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Modelling fat and protein concentration curves for Irish dairy cows

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The objective of this study was to acquire a well-fitting, single-equation model that would represent the fat and protein concentration curves of milk from Irish dairy cows. The dataset consisted of 16,086 records from both spring and autumn calving cows from both experimental and commercial herds. Many models cited in the literature to represent milk yield were examined for their suitability to model constituent curves. Models were tested for goodness-of-fit, adherence to the assumptions of regression analysis, and their ability to predict total fat and protein concentration for an entire lactation. Wilmink's model best satisfied these criteria. It had the best Mean Square Prediction Error (goodness-of-fit) value, it satisfied the assumptions of regression analysis (multicollinearity, heteroskedasticity, autocorrelation and normality of distribution), and it predicted the actual concentration of the constituents to within 0.01 percentage point.

Keywords: Analysis of residuals; lactation curves; milk constituents; modelling

Introduction

The concentration of fat or protein in the milk produced during a lactation can be represented by a curve, the shape of which normally mirrors a similar curve depicting milk yield (Pulina, 1990); the concentration of fat and protein in milk tends to decrease rapidly at the start of the lactation, and after falling to the minimum point increases slowly until the lactation is completed (Wood, 1976). The lowest point on the fat concentration curve lags

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approximately three weeks behind peak milk yield, but in the case of protein concentration, it reaches its lowest point at approximately the same time as the peak in milk yield (Schutz *et al.*, 1990; Stanton *et al.*, 1992).

Algebraic models explaining production levels of the constituents in milk have many uses in farm management, as well as in economic planning at farm level. Farm models which can be used for component, system and management research require accurate data from a variety of sources. The sub-models used within a farm system model need to be as accurate as possible in order for trustworthy farm models to be developed. The Moorepark Dairy Systems Model (MDSM) (Shalloo *et al.*, 2004) is a farm-simulation model that requires accurate representations of the fat and protein profiles of milk under Irish production circumstances. At present, the standard lactation curve (SLAC) method is incorporated into the MDSM. In Ireland, the SLAC method (Olori and Galesloot, 1999) is currently the preferred methodology to project partial lactation records for milk yield and its constituents. This is a method of interpolation consisting of a library of 2,160 equations for each constituent, from which the most appropriate equation is chosen. The SLAC method projects fat and protein yield, most of which is explained simply by the volume of milk produced. The projected fat and protein yield are converted to fat and protein percentages before being used in the MDSM farm model. It would be of great benefit to dairy scientists if a single equation model could explain as much of the variation in fat and protein as the 2,160 equations proposed by Olori and Galesloot (1999).

In this study a number of empirical algebraic models were investigated for potential use to predict the fat and protein concentration of milk. As in the case of modelling milk yield, the work of Wood (1967) provides the starting point for many studies involving empirical algebraic equations for representing the fat and protein concentration curves:

$$Y_n = an^{-b}e^{cn}$$

where

Y_n is the fat or protein concentration of milk produced in week n of lactation
a is a scaling factor associated with the average fat or protein concentration
b is related to pre-trough curvature and
c to post-trough curvature

Several studies (Cobby and Le Du, 1978; Dhanoa, 1981; Rowlands, Lucey and Russel, 1982; Ali and Schaeffer, 1987; Guo and Swalve, 1995) have found a poor fit to data on milk yield and its constituents when using this model. This poor fit may, in some instances, be due to environmental factors such as feed, weather and pregnancy status. The model of Wood (1967) is, however, still considered by many to be a basic reference for research on modelling lactation performance of livestock (Varona *et al.*, 1998).

