SCALE OF OPERATION, ALLOCATIVE INEFFICIENCIES AND SEPARABILITY OF INPUTS AND OUTPUTS IN AGRICULTURAL RESEARCH: EMBRAPA CASE STUDY

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RESUMO

Neste artigo consideram-se algumas propriedades de interesse da pesquisa agropecuária na Embrapa. São conduzidos testes estatísticos para estudar questões relacionadas à escala de operação das unidades produtivas, ineficiência alocativa e separabilidade de insumos e produtos. O processo de produção é avaliado via métodos não paramétricos com uso de modelos de Análise de Envoltória de Dados. O período de análise é 2002-2009. Conclui-se que a fronteira tecnológica da Embrapa opera sob retornos variáveis à escala, há ineficiência alocativa em subperíodos e é separável para insumos e produtos.

PALAVRAS-CHAVE: DEA, Eficiência, Pesquisa Agropecuária

Área principal: DEA Análise Envoltória de Dados

ABSTRACT

In this article we consider some properties of concern for research production at Embrapa. We address questions related to statistical tests for the scale of operation, the presence of allocative inefficiencies and separability of inputs and outputs. The production process is assessed by nonparametric methods with the use of a Data Envelopment Analysis frontier. The period of concern is 2002-2009. We conclude that Embrapa technology shows variable returns to scale, shows mild allocative inefficiencies in sub-periods, and is separable in inputs and outputs.

KEYWORDS: DEA, Efficiency, Agricultural Research.

Main area: DEA Data Envelopment Analysis

1. Introduction

The Brazilian Agricultural research Corporation (Embrapa) monitors, since 1996 the production process of 37 of its 42 research centers by means of a nonparametric production model. Measures of efficiency are computed using data envelopment analysis. For more details see Souza et al. (1997, 1999, 2007, 2010, 2011), Souza and Avila (2000).

Interest is on economic, technical and allocative measures of efficiency, computed under the assumption of cost minimization. Several important questions arise in the actual application of DEA in the monitoring process at Embrapa. Firstly there is the choice of whether aggregating or not the outputs. Embrapa has used for some time a weighted average of output variables as a single output variable in its production model. Aggregation assumes separability, a property not fully investigated in Embrapa production system. Aggregation, and separability, has been a longstanding subject of interest in the literature. See Berndt and Christensen (1973), Blackorby et al. (1977), Chambers and Färe (1993). Secondly, there is the assumption on the scale of operation. The assumption of constant returns to scale imposes harsher measures of efficiency in the evaluation process. Embrapa model seeks to reduce scale problems among research centers by measuring inputs and outputs on a per employee basis. A statistical test is in order to quantify differences related to the scale of operation after transformation to validate this procedure if constant returns is to be used as the final choice in the evaluation model. Finally, it is of importance for the institution to identify the sources of economic inefficiencies. Is it due to technical inefficiencies, to poor choice of input combinations or both?

Our approach to test for the presence of allocative inefficiencies, returns to scale, and separability of inputs and outputs for Embrapa production system follows closely to Banker and Natarajan (2004). The technological setting is similar to Banker et al. (2011).

Our discussion proceeds as follows. In Section 2 we describe Embrapa production system. In Section 3 we establish the technological nonparametric production setting that can be related to DEA and the parametric and nonparametric tests, which can be performed to the assessment of type of scale, allocative inefficiencies and separability. Crucial to validate the results put forward in this section are the consistency results of Banker (1993), Banker and Natarajan (2008) and the extensions of Souza and Staub (2007). In Section 4 we show empirical results based on the analysis by year. Finally, in Section 5 we summarize our results.

2. Embrapa Production Model

Embrapa research system comprises 37 research centers (DMUs), classified into three types (Ecological -13, Thematic - 9, Product - 15) and three sizes (Small - 11, Medium - 18, Large - 8). Input and output variables are defined from a set of performance indicators. This set comprises 28 outputs and 3 inputs.

We begin our discussion with the output. The output variables are classified into four categories: Scientific production; Production of technical publications; Development of technologies, products, and processes; Diffusion of technologies and image.

By scientific production we mean the publication of articles and book chapters aiming the academic world. We require each item to be specified with complete bibliographical reference.

The category of technical publications groups publications produced by research centers aiming, primarily, at agricultural businesses and agricultural production.

The category of development of technologies, products and processes groups indicators related to the effort made by a research unit to make its production available to society in the form of a final product. Only new technologies, products and processes are considered. Those must be already tested at the client's level in the form of prototypes, or through demonstration units, or be already patented.

