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about the occurrence of event A. It might be that event A did not materialize and the higher bid of the other agent might be an indicator for that (i.e. that the terminal payoff is non-zero). α therefore proceeds by using the terminal payoff of event $\neg A$ as a proxy for λ_β,

$$\lambda_{\alpha,2} = (1-c)\,\lambda_{\alpha,1} + c.$$

Let's assume that the responsiveness toward the market price is symmetric, i.e. also agent β adjusts with a fixed speed c. However, his initial bid turned out to be higher than the resulting market price which leaves them suspicious of having a too optimistic market assessment. He therefore adjusts his expectations 'downwards' according to

$$\lambda_{\beta,2} = (1-c)\,\lambda_{\beta,1}.$$

From this one can easily see that agent α's bid will still be lower than that one of agent β if

$$\lambda_{\alpha,1} - \lambda_{\beta,1} < \frac{c}{c-1}.$$

In this case, next period's market price will lie in between the individuals' bids of last period. As becomes clear, the price change is again informative to both agents and we therefore assume that they take the price change into consideration for their updating behaviour.

As prices are publicly observable by both agents, they are common knowledge. Let us denote the price change as

$$p_{t+1} - p_t = \Delta_{t+1}. \tag{3.8}$$

Note that this price change can be interpreted as some additional news. While it does not add any extra information to the market, it clearly entails extra information for each agent on the other agent's information set, which were not present before.

An elegant way to model the impact of this new information on beliefs is to use the DeGroot (2004) notion of a gamma distribution, i.e.

$$\gamma\left(\mu_\Delta \,|a, b\right), \tag{3.9}$$

where μ_Δ is the mean of the observable price change, and a and b are the mean and variance of the gamma distribution with $a > 1$ and $b > 0$. Consider that agents update their beliefs according to

$$\gamma\left(\mu_\Delta \,|a, b + c\Delta\right). \tag{3.10}$$

Given the belief process (3.9) and a current price p_t, we can say that

Proposition 3.3 *Agent i's optimal next period bid* $\lambda_{i,t+1}$ *is given by* $\lambda_{i,t+1} = \frac{(b-cp_t)^t}{(a-1-c)}$.

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Proof. Agent i needs to maximize $E\left[1-p_{t+1}\left|p_{t+1}\leq\lambda\right.\right]prob\left(p_{t+1}\leq\lambda\right)$. Agent $i's$ first-order condition is then given by

$$\lambda=E\left\{1-p_{t+1}\left|a,b,\Delta\right.\right\}=\frac{b+c\left(\lambda-p_t\right)}{a-1},$$

which has as a unique solution for λ for $a\neq 1$ and $c\neq 0$. ■

Note that from the optimal bid of agent i we can infer that expected positive payoffs are decreasing in the current price p_t. This is a fairly intuitive result: The higher today's price, the lower is the expected profit from investing in the asset if the potential gain (here positive payoff of 1) is limited. That is, the higher the current market price converges to the maximum liquidation value of the asset, the lower expected profits from investment are.

Given our parametrization we can now go one more step ahead to derive at an interesting result: Market crashes, defined as a consensus on the agents' willingness to sell the asset at the same time, can take place without changes in external conditions. The condition is that the variance of the mean expected payoff has to be higher than a measure of the responsiveness towards the other agent's information.

Proposition 3.4 *A market crash is taking place if*

$$b>\Gamma c, \tag{3.11}$$

and $\Gamma>0$.

Proof. In period $t+1$ both agents want to disinvest in the asset, if

$$\begin{aligned}\frac{b+c\left(\lambda-p_t\right)}{a-1} &< 0\Leftrightarrow\\ \Gamma c &< b,\end{aligned}$$

for $\Gamma=\left(\lambda-p_t\right)\frac{1-a}{a-1}>0$. ■

Note that Γ is positive, if the the bid is lower than the current market price, so we are in the midst of a downturn. If proposition (3.4) is satisfied, both agents want to disinvest in the asset due to non-positive expected returns. We interpret this results as a market crash.

This result deserves further interpretation. Firstly, we can say that - ceteris paribus - the lower the responsiveness towards the other agent's information (measured by the adjustment factor c) the more likely we are facing a situation in which we exhibit a market crash. As c is a measure for social learning behaviour, we can say that the less pronounced social learning

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behaviour in financial markets is, i.e. the more agents just look at prices and their own information for taking investment decisions, the more likely the market will face a crash. Social learning behaviour can thus be seen as an "insurance" for the market for crashes not to take place.

These findings do not come as a surprise. In our setup, agents are endowed with asymmetric and incomplete information. Pooling available private information via direct communication would yield a first-best market outcome, but we assume that this does not take place. Instead, agents can learn from the market price only. The more agents disregard the valuable information of other agents as partially mirrored in market prices, the more likely they are missing important information for rating the asset.

We can look at condition (3.11) also from another angle: b is the measure for variance of the expected price change. If prices are perceived to be very volatile, it is difficult for agents to obtain any valuable information from them. On the other hand, the lower the price variability, the easier it is for agents to extract privately held information from the aggregate statistic and market crashes are less likely taken place.

3.6 Discussion of Results and Concluding Remarks

The role of prices in aggregating and revealing privately held information is central to the study of allocative efficiency in any market. Despite this importance, economists (and even more policy makers) still know very little about the actual mechanism how these diverse bits of information are aggregated and disseminated. This proves particularly true in financial markets, where information is abundant and expectations of investors are at least partially endogenously generated by their own collective activity.

We have tried to shed some light on the question, whether market reversals or crashes can be explained without imposing news effects, sunspots or other external factors. We have done this in two steps. In the first step we took Geanakoplos and Polemarchakis' (1982) setup and showed that if agents are only able to observe each others' actions (instead of directly communication with each other as in Geanakoplos and Polemarchakis (1982)), we will get market crashes in finite time. The result can be explained by a "mute revision process", whereby agents are both asymmetrically and incompletely informed about the true state of nature and learn from each other and revise their beliefs. Only after a certain threshold of additional information is gained, this results in a potential change in action.

In the second case under consideration, we abstain from assuming that agents can observe each others' actions. Instead, we suppose that only an aggregate of individuals expectations

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becomes common knowledge, namely the market price. By allowing public signals that convey partial information about what others know, we incorporate an incentive for social learning into our investment model. Moreover, we restrict agents to learn *only* from market prices and not directly from each other. This seems to be the most sensible assumption for large markets, such as financial ones. We are able to derive a necessary condition for a market crash to take place. We can see that social learning behaviour can be regarded as a means of preventing market crashes. The less pronounced this social learning is, the more likely the market ends up in a crash. Or as Surowiecki (2005) would say: There exists some "wisdom of the crowds" and there is no a priori reason that learning from others will lead to herd-like behaviour.

A question that we did not tackle in our chapter is, whether we can find conditions on the agents' partitions under which market crashes are taken place or not. It is clear that if both agents would be symmetrically incompletely informed, no social learning could take place and therefore no dynamics in our model. It is the asymmetry of information which drives our results. We mentioned that in our setup no agent is strictly better informed than the other one. An important question to answer would how big the asymmetry needs to be in terms of market crashes to take place, and secondly whether there is some upper bound on asymmetry beyond which our results are not confirmed. Unfortunately we were not able to give any clear answer to this.[17]

We are also aware that the results we present, are at least partly contingent on the particular pricing rule of the market maker and other assumptions taken. It is thus more of a parametric example instead of a general characterization of situations in which financial market crashes take place without news. However, we believe that some of the results, e.g. that social learning behaviour matters as it refines the meet and thus allows for more precise judgements of markets, will translate into a more general setup. The applied simplification did, however, also have some good thing about it. In particular we were able to understand the interaction between private sector updating behaviour and social learning processes. Moreover, we are also able to say that improving the common knowledge state space, e.g. by disseminating market related data which are otherwise not given, is socially beneficial as it prevents some agents to embark on the wrong financial investment. In this sense we can conclude that when regarding financial markets, some news is in most cases better than no news.

[17] We introduced a topology on Ω and tried to find conditions on (a) the measure of entropy within the individual information partitions and (b) a cardinal measure for differences between information partitions necessary to create market crashes. We are not in the position yet to give an answer to these important questions.

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APPENDICES

A. The Assumption of a Representative Agent

Dating back to Alfred Marshall and Lionel Robbins, the representative agent assumption has been a landmark in the history of economic theory. However, its use and usefulness in macroeconomic theory has been questioned ever since its proposal (for an excellent summary of the arguments see Kirman (1993)).

In the representative agent hypothesis, to derive aggregate relationships for macroeconomic theory is generally performed by solving the representative agent‘s maximization problem and inserting aggregate values. Thus, individual behaviour is seen as equivalent to the aggregate behaviour. Hartley (1997) questions, what this kind of agent is: Some mean or median? And what does representative mean after all?

Representation is an abstract term, used ever since the middle ages. Historically, there exist different concepts of representation, such as symbolic, absolutist, national and corporate representation; the use in economic theory is closest to what is called identity representation. It basically means that the representing part equals the whole, which entails some notion of consensus.

While the representative agent assumption makes many problems tractable, however, many economic problems at hand cannot be dealt with at all, as they necessarily need some notion of heterogeneity. One possible alternative might therefore be to analyze a single agent‘s decision problem and then try to infer aggregate relationships without sticking to identical preferences and endowments. But aggregation faces serious problems. For macroeconomic relations to make sense, it is for instance required that a particular distribution of an aggregate variable is consistent and not subject to considerable change with respect to news. The most well-known consistency condition is the Gorman-form on preferences, which requires that Engel-curves are parallel straight lines for any set of prices. Note moreover that the Sonnenschein-Mantel results highlighted already 25 years ago that it is impossible to derive aggregate implications starting with general equilibrium models.

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So what might be the future of representative agent models? We would strongly believe that disaggregating the macroeconomic problem and modelling specific parts of the economy with more care could be one of the roads. Of course this means a dismantling of mainstream general equilibrium theory, but for the benefit of much stronger results.

B. Expectations and Learning Behaviour in Psychology

The case for using psychology in economic theory has been given long before Alan Greenspan's 'irrational excuberance' talk (1996). Kahneman et al. (1982) showed that the probabilistic assessments of agents often do not follow a rational judgement. Instead, "heuristic principles" are often used which try to reduce the complexity of the problem at hand, leading to systematic departures from rationality. Tversky and Kahneman (1974) attribute this partly to what they call the "law of small numbers", that is agents believe that the likelihood for some event in a small number of repetitions is roughly the same as the law of large numbers suggests. But there are other biases in agents decision making:

- Cognitive dissonance, i.e. adhering to a specific point of view despite counterfactual evidence.
- Anchoring, i.e. searching for reference points that provide assistance in the decision-making process, even if those are totally unrelated to the issue at hand.
- Status quo bias, i.e. people attach different values to identical goods depending on whether they want to acquire or sell them.
- Loss aversion, i.e. asymmetric attitudes to (possible) gains and losses.

According to Simon (1986), "[t]he rationality of economics is substantive rationality, while the rationality of psychology is procedural rationality".[18] Psychology, as the science of behaviour, experience and consciousness, does not have any normative standards and is oriented towards the explanation of actual behaviour. Because of this reason psychological approaches are a suitable supplement to economic modelling, in order to arrive at a meaningful representation of the expectations formation process. But how does the psychology literature deal with expectations formation and the learning issue?

While economists are mainly interested in studying repeated situations to explain changes in behaviour (that is: How does experience and social learning influence agents' behaviour),

[18]This is in line with Tversky and Kahneman's notion of 'framing', where expectations depend on the actual environment and circumstances and the expectations formation process is not a datum.

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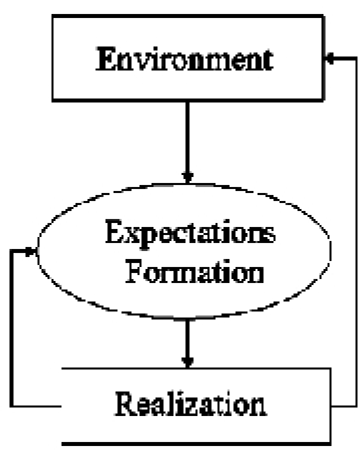
psychologists usually focus on cognitive processes with little interest in studying repeated situations only. Moreover, there exists a controversy in psychology over how best to theorize human learning behaviour. In principle it is a question of weighing up the sociocultural vis-a-vis the constructivist perspective of learning. Nonetheless there exists an agreement that epistemological and ontological issues are difficult to be separated in empirical examples, as learning entails transformations both of the agent himself and of the social world encompassing him. We will now briefly introduce main aspects of the psychological literature, namely motivational theories and social-psychology approaches.

Expected value theories of psychology are a subgroup of cognitive motivational theories (see for instance Lewin (1926). They are usually based on the Bernoulli principle: An agent chooses the alternative for which the expected utility is maximum. It is therefore not only personal preferences which lets the agent decide but also the influence of his environment. Memory and recall, recognition and attributes are therefore all relevant factors of human decision making.

Attribution theories refer to the influence of attributes and their influence via expectations on action. B. Weiner (1974) is one of the leading authors in this field; according to his theory of motivation and emotion, expectations are causally related to past results. That means that a form of 'feedback' is the main characterization underlying actions (see also box 2).



