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Bargaining and Efficiency in a Speculative Forward Market

H. Peter Møllgaard

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ECONOMICS DEPARTMENT

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Bargaining and Efficiency in a Speculative Forward Market[1]

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ABSTRACT: The 15-Day forward market for Brent crude oil is predominantly speculative. Transactions on this market thus contradict the assumptions that lead to zero speculation theorems. We set up a stochastic game model of a market with a small number of speculative traders that differ only with respect to the expected spot price and (possibly) with respect to risk aversion. Contracting is done after pairwise negotiations in random matches. The Markov perfect equilibrium of the model can mimic the 15-Day market and need not be efficient in the sense of belonging to the bilateral core.

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0. Introduction

How do the participants in the 15-Day market for Brent blend crude oil get to agree upon forward contracts?

The 15-Day market is a peculiar decentralized market institution in which a homogeneous commodity is traded forward. The main purpose of this paper is to model the trading procedure and to assess whether the outcome of this procedure is efficient (in a sense to be made precise).

There are other market institutions that could serve the same purposes. The standard futures market is a case in mind. In contrast to the 15-Day market, this market institution is centralized. It is *a priori* more likely to lead to efficient outcomes since market clearing is made easy by the presence of a clearing house and since information is conveyed on a continuous basis. The International Petroleum Exchange (IPE) of London enjoys these characteristics but trading of Brent blend stops too soon before maturity to be a good substitute for the forward market. This problem could certainly be remedied as has been the case elsewhere and for other commodities. (In)divisibility is also an issue: The 15-Day market operates in terms of cargoes of a standard size of 500,000 barrels, whereas the IPE contracts are specified in lots of 1,000 barrels.

Here the focus is on the trading *procedure* of the 15-Day market institution. The institutional features of this market will help specify a realistic model. These features are outlined in Section 1, which also illustrates a typical market outcome. We proceed by specifying the main characteristics of an oil trader in Section 2. Section 3 discusses a very simple model of two such traders to give us the feel for the problem. Section 4 goes on to the more complicated three person version of the game in which the problems of the general n-person model can be illustrated and exemplified. Section 5 generalizes to an $n \geq 2$ person, finite time horizon game of pairwise bargaining after random matches and analyses whether efficiency can be achieved under the more realistic specification of bargaining options. Section 6 concludes by discussing convergence to efficiency and by comparing the decentralized forward market with a centralized futures market.

The main conclusion is that the model catches some important circumstances affecting 15-Day traders and that it can mimic the observed outcome of the 15-Day forward market. The outcome is inefficient compared to a centralized market institution, but there is convergence to the efficient outcome as the time horizon gets longer. Measured against the standard model of a futures market, the 15-Day institution is inferior. Since a futures market (the IPE) exists and since the futures market institution has long been a well known way of organizing the economic functions that the 15-Day market accomplishes, it remains a paradox that the market participants accept such an inferior market institution.[1]

1. The Characteristics of the 15-Day Market

Traditionally thought of as London-based, the 15-Day market cannot be said to have a proper home. However, many traders have their offices in London and most price reporting services are located there. The early development of the markets for North Sea crude oil is described in detail in Mabro *et al.* (1986).

For the purpose of this paper, a short account of the **salient features of the 15-Day market** institution will suffice:

1) **Finite time horizon**: A contract for oil to be delivered in a given month[2] can only be traded during a given time interval. In order not to go into the complicated details regarding the timing of forward and spot transactions (on this, see Phlips (1992)), as a good approximation it is assumed that a given forward contract is traded during the last two and a half months prior to delivery. This allows around fifty trading days.

2) **Limited number of players**: "Though the number of market participants in 1980-85 exceeded 110, we found that the number of continually active players was of the order of 30-35, and that the 10-15 top participants accounted for most of the activity. The top five participants comprised four oil traders and one major oil company" (Mabro et al. (1986) p. xx).

3) **Pairwise negotiations**: The 15-Day market does not exist physically, but consists of a network of market participants trading via telephone.

4) **Standard contracts**: The contract is of a type where the quantity is prespecified (500.000 bl's) so that the contractors only have to fill in the agreed price. For each matching of two

---

[1] One explanation may be that the forward market automatically becomes the spot market in the sense that assignment of physical cargoes is done via chains of contracts that are left open when the forward market closes. This function could however be carried out in other ways. A centralization of this function would seem to make life easier for everybody involved in the market.

[2] The timing of delivery is not further specified. Only when the spot market opens will the delivery be known with more precision (namely within a three day delivery range). It is this assignment of different buyers to different slots that would be made easier with a centralized agency (cf. note 1).

traders this reduces the dimension of the bargaining problem from two to one: from (price x quantitity) to (price). The contract is binding in the sense that once it is telexed and therefore legally confirmed, the contractors cannot undo it before maturity and may thus end up with obligations to buy or sell oil on the spot market of the relevant forward month.[3]

5) **The clearing mechanism** is important for the same reason and involves two types of transactions: bookouts and daisy chains. A *bookout* is a situation where a number of players (at least two) can construct a circle of contracts (A sells to B, B to C, C to .., .. to A) and decide to close out by purely financial transfers. A *daisy chain* is similar but the chain is not closed to a circle: a cargo is passed on along a chain of traders and the last trader in the chain takes delivery. A daisy chain thus involves a cargo of physical oil at the end.

In addition to the institutional features we note that there is a great deal of speculation going on in the market. Bacon (1986) notes that:

> "this market has been *de facto* dominated by speculative deals" (p.5)

and

> "[a]n important factor is the dispersion of expectations held by traders: if expectations about price movements are widely spread then the number of deals can be high" (p. iii).

> "All this activity (...) was greatly amplified by the involvement of speculators in the market who, by taking views on likely future prices, tried to make a margin on buying or selling short and then covering their positions at a later date. ... [T]here was also considerable variation in the total amount of trading month by month. We have suggested that an important part of this volatility in quantity was related to changes in the dispersion of expectations held by those speculating in the market. When there was consensus on the likely price outcome the opportunities for trading decreased and when views were very disparate the number of deals increased." (pp. 48-49).

Take this as sufficient evidence that

[3] This feature is what leads to the frequent occurrence of squeezes, *cf.* Phlips and Møllgaard (1992).

1) differences in expectations exist;

2) differences in expectations drive the speculative trade;

3) speculative trade dominates the 15-Day market.

The assumption that differences in expectations drive speculative trade is in straight contradiction with the zero-speculation or zero-trade theorems (see Milgrom and Stokey (1982) and Tirole (1982) or for a survey Geanakoplos (1992)) that follow from the rational expectations hypothesis or from an assumption about common knowledge of common priors. That differences in expectations drive speculative trade therefore needs justification which is given in Møllgaard (1993). For the purpose of this paper, first note that the typical real world speculator seems to be much more confident about his own expectation (opinion) than game theory with common priors allows him to be. Second, even if people have the same information, this need not span a unique probability distribution leading to a unique spot price expectation (see also Kurz (1991)).

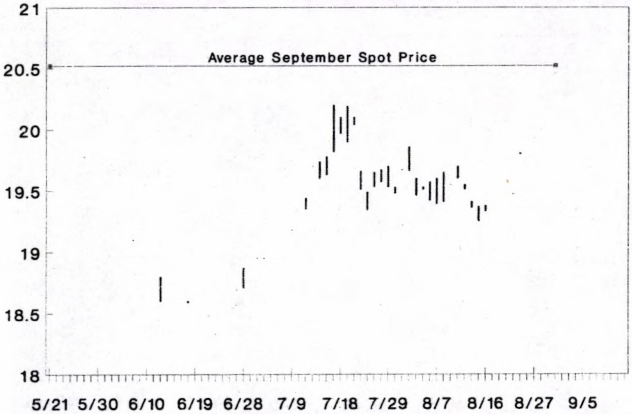
To characterize the forward market as entirely speculative is an obvious abstraction. In reality, there are four types of market participants, *viz.* non-integrated producers, non-integrated refineries, integrated oil companies and oil traders, but only the two first types of agents can be trusted to enter the market primarily for hedging purposes, and their overall significance is fairly limited. The physical production of Brent blend during a normal month is forty-two cargoes and hence hedging can explain no more than forty-two contracts[4]. The turnover on the 15-Day market is typically ten times this number. Integrated companies speculate about the future forward price and trade accordingly for tax purposes.[5] Oil traders live from speculation, so it is a fair claim that the forward market is predominantly speculative.

A typical outcome of the 15-Day market in terms of the number of contracts traded and the corresponding prices is illustrated by the trading of the September 1991 contract. Trading took place in the period from 21 June to 30 August 1991, see Figures 1 and 2. Figure 1 shows the price range of concluded deals on the different days of trading, while Figure 2

[4] This argument ignores possible hedging of other crudes for which Brent is a substitute. To a certain extent forward markets for other North Sea crudes exist, but they are relatively unimportant. It is well known from the theory of futures markets that you can hedge imperfectly in forward contracts for an imperfect substitute.

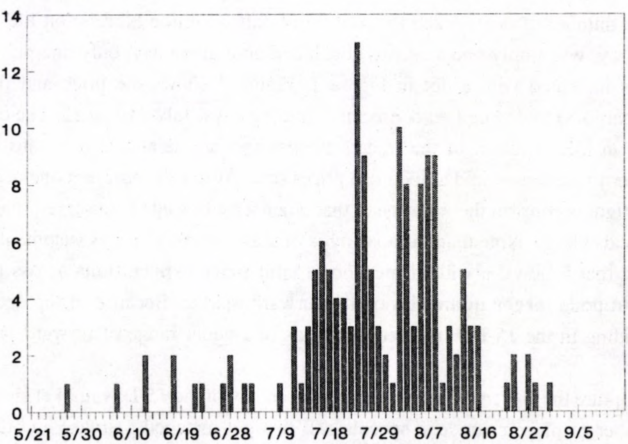
[5] See Bacon (1986) and Mabro et al. (1986) for a discussion of this. Clubley (1990, p. 33) notes that it is well known that the majors are speculative traders.

Figure 1: Selected Brent forward deals for September 1991 (High - Low)



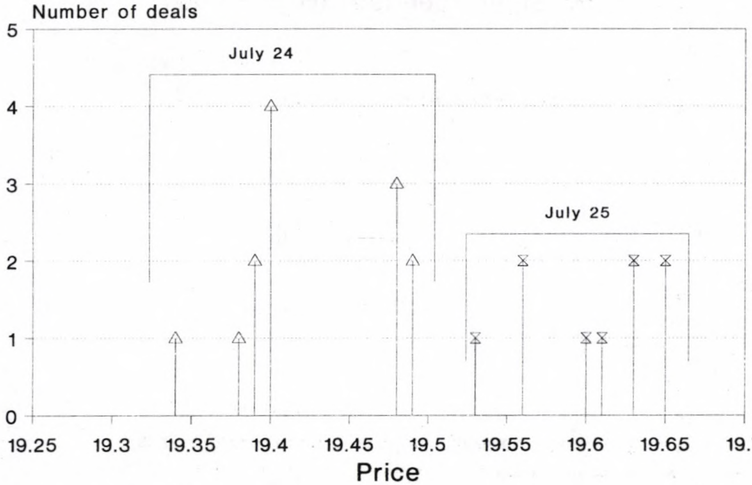
Source: Weekly Petroleum Argus

Figure 2: Number of September Deals



Source: Weekly Petroleum Argus

Figure 3: September contracts on July 24 and 25



Source: Weekly Petroleum Argus

indicates the number of deals (each for a cargo of half a million barrels) on the same days. Needless to say, when only one deal was concluded on a given day, only one price obtained. This price is indicated with a dot in Figure 1. Figure 3 shows the price and the quantity (number of cargoes) traded on two consecutive trading days, July 24 and 25. The dataset does not provide an identification of the traders involved in any deal and it is thus difficult to know whether two cargoes sold at identical prices on a given day represent one or two trades.

The figures confirm the impression that when a lot of trade is observed, then the price dispersion is also large. Note that no causality is implied, *a priori*, in this statement. However, in the model that follows, a wide dispersion of **spot price expectations** across traders will cause them to trade **larger quantities** on the forward market. Because of the decentralized nature of trading in the 15-Day market, this leads to a wider range of forward prices being observed.

To capture the aforementioned features our model of the 15-Day market should include a finite number of players (traders/ speculators) that get matched pairwise and then bargain about the price of a unit of an indivisible good. They are only matched a finite number of times and since they are speculators they will want to close whatever open position they might have at maturity.

The model is built up step by step increasing the number of interacting agents from one to two to three to n.

2. A Simple, Speculative Oil-Trader

Assume that the entire forward market is speculative. This means that no market participant has any interest in obtaining the underlying good, but only in buying cheap contracts on the forward market in order to sell them later at an expected higher price, or conversely, in selling expensive contracts to buy later at an expected lower price. Speculation is thus "sell high, buy low" with either happening first. Through most of the paper, it is aassumed that the forward market clears at maturity using the realization of the spot price. Accordingly, in this model, all contracts are supposed to be held till maturity. Some important problems regarding the clearing mechanism at maturity will be discussed later.

We employ two further assumptions of a technical nature: 1) The trader perceives the spot price as a normally distributed random variable; and 2) she is risk averse. To be specific, assume that her preferences can be represented by a utility function that is (negative) exponential in profits. The joint assumption of a normal spot price and an exponential utility function allows us to express the objective function according to the mean-variance model.

Thus the spot price, p^s, has a "subjective mean" p_I and is assumed to have a known variance $\sigma^2 = 1$ (this allows us to concentrate on differences in opinion about first order moments):

$$p^s \sim N(p_I, 1) \tag{1}$$

Trader I's **expected profit** from a forward transaction in period t is

$$\pi_t = (q_{I,t} - p_I) f_{I,t} \tag{2}$$

where $q_{I,t}$ is the price of the contract and $f_{I,t}$ the quantity sold. The expected profit from a given **contract** $c_{I,t} \equiv (q_{I,t}, f_{I,t})$ is positive if the forward price is higher than the mean spot price $(q_{I,t} > p_I)$ and if the trader sells forward $(f_{I,t} > 0)$. Conversely, if the forward price is lower than the expected spot price, the expected profit will be positive only if the trader goes long (*i.e.* buys forward: $f_{I,t} < 0$).

