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When You Don't Know Who The Optimists Are**

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# Futures Market Contracting

## When You Don't Know Who the Optimists Are<sup>1</sup>

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### Abstract

If traders on futures markets for exhaustible resources rationally update from consistent priors, a prediction of a zero aggregate volume of speculative activity is both inescapable and readily invalidated. We have built models of speculative futures trading based upon inconsistent priors, analyzing games of inconsistent incomplete information. These models have assumed that the inconsistent priors are themselves common knowledge. In this paper, we explore the game-theoretic implications of treating doubly inconsistent incomplete information, in that inconsistent priors are private information, and traders attach inconsistent assessments to the probability that a trader will be an optimist.

The result is not arbitrary: the logic of a separating equilibrium can be specified via backwards induction. It is unlikely that subgame-perfect equilibria will exhibit pooling. The volume of speculative trading is reduced by informational constraints, but a sense is specified in which no ex ante agreed-upon Pareto improvements over separating equilibrium behavior can satisfy the information constraints for take-it-or-leave-it contracts.

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## 1. Introduction

The standard literature on futures markets uses the assumption that all agents active in these markets have common priors. This assumption is a prerequisite for the construction of rational expectations models implying that the futures price is an unbiased predictor of the spot price at maturity. The present paper instead uses the more realistic assumption that agents have different priors: their beliefs are “inconsistent” in this sense and reflect differences in opinion, not in information. For concreteness, each agent is one of two *types*: either an “optimist” or a “pessimist.” The optimist expects the spot price at maturity to be high; the pessimist expects it to be low. As these expectations are due to differences in opinion, in our model, an optimist would not adjust his estimate of the spot price upon learning that another agent is a pessimist; his estimate has already taken into account the possibility that other agents might have expectations he views as incorrect.

In addition, we suppose that agents know only their own type and “don’t know who the optimists are.” They assign subjective probabilities to other agents’ types, and these probabilities are also inconsistent. There is thus a double inconsistency, in that agents have inconsistent beliefs about inconsistent beliefs of their trading partners. Our purpose is to show that a game-theoretic treatment of such a situation, which is described by Selten [1982] as one of inconsistent incomplete information, is possible and provides interesting insights into the working of futures markets.

We have studied futures markets for natural resources in circumstances featuring extractors with market power in the cash market (Phlips and Harstad [1991], Harstad and Phlips [1990], Phlips and Harstad [1992], listed in the order they were written). Those papers have assumed inconsistent priors, as set out in the first paragraph above; this has allowed us to provide explanations of some stylized facts about futures markets which are inconsistent with mainstream futures models, principally a nonzero volume of

rational trade. A key limitation of the analysis in those papers is the assumption that, while priors are inconsistent, they are common knowledge. Here we explore the complications that arise under the more reasonable assumption that the fact of inconsistent priors is common knowledge, but the priors themselves are private information. The complications are considerable, and to focus on this issue, we have sharply curtailed unrelated complications. When simplifying assumptions allowing us to sidestep other issues are introduced, we will refer the reader to one of the other papers in which the issue sidestepped here is addressed.

The model in this paper is basically the two-stage noncooperative game studied in Philips and Harstad [1991], but of course without the assumption that the players' (inconsistent) beliefs about the spot price at maturity are common knowledge. The final stage of the game is an extraction subgame played by two risk-averse extractors, producers  $\mathcal{A}$  and  $\mathcal{B}$ , who use Cournot strategies. This subgame is defined over two periods (the second period being the maturity date). The first stage is a futures market in which the two producers trade (in period 1) with a representative risk-neutral speculator (player  $\mathcal{S}$ ) in an "open cry" auction. The overall game is modeled in such a way that futures behavior can be separated from extraction subgame behavior. On the one hand, the double inconsistency of beliefs has no impact on the extraction subgame, so that the trading partner of a producer need not know this producer's beliefs in order to predict what the impact of a futures contract on the extraction game will be. On the other hand, the speculator can (and must) evaluate the impact of the extraction game on the expected profitability of his own futures position.

A first insight is that inconsistent incomplete information may cause players to miss opportunities for futures contracting. Subgame-perfect equilibria create speculative trading opportunities when the inconsistent beliefs are common knowledge



(see Phelps and Harstad [1991]). It is intuitively clear that, when players are uncertain about the other players' beliefs, some of these contracts may fail to materialize because a player incorrectly evaluates his trading partner's type. This is generally true when two trading partners are of the opposite types. It is also true when they are of the same type. Indeed, trade is possible among them under complete as well as incomplete information about the partner's type. Under incomplete information, however, the potential buyer may announce a futures price that is not acceptable to the potential seller when both are optimists, and the potential seller may announce an unacceptable price when both are pessimists, in situations where complete information would have led to profitable trade.

Second, subgame-perfect equilibria were found to reach Pareto-efficient outcomes when the players' types are common knowledge. Under the assumptions made here, the desired equilibrium outcomes are also located on contract curves. However, these curves are based on subjective probabilities. Consequently, the announced (and accepted) futures positions may be larger or smaller than the "objectively" efficient ones. In a world characterized by uncertainty, however, an allocation mechanism is appropriately judged by its efficiency relative to the information available at the time the mechanism is put in place. In that sense, the game analyzed here reaches an ex ante efficient outcome in subgame-perfect equilibrium. Nonetheless, ex post, objective inefficiencies occur.

We emphasize that the behavior we analyze below is fully rational, and indeed, assumes players correctly handle extremely complex calculations. It is simply behavior that is not conditioned on adjusting one's own beliefs for the beliefs of other traders.

## 2. Rules of the Game

A futures market for the natural resource is opened at the beginning of period 1. The game unfolds as follows:

Step 1: Nature determines whether a player is optimistic or pessimistic, and privately informs each player of his type. Nature also determines a “commitment order,” that is, a permutation of the order  $(\mathcal{A}, \mathcal{B}, \mathcal{F})$ , which is common knowledge.

Step 2: Each player simultaneously announces one contract offer, either a sale or a purchase offer. Each offer consists of a (price, quantity) pair, and is an all-or-nothing offer (neither price or quantity can be discussed). Once announced, the offers are irrevocable.

Step 3: The player designated *first* in the commitment order irrevocably accepts or rejects each contract offered by another player.

Step 4: The player designated *second* in the commitment order irrevocably accepts or rejects each contract offered by another player and not yet accepted.

Step 5: The player designated *third* in the commitment order irrevocably accepts or rejects each contract offered by another player and not yet accepted. The futures market then *closes*.

Step 6: The extraction subgame occurs, that is, producers  $\mathcal{A}$  and  $\mathcal{B}$  simultaneously determine what fraction of their stock of the resource to supply before the maturity date of the futures contracts. Technically, this ends the game.

Step 7: The market demand level is revealed and determines the spot price at maturity, given the extraction decisions in step 6.

Several aspects of this extensive form are highly simplified. Contracts are take-it-or-leave-it offers; Harstad and Philips [1990] explore the changes in results if an agent can partially accept terms offered, by accepting a smaller quantity at the offered price. A player is limited to announcing a sale or a purchase contract offer, but not both; on this issue, see Philips and Harstad [1991]. There are only two periods, so a trader does not have the opportunity to close out futures positions prior to maturity; Philips and Harstad [1992] focuses on this issue. For our purposes here, we regard realism of the model as comparatively inconsequential.<sup>2</sup>

### 3. Inconsistent Incomplete Information

There are two sources of uncertainty. The first is about the level of aggregate demand: each player has an expectation  $\hat{p}_i$  ( $i = \mathcal{A}, \mathcal{B}, \mathcal{F}$ ). These expectations are

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<sup>2</sup>It is worth commenting on a specified order in which players commit themselves to contracts. This may not be too unrealistic a stylization if one envisions a futures market in which, initially, potential buyers indicate that they may buy, but desire a low price, while potential sellers do the reverse. For some period of time, each side would maintain that the terms he announced are those which should be consummated, presumably by claiming that foregoing trade would be preferable to accepting his potential trading partner's terms. Such claims would be made whether true or not, and are calculably unlikely to be true, so one envisions these claims as not being credible. In the final few moments before the closing bell, the chance that one may not be heard accepting a contract if it is rejected now may suddenly lend credibility to whomever happens to have his mouth open and his potential trading partner's attention at that moment. We model this commitment as random. True, the current paper takes the unrealistic tack of assuming that traders know from the beginning who will randomly be put in this position; this is simply to set aside a significant complication in order to deal with the issue of private information on prior beliefs. We have explored the nature of the changes created when the commitment order does not become known until the moment of commitment, by invoking this difference when moving from our [1991] paper to our [1990] paper, which may be consulted. Alternatively, the reader can trust us that this paper is complicated enough without this uncertainty being introduced.

inconsistent, since drawn from different distributions. For simplicity, these expectations are of two types: either  $\hat{p}_i = \hat{p}^H$  or  $\hat{p}_i = \hat{p}^L < \hat{p}^H$ . In words, a player is either an optimist or a pessimist. All optimists have the same mean expectation  $\hat{p}^H$ . All pessimists have the same mean expectation  $\hat{p}^L$ . The values  $\hat{p}^H$  and  $\hat{p}^L$  are common knowledge.<sup>3</sup> An economic interpretation of  $\hat{p}$  is delayed until analysis of the extraction subgame in Section 4.

Second, each player knows his own type but does not know the types of the other players. They know that Nature determines whether a player is an optimist or a pessimist with probabilities<sup>4</sup>  $\lambda$  and  $(1 - \lambda)$ . However, each player has his own personal evaluation of  $\lambda, \lambda_i, i = \mathcal{A}, \mathcal{B}, \mathcal{J}$ .

If the players knew the true  $\lambda$ , their conditional subjective probabilities that any other player is an optimist or a pessimist would be equal to  $\lambda$  or  $1 - \lambda$ , respectively. To illustrate, consider the probability matrix of type combinations given in Table 1. An optimistic  $\mathcal{J}$  is represented as  $\mathcal{J}^H$ , and a pessimistic  $\mathcal{J}$  as  $\mathcal{J}^L$ , with types of  $\mathcal{A}$  and  $\mathcal{B}$  analogously represented. The sum of its 8 elements is 1. This matrix is not correlated if  $\lambda = 0.5$ . Otherwise, it is correlated. For example, with  $\lambda = 0.6$ , the values in Table 2 result.