Box 2 - The Interaction in the Expectations Formation Process

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The social-cognitive personality theory of Bandura (i.e. a kind of social learning theory) combines mental learning processes in interaction with the social environment of an actor. Bandura's theory entails two elementary propositions: Agents derive information for their decisions via observing others which interacts with self-efficacy. The latter term means that an agent has to figure out his own profile, including his ability to carry out alternative actions. The action of an agent is therefore a product of his evaluation of personal capabilities, his probabilistic assessment, and social learning behaviour. According to Bandura, expectations are checked and revised continuously with newly observed information coming along. The evaluation of personal capabilities, however, is more sticky and conditional on actual situations.

To conclude, approaches of motivational theories focus on the direct influence of decisions and actions via expectations. In many psychological theories, we meet the familiar premise of a 'rational man', who calculates expected utilities. Social psychology theories in contrast, highlight the importance of unconscious or even involuntary influences on agents' decisions. As the economics discipline, psychological research is fundamentally split between these two strands and bringing together the two lines of research seems to be an advisable way to go.

C. On Least Squares Estimation and Adaptive Learning

In the adaptive learning approach economic agents behave like econometricians when forecasting economic variables. The economy is in a temporary equilibrium in which the current state of the economy depends on expectations about the future. In this annex we want to build the bridge between least squares estimation and adaptive learning and show that both concepts could be viewed as analogous for a wide range of models.

For illustration consider a model with a reduced form according to ,

$$y_t = \beta_t x_t + \varepsilon_t, \tag{C.12}$$

where $\varepsilon \sim iid\left(0, \sigma_\eta^2\right)$. β is an unknown parameter. Using the orthogonality condition for least squares we are left with least squares estimator

$$\widehat{\beta}_t = \left(\frac{1}{T}\sum_{i=1}^{T} x_i x_i'\right)^{-1} \left(\frac{1}{T}\sum_{i=1}^{T} x_i y_i\right). \tag{C.13}$$

Consider agents having a perceived law of motion of the form

$$y_t = \widehat{\beta}_t x_t + \varepsilon_t. \tag{C.14}$$

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Note that the standard least squares regression can be represented in a recursive way for appropriate initial conditions. We get

$$\begin{aligned} \widehat{\beta}_t &= \beta_{t-1} + \gamma_t \frac{1}{R_t} x_t \left(y_t - x_t' \beta_{t-1} \right) \\ R_t &= R_{t-1} + \gamma_t \left(x_t x_t' - R_{t-1} \right), \end{aligned} \quad \text{(C.15)}$$

for initial conditions R_0, β_0 and γ_t being a suitable gain sequence.

Marcet and Sargent (1989) show that such dynamics fit into the framework of stochastic recursive algorithms that can be evaluated by means of stochastic approximation methods derived by Ljung (1977).[19] One can then associate a differential equation $T(\beta)$ with the recursive algorithm (C.15), s.t.

$$\frac{d\beta}{d\tau} = T(\beta(\tau)) - \beta(\tau). \quad \text{(C.16)}$$

So far, we only used a general representation for the recursive system (C.15) and did not specify the gain sequence. For a decreasing gain sequence with $\gamma_t = 1/t$ we get the OLS estimate with all observations having the same weight. For a constant gain $\gamma_t = \gamma$ it is still possible to derive an associated differential equation $T(\beta)$ provided that the gain sequence is a small positive constant;[20] in this case p^e_{t+1} becomes an exponentially weighted average of past prices, $p^e_{t+1} = \alpha \sum_{k=1}^{\infty} (1-\alpha)^{k-1} p_{t-k}$.

D. A One-Country Case with Decreasing Gain Parameter

Consider the linearized version of the actual law of motion of equation (1.6),

[19] Let β be a vector of unknown parameters and X_t a vector of exogenous state variables, s.t.we get a recursive algorithm according to

$$\beta_t = \beta_{t-1} + \gamma_t Q\left(t, \beta_{t-1}, X_t\right),$$

where γ_t is a nonstochastic, decreasing gain sequence with the following properties:

$$\sum_{t=1}^{\infty} \gamma_t = \infty, \text{ and } \sum_{t=1}^{\infty} \gamma_t^2 < \infty,$$

which is given for decreasing gain sequences s.t. $\gamma_t = 1/t$. Q denotes in which way expectations are revised.We can now associate a differential equation to the recursive algorithm above,

$$\frac{d\beta}{d\tau} = T(\beta),$$

where τ is notional time. Marcet and Sargent (1989) show that $T(\beta)$ is achieved by taking the limit $\lim_{t\to\infty} E_t Q(t, \beta, X_t)$, where expectations are taken over the invariant distribution of X_t for a constant β. Deriving the local stability conditions of the differential equation is then equivalent to the E-stability conditions.

[20] In this case β will be sensitive to shocks even in the limit and will not converge to a particular value but rather a limiting distribution (see Benveniste et al. (1990, Chapter 4) for details).

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$$\begin{aligned} p_{t+1}^{e} &= \left(1+\alpha_t\left(F'\left(p_t^e\right)-1\right)\right)p_t^e \\ &\left(1+\alpha_t\left(\frac{2\omega^2}{N\omega^1}-1\right)\right)p_t^e, \end{aligned}$$

where we substituted the assumption of a constant gain by a variable gain. For decreasing gain with $\lim_{t\to\infty}\alpha_t = 0$ and $\Sigma\alpha_t = \infty$. The steady state $\overline{p}$ is stable if $F'(\overline{p}) < 1$. Consider first the range $0 < F'(\overline{p}) < 1$. For an initial point $\widetilde{p}_0^e < \overline{p}$ and $F(\cdot)$ being a monotonically increasing between $\widetilde{p}_0^e$ and $\overline{p}$, we get $F(\overline{p}) > \overline{p}$ which implies $p_{t+1}^e > \overline{p}$. Since the gain parameter is assumed to be lower than one, it follows that $p_{t+1}^e \leq p_{t+1}^e > \overline{p}$. We now have to show that p_t actually converges to the steady state, that is $\lim_{t\to\infty} p_t = \widehat{p} = \overline{p}$. Suppose the opposite and let d denote the difference $F(\widehat{p}) - \widehat{p} > 0$. For t sufficiently large, we get

$$p_{t+1}^e \geq p_t^e + \alpha_t\left(\frac{d}{2}\right),$$

s.t. $p_t^e \geq p_t^e + \alpha_t\left(\frac{d}{2}\right)\sum_{i=t}^{t+s}\alpha_i$ is a contradiction as $\Sigma\alpha_t = \infty$. ■

E. Proof of Proposition 1.3 in Chapter 1

Choose an exchange rate $\widetilde{e} \neq e$, with the price sequence $\{\widetilde{p}_{1,t}, \widetilde{p}_{2,t}\}$, where $\widetilde{p}_{i,t} \neq p_{i,t}$ for $i = A, B$ such that

$$\frac{M_{1,t}}{p_{1,t}} + \frac{M_{2,t}}{p_{2,t}} = \frac{M_{1,t}}{\widetilde{p}_{1,t}} + \frac{M_{2,t}}{\widetilde{p}_{2,t}}. \tag{E.17}$$

Rearranging (E.17) yields

$$\widetilde{p}_{1,t} = \frac{M_{1,t} + \widetilde{e}M_{2,t}}{M_{1,t} + eM_{2,t}}p_{1,t}. \tag{E.18}$$

Thus we also get

$$\widetilde{p}_{2,t} = \frac{\widetilde{p}_{1,t}}{\widetilde{e}} \tag{E.19}$$

If the money supply is nonstochastic in both countries, the price sequences $\{\widetilde{p}_{1,t}, \widetilde{p}_{2,t}\}$

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and $\{p_{1,t}, p_{2,t}\}$ yield the same real rate of return:

$$\begin{aligned}
\widetilde{R}_{i,t} &= \frac{\widetilde{p}_{i,t}}{\widetilde{p}_{i,t+1}} \\
&= \frac{[M_{1,t} + \widetilde{e}M_{2,t}]\, p_{i,t}}{M_{1,t} + eM_{2,t}} \cdot \frac{M_{1,t+1} + eM_{2,t+1}}{[M_{1,t+1} + \widetilde{e}M_{2,t+1}]\, p_{i,t+1}} \\
&= \frac{[M_{1,t} + \widetilde{e}M_{2,t}]\, p_{i,t}}{M_{1,t} + eM_{2,t}} \cdot \frac{\theta M_{1,t} + e\theta M_{2,t}}{[\theta M_{1,t} + \widetilde{e}\theta M_{2,t}]\, p_{i,t+1}} \\
&= \frac{p_{i,t}}{p_{i,t+1}} = R_{i,t}, \text{ for } i = A, B. \qquad \text{(E.20)}
\end{aligned}$$

Since savings decisions depend on the rate of return, both price sequences imply the same savings decision. ■

F. Proof of Existence of the Monetary Equilibrium in Chapter 1

If money has a zero value in the equilibrium at time t, no agent will demand it. Moreover, if money is the only means of storage and its value is zero, every agent can only consume from his individual endowment. This case is generally termed the *non-monetary equilibrium*. Thus, for a monetary equilibrium to exist, money has to have a strictly positive value at the equilibrium. Reconsider our maximization problem of young agents in equations (1.13)-(1.15). Let δ_1 and δ_2 denote the nonnegative multipliers associated with the two constraints (1.14) and (1.15) respectively. Agents maximize their utility by choosing consumption and money holdings. The necessary and sufficient conditions for an optimum are thus given by

$$\frac{1}{c_t} - \delta_1 \leq 0 \qquad \text{(F.21)}$$

$$\frac{1}{c_{t+1}} - \delta_2 \leq 0 \qquad \text{(F.22)}$$

$$-\delta_1 + \delta_2 R^e_{1,t+1} \leq 0 \qquad \text{(F.23)}$$

$$-\delta_1 + \delta_2 R^e_{2,t+1} \leq 0. \qquad \text{(F.24)}$$

Let $v \equiv u_1/u_2$ define the marginal rate of substitution. By definition of a monetary equilibrium, we need that $p_t > 0$ and $M_{i,t} > 0$ at least at one t. This implies by equations (F.23) and (F.24) that $p_t > 0$ for all t.

Equations (F.23)-(F.24) hold with equality, as we claim that money holdings are positive. Equations (F.21)-(F.22) hold only with the weak inequality, as consumption at positive prices can be zero.

Therefore, by (F.21)-(F.24) we get for an optimizing choice that

$$\frac{\delta_2}{\delta_1} = \frac{1}{\upsilon}.. \tag{F.25}$$

Proposition 3.5 $\frac{\theta}{\upsilon} \leq 1$ *for* $i = 1, 2$ *is a necessary and sufficient condition for the existence of a monetary equilibrium.*

Proof of Necessity. Suppose that the contrary holds, i.e. $\frac{\theta}{\upsilon} > 1$. Using the definition of the money supply process and the conditions $M_{i,t} = N\overline{M}_{i,t}$, where $\overline{M}_{i,t}$ are average money holdings such that $\overline{M}_{i,t} = \Sigma M_{i,t}^h / N$ for all agents h of generation t, we end up at

$$\frac{p_{i,t+1}^e}{p_{i,t}} = \frac{M_{i,t+1} p_{i,t+1}^e}{\theta M_{i,t} p_{i,t}} = \frac{\overline{M}_{i,t+1} p_{i,t+1}^e}{\theta \overline{M}_{i,t} p_{i,t}} = \frac{q_{i,t+1}^e}{\theta q_{i,t}}, \tag{F.26}$$

where $q_{i,t} \equiv p_{i,t}\overline{M}_{i,t}$ are per capita money holdings. By using inequality $\frac{\theta}{\upsilon} > 1$ and condition (F.25), we are left with

$$\frac{q_{i,t+1}^e}{q_{i,t}} = \theta \frac{p_{i,t+1}^e}{p_{i,t}} = \theta \frac{\delta_2}{\delta_1} = \frac{\theta}{\upsilon} > 1. \tag{F.27}$$

Since $q_{i,t} = p_{i,t}\overline{M}_{i,t} < \omega^1$ for all t, no bounded $q_{i,t}$ can thus satisfy equation (F.27). ■

Proof of Sufficiency. We will show that $\frac{\theta}{\upsilon} \leq 1$ implies a monetary equilibrium with $q_{i,t}^h = q_{i,t}$ for all h and t. It is sufficient to find a positive $q_{i,t}$ sequence such that

$$\upsilon = \frac{q_{i,t+1}^e}{q_{i,t}} \geq \frac{1}{\upsilon}, \tag{F.28}$$

where $\upsilon \equiv \upsilon \left[\omega^1 - q_t, q_{t+1}\right]$.

Consider a $q \in (0, \omega^1)$ such that $q_{i,t} = q_{i,t+1} = q$ satisfies the equality of equation (F.28). The inequality is then implied by the assumption that $\frac{\theta}{\upsilon} \leq 1$. Since υ is continuous, $\lim_{q \to 0} \upsilon(q) = 0$ and $\lim_{q \to \omega^1} \upsilon(q) = \infty$, which implies that such a q exists. This closes our proof. ■

The interpretation of this result is straightforward. A monetary equilibrium exists if - for a given preference relation that supports savings behaviour - the money growth rate is not too high.