Let $h_I(\tau) \equiv (c_{I,1}, c_{I,2}, ..., c_{I,\tau})$ be trader I's history of trades from the first day of trading ($t = 1$) up to and including day τ. The expected profit that arises from this history is simply the sum of the single-deal expected profits, given that the trader's spot price expectation does not change over time:

$$\pi(h_I(\tau)) = \sum_{t=1}^{\tau}(q_{I,t} - p_I)f_{I,t} = V(h_I(\tau)) - p_I F(h_I(\tau)) \tag{3}$$

where $V(h_I(\tau)) = \sum_{t=1}^{\tau} q_{I,t} f_{I,t}$ is the (known) book value of the trader's forward position and $F(h_I(\tau)) = \sum_{t=1}^{\tau} f_{I,t}$ is the trader's net position. We consider a trading period short enough for discounting to be ignored. Given our assumptions, the objective function at time τ can be written

$$G(h_I(\tau);A_I,p_I) = \pi(h_I(\tau)) - \tfrac{A_I}{2}F^2(h_I(\tau)) \tag{4}$$

where A_I is the Arrow-Pratt measure of absolute risk aversion, here assumed to be constant. The trading stops at a known date T (≥ 1) and the *trader's objective* (TO) is thus:

$$Max_{\{h_I(T)\}}\ G(h_I(T);A_I,p_I) \tag{5}$$

Summing up, this section proposed the following assumptions:

A1 The typical forward market participant is a speculator.

A2 The forward market clears using the realization of the spot price at maturity.

A3 Each trader I perceives the spot price as a normal stochastic variable with mean p_I and variance 1.

A4 The trader has risk-averse preferences that can be presented by a utility function which is exponential in profits.

From these four assumptions we derived the trader's objective function which is quadratic in the net position F and we formulated the trader's objective (TO).

3. Two Traders with Differing Beliefs

Assume for the purpose of this section that the forward market consists of two traders. If they trade, it must be with each other. When they agree to a contract, they know that their spot price expectations differ. If they have identical spot price expectations, they do not want to trade since trading would expose them to a risk. We thus assume that agents are endowed with different spot price expectations. Given expectations and expected utility maximization, they will want to trade if expectations are sufficiently diverse. In the following, different "solutions" to the two players' problems are examined.

3.1 Axiomatic Approaches

Name the two agents I and J. Agent I expects the spot price to be p_I and J expects p_J; assume these expectations to be common knowledge. I and J agree that the spot price variance is one. They have risk preferences A_I and A_J respectively. Since they are the only two on the market, what one trader sells, the other must buy and we shall take f to signify I's short position (signed) which then is J's long position.

Arbitrarily assume $p_I > p_J$, set $T=1$ and/or suppress the time-index. The two TO's thereby become

$$Max_c \quad (q - p_I)f - \frac{A_I f^2}{2} \tag{6}$$

$$Max_c \quad -(q - p_J)f - \frac{A_J f^2}{2} \tag{7}$$

where $c = (q,f)$.

If I and J were price-takers, q would be exogenous to the traders and should somehow adjust to clear the market. Then the competitive equilibrium would be obtained, with

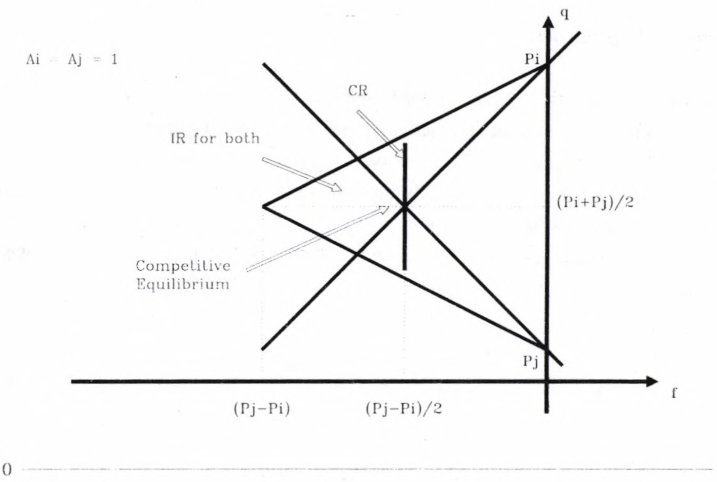
$$c = (q^*, f^*) = \left(\frac{A_J p_I + A_I p_J}{A_I + A_J} \, , \, \frac{p_J - p_I}{A_I + A_J} \right) . \tag{8}$$

Here I buys from J a position that is proportional to the difference in spot price expectations at a price that is a weighted average of their price expectations, the weights being the risk aversion constants: If I is more risk averse, J's spot price expectation gets a higher weight and vice versa. The competitive cquilibrium is illustrated in Figure 4.

With only two traders, or in general as long as the number of players is so small that any player perceives that she can influence the market outcome, price taking appears to be a highly artificial constraint on behaviour and the competitive solution does not carry much appeal, so we discard this approach.

Let us instead turn the problem upside down: Just studying the traders' objectives and without imposing further structure on the problem, what can be said about the set of contracts that they would agree to?

Figure 4: Competitive Equilibrium,
IR Contracts and Bilateral Core



The minimum requirement on any contract must be that it contributes non-negatively to the contractors' payoffs, that is, that it be *individually rational* (IR) *for both*.[6] This condition is satisfied by contracts in the set

$$\{(q,0)\} \;\; U \; \left\{(q,f){:}\ f<0\ ,\ p_J - \tfrac{A_J}{2}f \le q \le p_I + \tfrac{A_I}{2}f \right\} \qquad (9)$$

which is the vertical axis plus the big triangle in Figure 4. Any contract interior to the triangle is strictly preferred by both parties to no trade. A contract on the vertical axis trivially contributes zero to the expected payoff of both traders (zero volume, zero trade). A contract on the upper line ($q = p_I + ½A_I f$) of the triangle would contribute zero to I's expected payoff, whilst a contract on the lower line ($q = p_J$ - $½A_J f$) gives J zero expected payoff. We shall require that any contract be individually rational to both. This only amounts to saying that a trader only contracts if she finds it advantageous, which seems a natural and minimal requirement.

A stronger requirement would be that the outcome belong to the *bilateral core*. This will be our qualitative *efficiency* measure throughout the paper. The idea is that independently

[6] Or *participation constrained.*

of the organization of the market, in order for the market outcome to be called efficient, it should belong to the bilateral core.

Definition: The ***bilateral core*** is the set of contracts that is coalitionally rational, where the set of permissible coalitions is restricted to all singletons and all pairs.

In other words, the bilateral core is the set of contract allocations that cannot be blocked by coalitions of one or two agents and that satisfy participation constraints. Note that in general this is a different requirement than Pareto efficiency. In the two-person case the bilateral core is identical to the core and a strict subset of the Pareto efficient allocations. For now it suffices to note that individual rationality of an outcome means that no singletons can form a blocking coalition and our two person case therefore reduces to finding the contract curve in the set of IR contracts. This can be done by equalizing the marginal rate of substitution between price, q, and quantity, f, for the two players. The bilateral core in the two person case is:

$$\left\{ c\colon f = \frac{p_J - p_I}{A_I + A_J},\ p_J - \frac{A_J}{2} f \leq q \leq p_I + \frac{A_I}{2} f \right\}. \tag{10}$$

It is illustrated in Figure 4 as the vertical bar in the triangle. The quantity is the same as that of the competitive solution, but the price can be anywhere in the interval between the seller's (J's) reservation price and the buyer's (I's) reservation price.

The question is whether we can rely on economic principles to restrict the solution to the bilateral core. This is one of the recurring issues of the paper, so we should not expect an easy answer. Note that the competitive equilibrium belongs to the core, as should be. Another solution concept that has been applied to a problem of this type (see Brianza, Phlips and Richard (1990) p. 13) is the Nash-Bargaining point. Provided that the disagreement event is taken to be that no contract is telexed, the generalized (asymmetric) Nash bargaining solution which assigns weight ("bargaining power") α to I ($\alpha \in [0;1]$) can be written as

$$(q,f) = \left(w_I p_I + w_J p_J,\ \frac{p_J - p_I}{A_I + A_J} \right) \tag{11}$$

where $w_I = \frac{1}{2}\left[\frac{A_J}{A_I + A_J} + (1-\alpha)\right]$ and $w_J = \frac{1}{2}\left[\frac{A_I}{A_I + A_J} + \alpha\right]$. This generalized solution contains the original Nash point as the special case where $\alpha = ½$:

$$q = \frac{(3A_J+A_I)p_I + (3A_I+A_J)p_J}{4(A_I+A_J)}, \tag{12}$$

where the price expectation of the less risk averse agent gets the higher weight. This price will coincide with the competitive price (8) only if $A_I = A_J$. Otherwise, the difference between the competitive price and the symmetric Nash bargaining price will be:

$$\frac{1}{4}(p_I - p_J)\frac{A_J - A_I}{A_I + A_J} > 0 \quad \mathit{iff} \quad A_J > A_I \, , \tag{13}$$

so that the seller (J) is better off with competitive pricing than with Nash bargaining if she is more risk averse than the buyer (I) and *vice versa*.

The generalized solution (11) belongs to the bilateral core (10). Indeed, the bilateral core is equivalently described by (11) letting α run from zero to one.

The weights in w_I and w_J in (11) have the following interpretation in addition to what was said about (12): The stronger I is (*i.e.* the higher α is), the closer is the price of the contract to J's reservation price as given by (10). Conversely, the stronger J is, the closer q gets to I's reservation price. Generally, the stronger an agent is relative to the other, the more the forward price will reflect the other's spot price expectation.

Since the Nash bargaining approach is axiomatic, the solution by construction enjoys some nice properties: it is invariant to equivalent utility representations; it is symmetric if the problem is symmetric; it is independent of irrelevant alternatives; and it is Pareto efficient. The joint effect of these four axioms is even a *unique* outcome, but for our purpose we cannot use the approach since we would then assume what we want to show (efficiency of the outcome). Axiomatic approaches are silent on matters of timing and procedures and we therefore adopt a non-cooperative strategic bargaining approach to analyse whether and under which conditions the outcome of the given market institution is efficient.

3.2 Two Traders Bargaining Strategically

The strategic bargaining model as treated in Osborne and Rubinstein (1990) assumes that the players haggle over the price of a fixed quantity, taking turns in offering a price to the other and rejecting or accepting this offer. In the terminology of Rubinstein (1992), they attempt to partition a pie of a given size and once they agree on a partition, the game is over. Thus in our model, if the players are allowed to enter precisely one contract with a prespecified quantity, then the traders only haggle about the price. Given the finite time horizon, the price will either equal the seller's reservation price if the buyer is the last to make an offer, or the buyer's reservation price if the seller offers last. The one to make the final offer is the winner, and the winner takes all. It is a general feature of strategic bargaining models, that the offerer skims the cream off by making an offer equal to the receiver's reservation value (appropriately defined), leaving the receiver just indifferent between accepting and rejecting. This feature is exploited repeatedly throughout the paper.

However, the Osborne/Rubinstein (1990) approach is not immediately applicable to the problem at hand. First, contrary to Rubinstein's model and many other models, it is assumed here that players are infinitely patient but that the game has a finite time horizon. More importantly, we introduce indivisibilities and multiple trades: A contract c_t has a prespecified quantity. We take this as the indivisible unit for which the traders try to establish a price, and assume that in each period, a trader can enter at most one contract so that

$$f_t = \begin{cases} 1 & \text{if I sells a contract to J at t} \\ 0 & \text{if I and J cannot agree at t} \\ -1 & \text{if I buys a contract from J at t.} \end{cases} \tag{14}$$

Accordingly, if the two traders want to achieve a forward position of a given size, $F \geq 1$, at T, they will have to trade in at least F periods.

At any period, τ, agent I enters the period with accumulated expected payoff $G(h_I(\tau\text{-}1);A_I,p_I)$ and exits with accumulated payoff $G(h_I(\tau);A_I,p_I)$, so the incremental single period expected payoff is

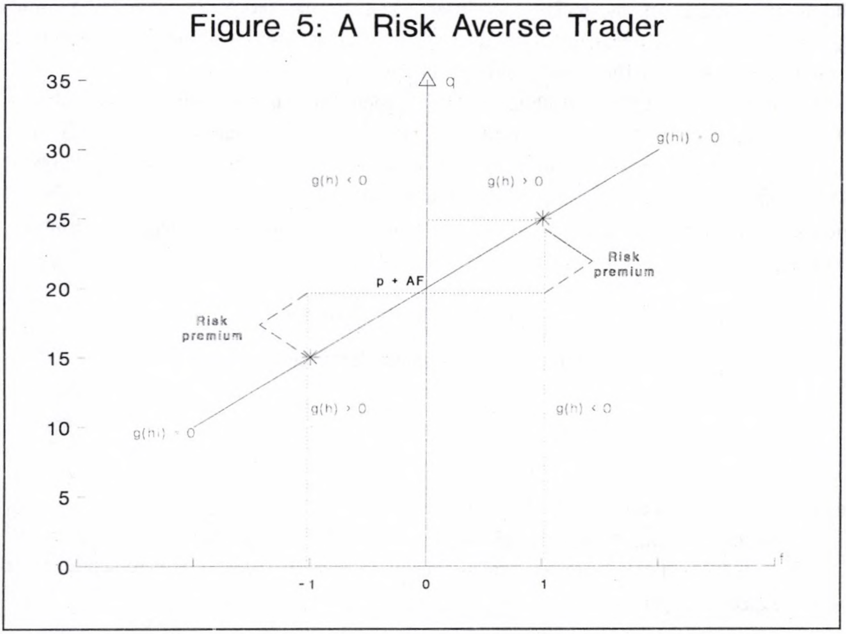
$$g(h_I(\tau);A_I,p_I) = f_\tau\left(q_\tau - p_I - \tfrac{A_I}{2}f_\tau - A_I F_I(h_I(\tau-1))\right), \tag{15}$$

which depends on history only through the accumulated net position. If no contracting is done in period τ, then $g(h_I(\tau);A_I,p_I) = 0$. If a contract is sold ($f_\tau = 1$) or bought ($f_\tau = -1$), the price must be individually rational, *i.e.*

$$\begin{aligned} f_\tau &= 1 \Rightarrow q_\tau \geq p_I + \tfrac{A_I}{2} + A_I F(h_I(\tau-1)) \equiv RS_I(\tau) \\ f_\tau &= -1 \Rightarrow q_\tau \leq p_I - \tfrac{A_I}{2} + A_I F(h_I(\tau-1)) \equiv RB_I(\tau). \end{aligned} \tag{16}$$

We shall use $RS_I(\tau)$ and $RB_I(\tau)$ as a convenient short-hand for I's myopic[7] reservation prices as a seller and as a buyer, respectively, and bear in mind that they evolve over time, depending in the shown way on I's forward position in the preceding period. If the trader is already net short ($F(h_I(\tau\text{-}1)) > 0$) this raises the selling reservation price but it also raises the maximum price at which she is willing to buy. This is so, since an additional unit sold

[7] Myopic since it does not take future payoffs into account. A forward looking reservation price would do this and thus require backward recursion or, equivalently, subgame perfection. The price (20) is a forward looking reservation price in the two-player case.



increases the riskiness of the overall position, whilst buying a unit reduces the short position and thereby the risk exposure. Figure 5 illustrates this for trader I and for a given τ: the intercept of the line with the q-axis is $p_I + A_I\ F(h_I(\tau\text{-}1))$. Only when $F(h_I(\tau\text{-}1)) = 0$ will the intercept equal the trader's spot price expectation.

