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## Multiple scattering of elastic waves in unidirectional composites with coated fibers

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### Abstract

A computational method for analyzing the two-dimensional multiple scattering of elastic waves in unidirectional fiber-reinforced media, consisting of elastic cylindrical fibers arranged in an infinitely extended elastic matrix, is presented. The method is based on the eigenfunction expansion of the displacement potentials and the numerical collocation technique to obtain the expansion coefficients. In the present study, the formulation is extended to incorporate the presence of a coating layer between the fibers and the matrix. As an example, the transmission characteristics of longitudinal as well as transverse waves in a unidirectional composite with square array of fibers are analyzed, and the phase velocities in the composite are evaluated. The effect of the coating layer on the wave transmission and stop-band formation behavior is also demonstrated.

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*Keywords:* Composite material; Elastic wave; Multiple scattering; Eigenfunction expansion; Collocation method; Stop band

### 1. Introduction

Multiple scattering of elastic waves in fiber-reinforced composite materials is an important subject regarding the design for dynamic loading as well as the ultrasonic nondestructive testing. From a theoretical point of view, many versions of the multiple scattering theory and micromechanical models have been applied to analyze the elastic wave propagation characteristics in fiber-reinforced composites, e.g. [1, 2]. Direct computational approaches were also employed to analyze the multiple scattering and the resulting wave propagation characteristics of SH (shear horizontal) waves in fiber-reinforced composites [3-8]. Recently, these approaches were extended to two-dimensional elastic waves in fiber-reinforced composites [9]. In many of the previous works, however, the fibers were assumed to be directly bonded to the matrix. In some composites, the fibers are often multilayered, for example

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having a coating layer [10]. In this study, the formulation of the previous work [9] is extended to incorporate the presence of a coating layer between the fibers and the matrix. As a numerical example, the effect of the coating layer on the propagation behavior of longitudinal as well as transverse waves in a fiber-reinforced composite is analyzed.

## 2. Formulation

The problem to be considered here is the time-harmonic, two-dimensional motion of an infinite isotropic medium (Lamé constants  $\lambda_1$ ,  $\mu_1$  and density  $\rho_1$ ) containing unidirectionally aligned  $N$  circular cylindrical isotropic elastic fibers (radius  $a$ , Lamé constants  $\lambda_2$ ,  $\mu_2$  and density  $\rho_2$ ) with isotropic elastic coating layer (outer radius  $b$ , Lamé constants  $\lambda_3$ ,  $\mu_3$  and density  $\rho_3$ ) as shown in Fig. 1. The wave speeds in each medium are given by  $c_{L\alpha} = \{(\lambda_\alpha + 2\mu_\alpha)/\rho_\alpha\}^{1/2}$  and  $c_{T\alpha} = (\mu_\alpha/\rho_\alpha)^{1/2}$ , and the wave numbers  $k_{L\alpha} = \omega/c_{L\alpha}$  and  $k_{T\alpha} = \omega/c_{T\alpha}$  ( $\alpha = 1, 2, 3$ ), where  $\omega = 2\pi f$  is the angular frequency ( $f$  is the frequency). The elastodynamic multiple scattering problem is formulated in terms of the displacement potentials  $\Phi(\mathbf{r})$  and  $\Psi(\mathbf{r})$  which satisfy the two-dimensional Helmholtz equation. First the exciting fields at the  $i$ th fiber are given by

$$\Phi^{i,\text{exc}}(\mathbf{r}) = \Phi^{\text{inc}}(\mathbf{r}) + \sum_{\substack{j=1 \\ j \neq i}}^N \Phi^{j,\text{sca}}(\mathbf{r}), \quad \Psi^{i,\text{exc}}(\mathbf{r}) = \Psi^{\text{inc}}(\mathbf{r}) + \sum_{\substack{j=1 \\ j \neq i}}^N \Psi^{j,\text{sca}}(\mathbf{r}), \quad (1)$$