For a given preference relation, we can moreover pin down the maximum $\overline{\theta}$ that satisfies the inequality $\frac{\theta}{\upsilon} \leq 1$ in a monetary equilibrium with adaptive learning behaviour. By using equations (1.13)-(1.15)we can deduce that $\overline{\theta}$ is bounded by

$$\upsilon = \frac{\left[\omega^2 - \omega^1 - \sqrt{10\omega^1\omega^2 + (\omega^1)^2 + 9(\omega^2)^2}\right]}{2\omega^2}.$$

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G. Proof of Proposition 1.4 in Chapter 1

In order to show that the recursive system (1.31) converges, we linearize (1.31) and find the eigenvalues of the Jacobian evaluated at the steady state. The Jacobian is given by

$$J(\overline{p}_1, \overline{p}_2) = \begin{pmatrix} 1 + \alpha(F_1(\overline{p}_1, \overline{p}_2) - 1) & \alpha F_2(\overline{p}_1, \overline{p}_2) \\ \alpha G_1(\overline{p}_1, \overline{p}_2) & 1 + \alpha(G_2(\overline{p}_1, \overline{p}_2) - 1) \end{pmatrix}, \tag{G.29}$$

where F_i, G_i are the partial derivatives of equation (1.30) with respect to price i.

If the eigenvalues of the Jacobian are less than one in modulus, stability is guaranteed. Checking these conditions is synonymous to

$$-2 < Tr\left(J\left(\overline{p}_1, \overline{p}_2\right)\right) < 2 \tag{G.30}$$

$$-1 < Det\left(J\left(\overline{p}_1, \overline{p}_2\right)\right) < 1, \tag{G.31}$$

where Tr denotes the trace of the matrix and Det the determinant.

To show (G.30) one can use the fact that

$$\begin{aligned} Tr\left(J\left(\overline{p}_1, \overline{p}_2\right)\right) &= 1 + \alpha\left(F_1\left(\overline{p}_1, \overline{p}_2\right) - 1\right) + 1 + \alpha\left(G_2\left(\overline{p}_1, \overline{p}_2\right) - 1\right) \\ &= 2 - 2\alpha + \alpha \underbrace{F_1\left(\overline{p}_1, \overline{p}_2\right)}_{>0} + \alpha \underbrace{G_2\left(\overline{p}_1, \overline{p}_2\right)}_{>0} > 0. \end{aligned}$$

Moreover,

$$Tr\left(J\left(\overline{p}_1, \overline{p}_2\right)\right) < 2$$

$\Leftrightarrow$

$$1 + \alpha\left(F_1\left(\overline{p}_1, \overline{p}_2\right) - 1\right) + 1 + \alpha\left(G_2\left(\overline{p}_1, \overline{p}_2\right) - 1\right) < 2,$$

which is given by our assumption that $F_1\left(\overline{p}_1, \overline{p}_2\right) < 1$ and $G_2\left(\overline{p}_1, \overline{p}_2\right) < 1$.

To see that condition (G.31) holds as well, let us firstly write down the determinate:

$$\begin{aligned} Det\left(J\left(\overline{p}_1, \overline{p}_2\right)\right) &= \left[1 + \alpha\left(F_1\left(\overline{p}_1, \overline{p}_2\right) - 1\right)\right]\left[1 + \alpha\left(G_2\left(\overline{p}_1, \overline{p}_2\right) - 1\right)\right] \\ &\quad - \alpha^2 F_2\left(\overline{p}_1, \overline{p}_2\right) G_1\left(\overline{p}_1, \overline{p}_2\right). \end{aligned}$$

Using the definitions of the functions G_1 and F_2, we can derive the following equation:

$$-\alpha^2 F_2\left(\overline{p}_1, \overline{p}_2\right) G_1\left(\overline{p}_1, \overline{p}_2\right) = -\alpha^2 \frac{M_2}{\overline{p}_2} \frac{\theta}{N\omega^1} \frac{M_1}{\overline{p}_1} \frac{\theta}{N\omega^1} < 0.$$

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Note that

$$\begin{aligned} F_2(\cdot) &= -\frac{\overline{p}_1}{\overline{p}_2^2}\frac{M_2\theta}{N\omega^1} < 0, \text{ and} \\ G_1(\cdot) &= -\frac{\overline{p}_2}{\overline{p}_1^2}\frac{M_1\theta}{N\omega^1} < 0. \end{aligned}$$

We now use again our assumption that $F_1(\overline{p}_1, \overline{p}_2) < 1$ and $G_2(\overline{p}_1, \overline{p}_2) < 1$, which gives us the solution that the determinant is less than one.

Moreover one can show by the same assumption that

$$1 > \alpha^2 \frac{M_2}{\overline{p}_2}\frac{\theta}{N\omega^1}\frac{M_1}{\overline{p}_1}\frac{\theta}{N\omega^1}.$$

As the first term in the determinant is positive by the same assumption, since

$$\begin{aligned} \frac{M_1}{\overline{p}_1}\frac{\theta}{N\omega^1} &< 1 \\ \frac{M_2}{\overline{p}_2}\frac{\theta}{N\omega^1} &< 1, \end{aligned}$$

the determinant has to be bigger than -1.

Note that this convergence behaviour is independent of the level of α.

For the more the more general case we consider $F_1(\cdot) < 1$ and $G_2(\cdot) < 1$ not to hold. One can show that condition (G.30) is still fulfilled in that case, while the condition for the determinant holds only for

$$\theta < \frac{N\omega^1}{\overline{S}} \frac{\alpha\left[\frac{2}{\omega^1}\left(\omega^2 - 2\right)\right] + \alpha^2\left[\left(\frac{\omega^2}{\omega^1}\right)^2 + 1\right]}{\alpha - \alpha^2 + \alpha^2\omega^2}, \tag{G.32}$$

where $\overline{S}$ denotes world aggregate savings.

As one can infer from (G.32), convergence now depends on both θ and α in the hyperbolic way exhibited in figure 3. Inequality (G.32) describes the stability frontier. For the parameter values employed in our simulation exercise, the stability frontier is depicted in figure 3.

Finally, we need to show that $\widetilde{\theta}_i < \overline{\theta}$ for all $i = 1.2$, which is equivalent to

$$\left(\omega^1 - \omega^2\right)\frac{\overline{p}_i}{M_i}\frac{N}{\theta} < \frac{\left[\omega^2 - \omega^1 - \sqrt{10\omega^1\omega^2 + \left(\omega^1\right)^2 + 9\left(\omega^2\right)^2}\right]}{2\omega^2}. \tag{G.33}$$

For our parameter choice of section 4 this inequality is fulfilled for $i = 1, 2$.■

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H Figures

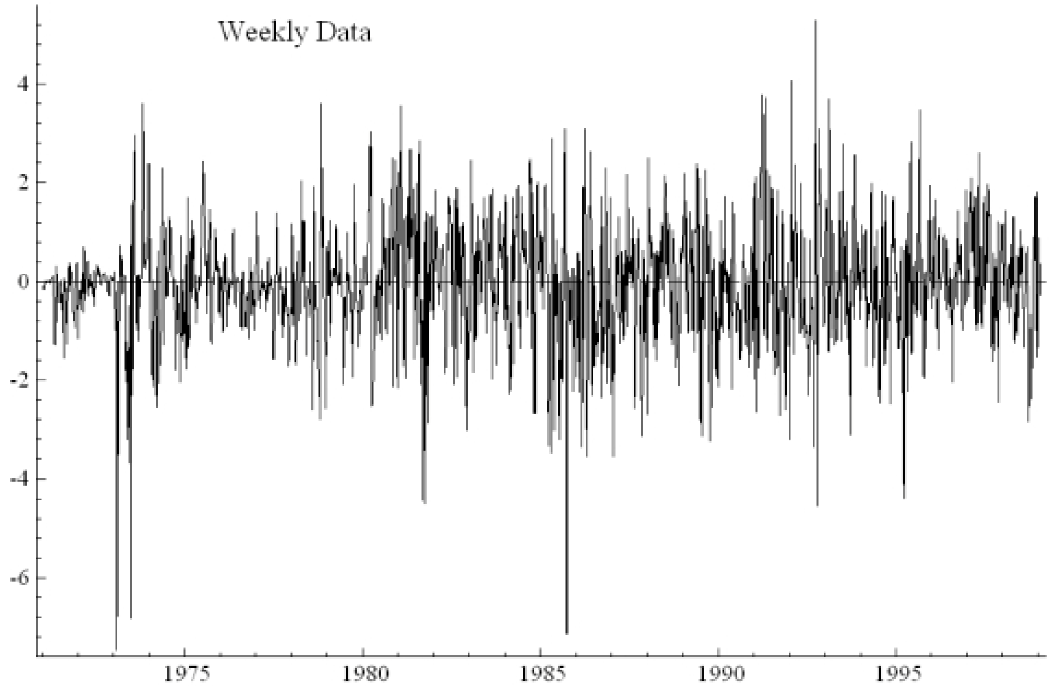

Figure 1a – Weekly Percentage Chang#es, Nominal DEM-USD Exchange Rate

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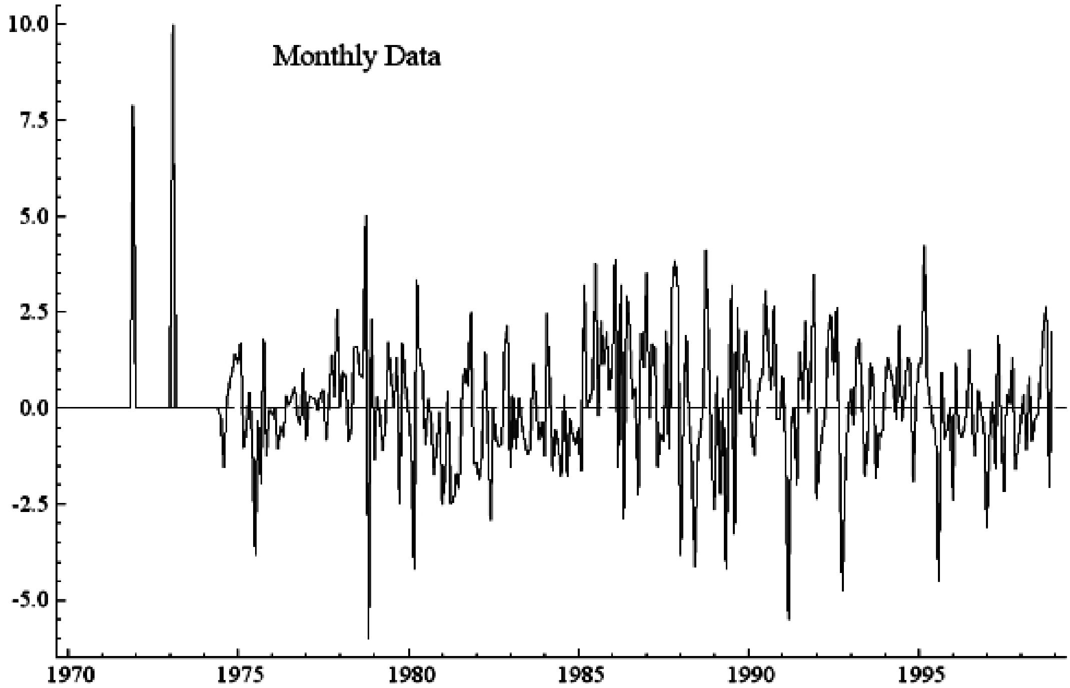

