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Endogenous Information Acquisition

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ABSTRACT. **This paper discusses a liquidity run model where investors optimally decide whether or not to acquire private information. This endogenizes the dichotomy "private information/no private information". The price of the information makes the equilibrium partitioning of the fundamentals two dimensional. For intermediate fundamentals multiplicity can be eliminated by the private information that investors *can have*. The dichotomy represents the information structures for low and high prices respectively. However, it presents a distorted view for intermediate prices and fundamentals for which unique equilibria without private information can occur. These results are preserved if the quality of the information is endogenized.**

KEYWORDS. **Bank runs, information acquisition, coordination games**

JEL CLASSIFICATION. **C73, D81, F34, G14**

Investors have incentives to be well-informed when they choose their portfolios. However, the actual quality of their information is also determined by its price. Liquidity run models after Diamond and Dybvig (1983) and Morris and Shin (2000), henceforth referred to as DD and MS respectively, take the information structure as given.[1] In DD models, investors do not have private information and can only base their decision whether to remain or withdraw on a common prior for the fundamentals of the investment. In MS models, investors can also base their decision on private information about these fundamentals, albeit incomplete. This difference in information structure has considerable implications for the equilibria. In DD models without private information there are two possible symmetric equilibria: all investors either remain or withdraw. Although for very good or very bad priors there is a unique equilibrium, multiplicity occurs for intermediate priors. In MS models with private information there is always a unique equilibrium if the private information is sufficiently precise. In this equilibrium investors with good private information remain while investors with bad information withdraw.

This paper presents a liquidity run model where private information acquisition is endogenous. Investors optimally decide whether to acquire private information taking its price as given. A trade-off between the price of information and its expected added value in terms of the investment return decides whether or not information is acquired. The price can be seen as monetary costs when the investors hire an investment agency or as a cost in terms of effort and time when they search for information themselves. The prior for the fundamentals together with the price of information determines whether in equilibrium investors acquire private information, which in turn determines the occurrence and extent of the liquidity run. We thereby address a comment on MS models raised by Rey (2000) that "costly and voluntary information acquisition should ideally be related to the other fundamentals of the economy." Hence, in addition to fundamental causes and self-fulfilling prophecies, we introduce the availability and quality of private information, or more general transparency, as an explanting factor for liquidity runs.

Although the ways in which liquidity runs occur in DD and MS models appear to be very different, both can arise in our model. When the price of information is very high, investors do not acquire information and we arrive in the DD world. When on contrary the price is very low, investors will always find it attractive to acquire information and we are in the MS world. For intermediate prices, the two worlds are blended which gives rise to a variety of possible equilibrium outcomes. The dichotomy "private information/no private information" thus presents a misleading simplification. The existence of an equilibrium for all prices and priors follows from complementarities in information acquisition. When investors base their decisions on private information, uncertainty about the investment return is increased, which makes private information more valuable.

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[1]We will interpret these models in a setting where investors have to decide whether or not to withdraw their money from a certain investment. Alternatively, we could have phrased our paper in terms of investors that have to decide on rolling-over the debt of a country. More general interpretations of the underlying coordination problem are possible as well.

Interestingly, even when the prior for the fundamentals is in the intermediate range for which multiplicity occurs in DD models, there can be a unique equilibrium in our model. Morris and Shin (2000) introduced private information to construct a hybrid equilibrium in which some investors remain and others withdraw to eliminate this DD multiplicity. This paper provides a two dimensional equilibrium partitioning of priors and prices in which each of the three equilibria can be unique for intermediate priors.

The main consequence of endogenous information acquisition is that the equilibria of the models with a fixed information structure are not necessarily equilibria in our model. The endogenous information acquisition eliminates a DD equilibrium in which the value of information is higher than its price. The MS equilibrium instead is eliminated if the value of information is lower than its price. For intermediate prices, the equilibrium where all investors remain without information is eliminated when the fundamentals are expected to be intermediate but on the bad side. The reason for this is that although the expected return is positive, there is a substantial probability that the realization of the fundamentals is such that the return is negative. This implies that information about the fundamentals is higher than its price. Likewise, when the fundamentals are expected to be intermediate but on the good side, the equilibrium where all investors withdraw without information is eliminated. The equilibrium where investors acquire information only exists when there is a high uncertainty about the sign of the return. This is the case for intermediate priors. This shows that for example in case of intermediate prices and intermediate but relatively bad priors for the fundamentals, there is a unique equilibrium in which all investors withdraw without information. Hence, the multiplicity that occurs for intermediate priors in DD models does not necessarily occur in our model. Even for these priors, an equilibrium without private information can be unique. Note that the multiplicity is not eliminated by imposing private information. The multiplicity can be eliminated by the private information that investors *can have*.

The equilibrium analysis shows that an increase in the precision of the private information or a decrease in its price have a similar effect on the information acquisition. Second, it shows that if bad fundamentals are likely, cheap information with a high precision (defining features of transparency) favors information acquisition and thereby helps to deter a liquidity run. When good fundamentals are likely, information acquisition is also favored but this leads to more withdrawing investors. Third, a high volatility favors information acquisition since the high uncertainty makes private information valuable. The effect on the run depends on whether the prior for the fundamentals is good or bad.

In Section 1 we discuss the basic model where the precision of the information is fixed and investors only have to decide whether or not to acquire information. A detailed analysis of the value of information and the equilibria is given in Section 2. In Section 3 we then discuss an extension of the model where investors can choose the precision of their information. The price of information is increasing in its precision which ensures that there is still a trade-off between the cost of a better precision and the added value. The results of the extended model and the

basic model are similar, if not strengthened. Section 4 discuss related literature and Section 5 concludes. The analysis of non-symmetric equilibria is deferred to Appendix A and proofs to Appendix B.

1. Model

There is a continuum of identical investors with total measure equal to 1. The utility function of the investors is simply a linear function of their money holdings. In period 0, all investors have put one unit in the same investment project. In period 1, investors receive new common information about the investment return that replaces any previous information and that causes investors to reconsider their investment. In this period they then simultaneously decide whether to remain invested or to withdraw the money. Investors who withdraw will be fully refunded. They can then use the money for a risk-free investment alternative with a normalized (net) return of 0 in the next period. Investors who remain invested until period 2 receive a (net) return of $\theta - \ell$, where θ is a random variable summarizing the fundamentals of the investment project and $\ell \in [0, 1]$ is the fraction of investors who withdrew in period 1.[2] The idea is that if investors withdraw, the project has to be downsized, which negatively affects the return of the remaining investors. Hence, the return function combines the fundamentals and the cost of premature liquidation. We assume that the fundamental θ is normally distributed with *ex ante* expectation $\hat{\theta} \in \mathbb{R}$ and precision $\alpha > 0$. These parameters are provided to the public only at the beginning of period 1.[3]

Before making the investment decision in period 1, but after learning $\hat{\theta}$ and α, investors can acquire private noisy information about the realization θ of the fundamental. This information helps to discriminate between good and bad realizations of the fundamental. Hence, by conditioning the investment decision on the information, an investor can remain invested if the fundamental is likely to be good, while she can leave otherwise. Since in expectation this will increase her return, the information is valuable.

Investors decide simultaneously whether or not to acquire noisy information. Since we want to analyze the effect of the price of the information, we let this price be exogenously determined. The price can be seen as a purely monetary cost of the information, but it may also reflect the efforts needed to collect the information. By focusing on noisy information instead of complete information, we want to capture the idea that different investors combine information from various noisy sources which leads to different private information. When investor i acquires information, she receives a realization of $x_i = \theta + \varepsilon_i$. The noise ε_i is independent across investors and distributed according to a normal distribution with zero mean and precision $\beta > 0$. In this section we assume that β is fixed, identical for all investors and public knowledge. In Section 3 we discuss the model where investors can acquire information with any precision. We assume

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[2]For the sake of clarity we do not truncate the return in case $\theta - \ell < -1$. This is not essential for the results.

[3]An alternative interpretation is that the fundamental has an improper uniform distribution over the real line. For a realization θ and normally distributed noise ε with zero mean and precision α, $\hat{\theta} = \theta + \varepsilon$ then represents imperfect public information about the *realization* θ.

that an investor does not know the information decisions of other investors when she makes her investment decision.[4]

Throughout the paper we will focus on symmetric equilibria, but in Appendix A we will discuss non-symmetric equilibria. For explanatory convenience we use the following rule when there is a mass zero of indifferent investors: an investor invests if she is indifferent between remaining and withdrawing and she acquires information if she is indifferent between acquiring and not acquiring. This is not essential for the results.

In case all investors decide not to acquire information, they have no means of coordination and we are in the Diamond-Dybvig world. Denote the two symmetric strategy profiles "no-information/all-remain" and "no-information/all-withdraw" by R and W respectively. Whether these profiles are equilibria depends on the value and the price of information.

In case all investors acquire information, we are in the Morris-Shin world. Whether an investor remains or withdraws depends on her private information. The related equilibrium candidate is characterized by a common switching point x^* such that investor i withdraws if and only if $x_i < x^*$, so if her information is worse than the switching point x^*.[5] An investor who receives the switching point as private information expects that the return of remaining invested is 0, so $\mathbb{E}[\theta - \ell | x^*] = 0$. For $\gamma_{MS} = \frac{\alpha^2}{\beta} \frac{\alpha+\beta}{\alpha+2\beta}$, it is shown in Morris and Shin (2000) that there is a unique switching point if $\gamma_{MS} < 2\pi$. Denote an equilibrium candidate "information/switching-point-x^*" by I.

In this section we derive detailed expressions for the maximum price investors are willing to pay for information. In the next section we will use these maximum prices to relate the equilibria with the *ex ante* expectation of the fundamental. We first introduce some notation and analyze the information structure in more detail. For given private information and candidate equilibrium, let the function $v : \emptyset \cup \mathbb{R} \times \{I, R, W\} \to \mathbb{R}$ denote the expected value of the investment opportunity for an investor who reacts optimally to the strategies of the other investors. We thus loosely interpret the profiles I, R and W as prescribing the strategies of all investors but one. For $Q \in \{I, R, W\}$ the function v is then defined as

$$v(\emptyset, Q) = \max\{0, \mathbb{E}[\theta - \ell | Q]\}, \tag{1}$$
$$v(x_i, Q) = \max\{0, \mathbb{E}[\theta - \ell | x_i, Q]\}. \tag{2}$$

Note that v is bounded, since $\mathbb{E}[\|X\|]$ is bounded when X has a normal distribution. We will denote information of which the value is not yet known with a capital letter, so the random variable X_i denotes the yet unknown information of investor i. Since she does not know the realization of the fundamental, for her the information X_i has a normal distribution with mean $\hat{\theta}$ and precision $\frac{1}{1/\alpha + 1/\beta} = \frac{\alpha\beta}{\alpha+\beta}$. Conversely, Bayesian updating shows that the density of θ

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[4]Assuming that these information decisions are known would not change the equilibrium analysis of Section 2. It would, however, introduce new non-symmetric equilibria.

