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Mathematical Models and Solution Approach for Staff Scheduling with Cross-Training at Call Centers

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MATHEMATICAL MODELS AND SOLUTION APPROACH FOR STAFF SCHEDULING WITH CROSS-TRAINING AT CALL CENTERS

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By

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ABSTRACT

Kilincli Taskiran, Gamze Ph.D., Engineering Ph.D. Program, Wright State University, 2015. Mathematical Models and Solution Approach for Staff Scheduling with Cross-Training at Call Centers.

Call centers face demand that varies throughout the week across multiple service categories and typically employ non-standard workforce schedules to meet this demand. In call centers, cross-training provides a buffer against fluctuation of demand between categories and is widely used. Full cross-training, however, is financially impractical in most cases, which has created a challenging problem in how to optimize a cross-trained workforce, i.e., a) what categories should be cross-trained, b) what portion of the workforce should be cross-trained, and c) how to schedule their weekly assignments. This problem is motivated by the need of a Fortune 50 company's technology support center to schedule its workforce with multiple service categories.

To solve this problem to its fullest extent, a mixed integer programming model that addresses staff assignment composition, shift scheduling, days off assignment, and break assignment across multi-skilled agents is proposed. The model is gigantic in size with thousands of general integer variables and is hard to solve. To improve computational efficiency, a two-phase sequential optimization approach is developed. The first phase is to find the optimal composition of the workforce to decide what categories should be cross-trained and when they should be deployed; the second phase is a staff scheduling model to find the size of the workforce with their skill sets and their shifts and weekly tours. The two-phase approach is an order of magnitude faster than the original model and is able to obtain better solutions orders of magnitude faster.

Experimental results with real data from the company clearly demonstrate the significance of cross-training; even partial limited cross-training, where 30% - 40% of the workforce is cross-trained with limited (two out of nine) skills per agent, results in considerable performance improvements. The model, when tested in the strategic analysis of the staff composition, suggested an estimated savings of 4% - 9% on staffing cost with an improved service level. Compared with other flexibility options such as part-time shifts, experiment results seem to suggest that cross-training could be a more effective approach to hedge against demand fluctuations when multiple service categories are involved.

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Dedicated to:

Defne Taskiran
My Lovely Daughter

CHAPTER 1 THE STAFF SCHEDULING PROBLEM AT CALL CENTERS

1.1 Introduction

Service organizations such as call centers typically face demand that varies throughout the day and the days of the week across multiple service categories, each requiring different skills. To cope with this time varying demand, call centers employ a flexible workforce assigned to various shifts and cross-train the workforce with multiple skills to balance workload across service categories. For service centers, staffing cost could comprise as much as 60% to 70% of the total cost, thus solution of the staff scheduling problem is critical to the profitability and competitiveness of these service centers – poor staff schedules can either lead to an over-staffing that incurs a high cost or an under-staffing that undermines the service quality and causes loss of business (Cleveland, 2009).

The typical staff scheduling problem, the process of determining the staff schedule, is typically solved in a hierarchical framework in several steps: a) determining the number of staff required to meet the service demand over the planning horizon through a simulation or queuing study, b) determining the size and composition of the workforce and constructing shifts and weekly tours, and c) assigning individual employees to shifts and weekly tours while taking into consideration absenteeism and variation in demand while maximizing employee preferences (Ernst et al., 2004).

The staff scheduling problem that arises from call centers has to address an additional complexity, the use of cross-training. Cross-training provides an efficient resource pooling mechanism to increase the flexibility of the call center in the face of uncertainty and variability in demand and supply, and is widely used in practice. Full cross-training, however, is almost financially impractical. To optimize staff scheduling with a cross-trained workforce, thus several important questions have to be addressed; i.e., a) what categories should be cross-trained, b) what portion of the workforce should be cross-trained, and c) how to schedule their weekly assignments across multiple skills/categories.

This thesis focuses on the solution of the staff scheduling with a cross-trained workforce problem at call centers, and aims to develop cross-training staff scheduling models to be used in a realistic staffing environment. In the design of these models, it is intended that the models should be able to serve strategic analysis that addresses the optimal mix of cross-trained agents and the daily and weekly schedules of all staff.

The proposed model is a mixed integer programming model that simultaneously addresses staffing, shift scheduling, days off assignment, lunch break assignment, and cross-training (CT) decision. Computationally, the two-phase sequential approach (TPSA) for the solution of the cross-training staff scheduling model (CTSSM) is also proposed. The first phase (P-I) is to find the optimal composition of the workforce to decide what categories should be cross-trained and when they should be deployed; the second phase (P-II) staff scheduling model is to find the size of the workforce and their skill sets, shifts and weekly tours. The two-phase approach is an order of magnitude faster than the original model and is able to obtain better solutions orders of magnitude faster.

Computational results based on the data of the support call center of a Fortune 50 retail company clearly demonstrate the significant benefit of cross-training across service categories. For example, if only 30% of all staff is cross-trained for two out of nine service groups, a reduction of 5% in overall cost could be achieved; additional cross-training adds little additional value. The proposed models have been used to provide a strategic guide to the employees' schedules for the call center; considerable cost savings around 4% - 9% is expected while providing better service levels.

Compared with other options such as flexible shifts, e.g. part time shifts, experiment results seem to suggest that cross-training could be a more effective approach to hedge against demand fluctuations when multiple service categories are involved.

1.2 Literature Review

The staff scheduling problem is a classical optimization problem and has seen various applications in call centers (Brigandi et al., 1994), hospitals (Bard and Purnomo, 2005), airport stations (Brusco and Jacobs, 1998), and postal facilities (Jarrah et al., 1994; Bard et al., 2003). For a recent review, please see VandenBergh et al. (2013).

Ernst et al. (2004) decomposed the staff scheduling process into sequential modules such as demand modeling, days off assignment, shift scheduling and break assignment, line of work or tour construction, task assignment and staff assignment. The development of a particular staff schedule may require only some of the modules and several modules may be combined into one procedure in many practical implementations. For example, tour scheduling combines shift scheduling and days off assignment and is typically seen in strategic staff scheduling systems.

In the case of a call center, the staff scheduling problem is composed of shift scheduling with break assignment, days off assignment, and cross-training policy analysis and assignment in multiple service categories, each with distinct skills. To distinguish call center staff scheduling with cross-training, the relevant research studies are divided into two subsections: single-skill workforce scheduling and multi-skill workforce scheduling with cross-training.

1.2.1 Single-Skill Workforce Scheduling

Shift scheduling: The earlier work on shift scheduling goes back to Dantzig (1954) where a set covering formulation was proposed. Segal (1974) addressed a shift scheduling problem for telephone operators who were required to be given a lunch break and two relief breaks during their shifts. He divided the day into ninety six 15-minute periods, used a network model to find solutions, and made the break assignment with a post-processing algorithm. Bechtold and Jacobs (1990) introduced the implicit modeling of breaks and derived three constraints that collectively ensured the feasibility of the break assignments.

Days off assignment: Burns and Carter (1985) provided a comprehensive solution to the days off assignment problem. They derived a set of lower bounds on the workforce size that took into account days off requirements as well as specific requirement for weekends off. Alfares (1997) proposed an efficient algorithm for the tour scheduling problem that assigns two consecutive days off to employees.

Tour scheduling: Jarrah et al. (1994) and Bard et al. (2003) presented a full-scale model of the tour scheduling problem (that includes shift scheduling, break assignment, and days off assignment) and examined several scenarios aimed at reducing the size of the workforce. Bard et al. (2007) addressed a staff planning and scheduling problem and developed a two-stage stochastic integer program. In the first stage, before the demand is known, the number of full-time and part-time employees is determined for the permanent workforce. In the second stage, the demand is revealed and workers are assigned to specific shifts during the week. Bard (2004) studied a hierarchical workforce scheduling with downgrading in postal facilities. In the downgrading analysis, a person in a higher skill category can be assigned to a job in a lower skill category.

1.2.2 Multi-Skill Workforce Scheduling with Cross-Training

Cross-training typically arises in production and service systems where workloads may be imbalanced across operations and cross-training enables workers to shift between operations and improve productivity. For studies of cross-training in serial production systems please see Hopp et al. (2004); in health care systems please see Wright and Mahar (2013), Paul and MacDonald (2014), and Gnanlet and Gilland (2014).

The studies of cross-training in call centers can be classified into two categories: single period cross-training policy analysis with constant arrival rate, and multiple period skill assignment with time dependent arrival rates. The former usually assumes a constant arrival rate and studies the pooling decision on which groups should be cross-trained while the latter assumes an arrival rate that changes over the time horizon, such as by hours of a day or days of a week, and aims to assign members of a cross-trained workforce to various departments over the planning horizon.

Single Period Cross-Training Assignment: Wallace and Whitt (2005) studied call center routing and staffing problems by exploiting limited cross-training and developed an algorithm to minimize the total staff subject to per-class performance constraints. Simulation experiments demonstrated that when each agent has only two skills in appropriate combinations, the performance is almost as good as when each agent has all skills.

Ahghari and Balcioglu (2009) studied customer contact centers that provide different types of services to customers who place phone calls or send e-mail messages to assess the performance improvement via cross-training the agents. Their numerical studies indicated that limited cross-training with two skills per agent results in considerable performance improvements. However, unbalanced cases where different classes of customers have the same arrival rate but different mean service times necessitate more cross-training at three skills per agent to have considerable improvement.

Tekin et al. (2009) examined pooling strategies for call centers and the solution of two fundamental issues: how many departments to pool and which departments to pool. The authors investigated the impact of different parameters, including mean service times, service time variability, and department size in deciding which departments to pool. The results showed that if the mean service times of the departments to be pooled are similar, pooling departments with the highest service time coefficient of variation reduces the expected delay the most.

Iravani et al. (2007) modeled inbound call centers as parallel queuing systems with flexible servers, and proposed a work sharing network model and used its average shortest path length metric to predict the more effective of two alternative cross-training structures in terms of customer waiting times. The results show that the average shortest path length metric of the small world network theory is a simple deterministic solution approach to the complex stochastic problem of designing effective workforce cross-training structures in call centers.

Multiple Period Cross-Training Assignment: Campbell (1999) developed a nonlinear generalized assignment model for allocating cross-trained workers at the beginning of a shift in a multi-department service environment. Campbell and Diaby (2002) later proposed a linear assignment heuristic to solve the problem. Results show that a small degree of cross-training can capture most of the benefits and beyond a certain amount additional cross-training adds little additional value, and the preferred amount of cross-training depends heavily on the level of demand variability. Brusco (2008) extended Campbell (1999)’s model to include several nonlinear assignment objectives to maximize overall utility and developed a branch-and-bound algorithm to evaluate cross-training policies.

Taking demand uncertainty into consideration, Campbell (2011) developed a two-stage stochastic program for scheduling and allocating cross-trained workers in multi-departments with random demands. The first stage corresponds to scheduling days off over a time horizon and the second stage is the recourse action that deals with allocating available workers at the beginning of a day to accommodate realized demand. Results show that cross-training can be more valuable than perfect information for demand, especially when demand uncertainty is high.

Easton (2011) studied how cross-training and workforce management decisions interact to affect labor costs and service levels in extended hour service operations with uncertain demand and employee attendance. Using a two-stage stochastic model, he first optimally staffs, cross-trains, schedules, and allocates workers across departments. He then simulates demand and attendance and re-allocates available cross-trained workers to best satisfy realized demand.

Avramidis et al. (2010) compared simulation-based algorithms for solving the agent scheduling problem, which is to minimize the total cost of agents under constraints on the expected service level per call type, per period, and aggregated. The problem is solved through a solution approach that combines simulation with integer or linear programming, with cut generation. Numerical experiments show that this approach was able to get better solutions than the standard approach, which could yield suboptimal solutions.

1.3 Contribution of this Dissertation

This study focuses on the development of staff scheduling models with cross-training and solution approaches to the efficient solution of these models, as typically seen in the call centers.

Cross-training is an integral part of a typical call center which typically handles several types of calls. Agents are typically trained to have different skills in various combinations. Most studies in literature are either focused on policy analysis, verified through simulation studies in single period or assignment over multiple periods, yet do not incorporate labor regulations and practice and thus limit their applications – it is the author's opinion that practical labor scheduling systems with cross-training require a holistic solution to staff scheduling models that includes cross-training decisions, such as the categories to be cross-trained, the composition of cross-training workforce (skills and sizes) and their schedules (shifts and days off assignments).

This study deals with the problem of designing effective workforce cross-training structures in conjunction with staffing and scheduling in call centers, and develops mathematical models and solution approach to be used in a realistic staffing environment. The contribution of the thesis includes:

a) The development of mathematical models for strategic and operational staff scheduling that integrate the cross-training, shift scheduling and days off assignment, and break assignment aspects of the problem.

b) The development of an effective and efficient two-phase sequential approach to the solution of the cross-training staff scheduling problem. The two-phase approach solves the problem in a sequential manner and is an order of magnitude faster than the original model.

c) The development of decision support systems that could be used in the solution of strategic and operational staff scheduling problems.

d) Extensive computational studies to evaluate the effectiveness of cross-training in a realistic environment. For example, experimental results seem to suggest that cross-training could be a more effective approach to hedge against demand fluctuations across multiple service categories.

1.4 Organization of the Dissertation

The remainder of the dissertation is organized as follows. In Chapter 2, the detailed mathematical model for the strategic cross-training staff scheduling problem is presented. In Chapter 3, the two-phase sequential approach designed to improve the computational performance of the cross-training staff scheduling model is presented along with the computational results using the data of the example call center. The preference-based operational cross-training staff scheduling model and its results are presented in Chapter 4. Chapter 5 presents experimental results for various cross-training configurations to gain managerial insights. Concluding remarks are given in Chapter 6. Call arrival data of the example call center is presented in Appendixes A and B; the model components, sample schedules obtained by the proposed models, and detailed experimental results are contained in Appendixes C, D, E, F, G, H, and I.

CHAPTER 2 STRATEGIC CROSS-TRAINING STAFF SCHEDULING MODEL

2.1 Call Center Staff Scheduling Problem

This research is motivated by the need to properly staff and schedule the agents within the technical support center of a leading grocery retailer in the United States. The retailer operates more than 2,500 supermarkets, 2,000 pharmacies, 1,000 fuel stores, and 700 convenience stores. The technical support center receives calls from store operations and is responsible for addressing various technical and operational issues related to point-of-sale machines, fuel stations, desktop or laptop computers, and various applications; as can be seen, the proper staffing of the technical support center is critical to the successful operation of the retailer. The call center operates 24 hours a day and 7 days a week, with an apparently very slow demand during early morning, where the demands at the stores are low and so is the volume of the incoming calls.

Incoming calls are categorized into nine service groups and routed based the skill required. In call centers, the calls have different requirements and the agents have different skills; modern automatic call distributors have the capability to assign calls to agents with the appropriate skills, known as skill-based routing (Wallace and Whitt, 2005). Based on queuing analysis, an Erlang-C formula is used to translate average call handling time and number of call arrivals or incoming call rate into the demand of agents for each half hour of the day. The call arrivals in all nine service groups for a week are presented in Appendix A.

The demand profiles of service groups, calculated from the Erlang-C formula, are illustrated in Figure 1. The details of nine service groups and the daily demand profiles are also presented in Appendix B. In the figure, each panel represents a different service group; the horizontal axes represent the time of the day, composed of 48 half-hour time periods with 1 representing 12:00 a.m. and 48 representing 11:30 p.m.; and the vertical axes represent the demand of staff throughout the day. The lines on each panel represent the demand on different days of the week.

As can be seen, the requirement for agents could be anywhere from 2 - 3 agents for low demand groups such as groups 8 and 9 to 9 - 10 agents for high demand groups such as groups 1 and 2. Within a day, demand for agents typically starts to increase at 6:00 a.m., period 13, and peaks from 10:00 a.m., period 20, to 4:00 p.m., period 32. The demands for service groups 2, 3, and 5 show variations between days where Saturdays and Sundays show lower demands than the weekdays as not all services are available and not all employees are working on the weekends.

Figure 2 presents average daily demands for a week for all nine service groups. In the figure, the vertical axis represents the number of agents whereas the horizontal axis represents the days of the week. Each line represents a service group. For example, for service group 1, the daily average of demand for Monday is around 4 to 5 agents, whereas it is less than 1 agent for service group 9.

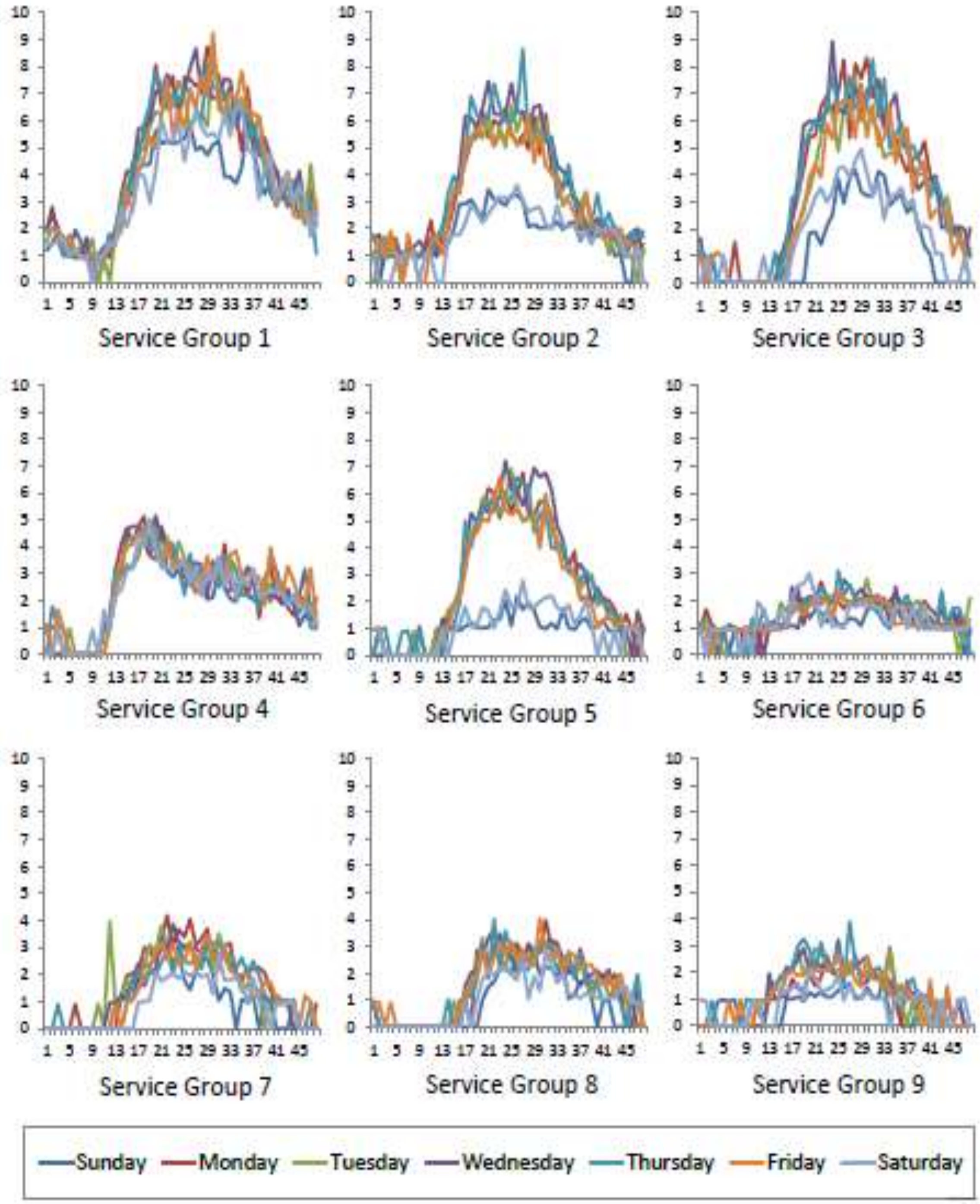

Figure 1: Demand Profiles of Service Groups

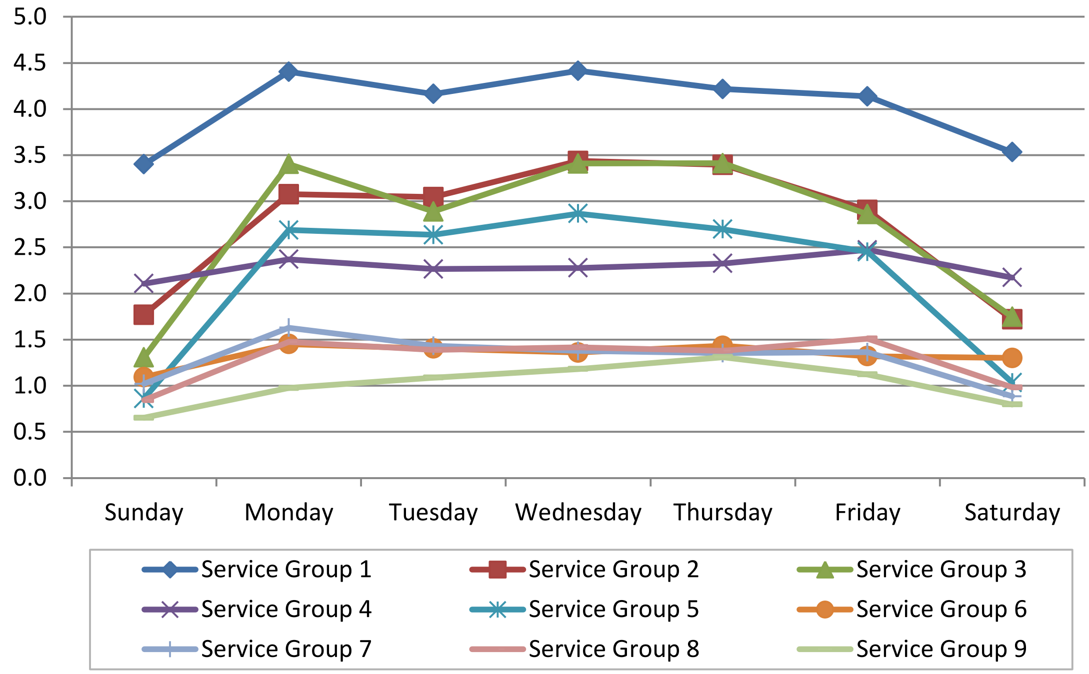

Figure 2: Average Daily Demands

2.2 Strategic Cross-Training Staff Scheduling Model

A staff scheduling model aims to find the optimal size and composition of the workforce and construct their weekly tours to satisfy a given demand.

The cross-training staff scheduling model, in the case of the call center considered in this study, is consisted of several sub-models: a) cross-training assignment, b) shift scheduling with break assignment, and c) days off assignment.

2.2.1 Full Cross-Training or Partial Limited Cross-Training

Cross-training is critical in call centers as it helps to buffer against unbalanced demand across different service categories. From a modeling perspective, the simplest form of cross-training is full cross-training, in which agents are trained to perform all required tasks; in practice, however, full cross-training is impractical due to the cost of cross-training, service quality penalties arising from cross-training, excessive agent stress, and employees' preferences and abilities. In practice, partial and limited cross-training has been widely adopted. Here, *partial cross-training* can be defined as having some percentage of the workforce that is cross-trained (Brusco, 2008). *Limited cross-training* can be defined as having agents with multiple skills, but only a few skills in appropriate combinations (Wallace and Whitt, 2005).

It is well known that "even a little flexibility goes a long way" (Aksin and Karaesmen, 2002) and the simulation experiments with single period steady arrival rate for each service group in call centers conducted by Wallace and Whitt (2005) and Ahghari and Balcioglu (2009) demonstrated that "when each agent has only two skills in appropriate combinations, the performance is almost as good as when each agent has all skills." Ahghari and Balcioglu (2009) also say that "however, unbalanced cases where each class of customers has the same arrival rate but different mean service times necessitate more cross-training at three skills per agent to have considerable improvement." In view of this, though all service categories are technically eligible for cross-training in the support center, the maximum number of skills for each agent is limited and computation studies with two, three, and four skills are employed for comparison purposes.

In this research, every agent is associated with a nonempty set, called a skill set, defining the types of calls the agent is cross-trained to serve. The set of skill sets is explicitly defined, where each member of the set is a combination of compatible skills such as one skill {*g*}, two skills {*g, g'*}, three skills {*g, g', g''*}, and four skills {*g, g', g'', g'''*}. In the case where there are two skills in a set, if the efficiency of a cross-trained agent in both skills is the same, skills {*g, g'*} and {*g', g*} become reciprocal and only {*g, g'*} is employed. If the efficiencies are different, which means a multi-skilled agent has priority levels for his/her skills, then the first skill is called the primary skill and the second one is called the secondary skill. In that case, both skill {*g, g'*} and skill {*g', g*} are employed to allow it to be possible to differentiate between primary and secondary skills.

In this study, the efficiency of a cross-trained agent in all his/her skills is set to 100% unless specified explicitly otherwise. Because there are nine service groups in the call center, there are 45 skill sets for cross-training with a maximum of two skills, 129 skill sets for cross-training with a maximum of three skills, and 255 skill sets for cross-training with a maximum of four skills, as presented in Table 1. All skill sets with two, three, and four skills are presented in Table 2.

In the experiments where the efficiency of a cross-trained staff in the secondary skill is allowed to vary, 81 skill sets are defined with a maximum of two skills cross-training where both {*g, g'*} and {*g', g*} skill sets are created. Similarly, 585 skill sets are defined for a maximum of three cross-trained skills and 3,609 skill sets for a maximum of four cross-trained skills.

Table 1: Number of Skill Sets with Two-Skill, Three-Skill, and Four-Skill CT

# of	Cross-Training Configuration				
	No CT	Two-Skill CT	Three-Skill CT	Four-Skill CT	Full CT
Skills	1	a maximum of 2	a maximum of 3	a maximum of 4	9
Skill Sets	9	45	129	255	1

Table 2: Skill Sets with Two, Three, and Four Skills

# of Skills in a Skill Set	# of Skill Sets	Skill Sets
One	9	{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}
Two	36	{1,2}, {1,3}, {1,4}, {1,5}, {1,6}, {1,7}, {1,8}, {1,9}, {2,3}, {2,4}, {2,5}, {2,6}, {2,7}, {2,8}, {2,9}, {3,4}, {3,5}, {3,6}, {3,7}, {3,8}, {3,9}, {4,5}, {4,6}, {4,7}, {4,8}, {4,9}, {5,6}, {5,7}, {5,8}, {5,9}, {6,7}, {6,8}, {6,9}, {7,8}, {7,9}, {8,9}
Three	84	{1,2,3}, {1,2,4}, {1,2,5}, {1,2,6}, {1,2,7}, {1,2,8}, {1,2,9}, {1,3,4}, {1,3,5}, {1,3,6}, {1,3,7}, {1,3,8}, {1,3,9}, {1,4,5}, {1,4,6}, {1,4,7}, {1,4,8}, {1,4,9}, {1,5,6}, {1,5,7}, {1,5,8}, {1,5,9}, {1,6,7}, {1,6,8}, {1,6,9}, {1,7,8}, {1,7,9}, {1,8,9}, {2,3,4}, {2,3,5}, {2,3,6}, {2,3,7}, {2,3,8}, {2,3,9}, {2,4,5}, {2,4,6}, {2,4,7}, {2,4,8}, {2,4,9}, {2,5,6}, {2,5,7}, {2,5,8}, {2,5,9}, {2,6,7}, {2,6,8}, {2,6,9}, {2,7,8}, {2,7,9}, {2,8,9}, {3,4,5}, {3,4,6}, {3,4,7}, {3,4,8}, {3,4,9}, {3,5,6}, {3,5,7}, {3,5,8}, {3,5,9}, {3,6,7}, {3,6,8}, {3,6,9}, {3,7,8}, {3,7,9}, {3,8,9}, {4,5,6}, {4,5,7}, {4,5,8}, {4,5,9}, {4,6,7}, {4,6,8}, {4,6,9}, {4,7,8}, {4,7,9}, {4,8,9}, {5,6,7}, {5,6,8}, {5,6,9}, {5,7,8}, {5,7,9}, {5,8,9}, {6,7,8}, {6,7,9}, {6,8,9}, {7,8,9}
Four	126	{1,2,3,4}, {1,2,3,5}, {1,2,3,6}, {1,2,3,7}, {1,2,3,8}, {1,2,3,9}, {1,2,4,5}, {1,2,4,6}, {1,2,4,7}, {1,2,4,8}, {1,2,4,9}, {1,2,5,6}, {1,2,5,7}, {1,2,5,8}, {1,2,5,9}, {1,2,6,7}, {1,2,6,8}, {1,2,6,9}, {1,2,7,8}, {1,2,7,9}, {1,2,8,9}, {1,3,4,5}, {1,3,4,6}, {1,3,4,7}, {1,3,4,8}, {1,3,4,9}, {1,3,5,6}, {1,3,5,7}, {1,3,5,8}, {1,3,5,9}, {1,3,6,7}, {1,3,6,8}, {1,3,6,9}, {1,3,7,8}, {1,3,7,9}, {1,3,8,9}, {1,4,5,6}, {1,4,5,7}, {1,4,5,8}, {1,4,5,9}, {1,4,6,7}, {1,4,6,8}, {1,4,6,9}, {1,4,7,8}, {1,4,7,9}, {1,4,8,9}, {1,5,6,7}, {1,5,6,8}, {1,5,6,9}, {1,5,7,8}, {1,5,7,9}, {1,5,8,9}, {1,6,7,8}, {1,6,7,9}, {1,6,8,9}, {1,7,8,9}, {2,3,4,5}, {2,3,4,6}, {2,3,4,7}, {2,3,4,8}, {2,3,4,9}, {2,3,5,6}, {2,3,5,7}, {2,3,5,8}, {2,3,5,9}, {2,3,6,7}, {2,3,6,8}, {2,3,6,9}, {2,3,7,8}, {2,3,7,9}, {2,3,8,9}, {2,4,5,6}, {2,4,5,7}, {2,4,5,8}, {2,4,5,9}, {2,4,6,7}, {2,4,6,8}, {2,4,6,9}, {2,4,7,8}, {2,4,7,9}, {2,4,8,9}, {2,5,6,7}, {2,5,6,8}, {2,5,6,9}, {2,5,7,8}, {2,5,7,9}, {2,5,8,9}, {2,6,7,8}, {2,6,7,9}, {2,6,8,9}, {2,7,8,9}, {3,4,5,6}, {3,4,5,7}, {3,4,5,8}, {3,4,5,9}, {3,4,6,7}, {3,4,6,8}, {3,4,6,9}, {3,4,7,8}, {3,4,7,9}, {3,4,8,9}, {3,5,6,7}, {3,5,6,8}, {3,5,6,9}, {3,5,7,8}, {3,5,7,9}, {3,5,8,9}, {3,6,7,8}, {3,6,7,9}, {3,6,8,9}, {3,7,8,9}, {4,5,6,7}, {4,5,6,8}, {4,5,6,9}, {4,5,7,8}, {4,5,7,9}, {4,5,8,9}, {4,6,7,8}, {4,6,7,9}, {4,6,8,9}, {4,7,8,9}, {5,6,7,8}, {5,6,7,9}, {5,6,8,9}, {5,7,8,9}, {6,7,8,9}
Full	1	{1,2,3,4,5,6,7,8,9}

2.2.2 Shift Scheduling and Break Assignment

To cope with demand fluctuation, it is typical for call centers to employ agents with various shifts of various lengths, with various start times and days off assignments. Here, it is defined that a full-time shift employee works 8 hours a day and 5 days a week, an extended shift employee works 10 hours a day and 4 days a week, and a part-time shift employee works 4, 5, 6, or 7 hours a day and 5 days a week.

Shift scheduling begins with the definition of all possible shifts and concludes with the number of staff that should be assigned to each shift to satisfy the demand on each day of the week. The shifts start at the beginning of every hour; each shift covers consecutive time periods equal to its length and cannot extend into the following day. The model includes 16 full-time shifts, 14 extended shifts, and 76 part-time shifts. Each employee has the same shift type and length with a constant start time in each day she/he is on duty. For reference, all shifts included in the model are demonstrated in Table 3, and coverage of all shifts for a day is presented in Appendix C. In the table, the shift lengths include the 1/2 hour lunch break where applicable.

Table 3: Shift Types, Start and End Times, and Lengths (1 period = 1/2 hour)

Shift Type	Start Time Periods	End Time Periods	Shift Lengths
Full-Time	1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 32	17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 48	8½ hours (17 periods)
Extended	1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 28	21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 48	10½ hours (21 periods)
Part-Time	1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41	8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48	4 hours (8 periods)
	1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39	10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48	5 hours (10 periods)
	1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 36	13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 48	6½ hours (13 periods)
	1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 34	15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 48	7½ hours (15 periods)

An auxiliary decision in the shift schedule is lunch break assignment. All shifts that are 6 hours or longer require a 1/2 hour lunch break. General practice is to create a break window, which is a set of consecutive time periods during which a break may be given, for each shift, and to assign a break within this predetermined window. The break is uncompensated and adds a 1/2 hour to the length of each eligible shift. In the model, the breaks are typically assigned sometime between the 9th and 12th periods of a shift giving a break window 4 time periods long. The implicit modeling of break allowances for each employee is possible with the appropriate variables and constraints as proposed by Bechtold and Jacobs (1990). However, this approach will only guarantee that there are a sufficient number of idle periods for each employee. Determining who takes which period off is an assignment problem and is handled by a post-processing break assignment algorithm.

2.2.3 Days Off Assignment

Full-time and part-time shifts are given two days off in a week, whereas extended shifts are given three days off in a week. The general consideration is that sufficient slack must be provided throughout the week so that the days off requirement is satisfied for every worker and there should be enough active workers for each day of the week to satisfy the daily demand. The days off policy followed here is any days off, which means that an employee's days off are not necessarily consecutive. Nevertheless, in the computational experiments, consecutive days off and any days off policies are compared to provide managerial insights for the days off assignment decision.

2.2.4 Mathematical Model for Cross-Training Staff Scheduling

The goal of the cross-training staff scheduling model is to find the size of workforce and their composition in various skill sets, and to assign them daily shifts and weekly tours. In the development of the models, the following notation is used.

Indices and Sets:

g	index for the set of service groups *G* where *G* = {1,. . . ,9}
w	index for the set of skill sets *W* where *WC* ∊ *W* is the set of multi-skill sets
d	index for days of a week *D* where *D* = {1,. . . ,7}
t	index for time periods (half-hour) *T* in a day where *T* = {1,. . . ,48}
f, e, p	index for full-time (*F*), extended (*E*), or part-time (*P*) shift types

W_g	set of skill sets that includes service group *g*, where $g \in G$
I, L	set of initial and last periods of break windows, in ascending order
FP_k^F, FP_k^E, FP_k^P	set of full-time, extended, or part-time shifts whose break window lies entirely between *ep* and *k*
BP_k^F, BP_k^E, BP_k^P	set of full-time, extended, or part-time shifts whose break window lies entirely between *k* and *lp*
PB	set of part-time shifts which require a lunch break

Parameters:

CF, CE, CP_p	weekly cost of a full-time, extended, or part-time shift employee
CU	penalty cost for an uncovered demand of staff in a time period
F_{ft}, E_{et}, P_{pt}	1 if full-time shift *f*, extended shift *e*, part-time shift *p* covers time period *t*; 0 otherwise
D_{gdt}	demand of staff for service group *g* on day *d* time period *t*
ep, lp	earliest, latest periods a break can begin for any permissible shifts
MaxP	maximum percentage of part-time shift employees in all staff
MaxC	maximum percentage of cross-trained employees in all staff

Decision Variables:

xf_{wf} , xe_{we}, xp_{wp}	number of employees who have skill *w* and who are assigned to full-time shift *f*, extended shift *e*, or part-time shift *p*

yf_{wfd}, ye_{wed}, yp_{wpd} — number of employees who have skill *w* and who are assigned to full-time shift *f,* extended shift *e*, or part-time shift *p* on day *d*

z_{wgdt} — number of active employees who have skill *w* and work for service group *g* $(g \in w)$ on day *d* in time period *t*

u_{gdt} — amount of demand uncovered by staff in service group *g* on day *d* in time period *t*

b_{wdt} — number of breaks on day *d* in time period *t* for an employee who has skill *w*

The mathematical model for the cross-training staff scheduling model, called the CTSSM, is presented below.

Minimize

$$\sum_{w \in W} \sum_{f \in F} CF\ xf_{wf} + \sum_{w \in W} \sum_{e \in E} CE\ xe_{we} + \sum_{w \in W} \sum_{p \in P} CP\ xp_{wp} + \sum_{g \in G} \sum_{d \in D} \sum_{t \in T} CU\ u_{gdt} \tag{1}$$

Subject to

a) Shift Scheduling with Cross-Training Assignment

$$\sum_{w \in W_g} z_{wgdt} + u_{gdt} \geq D_{gdt} \qquad \forall g \in G, d \in D, t \in T \tag{2}$$

$$\sum_{f \in F} F_{ft}\, yf_{wfd} + \sum_{e \in E} E_{et}\, ye_{wed} + \sum_{p \in P} P_{pt}\, yp_{wpd} - b_{wdt} = \sum_{g \in w} z_{wgdt}$$

$$\forall\, w \in W, d \in D, t \in T \qquad (3)$$

b) Days Off Assignment

$$xf_{wf} = 1/5 \sum_{d \in D} yf_{wfd} \qquad \forall\, w \in W, f \in F \qquad (4)$$

$$xe_{we} = 1/4 \sum_{d \in D} ye_{wed} \qquad \forall\, w \in W, e \in E \qquad (5)$$

$$xp_{wp} = 1/5 \sum_{d \in D} yp_{wpd} \qquad \forall\, w \in W, p \in P \qquad (6)$$

$$xf_{wf} \geq yf_{wfd} \qquad \forall\, w \in W, f \in F, d \in D \qquad (7)$$

$$xe_{we} \geq ye_{wed} \qquad \forall\, w \in W, e \in E, d \in D \qquad (8)$$

$$xp_{wp} \geq yp_{wpd} \qquad \forall\, w \in W, p \in P, d \in D \qquad (9)$$

c) Lunch Break Assignment

$$\sum_{t=ep}^{k} b_{wdt} - \sum_{f \in FP_k^F} yf_{wfd} - \sum_{e \in FP_k^E} ye_{wed} - \sum_{p \in FP_k^P} yp_{wpd} \geq 0$$

$$\forall\, k \in L, w \in W, d \in D \qquad (10)$$

$$\sum_{t=k}^{lp} b_{wdt} - \sum_{f \in BP_k^F} yf_{wfd} - \sum_{e \in BP_k^E} ye_{wed} - \sum_{p \in BP_k^P} yp_{wpd} \geq 0$$

$$\forall\, k \in I, w \in W, d \in D \qquad (11)$$

$$\sum_{t=ep}^{lp} b_{wdt} - \sum_{f \in F} yf_{wfd} - \sum_{e \in E} ye_{wed} - \sum_{p \in PB} yp_{wpd} = 0$$

$$\forall \; w \in W, d \in D \tag{12}$$

d) Flexibility Limitation

$$MaxC \left(\sum_{w \in W} \sum_{f \in F} xf_{wf} + \sum_{w \in W} \sum_{e \in E} xe_{we} + \sum_{w \in W} \sum_{p \in P} xp_{wp} \right)$$

$$\geq \sum_{w \in WC} \sum_{f \in F} xf_{wf} + \sum_{w \in WC} \sum_{e \in E} xe_{we} + \sum_{w \in WC} \sum_{p \in P} xp_{wp} \tag{13}$$

$$MaxP \left(\sum_{w \in W} \sum_{f \in F} xf_{wf} + \sum_{w \in W} \sum_{e \in E} xe_{we} + \sum_{w \in W} \sum_{p \in P} xp_{wp} \right) \geq \sum_{w \in W} \sum_{p \in P} xp_{wp} \tag{14}$$

e) Non-Negativity Requirements

$$xf_{wf}, xe_{we}, xp_{wp}, yf_{wfd}, ye_{wed}, yp_{wpd}, z_{wgdt}, b_{wdt} \geq 0 \; and \; integer,$$

$$u_{gdt} \geq 0 \qquad \forall \, w, g, f, e, p, d, t \tag{15}$$

The objective function (1) minimizes the total weekly cost that is composed of staff cost for full-time, extended, and part-time shift employees, and the penalty cost for uncovered demand (demand placed on the staff that cannot be satisfied).

Constraint (2) ensures that for each service group *g*, the staff requirement in each time period is met by active employees trained in skill set $w \epsilon W_g$ that are assigned to group *g*, but with the provision that shortages, tracked by u_{gdt}, are allowed. Here, an employee is said to be *on-duty* if his/her shift covers the time period under consideration; however, an employee is said to be *active* only if he/she is on duty yet not on break.

Constraint (3) keeps track of the number of on-duty employees for a skill set w and their composition of full-time, extended, and part-time shifts. Here, the 0-1 matrices (F, E, P) filter out the full-time, extended, and part-time shifts that cover the time period under consideration. For a skill set w on day d and in time period t, the difference between on-duty employees and employees on break, tracked by b_{wdt}, gives active employees. The use of breaks in this manner, rather than explicitly including breaks in the shift definition, ensures that each worker is assigned to the same shift every day while allowing his/her lunch break to vary by day.

Constraints (4) - (9) are used to calculate lower bounds on the number of employees required to meet the daily demand in each day of the week while taking into account days off requirements. The first set of these bounds is needed to ensure that there is enough coverage that full-time and part-time shift employees can take two days off and extended shift employees can take three days off in a week. The second set of lower bounds is necessary to assure that a sufficient number of workers exist to cover the day with the highest demand. Constraints (4), (5), and (6) correspond to the first lower bound whereas constraints (7), (8), and (9) correspond to the second lower bound for full-time, extended, and part-time shift employees, respectively. These bounds are derived from Burns and Carter (1985) and are sufficient to guarantee the required days off.

Constraints (10) - (12) are derived to assign lunch breaks to relevant shifts. These constraints were first introduced by Bechtold and Jacobs (1990) and are sufficient to guarantee that relevant shifts get the lunch breaks they need. To account for breaks, three constraints are needed. The first constraint (10) is referred to as the forward pass constraint. It ensures that the total number of possible breaks starting from period *ep*, the first period that can be taken as a break, up to a given period *k* exceeds the total number of employees who should have taken their breaks by that period. The employees included in the constraint are those whose break windows are fully covered through *k*, but not the ones who have the option of a break in some future period. The second constraint (11) is referred as the backward pass constraint and ensures that the total number of possible breaks starting from some specific period *k* through the period *lp*, the last period that can be taken as a break, exceeds the number of employees who are entitled to a break during this interval. In other words, there should be sufficient breaks in the future to satisfy the break requirement for the rest of the day. These two constraints are needed to provide every employee with a one-period break, but they are not sufficient to enforce the requirement that exactly one break is assigned to each shift which is entitled to one. The last constraint (12) is the balance equation, which is needed to ensure that every permissible shift is assigned exactly one break that is within its permitted time window.

Constraint (13) limits the maximum number of cross-trained employees. Constraint (14) limits the maximum number of part-time shift employees. Finally, constraint (15) satisfies the non-negativity and integer requirements of all of the decision variables.

The input of the model includes: a) staff cost per hour and penalty cost of an uncovered demand in a time period, b) shift definitions including start times and lengths, c) demand placed on the staff in each time period in each day of a week for all service groups, d) rules governing days off and lunch break assignment, e) part-time shift staff percentage, and f) cross-training staff percentage and cross-training compatibility of service groups. The output includes: a) number of staff assigned to each shift type, b) weekly schedule of each employee including days off and lunch break assignment, c) number of cross-trained workers and the service groups they are cross-trained for, d) amount of uncovered demand in each time period for each service group, and e) total weekly cost.

