THE EUROPEAN UNIVERSITY INSTITUTE

Department of Economics

Essays on the Determination and Formation of Prices in European Crude Oil Markets

H. Peter Møllgaard

May 1993

Thesis submitted for assessment with a view to obtaining the Degree of Doctor of the European University Institute.

Florence, May 1993

Essays on the Determination and Formation of Prices in European Crude Oil Markets

H. Peter Møllgaard

Thesis Committee:

Birgit Grodal
Ronald Harstad
Stephen Martin
Louis Phlips (supervisor)
Jacques-François Thisse
Antonio Villar

PREFACE

This thesis was written over three years, from May 1990 to May 1993, at the Economics Department of the European University Institute in Florence. The order of the chapters reflects, accidentally, more or less the order in which they were written, even though later corrections and amendments render a dating imprecise and perhaps uninteresting. .

The thesis was written under the supervision of Louis Phlips who deserves grateful acknowledgement for the abundant time he has invested in reading, discussing and criticizing the essays. If they are readable, he is to thank. He is the co-author of two of the essays, *Oil Futures and Strategic Stocks at Sea* (Chapter 2) and *Oil Stocks as a Squeeze Preventing Mechanism* (Chapter 3). The former appeared in a book edited by Louis Phlips and Lester D. Taylor: *Aggregation, Consumption and Trade - Essays in Honor of H.S. Houthakker* that was published by Kluwer Academic Publishers in Dordrecht in 1992. The latter is a working paper of the Economics Department of the European University Institute. For the thesis, I rewrote the introduction and made small changes.

Chapters 3 and 4 were to a large extent written while I was a visiting student at the Department of Economics at Stanford University. This visit was most kindly made possible by Peter J. Hammond who also provided extensive comments on different versions of Chapter 4. For this and for his hospitality he receives many thanks.

The final version of the thesis benefitted substantially from the many comments I received from Ronald Harstad and Stephen Martin.

Valeria Fichera provided data and information for Chapter 3: Grazie.

Dorothea K. Herreiner provided many valuable comments on Chapters 1, 3 and 4 at early stages when the notation was not yet comprehensible: Mange tak.

Mrudula Patel deserves warm thanks for chastising me into better performance on Chapter 3 and for her "constant comments."

Chapter 4 on *Bargaining and Efficiency in a Speculative Forward Market* has a very long history and innumerable commentators are thanked. The paper was presented in an extremely preliminary version in the first EUI workshop on price formation mechanisms on 11 June 1991. A more developed version was then presented in the EUI Students' Workshop on 10 December 1991 and a still incomplete draft in the Students' Workshop during "Summer in Tel Aviv", July 1992. Participants in all these workshops are thanked for comments and discussions as are Jørgen Rugholm Jensen, Alan P. Kirman, Mordecai Kurz and Robert Waldmann. Ronald Harstad and Ebbe Hendon deserve special thanks for extensive comments on different drafts.

I have enjoyed the collaboration of the Oxford Institute for Energy Studies where especially Paul Horsnell and Cristina Caffarra provided data and clarifying explanations at various stages.

Last, but not least, Jochen P. Lorentzen is thanked warmly for good company during four years of studying in Villa Schifanoia and for the running, hiking and 'solving-the-world's-problems' that has made this less of a tedious undertaking.

Badia Fiesolana, May 1993

Thesis Outline

Essays on the Determination and Formation of Prices in European Crude Oil Markets

Un'Opera in Quattro Atti

di

H. Peter Møllgaard

Overture

1. *Crescendo Storico*

The exploration and development of the Brent field and other fields in the North Sea commenced in the late 1960s as a response by the major oil companies and by Western governments to the cartelization of the petroleum exporting countries. This process was spurred by the first oil price shock in 1973. The first oil was produced in 1975. At that time there was no tradition for trading crude oil on markets. The majors and OPEC were used to post prices, and so there was no obvious organization for selling new non-OPEC oil and indeed it was not obvious that the oil should be marketed at all, given the dominance of integrated oil companies.

Yet a very peculiar market for Brent - the 15-Day market - developed over the years. Starting from occasional forward trading, a highly developed, organized forward market institution emerged. Its functioning resembles that of a traditional futures market except for the decentralized and non-regulated nature of trading. Compared to the standard *model* of futures trading, this particular market diverts in at least three important ways:

1. The spot market is highly concentrated and major oil companies trade in the market so the spot price can be manipulated;
2. There are frequent squeezes in the market: another example of manipulation;
3. Trading in the market has proved highly speculative.

Later again, in the mid-80s, a futures market, the International Petroleum Exchange of London, was in fact introduced. This market only gained importance as the Gulf crisis of 1990 evolved. At that time Brent had become an internationally important marker alongside with West Texas Intermediate and Dubai.

Today we have a unique situation of coexisting forward and futures markets for the same good. Including the spot market this makes three markets for the same good: Rather

DOI: 10.2870/54657

than having a 'missing market' there seems to be one market too many.

2. *Allegro Cantabile*

The following four "acts" deal with these issues. In the first act, "Strategic Inventories in Two Period Duopoly", the backdrop is entirely theoretical and a *leitmotif* is introduced that is going to reappear in different forms in the following two acts: In concentrated markets, how does the ability to store goods affect pricing? How can prices be manipulated? The main players are rival producers that use inventories in playing games with each other. Stocks make competition fiercer by providing a vehicle for committing to raise second-period output in the Cournot case or to lower the price in the Bertrand case. If the firms were to cooperate, they would not hold strategic inventories in this act.

The second act, "Oil Futures and Strategic Stocks at Sea", takes a broader view on the scenery including all three markets on three stages. Stage 1: The International Petroleum Exchange performs like a thick futures market which can be used to hedge against extrinsic uncertainty but also against the intrinsic uncertainty that is created by the forward market on stage 2. The forward market (the 15-Day market) is a small club of speculative traders, including the producers, that enter forward contracts knowing that this will affect the storage decision on stage 3. On stage 3, the spot market, we hear the *leitmotif* from the first act performed as a duet by a Cournot duopoly. As the act finishes we are left with a better understanding of how futures positions, forward contracts and stocks can be used to manipulate spot prices and how a foreseeing speculator knowing this will trade on the futures and forward market. The overall system is characterized by a good deal of complexity and certain themes need to be developed and enlarged. This is done in the last two acts.

In the third act, "Oil Stocks as a Squeeze Preventing Mechanism", we concentrate on the forward and spot markets and introduce a new player: The squeezer. The *leitmotif* is now played in a completely different key and in a certain sense backwards compared to act one. We recognize the protagonists: The producers. The plot is as follows: Squeezes are registered occasionally on the 15-Day market. The squeezer accumulates forward contracts and creates artificial scarcity by refusing to close out, exploiting imperfections in the decentralized market clearing. The artificial demand in turn creates a price surge and the possibility of a squeeze thus introduces additional uncertainty about the spot price. Squeezes therefore render the market institution less palatable to other market participants (traders and refineries) who may find other ways of accomplishing the economic functions of the forward market (for example an organized futures market). The producers thus have a long term interest in keeping market clearing smooth, for example by supplying stocks to squeezed traders. The extent to which such self-regulatory stocks should be held is analysed in the context of a repeated game and the conclusion is that unless the probability of a squeeze is very small, self-regulation is possible. This shows that there is a rôle for inventories if firms cooperate contrary to the result of the first act.

The fourth and final act is played by a set of players that we only met briefly in the second and third act: The traders. Here producers are ignored completely as the focus is on the process of trading in the forward market. The 15-Day market is predominantly speculative, a fact which contradicts the assumptions that lead to zero speculation theorems. We therefore set up a stochastic game model of a market with a small number of speculative traders that differ only with respect to the expected spot price and (possibly) with respect to risk aversion. Contracting is done after pairwise negotiations in random matches. The Markov perfect equilibrium of the model can mimic the 15-Day market and need not be efficient in the sense that it belongs to the bilateral core.

The third and fourth act show that the 15-Day market is very imperfect and that the organization of this market leads to more uncertainty than necessary. The point that was made in the second act is therefore emphasized: Trading in this peculiar forward market creates intrinsic uncertainty that could be avoided with a different organization of trade (for example an emerging regulated, centralized futures market). Considering these imperfections, it is remarkable that Brent is used as an international marker, *i.e.* in the price formation of a host of other crudes all over the world.

3. *Staccato* or: The Road to Hell is Paved with Good Intentions and Good Inconsistencies

If all the above sounded as *bel canto* to you, dear reader, this is where the dissonances appear: Once upon a time, when I embarked on this project, I thought the way to model the European oil markets using game theory would be to do bits and pieces in every chapter and then collect these bits and pieces to solve the puzzle in the end, with a grand final model encompassing all the other, preliminary studies. Dear reader, this isn't it! Rather, the different chapters provide different and somewhat contradictory views on the 15-Day market. Let me use the next few lines to point out the inconsistencies to you.

1. In Chapter 2 on price *determination*, it is taken for granted that the forward market leads to an efficient outcome. In Chapter 4 on price *formation*, this event is shown to happen with a positive probability (if time allows), but *not* with certainty. As already mentioned, this only emphasizes one point that is made in Chapter 2: Agents on the real futures market (the International Petroleum Exchange) may take positions partly as a hedge towards the uncertain outcome of the forward market.

2. In Chapter 2 the speculator is risk neutral, whereas in Chapters 3 and 4 speculators are risk averse. I think the latter is a more realistic assumption, although the risk

neutral speculator has a long tradition in financial economics. Traditionally, suppliers and demanders are seen as more risk averse than speculators, and the formulation where the former are *infinitely* risk averse and the latter risk *neutral* takes this view to its extreme (bar for risk *loving* speculators). Empirically, the magnitudes of risk aversion coefficients are under discussion and it is difficult to see why integrated oil companies should be more risk averse in their trading strategies than big Wall Street trading houses. The results in Chapter 2 are qualitatively robust to the introduction of risk aversion in the speculator's utility function, but quantities and prices in equilibrium will of course change accordingly.

3. In Chapter 2 the producers are assumed to engage in Cournot behaviour, which is somewhere in the middle of the scale between pure cooperation and pure competition. In Chapter 3 it is shown that the producers should cooperate albeit this emerges as the result of an infinite repetition of non-cooperative stage games. The two studies emphasize different aspects of oil stocks and should be seen as complementary. Maybe the reason why producers do not cooperate on squeeze preventing stocks is that the non-cooperative strategic stocks dominate. However, it does not seem quite reasonable to assume a non-cooperative business environment for the market. This connects us back to the theoretical discussion in Chapter 1 which analyses the strategic effect of stocks as a function of four different paradigms reflecting the assumed interaction between producers: Competition, Bertrand, Cournot and Cooperation. Note that the discussion of stocks is theoretical by force: European producers are very secretive when it comes to the size of their stocks - remarkably so, when compared to the excellent statistics available for American oil stocks. This shows that market participants attach importance to these statistics and therefore that they are important strategic variables, but it is at odds with the informational assumptions of the games we have analysed.

4. One inconsistency that is running through the thesis is the simplicity of the modelling compared to the complex real world. This is not a theme that will be elaborated here. Any theory needs some level of abstraction and I have tried to make reasonably realistic assumptions that are discussed throughout. This procedure has at times rendered the models more complex than they would have been if they were there as an intellectual exercise alone. The partial equilibrium approach is not without its pitfalls: In practical terms we have treated the system of North Sea oil markets as a more or less closed system ignoring possible effects from the world supply and demand for crude oil. This criticism bears particularly on Chapter 2 on the short run price determination that is supposed only to depend on factors within the market. This ignores that the price level of North Sea crude oil depends on a great many external factors.

5. In the last three essays it is assumed that spot prices follow a normal distribution. The advantage of this is that the distribution can be fully described by two parameters, mean and variance. Combined with an exponential utility function, the agents' preferences can then be represented by a function that is linear in mean and variance. This formulation is widely used, especially in financial economics, because of its simplicity. A drawback is that since the support of the normal distribution includes the entire real line, there is a positive probability that realized prices become negative — an obvious nonsense. The results carry through with more palatable distributions, *e.g.* a log-normal, but at an increase in complexity. The judgment was that since the use of the mean-variance model is well-known, the reader could be spared the extra nuisance implied by more complicated distributions.

4. *Formazione Determinata*

What is the difference between 'price determination' and 'price formation'?

To me, price determination is the ascertainment of the forces that determine equilibrium prices. In the game models of Chapters 1, 2 and 3, equilibrium is essentially a matter of consistency of beliefs at some level or other. No out-of-equilibrium dynamics are specified and therefore we are not informed *how* this equilibrium could be reached. The multi-stage models we concoct for this purpose have simultaneous move stage games. What we get is *in-formazione determinata*: equilibrium prices depend in this and that way on stocks, demand, the number of rivals and so forth.

Price formation, to the contrary, explains what prices you observe out of equilibrium and where dynamics take prices. This approach is taken up in Chapter 4 where we construct a bargaining type model with this in mind. At each stage, moves are sequential. We consider a truly dynamic model which is an extension of the usual steady state models of bargaining in markets. Market participants meet in random, pairwise matches. The state of the market evolves according to a Markov process that players can control only partly. If one expects a tatônnement leading to equilibrium we may disappoint. What we get is *formazione in-determinata*: the dynamic Markov perfect equilibrium may and may not take the market to the static equilibrium. The static equilibrium will only obtain after certain fortunate histories.

A Reader's guide

Each chapter has a breakdown into an introduction, sections on modelling and a concluding section that puts the modelling into perspective. References are found at the end of the thesis. The page before each chapter contains an outline of that chapter and an abstract.

It is beyond the scope of the thesis to give a comprehensive descriptive account of the North Sea crude oil markets. A main source has been *The Market for North Sea Crude Oil* from the Oxford Institute for Energy Studies (Mabro *et al.* (1986)) and the reader is referred to that and to the other references made in the text for further particulars. However in Chapters 2 through 4, the empirically relevant characteristics of the market are outlined (and stylized). This way I hope that the reader is not left with the feeling that something important is untold.

Microeconomics employs one vocabulary, game theory another, futures markets a third and the North Sea crude oil sector seems to have developed its own lingo. The latter is explained (I hope) to those who understand the three former. One question I am always asked is: Why on earth is it called the 15-Day market? 'The answer, my friend, is blowing in the wind.' — (Or see footnote 1 on page 67).

CHAPTER 1:

Strategic Inventories in Two Period Oligopoly

 DOI: 10.2870/54657

Chapter Outline

ABSTRACT: A general model of two-period duopoly is set up and it is shown how inventories can serve a strategic purpose, for example by providing a commitment to raise second period output. The strategic effect of inventories depends on the convexity of the cost function, on the cost of storage and on whether the "business regime" is competitive, Bertrand, Cournot or cooperative. It is shown how the model encompasses existing models in the literature. A closed form of the strategic inventories is then found for a parametrized two period, n-firm oligopoly

0. Introduction

This chapter is concerned with strategic inventories in two period oligopoly. By inventories I mean finished goods made to stock and stored at the manufacturer's level. These inventories typically account for around thirty percent of all manufacturers' stocks (see Phlips (1983) p. 68). This definition leaves out stocks of raw materials and goods-in-progress as well as inventories of goods made to order.

As to the motives for carrying inventories, the literature typically distinguishes between three such: the speculative motive, the precautionary motive (buffer stocks) and the transaction motive which leads to the familiar production smoothing. In addition to this, Phlips and Richard (1989) are concerned with the intertemporal discrimination motive in a dynamic oligopoly model with demand inertia, where the price a firm sets in one period affects the demand for its products in the next. Intertemporal price discrimination arises because firms equalize discounted marginal revenues over time.

Inventories are said to serve a *strategic* purpose if they are held with the explicit purpose of affecting the rival's decisions in later periods. As it will turn out, these strategic inventories are closely related to the convexity of the cost functions and cannot for that reason be studied independently of the transactions inventories. For the sake of realism and for completeness we will also include buffer stocks, but typically treat them as constant in which case they do not matter much. The speculative motive is completely ignored.

We solve our models using non-cooperative game theory and only shortly touch on the cooperative case in Section 1.5. On the other hand, we shall assume a "friendly environment" in the sense that firms are not actively trying to drive each other out of business, to merge or to take over. It may make most sense to think of the two firms and the two periods as a going-concern situation. We thereby disregard the strategic purpose of inventories in entry deterrence emphasized by Ware (1985). Ware shows that inventories can be used as a credible threat by the incumbent to dump prices below marginal costs in the post entry game since the opportunity cost of supplying a unit of inventory is zero.

In Section 1 we put forth a generalized framework of a duopoly with inventories and show how these can serve a strategic purpose. The generalization lies in the general treatment of the demand and cost function and in the way different "business regimes" are included by means of conjectural variations. Conjectural variations allow us to discuss the model under four different "business regimes": Competition, Bertrand price setting, Cournot quantity setting and Cooperation. We then show how this model encompasses existing models that typically assume Cournot conjectures, *i.e.* a regime of non-cooperative quantity setting.

Section 2 parametrizes the general model by postulating linear, homogeneous demand and quadratic cost functions, but generalizes the analysis to n-firm oligopoly to find closed form strategic inventories. Section 3 concludes.

1. Two Period Duopoly with Inventories

The purpose of this section, which constitutes the core of the paper, is to set up a general framework to analyse the strategic role of inventories in duopoly and to present a number of results under different assumptions about firms' beliefs regarding their rival's behaviour and the cooperative or non-cooperative character of the game.

We shall require our solutions to be subgame perfect and solve the game backwards, starting with the subgame in period two. This first step highlights the comparative statics of a duopoly with inventories. We make use of the concept of reaction functions to set up a rather general static second period game.

Treating inventories in a static framework is, of course, a contradiction in terms. Inventories should be treated as a state variable that implies that decisions on production and sales in the first period affect the outcome in the second.[1] We therefore move backwards one period and study the sales and production decisions of the first period.

It should be emphasized that inventories are the only source of dynamics in this model. In particular, demand is assumed to be intertemporally separable and independent in the two periods. This is not necessarily a natural assumption in a dynamic model with a storable good for at least three different reasons. First, if the good is durable, a high demand in the first period will tend to lower demand in the second period. Second, since the good is storable, consumers (which could be other firms) might find it worthwhile to carry out the storage themselves. Third, consumers might respond to price differences between different suppliers with a lag as in Phlips and Richard (1989).

The sections are organized as follows: In 1.1 we set up the model and develop a general notation. Section 1.2 describes the sales game of the second period in general terms. Section 1.3 describes the Nash and Stackelberg equilibria, and shows how asymmetry in the ability of firms to hold inventories can yield the Stackelberg outcome as a Cournot-equilibrium. Section 1.4 then describes how firms divide their period-1 production between sales and inventories and illustrates the strategic effect of stocks implicitly. Section 1.5 shows how our model is related to other non-cooperative models in the literature and comments on a cooperative model.

1.1 A General Two Period, Two Firm Model

This subsection provides a generalized framework for discussing the two period duopoly. Variables can have two subscripts: The first identifies the firm and the second

[1] Kirman and Sobel (1974) set up an oligopoly model as a stochastic game with Markov strategies.

denotes the period. Firms are indexed by i and j where $i, j = 1, 2, i \neq j$. Time is indexed by $t = 1,2$. Subscript "$\bullet$" symbolizes an aggregator.[2] The model consists of a set of demand functions, a set of cost functions regarding production, a set of identities connecting sales and production in the two periods, and a set of functions for the cost of storage.

The (inverse) demand for the goods of firm i is given by

$$p_{i,t} = p_{i,t}(q_{1,t}, q_{2,t}) > 0 \quad \text{with} \quad \frac{\partial p_{i,t}}{\partial q_{i,t}} < 0 \tag{1}$$

i.e. the price of firm i's product, $p_{i,t}$, is assumed to be a downward sloping function of the firms own sales, $q_{i,t}$. The products are assumed to be substitutes so that

$$\frac{\partial p_{i,t}}{\partial q_{j,t}} < 0 \ . \tag{2}$$

In many applications it is assumed that goods are homogeneous in which case the demand system can be written

$$p_{\bullet,t} = p_{\bullet,t}(q_{1,t} - q_{2,t}) \tag{3}$$

$$\frac{\partial p_{\bullet,t}}{\partial q_{\bullet,t}} < 0 \ . \tag{4}$$

The cost functions are supposed to depend on contemporaneous output only. We shall generally assume that variable costs, $C_{i,t}$, are an increasing, convex function of the amount produced, $x_{i,t}$:

$$C_{i,t} = C_{i,t}(x_{i,t}) \tag{5}$$

$$c_{i,t} = \frac{dC_{i,t}}{dx_{i,t}} \geq 0 \tag{6}$$

$$c_{i,t}' = \frac{d^2C_{i,t}}{dx_{i,t}^2} \geq 0 \tag{7}$$

[2] By this I mean a broadly defined aggregator: it can be used 1) as a placeholder; 2) to symbolize a sum over periods or agents; or 3) to symbolize a Cartesian product.

$$C_{i,t}(0) = 0 \,. \tag{8}$$

Inventories, $S_{i,t}$, are related to sales and production through the identity:

$$S_{i,t} - S_{i,t-1} \equiv x_{i,t} - q_{i,t}\,, \quad t=1,2\,. \tag{9}$$

Following Phlips and Thisse (1981) we assume that initial stocks, $S_{i,0}$, and terminal stocks, $S_{i,2}$, take the value, $\underline{S}_i$, at which cost of storage is minimal. The reason for such an $\underline{S}_i$ to exist is that in general it is neither costless to carry inventories, nor to experience stockouts. For a more elaborate argument, see Phlips (1981) pp. 106-7. The following change of variables will prove convenient:

$$s_{i,t} \equiv S_{i,t} - \underline{S}_i\,. \tag{10}$$

Identity (9) can then be written:

$$\begin{aligned} s_{i,1} &\equiv x_{i,1} - q_{i,1} \\ q_{i,2} &\equiv x_{i,2} - s_{i,1}\,. \end{aligned} \tag{11}$$

Again following Phlips and Thisse (1981) we assume that the cost of storage, $I_{i,1}$, can be described by a positive, strictly convex and twice continuously differentiable function which takes its minimum for $s_{i,1} = 0$, *i.e.*

$$I_{i,1} = I_i(s_{i,1}) > 0 \quad \text{for } s_{i,1} \geq -\underline{S}_i\,, \tag{12}$$

$$i_i \equiv \frac{dI_{i,1}}{ds_{i,1}} \begin{cases} < \\ = 0 \\ > \end{cases} \text{for } s_{i,1} \begin{cases} < \\ = 0\,, \\ > \end{cases} \tag{13}$$

$$i_i' \equiv \frac{d^2 I_{i,1}}{ds_{i,1}^2} > 0 \quad \text{for } s_{i,1} \geq -\underline{S}_i\,. \tag{14}$$

Equations (1-14) constitute the basic model of the duopoly. To find a solution to the system, we superimpose a game structure. This structure can be cooperative, but most of the time we shall assume it to be non-cooperative. In any case, firm *i* basically has four decisions to make: how much to produce and how much to sell in either period. A strategy for firm *i* thus in general consists of a vector

$$\tilde{\sigma}_i = \left(q_{i,1} \, , \, x_{i,1} \, , \, q_{i,2} \, , \, x_{i,2} \right) . \tag{15}$$

For a strategy to be feasible it has to satisfy non-negativity of all amounts and the identities (11). Given inventories in period 1, firm i thus only has one decision to make in period 2, and a feasible strategy can be represented by a three element vector:

$$\sigma_i = \left(x_{i,1}, s_{i,1}, x_{i,2}(s_{i,1}) \right) \geq \left(0, -\underline{S}_i, 0 \right), \quad i \neq 1,2. \tag{16}$$

Let the set of feasible strategies for firm i be called Σ_i. Pay-offs are total, discounted profits. For each of the players, profits in the two periods are given by:

$$\begin{aligned} \pi_{i,1} &= p_{i,1}(x_{1,1} - s_{1,1}, \, x_{2,1} - s_{2,1})(x_{i,1} - s_{i,1}) \\ &\quad - C_{i,1}(x_{i,1}) - F_{i,1} - I_{i,1}(s_{i,1}) \end{aligned} \tag{17}$$

$$\begin{aligned} \pi_{i,2} &= p_{i,2}(x_{1,2} - s_{1,1}, \, x_{2,2} + s_{2,1})(x_{i,2} - s_{i,1}) \\ &\quad - C_{i,2}(x_{i,2}) - F_{i,2} \, , \end{aligned} \tag{18}$$

where $F_{i,t}$ represents "fixed costs," *i.e.* costs that do not vary with the level of output or inventories. The pay-off of firm i is then

$$\pi_{i,\cdot} = \pi_{i,1} - \delta_i \pi_{i,2} \, , \tag{19}$$

where δ_i is firm i's discount factor. The game is now given by $\Gamma = \left(\Sigma_1, \Sigma_2, \pi_{1,\cdot}, \pi_{2,\cdot} \right)$.

In the non-cooperative game, the firms strive to maximize $\pi_{i,\cdot}$ given some beliefs[3] regarding the reaction of the other firm on the market. Such beliefs, or conjectural variations, were introduced by Bowley (1924). Following Dixit (1986) we formalize these conjectural variations by

$$\left(\frac{dq_{j,t}}{dq_{i,t}} \right)^e = v_{i,t}(q_{1,t}, q_{2,t}) \, . \tag{20}$$

$v_{i,t}$ thus tells us how firm i believes that firm j will react on a given change in its own sales. Four standard cases are readily interpreted as different specifications of the conjectures, $v_{i,t}$:

---

[3] or expectations: thus superscript e.

$$\text{a. Competitive Conjectures:} \quad v_{i,t} = -\frac{\partial p_{i,t}/\partial q_{i,t}}{\partial p_{i,t}/\partial q_{j,t}}$$

$$\text{b. Bertrand Conjectures:} \quad v_{i,t} = -\frac{\partial p_{j,t}/\partial q_{i,t}}{\partial p_{j,t}/\partial q_{j,t}} \tag{21}$$

$$\text{c. Cournot Conjectures:} \quad v_{i,t} = 0$$

$$\text{d. Constant Market Shares:} \quad v_{i,t} = \frac{q_{j,t}}{q_{i,t}}$$

Competitive conjectures are derived from requirement that *i* believe that her own price will not change if she changes sales. Taking account both of the direct effect and of the indirect effect via the rival's expected reaction yields (21.a). Bertrand conjectures (21.b) are derived from the requirement that *i* believe *j*'s price to be independent of her price. In the case of homogeneous goods both competitive and Bertrand conjectures are *-1*, so *i* expects *j* to match a change in output with an offsetting change keeping prices constant. Cournot conjectures are zero (21.c) and with constant market shares (21.d), firm *i* expects firm *j* to match any percentage change in output one to one. Note that the conjectures are listed in order of increased implied collusion. Furthermore it should be noted that if both firms hold constant market shares conjectures, the outcome will be identical to the outcome of the cooperative joint profit maximizing game if the demand functions satisfy Slutsky symmetry, *i.e.* if $\partial p_{i,t}/\partial q_{j,t} = \partial p_{j,t}/\partial q_{i,t}$. Slutsky symmetry is trivially satisfied with homogeneous goods.

In general conjectures could be constructed and combined in any conceivable way, but they are best interpreted "as if:" under the assumption that both[4] players hold competitive conjectures, the model produces an outcome *as if* there were a reason for the players to end up in competitive equilibrium. Given a small number of players, this may not be a natural assumption, but it serves as an illustration just as an Edgeworth box can be used to illustrate a competitive equilibrium of an exchange economy. In a one shot game, the natural assumption is Nash conjectures and that amounts to Bertrand conjectures if both firms are price setters or to Cournot conjectures if they are quantity setters. There is reason to be particularly wary of using combinations of conjectures, *e.g.* one player holding Cournot conjectures and the other consistent conjectures (*cf.* later), this leading to the Stackelberg outcome, since it forces a fundamentally sequential play into the strait-jacket of a simultaneous move game. Finally, constant market shares conjectures are not nice either, since we want to model an inherently non-cooperative environment and the story needed to achieve cooperation would involve infinitely repeated play - at odds with the assumption that there

[4] Read: *all* players in the *n*-person game.

are only two periods. Having said this, we continue using conjectural variations without further excuses but limit all interpretations to the four cases mentioned above where we can postulate that the model works "as if" and where we can use well developed intuition.[5]

These are the basic game theoretical settings. We will generally assume that the decision process of the firm can be described in the following way: In period 1 firm i decides on how much to produce in this period, *i.e.* on $x_{i,1}$, and on how much to sell in this period, *i.e.* on $q_{i,1}$, or equivalently, on how much to store for sale in the next period, *i.e.* $s_{i,1}$. These decisions then become common knowledge. In period 2, the only decision to make is how much to produce, since the inventories carried over from period 1 are given. In each period decisions are taken simultaneously. For behaviour to be subgame perfect, the game should be solved backwards thereby ensuring that the second period solution is consistent with *any* first period outcome. The next section thus solves the sales game of period 2.

1.2 The Period-2 Sales Game

The problem that firm 1, say, faces in period 2 is a purely static one:

$$\underset{\{x_{1,2}\}}{Max} \quad \pi_{1,2}\left(x_{1,2}, x_{2,2}^{e}; s_{1,1}, s_{2,1}\right) \tag{22}$$

where $x_{2,2}^{e}$ denotes firm 1's beliefs with respect to the output of firm 2. The perceived first order condition is

$$\frac{\partial \pi_{1,2}}{\partial x_{1,2}} = \left(\left(\frac{\partial p_{1,2}}{\partial q_{1,2}} + \frac{\partial p_{1,2}}{\partial q_{2,2}}\frac{dq_{2,2}}{dq_{1,2}}\right)\left(x_{1,2} + s_{1,1}\right) - p_{1,2}\right)\frac{dq_{1,2}}{dx_{1,2}} - c_{1,2} = 0. \tag{23}$$

In the case of an interior solution, sales increase with production at the margin so $dq_{i,2}/dx_{i,2} = 1$ and (23) can be written

$$\frac{\partial \pi_{1,2}}{\partial x_{1,2}} = \left(\frac{\partial p_{1,2}}{\partial q_{1,2}} + \frac{\partial p_{1,2}}{\partial q_{2,2}}v_{1,2}\right)\left(x_{1,2} + s_{1,1}\right) - p_{1,2} - c_{1,2} = 0\ . \tag{24}$$

To the contrary, in the case of a corner solution $dq_{i,2}/dx_{i,2} = 0$ and

[5] Note, however, that there is a conceptual difference between the empirical use of conjectural variations to measure the degree of imperfect competition in an industry and the theoretical use of the concept to model beliefs. See Tirole (1989) pp. 244-245 and Bresnahan (1987).

$$\frac{\partial \pi_{1,2}}{\partial x_{1,2}} = -c_{1,2} < 0 \,, \tag{25}$$

since increased production only increases variable costs, not sales revenue. The profit maximizing production is then zero and the perceived profit maximizing decision is to sell out of inventory alone. The corner solution obtains when, $\forall x_{1,2} > 0$,

$$p_{1,2}\left(s_{1,1},\ q^e_{2,2}\right)s_{1,1} - F_{1,2} > \pi_{1,2}\left(x_{1,2},\ x^e_{2,2};\ s_{1,1}, s_{2,1}\right) \tag{26}$$

This can happen if the firm's own level of inventory is very high, if the sales of the other firm is expected to be very high or if marginal costs are very high. Without further specification of the demand and cost functions we cannot say anything precise about the intervals for which the corner solution obtains.

Arvan (1985) shows that in a Cournot duopoly of identical firms producing a homogeneous good and facing a strictly concave inverse demand function, an upper level of inventories exists such that for all levels of inventory above this level, sales are equal to inventory and production thus zero.[6]

Here we concentrate on the case of interior solutions for both firms. The second order condition for firm 1 then is

$$\frac{\partial^2 \pi_{1,2}}{\partial x^2_{1,2}} = a_{1,2} + b_{1,2}v_{1,2} < 0, \qquad \text{where} \tag{27}$$

$$a_{1,2} = 2\frac{\partial p_{1,2}}{\partial q_{1,2}} + \frac{\partial p_{1,2}}{\partial q_{2,2}}v_{1,2} - c'_{1,2} - q_{1,2}\left(\frac{\partial^2 p_{1,2}}{\partial q^2_{1,2}} + \frac{\partial^2 p_{1,2}}{\partial q_{1,2}\partial q_{2,2}}v_{1,2} - \frac{\partial p_{1,2}}{\partial q_{2,2}}\frac{\partial v_{1,2}}{\partial q_{1,2}}\right) \tag{28}$$

and

$$b_{1,2} = \frac{\partial p_{1,2}}{\partial q_{2,2}} + q_{1,2}\left(\frac{\partial^2 p_{1,2}}{\partial q_{1,2}\partial q_{2,2}} + \frac{\partial^2 p_{1,2}}{\partial q^2_{2,2}}v_{1,2} - \frac{\partial p_{1,2}}{\partial q_{2,2}}\frac{\partial v_{1,2}}{\partial q_{2,2}}\right). \tag{29}$$

Observe that $a_{1,2}$ is the first partial derivative of the perceived marginal profit function w.r.t. $q_{1,2}$ ($x_{1,2}$) and $b_{1,2}$ similarly the first partial derivative w.r.t. $q_{2,2}$ ($x_{2,2}$). The slope of the "best

[6] Arvan (1985) also shows that an even higher level exists such that for all levels above this level it does not even pay to sell the whole inventory. Since we assume that the demand functions are known and that producing and holding inventory are costly activities this cannot maximize profit. We therefore disregard it.

response function" or reaction curve of firm 1 is then given by

$$r_{1,2} = -\frac{b_{1,2}}{a_{1,2}} . \tag{30}$$

Of course, firm 2's behaviour can be described by a set of equations like (27-29). Consistent conjectures can now be defined as $v_{2,2} = r_{1,2}$ and $v_{1,2} = r_{2,2}$. Note that here "consistency" is meant in terms of an untold dynamic story about how equilibrium is achieved. In terms of a one shot game, only Nash conjectures (*i.e.* Bertrand or Cournot) are consistent and consistency is indeed the main virtue of Nash conjectures. Inconsistency is seen only from the point of view of the untold dynamic process.[7]

In the case of homogeneous goods and linear demand that we are going to investigate in Section 2, it can be shown that $-1 < r_{i,2} < 0$ for both Cournot and Bertrand/Competitive conjectures.

Now, totally differentiating the first order conditions (24) we get the following set of equations:

$$\begin{pmatrix} a_{1,2} & b_{1,2} \\ b_{2,2} & a_{2,2} \end{pmatrix} d\vec{x}_{\cdot,2} = -\begin{pmatrix} a_{1,2} + c_{1,2}' & b_{1,2} \\ b_{2,2} & a_{2,2} - c_{2,2}' \end{pmatrix} d\vec{s}_{\cdot,1} \ , \tag{31}$$

where $d\vec{x}_{\cdot,2} = (dx_{1,2}, dx_{2,2})^T$ and $d\vec{s}_{\cdot,1} = (ds_{1,1}, ds_{2,1})^T$. The solution is

$$d\dot{x}_{\cdot,2} = -\left(E - \frac{1}{D_{\cdot,2}} \begin{pmatrix} a_{2,2}c_{1,2}' & r_{1,2}a_{1,2}c_{2,2}' \\ r_{2,2}a_{2,2}c_{1,2}' & a_{1,2}c_{2,2}' \end{pmatrix} \right) d\dot{s}_{\cdot,1} \ , \tag{32}$$

where E is the 2x2 identity matrix and $D_{\cdot,2}$ is the determinant of the left-hand side matrix in (31). Stability conditions imply that $a_{i,2} < 0$ and $D_{\cdot,2} > 0$ (see again Dixit (1986)). The diagonal elements of the matrix in (32) will therefore be negative. If $b_{i,2} < 0$, the off-diagonal elements will be positive and the reaction functions downward sloping (this is the "normal" case with Cournot conjectures, *cfr.* later).

Interpreting (32) is straightforward. An increase in the inventory level of, say, firm 1 by one unit leaving firm 2's inventory constant (*i.e.* $ds_{1,1}=1$ and $ds_{2,1}=0$), *ceteris paribus* decreases the production of firm 1 by one unit because of the identity

[7] A serious discussion of this interesting problem will take us too far from the main argument as it would involve (i) the literature on consistent conjectures (*e.g.* Breshnahan (1981) and Ulph (1983)), (ii) the literature on quantity *vs.* price competition (*e.g.* Kreps and Scheinkman (1983), Klemperer and Meyer (1986) and Benoit and Krishna (1987)) and at the limit; (iii) the literature on equilibrium selection (*e.g.* Harsanyi and Selten (1988)).

$$dq_{i,2} \equiv dx_{i,2} + ds_{i,1} \,. \tag{33}$$

The second term of $dx_{1,2}/ds_{1,1}$ can be written

$$-\frac{c'_{1,2}}{(a_{1,2} + b_{1,2}r_{2,2})} > 0 \,. \tag{34}$$

This term arises because inventory does not imply increased marginal cost since inventory costs are sunk in period 2, whereas production does. Note the similarity between the denominator in this expression and the second order condition (27). The denominator expresses the change in marginal profits as a result of a change in sales under consistent conjectures. The inverse of this multiplied by the second order derivative of the cost function, $c'_{1,2}$, then expresses the change in the profit maximizing sales in equilibrium given that output is lowered by one unit. This effect modifies the direct effect. In this sense, the derivative

$$\frac{dx_{i,2}}{ds_{i,1}} = -1 - \frac{c'_{i,2}}{a_{i,2} + b_{i,2}r_{j,2}} > -1 \quad i,j = 1,2,\ i \neq j, \tag{35}$$

shows that the relative advantage of additional inventories lies in second order gains seen from a period 2 perspective.

The effect on firm j's output of a change in firm i's stock is seen to be

$$\frac{dx_{j,2}}{ds_{i,1}} = \frac{-r_{j,2}c'_{i,2}}{a_{i,2} + b_{i,2}r_{j,2}} = r_{j,2}\frac{dq_{i,2}}{ds_{i,1}}. \tag{36}$$

This will be negative in the case of a downward sloping reaction function ($b_{2,2} < 0$). The story is much the same. We do not, of course, get the direct effect, but only the indirect effect via the change in the sales of the goods of firm 1.

As to the effect on prices, it follows directly that

$$\frac{dp_{i,2}}{ds_{i,1}} = \left(\frac{\partial p_{i,2}}{\partial q_{i,2}} + r_{j,2}\frac{\partial p_{i,2}}{\partial q_{j,2}}\right)\frac{-c'_{i,2}}{a_{i,2} + b_{i,2}r_{j,2}} \quad i,j = 1,2,\ i \neq j \,. \tag{37}$$

In general we cannot say anything about the sign. In the homogeneous good case we see that if $r_{j,2} > -1$, the effect will have the expected negative sign. In the homogeneous good case with linear demand, the effect on the price is thus negative for both Cournot and Bertrand/competitive conjectures.

The next section explores the comparative statics of inventories in the homogeneous good case under the assumptions of well-behaved Cournot reaction functions.

1.3 The Period-2 Game: Nash and Stackelberg Solutions

Consider the case of a homogeneous good, *i.e.* (3) and (4) apply. A sufficient condition for the existence (and stability) of a unique Cournot equilibrium is that the inverse demand function be strictly concave,[8] *i.e.*

$$\frac{\partial^2 p_{\bullet,2}}{\partial q_{\bullet,2}^2} < 0 \,. \tag{38}$$

If the two firms are identical with zero inventories, the Cournot-Nash equilibrium has $v_{i,2} = 0$ for $i = 1,2$, and the situation can be characterized by

$$\begin{aligned} a_{i,2} &= 2\frac{\partial p_{\bullet,2}}{\partial q_{\bullet,2}} + x_{i,2}\frac{\partial^2 p_{\bullet,2}}{\partial q_{\bullet,2}^2} - c_{i,2} < 0 \\ b_{i,2} &= \frac{\partial p_{\bullet,2}}{\partial q_{\bullet,1}} + q_{i,2}\frac{\partial^2 p_{\bullet,2}}{\partial q_{\bullet,2}^2} < 0 \\ & 0 > r_{i,2} > -1 \end{aligned} \tag{39}$$

Reaction functions are thus well behaved and the unique Nash-Cournot equilibrium can be found as the intersection (*N*) of the two best response functions in Figure 1.

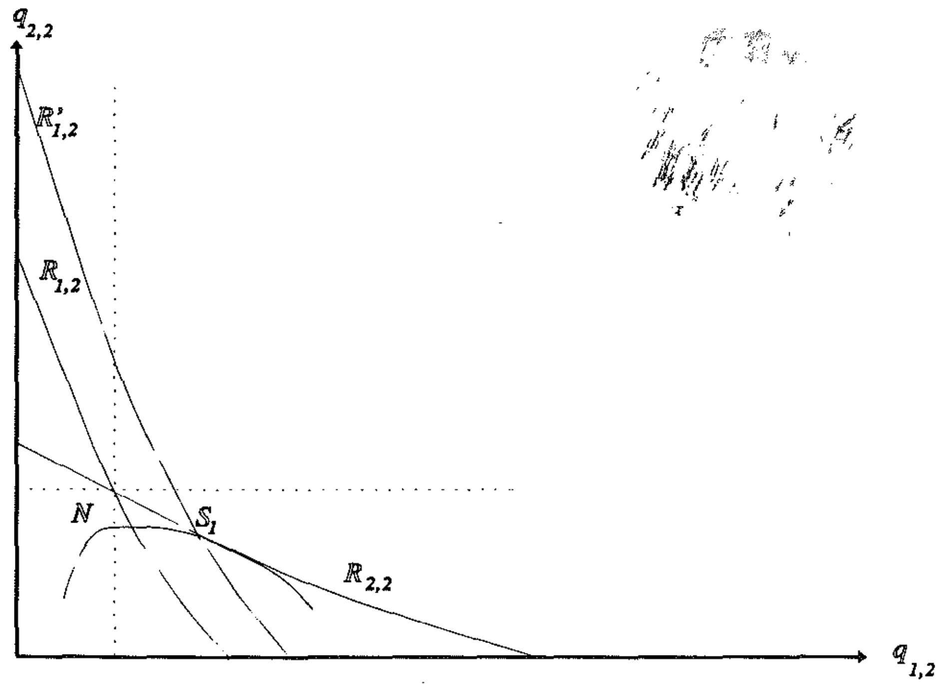
A Stackelberg equilibrium obtains in the case where the market behaves as if *e.g.* firm 1 has consistent conjectures ($v_{1,2} = r_{2,2}$) and firm 2 has Cournot conjectures ($v_{2,2} = 0$). This Stackelberg equilibrium is point S_1 in Figure 1. The Stackelberg leader earns higher and the follower lower profits in S_1 compared to *N*.

Now, assume that for some reason which is exogenous to the game, only firm 1 has the opportunity of carrying inventories. It can use this to shift its reaction curve to the right from $R_{1,2}$ to $R'_{1,2}$ thereby obtaining the Stackelberg outcome, S_1, as the Nash equilibrium. This is certainly profit maximizing in the second period subgame given that costs of producing and carrying inventory are sunk. Whether it will be profit maximizing in the overall game depends on whether the discounted increased second period profit is greater than or equal to the extra

[8] To be more precise, the sufficient condition is that the inverse demand function is strictly positive on an interval *(0,X)* on which it is twice continuously differentiable, strictly decreasing and concave, and that $p_{\cdot,t} = 0$ elsewhere. See Saloner (1987) p. 184.

 DOI: 10.2870/54657

Figure 1 Nash and Stackelberg Equilibria (N and S_1)



cost incurred in the first period to which we now turn.