[5]The model is thus a global game, see Carlsson and van Damme (1993) and Morris and Shin (2002).

conditional on the information x_i is proportional to

$$e^{-\frac{1}{2}\alpha(\theta - \hat{\theta})^2} e^{-\frac{1}{2}\beta(x_i - \theta)^2} \propto e^{-\frac{1}{2}(\alpha + \beta)(\theta - \frac{\alpha\hat{\theta}+\beta x_i}{\alpha+\beta})^2}. \tag{3}$$

So, $\theta|x_i$, the fundamental conditional on information x_i, has a normal distribution with mean $\frac{\alpha\hat{\theta}+\beta x_i}{\alpha+\beta}$ and precision $\alpha + \beta$.

Now consider the two Diamond-Dybvig candidate equilibria. The return in the R profile where all investors remain equals the *ex ante* expectation of the fundamental $\hat{\theta}$. In the original model investors do not have the possibility of acquiring information. The R profile is then an equilibrium if remaining gives a (weakly) higher return than withdrawing, so if and only if $\hat{\theta} \geq 0$. Likewise, the W profile where all investors withdraw is an equilibrium only for $\hat{\theta} \leq 1$ since the expected return equals $\hat{\theta} - 1$. Note especially that when the *ex ante* expectation of the fundamental is in the interval $[0, 1]$ both profiles are equilibria. In our model, investors do have the possibility to acquire information. Since this might be a profitable deviation, equilibria in the original model are not necessarily equilibria in our model. In order to determine whether a profile is an equilibrium, we need to confront the value of information with its price.

We first analyze the R profile where all investors remain. We can restrict the analysis to non-negative *ex ante* expectations of the fundamental, $\hat{\theta} \geq 0$, since the discussion of the original model showed that this is a necessary condition for the profile to be an equilibrium. Because it is necessarily (weakly) optimal for an investor without information to remain, this directly implies $v(\emptyset, R) = \hat{\theta}$. Suppose investor i deviates by acquiring private information and that she receives information x_i. She will invest if and only if $\frac{\alpha\hat{\theta}+\beta x_i}{\alpha+\beta} \geq 0$, so if $x_i \geq -\frac{\alpha}{\beta}\hat{\theta}$. The expected value of the investment opportunity when the value of the information is still unknown thus equals

$$\mathbb{E}[v(X_i, R)] = \mathbb{E}\left[\theta \,\Big|\, X_i \geq -\frac{\alpha}{\beta}\hat{\theta}\right] \mathbb{P}\left[X_i \geq -\frac{\alpha}{\beta}\hat{\theta}\right] \tag{4}$$

$$= \int_{-\frac{\alpha}{\beta}\hat{\theta}}^{\infty} \frac{\alpha\hat{\theta} + \beta x_i}{\alpha + \beta} \frac{\sqrt{\frac{\alpha\beta}{\alpha+\beta}}}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{\alpha\beta}{\alpha+\beta}(x_i - \hat{\theta})^2} \mathrm{d}x_i$$

$$= \hat{\theta}\Phi\left(\sqrt{\frac{\alpha + \beta}{\beta/\alpha}}\hat{\theta}\right) + \frac{\sqrt{\frac{\beta/\alpha}{\alpha+\beta}}}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{\alpha+\beta}{\beta/\alpha}\hat{\theta}^2},$$

where Φ denotes the normal cumulative density function. The maximum price investor i is willing to pay for information in the R profile is the difference between the value of the investment opportunity with and without information. The value of information as function of the *ex ante* expectation of the fundamental is thus given by $p^R(\hat{\theta}) = \mathbb{E}[v(X_i, R)] - \hat{\theta}$.

Similarly, consider the W profile where all investors withdraw. We can restrict the analysis to $\hat{\theta} \leq 1$, which necessarily implies $v(\emptyset, W) = 0$. Suppose that investor i has acquired information x_i. The negative externality of the withdrawing investors makes her invest if and only if $\frac{\alpha\hat{\theta}+\beta x_i}{\alpha+\beta} \geq 1$, so if $x_i \geq \frac{\alpha+\beta}{\beta} - \frac{\alpha}{\beta}\hat{\theta}$. We then find for the expected value of the investment opportunity given

unknown information

$$\mathbb{E}[v(X_i, W)] = \mathbb{E}\left[\theta - 1 \,\middle|\, X_i \geq \frac{\alpha + \beta}{\beta} - \frac{\alpha}{\beta}\hat{\theta}\right] \mathbb{P}\left[X_i \geq \frac{\alpha + \beta}{\beta} - \frac{\alpha}{\beta}\hat{\theta}\right] \tag{5}$$

$$= (\hat{\theta} - 1)\Phi\left(\sqrt{\frac{\alpha + \beta}{\beta/\alpha}}(\hat{\theta} - 1)\right) + \frac{\sqrt{\frac{\beta/\alpha}{\alpha+\beta}}}{\sqrt{2\pi}}e^{-\frac{1}{2}\frac{\alpha+\beta}{\beta/\alpha}(\hat{\theta} - 1)^2}.$$

Define $p^W(\hat{\theta}) = \mathbb{E}[v(X_i, W)]$, so that $p^W(\hat{\theta})$ is the value of information, or equivalently, the maximum price an investor is willing to pay for information in the W profile.

We continue by considering the Morris-Shin type I profile where all investors acquire information and take the investment decision according to a switching point strategy. In contrast with the R and W profiles, the value of information in the I profile is not only coming from the ability of discriminating between good and bad realizations of the fundamental. In the I profile investors base their decision whether to run or not on their information. Hence, private information allows to better predict the private information of other investors and is thus also useful for predicting the fraction of withdrawing investors.

We assume that $\gamma_{MS} < 2\pi$ so that there exists a unique switching point x^*. When investor i has no information, $v(\emptyset, I) = \max\{0, \mathbb{E}[\theta - \ell|I]\}$. Her expectation of the fundamental is $\hat{\theta}$. We can apply the law of large numbers (see Judd (1985)) to show that the fraction of withdrawing investors is equal to the probability that investor j receives information that is worse than the switching point, so

$$\mathbb{E}[\ell|I] = \mathbb{P}[X_j < x^*] = \Phi\left(\sqrt{\frac{\alpha\beta}{\alpha + \beta}}(x^* - \hat{\theta})\right). \tag{6}$$

Now suppose that investor i decides to acquire information. We first compute the expected value of the investment opportunity conditional on information x_i. We have already seen that $\theta|x_i$ has a normal distribution with mean $\frac{\alpha\hat{\theta}+\beta x_i}{\alpha+\beta}$ and precision $\alpha+\beta$. Since $X_j|x_i = (\theta+\varepsilon_j)|x_i = \theta|x_i + \varepsilon_j$, we know that $X_j|x_i$ has a normal distribution with the same mean but with precision $\frac{1}{1/(\alpha+\beta)+1/\beta} = \frac{\beta(\alpha+\beta)}{\alpha+2\beta}$. The fraction of withdrawing investors in the I equilibrium conditional on x_i is then given by

$$\mathbb{E}[\ell|x_i, I] = \mathbb{P}[X_j < x^*|x_i] = \Phi\left(\sqrt{\frac{\beta(\alpha + \beta)}{\alpha + 2\beta}}\left(x^* - \frac{\alpha\hat{\theta} + \beta x_i}{\alpha + \beta}\right)\right). \tag{7}$$

We now have a detailed expression for $v(x_i, I) = \max\{0, \mathbb{E}[\theta - \ell|x_i, I]\}$. It is straightforward to show that this value is increasing in x_i. Intuitively, when investor i receives better information she expects a better realization of the fundamental and a smaller fraction of withdrawing investors. The definition of the switching point gives that $\mathbb{E}[\theta - \ell|x^*, I] = 0$. Hence, investors only remain if their information is weakly better than x^*. The expected value of the investment opportunity when an investor has still unknown information thus equals

$$\mathbb{E}[v(X_i, I)] = \mathbb{E}[\theta - \ell|X_i \geq x^*, I]\mathbb{P}[X_i \geq x^*]. \tag{8}$$

The expected contribution of the fundamental to the value of the investment opportunity given the unknown information is

$$\mathbb{E}[\theta|X_i \geq x^*, I]\mathbb{P}[X_i \geq x^*] = \int_{x^*}^{\infty} \frac{\alpha\hat{\theta} + \beta x_i}{\alpha + \beta} \frac{\sqrt{\frac{\alpha\beta}{\alpha+\beta}}}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{\alpha\beta}{\alpha+\beta}(x_i - \hat{\theta})^2} \, \mathrm{d}x_i \tag{9}$$

$$= \hat{\theta}\Phi\left(\sqrt{\frac{\alpha\beta}{\alpha + \beta}}\left(\hat{\theta} - x^*\right)\right) + \frac{\sqrt{\frac{\beta/\alpha}{\alpha+\beta}}}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{\alpha\beta}{\alpha+\beta}(\hat{\theta} - x^*)^2}.$$

Using Equation (7), we find for the expected contribution of ℓ to the value of the investment opportunity given the unknown information

$$\mathbb{E}[\ell|X_i \geq x^*, I]\mathbb{P}[X_i \geq x^*] = \int_{x^*}^{\infty} \Phi\left(\sqrt{\frac{\beta(\alpha + \beta)}{\alpha + 2\beta}}\left(x^* - \frac{\alpha\hat{\theta} + \beta x_i}{\alpha + \beta}\right)\right) \tag{10}$$

$$\dots \times \frac{\sqrt{\frac{\alpha\beta}{\alpha+\beta}}}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{\alpha\beta}{\alpha+\beta}\left(x_i - \hat{\theta}\right)^2} \, \mathrm{d}x_i.$$

Given these findings, the value of information in the I profile as function of the *ex ante* expectation of the fundamental is now given by $p^I(\hat{\theta}) = \mathbb{E}[v(X_i, I)] - v(\emptyset, I)$.

2. Analysis

When investors have to decide whether or not to acquire information, they compare the value of information with its price. The R profile where all investors remain without acquiring information is an equilibrium if the value of information is *lower* than its price. The same condition should hold for the W profile where all investors withdraw without acquiring information to be an equilibrium. However, the I profile where all investors acquire information, is an equilibrium if the value of information is *higher* than its price. We first state a proposition about the values of information as function of $\hat{\theta}$ in the three profiles. The implications for the equilibrium follow directly from this proposition and are summarized in a corollary.

In Figure 1 the value of information for all three profiles is shown as function of the *ex ante* expectation of the fundamental.[6] The figure also indicates when the equilibria exist. The following proposition states that for all α and β such that $\gamma_{MS} < 2\pi$, the values of information $p^R(\hat{\theta})$, $p^W(\hat{\theta})$ and $p^I(\hat{\theta})$ behave as depicted in Figure 1 (the condition implies a unique switching point strategy so that the I profile is well defined).