The intercept and the two vertical lines indicate the set of IR contracts to I given the indivisible unit. A convenient fact is

$$RS_i(\tau) = RB_i(\tau) + A_i\ , \quad i \in \{I,J\},\ \forall\tau. \tag{17}$$

The following lemma commands full generality (that is to say it applies to all versions of the game independently of the number of players):

Lemma 1: $RS_I(\tau) \leq RB_J(\tau) \Rightarrow RS_J(\tau) > RB_I(\tau)$.

Proof: Follows directly from (17): $RS_I(\tau) = RB_I(\tau) + A_I \leq RS_J(\tau) = RB_J(\tau) - A_J$ $\Rightarrow RS_J(\tau) \geq RB_I(\tau) + A_I + A_J > RB_I(\tau)$ ■

Lemma 1 simply states that if there exists a contract where I sells to J that both players find IR, then there does not at the same time exist a contract that both players find

IR where J sells to I that both players find IR. The next lemma gives a dynamic extension of Lemma 1:

Lemma 2: Assume that two players, I and J, are matched in period τ-1. Then $RS_I(\tau\text{-}1) \leq RB_J(\tau\text{-}1) \Rightarrow RB_I(\tau) \leq RS_J(\tau)$.

Proof: If the two players do *not* enter a contract at τ-1, then $RS_I(\tau) = RS_I(\tau\text{-}1)$ and $RB_J(\tau) = RB_J(\tau\text{-}1)$ (because $F(h_I(\tau)) = F(h_I(\tau\text{-}1))$) and the conclusion follows directly from Lemma 1 (with strict inequality). If the players *do* enter a contract at τ-1, then $F(h_I(\tau\text{-}1)) = F(h_I(\tau\text{-}2)) + 1$ and $F(h_J(\tau\text{-}1)) = F(h_J(\tau\text{-}2)) - 1$, so $RS_I(\tau) = RS_I(\tau\text{-}1) + A_I$ and $RS_J(\tau) = RS_J(\tau\text{-}1) - A_J$. However, from (17) it then follows that $RB_I(\tau) = RS_I(\tau\text{-}1)$ and $RS_J(\tau) = RB_J(\tau\text{-}1)$. Since the condition of the Lemma is a weak inequality, the conclusion will also be a weak inequality. ■

Lemma 2 states that if, in period τ-1, there are gains from a trade where I sells to J, then there cannot be gains from a trade where J sells to I in period τ, given that the two agents are matched in τ-1. If they were not matched in τ-1 they could be matched to other agents in τ-1 and enter contracts that would disturb the conclusion. Here we concentrate on the two player case, where I and J are matched with each other in every period.

Remark: In the two player case, Lemma 2 implies that, in general, trade between I and J always goes in the same direction: I sells to J if $p_I < p_J$ and J sells to I if $p_I > p_J$.

Corollary: If $RS_I(\tau\text{-}1) = RB_J(\tau\text{-}1)$ and I and J enter a contract at τ-1, then $RB_I(\tau) = RS_J(\tau) = RS_I(\tau\text{-}1) = RB_J(\tau\text{-}1) = q_{I,\tau\text{-}1} \equiv q_{J,\tau\text{-}1}$.

Remark: If, in the two player case, the situation of the corollary to Lemma 2 occurs, then the net positions in τ-2 and τ-1 are

$$F(h_I(\tau-2)) = \frac{p_J - p_I}{A_I + A_J} - \frac{1}{2} \text{ and } F(h_I(\tau-1)) = \frac{p_J - p_I}{A_I + A_J} + \frac{1}{2}$$

which must both be integers. In this case, the agents can cycle back and forth between these two positions (recall I's position is always the negative of J's), each time trading at the competitive price and each trade adding zero to both trader's expected payoff. In fact, the two positions are defined by requiring that the marginal trade adds zero to both players expected payoff.

Generically, the positions mentioned in the remark will not be integers, and we have the following proposition determining the maximum value of the open interest which is then the maximum number of deals (ND) ($\lfloor x \rfloor$ denotes the integer part of x):

Proposition 1: Assume without loss of generality that $p_I > p_J$. Generically, the *upper bound on the open interest* is

$$F(h_J(\tau)) = -F(h_I(\tau)) \leq \left\lfloor \frac{p_I - p_J}{A_I + A_J} + \frac{1}{2} \right\rfloor \equiv ND. \tag{18}$$

Proof: Define $\delta \equiv \frac{p_I - p_J}{A_I + A_J} + \frac{1}{2} - \left\lfloor \frac{p_I - p_J}{A_I + A_J} + \frac{1}{2} \right\rfloor$, $\delta \in [0,1)$, and assume that the number of contracts already exchanged is $F(h_J(\tau)) = \frac{p_I - p_J}{A_I + A_J} + \frac{1}{2} - \delta$, which is then an integer. In this case, $RS_J(\tau+1) = q + (1-\delta)A_J > RB_I(\tau+1) = q - (1-\delta)A_I$ and
$RS_I(\tau+1) = q + \delta A_I \geq RB_J(\tau+1) = q - \delta A_J$,
where q is the competitive price defined in equation (8) of Section 3.1. Only if $\delta = 0$ can there be any trade. This is the special case discussed above. Otherwise, we have $RS_I(\tau+1) > RB_J(\tau+1)$ and no trade is possible in $\tau+1$ or thereafter, which implies that all gains from trade have been exploited. ■

Now we complete the description of the rules of the game by the assumption that *when I and J are matched in period τ, I makes an offer to J with probability 1/2 and J makes an offer to I with probability 1/2. An offer takes the form of a contract, i.e. essentially a price at which the offerer is willing to buy or sell one unit. The receiver of the offer can take it or leave it; accept it or reject it. Then the game moves on to period $\tau+1$.*[8]

The following proposition outlines the backward recursion principle by which the game at hand can be solved by studying the effect of all possible actions in τ on current period payoffs and on continuation payoffs (*i.e.* the expected value of the ensuing subgame) and letting the agents maximize over these actions:

[8] In terms of expected utility, the effect of the assumption that in any match either of the two agents has probability 1/2 of proposing to the other is the same as that of assuming Nash bargaining between the two agents in the match. This holds true also for the n-person game of Section 5 if the threat points are defined to be the continuation value to each of the agents of negotiations breaking down.

Proposition 2: Let $p_I > p_J$ and suppose that expected continuation payoffs are of the same form

$$v(\tau+1) = \frac{1}{2}\left(p_I - p_J - (A_I + A_J)(F(h_J(\tau))+\frac{1}{2})\right) \geq \frac{A_I+A_J}{4} \tag{19}$$

for both players. Then, if I is chosen to make an offer at τ, she will offer to buy a contract from J at the price

$$q_\tau = RS_J(\tau) + \frac{A_I + A_J}{2}, \tag{20}$$

which J will accept. If J is chosen to make an offer, she will offer to sell a contract to I at the price

$$q_\tau = RB_I(\tau) - \frac{A_I + A_J}{2}, \tag{21}$$

which I will accept. Thus, in both cases $F(h_I(\tau)) = F(h_I(\tau\text{-}1))\text{-}1$ and $F(h_J(\tau)) = F(h_J(\tau\text{-}1))+1$.

Ex ante, *i.e.* seen from period τ-1, the expected payoff to both players will then be

$$v(\tau) = \frac{1}{2}(p_I - p_J - (A_I + A_J)(F(h_J(\tau-1))+\frac{1}{2})). \tag{22}$$

Proof: $v(\tau+1) \geq (A_I + A_J)/4$ implies that $F(h_J(\tau)) \leq \dfrac{p_I - p_J}{A_I + A_J} - 1$, which ensures that there are IR contracts to be made in period τ. Consider the case in which I is chosen to make an offer to J. If J accepts, $F(h_J(\tau)) = F(h_J(\tau\text{-}1)) + 1$, if she rejects, $F(h_J(\tau)) = F(h_J(\tau\text{-}1))$. By hypothesis, J's expected continuation payoff in the case of acceptance is

$$v^a(\tau+1) = \frac{1}{2}(p_I - p_J - (A_I + A_J)(F(h_J(\tau-1)) + 1 + \frac{1}{2})),$$

whilst in the case of rejection it is

$$v^r(\tau+1) = \frac{1}{2}(p_I - p_J - (A_I + A_J)(F(h_J(\tau-1)) + \frac{1}{2})).$$

J then loses $v^r(\tau+1) - v^a(\tau+1) = (A_I + A_J)/2$ in terms of future payoff by accepting and I's offer should compensate J for this loss in order to induce acceptance. This can be done by offering a contract price such that $g(h_J(\tau)) = (A_I+A_J)/2 \Rightarrow q_\tau = RS_J(\tau)+(A_I+A_J)/2 \Rightarrow g(h_I(\tau)) = p_I - p_J + (A_I + A_J)(F(h_I(\tau\text{-}1)) - 1) \geq 0$ by the assumption that $v(\tau+1) \geq (A_I + A_J)/4$. This last assumption implies that it is not optimal for the offerer to construct an offer that is certain not to be accepted and wait for the other to make an offer. The case where J is chosen to make an offer to I follows a similar argument.

This shows that, seen from period τ-1, with probability 1/2 a player will expect to get $p_I - p_J + (A_I + A_J)(F(h_I(\tau\text{-}1)) - 1) \geq 0$, and with probability 1/2 she will get $(A_I+A_J)/2$ summing to:

$$Eg(h_i(\tau)) = (p_I - p_J + (A_I + A_J)(F(h_I(\tau\text{-}1)) - 1/2))/2$$

which is the same as (20) because $F(h_I) = -F(h_J)$. ■

Remark: The expected period-τ price will be:

$$Eq(\tau) = \frac{p_I + p_J}{2} + \frac{A_J - A_I}{2}(F(h_J(\tau-1)) - \frac{1}{2}). \tag{23}$$

If $A_I = A_J$, the expected price will be identical to the competitive price. If $A_I \neq A_J$, the expected price will show a trend, that will be positive if $A_J > A_I$ and negative in the other case. Particular realizations of the price will be as indicated in Proposition 2.

Equilibrium in this model is a situation in which I and J maximize $G(h_I(T))$ and $G(h_J(T))$ respectively. If the agents follow the recursive formula of Proposition 2, then equilibrium will be subgame perfect. There are two cases to be considered: If $T < ND$, there are not enough periods to allow the agents to reach the upper bound on the open interest, ND: They have to stop trading before all gains from trade are exploited. On the other hand, if $T \geq ND$, they can exhaust the gains from trade (under the restriction imposed by the indivisibility of a contract). The following lemma shows that they must exhaust all gains from trade if they can:

Lemma 3: Let $p_I > p_J$ and assume that $T \geq ND$. Then $F(h_J(T)) = ND$ is (generically) a necessary condition for equilibrium.

Proof: First assume that $F(h_J(T)) < ND$. Then $F(h_J(T)) \leq \dfrac{p_I - p_J}{A_I + A_J} - \dfrac{1}{2} - \delta$, where we used the fact that F is an integer. If $\delta \neq 0$, this implies that $RS_J(T) < RB_I(T)$ so that a contract could be set up which would give a non-negative incremental payoff to both traders and a strictly positive payoff to at least one, in contradiction with optimality. If $\delta = 0$, we could have the special case where the marginal contract gives both zero incremental profit. In this case $F(h_J(T)) = ND - 1$, is a necessary condition for equilibrium.

Hence $F(h_J(T)) \geq ND$. By Proposition 1, $F(h_J(T)) \leq ND$, so we have $F(h_J(T)) = ND$. ■

Proposition 2: Let $p_I > p_J$ and suppose that expected continuation payoffs are of the same form

$$v(\tau+1) = \frac{1}{2}\left(p_I - p_J - (A_I + A_J)(F(h_J(\tau))+\frac{1}{2})\right) \geq \frac{A_I+A_J}{4} \tag{19}$$

for both players. Then, if I is chosen to make an offer at τ, she will offer to buy a contract from J at the price

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Ex ante, *i.e.* seen from period τ-1, the expected payoff to both players will then be

$$v(\tau) = \frac{1}{2}(p_I - p_J - (A_I + A_J)(F(h_J(\tau-1))+\frac{1}{2})). \tag{22}$$

Proof: $v(\tau+1) \geq (A_I + A_J)/4$ implies that $F(h_J(\tau)) \leq \dfrac{p_I - p_J}{A_I + A_J} - 1$, which ensures that there are IR contracts to be made in period τ. Consider the case in which I is chosen to make an offer to J. If J accepts, $F(h_J(\tau)) = F(h_J(\tau\text{-}1)) + 1$, if she rejects, $F(h_J(\tau)) = F(h_J(\tau\text{-}1))$. By hypothesis, J's expected continuation payoff in the case of acceptance is

$$v^a(\tau+1) = \frac{1}{2}(p_I - p_J - (A_I + A_J)(F(h_J(\tau-1)) + 1 + \frac{1}{2})),$$

whilst in the case of rejection it is

$$v^r(\tau+1) = \frac{1}{2}(p_I - p_J - (A_I + A_J)(F(h_J(\tau-1)) + \frac{1}{2})).$$

J then loses $v^r(\tau+1) - v^a(\tau+1) = (A_I + A_J)/2$ in terms of future payoff by accepting and I's offer should compensate J for this loss in order to induce acceptance. This can be done by offering a contract price such that $g(h_J(\tau)) = (A_I+A_J)/2 \Rightarrow q_\tau = RS_J(\tau)+(A_I+A_J)/2 \Rightarrow g(h_I(\tau)) = p_I - p_J + (A_I + A_J)(F(h_I(\tau\text{-}1)) - 1) \geq 0$ by the assumption that $v(\tau+1) \geq (A_I + A_J)/4$. This last assumption implies that it is not optimal for the offerer to construct an offer that is certain not to be accepted and wait for the other to make an offer. The case where J is chosen to make an offer to I follows a similar argument.