2.2.5 Break Assignment Algorithm

The CTSSM determines the number of breaks allocated for each time period, yet no detailed assignment has been given. This can be done using the below assignment algorithm with the break assignment results of the CTSSM.

Indices, Sets, and Parameters:

SB set of employees whose shift is eligible for lunch break

BW_s break window of staff s's shift

WD_s working days of staff s

Decision Variables:

B_{sdt} 1 if staff *s* has a lunch break on day *d* in time period *t*; 0 otherwise

Minimize

$$\sum_{w \in W} \sum_{d \in D} \sum_{t \in T} b_{wdt} - \sum_{s \in SB} \sum_{d \in D} \sum_{t \in T} B_{sdt} \tag{16}$$

Subject to

$$\sum_{s \in SB} B_{sdt} = \sum_{w \in W} b_{wdt} \qquad \forall\, d \in D, t \in T \tag{17}$$

$$\sum_{t \in BW_s} B_{sdt} = 1 \qquad \forall\, s \in SB, d \in WD_s \tag{18}$$

The objective function (16) minimizes the difference between the total number of breaks assigned in the CTSSM and the total number of breaks that are going to be assigned to all employees in this model. Constraint (17) stipulates that total number of breaks assigned to employees whose shift is eligible for break assignment in a time period is equal to the number of breaks allocated to that time period by the CTSSM. Constraint (18) ensures that just one break is assigned within the break window in a working day to each employee whose shift is eligible for break assignment.

CHAPTER 3 COMPUTATIONAL IMPROVEMENT: TWO-PHASE SEQUENTIAL APPROACH AND COMPUTATIONAL RESULTS

3.1 Two-Phase Sequential Approach

The CTSSM, as defined by Equations (1) - (15), is a large-scale mixed integer program. To gain an appreciation of its size, consider that when the problem has nine service groups and limited cross-training with two out of nine skills per agent, the corresponding model has 80,685 variables and 69,851 constraints. The initial computational experiment shows that the model is computationally hard to solve; besides, the best bounds increase slowly, which makes it almost impossible to solve it optimally.

To improve computational efficiency, a two-phase sequential approach has been developed. The motivation of the TPSA is that though shift selection and break assignment are important – for example, breaks represent the loss of one twelfth of an employee's time and must be considered in the CTSSM – the major decisions for the cross-training model are a) which skill combination should each agent be cross-trained in and b) what time periods of each day and days of the week should that cross-trained agent be deployed. In view of this, the sequential approach solves the cross-training staff scheduling problem in two phases: the first phase is a cross-training with days off selection problem based on a cross-training time interval; given these cross-training decisions, the second phase is a staffing, shift scheduling, and tour scheduling problem, much reduced in size.

3.1.1 First Phase

In P-I, the concept of *cross-training interval* or *interval* is introduced, which represents the minimum length of the time span (interval) that management would like a cross-trained agent to be deployed for due to unbalanced demand. After consulting with the management team of the call center, this interval was set at 4 hours which is an aggregate of eight half-hour time periods, and each day was divided into 6 non-overlapping intervals. Setting a smaller interval would likely introduce scattered cross-training allocation of, for example, 10:00 a.m. to 12:00 p.m. and then 2:00 to 4:00 p.m. and is not preferable; setting a larger interval, on the other hand, limits the cross-training options and may result in excess assigned time where cross-training might not be needed. The use of overlapping intervals increases the size of the model, yet has no significant impact on the solution, and is thus not adopted.

In doing so, the goal of the P-I model becomes the selection of skill sets and interval combinations such that cross-training balances the unevenness in demand within the day and across the days of the week. In the development of the P-I model, the following notation is used.

Indices, Sets, and Parameters:

i index for intervals I where $I = \{1, \ldots, 6\}$

C_{it} 1 if interval i covers time period t; 0 otherwise

CI weekly cost of an employee who works during an interval

CU penalty cost for an uncovered demand of staff in a time period

D_{gdt} demand of staff for service group *g* on day *d* in time period *t*

MaxC maximum percentage of cross-trained employees in all staff

g index for the set of service groups *G* where *G* = {1,. . . ,9}

w index for the set of skill sets *W* where *WC* ϵ *W* is the set of multi-skill sets

d index for days of a week *D* where *D* = {1,. . . ,7}

t index for time periods (half-hour) *T* in a day where *T* = {1,. . . ,48}

W_g set of skill sets that includes service group *g*, where *g* ϵ *G*

Decision Variables:

m_{wid} number of staff with skill *w*, working in interval *i* on day *d*

n_{wi} number of staff with skill *w*, working in interval *i*

z_{wgdt} number of active staff who have skill *w* and work for service group *g (g* ϵ *w)* on day *d* in time period *t*

u_{gdt} amount of demand of staff uncovered in service group *g* on day *d* in time period *t*

Minimize

$$\sum_{w \in W} \sum_{i \in I} CI\, n_{wi} + \sum_{g \in G} \sum_{d \in D} \sum_{t \in T} CU\, u_{gdt} \tag{19}$$

Subject to

$$\sum_{w \in W_g} z_{wgdt} + u_{gdt} \geq D_{gdt} \qquad \forall\, g \in G, d \in D, t \in T \tag{20}$$

$$\sum_{i \in I} C_{it}\ m_{wid} = \sum_{g \in w} z_{wgdt} \qquad \forall\, w \in W, d \in D, t \in T \tag{21}$$

$$n_{wi}\ = 1/5 \sum_{d \in D} m_{wid} \qquad \forall\, w \in W, i \in I \tag{22}$$

$$n_{wi}\ \geq\ m_{wid} \qquad \forall\, w \in W, i \in I, d \in D \tag{23}$$

$$MaxC \left(\sum_{w \in W} \sum_{i \in I} n_{wi} \right) \geq \sum_{w \in WC} \sum_{i \in I} n_{wi} \tag{24}$$

$$m_{wid}, n_{wi}, z_{wgdt} \geq 0\ and\ integer,\ u_{gdt} \geq 0 \tag{25}$$

The objective function (19) minimizes the total cost that is composed of the staff cost and the penalty cost for uncovered demand. Treating each interval as a minimum length shift, constraints (20) - (21) ensure that the agents with relevant skill sets are sufficient to cover the demand required of each skill in each time period but with the provision that shortages are allowed. Constraints (22) - (23) ensure days off assignment and constraint (24) limits the amount of cross-training allowed for all agents. Constraint (25) satisfies the non-negativity requirements of the decision variables.

As can be seen, the detailed shift selection and break assignment decisions from the CTSSM are eliminated from P-I; these are addressed in P-II. As a result, the P-I model is much smaller. For example, for the nine service group problem with two-skill cross-training, the P-I model has 32,400 variables and 20,305 constraints, and is much easier to solve.

Table 4 presents the P-I results with two-skill cross-training for *MaxC* = 10% for the call center problem with nine service groups. The results for the other cross-training percentages are presented in Appendix D. In the table, the first column presents the intervals and the second column presents the P-I results for that interval. In the P-I result, the first parenthetical presents the skill set, and the second parenthetical presents the days of the week where that skill set is to be deployed, where "X" represents that the skill set is to be deployed and "O" represents that the skill set was not selected for that day. The seven days of a week are listed Sunday through Saturday. For example, in the first interval, skill set {1} is selected for all days in a week, whereas skill set {2} is selected only for Monday, Tuesday, Wednesday, Thursday, and Friday.

Table 4: Results of P-I of TPSA for *MaxC* = 10%

Interval	Skill Sets and Days (S,M,T,W,T,F,S)
1	{(1)(X,X,X,X,X,X,X)}, {(2)(O,X,X,X,X,X,O)}, {(2,3)(O,X,O,X,X,X,X)}, {(2,5)(X,O,X,O,X,X,X)}, {(4,9)(X,X,O,O,X,X,X)}, {(6)(X,X,O,O,X,X,X)}
2	{(1)(X,X,X,X,X,X,X)}, {(2)(X,X,X,X,X,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(5)(X,X,X,X,X,X,X)}, {(6)(X,X,X,X,X,X,X)}, {(7)(X,X,X,X,O,X,O)}, {(9)(X,X,X,X,X,X,X)}
3	{(1)(X,X,X,X,X,X,X)}, {(1,6)(X,O,O,X,X,X,X)}, {(2)(X,X,X,X,X,X,X)}, {(2,5)(X,O,X,X,X,X,O)}, {(3)(X,X,X,X,X,X,X)}, {(3,4)(O,X,X,X,X,X,O)}, {(3,6)(O,X,X,X,X,X,O)}, {(4)(X,X,X,X,X,X,X)}, {(4,8)(X,O,X,O,X,X,X)}, {(5)(X,X,X,X,X,X,X)}, {(6)(X,X,X,X,X,X,X)}, {(7)(X,X,X,X,X,X,X)}, {(8)(X,X,X,X,X,X,X)}, {(9)(X,X,X,X,X,X,X)}
4	{(1)(X,X,X,X,X,X,X)}, {(1,4)(O,X,X,X,X,X,X)}, {(1,5)(X,O,X,X,X,X,O)}, {(2)(X,X,X,X,X,X,X)}, {(2,3)(X,X,X,O,X,X,O)}, {(3)(X,X,X,X,X,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(5)(X,X,X,X,X,X,X)}, {(6)(X,X,X,X,X,X,X)}, {(7)(X,X,X,X,X,X,X)}, {(8)(X,X,X,X,X,X,X)}, {(9)(X,X,X,X,X,X,X)}
5	{(1)(X,X,X,X,X,X,X)}, {(1,9)(X,X,X,O,O,X,X)}, {(2)(X,X,X,X,X,X,X)}, {(2,9)(O,X,X,X,X,O,X)}, {(3)(X,X,X,X,X,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(4,5)(X,X,X,X,O,X,O)}, {(4,8)(X,X,O,X,X,O,X)}, {(5)(X,X,X,X,X,X,X)}, {(6)(X,X,X,X,X,X,X)}, {(7)(X,X,X,X,X,X,X)}, {(8)(X,X,X,X,X,X,X)}, {(9)(O,X,X,X,X,X,O)}
6	{(1)(X,X,X,X,X,X,X)}, {(1,3)(O,X,X,X,X,O,X)}, {(1,9)(O,X,O,X,X,X,X)}, {(2)(X,X,X,X,X,X,X)}, {(3)(O,X,X,X,X,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(5)(X,X,X,X,X,X,X)}, {(6)(X,X,O,X,O,X,X)}, {(6,8)(O,X,X,X,X,X,O)}, {(7)(O,X,O,X,X,X,X)}, {(8)(O,X,O,X,X,X,X)}, {(8,9)(X,X,X,X,O,O,X)}

3.1.2 Second Phase

A detailed staff scheduling model is solved with shift selection, and days off and break assignment in P-II; however, unlike the full CTSSM, it does not include all possible shifts and skill combinations, which comprise the majority of the variables and constraints. To fully utilize the solution provided by P-I, in P-II, only the skill - shift - day combinations that cover the skill - interval - day combinations obtained in P-I are defined.

More specifically, let $m_{\bar{w}\bar{i}\bar{d}} > 0$ be the solution from P-I which means skill $\bar{w}$ has been assigned to interval *i* on day $\bar{d}$, then full-time shift variable yf_{wfd} exists in P-II if and only if: a) the start time of shift *f* is earlier than the start time of interval *i*, b) the end time of shift *f* is later than the end time of interval *i*, c) $w = \bar{w}$, and d) $d = \bar{d}$. This is true for other extended and part-time shift variables ye_{wed} and yp_{wpd}, and all other skill related variables. For example, Figure 3 demonstrates the shift selection process for full-time shifts. As seen in the figure, if skill set *w* is utilized on day *d* in interval 3 in P-I of the TPSA, in P-II, only the full-time shift variables xf_{wf} and yf_{wfd} that correspond to full-time shifts (*f*) 5, 6, 7, 8, and 9 which completely cover interval 3 are created, but the variables that correspond to full-time shifts 1 to 4 and 10 to 16 which do not completely cover the interval are not created. This has dramatically reduced the size of the problem.

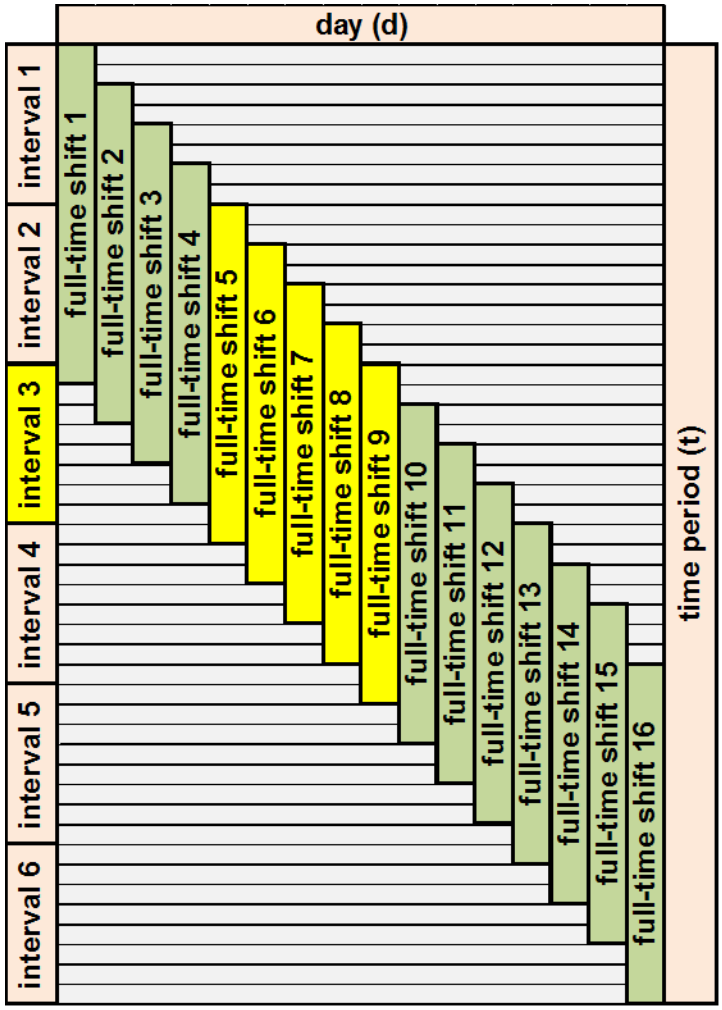

Figure 3: Shift Selection Process in P-II

3.1.3 Model Characteristics, Size, and Computation Time

The detailed P-II model is similar to the original CTSSM, yet is much reduced in size. For example, recall in the CTSSM, with nine service groups and two-skill cross-training, there are 45 skill sets and 106 shift types; as such, there are a total of 4,770 skill - shift combinations for all *MaxC* values (the sum of the *xf, xe,* and *xp* decision variables). In the TPSA, however, the total of skill - shift combinations in P-II are 917, 1,218, 1,439, 1,842, 2,187, 1,775, 1,998, 2,124, 2,041, and 1,956 for *MaxC* = 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90%, and 100%, respectively.

The breakdown of the model components for two-skill, three-skill, and four-skill cross-training with *MaxC =* 10% is presented in Table 5. For the CTSSM, the model size is constant for all partial cross-training ratios (*MaxC*). It bears mention that, for the TPSA, the model size of P-I is constant, yet the model size of P-II changes with different *MaxC* values; for different *MaxC* values, different skill - interval combinations could appear in the optimization result from P-I. Therefore, different skill – shift combinations are generated in P-II based on the P-I results. The model sizes for P-II for various *MaxC* values are presented in Table 6 for the call center problem with nine service groups.

As can be seen from the results in Table 5, when employing the TPSA with two-skill cross-training and *MaxC* = 10%, the size of the detailed staffing and scheduling with cross-training problem is reduced from 80,685 variables and 69,851 constraints to a P-I problem with 32,400 variables and 20,305 constraints and a P-II problem with 21,320 variables and 20,382 constraints; both problems are about 1/3 of the original CTSSM in both the number of variables and constraints.

For three-skill cross-training, the problem size reduces from 259,521 variables and 194,591 constraints to 121,104 variables and 52,561 constraints for the P-I problem and 23,475 variables and 20,250 constraints for the P-II problem for *MaxC* = 10%; the P-I problem has about 1/2 of the variables and 1/4 of the constraints of the CTSSM, while the P-II problem has about 1/10 of both the variables and constraints of the CTSSM.

For four-skill cross-training, the problem size reduces from 570,111 variables and 381,701 constraints to 296,496 variables and 100,945 constraints for the P-I problem and 25,705 variables and 20,836 constraints for the P-II problem for *MaxC* = 10%; the P-I problem has about 1/2 of the variables and 1/4 of the constraints of the CTSSM, while the P-II problem has about 1/20 of both the variables and constraints of the CTSSM.

Table 5: Breakdown of CTSSM and TPSA for Two, Three, Four-Skill CT (*MaxC* = 10%)

		Model Size								
		Two-Skill Cross-Training			Three-Skill Cross-Training			Four-Skill Cross-Training		
			TPSA			TPSA			TPSA	
Model Components		CTSSM	P-I	P-II	CTSSM	P-I	P-II	CTSSM	P-I	P-II
Variables	Shift (integer)									
	z_{wgdt}	27,216	27,216	7,950	111,888	111,888	10,281	281,232	281,232	12,344
	$yf_{wfd}, ye_{wed}, yp_{wpd}$	33,390	N/A	5,775	95,718	N/A	5,697	189,210	N/A	5,763
	$xf_{wf}, xe_{we}, xp_{wp}$	**4,770**	N/A	**917**	13,674	N/A	905	27,030	N/A	931
	m_{wid}	N/A	1,890	N/A	N/A	5,418	N/A	N/A	10,710	N/A
	n_{wi}	N/A	270	N/A	N/A	774	N/A	N/A	1,530	N/A
	Break (integer)									
	b_{wdt}	12,285	N/A	3,654	35,217	N/A	3,568	69,615	N/A	3,643
	Uncovered Demand									
	u_{gdt}	3,024	3,024	3,024	3,024	3,024	3,024	3,024	3,024	3,024
	Total	**80,685**	**32,400**	**21,320**	**259,521**	**121,104**	**23,475**	**570,111**	**296,496**	**25,705**
Constraints	Shift Scheduling									
	demand coverage	3,024	3,024	2,991	3,024	3,024	2,997	3,024	3,024	3,003
	shift assignment	15,120	15,120	5,444	43,344	43,344	5,388	85,680	85,680	5,636
	Days Off Assignment									
	daily demand	4,770	270	917	13,674	774	905	27,030	1,530	931
	highest demand	33,390	1,890	5,775	95,718	5,418	5,697	189,210	10,710	5,763
	Break Assignment	13,545	N/A	5,253	38,829	N/A	5,261	76,755	N/A	5,501
	Flexibility Limitation	2	1	2	2	1	2	2	1	2
	Total	**69,851**	**20,305**	**20,382**	**194,591**	**52,561**	**20,250**	**381,701**	**100,945**	**20,836**

Table 6: Model Sizes for P-II for Two, Three, and Four-Skill CT (*MaxC* = 10% - 100%)

		MaxC										
	CT	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	Avg.
Variables	2-Skill	21,320	28,227	33,172	41,974	47,345	41,271	45,199	46,840	46,375	43,756	**39,548**
	3-Skill	23,475	33,538	42,122	49,053	60,338	68,586	70,510	71,351	71,029	64,608	**55,461**
	4-Skill	25,705	35,801	47,046	59,100	64,259	74,061	72,502	86,765	99,299	100,110	**66,465**
Constraints	2-Skill	20,382	26,026	29,933	36,644	40,887	35,637	38,469	39,123	38,506	36,028	**34,164**
	3-Skill	20,250	27,037	32,125	36,479	44,251	49,267	50,693	51,110	52,143	48,093	**41,145**
	4-Skill	20,836	27,166	33,687	39,602	43,684	50,245	48,296	54,524	60,750	60,312	**43,910**

As mentioned, the model size of P-II changes with varying values of cross-training percentage (*MaxC*), whereas it is constant for the original CTSSM. As can be seen from Table 6, on average for *MaxC* = 10% to 100%, the P-II problem has 39,548 variables and 34,164 constraints in two-skill cross-training, 55,461 variables and 41,145 constraints in three-skill cross-training, and 66,465 variables and 43,910 constraints in four-skill cross-training. On average, in the two-skill cross-training models, the P-II problem has about 1/2 of both the variables and constraints of the original CTSSM which has 80,685 variables and 69,851 constraints. In the three-skill cross-training models, the P-II problem has about 1/5 of both the variables and constraints of the original CTSSM, which has 259,521 variables and 194,591 constraints. In the four-skill cross-training models, the P-II problem has about 1/9 of both the variables and constraints of the original CTSSM, which has 570,111 variables and 381,701 constraints. The comparisons of problem sizes for the CTSSM and P-I and P-II (average for *MaxC* = 10% - 100%) of the TPSA for different cross-training configurations are presented in Table 7.

Table 7: Problem Size Comparisons for CTSSM and P-I and P-II of TPSA for Case L1

	Comparison of CTSSM Problem			
	P-I Problem		P-II Problem	
	# of Variables	# of Constraints	# of Variables	# of Constraints
2-Skill CT	2/5	2/7	1/2	1/2
3-Skill CT	1/2	1/4	1/5	1/5
4-Skill CT	1/2	1/4	1/9	1/9

Even when *MaxC* = 100%, for the two-skill cross-training models, the maximum size of the P-II problem is still only 43,756 variables and 36,028 constraints, which is 1/2 of the variables and constraints of the original CTSSM, which has 80,685 variables and 69,851 constraints. For the three-skill cross-training models, the P-II problem has 64,608 variables and 48,093 constraints, which is 1/4 of the variables and constraints of the original CTSSM, which has 259,521 variables and 194,591 constraints. For the four-skill cross-training models, the P-II problem has 100,110 variables and 60,312 constraints, which is 1/6 of the variables and constraints of the original CTSSM, which has 570,111 variables and 381,701 constraints.

As it can be seen, an increase in the number of cross-training skills tends to increase the size of the CTSSM and thus computational difficulty. For cross-training with a maximum of three and four skills, the CTSSM becomes computationally intractable – the XPRESS Solve could not find any feasible solution in a day of computation for a CTSSM problem with a maximum of three skills cross-training and in two days of computation for a CTSSM problem with a maximum of four skills cross-training. On the other hand, an increase in the number of cross-training skills tends to increase the size of P-I and P-II, whereas an increase in the proportion of cross-trained workers tends to increase the size of P-II. Nevertheless, the TPSA was able to quickly find good solutions, which were proven to be feasible when seeded back to the CTSSM; as such, the TPSA is adopted in the rest of the managerial studies presented.

3.2 Computational Results for Two-Phase Sequential Approach

Extensive experiments have been conducted to evaluate the effectiveness of the TPSA as applied to the call center cross-training staff scheduling problem. All of the code was written in Xpress-Mosel, the modeling language of Xpress Optimization, and solved using its embedded Xpress-MP Solver. The computation was performed on an Intel Core i7 computer with a 3.4 GHz CPU and 16GB of RAM. The input data in the form of number of employees required in each time period of a week for each of the nine service groups were provided by the company.

The computational experiments were designed to compare the performance of the TPSA to the solution of the CTSSM defined by Equations (1) - (15). Using the demand data given by the company, various test cases were generated and the proposed methods were tested.

3.2.1 Test Case Generation

As shown in Figure 1, service groups 1, 2, 3, 4, and 5 show higher demands whereas service groups 6, 7, 8, and 9 show lower demands; service groups 2, 3, and 5 have weekday and weekend distinctions in volume whereas groups 1, 4, 6, 7, 8, and 9 do not.

In view of this, three sets of problems were generated based on: a) the size of demand, and b) distinctions in volume between weekdays and weekends.

The first test set has five small size problems labeled as Case *Sx*, each with three service groups:

- *Case S1*: Service groups 1, 2, and 3; all groups are similar in volume, but group 1 has no apparent weekday - weekend distinction in volume; groups 2 and 3 show a much lower demand in weekends than in weekdays.
- *Case S2*: Service groups 2, 3, and 5; all groups are similar in volume and all have weekday - weekend distinctions.
- *Case S3*: Service groups 1, 4, and 6; all groups have similar demand profiles, but the volume is uneven.
- *Case S4*: Service groups 4, 5, and 6; all groups have different demand profiles, but group 6 has lower demand and group 5 has a weekday - weekend distinction in volume.
- *Case S5*: Service groups 7, 8, and 9; all groups have similar demand profiles and volumes.

The second test set has four medium size problems labeled as Case *Mx*, each with six service groups:

- *Case M1*: Service groups 1, 4, 6, 7, 8, and 9; these groups have no apparent weekday - weekend distinctions, but have variation in volume.
- *Case M2*: Service groups 2, 3, 5, 7, 8, and 9; groups 2, 3, and 5 have weekday - weekend distinctions and have a larger volume that of groups 7, 8, and 9.

- *Case M3*: Service groups 1, 2, 3, 4, 5, and 6; groups 1, 4, and 6 have no weekday - weekend distinctions, groups 2, 3, and 5 have weekday - weekend distinctions, and groups 4, 5, and 6 are smaller in volume.
- *Case M4*: Service groups 1, 2, 3, 7, 8, and 9; groups 7, 8, and 9 are smaller in volume and have similar demand profiles.

The third test set has three large size problems labeled as Case *Lx*, each has the full nine service groups:

- *Case L1*: Service groups 1, 2, 3, 4, 5, 6, 7, 8, and 9 with various volumes in demand, some with weekday - weekend distinctions and some without (this is the original call center problem).
- *Case L2*: Service groups 1 to 9, yet the weekend demand of groups 2 and 3 are increased by 2 and group 5 is increased by 3, to make nine groups with no weekday - weekend distinctions.
- *Case L3*: Service groups 1 to 9, yet the weekend demand of group 1 is reduced to 1/2, and the weekday demand of groups 4, 6, 7, 8, and 9 are increased by 2 to make nine groups all with weekday - weekend distinctions.

The purpose is to evaluate cross-training at various sets with three (small), six (medium), and nine (large) service groups, and each set is composed of mixed demand patterns and weekday - weekend distinction combinations.

3.2.2 Test Case Results

Table 8, Table 9, and Table 10 present the objective function values for various cross-training ratios, *MaxC* = 10% to 100%, with two-skill cross-training for small, medium, and large test cases, which are then compared in Figure 4, Figure 5, and Figure 6. In the tables, "XPRESS" represents the commercial solver, Xpress, to the solution of the CTSSM and "TPSA" represents the two-phase sequential approach to the solution of the CTSSM. "Cost" represents the best solution obtained, "B.Bou." represents the best bound found in the branch and bound process, and "Gap" represents the solution gap between the best solution and the best bound when the search is terminated. In the figures, the horizontal axes represent the various cross-training ratios (*MaxC* = 10% - 100%) whereas the vertical axes represent the total weekly cost. The lines represent the results of XPRESS and TPSA for the solution of the CTSSM for the test cases.

For the small test cases, the computation time is set to 2 hours for XPRESS and 20 minutes for the TPSA (10 minutes for each of P-I and P-II), and the computational results are presented in Table 8 and Figure 4.

Table 8: Results for XPRESS and TPSA for Case S1, S2, S3, S4, and S5

Case	Approach	Result	MaxC									
			10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
S1	XPRESS	Cost	46,421	46,504	46,641	46,574	46,757	45,723	46,776	46,526	46,203	47,552
		B.Bou.	43,459	43,276	43,273	43,272	43,272	43,273	43,273	43,272	43,273	43,273
		Gap	6.38%	6.94%	7.22%	7.09%	7.45%	5.36%	7.49%	7.00%	6.34%	9.00%
	TPSA	Cost	**45,786**	**46,007**	**45,465**	**45,730**	**45,943**	46,056	**45,342**	**44,819**	**45,206**	**45,254**
		B.Bou.	43,584	43,305	43,299	43,297	43,302	43,297	43,297	43,297	43,298	43,298
		Gap	4.81%	5.87%	4.76%	5.32%	5.75%	5.99%	4.51%	3.40%	4.22%	4.32%
S2	XPRESS	Cost	37,478	35,824	36,343	36,429	35,983	35,249	36,511	35,496	35,669	35,887
		B.Bou.	34,050	33,797	33,780	33,780	33,780	33,780	33,781	33,780	33,780	33,781
		Gap	9.15%	5.66%	7.05%	7.27%	6.12%	4.17%	7.48%	4.84%	5.30%	5.87%
	TPSA	Cost	**36,480**	36,115	**35,319**	**36,363**	36,084	35,986	**35,057**	**35,253**	**35,556**	**35,559**
		B.Bou.	34,198	33,849	33,828	33,796	33,793	33,797	33,796	33,793	33,793	33,790
		Gap	6.25%	6.28%	4.22%	7.06%	6.35%	6.08%	3.60%	4.14%	4.96%	4.98%
S3	XPRESS	Cost	39,647	39,529	39,784	39,042	39,141	38,588	39,811	38,862	38,958	38,962
		B.Bou.	36,668	36,446	36,442	36,443	36,442	36,442	36,442	36,442	36,442	36,442
		Gap	7.51%	7.80%	8.40%	6.66%	6.90%	5.56%	8.46%	6.23%	6.46%	6.47%
	TPSA	Cost	**39,405**	39,628	39,920	39,494	**38,422**	**38,206**	**38,759**	**38,614**	**38,438**	**38,039**
		B.Bou.	36,918	36,517	36,506	36,506	36,511	36,506	36,509	36,506	36,507	36,508
		Gap	6.31%	7.85%	8.55%	7.56%	4.97%	4.45%	5.81%	5.46%	5.02%	4.02%
S4	XPRESS	Cost	30,037	29,032	30,114	28,638	29,284	29,721	29,310	29,466	29,246	29,395
		B.Bou.	27,301	26,965	26,906	26,906	26,904	26,904	26,906	26,905	26,908	26,905
		Gap	9.11%	7.12%	10.66%	6.05%	8.13%	9.48%	8.20%	8.69%	8.01%	8.47%
	TPSA	Cost	**29,308**	29,220	**29,230**	29,138	**28,099**	**28,232**	**28,527**	**28,694**	**28,866**	**28,451**
		B.Bou.	27,465	27,079	26,960	26,933	26,935	26,933	26,934	26,933	26,937	26,938
		Gap	6.29%	7.33%	7.76%	7.57%	4.14%	4.60%	5.59%	6.14%	6.68%	5.32%
S5	XPRESS	Cost	18,667	19,025	19,072	17,365	17,738	17,650	17,357	17,830	16,689	17,446
		B.Bou.	17,116	16,711	16,532	16,543	16,412	16,408	16,409	16,409	16,414	16,412
		Gap	8.31%	12.16%	13.32%	5.25%	7.48%	7.04%	5.46%	7.97%	1.65%	5.93%
	TPSA	Cost	**18,588**	**18,456**	**18,520**	18,262	**17,550**	**17,580**	**17,220**	**16,806**	16,915	**17,076**
		B.Bou.	17,367	16,984	16,668	16,567	16,527	16,517	16,471	16,495	16,441	16,485
		Gap	6.58%	7.97%	10.00%	9.28%	5.83%	6.05%	4.35%	1.85%	2.80%	3.47%

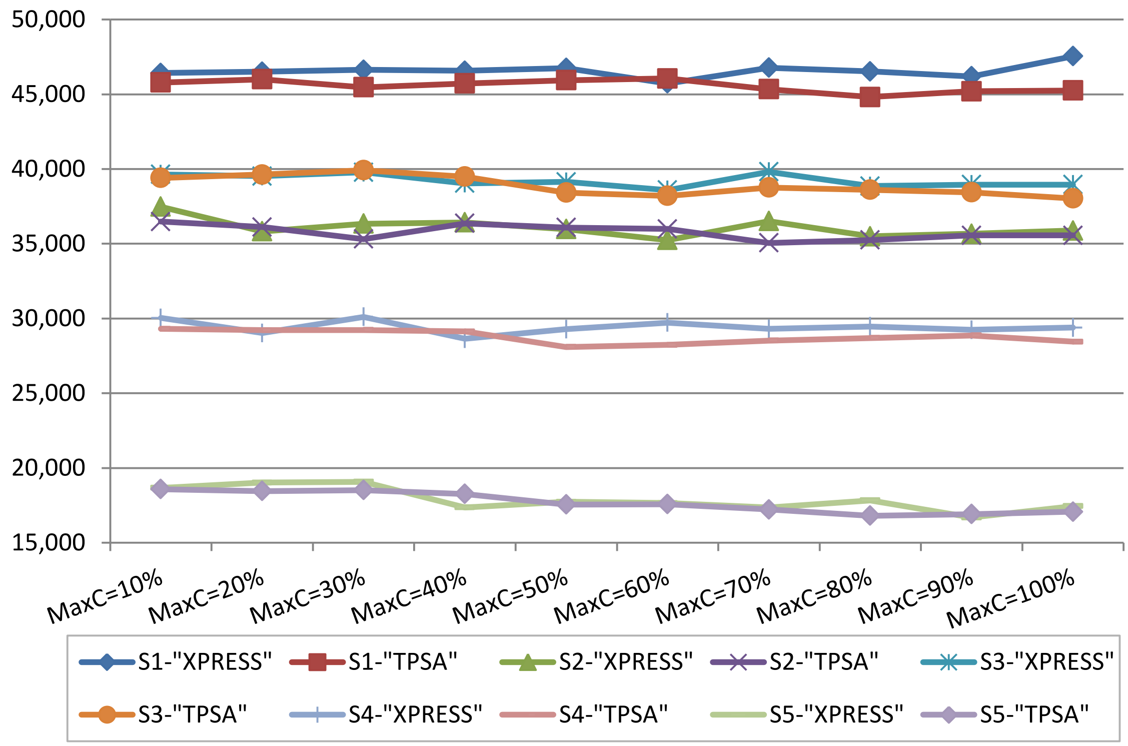

Figure 4: XPRESS and TPSA Comparisons for Small Test Cases

As can be seen, the TPSA gets comparable results to XPRESS in these small size problems. Out of the full 50 problems, the TPSA is able to get better results in 39 cases (bolded in the table); recall that these computational results for the TPSA are obtained in 20 minutes, as compared to 2 hours for XPRESS.

For the medium test cases, the computation time is set to 6 hours for XPRESS and 1 hour for the TPSA (1/2 hour for each of P-I and P-II), and the computational results are presented in Table 9 and Figure 5.

Table 9: Results for XPRESS and TPSA for Case M1, M2, M3, and M4

Case	Approach	Result	MaxC									
			10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
M1	XPRESS	Cost	59,198	58,404	57,315	58,110	56,640	58,610	57,152	56,535	57,051	56,545
		B.Bou.	53,391	52,341	52,010	51,977	51,976	51,977	51,976	51,976	51,976	51,976
		Gap	9.81%	10.38%	9.26%	10.56%	8.23%	11.32%	9.06%	8.06%	8.89%	8.08%
	TPSA	Cost	59,317	**58,236**	58,399	**55,862**	**55,211**	**55,661**	**55,998**	**55,806**	**56,728**	**55,393**
		B.Bou.	54,033	52,922	52,202	52,397	52,032	52,027	52,032	52,027	52,024	52,027
		Gap	8.91%	9.12%	10.62%	6.20%	5.76%	6.53%	7.08%	6.77%	8.29%	6.08%
M2	XPRESS	Cost	56,316	55,989	55,414	53,964	54,391	54,903	53,729	54,182	53,583	53,774
		B.Bou.	50,862	49,961	49,676	49,627	49,627	49,627	49,627	49,627	49,627	49,627
		Gap	9.68%	10.77%	10.35%	8.04%	8.76%	9.61%	7.63%	8.41%	7.38%	7.71%
	TPSA	Cost	**55,960**	56,532	55,746	54,577	**53,218**	**52,461**	**52,945**	**52,694**	53,844	**52,789**
		B.Bou.	51,316	50,370	50,683	49,941	49,693	49,681	49,679	49,673	49,708	49,663
		Gap	8.30%	10.90%	9.08%	8.50%	6.62%	5.30%	6.18%	5.73%	7.69%	5.93%
M3	XPRESS	Cost	75,894	76,710	76,981	75,910	75,675	76,388	76,153	74,401	73,915	75,324
		B.Bou.	70,382	69,677	69,633	69,633	69,633	69,633	69,632	69,633	69,633	69,633
		Gap	7.26%	9.17%	9.55%	8.27%	7.98%	8.84%	8.56%	6.41%	5.79%	7.56%
	TPSA	Cost	**74,524**	**74,653**	**74,798**	**74,725**	**74,214**	**74,464**	**74,649**	**73,505**	**73,690**	**73,421**
		B.Bou.	71,006	70,068	69,695	69,671	69,677	69,671	69,671	69,672	69,671	69,671
		Gap	4.72%	6.14%	6.82%	6.76%	6.11%	6.44%	6.67%	5.22%	5.45%	5.11%
M4	XPRESS	Cost	65,470	65,735	63,853	64,180	64,303	65,053	64,347	63,281	63,242	63,815
		B.Bou.	60,164	59,267	59,068	59,061	59,061	59,061	59,062	59,061	59,061	59,061
		Gap	8.10%	9.84%	7.49%	7.98%	8.15%	9.21%	8.21%	6.67%	6.61%	7.45%
	TPSA	Cost	65,751	**64,075**	**63,320**	**63,113**	**62,457**	**62,728**	**62,432**	**62,748**	63,496	**62,832**
		B.Bou.	60,722	59,632	59,909	59,140	59,103	59,091	59,098	59,094	59,092	59,109
		Gap	7.65%	6.93%	5.39%	6.30%	5.37%	5.80%	5.34%	5.82%	6.94%	5.92%

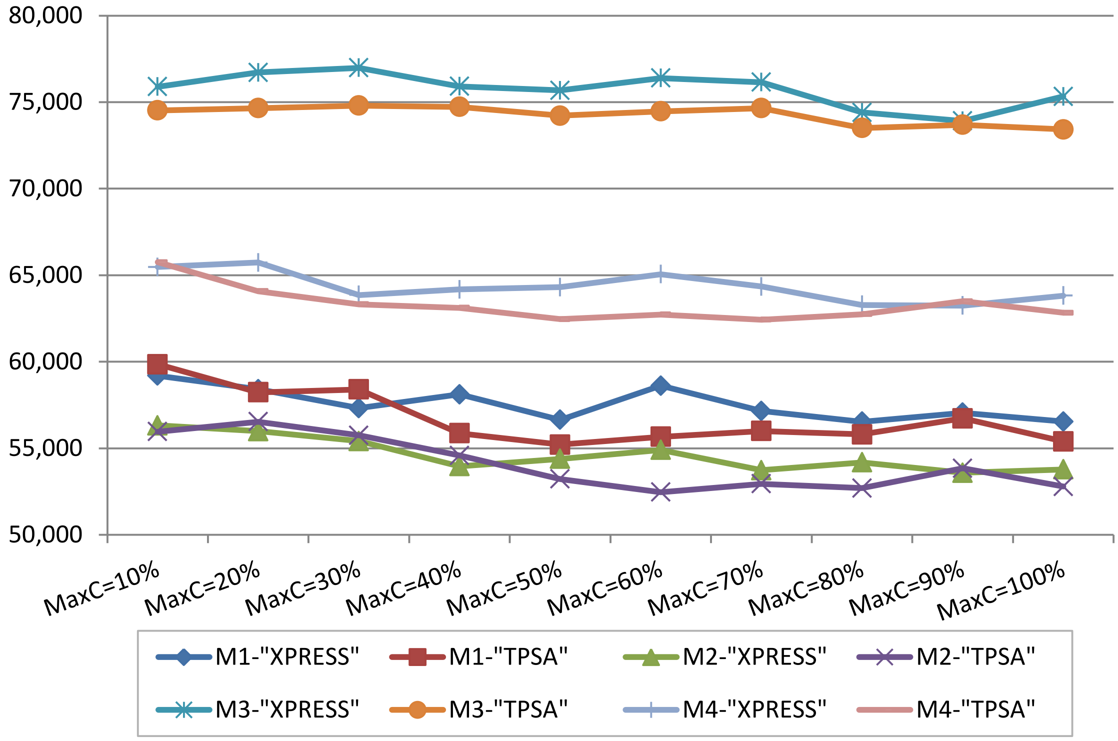

Figure 5: XPRESS and TPSA Comparisons for Medium Test Cases

For the medium size problems, the TPSA gets comparable results to XPRESS. Out of the 40 problems, the TPSA is able to get better results in 32 cases (bolded in the table); recall that these computational results for the TPSA are obtained in 1 hour, as compared to 6 hours for XPRESS.

For the large test cases, the computation time is set to 12 hours for XPRESS and 2 hours for the TPSA (1 hour for each of P-I and P-II), and the computational results are presented in Table 10 and Figure 6.