Figure 1b – Monthly Percentage Changes, Nominal DEM-USD Exchange Rate

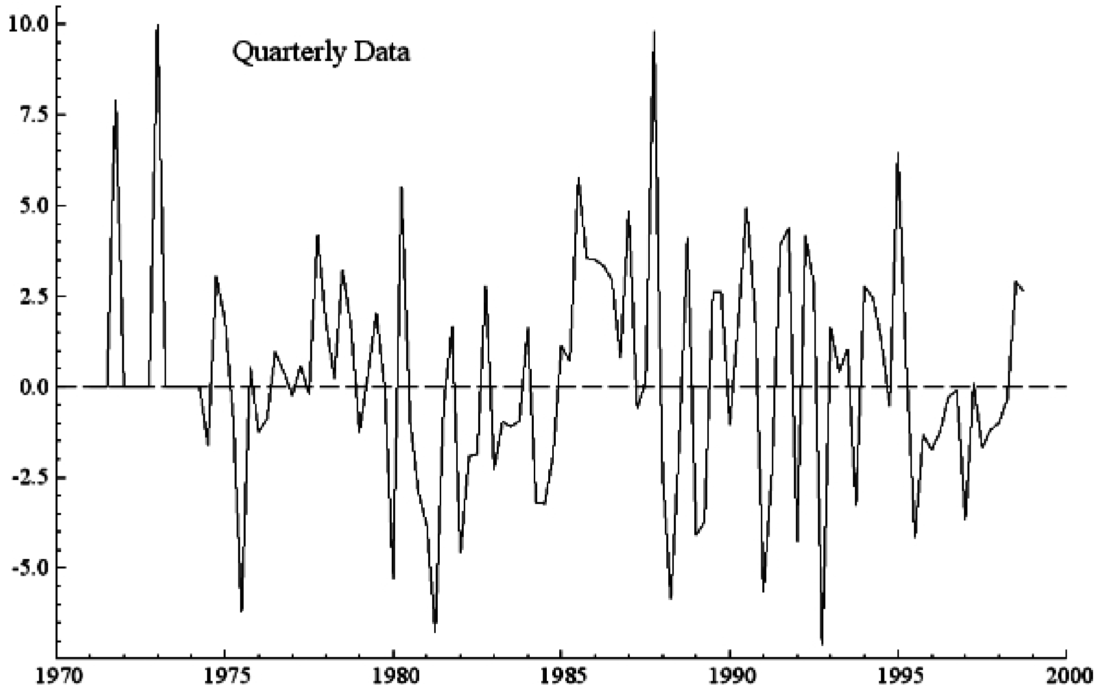

Figure 1c – Quarterly Percentage Changes, Nominal DEM-USD Exchange Rate

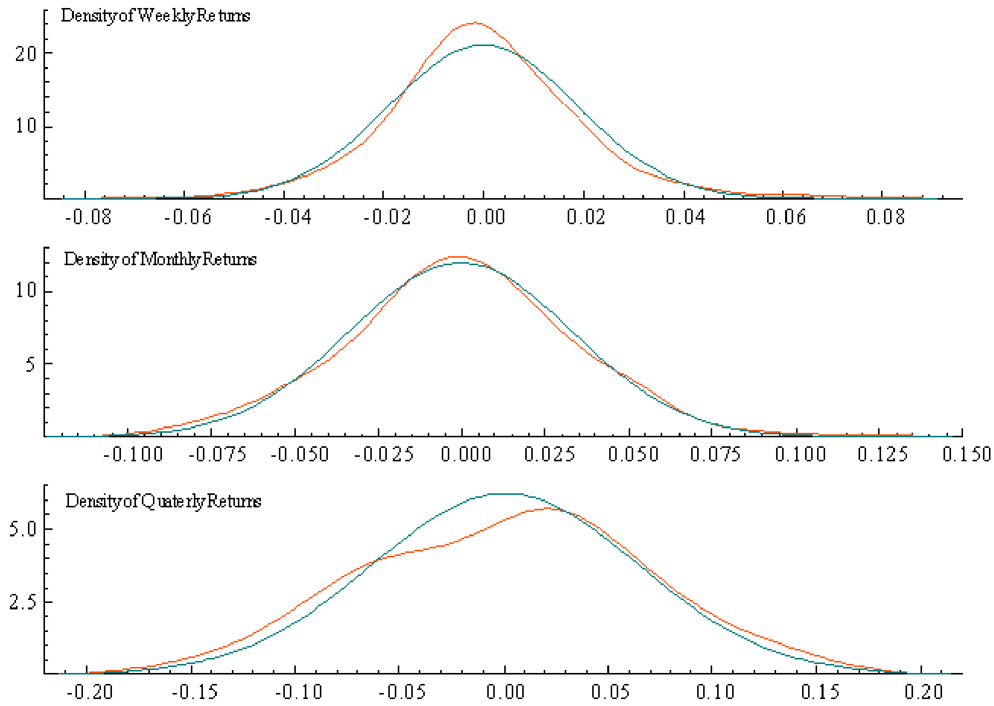

Figure 2 – Estimated Densities of Weekly, Monthly and Quaterly German Mark-U.S. Dollar Exchange Rate Returns

Note: Data are German mark-U.S. dollar exchange rates for 1979.1-1998.12; estimated densities are plotted against the normal distribution

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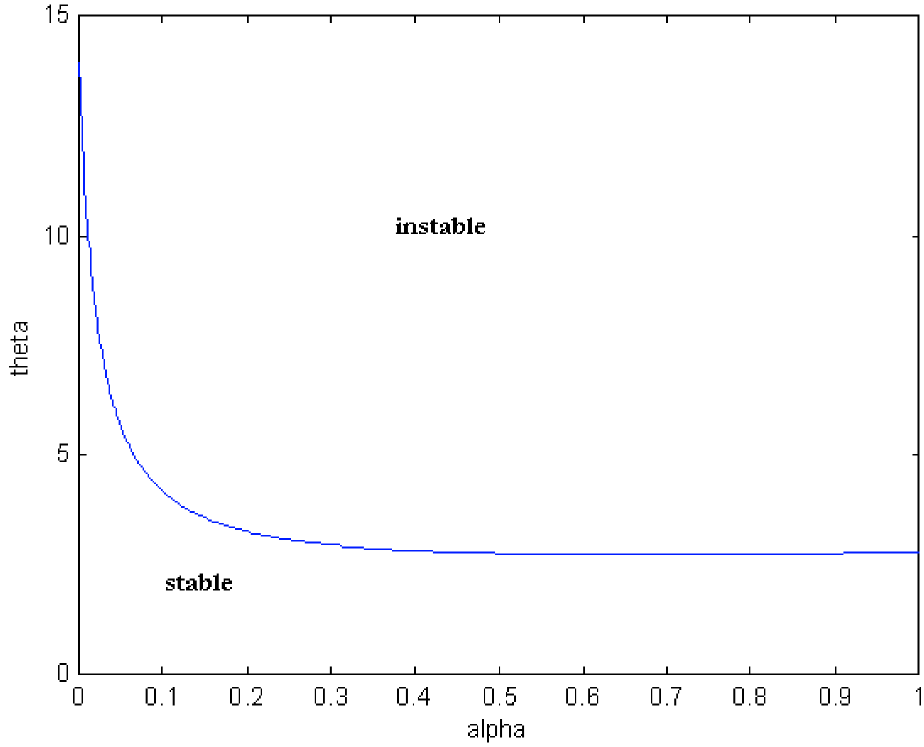

Figure 3 – Stability and Instability Regions with Adaptive Learning

(Parameter Values equal those of the simulation analysis of section 4, i.e.

$N = 30$, $w_1 = 200$, $w_2 = 50$, $M_{1,0} = 100$, $M_{2,0} = 110$).

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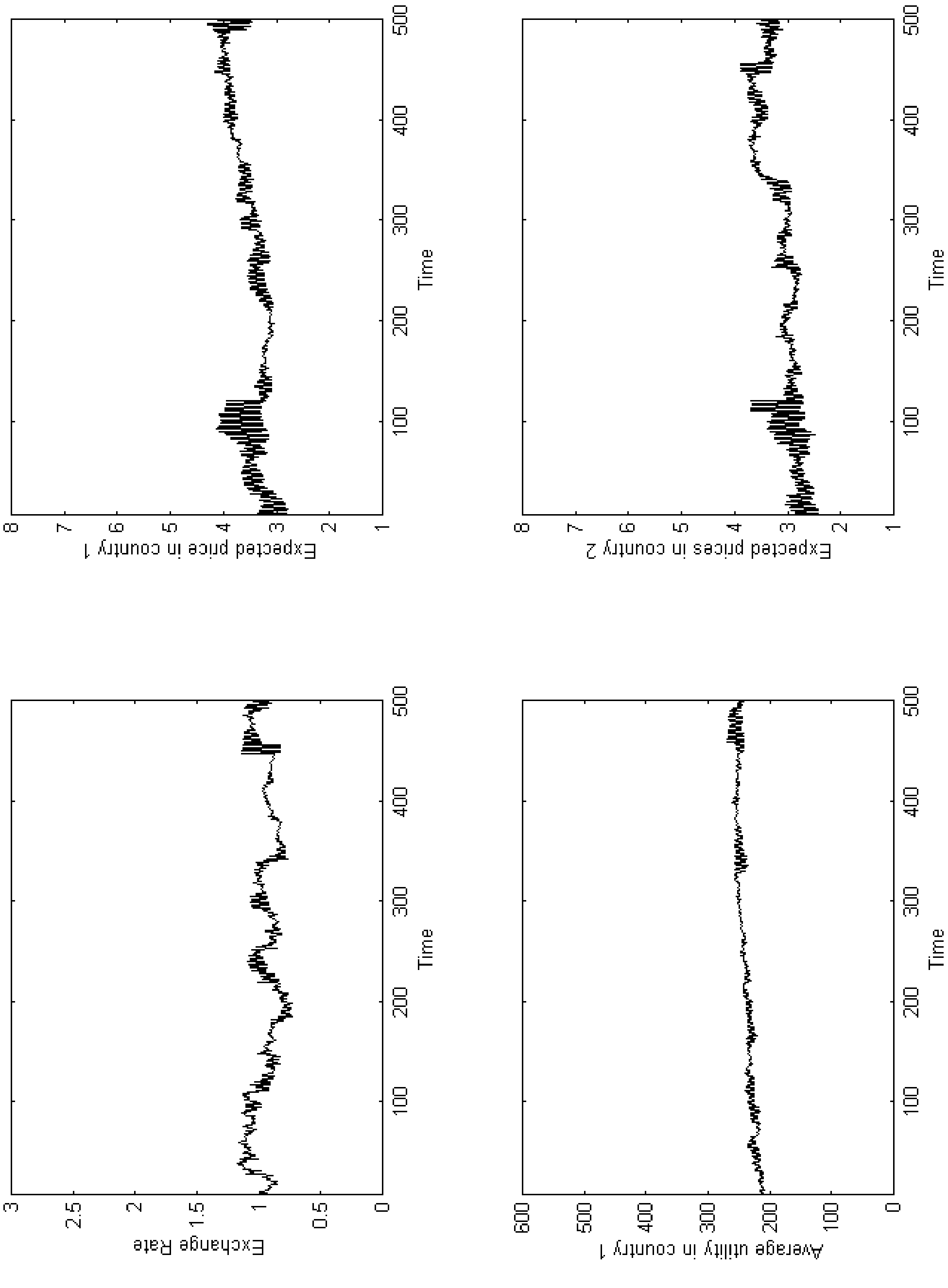

Figure 4 – Instability Region (θ = 3, α = 0.5)

(The y-axis of the average utility level is multiplied by 100)

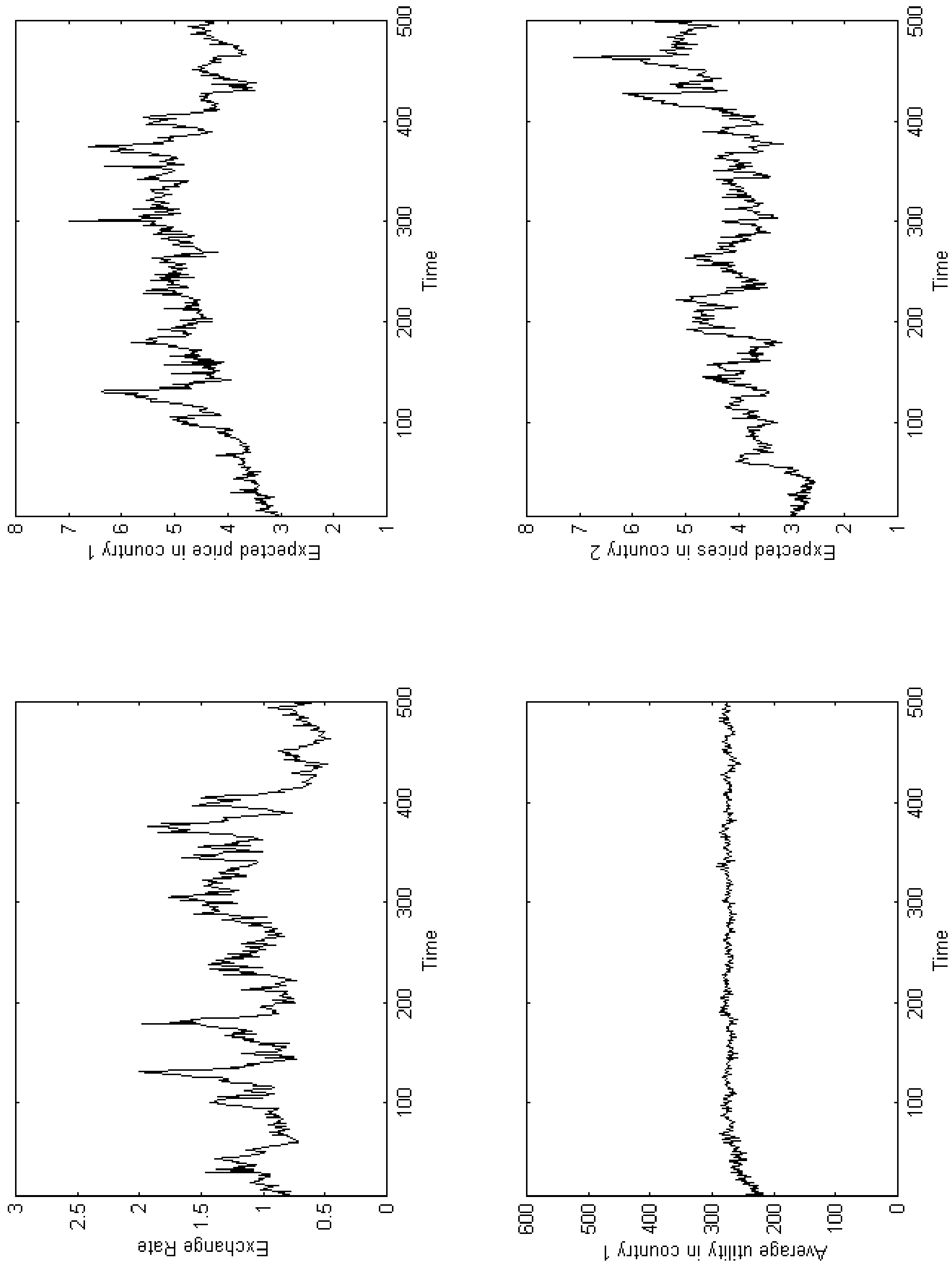

Figure 5 – Instability Region (θ = 7, α = 0.1)

