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THE USE OF SHADOW PRICES IN PROGRAMME EVALUATION

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I. INTRODUCTION

Some recent works on problems of economic development have emphasized one important proposition, i.e., that in underdeveloped countries one should use "shadow prices" of productive factors rather than their observed market prices in determining the priorities in an investment programme. 1/ By an investment programme we mean a design for determining an optimal product mix as well as an optimal technology for the productive sectors.

It is the purpose of this paper to discuss critically a range of issues connected with the use of shadow prices in programme evaluation. The issues are the following:

(a) What exactly do we mean by shadow prices.

(b) The problem of estimating shadow prices of the relevant productive factors.

(c) If there exist ways of determining them approximately even though an exact solution may be out of reach.

(d) What the conditions are under which shadow prices would enable an optimal assignment of priorities.

(e) And finally to examine if there are situations where although shadow prices do not lead in general to a proper assignment of priorities, yet within the context of an over-all optimal programme determined directly, they may still be used to choose between relevant alternatives within somewhat narrower specifications. To mention a conclusion reached much later in discussion, it would be noticed that in the more realistic situation with which we are likely to be faced, it is only an affirmative answer to question (e) which assigns a proper measure of importance to shadow prices in programme evaluation.

1/ J. Tinbergen, The Design of Development (Baltimore: Johns Hopkins Press, 1958). H.B. Chenery and P.G. Clark, Inter-Industry Economics (New York: John Wiley, 1959). H. B. Chenery, "Development Policies and Programmes", Economic Bulletin for Latin America, III, No. 1, March, 1958.

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So far as the estimation problems are concerned, we shall illustrate our argument with reference to the shadow prices of capital and foreign exchange, which figure in common discussion as two of the most important productive factors in the context of planning in underdeveloped areas. It may be thought a little surprising to use capital and foreign exchange as two separate factors. Because our usual definition of a factor of production runs in terms of a group of productive agents which have a very high elasticity of substitution among themselves, but between which and other productive agents, the elasticity of substitution is zero or nearly zero. On this basis, it may be questioned if capital and foreign exchange are such imperfect substitutes for each other as to be described as separate factors. It must be conceded that there is nothing a priori about this division. It is based on the assumption, a very realistic one for many underdeveloped countries, that possibilities of exporting and importing commodities at roughly unchanged prices are extremely low or roughly, non-existent. This means that substitution possibilities are very severely limited as to make it a convenient simplification to use them as separate factors.

II. THE CONCEPT AND RATIONALE OF SHADOW PRICES

In the language of programming, shadow prices are nothing but the Lagrange multipliers of a constrained optimization problem. An equivalent way of describing them is in terms of the optimal solution of the so-called symmetric "dual" problem. Their plain economic meaning is none other than that of marginal value productivity of the productive factors in an optimal situation when all alternative uses have been taken into account. The reason why shadow prices are considered to be important for an economist is that neo-classical theory of resource allocation tells us that the value of the national product at given prices of final commodities is maximized if productive factors are employed so as to equate their value productivities with their rentals.

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It so happens that the rules of the game associated with perfect competition also lead to an identical result, e.g., equivalence of marginal productivities with rentals. But the connection with institutional aspects of perfect competition in this context is incidental. What is, however, important is the use of prices as parameters in deciding how much to produce. Now, there are a variety of reasons why observed prices in an underdeveloped economy deviate from prices as calculated from the optimizing solution of a programming problem: (a) the institutional context of perfect competition is almost entirely absent; (b) there are structural shortages which do not respond to price changes. In some cases this is not an unmixed evil from the wider sociological point of view, for example, where marginal productivity of labor is zero, and the corresponding shadow price of labor should also be zero, but the market has to assign a non-zero wage level to labor just to keep them alive (c); connected with (b) there is the problem that prices do not reflect and hence do not transmit all the direct and indirect influences on the cost as well as on the demand side, which under smoother conditions, they would.

Now, it should be obvious that if our objective is to maximize the value of national income, then prices which should be regarded as pointers in planning investment are not the market prices, but what are called shadow prices.

There are, however, several questions which may be raised at this stage:

(a) How do we know these shadow prices.

(b) Even if we know them from an optimal programme in the sense discussed above, they may not be the appropriate ones, because the interest of the planner may lie not in maximizing current national income, but some other objective or a combination of objectives.