Fat and protein yield have been analysed in some studies (Wilmink, 1987; Stanton *et al.*, 1992; Gonzalo *et al.*, 1994; Schaeffer and Jamrozik, 1996; Jamrozik and Schaeffer, 1997; Olori and Galesloot, 1999; Garcia and Holmes, 2001; Vasconcelos *et al.*, 2003), while fat and protein concentration of milk has been used in others (Wood, 1967; Killen and Keane, 1978; Crosse, Van Heijst and

O'Donovan, 1988; Stanton *et al.*, 1992). The quality of milk is better explained by the fat and protein concentration and the MDSM uses concentrations rather than yields of fat and protein. The fat and protein concentration of milk has not been modelled in Ireland since the study of Crosse *et al*. (1988) which was limited to four experimental herds attached to Moorepark Research Centre over a 2-year period. Quinn, Killen and Buckley (2005) found that examining the residuals of regression analysis was beneficial in order to arrive at a well-fitting robust curve and therefore the residuals of regression analysis were examined in the present study of fat and protein concentration. The effect of seasonal factors, free of stage of lactation effects, are also examined.

Materials and Methods

Data

The data used comprised a total of 16,086 lactations from two data sets as described in Quinn *et al*. (2005). Dataset 1 comprised 14,198 lactations with monthly testday recordings from 79 commercial spring-calving dairy herds. Dataset 2 comprised 1,888 lactations with daily yields, from six experimental herds attached to Teagasc. Records with less than five recordings were deleted from Dataset 1 and lactations of less than 25 weeks duration were removed from Dataset 2. After editing, Datasets 1 and 2 consisted of 13,229 and 1,727 lactations respectively and they were amalgamated for ease of analysis. Abnormal fat and protein percentage values were then removed using the method described by Quinn, Killen and Buckley (unpublished). After removing the abnormal recordings, the combined dataset consisted of 156,365 recordings representing 14,956 lactations.

Models and Statistical Analysis

The models tested included those of Wood (1967), Wilmink (1987) and Guo and Swalve (1995) (see Table 1). Wood's model was tested in three different forms, as described by Cobby and Le Du (1978) and Quinn *et al*. (2005): in nonlinear form, linear form (in which Wood's equation is linearised by taking the natural logarithm of each side of the equation) and weighted linear form (in which the logarithm of the fat and protein concentrations are weighted proportionally to the square of the fat and protein concentration, respectively). The models of Ali and Schaeffer (1987) and Ali-B (Quinn *et al*., 2005) were inappropriate for modelling fat and protein concentrations as they are based on polynomial expressions that keep their concave

Table 1. Selection of models investigated

Year	Author	Model[a]
1967	Wood (nonlinear form)	$Y_n = an^{-b}e^{cn}$
1967	Wood (linear form)	$\ln(Y_n) = \ln(a) - b\ln(n) + cn$
1987	Wilmink	$Y_n = a + be^{-kn} + cn$
1995	Guo and Swalve	$Y_n = a + b\sqrt{n} + c\ln(n)$

[a]Y_n is the fat or protein concentration in milk in lactation week n.

shape. The model of Wilmink (1987) is biologically interpretable (Wilmink, 1987) and therefore easy to comprehend. Parameter *a* is associated with the level of concentration, *b* with concentration increase after the trough, *c* with the concentration decrease before the trough and *k* with the timing of the trough. When examining the model of Wilmink (1987), it was found that the value of the parameter *k* was consistent (with a value of 0.10) over lactation number, calving month and herd effect for both the fat and protein concentration. It was found, after carrying out an initial analysis of variance (Quinn, 2005) similar to that of Cunningham (1972) that calving month, lactation number, and herd had significant effects on the fat and protein concentration and that the effects of these factors had to be considered when performing the regression analysis.

The mean square prediction error (MSPE) was used as a measure of goodness-of-fit as outlined by Kvanli, Guynes and Pavur (1986) and the analysis of residuals was performed in the manner outlined by Quinn *et al*. (2005). The Durbin-Watson statistic was calculated for each model to test for autocorrelation of error terms, while White's test was used to examine the homoskedasticity assumption. A condition index was calculated to test for multicollinearity among explanatory variables (Belsley, Kuh and Welsch, 1980), while normality of the distribution of error terms was examined using the Kolmogorov-Smirnov test together with measures for kurtosis and skewness.