(The y-axis of the average utility level is multiplied by 100)

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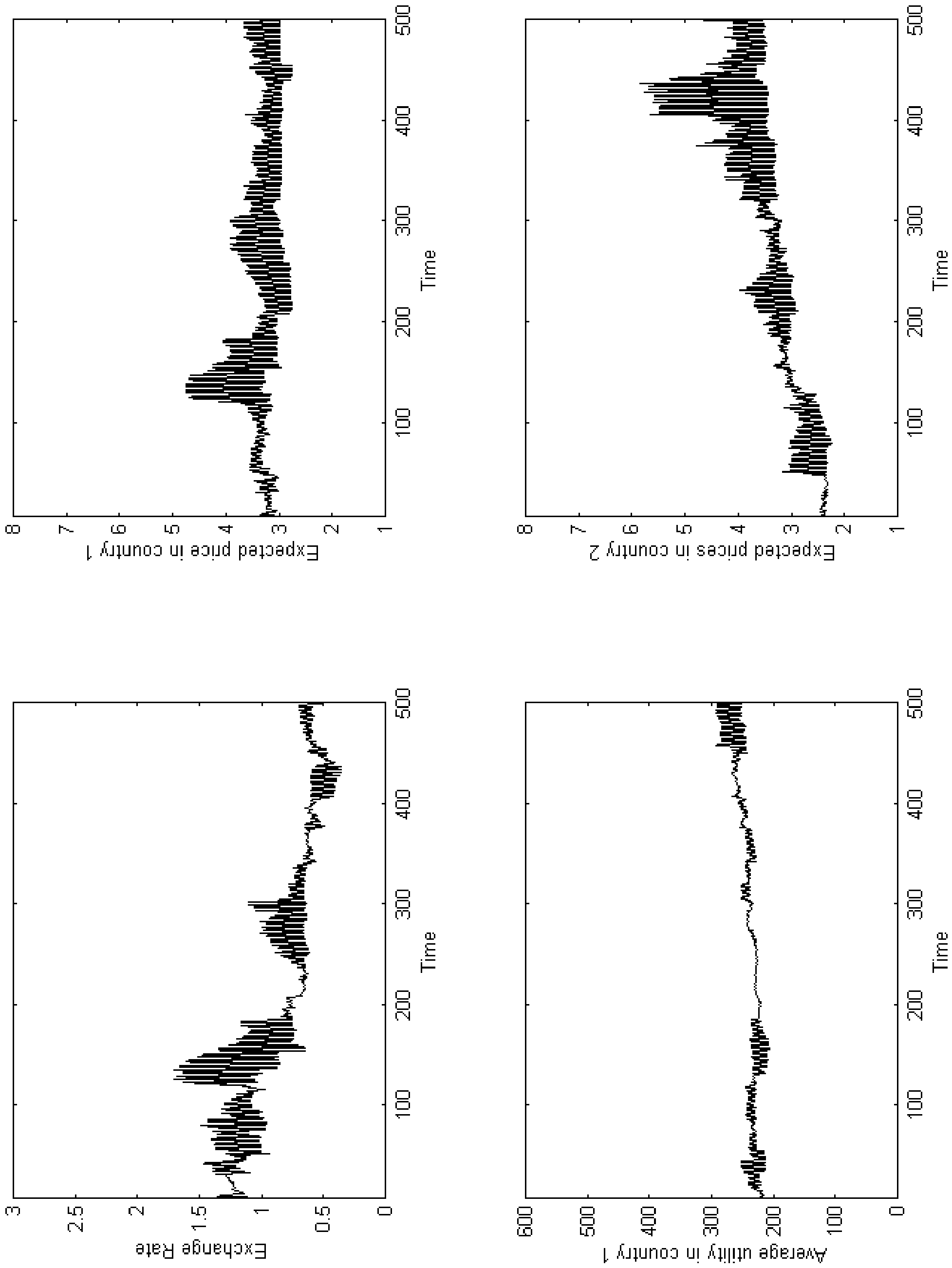

Figure 6 – Instability Region (θ = 7, α = 0.5)

(The y-axis of the average utility level is multiplied by 100)

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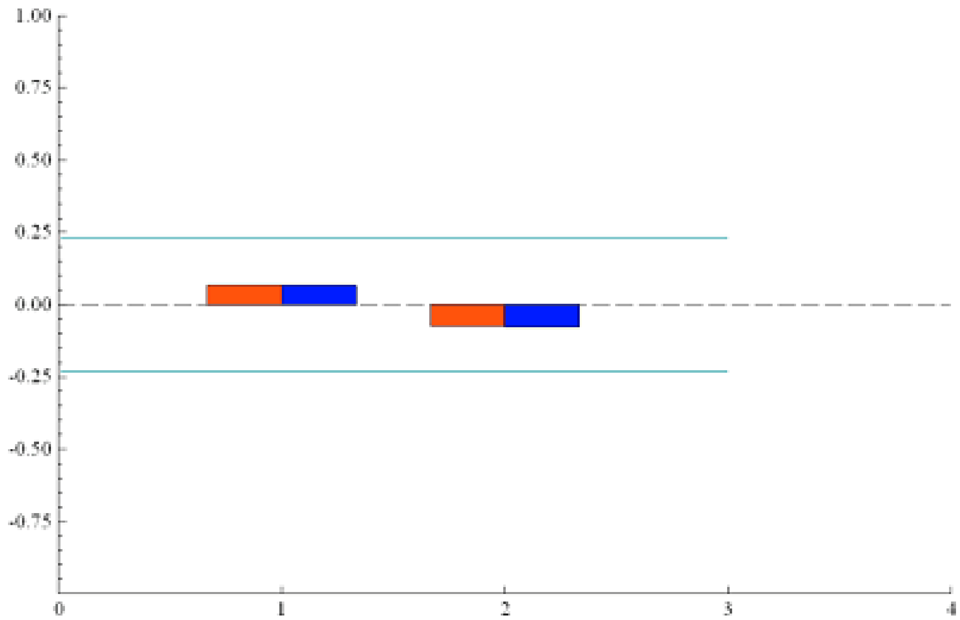

Figure 7 – Autocorrelation and Partial Autocorrelation Function of Simulated Exchange Rate Forecast Errors

Note: ACF (light grey), PCAF (dark grey); parameter values employed are those of figure 5.

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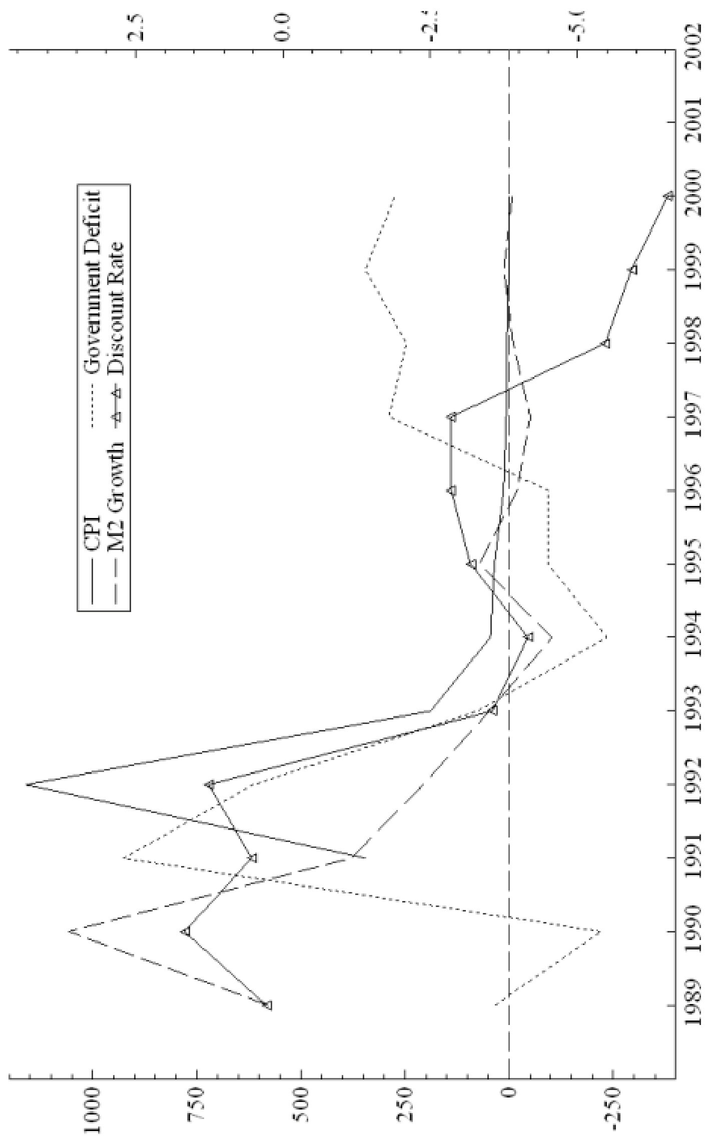
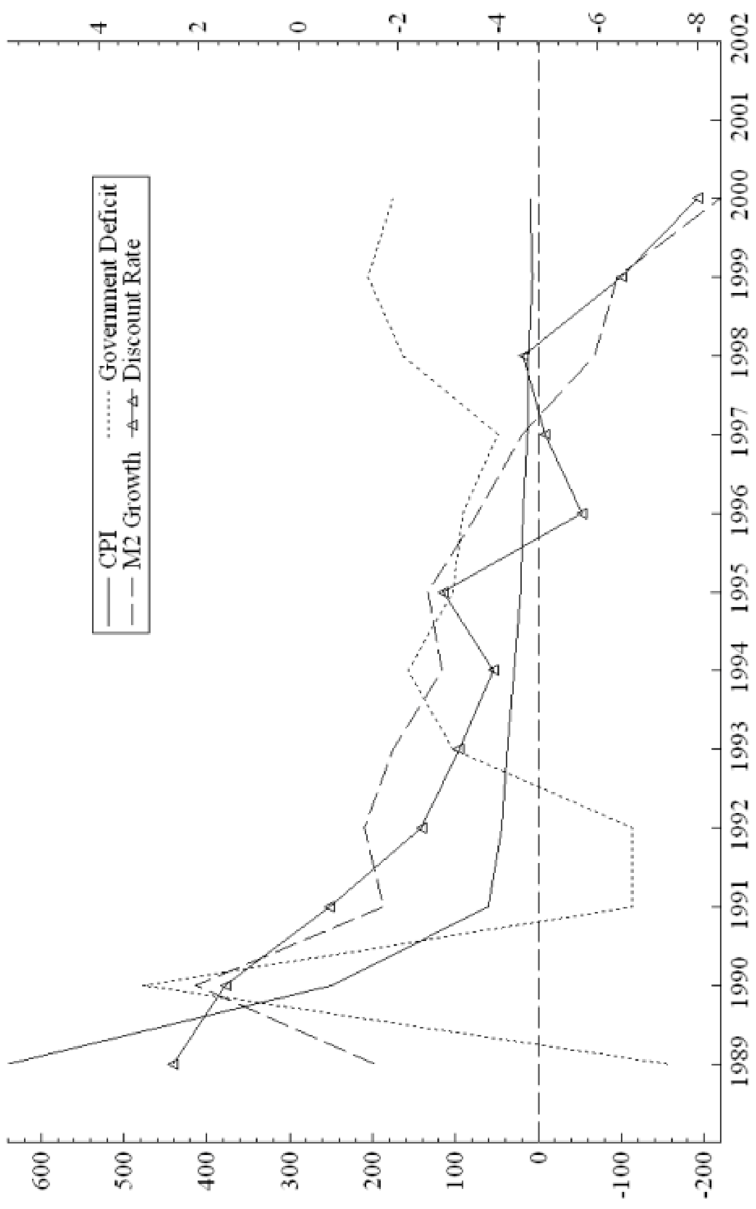
Figure 8 – Lithuania: CPI Rates, GDP, M2 and the Discount Rate



Note: Data are taken from the IMF International Financial Statistics. CPI is measured on the left y-axis, while all variables are measured on the right y-axis.