Table 10: Results for XPRESS and TPSA for Case L1, L2, and L3

Case	Approach	Result	MaxC									
			10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
L1	XPRESS	Cost	95,704	96,392	94,023	94,989	93,132	94,872	94,061	94,330	93,045	93,278
		B.Bou.	87,162	85,684	85,399	85,396	85,396	85,396	85,396	85,396	85,396	85,397
		Gap	8.93%	11.11%	9.17%	10.10%	8.31%	9.99%	9.21%	9.47%	8.22%	8.45%
	TPSA	Cost	**94,376**	**93,881**	**93,050**	**92,218**	**91,866**	**89,996**	**91,434**	**91,005**	**91,279**	**91,465**
		B.Bou.	87,981	86,435	85,738	85,530	85,462	85,440	85,440	85,434	85,435	85,434
		Gap	6.78%	7.93%	7.86%	7.26%	6.97%	5.06%	6.56%	6.12%	6.40%	6.59%
L2	XPRESS	Cost	102,956	102,494	102,125	100,845	100,326	101,338	99,254	99,656	99,070	100,150
		B.Bou.	93,828	92,167	91,766	91,744	91,744	91,744	91,744	91,744	91,744	91,744
		Gap	8.87%	10.08%	10.14%	9.03%	8.55%	9.47%	7.57%	7.94%	7.39%	8.39%
	TPSA	Cost	**102,128**	**99,957**	**99,877**	**98,120**	**96,935**	**97,296**	**97,391**	**97,668**	**97,736**	**98,795**
		B.Bou.	94,738	92,938	92,069	91,867	92,038	91,785	91,782	91,787	91,777	91,776
		Gap	7.24%	7.02%	7.82%	6.37%	5.05%	5.67%	5.76%	6.02%	6.10%	7.11%
L3	XPRESS	Cost	116,658	117,666	114,466	114,616	116,730	113,825	113,967	114,252	112,846	115,933
		B.Bou.	108,939	106,783	106,239	106,214	106,214	106,214	106,214	106,214	106,214	106,214
		Gap	6.62%	9.25%	7.19%	7.33%	9.01%	6.69%	6.80%	7.04%	5.88%	8.38%
	TPSA	Cost	**116,411**	**116,953**	**113,562**	**111,867**	**111,224**	**111,905**	**113,111**	**112,426**	**112,331**	**112,926**
		B.Bou.	109,756	107,657	107,153	106,721	106,255	106,255	106,241	106,249	106,245	106,242
		Gap	5.72%	7.95%	5.64%	4.60%	4.38%	5.05%	6.07%	5.49%	5.42%	5.92%

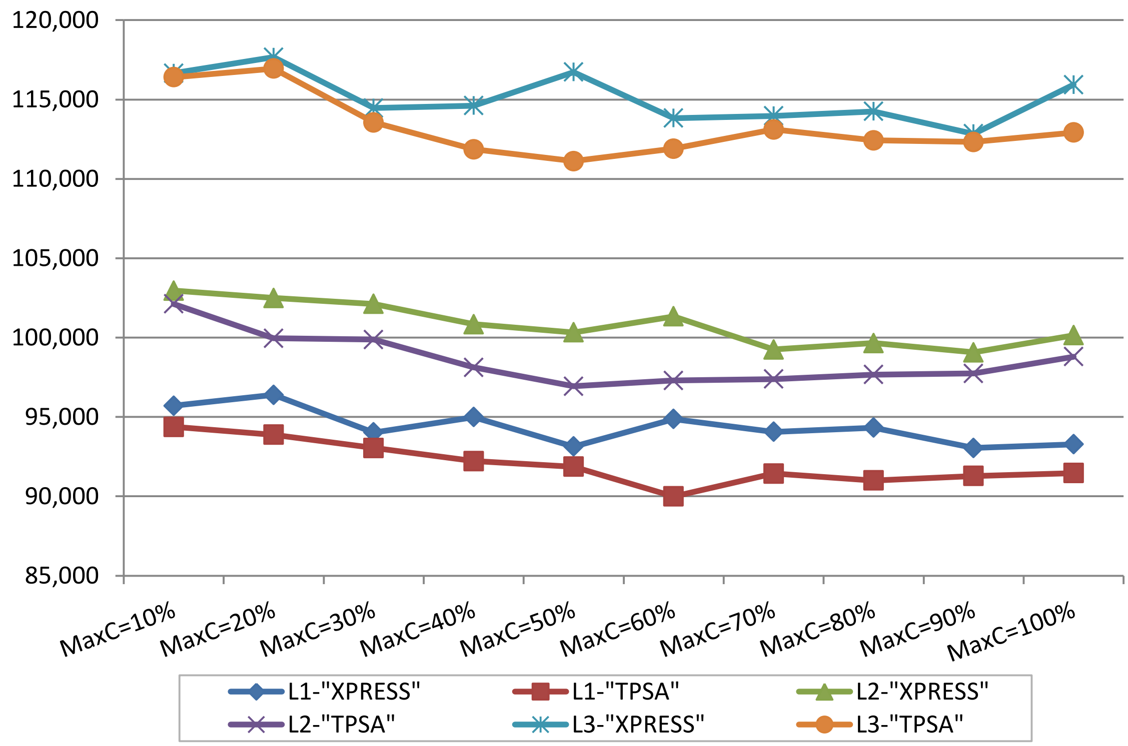

Figure 6: XPRESS and TPSA Comparisons for Large Test Cases

For the large size problems, the TPSA is able to get better solutions in all 30 problems (bolded in the table) with sometimes significant reductions in cost. The average cost for case L1 is $94,383 for XPRESS and $92,057 for the TPSA; the TPSA could find solutions that are nearly 2.5% better than XPRESS. To stress computation time again, the TPSA results are obtained in 2 hours and the XPRESS results are obtained in 12 hours.

Computationally, the TPSA has been able to quickly find good solutions, as compared with XPRESS. The TPSA decreases the size of the CTSSM, is able to get comparable or better solutions in a much shorter amount of time, and is applicable to real world cross-training and scheduling problems. Thus, the TPSA is adopted for the experimental studies in later stages.

As such, the detailed results of XPRESS and TPSA for case L1, which is the original call center problem, are given in Appendix E. The weekly schedule obtained by the TPSA for case L1 with *MaxC* = 10% is presented in Appendix F.

3.2.3 Summary of Results

From a modeling perspective, the TPSA differs from other models proposed in the literature in two ways. First, it does not give a complete pooling decision for the service groups in the beginning of scheduling. Second, it does not employ predetermined or random cross-training decisions or simple formulae or approaches (e.g. chaining) to assign secondary skills to agents. The current studies in the literature generally analyze the service groups and decide which groups to pool over the planning horizon (e.g. Tekin et al., 2009), employ predetermined (e.g. Cezik and L'Ecuyer, 2008; Avramidis et al., 2009 and 2010; Gurvich, 2010; Easton, 2011; Campbell, 2012; Roubos and Bhulai, 2012; Adan et al., 2013) or random cross-training configurations (e.g. Batta et al., 2007; Brusco, 2008; Campbell, 2011), or use a simple formula or take a simple approach to assign secondary skills to agents (e.g. Wallace and Whitt, 2005; Brusco, 2008; Ahgari and Balcioglu, 2009; Gnanlet and Gilland, 2014; Paul and MacDonald, 2014).

The TPSA solves an optimization problem with different demand profiles for each service group for different time intervals through each day of the week and decides a) which groups should be cross-trained and b) in which time periods and days they should be deployed. Based on this decision, it then finds the number of employees, their skill sets, and their shift schedules. Therefore, cross-training is not only a demand-based decision (certain demands balance each other) but also a time-based decision, especially in the case where there exists variation in demand for different time intervals of a day and days of the week.

For instance, as presented in Table 4, for the original nine groups call center problem (case L1), P-I of the TPSA with two-skill cross-training and *MaxC* = 10% chooses the skill sets {1,9}, {2,9}, {4,5}, {4,8} between the 33rd – 40th time periods (interval 5), but {1,3}, {1,9}, {6,8}, {8,9} between the 41st – 48th time periods (interval 6). It chooses the skill set {1,9} in both intervals but between the 33rd – 40th time periods on Sunday, Monday, Tuesday, Friday, and Saturday and between the 41st – 48th time periods on Monday, Wednesday, Thursday, Friday, and Saturday. This result shows that demand variation both in a day and between days of a week has an effect on cross-training decision, which in turn has an effect on skill set selection.

From a computational perspective, the TPSA creates an integrated cross-training decision by taking into account both demand and time combinations, thus decreasing the number of skill sets and shifts that are going to be evaluated in the detailed staffing and scheduling problem. Specifically, the P-I variable set n_{wi} represents the creation of variables which are then fixed in the P-II process; this allows significant reduction in the size of the problem and leads to superior solutions.

The size of the problem is mainly dependent on the number of service groups. The computation times get longer when the call center size increases. For instance, when there are more than two service groups, limited cross-training is possible, which increases the number of possible cross-training configurations and the complexity of decision making.

With the TPSA, the goal is to provide general, strategic insights into the selection of an effective skill pattern, and to determine the optimal workforce mix of flexible and specialized servers.

CHAPTER 4 OPERATIONAL CROSS-TRAINING STAFF SCHEDULING MODEL AND COMPUTATIONAL RESULTS

4.1 Operational Cross-Training Staff Scheduling Model

Because of the increasing importance of a flexible work environment that accommodates staff preferences, an operational model that assigns employee schedules for the coming weeks while satisfying employee preferences is proposed. The preference-based model aims to prepare a weekly schedule which mostly overlaps with individual preferences of employees, and thus helps to improve employees' morale and service quality.

The problem is to assign currently available employees to various shifts while satisfying demand, minimizing costs, and maximizing overlap between the weekly schedule and the preferred schedule of each employee. Because the primary purpose of staff scheduling is to minimize costs while satisfying customer demand, the overall objective of the preference-based model combines both cost minimization and preference satisfaction maximization.

The proposed model is divided into four parts: a) shift scheduling with constraints (2) – (3) and (27) – (31), b) days off scheduling with constraints (4) – (9), c) lunch break assignment with constraints (10) – (12), and d) staff preference weight calculations. The preference-based model is a multi-objective model; there are two different objective function components. A frequently employed method is to combine the two objectives into a single objective function by using a weight factor, and solve them simultaneously. In this case, however, choice of the weight factor becomes crucial in balancing these two performance measures. The available employees and their individual preferences and skill sets are used as the input data to the model. The model and its notation are presented below.

Indices, Sets, and Parameters:

s	index for the set of staff members *S*
GS_s	set of service groups staff member *s* is trained for
R	objective function coefficient for staff member preferences
PF_{swfd}, PE_{swed}, PP_{swpd}	preference weight of staff member *s* who has skill set *w* and is assigned to full-time shift *f*, extended shift *e*, or part-time shift *p* on day *d*

Decision Variables:

sf_{swfd}, se_{swed}, sp_{swpd}	1 if staff member *s* has skill set *w* and is assigned to full-time shift *f*, extended shift *e*, or part-time shift *p* on day *d*

Minimize

$$\sum_{g \in G} \sum_{d \in D} \sum_{t \in T} CU\, u_{gdt} -$$

$$R \sum_{s \in S} \sum_{w \in GS_s} \sum_{d \in D} \left(\sum PF_{swfd}\, sf_{swfd} + \sum PE_{swed}\, se_{swed} + \sum PP_{swpd}\, sp_{swpd} \right) \quad (26)$$

Subject to

Constraints (2) – (15) and

a) Staff - Shift Assignment

$$\sum_{s \in S} sf_{swfd} = yf_{wfd} \quad \forall\, w \in GS_s, f \in F, d \in D \tag{27}$$

$$\sum_{s \in S} se_{swed} = ye_{wed} \quad \forall\, w \in GS_s, e \in E, d \in D \tag{28}$$

$$\sum_{s \in S} sp_{swpd} = yp_{wpd} \quad \forall\, w \in GS_s, p \in P, d \in D \tag{29}$$

$$\sum_{s \in S} \sum_{w \in GS_S} \sum_{d \in D} \left(\sum_{f \in F} sf_{swfd} + \sum_{e \in E} se_{swed} + \sum_{p \in P} sp_{swpd} \right) = \sum_{w \in W} \sum_{d \in D} \left(\sum_{f \in F} yf_{wfd} + \sum_{e \in E} ye_{wed} + \sum_{p \in P} yp_{wpd} \right) \tag{30}$$

$$\sum_{w \in GS_s} \sum_{d \in D} \left(\sum_{f \in F} sf_{swfd} + \sum_{e \in E} se_{swed} + \sum_{p \in P} sp_{swpd} \right) = 1 \quad \forall\, s \in S \tag{31}$$

b) Non-Negativity Requirements

$$sf_{swfd}, se_{swed}, sp_{swpd} \in \{0,1\} \quad \forall\, s, w, f, e, p, d \tag{32}$$

Objective function (26) minimizes the penalty cost of uncovered demand while maximizing the weighted sum of staff preference weights. The choice of the parameter *R* implicitly defines the trade-off between satisfying the collective preferences of the staff and incurring additional cost by allowing for uncovered demand. In general, $CU >> R$ is set to ensure that costs are minimized before satisfying preferences, although it is a managerial decision to make a trade-off between service level and preferences.

Constraints (27) – (31) deal with the assignment of the available employees to the shifts. Constraints (27) – (29) ensure that the number of staff members assigned to a shift is equal to the number of employees necessary to satisfy the demand in that shift. Constraint (30) ensures that the total number of available staff members is equal to the total number of employees required to satisfy demand. Constraint (31) guarantees that each staff member is assigned exactly one shift type and corresponding shifts of fixed length and start time through the week. Finally, constraint (32) satisfies the non-negativity requirements of the decision variables, which are all binary.

Employees have various preferences such as taking weekends off, only working specific days or shifts, starting no earlier than a specific time, preferred work duration, preferences towards a specific service group, sharing shifts with somebody with whom they can carpool, or a preference to work together with or separate from a specific employee. For example, one of the employee's preferences is to have four 10-hour shifts on Thursday, Friday, Saturday, and Sunday, working from 6 a.m. to 4:30 p.m.

In this study, the preference weights are calculated based on seniority for each employee for preferred shift types, working and off days, and start times. Basically, the weights for weekly tours that are close to the preferred tours are maximized. It is also worth mentioning that by changing the preference weights, different weekly schedules can also be obtained.

Briefly, given the size, skills, and preferences of staff, the preference-based model is to construct weekly schedules that are as close to employees' ideal schedules as possible while satisfying a given weekly demand.

4.2 Computational Results of Preference-Based Model

In this section, the results of the preference model, which takes individual employees' preferences into account, are compared with the current weekly schedule employed by the company. For each service group, the number of employees and their weekly schedules are provided by the company. For illustration purposes, only the weekly schedule of one service group is presented here.

The results are presented in Table 11 and Table 12. Table 11 presents the total uncovered demand and its penalty cost for both the preference-based model and the company schedule. The company's weekly schedule has more uncovered demand than the weekly schedule obtained by the preference-based model. The results clearly demonstrate that the current daily schedules and weekly tours of the employees in the company schedule do not cover the demand well. The weekly schedule obtained by the preference-based model is able to provide much better coverage of demand and thus increases the service level and decreases the total cost while providing a schedule that is close to employee preferences.

In Table 12, the current schedule of the call center, the preferred schedule of the staff, and the proposed schedule obtained by the preference-based model are presented. The "Seniority" column represents the seniority weight of each staff member. The “ST” columns represent the start time of the shift and the “SMTWTFS” columns represent the working days. The “Match” column shows whether the proposed schedule fits with staff preferences. The match has a (+) when at least one off day is met with the preferred off days or the scheduled start time is within ±1 hour of the preferred start time, otherwise it has a (-).

Briefly, the analysis shows that it is possible to incorporate the preference-based model into the scheduling model and derive preferred schedules without significantly increasing cost and decreasing service level.

Table 11: Results for Company Schedule and Preference-Based Model for Group 1

	Company Schedule	Preference-Based Model
Staff Cost ($)	21,000	21,000
Penalty Cost ($)	11,227	6,003
Uncovered Demand (#)	265	143

Table 12: Current, Preferred, and Proposed Schedules for Service Group 1

Staff	Seniority	Current Schedule		Preferred Schedule		Proposed Schedule		Match
		ST	SMTWTFS	ST	SMTWTFS	ST	SMTWTFS	
1	14	07:00	XXXXXOO	07:00	XXXXXOO	06:00	XXXXOOO	++
2	13	11:00	OXXXXXO	11:00	OXXXXXO	11:00	OOXXXXO	++
3	12	09:00	OXXXXXO	09:00	OXXXXXO	08:00	OOXXXXO	++
4	11	08:30	OXXXXXO	08:00	OXXXXXO	08:00	OXXXXXO	++
5	11	09:00	OOXXXXX	09:00	OOXXXXX	09:00	OOOXXXX	++
6	10	10:00	XXXXXOO	09:00	XXXXXOO	08:00	XXXXOOO	++
7	9	09:00	OOOXXXX	09:00	OXXXXXO	09:00	OXXXXOO	++
8	9	07:30	XXXXOOO	07:00	XXXXOOO	07:00	XXXXOOO	++
9	9	08:30	OXXXXXO	08:00	OXXXXXO	08:00	OXXXXOO	++
10	8	07:30	XXXXOOO	08:00	OOOXXXX	07:00	OOOXXXX	++
11	8	08:00	OOOXXXX	08:00	OOOXXXX	07:00	OOOXXXX	++
12	7	11:30	OXXXXXO	10:00	OXXXXXO	10:00	OOXXXXO	++
13	6	08:30	OOXXXXX	08:00	OOXXXXX	08:00	OOOXXXX	++
14	5	12:00	OOXXXXX	10:00	OXXXXXO	11:00	XXOOOXX	+
15	5	13:30	OOXXXXX	13:00	OXXXXXO	13:30	OXXXXOO	++
16	5	06:30	OXXXXXO	09:00	OXXXXXO	09:00	OXXXXOO	++
17	5	09:30	XXOOOXX	09:00	OXXXXXO	09:00	XXXXOOO	++
18	4	07:00	XXOOXXX	08:00	OXXXXXO	06:00	XXXXXOO	+
19	4	09:00	XXOOXXX	09:00	XOOOXXX	13:30	XOOOXXX	+
20	4	13:00	XOOXXXX	12:00	OOXXXXX	13:30	OXXXXOO	+
21	4	08:00	OOXXXXX	08:00	OOXXXXX	06:00	OOXXXXX	+
22	4	09:00	XXXXXOO	11:00	OOXXXXX	13:30	OOOXXXX	+
23	3	14:30	OXXXXXO	09:00	OXXXXXO	06:00	XXOOXXX	-
24	2	10:00	OXXXXXO	10:00	OXXXXXO	15:30	XXXXOOX	-
25	1	13:00	OOXXXXX	13:00	OOXXXXX	15:30	XXXOOXX	-

4.3 Results of Post-Processing Break Assignment Algorithm

In this section, the lunch breaks obtained by the post-processing break assignment algorithm for the weekly schedule obtained by the preference-based model for service group 1 are presented in Table 13. The results indicate that the proposed break assignment algorithm assigns lunch breaks within the break window of each eligible shift in each working day.

Table 13: Lunch Breaks for Proposed Schedule for Service Group 1

Staff	Shift Start-End Times	Break Window	Breaks						
			Sun.	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.
1	06:00-16:30	10:00-12:00	10:00-10:30	11:30-12:00	11:30-12:00	10:00-10:30	off	off	off
2	11:00-21:30	15:00-17:00	off	off	15:00-15:30	16:30-17:00	16:30-17:00	16:00-16:30	off
3	08:00-18:30	12:00-14:00	off	off	13:30-14:00	13:30-14:00	12:00-12:30	12:00-12:30	off
4	08:00-18:30	12:00-14:00	off	12:30-13:00	13:30-14:00	12:00-12:30	13:30-14:00	off	off
5	09:00-19:30	13:00-15:00	off	off	off	14:30-15:00	14:30-15:00	13:00-13:30	13:30-14:00
6	08:00-18:30	12:00-14:00	13:30-14:00	13:30-14:00	13:00-13:30	12:00-12:30	off	off	off
7	09:00-19:30	13:00-15:00	off	13:00-13:30	14:30-15:00	14:30-15:00	13:00-13:30	off	off
8	07:00-17:30	11:00-13:00	11:00-11:30	11:30-12:00	11:00-11:30	12:00-12:30	off	off	off
9	08:00-18:30	12:00-14:00	off	12:00-12:30	13:30-14:00	13:30-14:00	13:30-14:00	off	off
10	07:00-17:30	11:00-13:00	off	off	off	12:30-13:00	11:00-11:30	12:30-13:00	12:30-13:00
11	07:00-17:30	11:00-13:00	off	off	off	11:00-11:30	12:00-12:30	11:30-12:00	12:30-13:00
12	10:00-20:30	14:00-16:00	off	off	15:30-16:00	15:00-15:30	15:00-15:30	15:30-16:00	off
13	08:00-18:30	12:00-14:00	off	off	off	13:00-13:30	12:30-13:00	13:30-14:00	13:30-14:00
14	11:00-21:30	15:00-17:00	15:00-15:30	16:00-16:30	off	off	off	16:00-16:30	16:00-16:30
15	13:30-24:00	17:30-19:30	off	18:00-18:30	19:00-19:30	17:30-18:00	19:00-19:30	off	off
16	09:00-19:30	13:00-15:00	off	14:30-15:00	13:00-13:30	14:30-15:00	14:30-15:00	off	off
17	09:00-19:30	13:00-15:00	14:30-15:00	13:30-14:00	14:30-15:00	14:30-15:00	off	off	off
18	06:00-14:30	10:00-12:00	10:30-11:00	11:30-12:00	11:30-12:00	10:00-10:30	11:30-12:00	off	off
19	13:30-24:00	17:30-19:30	17:30-18:00	off	off	off	19:00-19:30	17:30-18:00	17:30-18:00
20	13:30-24:00	17:30-19:30	off	18:30-19:00	18:00-18:30	19:00-19:30	17:30-18:00	off	off
21	06:00-14:30	10:00-12:00	off	off	10:00-10:30	11:00-11:30	10:00-10:30	11:00-11:30	10:00-10:30
22	13:30-24:00	17:30-19:30	off	off	off	19:00-19:30	18:30-19:00	19:00-19:30	17:30-18:00
23	06:00-14:30	10:00-12:00	11:30-12:00	11:30-12:00	off	off	10:30-11:00	10:00-10:30	11:30-12:00
24	15:30-24:00	19:30-21:30	20:30-21:00	19:30-20:00	21:00-21:30	19:30-20:00	off	off	21:00-21:30
25	15:30-24:00	19:30-21:30	21:00-21:30	19:30-20:00	19:30-20:00	off	off	21:00-21:30	21:00-21:30

4.4 System Implementation: Interface for Staff Scheduling

To manage the staffing and scheduling process of the call center, an interactive interface that provides an easy-to-use tool for the schedulers was built. The interface design also provides a solution to the preference bidding and re-optimization process. Bidding and re-scheduling are necessary to generate shifts that suit staff preferences so as to improve employee satisfaction. The system parameters can be managed easily and the results of the scheduling process can also be demonstrated via the interface. For the interface and its functions please see Appendix G.

CHAPTER 5 MANAGERIAL INSIGHTS FOR CROSS-TRAINING

In this section, a large set of experiments with various cross-training configurations are conducted to gain managerial insights for cross-training decisions. The experiments are conducted to answer the following questions:

a) How many skills should an agent have? Full cross-training is seldom feasible in practice; yet will limited cross-training be sufficient to gain reasonable benefits?

b) Will a decrease in efficiency in the secondary skills for the cross-trained agents significantly reduce the benefits of cross-training?

c) Will an increase in staffing cost for the cross-trained agents significantly reduce the benefits of cross-training?

d) Would cross-training be able to provide benefits that are either comparable or superior to other options such as flexible shifts in dealing with fluctuations in demand?

In the experiments, the TPSA is utilized to solve the problems and the computation time is set to 20 minutes (10 minutes for each of P-I and P-II) for small cases, 1 hour (1/2 hour for each of P-I and P-II) for medium cases, and 2 hours (1 hour for each of P-I and P-II) for large cases. In most of these studies, part-time shift percentage (*MaxP*) is set to 20% as suggested in the literature (Bard et al., 2003; Bard, 2004), any days off assignment is employed, two-skill cross-training is adopted, efficiency for the secondary skill is set to 100%, and cost increase for cross-trained staff is set to 0% unless otherwise explicitly specified.

5.1 Breadth of Limited Cross-Training

The first experiment aims to investigate the effect of breadth of limited cross-training as measured by the number of skills an employee is trained in (Brusco, 2008), and assesses the extent to which limited cross-training helps improve the performance measure; in this study, the total weekly cost.

Specifically, in this experiment, two-skill, three-skill, and four-skill cross-training strategies are being employed. The results are presented in Table 14 for small, medium, and large test cases. The detailed results for three-skill and four-skill cross-training and the comparisons for all test cases are given in Appendix H. The comparison of different cross-training configurations for the call center problem (case L1) is presented in Figure 7. In the graph, no cross-training (all employees are trained in only one skill) and full cross-training (all employees are trained in all nine skills) results are also presented to show lower and upper limits for the total cost. In the figure, vertical axis represents the total weekly cost, whereas horizontal axis represents the amount of cross-trained staff (*MaxC* = 10% - 100%).

Table 14: Limited Cross-Training Results for All Test Cases

Case	CT	MaxC									
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
S1	No	47,908									
	2-Skill	45,786	46,007	45,465	45,730	45,943	46,056	45,342	44,819	45,206	45,254
	Full	44,715									
S2	No	37,267									
	2-Skill	36,480	36,115	35,319	36,363	36,084	35,986	35,057	35,253	35,556	35,559
	Full	35,049									
S3	No	40,378									
	2-Skill	39,405	39,628	39,920	39,494	38,422	38,206	38,759	38,614	38,438	38,039
	Full	37,990									
S4	No	30,553									
	2-Skill	29,308	29,220	29,230	29,138	28,099	28,232	28,527	28,694	28,866	28,451
	Full	27,812									
S5	No	18,972									
	2-Skill	18,588	18,456	18,520	18,262	17,550	17,580	17,220	16,806	16,915	17,076
	Full	16,623									
M1	No	59,948									
	2-Skill	59,317	58,236	58,399	55,862	55,211	55,661	55,998	55,806	56,728	55,393
	3-Skill	58,050	56,312	55,636	55,370	56,823	55,403	55,291	54,905	55,190	55,629
	4-Skill	58,141	56,312	56,085	55,005	55,383	54,742	54,628	55,292	54,796	54,957
	Full	54,182									
M2	No	57,555									
	2-Skill	55,960	56,532	55,746	54,577	53,218	52,461	52,945	52,694	53,844	52,789
	3-Skill	56,692	54,538	52,801	52,824	52,446	53,601	52,603	52,565	52,831	51,755
	4-Skill	54,637	53,343	53,734	51,739	52,806	51,880	52,731	52,456	52,170	51,870
	Full	51,544									
M3	No	78,934									
	2-Skill	74,524	74,653	74,798	74,725	74,214	74,464	74,649	73,505	73,690	73,421
	3-Skill	76,865	74,091	73,969	74,533	73,842	74,810	74,324	73,182	72,277	73,084
	4-Skill	75,167	73,734	74,072	73,123	72,694	72,580	73,319	72,356	72,277	72,384
	Full	72,204									
M4	No	66,629									
	2-Skill	65,751	64,075	63,320	63,113	62,457	62,728	62,432	62,748	63,496	62,832
	3-Skill	64,336	62,205	62,295	62,639	63,063	62,475	62,763	62,600	61,818	62,526
	4-Skill	64,488	63,289	62,318	62,531	61,898	62,118	61,929	61,804	61,862	61,716
	Full	61,598									
L1	No	98,653									
	2-Skill	94,376	93,881	93,050	92,218	91,866	89,996	91,434	91,005	91,279	91,465
	3-Skill	92,982	91,688	91,093	89,548	91,958	91,945	90,380	90,299	90,260	89,535
	4-Skill	93,343	92,815	91,952	90,211	90,407	89,618	90,478	90,810	90,241	89,300
	Full	88,874									
L2	No	103,963									
	2-Skill	102,128	99,957	99,877	98,120	96,935	97,296	97,391	97,668	97,736	98,795
	3-Skill	102,258	100,433	98,975	97,299	96,989	96,897	95,935	96,374	96,464	95,980
	4-Skill	100,985	96,671	98,025	96,851	96,428	96,878	95,952	95,720	95,018	95,873
	Full	94,682									
L3	No	119,052									
	2-Skill	116,411	116,953	113,562	111,867	111,224	111,905	113,111	112,426	112,331	112,926
	3-Skill	116,212	112,239	111,768	112,002	112,066	111,920	111,367	111,658	111,252	110,365
	4-Skill	114,762	111,564	111,736	113,009	111,266	110,778	111,358	111,584	111,833	110,115
	Full	109,356									

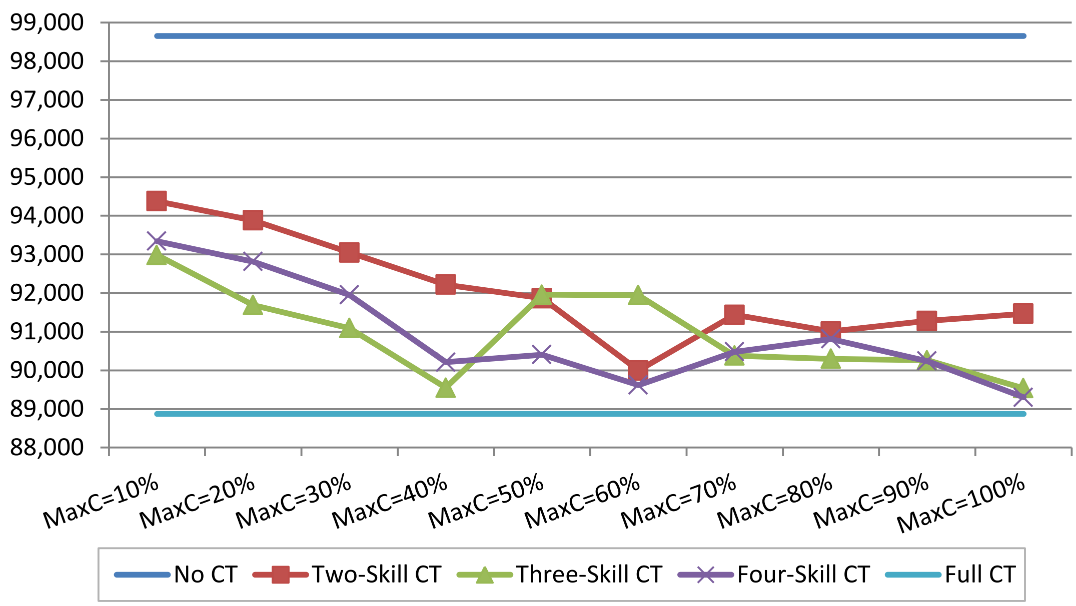

Figure 7: Limited Cross-Training Results for Case L1

These results, as expected, clearly demonstrate the potential effectiveness of limited cross-training. To see this, note that in Table 14, without cross–training, the costs for the twelve cases S1, S2, S3, S4, S5, and M1, M2, M3, M4, and L1, L2, L3 are $47,908, $37,267, $40,378, $30,553, $18,972, and $59,948, $57,555, $78,934, $66,629, and $98,653, $103,963, $119,052; with full cross-training, the costs for these twelve cases are $44,715, $35,049, $37,990, $27,812, $16,623, and $54,182, $51,544, $72,204, $61,598, and $88,874, $94,682, $109,356. The reductions in costs are roughly 6.7%, 6.0%, 5.9%, 9.0%, 12.4%, or 9.6%, 10.4%, 8.5%, 7.6%, or 9.9%, 8.9%, 8.1%; an average of an 8.6% cost reduction by employing full cross-training.

However, rather than having all agents cross-trained for all skills, which would be infeasible in practice, the results in Table 14 show that a dramatic sharp reduction in cost is typically observed at the lower percentage of cross-training, typically from *MaxC*

= 10% to 40%, for only two, three, or four skills. Reduction in costs could become very slow or almost flat at the higher percentage of cross-training, for example, from *MaxC* = 50% to 100%. These results suggest that minimal flexibility can provide great benefits. In other words, partial (*MaxC* < 100%) limited (with a maximum of two or three skills per agent in appropriate combinations) cross-training results in considerable performance improvement; additional benefits of having more skills or more agents cross-trained beyond a certain threshold are marginal.

To further illustrate the above observation, a detailed calculation of cost reduction under various partial and limited cross-training scenarios for case L1 is presented in Table 15. The table presents: a) the percent cost reductions (the top three rows) for various partial (*MaxC* < 100%) and limited (a maximum of two, three, or four skills) cross-training as compared with no cross-training, and b) cost reduction in percentage (the bottom three rows) future full cross-training could achieve compared to various partial and limited cross training.

Table 15: Cost Reductions: No CT, Partial Limited CT, and Full CT for Case L1

Comparison of		MaxC										Avg.
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	
No CT	2-Skill CT	4.3%	4.8%	5.7%	6.5%	6.9%	8.8%	7.3%	7.8%	7.5%	7.3%	**6.7%**
	3-Skill CT	5.7%	7.1%	7.7%	9.2%	6.8%	6.8%	8.4%	8.5%	8.5%	9.2%	**7.8%**
	4-Skill CT	5.4%	5.9%	6.8%	8.6%	8.4%	9.2%	8.3%	8.0%	8.5%	9.5%	**7.9%**
No CT	Full CT	9.9%										**9.9%**
2-Skill CT	Full CT	5.8%	5.3%	4.5%	3.6%	3.3%	1.2%	2.8%	2.3%	2.6%	2.8%	**3.4%**
3-Skill CT		4.4%	3.1%	2.4%	0.8%	3.4%	3.3%	1.7%	1.6%	1.5%	0.7%	**2.3%**
4-Skill CT		4.8%	4.2%	3.3%	1.5%	1.7%	0.8%	1.8%	2.1%	1.5%	0.5%	**2.2%**

a) Limited (a maximum of two, three, or four skills) cross-training provides significant cost reduction compared to no cross-training and is almost as good as full cross-training of all employees with all nine skills.

Compared to no cross-training, on average, partial (*MaxC* < 100%) limited cross-training provides a 6.7% cost reduction with two-skill cross-training, a 7.8% cost reduction with three-skill cross-training, and a 7.9% cost reduction with four-skill cross-training. If full cross-training is employed, the cost reduction is 9.9% compared to no cross-training. Full cross-training (cross-training all employees with all nine skills) only provides an average of an extra 3.4% cost reduction compared to two-skill cross-training, a 2.3% cost reduction compared to three-skill cross-training, and a 2.2% cost reduction compared to four-skill cross-training.

Note that four-skill cross-training with *MaxC* = 100% provides a cost of $89,300 and a 9.5% cost reduction compared to no cross-training, whereas full cross-training of all staff for all nine skills provides a cost of $88,874 and a 9.9% cost reduction compared to no cross-training. These results show that cross-training all employees for five more skills only reduces the weekly cost by (89,300 – 88,874) = $426.

b) Even partial (*MaxC* < 100%) limited (a maximum of two, three, or four skills) cross-training provides the majority of the cost reduction. In fact, even cross-training 10% of the workforce (*MaxC* = 10%) for two, three, or four skills provides considerable cost reduction.

For example, in the case of two-skill cross-training with *MaxC* = 10%, the cost is $94,376, which is (98,653 – 94,376) = $4,277 less than the no cross-training cost of

$98,653, and only (94,376 – 88,874) = $5,502 more than the full cross-training cost of $88,874. This result shows that cross-training only 10% of all employees with only two skills provides a savings halfway between the cost of no cross-training and that of full cross-training, and this cost reduction proves the efficiency of partial limited cross-training.

c) As the number of skills an agent can have increases, the percentage of multi-skilled workers necessary to achieve the largest benefit can be reduced. For case L1, while 50% of the employees need to be cross-trained with two skills to reduce the cost to $91,866, only 20% of the employees need to be cross-trained for a maximum of three skills to reach a similar cost of $91,688 and 30% of the employees need to be cross-trained for a maximum of four skills to reach a similar cost of $91,952.

   In general, increasing the number of skills an agent can have decreases the proportion of employees that needs to be cross-trained. After a certain percentage of cross-training (*MaxC* = 40% and more), two-skill, three-skill, and four-skill cross-training provide essentially equivalent results which are also close to the result of full cross-training.

The detailed calculations for all other test cases are not presented here due to space limitations, but the comparison results for each test case are presented in Appendix I. The summary of comparisons for all test cases is presented in Table 16, and the detailed comparisons for two-skill, three-skill, and four-skill cross-training with *MaxC* = 10% - 100% are presented in Table 17.

Table 16: Cost Reductions: No CT, Partial Limited CT, and Full CT for All Test Cases

Comparison of		Test Case												Avg.
		S1	S2	S3	S4	S5	M1	M2	M3	M4	L1	L2	L3	
No CT	2-Skill CT	4.9%	4.0%	3.7%	5.8%	6.7%	5.5%	6.0%	5.9%	5.0%	6.7%	5.2%	4.9%	**5.4%**
	3-Skill CT	N/A	N/A	N/A	N/A	N/A	6.8%	7.5%	6.1%	5.9%	7.8%	6.0%	5.9%	**6.6%**
	4-Skill CT	N/A	N/A	N/A	N/A	N/A	7.4%	8.4%	7.3%	6.4%	7.9%	6.9%	6.1%	**7.2%**
	Full CT	**6.7%**	**6.0%**	**5.9%**	**9.0%**	**12.4%**	**9.6%**	**10.4%**	**8.5%**	**7.6%**	**9.9%**	**8.9%**	**8.1%**	**8.6%**

Table 17: Cost Reductions: No CT and Partial Limited CT with *MaxC* = 10% - 100%

Comparison of		MaxC										Avg.
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	
No CT	2-Skill CT	2.8%	3.4%	4.0%	4.9%	6.3%	6.4%	6.3%	6.8%	6.1%	6.6%	**5.4%**
	3-Skill CT	2.9%	5.8%	6.7%	7.0%	6.5%	6.5%	7.2%	7.5%	7.7%	7.9%	**6.6%**
	4-Skill CT	4.0%	6.3%	6.3%	7.5%	7.6%	8.0%	7.7%	7.8%	8.1%	8.4%	**7.2%**

The average results for all test cases presented in Table 16 indicate that full cross-training provides 8.6% cost reduction compared to no cross-training. On the other hand, two-skill, three-skill, and four-skill cross-training provide 5.4%, 6.6%, and 7.2% cost reductions compared to no cross-training, respectively. Furthermore, the average results for various *MaxC* values presented in Table 17 indicate that these cost reductions can be obtained by cross-training only 40% - 50% of all staff in two-skill cross-training and 30% - 40% of all staff in three-skill and four-skill cross-training.

In summary, a low level of cross-training (either fewer people with more skills such as three-skill or four-skill cross-training for 10% to 20% of all staff or more people with fewer skills such as two-skill cross-training for 30% to 50% of all staff) usually suffices to deal effectively with fluctuations in the demand of different service groups, and provides the bulk of the benefits of full cross-training. Designing effective workforce cross-training structures in call centers is crucial, and the benefits of partial limited cross-training should be considered in conjunction with a good staffing and scheduling strategy.

5.2 Efficiency Loss in Secondary Skills

This experiment studies the effect of secondary skill efficiency in cross-training decisions; reduced server efficiency due to cross-training is taken into account and the optimal secondary skill efficiency is investigated.

Cross-trained agents may not be trained at 100% efficiency in all skills. Therefore, cross-training in multiple skills could lead to a loss of efficiency as compared to a server who is dedicated to one call type. Efficiency is measured as the ratio of time taken by a dedicated agent to do a task to the time taken by a cross-trained agent to do the same task.

In the experiments with two-skill cross-training, each agent has one primary skill with 100% efficiency and may have one secondary skill with varying degrees of efficiency from 10% to 100%. Efficiency losses are taken into account both in P-I for cross-training decision and in P-II for detailed staffing and scheduling. Incremental loss of efficiency (10% to 90%) is applied for the secondary skill and the results for case L1 are presented in Table 18 and compared in Figure 8.

In the table, the “Eff.Loss” column represents the loss of efficiency for the secondary skill of a cross-trained agent. The results with various efficiency losses are compared with the 0% efficiency loss case where a cross-trained agent is 100% efficient in all his/her skills (bolded in the table). Because *Eff.Loss* = 100% means 0% efficiency in the second skill, it is the same as having only one skill and therefore no cross-training.

Table 18: Efficiency Loss in Secondary Skill in Two-Skill CT for Case L1

	Eff.	MaxC									
CT	Loss	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Full	0%	88,874									
2-Skill	0%	**94,376**	**93,881**	**93,050**	**92,218**	**91,866**	**89,996**	**91,434**	**91,005**	**91,279**	**91,465**
	10%	95,962	95,434	94,642	93,857	92,689	91,951	93,706	93,753	92,287	92,330
	20%	**96,790**	**95,225**	**95,710**	**96,054**	**94,185**	**93,217**	**94,530**	**95,089**	**93,657**	**94,648**
	30%	94,769	94,777	96,898	96,512	95,084	96,026	94,759	98,037	96,080	94,828
	40%	97,815	96,265	96,415	98,243	97,629	96,961	97,475	95,983	95,899	95,414
	50%	97,104	97,536	97,551	98,163	97,211	96,282	95,828	96,642	95,608	96,412
	60%	97,850	96,311	98,076	97,898	97,760	96,415	98,065	96,866	95,417	97,039
	70%	97,305	98,404	98,438	97,128	96,203	98,452	97,990	98,185	97,853	98,526
	80%	95,526	96,740	98,075	96,510	95,990	96,648	96,508	97,596	95,891	98,449
	90%	97,658	96,639	95,845	97,670	96,944	96,822	97,106	96,628	95,838	96,827
No	100%	98,653									

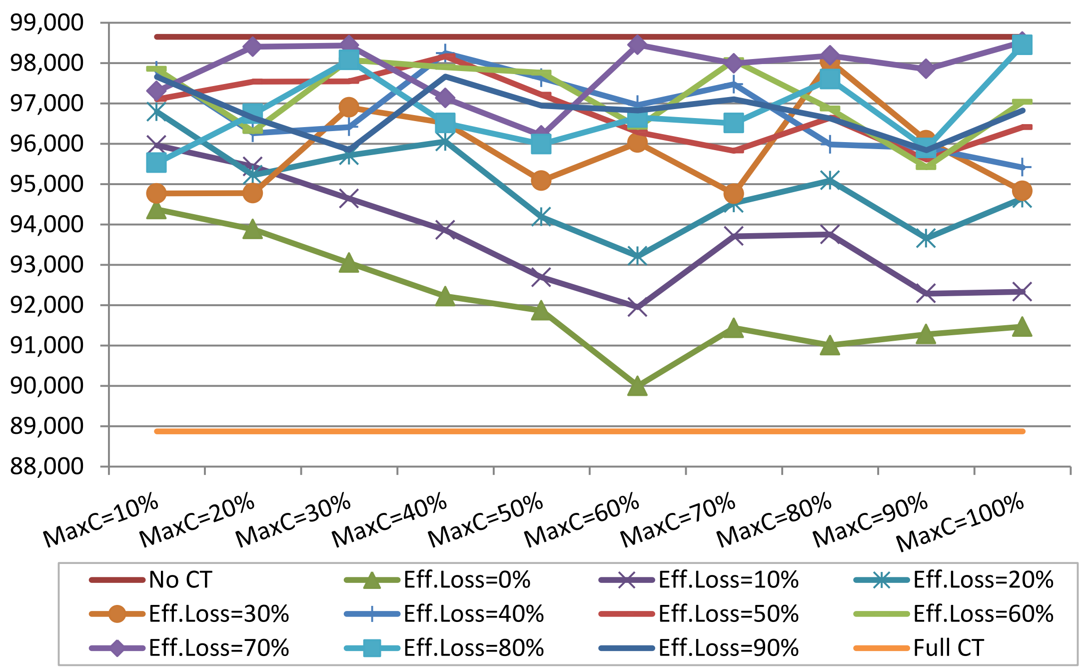

Figure 8: Efficiency Loss in Secondary Skill in Two-Skill CT for Case L1

These results demonstrate that the benefits of having flexible servers could vanish rapidly as the flexible servers' efficiency decreases. Note in the case of no cross-training, the total cost is $98,653 per week. In the case of full cross training (everyone has every skill), the total cost is $88,874 per week. The difference between these two bounds is nearly $10,000. For 0% efficiency loss in the second skill (*Eff.Loss* = 0%), the results were explained in the previous section and are not repeated here. These results provide a lower bound on the costs in various scenarios.

a) 10% loss of efficiency (*Eff.Loss* = 10%): If the efficiency loss is small, the benefits of cross-training could still be realized with a small percentage of cross-trained agents. For example, with a 10% efficiency loss in the second skill, *MaxC* = 50% provides a cost of $92,689 which is close to the result of $92,218 in the 0% efficiency loss case with *MaxC* = 40%; similarly, *MaxC* = 60% provides a cost of $91,951 which is close to the result of $91,866 in the 0% efficiency loss case with *MaxC* = 50%. These results show that the effect of a 10% efficiency loss diminishes when the amount of partial cross-training (*MaxC*) is increased, and it gives comparable results with the *Eff.Loss* = 0% case. In general, for all *MaxC* values, the *Eff.Loss* = 10% case provides similar results to the *Eff.Loss* = 0% case, without a noticeable cost increase.

b) 20% loss of efficiency (*Eff.Loss* = 20%): If the efficiency loss is 20% in the second skill, the cost of allowing all agents to be trained (*MaxC* = 100%) in two skills is $94,648, and is equivalent in cost to $94,376, the scenario with *MaxC* = 10% and *Eff.Loss* = 0% (in which only 10% of the staff is cross-trained for two skills without efficiency loss). This result proves that the effect of secondary skill efficiency on cost is much more significant than the amount of cross-training; for *Eff.Loss* = 20% and more, the increasing values of *MaxC* do not improve the result and never gives comparable results with the *Eff.Loss* = 0% case.