<sup>3</sup>The model must continue to draw back until some aspects of the rules are common knowledge; Harsanyi [1968] defends the practice used here. Of course, a player's beliefs could be viewed as still unknown in the presence of information about whether or not he was an optimist: there could be two distinct, nondegenerate distributions of  $\hat{p}_i$ , one for optimists and one for pessimists. Presumably the added complexity would still exhibit many of the qualitative features of the model presented here.

<sup>4</sup>The alternative assumption that Nature selects types out of a  $3 \times 2$  probability matrix, as in Selten [1982], would imply at least two more parameters.

Table 1. Probability Matrix of Type Combinations.

	$\mathcal{A}^H$	$\mathcal{A}^L$	$\mathcal{B}^H$	$\mathcal{B}^L$
$\mathcal{A}^H$	$\lambda^3$	$\lambda^2(1-\lambda)$	$\lambda^2(1-\lambda)$	$\lambda(1-\lambda)^2$
$\mathcal{A}^L$	$\lambda^2(1-\lambda)$	$\lambda(1-\lambda)^2$	$\lambda(1-\lambda)^2$	$(1-\lambda)^3$

Table 2. Probability Matrix of Type Combinations with  $\lambda = 0.6$ .

	$\mathcal{A}^H$	$\mathcal{A}^L$	$\mathcal{B}^H$	$\mathcal{B}^L$
$\mathcal{A}^H$	0.216	0.144	0.144	0.096
$\mathcal{A}^L$	0.144	0.096	0.096	0.064

However, the subjective probability, for  $\mathcal{Y}$  with  $\lambda = \lambda_s = 0.6$ , that a producer (say  $\mathcal{A}$ ) is an optimist is 0.6, whether  $\mathcal{Y}$  is an optimist ( $0.36 \div 0.6 = 0.6$ ) or a pessimist ( $0.24 \div 0.4 = 0.6$ ).

*Consistent* incomplete information is defined by the condition that all players know the true  $\lambda$  and therefore have the same subjective probabilities. *Inconsistent* incomplete information allows for the fact that each player may have a different theory about how Nature selects types and may, therefore, evaluate  $\lambda$  differently. Then we may have  $\lambda_s \neq \lambda_a \neq \lambda_b$ . It seems plausible that a speculator may have a different view about how Nature selects types than a producer and that symmetric duopolists have similar views. Notice, however, that the values  $\lambda_s, \lambda_a, \lambda_b$  and the corresponding

probability matrices are common knowledge in the inconsistent as well as in the consistent case.

#### 4. The Extraction Subgame

We start by modeling the last step of the game, namely step 6. The two producers have exogenous initial stocks  $s_{a0}$  and  $s_{b0}$ , which are common knowledge. In period 1, each extracts and sells  $q_{i1}$  units ( $i = \mathcal{A}, \mathcal{B}$ ). In period 2, that is, at maturity, they extract and sell the remainder

$$q_{i2} = s_{i0} - q_{i1}. \quad (1)$$

In both periods, the instantaneous inverse demand function is

$$p_t(q_{at} + q_{bt}) = \frac{\alpha}{\beta} - \frac{(q_{at} + q_{bt})}{\beta}, \quad t = 1, 2. \quad (2)$$

The intercept  $\alpha > 0$  is unknown. The slope coefficient  $\beta > 0$  is common knowledge; to simplify notation, we assume that physical units of the resource can be normalized so that  $\beta = 1$ .

Since we are interested in the futures market's impact on the extraction policy, we set both the cost of extraction and the interest rate to zero. Risk aversion implies the certainty equivalent payoffs

$$W_a = E_a[\pi_a] - \frac{M_a}{2}(s_{a0} - f_{ab} - f_{as})^2, \quad (3.1)$$

$$W_b = E_b[\pi_b] - \frac{M_b}{2}(s_{b0} + f_{ab} - f_{bs})^2, \quad (3.2)$$

where

$$\pi_a = p_2(q_{a2} + q_{b2})(q_{a2} - f_{ab} - f_{as}) + p_1(q_{a1} + q_{b1})q_{a1} + p_{ab}f_{ab} + p_{as}f_{as}, \quad (4.1)$$

$$\pi_b = p_2(q_{a2} + q_{b2})(q_{b2} + f_{ab} - f_{bs}) + p_1(q_{a1} + q_{b1})q_{b1} - p_{ab}f_{ab} + p_{bs}f_{bs}, \quad (4.2)$$

$f_{ab} > 0$  is a futures position such that  $\mathcal{A}$  sells to  $\mathfrak{B}$  ( $\mathcal{A}$  “goes short” and  $\mathfrak{B}$  “goes long”) at the futures price  $p_{ab}$  agreed between them, and similarly for  $f_{as}$  and  $f_{bs}$ . Of course,  $f_{ba} = -f_{ab}$  in this notation. The spot price,  $p_t$ , is equal, at maturity (period 2), to the futures price due to materialize: at maturity, all players will buy and sell futures only at the then valid spot price, so all futures positions are closed out at price  $p_2$  in period 2. This explains equations (4.1) and (4.2). Equations (3.1) and (3.2) use the mean-variance model (with constant absolute risk aversion parameters  $M_a, M_b$ ) and the fact that  $p_2(q_{a2} + q_{b2})$  and  $p_1(q_{a1} + q_{b1})$  have a common uncertain parameter  $\alpha$ . The variance of each producer’s belief about  $\alpha$  is normalized to 1, to simplify notation. There is no covariance term.

With given futures positions, simultaneous maximization of  $W_a$  in  $q_{a1}$  and  $W_b$  in  $q_{b1}$ —with  $q_{i2}$  eliminated by (1)—gives the unique Cournot equilibrium strategies

$$q_{a1} = \frac{s_{a0}}{2} - \frac{f_{ab} + f_{as}}{3} + \frac{f_{bs} - f_{ab}}{6}, \quad q_{a2} = \frac{s_{a0}}{2} + \frac{f_{ab} + f_{as}}{3} - \frac{f_{bs} - f_{ab}}{6}, \quad (5.1)$$

$$q_{b1} = \frac{s_{b0}}{2} - \frac{f_{bs} - f_{ab}}{3} + \frac{f_{ab} + f_{as}}{6}, \quad q_{b2} = \frac{s_{b0}}{2} - \frac{f_{bs} - f_{ab}}{3} + \frac{f_{ab} + f_{as}}{6}. \quad (5.2)$$

Producer  $\mathcal{A}$ ’s net short futures position is  $f_{ab} + f_{as}$ . Producer  $\mathfrak{B}$ ’s net short position is  $f_{bs} - f_{ab} = f_{bs} + f_{ba}$ . Each producer is seen to adjust the amount extracted prior to

maturity ( $q_{i1}$ ) for the net futures position he has taken and to partially counteract the adjustment his rival makes. If  $f_{ab} + f_{as} > 0$  and  $f_{bs} - f_{ab} < 2(f_{ab} + f_{as})$ , then  $\mathcal{A}$  makes his net short position more profitable by shifting extraction to at-maturity supply, driving down the spot price at maturity, and with it the value of the futures contracts he has (net) sold.

The equilibrium spot prices are

$$p_1 = \hat{p} + \frac{1}{6}(f_{as} + f_{bs}); \quad p_2 = \hat{p} - \frac{1}{6}(f_{as} + f_{bs}), \quad (6)$$

where  $(f_{as} + f_{bs})$  is the producers' combined net short position (or the speculator's net long position) and

$$\hat{p} = \alpha - \frac{s_{a0} + s_{b0}}{2}.$$

This parameter can now be given a straightforward economic interpretation:  $\hat{p}$  is the spot price that would prevail in both periods in Cournot equilibrium, if the producers were both inactive in the futures market. It is a natural benchmark. Since it is also a linear function of  $\alpha$ , it is convenient to express beliefs in terms of  $\hat{p}$  rather than  $\alpha$ . The players' beliefs are thus expressed as the mean expectations  $\hat{p}_a, \hat{p}_b$  and  $\hat{p}_s$ .

It should be noted that the equilibrium extraction rates do not depend upon the producers' beliefs about  $\alpha$ . A trading partner of a producer on the futures market can therefore predict how the producer will shift extraction in reaction to contracts accepted without knowing the producer's expectations. It remains to be seen whether he can predict if the producer will accept the contract.



### 5. Subgame-Perfect Acceptable Contracts

In order to study the subgame-perfect actions taken in steps 5 to 2, we first incorporate the extraction subgame's equilibrium strategies (5.1)-(5.2) and the equilibrium prices (6) in the payoff functions  $W_a$  and  $W_b$  of the producers and in the risk-neutral speculator's payoff

$$\phi_s = (p_2 - p_{as})f_{as} + (p_2 - p_{bs})f_{bs}. \quad (7)$$

The result is

$$W_a = s_{a0}\hat{p}_a + V_a, \quad (8.1)$$

$$W_b = s_{b0}\hat{p}_b + V_b, \quad (8.2)$$

$$V_s = (\hat{p}_s - p_{as})f_{as} + (\hat{p}_s - p_{bs})f_{bs} - \frac{1}{8}(f_{as} + f_{bs})^2, \quad (8.3)$$

where

$$V_a = (p_{ab} - \hat{p}_a)f_{ab} + (p_{as} - \hat{p}_a)f_{as} + \frac{1}{18}(f_{as} + f_{bs})^2 - \frac{M_a}{2}(s_{a0} - f_{ab} - f_{as})^2, \quad (9.1)$$

$$V_b = (\hat{p}_a - p_{ab})f_{ab} + (p_{bs} - \hat{p}_b)f_{bs} + \frac{1}{18}(f_{as} + f_{bs})^2 - \frac{M_b}{2}(s_{b0} + f_{ab} - f_{bs})^2. \quad (9.2)$$

We focus on  $V_a, V_b$  and  $V_s$ , because the expected value of the stock,  $s_{i0}\hat{p}_i$ , is unaffected by futures trading or by extraction rates.