 DOI: 10.2870/11573

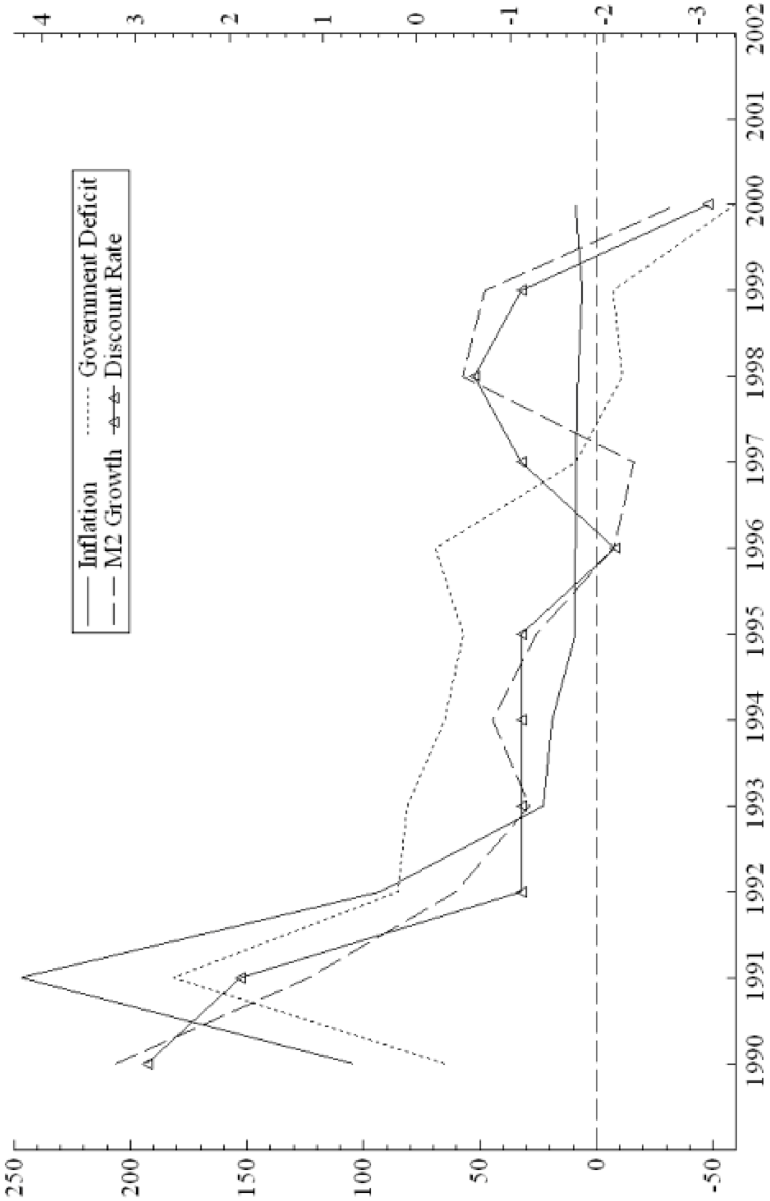
Figure 9 – Poland: CPI Rates, GDP, M2 and the Discount Rate



Note: Data are taken from the IMF International Financial Statistics. CPI is measured on the left y-axis, while all variables are measured on the right y-axis.

 DOI: 10.2870/11573

Figure 10 – Slovenia: CPI Rates, GDP, M2 and the Discount Rate



Note: Data are taken from the IMF International Financial Statistics. CPI is measured on the left y-axis, while all variables are measured on the right y-axis.

Figure 11 – Breakpoint Chow Test (Lithuania)

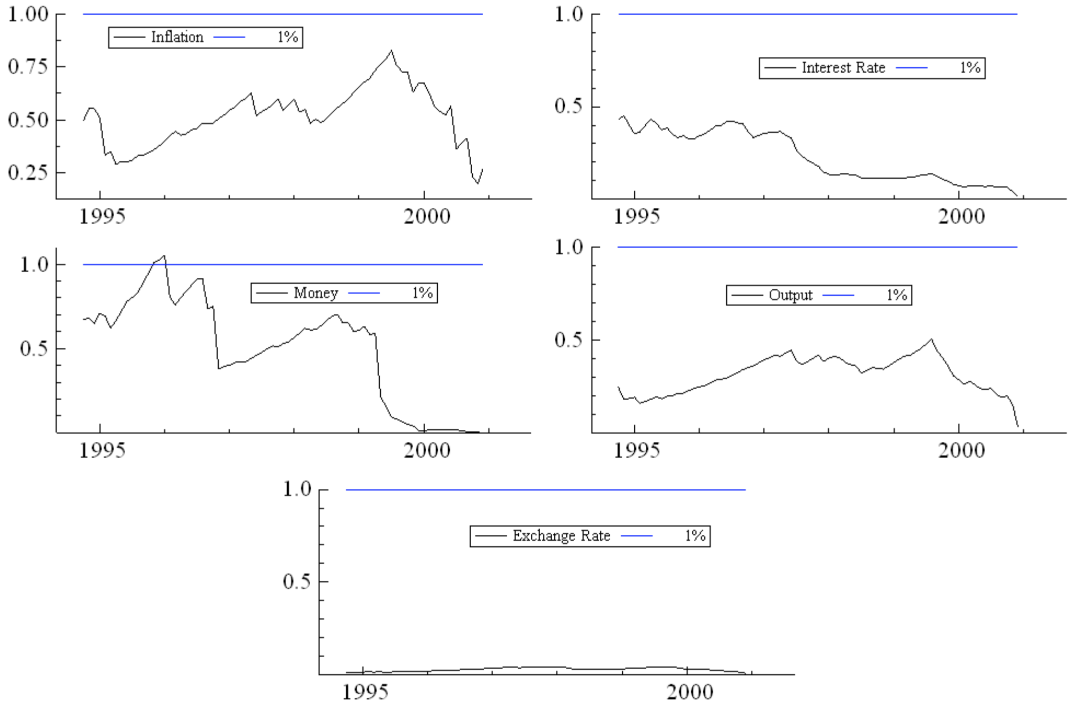


Figure 12 – Breakpoint Chow Test (Poland)

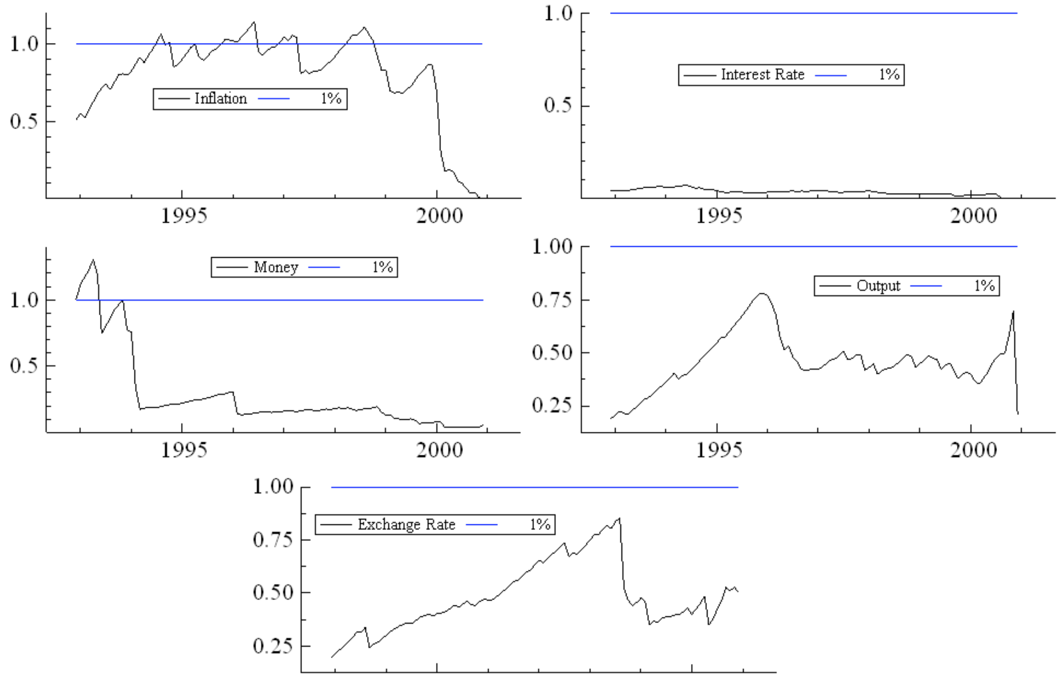


Figure 13 – Breakpoint Chow Test (Slovenia)

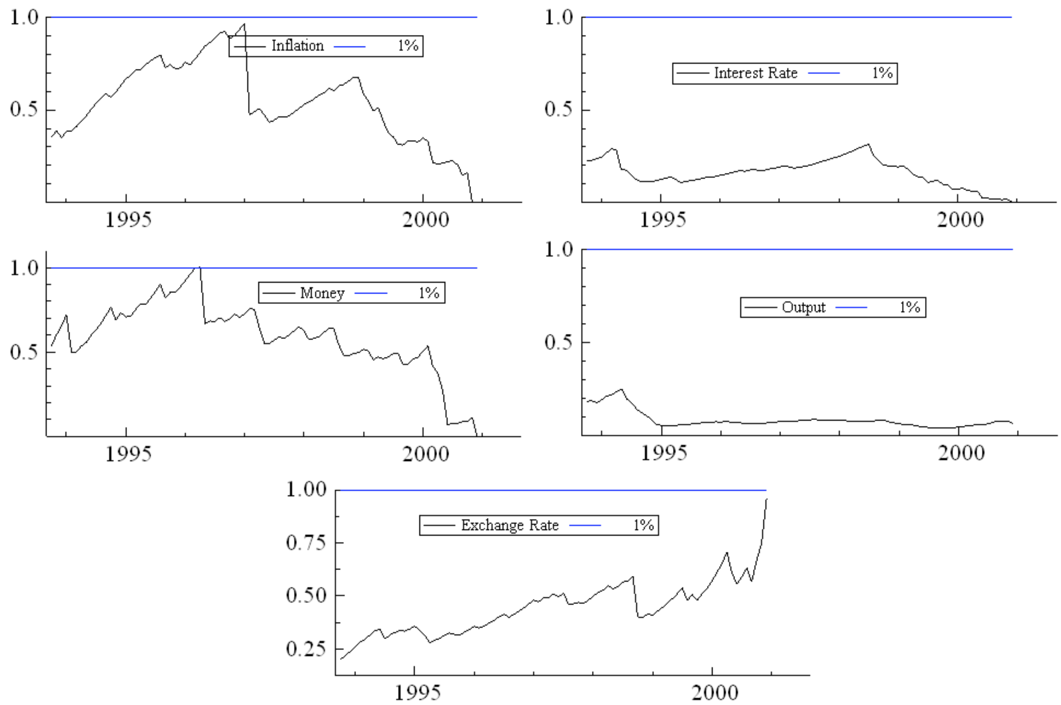
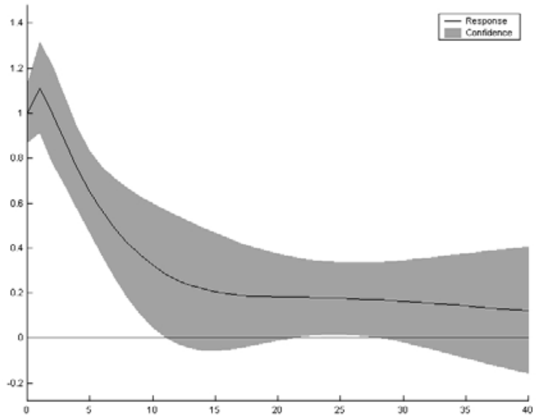
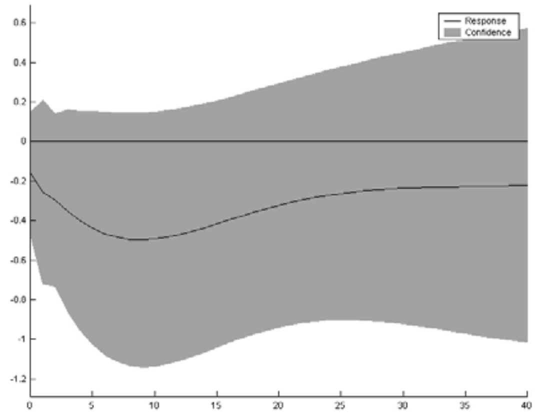


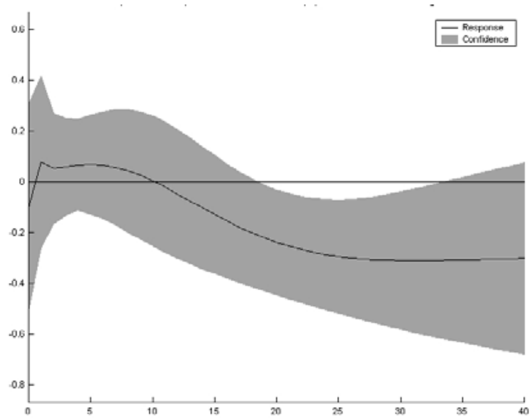
Figure 14 - Impulse Responses to a one-standard-deviation shock in the interest rate equation (Poland)



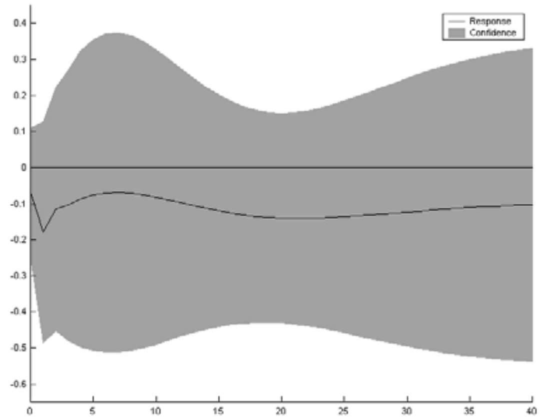
Interest Rate



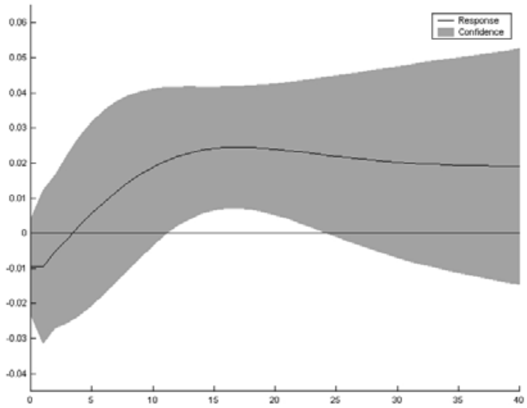
CPI



Output



Money



Exchange Rate

N.B. The vertical axes are approximately percentage deviation, except for the interest rate, while the horizontal axis is the number of months.