In the case with *MaxC* = 10% and *Eff.Loss* = 20%, the cost is $96,790, which is only (98,653 – 96,790) = $1,863 less than the upper bound of $98,653. In the case with *MaxC* = 10% and *Eff.Loss* = 0%, it is $94,376, which is (98,653 – 94,376) = $4,277 less than the upper bound of $98,653. These results show that when even only 10% of the workforce is cross-trained for two skills, if the efficiency loss in the second skill increases to only 20%, the total cost increases dramatically and comes close to the upper bound.

c) 30% and greater loss of efficiency (*Eff.Loss* = 30% - 90%): Similar results can be seen in the other cases with various *MaxC* values and increasing amounts of efficiency losses. In the case with *MaxC* = 50% and *Eff.Loss* = 30%, the cost is $95,084, which is only (98,653 – 95,084) = $3,569 less than the upper bound of $98,653. The case with *MaxC* = 50% and *Eff.Loss* = 0%, it is $91,866, which is (98,653 – 91,866) = $6,787 less than the upper bound of $98,653. This result proves that a 30% efficiency loss in the second skill deteriorates the solution and even moderate flexibility with 50% cross-training does not help to improve it.

The most flexible case with *MaxC* = 100% also provides similar results; the cost of the *MaxC* = 100% and *Eff.Loss* = 30% case is $94,828, which is (98,653 – 94,828) = $3,825 less than the upper bound, whereas the cost of the *MaxC* = 100% and *Eff.Loss* = 0% case is $91,465, which is (98,653 – 91,465) = $7,188 less than the upper bound. Even the highest level of flexibility does not completely prevent the negative effect of a 30% efficiency loss in the second skill on cost.

If the efficiency is reduced significantly, for example, 30% or more in this case, it is not advisable to conduct extensive cross-training. Due to this observation, it is not worthwhile to pursue the experiment with more than two skills and efficiency loss. Adding a third or a fourth skill brings extra losses in efficiency, and hence would not bring much advantage in the reduction of the staffing cost. To demonstrate this, the three-skill cross-training results for case L1 with *Eff.Loss* = 10% - 30% are presented in Table 19 and compared in Figure 9. The results for more than 30% efficiency loss are not demonstrated in the table because *Eff.Loss* = 30% represents 30% efficiency loss in the secondary skill and 60% efficiency loss in tertiary skill; as was presented in the two-skill case, more efficiency loss deteriorates the solution significantly.

Table 19: Efficiency Loss in Additional Skills in Three-Skill CT for Case L1

CT	Eff. Loss	MaxC 10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Full	0%	88,874									
3-Skill	0%	**92,982**	**91,688**	**91,093**	**89,548**	**91,958**	**91,945**	**90,380**	**90,299**	**90,260**	**89,535**
	10%	96,222	93,820	91,993	90,983	92,183	93,147	93,528	91,201	90,413	92,254
	20%	97,810	94,824	94,929	96,422	93,982	93,725	94,049	93,242	93,424	93,311
	30%	98,138	97,106	95,365	96,130	94,150	95,858	96,824	94,661	94,800	94,348
No	100%	98,653									

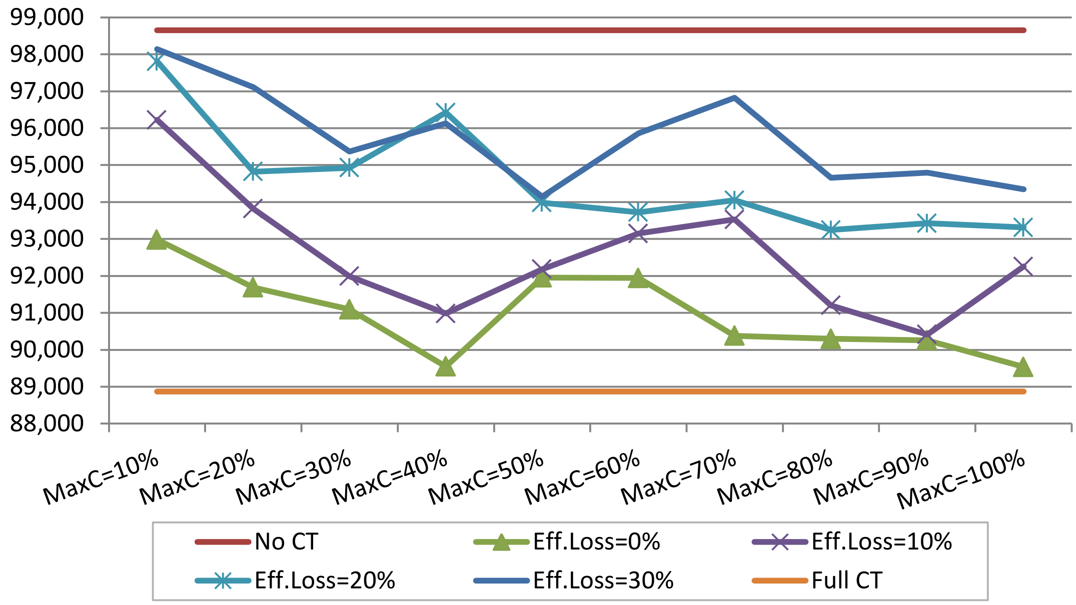

Figure 9: Efficiency Loss in Additional Skills in Three-Skill CT for Case L1

The results presented in Table 19 demonstrate that three-skill cross-training does not improve the solution in the presence of efficiency loss in additional skills. The results are similar with the results of the two-skill case; for example, for *Eff.Loss* = 10% and *MaxC* = 100%, the cost is \$92,254 for the three-skill case, whereas it is \$92,330 for the two-skill case.

In summary, the results presented in Figure 8 and Figure 9 clearly demonstrate that a small amount of cross-training still helps to reduce the cost even in the presence of efficiency loss, as seen by the reduction in cost under *MaxC* = 10% for example, however, due to efficiency loss, large amount of cross-training (such as *MaxC* = 30% and more) does not provide better results and is not beneficial.

To generalize the conclusions obtained for case L1, the results for 10%, 20%, and 30% efficiency losses in the secondary skill in two-skill cross-training for *MaxC* = 10%, 50%, and 100% are presented in Table 20 for small test cases and Table 21 for medium and large test cases. Similar to the results obtained for case L1, the results for all other test cases indicate that a 20% - 30% efficiency loss in the secondary skill deteriorates the solutions noticeably, and the total costs come close to the upper bound costs obtained with no cross-training.

For example, as presented in Table 20, for case S5, the cost for two-skill cross-training with *MaxC* = 100% is $17,076 for a 0% efficiency loss and $18,571 for a 30% efficiency loss in the secondary skill, whereas it is $18,972 for no cross-training (bolded in the table). A 30% increase in efficiency loss increases the cost by (18,571 – 17,076) = $1,495, and this cost is only (18,972 – 18,571) = $401 less than the upper bound cost obtained by no cross-training.

As presented in Table 21, for case M4, the cost of two-skill cross-training with *MaxC* = 100% is $62,832 for 0% efficiency loss and $65,576 for 30% efficiency loss in the secondary skill, whereas it is $66,629 for no cross-training (bolded in the table). A 30% increase in efficiency loss increases the cost by (65,576 – 62,832) = $2,744 and this cost is only (66,629 – 65,576) = $1,053 less than the upper bound cost obtained by no cross-training.

As another example, for case L2, the cost for two-skill cross-training with *MaxC* = 100% is $98,795 for 0% efficiency loss and $102,853 for 30% efficiency loss in the secondary skill, whereas it is $103,963 for no cross-training (bolded in the table). A 30% increase in efficiency loss increases the cost by (102,853 – 98,795) = $4,058 and this cost is only (103,963 – 102,853) = $1,110 less than the upper bound cost obtained by no cross-training.

Table 20: Efficiency Loss in Secondary Skill in Two-Skill CT for Small Cases

Case	CT	Eff. Loss	MaxC 10%	MaxC 50%	MaxC 100%
S1	Full	0%		44,715	
	2-Skill	0%	45,786	45,943	45,254
		10%	46,612	47,198	46,504
		20%	47,336	47,822	46,248
		30%	47,248	46,595	46,209
	No	100%		47,908	
S2	Full	0%		35,049	
	2-Skill	0%	36,480	36,084	35,559
		10%	37,224	35,487	36,202
		20%	36,862	36,811	35,898
		30%	37,149	37,181	36,611
	No	100%		37,267	
S3	Full	0%		37,990	
	2-Skill	0%	39,405	38,422	38,039
		10%	40,117	39,690	39,745
		20%	40,282	39,832	40,125
		30%	40,010	39,481	39,314
	No	100%		40,378	
S4	Full	0%		27,812	
	2-Skill	0%	29,308	28,099	28,451
		10%	29,721	29,926	29,900
		20%	29,665	30,068	29,580
		30%	30,479	29,762	30,297
	No	100%		30,553	
S5	Full	0%		16,623	
	2-Skill	0%	18,588	17,550	**17,076**
		10%	18,932	18,346	17,441
		20%	18,783	18,020	18,203
		30%	18,838	18,466	**18,571**
	No	100%		**18,972**	

Table 21: Efficiency Loss in Secondary Skill in Two-Skill CT for Medium and Large Cases

Case	CT	Eff. Loss	MaxC		
			10%	50%	100%
M1	Full	0%		54,182	
	2-Skill	0%	59,317	55,211	55,393
		10%	59,044	57,065	57,019
		20%	58,750	57,556	58,210
		30%	58,691	58,574	58,069
	No	100%		59,948	
M2	Full	0%		51,544	
	2-Skill	0%	55,960	53,218	52,789
		10%	55,456	55,261	54,437
		20%	56,353	55,120	55,654
		30%	56,614	56,533	54,815
	No	100%		57,555	
M3	Full	0%		72,204	
	2-Skill	0%	74,524	74,214	73,421
		10%	76,464	74,228	75,621
		20%	77,385	75,284	74,925
		30%	77,564	75,918	76,863
	No	100%		78,934	
M4	Full	0%		61,598	
	2-Skill	0%	65,751	62,457	**62,832**
		10%	65,355	63,994	63,997
		20%	65,520	64,476	64,589
		30%	66,268	65,526	**65,576**
	No	100%		**66,629**	
L2	Full	0%		94,682	
	2-Skill	0%	102,128	96,935	**98,795**
		10%	103,543	99,799	99,451
		20%	103,374	102,010	102,559
		30%	103,367	103,729	**102,853**
	No	100%		**103,963**	
L3	Full	0%		109,356	
	2-Skill	0%	116,411	111,224	112,926
		10%	117,837	115,626	115,688
		20%	118,320	117,548	115,812
		30%	118,443	118,588	116,807
	No	100%		119,052	

In summary, the impact of the server efficiency on the optimal fraction of flexible servers is quite significant. If cross-training leads to a significant loss in server efficiency, it is better to cross-train fewer agents because full flexibility is never optimal.

5.3 Cost Increase for Cross-Trained Staff

The previous results clearly demonstrate the benefits of partial limited cross-training, but it is also worthwhile to analyze the case where cross-training increases staff wages. Although cross-training of servers increases server flexibility, it could also increase the labor cost of an agent; this experiment tries to evaluate the trade-off between the additional cost due to cross-training and the savings due to the staff flexibility obtained via cross-training.

The increase in labor cost of an agent due to cross-training is taken into account for the cross-training decision in P-I and the staffing and scheduling decision in P-II of the TPSA. The results for case L1 with various increased staffing cost ratios for each additional skill in two-skill, three-skill, and four-skill cross-training are presented in Table 22 for *MaxC* = 10% - 100%. In the table, the "CICT" (cost increase for cross-training) column represents the wage increase for each additional skill for a cross-trained agent. For example, for *CICT* = 5%, if an agent has two skills, his/her wage is increased by 5%, whereas this modifier is increased to 10% for a three-skill agent and 15% for a four-skill agent – a 5% incremental increase of staffing cost for each additional skill. Again, in the case of no cross-training, the total cost is $98,653 per week. In the case of full cross-training (everyone has every skill) with no cost increase for additional skills, the total cost is $88,874 per week.

Table 22: Case L1 Results with Cost Increase for Cross-Trained Staff

		MaxC									
CT	CICT	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Full	0%	88,874									
2-Skill	0%	**94,376**	**93,881**	**93,050**	**92,218**	**91,866**	**89,996**	**91,434**	**91,005**	**91,279**	**91,465**
	5%	97,175	95,968	94,752	94,838	94,819	94,427	93,883	93,805	94,293	93,815
	10%	96,544	95,141	96,359	94,701	95,838	95,636	96,447	96,022	96,661	97,306
	15%	96,186	96,626	95,697	96,039	97,976	97,123	94,904	96,849	97,092	96,624
	20%	96,683	97,156	97,347	97,181	98,316	97,312	96,154	97,149	96,686	97,978
	25%	96,400	97,450	97,939	97,077	97,587	98,258	97,488	97,048	97,741	97,544
3-Skill	0%	**92,982**	**91,688**	**91,093**	**89,548**	**91,958**	**91,945**	**90,380**	**90,299**	**90,260**	**89,535**
	5%	96,422	94,521	93,992	92,679	93,707	94,143	93,746	94,502	92,412	93,687
	10%	96,850	95,353	94,848	96,230	95,079	96,488	95,102	95,273	95,842	94,580
	15%	97,299	96,947	98,046	95,524	97,700	96,609	96,118	97,030	95,401	95,091
4-Skill	0%	**93,343**	**92,815**	**91,952**	**90,211**	**90,407**	**89,618**	**90,478**	**90,810**	**90,241**	**89,300**
	5%	95,378	93,733	92,955	93,933	93,174	94,471	93,634	95,517	93,622	94,188
	10%	97,736	95,240	96,210	96,088	94,831	96,632	94,486	95,972	94,510	94,306
No	N/A	98,653									

The results seem to indicate that increasing the cost of flexibility also increases the cost of the weekly schedule for all cross-training configurations.

a) For two-skill cross-training with *MaxC* = 10%, the cost is $94,376 for *CICT* = 0%, whereas it is $97,175 for *CICT* = 5%. For *MaxC* = 100%, the cost is $91,465 for *CICT* = 0%, whereas it is $93,815 for *CICT* = 5%; note that the equivalent cost of $93,881 is obtained with only *MaxC* = 20% in the *CICT* = 0% case. These results indicate that even a 5% staffing cost increase for a two-skill agent deteriorates the solution and increases the total cost noticeably; Figure 10 presents all the results for two-skill cross-training.

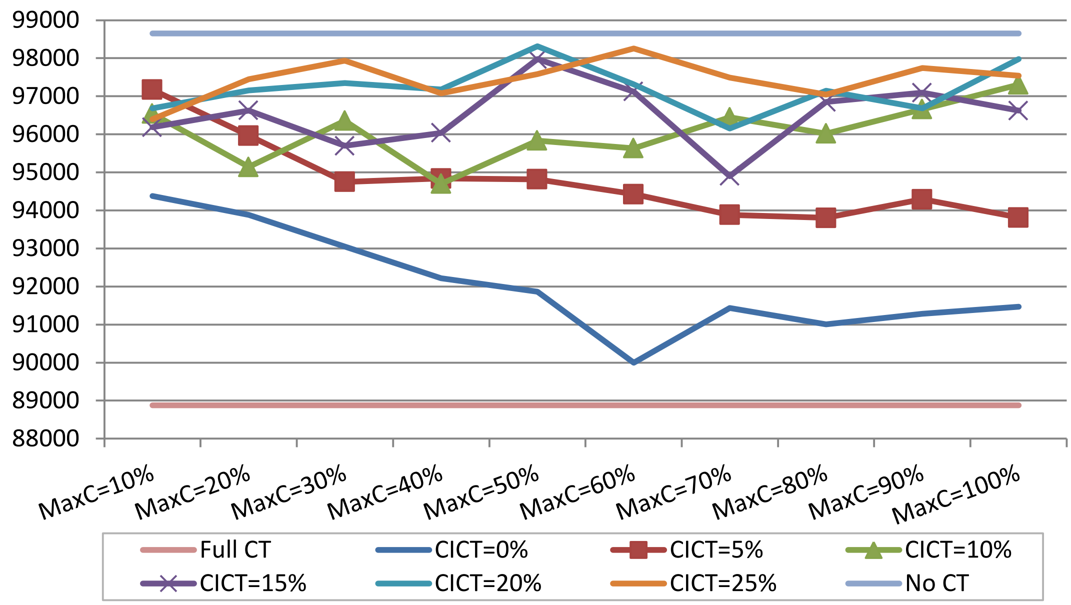

Figure 10: Case L1 Results with Cost Increase for Cross-Trained Staff in Two-Skill CT

As seen both in the table and in the figure, in two-skill cross-training, the total costs obtained in the *CICT* = 10%, 15%, 20%, and 25% cases are significantly higher than the costs of the *CICT* = 0% case, and are very close to the upper bound cost of $98,653 obtained with no cross-training. Furthermore, increasing the amount of cross-training (and therefore flexibility of the schedule) does not help to reduce these total costs; the *MaxC* = 10% and *MaxC* = 100% results are quite similar. For example, for *CICT* = 15%, the cost is $96,186 for *MaxC* = 10% and $96,624 for *MaxC* = 100%; these results are essentially equivalent and only around $2,000 less than the upper bound of $98,653.

b) For three-skill cross-training, even the cost obtained with the *CICT* = 5% and *MaxC* = 100% case ($93,687) is higher than the cost obtained with the *CICT* = 0% and *MaxC* = 10% case ($92,982). This result indicates that a 5% cost increase for each additional skill deteriorates the solution to where even 100% cross-training does not help to improve it. Figure 11 presents all of the results for three-skill cross-training. As seen in the figure, for *CICT* = 10%, the costs are very close to the upper bound cost of $98,653 obtained with no cross-training. For example, the *CICT* = 10% and *MaxC* = 10% case gives a cost of $96,850, which is only (98,653 – 96,850) = $1,803 less than the upper bound.

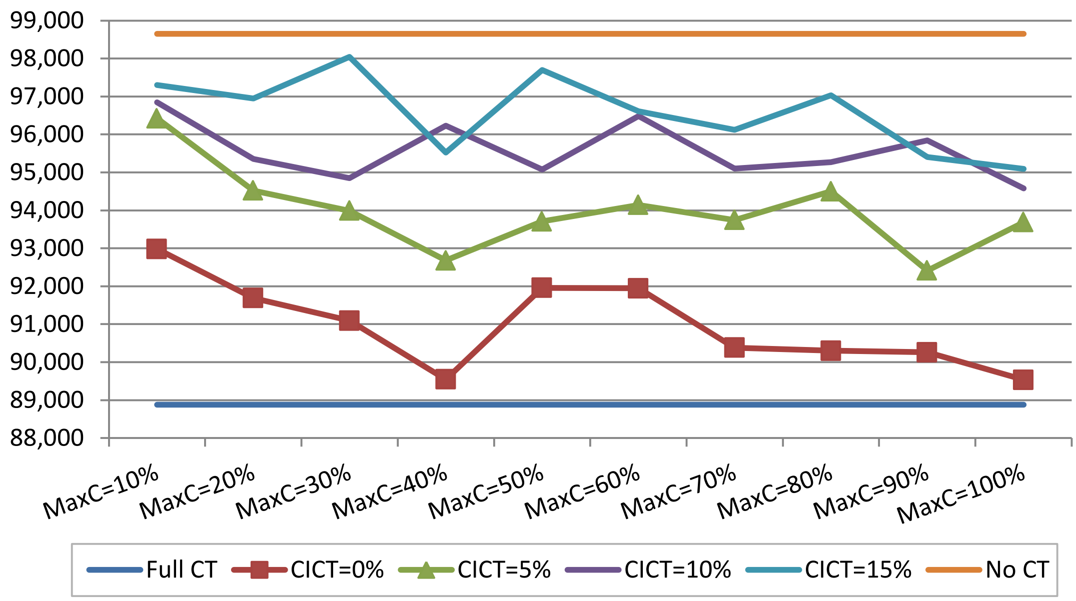

Figure 11: Case L1 Results with Cost Increase for Cross-Trained Staff in Three-Skill CT

c) For four-skill cross-training, even the case with *CICT* = 5% and *MaxC* = 100% gives a higher cost ($94,188) than the *CICT* = 0% and *MaxC* = 10% case ($93,343). Cost increase for each additional skill deteriorates the solution, and increasing the *MaxC* value does not help to improve it. Figure 12 presents all of the results for four-skill cross-training. As seen in the figure, for *CICT* = 10%, the costs are very close to the upper bound cost of $98,653 obtained with no cross-training. For example, the *CICT* = 10% and *MaxC* = 10% case gives a cost of $97,736, which is only (98,653 – 97,736) = $917 less than the upper bound.

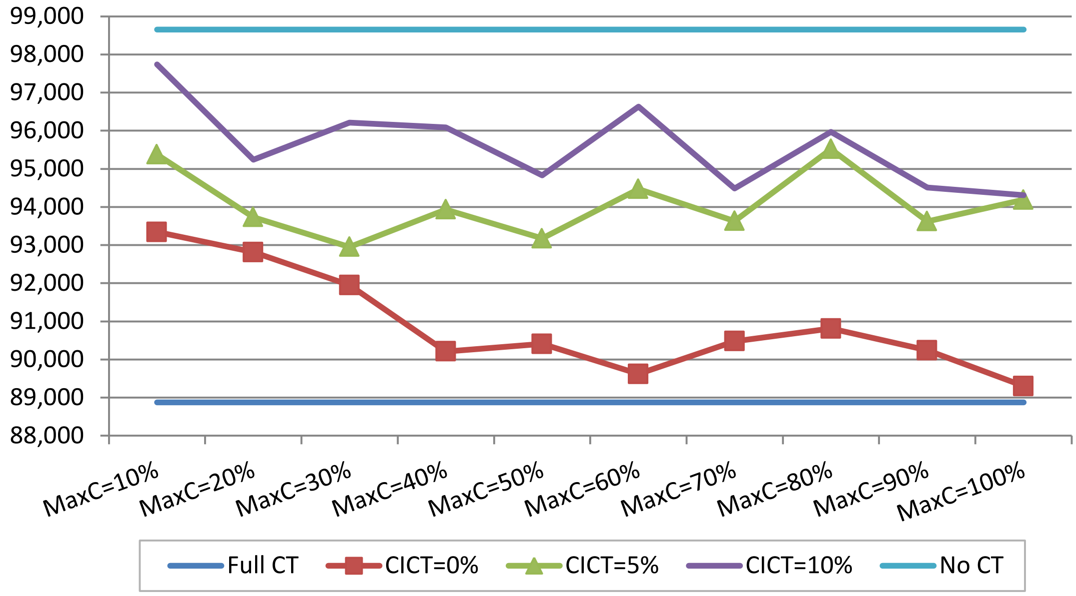

Figure 12: Case L1 Results for Cross-Training Cost Increase in Four-Skill CT

In summary, the results presented in Figure 10, Figure 11, and Figure 12 clearly demonstrate that a small amount of cross-training still helps to reduce the cost even in the presence of staffing cost increase for cross-training, as seen by the reduction in cost under *MaxC* = 10% for example, however, due to cost increase, a large amount of cross-training (such as *MaxC* = 30% and more) does not provide better results and is not beneficial.

As a last note, remember from the mathematical model section that the *MaxC* value gives the upper bound for the cross-training percentage in the workforce.

Therefore, actual cross-trained staff percentage may be lower in the proposed solution due to the trade-off between the staffing cost increase with cross-training and the amount of cross-training. The percentages of cross-trained staff in all staff are presented in Table 23 for all cross-training configurations to demonstrate the trade-off between cross-training cost and cross-training usage when cost increase is applied.

Table 23: Cross-Trained Staff Percentages for Case L1

CT	CICT	MaxC 10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
2-Skill	0%	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**69%**	**80%**	**87%**	**100%**
	5%	9%	20%	30%	31%	30%	30%	34%	35%	35%	34%
	10%	10%	20%	28%	25%	31%	27%	27%	27%	31%	29%
	15%	10%	20%	21%	24%	22%	26%	23%	23%	23%	27%
	20%	10%	17%	17%	18%	18%	19%	19%	19%	21%	23%
	25%	10%	17%	14%	12%	14%	13%	15%	16%	13%	17%
3-Skill	0%	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**79%**	**90%**	**92%**
	5%	9%	20%	30%	32%	33%	32%	33%	34%	36%	32%
	10%	10%	20%	30%	29%	30%	27%	29%	28%	26%	30%
	15%	9%	20%	22%	22%	24%	23%	20%	21%	21%	23%
4-Skill	0%	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**69%**	**79%**	**88%**	**93%**
	5%	9%	20%	30%	32%	32%	34%	33%	34%	33%	31%
	10%	9%	20%	29%	26%	28%	26%	27%	28%	30%	28%

The results presented in the table demonstrate that, when staffing cost for cross-trained staff (*CICT*) increases, the percentage of cross-trained staff in the solution decreases, especially for *MaxC* = 30% and more.

a) For two-skill cross-training with *MaxC* = 100%, if there is no cost increase for cross-trained staff (*CICT* = 0%), all agents are cross-trained for two skills (100%). However, for an increase in the cost of cross-trained staff, the optimum percentage of cross-trained agents drops from 100% to 34%, 29%, 27%, 23%, and 17% for *CICT* = 5%, 10%, 15%, 20% and 25%, respectively (please see the last column in Table 23).

For three-skill cross-training with *MaxC* = 100%, the percentage of cross-trained staff drops from 92% to 32%, 30%, and 23% for *CICT* = 5%, 10%, and 15%, respectively. Similarly, for four-skill cross-training with *MaxC* = 100%, the cross-training percentage drops from 93% to 31% and 28% for *CICT* = 5% and 10%, respectively.

b) For two-skill, three-skill, and four-skill cross-training with *MaxC* = 10% and 20%, the cross-training cost increase does not affect the cross-trained staff usage; for all *CICT* values, around 10% and 20% of the staff are cross-trained, respective to the values of *MaxC*. This result shows that it is still beneficial to cross-train up to 20% of the staff even the staffing cost is increased for cross-trained staff. For higher values of *MaxC* (30% and more), the cross-trained staff usage drops as the *CICT* value increases. For example, for two-skill cross-training with *MaxC* = 60%, the cross-trained staff usage is 60% for *CICT* = 0%, whereas it drops to half of that (30%) for *CICT* = 5%. Briefly, the need for flexibility decreases as the cost of flexibility increases.

It is natural to ask what a good configuration in practice would be when the percentage increase in cost is small. In Figure 13 and Figure 14, different cross-training configurations are compared for 5% and 10% staffing cost increases for additional skills to find the right cross-training configuration in the presence of a cross-training cost increase.

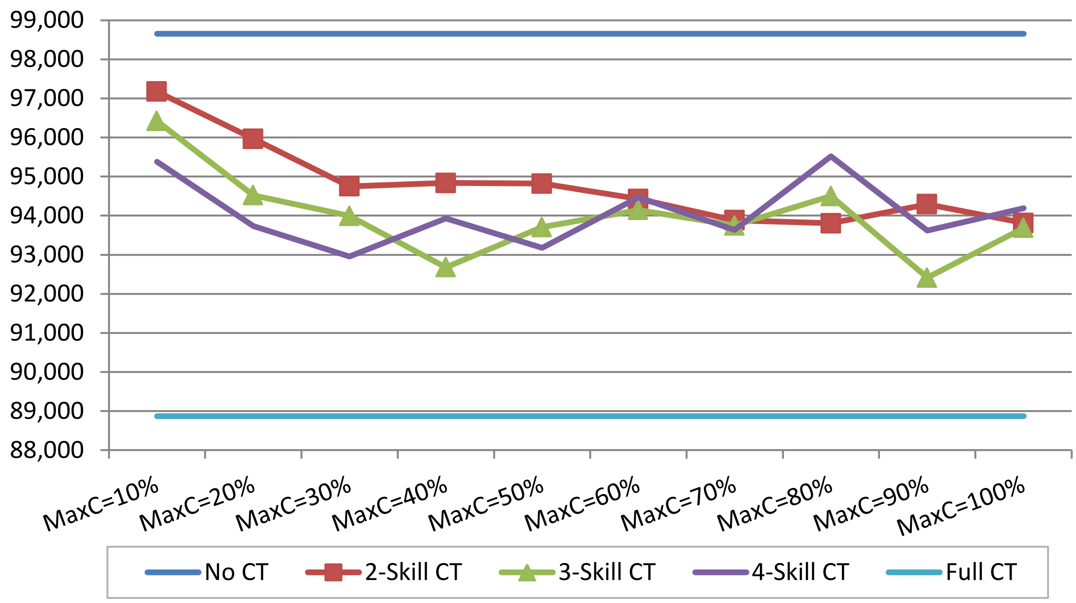

Figure 13: 5% Cost Increase for Cross-Trained Staff (*CICT* = 5%)

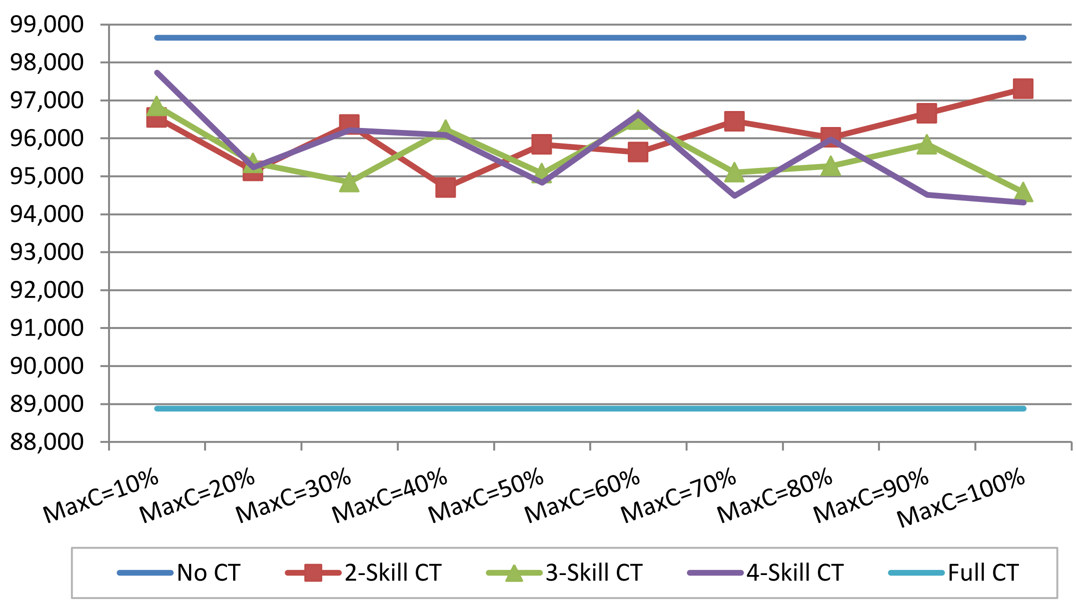

Figure 14: 10% Cost Increase for Cross-Trained Staff (*CICT* = 10%)

a) For a 5% cost increase for additional cross-training (*CICT* = 5%), Figure 13 demonstrates that any of two-skill, three-skill, or four-skill cross-training provides similar results. For *MaxC* = 10%, 20%, and 30%, four-skill cross-training gives better results than three-skill cross-training, and three-skill cross-training gives better results than two-skill cross-training, but the differences are not significant. For example, for *MaxC* = 10%, the cost is $97,175 for two-skill cross-training, $96,422 for three-skill cross-training, and $95,378 for four-skill cross-training. Especially for values of *MaxC* larger than 30%, all cross-training configurations give quite similar results (which are around $94,000), and their solution qualities do not improve significantly through *MaxC* = 100%. For example, for two-skill cross-training, the cost is $94,752 for *MaxC* = 30%, whereas $93,815 for *MaxC* = 100%; this is a difference of only $937. This result proves that even two-skill cross-training with *MaxC* = 30% provides good results in the presence of a 5% cost increase.

b) For a 10% cost increase for additional cross-training (*CICT* = 10%), Figure 14 demonstrates that two-skill, three-skill, and four-skill cross-training provide equivalent results for all cross-training percentages (*MaxC*). For example, for *MaxC* = 10%, the cost is $96,544 for two-skill cross-training, $96,850 for three-skill cross-training, and $97,736 for four-skill cross-training. For all cross-training configurations, especially for values of *MaxC* larger than 20%, the results do not improve significantly and become stable around $95,000 - $96,000. For example, for two-skill cross-training with *MaxC* = 20%, the cost is $95,141, whereas for four-skill cross-training with *MaxC* = 100%, the cost is $94,306; the cost reduction is only

$835. This result proves that even two-skill cross-training with *MaxC* = 20% provides good results in the presence of a 10% cost increase.

c) These results indicate that when there is added cost for additional flexibility, it is not effective to increase the cross-training breadth; limited cross-training with two skills is sufficient. Even for a 5% cost increase for each additional skill (*CICT* = 5%), the benefit for three-skill and four-skill cross-training comes close to that of two-skill cross-training, especially for values of *MaxC* greater than 30%, whereas they give essentially equivalent results in the 10% cost increase case (*CICT* = 10%).

Table 24 presents the number of staff in each skill for all cross-training configurations to demonstrate the effect of a cost increase on cross-training breadth (the number of skills), especially in three-skill and four-skill cross-training. The results indicate that when the cost of cross-trained agents increases (*CICT* = 5% and more), it is not desirable to have agents with more than two skills in three-skill and four-skill cross-training especially for values of *MaxC* greater than 30% (bolded in the table) because each additional skill brings an additional cost increase and this deteriorates the solution, especially for high cross-training percentages.

For example, for three-skill cross-training with *CICT* = 10%, the number of 3-skill agents is 8 for *MaxC* =10% and 5 for *MaxC* = 20%, whereas it is zero for *MaxC* = 30% to 100%. Similarly, in four-skill cross-training with *CICT* = 10%, the number of 3-skill agents is 5 for *MaxC* = 10% and 4 for *MaxC* = 20% and zero for *MaxC* = 30% to 100%, whereas the number of 4-skill agents is 2 for *MaxC* = 10% and zero for *MaxC* = 20% to 100%. These findings are consistent with the *CICT* = 5% results.

Table 24: Number of Cross-Trained Staff for Case L1

CT	CICT	# of Skills	MaxC									
			10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
2-Skill	0%	1	81	72	61	51	43	34	26	17	11	0
		2	9	18	26	34	43	51	59	67	74	86
	5%	1	77	72	61	59	61	62	57	57	59	59
		2	8	18	26	27	26	26	29	30	32	30
	10%	1	81	72	65	65	60	63	66	64	61	62
		2	9	18	25	22	27	23	24	24	27	25
	15%	1	81	70	70	67	70	65	67	69	68	66
		2	9	17	18	21	20	23	20	21	20	24
	20%	1	81	75	75	74	74	72	72	73	71	68
		2	9	15	15	16	16	17	17	17	19	20
	25%	1	81	72	77	77	76	77	73	76	77	75
		2	9	15	13	11	12	12	13	14	11	15
3-Skill	0%	1	81	68	61	51	42	35	26	18	9	7
		2	0	0	0	0	5	2	5	13	23	25
		3	9	17	26	34	37	50	55	55	54	53
	5%	1	80	72	61	58	58	61	58	58	56	59
		2	0	10	26	27	28	29	28	30	20	28
		3	**8**	**8**	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**0**
	10%	1	81	69	61	64	63	65	62	65	65	63
		2	1	12	26	26	27	24	25	25	23	27
		3	**8**	**5**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	15%	1	78	72	70	67	67	69	70	71	68	66
		2	4	17	20	19	21	21	18	19	18	20
		3	**4**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
4-Skill	0%	1	81	68	61	51	43	34	26	18	10	6
		2	0	0	0	0	0	2	2	1	3	7
		3	0	1	2	7	5	11	17	12	13	18
		4	9	16	24	27	38	38	40	55	59	54
	5%	1	78	70	61	60	59	59	58	59	60	59
		2	0	7	25	28	28	31	28	31	30	27
		3	**3**	**10**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
		4	**5**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	10%	1	79	68	64	66	63	67	62	61	61	64
		2	1	13	26	23	25	23	23	24	26	25
		3	**5**	**4**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
		4	**2**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**

These results indicate that if a large percentage of cross-training is allowed, then three-skill or four-skill agents are not necessary. On the other hand, if a small percentage is allowed, then three-skill and four-skill agents are needed because a three-skill or four-skill agent is more flexible than a two-skill agent.

To generalize the conclusions obtained for case L1, the results for 5% and 10% staffing cost increase for cross-trained agents in two-skill cross-training for *MaxC* = 10%, 50%, and 100% are presented in Table 25 and Table 26 for all other test cases. Similarly, the results for all other cases indicate that increasing the cost of flexibility also increases the cost of the weekly schedule, and the costs come close to upper bounds that are obtained for the no cross-training cases.

For example, for case S2, for two-skill cross-training with *MaxC* = 100%, the total cost is $37,172 for *CICT* = 5% and $37,008 for *CICT* = 10%. These costs are higher than the cost of *CICT* = 0% case which is $35,559, and very close to the upper bound cost of $37,267 for no cross-training.

Table 25: Cost Increase for Cross-Trained Staff in Two-Skill CT for Small Cases

Case	CT	CICT	MaxC		
			10%	50%	100%
S1	Full	0%		44,715	
	2-Skill	0%	45,786	45,943	45,254
		5%	46,702	45,908	46,111
		10%	46,369	47,224	45,841
	No	N/A		47,908	
S2	Full	0%		35,049	
	2-Skill	0%	36,480	36,084	**35,559**
		5%	36,836	36,876	**37,172**
		10%	37,166	37,057	**37,008**
	No	N/A		**37,267**	
S3	Full	0%		37,990	
	2-Skill	0%	39,405	38,422	38,039
		5%	39,976	38,789	39,860
		10%	39,871	39,457	38,988
	No	N/A		40,378	
S4	Full	0%		27,812	
	2-Skill	0%	29,308	28,099	28,451
		5%	30,419	29,298	28,910
		10%	29,844	29,447	29,384
	No	N/A		30,553	
S5	Full	0%		16,623	
	2-Skill	0%	18,588	17,550	17,076
		5%	18,971	18,659	18,270
		10%	18,963	18,645	18,524
	No	N/A		18,972	

For case M3, for two-skill cross-training with *MaxC* = 100%, the total cost is \$76,190 for *CICT* = 5% and \$77,356 for *CICT* = 10%. These costs are higher than the cost of *CICT* = 0% case which is \$73,421, and very close to the upper bound cost of \$78,934 for no cross-training. For case L3, for two-skill cross-training with *MaxC* = 100%, the total cost is \$117,644 for *CICT* = 5% and \$117,081 for *CICT* = 10%. These costs are much higher than the cost of *CICT* = 0% case which is \$112,926, and very close to the upper bound cost of \$119,052 for no cross-training.

Table 26: Cost Increase for Cross-Trained Staff for Medium and Large Cases

Case	CT	CICT	MaxC 10%	MaxC 50%	MaxC 100%
M1	Full	0%		54,182	
	2-Skill	0%	59,317	55,211	55,393
		5%	58,647	56,958	58,400
		10%	59,706	57,261	57,959
	No	N/A		59,948	
M2	Full	0%		51,544	
	2-Skill	0%	55,960	53,218	52,789
		5%	57,504	54,191	55,347
		10%	56,465	56,657	55,600
	No	N/A		57,555	
M3	Full	0%		72,204	
	2-Skill	0%	74,524	74,214	**73,421**
		5%	77,922	76,445	**76,190**
		10%	76,716	77,723	**77,356**
	No	N/A		**78,934**	
M4	Full	0%		61,598	
	2-Skill	0%	65,751	62,457	62,832
		5%	65,130	64,033	64,142
		10%	65,392	64,337	66,242
	No	N/A		66,629	
L2	Full	0%		94,682	
	2-Skill	0%	102,128	96,935	98,795
		5%	103,425	100,050	100,250
		10%	103,399	103,496	101,994
	No	N/A		103,963	
L3	Full	0%		109,356	
	2-Skill	0%	116,411	111,224	**112,926**
		5%	118,086	115,406	**117,644**
		10%	118,278	115,799	**117,081**
	No	N/A		**119,052**	

In conclusion, the above results demonstrate that additional pay for cross-training has an effect on optimal cross-training configuration. It is crucial to evaluate the trade-off between cost and level of cross-training. Total flexibility is not always optimal when the cost of adding flexibility is considered.

5.4 Efficiency Loss and Cost Increase Coexisting in Cross-Training

The previous experiments clearly demonstrate that an efficiency loss in the secondary skill or a cost increase for cross-trained staff alone deteriorates the solution and results in higher weekly costs. Additionally, this experiment studies the case in which both an efficiency loss and a cost increase exist together when partial limited cross-training is utilized. Table 27, Figure 15, and Figure 16 present the results for two-skill and three-skill cross-training for case L1.