What contract terms will the players aim at in the futures market? When announcing a particular offer (a price and a quantity), a player aims at obtaining contract terms that are most favorable to himself and yet acceptable to the player for

whom the announced contract is intended. Each player therefore has to determine which terms are just acceptable to the other side of the market, namely, the highest price acceptable to the other party if an offer to sell is being announced, and the lowest price acceptable to the other party if an offer to purchase is being announced. Player  $\mathfrak{B}$ , say, can do this by solving

$$V_a - V_a |_{f_{ab}=0} \geq 0.^5$$

If  $\mathfrak{B}$  wants to purchase from  $\mathcal{A}$  ( $f_{ab} > 0$ ), any price equal to or higher than the price that satisfies the equality is acceptable to  $\mathcal{A}$ , since

$$V_a - V_a |_{f_{ab}=0}$$

is the contribution of the futures position  $f_{ab}$  to  $\mathcal{A}$ 's expected profit and any nonnegative contribution will be preferable to not trading with  $\mathfrak{B}$ . The solution is

$$p_{ab} \begin{cases} \geq \\ < \end{cases} [\hat{p}_a - M_a(s_{a0} - f_{as})] + \frac{M_a}{2} f_{ab} \quad \text{as } f_{ab} \geq 0. \quad (10.1)$$

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<sup>5</sup>This equation assumes that  $\mathcal{A}$  accepts a contract which he is indifferent between accepting and rejecting. The reasons why subgame-perfect equilibrium requires this "accept when indifferent" principle in bargaining games with infinite strategy spaces are elaborated in Binmore [1992], §5.8. This equation also assumes that  $\mathcal{A}$  and  $\mathfrak{B}$  would otherwise not trade in the futures market. In other words, it assumes that  $\mathfrak{B}$  has not accepted and will not accept any contract offered by  $\mathcal{A}$ . For the situations below in which the equation is being relied on to characterize subgame-perfect equilibrium, this assumption fails only in pathological cases. Such pathologies can still be analyzed by the techniques used here; the only simplification lost is the linearity of what would otherwise be parallel quadratic curves in Figures 1 and 2.

The boldfaced italic subscript indicates the player for whom the price is acceptable. By the same reasoning,

$$p_{ab} \begin{matrix} \leq \\ > \end{matrix} [\hat{p}_b - M_b(s_{b0} - f_{bs})] - \frac{M_b}{2} f_{ab} \quad \text{as } f_{ab} \geq 0, \quad (10.2)$$

$$p_{as} \begin{matrix} \geq \\ > \end{matrix} [\hat{p}_a - M_a(s_{a0} - f_{ab}) - \frac{1}{4} f_{bs}] + (\frac{M_a}{2} - \frac{1}{18}) f_{as} \quad \text{as } f_{as} \geq 0, \quad (10.3)$$

$$p_{as} \begin{matrix} \leq \\ > \end{matrix} [\hat{p}_s - \frac{1}{3} f_{bs}] - \frac{1}{6} f_{as} \quad \text{as } f_{as} \geq 0, \quad (10.4)$$

$$p_{bs} \begin{matrix} \geq \\ > \end{matrix} [\hat{p}_b - M_b(s_{b0} + f_{ab}) - \frac{1}{4} f_{as}] + (\frac{M_b}{2} - \frac{1}{18}) f_{bs} \quad \text{as } f_{bs} \geq 0, \quad (10.5)$$

$$p_{bs} \begin{matrix} \leq \\ > \end{matrix} [\hat{p}_s - \frac{1}{3} f_{as}] - \frac{1}{6} f_{bs} \quad \text{as } f_{bs} \geq 0. \quad (10.6)$$

The difficulty for  $\mathfrak{B}$ , in evaluating  $p_{ab}$ , for example, is that  $\mathfrak{B}$  does not know whether  $\mathcal{A}$  is an optimist ( $\hat{p}_a = \hat{p}^H$ ) or a pessimist ( $\hat{p}_a = \hat{p}^L$ ).  $\mathfrak{B}$  must consider both possibilities and therefore define two boundary lines for the price-quantity combinations just acceptable to  $\mathcal{A}$ . The same is true when  $\mathcal{A}$  evaluates the terms acceptable to  $\mathfrak{B}$  according to (10.2):  $\mathcal{A}$  has to replace  $\hat{p}_b$  by  $\hat{p}^H$  and  $\hat{p}^L$ . For each of the possible contracts, a figure similar to Figure 1 can be drawn.

Figure 1 illustrates the case of a sale by  $\mathcal{A}$  to  $\mathfrak{Y}$  ( $f_{as} > 0$ ). The two upward-sloping lines represent the lowest selling prices  $\mathcal{A}$  can accept for  $\hat{p}_a = \hat{p}^H$  and (lower) for  $\hat{p}_a = \hat{p}^L$ . The two downward-sloping lines represent the highest prices at which  $\mathfrak{Y}$  is willing to buy for  $\hat{p}_s = \hat{p}^H$  and for  $\hat{p}_s = \hat{p}^L$ . The vertical distances between pairs of parallel lines is  $\hat{p}^H - \hat{p}^L$  (so necessarily the same for both pairs). The relative positioning of the pairs of lines reflects sizes of stocks, extent of risk aversion, and, most

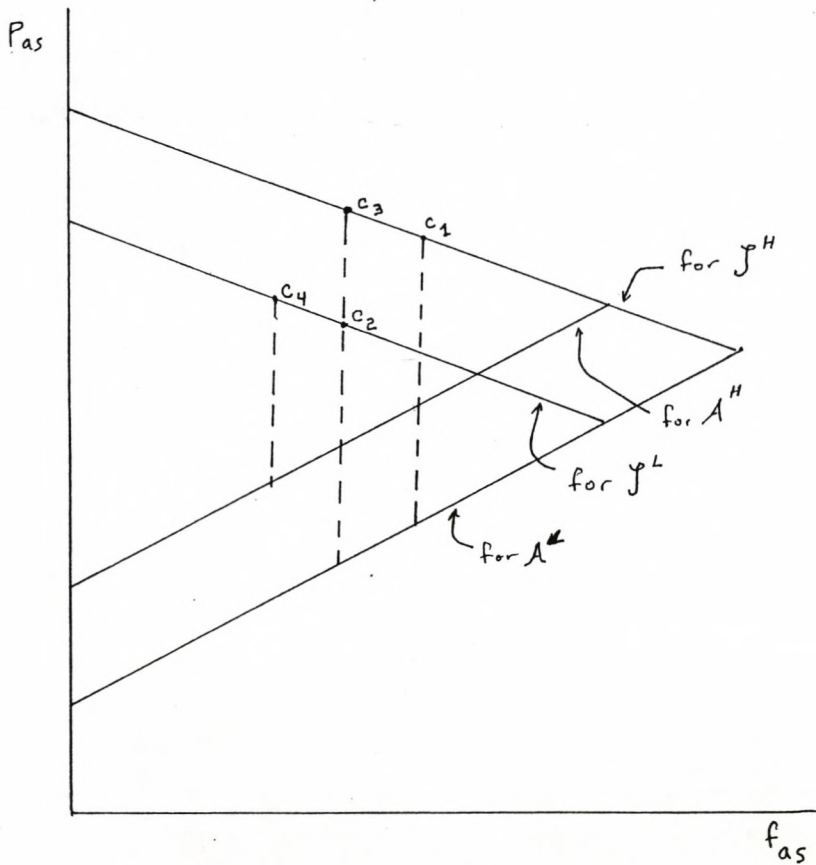


Figure 1

importantly, positions on contracts with the third player ( $\mathfrak{B}$ , in the case shown where a contract between  $\mathcal{A}$  and  $\mathcal{J}$  is being analyzed), as indicated in equations (10). When a sufficiently large share of the potential gains to trading futures between  $\mathcal{A}$  and  $\mathcal{J}$  have been usurped by contracts one or the other player reaches with  $\mathfrak{B}$ , this positioning may become tight enough to switch from Figure 1 to Figure 2. (We return to this issue in section 7.1 below.) If information limitations can be surmounted, trading is possible whenever an upward-sloping and a downward-sloping line cross, and for price-quantity combinations inside the triangle formed by these lines and the vertical axis.

## 6. Contract Acceptances in the $\mathcal{A}\mathfrak{B}\mathcal{J}$ Order

Our analysis throughout will be limited to consideration of pure strategies, presumably not a serious restriction when a continuum of actions is available at step 2. Description of subgame-perfect equilibrium, and the associated inferences about types, quickly gets complex. As the qualitative properties do not depend upon the commitment order, we will present the analysis only for the order  $\mathcal{A}\mathfrak{B}\mathcal{J}$ . This substantially reduces repetition and adds some useful concreteness; the cost is a suggestion of higher payoff for  $\mathcal{A}$  and lower payoff for  $\mathcal{J}$ , in general, than occurs across commitment orders. Where it aids concreteness, we will assume parameter configurations with the property that, were it common knowledge that all three players were of the same type (all optimists or all pessimists), then  $\mathcal{A}$  would take a net short position,  $\mathcal{J}$  a net long position, and  $\mathfrak{B}$  an intermediate position. This would be accomplished by giving  $\mathcal{A}$  a larger stock than  $\mathfrak{B}$ , and setting their risk postures close approximations to each other, relative to stocks. While these parameter configurations are not essential to the analysis, they simplify presentation, and fit with the intuitive value of imagining  $f_{ab}, f_{as}, f_{bs}$  all being positive.

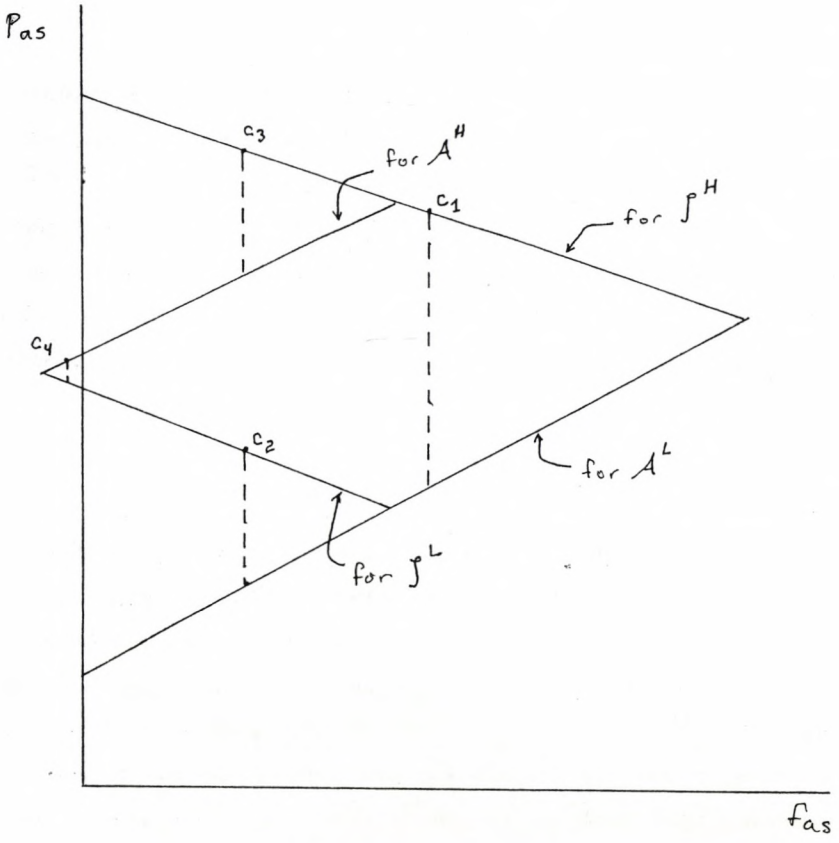


Figure 2

The analysis of subgame-perfect equilibrium proceeds, as is usual, backwards from Step 5. Before beginning, however, it is useful to note distinctions resulting from Step 2 behavior. It is common in games of incomplete information to distinguish between “pooling” and “separating” equilibria: in a pooling equilibrium, a player’s type is not revealed by his behavior, as both types choose the same strategy in equilibrium, typically because one type gains from hiding his identity. In a separating equilibrium, each type selects a different strategy, and no uncertainty about a player’s type remains after his action is observed.