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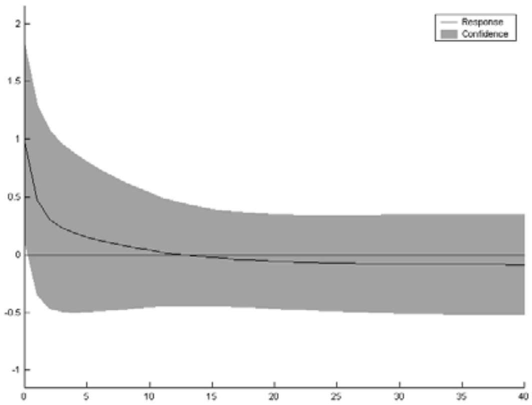
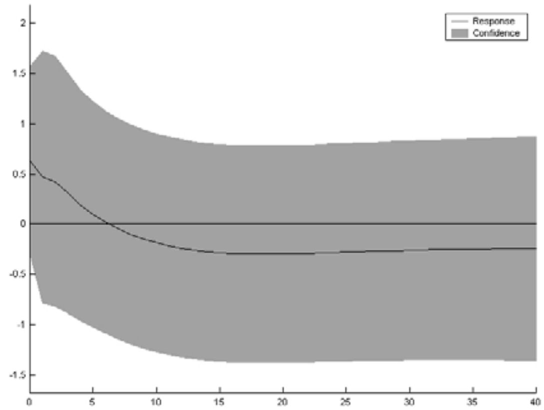
Figure 15 - Impulse Responses to a one-standard-deviation shock in the interest rate equation (Slovenia)



Interest Rate



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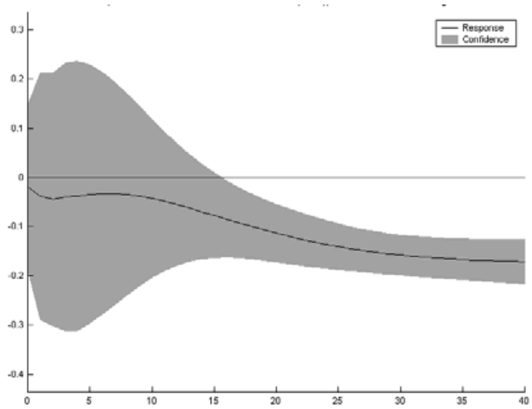

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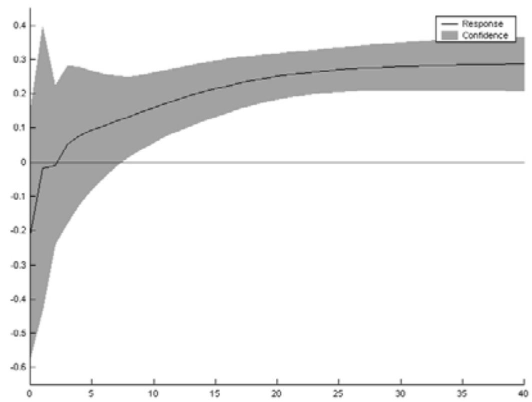
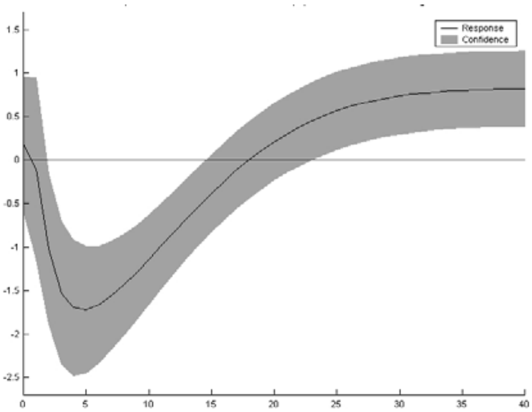


Money



Exchange Rate

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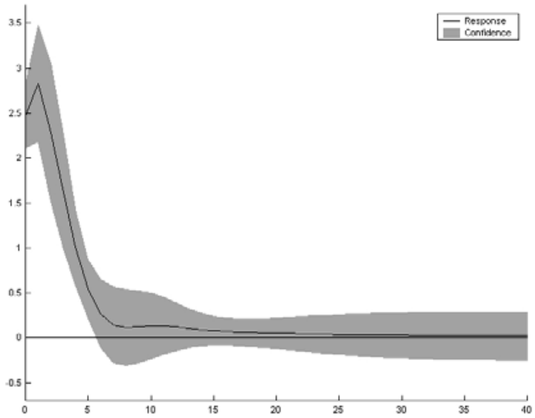
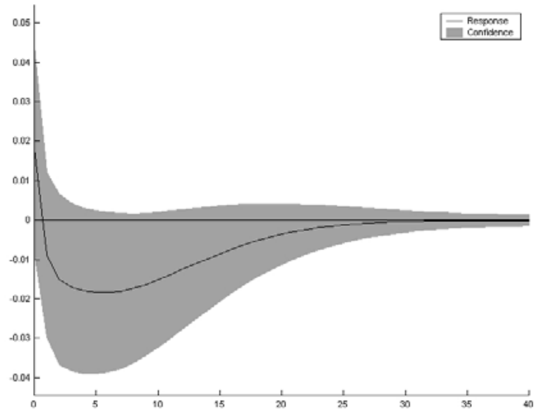
Figure 16 - Impulse Responses to a one-standard-deviation shock in the foreign interest rate equation (Lithuania)



Interest Rate



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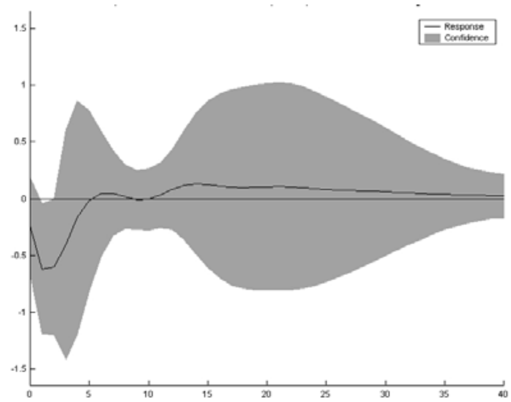

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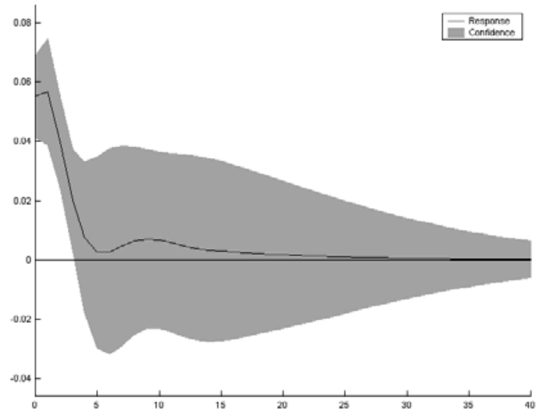
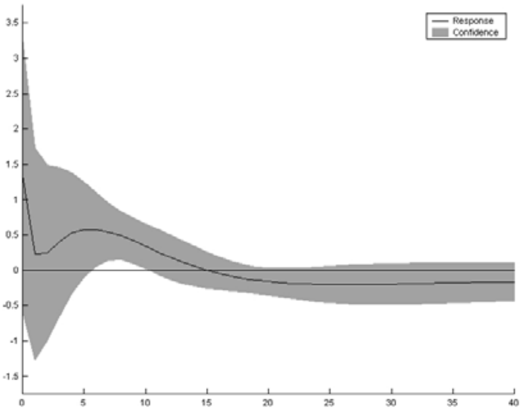


Money



Exchange Rate

N.B. The vertical axes are approximately percentage deviation, except for the interest rate, while the horizontal axis is the number of months.

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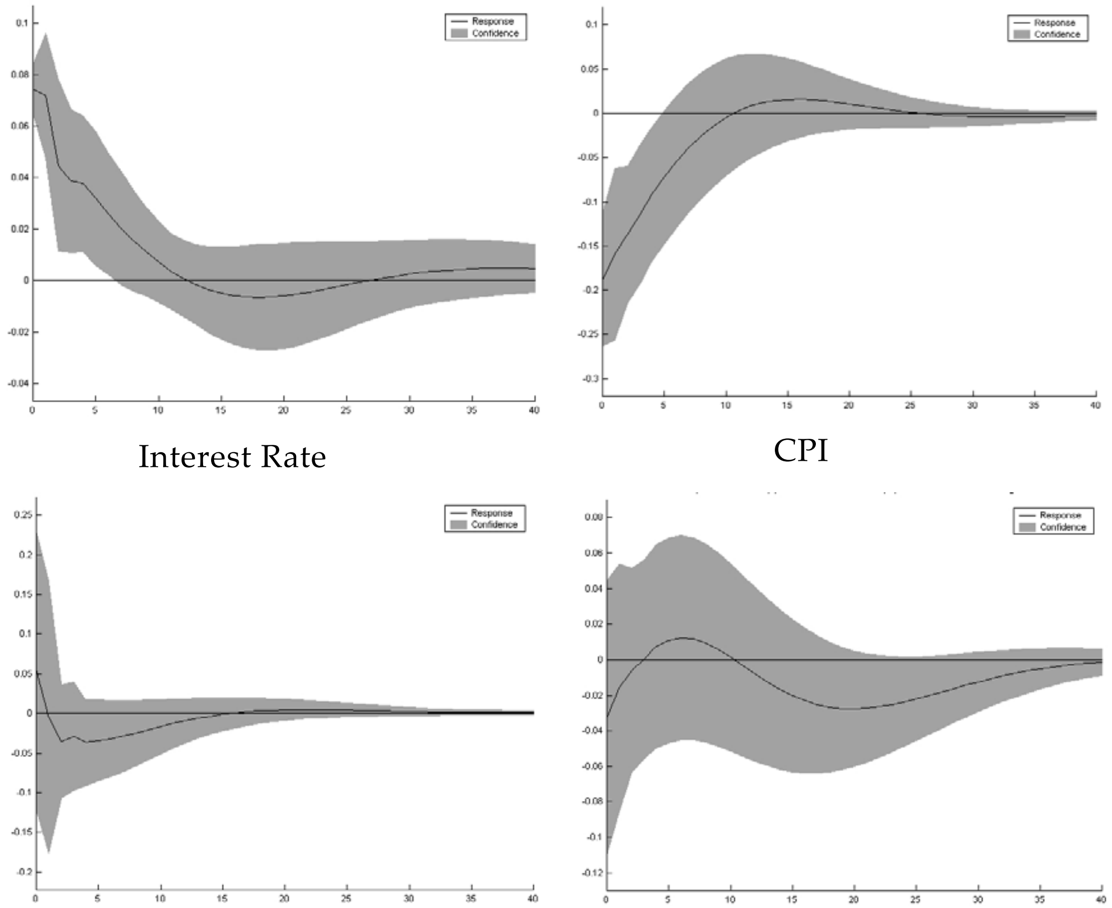
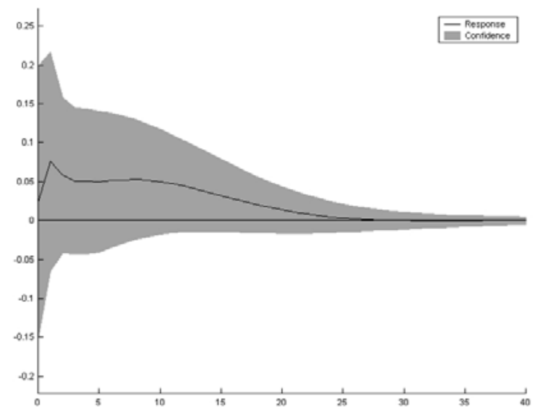
Figure 17 - Impulse Responses to a one-standard-deviation shock in the interest rate equation with expectations included (Poland)



Interest Rate CPI

Output Money



Exchange Rate

N.B. The vertical axes are approximately percentage deviation, except for the interest rate, while the horizontal axis is the number of months.

