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# Simple method for measuring the linewidth enhancement factor of semiconductor lasers

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# Simple method for measuring the linewidth enhancement factor of semiconductor lasers

## **Abstract**

A simple method for measuring the linewidth enhancement factor (LEF) of semiconductor lasers (SLs) is proposed and demonstrated in this paper. This method is based on the self-mixing effect when a small portion of optical signal intensity emitted by the SL reflected by the moving target re-enters the SL cavity, leading to a modulation in the SL's output power intensity, in which the modulated envelope shape depends on the optical feedback strength as well as the LEF. By investigating the relationship between the light phase and power from the well-known Lang and Kobayashi equations, it was found that the LEF can be simply measured from the power value overlapped by two SLs' output power under two different optical feedback strengths. Our proposed method is verified by both simulations and experiments. (C) 2015 Optical Society of America

## **Keywords**

semiconductor, enhancement, linewidth, measuring, method, simple, lasers, factor

## **Disciplines**

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# Simple method for measuring the linewidth enhancement factor of semiconductor lasers

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**A simple method for measuring the linewidth enhancement factor (LEF) of semiconductor lasers (SLs) is proposed and demonstrated in this paper. This method is based on the self-mixing effect when a small portion of optical signal intensity emitted by the SL reflected by the moving target re-enters the SL cavity, leading to a modulation in the SL's output power intensity, in which the modulated envelope shape depends on the optical feedback strength as well as the LEF. By investigating the relationship between the light phase and power from the well known Lang and Kobayashi (L-K) equations, it was found that the LEF can be simply measured from the power value overlapped by two SLs' output power under two different optical feedback strengths. Our proposed method is verified by both simulations and experiments. © 2015 Optical Society of America**

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## 1. INTRODUCTION

It is well known that semiconductor lasers (SLs) play a key role in the emerging field of optoelectronics, such as optical sensors, optical communication and optical disc system. For these applications the linewidth enhancement factor (LEF), also called as the alpha factor or  $\alpha$ -parameter, is a fundamental descriptive parameter of the SL that describes the characteristics of SLs, such as the spectral effects, the modulation response, the injection locking and the response to the external optical feedback [1, 2]. Therefore, the knowledge of the value of the LEF is of great importance for SL based applications.

It has been proved that LEF exhibits a strong dependence between the refractive index  $\eta$ , gain  $G$  and the injected carrier density  $N$ , and is defined as following [3-6]:

$$\alpha = \frac{\chi^R}{\chi^I} = -2 \frac{\omega}{c} \frac{\partial \eta / \partial N}{\partial G / \partial N} \quad (1)$$

where  $\chi$  is the complex electric susceptibility. Superscripts "R" and "I" denote the real and imaginary parts of  $\chi$ .  $\omega$  and  $c$  are the angular optical frequency of the SL and the speed of light respectively.

In the past three decades, various techniques [2-4, 7-17] are developed for measuring the LEF, and these techniques can be mainly classified into two categories based on the amount of injection current to the SL. For the first category, the injection current is below the threshold and in this situation, the LEF is regarded as a material

parameter and is measured according to the definition of the LEF in Eq. (1). In the second category the injection current is above or close to the threshold and a mathematical model for measuring the LEF was developed from the rate equations of the SL. In this situation, the LEF is considered as a model parameter or effective parameter which is detached from its physical origin to a certain extent [4, 18]. Among the techniques in the second category, optical feedback method is an emerging and promising technique which does not require high radio frequency or optical spectrum measurements, thus providing an ease of implementation and simplicity in system structure [3, 19].

The optical feedback technique is based on the self-mixing effect that occurs when a small fraction of light emitted by the SL reflected by the moving target re-enters the SL cavity, leading to both modulated amplitude and frequency of the SL power [20]. The modulated SL optical power is known as the self-mixing signal (SMS) which carries the information associated with the SLs' parameters.

Based on the optical feedback technique, various methods were proposed for measuring the LEF. In 2004, Yu et al. [3] proposed an approach which can obtain LEF by geometrically measuring the SMSs' waveform. However, this approach requires the SMS to have zero crossing points, which means the optical feedback level (denoted as  $C$ ) falls within a small range, i.e.,  $1 < C < 3$  which is difficult to achieve for some types of lasers. Additionally, the movement trace of the target must be away from and back to the SL at a constant speed, which is also difficult in practice. Later, in the following years, several approaches [13-15, 21] for measuring the LEF were developed, and these approaches are mainly based on the numerical optimization for

minimizing the cost functions in parameters. Similar to [3], these methods are also restricted to certain feedback levels, e.g, approaches in [13-15] requires a weak feedback level, i.e.,  $0 < C < 1$ , and method in [21] requires a moderate feedback level. Furthermore, these methods are quite time consuming due to large data samples to be processed. Recently, two different approaches [16, 17] were developed for measuring the LEF over a large range of  $C$  but they still face the problem of large amount of computation time.