Question (b) is, however, in a sense, not an important one, because the logic of using shadow prices is quite independent of the nature of the specific preference function that has been set up. Shadow prices in the programming interpretation are perfectly neutral with respect to the type

/of maximization

of maximization that is employed; although their interpretation as prices which would be realized under perfect competition, is not. But there is a somewhat related question, though a different one, which is not purely semantic. This is concerned with the empirical proposition that planners suggest, and given the power, carry out certain types of investment which yield results over finite though long periods of time. In certain extreme cases these projects do not yield results at all for some time to come. In evaluating such projects, to take into account only the impact on current national income is not appropriate. But if future experiences are to count, shadow prices calculated as of contemporary scarcities would not be proper. In planning for economic development, the endowments of the relevant primary factors are continually changing and their scarcity aspects are therefore shifting. Hence, what we need for such purposes is not merely the shadow price relating to one point of time, but the development of shadow prices over a period of time, i.e. the time path of shadow prices. Without such an estimate of the time path, there may arise a systematic bias against the use of long-run projects, if the "shadow prices" implied in maximizing current production were the only ones to be used.

Once, however, the values and time paths of these prices have been ascertained, there is no doubt they would greatly simplify the lack of assigning detailed priorities. Construction of adequate "benefit-cost" ratios for the investment projects is possible on the basis of these estimates only. They could then be employed to discriminate between projects, in view of all the interdependences existing at a point of time as well as over a period of time.

Granted what has been said above, we have to turn to question (a), which in a sense is the crucial one: how do we know these proper shadow prices? If they are known, then, the optimal pattern of capital accumulation is already known and vice-versa. Thus, we are not offering the planners anything immediately practical when we advise them to solve a problem in dynamic programming, however simplified its structure may be.[2/]

2/ S. Chakravarty: An Outline of a Method of Programme Evaluation, Center for International Studies, M.I.T., C/61-27.
R. Dorfman, P.A. Samuelson and R. Solow: Linear Programming and Economic Analysis, McGraw-Hill, 1958.

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At this stage, the argument for shadow prices rests on our ability to devise certain approximations, which do not require the solution of a full-scale dynamic programming problem. Thus we may first solve a programming model on a relatively very high degree of aggregation and determine the time path of prices of important groups of productive factors such as labour, capital and foreign exchange. Having attained these broad estimates, we may be justified in using them for purposes of assigning detailed priorities to the investment projects in various sectors.

Thus, the derivation of shadow prices on a more aggregative and hence approximative basis together with the decision rule to maximize net incomes or net discounted value of earnings at these prices would already go a long way to devising more efficient methods of programme evaluation.

An even more approximate procedure would be to use some general qualitative features of capital accumulation in an economy whose structural characteristics are well-known to make certain approximate estimates of ranges within which shadow prices of important productive factors might be expected to lie. This is attempted in our discussion of shadow rate of interest on the basis of the qualitative characteristics of a multi-sector growth process. Discussion on this point is meant only to suggest certain limits without pretending at quantitative exactitude.

Since the present practice in development programming is based almost exclusively on the current market prices of primary factors which are heavily out of line with their "intrinsic" values, even the use of such approximate shadow prices would lead to a more efficient resource allocation, provided the estimates are correct in a qualitative sense.

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III. THE PROBLEM OF ESTIMATION

(a) The Shadow Price of Foreign Exchange

It is a well-known observation that the shadow price of foreign exchange in many underdeveloped countries suffering from chronic balance-of-payments difficulties is substantially higher than the official rate of exchange. The reason for such maintained prices of foreign currency is that price elasticity of the exports and imports being quite low, the mechanism of letting price find its own level by equating the total demand for foreign currency to the total supply of foreign currency either does not work or works at the expense of income growth. Further, there is a widespread opinion that balance of payments difficulties of newly-developing countries are transitional in character, so that once certain structural changes have been well under way, excessive demand for imports or diversion of exports to home uses may cease, thus making it possible to approximate closely the equilibrium rate of exchange.[3/]

Thus while it is necessary to maintain an official rate of exchange different from the shadow rate, the shadow rate will still be the appropriate one to use in order to discriminate between alternative programmes or, in marginal cases, between alternative projects. Since sectors as well as the processes within any sector differ remarkably with respect to foreign exchange requirements, direct and cumulative, such discrimination is essential in order to satisfy the constraint relating to balance-of-payments equilibrium. If these constraints refer to different points of time, a time path of the shadow rate of exchange will be involved, rather than a single rate of exchange to be applied indefinitely. The standard procedure to determine the "shadow rate of exchange" at a point of time is to solve a programming problem of the following type:

3/ One may, however, argue for a devaluation of the home currency instead of letting the exchange rate seek its own level. This, however, runs into problems that are not entirely economic in character. Further, too frequent devaluations, depending on the variations in the import composition of the successive plans, will introduce nearly the same type of destabilizing influence as the method of floating exchange rates.