This shows that, seen from period τ-1, with probability 1/2 a player will expect to get $p_I - p_J + (A_I + A_J)(F(h_I(\tau-1)) - 1) \geq 0$, and with probability 1/2 she will get $(A_I+A_J)/2$ summing to:

$$Eg(h_i(\tau)) = (p_I - p_J + (A_I + A_J)(F(h_I(\tau-1)) - 1/2))/2$$

which is the same as (20) because $F(h_I) = -F(h_J)$. ■

Remark: The expected period-τ price will be:

$$Eq(\tau) = \frac{p_I + p_J}{2} + \frac{A_J - A_I}{2}(F(h_J(\tau-1)) - \frac{1}{2}). \tag{23}$$

If $A_I = A_J$, the expected price will be identical to the competitive price. If $A_I \neq A_J$, the expected price will show a trend, that will be positive if $A_J > A_I$ and negative in the other case. Particular realizations of the price will be as indicated in Proposition 2.

Equilibrium in this model is a situation in which I and J maximize $G(h_I(T))$ and $G(h_J(T))$ respectively. If the agents follow the recursive formula of Proposition 2, then equilibrium will be subgame perfect. There are two cases to be considered: If $T < ND$, there are not enough periods to allow the agents to reach the upper bound on the open interest, ND: They have to stop trading before all gains from trade are exploited. On the other hand, if $T \geq ND$, they can exhaust the gains from trade (under the restriction imposed by the indivisibility of a contract). The following lemma shows that they must exhaust all gains from trade if they can:

Lemma 3: Let $p_I > p_J$ and assume that $T \geq ND$. Then $F(h_J(T)) = ND$ is (generically) a necessary condition for equilibrium.

Proof: First assume that $F(h_J(T)) < ND$. Then $F(h_J(T)) \leq \frac{p_I - p_J}{A_I + A_J} - \frac{1}{2} - \delta$, where we used the fact that F is an integer. If $\delta \neq 0$, this implies that $RS_J(T) < RB_I(T)$ so that a contract could be set up which would give a non-negative incremental payoff to both traders and a strictly positive payoff to at least one, in contradiction with optimality. If $\delta = 0$, we could have the special case where the marginal contract gives both zero incremental profit. In this case $F(h_J(T)) = ND - 1$, is a necessary condition for equilibrium.

Hence $F(h_J(T)) \geq ND$. By Proposition 1, $F(h_J(T)) \leq ND$, so we have $F(h_J(T)) = ND$. ■

Proposition 3: Let $p_I > p_J$. Then a subgame perfect equilibrium exists in which I and J maximize $G(h_I(T))$ and $G(h_J(T))$ respectively. If $\boldsymbol{T \geq ND}$, in the first T - ND periods equilibrium has

$$F(h_I(\tau)) = 0 = F(h_J(\tau)) \tag{24}$$

$$v(\tau) = 0 \tag{25}$$

$$Eq(\tau) = \text{Undefined} \tag{26}$$

For both $\boldsymbol{T \leq ND}$ and $\boldsymbol{T \geq ND}$, define $u \equiv \tau - max\{T - ND , 0\}$. For $\tau \geq max\{T - ND , 0\}$ (*i.e.* $u \geq 0$, equilibrium has

$$F(h_I(\tau)) = -u = -F(h_J(\tau)) \tag{27}$$

$$v(\tau) = \frac{1}{2}\left(\left(p_I - \frac{A_I}{2}\right) - \left(p_J - \frac{A_J}{2}\right) - (u - 1)(A_I - A_J)\right) \tag{28}$$

$$Eq(\tau) = \frac{p_I + p_J}{2} + \frac{A_J - A_I}{2}\left(u - \frac{1}{2}\right). \tag{29}$$

Proof: We need to prove that we can apply to both cases the backward recursion of Proposition 2 by showing that in period T we get continuation payoffs of the form (19). If this is the case, then Proposition 2 showed in (22) that continuation payoffs in any earlier period take the same form. The rest of the proof follows from forward recursion from period 1 with $F(h_i(0)) \equiv 0$.

First consider the case in which $T \geq ND$. By Lemma 3 we know that $F(h_J(T)) = ND$. The trading must take place in the last ND periods. Before this, any trader can reject an offer hoping to become the first person to have an offer accepted sometime later, without sacrificing continuation payoffs, so it must be the case that $F(h_J(T\text{-}1)) = ND - 1$. Then reservation prices are $RS_J(T) = q^* - \delta A_J$ and $RB_I(T) = q^* - \delta A_I$. In period T, there is no continuation to worry about, so the winner takes all. Seen from period $T\text{-}1$, both players then expect to gain

$$\tfrac{1}{2}(RB_I(T) - RS_J(T)) = \delta\tfrac{A_I + A_J}{2}.$$

If $\delta \geq 1/2$ this satisfies the condition of Proposition 2 that $v(\tau+1) \geq (A_I + A_J)/4$. If $\delta < 1/2$, we regress one period. By assumption we have $F(h_J(T\text{-}2)) = ND - 2$, so $T\text{-}1$ reservation prices are $RS_J(T\text{-}1) = q^* - A_J(1 + \delta)$ and $RB_I(T\text{-}1) = q^* + A_I(1 + \delta)$. In case of rejection, the reservation prices remain unchanged in period T where the winner takes all, so in order to induce acceptance at $T\text{-}1$, the offerer should compensate the other with the amount

$$\frac{1}{2}(RB_I(T-1) - RS_J(T-1)) = \frac{1}{2}(1 + \delta)(A_I + A_J),$$

which will then be the $T\text{-}1$ incremental payoff to the acceptor. The offerer gets the same (the other half). Seen from period $T\text{-}2$, the continuation payoff is thus

$$v(T-1) = \frac{1}{2}(1 + \delta)(A_I + A_J) > \frac{A_I + A_J}{4},$$

so we can unravel the game from $T\text{-}1$ backwards.

We next consider the case in which $T < ND$. We then have $F(h_J(T\text{-}1)) = T\text{-}1$, so $RS_J(T) = p_J + A_J(T - 1/2)$ and $RB_I(T) = p_I - A_I(T - 1/2)$. Seen from $T\text{-}1$, continuation payoffs will be

$$v(T) = \frac{1}{2}(p_I - p_J - (A_I + A_J)(T - \frac{1}{2})).$$

$v(T) \geq (A_I + A_J)/4$ iff $T \leq (p_I - p_J)/(A_I + A_J)$, but this follows by the assumption that $T < ND$. ■

The equilibrium that is outlined in Propositions 1 through 3 can be interpreted as a Markov Perfect Equilibrium:[9] If we take as state variable (i) the forward positions of the previous period, $\tau\text{-}1$, (ii) the realization of who nature chooses to make an offer in the current period, τ, and (iii) time, τ, itself, then those are sufficient to determine what offer the agent should make and whether to accept or reject it in period τ, taking the expected value of future actions into account. Indeed, the payoff relevant history is summarized by $F(h_I(\tau\text{-}1)) = -F(h_J(\tau\text{-}1))$. Given a realization of the state variable, the offerer proposes a contract chosen such that the price exactly compensates the receiver for the loss in continuation payoff and leaves the receiver indifferent between accepting and rejecting. Equilibrium then requires her to accept. This is the essence of Proposition 2. Proposition 1 defines the ergodic states of the Markov chain that the Markov perfect equilibrium can be seen as: If the market reaches a state in which $F(h_J(\tau)) = ND$, then the two traders have no further gains from trade and will stop trading, so all possible future states will have this quality. Lemma 3 says that if possible ($T \geq ND$), the Markov chain should end up in an ergodic state. Proposition 3 then states that, if possible, the market ends up in an ergodic state, exhausting all gains from trade. This will happen in the last ND periods, so in the first $T\text{-}ND$ periods, nothing happens: The forward positions stay zero, there are no realizations of the price since all offers are rejected without any loss of continuation payoffs. In the last ND periods, the J's forward position is increasing by one contract each period (and I's is similarly decreasing) thus reaching ND when the game stops at T. This quantity is as efficient, *i.e.* as close to the bilateral core, as can be given the indivisibility. Prices are evolving around a trend (see (23)) if $A_I \neq A_J$ or take values randomly above and below the competitive price if $A_J = A_I$. All realized prices have to be individually

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[9] See Definition 4 and Proposition 7.

rational, so the equilibrium is efficient for $T \geq ND$. If $T < ND$, there is a trade in each period, so at the end of the game $F(h_J(T)) = T$ and prices evolve as in the other case. This equilibrium can by construction not be efficient, but given the time constraint the players get as close to efficiency as possible.

In short, the two trader game yields a unique Markov perfect equilibrium that is as efficient as possible given the time constraint. In this model the only uncertainty stems from who is going to make an offer in each period. The identity of the opponent is known with certainty. This is not the case with three or more players.

4. Three Traders with Differing Beliefs

This section extends the two-player results to three players as an appetizer to the n-person game. While the extension of the axiomatic approaches is relatively straightforward, the extension of the strategic approach to a dynamic, stochastic game proves to be trickier.

4.1 Axiomatic Approaches and the Phlips-Harstad Model

It is a matter of simple algebra to extend the static analysis of Section 3.1 to a three trader market, thereby obtaining a simple version of the model in Phlips and Harstad (1991): In our model the traders on the forward market have no power in the spot market, whereas in the Phlips/Harstad model they do. However, this simplification has no importance for the discussion of the efficiency of the forward market.

A straightforward extension of the notation is needed: The traders are called I, J and K and f_{ij}, $i=I,J$, $j=J,K$, $i \neq j$, denotes the amount that i sells to j at price q_{ij}. If f_{ij} is negative, then i buys from j as before. Suppressing the time dimension, the payoff function for, say, I now reads

$$G(\bullet; A_I, p_I) = (q_{IJ} - p_I)f_{IJ} + (q_{IK} - p_I)f_{IK} - \tfrac{A_I}{2}(f_{IJ} + f_{IK})^2. \tag{30}$$

The competitive equilibrium is readily found to be

$$
\begin{aligned}
q^* &= (A_J A_K P_I + A_I A_K P_J + A_I A_J P_K)/D \\
f_I^* &= (A_J(p_K - p_I) + A_K(p_J - p_I))/D \\
f_J^* &= (A_K(p_I - p_J) + A_I(p_K - p_J))/D \\
f_K^* &= (A_J(p_I - p_K) + A_I(p_J - p_K))/D
\end{aligned}
\tag{31}
$$

where $\quad D = A_I A_K + A_I A_J + A_J A_K$.

The set of contracts that are individually rational for any two players, say I and J, is described by

$$
\begin{aligned}
&\left\{(q_{IJ},f_{IJ}) \mid f_{IJ}>0\ ,\ p_J + \tfrac{A_J}{2}(f_J + f_{JK}) \geq q_{IJ} \geq p_I + \tfrac{A_I}{2}(f_I + f_{IK})\right\} U \\
&\left\{(q_{IJ},f_{IJ}) \mid f_{IJ}<0\ ,\ p_J + \tfrac{A_J}{2}(f_J + f_{JK}) \leq q_{IJ} \leq p_I + \tfrac{A_I}{2}(f_I + f_{IK})\right\} U \\
&\left\{(q_{IJ},f_{IJ}) \mid f_{IJ}=0\right\} ,
\end{aligned}
\tag{32}
$$

where $f_I \equiv f_{IJ} + f_{IK}$ and $f_J \equiv -f_{IJ} + f_{JK}$. As before, this set describes a triangle with the q-axis. The three subsets are mutually exclusive (either f_{IJ} <,> or = 0) so a given contract can belong to at most one of them. The new feature is that the sides of the triangle are subjected to parallel shifts by $A_J f_{JK}$ and $A_I f_{IK}$ respectively that is, the quantities of the two other contracts matter, because they enter jointly with f_{IJ} in the respective agents' risk evaluations. This requires that any trade with K be common knowledge, or, in other words, that the three contracts be coordinated.

Solving for the bilateral core we find that the quantities should satisfy the following system:

$$
\begin{pmatrix} p_J - p_I \\ p_K - p_I \\ p_K - p_J \end{pmatrix} = \begin{pmatrix} A_I + A_J & A_I & -A_J \\ A_I & A_I + A_K & A_K \\ -A_J & A_K & A_J + A_K \end{pmatrix} \begin{pmatrix} f_{IJ} \\ f_{IK} \\ f_{JK} \end{pmatrix}
\tag{33}
$$

This system exhibits linear dependence with one degree of freedom and the profile (f_{IJ}, f_{IK}, f_{JK}) is thus left undetermined. The three traders' *net* positions are, however, uniquely determined and equal to the competitive quantities above:

$$f_I^* = f_{IJ} + f_{IK}$$
$$f_J^* = f_{JK} - f_{IJ}, \tag{34}$$
$$f_K^* = -f_{IK} - f_{JK}$$

and prices should satisfy

$$\{(q_{IJ}, f_{IJ}) |\ f_{IJ}>0,\ q^* + \frac{A_J}{2} f_{IJ} \geq q_{IJ} \geq q^* - \frac{A_I}{2} f_{IJ}\} U$$
$$\{(q_{IJ}, f_{IJ}) |\ f_{IJ}<0,\ q^* + \frac{A_J}{2} f_{IJ} \leq q_{IJ} \leq q^* - \frac{A_I}{2} f_{IJ}\} U \tag{35}$$
$$\{(q_{IJ}, f_{IJ}) |\ f_{ij}=0\}.$$

Note that if one quantity is fixed, say f_{IJ}, then the two other quantities are uniquely determined by (34) and the IR requirements on forward prices are uniquely given by (35) and similar expressions for q_{IK} and q_{JK}.