where  $\Phi^{\text{inc}}(\mathbf{r})$  and  $\Psi^{\text{inc}}(\mathbf{r})$  are the incident fields propagating in the  $x_1$  direction, and  $\Phi^{j,\text{sca}}(\mathbf{r})$  and  $\Psi^{j,\text{sca}}(\mathbf{r})$  are the scattered fields by the  $j$ th fiber. The exciting and the scattered fields can be expressed as

$$\Phi^{i,\text{exc}}(\mathbf{r}) = \sum_{n=-\infty}^{\infty} E_n^i J_n(k_{L1} |\mathbf{r} - \mathbf{r}_i|) \exp(in\theta_i), \quad \Psi^{i,\text{exc}}(\mathbf{r}) = \sum_{n=-\infty}^{\infty} F_n^i J_n(k_{T1} |\mathbf{r} - \mathbf{r}_i|) \exp(in\theta_i), \quad (2a)$$

$$\Phi^{i,\text{sca}}(\mathbf{r}) = \sum_{n=-\infty}^{\infty} A_n^i H_n(k_{L1} |\mathbf{r} - \mathbf{r}_i|) \exp(in\theta_i), \quad \Psi^{i,\text{sca}}(\mathbf{r}) = \sum_{n=-\infty}^{\infty} B_n^i H_n(k_{T1} |\mathbf{r} - \mathbf{r}_i|) \exp(in\theta_i), \quad (2b)$$

where  $E_n^i$ ,  $F_n^i$ ,  $A_n^i$ ,  $B_n^i$  ( $n = 0, \pm 1, \pm 2, \dots$ ;  $i = 1, 2, \dots, N$ ) are unknown coefficients,  $J_n$  and  $H_n$  are the  $n$ th-order Bessel and Hankel functions of the first kind, respectively, and  $\theta_i$  is the polar angle of the position  $\mathbf{r}$  viewed from the position  $\mathbf{r}_i$  (center of the  $i$ th fiber). The fields refracted in the  $i$ th fiber and in the coating layer are expressed as

$$\Phi^{i,\text{ref}}(\mathbf{r}) = \sum_{n=-\infty}^{\infty} C_n^i J_n(k_{L2} |\mathbf{r} - \mathbf{r}_i|) \exp(in\theta_i), \quad \Psi^{i,\text{ref}}(\mathbf{r}) = \sum_{n=-\infty}^{\infty} D_n^i J_n(k_{T2} |\mathbf{r} - \mathbf{r}_i|) \exp(in\theta_i), \quad (3a)$$

$$\Phi^{i,\text{coat}}(\mathbf{r}) = \sum_{n=-\infty}^{\infty} \{a_n^i J_n(k_{L3} |\mathbf{r} - \mathbf{r}_i|) + b_n^i Y_n(k_{L3} |\mathbf{r} - \mathbf{r}_i|)\} \exp(in\theta_i), \quad (3b)$$

$$\Psi^{i,\text{coat}}(\mathbf{r}) = \sum_{n=-\infty}^{\infty} \{c_n^i J_n(k_{T3} |\mathbf{r} - \mathbf{r}_i|) + d_n^i Y_n(k_{T3} |\mathbf{r} - \mathbf{r}_i|)\} \exp(in\theta_i), \quad (3c)$$

where  $C_n^i$ ,  $D_n^i$ ,  $a_n^i$ ,  $b_n^i$ ,  $c_n^i$ ,  $d_n^i$  ( $n = 0, \pm 1, \pm 2, \dots$ ;  $i = 1, 2, \dots, N$ ) are unknown coefficients, and  $Y_n$  is the  $n$ th-order Bessel function of the second kind. The relations between the expansion coefficients in Eqs. (2) and (3) are obtained

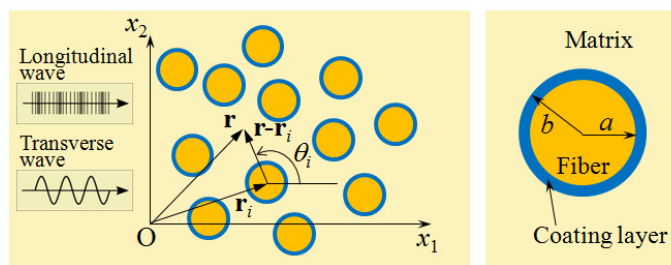


Fig. 1 A unidirectional composite with coated fibers, subjected to the incidence of plane longitudinal or transverse wave.