Proposition 1. *Assume that $\gamma_{MS} < 2\pi$.*

    (i) *The values of information in the I, R and W profile, p^I, p^R and p^W respectively, are positive.*

    (ii) *The value of information as function of the* ex ante *expectation of the fundamental $\hat{\theta}$ in the I profile is strictly increasing for $\hat{\theta} < \frac{1}{2}$ and strictly decreasing for $\hat{\theta} > \frac{1}{2}$, in the R profile it is strictly decreasing and in the W profile it is strictly increasing.*

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[6]The vertical lines starting at $(0, p^R(0))$ and $(1, p^W(1))$ are included to indicate that for prices higher than $p^R(0) = p^W(1)$ the R and W equilibrium cannot exist for $\hat{\theta} < 0$ and $\hat{\theta} > 1$ respectively.
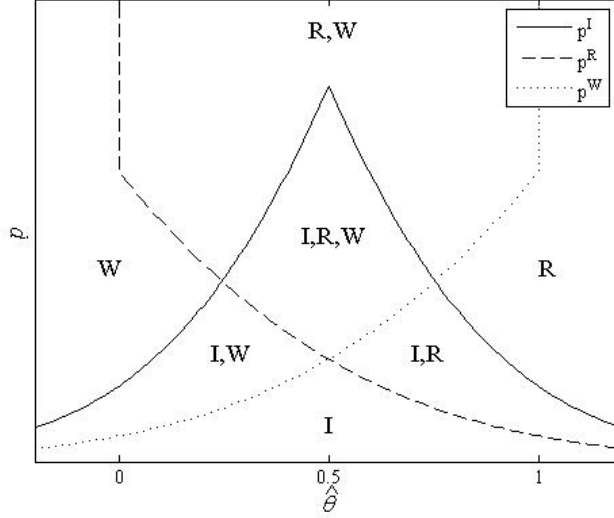
FIGURE 1. The value of information as function of the *ex ante* expectation of the fundamental and the equilibrium regions. $(\alpha = 1, \beta = 1)$

(iii) *There exists a threshold $\bar{\theta} \in (0, \frac{1}{2})$ such that if $\hat{\theta} \leq \bar{\theta}$ the value of information is highest in the R profile, if $\hat{\theta} \in [\bar{\theta}, 1 - \bar{\theta}]$ it is highest in the I profile and if $\hat{\theta} \geq 1 - \bar{\theta}$ it is highest in the W profile.*

(iv) *For **bad** ex ante expectations of the fundamental, $\hat{\theta} \leq \frac{1}{2}$, the value of information is lowest in the W profile, while for **good** ex ante expectations of the fundamental, $\hat{\theta} \geq \frac{1}{2}$, it is lowest in the R profile.*

(v) *The values of information in the R and W profiles are symmetric images around $\hat{\theta} = \frac{1}{2}$ in the sense that $p^R(\frac{1}{2} + d) = p^W(\frac{1}{2} - d)$, $d \geq -\frac{1}{2}$. The value of information in the I profile is symmetric around $\hat{\theta} = \frac{1}{2}$ in the sense that $p^I(\frac{1}{2} + d) = p^I(\frac{1}{2} - d)$, $d \geq 0$.*

Statement (i) claims that information always has a positive value. This implies that there always exist strictly positive prices for which the value of information is higher than its price.

From statement (ii) it follows that when the *ex ante* expectation of the fundamental increases, the value of information in the R profile decreases. Intuitively this follows from the fact that when $\hat{\theta}$ increases, the probability of making a loss in the R profile decreases, hence the price an investor is willing to pay for receiving information decreases. A symmetric argument gives that the foregone positive return in the W profile increases when $\hat{\theta}$ increases and that the value of information thus also increases. Statement (ii) also shows that p^I behaves similar to p^W for $\hat{\theta} < \frac{1}{2}$ and similar to p^R for $\hat{\theta} > \frac{1}{2}$. When $\hat{\theta} < \frac{1}{2}$ an investor in the I profile would withdraw when she has no information (we prove this in Lemma 6 in Appendix B). The foregone positive return increases when $\hat{\theta}$ becomes larger, so the price that an investor is willing to pay for information increases. When $\hat{\theta} > \frac{1}{2}$ an investor would remain when she has no information. The

expected return is increasing in $\hat{\theta}$ and does so faster than the return in case she would have had information. Hence, the maximum price an investor is willing to pay for information decreases.

Statement (iii) implies that when $\hat{\theta}$ is relatively low, information in the R profile is the most valuable. Intuitively, although the expected return is positive, it is not unlikely that the realized return will be negative. Investors want to be able to withdraw in these cases which makes the information valuable. Similarly, when the fundamental is likely to be good, the expected return of the investment in the W profile is negative but there are many realizations of the fundamental for which it is positive. Investors want to be able to distinguish these cases which gives information a high value. Finally, when $\hat{\theta}$ is intermediate in the I profile, the expected fraction of withdrawing investors is also intermediate. Information has a high value since it enables investors to predict which of the two is largest.

From statement (iv) we know that the value of information in the I profile is never the lowest of the three. The intuition why for $\hat{\theta} \leq \frac{1}{2}$ the value of information in the W profile is lower than in the I profile is straightforward. Since in the latter there is always a positive fraction of investors who remain, the expected return of remaining will always be higher than in the W profile.

Finally, statement (v) shows that the R and W profiles are not only symmetric in the sense that in the one all investors remain and in the other all investors withdraw, but that this symmetry goes further: $p^R(\frac{1}{2} + d) = p^W(\frac{1}{2} - d)$. The intuition is as follows. In the R profile an investor without information remains, while with information she can withdraw for very bad information. The value of information is thus minus the return in case of bad information. In the W profile the value of information is the return in case of good information. For fundamentals $\frac{1}{2} - d$ and $\frac{1}{2} + d$ these values are the same. Statement (v) also shows that $p^I(\frac{1}{2} + d) = p^I(\frac{1}{2} - d)$. Although somewhat more involved, the intuition is as before: for $\hat{\theta} = \frac{1}{2} - d$ information allows investors to remain when positive returns are expected, while for $\frac{1}{2} + d$ it allows investors to withdraw when negative returns are expected.

The R and W equilibrium only exist when the value of information is higher than its price. The I equilibrium instead only exists when the value is lower than the price. The existence of an equilibrium for all $\hat{\theta}$ now follows from the fact that $p^I(\hat{\theta}) > \min\{p^R(\hat{\theta}), p^W(\hat{\theta})\}$ (statement (iv) of Proposition 1). The intuition for $\hat{\theta} \leq \frac{1}{2}$ is as follows. In this case an investor without information should withdraw in the I equilibrium. The difference between $p^I(\hat{\theta})$ and $p^W(\hat{\theta})$ is thus only caused by different returns when information is acquired. In the I profile there are always some investors who remain. Conditional on the same private information, the expected return in the I profile is higher than in the W profile. An investor with private information is more willing to remain in the I profile than in the W profile (her switching point is lower). But this increases the return of other investors, who are in turn more willing to remain, etc. This implies that $p^I(\hat{\theta}) > p^W(\hat{\theta})$. A symmetric argument explains why $p^I(\hat{\theta}) > p^R(\hat{\theta})$ when $\hat{\theta} \geq \frac{1}{2}$. We conclude that the strategic complementarity in information acquisition ensures the existence of an equilibrium.

Let p denote the price of information. The following corollary reconciles the Diamond-Dybvig and the Morris-Shin world in a formal way.

Corollary 2. *Assume $\gamma_{MS} < 2\pi$.*

   (i) *The I profile is the unique equilibrium if and only if $p < \min\{p^R(\hat{\theta}), p^W(\hat{\theta})\}$.*
      The R and W profiles are the only equilibria candidates if and only if $p > p^I(\hat{\theta})$.

  (ii) *The R profile is the unique equilibrium if and only if both $p > p^I(\hat{\theta})$ and in addition $\hat{\theta} \leq 1$ implies $p < p^W(\hat{\theta})$).*
      The W profile is the unique equilibrium if and only if both $p > p^I(\hat{\theta})$ and in addition $\hat{\theta} \geq 0$ implies $p < p^R(\hat{\theta})$).

 (iii) *The sets $\{(\hat{\theta}, p) \in [0,1] \times (0, \infty) |\ Q$ is the unique equilibrium$\}$, $Q \in \{I, R, W\}$, are non-empty.*

In Appendix A we prove that if the I, R or W profile is the unique equilibrium among symmetric profiles, it is also the unique equilibrium if we allow for non-symmetric profiles.

Statement (i) relates the Diamond-Dybvig and the Morris-Shin world. If the price of information is sufficiently high, investors do not acquire private information. We arrive in the Diamond-Dybvig world where only the R equilibrium and the W can exist. If the price is sufficiently low, we arrive in the Morris-Shin world where investors acquire information and the I profile is the unique equilibrium. From statement (i) of Proposition 1 we know that $p^R(\hat{\theta})$ and $p^W(\hat{\theta})$ are strictly positive for every $\hat{\theta}$. This implies that for every $\hat{\theta}$ there are positive prices for which the I equilibrium is unique. Hence, although there are finite prices that guarantee that we are in the Diamond-Dybvig world regardless of the *ex ante* expectation of the fundamental, in fact any price higher than $p^I(\frac{1}{2})$ achieves this, we are only for sure in the Morris-Shin world when $p = 0$, so when the private information is for free.

From statement (ii) it follows that our model is doing more than embedding the two models. While in the original Diamond-Dybvig the R equilibrium exists for $\hat{\theta} \geq 0$ and the W equilibrium for $\hat{\theta} \leq 1$, in our model there are different conditions. This reflects that the possibility of acquiring information reduces the equilibrium regions.

Statement (iii) emphasizes the implications of statements (i) and (ii) for intermediate *ex ante* expectations of the fundamentals. For $\hat{\theta} \in [0,1]$ the R and W profiles are both equilibria if private information is not available. In our model, however, the I profile is the unique equilibrium for sufficiently low (but still positive) prices. Also, statements (iii) and (iv) of Proposition 1 show that there exists a threshold $\bar{\theta} \in (0, \frac{1}{2})$ such that when $\hat{\theta} \in (1 - \hat{\theta}, 1]$ we have $p^W(\hat{\theta}) > p^I(\hat{\theta}) > p^R(\hat{\theta})$. Hence, for $\hat{\theta} \in (1 - \bar{\theta}, 1]$ and $p \in (p^I(\hat{\theta}), p^W(\hat{\theta}))$ the R equilibrium is unique. Similarly, the W equilibrium is unique for $\hat{\theta} \in [0, \bar{\theta})$ and $p \in (p^I(\hat{\theta}), p^R(\hat{\theta}))$. Compared to the model without private information the W equilibrium is eliminated for $\hat{\theta}$ close to 1, while the R equilibrium is eliminated when $\hat{\theta}$ is close to 0. It is the intuitively more likely equilibrium that survives: R when the *ex ante* expectation of the fundamental is good, W when it is bad.