Phlips and Harstad (1991) propose a relatively simple non-cooperative game and show that the solution will satisfy (34) and (35). However, their game exhibits a continuum of subgame perfect equilibria and does not explain *how* the agents achieve coordination to pick one profile of three contracts that will satisfy (34).

4.2 Strategic Bargaining

The extension of the strategic bargaining game of Section 3.2 from two to three players is as follows: In any period τ, there is probability 1/3 that any two players are matched excluding the third from playing in that period. In each match, each of the two players then have probability 1/2 of being the one to make an offer, which the other can accept or reject. The game then moves on to period $\tau+1$.

This is a stochastic game: the history at τ, $h(\tau-1) \equiv (h_I(\tau-1), h_J(\tau-1), h_K(\tau-1))$ can be summarized in a state variable, $k(\tau)$ (as will be shown in due course):

$$k(\tau) \equiv (\boldsymbol{F}(h(\tau-1)\text{ , offerer , receiver , idle , }\tau)\text{ ,}$$
$$\text{where } \boldsymbol{F}(h(\tau-1)) \equiv (F(h_I(\tau-1)),\ F(h_J(\tau-1)),\ F(h_K(\tau-1))). \tag{36}$$

$k(\tau)$ consists of the profile of net-positions of the preceding period, $\boldsymbol{F}(h(\tau-1))$, the identity of the agent who is chosen to make an offer, together with the identity of the player who is chosen to receive the offer and the player that is idle at τ. It also includes time, τ, explicitly since the number of periods left may be important for the strategies. Note that there are redundancies in the state variable, since

$$\sum_i F(h_i(\tau-1)) = 0 \quad , \forall\tau, \tag{37}$$

and since the identity of the idle player is known by exclusion, but the notation (36) is maintained since it appears more intuitive and more symmetric. Expected payoffs at τ, $g(\tau) \equiv (g(h_I(\tau\text{-}1)),g(h_J(\tau\text{-}1)),g(h_K(\tau\text{-}1)))$, depend on $k(\tau)$ and on current actions, these being a contract offer, acceptance/rejection and a void action for the unmatched, idle player.

The state follows a Markov process in that the probability distribution on $k(\tau+1)$ is determined by $k(\tau)$ and the actions taken at τ. For example, assume that the state is

$$k(\tau) = (\boldsymbol{F}(h(\tau-1)) \ , \ I \ , \ K \ , \ J \ , \ \tau), \tag{38}$$

and that I offers to buy a contract from K in period τ. If K rejects (r), then $k(\tau+1)$ will be one of the following six states, each of which has probability 1/6:

$$(k(\tau+1)\,|\,k(\tau),(c_{I,\tau},r)) \in \begin{Bmatrix} (\boldsymbol{F}(h(\tau-1)),i,j,k,\tau) \\ \forall i,j,k \in \{I,J,K\}, i \neq j \neq k \end{Bmatrix} \tag{39}$$

in that the profile of net-positions remains unchanged in case of rejection. If K accepts (a), then $\boldsymbol{F}(h(\tau)) \equiv (F(h_I(\tau\text{-}1)) - 1,\ F(h_J(\tau\text{-}1)),\ F(h_K(\tau\text{-}1)) + 1)$ and the state will belong to:

$$(k(\tau+1)\,|\,k(\tau),(c_{I,\tau}\,, a)) \in \begin{Bmatrix} (\boldsymbol{F}(h(\tau)),i,j,k,\tau) \\ \forall i,j,k \in \{I,J,K\}, i \neq j \neq k \end{Bmatrix} \tag{40}$$

with transition function $Pr(k(\tau+1)\,|\, k(\tau),\ (c_{I,\tau},a)) = 1/6$. In the example the players' action spaces are $S_I=\{c_{I,\tau}\}$ (the right to propose a contract), $S_K = \{a,r\}$ (the right to accept it or reject it) and $S_J = \varnothing$ (the right to remain silent). In general, we have the following definition:

Definition 1: *(Action Spaces)* In any state $k(\tau) = (\boldsymbol{F}(h(\tau\text{-}1),i,j,k,\tau)$, the action spaces are $S_i = \{c_{i,\tau}\}$, $S_j = \{a,r\}$, $S_k = \varnothing$, (i,j,k) is any permutation of $\{I,J,K\}$. Let $S \equiv S_i{\times}S_j{\times}S_k$, with $s(\tau){\equiv}(s_i(\tau),s_j(\tau),s_k(\tau)) \in S$.

In principle strategies could depend on the entire history of the game. However, for any subgame starting in some period τ, only $k(\tau)$ matters. To show this, we adapt the following definitions from Fudenberg and Tirole (1991) (pp. 514-5): The future at τ is the current and future actions $\Phi(\tau) \equiv (s(\tau),\ s(\tau+1),\ ...,\ s(T)) \in S^{T-\tau}$ following choices of i, j and k by nature. In the following, h and h' denote two different histories.

Definition 2: *(Sufficient Partition of Histories)* A partition $\{H^{\tau}(\cdot)\}_{\tau=1,..,T}$ is *sufficient* if $\forall\tau$ and $\forall h(\tau\text{-}1), h'(\tau\text{-}1)$: $H^{\tau}(h(\tau\text{-}1)) = H^{\tau}(h'(\tau\text{-}1))$, the subgames starting at date τ after histories $h(\tau\text{-}1)$ and $h'(\tau\text{-}1)$ are strategically equivalent, *i.e.*

(i) The action spaces in the subgames are identical: $\forall i \in (I,J,K)$, $\forall t>0$ and $\forall s(\tau+t\text{-}1)$, $S_i(h(\tau\text{-}1), s(\tau),...,s(T)) = S_i(h'(\tau\text{-}1), s(\tau),...,s(T))$.

(ii) The players' payoffs conditional on $h(\tau\text{-}1)$ and $h'(\tau\text{-}1)$ are representations of the same preferences.

Definition 3: The *Payoff-Relevant History* is the coarsest sufficient partition.

From these two definitions we obtain Lemma 4:

Lemma 4: The payoff relevant history is summarized by $\boldsymbol{F}(h(\tau\text{-}1))$.

Proof: The claim is that a partition, H^{τ}, of histories leading to the same $\boldsymbol{F}(h(\tau\text{-}1))$ is sufficient. Let $h(\tau\text{-}1)$ and $h'(\tau\text{-}1)$ be two histories for which $\boldsymbol{F}(h(\tau\text{-}1)) = \boldsymbol{F}(h'(\tau\text{-}1))$. The action spaces are time invariant, so the first condition is satisfied. We now need to show that the payoffs conditional on any two histories that lead to the same profile of net positions at $\tau\text{-}1$ represent the same underlying preferences. The original preferences, $u_i(\pi) = -exp(-A_i\pi)$, are von Neumann-Morgenstern, so unique up to a linear transformation. What we need to show is thus that $\forall i \in \{I,J,K\}\ \exists(\lambda_i, \mu_i) \equiv \left(\lambda_i\left[h(\tau\text{-}1), h'(\tau\text{-}1)\right], \mu_i\left[h(\tau\text{-}1), h'(\tau\text{-}1)\right]\right)$ $(\lambda_i > 0)$ such that $\forall\Phi(\tau),\ G(h_i(\tau\text{-}1),\Phi(\tau)) = \lambda_i G(h_i'(\tau\text{-}1),\Phi(\tau)) + \mu_i$.

By the definition of $G(\cdot\,;\,\cdot)$, we have $G(h_i(\tau\text{-}1),\Phi(\tau)) = V(h_i(\tau\text{-}1)) + V(\Phi_i(\tau)) - p_i(F(h_i(\tau\text{-}1)) + F(\Phi_i(\tau))) + (A_i/2)\,(F(h_i(\tau\text{-}1)) + F(\Phi_i(\tau)))^2$, where we abuse the notation slightly, so that $V(\Phi_i(\tau))$ and $F(\Phi_i(\tau)))$ denote respectively i's book value and her (change in) net position arising from a given future $\Phi(\tau)$. It is easily seen that $\mu_i(h(\tau), h'(\tau), \cdot) = V(h_i(\tau\text{-}1)) - V((h_i'(\tau\text{-}1))$ and $\lambda_i(h(\tau\text{-}1), h'(\tau\text{-}1)) = 1$, so the payoffs conditional on any history that leads to $\boldsymbol{F}(h(\tau\text{-}1))$ represent the same underlying preferences:

$$G(h_i(\tau\text{-}1),\Phi(\tau)) = G(h_i'(\tau\text{-}1),\Phi(\tau)) + V(h(\tau\text{-}1)) - V(h'(\tau\text{-}1)).\blacksquare$$

Remark: The proof shows that payoffs after any two histories that lead to the same positions only differ with respect to the book value of the previously concluded contracts, which has no impact on future reservation prices and hence does not affect strategies in future subgames.

Definition 4: A *Markov perfect equilibrium* (MPE) is a profile of strategies s^* that is a subgame perfect equilibrium (SPE) and depends on only the payoff relevant history, *i.e.* $H(h(\tau\text{-}1)) = H(h'(\tau\text{-}1)) \Rightarrow \forall i,\ s_i^*(h(\tau-1))=s_i^*(h'(\tau-1))$.

In view of Lemma 4 we can let strategies depend on the profile of net positions, so that an MPE is a strategy profile s^* that is an SPE and that is measurable w.r.t. $F(h(\tau\text{-}1))$: $s_i^* = s_i^*(F(h(\tau-1))),\ \forall i$.

Let $K(\tau)$ denote the set of feasible states after a history leading to a set of forward positions, $\boldsymbol{F}(h(\tau\text{-}1))$. We have existence of MPE in the game:

Proposition 4: There exists a Markov perfect equilibrium for the game $\Gamma_3 \equiv (N \equiv \{I,J,K\},\ \tau=1,..,T,\ k(\tau)\in K(\tau),\ s(\tau)\in S,\ Pr(k(\tau+1)\,|k(\tau),s(\tau)),\ \{G(h_i(T))\}_{\forall i\in N})$.

Proof: The proof will be a special case of the *n*-person game of the later Section 5.2.■

Having claimed existence of equilibrium, we characterize the equilibrium as far as possible without knowing the essential parameters, T and (A_i, p_i), $i \in \{I,J,K\}$. In three propositions we show the counterpart to Proposition 1 for the two player case: there is a (generically unique) $\boldsymbol{F}(h(\tau\text{-}1))$ in the neighbourhood of (f_I^*, f_J^*, f_K^*) such that in all states after a history leading to $\boldsymbol{F}(h(\tau\text{-}1))$, there will be no trade. These absorbing states are in other words uniquely determined by a single combination of net positions that determine how close the players can hope to come to efficiency.

Proposition 5: Trading stops if the game reaches a state where

$$\begin{aligned} F(h_I(\tau-1)) &= f_I^* + \delta_I \\ F(h_J(\tau-1)) &= f_J^* + \delta_J \\ F(h_K(\tau-1)) &= f_K^* - (\delta_I + \delta_J) \end{aligned} \tag{41}$$

where (δ_I, δ_J) satisfy the following six inequalities:

$$-\frac{A_I+A_J}{2} \leq -A_I\delta_I + A_J\delta_J \leq \frac{A_I+A_J}{2}$$

$$-\frac{A_I+A_K}{2} \leq (A_I+A_K)\delta_I + A_K\delta_J \leq \frac{A_I+A_K}{2} \tag{42}$$

$$-\frac{A_J+A_K}{2} \leq A_K\delta_I + (A_J+A_K)\delta_J \leq \frac{A_J+A_K}{2}$$

Proof: If $RB_i(\tau) \leq RS_j(\tau)$, $\forall i,j\ ,\ i{\neq}j$, there are no further gains from trade. Manipulation of this system of inequalities yields (41) and (42). ■

Remark: Generically, all states like the one mentioned in the proposition are ergodic (absorbing): Once that kind of state has been reached, the market stays in it in all subsequent periods.

Remark: The proposition shows how close the market can get to the bilateral core: only if f_I^* and f_J^* are integers can the ergodic states be efficient ($\delta_I = \delta_J = 0$).

Figures 6.a and b show the range of (δ_I, δ_J) that satisfy the six inequalities for the two cases when $A_I = A_J = A_K = A$ (Figure 6.a) and $A_I = A_J/2 = A_K/3 = A$ (Figure 6.b). Note that it is not always the case that there is a non-binding inequality.