1.4 The Period-1 Game: the General Case

The result of Section 1.2 was basically that in period-2 equilibrium, the optimized profits (the value functions) were seen to be a function of the two commonly known inventory levels, *i.e.*

$$\pi^*_{1,2} = \pi_{1,2}(s_{1,1},\ s_{2,1})$$
$$\pi^*_{2,2} = \pi_{2,2}(s_{1,1},\ s_{2,1})\ . \qquad (40)$$

We assume that an interior solution exists for any inventory vector $(s_{1,1}, s_{2,1})$ that may result from the first period game. This may sound a bit heroic, and we shall discuss the assumption in Section 1.5. The reader will not have difficulties in verifying that a unit change in the inventory of firm 1 will affect its second period profit by
In period 1 the firms have to maximize overall profits by a simultaneous choice of production

Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

$$\frac{\partial \pi^*_{1,2}}{\partial s_{1,1}} = q_{1,2} \frac{\partial p_{1,2}}{\partial q_{2,2}} (r_{2,2} - v_{1,2}) \frac{a_{2,2}}{D_{\bullet,2}} c'_{1,2} + c_{1,2} \,. \tag{41}$$

and inventories:

$$\underset{\{s_{1,1}, x_{1,1}\}}{Max} \quad \pi_{1,\bullet} = \pi_{1,1} + \delta_1 \pi^*_{1,2} \,. \tag{42}$$

An interior solution will, if it exists, have to satisfy the first order conditions

$$\frac{\partial \pi_{1,\bullet}}{\partial s_{1,1}} = \frac{\partial \pi_{1,1}}{\partial s_{1,1}} - \delta_1 \frac{d\pi^*_{1,2}}{ds_{1,1}} = 0 \tag{43}$$

$$\text{and} \quad \frac{\partial \pi_{1,\bullet}}{\partial x_{1,1}} = \frac{\partial \pi_{1,1}}{\partial q_{1,1}} = 0 \,. \tag{44}$$

A similar set of equations exists for firm 2, and the subgame perfect equilibrium of Γ is fully characterized by the set of strategies $\left((q^*_{1,1}, s^*_{1,1}) \,,\, (q^*_{2,1}, s^*_{2,1})\right)$ that simultaneously solve this system, provided that the second order conditions are satisfied:

$$\frac{\partial^2 \pi_{1,\bullet}}{\partial x^2_{1,1}} = a_{1,1} + v_{1,1} b_{1,1} - c'_{1,1} < 0 \tag{45}$$

$$\frac{\partial^2 \pi_{1,\bullet}}{\partial s^2_{1,1}} = a_{1,1} - v_{1,1} b_{1,1} - i'_{1,1} - \delta_1 \frac{d^2 \pi^*_{1,2}}{ds^2_{1,1}} < 0 \,, \tag{46}$$

where

$$a_{1,1} = 2\frac{\partial p_{1,1}}{\partial q_{1,1}} + \frac{\partial p_{1,1}}{\partial q_{2,1}} v_{1,1} + q_{1,1} \left(\frac{\partial^2 p_{1,1}}{\partial q^2_{1,1}} + \frac{\partial^2 p_{1,1}}{\partial q_{1,1} \partial q_{2,1}} v_{1,1} - \frac{\partial p_{1,1}}{\partial q_{2,1}} \frac{\partial v_{1,1}}{\partial q_{1,1}} \right) \tag{47}$$

and

$$b_{1,1} = \frac{\partial p_{1,1}}{\partial q_{2,1}} + q_{1,1}\left(\frac{\partial^2 p_{1,1}}{\partial q_{1,1}\partial q_{2,1}} + \frac{\partial p_{1,1}}{\partial q_{2,1}}\frac{\partial v_{1,1}}{\partial q_{2,1}}\right). \tag{48}$$

Note that fulfilled second period second order conditions (27) help fulfil (46).

Subtracting the first order condition for $s_{1,1}$ from that for $q_{1,1}$ we get a condition that the solution must satisfy:

$$c_{1,1} + i_{1,1} - \delta_1 c_{1,2} = \delta_1 (r_{2,2} - v_{1,2})\, c'_{1,2} \frac{a_{2,2}}{D_{\cdot,2}} \frac{\partial p_{1,2}}{\partial q_{1,2}} \tag{49}$$

This equation shows the *strategic effect of inventories*: if the only purpose of inventories were to minimize overall costs, the condition would be that discounted marginal costs be equalized, or equivalently, that the left hand side of (49) be equal to zero.

Assuming stability conditions satisfied, this strategic term is seen to be positive only if $(r_{2,2} - v_{1,2}) > 0$, *i.e.* if $v_{1,2} < r_{2,2}$. But in general the term can have either sign. Under consistent conjectures, strategic considerations regarding inventories have zero effect since the firm can correctly manipulate the rival's sales using output from the second period. In this sense, the strategic stocks are seen to be based on inconsistent conjectures when these are deemed inconsistent with an unspecified dynamic process. However in a one shot situation it is perfectly legitimate for conjectures to be inconsistent. In general the term is therefore different from zero and is seen to depend on the curvature of the second period cost function, $c'_{1,2}$. In fact, with constant marginal costs, $c'_{1,2} = 0$, the *strategic* incentive to make inventories deviate from $\underline{S}_i$ ceases to exist.

We now turn to different special cases found in the literature in order to shed light on different aspects of the model.

1.5 Special Cases: A Survey of the Literature

Almost all studies of inventory in two period, two firm models under certainty can be seen as special cases of the model put forth in the preceding sections. We briefly review them and comment on them. As it turns out, the bulk of the studies assume Cournot behaviour.

Arvan (1985) provides in many ways the most complete and the most general study of dynamic *Cournot* duopoly with inventory. His focus is, contrary to ours, on the corner solutions in the second period game. As a result of the existence of corner solutions, the Nash value functions (*i.e.* (39) with $v_{1,2} = v_{2,2} = 0$) need not be concave in the inventories, but will typically experience a discontinuity for certain levels of $s_{1,1}$ and $s_{2,1}$. In this case, the first order conditions of the second period are not fulfilled, and the use of (40) in (42) is not valid.

Arvan next considers the case of entirely symmetric firms with identical cost and demand functions in the two periods. In the case of constant marginal costs he then argues for non-existence of equilibrium in a symmetric model *with* inventories:[9]

> "If there were a symmetric equilibrium where both firms carried inventory from the first period to the second, then both firms would be using inventory to act as a leader! This is not possible when the firms do not produce in the second period, either when inventory is exhausted or when it is redundant. It is also not possible when there is production in the second period, since marginal cost is constant. This rules out symmetric equilibrium with inventory." (Arvan (1985) p. 574).

This citation deserves a remark: We saw in Section 1.3 and Figure 1 that if, for some reason, only one of the firms were able to carry stocks from the first to the second period, it would use this opportunity to act as a leader. We also wondered whether this would be profitable in the overall game. In Arvan's model nothing prevents both firms from trying to act as leaders. However we also know that if marginal costs are constant, the strategic motive for holding inventories vanishes. Indeed if $c'_{1,2} = 0$, $\delta_1 = 1$ and $c_{1,1} = c_{1,2}$ it follows from (45-48) that $i_1=0$, but (13) then implies that $s_{1,1}=0$ or that inventory does not deviate from $\underline{S}_i$. So with constant marginal costs there will be no strategic inventories.

We therefore need cost functions to be convex. Arvan conjectures that

> "[i]n this case it is conceivable that symmetric equilibrium exists where both firms utilize inventory and also produce in period 2. For such an outcome to occur, costs must be sufficiently convex ... that there are ample leadership opportunities to cover storage cost" (Arvan (1985) p. 574, footnote 7).

We have shown that with with convex costs and 'inconsistent conjectures' his conjecture is true and that it generalizes: with competitive, Bertrand and Cournot business environments symmetric firms hold strategic inventories. We provide examples in Section 2.

Two other studies, Saloner (1987) and Allaz (1991), both assume Cournot conjectures and that sales do not take place in the first period. The focal points of the two studies are, however, widely different.

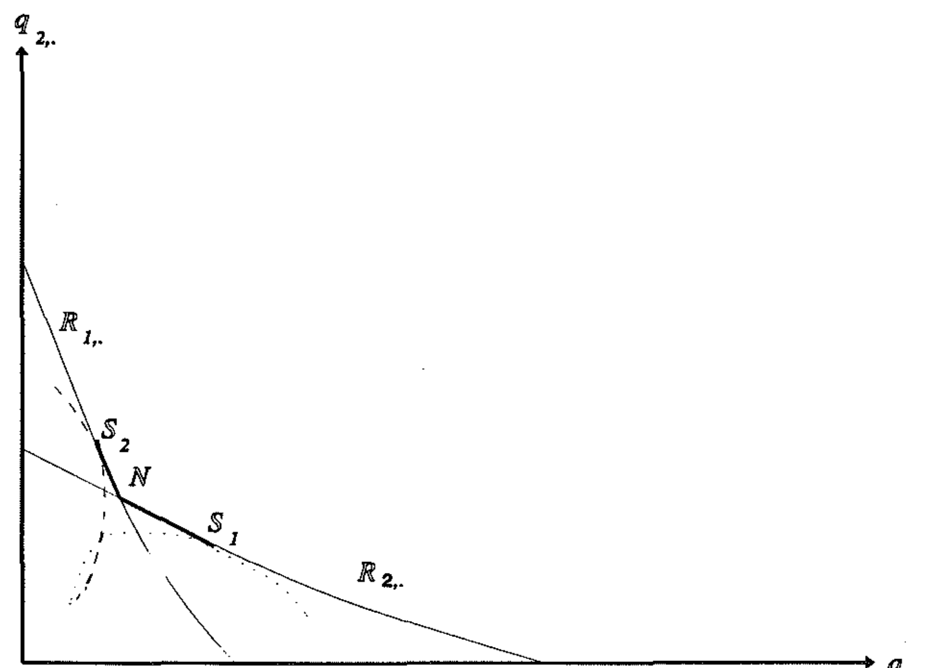
Saloner (1987) shows that under assumptions that normally lead to uniqueness of equilibrium, the existence of two production periods where first period productions become common knowledge in between the two periods leads to a continuum of possible equilibria. Saloner uses a parable, the origin of which is attributed to J.W. Friedman: Two competing fishermen go out in their boats and bring their catches back to the shore. There they observe the catch of their rival, and are allowed to go on another fishing trip before the market opens. Saloner does not use the term "inventory", but one would hope that first-period fish are stored in a cool place until the market opens! Storage costs are assumed zero and marginal cost

[9] In our model this should read "with deviations from $\underline{S}_i$."

constant. The inverse demand is assumed to satisfy all conditions for unique and stable single period Cournot equilibrium to occur.

Under these conditions, the fact that first period productions become common knowledge is shown to imply that any point on the outer envelope of the reaction functions between (and including) the two lowest Stackelberg equilibria may be a subgame perfect Nash equilibrium. Nothing more specific can be said. The range of possible equilibria is illustrated in Figure 2.

Figure 2 A Continuum of Equilibria



Allaz (1991) studies a model where each firm has two decision variables in the first period: futures and inventories. Futures are shown to serve a strategic purpose much in the same way as do inventories. Thus even under perfect foresight, firms could take positions on the futures markets in order to affect the outcome of the sales game in the second period. Furthermore it is shown that if futures trading is costless, strategic behaviour will be carried out using futures whilst cost minimization is achieved by means of inventories.

A study which is somewhat more remotely related to ours is that of Saloner (1986). In fact, in a certain sense, it turns our model on the head. Production only takes place in the first period, but sales can take place in both periods. The price in the second period is, however, determined by technical terms (obsolescence) and treated as a parameter. The assumption is that the firms can sell whatever they like at this technical price. The decisions of the first period are assumed to be made sequentially: Initially firms choose outputs, which then become common knowledge. Thereafter the sales decisions are made.

The model exhibits two interesting features: First, since the discounted second period price net of inventory cost is assumed to be less than (constant) marginal cost, supply of an additional unit for sale in the first period for a given level of production is less costly[10] than an additional unit of production. For a given production this allows the firms to pursue a more aggresive strategy on the market, thereby shoving the reaction function outwards. This more aggressive reaction function however experiences a kink at the actual production. The subgame perfect Nash equilibrium is the same as the equilibrium of the one shot game with the important qualification that conjectures are locally consistent which one may find a nice feature of the model. On the other hand, equilibrium inventories are zero so the only effect of introducing them as a strategic decision variable is to render conjectures consistent.

Second, it is shown that if one firm has a first mover advantage at the production stage, it will not achieve the Stackelberg equilibrium because it cannot commit itself to sell everything it produced in the first period. Instead it will end up at a point between the Nash and the Stackelberg equilibrium, with the Stackelberg equilibrium arising at the limit, when storage costs are prohibitive or when the decline in the price due to obsolescence is very large. Storage costs thus provide a commitment *not* to hold inventory, but rather to sell what it produced. This is a problem of time consistency: Producing the Stackelberg output does not commit the firm to actually sell it. If it reoptimizes it will sell less depending on the storage costs.

The cooperative study provided by Rotemberg and Saloner (1985) deserves a comment. Recall that if both firms share constant market shares conjectures and if Slutsky symmetry is satisfied, then the outcome will be collusive in the sense that joint profit will be maximized. In this sense, our model also encompasses the cooperative case. Rotemberg and Saloner see inventories as a means of punishing a deviator from a collusive understanding in a repeated game setting. The problem is to sustain collusion in the first place by threatening to revert to Cournot behaviour if the rival deviates. Again, inventories allow firms to employ a more aggressive strategy since costs are sunk, thus making the threat more credible. On the other hand, the higher the inventory, the higher is also the firms own incentive to cheat on the rival. The incentive constraints then imply two critical values of inventory: if the level of inventory falls between these two values, collusion can be sustained. If it falls outside the range, there is an incentive to deviate.

[10] In terms of opportunity costs.

2. Two Period Linear-Quadratic Oligopoly with Stocks

In this section, a closed form of the strategic stocks is found for an n-firm oligopoly. We parametrize the model by postulating homogeneous goods, linear demand and quadratic cost functions. By setting $n = 2$, an example of the general model of Section 1 is achieved. The Cournot, Bertrand/competitive and cooperative equilibrium stocks are calculated in a closed form. I show how stocks vary with the level and the slope of demand, the cost of storage, the number of firms and the concavity of the cost function.

2.1 The Parametrized n-person Model

I retain the notation of Section 1 with suitable extension of the definitions to a general oligopoly. The set of players is $N \equiv \{1,2, ..., n\}$. The inverse demand function is the same in both periods and linear in total sales:

$$p_{\cdot,t} = \alpha - \beta q_{\cdot,t} \quad \text{with } q_{\cdot,t} \equiv \sum_{\forall i \in N} q_{i,t} \,. \tag{50}$$

Cost functions are identical across all firms and periods and quadratic in current production:

$$C_{i,t} = \tfrac{1}{2}\gamma\, x_{i,t}^2 \quad , \gamma > 0. \tag{51}$$

Storage costs are linear in the amount stored and identical across firms:[11]

$$I_{i,1} = \iota\, s_{i,1}| \,. \tag{52}$$

Discounting is ignored: $\delta_i = 1$ for all i in N. We thus have a completely symmetric oligopoly with profit functions:

$$\begin{aligned} \pi_{i,1} &= p_{\cdot,1}q_{i,1} - C_{i,1} - I_{i,1} \\ \pi_{i,2} &= p_{\cdot,2}q_{i,2} - C_{i,2} \qquad \forall i \in N \\ \pi_{i,\cdot} &\equiv \pi_{i,1} + \pi_{i,2} \,. \end{aligned} \tag{53}$$

[11] This formulation is an approximation of (12-14). The second derivative is zero and though continuous, the function is not differentiable at $s_{i,1} = 0$, so care has to be taken to use the right derivative around zero. Alternatively on can think of the formulation as one in which $\underline{S}_i = 0$, for all i, in which case (52) is just a linear storage cost curve.

2.2 Cournot Conjectures[12]

The Period-2 Sales game:

The first order conditions can be written

$$\alpha 1' - (\beta M + \gamma E)\vec{x}_{\cdot,2} - \beta M \vec{s}_{\cdot,2} = 0', \tag{54}$$

where M is an $n \times n$ matrix with 2's in the diagonal and 1's elsewhere, $1'$ is an $n \times 1$ vector of 1's and E is the $n \times n$ identity matrix. $\vec{x}_{\cdot,2}$ and $\vec{s}_{\cdot,2}$ are the $n \times 1$ vectors of $x_{i,2}$'s and $s_{i,2}$'s respectively. Let $\Delta \equiv (\beta - \gamma)(\beta + \gamma + \beta n)$. $(\beta M + \gamma I)^{-1}$ is then an $n \times n$ matrix with $(\gamma + \beta n)/\Delta$ in the diagonal and $-\beta/\Delta$ off the diagonal. Manipulating the first order conditions leads to the following period-2 production decisions:

$$x_{i,2} = \frac{\alpha}{\beta(n-1) + \gamma} - \frac{\beta}{\beta + \gamma} s_{i,1} - \frac{\beta\gamma}{\Delta} s_{\cdot,1}, \tag{55}$$

from which it is seen that an increase in own stocks decreases period-2 production at least twice as much as an increase in rivals' stocks:

$$\frac{dx_{i,2}}{ds_{i,1}} = \frac{-\beta(\beta(n+1) + 2\gamma)}{\Delta} < \frac{dx_{i,2}}{ds_{j,1}} = \frac{-\beta\gamma}{\Delta}, \quad \forall i,j \in N,\ i \neq j . \tag{56}$$

Substituting (55) into the profit function (53) and taking the derivative with respect to the firms own decision variable, $s_{i,1}$, the marginal unit of inventories is seen to affect optimal second period profits by

$$\frac{d\pi^*_{i,2}}{ds_{1,1}} = \frac{\gamma}{\Delta^2}\Big[(\gamma+\beta)(\gamma+2\beta)(\gamma-n\beta)\alpha - \beta\gamma[(\beta-\gamma)^2+\beta\gamma(n+1)+2\beta^2 n]s_{\cdot,1} - \beta^2[(\beta-\gamma)(1+n\gamma)-(\gamma+\beta n)(1+n\beta)]s_{i,1}\Big]. \tag{57}$$

The Period-1 Production and Storage Game:

Subtracting the first order condition for $s_{i,1}$ from that for $q_{i,1}$ we get an expression similar to (49) that can be rearranged to yield

[12] The duopoly model with Cournot conjectures, linear demand and quadratic costs provides the basis for the model in Chapter 2.

$$x_{i,1} = \frac{1}{\gamma}\left[\frac{d\pi^{*}_{i,2}}{ds_{i,1}} - \iota\right] = \frac{1}{\gamma}\left[\mu_{\alpha}\alpha - \mu_i s_{i,1} - \mu_{\bullet} s_{\bullet,1} - \iota\right], \tag{58}$$

where μ_{α}, μ_i and $\mu_{\bullet}$ are the respective coefficients to α, $s_{i,1}$ and $s_{\bullet,1}$ in (57). Plugging this expression into the first order condition for $x_{i,1}$ and rearranging gives an equation in the $s_{i,1}$'s:

$$\eta_{\alpha}\alpha - \beta n\iota = \eta_2 s_{\bullet,1} - \eta_1 s_{i,1} \tag{59}$$

where the η's are the resulting coefficients:

$$\begin{aligned} \eta_{\alpha} &\equiv (\beta(n+1) + \gamma)\mu_{\alpha} - \gamma \\ \eta_i &\equiv \beta\gamma - \gamma\mu_i + \beta \\ \eta_{\bullet} &\equiv \beta\gamma - \gamma\mu_{\bullet} + \beta(\mu_i + (n+1)\mu_{\bullet} \ . \end{aligned} \tag{60}$$

The n equations (59) define a system in the $s_{i,t}$'s similar to (54):

$$(\eta_{\alpha}\alpha - \beta n\iota)I = \overline{M}\vec{s}_{\bullet,1} \tag{61}$$

where $\overline{M}$ is an $n\times n$ matrix with $\eta_i + \eta_{\bullet}$ on the diagonal and η_i off the diagonal. $\overline{M}^{-1}$ is an $n\times n$ matrix with $\frac{\eta_{\bullet} + (n-1)\eta_i}{\eta_{\bullet}(\eta_{\bullet} + n\eta_i)}$ on the diagonal and $\frac{-\eta_i}{\eta_{\bullet}(\eta_{\bullet} + n\eta_i)}$ off the diagonal. The solution to (61) is readily found to be

$$\begin{aligned} s^{*}_{i,1} &= \frac{\eta_{\alpha}\alpha - \beta n\iota}{\eta_i + n\eta_{\bullet}} \\ &= \frac{\Delta}{\beta}\,\frac{\gamma(\gamma + 2\beta)(\gamma - n\beta)\alpha - n\beta\Delta\iota}{\Xi} \qquad \forall i \in N, \end{aligned} \tag{62}$$

where

$$0 < \Xi \equiv 1 - \gamma\left[(n+1) + n\gamma(\beta+\gamma+\beta n)\left((\beta-\gamma)^2 - \beta\gamma(n-1) - 2\beta^2 n\right) - \beta(\beta n-\gamma)\left((\beta+\gamma)(1+\gamma n) - (\gamma+\beta n)(1-\beta n)\right)\right]. \tag{63}$$

Note that the first part of (62) could be found directly from (59) by postulating that since the oligopoly is completely symmetric, we must have $s_{i,1} = s_{\bullet,1}/n$, $\forall i \in N$. The strategic stocks are seen to depend positively on the demand intercept:

$$\frac{ds_{i,1}^{*}}{d\alpha} = \frac{\Delta\gamma(\gamma + \beta n)(\gamma + 2\beta)}{\Xi} > 0 \tag{64}$$

and negatively on the cost of storage:

$$\frac{ds_{i,1}^{*}}{d\iota} = \frac{-\beta n \Delta^2}{\Xi} < 0. \tag{65}$$

2.3 Bertrand or Competitive Conjectures

The Period-2 Sales Game:

The firms' production is optimal when price equals marginal cost or

$$\gamma x_{i,2} = p_{\bullet,2} \tag{66}$$

so total supply is

$$q_{\bullet,2} = np_{\bullet,2}/\gamma - s_{\bullet,1} \tag{67}$$

whilst demand is found by inverting the inverse demand curve (50). The market clearing price is then

$$p_{\bullet,2} = \gamma \frac{\alpha - \beta s_{\bullet,1}}{\gamma + \beta n} \quad , \tag{68}$$

which depends negatively on stocks. Interpreted as Bertrand price setting this means that the behaviour will be more aggressive and competition fiercer since the cost of producing inventories is sunk (as is the cost of storage). In other words, the marginal cost of supplying a unit from inventories is zero. Interpreted as competitive behaviour according to which the firms supposedly are price takers, (68) in principle just gives the market clearing price but in the period before they know that they can affect prices through inventories. This is somewhat paradoxical and one may therefore favour the Bertrand explanation, in which case (68) is the subgame perfect price.

Equilibrium profits depend on stocks as follows:

$$\frac{d\pi_{i,2}}{ds_{i,1}} = \frac{\gamma}{\gamma + \beta n}\left[\left(1 - \frac{\beta}{\gamma + \beta n}\right)(\alpha - \beta s_{\bullet,1}) - \beta s_{i,1}\right] . \tag{69}$$

The derivative is positive in symmetric equilibrium unless firms' inventories are very large compared to demand in which case a corner solution obtains anyway.[13] Note that the derivative (69) goes to zero as the number of players goes to infinity: In perfect competition where players form an atomistic supply side, there is no scope for strategic stocks.

The Period-1 Production and Storage Game:

The first order conditions for period 1 require that the marginal cost of storage should equal the marginal gain from inventories or $\iota = d\pi^*_{i,2}/ds_{i,1}$. From this we get optimal inventories:

$$s^*_{i,1} = \frac{1}{\gamma\beta}\frac{(\gamma - \beta(n-1))\gamma\alpha - (\gamma + \beta n)^2\iota}{(\gamma - \beta(n-1))n - (\gamma - \beta n)} \tag{70}$$

that again depend positively on demand (α) and negatively on the marginal cost of storage (ι).

2.4 Cooperation

If the firms cooperate they will not hold inventories in our model: $s^*_{i,1} = 0$. This is so since demand and cost functions are the same in the two periods and since storage is a costly undertaking. Everybody[14] is better off not engaging in the costly and unproductive storage activity. This result highlights two important points: First, if the firms are involved in Cournot or Bertrand competition then the possibility of carrying stocks leads to lower profits than if storage were not possible. The strategic weapon cannot be ignored once it is there and so competition gets fiercer than would otherwise be the case leading to a prisoners' dilemma and an inferior equilibrium. Second, in the cooperative equilibrium of Rotemberg and Saloner's (1985) repeated game (*cf.* Section 1.5), inventory is held in order to make the threat fiercer. This effect is lost in our simple two period model where cooperative behaviour is imposed exogenously on the model. In other words, our model does not explain how cooperation comes about.

---

[13] To be precise: if $s_{i,1} < \frac{n\alpha}{\beta}\frac{\gamma - \beta(n-1)}{\gamma(n-1) - \beta n} \approx \frac{\alpha}{\beta}\left(1 - \frac{\beta(n-2)}{\gamma - \beta}\right) \quad \forall i \in N$, then the derivative (69) is positive, but in the converse case prices are negative.

[14] *i.e.* among producers - not among consumers!

3. Conclusion

We set up a fairly general model of two period duopoly and showed how inventories can serve a strategic purpose. With Bertrand or Cournout conjectures, inventories provide a vehicle for committing to sell more in the second period. This renders competition fiercer and in the end the rivals end up in a suboptimal equilibrium as in the prisoners' dilemma. The strategic effect of inventories depends crucially on three factors:

1) convexity of the production cost function;

2) the cost of storage;

3) the competitive regime as expressed by the conjectures.

We discussed four different such regimes: Competition, Bertrand price setting, Cournot quantity setting and Cooperation.

We then proceeded to give a closed form example of strategic inventories in a parametrized model with linear, homogeneous demand and quadratic cost functions. While this set-up is more special, the model was rendered more general by treating an n-firm oligopoly rather than only duopoly. We found strategic inventories to be a linear function of the level of demand and of the unit cost of storage, and in addition to depend on the number of firms, the slope of the demand function and the convexity of the cost function. This applies to both Bertrand (competitive) price setting and to Cournot quantity setting. A cooperative oligopoly would avoid the cost of storage and not keep inventories for strategic purposes.

In the parametrized model with identical cost and demand functions in the two periods, the only reason to hold inventories is strategic. This therefore nicely illustrates that it is a costly option for non-cooperative rivals but also an option that they cannot afford to ignore once it is there. Cooperative players will ignore it.

CHAPTER 2:

Oil Futures and Strategic Stocks at Sea

Chapter Outline

ABSTRACT: A theoretical model explaining the determination of prices in the markets for North Sea crude oil is set up. Three markets are analysed in a three-stage game in which market concentration increases by each stage: In the first stage, the International Petroleum Exchange is modeled as a thick futures market. This market is also used to hedge against the uncertain outcome of the 15-Day forward market, modeled in the second stage. There, a small club of traders enter futures contracts knowing that this will affect the storage decision and thereby the spot price profile. The third stage models the spot-market as a two-period duopoly with inventories. The strategic effect of, and interaction between, inventories and futures positions is investigated.

0. Introduction

In his *The World Price of Oil* (1976), Henk Houthakker wondered why there existed no futures market for oil. Oil is a relatively standardized commodity that is stored under the ground or the sea by Mother Nature and storable above the ground and at sea. A standardized futures contract is easy to design. Let us quote (p. 2):

> "For oil, there has been nothing like the Chicago wheat market, where prices are set daily by the offers of producers and the bids of consumers, with considerable participation by merchants and speculators.
>
> Why does petroleum lack such a central market? In common with most commodities that are traded on futures markets, petroleum is storable. While not as homogeneous as copper, it is not more heterogeneous than wheat, and a serviceable standard contract would not be hard to design. Although transportation costs are relatively more important than for most centrally traded commodities, this would not seem an insuperable obstacle either. Perhaps the main reason for the failure of a central market is that for many years the industry has been dominated by integrated companies that handle oil from the well to the gasoline pump. Merchants, brokers, and other intermediaries are relatively unimportant; as a result, arm's length transactions have traditionally been less prevalent in petroleum than in many other raw materials. The significance of this point is that the integrated companies appear to be losing much of their control over crude oil, so that arm's-length transactions will become more common. In due course, an central market may emerge."

In the eighties, Houthakker's forecast became reality. In the US, the Nymex crude oil contract for West Texas Intermediate (WTI) was successfully launched in 1983, as a result of the increased volatility of oil after the Iranian revolution. In the meantime, North Sea oil was discovered. The need for "forward" trading of Brent oil was felt for the same reason and led (around 1986) to a standardized contract for Brent crude oil, on what began to be called the "Brent 15-Day market". Initially, the pairwise contracts were "forward" in the strict sense of the word, that is, the particular seller had to deliver to his particular trading partner. Gradually, this market developed into a futures market, that is, a market where a large proportion of the trade was for hedging and speculation purposes only. On top of it, the International Petroleum Exchange (IPE) launched - with mixed success - a classic crude oil futures contract[1], copied from the Nymex contract, in 1983. Today, two futures markets for Brent crude exist on top of each other: the 15-Day and the IPE. The latter has a centralized open outcry exchange, a clearing house and a growing number of participants, including the majors, oil traders and locals. The Brent 15-Day market, to the contrary, is in the hands of a club of producers and traders.

The emergence of these markets motivates the present chapter, which tries to simultaneously model the oligopolistic interplay of the majors who produce North Sea oil, for whom stocks at sea have a strategic role, and the presence of two futures markets for Brent crude on which these majors are also in a strategic situation, given their size and small

[1] Its modifications are described in Phlips (1992).

number. To put is simply, this paper is an attempt to combine the modelling of strategic stocks from Chapter 1 with an effort to give a game-theoretic explanation of how two futures markets for the same natural resource work when the corresponding spot market is controlled by a few producers. The basic approach is the one developed by Phlips and Harstad (1990 and 1991).

We confine the analysis to the minimum oligopoly model that allows for strategic interaction between suppliers, that is, a duopoly. The game proceeds in three stages. In the first stage, the producers meet with an anonymous futures market, which is supposed to mimic the characteristics of the IPE. Having determined the futures price and positions, we model the 15-Day market as the second-stage subgame, where oil companies trade bilateral futures contracts among themselves and with a speculator. These 15-Day contracts will depend on the positions already taken on the IPE. In the third stage, the companies then play a two-period extraction game. They each have a known total to extract over the two periods, but can use stocks of crude oil at sea and the extraction profile to manipulate prices so as to render their IPE and 15-Day positions more profitable. This comes about because the maturity futures price is taken to be the second-period spot price.

Time plays a role only in the two periods, 1 and 2, of the extraction game of the third stage. The two previous stages modelling the two futures markets allow the producers to precommit themselves to certain sales (or purchases) in the second period of the third stage. One can imagine that the IPE (in the first stage) opens and closes in period -0 leaving enough time for the producers to sell or buy IPE futures as they wish. When the IPE has closed, in period +0 the three market participants agree on their 15-Day contracts (in the second stage). When the 15-Day market closes, the extraction game of the third stage takes place. At this stage, the two producers decide on the optimal extraction, sales and storage profiles over the two periods. Only after this third stage will the stochastic demand be revealed and it is in this sense that time does not play a role in stages one and two modelling the futures markets: No relevant information is revealed during or between these two stages. This assumption allows us to focus on the strategic and speculative motives of futures market trading. We shall return to the interpretation of time in relation to the real world markets in section seven.

The rest of the chapter is organized as follows: Section 1 introduces the necessary notation for the spot market, which is described as a fairly traditional duopoly extracting an exhaustible resource. Everything is kept nicely linear or quadratic but the results carry through if these assumptions are substituted by appropriate convexity conditions. The following four sections then unravel the game backwards. Section 2 solves the Cournot duopoly for the extraction game. The production schedule is unaffected by the futures markets while the sales depend on the net position taken by the producers on the two futures markets. To close this gap, inventories must necessarily depend on these same net positions. Section 3 analyses the strategic use of stocks by changing the basic model of the spot market slightly so as to highlight the strategic effect of holding inventories. These strategic inventories are also found to depend on the futures positions as well as on the producers' beliefs regarding the future spot price. Section 4 then recedes to the second stage, modelling the 15-Day market. The set of contracts that are mutually beneficial to the market participants (*i.e.* the (bilateral) core) is characterized. This will be a large set depending on price expectations and the futures positions taken on the IPE. Section 5 takes us back to the first stage where the producers' optimal positions on the IPE are modelled. Given the potential multiplicity of outcomes of the imperfectly organized 15-Day market, the IPE serves as a vehicle for the producers to speculate and hedge not only against the uncertainty on the spot market but also against the non-uniqueness of the 15-Day market.

1. Setting the Stage: Duopoly, Storage and Futures

Since the play of all three stages focuses on the cash market we shall begin our story there. Two companies, A and B, supply a homogeneous product, crude oil, to a spot market. Extraction takes place in two periods, *1* and *2*, but each extractor $(i=A,B)$ has a known maximum $\bar{x}_i$.[2] First-period production can however be stored in tankers rather than being sold immediately.

Let the first (alphabetical) subscript refer to the companies and the second (numeric) to the period. q denotes sales and x production. We impose the constraints

$$q_{i1} + q_{i2} = x_{i1} + x_{i2} = \bar{x}_i, \qquad i = A,B \tag{1}$$

on total production and sales. Stocks at sea, s_i, are produced but not sold in the first period, that is, for $i = A,B$

$$s_i \equiv x_{i1} - q_{i1} \tag{2}$$

$$q_{i2} = x_{i2} + s_i, \tag{3}$$

so that the tankers have to be delivered in the second period.

Production and storage are not costless activities. We assume that production costs, C, are convex and, for simplicity, that they are quadratic in the number of barrels pumped:

---

[2] In the North Sea, the maximum production of Brent blend is approximately 21 million barrels per month for technical reasons. Each producer is entitled to a certain predetermined fraction of this according to ownership and participation. Thus, the maximum production is for this reason given in the short run. What determines capacity in the long run is a different matter which will not be pursued here.

$$C_{it} = \frac{1}{2} c_i x_{it}^2 \qquad (i = A,B; \quad t = 1,2). \tag{4}$$

The cost of storage, I, is taken to be linear in the number of barrels stored,

$$I_i = j s_i \qquad (i = A,B; \ j \geq 0), \tag{5}$$

and the profit of company i thus becomes

$$\Pi_i = p_1 q_{i1} + p_2 q_{i2} - (C_{i1} - C_{i2} + I_i) - (p^F - p_2) N_i \ , \tag{6}$$

where p_1 and p_2 are the spot prices in the two periods and p^F and N_i are the futures price and the net position resulting from the two first stages of the game.

To complete the description of the spot market, we propose linear inverse demand functions[3] to determine the spot prices:

$$p_t = \alpha - b q_t \quad \text{where} \quad q_t = q_{At} + q_{Bt}, \quad t = 1,2 \ , \tag{7}$$

and where the strength of demand, α, is a stochastic variable which is perceived by the agents as being distributed normally with unknown mean $\hat{\alpha}$ and known variance $Var(\alpha) = 1$:

$$\alpha \sim N(\hat{\alpha},1) \ . \tag{8}$$

The participants on the futures markets hold different beliefs $E_i(\alpha)$ on $\hat{\alpha}$: they assign probability one to their own belief (their subjective probability distribution) and probability zero to the beliefs of the other players. We are thus in a situation of inconsistent prior beliefs

[3] In earlier papers [Brianza, Phlips and Richard (1990); Phlips and Harstad (1991 and 1990)] the specification $q_t = \alpha' - \beta p_t$, leading to the inverse demand curve

$$p_t = \frac{\alpha'}{\beta} - \frac{1}{\beta} q_t \ ,$$

was chosen. While this gives us a natural interpretation of α' as the level of demand, the notational simpler version with $\alpha = \alpha'/\beta$ and $b = 1/\beta$ has been chosen here. The 'strength' of demand α and the 'level' of demand α' are related through $\alpha = \alpha' b$.

in the sense of Selten (1982).[4]

The risk averse producers' *ex ante* payoffs are modelled according to the mean-variance model:

$$W_i = E_i(\Pi_i) - \frac{K_i}{2} Var(\Pi_i) , \tag{9}$$

where K_i measures constant absolute risk aversion[5].

The expected profit is readily found by taking i's expectation of (6). The variance of the profit can be shown to be

$$Var(\Pi_i) = (\bar{x}_i - N_i)^2 . \tag{10}$$

We shall leave the modelling of the 15-Day market and the IPE for Sections 4 and 5. Here we follow many a good theatre play and first offer the solution to the final stage in which the strategic effects of stocks are highlighted.

2. Stage Three: Cournot Duopoly and the Extraction Game

First note that the variablitiy of profits arises from the unhedged part $(\bar{x}_i - N_i)$ of total extraction. It is unaffected by the time profile of sales or production. In order to decide on these time profiles, the companies maximize W_i with respect to x_{i1}, q_{i1}, x_{i2}, q_{i2} and s_i, which amounts to maximizing expected profits subject to the constaints (1-3).

Manipulation of the first order conditions leads to the following extraction schedule:

---

[4] It will take us too far from the main argument to discuss the origin of the differences in beliefs. We take it for an empirical fact that traders act on differences in subjective probability distributions (agreeing to disagree) even if they hold the same information (which they thus interpret differently), optimism and pessimism being inexplicable motivations for trading. Everyday futures and financial markets are crowded with busily trading agents that share the same information, thereby rejecting any zero-trade theorem. On optimistic and conservative standards of behaviour, see Greenberg (1990). On rejecting the rational expectations hypothesis, see Lovell (1986). On informational differences leading to different positions, see Stein (1987) and p. 27.

[5] This follows automatically if the underlying preferences are represented by a utility function that is exponential in profits and if profits follow the normal distribution. See for example Newbery and Stiglitz (1981), pp. 74-75. This utility function is used repeatedly in Chapter 3 and, especially, Chapter 4.

$$x_{i1} = \frac{c_2}{c_1 + c_2}\bar{x}_i - \frac{2j}{c_1 + c_2} = \delta_1 \bar{x}_i - \frac{2j}{c_1 + c_2}, \quad i = A, B \tag{11}$$

$$x_{i2} = \frac{c_1}{c_1 - c_2}\bar{x}_i - \frac{2j}{c_1 + c_2} = \delta_2 \bar{x}_i + \frac{2j}{c_1 + c_2} \quad i = A, B \tag{12}$$

where the first term represents cost smoothing:

$$\delta_1 = \frac{c_2}{c_1 + c_2} = \frac{c_2 x}{c_1 x + c_2 x} = \frac{\frac{1}{2}c_2 x^2}{\frac{1}{2}c_1 x^2 + \frac{1}{2}c_2 x^2} \tag{13}$$

$$\delta_2 = \frac{c_1}{c_1 + c_2} = \frac{c_1 x}{c_1 x - c_2 x} = \frac{\frac{1}{2}c_1 x^2}{\frac{1}{2}c_1 x^2 + \frac{1}{2}c_2 x^2} \tag{14}$$

The chosen quadratic form of the cost function implies that comparing the total cost of producing a given quantity, x, in one period to the total cost of producing the same amounts in both periods is equivalent to comparing the marginal cost, $c_t x$, in this period to the sum of marginal costs and again equivalent to just comparing increments, c_t, in the marginal cost. At any rate, (11) has the natural interpretation that the higher the cost of production in the second period is (the higher δ_1), the more should be produced in advance in period 1. *Vice versa* for second-period production: the higher the cost of first-period production is, relatively speaking, the more of it should be postponed to the cheaper second period. The second term of (11) and (12) compares the costliness of storage to that of production. As in Chapter 1, we find that the higher the cost of storage, the less should be produced in advance and the more should be postponed to the second period. Note that the optimal extraction policy does not depend on the futures position taken.

The sales schedule for company A, say, is found to be

$$q_{A1} = \frac{1}{2}\bar{x}_A - \frac{1}{3}N_A - \frac{1}{6}N_B + \frac{1}{6}\frac{j}{b} \quad \text{and} \tag{15}$$

$$q_{A2} = \frac{1}{2}\bar{x}_A + \frac{1}{3}N_A - \frac{1}{6}N_B - \frac{1}{6}\frac{j}{b} \tag{16}$$

and its stocks of crude oil can therefore be expressed as[6]

[6] Using (2-3), (11) and (15).

$$s_A = \frac{1}{2}\Delta\delta\bar{x}_A + \frac{1}{3}N_A - \frac{1}{6}N_B - \frac{1}{6}\gamma j, \quad \text{where} \tag{17}$$

$$\Delta\delta = \delta_1 - \delta_2 = \frac{c_2 - c_1}{c_1 - c_2} \quad \text{and} \tag{18}$$

$$\gamma = \frac{1}{b} + \frac{12}{(c_1 + c_2)}. \tag{19}$$

That sales depend on the net short futures positions in the manner shown in equations (15-16) is in perfect concordance with the findings in Phlips and Harstad (1991) (see their equation (6)). In addition, we find that sales depend on the term j/b which relates the marginal cost of storage to the slope of the inverse demand curve, and thus to the marginal revenue of the operation. We see that higher marginal cost of storage yields lower stocks and thus lower second-period sales. First-period sales will be correspondingly higher.

The size of inventory holdings is given by (17). The first term shows the cost-smoothing purpose of stocks. If $c_2 > c_1$ so that $\Delta\delta > 0$, a part of second-period sales should be produced in the first, less costly period and stored. This operation should take the cost of storage into account, as indicated by the fourth term, from which it is seen that higher cost of storage lowers the optimal level of stocks, as should be. The second term of (17) shows that if producer A has precommitted herself by taking a short position on the futures markets $(N_A > 0)$, then part of this quantity will optimally be met by sales from stocks. If the rival takes a similar position $(N_B > 0)$, this lowers the profitability of the operation, thereby also lowering the optimal level of stocks for company A. Aggregate stocks $(s_A + s_B)$ depend positively on the aggregate net position $(N_A + N_B)$.

Before discussing the strategic use of stocks, let us examine the expected spot prices:

$$E(p_1) = \hat{p} + \frac{1}{6}b(N_A + N_B) - \frac{1}{3}j \tag{20}$$

$$E(p_2) = \hat{p} - \frac{1}{6}b(N_A + N_B) - \frac{1}{3}j \tag{21}$$

$$\hat{p} \equiv \hat{\alpha} - \frac{1}{2}b(\bar{x}_A + \bar{x}_B). \tag{22}$$

$\hat{p}$ is the price that would be obtained in both periods if the futures markets and the storage facility did not exist. Net short futures positions $(N_i > 0)$ represent a binding commitment to furnish the second-period cash market with a certain amount of crude, and to the extent that this amount is already sold at the price p^F, the companies have a common interest to manipulate prices by lowering p_2 (and therefore raising p_1).

3. The Strategic Use of Stocks

The strategic use of stocks is in many ways similar to that of futures positions. Indeed, by undertaking a larger production in the first period and storing some of it for sale in the second period, the companies incur the extra cost of production and the cost of storage in the first period. In the second period these costs are sunk and the supply from stocks is costless. Stocks therefore represent a credible commitment to raise second-period sales, thereby offering a potential position as a Stackelberg leader.

In a more general model, the strategic effect of inventories would imply that total production were increased and that the average price were lower than if storage were impossible[7]. The producers are trapped in a suboptimal Nash equilibrium because both are trying to position themselves as leaders. Most of this strategic effect is so far lost in our model since the oil companies are required to produce and sell a given total (see (1)). What is left is the direct effect of storage on prices, whereby higher cost of storage discourages holding inventories and leads to higher immediate sales.

To illustrate this point we change the model slightly for the sake of this section. It is crucial that the companies be allowed to determine the size of total production. We therefore abandon the second equality of constraint (1) and only require total production to be sold by the end of period 2. To keep things simple we also assume that demand only materializes in the second period, which implies that everything that is produced in the first period is stored:

$$x_{i1} = s_i \tag{23}$$

$$q_{i2} = x_{i1} + x_{i2} \, . \tag{24}$$

The situation is more complicated than before since uncertainty now not only pertains to the average price that results from a known total production, but involves the determination of this total itself.

Maximization of W_i with respect to x_{i2} results in the following sales for company A:

[7] See Allaz (1991) and Chapter 1.

$$q_{A2} = s_A + \frac{1}{D}\Big\{b\big[\Delta(\alpha) - b(s_B - s_A) - b(N_B - N_A) - K_B(s_B - N_B) - K_A(s_A - N_A)\big] \\ - (b + c_2 + K_B)\big[E_A(\alpha) - b(s_A + s_B) - (b + K_A)(N_A - s_A)\big]\Big\} \tag{25}$$

where $D = (2b + c_2 + K_A)(2b + c_2 + K_B) - b^2 > 0$, $\Delta(\alpha)$ is the difference in opinion $(E_A(\alpha) - E_B(\alpha))$ on $\hat{\alpha}$, $(s_B - s_A)$ is the difference in stock levels and $(N_B - N_A)$ is similarly the difference in futures positions. $(s_i - N_i)$ represents the unhedged part of stocks, so that the two last terms in the first square brackets represent the different risk valuation of unhedged stocks.

The effect of an increase in the stocks of company A is seen to increase its own sales by

$$\frac{dq_{A2}}{ds_A} = \frac{1}{D}c_2(2b + c_2 + K_B) > 0 , \tag{26}$$

whereas it will decrease company B's sales by

$$\frac{dq_{B2}}{ds_A} = -\frac{1}{D}c_2 b < 0 , \tag{27}$$

so that total production and thus total sales are increased

$$\frac{dq}{ds_A} = \frac{1}{D}c_2(b - c_2 - K_b) > 0 , \tag{28}$$

while the spot price at maturity is lower,

$$\frac{dp_2}{ds_A} = -\frac{1}{D}bc_2(b + c_2 + K_B) < 0 , \tag{29}$$

than would otherwise be the case.