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Maximize a certain preference function, e.g. value of national income, subject to a specification of technology and a prescribed level of primary factors, including foreign exchange availability.[4] Such models have been extensively studied by Chenery, who normally expresses the preference function in terms of minimizing capital needed subject to final demand restrictions, technology and foreign exchange earnings. Chenery also includes import substitution as a built-in choice problem, even when alternative techniques are ruled out. When exports are not infinitely elastic, we have a problem in non-linear programming which has also been considered by him.[5] In keeping with what has been said in I, if the type of problem considered by Chenery in its static aspects is extended to take into account interdependences in time, in the form of usual recursion relationships that characterise a dynamic model, then, the corresponding preference function can be expressed in a large number of ways. Some details along these lines have been investigated in a somewhat different context.[6] But the upshot of the whole thing is to pose a problem having significant dimensions, although part of the dimensional difficulties may be reduced by taking advantage of block-triangularity, characterizing dynamic Leontief-type models. The way out of these difficulties from the computatorial point of view is the following:

4/ The more general approach including balance-of-payments deficit (or surplus), as well as the rate of growth of income in the social welfare function cannot be implemented unless we have some method of numerically estimating the relative rates of substitution between the different policy objectives. No very convenient method exists in this connexion, notwithstanding the contribution of Frisch. R. Frisch, "The Numerical Determination of the Coefficients of a Preference Function", Oslo, (mimeographed).

5/ Chenery, H.B. and Uzawa, H., Non-Linear Programming in Economic Development, in "Studies in Linear and Non-Linear Programming", edited by Arrow, Hurwicz, Uzawa, 1958.

6/ Dorfman, Samuelson, and Solow, op. cit., chapter 12.

/Develop a

Develop a programming model, linear or non-linear, which emphasizes heavily the sectors which engage in international trade either through earning foreign exchange or through consuming foreign exchange on a significant scale. Aggregate the remaining sectors considerably. Solve the resulting maximization problem and, then, compute the shadow rate of exchange from this approximate analysis. This preserves a certain notion of optimization, which we associate with shadow prices of primary factors.

The alternative to this procedure is to compute the demand for and supply of foreign exchange and then to determine the rate which equilibrates the two. We should be clear that since equilibrium is attainable at many different levels of income, there is no guarantee that this equilibrium rate of exchange is the same as the shadow rate of exchange defined in the preceding paragraphs. However, such an estimate may be useful to know as summarizing all the relevant information involved in a development plan bearing on the foreign exchange situation. This computation may be done on a very aggregative level, as well as on relatively disaggregated levels. Naturally, the accuracy of the estimates would improve, depending on how detailed the data happen to be.

What we elaborate in this paragraph is how all the components of demand and supply for foreign exchange may be. The following notations are employed in the formula for determining the shadow rate of exchange:

$\{e\}$ - Column vector of exports.
$\{e'\}$ - Is the corresponding row vector.
$\{w\}$ - Column vector of investment delivered by the sectors.
$\{\bar{w}\}$ - Column vector of investment received by the sectors.
$\{c\}$ - Column vector of final consumption.
$\bar{p}$ - Price level of goods produced at home.
$\{p\}$ - Vector of domestic prices.
p_m - Price of imports, here assumed to be homogeneous for simplicity.
k - The shadow rate of exchange.
m_1 - The quantity of raw materials imported.

/m_2 - The

m_2 - The quantity of investment goods imported.

m_3 - Import of consumer goods.

Coefficients:

$\{a\}$ - Leontief's matrix of flow coefficients.

$\{v_1\}$ - Row vector of imports per unit of gross output. These may also be called non-competitive import requirements per unit of output.

$\{v_2\}$ - Row vector of imports per unit of investment received. This gives the import composition of the investment programme.

v_3 - The functional dependence of imports of consumer goods on home consumption and the relative prices at home and abroad.

M - Total value of imports (measured in domestic prices).

E - Total value of exports (measured in domestic prices).

D - Permissible balance-of-payments deficit. This need not be a single number, but may only indicate a range within which the deficits should lie.