by applying the boundary conditions at the fiber-coating as well as the coating-matrix interfaces, and expressed in the following matrix form.

$$\begin{pmatrix} A_n^i & B_n^i & C_n^i & D_n^i & a_n^i & b_n^i & c_n^i & d_n^i \end{pmatrix}^T = [M] \begin{pmatrix} E_n^i \\ F_n^i \end{pmatrix}. \tag{4}$$

By rewriting the components of the  $8 \times 2$  matrix  $[M]$  as  $S_n = M_{11}$ ,  $T_n = M_{12}$ ,  $U_n = M_{21}$ ,  $V_n = M_{22}$ , the scattered and the exciting field coefficients are connected as

$$A_n^i = S_n E_n^i + T_n F_n^i, \quad B_n^i = U_n E_n^i + V_n F_n^i. \tag{5}$$

Substituting Eqs. (2) and (5) in Eq. (1) leads to the following equations which hold at arbitrary  $\mathbf{r}$  in the matrix.

$$\sum_{n=-\infty}^{\infty} E_n^i J_n(k_{L1} |\mathbf{r} - \mathbf{r}_i|) \exp(in\theta_i) = \Phi^{\text{inc}}(\mathbf{r}) + \sum_{\substack{j=1 \\ j \neq i}}^N \sum_{m=-\infty}^{\infty} (S_m E_m^j + T_m F_m^j) H_m(k_{L1} |\mathbf{r} - \mathbf{r}_j|) \exp(im\theta_j), \tag{6a}$$

$$\sum_{n=-\infty}^{\infty} F_n^i J_n(k_{T1} |\mathbf{r} - \mathbf{r}_i|) \exp(in\theta_i) = \Psi^{\text{inc}}(\mathbf{r}) + \sum_{\substack{j=1 \\ j \neq i}}^N \sum_{m=-\infty}^{\infty} (U_m E_m^j + V_m F_m^j) H_m(k_{T1} |\mathbf{r} - \mathbf{r}_j|) \exp(im\theta_j). \tag{6b}$$

In the numerical analysis, the infinite series in Eq. (6) are truncated at a finite but sufficiently large number, and the above equations are evaluated at collocation points, which are taken at the interface between the matrix and each coating layer. Then, the unknown coefficients can be determined as a solution of a linear system of equations.

### 3. Numerical example and results

As a numerical example, the elastic wave propagation in a unidirectional composite with Ti-alloy matrix ( $\lambda_1 = 103$  GPa,  $\mu_1 = 44.8$  GPa,  $\rho_1 = 5400$  kg/m<sup>3</sup>), SiC fibers ( $a = 67$   $\mu\text{m}$ ,  $\lambda_2 = 92.1$  GPa,  $\mu_2 = 177$  GPa,  $\rho_2 = 3200$  kg/m<sup>3</sup>) and carbon-rich coating layers ( $b = 70$   $\mu\text{m}$ ,  $\lambda_3 = 21.8$  GPa,  $\mu_3 = 4.6$  GPa,  $\rho_3 = 2100$  kg/m<sup>3</sup>) is analyzed. The coating layers are assumed to be perfectly bonded to the fibers and the matrix. For comparison, the composite without the coating layers is also considered, with the same material parameters for the matrix and fibers but the fiber radius 67  $\mu\text{m}$  replaced by 70  $\mu\text{m}$ . Although the method can be applied to arbitrary arrangement of coated fibers, in this study they are arranged in a square array with the volume fraction of coated fibers 25 %. By utilizing the periodicity of the fiber arrangement, only the fibers in a fundamental block [9] are explicitly accounted for, and the fiber arrangement in the matrix is assumed to consist of the repetition of this block in the  $x_2$  direction. As shown in Fig. 2, the present analysis employs the fundamental blocks containing two fiber rows in the  $x_2$  direction.