The expressions for the sales quantities (25) can be substituted back into the payoff functions (9) which then can be maximized to find the subgame-perfect first-period production (or, equivalently, stocks). These will depend on the beliefs about the strength of demand and on the futures positions taken at an earlier stage:

$$s_A = \nu_A E_A(\alpha) + \phi_A E_B(\alpha) + \psi_A N_A + \omega_A N_B \tag{30}$$

$$s_B = \phi_B E_A(\alpha) - \nu_B E_B(\alpha) + \omega_B N_A - \psi_B N_B \tag{31}$$

where the coefficients ν_i, ϕ_i, ψ_i and ω_i depend on the original parameters of the model, b, c_1,

c_2, K_A and K_B, as reported in the appendix.[8]

The main difference in comparison with the solution to the model where total production is given (see (11-19)) is that the beliefs regarding the strength of demand enter the production/storage/sales decisions explicitly (see (25)). Intuition suggests that production depends positively on the agent's own expectations regarding the strength of demand (*i.e.* $\nu_i > 0$) and negatively on the rival's (*i.e.* $\phi_i < 0$) but this may not be true for all parameter constellations. Another difference is that risk aversion affects decision making at the production level. This was not the case when total production was given, because in that case uncertainty regarding the strength of demand only affected profits through the average price $\bar{p}$ that pertained to the given quantity (see (20-22)), whereas here the total quantity can also be chosen freely. This renders the decision process much more complex. In a sense, the effect of changing the model to allow for strategic stocks has been to replace $\bar{x}_A$ and $\bar{x}_B$ in (11-16) by $E_A(\alpha)$ and $E_B(\alpha)$ in (25) and (30-31), and to render the corresponding coefficients more complex.

Summarizing, we have found that the production, sales and storage decisions of the two rivals of the duopoly cash market depend on the futures positions taken in advance. If there *is* a constraint on total production, then this enters the decisions, and storage mainly serves as a cost smoothing device. If the decision is *not* present (or not effective) then the agents' beliefs regarding the strength of demand are important, and the inventories serve a strategic purpose (in addition to the cost smoothing). The two companies are trapped in the non-cooperative Nash equilibrium of the prisoner's dilemma type: The effect of simultaneously positive stocks of crude oil will raise total production (30-31) and lower the spot price at maturity (29). The cooperative outcome, obtained if both agree not to store, would yield higher profits, but then each company has an incentive to defect (26), thereby making the other firm worse off (27). Since both firms simultaneously try to act as (Stackelberg) leaders by precommitting themselves to higher sales through inventories, a sub-optimal Nash equilibrium results.[9]

We now move on to describe the futures markets and find the futures positions that are so crucial for the sales, storage and production schedules.

---

[8] We subsume the cost of storage in the cost of production of the first period. Since first-period production is not sold, the storage and the production decisions are essentially one and the same.

[9] The incentive to try to act as a leader depends crucially on the convexity of the cost functions (*i.e.* on c_1 and c_2) as also noted by Arvan (1985). The more convex the cost function is (the higher c_1 and c_2 are), the higher are the strategic inventories. For this reason it is difficult to separate the cost smoothing and the strategic motive for holding stocks.

4. Stage Two: The 15-Day Market

As mentioned there are two futures markets for North Sea crude oil: the IPE and the 15-Day market. The 15-Day market, which is the subject of this section, is characterized by there being only few big participants in the market. The functioning of the market is described in Mabro *et al.* (1986), ch. 12, in Phlips (1992) and in Chapter 4. It is "an informal, self-regulating club of North Sea producers, oil traders, refiners and brokers, each of which is in the market for a variety of reasons" (Mabro *et al.* (1986), p. 169). These market participants bargain about standardized forward contracts via telephone and the agreed-upon contracts are then telexed. This bargaining can only be realistically modelled at the cost of a considerable increase in complexity[10]. We shall refrain from this here and simplify the model to highlight the rôle played by our two producers, A and B.

To be specific we assume that only three agents take part in the 15-Day market: A, B and an oil trader, S, who is not interested in the crude oil *per se* but only in buying (selling) crude oil on the 15-Day market with the expectation of being able to sell (buy) it on the spot market at a higher (lower) price. In the literature, such agents are referred to as speculators or, in a somewhat imprecise choice of words, as arbitrageurs. Here we call them speculators.

So far the two futures markets have been described crudely by an aggregate futures price p^F and the quantity, N_i, by which a producer i $(i = A,B)$ is net short. We now have to be more specific. Let F_i denote the net position of player i on the IPE and f_i the net position on the 15-Day market, *i.e.*

$$N_i = F_i + f_i \qquad i = A, B, S. \tag{32}$$

We concentrate on the 15-Day positions in this section. Each of these f_i's results from potentially four different contracts

$$f_i = f'_{ij} - f'_{ji} + f'_{ik} - f'_{ki}, \quad \text{where } i = A, B, S,\ i \neq j,\ j \neq k,\ i \neq k \text{ and all } f' \geq 0 \tag{33}$$

and where f'_{ik} signifies that i is selling to k. In our model, agents are trading on differences in beliefs regarding the future spot price. This being the case it will never happen that two contracts are set up where an agent i both sells to and buys from another agent at different prices, *cf.* Lemma 1 of Chapter 4. If the futures price is equal in the two contracts it will only be the net position that matters anyway. We shall therefore let the signed quantity

[10] This is the topic of Chapter 4.

$$f_{ij} = f'_{ij} - f'_{ji} \tag{34}$$

denote the net position of i *vis-à-vis* j. $f_{ij} > 0$ implies that i takes a short position (sells futures) in the contract with j. We take this to mean that $f'_{ij} = f_{ij}$ and $f'_{ji} = 0$. Then note that $f_{ji} = -f'_{ij} = -f_{ij}$, so that net positions on the 15-Day market can be written:

$$f_A = f_{AB} - f_{AS} \tag{35}$$

$$f_B = -f_{AB} - f_{BS} \tag{36}$$

$$f_S = -(f_{AS} - f_{BS}) \tag{37}$$

where $f_i > 0$ if the player is net short in total on the market. Note that $f_A + f_B + f_S = 0$.

The three agents, A, B, and S, trade on the expected spot price in period 2 and they all know that the spot price is formed according to (21-22). That is to say, they know that the producers' net positions will influence the spot price or, to put it more polemically, that the producers will manipulate the time profile of spot prices, raising the first-period price where all sales are cash and lowering the second-period price where part of the sales have been made in advance (assuming that they are net short, *i.e.* that $N_A + N_B > 0$). All this is common knowledge for the market participants and it will not discourage the speculator from trading on the market. What the market participants do not agree upon is the mean value of α. Each has his firm opinion $E_i(\alpha)$ on this. They thus expect three different mean spot prices $E_i(p_2) = p_2^i$ according to their different beliefs on $\hat{\alpha}$ and according to (21-22). It follows from (8) that they perceive p_2 as being normally distributed with mean p_2^i and unit variances.

4.1 The speculator's payoff

We assume that the speculator is risk neutral[11] and thus maximizes expected profit:

$$E_S(\Pi_S) = \left(p_2^S - p_{AS}^{15}\right) f_{AS} - \left(p_2^S - p_{BS}^{15}\right) f_{BS} \tag{38}$$

where p_{iS}^{15} is the price of the contract that S agrees upon with i. A contract is fully described by the pair (p_{ij}^{15}, f_{ij}). The expected profit can be rewritten using (21-22):

[11] To assume that he is risk averse does not change the analysis substantially.

$$-E_S(\Pi_S) = \left[\hat{p}_S - \tfrac{1}{6}bF + \tfrac{1}{3}j\right]f_S - p_{AS}^{15}f_{AS} + p_{BS}^{15}f_{BS} \tag{39}$$

where

$$\hat{p}_S = E_S(\hat{\alpha}) - \frac{1}{2}b\left(\bar{x}_A + \bar{x}_B\right) \tag{40}$$

and $F = F_A + F_B$ is the producers' joint net position on the IPE.

4.2 The two producers' payoffs

The producers' payoffs from trading on the futures markets are found by substituting (15-16) and (20-22) into (9) and subtracting the profit that would have been made on the spot market in the absence of the futures markets. This leaves us with the following expressions

$$W_A^{FUT} = (p^F - \hat{p}_A)N_A + \frac{1}{18}\left[b(N_A + N_B)^2 - j(7N_A - 2N_B)\right] - \frac{K_A}{2}(\bar{x}_A - N_A)^2 \tag{41}$$

$$W_B^{FUT} = (p^F - \hat{p}_B)N_B + \frac{1}{18}\left[b(N_A + N_B)^2 - j(2N_A - 7N_B)\right] - \frac{K_B}{2}(\bar{x}_B - N_B)^2. \tag{42}$$

The first term on the r.h.s. of these expressions illustrates the immediate gain from having a futures market: p^F is the average futures price and $\hat{p}_i$ is the spot price that i would expect in the absence of futures markets. If $p^F > \hat{p}_i$ then A should, *ceteris paribus*, take a net short position $N_A > 0$. The third term indicates the advantage of hedging the uncertain profit. In the case of complete hedging $\bar{x}_A = N_A$ and the term is effectively optimized. The second term shows the strategic effect on the spot market profits of having a futures market. This term occurs because taking futures positions provides a credible vehicle for precommitment of spot sales or purchases. Note that, for example, A's payoff varies proportionally with

$$\left(N_A - N_B\right)^2 - \frac{j}{b}\left(7N_A - 2N_B\right) \tag{43}$$

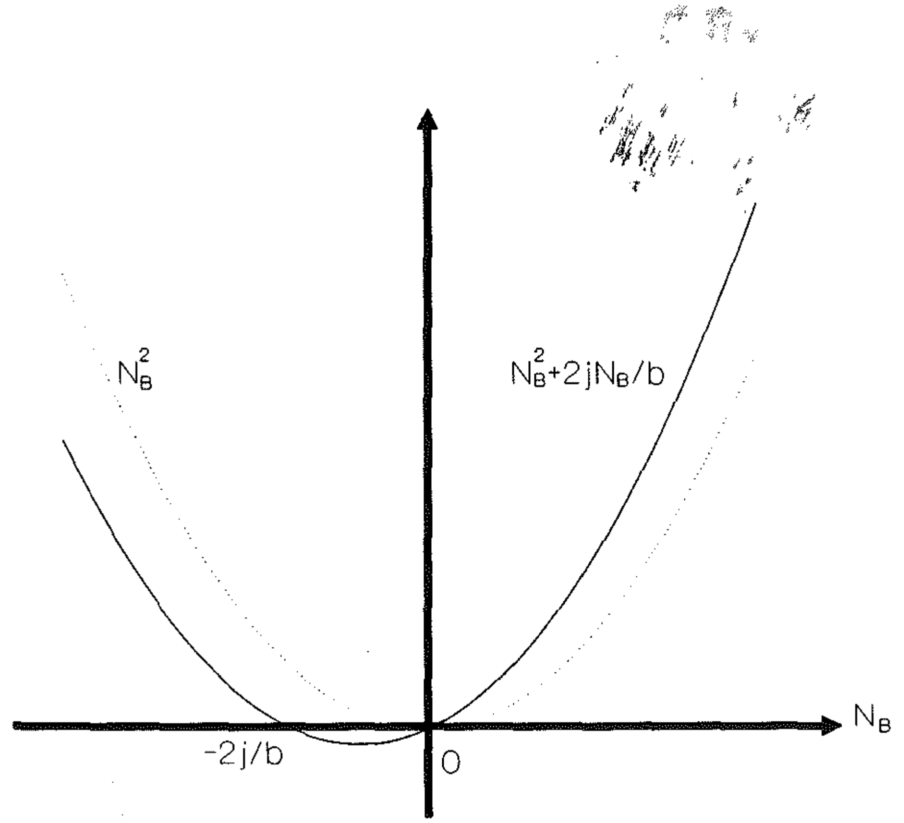
and that the futures markets affect A's output even if he does not participate, namely via the rival's net position on these markets. Indeed, if $N_A = 0$, W_A^{FUT} varies in proportion to

$$N_B^2 + 2\frac{j}{b}N_B\ . \tag{44}$$

B's net position will always affect A's payoff positively in the absence of costly storage ($j=0$).

This comes about because a producer with zero position responds to the rival's position by shifting sales to the period with a higher price. If storage is possible at a cost this is no longer true for all values of N_B, as illustrated by Figure 1.

Figure 1 The effect of a unilateral futures position on the rival's profit



If B is long in the range

$$0 > N_B > -2\frac{j}{b} \qquad (N_A = 0), \tag{45}$$

then A's optimal response on the spot market is to shift sales (see (15-16)) with N_B/b from period 1 where the price is then low (see (20)) to period 2 where the price is higher (21). Since the optimal extraction is independent of the futures positions (11-12), this operation can only be done by means of increased inventories (17). But increasing stocks has a cost, and so for small, long positions, B forces A to incur a loss since the cost of increasing the stocks dominates the extra expected revenue.[12]

The producers of course realize the strategic interdependence of their futures positions - an interdependence that stems from the effect of these positions on the equilibrium spot

[12] Note, however, that this result may be sensitive to the specification.

Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

prices and quantities. Indeed, when taking positions on the futures markets, the producers weigh the speculative, the strategic and the hedging motives according to (41-42).

The payoffs of the two futures markets can be split up according to the market that gives rise to them, or, put differently, we find the subgame perfect equilibria by maximizing the payoff of the second stage, the 15-Day market, for given positions on the IPE, before solving the first stage.

The ex-ante payoff stemming from the 15-Day market (including this market's effects on the spot market payoff) can be written:

$$\begin{aligned} W_A^{15} = {} & (p_{AB}^{15} - \hat{p}_A)f_{AB} + (p_{AS}^{15} - \hat{p}_A)f_{AS} - \tau_1(f_{AS} - f_{BS})^2 \\ & - \tau_{2A}f_{AS} + \tau_3 f_{BS} + \tau_{4A}f_{AB} - \frac{K_A}{2}(f_{AB} + f_{AS})^2 , \end{aligned} \tag{46}$$

$$\begin{aligned} W_B^{15} = {} & (\hat{p}_B - p_{AB}^{15})f_{AB} + (p_{BS}^{15} - \hat{p}_B)f_{BS} + \tau_1(f_{AS} + f_{BS})^2 \\ & + \tau_3 f_{AS} + \tau_{2B}f_{BS} - \tau_{4B}f_{AB} - \frac{K_B}{2}(f_{BS} - f_{AB})^2 , \end{aligned} \tag{47}$$

where

$$\tau_1 = \frac{1}{18}b > 0 \tag{48}$$

$$\tau_{2i} = \frac{1}{9}bF - \frac{7}{18}j + K_i(\overline{x}_i - F_i); \quad i=A,B \tag{49}$$

$$\tau_3 = \frac{1}{9}(bF - j) \tag{50}$$

$$\tau_{4i} = -\frac{1}{2}j - K_i(\overline{x}_i - F_i); \quad i = A,B. \tag{51}$$

In the following, superscript "15" is used to indicate that the variable has to do with the 15-Day market.

4.3 The contract curves

Now, what can we say about the solutions to the 15-Day stage, without imposing further structure on the game? We require that any contract (p_{ij}^{15},f_{ij}) belongs to the contract curve between i and j, *i.e.* it must be true that $MRS_i(p_{ij}^{15},f_{ij}) = MRS_j(p_{ij}^{15},f_{ij})$, where the marginal rates of substitution are given implicitly by (46-47) for the producers and (39) for the speculator. For example, the contract between the two producers should obey

$$MRS_A^{AB} = \frac{\dfrac{\partial W_A^{15}}{\partial p_{AB}^{15}}}{\dfrac{\partial W_A^{15}}{\partial f_{AB}}} = \frac{\partial f_{AB}}{\partial p_{AB}^{15}} = \frac{\dfrac{\partial W_B^{15}}{\partial p_{AB}^{15}}}{\dfrac{\partial W_B^{15}}{\partial f_{AB}}} = MRS_B^{AB}. \tag{52}$$

These requirements are fulfilled if $f_{ij} = 0$, that is, if the two participants do not enter a contract. Less trivially, if $f_{ij} \neq 0$, (52) leads to

$$(K_A - K_B)f_{AB} + K_A f_{AS} - K_B f_{BS} = (\hat{p}_B - \hat{p}_A) + K_A(\bar{x}_A - F_A) - K_B(\bar{x}_B - F_B) \tag{53}$$

$$-K_A f_{AB} - (K_A + \tfrac{2}{9}b)f_{AS} - \tfrac{2}{9}bf_{BS} = (\hat{p}_A - \hat{p}_S) - K_A(\bar{x}_A - F_A) - \tfrac{1}{18}(bF + j) \tag{54}$$

$$-K_B f_{AB} - \tfrac{2}{9}bf_{AS} + (K_B - \tfrac{2}{9}b)f_{BS} = (\hat{p}_S - \hat{p}_B) - K_B(\bar{x}_B - F_B) - \tfrac{1}{18}(bF + j), \tag{55}$$

which apply to the contracts (p_{AB}^{15}, f_{AB}), (p_{AS}^{15}, f_{AS}) and (p_{BS}^{15}, f_{BS}), repectively. Phlips and Harstad (1991) use an equivalent approach and find a similar system of equations.[13] The main differences are that the producers' positions on the IPE, F_A and F_B enter on the r.h.s. because they too can be used for hedging purposes; that their joint position on the IPE (F) enters because of the strategic effect of futures on the spot market (a feature which the two markets share); and that the cost of storage shows up since taking a futures position (on either market) changes optimal inventories (17).

The equational system (53-55) cannot be solved to obtain a unique set of three 15-Day contracts. There are two reasons for this. The first reason has to do with the fact that individual rationality points to a range of possible prices depending on the quantities. The second reason arises because (53-55) only determine the net positions f_A, f_B and f_S uniquely, not the decomposition on the three quantities f_{AB}, f_{AS} and f_{BS}. We discuss each point in turn.

4.4 Individual rationality

First, note that the futures prices, p_{ij}^{15}, do not appear in (53-55). All we can say about these prices is that they should be individually rational according to (39) and (46-47). Individual rationality simply states that any contract should contribute a non-negative amount to each player's payoff since a zero contribution can always be achieved by *not* entering the

[13] See their equations (13). In Chapter 4, Section 4.1 we discuss a simplified version of their model.

contract. This requirement puts the following bounds on the prices: For the (p_{AB}^{15}, f_{AB}) contract:

$$\hat{p}_A - K_A\left(\bar{x}_A - F_A - f_{AS} - \tfrac{1}{2}f_{AB}\right) - \tfrac{1}{2}j \\ \leq p_{AB}^{15} \leq \\ \hat{p}_B - K_B\left(\bar{x}_B - F_B - f_{BS} + \tfrac{1}{2}f_{AB}\right) + \tfrac{1}{2}j \quad \text{for } f_{AB} > 0; \tag{56}$$

for the (p_{AS}^{15}, f_{AS}) contract:

$$\hat{p}_A - \left(\tfrac{K_A}{2} - \tfrac{1}{18}b\right)f_{AS} - \tfrac{1}{9}b\left(F + f_{BS}\right) - K_A\left(\bar{x}_A - F_A - f_{AB}\right) + \tfrac{7}{18}j \\ \leq p_{AS}^{15} \leq \\ \hat{p}_S - \tfrac{1}{6}b\left(F + 2f_{AS} - f_{BS}\right) + \tfrac{1}{3}j \qquad \text{for } f_{AS} > 0; \tag{57}$$

and finally for the (p_{BS}^{15}, f_{BS}) contract:

$$\hat{p}_B - \tfrac{1}{9}b\left(F + f_{AS}\right) + \left(\tfrac{K_B}{2} - \tfrac{1}{18}b\right)f_{BS} - K_B\left(\bar{x}_B - F_B + f_{AB}\right) + \tfrac{7}{18}j \\ \leq p_{BS}^{15} \leq \\ \hat{p}_S - \tfrac{1}{6}b\left(F - f_{AS} + 2f_{BS}\right) - \tfrac{1}{3}j \qquad \text{for } f_{BS} > 0. \tag{58}$$

Note that the above three inequalities are true under the condition that f_{AB}, f_{AS} and f_{BS} be strictly positive. For each of them the inequality is reversed if the sign of the quantity is reversed. Observe also the following: The range of futures prices that are acceptable for the two players involved in a contract depends on the futures positions taken by the producers on the IPE, on the quantities of the other 15-Day contracts, on the expected spot price and on the cost of storage. This range may or may not be empty, depending on the values of these variables. In case the range is empty, this corresponds to the players agreeing on the contracts $(p_{ij}^{15}, f_{ij}) = (0,0)$ which is always a possibility. This just means that these two players do not find it profitable to enter a contract.

Equation (56) says that if A is selling to B ($f_{AB} > 0$) then A will require a higher minimal sales price

1) the more she has already hedged on the IPE (F_A),
2) the more she has already hedged in a contract with the speculator (f_{AS}),
3) the higher is her spot price expectation ($\hat{p}_A$), and
4) the higher is the cost of storage (j).

Buyer B will accept a higher maximal buying price

1) the more he went short on the IPE (F_B),
2) the more he went short in a contract with S (f_{BS}),

3) the higher is his expected spot price ($\hat{p}_B$), and

4) the higher is the cost of storage (j).

Note that the cost of storage raises the minimal selling price and the maximal buying price by the same amount, thus preserving the spread. Further, observe that the minimal selling price is increasing and the maximal buying price is decreasing in the quantity f_{AB} of the contract. This ensures that the individually rational contract must involve a finite quantity. Indeed, it can be shown that the contract must satisfy *either*

$$0 < f_{AB} < \frac{2}{K_A - K_B}\left(\hat{p}_B - \hat{p}_A + K_A(\bar{x}_A - F_A - f_{AS}) - K_B(\bar{x}_B - F_B - f_{BS})\right) \tag{59}$$

in the case of A selling to B, *or*

$$0 > f_{AB} > \frac{2}{K_A + K_B}\left(\hat{p}_B - \hat{p}_A + K_A(\bar{x}_A - F_A - f_{AS}) - K_B(\bar{x}_B - F_B - f_{BS})\right) \tag{60}$$

when B is selling to A.

The analysis of the contracts where the producers are selling to the speculator (57-58) follows the discussion above with the following two qualifications: Firstly, increased storage costs increase the producers' minimal selling price a bit more than the speculator's maximal buying price. Secondly, if $K_i/2 < \tau_I$, the minimal selling price will be falling in the size of the contract. However, the speculator's maximal buying price will decrease at a much faster rate in f_{iS}, thus still ensuring finite positions. The equivalents of (59-60) are

$$0 < |f_{AS}| < \frac{\left[\hat{p}_S - \hat{p}_A - \tau_I(F + f_{BS}) + K_A(\bar{x}_A - F_A - f_{AB}) - \frac{1}{18}j\right]}{\frac{K_A}{2} + \frac{5}{18}b} \tag{61}$$

$$0 < |f_{BS}| < \frac{\left[\hat{p}_S - \hat{p}_B - \tau_I(F + f_{AS}) + K_B(\bar{x}_B - F_B + f_{AB}) - \frac{1}{18}j\right]}{\frac{K_B}{2} + \frac{5}{18}b} \tag{62}$$

Equations (56-62) all imply individual rationality by securing that each contract adds positively to the payoffs. Each player can, however, secure himself a minimum payoff without participating in the 15-Day market at all. Individual rationality then implies that payoffs should satisfy:

$$W_A \geq \tau_1 f_{BS}^2 + \tau_3 f_{BS} \qquad (63)$$

$$W_B \geq \tau_1 f_{AS}^2 + \tau_3 f_{AS} \qquad (64)$$

$$E_S(\Pi_S) \geq 0 \ . \qquad (65)$$

Note, though, that the minimal payoff of producer i may become negative if the rival k takes a position $f_{kS} \in \,]0, -2(F + j/b)[$, where $]$ and $[$ are used to indicate that the interval is open.

4.5 Net 15-Day positions

The second observation on the system (53-55) is that it exhibits linear dependence in f_{AB}, f_{AS} and f_{BS} implying that the quantities of the contracts are indeterminate. This can be remedied partly by manipulating the system (53-55) using the identities (35-37) to find the *net* positions:

$$f_A^* = \Theta\Big[K_B(\hat{p}_S - \hat{p}_A) - \tfrac{2}{9}b(\hat{p}_B - \hat{p}_A) - (K_A K_B + \tfrac{2}{9}bK_A)(\bar{x}_A - F_A) - \tfrac{2}{9}bK_B(\bar{x}_B - F_B) - \tfrac{1}{18}K_B(bF + j)\Big] \qquad (66)$$

$$f_B^* = \Theta\Big[K_A(\hat{p}_S - \hat{p}_B) - \tfrac{2}{9}b(\hat{p}_B - \hat{p}_A) - (K_A K_B + \tfrac{2}{9}bK_B)(\bar{x}_B - F_B) - \tfrac{2}{9}bK_A(\bar{x}_A - F_A) - \tfrac{1}{18}K_A(bF + j)\Big] \qquad (67)$$

$$f_S^* = -\Theta\Big[K_A(\hat{p}_S - \hat{p}_B) - K_B(\hat{p}_S - \hat{p}_A) + K_A K_B(\bar{x}_A + \bar{x}_B - F) - \tfrac{1}{18}(K_A + K_B)(bF + j)\Big] \qquad (68)$$

$$\Theta^{-1} = K_A K_B + \tfrac{2}{9}b(K_A - K_B) \qquad (69)$$

Net positions are thus uniquely determined by the parameters of the spot market, the speculations about the spot price and the positions already taken on the IPE during the first stage, but the distribution on individual contracts cannot be found.

4.6 The bilateral core

The set of contracts described by equations (53-69) consists of those contracts that are individually and coalitionally rational at the same time and is thus basically the (bilateral) core. By substituting (66-69) into (53-55) the core can be described as the set of contracts $\{(p_{AB}^{15},f_{AB}),(p_{AS}^{15},f_{AS}),(p_{BS}^{15},f_{BS})\}$ that satisfy the following equations simultaneously:

$$f_{AB} + f_{AS} = f_A^* \tag{70}$$

$$-f_{AB} + f_{BS} = f_B^* \tag{71}$$

$$-(f_{AS} + f_{BS}) = f_S^* \tag{72}$$

$$g - \tfrac{1}{2}K_A f_{AB} < p_{AB}^{15} < g + \tfrac{1}{2}K_B f_{AB} \qquad \text{if } f_{AB} > 0 \tag{73}$$

$$g' - \tfrac{1}{2}(K_A - \tfrac{1}{9}b)f_{AS} < p_{AS}^{15} < g'' + \tfrac{1}{6}bf_{AS} \qquad \text{if } f_{AS} > 0 \tag{74}$$

$$g' - \tfrac{1}{2}(K_B - \tfrac{1}{9}b)f_{BS} < p_{BS}^{15} < g'' + \tfrac{1}{6}bf_{BS} \qquad \text{if } f_{BS} > 0 \tag{75}$$

where

$$g = \Theta\left[\tfrac{2}{9}b(K_B\hat{p}_A - K_A\hat{p}_B) + K_AK_B\hat{p}_S\right] - \tfrac{2}{9}\Theta bK_AK_B(\bar{x}_A + \bar{x}_B - F) - \tfrac{1}{18}\Theta K_AK_B(bF - j) + \tfrac{1}{2}j \tag{76}$$

$$g' = \Theta\left[\tfrac{1}{9}b(K_B\hat{p}_A + K_A\hat{p}_B) + \left(K_AK_B + \tfrac{1}{9}(K_A + K_B)\right)\hat{p}_S\right] -\tfrac{1}{9}\Theta bK_AK_B(\bar{x}_A - \bar{x}_B - F) + \tfrac{1}{18}\Theta\left(K_AK_B - \tfrac{1}{9}(K_A + K_B)\right)(bF + j) - \tfrac{1}{9}bF \tag{77}$$

$$g'' = \Theta\left[\tfrac{1}{3}b(K_B\hat{p}_A - K_A\hat{p}_B) + \left(K_AK_B - \tfrac{1}{9}(K_A - K_B)\right)\hat{p}_S\right] - \tfrac{1}{3}\Theta bK_AK_B(\bar{x}_A - \bar{x}_B - F) + \tfrac{1}{54}\Theta b(K_A + K_B)(bF + j) - \tfrac{1}{6}bF \tag{78}$$

The linear system for the quantities (70-72) has one degree of freedom implying that the quantities will be uniquely determined once one quantity is known. (Fix for example f_{AB}. Then (70) uniquely determines f_{AS}, whilst (71) uniquely determines f_{BS}. These quantities will be consistent with (70).)

Once the quantitites are known, (73-75) determine the ranges of prices acceptable.

Note that each of the inequalities is reversed if the corresponding quantity becomes negative. Also note that the lower bound appears to be decreasing and the upper bound increasing in the quantity. This somewhat surprising result has to be interpreted in the light of the constraints on quantities: If a contract becomes larger (say f_{AB}) and the price "spread" therefore increases then the other contracts become correspondingly smaller (f_{AS} decreases and f_{BS} increases one-to-one with f_{AB} but f_{AB} and f_{BS} have opposite signs in B's payoff function).

The three ranges that bound prices are determined by the g's given in equations (76-78). The first term in each of these consists of a weighted average of the three agents' expected prices, $\hat{p}_A$, $\hat{p}_B$ and $\hat{p}_S$, the weights being functions of K_A, K_B and b. The second term depends on the degree to which the producers have hedged their production on the IPE. In case they hedged fully on the IPE ($\bar{x}_A + \bar{x}_B = F$), the term drops out. In case they hedged less on the IPE, the bounds on prices will be lower, implying that it will be more costly for the producers to hedge on the 15-Day market. The last two terms of the g's depend on the producers' IPE position and on storage costs and basically incorporate the strategic effects of futures and storage on spot prices.

This core is never empty and always non-unique (in fact: infinitely large). We will not elaborate on the solution to the 15-Day stage here (that is done in Chapter 4), but simply note that, *a priori*, there is no means to pointing out a subset of the core as being more likely as a solution.

5. Stage One: The International Petroleum Exchange

The IPE, as described in the introduction, is a formal futures market with an open outcry exchange and a clearing house. We therefore assume that a single price, p^{IPE}, will be determined on this market. The question then is what positions F_A and F_B the producers should take.

The expected payoffs arising from the IPE are[14]

[14] Using (41), (42), (32) and (9). Recall that F is the joint position on the IPE. The τ's are given by (48-51).

$$W_A^{IPE} = (p^{IPE} - \hat{p}_A)F_A - \tau_1 F - \tfrac{1}{18} j(7F_A - 2F_B) \\ - \tfrac{1}{2}K_A(\bar{x}_A - F_A)^2 + E_A(W_A^{15}(F_A,F_B)) \tag{79}$$

$$W_B^{IPE} = (p^{IPE} - \hat{p}_B)F_B - \tau_1 F - \tfrac{1}{18} j(2F_A - 7F_B) \\ - \tfrac{1}{2}K_B(\bar{x}_B - F_B)^2 + E_B(W_B^{15}(F_A,F_B)) \tag{80}$$

where $E_A(W_A^{15}(F_A,F_B))$ and $E_B(W_B^{15}(F_A,F_B))$ are the payoffs that A and B expect to gain from the 15-Day market depending on what solution they expect to prevail. This could be formalized by claiming that they hold one subjective possibility distribution, h_i, on what the size of, say, f_{AB} will be and other subjective distributions, h_i^{AB}, h_i^{AS}, h_i^{BS} on what the price will be, conditional on the quantities. These probability distributions could be thought of as representing the way in which agents think the 15-Day market works.

A's expected value of f_{AB} will then be

$$E_A(f_{AB}) = \int_{-\infty}^{\infty} f_{AB} h_A \, df_{AB} \tag{81}$$

and her expectations with respect to the two other 15-Day quantities therefore

$$E_A(f_{AS}) = \int_{\infty}^{\infty} [f_A^*(F_A,F_B) - f_{AB}] h_A \, df_{AB} \tag{82}$$

and

$$E_A(f_{BS}) = \int_{-\infty}^{\infty} [f_B^*(F_A,F_B) + f_{AB}] h_A \, df_{AB} \; . \tag{83}$$

Her 15-Day price expectations will be,

$$E_A(p_{AB}^{15} \mid f_{AB}) = \int_{g-\frac{K_A}{2}f_{AB}}^{g-\frac{K_A}{2}f_{AS}} p_{AB}^{15}\, h_A^{AB}\, dp_{AB}^{15}\,, \tag{84}$$

$$E_A(p_{AS}^{15} \mid f_{AB}) = \int_{g-\frac{1}{2}(K_A-\frac{1}{9}b)(f_A'-f_{AB})}^{g+\frac{b}{6}(f_A'-f_{AB})} p_{AS}^{15}\, h_A^{AS}\, dp_{AS}^{15}\,, \tag{85}$$

and

$$E_A(p_{BS}^{15} \mid f_{AB}) = \int_{g-\frac{1}{2}(K_B-\frac{1}{9}b)(f_B'+f_{AB})}^{g''+\frac{b}{6}(f_A'-f_{AB})} p_{BS}^{15}\, h_A^{BS}\, dp_{BS}^{15}\,. \tag{86}$$

Note that the net positions on the 15-Day market f_i' depend on the position taken on on the IPE, so that the limits of the integrals in (84-86) depend on F_A and F_B.

The expected value to A of the 15-Day transactions can be found by taking the expectation of (46) using (81-85) and (66-69). This will give the payoff $E_A(W_A^{15}(F_A,F_B))$ which occurs in (79). A similar exercise can be done for B by substituting $(h_A,h_A^{AB},h_A^{AS},h_A^{BS})$ by $(h_B,h_B^{AB},h_B^{AS},h_B^{BS})$ in (81-86) and taking the expectation of (47). This will identify (W_A^{IPE},W_B^{IPE}) in (80). If the two sets of subjective probability distributions are common knowledge (to be precise: if A knows B's and B knows A's probability distributions and this is commonly known) and if the two players take the IPE positions simultaneously, a subgame perfect Nash equilibrium will result where A maximizes W_A^{IPE} with respect to F_A and B W_B^{IPE} with respect to F_B.

Note that when the two producers take positions on the IPE they do this for the same three motives as applies to the 15-Day market: A speculative, a strategic and a hedging motive are at play in (79-80) that readily compares to (46-47). In fact, the four first terms of (79-80) capture exactly these effects. But according to the fifth term, the producers realize that the position they take will have an effect on the unknown solution to the 15-Day game. So this adds another motive for trading on the IPE to the three well-known: optimization taking the non-unique outcomes of the 15-Day game into account.

6. The Model and the Oil Markets

The model that was put forth above is based on several abstractions compared to the real world. One important abstraction is related to the treatment of time. The model can be seen as a snapshot in a sense that will soon be made precise. The real world is rather a continuous series of rolling and overlapping snapshots. This section discusses how the model could be interpreted and what would be necessary to create a moving picture.

The interpretation of our game in terms of real world actions starts with the observation that the 15-Day market requires the producers to give the purchasers a 15-day notice before delivery. This notice specifies a three day range within which delivery will take place. The oil traded thereby goes from being undated to being dated: from being traded forward to being traded spot. These fifteen days correspond to our period 1 of the extraction game since the future and the forward markets for oil to be delivered in this period are closed. Cargoes that are lifted but not sold during this period represent an increase in stocks that can be sold in period 2.

In order to make period 2 of the extraction game correspond to the real world we adopt a strong abstraction: Assume that all cargoes of a given month are lifted within a given delivery range. In other words, the delivery month is collapsed into this range. Assume for concreteness that all September oil is to be delivered between the first and third of September of a given year. Period 2 of the production game could be interpreted as this period (September 1-3). This then would correspond to the maturity of the 15-Day contract. The fifteen days prior to September 1st (*i.e.* August 16th-31st) would constitute period 1.

The two futures markets are collapsed into points in time. We can interpret this by assuming that on the 15th of August the 15-Day market opens. This is technically the last day that forward oil can be traded for delivery on September 1-3. So the market closes before August 16, and will not reopen until period 2 where, by definition, the maturity price is identical to the spot price.

The IPE closes the trading of paper barrels referring to a given delivery month in the middle of the previous month, that is to say, well before the 15-Day market stops trading this delivery month. In fact, what is the first forward month on the IPE is normally the second forward month of the 15-Day market, and the maturity price of the IPE is the 15-Day price on the closing day. The latter fact is ignored in the model and the maturity price on both futures markets is chosen to be the spot price of the second period of the extraction game. The first feature is however modelled by letting the IPE precede the 15-Day game. In other words, we assume that the IPE opens and closes only once prior to maturity, on the 14th of August.

In reality, of course, the oil markets are much more dynamic than our model allows for. First, a sequence of extraction games are played and stocks are increased or decreased between them. Stocks therefore serve as a state variable in a dynamic game. This may change the strategic effect of inventories since it is no longer true that everything that was produced but not sold in one period has to be sold in the next. As mentioned in Chapter 1, Rotemberg and Saloner (1985) see inventories as a means to sustain high collusive prices by threatening to flood the market if a rival deviates.

The two futures markets are treated as one-shot situations in our model. In the real world, these markets are open every day and trade different contracts (up to six months ahead) simultaneously. This implies that there may be much more dynamic interaction going on than presented here. For example, informational intricacies have been ignored by assuming that the subjective probability distributions and all strategic features are common knowledge. This leaves the difference in subjective probability distributions unexplained. A natural explanation of this involves differences in information (asymmetric information or incomplete knowledge) or optimistic/pessimistic behaviour as noted in footnote 3. A model of a futures market for a storable good explaining the reasons for existence as well as the effects of asymmetric information is found in Stein (1987). His model, however, does not analyse the strategic aspects of inventories and of futures but concentrates on risk sharing and informational externalities.

Lastly it should be noted that the structure of the three markets has been simplified in the model. Obvious examples are the lack of explicit modelling of refineries, of intermediate products and of vertical integration. We are concerned with these problems in our current research.

Appendix to Section 3: Strategic Stocks

This appendix is offered as an aid to the reader who wants to obtain a full understanding of the model demonstrating the strategic use of stocks. Recall that we made the following assumptions:

(A.1) There is no exogenous upper bound to production, *i.e.* no $\bar{x}_i$, $i = A,B$.

(A.2) There is no demand in the first period, implying that the production of the first period has to be stored, and that sales (in the second period) equal total production over the two periods, *cf.* (23-24).

The profit functions are given by

$$\Pi_A = (s_A - x_{A2} - N_A)p_2 - \frac{c_1}{2}s_A^2 - \frac{c_2}{2}x_{A2}^2 - p^F N_A\,, \tag{87}$$

$$\Pi_B = (s_B - x_{B2} - N_B)p_2 - \frac{c_1}{2}s_B^2 - \frac{c_2}{2}x_{B2}^2 - p^F N_B\,, \tag{88}$$

Note that the cost of storage is subsumed in the period 1 cost function: The decision to produce in period one is essentially the same as the decision to store, and we do not need two cost variables to describe the decision. All other assumptions remain unchanged, *i.e.* the demand curve is stochastic, linear and downward sloping (7-8) and the producers' utilities from profits follow the mean-variance model (9).

The last decision the producers take regards second-period production: They take this decision by simultaneously maximizing their payoffs (9) with respect to their respective decision variables (x_{A2},x_{B2}) taking the stocks (s_A,s_B) and the net futures positions (N_A,N_B) as given. This results in the following second-period production for A (the expression for B is similar):

$$\begin{aligned} x_{A2} = {} & \frac{b}{D}\left[\Delta(\alpha) + b(s_B - s_A) - b(N_B - N_A) + K_B(s_B - N_B) - K_A(s_A - N_A)\right] \\ & + \frac{b + c_2 + K_B}{D}\left[E_A(\alpha) - b(s_A + s_B) - (b + K_A)(N_A - s_A)\right] \end{aligned} \tag{89}$$

which is comparable to (25) and where

$$D = (2b + c_2 + K_A)(2b + c_2 + K_B) - b^2 \tag{90}$$

$$\Delta(\alpha) = E_A(\alpha) - E_B(\alpha) \tag{91}$$

The multipliers (26-28) follow directly from (89) and (29) follows with the additional use of (7).

The second-to-last decision the producers take regards how much to produce in the first period or, equivalently, how much to store. Substituting the optimized second-period productions into the payoff functions and performing a simultaneous maximization of these with respect to the stocks (taking net futures positions as given), we obtain

$$s_A = \nu_A E_A(\alpha) + \phi_A E_B(\alpha) - \psi_A N_A + \omega_A N_B \tag{92}$$

$$s_B = \phi_B E_A(\alpha) + \nu_B E_B(\alpha) - \omega_B N_A + \psi_B N_B \tag{93}$$

where

$$\nu_A = (\rho_{B1}\delta_{A\alpha} - \delta_{B1}\rho_{A\alpha})/\underline{D} \quad ; \quad \nu_B = (\delta_{A1}\rho_{B\alpha} - \rho_{A1}\delta_{B\alpha})/\underline{D} \quad ; \tag{94}$$

$$\phi_A = (\rho_{B1}\delta_{B\alpha} - \delta_{B1}\rho_{B\alpha})/\underline{D} \quad ; \quad \phi_B = (\delta_{A1}\rho_{A\alpha} - \rho_{A1}\delta_{A\alpha})/\underline{D} \quad ; \tag{95}$$

$$\psi_A = (\rho_{B1}\delta_{N_A} - \delta_{B1}\rho_{N_A})/\underline{D} \quad ; \quad \psi_B = (\delta_{A1}\rho_{N_B} - \rho_{A1}\delta_{N_B})/\underline{D} \quad ; \tag{96}$$

$$\omega_A = (\rho_{B1}\delta_{N_B} - \delta_{B1}\rho_{N_B})/\underline{D} \quad ; \quad \omega_B = (\delta_{A1}\rho_{N_A} - \rho_{A1}\delta_{N_A})/\underline{D} \quad ; \tag{97}$$

$$\underline{D} = \rho_{A1}\delta_{B1} - \rho_{B1}\delta_{A1} \tag{98}$$

$$\rho_{A1} = -(c_1 - c_2)D + c_2^2(2b + c_2 + K_B)\left(1 + \frac{b^2}{D}\right) \tag{99}$$

$$\rho_{B1} = \left(2b + K_B - c_2\frac{b^2}{D}\right)bc_2 \tag{100}$$

$$\rho_{A\alpha} = c_2(2b - c_2 - K_B)\left(1 + \frac{b^2}{D}\right) > 0 \tag{101}$$

$$\rho_{B\alpha} = -bc_2\left(1 + \frac{b^2}{D}\right) < 0 \tag{102}$$

$$\rho_{N_A} = c_2(2b + c_2 + K_B)(b + K_A)\left(1 - \frac{b^2}{D}\right) - c_2 b^2 > 0 \tag{103}$$

$$\rho_{N_B} = -bc_2(b - K_B)\left(1 + \frac{b^2}{D}\right) < 0 \tag{104}$$

$$\delta_{A1} = bc_2\left(2b - K_A - c_2\frac{b^2}{D}\right) \tag{105}$$

$$\delta_{B1} = -(c_1 + c_2)D - c_2^2(2b - c_2 + K_B)\left(1 + \frac{b^2}{D}\right) \tag{106}$$

$$\delta_{A\alpha} = \rho_{B\alpha} < 0 \tag{107}$$

$$\delta_{B\alpha} = c_2(2b + c_2 + K_A)\left(1 + \frac{b^2}{D}\right) > 0 \tag{108}$$

$$\delta_{N_A} = -bc_2(b + K_A)\left(1 - \frac{b^2}{D}\right) < 0 \tag{109}$$

$$\delta_{N_B} = c_2(2b - c_2 + K_B)(b - K_B)\left(1 + \frac{b^2}{D}\right) - c_2 b^2 > 0\ . \tag{110}$$

The signs of the parameters are indicated where possible. The sign of the most important determinant, $\underline{D}$, is however undetermined but will generally be positive. Sufficient, but not necessary, conditions for non-negativity of $\underline{D}$ are, for example, that simultaneously $c_2 > 1$ and $(c_1 + c_2)^2 D > b^2 c_2 (1 + b^2/D)$. But these requirements are not easily interpreted.

CHAPTER 3:

Oil Stocks as a Squeeze Preventing Mechanism:

Is Self-Regulation Possible?

Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

Chapter Outline

ABSTRACT: Squeezes are registered occasionally in the forward market for Brent crude oil. The squeezer accumulates forward contracts and creates artificial demand by refusing to close out, exploiting imperfections in the decentralized market clearing. The artificial demand in turn creates a price surge and the possibility of a squeeze thus introduces uncertainty about the market outcome. Squeezes therefore render the market institution less palatable to other market participants (traders and refineries) who may find other ways of accomplishing the economic functions of the forward market, so the producers have a long term interest in keeping market clearing smooth by supplying stocks to those squeezed short. The extent to which such self-regulatory stocks should be held is analysed in the context of a repeated game. Unless the probability of a squeeze is very small, self-regulation should be possible.

Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

0. Introduction

A *squeeze* is, broadly defined, a market manipulation in which a market participant obtains market power by accumulating a position in forward trading that is large compared to supply in the underlying spot market. A *corner* is defined as the acquisition of one hundred percent (or more) of the supply. Kyle (1984) distinguishes between the two concepts by arguing that corners lead to the exertion of monopoly power on the spot market whereas squeezes exploit deficiencies in the clearing of forward trading. For the purpose of this chapter the distinction is a matter of degree, and as Kyle mentions and we shall argue later, it is of little importance for results whether manipulation occurs in forward trading or on the spot market.

Squeezes and corners are concepts that are normally brought to bear on *futures* markets where they are made possible by two features of the market:

★Trading is anonymous;

★The volume of trading is huge compared to that of the underlying spot market.