Finally, the category of diffusion of technologies and image encompasses production variables related to Embrapa effort to make its products known to the public and to market its image.

The input side of Embrapa production process is composed of three variables. Personnel expenditure, Operational Costs (consumption materials, travel and services less income from production projects), and Capital (measured by depreciation).

All output variables are measured as counts and normalized by the mean. Likewise, the inputs are normalized by the mean. It is possible to combine outputs considering a weighted average of all categories of production. The weights are user defined and reflect the administration perception of the relative importance of each variable to each research center or DMU. Weights were assigned for both individual indicators, as for the four aggregated production categories. Therefore it is possible to use either the four categories or a single aggregated output as the response variable. This is indeed done in the efficiency analysis carried out at Embrapa, where cost and technical measures of efficiency are computed assuming combined and separate outputs. With economic measures, the question of allocation of inputs becomes important.

DEA models implicitly assume that the DMUs are comparable. This is not strictly the case in Embrapa. To make them comparable it is necessary an effort to define an output measure adjusted for differences in operation and perceptions. At the level of the partial production categories we induced this measure allowing a distinct set of weights for each DMU and measuring combined production at a per employee basis.

A personnel score was created for each unit dividing its number of employees by the company's mean of this variable. Outputs and inputs were normalized by this score. This further established a common basis to compare research units (regarding scale) and avoided the incidence of spurious efficient units and zero output (shadow) prices, another common occurrence in multiple output models, and also a disturbing fact for management interpretation.

We see the use of ratios to define production variables in our application as unavoidable. Different denominators are used with the virtue of being independent of sizes of the units. This characteristic facilitates comparisons between units and allows the assumption of a common production function. In the context of a pure DEA analysis, the problem of efficiency comparisons may be solved imposing the BCC assumption. See Hollingsworth and Smith (2003) and Emrouznejad and Amin (2007). These authors state that when using ratio variables, the constant returns to scale assumption is not valid. In this context a comparison of CCR and BCC solutions is in order.

DEA models are known to be sensitive to outliers. In our application control of outliers is particularly import for output variables. In this context we use box plot fences to adjust the values of outlying observations. Values above $Q3 + 1.5(Q3 - Q1)$ are reduced to this value for any variable. Here $Q1$ and $Q3$ denote the first and third quartiles, respectively.

3. Technology Set and DEA Estimation

Here we follow Banker(1993) and Banker et al. (2011). Let $x_j > 0$ and $y_j > 0$, $j = 1,...,n$, be the observed input and output vectors in a sample of n observations generated from the underlying technology set $T = \{(x, y); \text{output } y \text{ can be produced from inputs } x\}$.

The efficiency of an observation (x_j, y_j) is defined by $\theta(x_j, y_j) = \inf_\eta \{\eta; (\eta x_j, y_j) \in T\}$.

Banker et al. (2011) assume the following minimal structure for the technology set T and the probability density function $f(\theta)$ for the inefficiency θ:

– The technology set is convex if $(x_1, y_1) \in T$, then $(x_2, y_2) \in T$ if $x_2 \geq x_1$ and $y_2 \geq y_1$.

– The support of $f(\theta)$ is the interval (0,1) if $\delta \in (0,1)$ then $\int_\delta^1 f(\theta)d\theta > 0$.

Under these assumptions one can show consistency and convergence in distribution for the variable returns to scale linear programming solution (Banker, 1993; Souza and Staub, 2007):

$\hat{\theta}(x_j, y_j) = \min_{\lambda,\eta} \eta$ subject to the conditions $Y\lambda \geq y_j$, $X\lambda \leq \eta\, x_j$, $\lambda 1 = 1$ and $\lambda \geq 0$.

Here $Y = (y_1,..., y_n)$ is the output matrix and $X = (x_1,..., x_n)$ is the input matrix.

One says that the technology T shows constant returns to scale if $(x, y) \in$ T implies $(kx, ky) \in$ T$, k > 0$. In this case, a consistent and asymptotically convergent in distribution estimator is obtained removing the convexity condition $\lambda 1 = 1$.

Banker and Natarajan (2004) and Banker et al. (2011) suggest three statistical tests to examine the assumption constant vs variable returns to scale. Two are based on specific assumptions on the density function (exponential and half-normal distributions) of the efficiency measure, and the third is a nonparametric test. Our choice is for the nonparametric test, which is based on the Smirnov-Kolmogorov two sample statistic.