In this letter, by investigating the relationship between the light phase and output power from the Lang and Kobayashi (L-K) equation, a simple method based on the optical feedback technique for measuring the LEF is proposed in order to lift the above mentioned limitations.

## 2. THEORY

Optical feedback technique for LEF measurement relies on a theoretical model based on the stationary solutions of the well known Lang and Kobayashi (L-K) rate equations [22] which describes the dynamics of an SL with optical feedback. The model mainly consists of the following two equations [20]:

$$\phi_F = \phi_0 - C \cdot \sin[\phi_F + \arctan(\alpha)] \quad (2)$$

$$g(\phi_0) = \cos(\phi_F) \quad (3)$$

where  $\phi_F$  and  $\phi_0$  are the phase corresponding to the perturbed and unperturbed laser frequency respectively and  $g(\phi_0)$  is the SMS.  $\phi_0$  is associated with the target movement trace  $L$  and is given as  $\phi_0 = 4\pi L/\lambda_0$ , where  $\lambda_0$  is the unperturbed laser wavelength. Clearly, there is a straightforward procedure to establish  $g(\phi_0)$  when  $L$  varies, i.e.,  $\phi_0 \rightarrow \phi_F \rightarrow g$ . During the establishment, another important parameter i.e., optical feedback level  $C$ , determines the shape of  $g(\phi_0)$ . For a target subjected to a simple harmonic vibration,  $g(\phi_0)$  is symmetric with a sinusoidal-like fringes when  $0 < C < 1$ . When  $C > 1$ ,  $g(\phi_0)$  shows asymmetric hysteresis sawtooth-like fringes. Figure 1 shows the relationship between  $\phi_0$  and  $\phi_F$  as well as  $\phi_0$  and  $g(\phi_0)$  for three different values of  $C$ , whereas  $\alpha$  is fixed to 4.0. The mechanism behind the relationship between  $g(\phi_0)$  and  $C$  has been well-established and presented in [23, 24].

From Eq. (2), it is interesting to notice that when  $\phi_0 = m\pi - \arctan(\alpha)$ , where  $m$  is an integer,  $\phi_F$  always equals to  $\phi_0$  no matter what value of  $C$  is, i.e.:

$$\phi_F = \phi_0 = m\pi - \arctan(\alpha) \quad (4)$$

The red solid line with circles in Fig. 1(a) shows the relationship of  $\phi_0 = \phi_F$ . Fig. 1(a) also shows the relationship between  $\phi_0$  and  $\phi_F$  for different  $C$  values, namely  $C = 0.5, 2.0, 4.0$ . It can be seen that these curves always intersect at the points corresponding to  $\phi_F = \phi_0 = m\pi - \arctan(\alpha)$ . Consequently, the SMSs  $g(\phi_0)$  with different  $C$  values also intersect at these points, as shown in Fig. 1(b). Note that the points of  $\phi_F = \phi_0 = k\pi - \arctan(\alpha)$ , where  $k$  is an odd number, corresponds to the unstable mode as it does not meet the condition of  $d\phi_F/d\phi_0 > 0$  [24], and this mode does not involve in the process of constitution of  $g(\phi_0)$ . Therefore, for the points  $\phi_F = \phi_0 = 2m\pi - \arctan(\alpha)$ , according to Eq. (3), we have:

$$g = \cos[2m\pi - \arctan(\alpha)] = \frac{1}{\sqrt{1 + \alpha^2}} \quad (5)$$

or

$$\alpha = \sqrt{\frac{1}{g^2} - 1} \quad (6)$$

Thus equation (6) provides us a simple approach to measure the LEF. For example, with two SMSs (denoted as  $g_1(\phi_0)$  and  $g_2(\phi_0)$ ) of different  $C$  values available, we can work out the intersect point by checking the zero-crossing point of  $g_1(\phi_0) - g_2(\phi_0)$ , and use the corresponding  $g$  to calculate  $\alpha$  according to Eq. (6).