These features allow the squeezer to quietly build up a substantial long position that they then refuse to close. By threatening to take delivery the squeezer drive up prices from the competitive level at which the futures were bought, thus rendering the whole operation profitable (see Anderson & Gilbert (1991) and Telser (1992)).

The same two features are also what makes futures markets an attractive way of trading by providing low transaction costs and high liquidity (*cf.* Telser (1981)), so futures exchanges have tried to regulate markets in order that squeezes not destroy these advantages. Regulatory measures include:

1) Delivery options, *i.e.* the possibility to deliver substitutes thereby expanding the size of the underlying spot market (see Duffie (1989) pp. 323-4);
2) Cash settlement, *i.e.* traders have the right (or obligation) to settle by a purely financial transaction (see Telser (1992) and Kyle (1984));
3) Position limits, *i.e.* the position of a single trader must not exceed a certain limit (See Kyle (1984));
4) Additional supply, *i.e.* somebody holds regulatory stocks that are sold in case of a squeeze.

Kyle (1984) concludes that cash settlement only transfers the problem from the futures market to the spot market in that the settlement price is related to the realized spot price and that the imposition of position limits is vulnerable to agents colluding in the squeeze. Delivery options and regulatory stocks seem to be the most effective ways of reducing squeezes in futures

Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

markets. Both expand supply and render a squeeze more costly and/or less likely.

Weiner (1992) reports on the infant oil futures markets in the United States in the 19th century. Interestingly, he notes that there were frequent squeezes even before the futures markets were organized as such:

> "While it is difficult to gauge the extent of trading of 'futures' before the advent of futures exchanges, it must have been extensive, judging from the reporting in the trade press on 'corners'." (Weiner (1992) p. 7)

This shows that the existence of a futures exchange is *not* a necessary condition for squeezes to occur. There being forward trade is, to the contrary, a necessary condition, but not all forward trading allows for this: for a squeeze to occur it has to be the case that the volume of the forward market exceeds the volume of the underlying spot market (Telser (1992)).

In 1949, Working predicted that the era of squeezes would soon be over:

> "Corners are, or were, consequences of an excessive freedom of enterprise which seems largely a thing of the past in American futures markets. The British grain trade has never permitted either corners or significant squeezes in its futures markets. Squeezes continue to occur in American futures markets, though they can and should be eliminated." (Working (1949))

But recently the organized British *forward* market for crude oil has permitted significant squeezes. On the 11th of January, 1988, the *Weekly Petroleum Argus* (WPA) reported:

> "Someone has got a lot of January Brent unsold. The name most frequently mentioned in this connection is Transworld Oil [TWO], a Netherlands based trader presided over by the buccaneering John Deuss who also owns a refinery in Philadelphia. Who if anyone is behind Mr. Deuss is a matter of speculation and *Argus* can report only that two traders out of three think there is an Arab Gulf state in the background.
>
> The story begins at the end of November when TWO begins to accumulate claims to Brent for January lifting. That seemed odd at the time. The Opec ministers were about to meet and it would have been hard to find anyone in the oil industry optimistic about the outcome. To take a long position in crude to be lifted after the ministers had finished their deliberations appeared foolhardy. [...]
>
> There was a precedent. In April TWO had succeeded in cornering 15-Day Brent for lifting at the end of the month and had collected a premium of up to a dollar and a half over dated crude. That manoeuvre was clearly profitable in itself. [...]
>
> When the manoeuvre was repeated in December the major Brent producers, Esso and Shell, let it be known that they would do all in their power to frustrate it. Some cargoes were released from their corporate systems and other similar North Sea crudes, together with Nigerian, were made available. But they failed to prevent a serious distortion from developing in the spectrum of oil prices."

This chapter is an attempt to understand the economics of self-regulating duopoly. The basic question is whether and to what extent oligopolistic producers will carry regulatory stocks in order that a squeezer not create artificial scarcity on the spot market. The last paragraph of the quotation from the WPA suggests that this type of self-regulation does not work.

Let us begin by explaining the jargon used in the quotation. First of all, "Brent" designates Brent Blend, a mixture of the production from seventeen separate oil fields in the North Sea. Much of the Arabian oil imported in Europe is priced with reference to the price of Brent, which explains why Arab states may have an interest in pushing this price up. Esso and Shell, who control the production of Brent, have a long-term interest in keeping the Brent market liquid.

On any given day, there are two prices for Brent. The "dated" price is for a cargo that is lifted or to be lifted into a vessel on a particular date: the dated price is thus what is called the spot price in standard literature. A "15-Day" price is for a "paper" cargo and can be "first-month" (for delivery on an unspecified date next month), or "second-month" (for delivery in the second month to come), etcetera. A paper cargo is a claim to 500.000 barrels of Brent to be lifted at an unspecified date in a particular month to come. Dates of lifting must be determined 15 days in advance.[1] For April liftings, these dates can thus be determined up to the 13th of April.[2] Consequently, during the first two weeks of April there exists simultaneously a 15-Day April price (for still undated April cargoes) and a dated April price. A premium for 15-Day April Brent over dated crude is thus the (positive) difference between the price of a paper April cargo and a dated April cargo.

Section 1 gives the evidence on the price distortions that resulted from the April 1987 and January 1988 squeezes and three subsequent squeezes. The distortions occurred on the 15-Day forward market and implied huge premia of first-month Brent over both second-month Brent and first-month West Texas Intermediate (WTI). The latter is an American crude that is a substitute for Brent and is traded on the New York Mercantile Exchange (NYMEX).

Section 2 sets up a model of a market with a potential squeeze. In Section 2.1 we examine the incentive to squeeze and for the producers to prevent the squeeze under the assumption that the producers coordinate their effort by setting up regulatory stocks. The opposing interests of the players are: Short-term profit to the squeezer against long-term profit to the producers. The question is whether and to what degree it pays to prevent a squeeze. In Section 2.2 we then analyse whether the oligopolists' decision to keep regulatory stocks is any different from the monopolist's (it is not) and whether this affects the scope for cooperation in the repeated game (it does).

Section 3 concludes by pondering why regulatory stocks are not held even though this study indicates they should be. The realism of the assumptions of the model is discussed, as

---

[1] To give the buyer time to charter a tanker and send it to Sullom Voe (Shetlands) where the loading takes place. Hence the name "15-Day market".

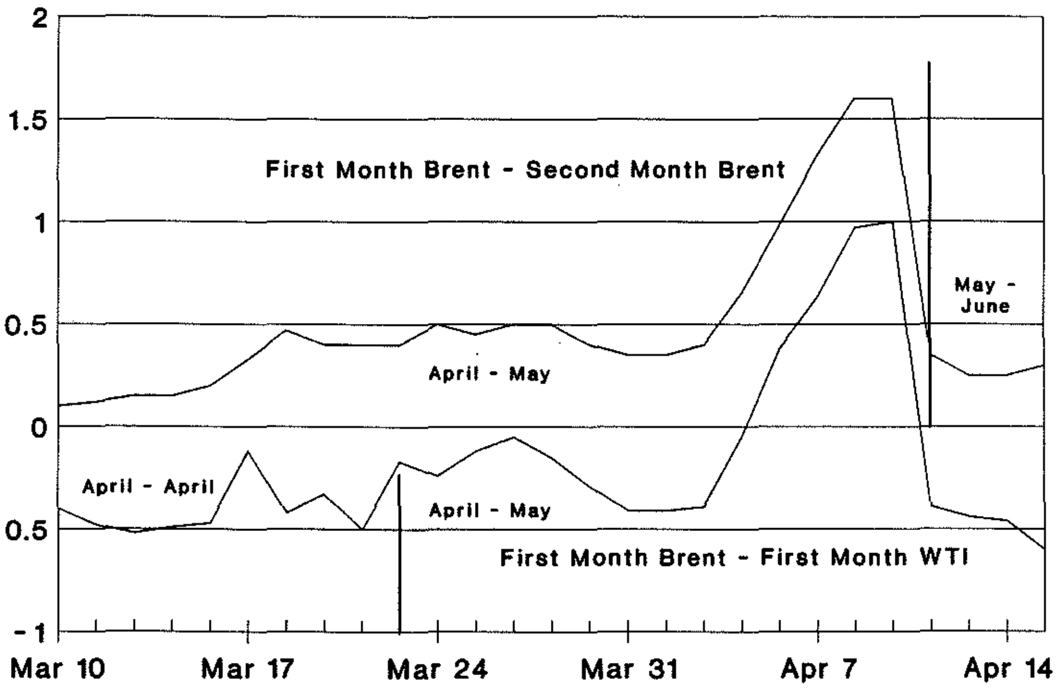
[2] Since a loading date is in fact a three-day period. With 30 days in the month, the last loading period starts April 28 and 28-15=13.

are variations of the model.

1. Squeezes on the 15-Day Brent Market

This section illustrates the price effects of the two squeezes mentioned in the introduction and gives an account of the major squeezes on the 15-Day market since then.

Figure 1: The April 1987 Squeeze
(10th of March to 15th of April 1987)



Source: Platt's and Nymex

Figure 1 covers the period from 10 March to 15 April 1987 and represents[3] the premium of first-month Brent over second-month Brent and the premium of first-month Brent over first-month WTI. From mid-March the premium of first over second-month Brent started to climb up to \$ 0.50. In the first week of April, it suddenly increased to \$ 1.60 within 5

[3] The sources are Platt's for Brent prices and Nymex for WTI prices. Note that for Brent, the months change as of the 10th of any particular month. Until 9 March, the first month is March. From 10 March to 9 April, the first month is April, and so on. For WTI, the first month is April until 20 March. As of 23 March, the first month is May.

days. That this was due to a squeeze and not to market fundamentals is confirmed by the fact that the premium of first-month Brent over first-month WTI followed the same pattern, although WTI is a close substitute to Brent.

How does such a squeeze arise? To close out a position on the 15-Day market, there are two possibilities. The first is a "bookout" whereby a number of participants agree to cancel their contracts with a cash settlement. These participants have contracts that can be arranged in a chain starting and ending with the same participant. A squeezer has never *sold* forward and thus will not appear in a bookout. The contracts that are cleared by a bookout are not pertinent to the squeeze.

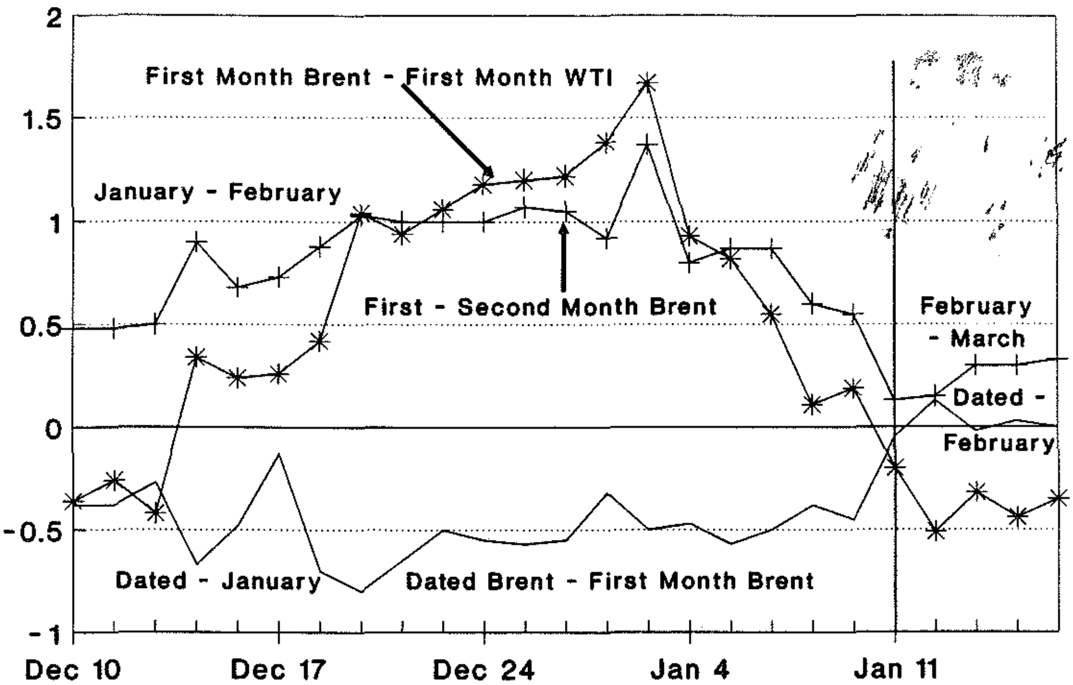
The second way of closing out a position is to pass on a 15-day notice to take delivery along a "daisy chain". The chain starts when a seller with entitlements to Brent serves a 15-day notice to take delivery to those who have buying contracts with him for April delivery. A buyer who receives such a notice can either accept it or pass it on to somebody who bought from him. The squeezer accepts the notice and thus builds up his stock of claims to Brent for April lifting.

What happens if a participant cannot close his position? If a participant without entitlement to Brent has sold a contract to the squeezer on the forward market and if the squeezer refuses to construct a bookout by selling the contract back to the participant, then the participant has a legal obligation to deliver Brent to the squeezer in April. The participant must buy a cargo on the spot market thereby creating an artificial demand for spot Brent. The squeezer has bought a significant amount of the entitlements to April Brent and thus more or less controls supply. With an inelastic supply and an artificially increased demand the spot price goes up. The other possibility is that the squeezer agrees to sell a paper cargo back to the short participant so that a bookout can be arranged. This time the squeezer sets the terms of the paper trade. This leaves the spot price unchanged but raises the 15-Day price. A mixture of these two types of squeezing is of course also a possibility.

The squeeze results from the fact that the market participants (apart from the squeezer) on average are net short and the squeezer is very long in the market. In case of a *corner*, the squeezer has *complete* control over the dated April cargoes, so that the sellers have to buy from him at a premium. Implicit in all this is the idea that the integrated producers are not willing to supply cargoes from the stocks they hold for refinery (or other) purposes.[4]

[4] Tax considerations may be involved: individual producers may have preferred to retain oil on their hands rather than selling it at arms-length at a higher price since they are taxed on the average of prices from the first day of the month preceding delivery (here 1st March) and ending on the middle day of the month of delivery, (WPA, 1st June 1987). This reasoning ignores the effect of producers' sales on the price, which is central in the analysis of section 2.

Figure 2: The January 1988 Squeeze (10th of December to 15th of January)



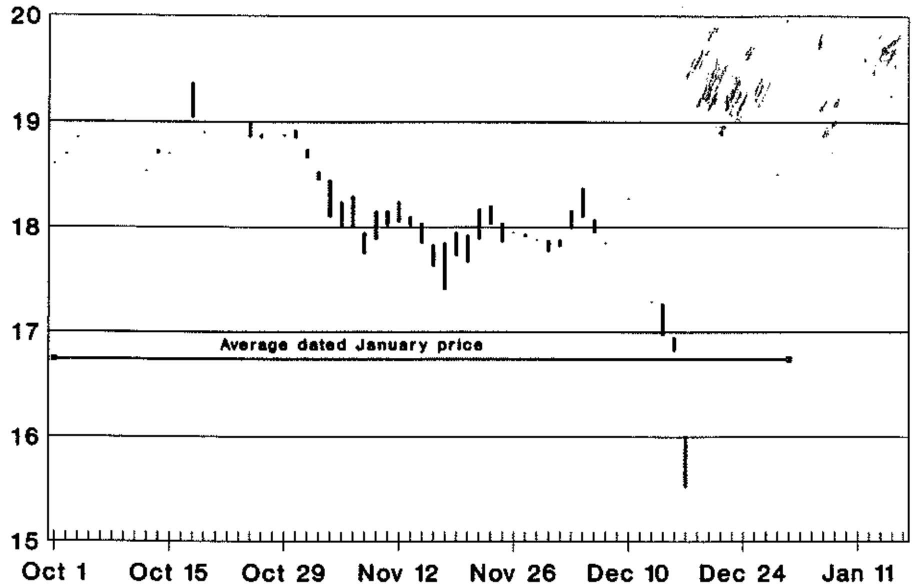
Source: Platt's and Nymex

In late november 1987, TWO began buying January cargoes at a price at parity or even at a discount to February cargoes. On 10 December 1987 (see Figure 2) a new price squeeze started, pushing the premium of first-month (January) over second-month (February) up to $ 1.37 within fifteen days. During the same time span the premium over first-month WTI climbed to $ 1.66, while dated Brent remained at a discount of about 50 cents per barrel. It is only in the second week of January that dated cargoes were sold at a small premium. So the squeeze happened on the 15-Day market, not on the spot market.

Figures 3-6 give detailed evidence on the daily deals of the January 1988 squeeze as published by the *Weekly Petroleum Argus*.[5] TWO is reported to have had control of almost all dated January cargoes in the second half of December. (Around the 10th of January it also owned the majority of the remaining undated January cargoes). Intuition suggests that TWO built up its long position at a time other longs were selling. Figure 4 shows that the majority of trading took place in the first three weeks of November. On one particular day, more than 50 deals were reported. This alone exceeds a month's production. Since the second half of December, that is, once the dating of January cargoes started, almost no deals were reported. That is the period during which, supposedly, TWO squeezed by refusing to close the market's open interest.

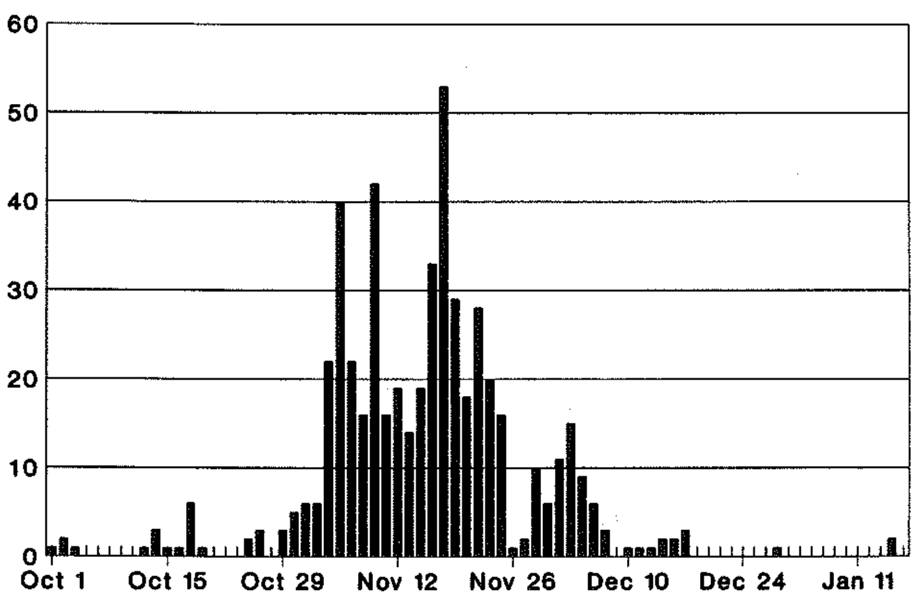
[5] Similar data do not exist for the April 1987 squeeze.

Figure 3: The January 1988 contract High/Low Price



Source: Weekly Petroleum Argus

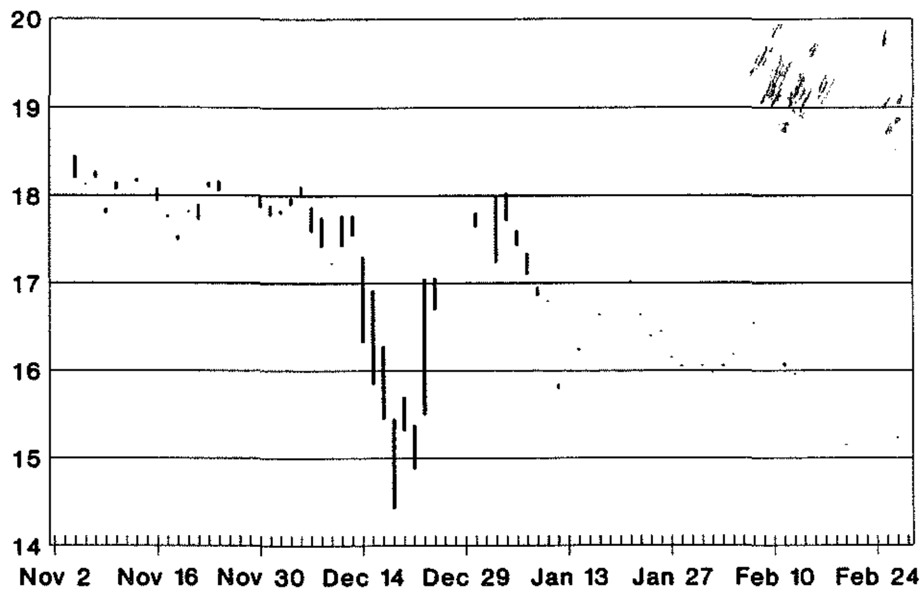
Figure 4: January 1988 contract Number of deals



Source: Weekly Petroleum Argus

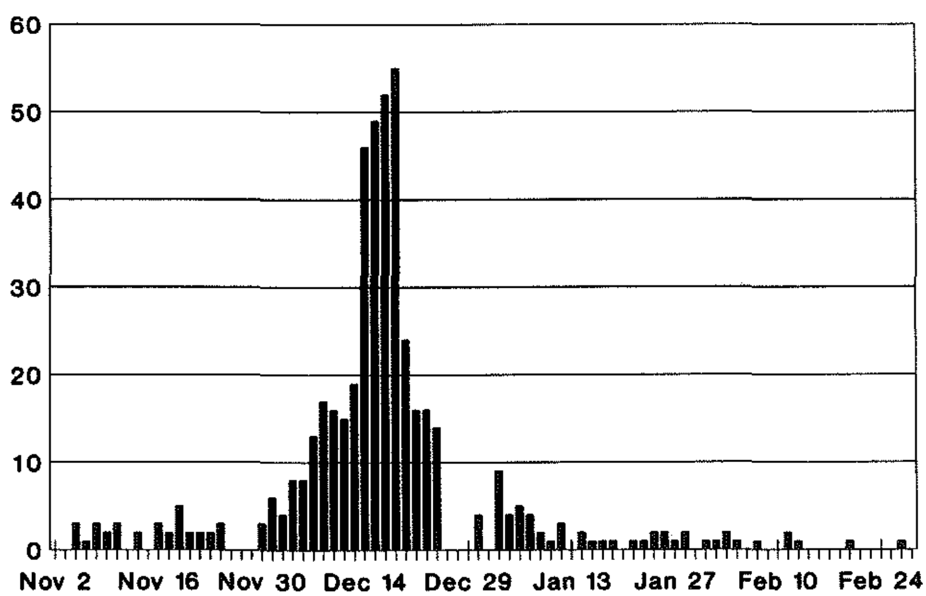
Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

Figure 5: February 1988 contract High/Low Price



Source: Weekly Petroleum Argus

Figure 6: February 1988 contract Number of deals



Source: Weekly Petroleum Argus

Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

Figure 3 shows that the period of active trading in November was also a period of moderately decreasing prices. On a given day the differences in price for deals made were of moderate size reflecting a near concurrence of spot price expectations. Figures 5 and 6 show similar data for the February 1988 contract. This pattern of trade is typical for the 15-Day market. The important thing to note is that a squeezer can easily hide in this kind of data: The trading is decentralized and nobody keeps track of the traders' positions, so the squeezer can quietly buy up contracts from different traders each of whom has a moderate short position.

In the first week of January the premia were falling (See Figure 2). By the 11th of January the squeeze was over. The fall of the premia may have been related to some extent to Esso's and Shell's announcement that they would supply stocks from their corporate systems. At any rate, TWO had to take delivery of 41 out of the 42 cargoes produced in January and did not push the dated premium to high levels.

It is worth noting that Esso in March 1988 proposed to its trading partners that sellers be given the option in the standard contract of substituting a number of other grades of crude oil from North Sea fields for Brent, in an attempt to deter a repetition of the squeeze. Substitution would incur a premium of 30 cents per barrel. Delivery options of this kind are as mentioned a standard way of avoiding squeezes in futures markets. After some discussion the proposal was not accepted. This alternative squeeze preventing proposal suggests that Esso had abandoned the idea that the main producers could deter squeezes by threatening to make stocks available.

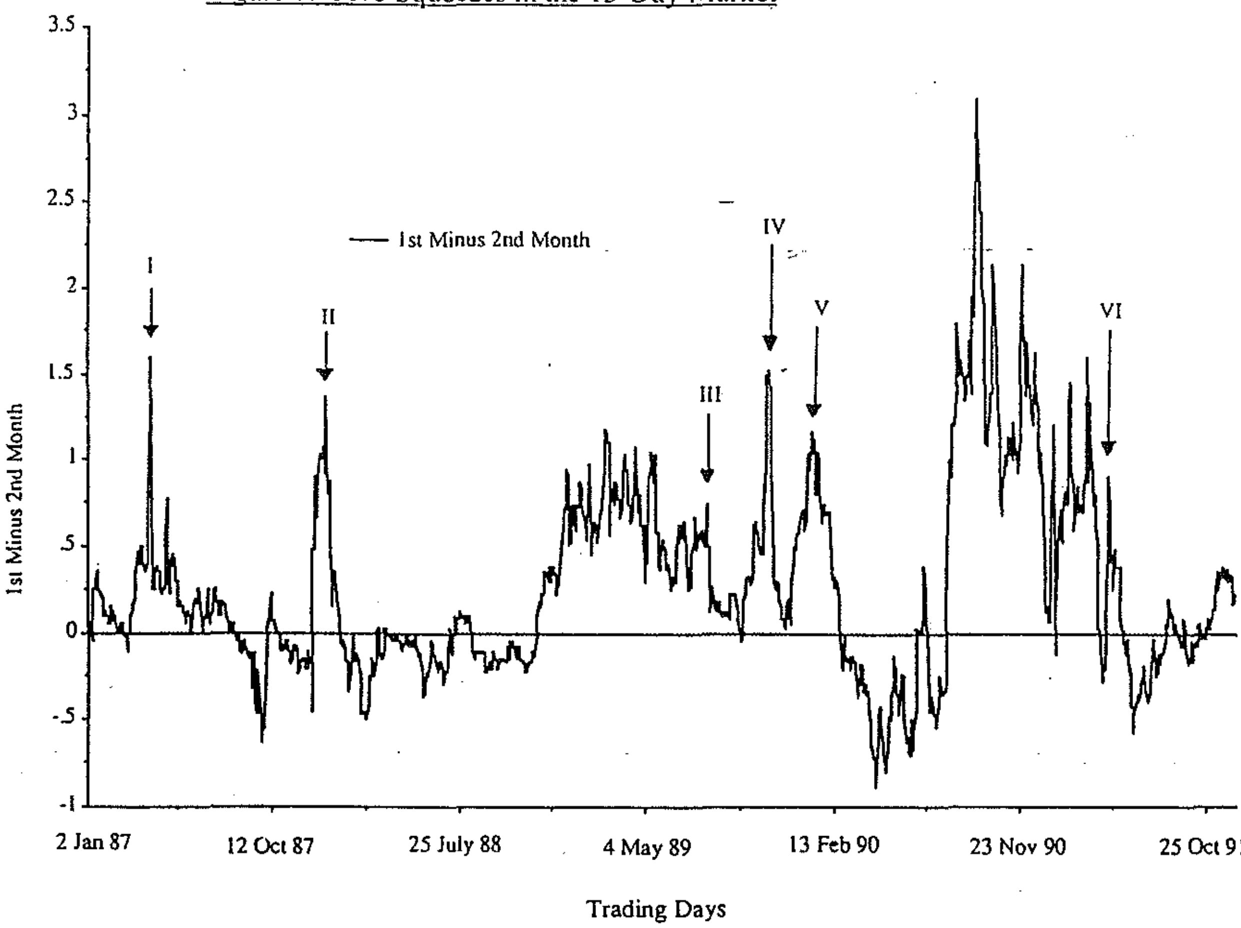
Figure 7 shows how the differential between *Platt*'s quotations of the average price of paper barrels for delivery in the first and second-month evolved from 1987 to 1991. The six peaks that can be identified (marked I-VI) are potential candidates for squeezes and in fact five of them (I-IV and VI) were. The huge peak between V and VI was caused by the Gulf Crisis that led to a maximum differential of 3.1 $/bl on 17 September 1992. The maxima of the six potential squeezes are given in Table 1.

TABLE 1: Five Squeezes

Peak	Date	Maximum Differential	Differential 2nd-Month
I	08/04/87	1.60 $/bl	8.9 %
II	31/12/87	1.37 $/bl	8.1 %
III	03/08/89	0.75 $/bl	4.6 %
IV	06/11/89	1.53 $/bl	8.1 %
V[1)]	10/01/90	1.17 $/bl	5.8 %
VI	12/04/91	0.90 $/bl	4.7 %

1) Due to unexpectedly strong demand for heating oil, cfr. text.

Figure 7: Five Squeezes in the 15-Day Market



Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

The two first peaks (I and II) are the two squeezes that were mentioned above and they both led to price surges that reached maxima of more than eight percent (measured as the differential over second-month price).[6] The third and fourth squeezes were supposedly carried out either by a Wall Street refiner or by a trading house. *Weekly Petroleum Argus* wrote about IV:

> **"The premium for November 15-Day Brent over December widened from 40 ¢/bl between Friday morning and Thursday evening.**
> This premium has talked wider all week [27/10 - 2/11/89] but in such secrecy that it appeared an attempt to force up the November quotations and price reporting services felt manipulated. However both numbers given above were confirmed and there is conjecture that one party sold short in expectation that the spread would close in. But it was eventually forced to cover at almost double the premium at which it bought [sold?]". (*Weekly Petroleum Argus*, 6/11/89 p. 8)

And: "... The price of North Sea crude is blurred by the pressure on November Brent which some feel is rubbing off on December prices. The possibility that an extra Brent cargo may be fitted into the November programme helped reduce November prices slightly but they remain out of line with the rest of the market. The monthly recurrence of so called squeezes since the summer, partly due to the reduction in Brent production, is angering traders who feel the future of the 15-Day market in jeopardy. ... (*Weekly Petroleum Argus* 13/11/89 p. 8).

The quotes show two important effects of a squeeze: first, a market participant that for speculative reasons had taken a short position had to cover at a large loss; and second, market participants are getting discontented with the market because of squeezes. Furthermore it shows that the possible availability of an additional cargo reduced the effect of the squeeze. These observations constitute a crucial part of our modelling of the market in Section 2.

Occurrence V was as mentioned not a squeeze. This can be seen from the fact that spot prices rose even more than first-month prices, and the cause of this was an increase in the demand for heating oil. *Platt's Oilgram Price Report* of 2/1/90 noted:

> "... With Brent now turning wet into the second-half of Jan, and with the absence of any squeeze-related play in Jan Brent, paper and wet Brent are acting in unison."

Thus there seems to be general agreement that this was not a squeeze.

Occurrence VI was a squeeze that was carried out by a large, Chinese trading house. The story was however not commented by the *Weekly Petroleum Argus*, nor by any other commentator that we know of and we are relying on confidential information in this case.

Summing up, we have identified five major squeezes in the 15-Day market for Brent crude oil in the period 1987 - 1991. The immediate effect of squeezes is that some market participants incur losses out of line with normal speculative gains and losses. The long-run effect is that market participants are worried to a degree where they feel that the future of the entire market is at stake. Finally, if extra cargoes (read: regulatory stocks) were available, the

---

[6] This measure may overestimate the size of the squeeze slightly if the market is in backwardation initially.

effects of squeezes could be dampened if not eliminated.

2. A Squeezer Round the Corner?

In this section we propose a model that captures the ideas that were presented above. We first model how the squeeze affects the market and what it costs to prevent a squeeze given that the producers can cooperate on holding regulatory stocks. We then move on to determining under what circumstances such regulatory stocks can be sustained in a non-cooperative repeated game.

Two points should be borne in mind throughout. The chapter is about regulatory stocks, and that only. This means that the only reason to hold stocks is a potential squeeze. In particular, the producers face no other uncertainty than that arising from the squeeze so buffer stocks are ruled out; the price that the producers receive is constant before and after a squeeze so speculative stocks are ruled out; the marginal cost of production is constant (zero) so there is no incentive to hold transaction stocks; the time horizon is infinite so strategic stocks in the sense of Chapter 2 are not an issue either.

The second point is that in this framework production always equals sales except in the period where a squeeze occurs in which case sales equal production plus stocks.

2.1 The Squeezer's Game

We ignore the presence of integrated oil companies and assume that the market participants belong to at most one of four groups:

1. refineries
2. traders
3. producers
4. squeezer.

The final demand for crude oil arises from the refineries, who buy the crude on the spot market. The producers hedge their entire production by selling it forward. The traders serve as intermediaries and buy forward from the producers to sell on the spot market. Thus, to simplify matters, we assume that the spot market consists of refineries, traders and possibly a squeezer, whereas the forward market consists of producers, traders and possibly a squeezer. Section 3 comments on the fact that integrated oil companies operate in the market.

Assume that the refineries' demand for crude oil takes a form that allows us to write the (inverse) demand that traders face on the spot market as a random variable that is linear in quantity, x_t:

$$\tilde{P}_t = (\tilde{\alpha}_t - RP) - x_t \,, \tag{1}$$

where $\tilde{\alpha}_t \sim N(\alpha_t, v)$ with $\alpha_t > 0$ and RP is some positive constant to be determined later. We have $E(\tilde{P}_t) \equiv P_t = \alpha_t + RP - x_t$. P_t can be thought of as the "market expectation" of the spot price. Traders are risk averse profit maximizers. In particular their utility function may be negatively exponential in profits ($u(\pi) = -e^{-A\pi}$) implying, since π follows a normal distribution, that they maximize $E(\pi) - AVar(\pi)/2$. Their expected payoff is thus

$$d(P_t - p_t) - d^2\frac{A}{2}Var\tilde{P}_t = d(P_t - p_t - d\frac{A}{2}v), \tag{2}$$

where p_t is the forard price, A is the Arrow-Pratt coefficient of constant absolute risk aversion and d is the net long position of the trader (d for "deals"). A trader's participation constraint is thus:

$$P_t - p_t \geq d\frac{A}{2}v \equiv RP. \tag{3}$$

As long as the participation constraint holds with inequality, there are payoffs to be made from trading in forward contracts, so the market should attract more traders implying that the forward price p_t be competed up to equality of (3). But this argument holds for a fixed d and the right hand side of (3) is minimized for $d = 1$ (given that a deal is indivisible and that $d > 0$), so it is in the interest of the producers to have at least as many traders in the market as the number of deals so that each of the traders buys one forward contract from the producers. Seen from the producers' point of view, the risk premium they have to pay - the cost of hedging - is decreasing in the number of traders in the market.[7] Note that (3) basically assumes a situation of normal backwardation in which the producers are so risk averse that they hedge completely. Given their (now certain) demand curve, they can go on and maximize profits in the usual way.

Equality of the constraint (3) with $d = 1$ implies that the producers face a certain demand function on the forward market of the simple form:

$$p_t = P_t - RP = \alpha_t - x_t \,. \tag{4}$$

In this subsection we assume that the producers can sustain a cooperative outcome in a non-cooperative repeated game (which we then study in Section 2.2). We also assume that the marginal cost of production is constant and normalized to zero. The n producers therefore

[7] A similar point is made in Working (1953). See especially the section entitled "The Cost of Hedging".

share monopoly profits in each period. The equilibrium is summarized in (5).

$$\begin{array}{lrcl} & \multicolumn{3}{c}{\underline{\text{Monopoly outcome}}} \\ & \multicolumn{3}{c}{\text{with no stocks}} \\ \text{Quantity:} & x_t & = & \dfrac{\alpha_t}{2} \\ \text{Price:} & p_t & = & \dfrac{\alpha_t}{2} \\ \text{Profit:} & \pi_t & = & \dfrac{\alpha_t^2}{4} \end{array} \tag{5}$$

The model above can explain a volume of trade on the forward market equal to the volume of production and it does not allow for squeezes since all traders are net long. As a matter of fact, the volume of trade on the 15-Day forward market is typically ten-fold the volume of production and squeezes do occur. It is argued in Chapter 4 that the volume on the forward market can be explained predominantly by speculation, implying that different traders enjoy different spot price expectations. This also explains why some traders short-sell contracts and thus how a squeezer can build up a large long position. We shall not pursue the issue of speculation here, but note that it may reduce the traders' aversion to risk that the market is liquid (has a high volume), since they can then, at any time, free themselves from whatever contract they have engaged in, incurring a moderate loss. Thus, the traders, as do the producers, favour a high number of trades.

The implicit assumption that we make and - more importantly - that the traders make, is that the market's open interest at maturity is cleared by financial transactions using the spot price which is determined by the (known) production and the realization of the refineries' demand curve (1). The traders do *not* expect a squeeze.

A *squeezer* is indistinguishable from a normal trader. The squeezer uses imperfections in the clearing of the forward market strategically to squeeze a temporary profit out of the markets. As mentioned, the volume in the forward market is bigger than the volume of the spot market, but the difference is *normally* closed by financial transactions in a *bookout* or a *daisy chain* as maturity approaches. However, in the 15-Day market, no participant is legally obliged to enter in the clearing and the squeezer uses this opportunity to squeeze the market: By offering a price in the high end of the spectrum, the squeezer obtains legal rights to a substantial part if not all of the physical cargoes. Those who sold cargoes forward without having them (everybody but the producers) and who therefore are genuinely short of oil are put in a difficult situation: they have to buy either on the spot market thereby crowding out the usual buyers or take the squeezer's terms on the forward market. This idea is modelled by assuming that a squeeze of size α^s equal to the open interest of the market at maturity affects the inverse demand in the following way:

$$P_t = \alpha_t - \alpha^s - x_t + RP^s = p_t + (\alpha^s + RP^s) \quad ; \quad \alpha^s > 0. \tag{6}$$

Here the assumption is that if the traders have to buy up paper cargoes on the forward market (from the squeezer) at a premium in order to satisfy their contractual obligations, they reduce their own risk premium (that may become negative):

$$RP^s = RP - \alpha^s \tag{7}$$

In this case we say that the squeeze was entirely on the 15-Day market, since we will note a price surge only on the 15-Day market as the traders close their positions.

The alternative is that the traders decide to deliver the oil, buying it on the spot market. This will raise the spot price by α^s and will in the end have the same effect on traders' profits, but this time the spot price is affected and so we say that the squeeze is on the spot market. Indeed, it is this possibility of buying spot oil that puts a limit on the terms that the squeezer can set if the squeeze is settled in terms of paper cargoes. One could, of course, imagine a combination: the squeeze could be settled partly in paper barrels and partly in wet barrels, but this does not affect the analysis. Note that, since the producers sold their entire production forward, their immediate profit is not affected.

To summarize: Within each period t, the producers first sell their production on the forward market to the traders. Some traders also sell forward contracts to other traders expecting financial settlement at maturity. The squeezer possibly buys a substantial number of forward contracts. Then the spot market opens. If the squeezer does *not* show up, those who bought forward from the producers clear the spot market with the refineries, and the resulting spot price is used in financial settlements of the remaining open interest on the forward market. If the squeezer *does* show up, short traders may appear on the spot market in desperate search for a possibility to fulfil their legal obligation. This would drive up the spot price. Alternatively, the short traders may seek a settlement with the squeezer on the forward market and this would drive up forward prices.

In the normal mode of functioning there is a friendly competitive environment on the forward market: friendly meaning that the imperfections of the clearing mechanism are not exploited in a squeezing game. When a squeeze occurs, an aggressive environment results: the imperfections of the clearing mechanism are exploited, traders and/or refiners are trapped and have to pay more for the crude oil than they expected and thus observe reduced profits or even losses.

A successful squeeze therefore results in a general dissatisfaction among the refiners or the intermediaries because of the malfunctioning and unpredictability of the market. They will tend to organize trade outside the market or in other markets or they get more averse to risk. After all, the squeeze proved that spot prices were more volatile than they thought initially and so a higher risk premium is required.

We assume (for simplicity of notation) that when a successful squeeze occurs and if it occurs on the spot market, α^s squeezed refineries leave the market in the following period to never come back, or, if the squeeze is on the forward market, that the traders require a risk premium that is α^s higher. One could again imagine a hybrid case, but the important thing to note is that in all periods following a squeeze, the producers are left with a demand curve with an intercept that is α^s lower. The inverse demand curve in all periods following a squeeze is:

$$p_{t-1} = \alpha_{t+1} - x_{t+1} = (\alpha_t - \alpha^s) - x_{t-1} \qquad (8)$$

The producers then have a long-run incentive to prevent a squeeze in order to keep the price of their hedged production from falling in the future; to keep the market liquid. They can achieve this by keeping stocks in order to match a squeeze if it occurs, *i.e.* by keeping stocks $s = \alpha^s$. The idea is that if the squeezer squeezes, then the producers throw their stocks on the market at the normal price to meet the artificially created demand. In a sense, the producers always keep a physical position of size α^s to match the squeezer's long paper position. We assume that the cost of storage is the interest r per \$ per barrel per period.

If they hold stocks to prevent a squeeze, *i.e.* $s = \alpha^s$, they have to subtract the interest on the value of the stocks $r\alpha^s p_t$ from revenue in each period and reoptimizing we find that compared to (5) the producers raise output and lower price a bit to take the cost of storage into account:

<u>Monopoly outcome</u>
with stocks

Quantity: $$x_t = \frac{\alpha_t - r\alpha^s}{2}$$

Price: $$p_t = \frac{\alpha_t - r\alpha^s}{2} \qquad (9)$$

Profit: $$\pi_t = \left(\frac{\alpha_t - r\alpha^s}{2}\right)^2.$$

If they fail to prevent a squeeze in period t (not holding stocks), profits will be $(\alpha_t-\alpha^s)^2/4$ in all future periods instead of $(\alpha_t/2)^2$. The discounted loss of *not* preventing a squeeze is therefore

$$\text{Loss} = \frac{(2\alpha_t - \alpha^s)\alpha^s}{4r} \qquad (10)$$

where we assume a discount rate of $1/(1+r)$ and that the effect of a squeeze in period t manifests from period $t+1$ onwards. The (reasonable) assumption underlying this modelling is that the producers' output is inflexible in period t so that production is decided upon before the squeezer reveals himself. Additional supply therefore has to come from the stocks if they exist. These stocks are supposedly made available immediately.

The price in case of a successful squeeze is the price of equation (6) taking the monopoly output as given:

$$p_t^s = \frac{\alpha_t}{2} + \alpha^s = p_t - \alpha^s, \tag{11}$$

so a price surge of size α^s will occur on the forward market towards maturity if the squeeze is happening there or on the spot market if that is where the squeeze pops up.

The (gross) payoff of a squeeze to the squeezer is the size of his long position, α^s, times the price difference between selling and buying, $p_t^s - p_t = \alpha^s$, *i.e.* $(\alpha^s)^2$. In the case of a squeeze, the squeezer has to dispose of the acquired oil elsewhere and he therefore suffers a loss, Θ, of squeezing independently of whether the squeeze is successful or not.

The payoffs are summarized in Table 2. The producers' payoff are reported as deviations from an eternal monopoly profit, with the value $((r+1)/r)(\alpha_t^2/4)$ today.

The producers' entry in the lower, right corner of the matrix is due to the assumption that if the squeezer squeezes and if the producers sell their stocks, the squeezer cannot squeeze again since he has now been identified as such and so the producers do not need to

Table 2: Payoff Matrix for Squeezer vs. Producers

Squeezer\Producers	No stocks	Stocks=α^s
No squeeze	$(0,0)$	$\left(0, -\frac{(1+r)\alpha^s(2\alpha_t - r\alpha^s)}{4}\right)$
Squeeze	$((\alpha^s)^2 - \Theta\ ,\ -\frac{(2\alpha_t - \alpha^s)\alpha^s}{4\ r})$	$(-\Theta, \alpha^s p_t)$

hold stocks any longer. On the other hand, the producers earn an additional profit ($\alpha^s p_t$) from satisfying the extra demand the squeezer has created.

There is no Nash equilibrium in pure strategies: if the producers do not carry stocks the squeezer will squeeze; if the squeezer squeezes, the producers will carry stocks; but then the squeezer will not squeeze and thus it does not pay for the producers to carry stocks.

Letting stocks be a continuous variable allows s to take a value above zero but below α^s. What happens if s takes an internal value? In the case of a squeeze, a number of the squeezed traders can fulfil their obligation by buying the producers stocks at price p_t and delivering to the squeezer. This modifies the squeeze to one of size $(\alpha^s\text{-}s)$ and thus affects prices by $(\alpha^s\text{-}s)$ and so the squeezer's profit, $\Delta(s)$, becomes a quadratic function of the stocks:

$$\begin{aligned} \Delta(s) &= (\alpha^s - s)^2 - \Theta \\ \Delta(s) &= 0 \Leftrightarrow s = \underline{s} = \alpha^s - \sqrt{\Theta} \quad . \end{aligned} \tag{12}$$

Observe that the squeezer's profit takes the value zero for $s=\underline{s} < \alpha^s$ if the cost, Θ, of squeezing is strictly positive. The square root of Θ translates the cost of squeezing into an equivalent number of cargoes. The presence of a cost of squeezing thus means that by keeping stocks equal to α^s the producers commit overkill, since stocks are costly and a squeeze will be unprofitable even if stocks are reduced by $\sqrt{\Theta}$.