Now we turn our attention to separability of inputs and outputs. We begin with complete input separability. Following Banker et al. (2011), the technology set under this assumption becomes $T^{Sinp} = \bigcap_{g=1}^{s} T_{Sep}^{inp_g}$; $T_{Sep}^{inp_g} = \left\{ (x = (x^g, x^{s-1}), y); x^g \text{ may produce } y \right\}$. Here s is the number of inputs and x^g is one component, and x^{s-1} the remaining components of the *s*-vector *x*.

For output separability one obtains $T^{Soutp} = \bigcap_{g=1}^{l} T_{Sep}^{out_g}$; $T_{Sep}^{out_g} = \left\{ (x, y = (y^g, y^{l-1}), y); x \text{ may produce } y^g \right\}$. Here *l* is the number of outputs and y^g is one component, and y^{l-1} the remaining components of the *l*-vector of the *l*-vector *y*.

Under the assumption of separability of inputs, the efficiency of firm *j* is given by $\theta^{Sinp}(x_j, y_j) = \inf_{\eta} \left\{ \eta; (\eta\, x_j, y_j) \in T^{Sinp} \right\}$. This is given by $\theta^{Sinp}(x_j, y_j) = \max_{g=1...s} \inf_{\eta} \left\{ \eta; (\eta\, x_j, y_j) \in T^{Sinp_g} \right\}$ or $\theta^{Sinp}(x_j, y_j) = \max_{g=1...s} \theta(x_j^g, y_j)$.

Under separability of inputs this quantity can be estimated calculating a DEA coefficient under constant or variable returns to scale considering, in turn, a DEA estimate $\hat{\theta}(x_j^g, y_j)$ for each input, and computing the maximum of these measurements. One obtains a similar estimate under output separability.

Under separability of outputs, a similar quantity can be estimated computing the DEA estimates $\hat{\theta}(x_j, y_j^g)$ for each output and computing the maximum of these measurements. The statistical assessment of separability is performed again via Smirnov-Kolmogorov test statistics.

Finally, the existence of allocative inefficiencies is investigated exploring the decomposition of economic (cost) efficiency into technical efficiency and allocative efficiency. If a firm is allocatively efficient, then technical and cost efficiency will be the same. That is the idea put forward in Banker and Natarajan (2004), where the two measurements are to be compared via a nonparametric statistical test like the Smirnov-Kolmogorov empirical distributions two sample test. We note here that technical efficiency information may be retrieved from cost data on inputs.

4. Empirical Results

Table 1 shows descriptive statistics for the efficiency measures of concern in our study. These are cost efficiency (BCC_1), technical efficiency under constant returns to scale (CCR_3), technical efficiency under variable returns to scale (BCC_3), allocative efficiency (ALLOC) and technical efficiencies computed under the assumptions of separability of inputs (SEP_X) and outputs (SEP_Y), respectively. Orientation in all DEA models is for inputs and technical efficiencies are computed, typically, with four outputs and three inputs. Cost efficiency is calculated with four outputs and one input (aggregated cost).

Looking at medians and quartiles we see large differences regarding the assumptions of scale. These differences are further highlighted in Figure 1, where one sees other quantiles under each assumption quite distinct. In the context of formal statistical test, only in 2006 the Smirnov-

Kolmogorov statistics shows a non significant p-value of 13,4%. Even in this case Figure 1 shows a distortion from the null hypothesis of no scale effect.

As for separability, we do not detect significant differences at the 5% level for the Smirnov-Kolmogorov statistics in none of the years. The assumption seems to hold for both inputs and outputs. The scatter in graphs of Figures 2 and 3 are closer to the reference lines than in Figure 1. The p-values for separability of inputs are 100%, 98,2%, 88,8%, 98,2%, 100%, 100%, and 98,2% for years 2002 to 2009, respectively, and 35,3%, 7,6%, 7,6%, 13,4%, 22,4%, 22,4%, 7,6%, and 7,6% for outputs, respectively in the same years. Results are stronger toward separability for inputs than for outputs. This is confirmed in Figures 2 and 3 where one sees a closer agreement with the reference lines among the quantiles for inputs than for outputs.