Table 27: Efficiency Loss and Cost Increase Results for Case L1

	Eff.		MaxC									
CT	Loss	CICT	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Full	0%	0%	88,874									
2-Skill	0%	0%	94,376	93,881	93,050	92,218	91,866	89,996	91,434	91,005	91,279	91,465
	10%	0%	95,962	95,434	94,642	93,857	92,689	91,951	93,706	93,753	92,287	92,330
	20%	0%	96,790	95,225	95,710	96,054	94,185	93,217	94,530	95,089	93,657	94,648
	0%	5%	97,175	95,968	94,752	94,838	94,819	94,427	93,883	93,805	94,293	93,815
	10%	**5%**	**95,319**	**95,037**	**95,453**	**94,499**	**96,716**	**94,690**	**95,202**	**95,155**	**95,405**	**95,539**
	20%	**5%**	**96,314**	**96,902**	**96,853**	**96,757**	**96,699**	**96,449**	**96,384**	**97,779**	**96,490**	**96,690**
	0%	10%	96,544	95,141	96,359	94,701	95,838	95,636	96,447	96,022	96,661	97,306
	10%	**10%**	**94,461**	**95,939**	**96,381**	**97,357**	**96,873**	**97,032**	**95,795**	**97,505**	**96,010**	**96,669**
	20%	**10%**	**96,400**	**97,964**	**96,621**	**97,558**	**97,997**	**98,648**	**97,140**	**97,704**	**97,377**	**96,509**
3-Skill	0%	0%	92,982	91,688	91,093	89,548	91,958	91,945	90,380	90,299	90,260	89,535
	10%	0%	96,222	93,820	91,993	90,983	92,183	93,147	93,528	91,201	90,413	92,254
	20%	0%	97,810	94,824	94,929	96,422	93,982	93,725	94,049	93,242	93,424	93,311
	0%	5%	96,422	94,521	93,992	92,679	93,707	94,143	93,746	94,502	92,412	93,687
	10%	**5%**	**96,719**	**95,993**	**95,554**	**94,250**	**95,478**	**95,390**	**94,922**	**94,042**	**95,995**	**95,693**
	20%	**5%**	**96,165**	**96,046**	**97,706**	**94,549**	**95,102**	**94,295**	**94,467**	**95,880**	**96,602**	**96,014**
	0%	10%	96,850	95,353	94,848	96,230	95,079	96,488	95,102	95,273	95,842	94,580
	10%	**10%**	**96,366**	**95,849**	**95,885**	**96,558**	**96,520**	**95,733**	**95,912**	**96,178**	**96,531**	**95,836**
	20%	**10%**	**97,486**	**98,639**	**98,299**	**97,036**	**96,110**	**96,550**	**96,764**	**97,108**	**97,861**	**96,531**
No	N/A	N/A	98,653									

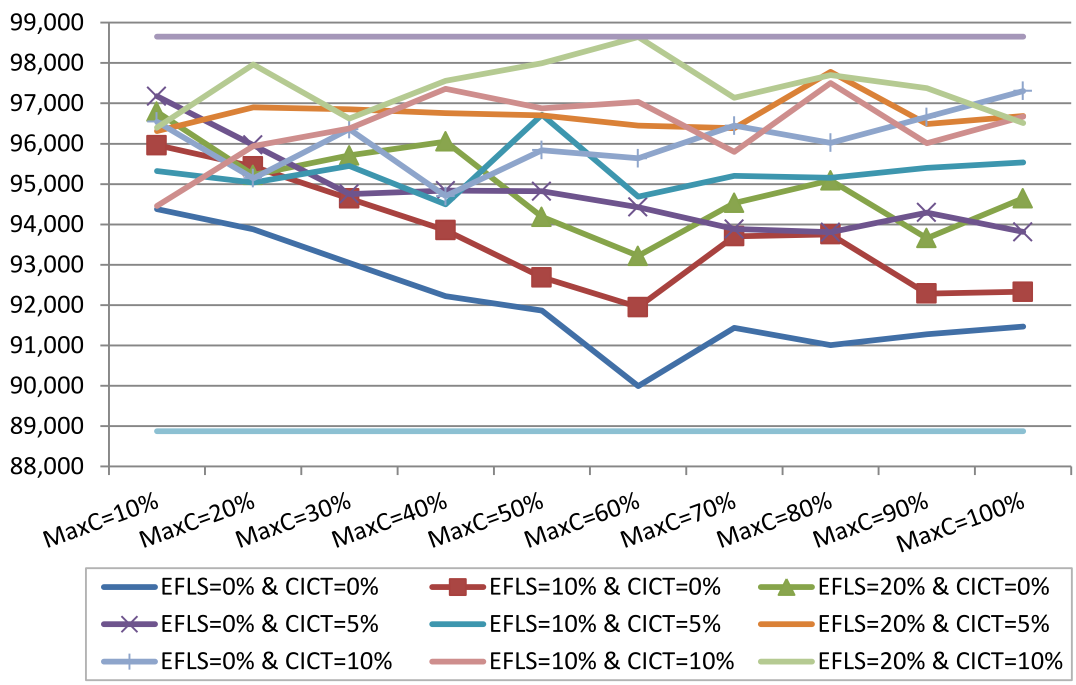

Figure 15: Case L1 Results for Efficiency Loss and Cost Increase in Two-Skill CT

In two-skill cross-training, for the baseline case (*Eff.Loss* = 0% and *CICT* = 0%), the average cost for *MaxC* = 10% - 100% is $92,057. On the other hand, when both efficiency loss and cost increase exist (*Eff.Loss* > 0% and *CICT* > 0%), the average costs for *MaxC* = 10% - 100% are around $95,000 - $98,000 (bolded in the table) and are very close to the no cross-training case ($98,653). The case with only a 10% cost increase (*Eff.Loss* = 0% and *CICT* = 10%) also gives similar results which indicate the negative effect of the cost increase for cross-trained agents on the solution. Furthermore, for these cases, increasing the amount of cross-trained agents (*MaxC*) does not decrease the total cost, which indicates that increasing the amount of cross-trained agents does not improve the solution.

For all other cases in which there is only an efficiency loss (*Eff.Loss* = 10% and 20%) or a cost increase (*CICT* = 5%), the average costs for *MaxC* = 10% - 100% are less than $95,000, although they are increased in cost compared to the baseline case. Furthermore, in these cases, increasing the amount of cross-trained agents (*MaxC*) decreases the cost, which indicates that it is still beneficial to increase the amount of cross-training.

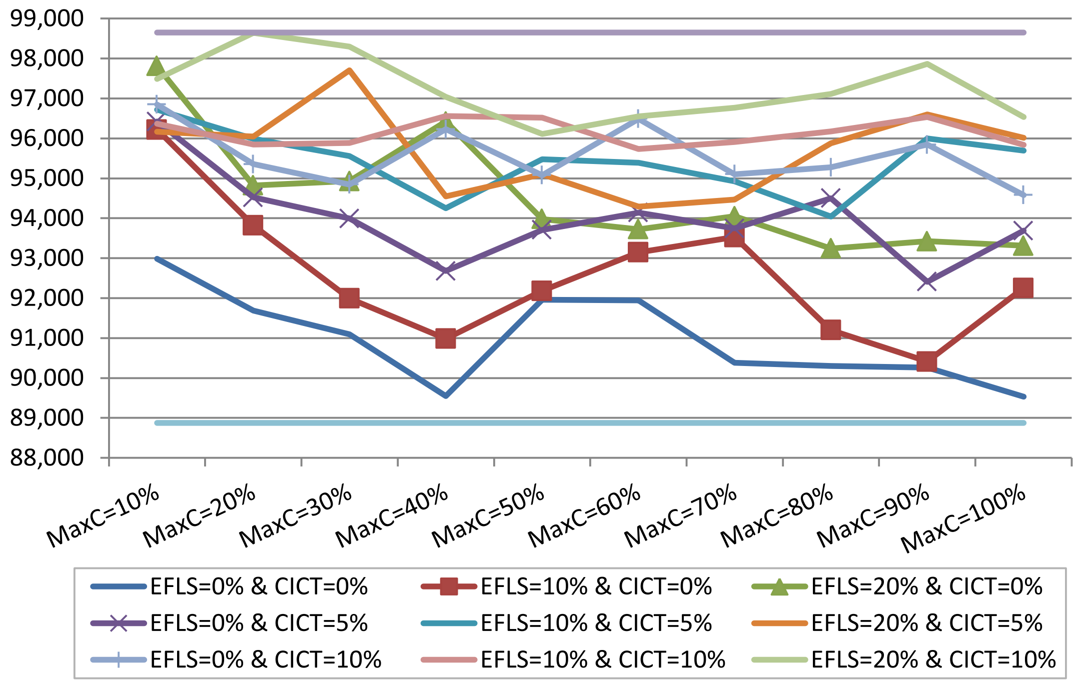

Figure 16: Case L1 Results for Efficiency Loss and Cost Increase in Three-Skill CT

Three-skill cross-training gives similar results to the two-skill case. When both efficiency loss and cost increase exist (*Eff.Loss* > 0% and *CICT* > 0%), for *MaxC* = 10% - 100%, the total costs are around $95,000 - $98,000 on average for three-skill cross-training. Similarly, the *Eff.Loss* = 0% and *CICT* = 10% case also provides similar results which indicate the negative effect of cost increases for cross-trained agents. For all of these cases, further amounts of cross-training (*MaxC*) do not improve the solution.

5.5 Flexibility: Cross-Training versus Part-Time Shifts

This experiment is designed to evaluate the effectiveness of cross-training and compare it to that of part-time shifts. It has been noted in the literature that both cross-training and part-time shifts (Mabert and Showalter, 1990; Jacobs and Betchold, 1993; Ernst et. al, 2004; Avramidis et. al, 2010; Maenhout and Vanhoucke, 2013) are commonly utilized to provide flexibility in the face of demand fluctuations.

To evaluate the flexibility benefits of cross-training as compared to that of part-time shifts, the results for no cross-training, two-skill, three-skill, and four-skill cross-training with varying cross-training percentages (*MaxC* = 10% - 100%), and full cross-training in conjunction with varying part-time shift percentages (*MaxP* = 0% - 100%) are presented in Table 28, and compared in Figure 17, Figure 18, and Figure 19 for case L1.

Table 28: Case L1 Results for Flexibility: Cross-Training versus Part-Time Shifts

		MaxP										
CT	MaxC	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
No	N/A	**98,493**	**98,404**	**98,653**	**97,688**	**97,229**	**96,635**	**96,725**	**96,798**	**96,724**	**96,564**	**96,345**
2-Skill	10%	**95,031**	95,500	94,376	96,261	95,570	96,083	94,331	95,197	94,973	95,765	93,762
	20%	**95,377**	96,144	93,881	95,548	93,445	94,125	93,745	95,331	92,943	93,154	93,592
	30%	**92,440**	94,251	93,050	92,651	94,407	92,257	91,156	90,688	92,027	93,791	91,911
	40%	**91,318**	92,269	92,218	92,965	92,003	91,747	92,205	91,637	92,720	91,185	92,241
	50%	**90,054**	90,967	91,866	93,070	92,631	91,558	90,621	90,633	90,903	90,668	91,037
	60%	**89,896**	91,936	89,996	91,833	90,464	90,612	90,657	89,871	89,152	89,470	89,836
	70%	**90,622**	91,531	91,434	90,803	91,544	90,162	91,282	89,909	90,064	89,646	89,456
	80%	**90,713**	91,588	91,005	90,786	89,881	90,299	90,374	90,936	90,254	89,766	89,845
	90%	**91,863**	91,429	91,279	90,726	90,527	90,555	90,825	90,075	90,349	90,143	90,342
	100%	**91,124**	91,545	91,465	91,303	90,602	90,158	90,064	90,368	90,014	89,520	90,424
3-Skill	10%	**94,441**	94,928	92,982	94,963	95,482	93,339	93,364	93,631	92,802	94,685	93,244
	20%	**92,030**	93,178	91,688	92,769	91,665	90,982	89,752	91,084	90,903	90,436	90,603
	30%	**89,662**	90,662	91,093	90,830	90,582	91,234	91,015	90,533	90,438	89,832	90,844
	40%	**91,002**	91,127	89,548	91,739	91,337	90,676	89,975	90,455	90,335	89,281	89,876
	50%	**90,646**	90,966	91,958	91,159	92,012	89,254	90,677	89,532	89,224	89,728	90,253
	60%	**90,049**	91,350	91,945	91,215	90,600	90,323	89,969	89,668	88,689	89,500	89,920
	70%	**90,899**	91,875	90,380	90,723	88,592	88,857	89,535	88,817	88,926	88,784	89,337
	80%	**89,883**	90,806	90,299	90,233	89,705	90,255	89,394	89,124	89,641	89,242	88,894
	90%	**90,654**	90,975	90,260	89,967	88,951	89,502	89,137	89,761	88,874	88,561	88,623
	100%	**89,059**	90,405	89,535	90,312	88,487	89,437	88,543	88,153	88,264	88,815	88,962
4-Skill	10%	**93,393**	94,416	93,343	94,064	94,824	93,836	93,576	92,803	92,857	93,113	92,724
	20%	**89,785**	92,094	92,815	91,250	90,321	90,833	90,670	91,471	89,818	90,698	89,676
	30%	**89,981**	91,062	91,952	90,100	91,031	91,136	90,657	90,814	89,794	90,273	91,180
	40%	**89,508**	90,290	90,211	90,261	89,936	90,017	89,862	90,171	89,760	89,920	90,143
	50%	**89,902**	90,891	90,407	90,048	89,092	89,523	89,800	89,449	88,723	89,803	89,282
	60%	**89,163**	90,517	89,618	90,248	89,759	89,190	89,243	89,653	88,910	89,351	88,927
	70%	**89,558**	90,802	90,478	89,858	89,286	89,192	89,217	88,868	88,343	89,059	89,524
	80%	**88,885**	90,150	90,810	89,266	88,090	89,004	88,888	89,114	88,646	88,120	87,895
	90%	**88,397**	89,781	90,241	89,413	88,454	89,020	88,287	88,517	88,951	88,773	88,237
	100%	**88,688**	90,257	89,300	88,425	88,036	89,463	88,108	88,613	87,965	88,455	88,374
Full	N/A	**88,258**	88,119	88,874	88,245	87,409	88,578	87,688	87,993	87,957	87,676	87,874

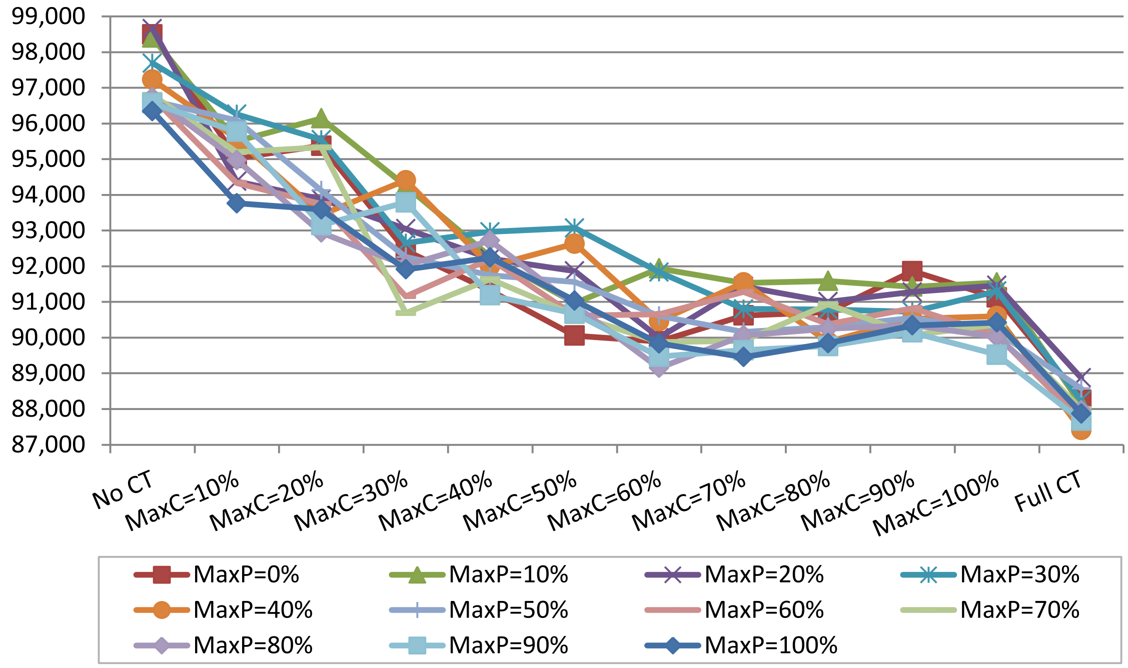

Figure 17: Case L1 Results for Flexibility: Part-Time Shifts versus Two-Skill CT

For two-skill cross-training, Figure 17 indicates that for any part-time shift percentage (*MaxP*), increasing the cross-training percentage (*MaxC*) decreases the total cost significantly. On the other hand, for any cross-training percentage (*MaxC*), the decrease in cost due to increasing the amount of part-time shifts is not so noticeable, especially in the presence of cross-training.

For example, for two-skill cross-training, the cost is $95,031 for *MaxP* = 0% and *MaxC* = 10%, $91,124 for *MaxP* = 0% and *MaxC* = 100%, $93,762 for *MaxP* = 100% and *MaxC* = 10%, and $90,424 for *MaxP* = 100% and *MaxC* = 100%. Increasing *MaxC* from 10% to 100% reduces the cost by (95,031 – 91,124) = $3,907 when *MaxP* = 0% and (93,762 – 90,424) = $3,338 when *MaxP* = 100%. On the other hand, increasing *MaxP* from 0% to 100% reduces the cost only by (95,031 – 93,762) = $1,269 when *MaxC* = 10% and (91,124 – 90,424) = $700 when *MaxC* = 100%.

As presented in Figure 18, the results for three-skill cross-training are similar to those of the two-skill case; increasing the amount of cross-training improves the solution much more significantly than increasing the amount of part-time shifts.

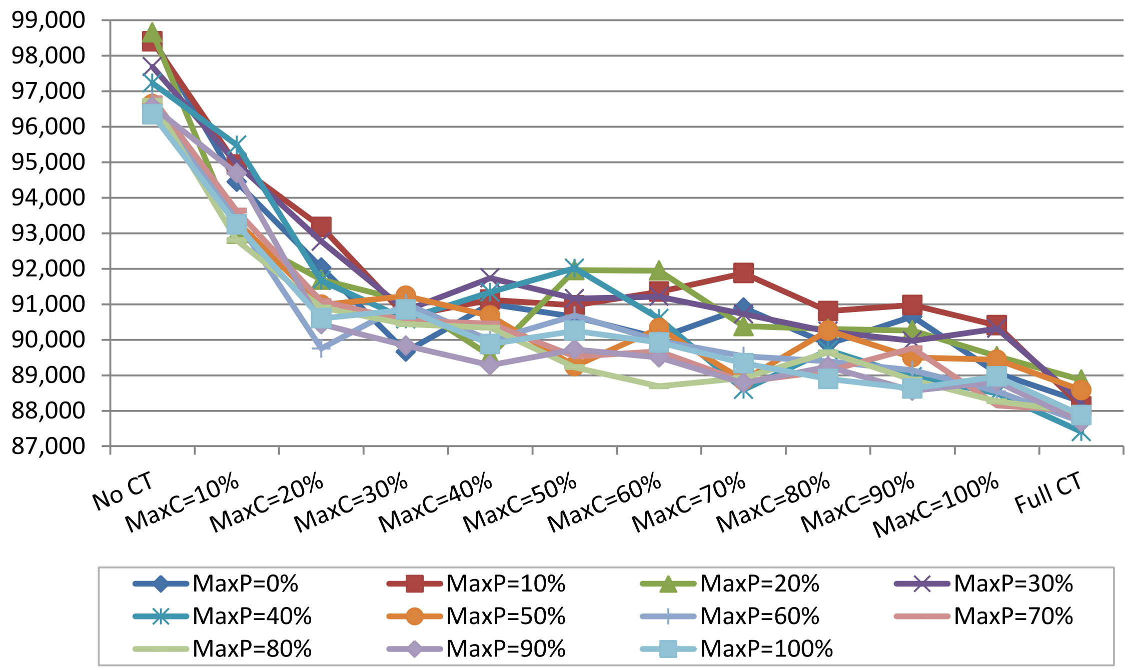

Figure 18: Case L1 Results for Flexibility: Part-Time Shifts versus Three-Skill CT

For example, for three-skill cross-training, the cost is $94,441 for *MaxP* = 0% and *MaxC* = 10%, $89,059 for *MaxP* = 0% and *MaxC* = 100%, $93,244 for *MaxP* = 100% and *MaxC* = 10%, and $88,962 for *MaxP* = 100% and *MaxC* = 100%. Increasing *MaxC* from 10% to 100% reduces the cost by (94,441 – 89,059) = $5,382 when *MaxP* = 0% and (93,244 – 88,962) = $4,282 when *MaxP* = 100%. On the other hand, increasing *MaxP* from 0% to 100% reduces the cost only by (94,441 – 93,244) = $1,197 when *MaxC* = 10% and (89,059 – 88,962) = $97 when *MaxC* = 100%.

Four-skill cross-training provides similar results with two-skill and three-skill cross-training. As presented in Figure 19, for four-skill cross-training, the cost is $93,393 for *MaxP* = 0% and *MaxC* = 10%, $88,688 for *MaxP* = 0% and *MaxC* = 100%, $92,724 for *MaxP* = 100% and *MaxC* = 10%, and $88,374 for *MaxP* = 100% and *MaxC* = 100%. Increasing *MaxC* from 10% to 100% reduces the cost by (93,393 – 88,688) = $4,705 when *MaxP* = 0% and (92,724 – 88,374) = $4,350 when *MaxP* = 100%. On the other hand, increasing *MaxP* from 0% to 100% reduces the cost only by (93,393 – 92,724) = $669 when *MaxC* = 10% and (88,688 – 88,374) = $314 when *MaxC* = 100%.

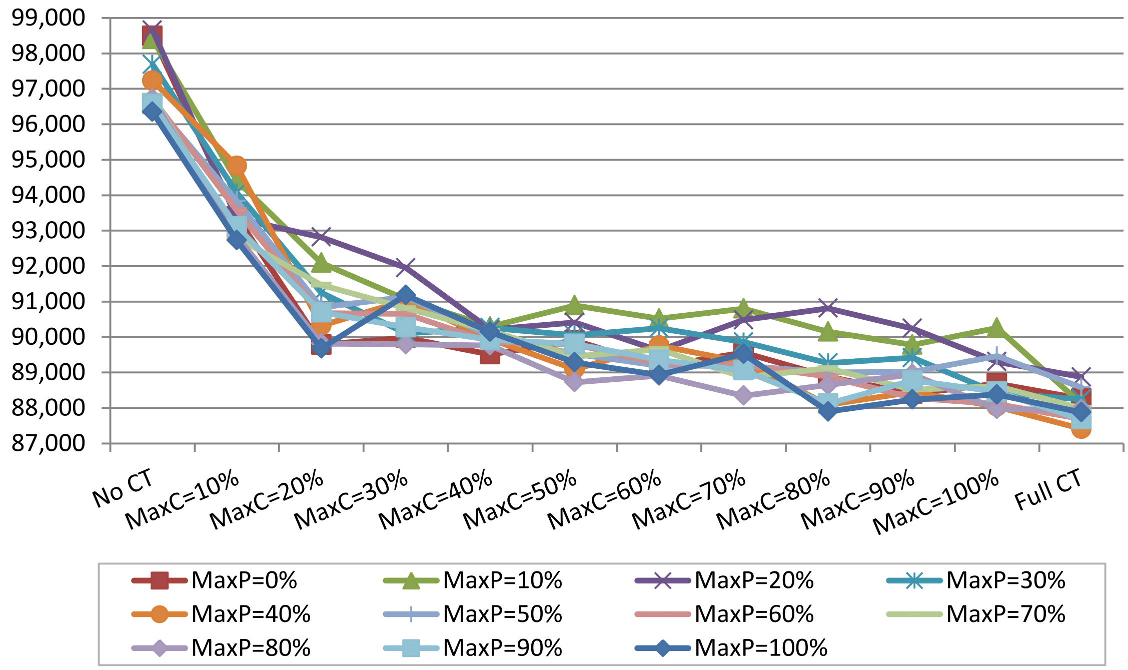

Figure 19: Case L1 Results for Flexibility: Part-Time Shifts versus Four-Skill CT

These results clearly indicate that increasing the percentage of part-time shifts (from *MaxP* = 0% to 100%) does not improve the solution noticeably for any cross-training percentage, whereas increasing the percentage of cross-training (from *MaxC* = 10% to 100% in two-skill, three-skill, and four-skill cross-training) noticeably decreases the total cost for any part-time shift percentage.

Comparing two-skill, three-skill, and four-skill cross-training, while two-skill cross-training shows a steady and constant reduction in cost, it can be noticed that sharper reductions in cost could be observed with three-skill and four-skill cross-training at a smaller percentage of cross-training (*MaxC* = 10% to 20%). This seems to be extremely beneficial in practice where employees are paid by their seniority and senior employees have more skills. Employing a few senior employees with more cross-trained skills would be sufficient.

For no and full cross-training, Figure 20 presents the total costs when the part-time shift percentage increases from 0% to 100%. These results demonstrate the flexibility provided by part-time shifts in two extreme cases for cross-training: a) no cross-training in which each agent has only one skill, and b) full cross-training in which all agents have all nine skills.

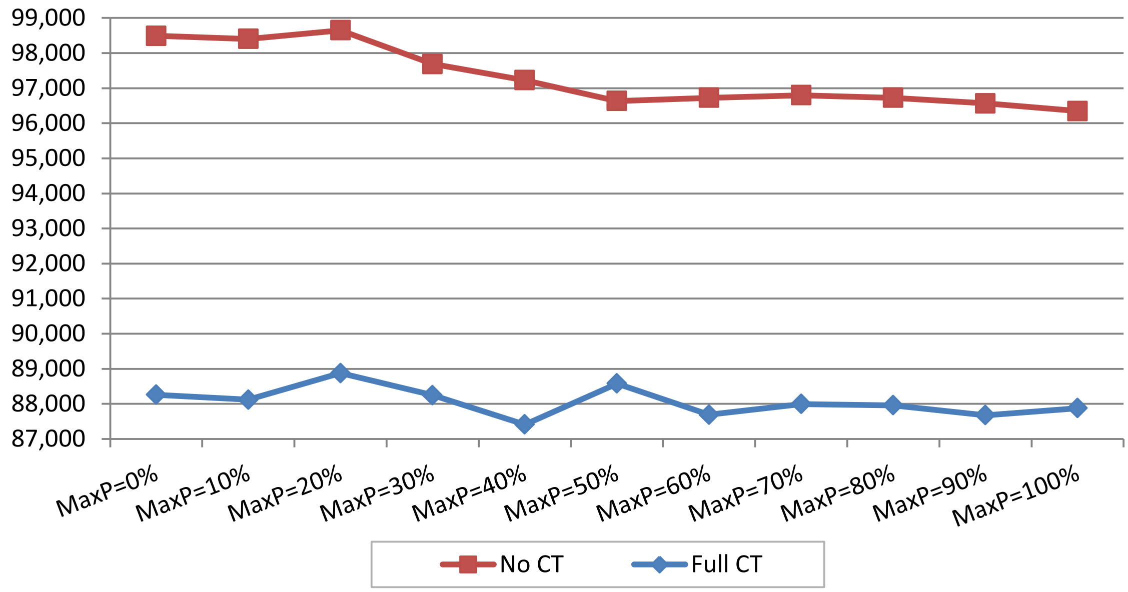

Figure 20: Case L1 Results for Flexibility: Part-Time Shifts versus No CT and Full CT

As presented in Figure 20, the results demonstrate that increasing the amount of part-time shifts does not reduce the cost significantly in either the case of no or full cross-training. For example, for full cross-training, the cost is $88,258 for *MaxP* = 0% and $87,874 for *MaxP* = 100%; the cost reduction is only $384. Even for no cross-training, the cost is $98,493 for *MaxP* = 0% and $96,345 for *MaxP* = 100%; the cost reduction is only $2,148. Even this cost reduction obtained in the no cross-training case is not comparable to the $4,000 - $5,000 reduction in cost obtained by increasing the amount of cross-trained staff in two-skill, three-skill, and four-skill cross-training. Furthermore, the reduction in cost is easily noticeable when full cross-training is employed instead of no cross-training; it is (98,493 – 88,258) = $10,235 when *MaxP* = 0%, and (96,345 – 87,874) = $8,471 when *MaxP* = 100%.

The above results indicate that though both the use of cross-training and the use of part time shifts reduce the total cost, cross-training has a much bigger impact on cost compared with part-time shifts. The results of two extreme cases (bolded in Table 28) prove this conclusion: a) no part-time shifts (*MaxP* = 0%) with varying amounts of cross-training (no cross-training, two-skill, three-skill, and four-skill cross-training with *MaxC* = 10% - 100%, and full cross-training), and b) no cross-training with varying amounts of part-time shifts (*MaxP* = 0% - 100%).

The results demonstrate that significant reduction in cost can be obtained by increasing the percentage of cross-trained agents even if part-time shifts are not employed. For example, in the case of no part-time shifts (*MaxP* = 0%), if two-skill cross-training is employed, the cost dramatically reduces from $98,493 with no cross-training to $95,031 with *MaxC* = 10%, $95,377 with *MaxC* = 20%, $92,440 with *MaxC* = 30%, and $91,318 with *MaxC* = 40%; this is almost equivalent to $88,258, the lowest possible bound with full cross-training.

The reduction in cost, on the other hand, is much lower when the amount of part-time shifts is increased but cross-training is not allowed. For example, in the case of no cross-training, if part-time shifts are not utilized (*MaxP* = 0%), the total cost is $98,493, whereas it is $98,404 for *MaxP* = 10% and $98,653 for *MaxP* = 20%. This is not nearly as significant as $95,031 with *MaxC* = 10% in two-skill cross-training. In fact, even this *MaxC* = 10% cross-training in a maximum of two skills with 0% part-time shifts ($95,031) gives better results than 100% part-time shifts with no cross-training ($96,345).

All other test cases demonstrate similar results with case L1. The results for various flexibility scenarios are presented in Table 29 for small cases and Table 30 for medium and large cases. Similarly, the results indicate that increasing the amount of cross-training decreases the cost noticeably compared to increasing the amount of part-time shifts.

For example, for case S1 (bolded in the table), when part-time shifts are not utilized, full cross-training provides a (48,990 – 44,397) = $4,593 cost reduction compared to no cross-training. Even two-skill cross-training with *MaxC* = 50% provides a (48,990 – 44,920) = $4,070 cost reduction compared to no cross-training. On the other hand, when cross-training is not utilized, 100% part-time shifts provides only a (48,990 - 48,026) = $964 cost reduction compared to 0% part-time shifts.

Similarly, for case M1 (bolded in the table), for *MaxP* = 0%, full cross-training provides a (60,020 – 54,632) = $5,388 cost reduction compared to no cross-training. Even two-skill cross-training with *MaxC* = 50% provides a (60,020 – 55,028) = $4,992 cost reduction and three-skill cross-training with *MaxC* = 50% provides a (60,020 – 56,426) = $3,594 cost reduction compared to no cross-training. On the other hand, for no cross-training, *MaxP* = 100% provides only a (60,020 – 59,643) = $377 cost reduction compared to *MaxP* = 0%.

In another instance, for case L3 (bolded in the table), for *MaxP* = 0%, full cross-training provides a (119,724 – 108,978) = $10,746 cost reduction compared to no cross-training. Even two-skill cross-training with *MaxC* = 50% provides a (119,724 – 111,448) = $8,276 cost reduction and three-skill cross-training with *MaxC* = 50% provides a

(119,724 – 111,985) = $7,739 cost reduction compared to no cross-training. On the other hand, for no cross-training, *MaxP* = 100% provides only a (119,724 – 119,165) = $559 cost reduction compared to *MaxP* = 0%.

Table 29: Small Cases Results for Flexibility

Case	MaxP	No CT	2-Skill CT MaxC 10%	2-Skill CT MaxC 50%	2-Skill CT MaxC 100%	Full CT
S1	0%	**48,990**	46,305	**44,920**	45,366	**44,397**
	100%	**48,026**	46,353	45,852	45,015	44,489
S2	0%	37,225	35,863	35,027	34,968	34,892
	100%	36,842	36,839	35,055	34,887	34,765
S3	0%	40,854	38,506	38,669	38,104	37,727
	100%	39,867	39,231	38,530	38,522	37,737
S4	0%	30,914	29,364	28,618	28,915	28,604
	100%	29,810	29,356	28,837	29,310	28,404
S5	0%	18,837	18,438	17,750	17,444	17,404
	100%	18,660	18,653	17,689	17,328	17,231

Table 30: Medium and Large Cases Results for Flexibility

Case	MaxP	No CT	2-Skill CT MaxC 10%	2-Skill CT MaxC 50%	2-Skill CT MaxC 100%	3-Skill CT MaxC 10%	3-Skill CT MaxC 50%	3-Skill CT MaxC 100%	Full CT
M1	0%	**60,020**	58,332	**55,028**	55,426	57,662	**56,426**	54,721	**54,632**
	100%	**59,643**	57,829	55,764	55,186	56,592	55,664	54,394	54,340
M2	0%	57,886	54,530	52,888	52,959	55,102	53,313	52,151	51,840
	100%	56,192	54,388	52,545	52,397	54,207	52,574	52,440	51,702
M3	0%	78,274	77,074	74,230	73,683	75,649	73,372	72,945	72,302
	100%	77,887	75,003	73,533	73,117	74,605	72,755	72,119	71,697
M4	0%	66,275	66,087	62,572	62,924	66,144	62,748	61,391	60,881
	100%	65,838	65,510	61,873	61,745	64,238	61,944	61,797	61,114
L2	0%	103,055	101,143	96,186	97,656	99,577	97,029	96,018	95,080
	100%	103,130	101,386	96,231	97,175	98,386	96,367	95,235	94,811
L3	0%	**119,724**	118,293	**111,448**	112,514	115,781	**111,985**	110,546	**108,978**
	100%	**119,165**	117,098	110,557	110,140	114,251	111,966	110,736	108,180

In summary, adding flexibility to the staff through cross-training is much more beneficial than adding flexibility to the schedule through part-time shifts. The benefits of cross-training increase with the use of part-time shifts, but the improvement is not significant. Cross-training increases staffing flexibility and enables service managers to better match available labor skills to time-varying demand.

5.6 Shift Flexibility: Part-Time Shifts versus Extended Shifts

This experiment investigates the impact of various shift types – full-time, extended, and part-time – on scheduling flexibility. The parameter *MinE* is used to guarantee the existence of extended shifts, whereas *MaxP* is used to limit part-time shifts. The below constraint (33) is added to the staffing and scheduling model, P-II of the TPSA, and it guarantees that at least *MinE* percent of full-time employees will be extended shift employees.

Parameter:

MinE minimum percentage of extended shift employees in all full-timers

a) Extended Shift Assignment

$$\sum_{w \in W} \sum_{e \in E} xe_{we} \geq MinE \left(\sum_{w \in W} \sum_{f \in F} xf_{wf} + \sum_{w \in W} \sum_{e \in E} xe_{we} \right) \quad (33)$$

The total weekly costs for case L1 obtained for various *MinE* and *MaxP* compositions under two-skill cross-training with *MaxC* = 100% are presented in Table 31 and compared in Figure 21. In the figure, the horizontal axis presents *MinE* values, whereas the vertical axis presents total cost; a line is drawn for each *MaxP* value.

Table 31: Results for Part-Time and Extended Shifts (*MaxC* = 100%) for Case L1

	MaxP										
MinE	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
0%	91,124	91,545	91,465	91,303	90,602	90,158	90,064	90,368	90,014	89,520	90,424
10%	91,620	91,479	90,536	91,151	90,009	90,479	89,860	89,801	90,324	90,513	90,335
20%	90,667	91,480	91,170	91,147	89,959	90,826	89,590	90,249	89,832	91,029	89,602
30%	90,095	91,653	91,312	91,969	90,114	91,457	90,057	90,225	89,567	90,988	89,800
40%	91,526	91,810	90,816	91,910	89,859	89,891	90,525	90,406	90,483	90,218	90,115
50%	91,626	92,829	92,791	90,951	90,681	90,789	90,250	90,508	89,669	90,200	90,523
60%	91,990	92,452	92,203	91,163	92,090	90,594	89,424	90,017	89,986	90,653	89,746
70%	93,092	92,227	92,619	92,076	91,602	91,703	91,335	90,413	90,504	90,607	89,426
80%	94,414	93,664	92,723	91,492	91,725	92,834	91,097	90,962	89,819	89,952	89,521
90%	96,928	95,136	92,768	91,838	92,381	91,844	90,852	89,959	89,604	89,981	90,394
100%	96,771	96,588	92,688	91,661	92,553	91,883	90,959	90,161	90,762	89,917	90,545

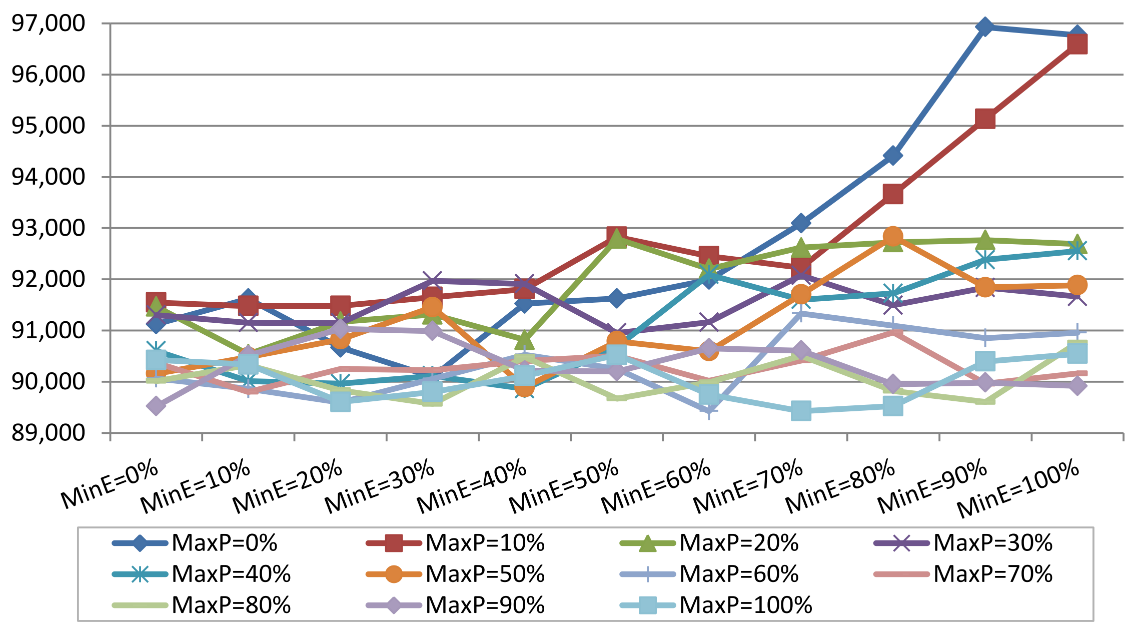

Figure 21: Results for Part-Time and Extended Shifts (*MaxC* = 100%) for Case L1

The results indicate that employing fewer extended shifts and more part-time shifts (lower *MinE* and higher *MaxP*) reduces the cost and increases the service level. In general, the *MaxP* = 0% case gives the highest costs whereas the *MaxP* = 100% case gives the lowest costs for all *MinE* values.

The above figure shows that, for the *MaxP* = 0% and 10% cases, the cost increases rapidly for *MinE* = 80% and more. For example, the cost is only $91,124 for *MinE* = 0% and *MaxP* = 0%, whereas it is $96,771 in the case that all full-timers have extended shifts and there are not any part-timers (*MinE* = 100% and *MaxP* = 0%). When part-timers are not allowed, assigning extended shifts to all full-timers increase the cost by nearly $5,500 even when 100% cross-training is allowed. This result shows that the use of excessive extended shifts deteriorates the solution quality in the absence of part-time shifts; even cross-training all staff (*MaxC* = 100%) with two skills does not help to improve it. A low part-time shift percentage (*MaxP* = 10%) also gives a similar result. For *MaxP* = 10%, the cost is only $91,545 for *MinE* = 0%, whereas it is $96,588 for *MinE* = 100%; the cost increase is nearly $5,000 due to the assignment of extended shifts to all full-timers.

Up to 60% extended shifts (*MinE* = 60%), the results are still good even when part-time shifts are not employed (*MaxP* = 0%), because 100% cross-training with two skills provides the necessary flexibility for the schedule. For example, for *MaxP* = 0%, the cost is $91,124 for *MinE* = 0% and $91,990 for *MinE* = 60%, whereas it is $93,092 for *MinE* = 70%, $94,414 for *MinE* = 80%, $96,928 for *MinE* = 90% and $96,771 for *MinE* = 100%. These results show that when part-time shifts are not allowed (*MaxP* = 0%), the cost increase due to increasing the amount of extended shifts (*MinE)* from 0% to 60% is still reasonable.

For the *MinE* = 0% case, the cost with *MaxP* = 0% is $91,124 and with *MaxP* = 100% is $90,424; there is only a slight difference of $700. On the other hand, for the *MinE* = 100% case, the cost with *MaxP* = 0% is $96,771 and with *MaxP* = 100% is $90,545; there is a difference of $6,000. This result also proves the negative effect of excessive extended shifts on cost when part-time shifts are not employed. However, with only 20% part-time shifts, good results can be obtained; for example, for the *MinE* = 100% and *MaxP* = 20% case, the cost is only $92,688. Therefore, to reduce the negative effect of excessive extended shifts on weekly cost, part-time shifts are still necessary even in the 100% cross-training (*MaxC* = 100%) in two-skills scenario.

Briefly, excessive use of extended shifts due to employee preferences significantly deteriorates the flexibility of the schedule for meeting fluctuations in demand; this increases cost and decreases service level. Employing a reasonable amount of part-time shifts helps to create flexible schedules at a lower cost and dramatically decreases the negative effect of having extended shift employees. It is not necessary to completely eliminate extended shifts, however, as doing so would reduce the feasible region and lead to inferior solutions as well.

An appropriate combination of full-time, extended, and part-time shifts creates flexible weekly schedules; if a limited number of employees are assigned to extended and part-time shifts, cost savings and service level improvements are possible.

5.7 Days Off Assignment: Any Days Off versus Consecutive Days Off

In this experiment, days off assignment alternatives (any days off and consecutive days off) are investigated. With the consecutive days off assignment, full-timers and part-timers are given two consecutive days off, and extended shift employees are given three consecutive days off. With the any days off assignment, the model assigns any days as off days.

For the consecutive days off assignment, a set of constraints are added to the P-II model of the TPSA.

Decision Variables:

zf_{wfd1d2} , $ze_{wed1d2d3}$, zp_{wpd1d2} number of employees who have skill w and are assigned to either full-time shift f with d_1 and d_2 consecutive days off, extended shift e with d_1, d_2, and d_3 consecutive days off, or part-time shift p with d_1 and d_2 consecutive days off

Minimize

Objective Function (1)

Subject to

Constraints (2) – (15) and

a) Consecutive Days Off Assignment

$$\sum_{\substack{d1,d2 \in D \\ d1,d2 \neq d}} zf_{wfd1d2} = yf_{wfd} \quad \forall\, w \in W, f \in F, d \in D \tag{34}$$

$$\sum_{\substack{d1,d2,d3 \in D \\ d1,d2,d3 \neq d}} ze_{wed1d2d3} = ye_{wed} \quad \forall\, w \in W, e \in E, d \in D \tag{35}$$

$$\sum_{\substack{d1,d2 \in D \\ d1,d2 \neq d}} zp_{wpd1d2} = yp_{wpd} \quad \forall\, w \in W, p \in P, d \in D \tag{36}$$

b) Non-Negativity Requirements

$$zf_{wfd1d2}, ze_{wed1d2d3}, zp_{wpd1d2} \geq 0 \; and \; integer \quad \forall\, w, f, e, p, d1, d2, d3 \tag{37}$$

The results for case L1 with any days off and consecutive days off (represented as “Cons.” in the table) assignments with two-skill cross-training for various cross-training percentages are demonstrated in Table 32 and compared in Figure 22.