For each model, the actual fat and protein concentrations were compared to those predicted using the parameters obtained through the regression analyses procedures. This was performed by first determining the parameter estimates for each lactation number which were then used to estimate the weekly fat and protein concentrations in milk. These weekly concentrations and the true milk yield values were used to calculate the predicted fat and protein yield for each lactation week before they were cumulated to calculate the total predicted fat and protein yields, and thus the overall predicted fat and protein concentration for the total lactation.

The effect of production month independent of stage of lactation was estimated to account for seasonal effects (Guthrie, 1994). For example, there is a stimulus to milk production from high availability and digestibility of grass in spring and a depressing effect due to the use of conserved forage in the winter (Cunningham, 1972; Killen and Keane, 1978; Quinn *et al*., 2005). The concentration of both fat and protein is also affected by these seasonal changes in feeding regime (Friggens *et al*., 1995; Kavanagh *et al*., 2003). This effect was calculated by comparing the weekly actual data with the predicted fat and protein concentrations and the deviations resulting from these comparisons were cumulated for each month of the year. The mean of these deviations were then computed for each month to arrive at an average effect of production month on fat and protein concentration. These affects were averaged over several seasons (1995 to 2002).

Results

Goodness-of-fit

Goodness-of-fit statistics, using the MSPE value, are presented in Table 2; they ranged from 0.221 to 0.233 for the models when fitted to fat concentration and 0.054 to 0.055 for protein concentration. There was no significant difference between the MSPE values for each model in fitting fat and protein concentration data (Table 2).

Table 2. Goodness-of-fit statistics of curves for fat and protein percentage

Model	Mean square prediction error	
	Fat	Protein
Wood (linear form)	0.224	0.055
Wood (weighted linear form)	0.233	0.054
Wood (nonlinear form)	0.222	0.054
Wilmink	0.221	0.055
Guo and Swalve	0.222	0.054

Analysis of residuals

The results of the analysis of the residuals are shown in Table 3. There was no first order autocorrelation present in any of the models analysed, for either fat or protein concentration, as the Durbin-Watson statistic, *d,* was found to be between d_u and $4 - d_u$ (where d_u is the upper critical bound as outlined by Mendoza (1999)). Also White's test (Sen and Srivastava, 1990) was non-significant (P > 0.05) for all models indicating that the homoskedasticity assumption was satisfied. Examination of the multicollinearity diagnostics for the model of Guo and Swalve (1995) revealed that moderate to strong multicollinearity was present, indicated by condition index values of 89.6 and 87.0, respectively (a condition index value of between 30 and 100 indicates moderate to strong multicollinearity (Belsley *et al*., 1980)). For fitting the model of Wood (1967), in its three forms, there was moderate multicollinearity present, with condition index values of approximately 45 for both fat and protein concentration. The lowest condition index value was associated with the model of Wilmink (1987), which had a weak presence of multicollinearity when fitted to either fat or protein concentration.

The assumption of normality was not violated either as indicated by the Kolmogorov-Smirnov test-statistic (P = 0.10 to 0.11) across all the models. Kurtosis varied between 0.77 and 0.99 for the residual related to the fat concentration models and between 0.58 and 1.32 for those related to

Table 3. Comparison of models for fat concentration

Test	Model				
	Wood (linear form)	Wood (weighted linear)	Wood (nonlinear)	Wilmink	Guo and Swalve
			Fat concentration		
R^2	0.28	0.30	0.29	0.29	0.29
Autocorrelation	None	None	None	None	None
Multicollinearity	Moderate (45.5)[a]	Moderate (46.5)	Moderate (46.0)	Weak (18.7)	Moderate – Strong (89.6)
Heteroskedasticity	None	None	None	None	None
Normality	Normal	Normal	Normal	Normal	Normal
Kurtosis	0.92	0.99	0.81	0.77	0.80
Skewness	–0.28	–0.38	0.08	0.06	0.09
			Protein concentration		
R^2	0.41	0.41	0.40	0.39	0.41
Autocorrelation	None	None	None	None	None
Multicollinearity	Moderate (44.3)	Moderate (45.5)	Moderate (45.4)	Weak (18.5)	Moderate – Strong (87.0)
Heteroskedasticity	None	None	None	None	None
Normality	Normal	Normal	Normal	Normal	Normal
Kurtosis	0.72	0.58	1.28	1.32	1.31
Skewness	0.11	–0.02	0.33	0.34	0.35

[a]( ) = condition index.

the protein concentration models. Skewness varied between –0.38 and 0.09, and –0.02 and 0.35, for the residuals related to the fat and protein models, respectively.