Now we exogenize the squeezer and assume that the producers perceive that there is an **exogenous probability**, σ, of a squeeze of size α^s in each period. This means that the producers do not try to influence the squeezer's behavior strategically, but that they rather observe the possibility of a squeeze and take it into account.

We choose a simple model of a squeeze probability: Assume that there is probability σ of a squeeze until the squeeze has occurred (if ever) whereafter the squeezer is identified and a squeeze cannot occur again.

Production can be changed from period to period and before a squeeze occurs it is chosen to maximize expected single period profit:

$$E(\Pi_t(x;\alpha_t,\sigma,r,s)) = (1 - \sigma)(\alpha_t - x)(x - rs) - \sigma(\alpha_t - x)(x - s) \quad . \tag{13}$$

The first term says that if a squeeze does not occur (an event that happens with probability 1-σ), stocks are held in vain and the producers have to pay the cost of storage, prs. The second term says that if a squeeze does occur, the producers' sales are increased by the size of the stocks at the going price.

The equilibrium that obtains *before* a squeeze is therefore

$$\begin{aligned} x &= \frac{1}{2}(\alpha_t - \gamma s) \\ p &= \frac{1}{2}(\alpha_t + \gamma s) \\ E(\Pi_t) &= \frac{1}{4}(\alpha_t + \gamma s)^2 \\ \text{where } \gamma &= \sigma - (1 - \sigma)r \ . \end{aligned} \tag{14}$$

If $\gamma > 0$, *i.e.* if $\sigma > \frac{r}{1+r}$, then the quantity is lower and the price is higher than in the situation in which a squeeze cannot occur, see (5). Conversely if $\gamma < 0$.

In the period *after* a squeeze, *i.e.* once uncertainty has been resolved, production is

Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

chosen to maximize

$$\Pi'(x;\alpha_t,\alpha^s,s) = (\alpha_t - \alpha^s + s - x_t)x_t \, , \tag{15}$$

since the new demand intercept is

$$\alpha_{t-i} = \alpha_t - (\alpha^s - s) \, , \quad \forall i > 0 \, . \tag{16}$$

The solution that obtains *after* a squeeze is therefore

$$\boxed{\begin{aligned} x &= \frac{1}{2}(\alpha_t - \alpha^s - s) \\ p &= \frac{1}{2}(\alpha_t - \alpha^s - s) \\ \Pi' &= \frac{1}{4}(\alpha_t - \alpha^s - s)^2 \, . \end{aligned}} \tag{17}$$

Seen from period t, the producers' expected, discounted profit takes the form of a complex, geometric progression:

$$\begin{aligned} E(\Pi) = {} & E(\Pi_t) \\ & - \frac{1}{1+r}[(1-\sigma)E(\Pi_t) + \sigma\Pi'] \\ & - \frac{1}{(1-r)^2}[(1-\sigma)^2 E(\Pi_t) + \sigma(1+(1-\sigma))\Pi'] \\ & - \frac{1}{(1-r)^3}[(1-\sigma)^3 E(\Pi_t) + \sigma(1+(1-\sigma)-(1-\sigma)^2)\Pi'] \\ & -\ldots \\ & = \frac{1+r}{\sigma+r}E(\Pi_t) + \frac{\sigma}{r}\frac{1+r}{\sigma-r}\Pi' \, . \end{aligned} \tag{18}$$

The first term in the square brackets multiplies the probability that a squeeze has not occurred up to a certain period with the expected stage game profit before a squeeze. The second term similarly multiplies the cumulated probability that a squeeze happened in a given period or before with the after-squeeze profits.

For later convenience, define

$$K(s;\alpha_t,\alpha^s,\sigma,r) \equiv 4[E(\Pi_t) + \frac{\sigma}{r}\Pi']$$
$$= (\alpha_t - \gamma s)^2 + \frac{\sigma}{r}(\alpha_t - \alpha^s + s)^2 \qquad (19)$$

and observe that K is proportional to $E(\Pi)$ and that these expressions therefore enjoy the same functional characteristics.

We employ the following assumptions in the sequel:

Assumption 1: The interest rate is positive, $r > 0$, and σ is a probability, *i.e.* $\sigma \in [0;1]$.

Assumption 2: $\alpha_t > \alpha^s > 0$.

Assumption 3: $0 \leq s \leq \alpha^s$.

Assumption 1 is hardly controversial. Assumption 2 is a joint assumption: Firstly it means that a squeeze raises prices by two hundred percent at the most, which does not seem restrictive given the size of the squeezes reported in Table 1. Secondly it implies that the market does not vanish completely in the periods following a squeeze if the squeeze is completely unprevented $(s = 0)$ so that the problem is still well defined after a squeeze. Assumption 3 requires stocks to be non-negative and not to exceed α^s. (If $s > \alpha^s$, $\alpha_{t+1} = \alpha_t - (\alpha^s - s) > \alpha_t$ which is not a tenable assumption.)

Let s^* denote optimal stocks. We immediately get the result that if a squeeze is totally unlikely to occur, it does not pay to hold stocks:

Proposition 1 (Certainly No Squeeze): $\sigma = 0 \Rightarrow s^* = 0$.

Proof: $K(s;\alpha_t,\alpha^s,0,r) = (\alpha_t - rs)^2$ takes a global minimum $= 0$ for $s = \alpha_t/r > 0$. $K(s)$ is real valued and continuous in s. The restriction of $K(s)$ to $[0;\alpha^s]$ therefore takes a maximum on this set. K is convex in s and the maximum thus is at $s = 0$ or $s = \alpha^s$. $K(0;\alpha_t,\alpha^s,0,r) = \alpha_t^2 > K(\alpha^s;\alpha_t,\alpha^s,0,r) = (\alpha_t - \alpha^s r)^2$ if $\alpha_t > \alpha^s r/2$. But if $\alpha^s r/2 > \alpha_t$, the corresponding price is negative, which does not make economic sense.■

Remark: Given Assumption 2, non-negativity of prices is only an issue for interest rates above 200 per cent. More general comments about non-negativity of prices are found in the Appendix.

Proposition 2: A **sufficient condition for** $s^* = \alpha^s$ is $\sigma > \frac{r}{1-r}$.

Proof: $\sigma > \frac{r}{1+r} \Rightarrow \gamma > 0 \Rightarrow \frac{dK(s)}{ds} > 0 \ , \forall s \ \Rightarrow K(\alpha^s) > K(0)$ ■

Remark 1: An equivalent formulation of Proposition 2 has $r < \frac{\sigma}{1+\sigma} \Rightarrow s^* = \alpha^s$.

Remark 2: $\frac{r}{1+r}$ is the inverse of the present value of an eternal sequence of *1*'s. In other words, the condition in Proposition 2 compares the probability of a squeeze with time preferences.

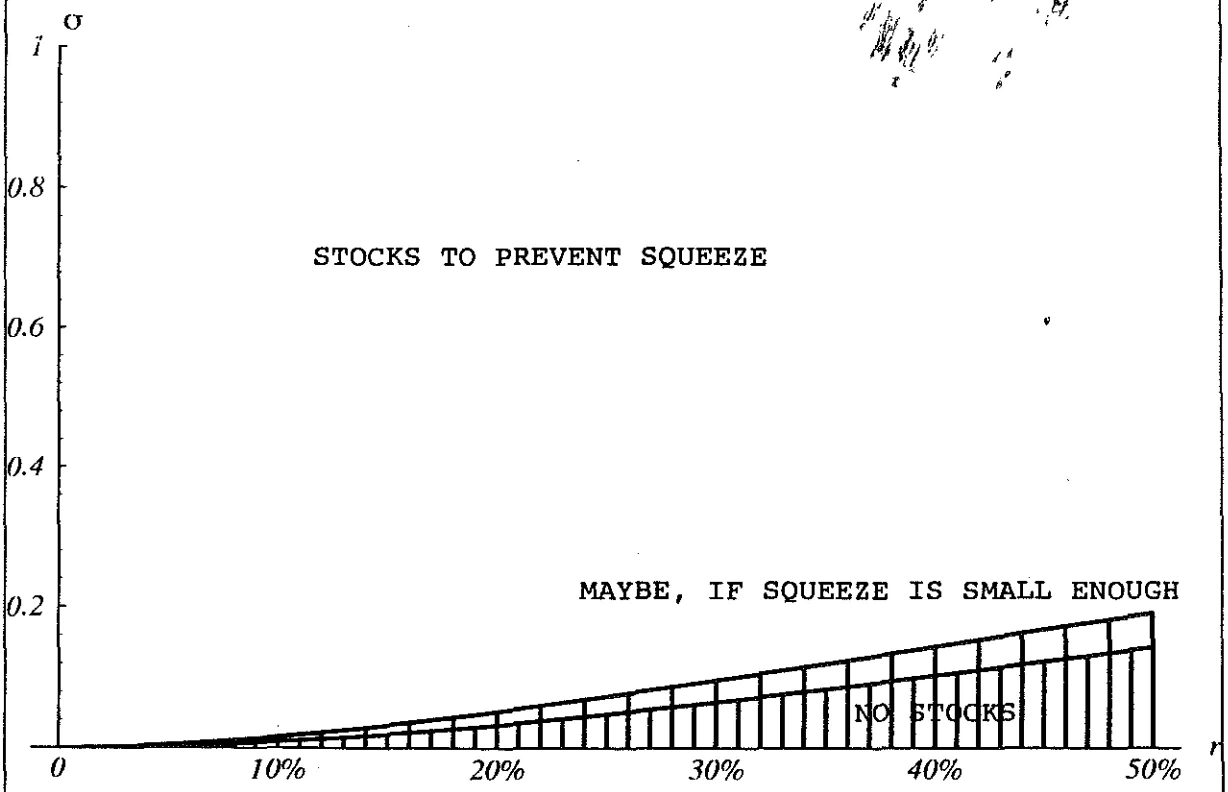
Corollary (Certain Squeeze): $\sigma = 1 \Rightarrow s^* = \alpha^s$.

Proof: Trivial: $\frac{r}{1+r} < 1 \ , \ \forall r \geq 0.$ ■

The necessary and sufficient conditions for $s^* = \alpha^s$ and the complementary conditions for $s = 0$ are given in **Proposition 3** of the Appendix. It is clear from the proof of Proposition 3 that the maximum must be obtained at $s = 0$ or at $s = \alpha^s$ since K is convex in s and since the domain is restricted to $[0;\alpha^s]$. Here we just indicate in Figure 8 the values of the parameters σ and r for which the necessary and sufficient conditions will always be fulfilled, where they can never be fulfilled and where the restrictions on α^s and α_t compared to σ and r are effectively binding. We have here chosen what we consider a normal range for r, namely $r \in [0;0.5]$. We think of our basic unit of time as one month, and month-to-month interest rates very rarely exceed one, let alone fifty, per cent. The **conclusion** is that only for small squeeze-probabilities and (very) high interest rates is it the case that a monopoly will *not* hold stocks.

To give an idea of the content of the Appendix (See A.1), let it suffice to be said that the parameter space is four dimensional and for each vector $(\alpha_t,\alpha^s,r,\sigma) \in \mathbb{R}^3_+ \times [0;1]$ it is possible to check whether stocks will equal zero or α^s. However, it turns out that the conditions can be expressed in terms of an inequality relating (α^s/α_t) to some function of σ and r, and so the problem is reduced to three dimensions: $(\frac{\alpha^s}{\alpha_t},\sigma,r) \in [0;1]^2 \times \mathbb{R}_+$, where we have used Assumption 2. The general necessary and sufficient condition for optimal stocks to be $s = \alpha^s$ is

Figure 8: Regions of the Parameter Space



$$\nu(\sigma,r)\frac{\alpha^{s}}{\alpha_{t}} + \mu(\sigma,r) > 0 \ , \tag{20}$$

where the functional forms of μ and ν are given in the Appendix (See A.1). Zero stocks are preferred whenever the inequality is reversed.

This finalizes the discussion of the profitability of stocks given that the producers cooperate. We now turn to a discussion of whether such cooperative outcomes is sustainable in a non-cooperative repeated game featuring squeezes and regulatory stocks.

2.2 The Producers' Repeated Game

It is well known from the literature on repeated games with observed actions that cooperative outcomes can be sustained as subgame perfect equilibria of repeated games, this leading to payoffs over and above the equilibrium payoffs of the stage game, *cf.* for an early example Friedman (1971). These results are known as 'folk theorems' and roughly maintain that *any* outcome that all players prefer to a Nash equilibrium of the stage game can be a subgame perfect equilibrium of the repeated game if the interest rate is sufficiently low. The cooperative behaviour is sustained by the threat to revert to the Nash equilibrium of the stage game if a deviation occurs and to play this equilibrium forever after. This punishment strategy is in itself (trivially) subgame perfect and it is thus a credible threat to lower the payoff of the deviator (and everybody else) in all periods following the deviation. If a potential deviator cares sufficiently about future payoffs, *i.e.* if she has a sufficiently high discount rate or a correspondingly low interest rate, then deviation is deterred by the threat. The deviator compares the immediate gain from deviating with the discounted loss from the eternal punishment starting the following period. There will be a threshold of the interest rate such that for all values below it, cooperation can be sustained. The questions that are treated in this section are whether a duopoly would hold stocks and how this and the possibility of a squeeze affect the scope for cooperation.

The analysis of the n-firm oligopoly case in which a squeeze may occur and stocks can be held follows similar considerations but gets somewhat more intricate because the interest rate and the probability of a squeeze enter the profit functions in a non-linear manner. Before a squeeze, the firms' profits are

$$\begin{aligned} E(\Pi_{it}(s_i)) &= (1-\sigma)(\alpha_t - x_t)(x_{it} - rs_i) - \sigma(\alpha_t - x_t)(x_{it} + s_i) \\ &= (\alpha_t - x_t)(x_{it} + \gamma s_i)\ , \quad i = 1,2,\ldots,n \end{aligned} \tag{21}$$

where $x_t \equiv \sum_{i=1}^{n} x_{it}$, s_i is firm i's stock ($s \equiv \sum_{i=1}^{n} s_i$), and equilibrium profits turn out to be

$$E(\Pi_{it}) = \left(\frac{\alpha_t + \gamma s}{n+1}\right)^2, \qquad \forall i. \tag{22}$$

After a squeeze, maximized profits are

$$\Pi_{it} = \left(\frac{\alpha_t - (\alpha^s - s)}{n+1}\right)^2, \qquad \forall i, \tag{23}$$

where we have used that the demand intercept equals α_t - $(\alpha^s - s)$ after a squeeze of size α^s that was met with sales, s, of stocks.

Importantly we get

Proposition 7: In equilibrium, the profits of the producers in n-firm oligopoly do not depend on their share of overall stocks (*i.e.* on s_i), only on overall stocks, s.

Proof: See (22-23). ■

Proposition 7 means that if non-cooperative oligopolists agree that a certain level of stocks would be optimal to (partially) prevent a squeeze, then it does not matter whether they split the stocks equally or whether, say, one of them holds all stocks. This is so, because in equilibrium (before a squeeze) prices are set to balance the cost of holding stocks with the possible gain from holding these stocks should a squeeze occur.

Furthermore, the producers will agree on the optimal level of stocks and that level will coincide with that of the monopolist:

Proposition 8: In equilibrium, a certain level of stocks, s, is optimal to a non-cooperating oligopolist if and only if it is optimal for the monopolist.

Proof: Expected, discounted profits are

$$E(\Pi_i) = \frac{1+r}{\sigma+r}\frac{1}{(n+1)^2}\left[(\alpha_t - \gamma s)^2 + \frac{\sigma}{r}(\alpha_t - \alpha^s + s)^2\right]$$

$$= \frac{1+r}{\sigma+r}\frac{1}{(n+1)^2}K(s;\alpha_t,\alpha^s,\sigma,r), \qquad \forall i$$

i.e. proportional to the $K(s)$ used to determine optimality of s for the monopoly (see (19)). ■

Remark: For a given s, single firm oligopoly profit is the fraction $(2/(n+1))^2$ $(n \geq 1)$ of the monopoly profit both in the short run (stage game) and in the long run (expected discounted profit).

Proposition 8 means that the oligopoly will hold either zero stocks or stocks equal to α^s, depending on the values of r and σ and possibly also of α^s/α_t, exactly as would the monopoly, so Figure 8 and the analysis in the Appendix can be applied without modification.

The question that now comes to mind is whether the possibility of a squeeze and the ability to hold stocks change the firms' incentive to deviate by increasing production. It does and quite significantly so. Observe that the profits mentioned in the proof of Proposition 8 are the result of the Nash equilibrium of the stage game. Thus if a deviator deviates from a situation of cooperation at time t, she will expect these profits from $t+1$. Since the discount rate is $1/(1+r)$, the discounted value at t is $(\sigma+r)^{-1}(n+1)^{-2}K(s)$. This is the **threat** that may or may not prevent deviation, depending on s, α^s, α_t, σ and r.

To get a bench-mark, firstly consider the standard model without the complications arising from squeezes and stocks. There is an incentive to deviate iff

$$r > \frac{1}{m} \equiv C_0(n), \tag{24}$$

where $m \equiv (n-1)^2/(4n)$. The critical value of r, $C_0(n)$, decreases from .89 for $n = 2$ to .49 for $n = 6$ to .33 for $n = 10$, as also seen in Table 3.

TABLE 3: Critical Values for r with $s = \alpha^s = \sigma = 0$

n	$C_0(n)$	$C_0(n)$
2	8/9	.89
3	3/4	.75
4	16/25	.64
5	5/9	.55
6	24/49	.49
7	7/16	.44
8	32/81	.40
9	9/25	.36
10	40/121	.33

In 1984 the concentration in the market for Brent has been calculated to correspond to 4.4 equal sized firms as measured by the inverse of the Herfindahl index (see Mabro (1986) pp. 40-45). The similar numbers for the British part of the North Sea and for the entire North Sea (*i.e.* including Norway's part) were 5.66 and 8.23 respectively. Since 1984, the British National Oil Corporation has been abolished and the Brent Blend has been redefined to include oil from other fields. Both of these events tended to decrease concentration (increase the equivalent number of equal sized firms) and for this reason, we have set $n = 6$

in the ensuing analysis. Note that a too high n tends to favour a non-cooperative outcome by lowering the critical value whereas a too low n has the opposite effect. The degree to which the critical value is over- or undervalued is indicated by Table 3. The critical value $C_0(6) = 24/49 = .49$ can be thought of as a benchmark in the following.

Now consider the incentive to deviate starting from a situation where a monopolist (and an oligopoly) would choose $\underline{s} = 0$. In this case, we require $s_i = 0$ for all firms and assume that each firm produces $1/n$'th of the monopoly output. There is an incentive to deviate if

$$r > \frac{-B - \sqrt{B^2 + 4\sigma k^2(m-1)^2/m}}{2(m-1)} \equiv C(\sigma,k,n;s=0), \tag{25}$$

where $B = m\sigma - \sigma k^2 - 1 + 1/m$ and where $k = \dfrac{\alpha_t - \alpha^s}{\alpha_t}$ is the percentage of the market that remains after a squeeze. Critical values for r, $C(\sigma,k,n=6;\ s = 0)$ are found in Table 4 below. It is seen that the possibility of an unprevented squeeze reduces the scope for cooperation unless either the probability or the size of the squeeze is zero $(k = 1)$. It is not surprising that the first row of Table 4 falls rapidly to zero as σ increases, since a possible squeeze of size $\alpha^s = \alpha_t$ eats away the entire market and all future profits. In the last row, the size of the squeeze is zero and its probability does not matter, wherefore we get the no-squeeze critical value of 24/49. The same applies for the first column where the probability is zero and the size does not matter. In all other cases, the value is lower than the benchmark value of 0.49. For a given size of the squeeze (a given k), the critical value of r falls dramatically as the probability of the squeeze increases - more so for big squeezes (small k's) than for small ones. Similarly, for a given probability of a squeeze, the critical value falls off rapidly as the size of the squeeze gets bigger (k goes from 1 to 0).

For $\sigma = 0.4$ and $k = 0.5$ we have the (six) oligopolists deviating if $r > 0.143$. But for this value of σ and for all r between 0 and 1.3973,[8] that is for interest rates up to 139.73 per cent, the oligopolists should hold stocks, $s = \alpha^s$, according to the necessary and sufficient conditions and in this case Table 5 is the one to look at. In Table 5 the critical value is found to be $C(0.5,0.5,n = 6;\ s = \alpha_s) = 0.435$, so that the duopolists should cooperate if the (month-to-month) interest rate does not exceed 43.5 per cent.

[8] That is for r such that $\sigma = \sigma_4 = 0.5$, see the appendix.

TABLE 4: Critical Values for r with $s = 0$, $n = 6$.

$k \setminus \sigma$	0	0.2	0.4	0.6	0.8	1
0	0.49	0.098	0	0	0	0
0.1	0.49	0.109	0.007	0.004	0.004	0.003
0.2	0.49	0.135	0.026	0.017	0.015	0.014
0.3	0.49	0.168	0.056	0.039	0.034	0.031
0.4	0.49	0.205	0.095	0.071	0.062	0.057
0.5	0.49	0.246	0.143	0.112	0.099	0.093
0.6	0.49	0.289	0.199	0.164	0.148	0.139
0.7	0.49	0.335	0.261	0.228	0.210	0.200
0.8	0.49	0.384	0.331	0.303	0.287	0.277
0.9	0.49	0.436	0.407	0.390	0.380	0.372
1	0.49	0.49	0.49	0.49	0.49	0.49

LEGEND TO TABLES 4 & 5: The tables show the critical value $C(\sigma, r, n=6\ ;\ s)$ for the interest rate: if r is greater than the value in a cell, there is an incentive to deviate and cooperation can not be sustained. k is the percentage of the market that would be left after a squeeze if the squeeze were unprevented, $k = \frac{\alpha_t - \alpha^s}{\alpha_t}$. σ is the probability of a squeeze.

TABLE 5: Critical Values for r with $s = \alpha^s$, $n = 6$.

$k \setminus \sigma$	0	0.2	0.4	0.6	0.8	1
0	0.49	(1.95)	0.366	0.200	0.129	0.093
0.1	0.49	(2.12)	0.380	0.217	0.143	0.103
0.2	0.49	(2.32)	0.395	0.236	0.159	0.117
0.3	0.49	0.682	0.409	0.258	0.179	0.133
0.4	0.49	0.614	0.422	0.282	0.202	0.153
0.5	0.49	0.577	0.435	0.309	0.229	0.178
0.6	0.49	0.551	0.448	0.339	0.262	0.210
0.7	0.49	0.531	0.459	0.373	0.303	0.251
0.8	0.49	0.515	0.470	0.409	0.353	0.306
0.9	0.49	0.501	0.480	0.448	0.414	0.382
1	0.49	0.49	0.49	0.49	0.49	0.49

LEGEND TO TABLE 5: The incentive to deviate describes a fourth-degree polynomial in r, which typically (but not always) has one negative and three positive roots. The numbers in Table 5 are the lowest of the positive roots. Numbers in parentheses correspond to the third positive root in that for these values of k and σ, there were two complex roots. See also text after (26).

If $s = \alpha^s$, the producers will deviate iff

$$\left(\frac{n+1}{4n}\right)^2 \left(\alpha_t + \gamma\alpha^s\right)^2 + \frac{1}{\sigma - r}\frac{1}{(n-1)^2}K(\alpha^s) > \frac{1}{n}\frac{1+r}{\sigma+r}\frac{1}{4}K(\alpha^s) \tag{26}$$

The first term is the deviator's expected profit in the period in which deviation takes place. The second term is the discounted value of the ensuing sequence of non-cooperative profits. The r.h.s. is $1/n$'th of expected discounted monopoly profits. (26) with equality describes a fourth degree polynomial in r (remember that γ is a function of σ and r) where, in general, there is one negative root and possibly three positive roots. A general analytical solution to (26) was not found and numerical methods were applied to generate Table 5, which gives the first (lowest) positive root with $n = 6$. For interest rates above the first positive root and below the second, cooperation cannot be sustained, but if the interest rate is between the second and the third positive root cooperation is again sustainable. For interest rates above the third positive root deviation is to be expected again. Only the first positive root is tabled here since the second and the third root both are well above 1. As an example take $k = 0.5$, $\sigma = 0.4$ and $n = 6$. Then the first positive root is 0.435 as seen in Table 5, the second is 2.698 and the third is 4.948.

In Table 5, the critical value of r is seen to vary much more with σ than with k: the size of a squeeze attempt matters less now that it is prevented.

Comparing Tables 4 and 5, it can be concluded that for all probabilities and all sizes of the squeeze, the case for cooperation is stronger with than without stocks. This follows partly from the fact that $K(0) < K(\alpha^s)$ for most of the reported values of σ unless r is very high, so that future profits are worth more with than without stocks. In fact, if the interest rate is below the critical values in Table 5 (ignoring the $\sigma = 0$ column), then in all but five cases the oligopolists should hold stocks. The five cases are the five first entries of the $\sigma = 0.2$ column. In these cases, if $r < 0.5196$,[9] producers will hold stocks and will cooperate. The general conclusion from this analysis is that if the producers cooperate, they should also hold stocks. In a few cases the converse is true: if they hold stocks, they should also cooperate.

A caveat is necessary here: Table 5 and inequality (26) are only valid in the completely symmetric case where the producers produce $1/n$'th of monopoly output *and* hold $1/n$'th of the stocks each. Only if the latter is also the case will each producer's expected profit be $1/n$'th of expected monopoly profit. At first sight this may seem at odds with Proposition 7. This, however is not the case: Proposition 7 describes a situation of Nash equilibrium in the stage game, whereas (26) describes the incentive to deviate from the cooperative scenario. Under the restriction that the n firms' stocks sum to α^s, the asymmetric case adds the following term to the right hand side of (26):

[9] r such that $\sigma_4(r) = 0.2$.

$$\left[\left(\frac{1}{2}\frac{1+r}{\sigma-r} - \gamma\frac{n-1}{4n}\right)(\alpha_t + \gamma\alpha^s) - \frac{1}{4}\gamma^2(s_D - \frac{\alpha^s}{n})\right]\left(s_D - \frac{\alpha^s}{n}\right). \tag{27}$$

s_D is the potential deviator's stock. (27) is the effect on cooperation of asymmetric stocks. If the effect is negative, deviation is more likely to occur. The term in the square brackets can be shown to be positive for all values of s_D between 0 and α^s,[10] so the effect shares the sign with $(s_D - \alpha^s/n)$: If a producer holds $s_D < \alpha^s/n$, he is less likely to cooperate, whilst a producer that holds $s_D > \alpha^s/n$ is more likely to do so. In particular, if one producer holds all of the stocks, he is less likely to deviate but all the other producers would be more inclined to do so. Whether they would in fact do so, depends on k, σ, r and n.[11]

Deviating by increasing production may not be an issue in the Brent market, where one producer (Shell UK) is in charge of organizing liftings (see Phlips (1992)). A production schedule is compiled well ahead of time and is negotiated and approved by the other producers. Production is thus "observed before it happens" and the response to a deviation could therefore be simultaneous rather than delayed a period, thus further discouraging deviation.

The overall conclusion of Section 2 is that unless the probability of a squeeze is very small or the (month-to-month) interest rate very high, **self-regulation by means of squeeze-preventing stocks should be possible.**

3. Conclusions and Extensions

The first four squeezes mentioned in Section 1 (see Table 1) occurred within a time range of three years (1987 - 1989), *i.e.* thirty-six months. A crude estimate of σ in this period is therefore $4/36 = 1/9 \approx 0.11$. For there to be any doubt that the producers should hold stocks to prevent the effects of a squeeze, the (month-to-month) interest rate should have been at least 33 pct. according to Proposition 4 of the Appendix.[12] According to the sufficient condition (see remark to Proposition 2), $r < (1/9)/(8/9) = 0.125$ is a sufficient condition for $s^* = \alpha^s$. This condition was definitely met in the mentioned period. (A 12.5 pct. monthly interest rate corresponds to an interest rate of approximately 310 pct. on a yearly basis). So why did the producers not introduce regulatory stocks (or other squeeze-preventing mechanisms, for that matter)?

---

[10] We assume that the price is positive so $\alpha_t + \gamma\alpha^s > 0$. Then note that $\gamma \leq 1$ and $(n-1)/(4n) < 1/4$ for all $n \geq 1$. The second, negative term takes its minimum for $s_D = \alpha^s$, but for this value it is a matter of manipulation to show that the entire expression is positive.

[11] For a more general treatment of asymmetric oligopolies, see Waldmann (1992).

[12] If $r = 0.332215$, $\sigma_4(r) = 0.111111$.

One partial answer is that they actually *did* release some cargoes in the December 1987 squeeze and a single cargo in the November 1989 squeeze was potentially made available, but these efforts were far from efficient - they "failed to prevent a serious distortion from developing in the spectrum of oil prices".

Another answer may be that the model may favour the incentive to keep stocks (or the model may be correct, but the producers do not realize this).

One feature of the model that may seem too strong is the assumption that $(\alpha^s - s)$ market participants (namely the squeezed traders/refineries) leave the market immediately after a squeeze has occurred. In reality, they might leave (or reduce the amount traded) gradually over time because it takes time for traders to develop other markets to work in or for refineries to find other crudes to substitute Brent. It may also very well be that the market participants get uptight about the market immediately after a squeeze and contemplate to leave it for a while, but then relax and continue trading. Indeed, it seems boundedly rational that the same company can squeeze twice within eight months: the traders, knowing the squeezer from the first squeeze, should think that trading with him again may well mean trouble. At any rate, if the producers think that the market will not lose "customers," their incentive to assuage the effects of a squeeze is of course non-existent.

On the other hand, the increasing popularity of the International Petroleum Exchange (IPE) of London may be a response to the malfunctioning of the 15-Day market. The IPE trades futures in units of 1.000 barrels of Brent blend, *i.e.* 1/500'th of the size of the 15-Day contract. The IPE is a traditional futures market and thus features all the regulations you expect from such a market. The producers have traditionally favoured the 15-Day market - but maybe their customers more and more prefer to trade on the IPE?

Further to this, our model supposes that a squeeze can only happen once. It actually happens frequently. Including this in the model would require a genuinely dynamic model (the demand intercept would be a non-increasing stochastic variable), but ceteris paribus our one-squeeze model would tend to underplay the role of stocks compared to a frequent-squeeze model.

Another feature of the model that may seem to favour stocks is the cost of storage. We have assumed that this cost only arises from the interest on the value of the stocks - what it costs to keep the crude off the market. In the real world, there may be substantial costs to keeping cargoes afloat or to renting tanks in Rotterdam. This would reduce the incentive to hold stocks. Stocks are furthermore used for a variety of other purposes, including strategic motives in games between the producers (on this, see Møllgaard and Phlips (1992)), and producers seem to be secretive about the size of their stocks. (Our model actually requires that the size of the stocks be common knowledge).

Finally, an argument that weighs against self-regulation is that some major producers are part of integrated oil companies: Squeezes may hurt independent refineries without affecting integrated refineries. In the long run, this will reduce competition among refineries and benefit "survivors": the integrated companies.

The model presupposes that the size of a potential squeeze and the probability of a squeeze are known. In reality, there might be more uncertainty involved. Introducing probability distributions over squeezes of any size (non-negative numbers of cargoes, say), would not alter the conclusions qualitatively.

The real world is of course much more complex and dynamic than the model of Section 2: demand and interest rates vary with time; the producers do not form a **symmetric** oligopoly; integrated oil companies complicate matters further. However, it is our belief that the model and the surrounding analysis shed light on the problem. The conclusion is clear: Self-regulation *is* possible.

Appendix: Necessary and Sufficient Conditions, Parameter Space and Non-Negativity

s^* designates optimal stocks.

Proposition 3: Necessary and sufficient conditions:

$$
\begin{aligned}
s^* &= \alpha^s \quad \Leftrightarrow \quad \nu\alpha^s + \mu\alpha_t \geq 0. \\
s^* &= 0 \quad \Leftrightarrow \quad \nu\alpha^s + \mu\alpha_t \leq 0,
\end{aligned}
\tag{A.1}
$$

where $\nu = \gamma^2 r - \sigma$ and $\mu = 2(\gamma r + \sigma)$.

Proof: $K(s)$ is globally convex in s:

$$\frac{d^2K(s)}{ds^2} = 2\left(\gamma^2 + \frac{\sigma}{r}\right) > 0, \quad \forall \sigma, r > 0. \tag{A.2}$$

Thus it suffices to compare the values of $K(s)$ at the lower and at the upper bounds for s, *i.e.* $K(0)$ and $K(\alpha^s)$. It is easily checked that

$\nu\alpha^s + \mu\alpha_t > 0 \Rightarrow K(\alpha^s) > K(0) \Rightarrow s^* = \alpha^s$; and $s^* = \alpha^s \Rightarrow K(\alpha^s) \geq K(0) \Rightarrow \nu\alpha^s + \mu\alpha_t \geq 0$.

$\nu\alpha^s + \mu\alpha_t < 0 \Rightarrow K(\alpha^s) < K(0) \Rightarrow s^* = 0$; and $s^* = 0 \Rightarrow K(\alpha^s) \leq K(0) \Rightarrow \nu\alpha^s + \mu\alpha_t \leq 0$.

$\nu\alpha^s + \mu\alpha_t = 0 \Rightarrow K(\alpha^s) = K(0) \Rightarrow \{s^* = 0 \vee s^* = \alpha^s\}$. ■

We state the following three lemmas without proof. (A parenthical remark on notation: $\sigma = \sigma(r)$ is taken to mean that $(r,\sigma) = (r,\sigma(r))$. Similarly, $\sigma \in \,]\underline{\sigma}(r), \sigma(r)[$ is taken to mean that (r,σ): $\underline{\sigma}(r) < \sigma < \sigma(r)$.

Lemma 1: $\{\sigma = \sigma_1 \vee \sigma = \sigma_2\} \Leftrightarrow \nu = 0$, where

$$
\begin{aligned}
\sigma_1 &= \frac{1 + 2r^2 - 2r^3 - \sqrt{1 + 4r^2 + 4r^3}}{2r(1 + r)^2} \\
\sigma_2 &= \frac{1 + 2r^2 - 2r^3 + \sqrt{1 + 4r^2 + 4r^3}}{2r(1 + r)^2}.
\end{aligned}
\tag{A.3}
$$

Lemma 2: $\sigma = \sigma_3 \Leftrightarrow \mu = 0$, where $\sigma_3 = \dfrac{r^2}{1 - r - r^2}$.

Lemma 3: $\{\sigma = \sigma_4 \vee \sigma = \sigma_5\} \Leftrightarrow -\dfrac{\mu}{\nu} = 1$, where

$$\sigma_4 = \frac{-\frac{1}{2} - r + r^3 - \sqrt{\frac{1}{4} + r + r^2 - r^3 + r^4}}{r(1 - r)^2}$$

$$\sigma_5 = \frac{-\frac{1}{2} - r + r^3 - \sqrt{\frac{1}{4} + r + r^2 - r^3 + r^4}}{r(1 - r)^2} \qquad \text{(A.4)}$$

Lemma 4: $\sigma_2 > \frac{r}{1 + r} > \sigma_4 > \sigma_3 > \sigma_1 > \sigma_5 \,, \quad \forall r > 0.$

(See Figure A.1)

Proof of Lemma 4:

1) $\sigma_2 > r/(1+r)$:

$$r > 0$$

$$\Rightarrow 1 - \sqrt{1 - 4r^2 + 4r^3} > 0$$

$$\Rightarrow 1 + 2r^2 + 2r^3 - \sqrt{1 + 4r^2 - 4r^3} > 2r^2(1 + r)$$

$$\Rightarrow \sigma_2 > \frac{r}{1-r} \quad \square$$

2) $r/(1+r) > \sigma_4$:

$$r > 0$$

$$\Rightarrow (1 - r - r^2)^2 = 1 + 2r - 3r^2 + 2r^3 + r^4 > \frac{1}{4} + r - r^2 + r^3 + r^4 \equiv a$$

$$\Rightarrow 1 + r - r^2 > \sqrt{a}$$

$$\Rightarrow r^2(1 - r) > -1 - r + r^3 - \sqrt{a}$$

$$\Rightarrow \frac{r}{1 + r} > \sigma_4 \quad \square$$

3) $\sigma_4 > \sigma_3$:

Let $b = (\frac{1}{2} + \frac{3}{2}r - \frac{3}{2}r^2 + r^3 + r^4)^2$
and $c = a(1 - r + r^2)^2$.

It is easily checked that

$$r>0 \;\Rightarrow\; c > b > 0 \;\Rightarrow\; \sqrt{c} > \sqrt{b} \;\Rightarrow\; \sigma_4 > \sigma_3 \quad \square$$

4) $\sigma_3 > \sigma_1$: Follows from expanding the expressions and cancelling terms.

5) $\sigma_1 > \sigma_5$:

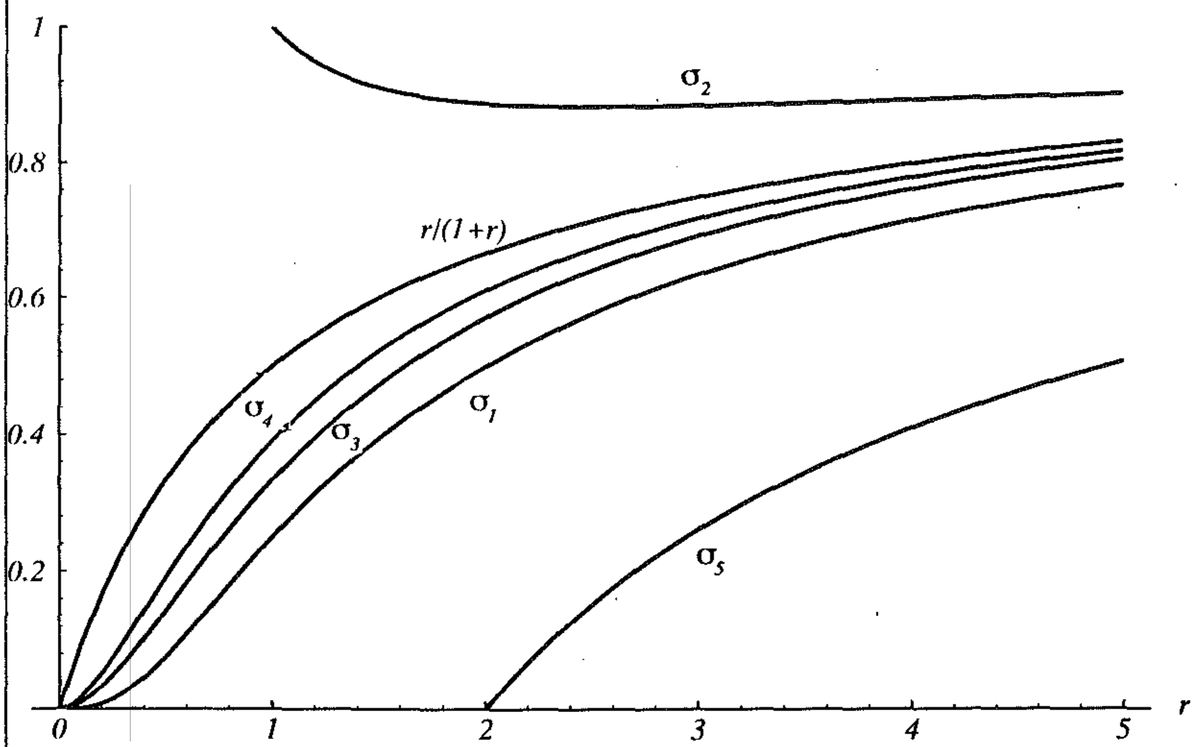
$$r > 0 \Rightarrow 1 + r^2 - \sqrt{\frac{1}{4} + r^2 + r^3} > r - \sqrt{\frac{1}{4} - r + r^2 + r^3 + r^4}$$

$$\Rightarrow 1 + 2r^2 + 2r^3 - \sqrt{1 + 4r^2 - 4r^3} > -1 - 2r - 2r^3 - 2\sqrt{\frac{1}{4} - r - r^2 + r^3 + r^4}$$

$$\Rightarrow \sigma_1 > \sigma_5 \quad \square$$

Transitivity of > completes the proof of Lemma 4. ■

Figure A1: The (r,σ)-Plane



Proposition 4: Division of the Parameter Space (See Figure A.2)

$$\forall\sigma \in [\sigma_4;1],\ s^* = \alpha^s. \tag{I}$$

$$\forall\sigma \in]\sigma_3;\sigma_4],\ 0<-\frac{\mu}{\nu}\leq 1 \Leftrightarrow \left\{ s^* = \alpha^s \Leftrightarrow \frac{\alpha^s}{\alpha_t} \leq -\frac{\mu}{\nu},\ s^* = 0 \Leftrightarrow \frac{\alpha^s}{\alpha_t} \geq -\frac{\mu}{\nu} \right\} \tag{II}$$

$$\forall\sigma\geq 0\ \ \sigma\in[\sigma_5,\sigma_3],\ s^* = 0. \tag{III}$$

$$\forall\sigma\geq 0\ \ \sigma\in[0;\sigma_5],\ 0\leq -\frac{\mu}{\nu}\leq 1 \Leftrightarrow \left\{ s^* = \alpha^s \Leftrightarrow \frac{\alpha^s}{\alpha_t} \geq -\frac{\mu}{\nu},\ s^* = 0 \Leftrightarrow \frac{\alpha^s}{\alpha_t} \leq -\frac{\mu}{\nu} \right\} \tag{IV}$$

Proof: The proof follows the division (I, II, III and IV) of the Proposition.

(I) We divide this region of the parameter space into two sub-regions:

(I.a) $\sigma \in [\sigma_4\ ;\ r/(1+r)]$

(I.b) $\sigma \in [r/(1+r)\ ;\ 1]$.

In region (I.a), $\gamma < 0$, $\nu < 0$, $\mu > 0$ and $-\mu/\nu \geq 1$. The necessary and sufficient condition (A.1) for stocks (*i.e.* $s^* = \alpha^s$) becomes

$$\frac{\alpha^s}{\alpha_t} \leq -\frac{\mu}{\nu}\ .$$

But the left hand side is by Assumption 2 smaller than or equal to one and the r.h.s. greater than or equal to one so the condition is always met.

Region (I.b) is exactly the sufficient condition of Proposition 2.

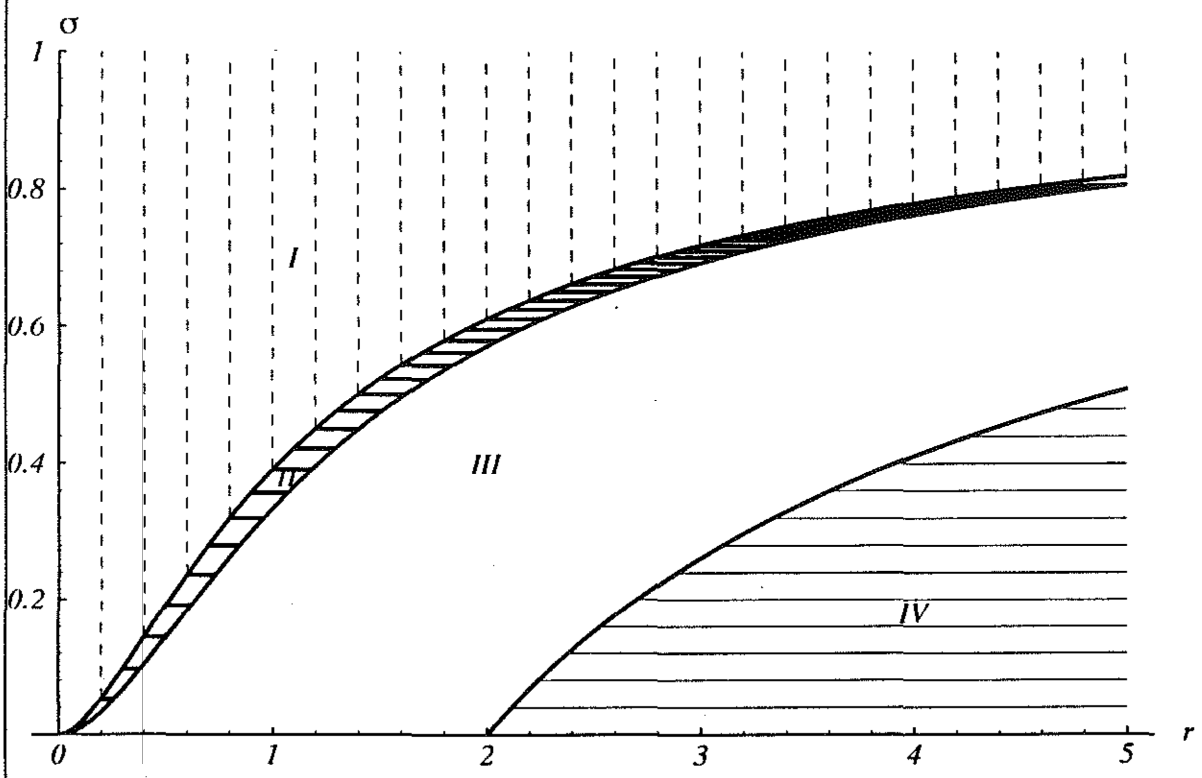
(II) In this region we have $\gamma < 0$, $\nu < 0$, $\mu > 0$ and $0 < -\mu/\nu \leq 1$ and so the necessary and sufficient condition is binding.