Intermediate cases can arise here, which we have not seen considered before. Specifically, one or two players could choose announcements which reveal their type, with the rest of the players choosing pooling announcements in Step 2. During Steps 3, 4 and 5, all players will face no uncertainty in predicting the behavior of a player who made a separating announcement, but may be unsure of which contracts, if any, will be acceptable to a player who chose a pooling announcement. To handle all possibilities, the following notation will be employed. Player  $i$ ’s announcement made in step 2 will be denoted  $c_i = (p_i, f_i)$ ,  $i = \mathcal{A}, \mathcal{B}, \mathcal{F}$ , where  $f_i$  is a short position:  $f_i > 0$  implies  $i$  is offering to sell futures at price  $p_i$ , while  $i$  is indicating a desire to go long (buy futures) at price  $p_i$  if  $f_i < 0$ . The information available in step 3 about players’ types as revealed in equilibrium announcements  $c = (c_a, c_b, c_s)$  is summarized by  $I = (I^{a\tau}, I^{b\tau}, I^{s\tau})$ , where  $I^{i\tau}$  takes the value  $I^{iH}$  if  $i$  made a separating announcement which revealed him to be an optimist,  $I^{iL}$  if  $i$  made a separating announcement which revealed him to be a pessimist, and  $I^{i0}$  if  $i$  made a pooling announcement which would be made in the equilibrium under analysis by both types of player  $i$ .

The next three subsections provide the detailed logic of backwards induction in the determination by players  $\mathcal{F}$ , then  $\mathcal{B}$ , then  $\mathcal{A}$  of which contracts to accept in steps 5,

4 and then 3. These subsections may not yield facile reading, but they are the heart of the analysis, in that step 2 decisions as to which contracts to announce at the beginning of the futures market are indecipherable without a clear understanding of how alternative announcements would affect which trades actually occur. Readers skeptical about slogging through the detail may wish to skip, at least initially, to the summary in subsection 6.4, page 21.

### 6.1. $\mathcal{Y}$ 's Contract Acceptances in Step 5

Any type of any player, whether a producer or not, has no reason to engage in pooling behavior once step 5 is reached: behavior in the extraction subgame is independent of all aspects of beliefs, given contracts accepted. At this point in the game,  $\mathcal{Y}$  faces no uncertainty about which other contracts will be accepted, or what his net short position would be if he were to refuse to accept any contracts in step 5. Thus, his step 5 decisions are governed strictly by substituting the correct value of  $\hat{p}_s$  into equations (10.4) and (10.6).

It will generally be the case (verified below) that  $\mathcal{Y}$  will find at most one outstanding contract which is acceptable to him.<sup>6</sup> If the outstanding contract is like  $c_2$  or  $c_4$  in Figure 1, it will be accepted by either type  $\mathcal{Y}^H$  or  $\mathcal{Y}^L$ , because being on the indifference line for  $\mathcal{Y}^L$  places the contract in the interior of the acceptance region for  $\mathcal{Y}^H$ . Similarly, a contract like  $c_2$  in Figure 2 will be accepted by either type of  $\mathcal{Y}$ . Contracts like  $c_1$  and  $c_3$  in either figure will be accepted by type  $\mathcal{Y}^H$ , who is just indifferent, but type  $\mathcal{Y}^L$  is strictly better off rejecting such contracts. Conversely, a contract like  $c_4$  in Figure 2, which shows a negative position because  $\mathcal{A}$  has offered to

<sup>6</sup>If both contracts are acceptable, an optimal decision to accept both will not depend upon which is accepted first. If either is acceptable in the event the other is rejected, but unacceptable in the event the other is accepted, a straightforward comparison of which contract places  $\mathcal{Y}$  on the higher indifference curve (considering impact on the extraction subgame) determines which is accepted.



buy futures from  $\mathcal{F}$ , is just acceptable to type  $\mathcal{F}^L$ , but will be rejected by type  $\mathcal{F}^H$  as implying selling at too low a price.

Given information  $I^{sH}$  or  $I^{sL}$  revealed by  $\mathcal{F}$ 's announcement, the information of the previous paragraph becomes common knowledge at the beginning of step 3, and commitments in steps 3 and 4 can be made knowing what will happen in step 5. Given information  $I^{s0}$ , however, the same announcement is made in equilibrium by both types of  $\mathcal{F}$ , and player  $\mathcal{A}$  commits in step 3 on the basis of a probabilistic assessment, as follows: contracts like  $c_1$  or  $c_3$  are expected to be accepted by  $S$  with probability  $\lambda_a$ , and a contract like  $c_4$  in Figure 2 is expected to be accepted by  $S$  with probability  $1 - \lambda_a$ . In step 4,  $\mathcal{B}$  commits on the basis of similar expectations, only using  $\lambda_b$  for the probability that  $c_1$  or  $c_3$  get accepted, and  $1 - \lambda_b$  for the probability that  $c_4$  in Figure 2 gets accepted. The essence of inconsistent incomplete information is that  $\mathcal{B}$  continues to use the same  $\lambda_b$  to evaluate  $\mathcal{F}$ 's type after learning what  $\mathcal{A}$  is doing. That types  $i^H$  and  $i^L$  use the same  $\lambda_i$  is simply the result of a parsimonious modeling assumption.

#### 6.2. $\mathcal{B}$ 's Contract Acceptances in Step 4

Assuming subgame-perfect play of the extraction subgame, each player's payoffs, as perceived by him at any time during the game, are a function of contracts accepted, and of his beliefs, but not of rivals' types. Since, as mentioned, the extraction subgame's play is independent of beliefs, no type of  $\mathcal{B}$  has any incentive to pool so as to hide his identity during step 4. Moreover,  $\mathcal{B}$ 's behavior in step 4 is unaffected by whether he has learned  $\mathcal{A}$ 's type.

Given information  $I^{sH}$  or  $I^{sL}$ ,  $\mathcal{B}$  can anticipate, for any pattern of open contracts that he might accept, what remaining open contracts  $\mathcal{F}$  will accept in step 5. Accordingly,  $\mathcal{B}$  can insert the proper values of the  $f$ 's into (10.2) and (10.5) to determine whether to accept contracts. One new slight complication arises: equations

(10) could indicate that an open contract  $c_i$  is acceptable to  $\mathcal{B}$  if and only if an open contract  $c_j$  will [alternatively, will not] be accepted by  $\mathcal{F}$ ,  $\mathcal{B} \neq i \neq j$  (but possibly  $j = \mathcal{B}$ ), yet  $\mathcal{B}$  knows that  $\mathcal{F}$  will not [will] accept contract  $c_j$  if  $\mathcal{B}$  accepts  $c_i$ . This is a finite-strategy version of a familiar Stackelberg leader problem, solved by a straightforward comparison of  $\mathcal{B}$ 's payoff when  $c_i$  but not  $c_j$  is [  $c_i$  and  $c_j$  are ] accepted with  $\mathcal{B}$ 's payoff when  $c_j$  but not  $c_i$  is [neither  $c_i$  or  $c_j$  are] accepted. Such a complication shows up as a distance-maintaining parallel shift in the two lines in a diagram such as Figure 1 or 2 that relate to minimally acceptable contracts for  $\mathcal{B}$  (or in an adjustment of the constant (with respect to a  $c_i$  being considered) term in an equation (10).

A more interesting complication arises given information  $I^{s0}$ . Here,  $\mathcal{F}$  has made a pooling announcement, and  $\mathcal{B}$  is uncertain of  $\mathcal{F}$ 's type, attaching a subjective probability  $\lambda_b$  to the probability that  $\mathcal{F}$  is an optimist. Thus, in equations (10.2) and (10.5),  $\mathcal{B}$  substitutes in values for the  $f$ 's corresponding to contracts  $\mathcal{A}$  accepted and contracts which either type of  $\mathcal{F}$  will accept. Contracts like  $c_1$  or  $c_3$  in either figure are entered probabilistically: if  $\mathcal{A}$  refused to accept  $\mathcal{F}$ 's offered contract, and offered  $(p_a, f_a)$  represented by  $c_1$  or  $c_3$ , which  $\mathcal{B}$  did not intend to accept,  $\mathcal{B}$  would substitute the quantity  $\lambda_b f_a$  for  $f_{as}$  in (10.5) in order to decide whether to accept  $\mathcal{F}$ 's offered contract. Similarly, if  $\mathcal{A}$  refused  $\mathcal{F}$  and offered  $(p_a, f_a)$  represented by  $c_4$  in Figure 2,  $\mathcal{B}$  would substitute the quantity  $(1 - \lambda_b)f_a$  for  $f_{as}$  in (10.5). The Stackelberg leader problem also complicates these comparisons: it could be that  $\mathcal{B}$  has to choose between accepting  $c_s$  followed by  $\mathcal{F}$  rejecting  $c_a$  versus rejecting  $c_s$  followed by  $\mathcal{F}$  accepting  $c_a$  with probability  $1 - \lambda_b$ .

### 6.3. $\mathcal{A}$ 's Contract Acceptances in Step 3

As in the cases of  $\mathcal{F}$  and  $\mathcal{B}$ , in subgame-perfect equilibrium, no type of  $\mathcal{A}$  has any incentive to pool so as to hide his identity during step 3. Also analogous to the



least probability  $\bar{\lambda}$ . It is common knowledge that, whether or not  $c_i$  is accepted by  $\mathfrak{B}$ ,  $\mathfrak{F}^H$  will accept  $c_j$  and  $\mathfrak{F}^L$  will reject. For  $\lambda_b > \bar{\lambda} > \lambda_a$ ,  $\mathcal{A}$  can anticipate that  $\mathfrak{B}$  will accept  $c_i$  and can also confidently anticipate that  $\mathfrak{B}$  will on average regret this decision. (After all,  $\mathcal{A}$  believes that  $\lambda_a$  is the correct prior, otherwise he would have updated it upon learning that  $\mathfrak{B}$  believes prior  $\lambda_b$ .) This is not possible if prior beliefs over the likelihood of a player being optimistic are consistent.