The problem then consists in determining the value or values of 'k' so that the balance of payments deficits are confined to a certain pre-assigned range determined by possibilities regarding foreign aid. Since the estimates are seldom precise, it is useful to work out alternative values of 'k' corresponding to a whole range of possibilities relating to 'D'. In principle, we can solve the various numerical situations to get a step-function relating the shadow rate of exchange to the parameter 'D' assumed variable over a certain range. Assuming, however, that the plan specifies a set of values of $\{e\}$, $\{w\}$, and $\{c\}$, and the coefficients are inflexible, then 'k' is the only variable to adapt itself to such predetermined magnitudes. It will, however, be desirable to determine the sensitivity of 'k' to adjustment in some of the physical magnitudes which are subject to some degree of control, e.g. $\{w\}$ which gives the import composition of investment or $\{c\}$, the import of consumer goods. We have the following final equation for this purpose:

$$/\overline{D} = M - E$$

$$\bar{D} = M - E$$

$$= kp_m m - e'p$$

$$= kp_m (m_1 + m_2 + m_3) - e'p$$

$$\bar{D} = kp_m \left\{ \{v_1\} (I - a)^{-1} (e + w + c) + v_2 \{\bar{w}\} + v_3 (c, \bar{p} - p_m) \right\}$$

$$\approx \left\{ p_1 e_1 + p_2 e_2 + \ldots + p_n e_n \right\}$$

We give 'n' export quantities for generality, but some of these will be identically equal to zero, since we have sectors which do not export anything, like services for example. The dimensionalities in matrix multiplication are also properly observed in as much as $\{v_1\}$ is (1 x n), $(I - a)^{-1}$ is (n x n), $(e + w + c)$ is (n x 1). Thus the whole expression is (1 x 1) and may be multiplied by 'p_m' to get the value in foreign currency of the required amount of imports of raw materials.

$\{\bar{w}\}$ and $\{w\}$ are connected by the following matrix equation: $\{w\} = [w]\{\bar{w}\}$ where $[w]$ is the matrix of investment coefficients.[7]

Each 'p_n' may be written in the following way: (2) $p_i = A_{oi} kp_m$ i = 1 . . . n + other terms, indicating the influence of whatever other primary factors are assumed to be important. Thus we have (n + 1) equations to determine the (n + 1) unknowns, the shadow rate of exchange, 'k' and 'n' domestic prices. This circularity arises because the production of domestic goods needs imports, and as such prices of domestic goods are dependent on prices of imports as expressed in domestic currency.

The above analysis may be easily extended to take into account the heterogeneity of imports, and thus we need not assume only one composite type of imports which is capable of being used for various functional purposes. The extension is of merely algebraic nature and is thus relegated to an appendix.

It should be apparent from the above discussion that exports for this purpose have been assumed to be exogenously prescribed. This is a simplification, although of a nature that is not difficult to justify,

7/ For a discussion of this matrix, see S. Chakravarty, The Logic of Investment Planning, Chapter V, North Holland Publishing Company.

/especially when

especially when price elasticity of exports is very low or low in relation to the other factors involved. These other factors involve the level of world demand as determined by rising world incomes, as well as the domestic expansion of demand for export commodities. If the price elasticities are assumed to be significant, then this may also be taken account of by a further complication in analysis. But, then, to retain manageability we should have to restrict the number of sectors very considerably.

(b) The Shadow Rate of Interest

The shadow rate of interest is commonly regarded as a concept more difficult than the shadow rate of foreign exchange. One reason for this is that in the case of foreign exchange we are concerned exclusively with flow magnitudes; so much imports representing a flow demand for foreign currency and so much exports representing a flow supply of foreign currency. The shadow rate of exchange equilibrates the demand and supply of foreign currency. With the shadow rate of interest, however, we are concerned with relations between stock and flow, and a very large variety of stocks at that. Further, these stocks have different degrees of durability. All these become extremely complicated if we want to get one single measure of these stocks, as we normally do in talking about "the amount of capital" and "the rate of interest".

The presence of double index number ambiguity, one due to cross-sectional aspects and the other due to longitudinal or intertemporal aspects of capital, makes the interpretation of this single measure somewhat dubious. Nonetheless, it has heuristic significance, as more rigorous models involving multiple capital goods seem to indicate.[8/] The logically rigorous way of deriving these interest rates, one for each stock, which under certain circumstances equal each other, is to specify the decision problem as one in dynamic programming, with appropriate initial and boundary conditions. Choice of natural boundary conditions is not an easy question. For absence of "compactness" in the policy space, infinity does not serve as a proper boundary condition in most economic problems extending over time.[9/]

8/ Samuelson, P.A. and Solow, R.M., "A Complete Capital Model Involving Heterogeneous Capital Goods", Quarterly Journal of Economics, November 1956.

9/ For a discussion of this point see: S. Chakravarty, On the Existence of an Optimum Savings Program, CENIS, 0/60-11.