There are statistically significant allocative inefficiencies for almost all years. Corresponding p-values for Smirnov Kolmogorov test statistics are 1%, 0.4%, 0.03%, 7,6%, 13,4% ,7,6% , 7,6%, and 4% for years 2002-2009, respectively. On the other hand, however, it should be pointed out that the annual medians of allocative efficiencies are all above 90% (exception of 2004 with 88%), indicating proper choices of input mixes. In this case the Smirnov-Kolmogorov test statistics seems to be detecting small deviations from the null.

5. Summary and Conclusions

For Embrapa research production model we investigated the properties of returns to scale, proper choice of input mixes, and separability of inputs and outputs.

The assumption of constant returns to scale is rejected leading to the more flexible variable returns and higher values of the DEA measures of efficiency. The scale adjustments carried out by the company measuring production to a per employee basis scale problems were not fully succeeded to overcome scale of operation differences.

Allocative efficiency is very high for all years, although one notices, in sub-periods, statistically significant differences relative to a variable returns cost technology frontier.

Inputs and outputs are separable. This implies that aggregation is justifiable on both sides of production. The implications of this result to Embrapa are important. For inputs, separability means that the influence of each of the inputs on the output is independent of other inputs, emphasizing the need for controlling input effects. For outputs, it implies that the same efficiency level could be obtained considering as production response variables the output projections. In this context combined output weighted averages may be computed in Embrapa to impose the administration perceptions in the production process and to reduce any biases noticed in the process in the direction of an unwanted group of variables. This justifies the use of combined outputs by the company in the evaluation process.

The differences between the use of separate and combined outputs can be seen in the median evolution through 2002-2009. For separate outputs the figures are 1.00, 0.99, 0.97, 1.00, 1.00, 0.97, 1,00, 0.98, respectively, and for a weighted average combined output the figures are significantly lower: 0.90, 0.85, 0.87, 0.89, 0.91, 0.87, 0.88, and 0.88.

6. Acknowledgment

Table 1: 5 Number Summaries for Cost Efficiency (BCC_1), Technical Efficiency under Constant returns to Scale (CCR_3), Variable Returns to Scale (BCC_3), Allocative efficiency (ALLOC) and Technical Efficiencies   under Separability for Inputs (SEP_X) and Outputs (SEP_Y).