Having used  $\lambda_b$  to make the calculations about how likely  $\mathfrak{B}$  believes it is that  $\mathfrak{F}$  will accept various contracts, and thus coming to anticipate  $\mathfrak{B}$ 's behavior,  $\mathcal{A}$  now "corrects" these calculations by substituting in  $\lambda_a$  in order to come up with his best estimate of the odds of  $\mathfrak{F}$  accepting these contracts. He then substitutes these probabilistic values of  $\mathfrak{F}$ 's acceptances together with the deterministic values of  $\mathfrak{B}$ 's acceptances into equations (10.1) and (10.3) to determine which of his contracts to sign in step 3, now engaging in precisely the same kind of Stackelberg leader problem discussed above.

Finally consider information  $(I^{b0}, I^{s0})$ , i. e., pooling announcement by both  $\mathfrak{B}$  and  $\mathfrak{F}$ .  $\mathcal{A}$  calculates, using  $\lambda_b$ , the predictions  $\mathfrak{B}$  would make as to the likelihood of  $\mathfrak{F}$  accepting any particular contract; these predictions are independent of  $\mathfrak{B}$ 's type. Next,  $\mathcal{A}$  calculates, separately for  $\mathfrak{B}^H$  and  $\mathfrak{B}^L$ , which contracts that type of player  $\mathfrak{B}$  will accept. Then  $\mathcal{A}$  attaches probability  $\lambda_a$  to the acceptance of any contract  $\mathfrak{B}^H$  would accept and probability  $(1 - \lambda_a)$  to the acceptance of any contract  $\mathfrak{B}^L$  would accept. Still using  $\lambda_a$  as his prior, he now conditions his probabilities that  $\mathfrak{F}$  will accept contracts on  $\mathfrak{B}$ 's probabilistic behavior only in that  $\mathfrak{B}$  must either have been the offeror or must have rejected a contract for it to be available to  $\mathfrak{F}$ .

The resulting version of the Stackelberg leader problem has an interesting probabilistic character to it.  $\mathcal{A}$  could find, for example, that if he accepts contract  $c_i$ ,

neither type of  $\mathfrak{B}$  will accept contract  $c_j$ , while if  $\mathcal{A}$  rejects contract  $c_i$ ,  $\mathfrak{B}^H$  but not  $\mathfrak{B}^L$  will accept contract  $c_j$ . As before,  $\mathcal{A}$  makes this decision simply by comparing the expected utility in subgame-perfect equilibrium continuation of having  $c_i$  accepted and  $c_j$  rejected for certain versus having  $c_i$  rejected for certain and  $c_j$  accepted with probability  $\lambda_a$ ; this comparison is a straightforward variation on equations (10).

#### 6.4. Summary of Futures Contract Acceptance Behavior

In the order of commitment randomly determined by nature, players decide which, if any, of the open contracts they wish to accept. In subgame-perfect equilibrium, they do this by anticipating, to the extent possible, how the contract acceptance behavior following their acceptance decisions will respond to their decisions. Anticipation can only be determinate if the player whose acceptance behavior is being predicted exhibited separating equilibrium announcements, so that rivals have learned his type from the contract he offered.

PROPOSITION 1: Under doubly inconsistent incomplete information, any set of contract announcements yields a unique subgame-perfect equilibrium continuation.

Uniqueness depends upon the convention that a player indifferent between accepting and rejecting a contract accepts. The point, though, is that doubly inconsistent incomplete information does not create a game in which any behavior can be rationalized. Contract acceptance behavior is as uniquely determined by the rationality postulates supporting subgame-perfect equilibrium as in a game of consistent incomplete information. Probabilistically, it is also just as determinable as if information were consistent.

PROPOSITION 2: In subgame-perfect equilibrium continuation following any set of contract announcements, no player alters his contract acceptance behavior to avoid revealing his type.

Separating equilibrium behavior in steps 3–5 stems from two sources: independence of extraction behavior from beliefs, and independence of payoffs from rivals' types, given the contracts that are accepted by rivals.

PROPOSITION 3: Contract acceptance behavior of any player  $i$  when any rivals deciding after  $i$  have revealed their types is essentially a matter of anticipating which contracts will be accepted following any behavior by  $i$ , and then simply deciding whether  $i$  would prefer the pattern of acceptances that includes his acceptance of an open contract to the pattern that includes his rejection of that contract.

PROPOSITION 4: Contract acceptance behavior of any player  $i$  when some rival deciding after  $i$  has not revealed his type involves anticipating the subjectively expected pattern of acceptances following any behavior by  $i$ , and then simply deciding whether  $i$  would prefer the stochastic pattern of acceptances that includes his acceptance of an open contract to the pattern that includes his rejection of that contract. When the player committing first makes this determination, he anticipates the behavior of the player committing second by anticipating the behavior of the player committing third using the second player's prior beliefs (with which he disagrees), and then using his own prior beliefs to predict both rivals' behavior.

### 7. Step 2 Announcements in the $ABF$ Order

PROPOSITION 5: In any subgame-perfect equilibrium, the player committing first makes a separating announcement in step 2, revealing his type.

Since the announcements are simultaneous,  $\mathcal{A}$ 's rivals in the  $\mathcal{A}\mathcal{B}\mathcal{F}$  order cannot gain in step 2 from learning  $\mathcal{A}$ 's type. Proposition 2 has already shown that  $\mathcal{A}$  will choose to separate, revealing his type, in step 3 before his rivals take their next actions. So in a pooling announcement, at least one type of player  $\mathcal{A}$  would be sacrificing payoff.

Initially, the next subsection also assumes that players  $\mathcal{B}$  and  $\mathcal{F}$  make separating announcements. The following subsection discusses the existence of a separating equilibrium. Subsection 7.3 considers the scope for semi-pooling equilibria in which at least one of  $\mathcal{B}$  and  $\mathcal{F}$  hides his type at step 2.

### 7.1. Subgame-Perfect Equilibrium Announcements with Type Revelation

Some further notation will actually help to simplify matters. Let  $I^1$  denote the information in the event under discussion, the union across types in which all players make separating announcements in step 2. Let  $c_{-i} = (c_j, c_k)$  be the vector of contracts offered by player  $i$ 's two rivals. Any player  $i$ , prior to step 2, can calculate how the rest of the game will be played following separating actions  $(c_i, c_{-i})$ : the backwards-inductive process of sections 4 and 6 can be applied. Let  $r_i(c_i, c_{-i})$  denote this calculable sequence of subgame-perfect equilibrium actions (in the separating equilibrium case,  $r_i$  depends upon  $i$  only in the order of the arguments).

At the beginning of step 2, type  $t$  of player  $i$  is considering what contract—a futures price and a take-it-or-leave-it futures position—to announce as the market opens. When this decision is made,  $t$  knows his own type, but not that of the other players, since announcements are simultaneous. If equilibrium calls for player  $j^H$  to choose (pure) action  $c_j^H$  and player  $j^L$  to choose action  $c_j^L$ , then player  $i$  views this as a  $\lambda_i$  probability that action  $c_j^H$  will be taken and a  $(1 - \lambda_i)$  probability that action  $c_j^L$  will be taken, as if player  $j$  were using a mixed strategy. However, the remaining player  $k$  attaches different mixing probabilities,  $\lambda_k$  and  $(1 - \lambda_k)$ , to these actions. Note the

probabilities  $i$  attaches do not depend on  $i$ 's type. For any player  $i$  and fixed actions for each type of each other player, let

$$\chi_{-i} = [(c_j^H, c_k^H), \lambda_i^2; (c_j^H, c_k^L), \lambda_i(1 - \lambda_i); (c_j^L, c_k^H), \lambda_i(1 - \lambda_i); (c_j^L, c_k^L), (1 - \lambda_i)^2],$$

which is the (mixed) action profile facing  $i$ . Based upon  $r_i$ , let  $\rho_i(c_i, \chi_{-i})$  be the probabilistic sequence of subgame-perfect actions following  $c_i, \chi_{-i}$ . Substituting  $r_i$  into  $V_i$  for each  $c_{-i}$  in the carrier of  $\chi_{-i}$ , a function  $G_i^t$  can specify the expected payoff  $G_i^t(c_i, \chi_{-i} | \rho_i, I^1)$  that would result if  $t$  responded to  $\chi_{-i}$  with  $c_i$ . The functional form of  $G_i^t$  is messy and unenlightening, requiring further notation not otherwise needed.

Then let  $g_i^t(\chi_{-i} | \rho_i, I^1)$  be the set of best responses:

$$\hat{c}_i^t \in g_i^t(\chi_{-i} | \rho_i, I^1) \Rightarrow G_i^t(\hat{c}_i^t, \chi_{-i} | \rho_i, I^1) \geq G_i^t(c, \chi_{-i} | \rho_i, I^1) \forall c.$$

Subgame-perfect equilibrium announcements then have the usual specification: a vector  $(\hat{c}_i^t)_i = \mathcal{A}, \mathfrak{B}, \mathfrak{Y}, t = H, L$  such that

$$\hat{c}_i^t \in g_i^t(\hat{\chi}_{-i} | \rho_i, I^1) \text{ for all } i \text{ and } t.$$

For the moment, existence of separating equilibrium is assumed; discussion of this issue is rejoined later in this section.

Our concern is with the logic of what announcements to make. We initially consider player  $\mathcal{A}$ , and consider the possibility that  $\mathcal{A}$  will sell futures to  $\mathfrak{Y}$ , with  $\chi_{-a}$  and the functional form of  $\rho_a$  given. The reader's attention is returned briefly to Figures 1 and 2. Recall that the slopes of lines in these figures are determined by



exogenous parameters (in particular, by the accepting player's risk tolerance, see (10)). When the gains from trade are sufficiently large (due to unhedged stocks and trades with the unshown player) relative to the difference in beliefs between an optimist and a pessimist, Figure 1 results; when they are relatively insufficient, the situation is as depicted in Figure 2. We will explore in some detail  $\mathcal{A}$ 's choice of a futures contract to announce for the situation depicted in Figure 1.