Figure 6A: The area where the requirements (42) are met for $A_I = A_J = A_K$

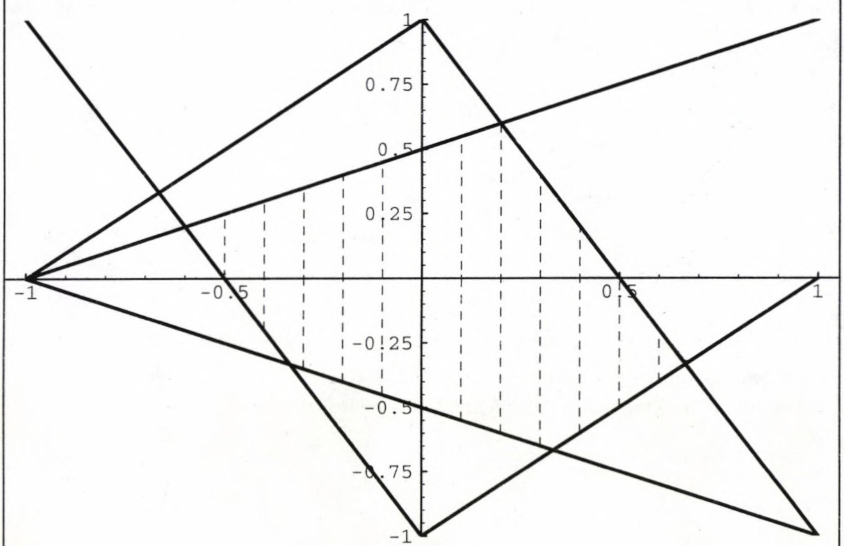
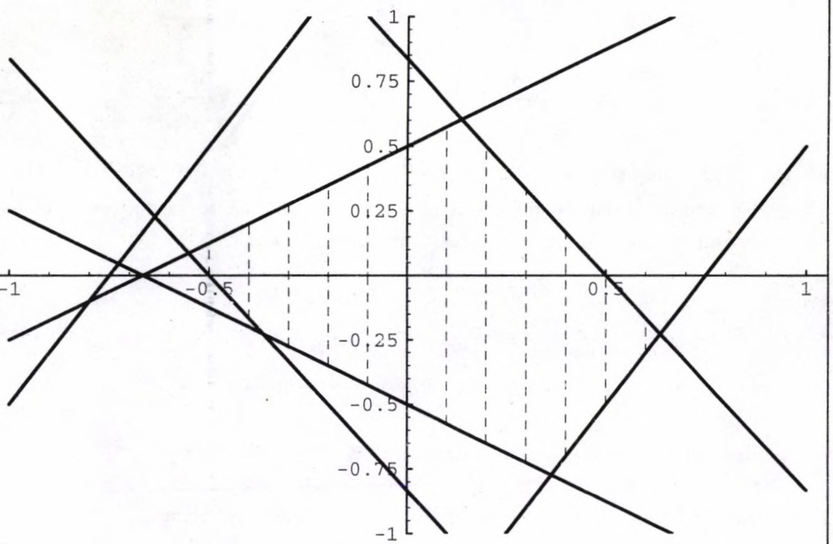


Figure 6B: The area where the requirements (42) are met for $A_I = A_J/2 = A_K/3$



Proposition 6: The only ergodic states *(F(h(τ-1),i,j,k)* are those that satisfy Proposition 5.

Proof: In order for a state to be ergodic at date $\tau \leq T$, we need $RB_i(\tau) \leq RS_j(\tau)$, $\forall i,j,\ i \neq j$. This leads to the following set of inequalities:

$$p_I - p_J - \frac{A_I + A_J}{2} \leq A_J F_J - A_I F_I \leq p_I - p_J + \frac{A_I + A_J}{2}$$

$$p_I - p_K - \frac{A_I + A_K}{2} \leq A_K F_K - A_I F_I \leq p_I - p_K + \frac{A_I + A_K}{2} \tag{43}$$

$$p_J - p_K - \frac{A_J + A_K}{2} \leq A_K F_K - A_J F_J \leq p_J - p_K + \frac{A_J + A_K}{2}$$

where (F_I, F_J, F_K) are net positions at τ-1 $(F_I+F_J+F_K=0)$. Let $F_I = f_I^* + \delta_I$ and $F_J = f_J^* + \delta_J$, where the only restriction on the δ's is that when they are added to the corresponding efficient positions, the result be an integer. Combining (33) and (34), we get

$$A_J F_J - A_I F_I = p_I - p_J - A_I \delta_I + A_J \delta_J$$

$$A_K F_K - A_I F_I = p_I - p_K - (A_I + A_K)\delta_I - A_K \delta_J, \tag{44}$$

$$A_K F_K - A_J F_J = p_J - p_K - A_K \delta_I - (A_J + A_K)\delta_J$$

which combined with (43) gives the desired result.■

Proposition 7: The positions in the ergodic states are (generically) unique.

Proof: Let $\boldsymbol{F}(h(\tau\text{-}1)) = (F_I, F_J, F_K)$ be the forward positions of an absorbing state. The pair (F_I, F_J) thus satisfies

$$p_I - p_J - \frac{A_I + A_J}{2} \leq A_J F_J - A_I F_I \leq p_I - p_J + \frac{A_I + A_J}{2}.$$

There are six candidates for ergodic positions in the neighbourhood of $\boldsymbol{F}(h(\tau\text{-}1))$, namely $(F_I, F_J, F_K) + \bar{x}$ where $\bar{x} \in \{(1,-1,0),(-1,1,0),(1,0,-1),(-1,0,1),(0,1,-1),(0,-1,1)\}$. Take the first element. If $A_J F_J - A_I F_I < p_I - p_J + (A_I + A_J)/2$, we have $A_J(F_J\text{-}1) - A_I(F_I\text{+}1) < p_I - p_J - (A_I + A_J)/2$, so $(F_I, F_J, F_K) + (1,-1,0)$ is not an ergodic position. If $A_J F_J - A_I F_I = p_I - p_J + (A_I + A_J)/2$, we have $A_J(F_J\text{-}1) - A_I(F_I\text{+}1) = p_I - p_J - (A_I + A_J)/2$, so $(F_I, F_J, F_K) + (1,-1,0)$ is also an ergodic position, and there can be a cycle back and forth between the two, this adding nothing to the expected payoffs of I and J. The argument for the other five candidates is similar.■

4.3 An Example of a Markov Perfect Equlibrium

To get a feel for the nature of the Markov perfect equilibrium, consider the following simple numerical example. Assume that the basic parameters of the model are chosen to be:

Spot price expectations:	$p_I = 20.0$,	$p_J = 19.0$,	$p_K = 18.0$
Risk aversion constants:	$A_I = 0.2$,	$A_J = 0.4$,	$A_K = 0.6$
Time horizon:	$T = 3$		

The competitive equilibrium is found to be:

$$q^* = \frac{213}{11} = 19\frac{4}{11},\ (f_I^*, f_J^*, f_K^*) = (-3\frac{2}{11}, \frac{10}{11}, 2\frac{3}{11}),$$

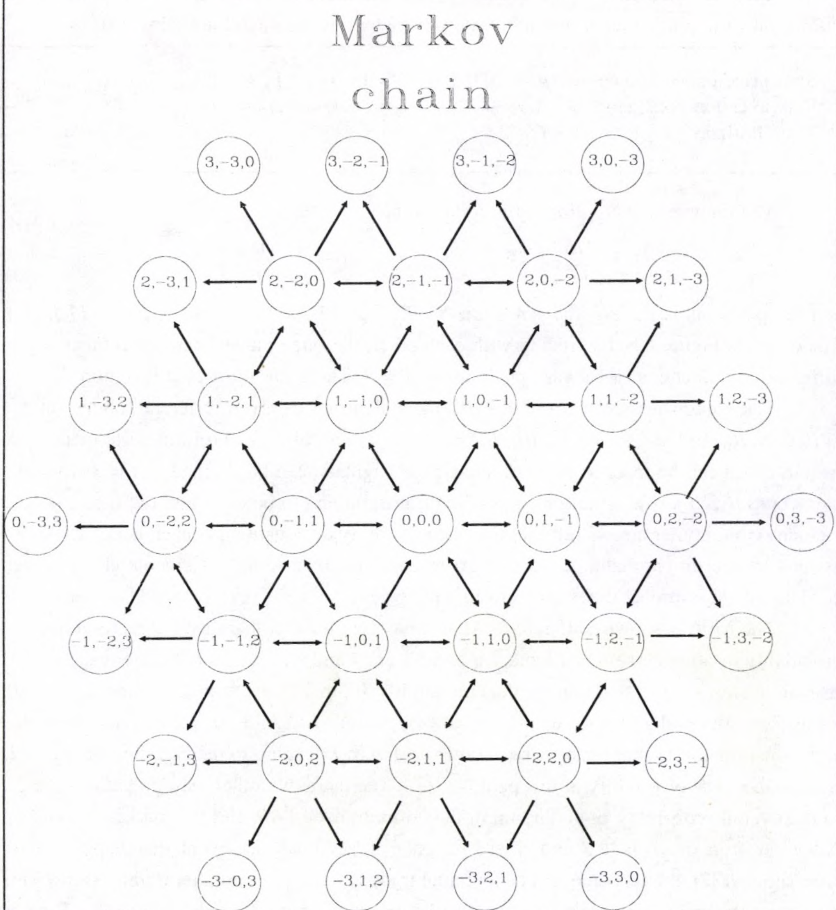
and the positions in the ergodic states are $(F_I, F_J, F_K) = (-3,1,2)$ (*i.e.* $(\delta_I, \delta_K) = (2/11, 1/11)$). The case fits Figure 6.b. By trading with each other, the three players can reach thirty-seven different combinations of forward positions at $T = 3$. These are illustrated in Figure 7.

The game starts with $\boldsymbol{F}(h(0)) \equiv (0,0,0)$ and can go to seven different "nodes" at $\tau = 1$: $(0,0,0)$, $(0,-1,1)$, $(-1,0,1)$, $(-1,1,0)$, $(0,1,-1)$, $(1,0,-1)$ and $(1,-1,0)$. From these an additional twelve nodes can be reached at $\tau = 2$ and further eighteen can be reached at $\tau = 3$ when the game ends. Each arrow represents a possible transition and to be complete the figure should include semi-circular arrows representing the possibility of remaining at each node. The MPE assigns equilibrium probabilities to each arrow conditional on being at a given node at a given τ. Many of these probabilities are zero in equilibrium.

The MPE is shown in Figure 8. All arrows there have probability $1/3$ representing the probability of a match between I and J, I and K, and J and K at each node (after each payoff relevant history). At $(0,0,0)$ there is thus probability $1/3$ that I buys from J leading to $(-1,1,0)$. From there, the probability is again $1/3$ that I buys from K leading to $(-2,1,1)$ and from this node, the probability that I can strike another deal with K leading to the ergodic state $(-3,1,2)$ is again $1/3$. The probability of this path is $(1/3)^3$. There are two other ways to get to $(-3,1,2)$, so the overall probability of ending up in the desired node is $1/9$. But the market can end up rather far from it also: If J and K are matched three times (an event that happens with probability $1/27$), the outcome will be an initial trade to $(0,-1,1)$ whereafter trading stops. This is because the gains from trade between J and K are exhausted after only one trade: All paths in Figure 8 include at most one horizontal arrow.

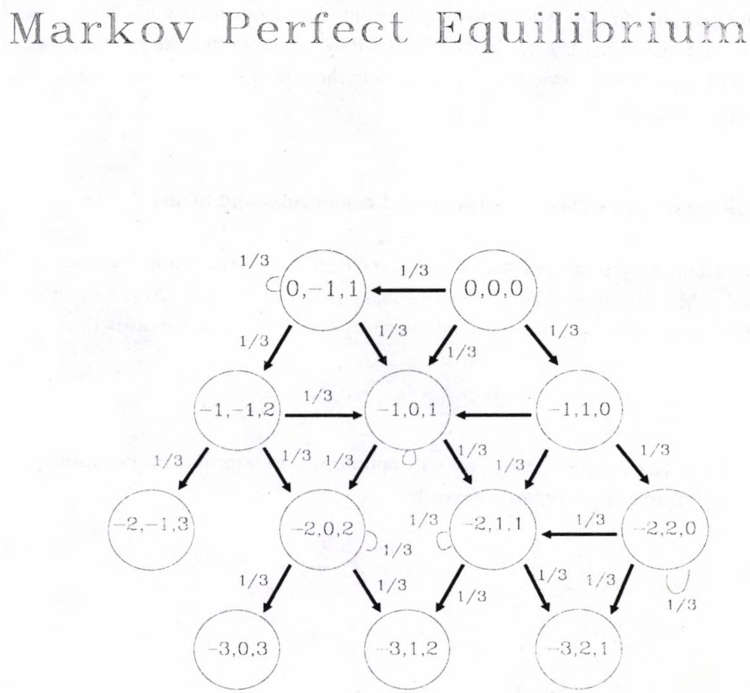
The expected payoff at $\tau = 0$ is $(0.6282, 0.3065, 0.5125)$ compared to $(1.0124, 0.1653, 1.5496)$ in the competitive equilibrium.

Figure 7: Markov Lattice



Horizontal arrows: Trade between *J* and *K*; Left-down/Up-Right Arrows: Trade between *I* and *K*; Right-Down/Up-Left Arrows: Trade between *I* and *J*.

Figure 8: MPE



Markov Perfect Equilibrium
0,0,0
0,−1,1
−1,−1,2
−1,0,1
−1,1,0
−2,−1,3
−2,0,2
−2,1,1
−2,2,0
−3,0,3
−3,1,2
−3,2,1
1/3

5. *n* Traders

We now turn to the $n \geq 2$ player case. A typical player is called I, $I \in N \equiv \{1,2,...,n\}$. As in the two preceding sections we first treat the axiomatic approaches in which the time dimension is suppressed and find the competitive equilibrium (the futures market equilibrium) and the bilateral core (the efficiency standard). We then move on to a full scale strategic bargaining model with a proof of existence of equilibrium in the n-person game and a characterization of the ergodic states.

5.1: Axiomatic approaches: Futures market equilibrium and bilateral core

As a benchmark, first assume that the players form a standard futures market which is assumed to be competitive, yielding a single market clearing price, q^*. Also assume that their different price expectations are common knowledge. Each player's payoff is then given by

$$G(\cdot;A_I,p_I) = (q^* - p_I)f_I - \frac{A_I}{2}f_I^2 . \tag{45}$$

If q^* is taken as given, the agent only optimizes with respect to the net position, f_I, and the (futures) market equilibrium is given by

$$f_I^* = \frac{q^* - p_I}{A_I} = \frac{1}{n}\frac{A^m}{A_I}\sum_{i=1}^{n}\frac{p_i - p_I}{A_i} \qquad \forall I \in N \tag{46}$$

$$q^* = \frac{\sum_{i=1}^{n}\frac{p_i}{A_i}}{\sum_{i=1}^{n}\frac{1}{A_i}} = \frac{A^m}{n}\sum_{i=1}^{n}\frac{p_i}{A_i} , \tag{47}$$

where A^m is the harmonic mean of the risk aversion coefficients ($1/A^m$ is the arithmetic mean risk tolerance). A_I/A^m is the individual risk aversion relative to the market risk aversion. The market clearing price is *fully revealing* in the sense that it is the weighted mean opinion of the spot price, the weights being the agent's risk tolerance relative to the sum of the risk tolerances. This is the "market's spot price expectation" and it fully reflects the market participants' opinions weighted by their willingness to bet on them.