|		BCC_1	CCR_3	BCC_3	ALLOC	SEP_X	SEP_Y
	Min	0,4618	0,3739	0,6673	0,5802	0,6673	0,6462
	Q1	0,6722	0,6412	0,8252	0,7938	0,8252	0,8252
2002	Median	0,8388	0,8432	1,0000	0,9340	1,0000	0,9579
	Q3	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	Max	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	Min	0,4246	0,2619	0,6748	0,6208	0,6748	0,6466
	Q1	0,7312	0,5691	0,8785	0,8323	0,8683	0,8555
2003	Median	0,8405	0,8408	0,9866	0,9273	0,9255	0,9202
	Q3	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	Max	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	Min	0,6355	0,2719	0,7046	0,6493	0,6850	0,6687
	Q1	0,7228	0,6538	0,8872	0,8293	0,8703	0,8585
2004	Median	0,8144	0,8770	0,9749	0,8810	0,9410	0,9286
	Q3	0,9283	1,0000	1,0000	0,9858	1,0000	1,0000
	Max	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	Min	0,3032	0,2831	0,7763	0,3906	0,7763	0,7736
	Q1	0,8022	0,6949	0,9323	0,8575	0,8560	0,8682
2005	Median	0,8927	0,9103	1,0000	0,9398	1,0000	0,9815
	Q3	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	Max	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	Min	0,5916	0,3548	0,7661	0,7121	0,7661	0,7661
	Q1	0,8169	0,7800	0,8687	0,9307	0,8584	0,8584
2006	Median	0,9773	0,9371	1,0000	0,9831	1,0000	1,0000
	Q3	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	Max	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	Min	0,4150	0,3936	0,7275	0,5524	0,7275	0,7275
	Q1	0,7370	0,6091	0,8400	0,8539	0,8333	0,8274
2007	Mediana	0,8567	0,8009	0,9680	0,9272	0,9327	0,9129
	Q3	0,9735	1,0000	1,0000	0,9780	1,0000	1,0000
	Max	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	Min	0,2998	0,3872	0,6575	0,4560	0,6575	0,6470
	Q1	0,7725	0,6413	0,8532	0,8745	0,8411	0,8311
2008	Median	0,8785	0,8429	1,0000	0,9178	0,9722	0,9748
	Q3	1,0000	0,9731	1,0000	1,0000	1,0000	1,0000
	Max	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	Min	0,3447	0,5066	0,6354	0,5424	0,6354	0,6354
	Q1	0,8070	0,7135	0,9085	0,8830	0,8919	0,8817
2009	Median	0,9130	0,9068	0,9796	0,9701	0,9649	0,9694
	Q3	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
	Max	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
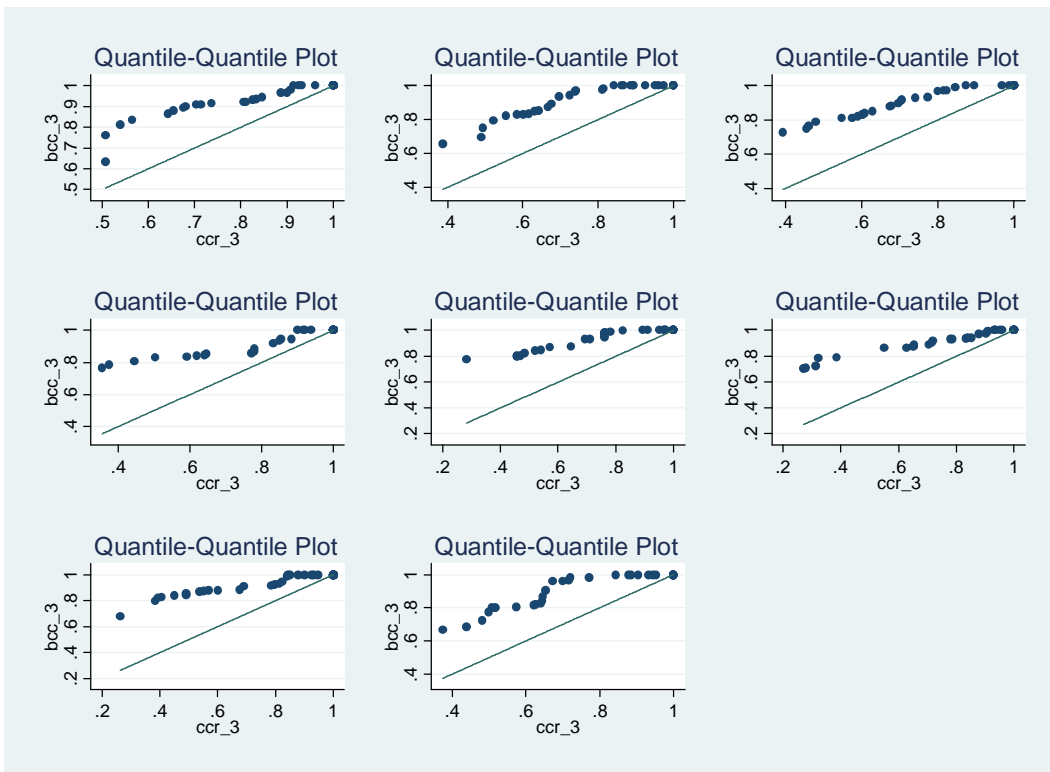
Figure 1: Quantile-Quantile Plots of technical efficiency measures under variable returns to scale (BCC_3) and constant returns to scale (CCR_3) by year – 2009 to 2002 in row order.
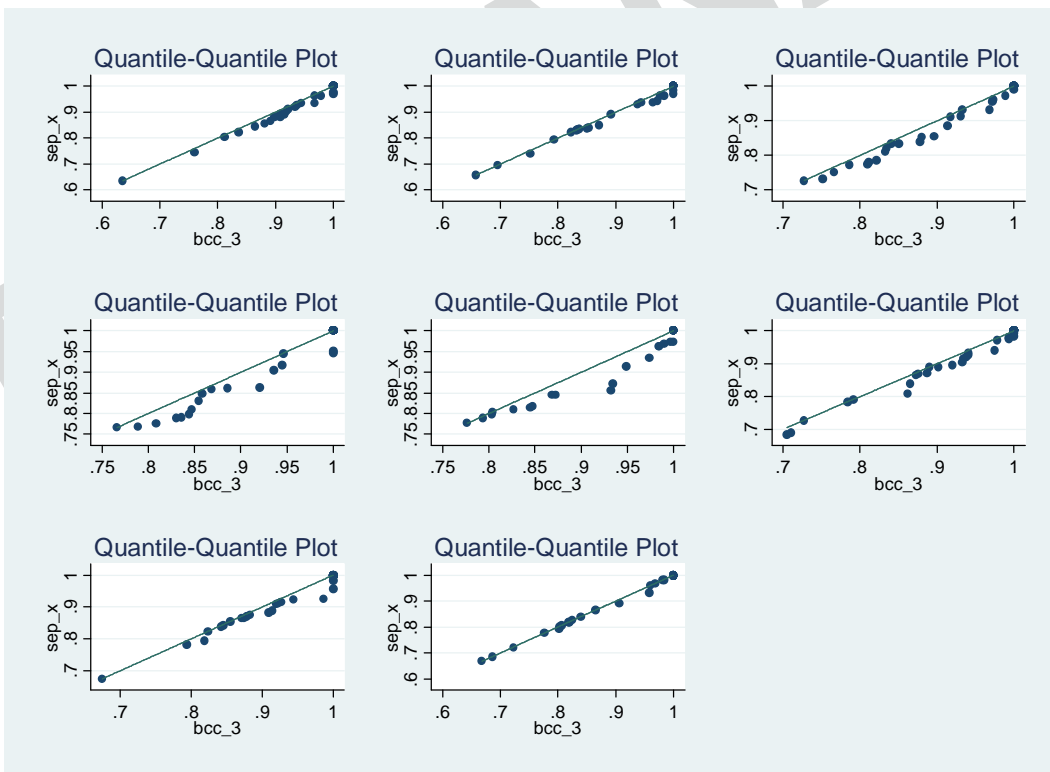