/All these

All these theoretical considerations are, however, poor consolation for the planner, if the policy maker is concerned with rationing out scarce capital amongst a number of competing projects. True enough that if we know the solution to a full-fledged dynamic programming problem, we know at the same time the shadow rates of interest, because the optimum programme of capital accumulation determines the shadow rates of interest. In that context, they may be used to decentralize decision making by permitting simple decision rules to be specified. But when that is not feasible, we still need a kind of computational shorthand in order to rank projects. Whatever approximations we may devise for computing the shadow rate of interest, even though they are correct in only a qualitative sense, will be more useful than relying on the observed market rate of interest in economies characterized by market imperfections, etc.

In the subsequent paragraphs, certain considerations relating to the shadow rate of interest are discussed under the following sets of assumptions.

a) Where capital stocks are growing at the same proportionate rate and the production functions are linear and homogeneous;

b) Where the relative rates of growth of the capital stocks are different, but we still maintain the linear homogeneity assumption;

c) Where the production functions are no longer assumed to satisfy the linear homogeneity conditions, and the equiproprotionate rate of growth of all the sector does not hold.

We shall discuss these various cases in the order presented above.

a) The situation (a) may be further subdivided into the following two cases: (i) where there is no final demand; and (ii) where the system admits of final demand, i.e. not all the net product is reinvested. An illustration of case (i) is the closed dynamic model enunciated by Von Neumann in the early thirties. The specific set-up of the Von Neumann model is well known and does not require any repetition. Von Neumann stated as the main conclusion of his investigation the now famous equality between the rate of interest and the maximum rate of balanced growth that

/the system

the system can perform. The maximum rate of balanced growth is known to satisfy the criterion of intertemporal efficiency. But, as such, it is one among an infinite number of efficient paths. But what Solow and Samuelson have shown is that for situations referring to sufficiently distant points of time, and preference function involving terminal stocks of different commodities, the maximal rate of steady growth is the best way in which the system may be allowed to grow, expecting for a finite number of time periods. The length of the period for which the system is allowed to deviate from the Von Neumann model of growth is independent of the time horizon. Admittedly, this is true for "closed systems", e.g. systems admitting no autonomous consumption. But as a first approximation for economies on a very low level of real income, a closed model, particularly one such as Von Neumann's, which allows for different patterns of consumption in the same way that it includes different techniques for producing a particular commodity, may not be entirely dismissed out of hand. Hence, the above consideration is not entirely irrelevant from the empirical viewpoint, although from the purely logical point of view, its special nature should be clearly understood.

The Von Neumann model of a closed expanding economy has been generalized by Solow and Malinvaud, who relax the assumption that all the net product is reinvested. In other words, they assume the savings coefficient to be less than unity. Despite differences in presentation, the relationship between the rate of interest and the rate of growth given by the above authors is the same.

The following expression of the relationship is due to Solow[10/] who considers both the capitalists and the wage earners to be saving constant proportions of their incomes:

10/ R.M. Solow, Notes Towards a Wicksellian Theory of Distributive share (mimeographed).

/ρ=

$$\rho = \frac{g}{\bar{\sigma}_R + \frac{1-D}{D}\bar{\sigma}_W}$$

where ρ is the rate of interest

g is the rate of growth

$\bar{\sigma}_R$ is the savings coefficient for profit receivers

$\bar{\sigma}_W$ is the savings coefficient for wage earners

D is the share of profit income in total income

It is evident that the $\rho \geq g$ according as the denominator is ≤ 1. Now the denominator may be written as follows:

$$\frac{D\bar{\sigma}_R + (1-D)\bar{\sigma}_W}{D}$$

The expression $D\bar{\sigma}_R + (1-D)\bar{\sigma}_W$ is nothing other than the weighted average of the two savings coefficients or the savings coefficient for the economy as a whole. Thus we may write $\rho = \frac{g}{s/D}$ where 's' is the global savings ratio. That this relationship is merely a generalization of the Von Neumann result may be seen easily. On the specific Von Neumann assumption that $\bar{\sigma}_R = 1$ and $\bar{\sigma}_W = 0$, the above formula indicates $\rho = g$. When $\bar{\sigma}_W$ is allowed to assume positive values, there are other constellations of the coefficients for which equality holds. Although the formula indicates the theoretical possibility that the rate of interest may be lower than the rate of growth, whatever empirical evidence we have rules out this as a realistic case. Thus we may be justified to consider the equality as the limiting case.