It is interesting to look more carefully at the two ways the multiplicity of the original Diamond-Dybvig model with $\hat{\theta} \in [0,1]$ disappears for some combinations of $\hat{\theta}$ and p. First, for low prices of information investors *have* private information, which replaces the multiple equilibria with a unique hybrid switching point equilibrium. Ruling out multiplicity was in fact the very reason that private information was introduced in Morris and Shin (2000). Second, for low but not very low prices and $\hat{\theta}$ close to 0 or 1, the original multiplicity disappears since investors *can have* private information. It is the sheer possibility of being able to acquire information that eliminates one of the equilibria.

For intermediate prices the Diamond-Dybvig and Morris-Shin worlds are blended. For example, when $\hat{\theta} \leq \frac{1}{2}$ both the I and the W equilibrium occur when the price is between $p^W(\hat{\theta})$ and $p^I(\hat{\theta})$. For $\hat{\theta}$ close to $\frac{1}{2}$ even the R equilibrium joins and the multiplicity icnreases. Hence, sunspots are not ruled out when the price is not convincingly low or high. But the jump is not necessarily extreme in the sense that all investors suddenly change behavior. The hybrid I equilibrium where some investors remain and others withdraw can smooth a jump.

Our model presents a partitioning of the *ex ante* expectation of the fundamental in two dimensions. Liquidity run models without private information typically have a one dimensional partitioning where a run occurs when $\hat{\theta}$ is bad, no run occurs when $\hat{\theta}$ is good, and both a run or no run can occur for intermediate $\hat{\theta}$. Liquidity run models with private information typically have the trivial partitioning of a unique hybrid equilibrium where a fraction of the investors withdraws for all $\hat{\theta}$. The extent of the run is then decreasing in $\hat{\theta}$. In our model these one-dimensional partitionings arise for high prices ($p > p^I(\frac{1}{2})$) or for free private information ($p = 0$). In general, the partitioning of $\hat{\theta}$ depends on the price of information and concerns three different equilibria. The *ex ante* expectation of the fundamental and the price together determine whether or not a run occurs and its extent.

In the original Diamond-Dybvig model with $\hat{\theta} \in [0,1]$, not only the *ex ante* expectation of the fundamental, but also its precision does not play a role in the equilibrium selection. However, the availability of private information makes that besides β also α has an effect on the maximum prices. Since a high transparency is associated with a high precision of private information available for a low price, we can expect that an increase in β has the same effect as a decrease in the price of information, which suggests that the value of information increases. When α decreases, the relative importance of the private information increases, so intuitively we expect results similar to an increase in β. The following proposition makes this precise.

Proposition 3. *Assume $\gamma_{MS} < 2\pi$. The value of information in the R, W and I profiles is decreasing in α and increasing in β.*

When the precision of the fundamental decreases or the precision of private information increases, an investor with information is better able to distinguish the cases where the return will be positive from the cases where it will be negative. This ensures that in the R and the W profiles she will receive a higher return and that the value of information increases. In the

I profile an increase in the precision of private information or a decrease in the precision of the fundamental has the same effect. For *ex ante* bad fundamentals ($\hat{\theta} < \frac{1}{2}$) the intuition is straightforward: better information is positive since it allows investors to better distinguish good *ex post* fundamentals which makes investors more willing to invest. The expected return when acquiring information thus increases while without information the investor will still withdraw. This shows that the value of information increases. For *ex ante* good fundamentals ($\hat{\theta} > \frac{1}{2}$) the intuition is not so clear: investors can also better distinguish the cases of good and bad realizations of the fundamentals. Since without private information it is optimal to remain, investors are now more inclined to withdraw. This reduces the expected return when acquiring information, but it also reduces the expected return when not acquiring information. Since the value of information is symmetric in $\hat{\theta} = \frac{1}{2}$, we know that the second effect is stronger.

Combining Proposition 3 and Corollary 2 gives insight in how the equilibria depend on the precisions of the private information and the fundamental. When the investment project is likely to have a bad fundamental, $\hat{\theta} < \frac{1}{2}$, relatively cheap information with a high precision, so a high transparency, will help to attract investors. The reason is simple. Information with a high precision has a high value. When its price is low, investors are inclined to acquire information. We expect the I equilibrium to exist and not the W equilibrium. A low volatility of the fundamental has an opposite effect. When a bad fundamental is likely, a low volatility of the fundamental makes a bad realization very likely. Information is not very useful in the W profile and we expect the W equilibrium to exist. A very volatile fundamental makes private information very useful and we expect that the I profile is an equilibrium. A decrease in α and an increase in β thus favor information acquisition. When the price is not too high, the information acquisition favors the I equilibrium at the expense of the W equilibrium. Since in the I equilibrium some investors remain, the run is less severe than in the W equilibrium. This reflects the effect of a decrease in α or an increase in β in the I equilibrium itself, where investors are more willing to remain when private information becomes relatively more important (see the proof of the proposition).

When the *ex ante* expectation of the fundamental is relatively good, $\hat{\theta} > \frac{1}{2}$, a decrease in α and an increase in β favor information acquisition for the same reason. However, the effect on the run is opposite. When the price is not too high, the information acquisition favors the I equilibrium at the expense of the R equilibrium. More investors withdraw and the run is more severe. Again this reflects the effects inside the I equilibrium.

For global game models where agents have private information, it is common practice to discuss the limiting case where the private information becomes arbitrarily precise. Combining Proposition 3 and Corollary 2 shows that when β increases to infinity, the price range for which the I profile is unique expands. For this limiting case the constraint on the price of information becomes less severe, and for not too high prices we indeed expect the agents to acquire private information.

Instead of changing the precision β of the private information, we could also allow investors to choose between information with precision β and information with precision $\beta_2 > \beta$ for a higher price. Since there is a new deviation possible, the R and W equilibrium regions will be smaller. The effect on the I equilibrium region is unclear. Although the new deviation makes the region of the I equilibrium with precision β smaller, the additional region of the I equilibrium with precision β_2 may offset this effect. In the next section we generalize this extension and discuss a model where investors can choose information with any precision.

3. EXTENSION

In this section we loosen the restriction that investors can only acquire information with an exogenously given precision β. We endogenize the precision by letting investors choose their preferred precision. The price of information is a linear function of its precision, so information with precision β costs $\beta\hat{p}$, where $\hat{p} > 0$ denotes the price of information per unit precision. Modelling the information acquisition in this way reflects the possibility of investors to buy $\beta \in [0, \infty)$ units of information with unit precision and price $\hat{p}$ each. Note that the cost of information is a convex function of the variance: information with half the variance costs double. We conjecture that qualitatively similar results are obtained for more general structures if the price as function of the precision is increasing and convex.

To indicate that the expected value of the investment opportunity conditional on information x_i depends on the precision β of this information, we will write $v(x_i, Q; \beta)$, $Q \in \{I, R, W\}$. Since an investor without private information is identical to an investor with unrelated information, which is information with zero precision, we slightly abuse notation by letting $\beta = 0$ refer to this case (i.e. $\mathbb{E}[v(X_i, Q; 0] = \mathbb{E}[v(\emptyset, Q)])$.

First consider the R profile with $\hat{\theta} \geq 0$. The problem of investor i is

$$\max_{\beta \geq 0} \mathbb{E}[v(X_i, R; \beta)] - \beta\hat{p}, \tag{11}$$

where Equation (4) gives an expression for $\mathbb{E}[v(X_i, R; \beta)]$ if $\beta > 0$. Since an investor will remain when her information is better than $-\frac{\alpha}{\beta}\hat{\theta}$, we see that if the precision goes to zero, she will always remain. Hence, $\lim_{\beta \to 0} \mathbb{E}[v(X_i, R; \beta)] - \beta\hat{p} = \mathbb{E}[v(X_i, R; 0)]$ and the objective function is continuous in β. Since the maximum return of investing is bounded, we know that when β becomes very large, the total return becomes negative. Hence, instead of maximizing over $[0, \infty)$ we can maximize over a certain closed interval $[0, \bar{\beta}]$ without affecting the outcome, so the maximum is well-defined.

We are interested in the maximum price per unit precision for which an investor acquires information. In Section 1 we derived the maximum price that an investor is willing to pay for information with precision β. To explicitly indicate the dependence on β we denote this maximum price by $p^R(\hat{\theta}; \beta)$. Hence, when the price per unit precision equals $\frac{p^R(\hat{\theta}; \beta)}{\beta}$ we know that investor i acquires information. The maximum price per unit precision for which an investor wants to acquire information is then given by $\hat{p}^R(\hat{\theta}) = \max_{\beta > 0} \frac{p^R(\hat{\theta}; \beta)}{\beta}$. To prove that this

maximum exists we use Equation (4) to obtain

$$\lim_{\beta \to 0} \frac{p^R(\hat{\theta}; \beta)}{\beta} = \lim_{\beta \to 0} \frac{\sqrt{\frac{1}{\alpha\beta(\alpha+\beta)}}}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{\alpha+\beta}{\beta/\alpha}\hat{\theta}^2} = 0. \tag{12}$$

Since the return is always finite the maximum price per unit precision goes to zero for $\beta \to \infty$, which shows that the maximum is well defined. We will refer to $\hat{p}^R(\hat{\theta})$ as the standardized value of information. When the price of information per unit precision is higher than the standardized value of information $\hat{p}^R(\hat{\theta})$, an investor will not acquire information and hence the R equilibrium exists. Since Proposition 1 states that all $\frac{p^R(\hat{\theta};\beta)}{\beta}$ decrease in $\hat{\theta}$, their upper envelope $\hat{p}^R$ is decreasing in $\hat{\theta}$ as well.

Now consider the W profile with $\hat{\theta} \leq 1$. Investor i needs to solve

$$\max_{\beta \geq 0} \mathbb{E}[v(X_i, W; \beta)] - \beta\hat{p}, \tag{13}$$

and an expression for $\mathbb{E}[v(X_i, W; \beta)]$ if $\beta > 0$ is given in Equation (5). In the same way as for the R profile it can be proved that the maximum is well-defined. Similarly to the R profile the standardized value of information is given by $\hat{p}^W(\hat{\theta}) = \max_{\beta>0} \frac{p^W(\hat{\theta};\beta)}{\beta}$. The W equilibrium only exists if there is no positive precision for which the standardized value of information is larger than its price. From Proposition 1 it follows that $\hat{p}^W$ is increasing in $\hat{\theta}$ and that $\hat{p}^R$ and $\hat{p}^W$ are symmetric images around $\hat{\theta} = \frac{1}{2}$.