Table 32: Case L1 Results for Any and Consecutive Days Off in Two-Skill CT

Days Off	Result	MaxC									
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Any	Cost	94,376	93,881	93,050	92,218	91,866	89,996	91,434	91,005	91,279	91,465
	B.Bou.	87,981	86,435	85,738	85,530	85,462	85,440	85,440	85,434	85,435	85,434
	Gap	6.78%	7.93%	7.86%	7.26%	6.97%	5.06%	6.56%	6.12%	6.40%	6.59%
Cons.	Cost	95,442	94,517	93,374	93,639	92,284	90,939	90,184	89,481	91,139	89,883
	B.Bou.	88,573	86,904	85,907	85,617	85,492	85,499	85,497	85,492	85,510	85,491
	Gap	7.20%	8.06%	8.00%	8.57%	7.36%	5.98%	5.20%	4.46%	6.18%	4.89%

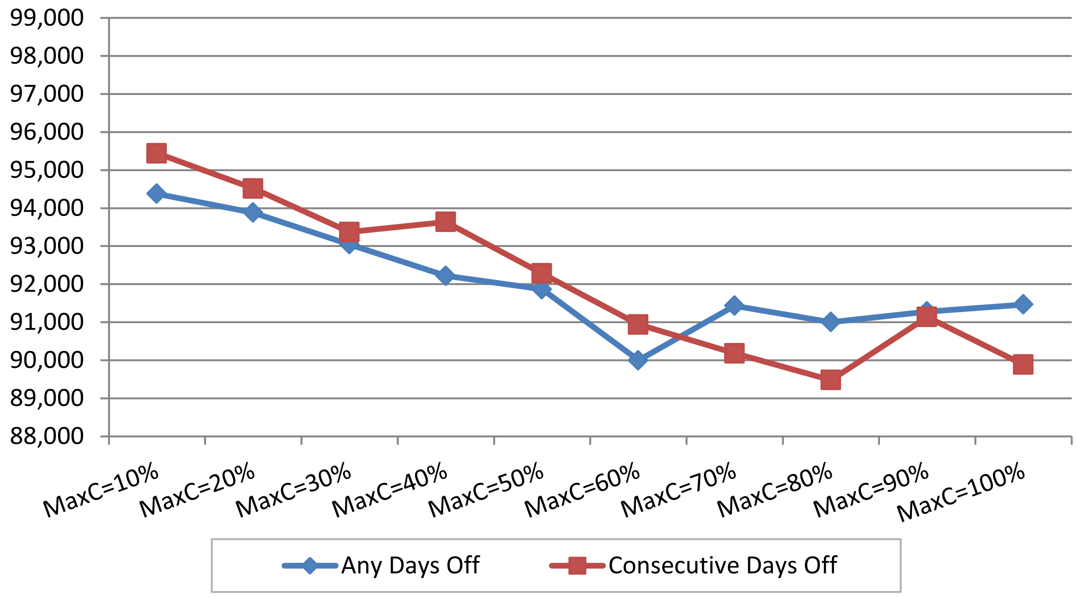

Figure 22: Case L1 Results for Any and Consecutive Days Off in Two-Skill CT

The above results show that any and consecutive days off assignment policies give similar results under all cross-training percentages. For two-skill cross-training, on average for *MaxC* = 10% - 100%, any days off assignment provides a cost of $92,057 and consecutive days off assignment provides a cost of $92,088, which is only 0.1% more than the average cost of any days off assignment. These results prove that there is not any noticeable cost difference between these two days off assignment policies.

The results for three-skill and four-skill cross-training for any and consecutive days off assignments are presented in Table 33 and compared in Figure 23 and Figure 24 for case L1. Similar with two-skill cross-training, the results for three-skill and four-skill cross-training indicate that there is not any noticeable cost difference between any and consecutive days off assignment policies.

Table 33: Case L1 Results for Any and Consecutive Days Off in Three and Four-Skill CT

CT	Days Off	MaxC									
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
3-Skill	Any	92,982	91,688	91,093	89,548	91,958	91,945	90,380	90,299	90,260	89,535
	Cons.	96,283	94,455	93,413	91,191	90,625	90,368	91,335	90,844	90,351	89,630
4-Skill	Any	93,343	92,815	91,952	90,211	90,407	89,618	90,478	90,810	90,241	89,300
	Cons.	96,114	93,645	92,502	92,236	91,234	91,535	90,545	90,537	90,261	88,981

For three-skill cross-training, the average cost for *MaxC* = 10% - 100% is $90,969 with any days off assignment and $91,850 with consecutive days off assignment; consecutive days off assignment increases the cost only 1.0% compared to any days off assignment. For four-skill cross-training, the average cost for *MaxC* = 10% - 100% is $90,918 with any days off assignment and $91,759 with consecutive days off assignment; consecutive days off assignment increases the cost only 0.9% compared to any days off assignment.

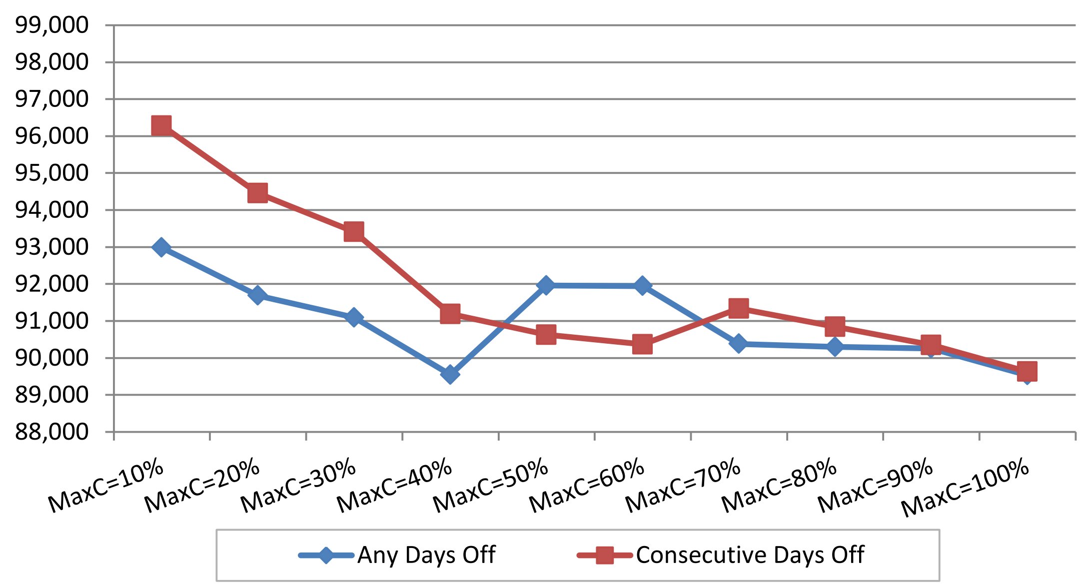

Figure 23: Case L1 Results for Any and Consecutive Days Off in Three-Skill CT

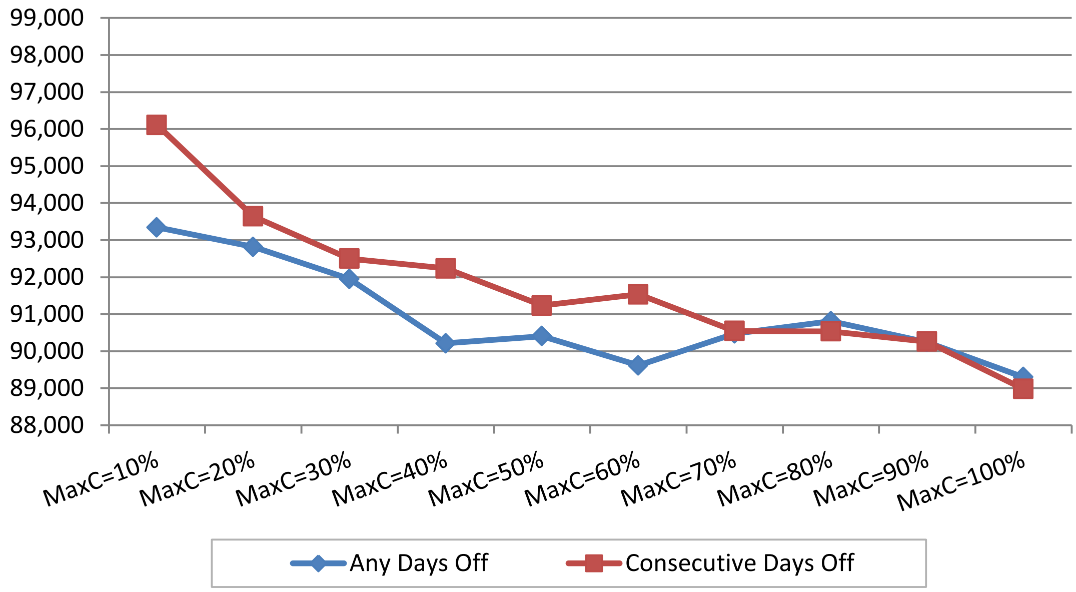

Figure 24: Case L1 Results for Any and Consecutive Days Off in Four-Skill CT

To generalize the above conclusion for case L1, for all test cases, the results with no cross-training, two-skill cross-training with *MaxC* = 10%, 50%, and 100%, and full cross-training for any days off and consecutive days off assignments are presented in Table 34.

For example, for case L1, for no cross-training, the total cost is $98,653 (Best Bound: 90,342 and Gap: 8.42%) with any days off, whereas it is $98,439 (Best Bound: 91,077 and Gap: 7.48%) with consecutive days off; for full cross-training, the total cost is $88,874 (Best Bound: 85,396 and Gap: 3.91%) with any days off, whereas it is $89,262 (Best Bound: 85,462 and Gap: 4.26%) with consecutive days off (bolded in the table). For case L1, the consecutive days off assignment does not increase the cost noticeably in the case of no cross-training and full cross-training.

Table 34: Results for Any and Consecutive Days Off Assignments for All Test Cases

Case	Days Off	No CT	2-Skill CT MaxC 10%	2-Skill CT MaxC 50%	2-Skill CT MaxC 100%	Full CT
S1	Any	47,908	45,786	45,943	45,254	44,715
	Cons.	47,339	46,936	45,379	46,680	45,829
S2	Any	37,267	36,480	36,084	35,559	35,049
	Cons.	37,192	36,997	36,030	35,497	35,968
S3	Any	40,378	39,405	38,422	38,039	37,990
	Cons.	40,767	39,285	39,551	39,281	38,055
S4	Any	30,553	29,308	28,099	28,451	27,812
	Cons.	30,761	29,537	28,923	28,757	29,358
S5	Any	18,972	18,588	17,550	17,076	16,623
	Cons.	19,919	19,070	17,761	16,915	17,530
M1	Any	59,948	59,317	55,211	55,393	54,182
	Cons.	61,605	58,219	56,142	56,092	55,973
M2	Any	57,555	55,960	53,218	52,789	51,544
	Cons.	58,779	56,253	53,155	52,547	52,176
M3	Any	78,934	74,524	74,214	73,421	72,204
	Cons.	77,387	76,352	73,861	74,526	71,941
M4	Any	66,629	65,751	62,457	62,832	61,598
	Cons.	67,242	65,202	62,943	63,538	61,761
L1	Any	**98,653**	94,376	91,866	91,465	**88,874**
	Cons.	**98,439**	95,442	92,284	89,883	**89,262**
L2	Any	103,963	102,128	96,935	98,795	94,682
	Cons.	105,941	102,375	97,360	96,181	95,532
L3	Any	119,052	116,411	111,224	112,926	109,356
	Cons.	119,742	118,614	112,371	110,751	108,657

The results presented in Table 34 indicate that, compared to any days off assignment, on average, consecutive days off assignment increases the cost only 1.8% for small test cases, 1.3% for medium test cases, and 0.6% for large test cases. Consecutive days off assignment on average increases the cost by 1.2% in the case of no cross-training, 1.0% in the case of two-skill cross-training, and 1.9% in the case of full cross-training. For all test cases, on average, consecutive days off assignment increases the total weekly cost only 1.3% compared to any days off assignment.

The results clearly indicate that assigning consecutive off days does not increase the total cost significantly, whereas it provides better weekly schedules and improves employees' morale. Contrary to what literature has suggested, it was found that consecutive or any days off assignment has little effect to the overall cost, due perhaps to the relatively stable daily demand of the call center across the week.

CHAPTER 6 CONCLUSIONS

This study deals with the problem of designing effective workforce cross-training structures to supplement the process of staff scheduling in call centers. For the solution of this problem, a cross-training staff scheduling model that incorporates cross-training decision optimization with shift scheduling, days off assignment, and lunch break assignment is proposed. To improve computational time, a two-phase sequential approach which finds good feasible results in less time is proposed for the solution of the strategic model. There are no studies in the literature that cover all of the features of staff scheduling while optimizing cross-training configuration; generally, cross-training decision is given a priori with a random or simple approach, or as a complete pooling decision, which ignores staffing and scheduling aspects of a service center.

The proposed mathematical models and the solution approach are employed to conduct a set of computational experiments to evaluate the value of cross-training as a source of staff flexibility in service operations. Using demand data provided by the support center of a Fortune 50 retailer company, various parameters have been analyzed to evaluate cross-training policies (full, partial, and limited), cross-training breadth (two, three, and four skills), varying efficiency levels for the secondary skill; cost increases for cross-trained staff; shift types (full-time, extended, and part-time), and days off assignments (any and consecutive). Mainly, several managerial insights for cross-training service agents are developed: number of agents cross-trained, which agents are cross-trained for which skills, number of additional skills, and efficiency of the secondary skills.

The results of the experiments indicate that the use of cross-training, though it might increase the complexity of the problem, offers much advantage in providing demand coverage, and in avoiding overstaffing by maintaining server flexibility. When workers are cross-trained, the added flexibility increases the manager's ability to meet real-time staffing requirements for cases where the service capacity does not match actual demand.

With only a fraction of the workforce being cross-trained for only two out of nine service groups, cross-training offers the risk pooling effect and could dramatically reduce staff cost and penalty costs related to insufficient service, while increasing customer service level. A moderate level of cross-training provides good staff schedules that perfectly match demand while improving employee satisfaction.

In conclusion, the proposed models and solution approach provide better, faster, and more comprehensive solutions and strategic assistance to the call center in the composition and scheduling of staff. Cross-training for compatible service groups offers a significant strategic advantage in staff scheduling and is worth much consideration. Even though this study emphasizes the application of cross-training to call center staffing and scheduling, the proposed methods can be applied to many service or manufacturing operations.

APPENDIX A: Call Arrivals

Table 35: Call Arrivals for a Week for Service Group 1

	Sunday		Monday		Tuesday		Wednesday		Thursday		Friday		Saturday	
Time	C*	AHT*	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT
00:00-00:30	1	598	2	799	1	824	2	1063	2	831	3	583	2	673
00:30-01:00	3	411	3	1355	2	1295	3	1101	3	806	2	1315	1	803
01:00-01:30	4	430	2	542	2	918	2	938	3	581	3	800	2	556
01:30-02:00	4	398	1	398	1	709	1	402	2	347	1	776	4	368
02:00-02:30	4	237	2	671	3	325	1	525	1	285	3	555	1	528
02:30-03:00	5	441	2	272	1	1111	1	98	2	332	2	469	3	353
03:00-03:30	2	703	2	429	1	1634	1	576	1	177	1	251	1	371
03:30-04:00	1	161	3	215	1	677	2	617	2	367	1	120	1	429
04:00-04:30	1	272	1	673	1	1050	0	0	1	223	1	673	0	0
04:30-05:00	2	294	0	0	0	0	1	446	1	191	1	370	1	476
05:00-05:30	4	285	2	401	1	241	1	233	1	316	1	310	2	374
05:30-06:00	2	400	1	196	0	0	2	714	2	695	2	500	2	501
06:00-06:30	7	330	4	441	5	379	5	279	4	422	4	311	4	284
06:30-07:00	5	442	8	640	7	629	8	489	8	534	6	318	7	559
07:00-07:30	10	451	9	763	9	650	9	531	9	533	9	614	7	353
07:30-08:00	12	459	11	655	9	624	9	728	10	848	7	507	9	466
08:00-08:30	15	497	14	680	14	675	15	732	12	711	9	764	11	634
08:30-09:00	15	526	16	667	16	576	19	614	16	724	14	603	11	584
09:00-09:30	17	482	18	724	15	748	17	685	19	728	18	583	10	440
09:30-10:00	20	550	20	878	16	803	23	656	23	723	15	626	15	499
10:00-10:30	19	521	23	603	19	650	24	629	20	769	21	605	19	639
10:30-11:00	18	550	23	662	29	521	24	686	20	676	19	851	16	649
11:00-11:30	17	577	23	724	23	600	22	679	21	690	19	555	16	620
11:30-12:00	14	684	20	603	22	719	21	646	20	705	20	648	16	744
12:00-12:30	17	593	21	791	21	687	21	720	24	624	20	571	13	642
12:30-13:00	17	688	24	665	24	485	27	635	22	691	19	793	14	779
13:00-13:30	17	537	22	687	21	606	24	810	20	654	23	587	19	684
13:30-14:00	17	543	23	684	21	586	23	644	21	588	22	759	16	695
14:00-14:30	13	654	28	696	19	609	23	662	22	793	22	666	13	790
14:30-15:00	14	699	23	742	23	816	22	640	24	635	26	795	14	786
15:00-15:30	15	653	25	678	22	681	20	725	22	707	17	769	17	595
15:30-16:00	13	493	19	755	22	555	19	826	20	746	16	787	17	728
16:00-16:30	11	593	21	718	17	799	18	886	21	570	22	675	17	642
16:30-17:00	10	609	15	826	16	784	20	579	22	639	18	757	15	872
17:00-17:30	9	842	14	651	17	848	17	780	16	812	20	852	15	881
17:30-18:00	11	1011	15	1029	13	923	16	720	13	790	16	805	10	888
18:00-18:30	10	926	13	766	13	836	14	838	15	771	13	935	10	850
18:30-19:00	8	846	15	642	12	1007	12	703	9	1055	13	967	9	549
19:00-19:30	8	689	12	683	9	985	13	810	13	596	12	703	9	780
19:30-20:00	8	786	8	750	9	710	10	836	8	632	11	669	9	945
20:00-20:30	4	902	9	841	7	625	8	902	6	673	8	516	8	604
20:30-21:00	8	713	7	691	7	771	7	805	8	772	9	688	9	613
21:00-21:30	6	879	9	801	7	868	8	484	8	539	6	1190	8	828
21:30-22:00	5	954	5	895	5	1125	4	1174	6	827	5	643	6	767
22:00-22:30	4	700	5	1135	4	971	6	1232	7	861	5	616	6	948
22:30-23:00	4	1074	3	818	3	834	3	1174	4	1072	6	718	6	706
23:00-23:30	4	830	4	1276	6	1210	4	732	5	860	3	749	3	454
23:30-00:00	3	1234	3	889	3	1004	3	1345	3	294	5	943	3	1141

* C: Number of Calls, AHT: Average Handling Time in Seconds

Table 36: Call Arrivals for a Week for Service Group 2

	Sunday		Monday		Tuesday		Wednesday		Thursday		Friday		Saturday	
Time	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT
00:00-00:30	0	0	1	937	0	0	1	971	2	779	2	262	1	530
00:30-01:00	0	0	2	1026	1	589	0	0	1	756	2	652	0	0
01:00-01:30	2	725	2	497	2	877	1	465	1	242	1	559	0	0
01:30-02:00	1	502	2	863	1	1108	1	1668	1	593	2	910	0	0
02:00-02:30	1	546	1	2432	2	723	1	1576	1	615	1	469	1	222
02:30-03:00	0	0	0	0	0	0	1	284	2	467	0	0	1	785
03:00-03:30	2	287	1	744	1	704	1	744	1	656	2	904	1	653
03:30-04:00	2	415	1	375	2	367	2	472	1	526	2	395	1	793
04:00-04:30	1	733	1	222	1	453	2	730	2	292	0	0	0	0
04:30-05:00	1	411	2	573	1	606	2	537	2	350	0	0	1	840
05:00-05:30	1	671	3	968	1	486	1	1099	2	342	3	492	1	552
05:30-06:00	2	368	1	602	3	576	1	277	2	769	2	925	0	0
06:00-06:30	2	430	2	668	4	522	4	522	3	725	3	246	0	0
06:30-07:00	3	351	5	399	5	570	6	610	9	630	4	716	2	632
07:00-07:30	4	495	7	541	8	665	8	607	10	691	8	736	2	883
07:30-08:00	7	584	10	654	10	533	11	643	9	609	9	579	4	460
08:00-08:30	7	585	14	576	16	586	18	684	16	562	14	676	5	599
08:30-09:00	6	723	17	609	20	491	23	544	19	743	17	635	6	665
09:00-09:30	9	426	17	615	21	558	21	543	23	555	21	522	5	684
09:30-10:00	7	427	20	640	21	488	23	578	19	633	20	588	8	398
10:00-10:30	9	632	22	480	23	547	26	618	20	629	21	550	8	450
10:30-11:00	9	523	20	598	18	516	23	557	24	642	23	419	8	518
11:00-11:30	6	744	21	545	20	561	20	631	24	555	18	580	8	597
11:30-12:00	9	451	24	555	19	489	23	554	23	531	23	474	9	526
12:00-12:30	7	738	22	508	22	620	26	613	19	624	19	583	10	506
12:30-13:00	8	667	18	542	22	500	22	584	22	599	20	494	9	632
13:00-13:30	9	516	22	492	22	592	21	612	25	764	19	516	9	513
13:30-14:00	5	523	19	499	18	610	19	608	19	557	18	675	9	406
14:00-14:30	5	547	21	587	16	629	21	653	19	585	17	492	7	533
14:30-15:00	5	433	14	611	18	717	20	674	16	685	17	653	7	528
15:00-15:30	4	560	18	715	18	640	19	519	17	601	14	575	7	407
15:30-16:00	5	506	13	537	17	572	17	608	15	706	14	515	6	397
16:00-16:30	5	562	13	505	13	503	9	814	9	538	12	501	6	688
16:30-17:00	3	642	10	575	7	532	10	729	7	794	9	494	5	534
17:00-17:30	5	591	7	864	7	674	10	637	10	822	7	648	3	852
17:30-18:00	3	842	10	503	8	646	8	590	7	633	6	602	4	596
18:00-18:30	3	440	3	651	4	867	5	782	6	646	7	373	3	555
18:30-19:00	2	848	7	405	7	814	5	553	5	408	7	799	3	652
19:00-19:30	3	626	3	385	4	506	7	372	5	562	4	649	2	470
19:30-20:00	2	650	4	738	4	637	6	518	7	748	5	480	3	666
20:00-20:30	3	688	4	668	3	636	4	452	4	644	3	429	3	700
20:30-21:00	1	1468	4	590	2	280	4	466	6	546	3	523	4	580
21:00-21:30	1	420	3	448	3	971	3	513	4	469	3	597	2	1181
21:30-22:00	1	598	1	630	1	586	2	657	2	856	3	839	1	960
22:00-22:30	0	0	1	342	1	423	2	594	1	613	1	847	1	472
22:30-23:00	0	0	2	600	1	561	2	938	1	1655	1	1018	1	385
23:00-23:30	1	1464	2	550	0	0	3	745	3	509	1	736	1	1859
23:30-00:00	1	1116	1	1028	1	736	2	848	3	578	0	0	0	0

Table 37: Call Arrivals for a Week for Service Group 3

	Sunday		Monday		Tuesday		Wednesday		Thursday		Friday		Saturday	
Time	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT
00:00-00:30	0	0	1	841	0	0	1	2120	1	1084	1	547	0	0
00:30-01:00	0	0	0	0	0	0	0	0	1	740	0	0	1	212
01:00-01:30	1	982	0	0	0	0	0	0	1	260	1	138	0	0
01:30-02:00	0	0	0	0	0	0	0	0	0	0	1	1532	0	0
02:00-02:30	0	0	0	0	0	0	0	0	0	0	1	242	1	1051
02:30-03:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
03:00-03:30	0	0	1	1459	0	0	0	0	0	0	0	0	0	0
03:30-04:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
04:00-04:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
04:30-05:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
05:00-05:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
05:30-06:00	0	0	0	0	0	0	0	0	0	0	0	0	1	788
06:00-06:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
06:30-07:00	0	0	0	0	0	0	0	0	1	1037	0	0	0	0
07:00-07:30	0	0	1	695	1	660	1	626	0	0	0	0	0	0
07:30-08:00	0	0	0	0	0	0	2	710	1	1337	0	0	0	0
08:00-08:30	0	0	5	663	2	834	4	747	4	1245	3	433	1	782
08:30-09:00	0	0	5	1003	3	836	6	988	5	978	4	751	1	1212
09:00-09:30	0	0	9	837	6	695	11	1023	11	856	5	656	7	408
09:30-10:00	1	1357	15	697	8	705	13	874	12	905	9	701	5	700
10:00-10:30	2	809	15	718	10	761	14	814	11	1039	10	826	7	742
10:30-11:00	2	575	15	911	13	794	17	720	13	834	10	632	8	749
11:00-11:30	4	547	16	903	19	741	14	936	15	874	12	938	8	573
11:30-12:00	6	588	16	805	16	766	18	1117	16	1044	14	750	7	824
12:00-12:30	8	827	16	895	12	731	13	1016	13	978	22	621	10	757
12:30-13:00	8	514	20	890	18	735	16	769	18	750	20	710	11	675
13:00-13:30	8	760	18	579	16	756	17	840	20	818	17	825	9	715
13:30-14:00	9	810	24	733	20	820	18	825	16	808	15	699	12	711
14:00-14:30	8	715	21	786	17	696	17	741	18	768	21	739	10	921
14:30-15:00	7	752	21	870	16	826	16	910	18	745	16	670	11	650
15:00-15:30	7	696	20	702	16	755	20	753	20	903	18	766	7	732
15:30-16:00	6	1080	21	762	15	717	17	784	17	707	16	609	9	721
16:00-16:30	8	800	16	750	13	640	16	692	21	769	15	704	5	670
16:30-17:00	9	454	15	668	13	667	18	756	14	647	16	837	8	678
17:00-17:30	5	984	15	715	13	687	18	806	16	691	15	536	9	612
17:30-18:00	5	947	13	662	12	772	14	761	14	813	15	711	8	685
18:00-18:30	4	841	9	751	14	787	12	745	17	529	16	714	5	808
18:30-19:00	3	1195	13	663	10	718	10	735	15	624	9	635	5	865
19:00-19:30	3	761	10	807	10	651	9	608	12	719	10	741	4	528
19:30-20:00	2	457	14	706	11	618	10	592	12	599	12	669	3	1010
20:00-20:30	1	763	9	762	7	791	11	581	10	513	7	463	4	557
20:30-21:00	0	0	9	651	8	726	9	747	11	601	6	597	1	570
21:00-21:30	0	0	7	619	4	923	4	530	9	551	6	588	1	714
21:30-22:00	0	0	6	662	6	808	5	531	4	547	5	772	0	0
22:00-22:30	0	0	4	560	2	653	2	691	4	500	2	286	0	0
22:30-23:00	0	0	4	578	3	608	4	676	2	943	2	596	0	0
23:00-23:30	0	0	1	621	1	1150	2	915	2	546	1	573	1	407
23:30-00:00	0	0	2	1062	1	380	1	516	2	597	0	0	0	0

Table 38: Call Arrivals for a Week for Service Group 4

	Sunday		Monday		Tuesday		Wednesday		Thursday		Friday		Saturday	
Time	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT
00:00-00:30	1	1065	2	457	0	0	1	629	0	0	1	1178	0	0
00:30-01:00	0	0	1	509	0	0	0	0	1	1985	0	0	0	0
01:00-01:30	0	0	1	320	0	0	0	0	1	943	1	1705	1	431
01:30-02:00	0	0	0	0	0	0	0	0	1	260	1	255	0	0
02:00-02:30	0	0	0	0	1	361	0	0	0	0	0	0	0	0
02:30-03:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
03:00-03:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
03:30-04:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
04:00-04:30	0	0	0	0	0	0	0	0	0	0	0	0	1	235
04:30-05:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
05:00-05:30	1	422	1	545	0	0	2	345	1	680	0	0	1	1022
05:30-06:00	1	369	3	388	2	394	1	451	2	359	2	387	2	251
06:00-06:30	8	410	8	623	6	510	8	532	9	569	7	462	8	367
06:30-07:00	7	576	9	813	7	685	6	811	6	857	8	674	7	473
07:00-07:30	9	537	10	829	8	640	10	862	10	789	11	590	10	498
07:30-08:00	7	697	9	791	9	557	9	934	9	872	9	803	7	703
08:00-08:30	7	925	9	925	10	599	11	807	11	753	10	808	11	643
08:30-09:00	7	967	10	945	10	817	10	786	10	872	9	903	8	891
09:00-09:30	6	974	8	742	9	738	10	720	6	1069	9	923	10	944
09:30-10:00	6	931	9	814	10	835	10	933	7	1142	7	827	7	909
10:00-10:30	7	1147	8	1004	8	661	11	688	7	755	6	890	7	717
10:30-11:00	5	871	10	778	8	968	8	843	9	838	8	930	7	732
11:00-11:30	4	1087	8	781	7	677	8	687	7	1014	9	629	7	704
11:30-12:00	5	1019	6	916	8	916	7	630	6	891	8	728	8	639
12:00-12:30	4	733	5	747	7	817	6	862	4	659	7	746	6	889
12:30-13:00	5	675	6	757	8	618	5	713	7	832	6	799	5	730
13:00-13:30	4	812	5	765	7	629	6	848	4	957	3	1042	5	1053
13:30-14:00	4	811	5	924	6	739	6	908	7	734	6	633	4	761
14:00-14:30	4	516	5	875	6	814	6	446	6	681	8	757	4	1039
14:30-15:00	4	605	5	764	5	793	7	836	8	653	6	599	4	916
15:00-15:30	6	805	5	721	6	987	6	732	4	848	6	815	5	1146
15:30-16:00	5	614	7	1057	6	702	4	718	7	862	6	626	5	618
16:00-16:30	3	825	5	689	5	988	3	943	11	553	9	687	6	770
16:30-17:00	2	883	4	697	4	577	4	828	7	647	9	684	5	519
17:00-17:30	4	996	4	899	4	739	4	888	5	616	6	657	6	721
17:30-18:00	3	790	3	971	6	862	4	1229	4	760	5	870	5	732
18:00-18:30	3	740	4	1153	4	1039	4	903	4	636	7	637	5	1063
18:30-19:00	3	593	2	639	2	836	2	647	2	663	4	977	2	1199
19:00-19:30	2	1105	3	1121	4	1070	3	809	5	677	2	914	2	1262
19:30-20:00	4	1416	5	1173	3	609	3	907	3	1105	6	1083	3	994
20:00-20:30	5	1003	2	1036	3	1144	2	1139	4	736	5	841	2	1463
20:30-21:00	2	1345	2	1218	3	1224	2	787	3	1157	4	701	3	1140
21:00-21:30	2	1361	2	667	2	1126	3	906	3	815	4	1268	2	960
21:30-22:00	3	818	1	1033	3	927	2	912	4	731	3	1306	3	1088
22:00-22:30	2	1109	2	1402	3	897	1	808	1	913	2	935	3	613
22:30-23:00	3	1781	2	1437	2	743	1	781	2	1085	1	1390	2	976
23:00-23:30	1	792	2	859	1	1286	1	722	2	296	2	2355	2	671
23:30-00:00	1	1037	1	484	2	663	1	2915	1	404	1	638	1	810

Table 39: Call Arrivals for a Week for Service Group 5

	Sunday		Monday		Tuesday		Wednesday		Thursday		Friday		Saturday	
Time	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT
00:00-00:30	0	0	1	697	0	0	0	0	0	0	1	904	1	843
00:30-01:00	1	678	1	290	1	913	0	0	1	295	1	776	1	684
01:00-01:30	0	0	0	0	0	0	0	0	1	752	0	0	0	0
01:30-02:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
02:00-02:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
02:30-03:00	0	0	0	0	0	0	0	0	1	127	0	0	0	0
03:00-03:30	0	0	0	0	0	0	0	0	1	252	0	0	0	0
03:30-04:00	0	0	0	0	1	136	0	0	0	0	0	0	0	0
04:00-04:30	0	0	0	0	0	0	1	1144	1	182	0	0	0	0
04:30-05:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
05:00-05:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
05:30-06:00	1	351	1	249	1	786	0	0	0	0	0	0	0	0
06:00-06:30	0	0	0	0	2	387	2	618	0	0	1	291	1	913
06:30-07:00	1	367	2	661	3	503	1	608	2	412	1	295	0	0
07:00-07:30	1	473	3	569	5	357	3	486	3	287	3	554	1	221
07:30-08:00	1	188	6	486	9	393	8	378	4	577	4	719	2	463
08:00-08:30	3	251	13	444	14	460	14	421	14	685	10	486	3	646
08:30-09:00	3	331	14	578	16	476	18	562	13	544	13	660	1	656
09:00-09:30	2	288	12	725	19	467	17	552	14	682	16	539	3	307
09:30-10:00	2	273	17	604	18	656	24	472	16	637	16	585	2	778
10:00-10:30	3	271	18	689	18	611	18	573	18	647	16	595	4	346
10:30-11:00	3	295	22	555	20	587	19	594	18	565	16	698	1	531
11:00-11:30	3	308	17	572	22	479	20	592	17	682	22	636	2	671
11:30-12:00	4	562	20	625	18	617	22	676	19	605	17	624	3	1021
12:00-12:30	3	281	17	616	19	753	22	572	16	703	14	702	3	660
12:30-13:00	5	534	21	573	17	626	21	584	19	722	18	576	3	519
13:00-13:30	3	437	18	769	16	592	19	575	17	761	16	588	5	713
13:30-14:00	4	569	14	696	15	624	20	612	24	522	16	601	3	564
14:00-14:30	3	530	14	697	16	627	20	708	15	552	16	598	3	544
14:30-15:00	1	512	16	725	15	533	18	753	15	683	13	519	3	771
15:00-15:30	1	342	16	697	17	682	23	610	21	511	14	806	6	489
15:30-16:00	2	435	15	663	13	662	20	630	10	664	13	691	5	578
16:00-16:30	1	334	10	685	10	724	14	641	11	620	11	675	4	433
16:30-17:00	2	821	11	623	11	553	13	577	11	664	9	561	5	291
17:00-17:30	2	395	10	517	8	647	8	570	11	444	10	553	3	561
17:30-18:00	1	434	9	708	8	535	7	647	7	575	6	712	2	552
18:00-18:30	1	868	6	432	5	601	8	650	8	479	6	390	2	552
18:30-19:00	1	828	5	590	5	809	7	663	7	725	4	684	3	798
19:00-19:30	1	523	4	637	6	689	4	431	5	839	3	707	2	490
19:30-20:00	1	555	5	577	3	619	5	656	5	657	2	321	0	0
20:00-20:30	1	921	3	771	3	1021	4	587	3	692	2	584	1	218
20:30-21:00	1	290	3	693	2	604	3	369	2	813	2	797	0	0
21:00-21:30	1	200	3	586	2	499	2	704	3	459	2	440	1	508
21:30-22:00	1	368	1	460	1	940	1	289	1	832	1	684	0	0
22:00-22:30	0	0	1	1482	0	0	1	334	1	166	1	713	1	758
22:30-23:00	0	0	1	481	0	0	0	0	2	389	1	961	1	494
23:00-23:30	0	0	0	0	1	922	1	2124	1	392	1	694	1	284
23:30-00:00	1	372	1	602	1	238	1	367	0	0	0	0	0	0

Table 40: Call Arrivals for a Week for Service Group 6

	Sunday		Monday		Tuesday		Wednesday		Thursday		Friday		Saturday	
Time	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT
00:00-00:30	1	585	1	263	1	243	1	230	1	637	1	315	1	314
00:30-01:00	0	0	2	674	1	1089	1	470	1	54	1	618	0	0
01:00-01:30	1	463	1	573	1	113	0	0	2	312	0	0	1	230
01:30-02:00	0	0	1	263	1	177	1	184	1	144	1	152	1	210
02:00-02:30	1	137	1	70	0	0	1	244	1	347	1	291	1	321
02:30-03:00	1	745	1	507	0	0	0	0	0	0	1	307	1	191
03:00-03:30	1	316	1	132	0	0	1	308	0	0	1	170	1	192
03:30-04:00	1	119	1	114	0	0	0	0	0	0	1	100	0	0
04:00-04:30	1	451	0	0	1	203	1	461	0	0	1	659	1	145
04:30-05:00	1	434	0	0	1	563	1	157	1	1234	1	354	0	0
05:00-05:30	0	0	1	284	1	241	0	0	2	278	1	436	2	1090
05:30-06:00	2	443	0	0	1	690	0	0	2	881	1	231	2	888
06:00-06:30	2	373	1	263	2	223	2	351	1	382	2	127	1	432
06:30-07:00	2	305	1	305	1	220	2	492	3	213	2	249	1	191
07:00-07:30	2	187	3	320	5	378	4	204	3	265	2	119	5	280
07:30-08:00	3	179	4	186	5	289	4	143	2	428	5	135	5	170
08:00-08:30	4	215	6	344	6	286	6	530	5	285	5	342	8	318
08:30-09:00	5	113	9	397	6	299	7	293	7	234	7	243	9	377
09:00-09:30	6	306	5	199	9	197	8	294	6	312	8	161	11	332
09:30-10:00	7	172	9	310	6	289	6	404	10	290	7	243	13	354
10:00-10:30	5	267	8	315	11	230	10	176	9	203	10	273	7	415
10:30-11:00	5	265	9	411	10	196	9	291	11	305	12	212	4	158
11:00-11:30	5	196	12	244	11	240	10	237	10	259	13	187	6	268
11:30-12:00	6	220	11	187	11	273	9	245	6	178	8	136	5	131
12:00-12:30	6	154	12	183	11	299	12	226	13	366	11	310	4	223
12:30-13:00	5	185	10	173	12	194	12	323	12	269	11	246	4	129
13:00-13:30	6	180	9	255	9	217	11	313	12	314	9	225	7	274
13:30-14:00	6	182	12	177	11	257	10	297	10	244	7	239	6	281
14:00-14:30	2	328	8	302	10	202	11	299	9	264	10	214	5	378
14:30-15:00	4	317	10	237	10	400	12	240	9	269	9	264	6	292
15:00-15:30	7	231	12	243	10	229	10	258	8	196	9	260	6	287
15:30-16:00	4	323	9	340	9	246	7	252	8	224	10	287	5	327
16:00-16:30	5	196	8	231	7	223	8	311	9	254	10	227	5	196
16:30-17:00	3	127	7	305	9	206	7	142	7	223	7	288	3	248
17:00-17:30	4	329	7	428	6	286	7	451	9	190	6	161	3	489
17:30-18:00	3	511	7	264	4	244	6	195	6	344	7	145	4	398
18:00-18:30	4	202	5	384	6	284	5	140	4	703	5	226	2	218
18:30-19:00	2	481	4	218	5	362	5	177	4	321	4	184	3	608
19:00-19:30	2	142	5	309	4	484	5	188	4	269	2	273	3	110
19:30-20:00	2	453	8	286	3	175	5	166	2	176	6	143	4	438
20:00-20:30	2	214	4	384	3	343	3	302	1	261	4	166	3	276
20:30-21:00	1	284	3	423	2	221	4	162	3	263	2	329	2	322
21:00-21:30	2	340	3	187	2	265	3	466	3	994	4	206	1	192
21:30-22:00	2	510	3	281	2	304	2	219	2	404	1	355	1	88
22:00-22:30	2	570	3	349	2	415	1	320	2	705	1	777	2	186
22:30-23:00	1	428	1	693	0	0	2	724	2	833	1	301	1	164
23:00-23:30	0	0	1	346	1	759	1	953	1	986	1	1335	1	239
23:30-00:00	1	71	1	277	2	1460	0	0	0	0	1	871	1	631

Table 41: Call Arrivals for a Week for Service Group 7

	Sunday		Monday		Tuesday		Wednesday		Thursday		Friday		Saturday	
Time	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT
00:00-00:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
00:30-01:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
01:00-01:30	0	0	0	0	0	0	0	0	1	191	0	0	0	0
01:30-02:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
02:00-02:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
02:30-03:00	0	0	1	231	0	0	0	0	0	0	0	0	0	0
03:00-03:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
03:30-04:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
04:00-04:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
04:30-05:00	0	0	0	0	1	249	0	0	0	0	0	0	0	0
05:00-05:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
05:30-06:00	0	0	0	0	1	10490	1	458	0	0	0	0	0	0
06:00-06:30	1	644	0	0	1	131	1	455	0	0	1	291	0	0
06:30-07:00	1	530	1	684	1	610	1	1370	0	0	0	0	0	0
07:00-07:30	2	481	3	788	2	798	2	435	1	398	1	347	0	0
07:30-08:00	3	542	5	449	2	483	2	456	3	492	2	352	0	0
08:00-08:30	5	580	4	516	3	559	4	492	4	668	2	543	1	408
08:30-09:00	5	617	8	579	6	505	4	301	5	317	3	632	1	353
09:00-09:30	7	473	6	651	8	612	5	560	6	445	4	675	3	306
09:30-10:00	5	678	6	529	6	614	6	470	6	398	6	658	4	603
10:00-10:30	6	699	8	613	10	638	9	611	6	519	7	692	3	479
10:30-11:00	8	613	10	706	8	613	7	561	8	538	7	487	5	409
11:00-11:30	5	594	10	488	7	389	11	595	5	523	8	615	5	476
11:30-12:00	5	789	10	629	7	535	10	524	7	629	7	439	4	448
12:00-12:30	3	493	9	627	8	557	8	556	6	524	7	479	5	516
12:30-13:00	2	454	12	600	7	678	7	600	8	403	7	687	4	522
13:00-13:30	5	529	8	596	8	443	8	536	6	594	6	491	5	527
13:30-14:00	4	790	8	710	7	528	6	598	6	505	8	540	4	516
14:00-14:30	4	880	9	666	8	627	7	774	8	632	10	574	4	575
14:30-15:00	5	487	7	463	5	522	8	564	8	647	6	613	2	418
15:00-15:30	2	301	8	456	7	823	4	825	7	674	5	596	4	924
15:30-16:00	2	556	6	724	5	662	5	702	5	696	7	693	2	789
16:00-16:30	2	453	7	707	5	532	4	737	5	724	5	569	3	656
16:30-17:00	0	0	3	536	4	515	3	765	5	577	3	660	2	896
17:00-17:30	1	693	3	551	2	558	3	552	6	579	4	600	2	619
17:30-18:00	1	561	5	574	3	469	4	770	4	589	3	631	1	242
18:00-18:30	2	481	3	841	1	616	1	795	4	851	3	473	2	1003
18:30-19:00	0	0	3	880	1	723	2	519	3	744	2	443	2	427
19:00-19:30	1	444	3	835	0	0	1	621	2	594	2	729	1	585
19:30-20:00	1	711	2	519	1	620	1	800	1	513	2	644	0	0
20:00-20:30	0	0	1	686	1	498	1	837	1	207	1	593	1	285
20:30-21:00	0	0	1	541	1	402	1	979	1	224	1	559	1	513
21:00-21:30	0	0	1	256	1	894	1	451	1	387	1	741	1	332
21:30-22:00	1	859	1	636	0	0	0	0	1	758	1	383	1	516
22:00-22:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22:30-23:00	0	0	0	0	0	0	0	0	0	0	1	685	0	0
23:00-23:30	0	0	0	0	0	0	0	0	1	174	1	577	0	0
23:30-00:00	0	0	1	428	0	0	0	0	0	0	0	0	0	0