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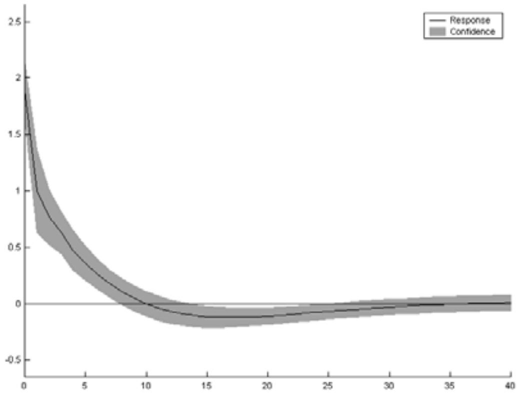
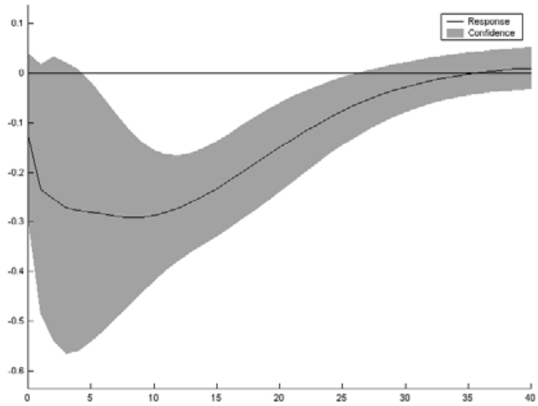
Figure 18 - Impulse Responses to a one-standard-deviation shock in the interest rate equation with expectations included (Slovenia)



Interest Rate



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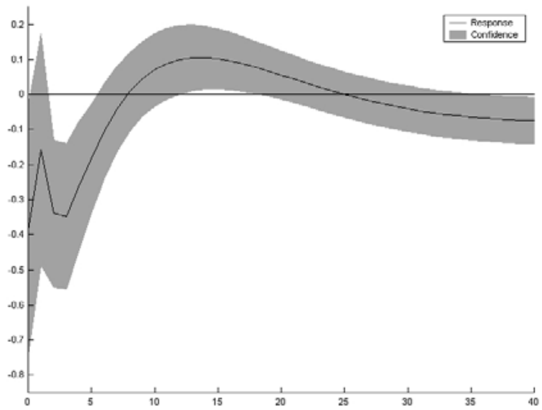

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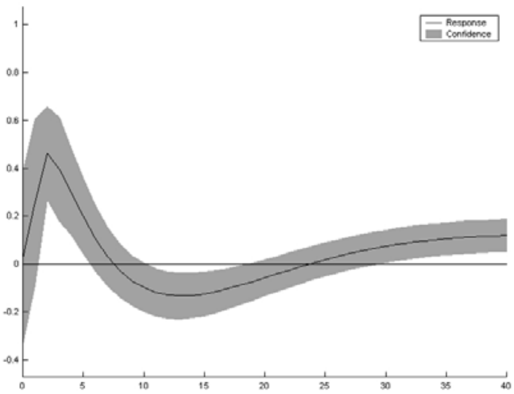
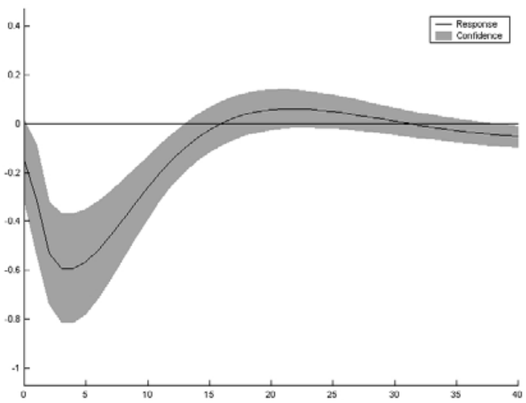


Money



Exchange Rate

N.B. The vertical axes are approximately percentage deviation, except for the interest rate, while the horizontal axis is the number of months.

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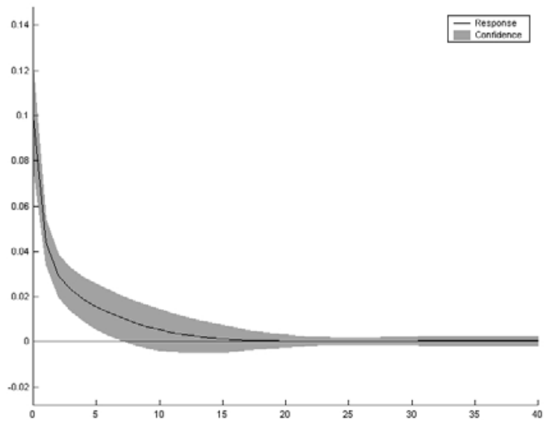
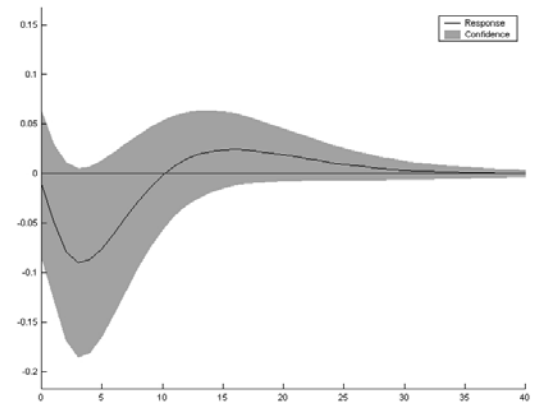
Figure 19 - Impulse Responses to a one-standard-deviation shock in the foreign interest rate equation with expectations included (Lithuania)



Interest Rate



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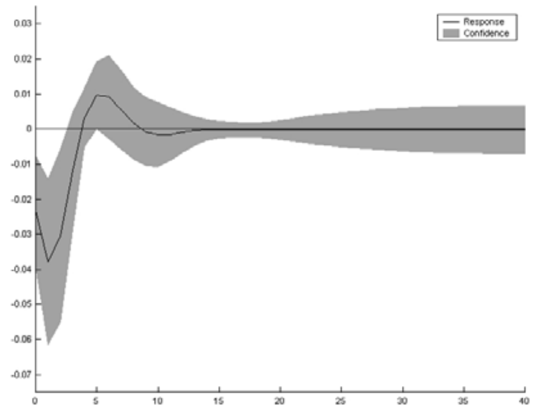

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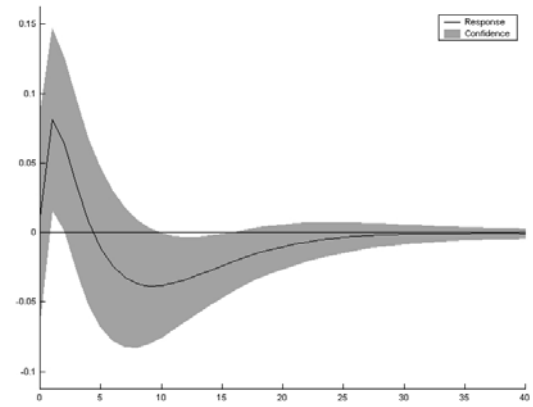
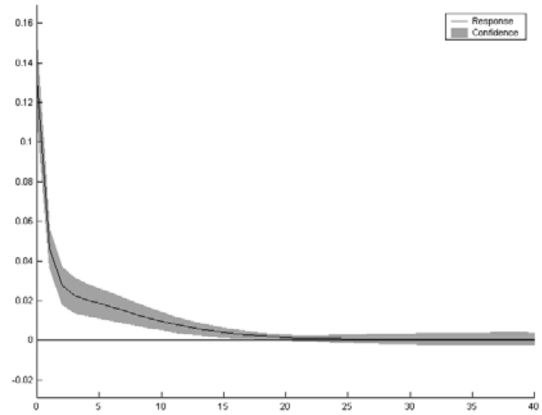


Money



Exchange Rate

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I Tables

	standard deviation
weekly data	
1979-1998	0.012
monthly data	
1979-1998	0.033
quarterly data	
1979-1998	0.064

Table 1 - Volatility of the Δ Log German Mark- U.S. Dollar Nominal Exchange Rate (1979-1998)

Source: IMF *International Financial Statistics*, Federal Reserve Statistical Release H.10, various volumes; data are end of period spot rates.

	standard deviation
monthly data	
German to U.S. interest rate differential	0.0293
log German to U.S. industrial production	0.0942
log German to U.S. prices	0.0880
quarterly data	
German to U.S. interest rate differential	0.0291
log German to U.S. industrial production	0.0935
log German to U.S. prices	0.0881
log German to U.S. M0	0.1039
log German to U.S. national income	0.1012

Table 2 – Standard Deviation of Δ Log Fundamentals (1979-1998)

Source: All data from IMF *International Financial Statistics*, various volumes; interest rates are end of period discount rates, prices are CPI averages (base year=1995), M0 consists of the components money and quasi-money; national income is GNI.

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weekly data	Skewness	0.031079
	Excess Kurtosis	5.8261
	Normality test: Chi^2(2) =	266.20 [0.0000]**
monthly data	Skewness	0.11975
	Excess Kurtosis	3.3322
	Normality test: Chi^2(2) =	74.136 [0.0485]*
quaterly data	Skewness	-0.072551
	Excess Kurtosis	0.0507
	Normality test: Chi^2(2) =	18.606 [0.0001]**

Table 3a – Normality Tests for the German Mark-U.S. Dollar Exchange Rate Returns (1979-1998)

Source: IMF *International Financial Statistics*, various volumes; Federal Reserve Statistical Release H.10; two stars indicate that the test is significant at the 1% level; one stars indicates that the test is significant at the 5% level

	ADF test statistics
weekly data	
log spot rate[1]	t-adf -0.187
Δ log spot rate[1]	t-adf -4.092**
monthly data	
log spot rate[2]	t-adf -0.518
Δ log spot rate[2]	t-adf -14.19**
quaterly data	
log spot rate[3]	t-adf -0.302
Δ log spot rate[3]	t-adf -7.927**

Table 3b – Stationarity Tests (ADF) for the German Mark-U.S. Dollar Exchange Rate (1979-1998)

Source: IMF *International Financial Statistics*, various volumes
[1]Critical values (5%=-1.95 1%=-2.65); [2] Critical values (5%=-1.94 1%=-2.57);
[3] Critical values (5%=-1.94 1%=-2.59)

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	standard deviation
$\theta = 2.2, \alpha = 0.5$	0.0196
$\theta = 7, \alpha = 0.1$	0.0201
$\theta = 7, \alpha = 0.5$	0.0216

Table 4 - Volatility of the Simulated Exchange Rate Series (non-convergence case)

Note: We employ standard values of chapter 4 for all other parameters.

simulated data ($\theta = 7, \alpha = 0.5$)		
	Skewness	0.019812
	Excess Kurtosis	3.6170
	Normality test: Chi^2(2) =	142.39 [0.0000]**

Table 5 – Normality Tests for the Simulated Exchange Rate Returns

Note: Two stars indicate that the test is significant at the 1% level.

	ADF test statistics
simulated data ($\theta = 7, \alpha = 0.5$)	
log rate[1]	t-adf -0.992
Δ log rate[1]	t-adf -44.86**

Table 6 – Stationarity Tests (ADF) for the Simulated Exchange Rate Data

[1] Critical values (5%=-1.94 1%=-2.57)

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	γ	t-value	F-test
simulated data	0.0901	-3.15*	9.921 [0.002]**
($\theta = 7$, $\alpha = 0.5$)	(0.045)		

Table 7 – OLS Regression of Forecast Error on Lagged Forecast Error

Note: We omit the estimates for the constant term here to conserve space. Two stars indicate that the test is significant at the 1% level, one star that it is significant at the 5% level. Standard errors are given in the curved paranthesis.

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	Bulgaria	Croatia	Czech Republic	Estonia	Hungary	Latvia
1989	10	n.a.	1,5	n.a.	18,1	n.a.
1990	72,5	136	**18,4**	n.a.	**33,4**	n.a.
1991	**338,9**	249,8	52	303,8	32,3	**262,4**
1992	79,4	938,2	12,7	**953,5**	21,6	959
1993	63,8	**1517,5**	18,2	35,6	21,1	35,6
1994	121,9	-3	9,7	42	21,2	42
1995	32,9	3,8	7,9	29	28,3	29
1996	310,8	3,4	8,6	15	19,8	15
1997	578,6	3,8	10	12,5	18,4	12,5
1998	22,2	5,4	10,7	8,1	14,3	4,7
1999	0,7	4,2	2,1	3,3	10	2,4
2000	9,9	6,2	3,9	4	9,8	2,6

	Lithuania	Poland	Romania	Slovak Republic	Slovenia	Russia
1989	n.a.	**639,5**	0,6	1,5	**2772**	n.a.
1990	n.a.	249	37,7	18,4	104,6	n.a.
1991	345	60,4	222,8	**58,3**	247,1	161
1992	**1161,1**	44,3	199,2	9,1	92,9	**2506,1**
1993	188,8	37,6	**295,5**	25,1	22,8	840
1994	45	29,4	61,7	11,7	19	204,4
1995	35,5	21,6	27,8	7,2	9	128,6
1996	13,1	18,5	56,9	5,4	9	21,8
1997	7	13,2	151,4	6,4	8,8	10,9
1998	5,1	11,8	59,1	6,7	7,9	27,6
1999	0,8	7,3	45,8	10,6	6,1	86,1
2000	1	10,1	45,7	12	8,9	20,8

Table 8 – Changes in Annual Consumer Price Inflation Rates (in %)

Source: EBRD Transition Reports, various volumes.

Rau-Göhring, Matthias (2007),Three Implications of Learning Behaviour for Price Processes
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	Lag order according to AIC
Lithuania	2
Poland	2 or 3
Slovenia	2 or 3

Table 9 – Determination of VAR Lag Length

	Lithuania	Poland	Slovenia
Data Sample	01/93 – 12/00	01/91 – 12/00	01/92 – 12/00
Mean	94.095	66.677	16.197
Standard Deviation	189.37	31.241	12.972
Skewness	2.5395	-0.22673	2.2898
Excess Kurtosis	5.2286	-1.3431	7.2623
Normality Test	364.14 [0.0000]**	6.3789 [0.0412]*	80.379 [0.0000]**

Table 10 - Descriptive Statistics of the Inflation Rates in Lithuania, Poland and Slovenia

Note: One star denotes a rejection at the 5% level, two stars at the 1% level.

	κ_t
Lithuania	0.142
Poland	0.045
Slovenia	0.034

Table 11 – Kalman Gain in Lithuania, Poland and Slovenia

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