Recall that the vertical intercepts for lines in Figure 1, indicating contracts that are minimally acceptable to  $\mathcal{A}^H$  (higher upward-sloping line),  $\mathcal{A}^L$  (lower upward-sloping line),  $\mathcal{P}^H$  (higher downward-sloping line) and  $\mathcal{P}^L$  (lower downward-sloping line), all depend upon the futures positions the two players are exchanging with  $\mathcal{B}$ , which are not otherwise shown in Figure 1. However, at step 2 in a separating equilibrium,  $\mathcal{B}$ 's type is not known, and thus these lines are viewed by  $\mathcal{A}$  as each taking on one location with probability  $\lambda_a$ , corresponding to  $c_b^H$  (or simply to  $\mathcal{B}^H$ ), and another location with probability  $1 - \lambda_a$ , corresponding to  $c_b^L$  (to  $\mathcal{B}^L$ ), as illustrated in Figure 3.

In Figure 3, the two solid upward-sloping lines, CT and DF, correspond to the two positions of loci of contracts minimally acceptable to  $\mathcal{A}^L$ , for  $\mathcal{B}^H$  and  $\mathcal{B}^L$ ; so also the two solid downward-sloping lines, AB and LZ, bound  $\mathcal{P}$ 's acceptable contracts, for  $\mathcal{B}^H$  and  $\mathcal{B}^L$ . Similarly, the dashed upward-sloping lines, JQ and HP, are minimally acceptable contracts for  $\mathcal{A}^H$ , given  $\mathcal{B}^H$  and  $\mathcal{B}^L$ ; the dashed downward-sloping lines, KT and GF (atop hatched trapezoids), bound contracts acceptable to  $\mathcal{P}^H$ , given  $\mathcal{B}^H$  and  $\mathcal{B}^L$ . As a set, a pair of dashed lines must lie above the corresponding set of solid lines in Figure 3, but the relation between the higher dashed line and the lower solid line varies across parameter constellations. Figure 3 draws  $\mathcal{A}$ 's minimally acceptable contracts as more strongly influenced by the direct affect of  $\mathcal{A}$ 's type than by the indirect affect of  $\mathcal{B}$ 's type through  $\mathcal{B}$ 's separating equilibrium contract announcement.



For contrast,  $\mathcal{Y}$ 's just acceptable contracts are drawn as more heavily influenced by the type dependence of  $\mathcal{B}$ 's announcements than by  $\mathcal{Y}$ 's type. A priori, either arrangement is possible for either player.

The directional impact of  $\mathcal{B}$ 's type is not clear a priori, either. For example, line CT, for  $\mathcal{A}^L$ , and line JQ, for  $\mathcal{A}^H$ , have to refer to the same type of player  $\mathcal{B}$ ; which type depends on parameters, in murky ways. To aid in following the discussion, we will maintain for this section the mnemonic assumption that each line for  $\mathcal{B}^H$  is higher than the corresponding line for  $\mathcal{B}^L$ . Thus, for  $\mathcal{A}^L$ ,  $\mathcal{A}^H$ ,  $\mathcal{Y}^L$ ,  $\mathcal{Y}^H$ , lines CT, JQ, AB, and KT correspond to type  $\mathcal{B}^L$  (so  $\mathcal{A}$  considers them relevant with probability  $1 - \lambda_a$ ), while lines DF, HP, LZ, and GF correspond to type  $\mathcal{B}^H$  (so  $\mathcal{A}$  considers them relevant with probability  $\lambda_a$ ).

The situation in step 2 has  $\mathcal{A}$  deciding what contract offer to announce, knowing that he gets the chance to commit himself to rejecting any contracts he wishes to reject in step 3, before the other players have a similar commitment opportunity. Thus, for the moment, we are considering what contract to announce, assuming  $\mathcal{A}$  can make  $\mathcal{Y}$  a take-it-or-leave-it offer. Naturally,  $\mathcal{A}$  will announce an offer that is acceptable to himself; Figure 3 indicates regions in which offers will be acceptable to  $\mathcal{Y}$  under different states (types of  $\mathcal{B}$  and  $\mathcal{Y}$ ). Contracts in the triangle ABC will be accepted by  $\mathcal{Y}$  no matter what  $\mathcal{Y}$ 's or  $\mathcal{B}$ 's type, i. e., accepted with probability one. Contracts in the horizontally hatched trapezoid KABT will be accepted by  $\mathcal{Y}$  if either  $\mathcal{Y}$  or  $\mathcal{B}$  is an optimist, i. e., accepted with probability  $1 - (1 - \lambda_a)^2 = \lambda_a(2 - \lambda_a)$ , using  $\mathcal{A}$ 's subjective probabilities. Contracts in the unshaded trapezoid LKXZ will be accepted by both types of  $\mathcal{Y}$  given  $\mathcal{B}^H$ , and rejected by both types given  $\mathcal{B}^L$ , so the probability of acceptance is viewed by  $\mathcal{A}$  as  $\lambda_a$ . Finally, contracts in the vertically hatched trapezoid

GLZF will be accepted only if  $\mathcal{J}$  and  $\mathcal{B}$  are both optimists, an event  $\mathcal{A}$  views as having probability  $\lambda_a^2$ .

7.1.1. *Offers with the Same Probability of Acceptance*

PROPOSITION 6: If the second ( $j$ ) and third ( $k$ ) players in the commitment order make separating announcements (as does the first player  $i$ ), then  $i$ :

- [a] examines each combination of rivals' types for which an offer is accepted by  $k$  with some given probability and determines the conditionally efficient contract for each combination;
- [b] determines, among these efficient contracts with  $k$ , the one that has the highest expected profit;
- [c] repeats steps [a] and [b] to determine an efficient offer to  $j$ ;
- [d] finally selects the offer (directed to  $j$  or  $k$ ) that provides the highest expected profit.

We initially consider the question of what take-it-or-leave-it offer  $\mathcal{A}$  should make to  $\mathcal{J}$  if  $\mathcal{A}$  is artificially restricted to a collection of offers which will all be accepted with the same probability, and then remove this restriction. For concreteness, Figure 4 presents the case of offers which will be accepted with probability one. In Figure 4, AB (carried over from Figure 3) is the upper boundary on contracts that will be acceptable even to  $\mathcal{J}^L$ , even if  $\mathcal{B}$  is type  $\mathcal{B}^L$ , thus, with probability one.  $\mathcal{A}$  knows his own type, so only the two upward-sloping lines relevant to that type have been carried over to Figure 4: CB bounds contracts acceptable to  $\mathcal{A}$  given  $\mathcal{B}^L$ , while DF bounds contracts acceptable to  $\mathcal{A}$  given  $\mathcal{B}^H$ . Of course, for any given futures position  $\mathcal{A}$  might announce (on the horizontal axis), he prefers to announce as high a price as will be acceptable, so the contracts under consideration lie along the line segment AB.

If  $(\mathcal{B}^L, \mathcal{J}^L)$  were the type profile, these contracts would all provide  $\mathcal{A}$  with the entire gains from exchange, so  $\mathcal{A}$ 's preferred contract would be the efficient contract.

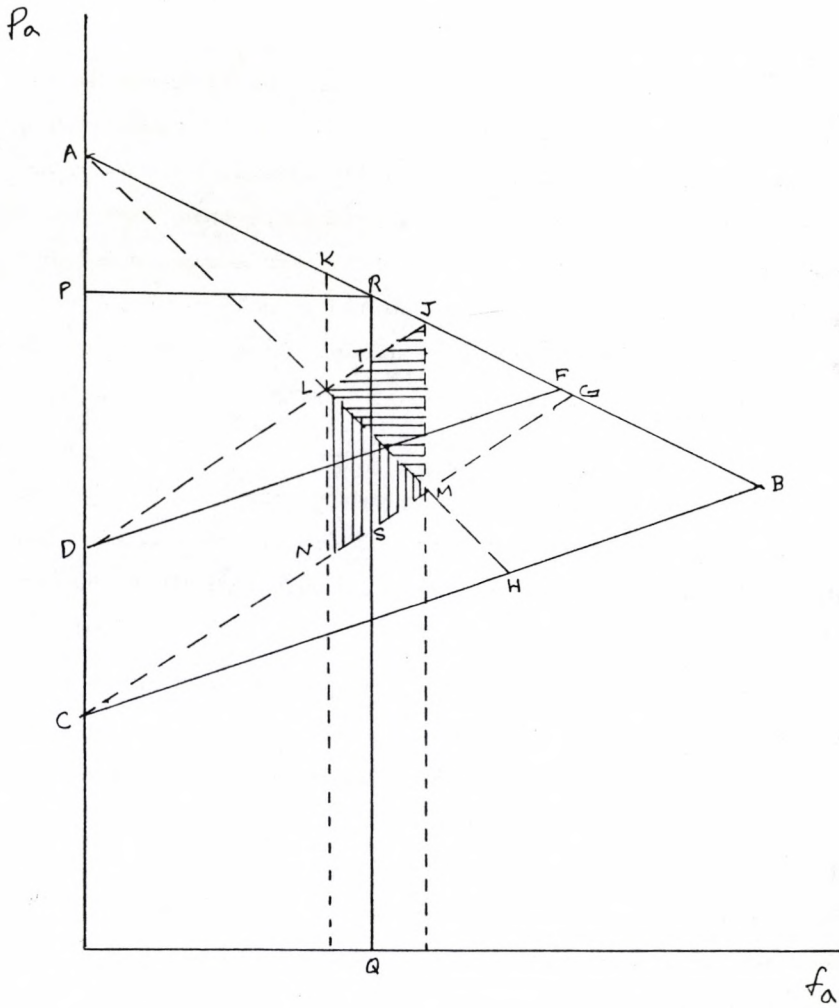


Figure 4

Efficiency differentials can simply be determined by the difference between the price that would be set by  $\mathcal{F}$ 's marginal willingness to pay and the price that would be set by  $\mathcal{A}$ 's marginal willingness to accept, if each were to view the price as immutable. Figure 4 shows the demand curve of type  $\mathcal{F}^L$ , given  $\mathcal{B}^L$ , for futures positions bought from  $\mathcal{A}$  (treating price as immutable, still): the dashed line AH which bisects the angle CAB; it also shows the supply curve of  $\mathcal{A}$  for futures sales to  $\mathcal{F}$ , given  $\mathcal{B}^L$ : the dashed line CG which bisects the angle ACB. These demand and supply curves intersect at M, so the contract that would extract the most surplus for  $\mathcal{A}$ , given  $\mathcal{B}^L$ , is the price/quantity pair at J.