Estimates of the models used in this study are in Table 4. It can be seen that the values of the *b* and *c* parameters for the three forms of Wood's model applied to fat concentration, are very similar to each other. Likewise for protein concentration, the values of the parameter estimates are almost identical between the three estimation methods for this model.

The actual average fat and protein concentrations were 3.94% and 3.41%, respectively (Table 5). The models of Wilmink (1987) and Guo and Swalve (1995) yielded estimates for fat and protein concentration that were within 0.01 percentage point of the actual average. The model of Wood (1967), in linear form, was the poorest at estimating the fat and protein concentrations (2.69 and 2.59%, respectively).

Table 4. Expected curves for fat and protein concentration

Model	Estimated equation[a]
	Fat concentration
Wood (linear)	$Y_n = \exp(1.51 - 0.13\ln(n) + 0.01n)$
Wood (weighted linear)	$Y_n = \exp(1.52 - 0.12\ln(n) + 0.01n)$
Wood (nonlinear)	$Y_n = 4.72n^{-0.13}e^{0.01n}$
Wilmink	$Y_n = 2.76 + 1.66e^{-0.10n} + 0.040n$
Guo and Swalve	$Y_n = 4.40 + 0.69\sqrt{n} - 1.22\ \log(n)$
	Protein concentration
Wood (linear)	$Y_n = \exp(1.22 - 0.05\ln(n) + 0.01n)$
Wood (weighted linear)	$Y_n = \exp(1.23 - 0.06\ln(n) + 0.01n)$
Wood (nonlinear)	$Y_n = 3.55n^{-0.05}e^{0.01n}$
Wilmink	$Y_n = 2.78 + 0.61e^{-0.10n} + 0.025n$
Guo and Swalve	$Y_n = 3.25 + 0.41\sqrt{n} - 0.57\log(n)$

[a]Y_n is the fat or protein concentration in lactation week *n*.

Table 5. Comparison of estimated total yield with actual total yield

Model	Yield (kg)	Concentration (%)
	Fat concentration	
Actual total fat	224.77	3.94
Wood (linear)	152.02	2.69
Wood (weighted linear)	163.10	2.85
Wood's (nonlinear)	164.28	2.87
Wilmink	223.85	3.93
Guo and Swalve	224.3	3.94
	Protein concentration	
Actual total protein	194.37	3.41
Wood (linear)	147.74	2.59
Wood (weighted linear)	148.02	2.60
Wood's (nonlinear)	157.57	2.77
Wilmink	194.10	3.40
Guo and Swalve	194.15	3.40

While all of the models tested were similar in terms of goodness-of-fit, when the residuals were examined it was found that the model of Wilmink (1987) was significantly better than the others with weak condition index values when applied to both fat and protein concentration (18.7 and 18.5, respectively). The variance of the residuals was found to be constant over all observations and there was found to be no significant deviation from normality in the distribution of the residuals. In addition, the Wilmink (1987) model was the most satisfactory at fitting the data, it is also the best at predicting the fat and protein concentrations over an entire lactation (to within 0.01 percentage point of the actual percentages).

The effects of production month, independent of stage of lactation, using the model of Wilmink (1987), are shown in Table 6. From September to March, excluding December and January, the model of Wilmink (1987) underestimated the fat concentration in milk by between 0.1 and 4.3 percentage points, with greatest variations occurring in October and November. Similarly, the greatest variations in the protein concentration of milk due to production month were found to occur between September and February. The effects shown in Table 6 were added to the model of Wilmink (1987) to obtain adjusted weekly fat and protein concentrations which account for the seasonal variation attributable to production month. Figures 1 and 2 show the average actual and predicted fat and protein concentration curves, respectively. Examples of how the predicted fat and protein model changes for cows of different calving months are shown in Figures 3 and 4.