(III) We divide this region into two subregions:

(III.a) $\sigma \in [\sigma_1\ ;\ \sigma_3]$

(III.b) $\sigma \in [\sigma_5\ ;\ \sigma_1[$.

Figure A2: Regions in the (r,σ)-Plane



Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

(III.a): Here we have $\nu \leq 0$, $\mu \leq 0$ and the necessary and sufficient condition for $s^* = \alpha^s$ becomes

$$\frac{\alpha^s}{\alpha_t} < -\frac{\mu}{\nu} < 0$$

for $\nu < 0$ (off σ_1) and $\mu \geq 0$ for $\nu = 0$ (on σ_1), both of which are impossible. So we have $s^* = 0$.

(III.b): In this region $\nu > 0$, $\mu < 0$ and $-\mu/\nu \geq 1$, so the necessary and sufficient condition for $s^* = \alpha^s$ is $\alpha^s/\alpha_t > -\mu/\nu$, which is impossible by Assumption 2.

(IV): $\nu > 0$, $\mu < 0$ and $0 \leq -\mu/\nu \leq 1$, so the necessary and sufficient condition for $s^* = \alpha^s$ is $\alpha^s/\alpha_t > -\mu/\nu$, which is binding. Note, however, that $s^* = \alpha^s$ gives rise to a negative price, *cf.* below. ■

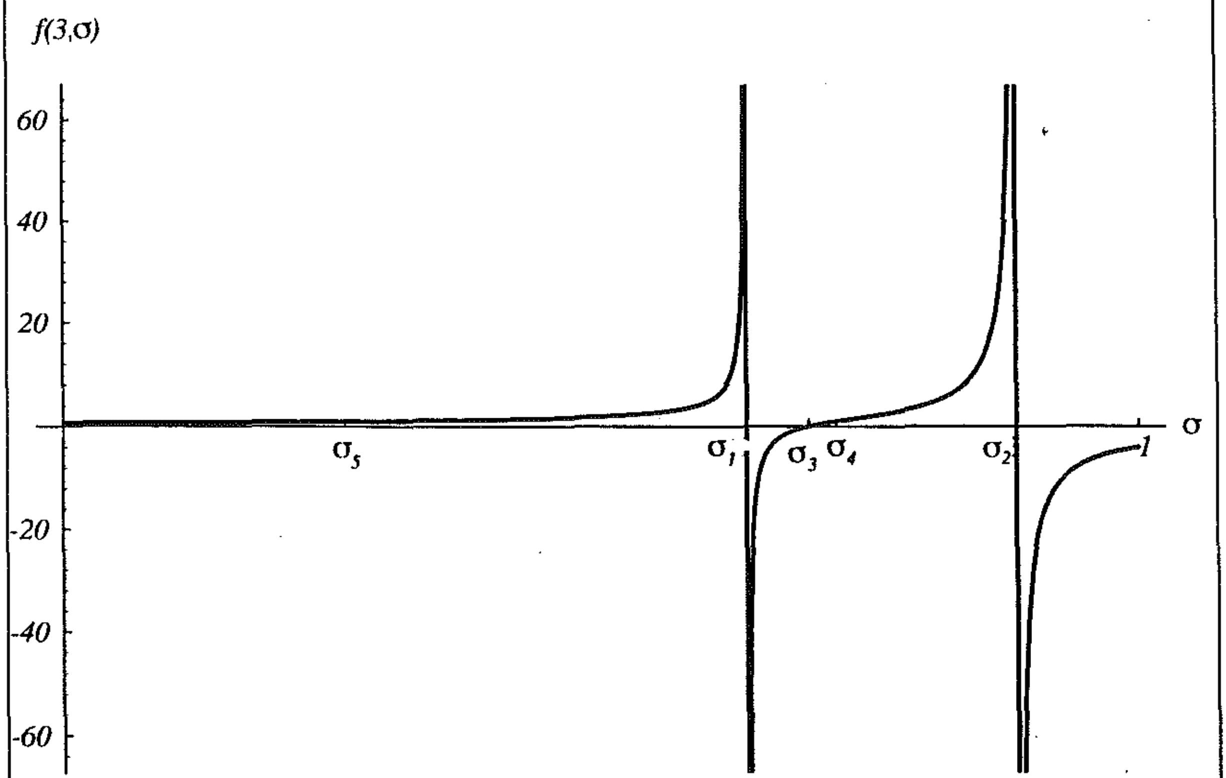
Remark: The reader may by now wonder what happened to σ_2. It is hidden in region (I.b):

$$\forall \sigma \leq 1: \ \sigma \in [\sigma_2;1], \ \nu > 0, \ \mu > 0 \text{ and } \nu\alpha^s + \mu\alpha_t$$

will always hold true. For $\sigma \in [r/(1+r); \sigma_2[$ we have $\nu < 0$, $\mu > 0$ and $-\mu/\nu > 1$ so the condition for $s^* = \alpha^s$ is always satisfied.

Remark: The function $-\mu/\nu = f(r,\sigma)$ has singularities along σ_1 and σ_2, where $\nu = 0$. Fix r at, say, $r = 3$, and consider the function as σ goes from 0 to 1 (See Figure A.3): As σ goes from 0 to σ_5, $f(3,\sigma)$ goes from $2/3$ to 1. As σ goes from σ_5 to σ_1, $f(3,\sigma)$ goes from 1 to ∞. As σ goes from σ_1 to σ_3, $f(3,\sigma)$ goes from $-\infty$ to 0. As σ goes from σ_3 to σ_4, $f(3,\sigma)$ goes from 0 to 1. As σ goes from σ_4 to σ_2, f goes from 1 to ∞. Finally, as σ goes from σ_2 to 1, f goes from $-\infty$ to -4.

Figure A3: f(r = 3, σ)



Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

Non-Negativity:

Proposition 5: Production is always positive.

Proof:

$$x = \frac{1}{2}(\alpha_t - \gamma s) > 0 \quad \Leftrightarrow \quad \alpha_t > \gamma s \quad \Leftrightarrow \quad \alpha_t - \sigma s > -(1 - \sigma)rs \,,$$

but the l.h.s. is always positive and the r.h.s. always non-positive.■

Lemma 5: $\forall(\sigma,r)\in[0,1]\times\mathbb{R}_+,\ \gamma < -1:\ -\frac{1}{\gamma} < -\frac{\mu}{\nu}.$

Proof: $\left\{\gamma < -1 \Leftrightarrow \sigma < \frac{r-1}{r+1}\right\} \Rightarrow \{\eta > 0 \,, \mu < 0\} \Rightarrow \left\{-\frac{1}{\gamma} < -\frac{\mu}{\eta} \Leftrightarrow \eta < \gamma\mu\right\}$

$$\sigma < \frac{r^2}{2+r-r^2} \Rightarrow \gamma r - 2\sigma < 0 \Rightarrow \gamma(\gamma r + 2\sigma) > 0 > -\sigma \Rightarrow \gamma\mu > \eta.$$

But the condition was that $\sigma < \frac{r-1}{r-1} < \frac{r^2}{2+r+r^2}$, which then proves sufficient.■

Proposition 6: In region (IV) (*i.e.* $\{(\sigma,r)\in[0;1]\times[2;\infty]:\ \sigma \le \sigma_5(r)\}$), $s^* = \alpha^s$ implies a negative price.

Proof: By Proposition 5 we have that $s^* > \alpha^s \Leftrightarrow \frac{\alpha^s}{\alpha_t} > -\frac{\mu}{\eta}$.. Non-negativity of prices

$p = \frac{1}{2}(\alpha_t + \gamma\alpha^s) > 0 \Rightarrow \frac{\alpha^s}{\alpha_t} < -\frac{1}{\gamma}$ $(\gamma < 0)$, but by Lemma 5 we have $-\frac{1}{\gamma} < -\frac{\mu}{\eta}$, so prices are negative if $\frac{\alpha^s}{\alpha_t} > -\frac{\mu}{\eta}$.■

Remark: The reason for this oddity is that the expected sales less the expected real cost of storage become largely negative due to very high interest rates (r > 200 %) and very large stocks. Multiplied by a negative price, profits become positive. The choice of $s = 0$ always leads to positive prices, quantities and profits.

CHAPTER 4:

Bargaining and Efficiency in a Speculative Forward Market

 DOI: 10.2870/54657

Chapter Outline

ABSTRACT: The 15-Day forward market for Brent crude oil is predominantly speculative. Transactions on this market thus contradict the assumptions that lead to zero speculation theorems. We set up a stochastic game model of a market with a small number of speculative traders that differ only with respect to the expected spot price and (possibly) with respect to risk aversion. Contracting is done after pairwise negotiations in random matches. The Markov perfect equilibrium of the model can mimic the 15-Day market and need not be efficient in the sense of belonging to the bilateral core.

0. Introduction

How do the participants in the 15-Day market for Brent blend crude oil get to agree upon forward contracts?

The 15-Day market is a peculiar decentralized market institution in which a homogeneous commodity is traded forward. The main purpose of this chapter is to model the trading procedure and to assess whether the outcome of this procedure is efficient (in a sense to be made precise).

There are other market institutions that could serve the same purposes. The standard futures market is a case in mind. In contrast to the 15-Day market, this market institution is centralized. It is *a priori* more likely to lead to efficient outcomes since market clearing is made easy by the presence of a clearing house and since information is conveyed on a continuous basis. The International Petroleum Exchange (IPE) of London enjoys these characteristics but trading of Brent blend stops too soon before maturity to be a good substitute for the forward market. This problem could certainly be remedied as has been the case elsewhere and for other commodities. (In)divisibility is also an issue: The 15-Day market operates in terms of cargoes of a standard size of 500,000 barrels, whereas the IPE contracts are specified in lots of 1,000 barrels.

Here the focus is on the trading *procedure* of the 15-Day market institution. The institutional features of this market will help specify a realistic model. These features are outlined in Section 1, which also illustrates a typical market outcome. We proceed by specifying the main characteristics of an oil trader in Section 2. Section 3 discusses a very simple model of two such traders to give us the feel for the problem. Section 4 goes on to the more complicated three person version of the game in which the problems of the general n-person model can be illustrated and exemplified. Section 5 generalizes to an $n \geq 2$ person, finite time horizon game of pairwise bargaining after random matches and analyses whether efficiency can be achieved under the more realistic specification of bargaining options. Section 6 concludes by discussing convergence to efficiency and by comparing the decentralized forward market with a centralized futures market.

The main conclusion is that the model catches some important circumstances affecting 15-Day traders and that it can mimic the observed outcome of the 15-Day forward market. The outcome is inefficient compared to a centralized market institution, but there is convergence to the efficient outcome as the time horizon gets longer. Measured against the standard model of a futures market, the 15-Day institution is inferior. Since a futures market (the IPE) exists and since the futures market institution has long been a well known way of organizing the economic functions that the 15-Day market accomplishes, it remains a paradox that the market participants accept such an inferior market institution.[1]

1. The Characteristics of the 15-Day Market

Traditionally thought of as London-based, the 15-Day market cannot be said to have a proper home. However, many traders have their offices in London and most price reporting services are located there. The early development of the markets for North Sea crude oil is described in detail in Mabro *et al.* (1986).

For the purpose of this chapter, a short account of the **salient features of the 15-Day market** institution will suffice:

1) **Finite time horizon**: A contract for oil to be delivered in a given month[2] can only be traded during a given time interval. In order not to go into the complicated details regarding the timing of forward and spot transactions (on this, see Phlips (1992)), as a good approximation it is assumed that a given forward contract is traded during the last two and a half months prior to delivery. This allows around fifty trading days.

2) **Limited number of players**: "Though the number of market participants in 1980-85 exceeded 110, we found that the number of continually active players was of the order of 30-35, and that the 10-15 top participants accounted for most of the activity. The top five participants comprised four oil traders and one major oil company" (Mabro et al. (1986) p. xx).

3) **Pairwise negotiations**: The 15-Day market does not exist physically, but consists of a network of market participants trading via telephone.

4) **Standard contracts**: The contract is of a type where the quantity is prespecified (500.000 bl's) so that the contractors only have to fill in the agreed price. For each matching of two

[1] One explanation may be that the forward market automatically becomes the spot market in the sense that assignment of physical cargoes is done via chains of contracts that are left open when the forward market closes. This function could however be carried out in other ways. A centralization of this function would seem to make life easier for everybody involved in the market.

[2] The timing of delivery is not further specified. Only when the spot market opens will the delivery be known with more precision (namely within a three day delivery range). It is this assignment of different buyers to different slots that would be made easier with a centralized agency (cf. note 1).

traders this reduces the dimension of the bargaining problem from two to one: from (price x quantitity) to (price). The contract is binding in the sense that once it is telexed and therefore legally confirmed, the contractors cannot undo it before maturity and may thus end up with obligations to buy or sell oil on the spot market of the relevant forward month.[3]

5) **The clearing mechanism** is important for the same reason and involves two types of transactions: bookouts and daisy chains. A *bookout* is a situation where a number of players (at least two) can construct a circle of contracts (A sells to B, B to C, C to .., .. to A) and decide to close out by purely financial transfers. A *daisy chain* is similar but the chain is not closed to a circle: a cargo is passed on along a chain of traders and the last trader in the chain takes delivery. A daisy chain thus involves a cargo of physical oil at the end.

In addition to the institutional features we note that there is a great deal of speculation going on in the market. Bacon (1986) notes that:

> "this market has been *de facto* dominated by speculative deals" (p.5)

and

> "[a]n important factor is the dispersion of expectations held by traders: if expectations about price movements are widely spread then the number of deals can be high" (p. iii).

> "All this activity (...) was greatly amplified by the involvement of speculators in the market who, by taking views on likely future prices, tried to make a margin on buying or selling short and then covering their positions at a later date. ... [T]here was also considerable variation in the total amount of trading month by month. We have suggested that an important part of this volatility in quantity was related to changes in the dispersion of expectations held by those speculating in the market. When there was consensus on the likely price outcome the opportunities for trading decreased and when views were very disparate the number of deals increased." (pp. 48-49).

Take this as sufficient evidence that

1) differences in expectations exist;

2) differences in expectations drive the speculative trade;

3) speculative trade dominates the 15-Day market.

The assumption that differences in expectations drive speculative trade is in straight contradiction with the zero-speculation or zero-trade theorems (see Milgrom and Stokey (1982) and Tirole (1982) or for a survey Geanakoplos (1992)) that follow from the rational

[3] This feature is what leads to the frequent occurrence of squeezes, *cf.* Chapter 2.

Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

expectations hypothesis or from an assumption about common knowledge of common priors. That differences in expectations drive speculative trade therefore needs justification which is given in Møllgaard (1993). For the purpose of this chapter, first note that the typical real world speculator seems to be much more confident about his own expectation (opinion) than game theory with common priors allows him to be. Second, even if people have the same information, this need not span a unique probability distribution leading to a unique spot price expectation (see also Kurz (1991)).

To characterize the forward market as entirely speculative is an obvious abstraction. In reality, there are four types of market participants, *viz.* non-integrated producers, non-integrated refineries, integrated oil companies and oil traders, but only the two first types of agents can be trusted to enter the market primarily for hedging purposes, and their overall significance is fairly limited. The physical production of Brent blend during a normal month is forty-two cargoes and hence hedging can explain no more than forty-two contracts[4]. The turnover on the 15-Day market is typically ten times this number. Integrated companies speculate about the future forward price and trade accordingly for tax purposes.[5] Oil traders live from speculation, so it is a fair claim that the forward market is predominantly speculative.

A typical outcome of the 15-Day market in terms of the number of contracts traded and the corresponding prices is illustrated by the trading of the September 1991 contract. Trading took place in the period from 21 June to 30 August 1991, see Figures 1 and 2. Figure 1 shows the price range of concluded deals on the different days of trading, while Figure 2 indicates the number of deals (each for a cargo of half a million barrels) on the same days. Needless to say, when only one deal was concluded on a given day, only one price obtained. This price is indicated with a dot in Figure 1. Figure 3 shows the price and the quantity (number of cargoes) traded on two consecutive trading days, July 24 and 25. The dataset does not provide an identification of the traders involved in any deal and it is thus difficult to know whether two cargoes sold at identical prices on a given day represent one or two trades.

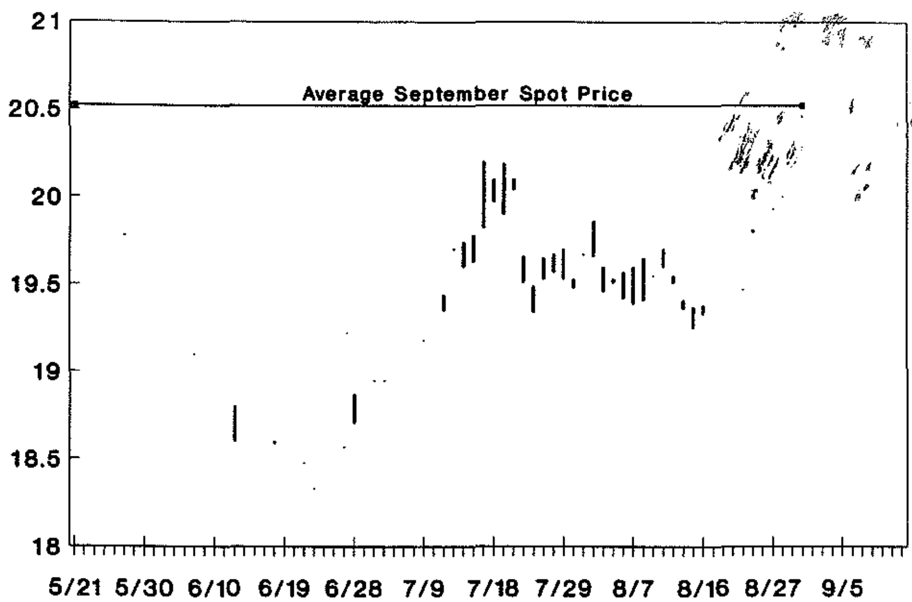
The figures confirm the impression that when a lot of trade is observed, then the price dispersion is also large. Note that no causality is implied, *a priori*, in this statement. However, in the model that follows, a wide dispersion of **spot price expectations** across traders will cause them to trade **larger quantities** on the forward market. Because of the decentralized nature of trading in the 15-Day market, this leads to a wider range of forward prices being

---

[4] This argument ignores possible hedging of other crudes for which Brent is a substitute. To a certain extent forward markets for other North Sea crudes exist, but they are relatively unimportant. It is well known from the theory of futures markets that you can hedge imperfectly in forward contracts for an imperfect substitute.

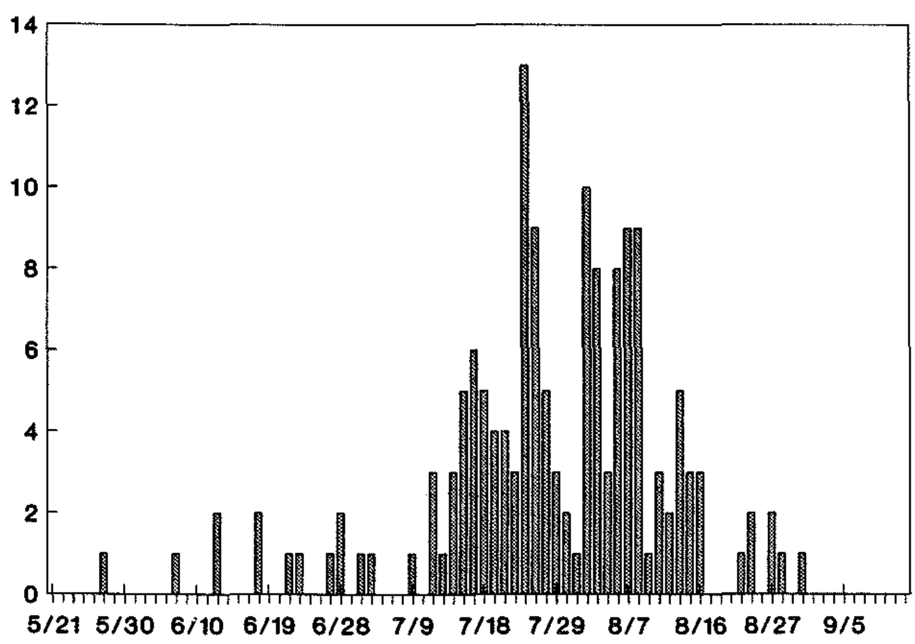
[5] See Bacon (1986) and Mabro et al. (1986) for a discussion of this. Clubley (1990, p. 33) notes that it is well known that the majors are speculative traders.

Figure 1: Selected Brent forward deals for September 1991 (High - Low)



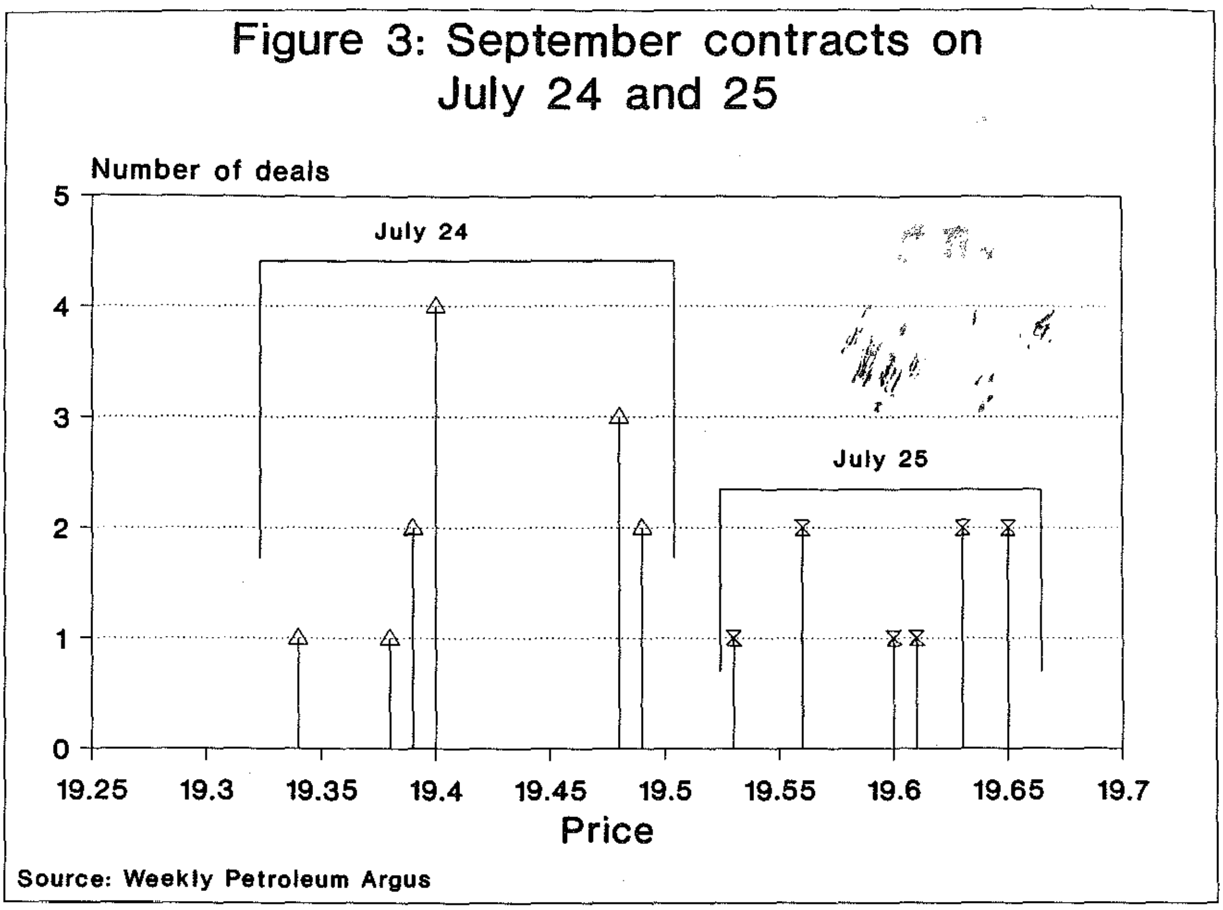
Source: Weekly Petroleum Argus

Figure 2: Number of September Deals



Source: Weekly Petroleum Argus

Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657



observed.

To capture the aforementioned features our model of the 15-Day market should include a finite number of players (traders/ speculators) that get matched pairwise and then bargain about the price of a unit of an indivisible good. They are only matched a finite number of times and since they are speculators they will want to close whatever open position they might have at maturity.

The model is built up step by step increasing the number of interacting agents from one to two to three to *n*.

2. A Simple, Speculative Oil-Trader

Assume that the entire forward market is speculative. This means that no market participant has any interest in obtaining the underlying good, but only in buying cheap contracts on the forward market in order to sell them later at an expected higher price, or conversely, in selling expensive contracts to buy later at an expected lower price. Speculation is thus "sell high, buy low" with either happening first. Through most of the chapter, it is aassumed that the forward market clears at maturity using the realization of the spot price. Accordingly, in this model, all contracts are supposed to be held till maturity. Some important problems regarding the clearing mechanism at maturity will be discussed later.

We employ two further assumptions of a technical nature: 1) The trader perceives the spot price as a normally distributed random variable; and 2) she is risk averse. To be specific, assume that her preferences can be represented by a utility function that is (negative) exponential in profits. The joint assumption of a normal spot price and an exponential utility function allows us to express the objective function according to the mean-variance model.

Thus the spot price, p^s, has a "subjective mean" p_I and is assumed to have a known variance $\sigma^2 = 1$ (this allows us to concentrate on differences in opinion about first order moments):

$$p^s \sim N(p_I, 1) \tag{1}$$

Trader I's **expected profit** from a forward transaction in period t is

$$\pi_t = (q_{I,t} - p_I) f_{I,t} \tag{2}$$

where $q_{I,t}$ is the price of the contract and $f_{I,t}$ the quantity sold. The expected profit from a given **contract** $c_{I,t} \equiv (q_{I,t}, f_{I,t})$ is positive if the forward price is higher than the mean spot price $(q_{I,t} > p_I)$ and if the trader sells forward $(f_{I,t} > 0)$. Conversely, if the forward price is lower than the expected spot price, the expected profit will be positive only if the trader goes long (*i.e.* buys forward: $f_{I,t} < 0$).

Let $h_I(\tau) \equiv (c_{I,1}, c_{I,2}, ..., c_{I,\tau})$ be trader I's history of trades from the first day of trading ($t = 1$) up to and including day τ. The expected profit that arises from this history is simply the sum of the single-deal expected profits, given that the trader's spot price expectation does not change over time:

$$\pi(h_I(\tau)) = \sum_{t=1}^{\tau} (q_{I,t} - p_I) f_{I,t} = V(h_I(\tau)) - p_I F(h_I(\tau)) \tag{3}$$

where $V(h_I(\tau)) = \sum_{t=1}^{\tau} q_{I,t} f_{I,t}$ is the (known) book value of the trader's forward position and $F(h_I(\tau)) = \sum_{t=1}^{\tau} f_{I,t}$ is the trader's net position. We consider a trading period short enough for discounting to be ignored. Given our assumptions, the objective function at time τ can be written

$$G(h_I(\tau); A_I, p_I) = \pi(h_I(\tau)) - \frac{A_I}{2} F^2(h_I(\tau)) \tag{4}$$

where A_I is the Arrow-Pratt measure of absolute risk aversion, here assumed to be constant. The trading stops at a known date T (≥ 1) and the *trader's objective* (TO) is thus:

$$Max_{\{h_I(T)\}} \; G(h_I(T);A_I,p_I) \tag{5}$$

Summing up, this section proposed the following assumptions:

A1 The typical forward market participant is a speculator.

A2 The forward market clears using the realization of the spot price at maturity.

A3 Each trader I perceives the spot price as a normal stochastic variable with mean p_I and variance 1.

A4 The trader has risk-averse preferences that can be presented by a utility function which is exponential in profits.

From these four assumptions we derived the trader's objective function which is quadratic in the net position F and we formulated the trader's objective (TO).

3. Two Traders with Differing Beliefs

Assume for the purpose of this section that the forward market consists of two traders. If they trade, it must be with each other. When they agree to a contract, they know that their spot price expectations differ. If they have identical spot price expectations, they do not want to trade since trading would expose them to a risk. We thus assume that agents are endowed with different spot price expectations. Given expectations and expected utility maximization, they will want to trade if expectations are sufficiently diverse. In the following, different "solutions" to the two players' problems are examined.

3.1 Axiomatic Approaches

Name the two agents I and J. Agent I expects the spot price to be p_I and J expects p_J; assume these expectations to be common knowledge. I and J agree that the spot price variance is one. They have risk preferences A_I and A_J respectively. Since they are the only two on the market, what one trader sells, the other must buy and we shall take f to signify I's short position (signed) which then is J's long position.

Arbitrarily assume $p_I > p_J$, set $T=1$ and/or suppress the time-index. The two TO's thereby become

$$Max_c \quad (q - p_I)f - \frac{A_I f^2}{2} \tag{6}$$

$$Max_c \quad -(q - p_J)f - \frac{A_J f^2}{2} \tag{7}$$

where $c = (q,f)$.

If I and J were price-takers, q would be exogenous to the traders and should somehow adjust to clear the market. Then the competitive equilibrium would be obtained, with

$$c = (q^*,f^*) = \left(\frac{A_J p_I + A_I p_J}{A_I - A_J} , \frac{p_J - p_I}{A_I - A_J}\right). \tag{8}$$

Here I buys from J a position that is proportional to the difference in spot price expectations at a price that is a weighted average of their price expectations, the weights being the risk aversion constants: If I is more risk averse, J's spot price expectation gets a higher weight and vice versa. The competitive equilibrium is illustrated in Figure 4.

With only two traders, or in general as long as the number of players is so small that any player perceives that she can influence the market outcome, price taking appears to be a highly artificial constraint on behaviour and the competitive solution does not carry much appeal, so we discard this approach.

Let us instead turn the problem upside down: Just studying the traders' objectives and without imposing further structure on the problem, what can be said about the set of contracts that they would agree to?

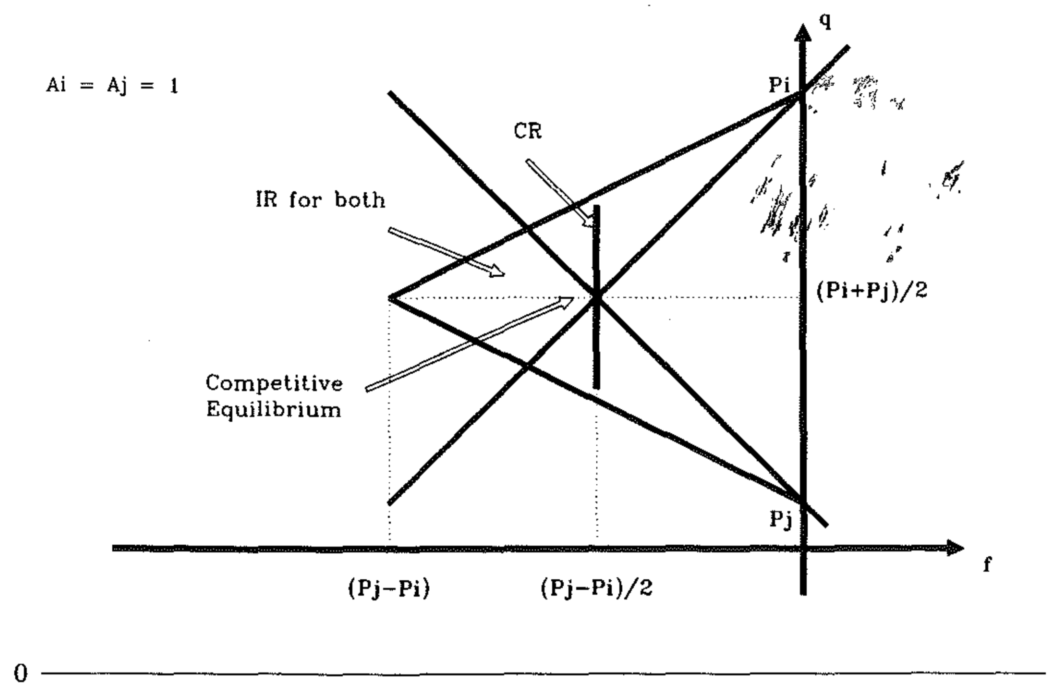
The minimum requirement on any contract must be that it contributes non-negatively to the contractors' payoffs, that is, that it be *individually rational* (IR) *for both*.[6] This condition is satisfied by contracts in the set

$$\{(q,0)\} \;\; U \; \left\{(q,f): f<0 \; , \; p_J - \tfrac{A_J}{2}f \le q \le p_I + \tfrac{A_I}{2}f\right\} \tag{9}$$

which is the vertical axis plus the big triangle in Figure 4. Any contract interior to the triangle is strictly preferred by both parties to no trade. A contract on the vertical axis trivially contributes zero to the expected payoff of both traders (zero volume, zero trade). A contract on the upper line ($q = p_I + \frac{1}{2}A_I f$) of the triangle would contribute zero to I's expected payoff, whilst a contract on the lower line ($q = p_J - \frac{1}{2}A_J f$) gives J zero expected payoff. We shall require that any contract be individually rational to both. This only amounts to saying that a

[6] Or *participation constrained.*

Figure 4: Competitive Equilibrium, IR Contracts and Bilateral Core



trader only contracts if she finds it advantageous, which seems a natural and minimal requirement.

A stronger requirement would be that the outcome belong to the *bilateral core*. This will be our qualitative *efficiency* measure throughout the chapter. The idea is that independently of the organization of the market, in order for the market outcome to be called efficient, it should belong to the bilateral core.

Definition: The ***bilateral core*** is the set of contracts that is coalitionally rational, where the set of permissible coalitions is restricted to all singletons and all pairs.

In other words, the bilateral core is the set of contract allocations that cannot be blocked by coalitions of one or two agents and that satisfy participation constraints. Note that in general this is a different requirement than Pareto efficiency. In the two-person case the bilateral core is identical to the core and a strict subset of the Pareto efficient allocations. For now it suffices to note that individual rationality of an outcome means that no singletons can form a blocking coalition and our two person case therefore reduces to finding the contract curve in the set of IR contracts. This can be done by equalizing the marginal rate of substitution between price, q, and quantity, f, for the two players. The bilateral core in the two person case is:

$$\left\{ c: f = \frac{p_J - p_I}{A_I + A_J},\ p_J - \frac{A_J}{2}f \le q \le p_I - \frac{A_I}{2}f \right\}. \tag{10}$$

It is illustrated in Figure 4 as the vertical bar in the triangle. The quantity is the same as that of the competitive solution, but the price can be anywhere in the interval between the seller's (J's) reservation price and the buyer's (I's) reservation price.

The question is whether we can rely on economic principles to restrict the solution to the bilateral core. This is one of the recurring issues of the chapter, so we should not expect an easy answer. Note that the competitive equilibrium belongs to the core, as should be. Another solution concept that has been applied to a problem of this type (see Brianza, Phlips and Richard (1990) p. 13) is the Nash-Bargaining point. Provided that the disagreement event is taken to be that no contract is telexed, the generalized (asymmetric) Nash bargaining solution which assigns weight ("bargaining power") α to I ($\alpha \in [0;1]$) can be written as

$$(q,f) = \left(w_I p_I + w_J p_J ,\ \frac{p_J - p_I}{A_I - A_J} \right) \tag{11}$$

where $w_I = \frac{1}{2}\left[\frac{A_J}{A_I + A_J} - (1-\alpha)\right]$ and $w_J = \frac{1}{2}\left[\frac{A_I}{A_I - A_J} - \alpha\right]$. This generalized solution contains the original Nash point as the special case where $\alpha = ½$:

$$q = \frac{(3A_J + A_I)p_I + (3A_I + A_J)p_J}{4(A_I + A_J)}, \tag{12}$$

where the price expectation of the less risk averse agent gets the higher weight. This price will coincide with the competitive price (8) only if $A_I = A_J$. Otherwise, the difference between the competitive price and the symmetric Nash bargaining price will be:

$$\frac{1}{4}(p_I - p_J)\frac{A_J - A_I}{A_I - A_J} > 0 \quad \textit{iff} \quad A_J > A_I , \tag{13}$$

so that the seller (J) is better off with competitive pricing than with Nash bargaining if she is more risk averse than the buyer (I) and *vice versa*.

The generalized solution (11) belongs to the bilateral core (10). Indeed, the bilateral core is equivalently described by (11) letting α run from zero to one.

The weights in w_I and w_J in (11) have the following interpretation in addition to what was said about (12): The stronger I is (*i.e.* the higher α is), the closer is the price of the contract to J's reservation price as given by (10). Conversely, the stronger J is, the closer q gets to I's reservation price. Generally, the stronger an agent is relative to the other, the more the forward price will reflect the other's spot price expectation.

Since the Nash bargaining approach is axiomatic, the solution by construction enjoys some nice properties: it is invariant to equivalent utility representations; it is symmetric if the problem is symmetric; it is independent of irrelevant alternatives; and it is Pareto efficient. The joint effect of these four axioms is even a ***unique*** outcome, but for our purpose we cannot use the approach since we would then assume what we want to show (efficiency of the outcome). Axiomatic approaches are silent on matters of timing and procedures and we therefore adopt a non-cooperative strategic bargaining approach to analyse whether and under which conditions the outcome of the given market institution is efficient.

3.2 Two Traders Bargaining Strategically

The strategic bargaining model as treated in Osborne and Rubinstein (1990) assumes that the players haggle over the price of a fixed quantity, taking turns in offering a price to the other and rejecting or accepting this offer. In the terminology of Rubinstein (1992), they attempt to partition a pie of a given size and once they agree on a partition, the game is over. Thus in our model, if the players are allowed to enter precisely one contract with a prespecified quantity, then the traders only haggle about the price. Given the finite time horizon, the price will either equal the seller's reservation price if the buyer is the last to make an offer, or the buyer's reservation price if the seller offers last. The one to make the final offer is the winner, and the winner takes all. It is a general feature of strategic bargaining models, that the offerer skims the cream off by making an offer equal to the receiver's reservation value (appropriately defined), leaving the receiver just indifferent between accepting and rejecting. This feature is exploited repeatedly throughout the chapter.

However, the Osborne/Rubinstein (1990) approach is not immediately applicable to the problem at hand. First, contrary to Rubinstein's model and many other models, it is assumed here that players are infinitely patient but that the game has a finite time horizon. More importantly, we introduce indivisibilities and multiple trades: A contract c_t has a prespecified quantity. We take this as the indivisible unit for which the traders try to establish a price, and assume that in each period, a trader can enter at most one contract so that

$$f_t = \begin{cases} 1 & \text{if I sells a contract to J at t} \\ 0 & \text{if I and J cannot agree at t} \\ -1 & \text{if I buys a contract from J at t.} \end{cases} \tag{14}$$

Accordingly, if the two traders want to achieve a forward position of a given size, $F \geq 1$, at T, they will have to trade in at least F periods.

At any period, τ, agent I enters the period with accumulated expected payoff $G(h_I(\tau-1);A_I,p_I)$ and exits with accumulated payoff $G(h_I(\tau);A_I,p_I)$, so the incremental single period expected payoff is

$$g(h_I(\tau);A_I,p_I) = f_\tau\left(q_\tau - p_I - \frac{A_I}{2}f_\tau - A_I F_I(h_I(\tau-1))\right), \tag{15}$$

which depends on history only through the accumulated net position. If no contracting is done in period τ, then $g(h_I(\tau);A_I,p_I) = 0$. If a contract is sold ($f_\tau = 1$) or bought ($f_\tau = -1$), the price must be individually rational, *i.e.*

$$\begin{aligned} f_\tau &= \ \ 1 \Rightarrow q_\tau \geq p_I + \frac{A_I}{2} + A_I F(h_I(\tau-1)) \equiv RS_I(\tau) \\ f_\tau &= -1 \Rightarrow q_\tau \leq p_I - \frac{A_I}{2} + A_I F(h_I(\tau-1)) \equiv RB_I(\tau). \end{aligned} \tag{16}$$

We shall use $RS_I(\tau)$ and $RB_I(\tau)$ as a convenient short-hand for I's myopic[7] reservation prices as a seller and as a buyer, respectively, and bear in mind that they evolve over time, depending in the shown way on I's forward position in the preceding period. If the trader is already net short ($F(h_I(\tau-1)) > 0$) this raises the selling reservation price but it also raises the maximum price at which she is willing to buy. This is so, since an additional unit sold increases the riskiness of the overall position, whilst buying a unit reduces the short position and thereby the risk exposure. Figure 5 illustrates this for trader I and for a given τ: the intercept of the line with the q-axis is $p_I + A_I\, F(h_I(\tau-1))$. Only when $F(h_I(\tau-1)) = 0$ will the intercept equal the trader's spot price expectation.

The intercept and the two vertical lines indicate the set of IR contracts to I given the indivisible unit. A convenient fact is

$$RS_i(\tau) = RB_i(\tau) + A_i\,, \quad i \in \{I,J\},\ \forall\tau. \tag{17}$$

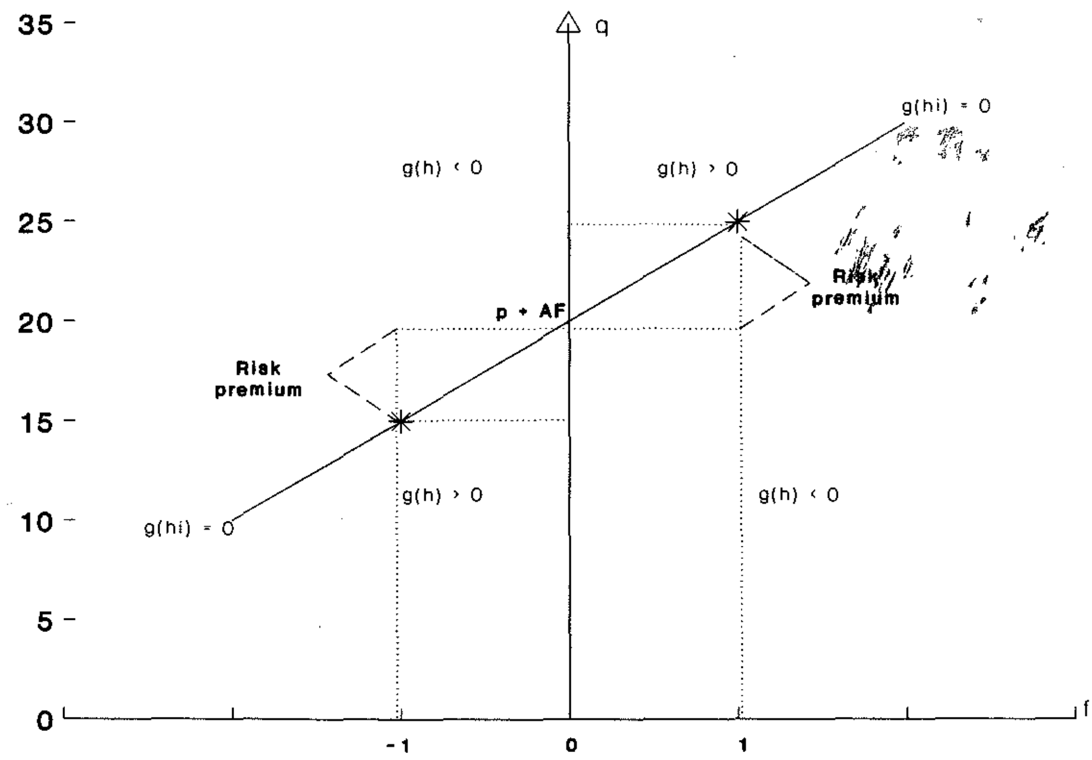
The following lemma commands full generality (that is to say it applies to all versions of the game independently of the number of players):

Lemma 1: $RS_I(\tau) \leq RB_J(\tau) \Rightarrow RS_J(\tau) > RB_I(\tau)$.

Proof: Follows directly from (17): $RS_I(\tau) = RB_I(\tau) + A_I \leq RS_J(\tau) = RB_J(\tau) - A_J$ $\Rightarrow RS_J(\tau) \geq RB_I(\tau) + A_I - A_J > RB_I(\tau)$ ■

[7] Myopic since it does not take future payoffs into account. A forward looking reservation price would do this and thus require backward recursion or, equivalently, subgame perfection. The price (20) is a forward looking reservation price in the two-player case.

Figure 5: A Risk Averse Trader



Lemma 1 simply states that if there exists a contract where I sells to J that both players find IR, then there does not at the same time exist a contract where J sells to I that both players find IR. The next lemma gives a dynamic extension of Lemma 1:

Lemma 2: Assume that two players, I and J, are matched in period τ-1. Then $RS_I(\tau\text{-}1) \leq RB_J(\tau\text{-}1) \Rightarrow RB_I(\tau) \leq RS_J(\tau)$.