Note that the accuracy of the proposed method depends on the level of noise in the SMS. In order to reduce the noise, we can perform the following two ways to obtain the value of the LEF:

- (1) When two sets of SMSs are available, we may obtain multiple zero-crossing points of  $g_1(\phi_0) - g_2(\phi_0)$ , each will yield an  $\alpha$  value, and we take the average of them as the final result. This will lead to a better estimation of the  $\alpha$ .
- (2) When more than two sets of SMSs of different  $C$  values are available, e.g.,  $g_0(\phi_0), g_1(\phi_0), \dots, g_{p+1}(\phi_0)$ , where  $p$  is an even number. We can divide the SMSs into pairs, each of which can give the value of  $\alpha$  from the intersect point, and the average of them will lead to a more accurate estimation of the  $\alpha$ . This can be done by checking the zero-crossing point of the following:

$$\frac{2}{p} \sum_{i=0}^{p/2} [g_{2i}(\phi_0) - g_{2i+1}(\phi_0)] \quad (7)$$

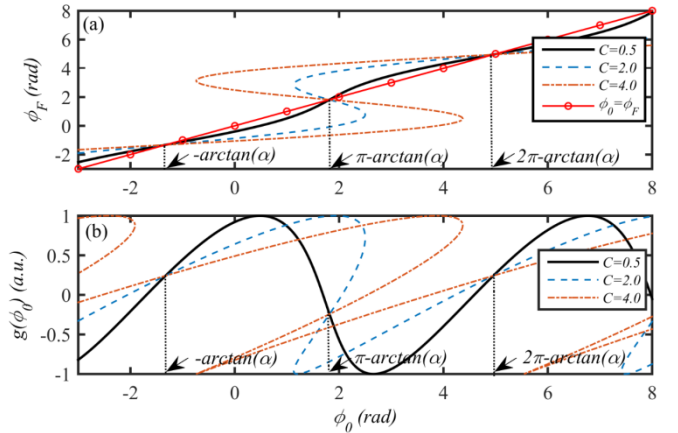


Fig. 1. The relationship between (a)  $\phi_0$  and  $\phi_F$  as well as (b)  $\phi_0$  and  $g(\phi_0)$  for a fixed value of  $\alpha = 4.0$ .

## 3. SIMULATION

To verify our proposed approach, we firstly carry out the computer simulations to generate SMSs. Without loss of generality,  $\lambda_0$  is set to be  $830nm$ , and the external target is assumed to be subjected to a simple harmonic vibration, i.e.,  $L(n) = L_0 + \Delta L(n) = L_0 + \Delta L \cos(2\pi \frac{f_0}{f_s} n)$ , where  $L_0 = 0.25m$ ,  $\Delta L = 1.18\mu m$ ,  $f_0 = 200Hz$  and  $f_s = 100kHz$  are respectively the initial external cavity length, vibration amplitude, vibration frequency and sampling frequency.  $n$  is the discrete time series. Thus  $\phi_0(n)$  can be obtained as  $\phi_0(n) = 3.8 \times 10^6 + 5.7\pi \cos(0.0126n)$ . Then the SMS can be obtained from Eqs. (2) and (3) [24] for a given set values of  $C$  and  $\alpha$ . Assuming two sets of the SMSs are available, Fig. 2 shows the vibration trace as well as two SMSs for two different values of  $C$  when  $\alpha$  is

preset to 4.0. Note that the SMSs are superposed by a noise signal with SNR=20dB.

From Fig. 2(b), we can see that there are ten intersect points between the two SMSs, as indicated by the large black dots which correspond to the condition of  $\phi_f = \phi_0 = 2m\pi - \arctan(\alpha)$ . Then the value of  $\alpha$  can be obtained using the first option mentioned in the Section 2. The accuracy is considered as the standard deviation (denoted as  $\sigma$ ) of all the calculated data. The calculated results of  $\hat{\alpha}$  and  $\sigma$  are respectively 3.81 and 6.5%.

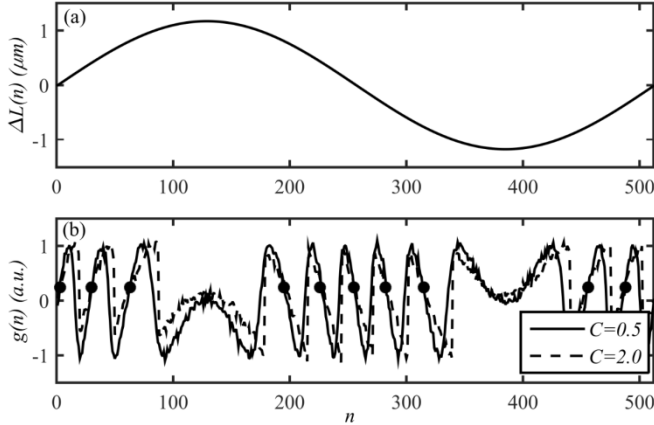


Fig. 2. Simulation results of SMSs for two different  $C$  values when  $\alpha = 4.0$ . (a) the vibration trace (b) two SMSs for  $C = 0.5$  (solid line) and  $C = 2.0$  (dotted line).