Table 42: Call Arrivals for a Week for Service Group 8

	Sunday		Monday		Tuesday		Wednesday		Thursday		Friday		Saturday	
Time	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT
00:00-00:30	0	0	1	414	0	0	1	642	1	127	1	194	0	0
00:30-01:00	0	0	0	0	0	0	0	0	0	0	1	356	0	0
01:00-01:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
01:30-02:00	0	0	0	0	0	0	0	0	0	0	1	313	0	0
02:00-02:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
02:30-03:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
03:00-03:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
03:30-04:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
04:00-04:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
04:30-05:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
05:00-05:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
05:30-06:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0
06:00-06:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
06:30-07:00	0	0	0	0	0	0	0	0	1	709	0	0	0	0
07:00-07:30	0	0	0	0	1	621	0	0	0	0	0	0	0	0
07:30-08:00	0	0	1	562	0	0	0	0	0	0	1	491	0	0
08:00-08:30	0	0	1	575	2	372	2	723	2	525	2	446	1	118
08:30-09:00	0	0	0	0	2	569	3	683	2	446	3	554	0	0
09:00-09:30	0	0	5	740	5	578	4	592	2	670	4	459	2	306
09:30-10:00	1	579	7	532	6	879	7	615	6	829	5	727	3	527
10:00-10:30	2	698	7	785	5	563	5	585	3	636	4	473	4	688
10:30-11:00	2	692	7	828	7	696	6	681	6	1151	5	643	3	688
11:00-11:30	4	708	4	799	10	391	6	864	4	611	8	607	4	804
11:30-12:00	3	812	6	763	6	480	6	600	7	843	5	711	4	605
12:00-12:30	3	654	5	799	6	825	6	582	4	983	7	707	3	687
12:30-13:00	5	845	6	740	5	455	5	509	4	815	7	522	2	863
13:00-13:30	4	771	5	644	5	672	4	653	5	580	7	582	5	734
13:30-14:00	3	1011	6	776	7	520	7	682	5	570	6	534	2	370
14:00-14:30	5	777	5	627	5	645	6	629	6	756	5	651	4	527
14:30-15:00	4	560	6	630	5	918	6	608	6	657	7	1033	2	644
15:00-15:30	4	687	11	636	6	786	5	939	8	612	6	611	5	753
15:30-16:00	5	584	7	621	6	766	7	747	5	764	5	639	3	1028
16:00-16:30	4	452	6	656	6	833	7	627	4	633	6	649	3	492
16:30-17:00	3	1014	6	544	4	655	6	607	6	530	4	565	3	644
17:00-17:30	1	1069	5	483	6	660	5	509	5	729	6	694	2	499
17:30-18:00	1	700	5	492	4	468	3	589	3	577	7	528	2	568
18:00-18:30	1	1334	5	848	4	712	3	432	4	1048	4	611	1	577
18:30-19:00	2	813	3	1019	4	588	4	422	3	475	3	896	1	1136
19:00-19:30	1	482	2	511	2	459	3	665	3	479	3	906	1	836
19:30-20:00	0	0	3	532	2	559	4	652	2	585	4	495	1	587
20:00-20:30	1	665	3	640	2	896	3	506	1	211	3	441	1	492
20:30-21:00	1	484	2	653	2	473	3	757	3	290	3	667	1	580
21:00-21:30	0	0	2	1133	1	796	1	712	2	531	2	442	1	968
21:30-22:00	0	0	2	611	2	819	1	804	1	470	2	992	2	518
22:00-22:30	0	0	1	394	1	533	0	0	0	0	1	432	2	576
22:30-23:00	0	0	1	410	0	0	0	0	1	508	1	441	1	375
23:00-23:30	0	0	0	0	0	0	1	1275	1	2574	1	333	1	174
23:30-00:00	0	0	0	0	0	0	0	0	0	0	0	0	1	311

Table 43: Call Arrivals for a Week for Service Group 9

	Sunday		Monday		Tuesday		Wednesday		Thursday		Friday		Saturday	
Time	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT	C	AHT
00:00-00:30	0	0	0	0	0	0	0	0	0	0	0	0	1	499
00:30-01:00	0	0	0	0	0	0	0	0	0	0	0	0	1	291
01:00-01:30	0	0	0	0	0	0	0	0	1	258	0	0	0	0
01:30-02:00	1	181	0	0	0	0	0	0	0	0	0	0	0	0
02:00-02:30	1	302	0	0	0	0	0	0	0	0	0	0	0	0
02:30-03:00	1	219	0	0	0	0	0	0	0	0	1	469	0	0
03:00-03:30	1	346	0	0	0	0	0	0	0	0	0	0	0	0
03:30-04:00	0	0	0	0	0	0	0	0	0	0	1	387	0	0
04:00-04:30	0	0	0	0	0	0	0	0	1	568	0	0	0	0
04:30-05:00	1	182	0	0	0	0	0	0	1	361	0	0	1	445
05:00-05:30	1	370	1	218	1	338	0	0	1	278	1	432	1	443
05:30-06:00	0	0	0	0	0	0	0	0	1	186	1	388	0	0
06:00-06:30	0	0	1	466	1	1120	2	1061	1	1112	1	536	0	0
06:30-07:00	0	0	1	562	2	749	1	398	1	770	2	616	0	0
07:00-07:30	0	0	1	955	2	635	2	383	3	701	3	361	2	614
07:30-08:00	1	470	2	394	4	576	3	898	4	515	4	530	2	588
08:00-08:30	1	329	2	664	4	600	3	796	5	572	5	481	2	584
08:30-09:00	1	740	3	413	6	699	5	673	7	628	3	548	1	967
09:00-09:30	1	201	2	426	4	487	6	627	7	778	4	492	2	339
09:30-10:00	2	449	4	541	5	632	5	481	6	678	7	423	3	885
10:00-10:30	1	545	4	294	5	759	4	722	4	595	5	467	3	656
10:30-11:00	1	634	3	541	6	626	4	635	7	711	4	619	2	615
11:00-11:30	1	475	5	723	5	586	6	558	6	516	5	523	2	551
11:30-12:00	1	614	3	731	4	563	5	418	4	603	6	492	2	709
12:00-12:30	2	403	3	549	6	431	7	703	5	801	6	591	2	858
12:30-13:00	2	528	4	493	4	593	4	489	2	543	4	553	2	156
13:00-13:30	2	765	4	599	4	542	5	635	9	709	5	676	2	578
13:30-14:00	2	316	5	770	4	750	5	557	5	665	4	380	2	368
14:00-14:30	1	385	2	591	4	374	5	674	4	496	3	348	1	475
14:30-15:00	2	530	3	438	4	559	5	703	3	566	4	824	3	377
15:00-15:30	1	557	4	626	3	602	3	682	3	613	3	607	1	448
15:30-16:00	1	562	3	565	2	774	4	386	5	564	2	829	1	446
16:00-16:30	2	1033	2	719	2	481	3	609	3	507	2	972	1	717
16:30-17:00	0	0	3	1441	3	1442	2	727	2	471	1	786	0	0
17:00-17:30	1	1406	1	766	2	654	1	512	3	354	1	1137	1	290
17:30-18:00	0	0	0	0	1	408	2	580	2	338	2	566	1	1633
18:00-18:30	0	0	2	684	0	0	1	681	2	768	2	626	1	774
18:30-19:00	0	0	1	326	1	373	2	435	2	677	1	854	1	494
19:00-19:30	1	467	1	525	0	0	1	633	1	474	1	158	1	690
19:30-20:00	1	549	1	349	1	633	1	644	1	1012	0	0	1	475
20:00-20:30	0	0	0	0	0	0	1	824	1	649	1	1404	0	0
20:30-21:00	0	0	0	0	1	632	0	0	0	0	0	0	0	0
21:00-21:30	1	317	1	334	0	0	0	0	1	521	0	0	1	230
21:30-22:00	0	0	0	0	0	0	1	854	0	0	1	1403	0	0
22:00-22:30	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22:30-23:00	0	0	1	841	0	0	1	422	0	0	0	0	1	571
23:00-23:30	0	0	0	0	0	0	0	0	0	0	0	0	1	1003
23:30-00:00	0	0	0	0	0	0	0	0	0	0	0	0	0	0

APPENDIX B: Service Groups and Daily Demand Profiles

Table 44: Service Groups

Service Group	Name
1	POS - - Point of sale
2	ISN - - In-store network
3	EPRN - - Pharmacy applications - I
4	FUEL - - Fuel center
5	DESKTOP - - Laptop and desktop computers
6	DEFAULT - - Anything not covered
7	STORE - - Store applications
8	NDC - - Pharmacy applications - II
9	SUPPLY - - Supply chain

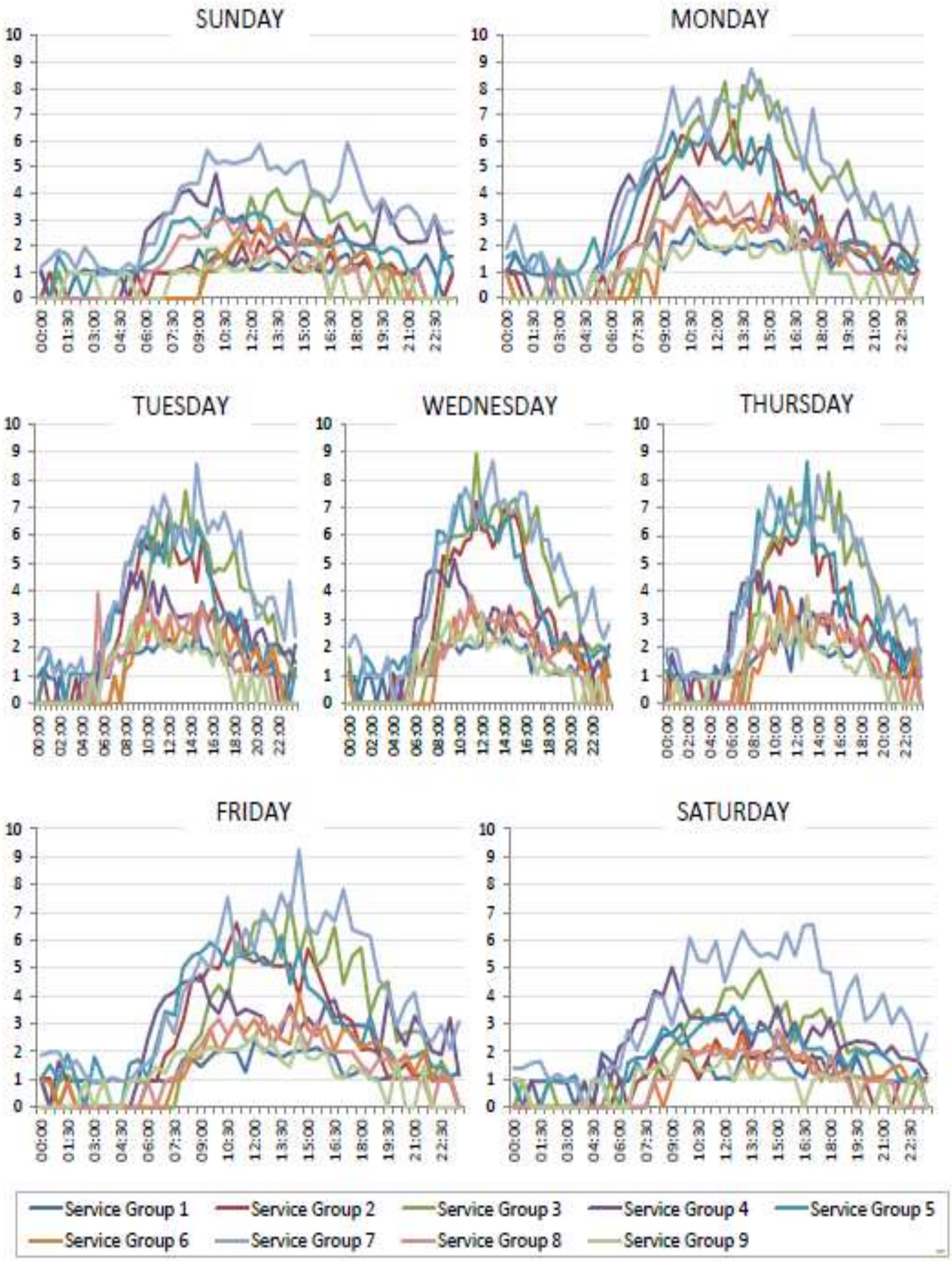

Figure 25: Daily Demand Profiles for Service Groups

APPENDIX C: Daily Coverage of Shifts

Figure 26 presents the coverage of all 106 different shifts employed in this study for a day. In the figure, the horizontal axis represents 48 half-hour time periods in a day with 1 representing 12:00 a.m. and 48 representing 11:30 p.m. The red lines represent 16 full-time shifts, each 8.5 hours long and including a half-hour lunch break, starting at the beginning of each hour. The blue lines represent 14 extended shifts, each 10.5 hours long and including a half-hour lunch break, starting at the beginning of each hour. The green lines represent 76 part-time shifts: 21 shifts with a length of 4 hours, 20 shifts with a length of 5 hours, 18 shifts with a length of 6.5 hours including a half-hour lunch break, and 17 shifts with a length of 7.5 hours including a half-hour lunch break. These proposed shifts overlap, thus creating more alternatives for shift scheduling, especially between 4 a.m. and 8 p.m., when the demand placed on service groups is at its highest.

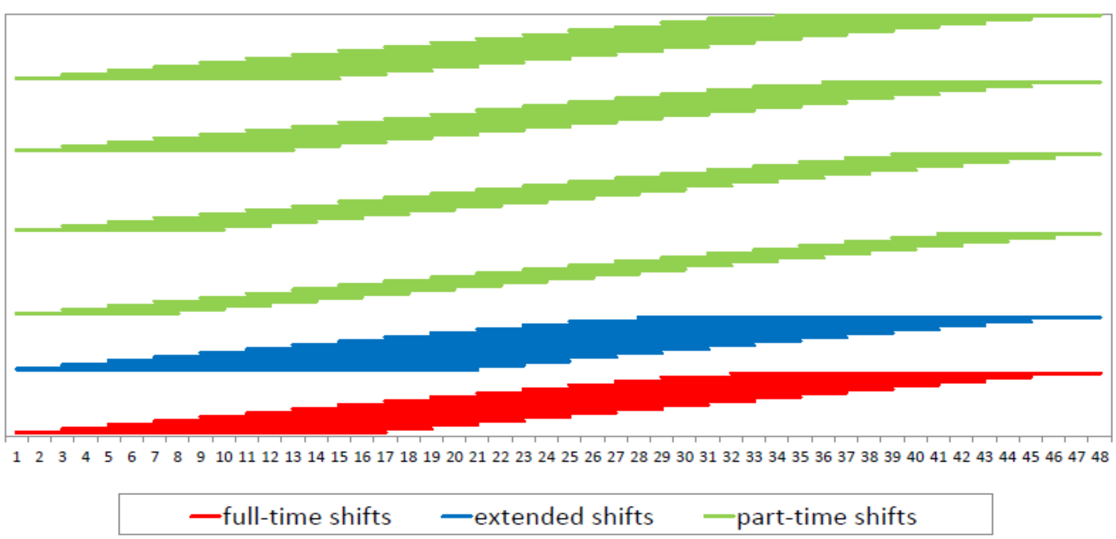

Figure 26: Coverage of All Shift Types for a Day

APPENDIX D: P-I Results of TPSA

Table 45: Results of P-I of TPSA for *MaxC* = 20%

Interval	Skill Sets and Days (S,M,T,W,T,F,S)
1	{(1)(X,O,X,X,X,X,O)}, {(1,2)(X,O,O,X,X,X,X)}, {(1,3)(X,X,O,O,X,X,X)}, {(1,9)(X,X,X,O,O,X,X)}, {(2)(X,X,X,X,X,O,O)}, {(2,5)(O,X,X,O,X,X,X)}, {(4,6)(X,X,X,O,X,X,O)}, {(6)(O,X,O,X,X,X,X)}
2	{(1)(X,X,X,X,X,X,X)}, {(1,9)(X,O,O,X,X,X,X)}, {(2)(X,X,X,X,X,X,X)}, {(2,3)(O,X,O,X,X,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(5)(X,X,X,X,X,X,X)}, {(6)(X,X,X,X,X,X,X)}, {(7)(X,X,X,X,X,X,O)}, {(9)(O,X,X,X,X,X,O)}
3	{(1)(X,X,X,X,X,X,X)}, {(1,2)(O,X,X,X,O,X,X)}, {(1,6)(O,X,O,X,X,X,X)}, {(1,7)(O,X,X,X,X,O,X)}, {(2)(X,X,X,X,X,X,X)}, {(3)(X,X,X,X,X,X,X)}, {(3,4)(O,X,X,X,O,X,X)}, {(3,6)(O,O,X,X,X,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(4,5)(X,X,O,X,X,X,O)}, {(4,8)(X,O,X,O,X,X,X)}, {(5)(X,X,X,X,X,X,X)}, {(6)(X,X,X,X,X,X,X)}, {(7)(X,X,X,X,X,X,X)}, {(8)(X,X,X,X,X,X,X)}, {(9)(X,X,X,X,X,X,X)}
4	{(1)(X,X,X,X,X,X,X)}, {(1,3)(X,X,X,O,X,X,O)}, {(1,4)(O,X,O,X,X,X,X)}, {(1,9)(O,X,X,X,O,X,X)}, {(2)(X,X,X,X,X,X,X)}, {(2,6)(X,O,X,O,X,X,X)}, {(2,7)(X,X,X,O,X,O,X)}, {(2,8)(X,X,X,X,X,O,O)}, {(3)(X,X,X,X,X,X,X)}, {(3,5)(X,X,X,X,X,O,O)}, {(3,8)(O,X,X,X,O,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(5)(X,X,X,X,X,X,X)}, {(5,6)(O,O,X,X,X,X,X)}, {(6)(X,X,X,X,X,X,X)}, {(7)(X,X,X,X,X,X,X)}, {(7,9)(X,X,X,O,X,X,O)}, {(8)(X,X,X,X,X,X,X)}, {(9)(X,X,X,X,X,X,X)}
5	{(1)(X,X,X,X,X,X,X)}, {(1,3)(X,X,X,O,X,X,O)}, {(1,4)(X,X,X,O,O,X,X)}, {(1,9)(X,X,X,X,O,O,X)}, {(2)(X,X,X,X,X,X,X)}, {(2,9)(O,X,X,O,X,X,X)}, {(3)(X,X,X,X,X,X,X)}, {(3,5)(X,O,X,X,X,X,O)}, {(4)(X,X,X,X,X,X,X)}, {(4,8)(X,X,O,X,X,X,O)}, {(5)(X,X,X,X,X,X,X)}, {(6)(X,X,X,X,X,X,X)}, {(6,7)(X,X,X,X,O,O,X)}, {(7)(O,X,X,X,X,X,X)}, {(8)(X,X,X,X,X,X,X)}, {(9)(O,X,X,X,X,X,O)}
6	{(1)(X,X,X,X,X,X,X)}, {(1,2)(O,O,X,X,X,X,X)}, {(1,8)(X,X,X,O,X,X,O)}, {(2)(O,X,X,X,X,X,O)}, {(2,8)(X,X,O,X,O,X,X)}, {(2,9)(X,X,O,X,X,O,X)}, {(3)(O,X,X,X,X,X,X)}, {(3,6)(O,X,X,X,X,O,X)}, {(4)(2,X,X,X,X,X,X)}, {(4,7)(X,O,X,X,X,X,O)}, {(5)(O,X,X,X,X,O,X)}, {(5,6)(X,X,O,X,X,X,O)}, {(6)(X,O,X,X,X,X,O)}, {(7)(O,X,O,X,X,X,X)}, {(8)(O,X,X,X,X,O,X)}

Table 46: Results of P-I of TPSA for *MaxC* = 30%

Interval	Skill Sets and Days (S,M,T,W,T,F,S)
1	{(1,2)(X,X,X,X,O,O,X)}, {(1,4)(X,O,X,O,X,X,X)}, {(1,8)(O,X,X,X,X,X,O)}, {(1,9)(X,X,O,O,X,X,X)}, {(2,4)(X,X,O,X,X,X,O)}, {(2,7)(O,X,X,X,X,X,O)}, {(3,6)(O,X,O,X,X,X,X)}, {(5,6)(X,O,X,O,X,X,X)}
2	{(1)(X,X,X,X,X,X,X)}, {(1,7)(X,X,X,O,O,X,X)}, {(1,9)(X,O,X,X,X,O,X)}, {(2)(X,X,X,X,X,X,X)}, {(2,3)(O,X,X,X,X,O,X)}, {(4)(X,X,X,X,X,X,X)}, {(4,6)(X,O,X,X,X,O,X)}, {(5,6)(O,X,X,X,X,O,X)}, {(5,7)(X,X,X,X,O,X,O)}, {(6)(X,O,X,X,O,X,X)}, {(7,8)(X,X,X,X,X,O,O)}, {(9)(O,X,X,X,X,X,O)}
3	{(1)(X,X,X,X,X,X,X)}, {(1,4)(X,X,X,O,O,X,X)}, {(1,7)(O,X,X,X,O,X,X)}, {(2)(X,X,X,X,X,X,X)}, {(2,3)(O,X,O,X,X,X,X)}, {(3)(X,X,X,X,X,X,X)}, {(3,4)(X,X,X,X,O,X,O)}, {(3,6)(O,X,X,X,X,O,X)}, {(3,9)(O,O,X,X,X,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(4,5)(X,O,O,X,X,X,X)}, {(4,8)(O,X,X,O,X,X,X)}, {(5)(X,X,X,X,X,X,X)}, {(6)(X,X,X,X,X,X,X)}, {(6,7)(O,O,X,X,X,X,X)}, {(7)(X,X,X,X,X,X,X)}, {(8)(X,X,X,X,X,X,X)}, {(8,9)(X,X,X,X,X,O,O)}, {(9)(X,X,X,X,X,X,X)}
4	{(1)(X,X,X,X,X,X,X)}, {(1,3)(X,X,O,O,X,X,X)}, {(1,5)(O,X,X,X,X,X,O)}, {(1,7)(X,O,X,X,O,X,X)}, {(1,9)(X,O,X,O,X,X,X)}, {(2)(X,X,X,X,X,X,X)}, {(2,4)(O,X,X,X,X,X,O)}, {(2,7)(X,O,X,O,X,X,X)}, {(2,8)(O,O,X,X,X,X,X)}, {(3)(X,X,X,X,X,X,X)}, {(3,4)(X,X,X,O,X,O,X)}, {(3,6)(O,X,X,X,X,X,O)}, {(3,8)(X,X,X,X,O,X,O)}, {(4)(X,X,X,X,X,X,X)}, {(4,5)(X,X,X,X,O,X,O)}, {(5)(X,X,X,X,X,X,X)}, {(5,9)(O,X,X,X,X,X,O)}, {(6)(X,X,X,X,X,X,X)}, {(6,7)(X,X,O,O,X,X,X)}, {(7)(X,X,X,X,X,X,X)}, {(8)(X,X,X,X,X,X,X)}, {(9)(X,X,X,X,X,X,X)}
5	{(1)(X,X,X,X,X,X,X)}, {(1,3)(X,X,X,X,O,O,X)}, {(1,4)(X,X,X,O,X,X,O)}, {(1,9)(X,X,X,O,O,X,X)}, {(2)(X,X,X,X,X,X,X)}, {(2,4)(X,X,O,X,X,X,O)}, {(2,8)(X,X,X,O,O,X,X)}, {(2,9)(O,X,X,X,X,O,X)}, {(3)(X,X,X,X,X,X,X)}, {(3,5)(X,O,X,X,O,X,X)}, {(3,8)(O,X,O,X,X,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(5)(X,X,X,X,X,X,O)}, {(5,7)(O,X,X,O,X,X,X)}, {(6)(X,X,X,X,X,X,X)}, {(6,8)(X,O,X,X,O,X,X)}, {(7)(X,X,X,X,X,X,X)}, {(8)(X,X,X,X,X,X,O)}, {(9)(O,X,X,X,X,X,O)}
6	{(1)(X,X,X,X,X,X,X)}, {(1,2)(X,X,X,X,O,X,O)}, {(1,6)(X,X,O,X,O,X,X)}, {(1,8)(X,O,O,X,X,X,X)}, {(1,9)(X,X,X,X,O,O,X)}, {(2)(X,X,X,X,X,X,X)}, {(3)(O,X,X,X,X,X,O)}, {(3,6)(O,X,X,O,X,X,X)}, {(3,8)(O,X,X,X,X,O,X)}, {(4)(X,X,X,X,X,X,X)}, {(4,5)(X,X,X,O,O,X,X)}, {(5)(O,X,X,X,X,X,O)}, {(6)(X,X,X,X,X,O,O)}, {(7)(O,X,O,X,X,X,X)}, {(8)(O,X,X,X,O,X,X)}

Table 47: Results of P-I of TPSA for *MaxC* = 40%

Interval	Skill Sets and Days (S,M,T,W,T,F,S)
1	{(1,2)(X,O,X,X,O,X,X)}, {(1,5)(X,X,X,O,X,O,X)}, {(1,9)(O,O,X,X,X,X,X)}, {(2,4)(X,X,X,X,O,X,O)}, {(3,6)(X,X,O,X,X,O,X)}, {(4,6)(X,O,X,X,O,X,X)}
2	{(1)(X,X,X,X,X,X,X)}, {(1,9)(O,X,X,X,X,X,O)}, {(2)(X,X,X,X,X,X,X)}, {(2,7)(O,X,X,X,O,X,X)}, {(3,6)(O,X,X,X,X,O,X)}, {(4)(X,X,X,X,X,X,X)}, {(4,6)(O,X,O,X,X,X,X)}, {(4,7)(X,X,X,X,O,X,O)}, {(5,6)(O,X,O,X,X,X,X)}, {(5,7)(X,X,X,X,X,O,O)}, {(9)(O,X,X,X,X,X,O)}
3	{(1)(X,X,X,X,X,X,X)}, {(1,3)(X,X,X,X,X,X,X)}, {(1,4)(O,X,X,X,O,X,X)}, {(1,6)(O,X,X,O,X,X,X)}, {(1,7)(X,X,X,X,O,O,X)}, {(1,8)(X,O,O,X,X,X,X)}, {(2)(X,X,X,X,X,X,X)}, {(2,3)(O,X,X,X,O,X,X)}, {(2,4)(X,O,X,X,X,O,X)}, {(2,6)(X,O,X,X,O,X,X)}, {(3)(O,X,X,X,X,X,X)}, {(3,5)(X,X,O,X,X,O,X)}, {(3,9)(X,O,X,X,X,X,O)}, {(4)(X,X,X,X,X,X,X)}, {(4,5)(X,X,O,O,X,X,X)}, {(4,8)(X,X,X,X,O,O,X)}, {(5)(X,X,X,X,X,X,X)}, {(5,8)(X,X,O,O,X,X,X)}, {(6)(X,X,X,X,X,X,X)}, {(6,7)(O,X,O,X,X,X,X)}, {(6,9)(O,X,X,X,O,X,X)}, {(7)(X,X,X,X,X,X,X)}, {(8)(O,X,X,X,X,X,X)}, {(8,9)(X,X,O,X,X,O,X)}, {(9)(X,X,X,X,X,X,X)}
4	{(1)(X,X,X,X,X,X,X)}, {(1,2)(X,O,X,X,X,O,X)}, {(1,4)(X,X,X,O,X,X,O)}, {(1,7)(X,X,X,X,O,O,X)}, {(1,9)(O,O,X,X,X,X,X)}, {(2)(X,X,X,X,X,X,X)}, {(2,5)(O,O,X,X,X,X,X)}, {(2,6)(X,X,O,X,X,X,O)}, {(2,7)(O,O,X,X,X,X,X)}, {(2,8)(O,X,X,X,X,O,X)}, {(3)(X,X,X,X,X,X,X)}, {(3,4)(X,O,X,X,X,O,X)}, {(3,5)(O,X,X,O,X,X,X)}, {(3,6)(O,X,X,O,X,X,X)}, {(3,7)(X,O,X,O,X,X,X)}, {(3,9)(X,X,O,O,X,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(4,5)(X,X,O,X,X,X,O)}, {(4,6)(X,O,O,X,X,X,X)}, {(5)(X,X,X,X,X,X,X)}, {(5,6)(X,X,X,X,X,O,O)}, {(5,7)(X,X,O,O,X,X,X)}, {(5,9)(X,O,X,X,X,X,O)}, {(6)(X,X,X,X,X,X,X)}, {(6,9)(X,O,X,O,X,X,X)}, {(7)(X,X,X,X,X,X,X)}, {(7,8)(O,X,X,X,X,X,O)}, {(8)(X,X,X,X,X,X,X)}, {(9)(O,X,X,X,X,X,X)}
5	{(1)(X,X,X,X,X,X,X)}, {(1,3)(X,X,O,O,X,X,X)}, {(1,8)(X,X,O,O,X,X,X)}, {(1,9)(X,O,X,X,X,O,X)}, {(2)(X,X,X,X,X,X,X)}, {(2,3)(O,X,X,X,X,X,O)}, {(2,6)(O,O,X,X,X,X,X)}, {(3)(X,X,X,X,X,X,X)}, {(3,5)(X,X,X,O,O,X,X)}, {(3,6)(X,X,X,X,X,O,O)}, {(3,8)(X,X,X,X,O,O,X)}, {(4)(X,X,X,X,X,X,X)}, {(4,5)(O,X,X,X,X,O,X)}, {(4,6)(X,X,X,X,O,O,X)}, {(4,7)(X,X,X,O,O,X,X)}, {(4,8)(X,X,X,X,O,O,X)}, {(4,9)(X,O,X,X,X,X,O)}, {(5)(O,X,X,X,X,X,X)}, {(5,9)(X,X,X,X,O,O,X)}, {(6)(X,X,X,X,X,X,X)}, {(6,7)(X,X,X,X,O,O,X)}, {(7)(X,X,X,X,X,X,X)}, {(7,8)(X,O,X,O,X,X,X)}, {(7,9)(X,O,X,X,X,X,O)}, {(8)(O,X,X,X,X,X,X)}, {(8,9)(X,O,X,X,X,O,X)}
6	{(1)(X,X,X,X,X,X,X)}, {(1,4)(X,X,O,X,O,X,X)}, {(1,7)(X,X,X,X,O,X,O)}, {(1,9)(X,X,X,O,O,X,X)}, {(2)(X,X,X,X,X,O,X)}, {(2,3)(X,X,X,X,X,O,O)}, {(2,4)(X,O,X,X,O,X,X)}, {(2,6)(O,O,X,X,X,X,X)}, {(3)(O,X,X,X,X,X,X)}, {(3,4)(X,X,X,O,O,X,X)}, {(3,6)(X,X,O,X,X,X,O)}, {(3,7)(X,X,X,O,X,X,O)}, {(3,8)(O,X,X,X,X,O,X)}, {(4)(X,X,X,X,X,X,O)}, {(4,5)(X,X,X,X,O,O,X)}, {(4,8)(X,O,X,X,X,O,X)}, {(5)(O,X,X,X,X,X,O)}, {(5,6)(X,X,X,O,X,O,X)}, {(5,7)(O,O,X,X,X,X,X)}, {(6)(O,X,X,X,X,X,O)}, {(6,7)(X,O,X,X,O,X,X)}, {(8)(O,X,X,O,X,X,X)}, {(8,9)(O,X,X,X,O,X,X)}

Table 48: Results of P-I of TPSA for *MaxC* = 50%

Interval	Skill Sets and Days (S,M,T,W,T,F,S)
1	{(1,2)(X,O,X,X,X,O,X)}, {(1,3)(X,X,O,X,X,X,O)}, {(1,4)(X,O,X,X,X,O,X)}, {(1,8)(O,X,X,X,X,X,O)}, {(1,9)(O,O,X,X,X,X,X)}, {(2,4)(X,X,X,O,O,X,X)}, {(2,6)(X,O,X,X,X,X,O)}, {(2,7)(O,X,X,X,X,X,O)}, {(2,9)(O,X,X,O,X,X,X)}, {(4,6)(X,O,X,X,X,O,X)}, {(5,6)(X,X,X,O,X,O,X)}, {(6,9)(X,X,O,X,O,X,X)}
2	{(1)(X,X,X,X,X,X,X)}, {(1,2)(X,X,X,X,O,O,X)}, {(1,3)(X,O,X,O,X,X,X)}, {(1,6)(X,O,X,O,X,X,X)}, {(1,9)(X,O,X,O,X,X,X)}, {(2)(X,X,X,X,X,X,O)}, {(2,5)(O,X,O,X,X,X,X)}, {(2,7)(O,X,X,X,X,X,O)}, {(2,9)(X,X,X,X,O,X,O)}, {(4)(X,X,X,X,X,X,X)}, {(4,6)(O,O,X,X,X,X,X)}, {(5,6)(X,X,X,O,X,O,X)}, {(5,9)(O,X,X,X,X,X,O)}, {(6,7)(X,X,X,O,X,O,X)}, {(6,9)(X,X,O,X,X,X,O)}, {(9)(O,X,X,X,X,X,O)}
3	{(1)(X,X,X,X,X,X,X)}, {(1,2)(X,X,X,O,O,X,X)}, {(1,3)(O,X,X,O,X,X,X)}, {(1,4)(X,X,O,X,X,O,X)}, {(1,5)(X,X,X,O,O,X,X)}, {(1,9)(X,O,X,X,X,X,O)}, {(2)(X,X,X,X,X,X,X)}, {(2,4)(X,O,X,X,X,X,O)}, {(2,6)(X,X,X,X,X,O,O)}, {(2,7)(X,O,X,X,X,X,O)}, {(2,8)(O,X,X,O,X,X,X)}, {(3)(O,X,X,X,X,X,X)}, {(3,4)(X,X,X,X,X,O,O)}, {(3,5)(X,X,O,X,O,X,X)}, {(3,6)(X,X,X,O,X,O,X)}, {(3,7)(X,X,X,X,X,O,O)}, {(3,9)(O,O,X,X,X,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(4,5)(X,X,X,X,X,O,X)}, {(4,8)(O,X,X,X,X,X,O)}, {(4,9)(X,X,X,X,X,O,O)}, {(5)(X,X,X,X,X,X,X)}, {(5,7)(X,X,X,O,X,O,X)}, {(5,8)(O,X,X,X,O,X,X)}, {(6)(X,X,X,O,O,X,X)}, {(6,7)(O,X,X,X,X,X,O)}, {(6,8)(O,X,X,X,X,O,X)}, {(7)(X,X,X,X,X,X,X)}, {(7,9)(O,X,X,X,X,X,O)}, {(8)(O,X,X,X,X,X,X)}, {(8,9)(X,X,X,O,O,X,X)}, {(9)(O,X,X,X,O,X,X)}
4	{(1)(X,X,X,X,X,X,X)}, {(1,2)(X,O,O,X,X,X,X)}, {(1,3)(X,X,X,O,O,X,X)}, {(1,5)(X,O,X,X,X,X,O)}, {(1,6)(O,X,X,X,O,X,X)}, {(1,8)(X,X,X,X,O,O,X)}, {(2)(X,X,X,X,X,X,X)}, {(2,3)(O,X,X,X,X,O,X)}, {(2,4)(X,O,X,X,O,X,X)}, {(2,5)(O,X,X,X,O,X,X)}, {(2,7)(X,X,O,X,X,O,X)}, {(2,8)(O,X,X,X,X,O,X)}, {(2,9)(X,X,X,X,X,O,O)}, {(3)(X,X,X,X,X,X,X)}, {(3,4)(X,O,X,O,X,X,X)}, {(3,7)(X,X,X,O,O,X,X)}, {(3,8)(O,X,X,O,X,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(4,5)(X,O,X,X,X,O,X)}, {(4,6)(X,X,X,X,O,X,O)}, {(4,7)(X,X,O,O,X,X,X)}, {(4,9)(X,O,X,X,O,X,X)}, {(5)(O,X,X,X,X,X,X)}, {(5,6)(O,X,X,X,X,X,O)}, {(5,9)(X,O,O,X,X,X,X)}, {(6)(X,X,X,X,X,X,X)}, {(6,8)(O,X,X,X,X,X,O)}, {(7)(X,X,X,X,X,X,X)}, {(7,8)(X,X,X,O,X,X,O)}, {(7,9)(X,X,X,X,X,O,O)}, {(8)(X,X,X,X,X,X,X)}, {(8,9)(X,O,O,X,X,X,X)}, {(9)(X,X,X,X,X,X,X)}
5	{(1)(X,X,X,X,X,X,X)}, {(1,2)(X,O,X,O,X,X,X)}, {(1,6)(X,X,X,O,X,O,X)}, {(1,7)(X,X,X,X,O,X,O)}, {(2)(X,X,X,X,X,X,X)}, {(2,4)(O,O,X,X,X,X,X)}, {(2,8)(X,O,X,O,X,X,X)}, {(2,9)(X,X,X,X,O,O,X)}, {(3)(X,X,X,X,X,X,X)}, {(3,4)(O,X,X,X,O,X,X)}, {(3,5)(X,X,X,X,X,O,O)}, {(3,6)(X,X,O,O,X,X,X)}, {(3,7)(X,X,O,X,X,X,O)}, {(3,8)(X,O,X,O,X,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(4,5)(X,X,O,X,X,O,X)}, {(4,6)(X,X,X,X,X,O,O)}, {(4,7)(X,X,O,X,X,O,X)}, {(4,9)(X,O,X,X,X,X,O)}, {(5)(O,X,X,X,X,X,X)}, {(5,6)(X,X,X,X,O,O,X)}, {(5,7)(X,X,O,X,X,O,X)}, {(5,8)(O,X,O,X,X,X,X)}, {(5,9)(X,X,O,X,X,X,O)}, {(6)(O,X,X,O,X,X,X)}, {(6,8)(X,O,X,X,O,X,X)}, {(7)(O,X,X,X,X,X,X)}, {(7,8)(X,X,X,X,O,O,X)}, {(8)(O,X,X,X,X,X,X)}
6	{(1)(X,X,X,X,X,X,X)}, {(1,3)(X,X,O,O,X,X,X)}, {(1,4)(X,X,O,O,X,X,X)}, {(1,6)(X,X,X,X,O,X,O)}, {(1,7)(X,X,X,X,O,X,O)}, {(1,8)(X,X,O,X,X,O,X)}, {(1,9)(X,X,X,X,O,X,O)}, {(2,4)(X,O,X,O,X,X,X)}, {(2,5)(O,X,O,X,X,X,X)}, {(2,6)(X,O,X,X,X,X,O)}, {(2,7)(X,X,X,O,X,X,O)}, {(2,8)(X,O,O,X,X,X,X)}, {(3)(O,X,X,X,X,X,O)}, {(3,4)(X,X,X,O,X,O,X)}, {(3,5)(O,X,X,O,X,X,X)}, {(3,8)(O,X,X,X,O,X,X)}, {(3,9)(O,X,X,O,X,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(4,5)(X,X,X,O,O,X,X)}, {(4,6)(X,X,X,O,X,O,X)}, {(5)(O,X,X,X,X,X,O)}, {(6)(O,X,X,X,X,X,O)}, {(7,8)(O,X,X,X,O,X,X)}

Table 49: Results of P-I of TPSA for *MaxC* = 60%

Interval	Skill Sets and Days (S,M,T,W,T,F,S)
1	{(1,4)(X,O,X,X,X,X,O)}, {(1,6)(X,X,X,X,O,O,X)}, {(1,8)(O,X,X,X,X,X,O)}, {(1,9)(X,X,O,O,X,X,X)}, {(2,3)(O,X,X,X,X,X,O)}, {(2,4)(O,X,O,X,X,X,X)}, {(2,5)(X,X,X,O,X,X,O)}, {(3,6)(X,X,O,O,X,X,X)}
2	{(1)(X,X,X,X,X,X,O)}, {(1,2)(X,X,X,X,X,O,O)}, {(1,3)(O,X,X,X,X,O,X)}, {(1,4)(X,X,O,X,O,X,X)}, {(1,7)(X,X,X,X,O,O,X)}, {(1,9)(O,X,X,X,X,X,O)}, {(2)(X,X,X,X,X,X,O)}, {(2,5)(O,X,X,X,X,O,X)}, {(2,8)(O,O,X,X,X,X,X)}, {(4)(O,X,X,X,X,X,O)}, {(4,5)(X,X,X,O,X,O,X)}, {(4,6)(X,O,X,O,X,X,X)}, {(4,9)(X,X,X,X,X,X,X)}, {(5,7)(O,X,X,X,X,X,O)}, {(6,7)(X,X,X,X,X,X,X)}
3	{(1)(X,X,X,X,X,X,X)}, {(1,2)(X,X,O,X,O,X,X)}, {(1,3)(X,X,X,X,X,O,O)}, {(1,5)(X,O,X,X,X,O,X)}, {(1,7)(O,X,X,O,X,X,X)}, {(2)(X,X,X,X,X,X,O)}, {(2,3)(X,X,X,X,X,X,X)}, {(2,4)(O,X,X,X,X,O,X)}, {(2,5)(X,X,O,O,X,X,X)}, {(2,7)(O,X,O,X,X,X,X)}, {(3)(O,X,X,X,X,X,X)}, {(3,6)(O,X,X,O,X,X,X)}, {(3,9)(X,X,X,X,X,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(4,5)(X,X,X,X,O,O,X)}, {(4,6)(X,O,X,X,X,X,O)}, {(4,7)(X,O,X,X,X,O,X)}, {(4,8)(X,X,O,X,O,X,X)}, {(5)(O,X,X,X,X,X,X)}, {(5,6)(O,X,X,X,X,O,X)}, {(5,9)(O,X,X,X,X,X,O)}, {(6,7)(X,X,O,X,O,X,X)}, {(7)(X,X,X,X,X,O,O)}, {(7,8)(X,X,X,X,X,X,O)}, {(8)(O,X,X,X,X,X,X)}, {(8,9)(X,X,O,X,X,X,O)}
4	{(1)(X,X,X,X,X,X,X)}, {(1,4)(X,X,X,O,O,X,X)}, {(1,5)(O,O,X,X,X,X,X)}, {(1,7)(X,X,X,O,X,X,O)}, {(1,8)(X,O,O,X,X,X,X)}, {(2)(O,X,X,X,X,X,X)}, {(2,3)(O,X,O,X,X,X,X)}, {(2,6)(X,O,X,X,O,X,X)}, {(2,7)(X,O,X,X,X,O,X)}, {(2,8)(X,X,O,X,X,O,X)}, {(3)(X,X,X,X,X,X,X)}, {(3,6)(X,X,X,O,X,O,X)}, {(3,7)(X,O,X,X,O,X,X)}, {(3,9)(X,X,X,X,O,O,X)}, {(4)(O,X,X,X,X,O,X)}, {(4,5)(X,X,X,O,X,O,X)}, {(4,6)(X,X,X,X,X,X,O)}, {(4,7)(O,X,O,X,X,X,X)}, {(5)(O,X,X,X,X,X,O)}, {(5,6)(O,X,X,X,X,X,O)}, {(5,9)(X,X,X,X,O,O,X)}, {(6,7)(X,X,X,X,O,X,O)}, {(6,9)(X,X,O,X,X,O,X)}, {(7)(O,X,O,X,X,X,X)}, {(7,9)(O,X,X,O,X,X,X)}, {(8)(X,X,X,X,X,X,X)}, {(8,9)(X,X,X,X,X,X,X)}
5	{(1)(X,X,X,X,X,X,X)}, {(1,4)(X,X,X,X,X,X,X)}, {(1,5)(X,X,X,X,O,X,X)}, {(2)(O,X,X,X,X,X,O)}, {(2,3)(X,X,X,X,X,X,X)}, {(2,4)(X,X,X,X,O,O,X)}, {(2,5)(O,X,X,X,X,X,X)}, {(2,6)(O,X,X,X,X,X,O)}, {(2,9)(X,X,X,X,X,X,X)}, {(3)(X,X,X,X,X,X,O)}, {(3,4)(X,X,X,X,X,X,X)}, {(3,6)(X,O,X,X,X,O,X)}, {(3,9)(O,X,X,X,X,X,X)}, {(4,6)(O,X,X,X,X,X,O)}, {(4,8)(X,X,O,X,X,X,O)}, {(5)(O,X,X,X,X,X,O)}, {(5,8)(X,X,X,O,O,X,X)}, {(6,7)(X,X,X,X,X,X,X)}, {(6,9)(X,X,O,X,X,X,O)}, {(7)(O,X,X,X,X,X,X)}, {(8)(X,X,X,X,X,X,X)}
6	{(1,2)(X,X,X,O,X,X,O)}, {(1,3)(X,X,X,X,X,X,X)}, {(1,4)(X,X,X,X,X,X,X)}, {(1,5)(O,X,O,X,X,X,X)}, {(1,6)(X,X,X,X,X,O,O)}, {(1,7)(X,X,X,O,X,O,X)}, {(2,3)(O,X,X,X,X,X,X)}, {(2,8)(X,X,X,O,O,X,X)}, {(3,7)(O,X,X,X,X,X,O)}, {(3,9)(O,X,X,X,X,X,O)}, {(4,5)(X,X,X,X,X,X,X)}, {(4,8)(O,X,O,X,X,X,X)}, {(4,9)(X,O,O,X,X,X,X)}, {(5,6)(O,X,X,O,X,X,X)}, {(6,7)(O,X,X,X,O,X,X)}, {(6,8)(X,O,X,X,X,X,O)}, {(8,9)(O,X,O,X,X,X,X)}