For the square arrangement of  $80 \times 2$  fibers in the fundamental block as shown in Fig. 2 (a), the wave fields are computed for the incidence of either plane longitudinal (P) or plane transverse (SV) waves. From the obtained wave fields, the wavelengths in the fiber-reinforced region can be evaluated, which then gives the phase velocities in the

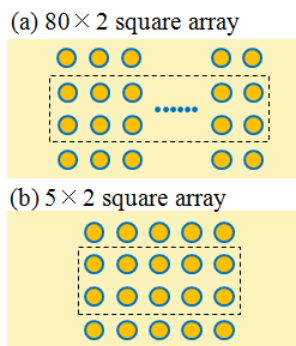


Fig. 2 Fundamental blocks with (a)  $80 \times 2$  and (b)  $5 \times 2$  square array of fibers.

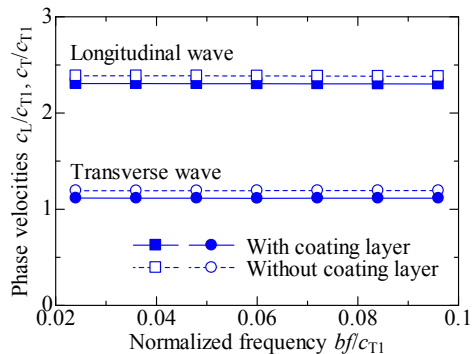


Fig. 3 Phase velocities of longitudinal and transverse waves in the composite.

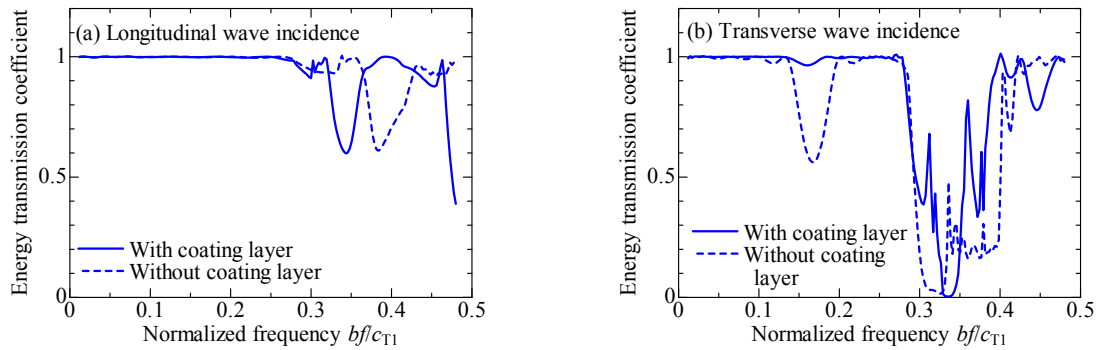


Fig. 4 Energy transmission spectra of (a) longitudinal and (b) transverse waves when the fibers are with or without coating.

composite. The normalized phase velocities of P and SV waves in the composite ( $c_L/c_{T1}$  and  $c_T/c_{T1}$ ) in a relatively low frequency range are shown in Fig. 3 with and without the presence of coating layers. As expected, the presence of coating layer reduces the phase velocities to a certain extent.

The square arrangement of  $5 \times 2$  fibers in the fundamental block as shown in Fig. 2 (b) is also analyzed, and the energy transmission coefficient of P and SV waves are determined. The frequency dependence of the energy transmission coefficients is shown in Fig. 4 for P and SV waves, with and without the coating layers. It is noted that these results are in good agreement with the previous finite element simulations [11]. The square array of fibers causes the reduction of energy transmission at certain frequencies due to the Bragg reflection. Stop-band formation is observed in the numerical results with only five layers of fibers in the  $x_1$  direction. In Fig. 4 (a), a stop band of the P wave at around  $bfc_{T1} = 0.38$  in the absence of coating shifts to lower frequency in the presence of coating, which is likely due to the lower wave speed of the coated-fiber composite. In Fig. 4 (b), while a clear stop band appears for the SV wave at around  $bfc_{T1} = 0.16$  in the absence of coating layer, it becomes very shallow in the presence of coating layer. This can be explained by the relatively low stiffness of the coating, which moderates the magnitude of wave scattering by each fiber.

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