For the I profile we focus on investor i. We assume that all other investors choose precision $\beta_j > 0$ and act according to a switching point strategy where the switching point x_j^* satisfies $\mathbb{E}[\theta - \ell | x_j^*] = 0$. The uniqueness condition is not satisfied for very small values of β_j. For these precisions we restrict the set of allowed strategies to switching point strategies.[7] In order to find her optimal precision, investor i has to solve the problem

$$\max_{\beta \geq 0} \mathbb{E}[v(X_i, I; \beta)] - \beta\hat{p}, \tag{14}$$

where an expression for $\mathbb{E}[v(X_i, I; 0)] = v(\emptyset, I)$ is given in Section 1. Of course, in equilibrium we should have that the maximizing β is equal to β_j. In order to determine the existence of an equilibrium we need more detailed expressions. As before, $v(x_i, I; \beta)$ is increasing in x_i. Thus for every $\beta > 0$ there exists a unique $x^*(\beta)$ such that investor i leaves if $x_i < x^*(\beta)$ and remains otherwise (we suppress the dependency of $x^*(\beta)$ on β_j). Since the information of another investor given information $x^*(\beta)$ is normally distributed with mean $\frac{\alpha\hat{\theta}+\beta x^*(\beta)}{\alpha+\beta}$ and precision $\frac{\beta_j(\alpha+\beta)}{\alpha+\beta+\beta_j}$, the value of $x^*(\beta)$ for $\beta > 0$ is determined by

$$\frac{\alpha\hat{\theta} + \beta x^*(\beta)}{\alpha + \beta} = \Phi\left(\sqrt{\frac{\beta_j(\alpha+\beta)}{\alpha+\beta+\beta_j}}\left(x_j^* - \frac{\alpha\hat{\theta} + \beta x^*(\beta)}{\alpha + \beta}\right)\right). \tag{15}$$

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[7]Alternatively, for a given precision of the fundamental α we could have restricted the set of allowed private precisions to $\{0\} \cup (\beta(\alpha), \infty)$, where $\beta(\alpha) = \frac{\alpha}{8\pi}(\alpha - 2\pi + \sqrt{(\alpha - 2\pi)^2 + 4\pi\alpha}) > 0$ is the unique precision of private information such that $\gamma_{MS} = \sqrt{2\pi}$.

For $\beta > 0$ we then know that

$$\mathbb{E}[v(X_i, I; \beta)] = \int_{x^*(\beta)}^{\infty} \left(\frac{\alpha\hat{\theta} + \beta x_i}{\alpha + \beta} - \Phi\left(\sqrt{\frac{\beta_j(\alpha + \beta)}{\alpha + \beta + \beta_j}} \left(x_j^* - \frac{\alpha\hat{\theta} + \beta x_i}{\alpha + \beta} \right) \right) \right) \tag{16}$$

$$\dots \times \frac{\sqrt{\frac{\alpha\beta}{\alpha+\beta}}}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{\alpha\beta}{\alpha+\beta}\left(x_i - \hat{\theta} \right)^2} \, dx_i.$$

We denote by $\hat{p}^I(\hat{\theta})$ the maximum price per unit precision for which investor i is willing to acquire information in an I equilibrium as a function of $\hat{\theta}$ and refer to $\hat{p}^I(\hat{\theta})$ as the standardized value of information. Since the free variable β appears eleven times in Equation (16), once even as the argument of an implicitly defined variable, we did not succeed in finding an explicit expression for the optimal β. However, for low unit prices an equilibrium exists, and we give a sketch of the proof for $\hat{\theta} < \frac{1}{2}$ (the proposition below states that $\hat{p}^I$ is symmetric in $\hat{\theta} = \frac{1}{2}$). Similar to the R profile we can prove that the maximization problem is well defined. We can then see this maximization problem as a mapping from β_j to β. For β_j close to 0, there is an equilibrium where almost all the investors withdraw. Comparison with the W profile shows that for low enough unit prices investor i wants precision $\beta > \beta_j$. Since the return is bounded, we know that for very large β_j investor i wants precision $\beta < \beta_j$. When the mapping is continuous a fixed point is guaranteed, which shows the existence of an equilibrium for low unit prices.

Numerical results show that the chosen precision is decreasing in the price and $|\hat{\theta} - \frac{1}{2}|$. The first is obvious. The latter follows from the fact that when $\hat{\theta}$ is close to $\frac{1}{2}$ the uncertainty about the sign of the return is highest and investors want to have precise information. For high prices the mapping from β_j to β is not necessarily continuous. The reason is that the maximum of the expected return as function of β decreases when the price of information increases. When the peak drops below zero, the guaranteed return of zero when withdrawing implies that there is a sudden jump from a positive precision to zero precision. Indeed, when the price equals $\hat{p}^I(\hat{\theta})$, both a zero precision (no information) and a positive precision are optimal responses. Similar to the basic model there is an equilibrium where investors acquire informative information when the price equals $p^I(\hat{\theta})$. The strategic complementarities in information acquisition imply that the W profile is an equilibrium for prices below $p^I(\hat{\theta})$ which guarantees the existence of an equilibrium.

In Figure 2 the standardized value of information for all three profiles is shown as function of the *ex ante* expectation of the fundamental. The following proposition states that for all α and β the standardized value of information as function of $\hat{\theta}$ in the R and the W profiles behave as depicted in the figure, and that the standardized value of information in the I profile is symmetric.

Proposition 4.

(i) *The standardized values of information as function of the* ex ante *expectation of the fundamental $\hat{\theta}$ in the R and the W profile, $\hat{p}^R$ and $\hat{p}^W$, are strictly decreasing respectively increasing.*
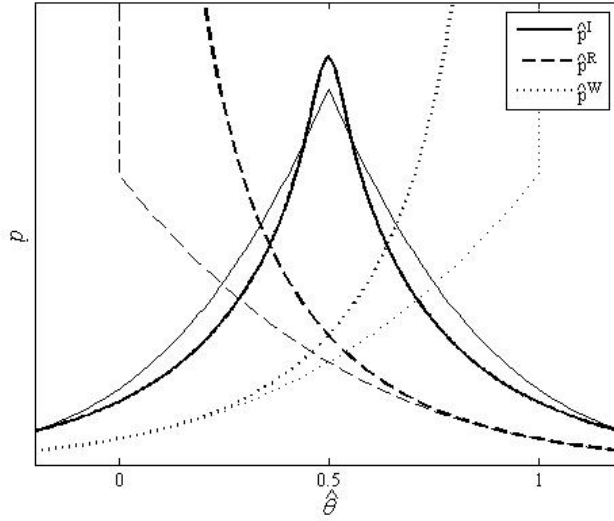
FIGURE 2. The standardized value of information as function of the *ex ante* expectation of the fundamental. ($\alpha = 1$, the thin lines show the value of information for $\beta = 1$)

(ii) *The standardized value of information is higher in the R profile than in the W profile if and only if $\hat{\theta} < \frac{1}{2}$.*

(iii) *The standardized values of information in the R and W profiles are symmetric images around $\hat{\theta} = \frac{1}{2}$ in the sense that $\hat{p}^R(\frac{1}{2} + d) = \hat{p}^W(\frac{1}{2} - d)$, $d \geq -\frac{1}{2}$. The standardized value of information in the I profile is symmetric around $\hat{\theta} = \frac{1}{2}$ in the sense that $\hat{p}^I(\frac{1}{2} + d) = \hat{p}^I(\frac{1}{2} - d)$, $d \geq 0$.*

Comparing this proposition to Proposition 1 shows that the standardized values of information in the R and W profiles behave similar to their non-standardized counterparts. Since for all β we know that $\hat{p}^R(\hat{\theta}) \geq \frac{p^R(\hat{\theta};\beta)}{\beta}$ and $\hat{p}^W(\hat{\theta}) \geq \frac{p^W(\hat{\theta};\beta)}{\beta}$, the regions where the R and the W equilibria are eliminated are expanded. Loosely speaking, the statements made in Corollary 2 are strengthened if the precision is free to choose. Figure 2 suggests that the statements for the I profile are qualitatively the same. Numerical analysis shows that it depends on α and β whether or not $\hat{p}^I$ is higher than p^I/β at the tails and/or the peak. However, the form of $\hat{p}^I$ remains roughly similar. The reason why $\hat{p}^I$ increases for $\hat{\theta} < \frac{1}{2}$ and decreases for $\hat{\theta} > \frac{1}{2}$ is the same as before: the closer the *ex ante* expectation of the fundamental to $\frac{1}{2}$, so the more uncertainty about the sign of the return, the more valuable the information is. We conclude that the statements of Corollary 2 are robust, if not strengthened, when the investors can choose the precision of their information.

4. Related Literature

The literature on liquidity runs that emerged from the Diamond-Dybvig model is vast. An excellent survey is given by Gorton and Winton (2003). In a recent paper, Bernardo and Welch (2004) tailor the model to describe financial market runs. In their liquidity run model the multiplicity that typically arises for intermediate priors in Diamond-Dybvig settings is eliminated by share prices and risk averse market-makers. Goldstein and Pauzner (2005) introduce costless noisy private information in the original model with patient and impatient investors. This eliminates the multiplicity for intermediate fundamentals and allows for computing the probability of a runs.

This Morris-Shin way of modelling agents with incomplete private information in a global game setting is widely followed to analyze the impact of private information in these kind of coordination games. The multiplicity that can occur in the absence of private information is eliminated by the introduction of a hybrid equilibrium. The partitioning of the fundamental thus only consists of regions with unique equilibria, see Sbracia and Zaghini (2001) and Metz (2002). The prediction of a unique equilibrium for sufficiently precise information is confirmed by an experimental study discussed by Heinemann, Nagel and Ockenfels (2004). In response to comments on the original model, see Atkeson (2000) and Rey (2000), several more realistic models have been developed that preserve this uniqueness result. For example, Tarashev (2003) introduces an interest rate in a currency crises model. The interest rate informs the investors about the actions (and thus about the information) of other investors without leading to common perfect information and multiple equilibria. Angeletos and Werning (2005) show that multiplicity can also be eliminated in case a secondary market exists where an asset price imperfectly aggregates private information. Carlson and Hale (2005) show that the introduction of a rating agency that provides free public information makes investments less responsive to changes in the fundamentals but do not significantly alter the character of the equilibria.

Interestingly, the presumption that multiplicity is an artifact of common knowledge that can be eliminated by the introduction of private information starts to become challenged in more elaborated models. Hellwig, Mukherji and Tsyvinsky (2005) also discuss the role of interest rates in self-fulfilling currency crises. Their main finding is that the multiplicity emerges even if private information is available. It is interesting to note that they derive public knowledge from private knowledge, while in our model information acquisition is determined by public information. However, both models suggest that the dichotomy "private information/no private information" in relation to "uniqueness/multiplicity" gives a simplified picture.