Proof: If the two players do *not* enter a contract at τ-1, then $RS_I(\tau) = RS_I(\tau\text{-}1)$ and $RB_J(\tau) = RB_J(\tau\text{-}1)$ (because $F(h_I(\tau)) = F(h_I(\tau\text{-}1))$) and the conclusion follows directly from Lemma 1 (with strict inequality). If the players *do* enter a contract at τ-1, then $F(h_I(\tau\text{-}1)) = F(h_I(\tau\text{-}2)) + 1$ and $F(h_J(\tau\text{-}1)) = F(h_J(\tau\text{-}2)) - 1$, so $RS_I(\tau) = RS_I(\tau\text{-}1) + A_I$ and $RS_J(\tau) = RS_J(\tau\text{-}1) - A_J$. However, from (17) it then follows that $RB_I(\tau) = RS_I(\tau\text{-}1)$ and $RS_J(\tau) = RB_J(\tau\text{-}1)$. Since the condition of the Lemma is a weak inequality, the conclusion will also be a weak inequality. ■

Lemma 2 states that if, in period τ-1, there are gains from a trade where I sells to J, then there cannot be gains from a trade where J sells to I in period τ, given that the two agents are matched in τ-1. If they were not matched in τ-1 they could be matched to other agents in τ-1 and enter contracts that would disturb the conclusion. Here we concentrate on the two player case, where I and J are matched with each other in every period.

Remark: In the two player case, Lemma 2 implies that, in general, trade between I and J always goes in the same direction: I sells to J if $p_I < p_J$ and J sells to I if $p_I > p_J$.

Corollary: If $RS_I(\tau\text{-}1) = RB_J(\tau\text{-}1)$ and I and J enter a contract at $\tau\text{-}1$, then $RB_I(\tau) = RS_J(\tau) = RS_I(\tau\text{-}1) = RB_J(\tau\text{-}1) = q_{I,\tau-1} \equiv q_{J,\tau-1}$.

Remark: If, in the two player case, the situation of the corollary to Lemma 2 occurs, then the net positions in τ-*2* and τ-*1* are

$$F(h_I(\tau-2)) = \frac{p_J - p_I}{A_I + A_J} - \frac{1}{2} \text{ and } F(h_I(\tau-1)) = \frac{p_J - p_I}{A_I - A_J} + \frac{1}{2}$$

which must both be integers. In this case, the agents can cycle back and forth between these two positions (recall I's position is always the negative of J's), each time trading at the competitive price and each trade adding zero to both trader's expected payoff. In fact, the two positions are defined by requiring that the marginal trade adds zero to both players expected payoff.

Generically, the positions mentioned in the remark will not be integers, and we have the following proposition determining the maximum value of the open interest which is then the maximum number of deals (*ND*) ($\lfloor x \rfloor$ denotes the integer part of x):

Proposition 1: Assume without loss of generality that $p_I > p_J$. Generically, the *upper bound on the open interest* is

$$F(h_J(\tau)) = -F(h_I(\tau)) \leq \left\lfloor \frac{p_I - p_J}{A_I + A_J} + \frac{1}{2} \right\rfloor \equiv ND. \tag{18}$$

Proof: Define $\delta \equiv \frac{p_I - p_J}{A_I - A_J} + \frac{1}{2} - \left\lfloor \frac{p_I - p_J}{A_I + A_J} + \frac{1}{2} \right\rfloor$, $\delta \in [0,1)$, and assume that the number of

contracts already exchanged is $F(h_J(\tau)) = \frac{p_I - p_J}{A_I + A_J} - \frac{1}{2} - \delta$, which is then an integer. In

this case, $RS_J(\tau+1) = q + (1\text{-}\delta)A_J > RB_I(\tau+1) = q - (1\text{-}\delta)A_I$ and
$RS_I(\tau+1) = q + \delta A_I \geq RB_J(\tau+1) = q - \delta A_J$,
where q is the competitive price defined in equation (8) of Section 3.1. Only if $\delta=0$ can there be any trade. This is the special case discussed above. Otherwise, we have $RS_I(\tau+1) >$

$RB_J(\tau+1)$ and no trade is possible in $\tau+1$ or thereafter, which implies that all gains from trade have been exploited. ■

Now we complete the description of the rules of the game by the assumption that *when I and J are matched in period* τ*, I makes an offer to J with probability 1/2 and J makes an offer to I with probability 1/2. An offer takes the form of a contract, i.e. essentially a price at which the offerer is willing to buy or sell one unit. The receiver of the offer can take it or leave it; accept it or reject it. Then the game moves on to period* $\tau+1$.[8]

The following proposition outlines the backward recursion principle by which the game at hand can be solved by studying the effect of all possible actions in τ on current period payoffs and on continuation payoffs (*i.e.* the expected value of the ensuing subgame) and letting the agents maximize over these actions:

Proposition 2: Let $p_I > p_J$ and suppose that expected continuation payoffs are of the same form

$$v(\tau+1) = \frac{1}{2}\left(p_I - p_J - (A_I - A_J)(F(h_J(\tau))+\frac{1}{2})\right) \geq \frac{A_I - A_J}{4} \tag{19}$$

for both players. Then, if I is chosen to make an offer at τ, she will offer to buy a contract from J at the price

$$q_\tau = RS_J(\tau) + \frac{A_I + A_J}{2}, \tag{20}$$

which J will accept. If J is chosen to make an offer, she will offer to sell a contract to I at the price

$$q_\tau = RB_I(\tau) - \frac{A_I + A_J}{2}, \tag{21}$$

which I will accept. Thus, in both cases $F(h_I(\tau)) = F(h_I(\tau-1))-1$ and $F(h_J(\tau)) = F(h_J(\tau-1))+1$.

Ex ante, *i.e.* seen from period $\tau-1$, the expected payoff to both players will then be

[8] In terms of expected utility, the effect of the assumption that in any match either of the two agents has probability 1/2 of proposing to the other is the same as that of assuming Nash bargaining between the two agents in the match. This holds true also for the *n*-person game of Section 5 if the threat points are defined to be the continuation value to each of the agents of negotiations breaking down.

$$v(\tau) = \frac{1}{2}(p_I - p_J - (A_I - A_J)(F(h_J(\tau-1)) - \frac{1}{2})). \qquad (22)$$

Proof: $v(\tau+1) \geq (A_I + A_J)/4$ implies that $F(h_J(\tau)) \leq \dfrac{p_I - p_J}{A_I + A_J} - 1$, which ensures that there are IR contracts to be made in period τ. Consider the case in which I is chosen to make an offer to J. If J accepts, $F(h_J(\tau)) = F(h_J(\tau-1)) + 1$, if she rejects, $F(h_J(\tau)) = F(h_J(\tau-1))$. By hypothesis, J's expected continuation payoff in the case of acceptance is

$$v^a(\tau+1) = \frac{1}{2}(p_I - p_J - (A_I - A_J)(F(h_J(\tau-1)) - 1 + \frac{1}{2})),$$

whilst in the case of rejection it is

$$v^r(\tau+1) = \frac{1}{2}(p_I - p_J - (A_I - A_J)(F(h_J(\tau-1)) + \frac{1}{2})).$$

J then loses $v^r(\tau+1) - v^a(\tau+1) = (A_I + A_J)/2$ in terms of future payoff by accepting and I's offer should compensate J for this loss in order to induce acceptance. This can be done by offering a contract price such that $g(h_J(\tau)) = (A_I+A_J)/2 \Rightarrow q_\tau = RS_J(\tau)+(A_I+A_J)/2 \Rightarrow g(h_I(\tau)) = p_I - p_J + (A_I + A_J)(F(h_I(\tau-1)) - 1) \geq 0$ by the assumption that $v(\tau+1) \geq (A_I + A_J)/4$. This last assumption implies that it is not optimal for the offerer to construct an offer that is certain not to be accepted and wait for the other to make an offer. The case where J is chosen to make an offer to I follows a similar argument.

This shows that, seen from period $\tau-1$, with probability 1/2 a player will expect to get $p_I - p_J + (A_I + A_J)(F(h_I(\tau-1)) - 1) \geq 0$, and with probability 1/2 she will get $(A_I+A_J)/2$ summing to:

$$Eg(h_I(\tau)) = (p_I - p_J + (A_I + A_J)(F(h_I(\tau-1)) - 1/2))/2$$

which is the same as (20) because $F(h_I) = -F(h_J)$. ■

Remark: The expected period-τ price will be:

$$Eq(\tau) = \frac{p_I + p_J}{2} + \frac{A_J - A_I}{2}(F(h_J(\tau-1)) - \frac{1}{2}). \qquad (23)$$

If $A_I = A_J$, the expected price will be identical to the competitive price. If $A_I \neq A_J$, the expected price will show a trend, that will be positive if $A_J > A_I$ and negative in the other case. Particular realizations of the price will be as indicated in Proposition 2.

Equilibrium in this model is a situation in which I and J maximize $G(h_I(T))$ and $G(h_J(T))$ respectively. If the agents follow the recursive formula of Proposition 2, then equilibrium will be subgame perfect. There are two cases to be considered: If $T < ND$, there are not enough periods to allow the agents to reach the upper bound on the open interest, ND: They have to stop trading before all gains from trade are exploited. On the other hand, if $T \geq ND$, they can exhaust the gains from trade (under the restriction imposed by the indivisibility of a contract). The following lemma shows that they must exhaust all gains from trade if they can:

Lemma 3: Let $p_I > p_J$ and assume that $T \geq ND$. Then $F(h_J(T)) = ND$ is (generically) a necessary condition for equilibrium.

Proof: First assume that $F(h_J(T)) < ND$. Then $F(h_J(T)) \leq \frac{p_I - p_J}{A_I + A_J} - \frac{1}{2} - \delta$, where we used the fact that F is an integer. If $\delta \neq 0$, this implies that $RS_J(T) < RB_I(T)$ so that a contract could be set up which would give a non-negative incremental payoff to both traders and a strictly positive payoff to at least one, in contradiction with optimality. If $\delta = 0$, we could have the special case where the marginal contract gives both zero incremental profit. In this case $F(h_J(T)) = ND - 1$, is a necessary condition for equilibrium.

Hence $F(h_J(T)) \geq ND$. By Proposition 1, $F(h_J(T)) \leq ND$, so we have $F(h_J(T)) = ND$. ■

Proposition 3: Let $p_I > p_J$. Then a subgame perfect equilibrium exists in which I and J maximize $G(h_I(T))$ and $G(h_J(T))$ respectively. If $\boldsymbol{T \geq ND}$, in the first T - ND periods equilibrium has

$$F(h_I(\tau)) = 0 = F(h_J(\tau)) \tag{24}$$

$$v(\tau) = 0 \tag{25}$$

$$Eq(\tau) = \text{Undefined} \tag{26}$$

For both $\boldsymbol{T \leq ND}$ and $\boldsymbol{T \geq ND}$, define $u \equiv \tau - max\{T - ND , 0\}$. For $\tau \geq max\{T - ND , 0\}$ (*i.e.* $u \geq 0$), equilibrium has

$$F(h_I(\tau)) = -u = -F(h_J(\tau)) \tag{27}$$

$$v(\tau) = \frac{1}{2}\left(\left(p_I - \frac{A_I}{2}\right) - \left(p_J - \frac{A_J}{2}\right) - (u - 1)(A_I - A_J)\right) \tag{28}$$

$$Eq(\tau) = \frac{p_I - p_J}{2} - \frac{A_J - A_I}{2}\left(u - \frac{1}{2}\right). \tag{29}$$

Proof: We need to prove that we can apply to both cases the backward recursion of Proposition 2 by showing that in period T we get continuation payoffs of the form (19). If this is the case, then Proposition 2 showed in (22) that continuation payoffs in any earlier period take the same form. The rest of the proof follows from forward recursion from period 1 with $F(h_i(0)) \equiv 0$.

First consider the case in which $T \geq ND$. By Lemma 3 we know that $F(h_J(T)) = ND$. The trading must take place in the last ND periods. Before this, any trader can reject an offer hoping to become the first person to have an offer accepted sometime later, without sacrificing continuation payoffs, so it must be the case that $F(h_J(T-1)) = ND - 1$. Then reservation prices are $RS_J(T) = q^* - \delta A_J$ and $RB_I(T) = q^* - \delta A_I$. In period T, there is no continuation to worry about, so the winner takes all. Seen from period T-1, both players then expect to gain

$$\frac{1}{2}(RB_I(T) - RS_J(T)) = \delta\frac{A_I - A_J}{2}.$$

If $\delta \geq 1/2$ this satisfies the condition of Proposition 2 that $v(\tau+1) \geq (A_I + A_J)/4$. If $\delta < 1/2$, we regress one period. By assumption we have $F(h_J(T-2)) = ND - 2$, so T-1 reservation prices are $RS_J(T-1) = q^* - A_J(1 + \delta)$ and $RB_I(T-1) = q^* + A_I(1 + \delta)$. In case of rejection, the reservation prices remain unchanged in period T where the winner takes all, so in order to induce acceptance at T-1, the offerer should compensate the other with the amount

$$\frac{1}{2}(RB_I(T-1) - RS_J(T-1)) = \frac{1}{2}(1 - \delta)(A_I + A_J),$$

which will then be the T-1 incremental payoff to the acceptor. The offerer gets the same (the other half). Seen from period T-2, the continuation payoff is thus

$$v(T-1) = \frac{1}{2}(1 - \delta)(A_I - A_J) > \frac{A_I + A_J}{4},$$

so we can unravel the game from T-1 backwards.

We next consider the case in which $T < ND$. We then have $F(h_J(T-1)) = T$-1, so $RS_J(T) = p_J + A_J(T - 1/2)$ and $RB_I(T) = p_I - A_I(T - 1/2)$. Seen from T-1, continuation payoffs will be

$$v(T) = \frac{1}{2}(p_I - p_J - (A_I + A_J)(T - \frac{1}{2})).$$

$v(T) \geq (A_I + A_J)/4$ iff $T \leq (p_I - p_J)/(A_I + A_J)$, but this follows by the assumption that $T < ND$. ■

The equilibrium that is outlined in Propositions 1 through 3 can be interpreted as a

Markov Perfect Equilibrium:[9] If we take as state variable (i) the forward positions of the previous period, τ-1, (ii) the realization of who nature chooses to make an offer in the current period, τ, and (iii) time, τ, itself, then those are sufficient to determine what offer the agent should make and whether to accept or reject it in period τ, taking the expected value of future actions into account. Indeed, the payoff relevant history is summarized by $F(h_I(\tau-1)) = -F(h_J(\tau-1))$. Given a realization of the state variable, the offerer proposes a contract chosen such that the price exactly compensates the receiver for the loss in continuation payoff and leaves the receiver indifferent between accepting and rejecting. Equilibrium then requires her to accept. This is the essence of Proposition 2. Proposition 1 defines the ergodic states of the Markov chain that the Markov perfect equilibrium can be seen as: If the market reaches a state in which $F(h_J(\tau)) = ND$, then the two traders have no further gains from trade and will stop trading, so all possible future states will have this quality. Lemma 3 says that if possible ($T \geq ND$), the Markov chain should end up in an ergodic state. Proposition 3 then states that, if possible, the market ends up in an ergodic state, exhausting all gains from trade. This will happen in the last ND periods, so in the first T-ND periods, nothing happens: The forward positions stay zero, there are no realizations of the price since all offers are rejected without any loss of continuation payoffs. In the last ND periods, the J's forward position is increasing by one contract each period (and I's is similarly decreasing) thus reaching ND when the game stops at T. This quantity is efficient, *i.e.* in the bilateral core, given the indivisibility. Prices are evolving around a trend (see (23)) if $A_I \neq A_J$ or take values randomly above and below the competitive price if $A_J = A_I$. All realized prices have to be individually rational, so the equilibrium is efficient for $T \geq ND$. If $T < ND$, there is a trade in each period, so at the end of the game $F(h_J(T)) = T$ and prices evolve as in the other case. This equilibrium can by construction not be efficient, but given the time constraint the players get as close to efficiency as possible.

In short, the two trader game yields a unique Markov perfect equilibrium that is as efficient as possible given the time constraint. In this model the only uncertainty stems from who is going to make an offer in each period. The identity of the opponent is known with certainty. This is not the case with three or more players.

4. Three Traders with Differing Beliefs

This section extends the two-player results to three players as an appetizer to the n-person game. While the extension of the axiomatic approaches is relatively straightforward, the extension of the strategic approach to a dynamic, stochastic game proves to be trickier.

---

[9] See Definition 4 and Proposition 7.

4.1 Axiomatic Approaches and the Phlips-Harstad Model

It is a matter of simple algebra to extend the static analysis of Section 3.1 to a three trader market, thereby obtaining a simple version of the model in Phlips and Harstad (1991): In our model the traders on the forward market have no power in the spot market, whereas in the Phlips/Harstad model they do. However, this simplification has no importance for the discussion of the efficiency of the forward market.

A straightforward extension of the notation is needed: The traders are called I, J and K and f_{ij}, $i=I,J$, $j=J,K$, $i\neq j$, denotes the amount that i sells to j at price q_{ij}. If f_{ij} is negative, then i buys from j as before. Suppressing the time dimension, the payoff function for, say, I now reads

$$G(\bullet;A_I,p_I) = (q_{IJ} - p_I)f_{IJ} + (q_{IK} - p_I)f_{IK} - \frac{A_I}{2}(f_{IJ} + f_{IK})^2. \tag{30}$$

The competitive equilibrium is readily found to be

$$\begin{aligned} q^* &= (A_J A_K p_I + A_I A_K p_J + A_I A_J p_K)/D \\ f_I^* &= (A_J(p_K - p_I) + A_K(p_J - p_I))/D \\ f_J^* &= (A_K(p_I - p_J) + A_I(p_K - p_J))/D \\ f_K^* &= (A_J(p_I - p_K) + A_I(p_J - p_K))/D \end{aligned} \tag{31}$$

where $D = A_I A_K + A_I A_J + A_J A_K$.

The set of contracts that are individually rational for any two players, say I and J, is described by

$$\begin{aligned} &\left\{(q_{IJ},f_{IJ}) \mid f_{IJ}>0 \ , \ p_J + \tfrac{A_J}{2}(f_J + f_{JK}) \geq q_{IJ} \geq p_I - \tfrac{A_I}{2}(f_I + f_{IK})\right\} \cup \\ &\left\{(q_{IJ},f_{IJ}) \mid f_{IJ}<0 \ , \ p_J + \tfrac{A_J}{2}(f_J + f_{JK}) \leq q_{IJ} \leq p_I - \tfrac{A_I}{2}(f_I + f_{IK})\right\} \cup \\ &\left\{(q_{IJ},f_{IJ}) \ \ f_{IJ} = 0\right\} , \end{aligned} \tag{32}$$

where $f_I \equiv f_{IJ} + f_{IK}$ and $f_J \equiv -f_{IJ} + f_{JK}$. As before, this set describes a triangle with the q-axis. The three subsets are mutually exclusive (either f_{IJ} <,> or = 0) so a given contract can belong to at most one of them. The new feature is that the sides of the triangle are subjected to parallel shifts by $A_J f_{JK}$ and $A_I f_{IK}$ respectively that is, the quantities of the two other contracts matter, because they enter jointly with f_{IJ} in the respective agents' risk evaluations. This requires that any trade with K be common knowledge, or, in other words, that the three contracts be coordinated.

Solving for the bilateral core we find that the quantities should satisfy the following system:

$$\begin{pmatrix} p_J - p_I \\ p_K - p_I \\ p_K - p_J \end{pmatrix} = \begin{pmatrix} A_I + A_J & A_I & -A_J \\ A_I & A_I - A_K & A_K \\ -A_J & A_K & A_J + A_K \end{pmatrix} \begin{pmatrix} f_{IJ} \\ f_{IK} \\ f_{JK} \end{pmatrix} \tag{33}$$

This system exhibits linear dependence with one degree of freedom and the profile (f_{IJ}, f_{IK}, f_{JK}) is thus left undetermined. The three traders' *net* positions are, however, uniquely determined and equal to the competitive quantities above:

$$\begin{aligned} f_I^* &= f_{IJ} + f_{IK} \\ f_J^* &= f_{JK} - f_{IJ}, \\ f_K^* &= -f_{IK} - f_{JK} \end{aligned} \tag{34}$$

and prices should satisfy

$$\begin{gathered} \{(q_{IJ}, f_{IJ}) \;\; f_{IJ}>0,\; q^* + \frac{A_J}{2} f_{IJ} \geq q_{IJ} \geq q^* - \frac{A_I}{2} f_{IJ}\} \cup \\ \{(q_{IJ}, f_{IJ}) \;\; f_{IJ}<0,\; q^* + \frac{A_J}{2} f_{IJ} \leq q_{IJ} \leq q^* - \frac{A_I}{2} f_{IJ}\} \cup \\ \{(q_{IJ}, f_{IJ}) \mid f_{ij}=0\}. \end{gathered} \tag{35}$$

Note that if one quantity is fixed, say f_{IJ}, then the two other quantities are uniquely determined by (34) and the IR requirements on forward prices are uniquely given by (35) and similar expressions for q_{IK} and q_{JK}.

Phlips and Harstad (1991) propose a relatively simple non-cooperative game and show that the solution will satisfy (34) and (35). However, their game exhibits a continuum of subgame perfect equilibria and does not explain *how* the agents achieve coordination to pick one profile of three contracts that will satisfy (34).

4.2 Strategic Bargaining

The extension of the strategic bargaining game of Section 3.2 from two to three players is as follows: In any period τ, there is probability 1/3 that any two players are matched excluding the third from playing in that period. In each match, each of the two

 DOI: 10.2870/54657

similar expressions for q_{IK} and q_{JK}.

Phlips and Harstad (1991) propose a relatively simple non-cooperative game and show that the solution will satisfy (34) and (35). However, their game exhibits a continuum of subgame perfect equilibria and does not explain *how* the agents achieve coordination to pick one profile of three contracts that will satisfy (34).

4.2 Strategic Bargaining

The extension of the strategic bargaining game of Section 3.2 from two to three players is as follows: In any period τ, there is probability 1/3 that any two players are matched excluding the third from playing in that period. In each match, each of the two players then have probability 1/2 of being the one to make an offer, which the other can accept or reject. The game then moves on to period $\tau+1$.

This is a stochastic game: the history at τ, $h(\tau-1) \equiv (h_I(\tau-1),h_J(\tau-1),h_K(\tau-1))$ can be summarized in a state variable, $k(\tau)$ (as will be shown in due course):

$$k(\tau) \equiv (F(h(\tau-1)) \text{ , offerer , receiver , idle , } \tau) \text{ ,}$$
$$\text{where } F(h(\tau-1)) \equiv (F(h_I(\tau-1)),\ F(h_J(\tau-1)),\ F(h_K(\tau-1))). \tag{36}$$

$k(\tau)$ consists of the profile of net-positions of the preceding period, $F(h(\tau-1))$, the identity of the agent who is chosen to make an offer, together with the identity of the player who is chosen to receive the offer and the player that is idle at τ. It also includes time, τ, explicitly since the number of periods left may be important for the strategies. Note that there are redundancies in the state variable, since

$$\sum_i F(h_i(\tau-1)) = 0 \quad , \forall\tau, \tag{37}$$

and since the identity of the idle player is known by exclusion, but the notation (36) is maintained since it appears more intuitive and more symmetric. Expected payoffs at τ, $g(\tau) \equiv (g(h_I(\tau-1)),g(h_J(\tau-1)),g(h_K(\tau-1)))$, depend on $k(\tau)$ and on current actions, these being a contract offer, acceptance/rejection and a void action for the unmatched, idle player.

The state follows a Markov process in that the probability distribution on $k(\tau+1)$ is determined by $k(\tau)$ and the actions taken at τ. For example, assume that the state is

$$k(\tau) = (F(h(\tau-1)) \text{ , } I \text{ , } K \text{ , } J \text{ , } \tau), \tag{38}$$

and that I offers to buy a contract from K in period τ. If K rejects (r), then $k(\tau+1)$ will be one of the following six states, each of which has probability 1/6:

$$(k(\tau+1)\,|\,k(\tau),(c_{I,\tau},r)) \in \begin{Bmatrix} (F(h(\tau-1)),i,j,k,\tau) \\ \forall i,j,k \in \{I,J,K\}, i \neq j \neq k \end{Bmatrix} \tag{39}$$

in that the profile of net-positions remains unchanged in case of rejection. If K accepts (*a*), then $F(h(\tau)) \equiv (F(h_I(\tau\text{-}1)) - 1, F(h_J(\tau\text{-}1)), F(h_K(\tau\text{-}1)) + 1)$ and the state will belong to:

$$(k(\tau+1)\,|\,k(\tau),(c_{I,\tau},a)) \in \begin{Bmatrix} (F(h(\tau)),i,j,k,\tau) \\ \forall i,j,k \in \{I,J,K\}, i \neq j \neq k \end{Bmatrix} \tag{40}$$

with transition function $Pr(k(\tau+1)\,|\,k(\tau), (c_{I,t},a)) = 1/6$. In the example the players' action spaces are $S_I = \{c_{I,\tau}\}$ (the right to propose a contract), $S_K = \{a,r\}$ (the right to accept it or reject it) and $S_J = \varnothing$ (the right to remain silent). In general, we have the following definition:

Definition 1: *(Action Spaces)* In any state $k(\tau) = (F(h(\tau\text{-}1),i,j,k,\tau)$, the action spaces are $S_i = \{c_{i,}\}$, $S_j = \{a,r\}$, $S_k = \varnothing$, (i,j,k) is any permutation of $\{I,J,K\}$. Let $S \equiv S_i \times S_j \times S_k$, with $s(\tau) \equiv (s_i(\tau),s_j(\tau),s_k(\tau)) \in S$.

In principle strategies could depend on the entire history of the game. However, for any subgame starting in some period τ, only $k(\tau)$ matters. To show this, we adapt the following definitions from Fudenberg and Tirole (1991) (pp. 514-5): The future at τ is the current and future actions $\Phi(\tau) \equiv (s(\tau), s(\tau+1), ..., s(T)) \in S^{T-\tau}$ following choices of *i*, *j* and *k* by nature. In the following, *h* and *h′* denote two different histories.

Definition 2: *(Sufficient Partition of Histories)* A partition $\{H^\tau(\cdot)\}_{\tau=1,..,T}$ is *sufficient* if $\forall\tau$ and $\forall h(\tau-1),h'(\tau-1)$: $H^\tau(h(\tau\text{-}1)) = H^\tau(h'(\tau\text{-}1))$, the subgames starting at date τ after histories $h(\tau\text{-}1)$ and $h'(\tau\text{-}1)$ are strategically equivalent, *i.e.*

(i) The action spaces in the subgames are identical: $\forall i \in (I,J,K)$, $\forall t>0$ and $\forall\, s(\tau+t\text{-}1)$, $S_i(h(\tau\text{-}1),s(\tau),...,s(T)) = S_i(h'(\tau\text{-}1),s(\tau),...,s(T))$.

(ii) The players' payoffs conditional on $h(\tau\text{-}1)$ and $h'(\tau\text{-}1)$ are representations of the same preferences.

Definition 3: The *Payoff-Relevant History* is the coarsest sufficient partition.

From these two definitions we obtain Lemma 4:

Lemma 4: The payoff relevant history is summarized by $F(h(\tau\text{-}1))$.

Proof: The claim is that a partition, H^{τ}, of histories leading to the same $F(h(\tau\text{-}1))$ is sufficient. Let $h(\tau\text{-}1)$ and $h'(\tau\text{-}1)$ be two histories for which $F(h(\tau\text{-}1)) = F(h'(\tau\text{-}1))$. The action spaces are time invariant, so the first condition is satisfied. We now need to show that the payoffs conditional on any two histories that lead to the same profile of net positions at $\tau\text{-}1$ represent the same underlying preferences. The original preferences, $u_i(\pi) = -exp(-A_i\pi)$, are von Neumann-Morgenstern, so unique up to a linear transformation. What we need to show is thus that $\forall i \in \{I,J,K\}\ \exists\,(\lambda_i, \mu_i) \equiv (\lambda_i[h(\tau\text{-}1),h'(\tau\text{-}1)],\ \mu_i[h(\tau\text{-}1),h'(\tau\text{-}1)])\ (\lambda_i > 0)$ such that $\forall \Phi(\tau),\ G(h_i(\tau\text{-}1),\Phi(\tau)) = \lambda_i G(h_i'(\tau\text{-}1),\Phi(\tau)) + \mu_i$.

By the definition of $G(\cdot\,;\,\cdot)$, we have $G(h_i(\tau\text{-}1),\Phi(\tau)) = V(h_i(\tau\text{-}1)) + V(\Phi_i(\tau)) - p_i(F(h_i(\tau\text{-}1)) + F(\Phi_i(\tau))) + (A_i/2)\,(F(h_i(\tau\text{-}1)) + F(\Phi_i(\tau)))^2$, where we abuse the notation slightly, so that $V(\Phi_i(\tau))$ and $F(\Phi_i(\tau)))$ denote respectively i's book value and her (change in) net position arising from a given future $\Phi(\tau)$. It is easily seen that $\mu_i(h(\tau),h'(\tau),\cdot) = V(h_i(\tau\text{-}1)) - V((h_i'(\tau\text{-}1))$ and $\lambda_i(h(\tau\text{-}1),h'(\tau\text{-}1)) = 1$, so the payoffs conditional on any history that leads to $F(h(\tau\text{-}1))$ represent the same underlying preferences:

$$G(h_i(\tau\text{-}1),\Phi(\tau)) = G(h_i'(\tau\text{-}1),\Phi(\tau)) + V(h(\tau\text{-}1)) - V(h'(\tau\text{-}1)).\ \blacksquare$$

Remark: The proof shows that payoffs after any two histories that lead to the same positions only differ with respect to the book value of the previously concluded contracts, which has no impact on future reservation prices and hence does not affect strategies in future subgames.

Definition 4: A *Markov perfect equilibrium* (MPE) is a profile of strategies s^* that is a subgame perfect equilibrium (SPE) and depends on only the payoff relevant history, *i.e.* $H(h(\tau\text{-}1)) = H(h'(\tau\text{-}1)) \Rightarrow \forall i,\ s_i^*(h(\tau\text{-}1)) = s_i^*(h'(\tau\text{-}1))$.

In view of Lemma 4 we can let strategies depend on the profile of net positions, so that an MPE is a strategy profile s^* that is an SPE and that is measurable w.r.t. $F(h(\tau\text{-}1))$: $s_i^* = s_i^*(F(h(\tau\text{-}1))),\ \forall i$.

Let $K(\tau)$ denote the set of feasible states after a history leading to a set of forward positions, $F(h(\tau\text{-}1))$. We have existence of MPE in the game:

Proposition 4: There exists a Markov perfect equilibrium for the game $\Gamma_3 \equiv (N \equiv \{I,J,K\},\ \tau=1,..,T,\ k(\tau)\in K(\tau),\ s(\tau)\in S,\ Pr(k(\tau\text{-}1)\ k(\tau),s(\tau)),\ \{G(h_i(T))\}_{\forall i\in N})$.

Proof: The proof will be a special case of the n-person game of the later Section 5.2. $\blacksquare$

Having claimed existence of equilibrium, we characterize the equilibrium as far as possible without knowing the essential parameters, T and (A_i, p_i), $i \in \{I,J,K\}$. In three propositions we show the counterpart to Proposition 1 for the two player case: there is a (generically unique) $F(h(\tau\text{-}1))$ in the neighbourhood of (f_I^*, f_J^*, f_K^*) such that in all states after a history leading to $F(h(\tau\text{-}1))$, there will be no trade. These absorbing states are in other words uniquely determined by a single combination of net positions that determine how close the players can hope to come to efficiency.

Proposition 5: Trading stops if the game reaches a state where

$$\begin{aligned} F(h_I(\tau-1)) &= f_I^* + \delta_I \\ F(h_J(\tau-1)) &= f_J^* + \delta_J \\ F(h_K(\tau-1)) &= f_K^* - (\delta_I - \delta_J) \end{aligned} \qquad (41)$$

where (δ_I, δ_J) satisfy the following six inequalities:

$$\begin{aligned} -\frac{A_I + A_J}{2} &\leq -A_I\delta_I - A_J\delta_J \leq \frac{A_I - A_J}{2} \\ -\frac{A_I + A_K}{2} &\leq (A_I + A_K)\delta_I - A_K\delta_J \leq \frac{A_I + A_K}{2} \\ -\frac{A_J - A_K}{2} &\leq A_K\delta_I - (A_J - A_K)\delta_J \leq \frac{A_J + A_K}{2} \end{aligned} \qquad (42)$$

Proof: If $RB_i(\tau) \leq RS_j(\tau)$, $\forall i,j$, $i \neq j$, there are no further gains from trade. Manipulation of this system of inequalities yields (41) and (42). ■

Remark: Generically, all states like the one mentioned in the proposition are ergodic (absorbing): Once that kind of state has been reached, the market stays in it in all subsequent periods.

Remark: The proposition shows how close the market can get to the bilateral core: only if f_I^* and f_J^* are integers can the ergodic states be efficient ($\delta_I = \delta_J = 0$).

Figures 6.a and b show the range of (δ_I, δ_J) that satisfy the six inequalities for the two cases when $A_I = A_J = A_K = A$ (Figure 6.a) and $A_I = A_J/2 = A_K/3 = A$ (Figure 6.b). Note that it is not always the case that there is a non-binding inequality.

Figure 6A: The area where the requirements (42) are met for $A_I = A_J = A_K$

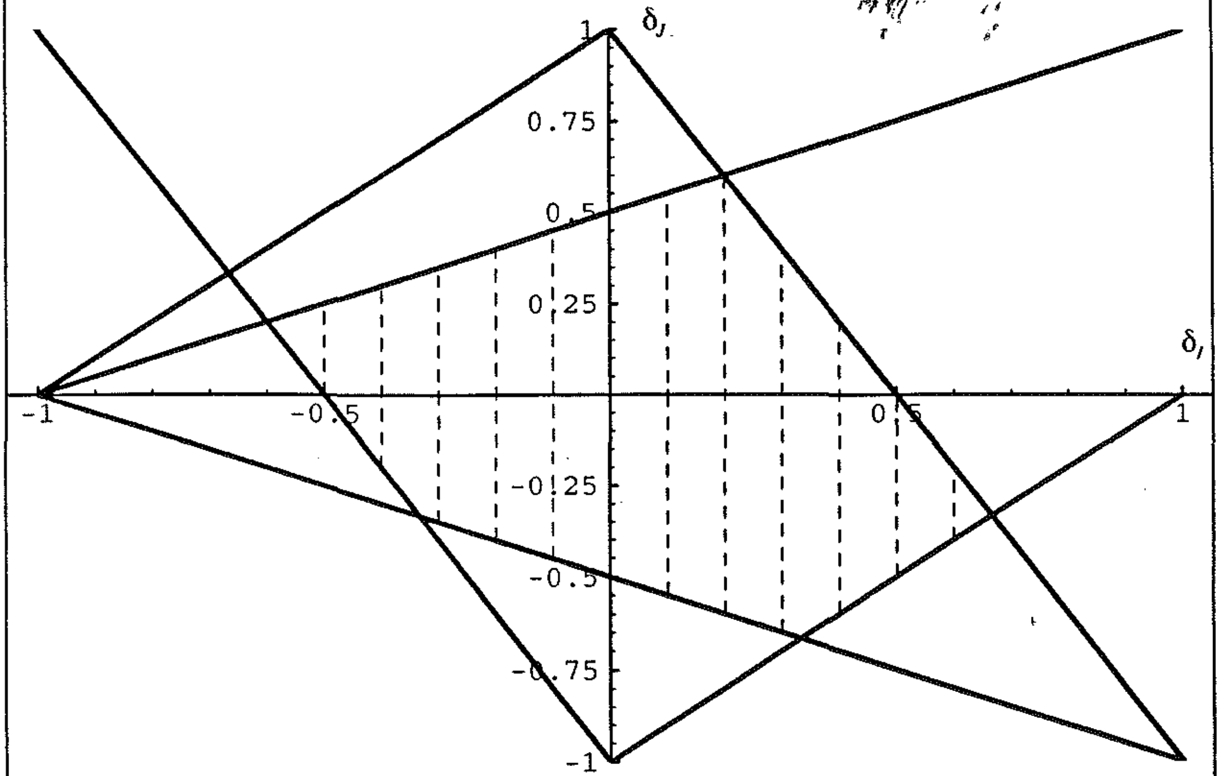
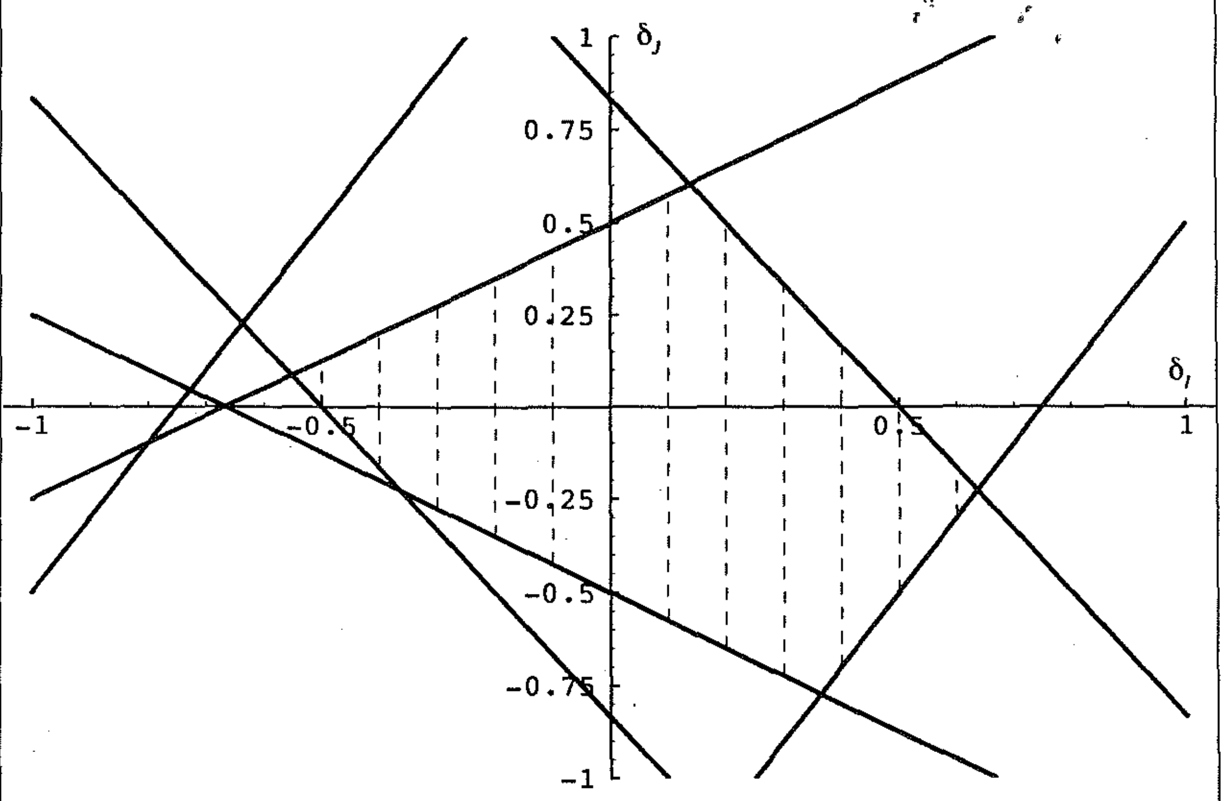


Figure 6B: The area where the requirements to (42) are met for $A_I = A_J/2 = A_K/3$



Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

Proposition 6: The only ergodic states $(F(h(\tau\text{-}1),i,j,k)$ are those that satisfy Proposition 5.

Proof: In order for a state to be ergodic at date $\tau \leq T$, we need $RB_i(\tau) \leq RS_j(\tau)$, $\forall i,j,\ i \neq j$. This leads to the following set of inequalities:

$$p_I - p_J - \frac{A_I+A_J}{2} \leq A_JF_J - A_IF_I \leq p_I - p_J + \frac{A_I+A_J}{2}$$

$$p_I - p_K - \frac{A_I+A_K}{2} \leq A_KF_K - A_IF_I \leq p_I - p_K + \frac{A_I+A_K}{2} \qquad (43)$$

$$p_J - p_K - \frac{A_J+A_K}{2} \leq A_KF_K - A_JF_J \leq p_J - p_K + \frac{A_J+A_K}{2}$$

where (F_I,F_J,F_K) are net positions at $\tau\text{-}1$ $(F_I+F_J+F_K=0)$. Let $F_I = f_I^* + \delta_I$ and $F_J = f_J^* + \delta_J$, where the only restriction on the δ's is that when they are added to the corresponding efficient positions, the result be an integer. Combining (33) and (34), we get

$$A_JF_J - A_IF_I = p_I - p_J - A_I\delta_I + A_J\delta_J$$

$$A_KF_K - A_IF_I = p_I - p_K - (A_I+A_K)\delta_I - A_K\delta_J, \qquad (44)$$

$$A_KF_K - A_JF_J = p_J - p_K - A_K\delta_I - (A_J-A_K)\delta_J$$

which combined with (43) gives the desired result.■

Proposition 7: The positions in the ergodic states are (generically) unique.

Proof: Let $F(h(\tau\text{-}1)) = (F_I,F_J,F_K)$ be the forward positions of an absorbing state. The pair (F_I ,F_J) thus satisfies

$$p_I - p_J - \frac{A_I+A_J}{2} \leq A_JF_J - A_IF_I \leq p_I - p_J + \frac{A_I+A_J}{2}.$$

There are six candidates for ergodic positions in the neighbourhood of $F(h(\tau\text{-}1))$, namely $(F_I,F_J,F_K) + \bar{x}$ where $\bar{x} \in \{(1,-1,0),(-1,1,0),(1,0,-1),(-1,0,1),(0,1,-1),(0,-1,1)\}$. Take the first element. If $A_JF_J - A_IF_I < p_I - p_J + (A_I + A_J)/2$, we have $A_J(F_J\text{-}1) - A_I(F_I\text{+}1) < p_I - p_J - (A_I + A_J)/2$, so $(F_I,F_J,F_K) + (1,-1,0)$ is not an ergodic position. If $A_JF_J - A_IF_I = p_I - p_J + (A_I + A_J)/2$, we have $A_J(F_J\text{-}1) - A_I(F_I\text{+}1) = p_I - p_J - (A_I + A_J)/2$, so $(F_I,F_J,F_K) + (1,-1,0)$ is also an ergodic position, and there can be a cycle back and forth between the two, this adding nothing to the expected payoffs of I and J. The argument for the other five candidates is similar.■

Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

4.3 An Example of a Markov Perfect Equlibrium

To get a feel for the nature of the Markov perfect equilibrium, consider the following simple numerical example. Assume that the basic parameters of the model are chosen to be:

Spot price expectations:	$p_I = 20.0,$	$p_J = 19.0,$	$p_K = 18.0$
Risk aversion constants:	$A_I = 0.2,$	$A_J = 0.4,$	$A_K = 0.6$
Time horizon:	$T = 3$		

The competitive equilibrium is found to be:

$$q^* = \frac{213}{11} = 19\frac{4}{11}, \; (f_I^*, f_J^*, f_K^*) = (-3\frac{2}{11}, \frac{10}{11}, 2\frac{3}{11}),$$

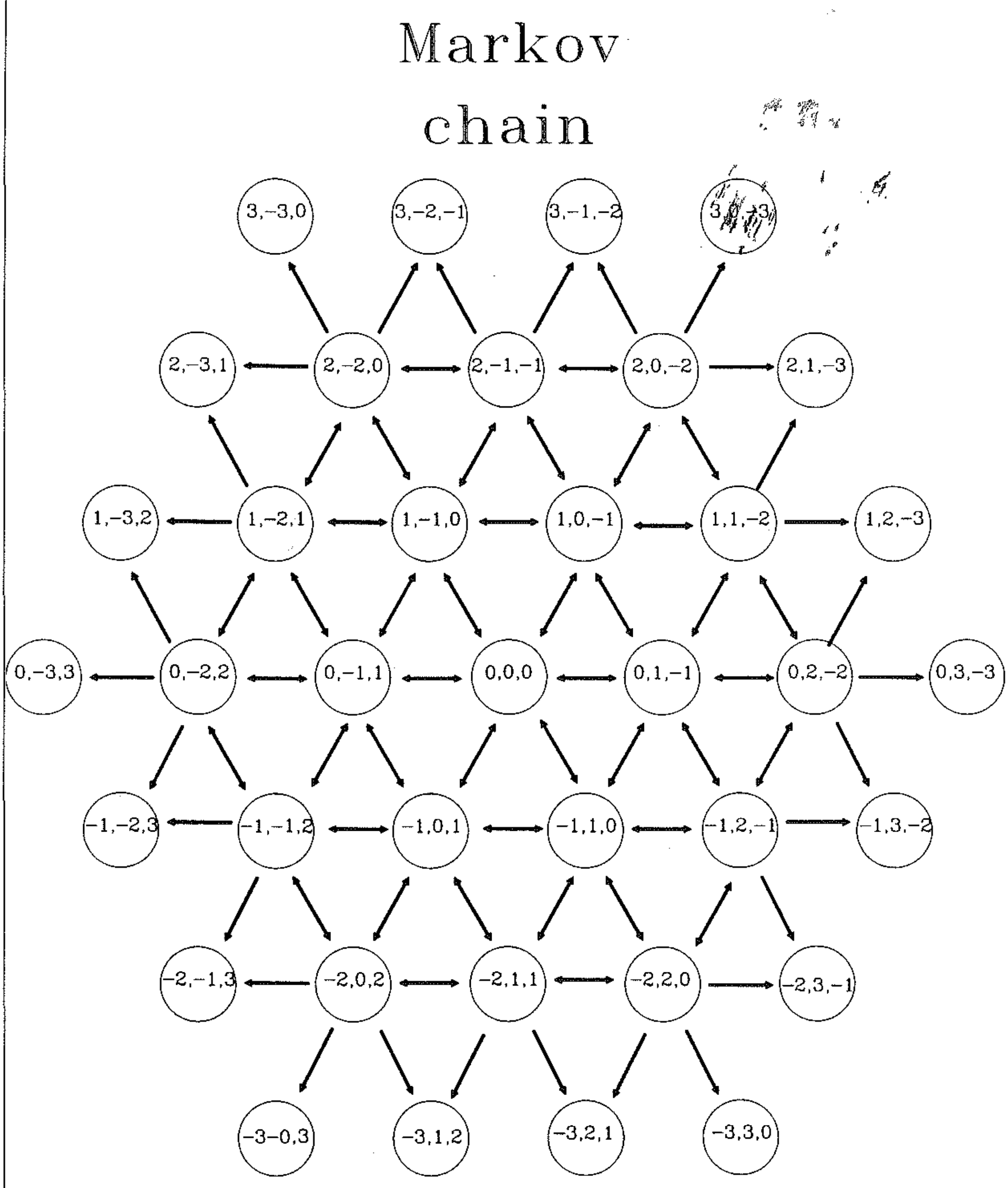
and the positions in the ergodic states are $(F_I, F_J, F_K) = (-3,1,2)$ (*i.e.* $(\delta_I, \delta_K) = (2/11, 1/11)$). The case fits Figure 6.b. By trading with each other, the three players can reach thirty-seven different combinations of forward positions at $T = 3$. These are illustrated in Figure 7.