Next suppose  $\mathcal{B}^H$ , but still with contracts restricted to AB. Given this restriction, it is still appropriate to treat AH as the demand curve to measure surplus extraction, but now the relevant supply curve is DJ, which bisects ADF. It intersects AH at L, so K is the contract which will yield the highest addition to payoff given  $\mathcal{B}^H$ .

Suppose  $\mathcal{A}$  were to announce K and  $\mathcal{B}$ 's type were revealed to be  $\mathcal{B}^L$ . In this event, how much lower will  $\mathcal{A}$ 's payoff be than if he had announced J? Simply by the amount of the inefficiency associated with K, because both contracts were fully surplus-extracting. That conditional inefficiency is the area of the vertically hatched triangle LMN. Similarly, if J were announced and then  $\mathcal{B}^H$  revealed,  $\mathcal{A}$ 's payoff would be lower than having announced K by the area of the horizontally hatched triangle JLM. Since these two triangles are congruent, in the binary choice between J and K,  $\mathcal{A}$  prefers K if and only if  $\lambda_a \geq 1/2$ .

Consider  $\mathcal{A}$  announcing a contract  $c$  along AB incrementally to the right of K. With probability  $\lambda_a$ ,  $c$  reduces  $\mathcal{A}$ 's payoff relative to K by the height of the horizontally hatched triangle just below  $c$ , while with probability  $1 - \lambda_a$ ,  $c$  increases  $\mathcal{A}$ 's payoff by the height of the vertically hatched triangle. The ex ante efficient contract along AB,

then, trades off these gains and losses to equate their marginal expectation, and is  $c^* = \lambda_a K + (1 - \lambda_a)J$ . Figure 4 illustrates for the case  $\lambda_a = 0.6$ : below R, the height of the line LM above S is 0.6 times the height ST, so R (a price P and futures position Q) is  $\mathcal{A}$ 's preferred choice among contracts that will be accepted with probability one.

The preferred contract among those along line KT in Figure 3, contracts that will be accepted if either type  $\mathcal{Y}^H$  or type  $\mathcal{B}^H$  materializes, is determined similarly, with the added complication that these contracts are accepted with probability one given  $\mathcal{B}^H$ , but only with probability  $\lambda_a$  (that  $\mathcal{Y}$  is  $\mathcal{Y}^H$ ) given  $\mathcal{B}^L$ . Thus, if AB in Figure 4 represented KT from Figure 3, the area of triangle JLM would have to be multiplied by  $\lambda_a$  before comparison with LMN, and the corresponding  $c^*$  would be  $\lambda_a(2 - \lambda_a)K + (1 - \lambda_a)^2J$ . Contracts along LZ and those along GF in Figure 3 would be rejected unless  $\mathcal{B}$  is  $\mathcal{B}^H$ , so the analysis corresponding to Figure 4 would simply yield contracts corresponding to K as maximizing the only relevant efficiency calculation.

### 7.1.2. Offers with Different Probabilities of Acceptance

Suppose the preferred contract for  $\mathcal{A}$  among each group of equally likely acceptances has been found, and consider the binary comparison among them. Line AB in Figure 5 (carried over from Figures 3 and 4) represents the set of contracts that will be accepted with probability one. Repeating Figure 4's analysis, the demand curve AH is intersected with supply curves CG and DJ, yielding conditionally efficient positions shown by dotted vertical lines (but unlabeled). The efficient convex combination for  $\lambda_a = 0.6$  has again been labeled R. To keep the diagram relatively simple, the contract that will be compared with R is the preferred contract among those that will be accepted with probability  $\lambda_a$ . In keeping with the conventions assumed in drawing Figure 3, these contracts, along LZ carried over from Figure 3, are accepted by either type of  $\mathcal{Y}$  given  $\mathcal{B}^H$ , but rejected by both types of  $\mathcal{Y}$  given  $\mathcal{B}^L$ . Thus,  $\mathcal{A}$  can assume

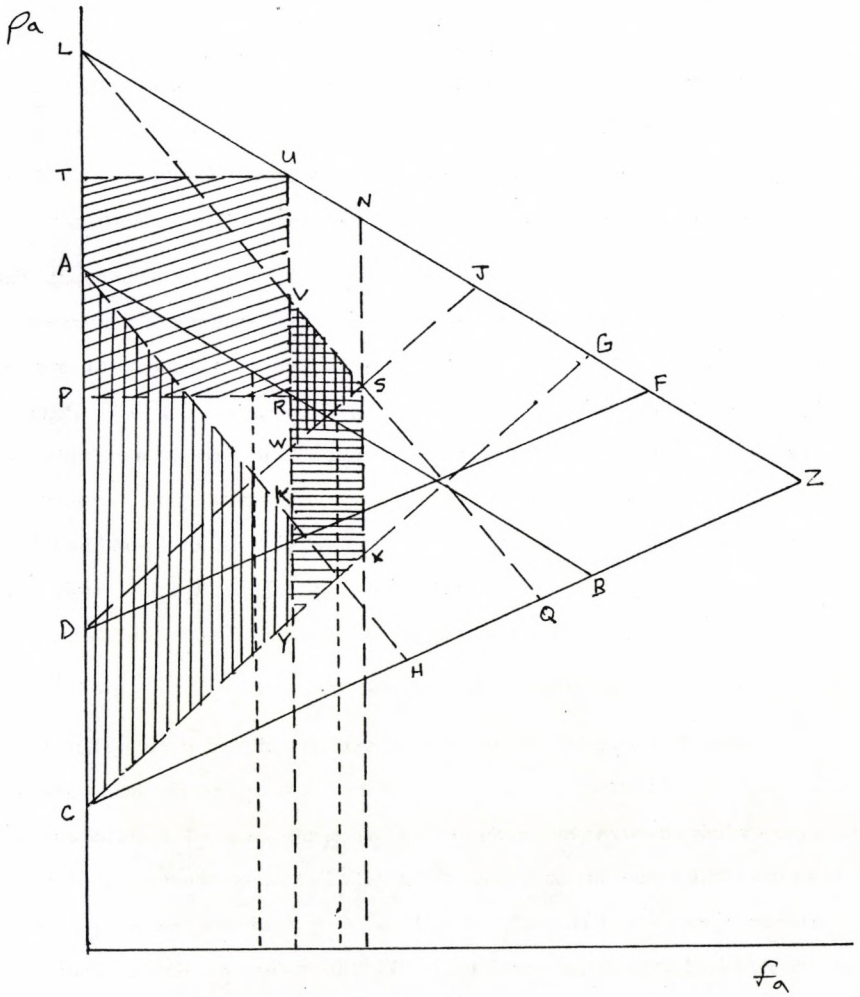


Figure 5



$\mathfrak{B}^H$ , hence supply curve DJ, in deciding which of these contracts is efficient. It intersects the relevant demand curve LQ at S, so N is the most preferred of these contracts.

The point of Figure 5 is the calculation of which of these two contracts yields the higher expected payoff given subgame-perfect equilibrium continuation  $\rho_{-a}$  (which has been reflected in the vertical intercepts). By way of convenient reference, we will let 0 represent the expected payoff associated with  $\mathcal{A}$ 's announcement being rejected (recall that  $c_{-a}$  is being held fixed, and behavior for the rest of the game becomes common knowledge at the end of step 2). With probability  $1 - \lambda_a$ ,  $\mathfrak{B}$  will be  $\mathfrak{B}^L$ , and R would be accepted by  $\mathcal{F}$  while N would be rejected. The profitability of R given  $\mathfrak{B}^L$  is the area under the relevant demand curve AH and above the relevant supply curve CG from the axis over to U; this is the area of the vertically hatched trapezoid AKYC. With probability  $\lambda_a$ ,  $\mathfrak{B}$  will be  $\mathfrak{B}^H$ , and both contracts will be accepted, so it remains to calculate the excess profitability to  $\mathcal{A}$  of N over R given  $\mathfrak{B}^H$ . This can be shown as the sum of two areas, by comparison with the contract U, which would sell the same futures position to  $\mathcal{F}$  as R, but at a price high enough to extract all surplus from type  $\mathcal{F}^L$ . The excess payoff from U over R is a pure revenue affect, the area of the obliquely hatched rectangle RUTP. Relative to U, N gains by extracting the fully efficient level of surplus; this is the area of the cross-hatched triangle VSW. So N is preferred to R if  $\lambda_a$  times the sum of areas RUTP and VSW exceeds  $(1 - \lambda_a)$  times area AKYC. (For the parameters underlying Figure 5,  $\lambda_a = 0.6$  is not quite large enough to yield N preferred to R.)<sup>7</sup>

An exactly analogous logic can be applied to contract offers that  $\mathcal{A}$  might offer to  $\mathfrak{B}$ , and then the contract which is the best  $\mathcal{A}$  could offer  $\mathfrak{B}$  is compared in additional payoff to the best contract  $\mathcal{A}$  could offer  $\mathcal{F}$ . To be complete, a similar analysis should

be made for the expected profitability of contracts which would have an interior probability of acceptance by  $\mathfrak{B}$  and a positive probability of acceptance by  $\mathfrak{Y}$  conditional upon rejection by  $\mathfrak{B}$ ; this would be similar though messier, and would arise in equilibrium only for unusual parameter constellations. While in principal a contract announced at step 2 is being offered to the market,

PROPOSITION 7: Proposition 6 implies that targeting any announcement to the player with whom the greatest gains from trade are at stake is typically best response behavior.

None of the logic explained via Figures 4 and 5 depended upon  $\mathcal{A}$ 's type, except to include the relevant upward-sloping lines from Figure 3. Also, with probabilistic continuation  $\rho_{-i}$  specified, none of the logic depended upon the identities or roles of the players. For example, diagrams corresponding to Figures 3-5 but contemplating the announcement  $\mathfrak{Y}$  might make of a contract to buy futures, targeted for  $\mathfrak{B}$ , will have  $\mathfrak{Y}$  pricing a position so that any lower price has a lower acceptance probability, and will presumably show a more dramatic shift in response to  $\mathcal{A}$ 's type, since  $\mathcal{A}$  commits first, but will otherwise follow the lines discussed. Thus, our layout of the logic of separating equilibrium is complete.