From the data given by S.J. Patel, (Indian Economic Review, February 1956) it appears that 's/D' in India may lie somewhere between .5 and .3 depending on how one classifies income in the household sectors. Thus, if we assume a maximal rate of steady growth of income at 4 per cent, the rate of interest lies between 8 per cent and 12 per cent. It is obvious that with a larger rate of growth, the equilibrium value of the rate of interest goes up, or with a higher rate of savings, it falls.

The use of the above formula may enable us to calculate limits for the shadow rate of interest if our a priori knowledge strongly indicates that the "real scarcity" of capital is greater than would be indicated

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by the currently ruling rate of interest. In that case, the limits are given by the current rate, on the one hand, and by the formula connecting the rate of interest with the maximal rate of steady growth, on the other. The maximal rate of steady growth may be calculated on a first approximation from the set of data usually given in the two Leontief matrices. With the Leontief assumptions the maximal rate of steady growth is determined by the Frobenius root of the matrix $B(I-A)^{-1}$, which is naturally non-negative. For more general situations, the computational difficulties would be much greater.[11/]

There are two points that one should remember in this context:

(a) The rate of interest as calculated on the above approach is not "the rate of interest" as usually understood in connection with the capital or money market. This should be obvious, because the model does not introduce uncertainty and corresponding distinction between various types of assets.

(b) The rate of interest as deduced from the Solow formula is different from the pure rate of time discount. It takes into account both productivity and thrift. The influence of productivity is taken into account in the numerator, while the savings coefficient subsumes the influence of thrift. Behind thrift lies the factor of time preference. The rate of pure time discount that is involved may be estimated if we assume that the observed savings rate is the result of an operational decision to maximize the sum of discounted values of consumption over a period of time. This is similar to the famous Ramsey model of optimal savings. The difference consists in introducing a nonzero rate of time discount which Ramsey would have found ethically inappropriate, and in the further restriction that is involved in reducing the 'path maximum' problem to a 'point maximum' problem. By a 'point maximum' problem we mean the problem of maximizing an integral of discounted utilities, by a once-for-all choice of savings rate. The period of time may be finite or infinite,

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11/ In using the formula for the generalized Von Neumann situation, we should consider whether the decision on the savings rate is an optimal one or not. If no optimality considerations may be adduced for the savings coefficient, the rate of interest calculated from the Solow expression woul not measure the intrinsic scarcity of capital.

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depending on the planner's point of view. In the finite case, there should be a provision for terminal equipment. Then, for every savings rate, we can find the underlying rate of time preference.

This problem has been investigated by Tinbergen.[12/] He gives a number of equilibrium relations involving the rate of time discount, the savings rate, and the capital coefficient, each based on a specific hypothesis relating to the utility function. The utility function underlying the simplest problem is in his case a logarithmic one. It should, however, be noted that our problem here is the logical inverse to Tinbergen's problem. He is interested in finding out the optimum rate of savings corresponding to any given values of the capital-coefficient, and time preference In our case, we want to know the underlying time preference, assuming that the savings rate is already an optimal one, other parameters remaining the same.

The Tinbergen result can be generalized by introducing more general types of production functions and utility functions other than the logarithmic or hyperbolic ones considered by him. There is scope for much further investigations along these lines.

(b) We now consider the situation when all the sectors are not assumed to grow at the same proportionate rate, but all the relevant production functions have the needed convexity properties.

In this case, the relative prices and the interest rate are no longer constant. Further, since the rate of growth is not a unique number characterizing the entire process, we have to deal with the constantly changing moving equilibria, as it were, and the relation in which the growth rate stands to the rate of interest would therefore be continually shifting. Further, 'the growth rate' in this case is itself a somewhat ambiguous concept. Also, the various own rates of interest do not any longer equal the own rate of interest for the numeraire commodity. It therefore inescapably appears that we could say very little on the question without going the whole hog of solving a problem in dynamic programming. In principle, an optimal solution is always possible in case (b). But to do that we have to specify first the appropriate terminal conditions, the

12/ J. Tinbergen, "The Optimum Rate of Savings", Economic Journal, 1956.

/initial stocks

initial stocks and the time profile of consumption over the entire period. Having done that, we have to apply the usual techniques of maximization over time. Such problems have been considered in the earlier paper entitled "A Complete Model of Programme Evaluation." For a general reference, see Dorfman, Samuelson, and Solow, Linear Programming and Economic Analysis, Chapter 12.