The game starts with $F(h(0)) \equiv (0,0,0)$ and can go to seven different "nodes" at $\tau = 1$: *(0,0,0)*, *(0,-1,1)*, *(-1,0,1)*, *(-1,1,0)*, *(0,1,-1)*, *(1,0,-1)* and *(1,-1,0)*. From these an additional twelve nodes can be reached at $\tau = 2$ and further eighteen can be reached at $\tau = 3$ when the game ends. Each arrow represents a possible transition and to be complete the figure should include semi-circular arrows representing the possibility of remaining at each node. The MPE assigns equilibrium probabilities to each arrow conditional on being at a given node at a given τ. Many of these probabilities are zero in equilibrium.

The MPE is shown in Figure 8. All arrows there have probability *1/3* representing the probability of a match between *I* and *J*, *I* and *K*, and *J* and *K* at each node (after each payoff relevant history). At *(0,0,0)* there is thus probability *1/3* that *I* buys from *J* leading to *(-1,1,0)*. From there, the probability is again *1/3* that *I* buys from *K* leading to *(-2,1,1)* and from this node, the probability that *I* can strike another deal with *K* leading to the ergodic state *(-3,1,2)* is again *1/3*. The probability of this path is $(1/3)^3$. There are two other ways to get to *(-3,1,2)*, so the overall probability of ending up in the desired node is *1/9*. But the market can end up rather far from it also: If *J* and *K* are matched three times (an event that happens with probability *1/27*), the outcome will be an initial trade to *(0,-1,1)* whereafter trading stops. This is because the gains from trade between *J* and *K* are exhausted after only one trade: All paths in Figure 8 include at most one horizontal arrow.

The expected payoff at $\tau = 0$ is *(0.6282, 0.3065, 0.5125)* compared to *(1.0124, 0.1653, 1.5496)* in the competitive equilibrium.

Figure 7: Markov Lattice

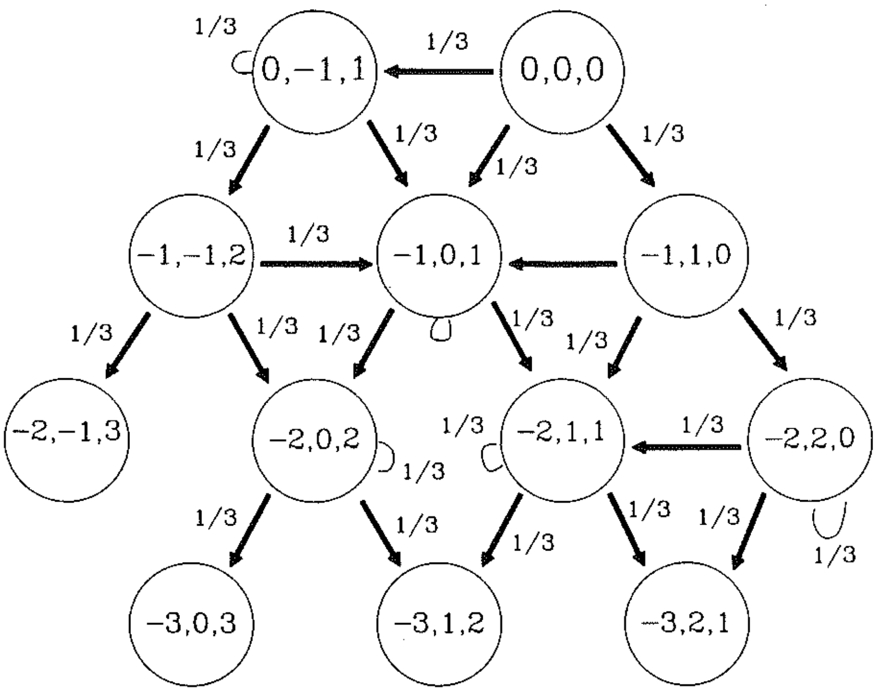


Horizontal arrows: Trade between *J* and *K*; Left-down/Up-Right Arrows: Trade between *I* and *K*; Right-Down/Up-Left Arrows: Trade between *I* and *J*.

Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

Figure 8: MPE

Markov Perfect Equilibrium



Mollgaard, Peter (1993), Essays on the Determination and Formation of Prices in European Crude Oil Markets
European University Institute DOI: 10.2870/54657

5. *n* Traders

We now turn to the $n \geq 2$ player case. A typical player is called I, $I \in N \equiv \{1,2,...,n\}$. As in the two preceding sections we first treat the axiomatic approaches in which the time dimension is suppressed and find the competitive equilibrium (the futures market equilibrium) and the bilateral core (the efficiency standard). We then move on to a full scale strategic bargaining model with a proof of existence of equilibrium in the n-person game and a characterization of the ergodic states.

5.1: Axiomatic approaches: Futures market equilibrium and bilateral core

As a benchmark, first assume that the players form a standard futures market which is assumed to be competitive, yielding a single market clearing price, q^*. Also assume that their different price expectations are common knowledge. Each player's payoff is then given by

$$G(\cdot;A_I,p_I) = (q^* - p_I)f_I - \frac{A_I}{2}f_I^2 . \qquad (45)$$

If q^* is taken as given, the agent only optimizes with respect to the net position, f_I, and the (futures) market equilibrium is given by

$$f_I^* = \frac{q^* - p_I}{A_I} = \frac{1}{n}\frac{A^m}{A_I}\sum_{i=1}^{n}\frac{p_i - p_I}{A_i} \qquad \forall I \in N \qquad (46)$$

$$q^* = \frac{\sum_{i=1}^{n}\frac{p_i}{A_i}}{\sum_{i=1}^{n}\frac{1}{A_i}} = \frac{A^m}{n}\sum_{i=1}^{n}\frac{p_i}{A_i} , \qquad (47)$$

where A^m is the harmonic mean of the risk aversion coefficients ($1/A^m$ is the arithmetic mean risk tolerance). A_I/A^m is the individual risk aversion relative to the market risk aversion. The market clearing price is *fully revealing* in the sense that it is the weighted mean opinion of the spot price, the weights being the agent's risk tolerance relative to the sum of the risk tolerances. This is the "market's spot price expectation" and it fully reflects the market participants' opinions weighted by their willingness to bet on them.

Now assume that the market is one in which the participants enter bilateral contracts, but still disregard the time dimension. The payoff function then becomes

$$G(\cdot\,;A_I,p_I) = \sum_{\forall i \in N\setminus\{I\}} (q_{Ii} - p_I)f_{Ii} - \frac{A_I}{2}f_I^2 \tag{48}$$

$$\text{where } f_I = \sum_{\forall i \in N\setminus\{I\}} f_{Ii} \qquad ,\forall I \in N.$$

To belong to the bilateral core, the quantities have to fulfil

$$p_J - p_I = A_I f_I - A_J f_J \qquad \forall I,J, \tag{49}$$

which is satisfied only by f_I^*, $\forall I$. Prices should satisfy individual rationality given these quantities, *i.e.* contracts should belong to

$$\left\{(q_{ij},f_{ij}) \;\; f_{ij} > 0, \;\; q^* + \frac{A_j}{2}f_{ij} \geq q_{ij} \geq q^* - \frac{A_i}{2}f_{ij}\right\} \cup$$
$$\left\{(q_{ij},f_{ij}) \;\; f_{ij} < 0, \;\; q^* + \frac{A_j}{2}f_{ij} \leq q_{ij} \leq q^* - \frac{A_i}{2}f_{ij}\right\} \cup \tag{50}$$
$$\left\{(q_{ij},f_{ij}) \;\; f_{ij} = 0\right\} \qquad \forall i,j \in N, \; i \neq j.$$

The bilateral core thus consists of n contracts where the quantities and prices satisfy (46) and (50). Note that the market equilibrium (46-47) belongs to the core.

5.2 Strategic bargaining: the decentralized market game

The stochastic game Γ_n is defined by

- ☺ a set of players, $N \equiv \{1,...,n\}$
- ☺ a time horizon $T \geq 1$: $\tau = 1,...,T$
- ☺ a set of states $k(\tau) \in K(\tau)$
- ☺ a set of actions $s(\tau) \in S(\tau)$
- ☺ a transition function: $Pr[k(\tau+1)] = Pr[k(\tau+1) \mid k(\tau)\;,s(\tau)]$
- ☺ a payoff function for each $I \in N$: $G(h_I(T); A_I,p_I)$

The state variable $k(\tau)$ consists of an n-vector of forward positions, $F(h(\tau\text{-}1))$, a realization of the matching technology $M(\tau)$ and the time index, τ:

$$k(\tau) = (F(h(\tau-1)),\ M[N](\tau),\ \tau)\ , \qquad (51)$$

and $K(\tau)$ is the set of states that is feasible after any history leading to a given $F(h(\tau\text{-}1))$.

The vector of forward positions for the n players at the beginning of period τ is defined in an obvious extension of the notation for the three player case:

$$F(h(\tau-1)) = (F(h_1(\tau-1),\ \dots,\ F(h_n(\tau-1))). \qquad (52)$$

The *matching technology* is called M and a particular realization at time τ is denoted by $M(\tau)$. Let $<O,R>$ denote an ordered pair and let S be a set of integers with $\#S = 2\kappa$, where κ is a positive integer. Finally, let M_S denote the set of κ ordered pairs exhausting S, *i.e.* an element $\mu \in M_S$ is a collection $\{<O_1,R_1>, <O_2,R_2>, \dots, <O_\kappa,R_\kappa>\}$ such that $O_i,R_i \in S$, $\forall i = 1, ., \kappa$, $O_i \neq O_j$ and $R_i \neq R_j$ $\forall i,j = 1, ., \kappa$ w. $i \neq j$ and $O_i \neq R_j$, $\forall i,j = 1, ., \kappa$.

If $n \equiv \#N$ is even, the matching technology simply maps from the set of players to a set of $n/2$ ordered pairs:

$$M\colon N \rightarrow M_N\ . \qquad (53)$$

In other words, an outcome of the matching technology chooses $n/2$ matches *and* in each match, m ($m \in \mu \in M_N$), the identity of the offerer, O_m, and of the receiver of the offer, R_m. If n is even, an outcome of M becomes $M(\tau) = (<O_1,R_1>, <O_2,R_2>, \dots, <O_{n/2},R_{n/2}>)$ with O_i, $R_i \in N$, $i = 1, \dots, n$, $O_i \neq O_j$, $R_i \neq R_j$, $\forall i,j = 1,.,n/2$, $i \neq j$, and $O_i \neq R_j$, $\forall i,j$. In the "even" case, the matching technology maps from the set of players to the set of possible two-permutations of this set. The probability of a particular outcome of the map is

$$Pr\left[M(\tau) = \mu \in M_N\right] = \frac{(n-2)!}{n!} = \frac{1}{n(n-1)} \qquad (54)$$

since all outcomes are equally likely by assumption.

If $n \equiv \#N$ is odd, M maps from N to the space of $(n\text{-}1)/2$ ordered pairs plus the identity of the idle player:

$$M\colon N \rightarrow M_{N\setminus\{Idle\}} \times N\ . \qquad (55)$$

An outcome of the matching technology now determines the identity of the idle player and $(n\text{-}1)/2$ matches exhausting the $n\text{-}1$ non-idle players and within each match, m ($m \in \mu \in M_{N\setminus\{Idle\}}$), the identity of the offerer, O_m, and of the receiver of the offer, R_m. For n odd, an outcome of M becomes

$$M(\tau) = (\mu,\ Idle) = (\{<O_1,R_1>, <O_2,R_2>, \dots, <O_{(n-1)/2},R_{(n-1)/2}>\},\ Idle).$$

The probability of a particular outcome of the map, M, is

$$Pr\left[M(\tau) = (\mu, Idle) \in M_{N\setminus\{Idle\}} \times N\right] = \frac{(n-3)!}{n!} = \frac{1}{n(n-1)(n-2)} \qquad (56)$$

since for each player there is probability $1/n$ of being idle at τ and for the $n\text{-}1$ remaining players, there are $(n\text{-}1)!/(n\text{-}3)!$ two-permutations.

Given an outcome of M, in each match $m \in \mu$, the offerer, O, has action space $S_O = \{c_{m,.}\}$, that is, she can propose to buy or sell a contract at a price of her choice, or she can choose not to propose a deal, so $S_O = \{(q_{.,.}, 1), (q_{.,.}, -1), (q_{.,.}, 0): q \in \mathbb{R}_+\}$. The receiver in m can accept (a) or reject (r), so $S_R = \{a, r\}$. A possible idle player has $S_{Idle} = \varnothing$. The strategy space of a player thus depends on whether the outcome of the macthing technology makes her offerer, receiver or idle. In any state $k(\tau)$, action spaces for the market in that state are

$$S = (S_O \times S_R)^{\frac{n}{2}} \qquad \text{if } n \text{ is even}$$
$$S = (S_O \times S_R)^{\frac{n-1}{2}} \times \varnothing \qquad \text{if } n \text{ is odd.} \qquad (57)$$

Given today's state, $k(\tau)$, and today's actions, $s(\tau)$, a new profile of net positions appears: $F(h(\tau))$. Given this, the probability of a state

$$k(\tau+1) = (F(h(\tau)), M(\tau-1) = \mu, \tau-1) \qquad (58)$$

will be $(n\text{-}2)!/n!$ if n is even and $(n\text{-}3)!/n!$ if n is odd. These are the transition probabilities. When the players have chosen the trades leading to a new F (the first element of the state variable at $\tau+1$, then nature chooses a new outcome of the matching technology, $M(\tau+1)$ (the second element of the state variable), and one period has passed so $t = \tau+1$ (the third entry of the state variable). The sequence of events in each period τ is as follows:

---

0. The realization of $k(\tau) = (F(h(\tau\text{-}1)), M(\tau), \tau)$ becomes common knowledge (c.k.).
1. The Offerers simultaneously propose a contract (*not* c.k.).
2. The Receivers simultaneously accept or reject (*not* c.k.).
3. The Idle player (simultaneously with 1 and 2) does nothing (c.k.).
4. The actions in 1, 2 and 3 lead to $F(h(\tau))$.
5. Nature chooses $M(\tau+1)$ and $k(\tau+1) = (F(h(\tau)), M(\tau+1), \tau+1)$ becomes c.k.

Lemma 4 still applies: the payoff relevant history is summarized by the forward positions at the beginning of the period, $F(h(\tau\text{-}1))$. Indeed, the proof is independent of the number of players. We can also extend the essential proof of existence for the three person game, Γ_3, to the n-person game, Γ_n:

Proposition 8: There exists a Markov perfect equilibrium for the game Γ_n.

Proof: If the set of actions S were finite, the proof would be a trivial adaption to Γ_n of the more general Theorem 13.2 in Fudenberg and Tirole (1991) (p. 515). However there are no restrictions on the price an offerer can propose apart from positivity, so S_O is infinite. But there are restrictions on the quantities that the offerer can propose, *viz.* to buy one unit, to sell one unit or not to do anything, and thus $S_O = \{c_{m,\cdot} \mid q_{m,\cdot} \in \mathbb{R}_+ \wedge f_{m,\cdot} \in \{-1,0,1\}\}$. This means that there is a finite number of feasible states that can be reached during the game and subgame perfect equlibrium requires that the strategy be optimal at any state, be they reached or not.

At T, in any feasible state $k(T)$, O_m will propose a contract to R_m that *either* is equal to R_m's reservation price if there are gains from trade to be made and which R_m then will accept *or* is IR to O_m if not, and then R_m will reject. Reservation prices are functions of the state via $F(h(T\text{-}1))$.

At $T\text{-}1$, the subgame starting there can be solved with knowledge of what is going to happen at T in each of the states possible at T and of the transition function $Pr(k(T)\,|\,k(T\text{-}1), s(T\text{-}1))$. Continuing the backward recursion, an MPE appears.■

We also get the n-person equivalent of Propositions 4 - 6 establishing existence and (generic) uniqueness of the absorbing states for the three person game:

Proposition 9: Trading stops if the game reaches a state in which

$$F(h_i(\tau-1)) = F_i^* + \delta_i\,, \quad \forall i \in N, \tag{59}$$

where $\forall i,j$ the δ_i's satisfy

$$-\frac{1}{2} \leq \frac{A_j}{A_i + A_j}\delta_j - \frac{A_i}{A_i - A_j}\delta_i \leq \frac{1}{2} \tag{60}$$

and

$$\sum_{\forall i \in N} \delta_i = 0. \tag{61}$$

Proof: If $RB_i(\tau) \leq RS_j$, $\forall i,j \in N$, $i \neq j$, there are no further gains from trade. Solving this system of inequalities yields the desired result.■

Remark: The δ_i's are again to be understood as the real numbers that when added to the respective efficient quantities of the bilateral core yield integers. Weighed by the relative risk aversion the difference between the δ's of any two players should numerically be less than 1/2.

Proposition 10: The only ergodic states $k(\tau)$ are those that satisfy proposition 9.

Proof: In any ergodic state we need $RB_i(\tau) \leq RS_j(\tau)$ and $RB_j(\tau) \leq RS_i(\tau)$ for all i and j. This leads to the following set of inequalities:

$$p_i - p_j - \frac{A_i + A_j}{2} \leq A_j F_j - A_i F_i \leq p_i - p_j - \frac{A_i - A_j}{2} \tag{62}$$

where $F = \{F_1, F_2, ..., F_i, ..., F_n\} \equiv F(h(\tau\text{-}1))$ is the profile of net positions at $\tau\text{-}1$. Let $F_i = F_i^* + \delta_i$, $\forall\, i \in N$ where the only restriction on the δ's is that when added to the respective efficient position, the result must be an integer. From (49) we then get

$$A_j F_j - A_i F_i = A_j F_j^* - A_i F_i^* + A_j \delta_j - A_i \delta_i = p_i - p_j - A_j \delta_j - A_i \delta_i$$

which combined with (62) gives the desired result.■

Proposition 11: The positions in the ergodic states are (generically) unique.

Proof: Let $F(h(\tau\text{-}1)) = (F_1, ..., F_n)$ be the forward positions of an ergodic state. $F(h(\tau))$ thus satisfies (62) for all $i,j \in N$. Let $\bar{x}$ be an n-vector of integers such that $e'\bar{x} = 0$ where $e \equiv (1, 1, ..., 1)$ is an n-vector too. Let $\bar{x} \neq 0$. There must then be at least one element of $\bar{x}$ that is positive and at least one that is negative. Pick a positive element, x_i, and a negative, x_j, with the indices, i and j corresponding to the "name" of the agent. We have $x_i \geq 1$ and $x_j \leq -1$, and now have to show that $F(h(\tau\text{-}1))$ cannot be an ergodic state. From this state we have $A_j(F_j - x_j) - A_i(F_i + x_i) = A_j F_j - A_i F_i + (A_j x_j - A_i x_i)$. We know that $A_j x_j - A_i x_i \leq -(A_j + A_i)$, so if we have strict inequality of (62), then $A_j(F_j - x_j) - A_i(F_i - x_i) < p_i - p_j - \frac{A_i - A_j}{2}$, so $F(h(\tau\text{-}1)) + \bar{x}$ is not ergodic. If we have equality of (62) by coincidence and if $(x_i\ ,\ x_j\ ) = (1,\ -1)$, then $A_j(F_j - x_j) - A_i(F_i - x_i) = p_i - p_j - \frac{A_i - A_j}{2}$, so the state may still be ergodic.■

A Markov perfect equilibrium of Γ_n can be interpreted along the lines of the two and three person markets, Γ_2 and Γ_3, which indeed are special cases. But for $n > 3$ a new feature appears: the possibility of *multiple* Markov perfect equilibria stemming from the possibility of multiple Nash equilibria at each state: when there is more than one simultaneous match in a given period, optimality of a decision within a given match may hinge on the outcome of other matches at that time. The outcome of the matching technology is common knowledge, but the actions are not, and since the play across matches is simultaneous, problems of the following sort may arise: The optimality of any pairs' decision hinges on what the entire trade vector looks like. A trade vector is an n-vector of possible $\{-1, 0, 1\}$ where the sum of the elements is zero. The *entire* vector determines the state in the next period, which in turn determines expected continuation payoffs. So it may be optimal for one pair of agents to agree on a contract if and only if another pair agrees on a contract, and *vice versa*.

Whether multiple MPE occurs is a matter of choice of parameters, but in general the set-up allows for that.[10] This introduces the possibility of coordination failure (failure to coordinate on the same Nash equilibrium) on top of the other inefficiencies of the market (stemming from indivisibilities and from not being able to control the matching process). The next section looks at whether one can expect convergence of the process to the efficient outcome as one parameter, the time horizon, T goes to infinity.

6. Convergence and Decentralized *vs.* Centralized Trade

In this section we first (6.1) discuss convergence of the Markov perfect equilibrium of Γ_n to the ergodic states, which is the closest the market can come to efficiency. Then (6.2) we compare Γ_n to other models of decentralized trade and conclude (6.3) with a comment on forward markets compared to futures markets.

6.1 Inefficiency and Convergence to Efficiency

Compared to a competitive standard, the outcome of Γ_n ($n > 2$) is inefficient: there is always a positive probability that the game ends with a set of forward positions that does not belong to the ergodic states. The heuristic proof of this point is simple: re-order the players from 1 through n according to the value of p_i/A_i, so p_1/A_1 is lowest and p_n/A_n is highest. The event that 1 and 2, 3 and 4, ..., $n-1$ and n (or if n is odd: $n-2$ and $n-1$) are matched in every

[10] It would seem that for $n \geq 4$, the game generically possesses multiple equilibria, so that the parameter space in terms of $\{A_i, p_i\}$ and T for which this happens is topologically large. How to show this formally is not clear to me.

period $\tau = 1, . , T$ has a positive probability (that is decreasing in T, however). Obviously to be efficient, the long positions at T should be concentrated at high values of i ($i \in \{1,.,n\}$) whereas short positions should be concentrated at low values of i. However, this is impossible given the outlined event.

Generally, there are several sequences of matchings in $\tau = 1,.,T$ that can lead to efficient outcomes (as in the three trader example of Section 4.3) but also several sequences for which the efficient positions can not possibly be reached. What the probabilities are and how close the market can get to the efficient positions depends on how large these positions are and how they are distributed, *i.e.* on $\{p_i, A_i\}$ ($\forall i \in N$), on the one hand and on the matching probabilities (*i.e.* on n) and the time horizon, T, on the other. Indeed, one would expect the following **conjecture** to hold true:

> *In a Markov Perfect Equilibrium of Γ_n the efficient positions are reached almost surely as $T \rightarrow \infty$.*

A very rough and heuristic proof of this goes as follows:[11] we have already shown that there is a generically unique set of ergodic states for a given T.[12] If the process converges to anything, it must be to the ergodic states. To show convergence implies to show that the process is a contraction - but it must be, since any trade reduces the aggregate gains from trade left for the future.

For any given T, the MPE of Γ_n describes a Markov chain. The reason why standard techniques of showing convergence of Markov chains do not work here is that as T increases, the transition probabilities among existing states may change *and* the size of the state space increase: there are a number of new feasible states that were not reachable for a smaller T. As $T \rightarrow \infty$, the state space becomes infinity too (and actually this happens at a faster rate).

At the risk of belabouring the last point, the problem is that even though for any (finite) T the state space is finite, if a period $T+1$ is introduced, this increases the number of feasible states by more than one to one. To see this, consider the example of Section 4.3 and Figure 8: if a period $T=4$ is introduced, twenty-four new feasible profiles of forward positions pop up. For the three person case, the number of feasible nodes as a function of T is:

[11] Another argument is that Gale (1987) gets convergence to the competitive equilibrium in a somewhat similar set-up. The difference is that Gale has an infinite time horizon and that he then studies convergence of equilibrium as a function of parameters like the time preferences (as the traders get more patient, the equilibrium converges to the competitive equilibrium).

[12] Ignoring that all states that are reached at T are ergodic in the sense that the game ends, so the probability of getting out of such a state is trivially zero.

$$1 - 6\sum_{i=1}^{T} i \tag{63}$$

which clearly increases six times faster than T. In the more general n-person game, each node that is reachable at T-1 has number of *neighbours*:

$$\sum_{i=1}^{n/2} \frac{n!}{(n-2i)!}\left(\frac{1}{i!}\right)^2 \tag{64}$$

for n even and

$$\sum_{i=1}^{\frac{n-1}{2}} \frac{n!}{(n-2i)!}\left(\frac{1}{i!}\right)^2 \tag{65}$$

for n odd. For example, if there are twenty players ($n = 20$) each node reachable at T-1 has 377,379,368 neighbours! For a larger number of players, the state space increases faster than for the three player case, thus emphasizing the point that the number of states grows rapidly as $T \to \infty$.

The conjecture is so much more remarkable considering that it requires the equilibrium to be a contraction in an expanding state space. The reasoning behind this is that the agents will not use the extra states (given that the ergodic states were already reachable), but will rather use the extra time available to get to more attractive nodes (forward positions) among the already existing ones: returning to the example and Figure 8, in the three player case $F(h(3)) = (0,-1,1)$ could be an equilibrium outcome if J and K are matched for three consecutive periods. This event has probability $(1/3)^3 = 1/27$ *ex ante*. If the transition probabilities that are indicated in Figure 8 remain the same as T increases, then the *ex ante* probability of this deadlock decreases rapidly (exponentially) with T. The conjecture thus implies that when T increases, there will be few or no new nodes in Figure 8 and some nodes may even drop out because the probability of getting to a more attractive state increases. In other words, as the state space ramifies, Markov perfect equilibria effectively prune away new branches.

6.2 Models of decentralized trade

The model presented in Sections 2 through 6 is one of decentralized trade. It obviously has links to other models of decentralized trade. This section seeks to make the connection and to show the differences.

Gale (1988) makes an important distinction between models of *ex ante* pricing where all prices are posted and known by all relevant parties before the agents get together to trade, and models of *ex post* pricing where the agents get together before prices are quoted. Our model belongs to the *ex post* category in that the players are matched before contracts are proposed.

In the framework of *ex ante* pricing, three papers by Ostroy and Starr (1974) and Starr (1976; 1986) treat points similar to ours: in a general equilibrium setting, the question addressed is whether a competitive equilibrium can be implemented by a decentralized trading process. The equilibrium prices (and quantities) are determined by a Walrasian auctioneer, but the agents are not allowed to hand the net trade vector over to the auctioneer, so there is no centralized clearing. Instead, the agents have to sort the equilibrium out themselves in pairwise meetings. In each round of trading, every trader meets every other trader in an arbitrary order, so the only uncertainty is with respect to the order of matches, not with respect to whether they get matched or not. Within this set-up, Ostroy and Starr show that in a barter economy it is in general *not* possible to find a decentralized procedure that achieves the competitive equilibrium, but in a monetary economy such a decentralized mechanism exists. Starr (1976) then shows that if barter trading can implement the competitive equilibrium, then there is a monetary trading procedure that does this a good deal faster. In fact, the non-monetary procedure may take forever to converge. Finally, Starr (1986) allows for 'short sales' and discusses convergence in different credit economies (commodity credit, trade credit and bank credit). None of these institutional set-ups do as well as the monetary economy in terms of convergence and even existence of a convergent procedure.

Our model could be seen as a monetary economy with one good (a contract) and with *ex ante* pricing and indeed with the Ostroy and Starr matching technology and trading procedure, the competitive equilibrium could quickly and certainly be obtained in decentralized trading. Another way of re-interpreting our model in terms of Ostroy and Starr is that potentially there are $\sum_{i=1}^{n}(n-i) = n^2 - \sum_{i=1}^{n} i$ different goods in the forward market: a contract between I and J is different from a contract between I and K and both are different from a contract between J and K and so forth. This is especially important for the clearing procedure in the 15-Day market, where having a long contract with one trader and a short with another does not mean that these trades net out: A trader may still have to honour both contracts and may thus have to bother about taking and arranging delivery. This is why most traders in the 15-Day market close their positions in book-outs and in daisy chains and it is also why the assumption that the forward market clears at maturity is not realistic for this market. Maturity (in the spot market) is not a point in time but rather a whole month and traders want to realize gains and losses *before* maturity in order not to deal with problems of delivery. This renders the market even more inefficient seen from the traders' point of view. The point that is made here, is that not only is the trading on the forward market decentralized, but so is the 'clearing at maturity' which forces the traders to *clear before maturity!* Clearly a centralization of this clearing mechanism along the lines of a clearing house in a futures market is both feasible and more efficient. Note that the market is only one step short of this, since one major producer organizes the liftings (actual deliveries) for the entire market.

In the framework of *ex post* pricing, there is a huge and growing literature on strategic bargaining applied to markets. An unsurpassed treatment is Osborne and Rubinstein (1990). That our model is inspired by, and in line with, this literature, will be clear from the following quote:

> "Bargaining theory provides a natural framework within which to study price formation in markets where transactions are made in a decentralized manner via interactions between pairs of agents rather than being organized centrally through the use of a formal trading institution like an auctioneer. One might describe the aim of investigations in this area as that of providing "mini-micro" foundations for the microeconomic analysis of markets and, in particular, of determining the range of validity of the Walrasian paradigm. Such a program represents something of a challenge for game theorists in that its success will presumably generate new solution concepts for market situations intermediate between those developed for bilateral bargaining and the notion of Walrasian equilibrium." (Binmore, Osborne and Rubinstein (1992)).

The original paper in this strain of literature is Rubinstein and Wolinsky (1985) in which *steady states* of a market are investigated. The market is partitioned into buyers and sellers and they are matched in a stochastic process that renews the pairings every period and that is not in the control of the agents. The matching technology of our model can be seen as a special case of this. In the Rubinstein/Wolinsky framework, every seller has one unit for sale and each buyer wants one unit. Once a match has concluded a deal, the pair leaves the market and is then replaced with a new pair, thus keeping stocks of agents constant. The time horizon is infinite, but traders have impatient von Neumann-Morgenstern utility functions and thus an incentive to conclude a deal rather sooner than later. An important conclusion that arises from this model is that the market equilibrium of the decentralized market need not be competitive. That is, a model that (contrary to the competitive equilibrium) explains the *formation* of prices in equilibrium does not necessarily support a competitive outcome. This result has triggered a number of studies of when and why convergence to the competitive equilibrium arises (notably Gale (1986a,b; 1987) and McLennan and Sonnenschein (1991)).

Contrary to these studies an important feature of our model is the finite time horizon, imposed since the forward contracts eventually mature and since this is clearly perceived by the traders. On the other hand, time preferences are not important (traders are infinitely patient), so the incentive to conclude deals is that time runs out. A third difference is that our equilibrium is inherently dynamic,[13] whereas most other models of bargaining and markets concentrate on steady states.

---

[13] This is also the case in Gale (1987) and in Binmore and Herrero (1988) but they work with an infinite time horizon.

6.3 Forward *vs.* Futures Markets - and the Future

The model that was presented in this paper describes a decentralized, speculative forward market and compares the market equilibrium with that of a centralized, competitive market that can be thought of as a futures market. It was shown that the decentralized market is inferior to the centralized market in that the random matching makes it difficult to coordinate on an efficient outcome: There is always a positive probability of not reaching the efficient outcome but this probability drops as the traders get more time to complete their affairs.

Matching is not entirely random in reality, but it is not entirely under the control of the traders either. Endogenized matching would lead to more complicated transition probabilities since matching behaviour should be explained by equilibrium strategies (*cf.* Herreiner (1993)). It is not clear how this would affect Markov perfect equilibrium, but a conjecture is that, given the assumption of common knowledge of different priors, there would be faster convergence to the efficient outcome, since traders with very different beliefs have a common interest in getting together.

The informational requirements that underlie both the model of decentralized trade and that of centralized trade are very severe: we have a game of complete information, so the different spot price expectations and the different risk aversions are common knowledge. In reality, agents face incomplete information. This leads to problems for the agents such as identifying who the optimists are (*cf.* Harstad and Phlips (1993)) and, without further specifications of the agents' knowledge, almost certainly to results of (generic) non-existence of Markov perfect equilibrium or to a situation in which any outcome can be rationalized as the MPE for appropriate choices of beliefs. Subgame rationalizability may be all one can hope for. The way to model this may be to let strategies be part of a controlled process in which the traders try to learn the spot price expectations of the other traders and to make money at the same time. This is a standard learning problem, but with a finite time horizon.

Another informational intricacy stems from the once-and-for-all nature of the spot price expectations. New information is likely to appear during trading so agents change their mind while trading, revaluing the book value of completed contracts and of future strategies. This could be modeled within the framework of common knowledge of different priors by exposing the whole vector of price expectations to (*e.g.* additive or multiplicative) random shocks, thus generating the erratic behaviour observed in Figures 1 and 2.

While these modifications (endogenized matching, incomplete information and continuous information on spot prices) are interesting in their own right, it should be clear that these problems by no means are assuaged by decentralized trading. It is easier to see how an equilibrium with incomplete, imperfect information and a continuous flow of data can lead to an efficient outcome in a centralized futures market than in a decentralized forward market - and in a futures market, matching is not an issue. It therefore remains a paradox that the participants in the 15-Day market accept such an inferior institution.

REFERENCES

Allaz, B. (1991) "Duopoly, Inventories and Futures Markets" in L. Phlips (ed.) *Commodity, Futures and Financial Markets* Kluwer Academic Publishers, Dordrecht: 249-271.

Anderson, R.W. and Gilbert, C.L. (1991) "Four Articles on Commodities: Commodity Agreements, Commodity Markets, Cornering the Market and Cross Hedging" *Discussion Paper No. 9116*, Université Catholique de Louvain.

Arvan, L. (1985) "Some Examples of Dynamic Cournot Duopoly with Inventory" *Rand Journal of Economics* 16 (4): 569-5078.

Bacon, R. (1986) "The Brent Market - An Analysis of Recent Developments" *Working Paper WPM8* Oxford Institute for Energy Studies, Oxford.

Benoit, J.-P. and Krishna, V. (1987) "Dynamic Duopoly: Prices and Quantities" *Review of Economic Studies* 54: 23-35.

Binmore, K.; Osborne, M.J. and Rubinstein, A. (1992) "Noncooperative Models of Bargaining" in R.J. Aumann and S. Hart (ed.s) *Handbook of Game Theory with Economic Applications - Volume 1* North-Holland, Amsterdam, ch. 7: 179-226.

Bowley, A. (1924) *The Mathematical Groundwork of Economics* Oxford University Press.

Bresnahan, T.F. (1981) "Duopoly Models with Consistent Conjectures" *American Economic Review* 71(5): 934-945.

Bresnahan, T.F. (1989) "Empirical Studies of Industries with Market Power" in R. Schmalensee and R. Willig (ed.s) *Handbook of Industrial Organization - Volume II*, North-Holland, Amsterdam, ch. 17: 1011-1057.

Brianza, T., Phlips, L. and Richard, J.F. (1990) "Futures Markets, Speculation and Monopoly Pricing" *EUI Working Paper n° 90/8* European University Institute, Florence.

Clubley, S. (1990) *Trading in Oil Futures* (2nd edition), Woodhead Faulkner, New York.

Dixit, A. (1986) "Comparative Statics for Oligopoly" *International Economic Review* 27 (1): 107-22.

Duffie, D. (1989) *Futures Markets* Prentice-Hall, London.

Friedman, J. (1971) "A Noncooperative Equilibrium for Supergames" *Review of Economic Studies* 38: 1-12.

Fudenberg, D. and Tirole, J. (1991) *Game Theory* MIT Press, Cambridge (Ma.).

Gale, D. (1986a) "Bargaining and Competition Part I: Characterization" *Econometrica* 54(4): 786-806.

Gale, D. (1986b) "Bargaining and Competition Part II: Existence" *Econometrica* 54(4): 807-818.

Gale, D. (1987) "Limit Theorems for Markets with Sequential Bargaining" *Journal of Economic Theory* 43: 20-54.

Gale, D. (1988) "Price Setting and Competition in a Simple Duopoly Model" *Quarterly Journal of Economics* 103(4): 729-39.

Geanakoplos, J. (1992) "Common Knowledge" *Journal of Economic Perspectives* 6(4): 53-82.

Greenberg, J. (1990) *The Theory of Social Situations - An Alternative Game-Theoretic Approach* Cambridge University Press, Cambridge.

Harsanyi, J. and Selten, R. (1988) *A General Theory of Equilibrium Selection in Games* MIT Press, Cambridge, Ma..

Harstad, R.M. and Phlips, L. (1993) "Futures Market Contracting When You Dont' Know Who the Optimists Are" *EUI Working Paper 93/3* European University Institute, Florence.

Herreiner, D.K. (1993) "Searchers and Loyalists - Endogenized Matching" *draft* version February 1993, European University Institute, Florence.

Houthakker, H.S. (1976) *The World Price of Oil: A Medium-Term Analysis* American Enterprise Institute for Public Policy Research, Washington.

Kirman, A.P. and M.J. Sobel (1974) "Dynamic Oligopoly with Inventories" *Econometrica* 42 (2): 279-287.

Klemperer, P. and Meyer, M. "Price Competition *vs.* Quantity Competition: the Role of Uncertainty" *Rand Journal of Economics* 17(4): 404-415.

Kreps, D. and Scheinkman, J. (1983) "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes" *Bell Journal of Economics* 14(2): 326-337.

Kurz, M. (1990) "On the Structure and Diversity of Rational Beliefs" *mimeo* (Revised, November 1991), Stanford University, California.

Kurz, M. (1991) "On Rational Belief Equilibria" *mimeo* (Revised, October 1991), Stanford University, California.

Kyle, A.S. (1984) "A Theory of Futures Market Manipulations" in R.W. Anderson (ed.) *The Industrial Organization of Futures Markets*, Lexington Books, Lexington: 141-75.

Lovell, M.C. (1986) "Tests of the Rational Expectations Hypothesis" *American Economic Review* 76: 110-124.

Mabro, R. *et al.* (1986) *The Market for North Sea Crude Oil* Oxford Institute for Energy Studies, Oxford University Press, Oxford.

McLennan, A. and Sonnenschein, H. (1991) "Sequential Bargaining as a Noncooperative Foundation for Walrasian Equilibrium" *Econometrica* 59(5): 1395-1424.

Milgrom, P. and Stokey, N. (1982) "Information, Trade and Common Knowledge" *Journal of Economic Theory* 26: 177-227.

Møllgaard, P. (1993) "Speculation, Expectations and Learning: a Review Essay", *good in progress* European University Institute, Florence.

Newbery, D.M.G. and Stiglitz, J.E. (1981) *The Theory of Commodity Price Stabilization - A Study in the Economics of Risk* Clarenden Press, Oxford.

Osborne, M.J. and Rubinstein, A. (1990) *Bargaining and Markets* Academic Press, San Diego, California.

Ostroy, J.M. and Starr, R.M. (1974) "Money and the Decentralization of Exchange" *Econometrica* 42(6): 1093-1113.

Phlips, L. (1983) *The Economics of Price Discrimination* Cambridge University Press.

Phlips, L. (1992) "Oligopolistic Pricing of Crude Oil Futures" in Stein, J.L. and Goss, B.A. (ed.s) *Supplement to The Economic Record, 1992, Special Issue on Futures Markets.*

Phlips, L. and Harstad, R.M. (1990) "Oligopolistic Manipulation of Spot Markets and the Timing of Futures Market Speculation" *EUI Working Paper n° 90/20* European University Institute, Florence.

Phlips, L. and Harstad, R.M. (1991) "Interaction Between Resource Extraction and Futures Markets: A Game-Theoretic Analysis" in R. Selten (ed.) *Game Equilibrium Models II*, Springer-Verlag, Heidelberg: 289-307.

Phlips, L. and Richard, J.-F. (1989) "A Dynamic Oligopoly Model with Demand Inertia and Inventories" *Mathematical and Social Sciences* 18 (1): 1-33.

Phlips, L. and Thisse, J.-F. (1981) "Pricing, Distribution and the Supply of Storage", *European Economic Review* 15: 225-243.

Rotemberg, J.J. and Saloner, G. (1985) "Strategic Inventories and The Excess Volatility of Production" *Sloan School of Management Working Paper* No. 1650-85, MIT (Ma.).

Rubinstein, A. (1982) "Perfect Equilibrium in a Bargaining Model" *Econometrica* 50(1): 97-109.

Saloner, G. (1986) "The Role of Obsolescence and Inventory Costs in Providing Commitment" *International Journal of Industrial Organization* 4: 333-345.

Saloner, G. (1987) "Cournot Duopoly with Two Production Periods" *Journal of Economic Theory* 42: 183-7.

Selten, R. (1982) "Einführung in die Theorie der Spiele mit unvollständiger Information" in *Information in der Wirtschaft* Schriften des Vereins für Socialpolitik, Gesellschaft für Wirtschafts - und Sozialwissenschaften, N.F. Band 126, Duncker & Humblodt, Berlin: 81-147.

Starr, R.M. (1976) "Decentralized Nonmonetary Trade" *Econometrica* 44(5): 1087-1089.

Starr, R.M. (1986) "Decentralized Trade in a Credit Economy" in W.P. Heller, R.M. Starr, and D.A. Starret (ed.s) *Equilibrium Analysis - Essays in Honor of Kenneth J. Arrow* Cambridge University Press, Cambridge: 105-120.

Stein, J.C. (1987) "Informational Externalities and Welfare-Reducing Speculation" *Journal of Political Economy* 95(6): 1124-45.

Telser, L.G. (1981) "Why There Are Organized Futures Markets" *Journal of Law and Economics* 24: 1-22.

Telser, L.G. (1992) "Corners in Organized Futures Markets" in L. Phlips and L.D. Taylor *Aggregation, Consumption and Trade — Essays in Honor of H.S. Houthakker* Kluwer Academic Publishers, Dordrecht: 159-167.

Tirole, J. (1982) "On the Possibility of Speculation under Rational Expectations" *Econometrica* 50: 1163-1182.

Tirole, J. (1989) *The Theory of Industrial Organization* MIT Press, Cambridge, Ma..

Ulph, D. (1983) "Rational Conjectures in the Theory of Oligopoly" *International Journal of Industrial Organization* 1: 131-154.

Waldmann, R. (1992) "Asymmetric Oligopoly" *EUI Working Paper 92/63* European University Institute, Florence.

Ware, R. (1985) "Inventory Holding as a Strategic Weapon to Deter Entry" *Economica* 52: 93-101.

Weekly Petroleum Argus (1991/2) "Weekly Petroleum Argus - Oil Price Reporting and Analysis" Vol. XXI-XXII.

Weiner, R.J. (1992) "Origins of Futures Trading: The Oil Exchanges in the 19th Century" *mimeo*, Brandeis University.

Working, H. (1949) "The Investigation of Economic Expectations" *American Economic Review* 39 (3) (Papers and Proceedings): 151-166.

Working, H. (1962) "New Concepts Concerning Futures Markets and Prices" *American Economic Review* 52: 431-459.