Several papers focus on information acquisition in settings with strategic complementarities. In the model of Nikitin (2004), investors can acquire complete information about the return of all investment opportunities. Although the three equilibria resemble the equilibria in our model, the model is rather complex and the equilibrium analysis necessarily only focuses on explaining that the three equilibria can occur instead of elaborating on how the interaction of fundamentals and prices effect the existence of these equilibria. Hellwig and Veldkamp (2005) mainly analyzes

the effect of costly, private information on beauty contest models. Although the structure of the model is similar to the structure of our model, only the squared distance to a realized random variable matters for the payoffs. This is a key difference since it makes the expected value of the random variable irrelevant for the equilibrium analysis, while this is at the heart of our analysis.

The equilibrium analysis of our model showed that an increase in the relative precision of private information has different effects for good and bad priors for the fundamentals. This is a common feature in the global games literature, see for example Metz (2002) and Sbracia and Zaghini (2001). Prati and Sbracia (2002) provide empirical evidence for this prediction.

5. Conclusion

This paper discusses a stylized model of liquidity runs where private information acquisition of investors is endogenous. Investors decide optimally whether or not to acquire private information taking its price as given. This provides a two dimensional equilibrium partitioning of the prior for the fundamentals and the price of information. The existence of an equilibrium for all prices follows from strategic complementarities in information acquisition. The multiplicity that occurs for intermediate priors in models without private information can be eliminated by the information that investors *can have*. Even equilibria without private information can be unique for intermediate priors. These results are qualitatively preserved when information of any precision is available for a price that is linearly increasing in the precision.

Endogenous information acquisition does affect the occurrence and the extent of the run. Only when the price of private information is high or very low, the information structure can be taken as fixed without affecting the results. For intermediate prices the artificial dichotomy "private information/no private information" is treacherous. This is most clear when priors are intermediate. In this case, regardless of whether the prior is relatively good or bad, both a full run or no run can occur when private information is not available, while a partial run occurs in case the fixed information structure contains private information. However, in our model with endogenous information acquisition, no run occurs when the prior is relatively good and a full run occurs when the prior is relatively bad.

A promising direction for future research is to embed the model in a dynamic context. The unique equilibria for some price-prior combinations together with the multiple equilibria that occur for other combinations suggest a role for hysteresis. Specifically, for countries with improving fundamentals this implies a lock-in effect since the fundamental has to improve considerably before investors become sufficiently interested to acquire information and consider investing.

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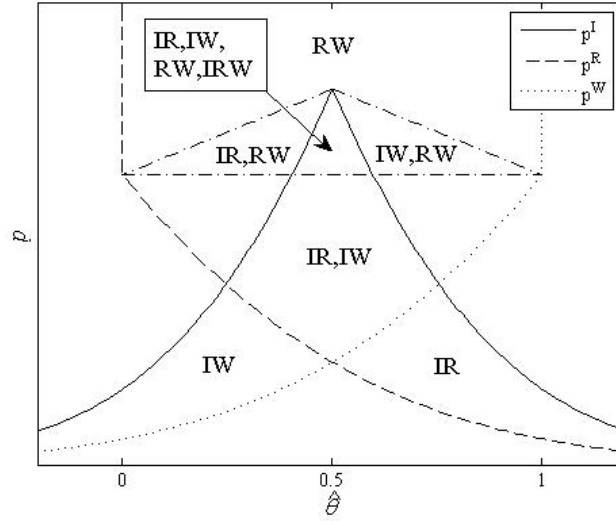## Appendix A - Non-Symmetric Equilibria

In the model of Diamond and Dybvig (1983) there exists a non-symmetric equilibrium (a mixed strategy equilibrium). In the original paper this is not analyzed since it is not economically meaningful. Here we show that all non-symmetric equilibria of our model have similar characteristics. We assume that $\gamma_{MS} < 2\pi$. This guarantees that in case an investor acquires information, the existence of dominant regions implies that we can focus on strategies where investors with information act according to a switching point strategy.

The discussion of non-symmetric equilibria below is reflected in Figure 3. Confronting this figure with Figure 1 gives the following proposition.

Proposition 5. *Assume $\gamma_{MS} < 2\pi$. If there is a unique equilibrium among the symmetric profiles, then there does not exist a non-symmetric equilibrium.*

The intuition is straightforward: a non-symmetric equilibrium can only arise if there are at least two equilibria that can be mixed. The details are more complicated and discussed below.

In the non-symmetric equilibrium of the original model with $\hat{\theta} \in [0, 1]$ some investors withdraw and others remain. In the setting of our model, a fraction $\hat{\theta}$ of the investors withdraws and

FIGURE 3. The non-symmetric equilibria regions. $(\alpha = 1, \beta = 1)$

a fraction $1-\hat{\theta}$ remains. Denote this non-symmetric equilibrium by RW. Since $\mathbb{E}[\theta - \ell | RW] = 0$ investors are indeed indifferent between remaining and leaving. The reason why this equilibrium is not economically meaningful is that the extent of the run is increasing in the *ex ante* expectation of the fundamental. This equilibrium does not exist for all prices. Since $\mathbb{E}[\theta - \ell | RW]$ has a normal distribution with zero mean and precision α, we know that for all $\hat{\theta}$ the value of information is given by $p^R(0) = p^W(1)$. For prices of at least this value the RW equilibrium exists.

Denote by IW a profile where a fraction $\lambda \in (0,1)$ of the investors acquires information and has a switching point strategy while the remaining fraction $1 - \lambda$ leaves without acquiring information. Denote by $v(x_i, IW)$ the value of the investment opportunity given information x_i. In equilibrium an investor without information should weakly prefer to withdraw, so $\mathbb{E}[\theta - \lambda \ell | IW] - (1 - \lambda) \leq 0$, and the value of unknown information should equal the price p, so $\mathbb{E}[v(X_i, IW)] = p$. The switching point x^* is determined by $\mathbb{E}[\theta - \lambda \ell | x^*, IW] - (1 - \lambda) = 0$. An investor who acquires information remains with positive probability while without information she withdraws for sure. Hence, the fraction of remaining investors is increasing in λ which implies that the switching point x^* is decreasing in λ. The value of the investment opportunity given information x_i in this equilibrium is given by $v(x_i, IW) = \max\{0, \mathbb{E}[\theta - \lambda \ell | x_i, IW] - (1 - \lambda)\}$. When x^* decreases, $v(x_i, IW)$ is positive for a larger range of private information and for every x_i in this range the expected fraction of withdrawing investors is less. Hence $\mathbb{E}[v(X_i, IW)]$ is decreasing in x^* and thus it is increasing in λ. In the proof of Lemma 6 in Appendix B it is shown that x^* is decreasing in $\hat{\theta}$. This implies that $\mathbb{E}[v(X_i, IW)]$ is increasing in $\hat{\theta}$. This shows that for a fixed price, λ should decrease in $\hat{\theta}$. So, when the *ex ante* expectation of the fundamental

improves, more investors leave. Similarly, when p increases, λ also increases. Hence, for higher prices more investors want information.

For $\hat{\theta} \leq \frac{1}{2}$, the IW equilibrium only exists in the interior of the region where both the I and the W equilibrium exist. Consider $\hat{\theta} < \frac{1}{2}$. In the last paragraph we have seen that λ is increasing in p. There is only one price for which $\lambda = 0$, and this price is given by $p^W(\hat{\theta})$. Similarly, $p^I(\hat{\theta})$ is the unique price for which $\lambda = 1$ and the value of the investment opportunity equals its price. It follows that only for $p \in (p^W(\hat{\theta}), p^I(\hat{\theta}))$ the IW profile exists with $\lambda \in (0,1)$. When λ increases, the expected fraction of remaining investors increases as well, hence the return of an investor who does not acquire information and remains is increasing in λ. But Lemma 6 of Appendix B shows that when $p = p^I(\hat{\theta})$ an investor without information should withdraw. Thus, also for $p \in (p^W(\hat{\theta}), p^I(\hat{\theta}))$ an investor without information should withdraw. We conclude that for these prices the IW profile is a non-symmetric equilibrium.

For $\hat{\theta} > \frac{1}{2}$ the region where the IW equilibrium exists is more complex. In the I equilibrium the value of the investment opportunity given unknown information is $\mathbb{E}[v(X_i, I)]$. We know that for $\hat{\theta} \leq \frac{1}{2}$ this value equals $p^I(\hat{\theta})$. Similar reasoning to above now gives that the IW profile only exists with $\lambda \in (0,1)$ if $p \in (p^W(\hat{\theta}), \mathbb{E}[v(X_i, I)])$. The expected return of an investor who remains without acquiring information is strictly increasing in λ. For prices close to $p^W(\hat{\theta})$ it is negative. Similar to the proof that $p^I(\hat{\theta})$ is increasing in $\hat{\theta}$ for $\hat{\theta} < \frac{1}{2}$, we can prove that $\mathbb{E}[v(X_i, I)]$ is increasing in $\hat{\theta}$. Since this implies that $\mathbb{E}[v(X_i, I)] > p^I(\hat{\theta})$ an investor who remains without acquiring information expects a positive return when the price is close to $\mathbb{E}[v(X_i, I)]$. There thus exist a unique $\bar{p}^{IW}(\hat{\theta}) \in (p^W(\hat{\theta}), \mathbb{E}[v(X_i, I)])$ such that the expected return equals zero. We know that $\bar{p}^{IW}(\frac{1}{2}) = p^I(\frac{1}{2})$ and that $\bar{p}^{IW}(1) = p^W(1)$ (note that $p^I(\frac{1}{2}) > p^W(1) = p^R(0)$ since the uncertainty about the behavior of the other investors increases the value of information in the I equilibrium). For $p \in (p^W(\hat{\theta}), \bar{p}^{IW}(\hat{\theta}))$ the IW equilibrium exists with $\lambda \in (0,1)$.

Denote by IR a profile where a fraction $\lambda \in (0,1)$ of the investors acquires information and has a switching point strategy while the remaining fraction $1 - \lambda$ remains without acquiring information. The switching point is then determined by $\mathbb{E}[\theta - \lambda \ell | x^*] = 0$. Arguments similar to above show that the switching point x^* is now increasing in λ. This profile can only be an equilibrium if the price of information satisfies $\mathbb{E}[v(X_i, IR)] - v(\emptyset, IR) = p$. Due to symmetry with the IW case, we know that the left hand side is increasing in λ. Also due to symmetry, the left hand side is decreasing in $\hat{\theta}$. We thus get that an increase in $\hat{\theta}$ or an increase in p lead to an increase in λ. So, when the *ex ante* expectation of the fundamental improves less investors remain and when the price of information increases more investors acquire information.