IV. THE CALCULATION OF PRIORITIES

In this section we consider the method of calculating priorities in an investment programme by using shadow prices. We must bear in mind that while we calculate the benefit-cost ratios for a single project, we do it as of a given programme, and not for the project in isolation. This follows out of the fact that the projects are necessarily interlinked, and imply certain assumptions about the rest of the economy. Thus one project may be chosen from a set of competing projects, if the rest of the programmes may be assumed to be relatively unaffected by this choice.

We may also consider a more generalized situation where there is a technically nonseparable collection of projects which can be singled out for piecemeal decision making. Now in this case this whole collection has to be treated as one unit and the benefit-cost calculations have to be calculated for this one unit as a whole. The word 'technical nonseparability' is important in this connection. The assumption of linear homogeneity is crucial to the applicability of the shadowprice argument, as usually understood. This is because the logic of applying shadow prices is, in essence, an argument piecemeal decision-making. Piecemeal decision-making in situations characterized by increasing returns or significant external economies either leads to insufficient output or to no production at all. This causes difficulties with respect to the remainder of the programme where the outputs of the above sectors serve as technologically necessary ingredients. Thus, it appears that in these situations the better procedure is to solve the entire problem simultaneously as an example of coordinated decision-making.[13/]

13/ Possibilities of decentralized decision-making in situations characterized by the absence of classical homogeneity or independence assumption have been investigated in the important paper by Arrow, K.J. and Hurwicz, Lionel, in "Essays in Honour of Harold Hotelling," edited by R.W. Pfouts, Chapel Hill, North Carolina.

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The advantage of the shadow price technique becomes considerably greater if the complex of planning problems may be assumed to be decomposable into the following stages:

a) How much to invest in total over a number of years;

b) How to distribute the total investment resources among different sectors of the economy;

c) How to choose the best method of utilizing the resources allocated to a sector.

If the stages are strictly consecutive, we may think that the decision on level (b) is reached on the basis of maximizing income over a period of time subject to all the interdependencies in production, investment and consumption. This would roughly indicate how much to invest in each sector. If there are sectors like social overhead capital where investment is made on grounds independent of any maximization process, then we should consider the remaining sub-set of sectors for our decision purposes.

The decision on stage (c) can be reached on the basis of utilizing a shadow rate of interest and for a given time profile of production, on the requirement that the costs are minimized.

In theory as well as practice, the stages may not be that distinct, in which case decisions on (b) and (c) may have to be reached simultaneously. The shadow rate technique should then be replaced by the general methods of dynamic programming.

Now let us consider the problem quantitatively. We use the following notations:

$W_i(t)$ - The investment in the project per unit time.

$F_i(t)$ - The foreign component of investment per unit time. $F_i = aW_i$ where $0 \leq a \leq 1$.

g - The length of the gestation period.

n - The length of the operating period.

r - The shadow rate of interest.

k - The shadow rate of exchange.

$D(t)$ - The current operating expenses of a project.

Then the cost of a project may be calculated as follows:

We have $F_i = aW_i$

/Therefore H_i

Therefore $H_i = (1-a)W_i$ where H_i is the domestic component of investment.

Since we value the foreign investment component at the shadow exchange rate, we have:

$$kaW_i + (1 - a)\, W_i = W_i\, (ka + 1 - a)$$

$$= W_i \left\{1 - a\, (1 - k)\right\}$$

Let us assume that we know the timeshape on construction effort: W(t). Then the cost of investment in the project may be calculated as:

$$C = \sum_{t=g}^{o} W(t) \left\{1 - a\, (1 - k)\right\} (1 + r)^{-t} + \sum_{o}^{n} D\,(t)\, (1 + r)^{-t}$$

The first term on the left-hand side indicates the investment that is made during the gestation period of the project and the second part indicates the cost that is incurred during the exploitation period. Now the decision rule consists in minimizing "C" for a given time profile of 'output.' To put it differently the projects to be compared are those which give the same time profile of output, given by the over-all planning problem. Out of these projects, the one will be chosen which minimizes total cost, over the combined gestation and exploitation period of the project.

V. CONCLUSION

In this section we may briefly review the conclusions reached in the earlier sections and indicate the relevance of the shadow price concept with respect to a few practical problems encountered in Indian planning.