Discussion

The aim of this study was to identify a model that satisfactorily represented variation in the fat and protein concentration of milk produced throughout a lactation. It was found that there was no significant difference between the MSPE values of the models investigated; however, when the residuals were examined it was found that the model of Wilmink (1987) was significantly better than the others with weak condition index values for both fat and protein concentrations. The other assumptions of regression analysis were satisfied – although there were some issues with the kurtosis and skewness values. The measures of kurtosis and skewness when fitting the various models to fat concentration indicated that there might be issues in the cases of the linear and weighted linear forms of the model of Wood (1967). In the case of the models for protein concentration, kurtosis is possibly a problem in some instances, but of overriding importance was the finding from the Kolmogorov-Smirnov test which suggests adherence to the normality assumption in all cases. In addition, the models were examined for their ability to predict the fat and protein concentrations over an entire lactation and the model of Wilmink (1987) also proved to be very accurate. Thus, the model of Wilmink was investigated further by calculating the variations attributable to production month.

Table 6. Seasonal deviations for predicted values using the model of Wilmink, independent of stage of lactation

Month	Component	
	Fat concentration (%)	Protein concentration (%)
January	–0.08	–2.53
February	2.02	2.68
March	0.81	0.63
April	–0.04	1.23
May	–2.30	1.58
June	–2.95	0.45
July	–3.22	0.00
August	–1.48	–0.65
September	0.07	2.43
October	4.28	5.14
November	4.10	1.61
December	–0.50	–3.27

From the data used in this study, it would appear that the fat and protein concentrations are less subject to seasonal variation when compared to whole milk production. Deviations ranged from –3.3 to 5.1 for fat and protein concentration, whereas they ranged from –8.6 to 8.9 for milk production (Quinn *et al*., 2005). This may be explained by the fact that differences in feeding regimes tend to impact more on milk volume rather than the milk constituents (Friggens *et al*., 1995; Kavanagh *et al*., 2003). The trough for fat percentage was found to lag approximately 3 weeks behind the peak for milk yield, while that for protein concentration coincided with peak milk yield. This corresponds with the findings of Schutz *et al*. (1990) and Stanton *et al*. (1992). The seasonal trends, for both fat and protein concentration, are similar to the findings of Killen and Keane (1978); there is a stimulus to fat production in the winter months and a depression in early

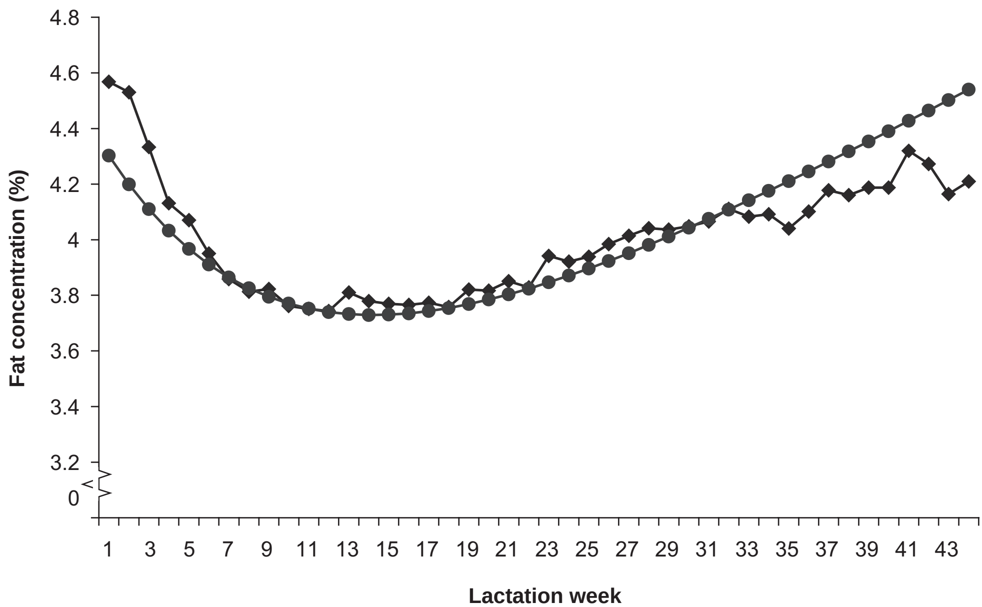

Figure 1: Actual average (—◆—) and estimated average (—●—) fat concentration (using the model of Wilmink).