From the symmetry with respect to the IW profile, we know that for $\hat{\theta} \geq \frac{1}{2}$ the IR equilibrium only exists in the interior of the region where both the I and R equilibrium exist. For $\hat{\theta} < \frac{1}{2}$, there exist $\bar{p}^{IR}(\hat{\theta})$ such that the expected return of an investor who remains without acquiring information is zero. It follows that $\bar{p}^{IR}(0) = p^R(0)$, $\bar{p}^{IR}(\frac{1}{2}) = p^I(\frac{1}{2})$. The IR equilibrium with $\lambda \in (0,1)$ only exists for $p \in (p^R(\hat{\theta}), \bar{p}^{IR}(\hat{\theta}))$.

Denote by IRW a profile where a fraction of the investors remains without acquiring information, a fraction withdraws without acquiring information and the remaining investors acquire information. This equilibrium only exists in the interior of the region where the IR, the IW and the RW equilibria exist. So, for $\hat{\theta} \leq \frac{1}{2}$ only if $p \in (p^R(0), \bar{p}^{IR}(\hat{\theta}))$ and for $\hat{\theta} \geq 0$ only if $p \in (p^W(1), \bar{p}^{IW}(\hat{\theta}))$. For lower prices more investors want to acquire information, which increases the uncertainty and thus makes information more valuable. For higher prices investors do not want to acquire information. For a worse *ex ante* expectation of the fundamental investors prefer to withdraw, for a better one they prefer to stay. For reasons explained above, the expected fraction of withdrawing investors is increasing in the *ex ante* expectation of the fundamental, while the fraction of investors who acquire information is increasing in the price.

APPENDIX B - PROOFS

We first prove the following lemma where $x^*(\hat{\theta})$ denotes the switching point as function of the *ex ante* expectation of the fundamental.

Lemma 6.

  (i) $v(\emptyset, I) = 0$ *if* $\hat{\theta} \leq \frac{1}{2}$ *and* $v(\emptyset, I) = \mathbb{E}[\theta - \ell | I]$ *if* $\hat{\theta} \geq \frac{1}{2}$
  (ii) $x^*(\frac{1}{2} + d) = 1 - x^*(\frac{1}{2} - d)$

Note that (i) states that an investor without information in the I equilibrium should withdraw if and only if $\hat{\theta} < \frac{1}{2}$.

Proof of Lemma 6.
(i) Define $A(\hat{\theta}, \alpha, x_i, \beta) = \frac{\alpha\hat{\theta} + \beta x_i}{\alpha + \beta} - \Phi(\sqrt{\gamma_{MS}}(\frac{\alpha\hat{\theta} + \beta x_i}{\alpha + \beta} - \hat{\theta}))$. In equilibrium we should have $A(\hat{\theta}, \alpha, x^*, \beta) = 0$. We compute the following derivatives

$$\frac{\partial}{\partial \hat{\theta}} A = \frac{\alpha}{\alpha + \beta} \left(1 + \frac{\beta}{\alpha} \frac{\sqrt{\gamma_{MS}}}{\sqrt{2\pi}} e^{-\frac{1}{2}\gamma_{MS}\left(\frac{\alpha\hat{\theta} + \beta x^*}{\alpha + \beta} - \hat{\theta}\right)^2}\right) > 0, \qquad (17)$$

$$\frac{\partial}{\partial x^*} A = \frac{\beta}{\alpha + \beta} \left(1 - \frac{\sqrt{\gamma_{MS}}}{\sqrt{2\pi}} e^{-\frac{1}{2}\gamma_{MS}\left(\frac{\alpha\hat{\theta} + \beta x^*}{\alpha + \beta} - \hat{\theta}\right)^2}\right) > 0, \qquad (18)$$

where the inequality in the last line is implied by the condition $\gamma_{MS} < 2\pi$. Now the implicit function theorem gives $\frac{\partial}{\partial \hat{\theta}} x^*(\hat{\theta}) = -\frac{\partial A/\partial \hat{\theta}}{\partial A/\partial x^*} < 0$. Using Equation (6) we obtain

$$\frac{\partial}{\partial \hat{\theta}} \mathbb{E}[\ell | I] = \frac{\sqrt{\frac{\alpha\beta}{\alpha+\beta}}}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{\alpha\beta}{\alpha+\beta}\left(x^*(\hat{\theta}) - \hat{\theta}\right)^2} \left(\frac{\partial}{\partial \hat{\theta}} x^*(\hat{\theta}) - 1\right) < 0. \qquad (19)$$

But then $\mathbb{E}[\theta - \ell | I]$ is increasing in $\hat{\theta}$. Note that $x^*(\frac{1}{2}) = \frac{1}{2}$ and thus $\mathbb{E}[\theta - \ell | I] = 0$ if $\hat{\theta} = \frac{1}{2}$. This implies that $\mathbb{E}[\theta - \ell | I] \leq 0$ if and only if $\hat{\theta} \leq \frac{1}{2}$.
  (ii) The definition of $x^*(\hat{\theta})$ gives for $\frac{1}{2} - d$

$$\frac{\alpha(\frac{1}{2} - d) + \beta x^*(\frac{1}{2} - d)}{\alpha + \beta} = \Phi\left(\frac{\beta(\alpha + \beta)}{\alpha + 2\beta}\left(x^*(\frac{1}{2} - d) - \frac{\alpha(\frac{1}{2} - d) + \beta x^*(\frac{1}{2} - d)}{\alpha + \beta}\right)\right) \qquad (20)$$

Now subtract both sides from 1 to arrive at

$$\frac{\alpha(\frac{1}{2} + d) + \beta(1 - x^*(\frac{1}{2} - d))}{\alpha + \beta} \tag{21}$$
$$= \Phi\left(\frac{\beta(\alpha + \beta)}{\alpha + 2\beta}\left((1 - x^*(\frac{1}{2} - d)) - \frac{\alpha(\frac{1}{2} + d) + \beta(1 - x^*(\frac{1}{2} - d))}{\alpha + \beta}\right)\right).$$

This last line is exactly the definition of $x^*(\frac{1}{2} + d)$.

Proof of Proposition 1.

(v) The statement about p^R and p^W follows directly from the definitions of p^R and p^W and Equations (4) and (5). The proof of the statement about p^I is more involved. We will relate the variables when $\hat{\theta} = \frac{1}{2} - d$ to the variables when $\hat{\theta} = \frac{1}{2} + d$. We add the value of $\hat{\theta}$ as a subscript to $\mathbb{E}$ and $\mathbb{P}$ in order to explicitly denote the dependence of expectations and probabilities on $\hat{\theta}$.

From Lemma 6 we know that $x^*(\frac{1}{2} + d) = 1 - x^*(\frac{1}{2} - d)$, which implies that $x^*(\frac{1}{2} + d) - (\frac{1}{2} + d) = -(x^*(\frac{1}{2} - d) - (\frac{1}{2} - d))$. Use Equation (9) to get that

$$\mathbb{E}_{\frac{1}{2} - d}[\theta | X_i \geq x^*(\frac{1}{2} - d), I]\mathbb{P}_{\frac{1}{2} - d}[X_i \geq x^*(\frac{1}{2} - d)] \tag{22}$$
$$\ldots - \mathbb{E}_{\frac{1}{2} + d}[\theta | X_i \geq x^*(\frac{1}{2} + d), I]\mathbb{P}_{\frac{1}{2} + d}[X_i \geq x^*(\frac{1}{2} + d)]$$
$$= (\frac{1}{2} - d)\mathbb{P}_{\frac{1}{2} + d}[X_i < x^*(\frac{1}{2} + d)] - (\frac{1}{2} + d)\mathbb{P}_{\frac{1}{2} + d}[X_i \geq x^*(\frac{1}{2} + d)]$$
$$= -(\frac{1}{2} + d) + \mathbb{P}_{\frac{1}{2} + d}[X_i < x^*(\frac{1}{2} + d)].$$

Now note that $\mathbb{E}_{\frac{1}{2} - d}[\ell | X_i \geq \theta_s^*(\frac{1}{2} - d), I]\mathbb{P}_{\frac{1}{2} - d}[X_i \geq \theta_s^*(\frac{1}{2} - d)] = \mathbb{P}_{\frac{1}{2} - d}[X_j < x^*(\frac{1}{2} - d) \wedge X_i > x^*(\frac{1}{2} - d)]$. This in turn equals

$$\mathbb{E}_{\frac{1}{2} - d}[\mathbb{P}[X_j < x^*(\frac{1}{2} - d)|\theta]\mathbb{P}[X_i \geq x^*(\frac{1}{2} - d)|\theta]] \tag{23}$$
$$= \mathbb{E}_{\frac{1}{2} + d}[\mathbb{P}[X_j < 1 - x^*(\frac{1}{2} + d)|1 - \theta]\mathbb{P}[X_i < 1 - x^*(\frac{1}{2} + d)|1 - \theta]]$$
$$= \mathbb{E}_{\frac{1}{2} + d}[\mathbb{P}[X_j \geq x^*(\frac{1}{2} + d)|\theta]\mathbb{P}[X_i < x^*(\frac{1}{2} + d)|\theta]].$$

where we used that $X_j|\theta$ and $X_i|\theta$ are independent, that the involved precisions do not change and that the normal distribution is symmetric. When we interchange i and j, the expression in the last line of Equation (23) is equal to $\mathbb{E}_{\frac{1}{2} + d}[\ell | X_i \geq \theta_s^*(\frac{1}{2} + d), I]\mathbb{P}_{\frac{1}{2} + d}[X_i \geq \theta_s^*(\frac{1}{2} + d)]$. We conclude that the expected contribution of ℓ to the return of an investor is the same for $\hat{\theta} = \frac{1}{2} - d$ and $\hat{\theta} = \frac{1}{2} + d$.

Combining this finding and Equation (22) we get

$$\mathbb{E}_{\frac{1}{2} - d}[v(X_i, I)] = \mathbb{E}_{\frac{1}{2} + d}[v(X_i, I)] - \left((\frac{1}{2} + d) - \mathbb{P}_{\frac{1}{2} + d}[X_i < x^*(\frac{1}{2} + d)]\right). \tag{24}$$

From the lemma we know that $v(\emptyset, I) = 0$ for $\hat{\theta} \leq \frac{1}{2}$ and $v(\emptyset, I) = \mathbb{E}[\theta - \ell | I]$ for $\hat{\theta} \geq \frac{1}{2}$. By recognizing that the last term of Equation (24) is exactly the expected return of an investor without information who remains in the I equilibrium, we finally get $p^I(\frac{1}{2} + d) = p^I(\frac{1}{2} - d)$.