Now assume that the market is one in which the participants enter bilateral contracts, but still disregard the time dimension. The payoff function then becomes

$$G(\cdot;A_I,p_I) = \sum_{\forall i \in N\setminus\{I\}} (q_{Ii} - p_I)f_{Ii} - \frac{A_I}{2}f_I^2$$

$$\text{where } f_I = \sum_{\forall i \in N\setminus\{I\}} f_{Ii} \qquad ,\forall I \in N. \tag{48}$$

To belong to the bilateral core, the quantities have to fulfil

$$p_J - p_I = A_I f_I - A_J f_J \qquad \forall I,J, \tag{49}$$

which is satisfied only by f_I^*, $\forall I$. Prices should satisfy individual rationality given these quantities, *i.e.* contracts should belong to

$$\left\{(q_{ij},f_{ij}) \mid f_{ij} > 0,\ \ q^* + \frac{A_j}{2}f_{ij} \geq q_{ij} \geq q^* - \frac{A_i}{2}f_{ij}\right\} \cup$$
$$\left\{(q_{ij},f_{ij}) \mid f_{ij} < 0,\ \ q^* + \frac{A_j}{2}f_{ij} \leq q_{ij} \leq q^* - \frac{A_i}{2}f_{ij}\right\} \cup \tag{50}$$
$$\left\{(q_{ij},f_{ij}) \mid f_{ij} = 0\right\} \qquad \forall i,j \in N,\ i \neq j.$$

The bilateral core thus consists of n contracts where the quantities and prices satisfy (46) and (50). Note that the market equilibrium (46-47) belongs to the core.

5.2 Strategic bargaining: the decentralized market game

The stochastic game Γ_n is defined by

- ☺ a set of players, $N \equiv \{1,...,n\}$
- ☺ a time horizon $T \geq 1$: $\tau = 1,...,T$
- ☺ a set of states $k(\tau) \in K(\tau)$
- ☺ a set of actions $s(\tau) \in S(\tau)$
- ☺ a transition function: $Pr[k(\tau+1)] = Pr[k(\tau+1) \mid k(\tau)\ ,\ s(\tau)]$
- ☺ a payoff function for each $I \in N$: $G(h_I(T);\ A_I,p_I)$

The state variable $k(\tau)$ consists of an n-vector of forward positions, $\boldsymbol{F}(h(\tau-1))$, a realization of the matching technology $M(\tau)$ and the time index, τ:

$$k(\tau) = (\boldsymbol{F}(h(\tau-1)),\ M[N](\tau),\ \tau)\ , \tag{51}$$

and $K(\tau)$ is the set of states that is feasible after any history leading to a given $\boldsymbol{F}(h(\tau-1))$.

The vector of forward positions for the n players at the beginning of period τ is defined in an obvious extension of the notation for the three player case:

$$\boldsymbol{F}(h(\tau-1)) = (F(h_1(\tau-1), \ldots, F(h_n(\tau-1))). \tag{52}$$

The *matching technology* is called M and a particular realization at time τ is denoted by $M(\tau)$. Let $<O,R>$ denote an ordered pair and let S be a set of integers with $\#S = 2\kappa$, where κ is a positive integer. Finally, let M_S denote the set of κ ordered pairs exhausting S, *i.e.* an element $\mu \in M_S$ is a collection $\{<O_1,R_1>, <O_2,R_2>, \ldots, <O_\kappa,R_\kappa>\}$ such that $O_i,R_i \in S$, $\forall i = 1, ., \kappa$, $O_i \neq O_j$ and $R_i \neq R_j$ $\forall i,j = 1, ., \kappa$ w. $i \neq j$ and $O_i \neq R_j$, $\forall i,j = 1, ., \kappa$.

If $n \equiv \#N$ is even, the matching technology simply maps from the set of players to a set of $n/2$ ordered pairs:

$$M: N \to M_N \quad . \tag{53}$$

In other words, an outcome of the matching technology chooses $n/2$ matches *and* in each match, m ($m \in \mu \in M_N$), the identity of the offerer, O_m, and of the receiver of the offer, R_m. If n is even, an outcome of M becomes $M(\tau) = (<O_1,R_1>, <O_2,R_2>, \ldots, <O_{n/2},R_{n/2}>)$ with O_i, $R_i \in N$, $i = 1, \ldots, n$, $O_i \neq O_j$, $R_i \neq R_j$, $\forall i,j = 1,.,n/2$, $i \neq j$, and $O_i \neq R_j$, $\forall i,j$. In the "even" case, the matching technology maps from the set of players to the set of possible two-permutations of this set. The probability of a particular outcome of the map is

$$Pr\Big[M(\tau) = \mu \in M_N\Big] = \frac{(n-2)!}{n!} = \frac{1}{n(n-1)} \tag{54}$$

since all outcomes are equally likely by assumption.

If $n \equiv \#N$ is odd, M maps from N to the space of $(n-1)/2$ ordered pairs plus the identity of the idle player:

$$M: N \to M_{N\setminus\{Idle\}} \times N \quad . \tag{55}$$

An outcome of the matching technology now determines the identity of the idle player and $(n-1)/2$ matches exhausting the $n-1$ non-idle players and within each match, m ($m \in \mu \in M_{N\setminus\{Idle\}}$), the identity of the offerer, O_m, and of the receiver of the offer, R_m. For n odd, an outcome of M becomes

$$M(\tau) = (\mu, Idle) = (\{<O_1,R_1>, <O_2,R_2>, \ldots, <O_{(n-1)/2},R_{(n-1)/2}>\}, Idle).$$

The probability of a particular outcome of the map, M, is

$$Pr\Big[M(\tau) = (\mu, Idle) \in M_{N\setminus\{Idle\}} \times N\Big] = \frac{(n-3)!}{n!} = \frac{1}{n(n-1)(n-2)} \tag{56}$$

since for each player there is probability $1/n$ of being idle at τ and for the n-1 remaining players, there are $(n\text{-}1)!/(n\text{-}3)!$ two-permutations.

Given an outcome of M, in each match $m \in \mu$, the offerer, O, has action space $S_O = \{c_{m,.}\}$, that is, she can propose to buy or sell a contract at a price of her choice, or she can choose not to propose a deal, so $S_O = \{(q_{.,.},1), (q_{.,.},-1), (q_{.,.},0): q_{...} \in \mathbb{R}_+\}$. The receiver in m can accept (a) or reject (r), so $S_R = \{a,r\}$. A possible idle player has $S_{Idle} = \varnothing$. The strategy space of a player thus depends on whether the outcome of the macthing technology makes her offerer, receiver or idle. In any state $k(\tau)$, action spaces for the market in that state are

$$\begin{aligned} S &= (S_O \times S_R)^{\frac{n}{2}} && \text{if } n \text{ is even} \\ S &= (S_O \times S_R)^{\frac{n-1}{2}} \times \varnothing && \text{if } n \text{ is odd.} \end{aligned} \tag{57}$$

Given today's state, $k(\tau)$, and today's actions, $s(\tau)$, a new profile of net positions appears: $\boldsymbol{F}(h(\tau))$. Given this, the probability of a state

$$k(\tau+1) = (\boldsymbol{F}(h(\tau)),\ M(\tau+1) = \mu,\ \tau+1) \tag{58}$$

will be $(n\text{-}2)!/n!$ if n is even and $(n\text{-}3)!/n!$ if n is odd. These are the transition probabilities. When the players have chosen the trades leading to a new $\boldsymbol{F}$ (the first element of the state variable at $\tau+1$, then nature chooses a new outcome of the matching technology, $M(\tau+1)$ (the second element of the state variable), and one period has passed so $t = \tau+1$ (the third entry of the state variable). The sequence of events in each period τ is as follows:

0. The realization of $k(\tau) = (\boldsymbol{F}(h(\tau\text{-}1)), M(\tau), \tau)$ becomes common knowledge (c.k.).
1. The Offerers simultaneously propose a contract (*not* c.k.).
2. The Receivers simultaneously accept or reject (*not* c.k.).
3. The Idle player (simultaneously with 1 and 2) does nothing (c.k.).
4. The actions in 1, 2 and 3 lead to $\boldsymbol{F}(h(\tau))$.
5. Nature chooses $M(\tau+1)$ and $k(\tau+1) = (\boldsymbol{F}(h(\tau)), M(\tau+1), \tau+1)$ becomes c.k.

Lemma 4 still applies: the payoff relevant history is summarized by the forward positions at the beginning of the period, $\boldsymbol{F}(h(\tau\text{-}1))$. Indeed, the proof is independent of the number of players. We can also extend the essential proof of existence for the three person game, Γ_3, to the *n*-person game, Γ_n:

Proposition 8: There exists a Markov perfect equilibrium for the game Γ_n.

Proof: If the set of actions S were finite, the proof would be a trivial adaption to Γ_n of the more general Theorem 13.2 in Fudenberg and Tirole (1991) (p. 515). However there are no restrictions on the price an offerer can propose apart from positivity, so S_O is infinite. But there are restrictions on the quantities that the offerer can propose, *viz.* to buy one unit, to sell one unit or not to do anything, and thus $S_O = \{c_{m,\cdot} \mid q_{m,\cdot} \in \mathbb{R}_+ \wedge f_{m,\cdot} \in \{-1,0,1\}\}$. This means that there is a finite number of feasible states that can be reached during the game and subgame perfect equlibrium requires that the strategy be optimal at any state, be they reached or not.

At *T*, in any feasible state $k(T)$, O_m will propose a contract to R_m that *either* is equal to R_m's reservation price if there are gains from trade to be made and which R_m then will accept *or* is IR to O_m if not, and then R_m will reject. Reservation prices are functions of the state via $\boldsymbol{F}(h(T\text{-}1))$.

At *T-1*, the subgame starting there can be solved with knowledge of what is going to happen at *T* in each of the states possible at *T* and of the transition function $Pr(k(T) \mid k(T\text{-}1), s(T\text{-}1))$. Continuing the backward recursion, an MPE appears.■

We also get the *n*-person equivalent of Propositions 4 - 6 establishing existence and (generic) uniqueness of the absorbing states for the three person game:

Proposition 9: Trading stops if the game reaches a state in which

$$F(h_i(\tau\text{-}1)) = F_i^* + \delta_i\,, \qquad \forall i \in N, \tag{59}$$

where $\forall i,j$ the δ_i's satisfy

$$-\frac{1}{2} \leq \frac{A_j}{A_i + A_j}\delta_j - \frac{A_i}{A_i + A_j}\delta_i \leq \frac{1}{2} \tag{60}$$

and

$$\sum_{\forall i \in N} \delta_i = 0. \tag{61}$$

Proof: If $RB_i(\tau) \leq RS_j$, $\forall i,j \in N$, $i \neq j$, there are no further gains from trade. Solving this system of inequalities yields the desired result.■

Remark: The δ_i's are again to be understood as the real numbers that when added to the respective efficient quantities of the bilateral core yield integers. Weighed by the relative risk aversion the difference between the δ's of any two players should numerically be less than 1/2.

Proposition 10: The only ergodic states $k(\tau)$ are those that satisfy proposition 9.

Proof: In any ergodic state we need $RB_i(\tau) \leq RS_j(\tau)$ and $RB_j(\tau) \leq RS_i(\tau)$ for all i and j. This leads to the following set of inequalities:

$$p_i - p_j - \frac{A_i + A_j}{2} \leq A_j F_j - A_i F_i \leq p_i - p_j + \frac{A_i + A_j}{2} \tag{62}$$

where $\boldsymbol{F} = \{F_1, F_2, ..., F_i, ..., F_n\} \equiv \boldsymbol{F}(h(\tau\text{-}1))$ is the profile of net positions at $\tau\text{-}1$. Let $F_i = F_i^* + \delta_i$, $\forall\ i \in N$ where the only restriction on the δ's is that when added to the respective efficient position, the result must be an integer. From (49) we then get

$$A_j F_j - A_i F_i = A_j F_j^* - A_i F_i^* + A_j \delta_j - A_i \delta_i = p_i - p_j + A_j \delta_j - A_i \delta_i$$

which combined with (62) gives the desired result.■

Proposition 11: The positions in the ergodic states are (generically) unique.

Proof: Let $\boldsymbol{F}(h(\tau\text{-}1)) = (F_1, ..., F_n)$ be the forward positions of an ergodic state. $\boldsymbol{F}(h(\tau))$ thus satisfies (62) for all $i,j \in N$. Let $\bar{x}$ be an n-vector of integers such that $e'\bar{x} = 0$ where $e \equiv (1, 1, ..., 1)$ is an n-vector too. Let $\bar{x} \neq 0$. There must then be at least one element of $\bar{x}$ that is positive and at least one that is negative. Pick a positive element, x_i, and a negative, x_j, with the indices, i and j corresponding to the "name" of the agent. We have $x_i \geq 1$ and $x_j \leq -1$, and now have to show that $\boldsymbol{F}(h(\tau\text{-}1))$ cannot be an ergodic state. From this state we have $A_j(F_j + x_j) - A_i(F_i + x_i) = A_j F_j + A_i F_i + (A_j x_j - A_i x_i)$. We know that $A_j x_j - A_i x_i \leq -(A_j + A_i)$, so if we have strict inequality of (62), then $A_j(F_j + x_j) - A_i(F_i + x_i) < p_i - p_j - \frac{A_i + A_j}{2}$, so $\boldsymbol{F}(h(\tau\text{-}1)) + \bar{x}$ is not ergodic. If we have equality of (62) by coincidence and if $(x_i, x_j) = (1, -1)$, then $A_j(F_j + x_j) - A_i(F_i + x_i) = p_i - p_j - \frac{A_i + A_j}{2}$, so the state may still be ergodic.■

A Markov perfect equilibrium of Γ_n can be interpreted along the lines of the two and three person markets, Γ_2 and Γ_3, which indeed are special cases. But for $n > 3$ a new feature appears: the possibility of *multiple* Markov perfect equilibria stemming from the possibility of multiple Nash equilibria at each state: when there is more than one simultaneous match in a given period, optimality of a decision within a given match may hinge on the outcome of other matches at that time. The outcome of the matching technology is common knowledge, but the actions are not, and since the play across matches is simultaneous, problems of the following sort may arise: The optimality of any pairs' decision hinges on what the entire trade vector looks like. A trade vector is an n-vector of possible $\{-1, 0, 1\}$ where the sum of the elements is zero. The *entire* vector determines the state in the next period, which in turn determines expected continuation payoffs. So it may be optimal for one pair of agents to agree on a contract if and only if another pair agrees on a contract, and *vice versa*.