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<sup>7</sup>For the comparison between the best contract accepted for sure and the best contract accepted if either rival is an optimist, an analysis similar to Figure 5 would also have to take into account an efficiency advantage of N over R in the event  $(\mathfrak{B}^L, \mathfrak{Y}^H)$  that would correspond to the vertically hatched trapezoid VSXY. Suppose the alternative being compared to R were the best contract among those that would only be accepted if both rivals were optimists; let this be represented by N in Figure 5. Then for N to be preferred to R requires that  $\lambda_a^2$  times the sum of areas RUTP and VSW exceed  $(1 - \lambda_a^2)$  times area AKYC. The underlying logic that would be applied in the event that configurations in Figure 3 were changed, or that Figure 2 rather than Figure 1 might result for some type of player  $\mathfrak{B}$ , is unchanged in its essentials.

## 7.2. Remarks on Existence of a Separating Equilibrium

To our knowledge, the literature on existence of a separating equilibrium has not offered a proof covering the case of an extensive form with three players simultaneously choosing from an unordered continuum of signals prior to a sequential-move subgame; this paper will not change that situation. By and large, adding realism to the deliberately demonstrative model considered here would surely not ease these complications. Furthermore, an added complication of this model is that best-response correspondences, while they are upper-hemicontinuous, fail to be convex-valued: there is a value for  $\lambda_a$  which makes  $\mathcal{A}$  indifferent between N and R in Figure 5, but in this case,  $\mathcal{A}$  strictly prefers either N or R to any convex combination which would only be accepted if  $\mathcal{B}$ 's type were  $\mathcal{B}^H$  (i. e., in the unshaded trapezoid in Figure 3). Convex-valued best responses could perhaps be attained by a continuum of types (as in the simpler situation considered by Mailath [1988]), but this seems likely to introduce new difficulties in the treatment of inconsistent incomplete information. It is also possible that the nonconvexities constitute the sort of "controlled discontinuities" for which Dasgupta and Maskin [1986] offer existence theorems, but only in finite strategic form games.

Competing our tour of standard assumptions of existence proofs, we remark that compactness is not a problem in this model. Since the  $V_i$  functions all involve quadratic terms in futures positions with negative coefficients, there must exist a finite number which is a nonbinding absolute bound on the  $f_i$  coefficient of any best response. With the  $f_i$ 's artificially but harmlessly bounded, it is then similarly possible to obtain a nonbinding, finite upper bound on the  $p_i$  component of best responses. Without loss of generality, the strategy sets can be altered to incorporate these bounds.

The fact we found a one-parameter family of subgame-perfect equilibria in our similar model where inconsistent beliefs were common knowledge (Phlips and Harstad [1991]) is also encouraging for the existence of a separating equilibrium here.

### *7.3. Remarks on Existence, Nature and Plausibility of a Semi-Pooling Equilibrium*

Proposition 5 has shown the nonexistence of completely pooling equilibrium, as the player who commits first does not pool. Thus, the issue discussed here is the possibility of a semi-pooling equilibrium, in which both types would make the same contract offer announcement, and then separate in their contract acceptance behavior, for one or both of the players committing last. In this section, for this model, that is what the term “pooling equilibrium” will mean.

The canonical signaling games (see van Damme [1987], ch. 10, for an introduction) achieve pooling equilibria by having the receiving player’s payoff depend directly on the signaling player’s type, and by having the receiver respond to an out-of-equilibrium message based upon an assumption about sender’s type that is unfortunate for one type of sender. As a result, that type of sender’s predominant interest is in imitating the equilibrium message of the other type of sender.

It does not require the sort of subtle thinking debated in the refinements literature (van Damme [1987] and references cited) to restrict acceptance behavior following out-of-equilibrium contract offer announcements. A player’s payoff depends upon a rivals’ type only through the contracts that are accepted; no player changes his own contract acceptance behavior directly as a result of his belief about a rival’s type. The only possible impact is that during step 3 or 4, a player may alter his contract acceptance behavior in response to a belief that a rival’s type will lead to particular acceptance behavior in following steps; in what follows, we call this the “pooling impact.” Acceptance behavior that serves directly to reduce the payoff of a rival whose

type is uncertain, or is believed to be revealed by an out-of-equilibrium move is clearly disequilibrium behavior in this model.

Thus, there is no particular incentive for types of a player to pool in order to avoid revealing their types, other than the pooling impact. Its possible benefit is limited by the binary nature of the sequential decisions. It seems most likely, in a pooling equilibrium scenario, that a pooling impact beneficial to one type of a player is harmful to that player's other type. It follows that type  $t'$  will not benefit from an effort of type  $t$  to imitate type  $t'$ , so  $t'$  will only alter his contract announcement from a separating equilibrium choice if and only if doing so separates. Hence,  $t'$ 's benefit from the pooling impact would have to increase his equilibrium payoff more than the decrease that resulted from announcing the contract that was a best response for  $t'$  rather than for  $t$ . We have no proof that such pooling equilibria exist only for unlikely parameter constellations, but this seems the most natural conclusion to conjecture.

Should both a separating and a pooling equilibrium exist, refinements that are only well-defined for finite games or single-signaler, single-receiver games will not directly offer useful distinctions. However, we can envision an argument of the following sort. Any pooling equilibrium supported by beliefs that an out-of-equilibrium message was sent by the type benefiting from the pooling impact, when that message could be a sensible announcement for the type who does not benefit from the pooling arrangement is likely to seem less plausible than the separating equilibrium, under the same sort of arguments raised in refining pooling equilibria of finite signaling games.

## 8. Market Efficiency

We briefly discuss the efficiency of trading in this model, assuming that behavior is characterized by a subgame-perfect separating equilibrium. Initially consider the

singly-inconsistent case: whether each player is optimistic is his own private information, but the subjective prior odds are consistent:  $\lambda_a = \lambda_b = \lambda_s$ . Each player is offering a take-it-or-leave-it contract which maximizes his share of gains from exchange, given the behavior of the others and equilibrium continuation. Thus, each has incentives to take efficiency into account. In particular, a social planner faced with the same informational constraints as the players could not suggest ex ante an alternative pattern of behavior that would yield a higher expected payoff for one type of one player without yielding a lower expected payoff for some type of some player (possibly the other type of the same player). In this sense, the singly-inconsistent case reaches an outcome that is informationally constrained, ex ante efficient.

It is difficult to arrive at a satisfactory definition of an efficient outcome for the doubly inconsistent case, the general case analyzed above. Players disagree about the strength of demand for an exhaustible resource at the maturity date of a futures market; neither optimists or pessimists revise their beliefs even if certain that others hold different beliefs. This is essential for rational players to strictly prefer some speculative trades to foregoing any speculative trade which is acceptable to the trading partner. In addition, here, we have allowed players to have inconsistent prior beliefs about the probability that another player is an optimist;  $\mathcal{A}$  does not revise his estimate of the odds that  $\mathcal{F}$  is an optimist upon learning that  $\mathcal{B}$  attaches higher odds.

One could argue that the efficiency of the contract offers made be evaluated in terms of the "true" odds  $\lambda$ , treating any  $\lambda_a$ ,  $\lambda_b$ ,  $\lambda_s$  as mistaken unless it happens to agree with the true  $\lambda$ . It would then be possible to create an alternative for a social planner to suggest, that would generate an alternative outcome such that, if players used the true  $\lambda$  to evaluate payoffs and guide actions, this alternative would be subgame-perfect and Pareto-preferred.

We regard such an efficiency analysis as nonsense. If players are mistaken in their beliefs about a true parameter of the model, then the relevance of such rationality-based machinery as subgame perfection is unclear. Our basic philosophical position, moreover, is that no such "true"  $\lambda$  exists: the players have fundamental differences in opinion, the  $\lambda_i$ 's are primitives, and there is no sense in which any  $\lambda_i$  can be considered "incorrect" while anything else can be considered "correct."

Let the rationality of each player  $i$  evaluating uncertainty over types via  $\lambda_i$  be accepted for the purposes of a welfare analysis that is at least a distant analogue to efficiency. Then the following sort of characterization holds for the separating equilibrium. There is no alternative pattern of offer and acceptance behavior which a social planner, subject to the same informational constraints and to subgame-perfect equilibrium actions guided by the individual  $\lambda_i$ 's, could suggest that would yield an outcome in which some type of some player viewed himself as attaining a higher expected payoff, without the other type of the same player or some type of another player viewing himself as attaining a lower expected payoff.

In sum, despite doubly inconsistent incomplete information, a logic of specific rational behavior that was straightforward, step-by-step, emerges. It predicts a nonzero volume of purely speculative activity, roughly akin to *ex ante* efficiency. Inconsistent priors increase the difficulties of calculating equilibrium strategies by at least an order of magnitude; doubly inconsistent priors add little further difficulty. No Pandora's box is opened.

## References

- Binmore, Ken [1992], *Fun and Games*, D. C. Heath, Lexington, Ma.
- van Damme, Eric [1987], *Stability and Perfection of Nash Equilibria*, Springer-Verlag, Heidelberg.
- Dasgupta, Partha, and Eric Maskin [1986], "The Existence of Equilibrium in Discontinuous Economic Games. 1: Theory," *Review of Economic Studies* 53, 1-26.
- Debreu, Gerard [1952], "A Social Equilibrium Existence Theorem," *Proceedings of the National Academy of Sciences* 38, 886-93.
- Harsanyi, John C. [1968], "Games with Incomplete Information Played by 'Bayesian' Players, Parts I, II, and III," *Management Science* 14, 159-82, 320-32, 468-502.
- Harstad, Ronald M., and Louis Philips [1990], "Perfect Equilibria of Speculative Futures Markets," pp. 157-174 in J. J. Gabszewicz, J.-F. Richard and L. A. Wolsey, *Economic Decision-Making: Games, Econometrics and Optimisation*, North-Holland, Amsterdam.
- Mailath, George J. [1988], "An Abstract Two-Period Game with Simultaneous Signaling—Existence of Separating Equilibria," *Journal of Economic Theory* 46, 373-94.
- Philips, Louis, and Ronald M. Harstad [1991], "Interaction Between Resource Extraction and Futures Markets: A Game-Theoretic Analysis," pp. 289-387 in Volume II: Methods, Morals and Markets, of R. Selten (ed.), *Game Equilibrium Models*, Springer-Verlag, Berlin.
- Philips, Louis, and Ronald M. Harstad [1992], "Oligopolistic Manipulation of Spot Markets and the Timing of Futures Market Speculation," working paper, European University Institute, Florence, Italy.
- Selten, Reinhard [1982], "Einführung in die Theorie der Spiele mit unvollständiger Information," pp. 81-147 in *Information in der Wirtschaft*, Schriften des Vereins für Sozialpolitik, Gesellschaft für Wirtschafts- und Sozialwissenschaften, N. F. Band 126, Duncker and Humblot, Berlin.





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