(ii) Due to the symmetry of p^R and p^W which was proved in (v), we only have to prove the statement for p^R. Using Equation (4) we obtain

$$\frac{\partial}{\partial \hat{\theta}} p^R(\hat{\theta}) = \Phi\left(\sqrt{\frac{\alpha + \beta}{\beta/\alpha}} \hat{\theta}\right) - 1 < 0. \tag{25}$$

Due to symmetry we only have to prove the statement about p^I for $\hat{\theta} \leq \frac{1}{2}$. From the lemma we know that $p^I(\hat{\theta}) = \mathbb{E}[v(X_i, I)]$ for $\hat{\theta} \leq \frac{1}{2}$. The definition of $p^I(\hat{\theta})$ shows that a change in $\hat{\theta}$ has an effect via the expected return in case the information is better than the switching point, and via the expected return due to a different investment decision in case the information equals the switching point. For the first effect we find

$$\frac{\partial}{\partial \hat{\theta}} \mathbb{E}[\theta - \ell | x_i, I] = \frac{\alpha}{\alpha + \beta} - \frac{\sqrt{\frac{\beta(\alpha+\beta)}{\alpha+2\beta}}}{\sqrt{2\pi}} e^{-\frac{1}{2} \frac{\beta(\alpha+\beta)}{\alpha+2\beta} \left(x^*(\hat{\theta}) - \frac{\alpha\hat{\theta}+\beta x_i}{\alpha+\beta}\right)^2}$$
$$\ldots \times \left(\frac{\partial}{\partial \hat{\theta}} x^*(\hat{\theta}) - \frac{\alpha}{\alpha + \beta}\right) > 0. \tag{26}$$

We conclude that $\frac{\partial}{\partial \hat{\theta}} v(x_i, I) > 0$ for $\hat{\theta} < \frac{1}{2}$, so for the values of the information for which the investor already would have remained, her return will be higher. The proof is finished when noting that $\frac{\partial}{\partial \hat{\theta}} x^*(\hat{\theta}) < 0$ implies that the range of information for which an investor expects a positive return increases.

(iv) Due to symmetry it suffices to prove the statement only for $\hat{\theta} < \frac{1}{2}$. The symmetry of p^R and p^W and (ii) imply that $p^R(\hat{\theta}) > p^W(\hat{\theta})$. Given (ii) we have to prove that $p^I(\hat{\theta}) > p^W(\hat{\theta})$. From the lemma we know that $p^I(\hat{\theta}) = \mathbb{E}[v(X_i, I)]$ for $\hat{\theta} < \frac{1}{2}$. Since there is always a strictly positive fraction of investors who remain in the I equilibrium, we have $\mathbb{E}[\theta - \ell | x_i, I] > \mathbb{E}[\theta - \ell | x_i, W]$ and thus $x^* < \frac{\alpha+\beta}{\beta} - \frac{\alpha}{\beta} \hat{\theta}\}$. Hence, $v(x_i, I) \geq v(x_i, W)$ for all x_i while a strict inequality holds for $x_i > x^*$. Since these values have a positive probability mass, we have $\mathbb{E}[v(X_i, I)] > \mathbb{E}[v(X_i, W)]$ which implies that $p^I(\hat{\theta}) > p^W(\hat{\theta})$ for $\hat{\theta} < \frac{1}{2}$.

(i) Due to (iv) and (v) we only have to proof that $p^W(\hat{\theta}) > 0$ for $\hat{\theta} \leq \frac{1}{2}$. We know that $v(x_i, W) > 0$ if $x_i > \frac{\alpha+\beta}{\beta} - \frac{\alpha}{\beta} \hat{\theta}$, which happens with positive probability. Since $v(x_i, W) = 0$ otherwise, we know that $p^W(\hat{\theta}) = \mathbb{E}[v(X_i, W)] > 0$.

(iii) Due to symmetry it suffices to prove the statement only for $\hat{\theta} \leq \frac{1}{2}$. The symmetry of p^R and p^W and (ii) imply that $p^R(\hat{\theta}) > p^W(\hat{\theta})$ for $\hat{\theta} < \frac{1}{2}$. Given (ii) and (iv) we have to prove that $p^R(0) > p^I(0)$. But this holds since from arguments similar to the ones used in (iii) it follows that $\mathbb{E}[v(X_i, R)] > \mathbb{E}[v(X_i, I)]$.

Proof of Proposition 3.
Due to symmetry of p^R and p^W it suffices to prove the statement for p^R. Since $p^R(\hat{\theta}) =$

$\mathbb{E}[v(X_i, R)] - \hat{\theta}$, we can use Equation (4) to obtain

$$\frac{\partial}{\partial \alpha} p^R(\hat{\theta}) = -\frac{1}{2} \frac{\beta(2\alpha + \beta)}{\alpha^2(\alpha + \beta)^2} \frac{\sqrt{\frac{\alpha+\beta}{\beta/\alpha}}}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{\alpha+\beta}{\beta/\alpha}\hat{\theta}^2} < 0, \tag{27}$$

$$\frac{\partial}{\partial \beta} p^R(\hat{\theta}) = \frac{1}{2} \frac{1}{(\alpha + \beta)^2} \frac{\sqrt{\frac{\alpha+\beta}{\beta/\alpha}}}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{\alpha+\beta}{\beta/\alpha}\hat{\theta}^2} > 0. \tag{28}$$

Due to symmetry of p^I in $\hat{\theta} = \frac{1}{2}$ it suffices to give the proof for $\hat{\theta} \leq \frac{1}{2}$, and due to continuity it even suffices to only consider $\hat{\theta} < \frac{1}{2}$. From the lemma we know that an investor without information withdraws. The value of information is then given by $\mathbb{E}[v(X_i, I)]$. For reasons explained in the proof of Proposition 1 (i) this value is decreasing in the switching point x^*. To compute the derivative of x^* to α and β we use the function A as defined in the proof of Proposition 1. There we found that $\frac{\partial}{\partial x^*} A > 0$. We then need to compute

$$\frac{\partial}{\partial \alpha} A = \frac{\alpha(\hat{\theta} - x^*)}{(\alpha + \beta)^2} \left(1 + \frac{1}{2} \frac{3\alpha\beta + 4\beta^2}{\alpha^2 + 2\beta\alpha} \frac{\sqrt{\gamma_{MS}}}{\sqrt{2\pi}} e^{-\frac{1}{2}\gamma_{MS}\left(\frac{\alpha\hat{\theta}+\beta x^*}{\alpha+\beta} - \hat{\theta}\right)^2} \right), \tag{29}$$

$$\frac{\partial}{\partial \beta} A = \frac{\alpha(x^* - \hat{\theta})}{(\alpha + \beta)^2} \left(1 - \frac{1}{2} \frac{\alpha^2 - 2\beta^2}{\alpha^2 + 2\alpha\beta} \frac{\sqrt{\gamma_{MS}}}{\sqrt{2\pi}} e^{-\frac{1}{2}\gamma_{MS}\left(\frac{\alpha\hat{\theta}+\beta x^*}{\alpha+\beta} - \hat{\theta}\right)^2} \right). \tag{30}$$

Since $\gamma_{MS} < 2\pi$ and $x^* > \hat{\theta}$ when $\hat{\theta} < \frac{1}{2}$ (see the proof of the lemma), we have $\frac{\partial}{\partial \alpha} A < 0$ and $\frac{\partial}{\partial \beta} A > 0$. The implicit function theorem gives that $\frac{\partial}{\partial \alpha} x^* > 0$ and $\frac{\partial}{\partial \beta} x^* < 0$ which finishes the proof.

Proof of Proposition 4.
(i) This is proved in the text.

(ii) This follows from the symmetry of $\hat{p}^R$ and $\hat{p}^W$ and the fact that $\hat{p}^R$ decreases.

(iii) The first statement is proved in the text. The statement about $\hat{p}^I$ is more complicated. Taking β_j as given, we have to show that the maximization problems for $\hat{\theta} = \frac{1}{2} + d$ and for $\hat{\theta} = \frac{1}{2} - d$ are identical up to a constant. From the lemma we know that $x^*(\frac{1}{2} + d) = 1 - x^*(\frac{1}{2} - d)$. Using this and Equation (15) it follows that $x_j^*(\frac{1}{2} + d; \beta_j) = 1 - x_j^*(\frac{1}{2} - d; \beta_j)$ and $x^*(\frac{1}{2} + d; \beta) = 1 - x^*(\frac{1}{2} - d; \beta)$, where we explicitly denote the dependence on $\hat{\theta}$ and either β_j or β.

The derivation in Equation (22) still holds when we explicitly denote the dependence of x^* on β.

The derivation in Equation (23) shows that $\mathbb{E}_{\frac{1}{2} - d}[\ell | X_i \geq x^*(\frac{1}{2} - d; \beta), I] \mathbb{P}_{\frac{1}{2} - d}[X_i \geq x^*(\frac{1}{2} - d; \beta)] = \mathbb{P}_{\frac{1}{2} + d}[X_j \geq x_j^*(\frac{1}{2} + d; \beta_j) \wedge X_i < x^*(\frac{1}{2} + d; \beta)]$. Using the fact that $\mathbb{P}[A^c \cap B^c] =$

$1 - \mathbb{P}[A] - \mathbb{P}[B] + \mathbb{P}[A \cap B]$ then gives

$$\mathbb{E}_{\frac{1}{2} - d}[\ell | X_i \geq x^*(\tfrac{1}{2} - d; \beta), I]\mathbb{P}_{\frac{1}{2} - d}[X_i \geq x^*(\tfrac{1}{2} - d; \beta_j)] \tag{31}$$

$$= 1 - \mathbb{P}_{\frac{1}{2} + d}[X_j < x_j^*(\tfrac{1}{2} + d; \beta_j)] - \mathbb{P}_{\frac{1}{2} + d}[X_i \geq x^*(\tfrac{1}{2} + d; \beta)]$$

$$\ldots + \mathbb{E}_{\frac{1}{2} + d}[\ell | X_i \geq x^*(\tfrac{1}{2} + d; \beta), I]\mathbb{P}_{\frac{1}{2} + d}[X_i \geq x^*(\tfrac{1}{2} + d; \beta)].$$

We thus find that

$$\mathbb{E}_{\frac{1}{2} - d}[v(X_i, I, \beta)] - \mathbb{E}_{\frac{1}{2} + d}[v(X_i, I, \beta)] \tag{32}$$

$$= -(\tfrac{1}{2} + d) + \mathbb{P}_{\frac{1}{2} + d}[X_i < x^*(\tfrac{1}{2} + d; \beta)]$$

$$\ldots - 1 + \mathbb{P}_{\frac{1}{2} + d}[X_j < x_j^*(\tfrac{1}{2} + d; \beta_j)] + \mathbb{P}_{\frac{1}{2} + d}[X_i \geq x^*(\tfrac{1}{2} + d; \beta)]$$

$$= -(\tfrac{1}{2} + d) + \mathbb{P}_{\frac{1}{2} + d}[X_j < x_j^*(\tfrac{1}{2} + d; \beta_j)].$$

This expression is independent of β which finishes the proof.