Similarly to the above case, computer simulations have been performed for various preset values of  $\alpha$  and the corresponding estimated values of  $\hat{\alpha}$  are presented in Table I. In Table I, we also present the  $\hat{\alpha}$  values calculated using the approach in [3]. Note that the approach in [3] is only valid when the feedback level is moderate, i.e.,  $1 < C < 3$  and the value we choose for the results in Table I is  $C = 2.5$ . Also from Table I, we can see that even for the value of  $C$  falling in the range of  $1 < C < 3$ , the approach in [3] is still not valid when  $\alpha$  is equal or less than 1.0 because there is no zero crossing points of SMSs, which is essential for utilizing the approach in [3]. From Table I, it can be seen that our proposed method is more accurate and has wider practical utility.

#### 4. EXPERIMENT

The experimental set-up for verifying our proposed approach is presented in Fig. 3. The light emitted by the SL is focused by a lens and then hits on the target which is a loudspeaker vibrating harmonically. The SMS is detected by the Photo Diode (PD) packaged at the back of

the SL and is acquired by a data acquisition unit (DAU). The optical feedback level is adjusted by the attenuator inserted between the lens and loudspeaker. The attenuator we used is NDC-50C-2M-B (provided by the Thorlabs) which is continuously variable density filter with angular graduations mounted on a rotating axle.

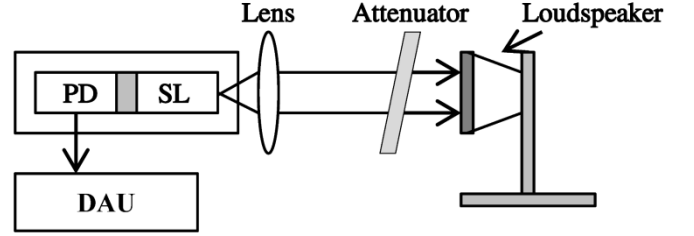


Fig. 3. Experimental set-up for measuring the LEF.

In the experiment, the SL is a single mode quantum well laser from Hitachi (HL8325G). The SL is biased with a DC current of 70mA and its temperature is stabilized at  $25 \pm 0.1^\circ\text{C}$  which corresponds to a threshold current of 45mA and a wavelength of 830nm. The loudspeaker is placed 0.25m away from the SL and driven by a sinusoidal signal with a frequency of 200Hz and peak-peak voltage of 200mV.

Figure 4 shows the two SMSs acquired by the experimental set-up shown in Fig. 3. The  $C$  values of the two SMSs are respectively 0.82 (solid line in Fig. 4) and 2.94 (dashed line in Fig. 4). Note that the value of  $C$  can be obtained from the waveform of an SMS using the method reported in [24, 25]. Using the first way described in section 2, we are able to obtain  $\hat{\alpha}$  as 3.19 and  $\sigma$  as 4.05%, whereas  $\hat{\alpha}$  and  $\sigma$  are respectively calculated as 3.01 and 6.80% using the approach in [3].

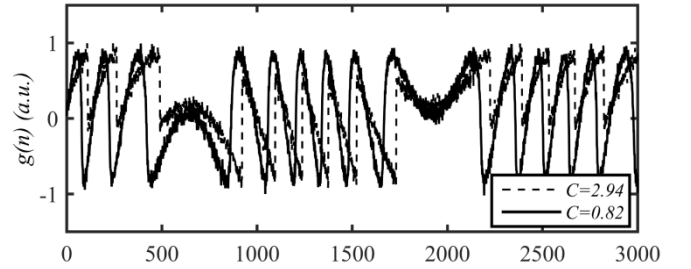


Fig. 4. Experimental signals of SMSs for two different  $C$  values.

Table I. Simulation results of the LEF obtained using the first way described in Section 2 where the SMSs are superposed a noise signal with SNR=20dB

Preset $\alpha$		0.1	0.5	1.0	2.0	4.0	6.0	8.0
Estimated $\hat{\alpha} / \sigma$ using the approach in	this paper	0.19/7.8%	0.37/6.9%	0.78/7.1%	1.83/5.1%	3.81/6.5%	6.14/4.1%	7.91/3.8%
	[3]	N.A	N.A	N.A	1.78/4.5%	3.71/7.1%	5.81/5.6%	8.21/5.1%

#### 5. CONCLUSION

A new and simple method for measuring the LEF based on the optical feedback technique by investigating the relationship between the light phase and power is demonstrated. The proposed method is superior over the existing methods due to the following two aspects: (i) the proposed method eliminates the feedback level restrictions used in previous methods, thus allowing the experimental design

simpler, (ii) the LEF can be simply determined from the intersect point of two different SMSs' waveforms, thus providing a fast and easy measurement technique for the LEF.

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