Whether multiple MPE occurs is a matter of choice of parameters, but in general the set-up allows for that.[10] This introduces the possibility of coordination failure (failure to coordinate on the same Nash equilibrium) on top of the other inefficiencies of the market (stemming from indivisibilities and from not being able to control the matching process). The next section looks at whether one can expect convergence of the process to the efficient outcome as one parameter, the time horizon, T goes to infinity.

6. Convergence and Decentralized *vs.* Centralized Trade

In this section we first (6.1) discuss convergence of the Markov perfect equilibrium of Γ_n to the ergodic states, which is the closest the market can come to efficiency. Then (6.2) we compare Γ_n to other models of decentralized trade and conclude (6.3) with a comment on forward markets compared to futures markets.

6.1 Inefficiency and Convergence to Efficiency

Compared to a competitive standard, the outcome of Γ_n $(n > 2)$ is inefficient: there is always a positive probability that the game ends with a set of forward positions that does not belong to the ergodic states. The heuristic proof of this point is simple: re-order the players from *1* through n according to the value of p_i/A_i, so p_1/A_1 is lowest and p_n/A_n is highest. The event that *1* and 2, *3* and *4*, ..., n-*1* and n (or if n is odd: n-*2* and n-*1*) are matched in every period

[10] It would seem that for $n \geq 4$, the game generically possesses multiple equilibria, so that the parameter space in terms of $\{A_i, p_i\}$ and T for which this happens is topologically large. How to show this formally is not clear to me.

Proof: If $RB_i(\tau) \leq RS_j$, $\forall i,j \in N,\ i \neq j$, there are no further gains from trade. Solving this system of inequalities yields the desired result.■

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where $F = \{F_1, F_2, ..., F_i, ..., F_n\} \equiv F(h(\tau\text{-}1))$ is the profile of net positions at τ-1. Let $F_i = F_i^* + \delta_i,\ \forall\ i \in N$ where the only restriction on the δ's is that when added to the respective efficient position, the result must be an integer. From (49) we then get

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[10] It would seem that for $n \geq 4$, the game generically possesses multiple equilibria, so that the parameter space in terms of $\{A_i, p_i\}$ and T for which this happens is topologically large. How to show this formally is not clear to me.

$\tau = 1, . , T$ has a positive probability (that is decreasing in T, however). Obviously to be efficient, the long positions at T should be concentrated at high values of i ($i \in \{1,.,n\}$) whereas short positions should be concentrated at low values of i. However, this is impossible given the outlined event.

Generally, there are several sequences of matchings in $\tau = 1,.,T$ that can lead to efficient outcomes (as in the three trader example of Section 4.3) but also several sequences for which the efficient positions can not possibly be reached. What the probabilities are and how close the market can get to the efficient positions depends on how large these positions are and how they are distributed, *i.e.* on $\{p_i, A_i\}$ ($\forall i \in N$), on the one hand and on the matching probabilities (*i.e.* on n) and the time horizon, T, on the other. Indeed, one would expect the following **conjecture** to hold true:

> *In a Markov Perfect Equilibrium of Γ_n the efficient positions are reached almost surely as $T \rightarrow \infty$.*

A very rough and heuristic proof of this goes as follows:[11] we have already shown that there is a generically unique set of ergodic states for a given T.[12] If the process converges to anything, it must be to the ergodic states. To show convergence implies to show that the process is a contraction - but it must be, since any trade reduces the aggregate gains from trade left for the future.

For any given T, the MPE of Γ_n describes a Markov chain. The reason why standard techniques of showing convergence of Markov chains do not work here is that as T increases, the transition probabilities among existing states may change *and* the size of the state space increase: there are a number of new feasible states that were not reachable for a smaller T. As $T \rightarrow \infty$, the state space becomes infinity too (and actually this happens at a faster rate).

At the risk of belabouring the last point, the problem is that even though for any (finite) T the state space is finite, if a period $T+1$ is introduced, this increases the number of feasible states by more than one to one. To see this, consider the example of Section 4.3 and Figure 8: if a period $T=4$ is introduced, twenty-four new feasible profiles of forward positions pop up. For the three person case, the number of feasible nodes as a function of T is:

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[11] Another argument is that Gale (1987) gets convergence to the competitive equilibrium in a somewhat similar set-up. The difference is that Gale has an infinite time horizon and that he then studies convergence of equilibrium as a function of parameters like the time preferences (as the traders get more patient, the equilibrium converges to the competitive equilibrium).

[12] Ignoring that all states that are reached at T are ergodic in the sense that the game ends, so the probability of getting out of such a state is trivially zero.

$$1 + 6\sum_{i=1}^{T} i \tag{63}$$

which clearly increases six times faster than *T*. In the more general *n*-person game, each node that is reachable at *T-1* has number of *neighbours*:

$$\sum_{i=1}^{n/2} \frac{n!}{(n-2i)!}\left(\frac{1}{i!}\right)^2 \tag{64}$$

for *n* even and

$$\sum_{i=1}^{\frac{n-1}{2}} \frac{n!}{(n-2i)!}\left(\frac{1}{i!}\right)^2 \tag{65}$$

for *n* odd. For example, if there are twenty players ($n = 20$) each node reachable at *T-1* has 377,379,368 neighbours! For a larger number of players, the state space increases faster than for the three player case, thus emphasizing the point that the number of states grows rapidly as $T \to \infty$.

The conjecture is so much more remarkable considering that it requires the equilibrium to be a contraction in an expanding state space. The reasoning behind this is that the agents will not use the extra states (given that the ergodic states were already reachable), but will rather use the extra time available to get to more attractive nodes (forward positions) among the already existing ones: returning to the example and Figure 8, in the three player case $\boldsymbol{F}(h(3)) = (0,-1,1)$ could be an equilibrium outcome if *J* and *K* are matched for three consecutive periods. This event has probability $(1/3)^3 = 1/27$ *ex ante*. If the transition probabilities that are indicated in Figure 8 remain the same as *T* increases, then the *ex ante* probability of this deadlock decreases rapidly (exponentially) with *T*. The conjecture thus implies that when *T* increases, there will be few or no new nodes in Figure 8 and some nodes may even drop out because the probability of getting to a more attractive state increases. In other words, as the state space ramifies, Markov perfect equilibria effectively prune away new branches.

6.2 Models of decentralized trade

The model presented in Sections 2 through 6 is one of decentralized trade. It obviously has links to other models of decentralized trade. This section seeks to make the connection and to show the differences.

Gale (1988) makes an important distinction between models of *ex ante* pricing where all prices are posted and known by all relevant parties before the agents get together to trade, and models of *ex post* pricing where the agents get together before prices are quoted. Our model belongs to the *ex post* category in that the players are matched before contracts are proposed.

In the framework of *ex ante* pricing, three papers by Ostroy and Starr (1974) and Starr (1976; 1986) treat points similar to ours: in a general equilibrium setting, the question addressed is whether a competitive equilibrium can be implemented by a decentralized trading process. The equilibrium prices (and quantities) are determined by a Walrasian auctioneer, but the agents are not allowed to hand the net trade vector over to the auctioneer so there is no centralized clearing. Instead, the agents have to sort the equilibrium out themselves in pairwise meetings. In each round of trading, every trader meets every other trader in an arbitrary order, so the only uncertainty is with respect to the order of matches, not with respect to whether they get matched or not. Within this set-up, Ostroy and Starr show that in a barter economy it is in general *not* possible to find a decentralized procedure that achieves the competitive equilibrium, but in a monetary economy such a decentralized mechanism exists. Starr (1976) then shows that if barter trading can implement the competitive equilibrium, then there is a monetary trading procedure that does this a good deal faster. In fact, the non-monetary procedure may take forever to converge. Finally, Starr (1986) allows for 'short sales' and discusses convergence in different credit economies (commodity credit, trade credit and bank credit). None of these institutional set-ups do as well as the monetary economy in terms of convergence and even existence of a convergent procedure.

Our model could be seen as a monetary economy with one good (a contract) and with *ex ante* pricing and indeed with the Ostroy and Starr matching technology and trading procedure, the competitive equilibrium could quickly and certainly be obtained in decentralized trading. Another way of re-interpreting our model in terms of Ostroy and Starr is that potentially there are $\sum_{i=1}^{n}(n-i) = n^2 - \sum_{i=1}^{n} i$ different goods in the forward market: a contract between I and J is different from a contract between I and K and both are different from a contract between J and K and so forth. This is especially important for the clearing procedure in the 15-Day market, where having a long contract with one trader and a short with another does not mean that these trades net out: A trader may still have to honour both contracts and may thus have to bother about taking and arranging delivery. This is why most traders in the 15-Day market close their positions in book-outs and in daisy chains and it is also why the assumption that the forward market clears at maturity is not realistic for this market. Maturity (in the spot market) is not a point in time but rather a whole month and traders want to realize gains and losses *before* maturity in order not to deal with problems of delivery. This renders the market even more inefficient seen from the traders' point of view. The point that is made here, is that not only is the trading on the forward market decentralized, but so is the 'clearing at maturity' which forces the traders to *clear before maturity!* Clearly a centralization of this clearing mechanism along the lines of a clearing house in a futures market is both feasible and more efficient. Note that the market is only one step short of this, since one major producer organizes the liftings (actual deliveries) for the entire market.

In the framework of *ex post* pricing, there is a huge and growing literature on strategic bargaining applied to markets. An unsurpassed treatment is Osborne and Rubinstein (1990). That our model is inspired by, and in line with, this literature, will be clear from the following quote:

> "Bargaining theory provides a natural framework within which to study price formation in markets where transactions are made in a decentralized manner via interactions between pairs of agents rather than being organized centrally through the use of a formal trading institution like an auctioneer. One might describe the aim of investigations in this area as that of providing "mini-micro" foundations for the microeconomic analysis of markets and, in particular, of determining the range of validity of the Walrasian paradigm. Such a program represents something of a challenge for game theorists in that its success will presumably generate new solution concepts for market situations intermediate between those developed for bilateral bargaining and the notion of Walrasian equilibrium." (Binmore, Osborne and Rubinstein (1992)).

The original paper in this strain of literature is Rubinstein and Wolinsky (1985) in which *steady states* of a market are investigated. The market is partitioned into buyers and sellers and they are matched in a stochastic process that renews the pairings every period and that is not in the control of the agents. The matching technology of our model can be seen as a special case of this. In the Rubinstein/Wolinsky framework, every seller has one unit for sale and each buyer wants one unit. Once a match has concluded a deal, the pair leaves the market and is then replaced with a new pair, thus keeping stocks of agents constant. The time horizon is infinite, but traders have impatient von Neumann-Morgenstern utility functions and thus an incentive to conclude a deal rather sooner than later. An important conclusion that arises from this model is that the market equilibrium of the decentralized market need not be competitive. That is, a model that (contrary to the competitive equilibrium) explains the *formation* of prices in equilibrium does not necessarily support a competitive outcome. This result has triggered a number of studies of when and why convergence to the competitive equilibrium arises (notably Gale (1986a,b; 1987) and McLennan and Sonnenschein (1991)).

Contrary to these studies an important feature of our model is the finite time horizon, imposed since the forward contracts eventually mature and since this is clearly perceived by the traders. On the other hand, time preferences are not important (traders are infinitely patient), so the incentive to conclude deals is that time runs out. A third difference is that our equilibrium is inherently dynamic,[13] whereas most other models of bargaining and markets concentrate on steady states.

6.3 Forward *vs.* Futures Markets - and the Future

The model that was presented in this paper describes a decentralized, speculative forward market and compares the market equilibrium with that of a centralized, competitive market that can be thought of as a futures market. It was shown that the decentralized market is inferior to the centralized market in that the random matching makes it difficult to coordinate on an efficient outcome: There is always a positive probability of not reaching the efficient outcome but this probability drops as the traders get more time to complete their affairs.

Matching is not entirely random in reality, but it is not entirely under the control of the traders either. Endogenized matching would lead to more complicated transition probabilities since matching behaviour should be explained by equilibrium strategies (*cf.* Herreiner (1993)). It is not clear how this would affect Markov perfect equilibrium, but a conjecture is that, given the assumption of common knowledge of different priors, there would be faster convergence to the efficient outcome, since traders with very different beliefs have a common interest in getting together.

The informational requirements that underlie both the model of decentralized trade and that of centralized trade are very severe: we have a game of complete information, so the different spot price expectations and the different risk aversions are common knowledge. In reality, agents face incomplete information. This leads to problems for the agents such as identifying who the optimists are (*cf.* Harstad and Phlips (1993)) and, without further specifications of the agents' knowledge, almost certainly to results of (generic) non-existence of Markov perfect equilibrium or to a situation in which any outcome can be rationalized as the MPE for appropriate choices of beliefs. Subgame rationalizability may be all one can hope for. The way to model this may be to let strategies be part of a controlled process in which the traders try to learn the spot price expectations of the other traders and to make money at the same time. This is a standard learning problem, but with a finite time horizon.

Another informational intricacy stems from the once-and-for-all nature of the spot price expectations. New information is likely to appear during trading so agents change their mind while trading, revaluing the book value of completed contracts and of future strategies. This could be modeled within the framework of common knowledge of different priors by exposing the whole vector of price expectations to (*e.g.* additive or multiplicative) random shocks, thus generating the erratic behaviour observed in Figures 1 and 2.

While these modifications (endogenized matching, incomplete information and

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[13] This is also the case in Gale (1987) and in Binmore and Herrero (1988) but they work with an infinite time horizon.

continuous information on spot prices) are interesting in their own right, it should be clear that these problems by no means are assuaged by decentralized trading. It is easier to see how an equilibrium with incomplete, imperfect information and a continuous flow of data can lead to an efficient outcome in a centralized futures market than in a decentralized forward market - and in a futures market, matching is not an issue. It therefore remains a paradox that the participants in the 15-Day market accept such an inferior institution.

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