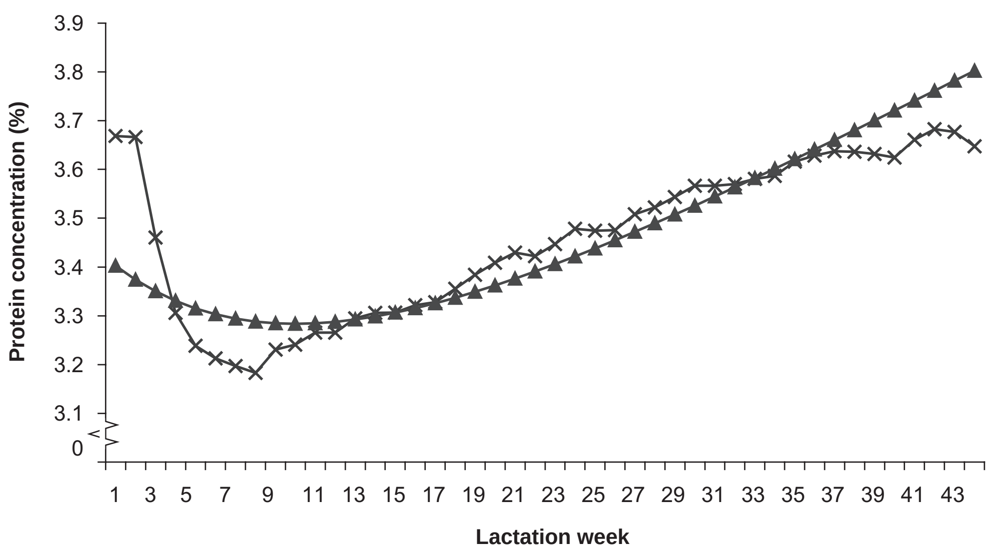

Figure 2: Actual average (—×—) and estimated average (—▲—) protein concentration (using the model of Wilmink).

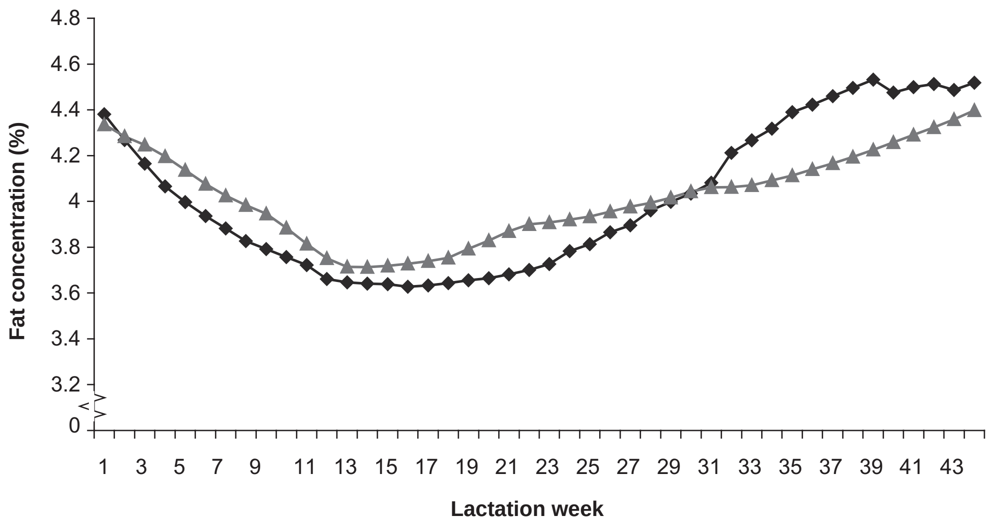

Figure 3: Estimated average fat concentration for a typical February (–◆–) and September (–▲–) calving cow.