Figure 2: Quantile – Quantile Plots for investigation of input separability. SEP_X is technical efficiency under input separability and variable returns to scale and BCC_3 is technical efficiency under variable returns to scale.
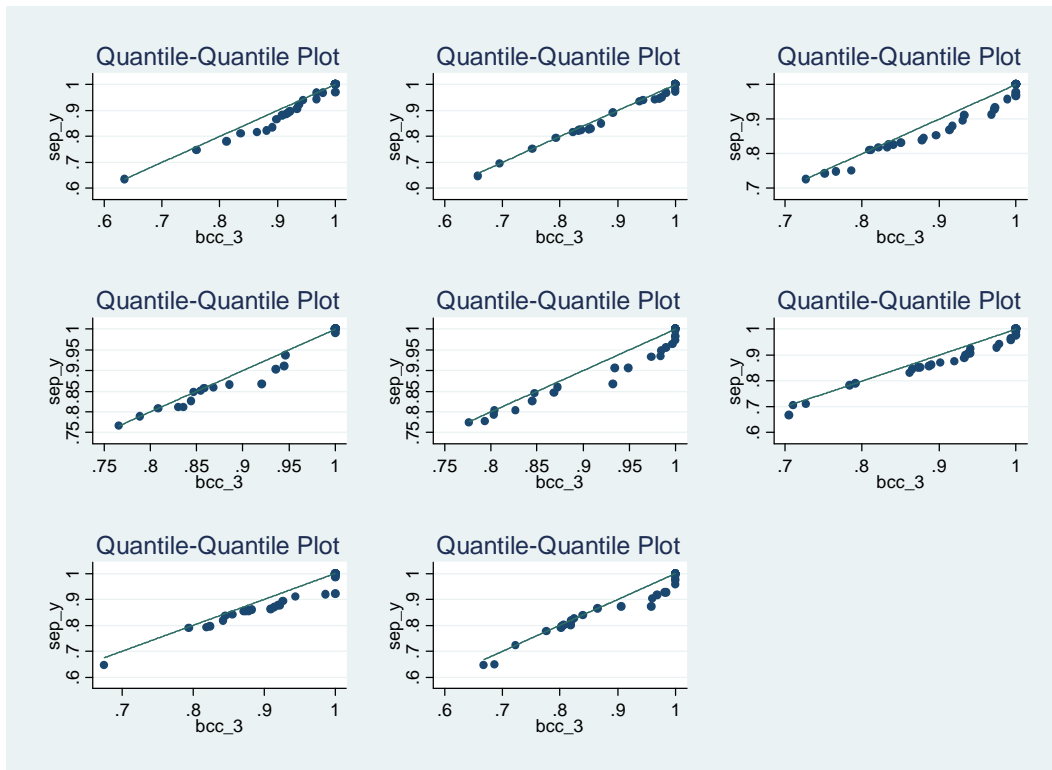
Figure 3: Quantile – Quantile Plots for investigation of output separability. SEP_Y is technical efficiency under output separability and variable returns to scale and BCC_3 is technical efficiency under variable returns to scale.

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