Briefly stated, our discussion has clearly indicated that the technique of using shadow prices serves as a useful computational short-hand in devising a relatively "efficient" system of programme evaluation. The qualification on "efficiency" arises because in the presence of non-convexities in the production processes of certain sectors, the shadow price device does not enable one to reach the "efficient" constellation of the system. The advantage from using shadow prices holds good even though the shadow prices we use are not exact, but merely approximations,

/although it

although it is important that they should be in the right direction. Given the data, the calculation of the shadow rate of exchange does not raise great difficulties. The simplified procedure indicated in this paper, or the more elaborate linear programming method discussed by Chenery may be usefully employed. With respect to the shadow rate of interest, the conceptual difficulties are greater. But if we use the approximation procedure outlined earlier in this paper, we get a range of 8 per cent to 12 per cent for the shadow rate of interest under Indian conditions. The exact shadow rate of interest may be higher than this, but it is unlikely that this would be lower than given by this range. This already gives us a basis for how to judge projects which are economic only if the rate of interest is 4 per cent or 4 1/2 per cent.

The relevance of the shadow prices to practical problems may be understood if we take into account the problem of choosing between importing fertilizer, or setting up a fertilizer plant, or a machinery for manufacturing fertilizer producing equipment. In the simple Austrian models, where choice is confined to a pair of alternatives, the cost of one is the opportunity foregone with the other projects. This is difficult to apply if there exists a manifold of possibilities for each unit of investment. Under such conditions, the opportunity cost of a unit of investment is measured by its shadow rate of interest. Similarly, the cost of a unit of import should be valued at the shadow rate of exchange, rather than at the official rate. Now, if we take, for example, a shadow rate of exchange of Rs. 6 to a dollar and a rate of interest lying between 8 per cent and 12 per cent, we may calculate the cost of each type of project, over the gestation period, given the time shape of the construction effort. Further, with a given time profile of 'output,' in this case agricultural production, we can calculate the total costs for each project, e.g., investment costs and operating costs. Naturally, with other things remaining the same, the project with the lowest cost should be chosen.

/The same

The same line of reasoning may be applied to other problems such as the choice between various types of power stations. An interesting contribution in this regard is the paper of Professor P.N. Rosenstein-Rodan on the contribution of atomic energy to India's development programme.14/

All this is to suggest the fruitfulness of the shadow price method in practical policy making, if appropriate qualifications are borne in mind.

14/ P.N. Rosenstein-Rodan, Contribution of Atomic Energy to a Power Programme, C/59-15.

/Appendix

Appendix

The Shadow Rate of Exchange: The General Case

This appendix deals with the case of how to determine the shadow rate of exchange where imports consist of different types of goods.

The price of each domestic commodity in domestic currency is given by the following equation:

$$P_i = k\,(A_{n+1,\,i}\,p_{n+1} + A_{n+2,\,i}\,p_{m_{n+2}} + \cdots + A_{n+j,\,i}\,p_{m_{n+j}}) \quad (i=1, 2\cdots n)$$

+ contribution of other primary factors.

Here $A_{n+1,\,i}$ is the cumulative coefficient of the first import commodity in the production of i^{th} domestic commodity. We have 'n' such equations for 'n' domestic commodities.

In addition we have the equation relating to the permissible balance of payments deficit:

$$C = k\left[\left\{(p_{n+j})'\,[v_1]\,[I-a]^{-1}\right\}(e + w + c) + (p_{n+j})'\,[v_2]\,\{w\} + (p_{n+j})'\left\{v_3\,(c, \{p_{n+j}\}', \{p_i\}')\right\}\right] - (p)'\,(e)$$

Thus we have $(n + 1)$ equations to determine $(n + 1)$ prices, 'n' domestic prices and one shadow rate of exchange.

The dimensionalities of above matrices and column vectors are as follows:

(i) $(p_{n+j})'$ is a row vector of the dimension $(1 \times j)$.

(ii) $[v_1]$ is a matrix of dimensions $(j \times n)$.

(iii) $[I-a]^{-1}$ is a matrix of dimension of $(n \times n)$. Thus the product has dimension $(1 \times n)$, hence a row vector.

(iv) $(e + w + c)$ is a column vector of dimensions $(n \times 1)$. Thus the first term in brackets is a scalar, indicating the total amount spent on imports of raw materials.

(v) $[v_2]$ is a matrix of dimensions $(j \times n)$.

(vi) $\{w\}$ is a column vector of dimensions $(n \times 1)$.

/(vii) The

(vii) The second term in brackets is (1 x 1), also a scalar, indicating the amount spent on imports of investment goods.

(viii) $v_3\ (c,p_{n+j})'$, $(p_i)'$ is a column vector of dimensions (j x 1). The third term is also a scalar, indicating the amount spent on imports of consumer goods.

(ix) $(p)'(e)$ is also a scalar since (p') is (1 x n) and (e) is (n x 1).

In this case, exports have been exogenously determined. We may also consider the more general case, where exports are determined from within the above set of calculations. This, however, requires a more complicated approach.