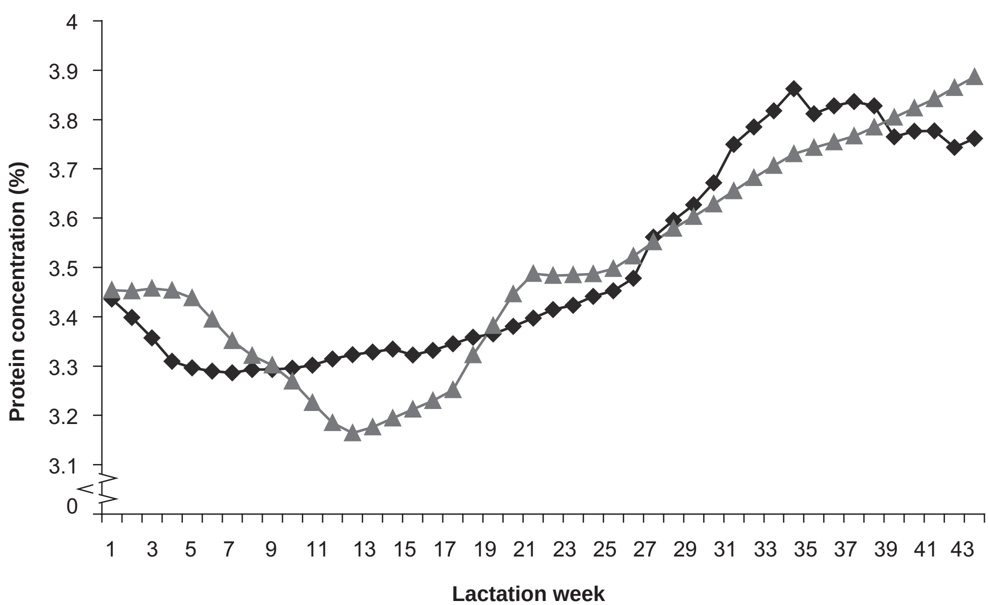

Figure 4: Estimated average protein concentration for a typical February (–◆–) and September (–▲–) calving cow.

summer that is partially due to the availability of grass and grazing conditions.

There are discrepancies between the predicted and actual fat concentration (Figures 1 and 2) especially at the early and late stages of lactation. These discrepancies are always difficult to interpret as there are many factors influencing the output of a cow at these periods (Wood, 1976; Killen and Keane, 1978) and there is high variability in the concentrations of fat and protein (Quinn *et al*., unpublished) at these times. The natural drop at the end of lactation (Killen and Keane, 1978; Crosse *et al*., 1988; Stanton *et al.*, 1992; Buckley, 1998) in fat and protein concentration (Figures 1 and 2) is often due to nutritional regime at the time of drying off. As the model used to predict the fat and protein concentrations is three dimensional, a limitation of the model is not being able to cope with the drop at the end of lactation. However, if the dimensions of the model were increased then it would be more difficult to interpret and it would possibly give rise to multicollinearity. The models predict that a cow calving in February will have slightly different fat and protein concentration profiles compared with a cow calving in September resulting in predicted cumulative fat and protein concentrations of 3.93% and 3.36%, respectively for February calvers compared with 3.99% and 3.08%, for September calvers. The higher predicted milk protein concentration for a February calved cow is not surprising. This is attributable to a higher proportion of pasture in the diet (Rook and Rowland, 1959; Dillon *et al.*, 1995; Roche *et al.*, 1996; Dillon *et al.*, 2001; Kennedy *et al.,* 2005).

Conclusions

The Wilmink model is appropriate for use by bio-economists who are constantly updating and recreating the regression parameters for different scenarios depending on factors such as herd, feeding system and environment. It is a single equation model and therefore less complicated to use than the 2,160-equation SLAC model.

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