Table 50: Results of P-I of TPSA for *MaxC* = 70%

Interval	Skill Sets and Days (S,M,T,W,T,F,S)
1	{(1,2)(X,X,O,X,X,O,X)}, {(1,4)(X,X,O,O,X,X,X)}, {(1,8)(O,X,X,X,X,X,O)}, {(1,9)(X,O,X,X,O,X,X)}, {(2,6)(X,X,X,O,X,X,O)}, {(2,7)(O,X,X,X,X,X,O)}, {(3,6)(O,X,O,X,X,X,X)}, {(5,6)(X,O,X,O,X,X,X)}
2	{(1)(X,X,O,X,O,X,X)}, {(1,4)(X,X,X,X,X,X,O)}, {(1,6)(X,X,X,X,X,O,X)}, {(1,7)(O,X,X,X,X,X,O)}, {(1,9)(O,X,X,X,O,X,X)}, {(2,3)(O,X,X,O,X,X,X)}, {(2,5)(X,X,O,X,O,X,X)}, {(2,6)(X,X,X,X,X,X,O)}, {(2,9)(X,X,X,X,X,O,O)}, {(3,7)(X,X,X,X,X,O,O)}, {(4,7)(O,X,X,X,X,X,X)}, {(4,8)(X,X,X,O,X,O,X)}, {(4,9)(O,X,O,X,X,X,X)}, {(5,6)(O,X,X,X,X,X,O)}, {(5,9)(O,X,X,X,X,X,O)}
3	{(1)(X,X,X,X,X,X,X)}, {(1,2)(O,O,X,X,X,X,X)}, {(1,3)(X,X,X,X,X,O,O)}, {(1,4)(X,X,X,O,O,X,X)}, {(1,5)(O,X,X,O,X,X,X)}, {(1,6)(O,X,X,X,X,O,X)}, {(1,7)(X,X,X,O,X,X,X)}, {(2)(O,X,X,X,X,X,X)}, {(2,3)(O,X,X,X,X,X,O)}, {(2,4)(X,X,O,X,X,O,X)}, {(2,5)(X,X,O,X,X,X,O)}, {(2,6)(X,X,X,X,X,O,X)}, {(3)(O,X,X,X,X,O,X)}, {(3,4)(O,X,O,X,X,X,X)}, {(3,5)(X,X,X,X,X,X,X)}, {(3,6)(O,X,O,X,X,X,X)}, {(3,9)(X,X,X,O,X,X,O)}, {(4)(O,X,X,X,X,X,X)}, {(4,5)(X,X,X,O,X,O,X)}, {(4,8)(X,X,X,X,X,X,O)}, {(4,9)(X,O,X,X,X,X,O)}, {(5)(O,X,X,X,X,X,O)}, {(5,7)(X,X,O,X,X,O,X)}, {(5,8)(O,X,X,X,X,X,X)}, {(5,9)(O,X,X,X,X,X,O)}, {(6,8)(X,X,X,O,X,O,X)}, {(6,9)(X,O,X,X,O,X,X)}, {(7)(X,X,X,X,X,O,O)}, {(7,8)(O,X,X,X,X,X,O)}, {(7,9)(O,X,O,X,X,X,X)}, {(8,9)(O,X,X,X,X,O,X)}
4	{(1)(O,X,X,X,X,X,X)}, {(1,2)(X,X,X,X,X,O,O)}, {(1,3)(X,X,X,X,O,X,O)}, {(1,4)(X,X,X,O,O,X,X)}, {(1,5)(X,X,X,O,X,X,O)}, {(1,7)(X,O,X,X,X,O,X)}, {(1,9)(X,X,X,X,X,X,X)}, {(2)(O,X,X,X,X,X,X)}, {(2,5)(O,X,X,X,X,O,X)}, {(2,6)(X,X,X,X,X,X,X)}, {(2,7)(X,X,X,X,X,X,O)}, {(2,8)(X,X,O,X,X,X,O)}, {(2,9)(O,X,O,X,X,X,X)}, {(3)(X,X,X,X,X,X,X)}, {(3,4)(O,X,X,O,X,X,X)}, {(3,5)(X,O,X,X,X,O,X)}, {(3,6)(O,X,X,O,X,X,X)}, {(3,7)(X,X,X,O,O,X,X)}, {(4)(O,X,O,X,X,X,X)}, {(4,6)(X,O,O,X,X,X,X)}, {(4,8)(X,O,X,X,O,X,X)}, {(4,9)(X,X,X,X,X,O,O)}, {(5)(O,X,X,X,X,X,X)}, {(5,6)(O,X,X,O,X,X,X)}, {(5,9)(X,X,X,X,X,O,O)}, {(6,7)(X,X,X,X,O,X,O)}, {(7)(O,X,X,X,X,X,O)}, {(7,9)(X,O,O,X,X,X,X)}, {(8)(1,X,X,X,X,X,X)}, {(8,9)(X,O,X,X,X,O,X)}
5	{(1)(X,X,X,X,X,X,X)}, {(1,2)(X,O,X,O,X,X,X)}, {(1,3)(X,X,O,X,X,X,O)}, {(1,6)(X,X,X,O,X,O,X)}, {(1,7)(X,X,O,X,O,X,X)}, {(1,8)(X,X,X,O,X,X,O)}, {(2)(O,X,X,X,X,X,O)}, {(2,3)(X,X,X,O,X,O,X)}, {(2,5)(X,O,O,X,X,X,X)}, {(2,8)(X,X,X,X,O,O,X)}, {(3)(O,X,X,X,X,X,O)}, {(3,8)(X,X,X,X,X,X,X)}, {(3,9)(O,X,X,X,O,X,X)}, {(4)(X,X,X,X,X,X,X)}, {(4,5)(O,X,X,X,X,X,X)}, {(4,7)(X,X,X,X,X,X,X)}, {(4,8)(X,O,X,X,O,X,X)}, {(4,9)(X,O,X,X,X,X,O)}, {(5,6)(X,X,O,X,X,X,O)}, {(5,9)(X,X,X,X,X,X,X)}, {(6)(O,O,X,X,X,X,X)}, {(6,8)(X,X,X,X,X,O,O)}, {(7,8)(O,X,X,O,X,X,X)}, {(8)(O,X,X,X,X,X,O)}
6	{(1)(X,X,X,X,O,X,X)}, {(1,2)(O,X,X,X,X,O,X)}, {(1,3)(O,X,X,X,X,X,X)}, {(1,4)(X,X,O,X,X,X,O)}, {(1,5)(X,X,O,X,X,X,O)}, {(1,8)(X,X,X,O,X,O,X)}, {(2,4)(X,O,X,X,X,O,X)}, {(2,5)(O,O,X,X,X,X,X)}, {(2,7)(X,X,X,O,X,X,O)}, {(3,4)(O,X,X,X,X,X,O)}, {(3,5)(O,X,X,X,O,X,X)}, {(3,6)(X,X,X,X,X,X,X)}, {(4,6)(X,O,X,X,O,X,X)}, {(4,9)(X,X,X,O,O,X,X)}, {(5,8)(O,X,X,X,X,X,O)}, {(7,8)(O,X,O,X,X,X,X)}, {(7,9)(O,X,O,X,X,X,X)}

Table 51: Results of P-I of TPSA for *MaxC* = 80%

Interval	Skill Sets and Days (S,M,T,W,T,F,S)
1	{(1)(X,X,O,X,X,O,X)}, {(1,6)(X,O,X,X,O,X,X)}, {(1,8)(O,X,X,X,X,X,O)}, {(1,9)(X,X,O,O,X,X,X)}, {(2,4)(X,X,X,O,X,X,O)}, {(2,5)(O,X,X,O,X,X,X)}, {(2,6)(O,X,X,X,X,X,O)}, {(3,5)(X,X,O,O,X,X,X)}, {(4,6)(X,X,O,O,X,X,X)}
2	{(1)(X,X,X,X,X,X,X)}, {(1,2)(X,X,O,X,X,O,X)}, {(1,4)(O,X,O,X,X,X,X)}, {(1,7)(X,X,X,X,O,X,O)}, {(2,4)(O,X,X,X,X,O,X)}, {(2,6)(X,X,X,X,X,X,X)}, {(2,7)(X,X,X,X,X,X,O)}, {(2,8)(X,X,X,X,X,O,O)}, {(3,4)(X,X,X,X,X,X,X)}, {(3,5)(O,X,X,X,X,X,O)}, {(4,5)(X,X,O,X,O,X,X)}, {(4,6)(X,X,X,X,X,X,O)}, {(4,9)(O,X,X,X,X,O,X)}, {(5,6)(O,X,X,X,X,O,X)}, {(7,9)(X,X,X,X,X,X,O)}
3	{(1)(X,X,X,X,X,X,X)}, {(1,2)(O,X,X,X,O,X,X)}, {(1,4)(X,X,X,O,X,X,X)}, {(1,5)(O,X,X,X,X,O,X)}, {(1,6)(O,O,X,X,X,X,X)}, {(1,7)(X,X,O,X,X,X,O)}, {(1,8)(X,O,O,X,X,X,X)}, {(1,9)(X,O,X,X,O,X,X)}, {(2)(X,X,X,X,X,O,X)}, {(2,3)(O,X,X,X,X,X,O)}, {(2,4)(X,X,O,X,X,X,O)}, {(2,5)(O,X,X,X,X,X,O)}, {(2,7)(O,X,X,X,X,X,O)}, {(2,8)(O,X,X,X,X,X,O)}, {(2,9)(O,X,X,O,X,X,X)}, {(3)(O,X,X,X,X,X,X)}, {(3,5)(X,X,X,X,X,X,O)}, {(3,6)(O,X,X,X,X,X,O)}, {(3,7)(X,X,X,X,X,X,X)}, {(3,9)(O,X,X,X,X,O,X)}, {(4)(X,X,O,X,X,O,X)}, {(4,5)(O,X,X,X,O,X,X)}, {(4,6)(X,X,O,X,X,O,X)}, {(4,7)(X,X,X,X,O,X,X)}, {(4,8)(O,X,O,X,X,X,X)}, {(4,9)(X,X,X,O,X,X,O)}, {(5)(O,X,X,X,X,O,X)}, {(5,6)(X,X,X,O,X,X,O)}, {(5,7)(O,X,X,O,X,X,X)}, {(5,8)(O,X,X,X,X,X,O)}, {(5,9)(X,O,X,X,X,X,O)}, {(6,7)(O,X,X,X,O,X,X)}, {(6,8)(X,O,X,X,X,X,O)}, {(6,9)(O,X,O,X,X,X,X)}, {(7,8)(X,X,O,X,X,X,O)}, {(8,9)(O,X,X,X,X,O,X)}
4	{(1)(X,X,X,X,X,X,X)}, {(1,2)(X,X,X,X,O,X,O)}, {(1,3)(O,X,X,X,X,O,X)}, {(1,4)(O,X,X,X,O,X,X)}, {(1,5)(O,X,X,X,X,O,X)}, {(1,7)(X,X,X,X,O,X,O)}, {(1,9)(X,O,X,O,X,X,X)}, {(2)(O,X,X,X,X,X,X)}, {(2,4)(X,X,X,X,O,O,X)}, {(2,5)(X,O,X,X,X,X,O)}, {(2,6)(X,X,O,X,X,X,O)}, {(2,7)(X,X,O,O,X,X,X)}, {(2,8)(O,O,X,X,X,X,X)}, {(2,9)(X,X,X,X,X,X,O)}, {(3)(X,X,X,X,X,X,O)}, {(3,4)(X,O,O,X,X,X,X)}, {(3,7)(O,X,X,X,X,X,O)}, {(3,8)(X,X,X,O,X,X,X)}, {(3,9)(O,X,X,X,O,X,X)}, {(4,5)(X,O,O,X,X,X,X)}, {(4,6)(O,X,X,X,X,X,O)}, {(4,7)(X,X,X,X,X,O,O)}, {(4,8)(X,X,X,O,X,O,X)}, {(4,9)(O,X,X,X,O,X,X)}, {(5)(O,X,X,X,X,X,O)}, {(5,6)(X,X,O,X,X,O,X)}, {(5,9)(O,X,O,X,X,X,X)}, {(6,7)(X,O,X,O,X,X,X)}, {(6,8)(X,X,X,X,X,X,X)}, {(6,9)(O,X,X,X,X,X,O)}, {(7,8)(X,X,X,X,X,X,O)}, {(7,9)(X,X,O,X,X,O,X)}
5	{(1)(X,X,X,X,X,X,X)}, {(1,3)(X,X,X,X,X,X,X)}, {(1,5)(X,X,X,X,X,O,O)}, {(1,6)(O,X,X,X,O,X,X)}, {(1,7)(O,O,X,X,X,X,X)}, {(1,8)(X,X,O,O,X,X,X)}, {(1,9)(X,O,X,X,X,X,O)}, {(2)(O,X,X,X,X,X,O)}, {(2,4)(X,X,X,X,X,X,X)}, {(2,5)(X,X,X,X,X,O,O)}, {(2,8)(X,X,X,X,X,X,X)}, {(3,4)(X,O,X,X,X,X,X)}, {(3,6)(X,X,X,X,X,X,O)}, {(3,9)(O,X,O,X,X,X,X)}, {(4,6)(O,X,O,X,X,X,X)}, {(4,7)(X,X,O,O,X,X,X)}, {(4,8)(X,X,X,X,X,O,O)}, {(4,9)(O,X,X,X,O,X,X)}, {(5,6)(X,X,X,X,X,X,O)}, {(5,7)(X,X,X,X,X,X,X)}, {(5,8)(O,X,X,X,O,X,X)}, {(5,9)(O,X,X,X,X,O,X)}, {(6,8)(X,O,X,X,X,O,X)}, {(7,8)(O,X,X,X,O,X,X)}, {(7,9)(O,X,X,X,X,X,O)}, {(8,9)(X,X,O,X,X,X,O)}
6	{(1,3)(X,X,X,X,X,X,X)}, {(1,4)(X,X,X,X,X,X,X)}, {(1,6)(X,X,X,O,X,X,O)}, {(1,8)(O,X,O,X,X,X,X)}, {(1,9)(X,O,X,O,X,X,X)}, {(2,3)(X,X,O,X,X,O,X)}, {(2,4)(X,X,X,X,X,X,X)}, {(2,6)(O,X,X,X,X,X,O)}, {(3)(O,X,X,X,X,X,O)}, {(3,4)(O,X,X,O,X,X,X)}, {(3,6)(O,O,X,X,X,X,X)}, {(4,5)(O,X,X,X,X,X,O)}, {(4,7)(X,X,O,X,X,X,O)}, {(5,6)(X,X,X,X,O,O,X)}, {(5,7)(O,X,X,O,X,X,X)}, {(5,8)(X,X,X,X,O,X,O)}, {(8,9)(O,X,X,X,X,X,X)}

Table 52: Results of P-I of TPSA for *MaxC* = 90%

Interval	Skill Sets and Days (S,M,T,W,T,F,S)
1	{(1)(X,X,X,X,X,X,O)}, {(1,3)(X,X,O,X,X,X,X)}, {(1,4)(X,O,X,X,X,X,O)}, {(1,5)(X,O,X,O,X,X,X)}, {(1,6)(O,X,X,X,X,X,X)}, {(2,4)(O,X,X,X,X,X,X)}, {(2,9)(X,X,X,O,X,X,X)}, {(6,8)(X,X,O,X,X,X,X)}
2	{(1)(X,X,O,X,X,X,X)}, {(1,4)(X,X,O,X,X,X,X)}, {(1,6)(X,X,X,X,X,X,X)}, {(1,7)(X,X,X,X,X,X,O)}, {(1,9)(O,X,X,X,X,X,X)}, {(2,3)(X,X,X,X,X,X,X)}, {(2,4)(O,X,O,X,X,X,X)}, {(2,5)(O,X,X,X,X,X,X)}, {(2,7)(X,X,X,X,X,O,O)}, {(2,9)(O,X,X,X,X,X,O)}, {(3,4)(O,X,X,O,X,X,X)}, {(4,6)(X,X,X,O,X,O,X)}, {(4,7)(X,O,X,X,X,X,O)}, {(4,8)(X,X,X,X,X,O,O)}, {(4,9)(O,X,X,X,O,X,X)}, {(5,6)(O,X,X,X,X,X,X)}, {(5,7)(X,X,X,X,X,X,O)}, {(6,9)(X,X,X,O,X,X,X)}
3	{(1)(X,X,X,O,X,O,X)}, {(1,2)(X,X,X,X,X,X,O)}, {(1,3)(X,X,X,X,X,X,X)}, {(1,4)(O,O,X,X,X,X,X)}, {(1,5)(X,X,X,X,O,O,X)}, {(1,9)(O,X,O,X,X,X,X)}, {(2,3)(X,X,X,X,X,O,X)}, {(2,4)(X,X,O,X,X,X,X)}, {(2,5)(X,X,X,X,X,X,O)}, {(2,6)(O,X,X,X,X,X,X)}, {(2,7)(X,X,X,O,X,X,O)}, {(2,8)(O,X,X,X,X,X,O)}, {(3,4)(X,X,X,X,X,X,X)}, {(3,5)(O,X,X,O,X,X,X)}, {(3,6)(O,X,X,X,X,X,X)}, {(3,7)(O,X,X,X,X,X,O)}, {(4)(O,X,X,X,X,X,O)}, {(4,5)(O,X,X,X,X,X,X)}, {(4,6)(X,X,O,X,O,X,X)}, {(4,7)(O,O,X,X,X,X,X)}, {(5,8)(X,X,X,X,X,X,X)}, {(5,9)(X,X,O,X,X,X,X)}, {(6,7)(X,X,X,X,O,O,X)}, {(6,8)(O,X,X,X,X,O,X)}, {(6,9)(X,X,X,X,X,O,O)}, {(7,8)(X,O,X,X,X,X,O)}, {(7,9)(X,X,X,X,X,X,X)}, {(8,9)(X,X,X,X,X,X,O)}
4	{(1)(O,X,X,X,X,X,X)}, {(1,2)(X,X,X,X,O,X,O)}, {(1,3)(X,X,O,X,X,O,X)}, {(1,4)(X,X,X,X,X,X,O)}, {(1,6)(X,X,O,X,O,X,X)}, {(1,7)(X,X,O,O,X,X,X)}, {(1,9)(X,O,X,X,O,X,X)}, {(2)(O,X,X,X,X,X,O)}, {(2,3)(X,X,X,X,X,X,X)}, {(2,4)(O,X,X,X,X,X,X)}, {(2,5)(X,X,X,X,X,X,X)}, {(2,7)(X,O,X,X,X,O,X)}, {(2,8)(O,X,X,X,X,O,X)}, {(3)(O,X,X,X,X,X,O)}, {(3,5)(X,X,X,X,O,X,O)}, {(3,7)(X,X,O,X,X,X,O)}, {(3,8)(X,O,X,X,X,O,X)}, {(4,5)(O,X,O,X,X,X,X)}, {(4,6)(X,X,X,O,X,X,X)}, {(4,7)(X,X,O,X,X,O,X)}, {(4,8)(X,X,X,O,X,X,O)}, {(5)(O,X,X,X,X,X,O)}, {(5,6)(O,X,X,X,X,O,X)}, {(5,8)(X,X,X,X,O,X,O)}, {(5,9)(X,X,X,X,X,O,O)}, {(6,7)(X,X,X,X,X,X,O)}, {(6,8)(X,X,X,X,O,O,X)}, {(6,9)(X,X,O,X,X,X,X)}, {(7,8)(X,X,X,X,X,X,X)}, {(7,9)(O,X,X,O,X,X,X)}, {(8,9)(O,O,X,X,X,X,X)}
5	{(1)(O,X,X,X,X,X,O)}, {(1,2)(X,X,X,X,X,X,X)}, {(1,3)(X,O,X,X,O,X,X)}, {(1,4)(X,X,X,X,X,O,X)}, {(1,5)(X,O,X,X,O,X,X)}, {(1,6)(X,X,O,X,O,X,X)}, {(1,8)(X,X,O,O,X,X,X)}, {(1,9)(O,X,X,O,X,X,X)}, {(2,3)(X,X,X,O,X,X,X)}, {(2,5)(O,X,X,X,X,O,X)}, {(2,6)(O,X,X,X,X,X,O)}, {(2,7)(X,X,X,X,X,X,O)}, {(2,9)(X,X,X,X,X,O,X)}, {(3,4)(X,X,X,X,X,X,X)}, {(3,5)(O,X,X,X,X,X,X)}, {(3,7)(X,X,X,X,X,X,O)}, {(3,8)(X,X,X,X,X,O,O)}, {(3,9)(X,X,X,X,O,O,X)}, {(4,5)(X,X,X,X,X,X,O)}, {(4,6)(O,X,X,O,X,X,X)}, {(4,7)(X,X,O,X,X,X,X)}, {(4,8)(X,O,X,X,X,X,O)}, {(5,7)(O,X,X,O,X,X,X)}, {(5,8)(X,X,X,X,O,X,O)}, {(6,7)(X,O,O,X,X,X,X)}, {(6,8)(O,X,X,X,X,X,X)}, {(6,9)(X,O,X,X,X,X,O)}, {(7,8)(O,X,X,X,O,X,X)}, {(8,9)(O,X,X,X,X,X,O)}
6	{(1)(O,X,X,X,X,X,O)}, {(1,2)(X,X,X,X,X,X,X)}, {(1,3)(X,X,X,X,X,O,X)}, {(1,5)(X,X,X,X,X,X,X)}, {(1,6)(X,X,X,X,X,X,X)}, {(2)(O,X,X,X,O,X,X)}, {(2,4)(X,X,X,X,X,X,O)}, {(3,4)(O,X,X,X,X,X,O)}, {(3,6)(O,X,X,X,X,X,O)}, {(3,9)(O,X,X,O,X,X,X)}, {(4,5)(X,X,X,O,X,X,X)}, {(4,7)(X,X,O,X,X,X,O)}, {(4,9)(X,O,X,X,X,X,X)}, {(5,8)(X,X,X,X,X,X,X)}, {(6,7)(O,X,X,O,X,X,X)}, {(6,8)(X,X,X,X,O,X,O)}, {(7,8)(O,X,O,X,X,X,X)}

Table 53: Results of P-I of TPSA for *MaxC* = 100%

Interval	Skill Sets and Days (S,M,T,W,T,F,S)
1	{(1)(O,X,X,X,X,X,O)}, {(1,3)(X,X,O,O,X,X,X)}, {(1,6)(X,O,X,X,X,O,X)}, {(1,8)(O,X,X,X,X,X,O)}, {(1,9)(X,X,O,O,X,X,X)}, {(2,4)(O,X,O,X,X,X,X)}, {(2,5)(X,O,X,O,X,X,X)}, {(4,6)(X,X,X,O,X,X,O)}
2	{(1)(X,O,X,O,X,X,X)}, {(1,3)(O,X,O,X,X,X,X)}, {(1,4)(X,X,X,O,O,X,X)}, {(1,6)(X,X,X,X,X,O,O)}, {(1,8)(X,X,O,X,X,X,O)}, {(2,3)(O,X,X,X,X,X,O)}, {(2,4)(X,O,X,X,X,O,X)}, {(2,5)(O,X,X,X,X,O,X)}, {(2,6)(O,X,X,X,X,X,O)}, {(2,7)(X,X,X,X,O,X,O)}, {(3,4)(O,X,O,X,X,X,X)}, {(4,5)(X,X,X,X,X,O,O)}, {(4,6)(O,X,X,X,O,X,X)}, {(4,8)(O,X,X,X,X,X,O)}, {(4,9)(X,O,O,X,X,X,X)}, {(5,6)(X,X,O,O,X,X,X)}, {(5,7)(X,X,X,X,X,O,O)}, {(5,9)(X,X,X,O,X,X,O)}, {(7,9)(X,X,X,X,X,X,O)}
3	{(1)(X,O,X,X,X,X,O)}, {(1,2)(X,X,X,X,X,O,X)}, {(1,3)(O,X,X,X,X,X,X)}, {(1,4)(X,X,X,X,O,X,O)}, {(1,5)(O,X,X,X,X,X,X)}, {(1,6)(X,X,X,X,O,O,X)}, {(1,7)(O,X,X,X,X,O,X)}, {(1,8)(X,X,X,O,X,X,O)}, {(1,9)(X,X,X,X,X,X,X)}, {(2)(O,X,X,X,X,X,O)}, {(2,3)(O,X,X,X,X,X,O)}, {(2,4)(X,X,O,X,O,X,X)}, {(2,5)(X,O,X,X,X,O,X)}, {(2,6)(X,X,X,O,X,X,X)}, {(2,7)(X,X,X,X,X,X,O)}, {(2,8)(X,X,X,X,X,X,X)}, {(3,4)(X,X,X,X,X,X,X)}, {(3,5)(X,X,X,X,X,X,O)}, {(3,7)(O,X,X,O,X,X,X)}, {(3,9)(X,O,X,O,X,X,X)}, {(4,6)(O,X,O,X,X,X,X)}, {(4,7)(O,X,X,X,X,X,X)}, {(4,8)(X,O,X,X,O,X,X)}, {(5,6)(O,X,X,X,X,X,O)}, {(5,7)(X,X,X,X,X,O,X)}, {(5,9)(X,X,X,X,X,X,O)}, {(6,9)(O,X,X,X,X,X,O)}, {(8,9)(O,X,X,X,X,X,X)}
4	{(1)(O,X,X,X,X,X,O)}, {(1,2)(X,X,X,X,X,X,X)}, {(1,3)(X,X,X,O,O,X,X)}, {(1,4)(X,X,X,X,X,X,X)}, {(1,5)(X,X,X,X,X,X,X)}, {(1,7)(O,X,X,X,X,O,X)}, {(1,8)(X,X,X,X,X,X,X)}, {(2)(O,X,X,X,X,X,O)}, {(2,3)(O,O,X,X,X,X,X)}, {(2,4)(X,O,X,X,X,O,X)}, {(2,7)(X,X,O,X,X,X,O)}, {(2,8)(O,X,X,X,X,X,O)}, {(2,9)(X,X,X,X,O,O,X)}, {(3,4)(O,X,X,X,X,O,X)}, {(3,5)(X,X,O,X,X,X,O)}, {(3,6)(X,X,X,X,X,X,X)}, {(3,7)(X,X,X,X,X,X,X)}, {(3,8)(X,X,X,X,X,X,O)}, {(4,5)(X,X,O,X,X,X,X)}, {(4,7)(X,O,X,X,X,X,O)}, {(4,9)(O,X,X,X,X,X,X)}, {(5,6)(O,O,X,X,X,X,X)}, {(5,9)(X,X,X,X,X,X,X)}, {(6,8)(X,X,O,X,X,X,X)}, {(6,9)(X,X,X,X,X,O,O)}, {(7,9)(O,X,X,X,X,X,X)}, {(8,9)(X,O,X,X,X,X,O)}
5	{(1)(O,X,X,X,X,O,X)}, {(1,2)(O,O,X,X,X,X,X)}, {(1,3)(X,X,X,X,X,O,O)}, {(1,4)(X,O,X,X,O,X,X)}, {(1,5)(O,X,X,X,X,X,X)}, {(1,6)(X,X,O,X,X,O,X)}, {(1,7)(X,X,X,X,X,X,O)}, {(1,8)(X,X,O,O,X,X,X)}, {(1,9)(X,X,X,X,O,X,O)}, {(2)(O,X,X,X,X,X,O)}, {(2,3)(O,X,X,X,O,X,X)}, {(2,4)(X,X,X,X,X,O,O)}, {(2,6)(O,X,X,O,X,X,X)}, {(2,7)(X,O,X,X,X,X,O)}, {(2,8)(X,X,O,X,O,X,X)}, {(3,4)(X,X,X,X,X,X,X)}, {(3,5)(X,X,X,X,X,X,X)}, {(3,7)(X,X,X,O,X,X,X)}, {(3,8)(O,X,X,O,X,X,X)}, {(3,9)(X,X,O,X,O,X,X)}, {(4,5)(X,X,X,X,X,O,O)}, {(4,6)(X,X,X,X,O,O,X)}, {(4,8)(X,O,O,X,X,X,X)}, {(4,9)(O,X,X,O,X,X,X)}, {(5,7)(X,X,X,X,X,X,X)}, {(5,9)(O,X,X,X,X,X,O)}, {(6,7)(X,O,X,X,X,X,O)}, {(8,9)(O,X,X,X,X,X,X)}
6	{(1)(X,X,X,X,X,O,X)}, {(1,3)(X,X,X,X,O,O,X)}, {(1,4)(X,O,X,O,X,X,X)}, {(1,7)(O,X,X,X,X,X,O)}, {(1,8)(X,X,O,X,O,X,X)}, {(1,9)(X,X,X,X,X,X,X)}, {(2,4)(X,X,O,X,X,X,X)}, {(2,5)(O,X,O,X,X,X,X)}, {(3,5)(O,X,X,X,X,X,O)}, {(3,6)(O,X,X,X,X,X,O)}, {(3,8)(O,X,X,X,X,O,X)}, {(3,9)(O,X,X,X,X,X,O)}, {(4,5)(X,O,X,X,O,X,X)}, {(4,6)(O,X,O,X,X,X,X)}, {(4,8)(X,X,X,X,O,X,O)}, {(5,6)(X,X,X,O,X,X,O)}, {(6,7)(X,X,X,O,X,O,X)}, {(7,8)(O,X,X,X,X,X,O)}

APPENDIX E: XPRESS and TPSA Results for the Call Center Problem

Table 54: TPSA Results with Two-Skill CT for Case L1

	MaxC									
Result	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Total Cost ($)	94,376	93,881	93,050	92,218	91,866	89,996	91,434	91,005	91,279	91,465
Staff Cost	71,505	71,190	68,880	67,515	68,250	68,145	67,935	68,040	68,565	68,775
Penalty Cost	22,871	22,691	24,170	24,703	23,616	21,850	23,499	22,965	22,714	22,690
Uncovered Demand (#)	545	540	575	588	562	520	560	547	541	540
Total Staff (#)	90	90	87	85	86	85	85	84	85	86
Full-Time Stf.	39	38	37	40	48	46	49	53	50	49
Extended Stf.	33	34	33	28	21	22	19	18	20	20
Part-Time Stf.	18	18	17	17	17	17	17	13	15	17
CT Staff (#)	9	18	26	34	43	51	59	67	74	86
Full-Time CT Stf.*	2	5	7	13	21	25	30	41	42	49
Extended CT Stf.	6	11	17	16	15	15	18	17	17	20
Part-Time CT Stf.	1	2	2	5	7	11	11	9	15	17
CT Staff Percentage	10%	20%	29.9%	40%	50%	60%	69.4%	79.8%	87.1%	100%

*CT Stf.: Cross-Trained Staff

Table 55: XPRESS Results with Two-Skill CT for Case L1

	MaxC									
Result	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Total Cost ($)	95,704	96,392	94,023	94,989	93,132	94,872	94,061	94,330	93,045	93,278
Staff Cost	68,250	70,245	71,085	69,615	70,035	71,400	70,770	70,035	68,250	71,505
Penalty Cost	27,454	26,147	22,937	25,373	23,097	23,472	23,291	24,294	24,795	21,773
Uncovered Demand (#)	654	623	546	604	550	559	555	578	590	518
Total Staff (#)	87	91	90	88	88	90	89	88	86	90
Full-Time Stf.	49	44	52	55	53	57	56	53	51	54
Extended Stf.	21	29	20	16	19	15	16	18	18	18
Part-Time Stf.	17	18	18	17	16	18	17	17	17	18
CT Staff (#)	8	18	27	35	44	54	62	70	77	84
Full-Time CT Stf.	6	11	21	25	26	32	38	43	48	49
Extended CT Stf.	2	5	2	5	9	10	11	13	13	17
Part-Time CT Stf.	0	2	4	5	9	12	13	14	16	18
CT Staff Percentage	9.2%	19.8%	30%	39.8%	50%	60%	69.7%	79.5%	89.5%	93.3%

Table 56: Number of Staff in Each Skill Set in TPSA Results for Case L1

Skill Set	MaxC									
	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
{1}	15	16	12	13	10	10	8	6	4	0
{1,2}	0	1	1	0	2	1	4	2	5	4
{1,3}	1	0	1	0	1	3	3	4	5	4
{1,4}	0	1	0	2	3	3	2	4	2	4
{1,5}	1	0	1	0	1	3	2	2	4	2
{1,6}	1	0	1	0	2	1	2	2	3	2
{1,7}	0	1	1	2	1	2	2	2	0	3
{1,8}	0	0	0	2	2	1	1	3	1	3
{1,9}	1	1	2	2	2	0	2	2	3	6
{2}	12	10	9	8	6	5	3	5	1	0
{2,3}	0	0	0	2	1	2	2	1	3	4
{2,4}	0	0	2	2	1	2	2	4	3	4
{2,5}	0	0	0	1	2	2	1	1	3	2
{2,6}	0	0	0	1	2	2	2	1	1	1
{2,7}	0	0	1	0	1	1	1	2	3	2
{2,8}	0	2	1	1	1	2	2	2	1	4
{2,9}	1	1	1	0	1	1	1	2	1	1
{3}	12	10	9	7	8	7	5	2	2	0
{3,4}	1	0	1	1	2	1	1	2	4	4
{3,5}	0	1	1	1	1	0	2	2	2	4
{3,6}	0	2	2	1	1	1	3	2	1	3
{3,7}	0	0	0	1	1	1	1	2	1	2
{3,8}	0	1	1	1	1	0	2	2	1	3
{3,9}	0	0	1	1	2	2	1	3	2	0
{4}	10	10	8	6	5	4	3	1	1	0
{4,5}	0	0	2	2	2	3	3	1	3	1
{4,6}	0	0	0	1	2	2	1	1	2	2
{4,7}	0	1	0	1	1	1	2	3	2	1
{4,8}	1	2	1	2	0	1	2	1	2	3
{4,9}	0	0	0	0	1	1	2	1	1	2
{5}	10	9	8	7	6	4	3	3	3	0
{5,6}	0	2	0	1	1	1	1	2	1	3
{5,7}	0	0	1	0	0	0	1	1	0	3
{5,8}	0	0	0	1	0	1	1	2	2	0
{5,9}	0	0	1	1	0	3	3	1	0	3
{6}	6	4	4	2	1	0	0	0	0	0
{6,7}	0	1	1	1	0	2	1	1	1	1
{6,8}	1	0	1	0	2	1	1	2	3	1
{6,9}	0	0	0	1	0	1	1	1	2	1
{7}	6	5	4	4	3	2	2	0	0	0
{7,8}	0	0	0	1	1	1	0	1	2	0
{7,9}	0	1	0	0	1	1	0	1	2	2
{8}	6	4	4	2	2	2	2	0	0	0
{8,9}	1	0	1	1	1	1	1	1	2	1
{9}	4	4	3	2	2	0	0	0	0	0
Total	**90**	**90**	**87**	**85**	**86**	**85**	**85**	**84**	**85**	**86**

Table 57: Number of Staff in Each Skill Set in XPRESS Results for Case L1

Skill Set	MaxC									
	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
{1}	16	16	14	13	11	11	8	4	2	1
{1,2}	0	1	2	1	3	2	2	3	6	4
{1,3}	0	0	0	2	1	1	2	3	3	5
{1,4}	0	0	0	1	1	2	2	3	4	3
{1,5}	1	1	2	2	2	1	2	5	3	3
{1,6}	0	0	0	1	2	2	2	2	3	4
{1,7}	1	1	2	1	1	2	2	1	1	2
{1,8}	0	1	1	1	1	1	2	2	2	2
{1,9}	0	1	1	1	1	2	2	4	3	3
{2}	11	10	9	8	7	5	4	4	0	1
{2,3}	0	1	2	1	1	2	3	3	3	3
{2,4}	0	0	2	1	2	3	2	2	3	4
{2,5}	0	1	0	1	1	3	2	3	3	2
{2,6}	1	1	0	1	1	1	2	2	2	3
{2,7}	0	0	0	1	1	1	1	2	3	2
{2,8}	0	0	0	1	1	1	2	1	3	2
{2,9}	1	0	0	0	0	2	1	1	1	2
{3}	12	11	10	8	6	6	4	2	3	1
{3,4}	0	1	1	0	2	2	2	4	2	3
{3,5}	0	2	0	1	2	1	2	2	3	2
{3,6}	0	0	2	2	1	2	2	1	2	1
{3,7}	0	0	1	1	1	1	1	2	2	3
{3,8}	0	0	0	0	2	3	2	2	1	3
{3,9}	1	1	1	1	2	1	2	1	2	2
{4}	11	9	8	7	5	3	5	2	0	0
{4,5}	0	0	0	1	1	2	1	3	3	3
{4,6}	0	0	0	1	1	1	1	2	2	3
{4,7}	0	0	1	1	1	2	1	2	2	2
{4,8}	1	2	2	2	2	2	2	2	1	3
{4,9}	0	1	1	2	1	1	2	2	2	1
{5}	9	9	8	6	5	3	2	1	2	1
{5,6}	1	0	2	0	1	1	2	1	1	3
{5,7}	0	0	0	0	1	2	1	2	2	2
{5,8}	0	0	1	1	1	1	2	1	1	2
{5,9}	0	0	0	1	1	2	2	1	2	1
{6}	4	4	5	3	3	2	0	2	0	0
{6,7}	1	1	0	1	2	1	2	1	2	2
{6,8}	0	1	1	2	0	1	1	0	1	1
{6,9}	0	1	0	0	1	0	1	1	1	1
{7}	6	5	4	3	3	2	2	0	1	1
{7,8}	0	0	1	1	1	1	1	1	1	1
{7,9}	0	0	0	1	0	1	1	1	0	1
{8}	6	5	3	3	2	2	2	2	1	0
{8,9}	0	0	1	0	1	0	2	1	1	0
{9}	4	4	2	2	2	2	0	1	0	1
Total	**87**	**91**	**90**	**88**	**88**	**90**	**89**	**88**	**86**	**90**

APPENDIX F: Weekly Schedule for the Call Center

Table 58: Weekly Schedule for Case L1 with TPSA for Two-Skill CT and *MaxC* = 10%

Skill Set	Number of Employees	Shift Type					
		Full-Time		Extended		Part-Time	
		Shift (Periods)	Days (SMTWTFS)	Shift (Periods)	Days (SMTWTFS)	Shift (Periods)	Days (SMTWTFS)
{1}	15	13 - 29	XXOXXOX	13 - 33	OXXXOXO	13 - 25	XXXOXOX
		15 - 31	XXXOXXO	17 - 37	XXOXOXO	36 - 48	XXOXXXO
		15 - 31	OXXXXXO	17 - 37	OOOXXXX		
		19 - 35	OXXXXOX	19 - 39	XOXOXXO		
		32 - 48	XXXXXOO	21 - 41	OXXOOXX		
		32 - 48	OOXXXXX	23 - 43	OXOXXOX		
				28 - 48	XOXOOXX		
{1,3}	1	-		28 - 48	OXXXOOX	-	
{1,5}	1	-		23 - 43	XOOXXXO	-	
{1,6}	1	13 - 29	XOOXXXX	-		-	
{1,9}	1	32 - 48	OXXXXXO	-		-	
{2}	12	13 - 29	XXXXXOO	13 - 33	OOXXXOX	13 - 27	XXOOXXX
		15 - 31	XXXXOXO	19 - 39	OXXXOXO	21 - 35	XXOXXOX
		15 - 31	OXXXXXO	28 - 48	OXXOXXO	36 - 48	XXOXXOX
		15 - 31	OOXXXXX				
		17 - 33	OXXXXXO				
		32 - 48	XOOXXXX				
{2,9}	1	-		23 - 43	OXXOXOX	-	
{3}	12	15 - 31	OXXXXOX	19 - 39	XOXXXOO	17 - 24	OXXXXXO
		17 - 33	OXXXXXO	21 - 41	OXXOOXX	39 - 48	OXXXXXO
		19 - 35	XXOXOXX	23 - 43	OXOXXXO		
		21 - 37	OXXXXOX				
		23 - 39	XOXXXXO				
		25 - 41	XXOOXXX				
		29 - 45	OXXXXXO				
{3,4}	1	-		13 - 33	OXOXXXO	-	
{4}	10	13 - 29	OXXOXXX	13 - 33	XXXXOOO	13 - 25	XXXXXOO
		29 - 45	OXXOXXX	15 - 35	XOOXOXX	13 - 25	OXOXXXX
				19 - 39	OXXOXOX	34 - 48	XXOXXXO
						36 - 48	XOXXOXX
						39 - 48	XXXOXOX
{4,8}	1	-		13 - 33	XOXOXXO	-	
{5}	10	13 - 29	XXOXXOX	13 - 33	OXXXOXO	17 - 31	OXXXXXO
		15 - 31	OXXXXXO	17 - 37	OOXOXXX	36 - 48	XOXOXXX
		19 - 35	OXXXXXO				
		21 - 37	XXXXXOO				
		27 - 43	OXXXXOX				
		32 - 48	OXXXXXO				
{6}	6	13 - 29	OXXXXOX	28 - 48	XOXOOXX	13 - 25	XXXOOXX
		15 - 31	XXOXXOX				
		21 - 37	OXXXXXO				
		32 - 48	XXOXXOX				
{6,8}	1	-		-		34 - 48	OXXXXXO
{7}	6	13 - 29	OXXXXXO	13 - 33	XXXXOOO	17 - 24	XOXOXXX
		29 - 45	OOXXXXX	19 - 39	OXOXOXX		
				21 - 41	XXXOXOO		
{8}	6	15 - 31	OXXXXXO	19 - 39	XOXOXXO	17 - 24	OXXXXOX
		19 - 35	OXOXXXX	23 - 43	XXOXOXO		
				28 - 48	OXXOXOX		
{8,9}	1	-		28 - 48	XOXXOOX	-	
{9}	4	13 - 29	OOXXXXX	13 - 33	OXXXXOO	-	
		15 - 31	XOXXOXX	15 - 35	XXOOXXO		

APPENDIX G: Interface

The interface has various sections: a) all shifts with their start times, b) any days off and consecutive days off assignment options, c) maximum part-time shift percentage and minimum extended shift percentage, d) weight ratio for the trade-off between cost and preferences, and e) demand file and staff preference file. These system parameters can be selected, fixed, changed, and managed easily using the interface, and then the staffing and scheduling can be processed, bidding can be applied, and results can be re-optimized.

The results of the scheduling process are also demonstrated on the interface: a) individual schedules with their working days, and start and end times, b) graph of daily and weekly schedules for employees, c) total cost, staff cost, and penalty cost for uncovered demand, d) amount of full-time, extended, and part-time shift staff, e) amount of uncovered demand, f) amount of consecutive and non-consecutive off days for employees, and g) total amount of satisfied demand throughout the week.

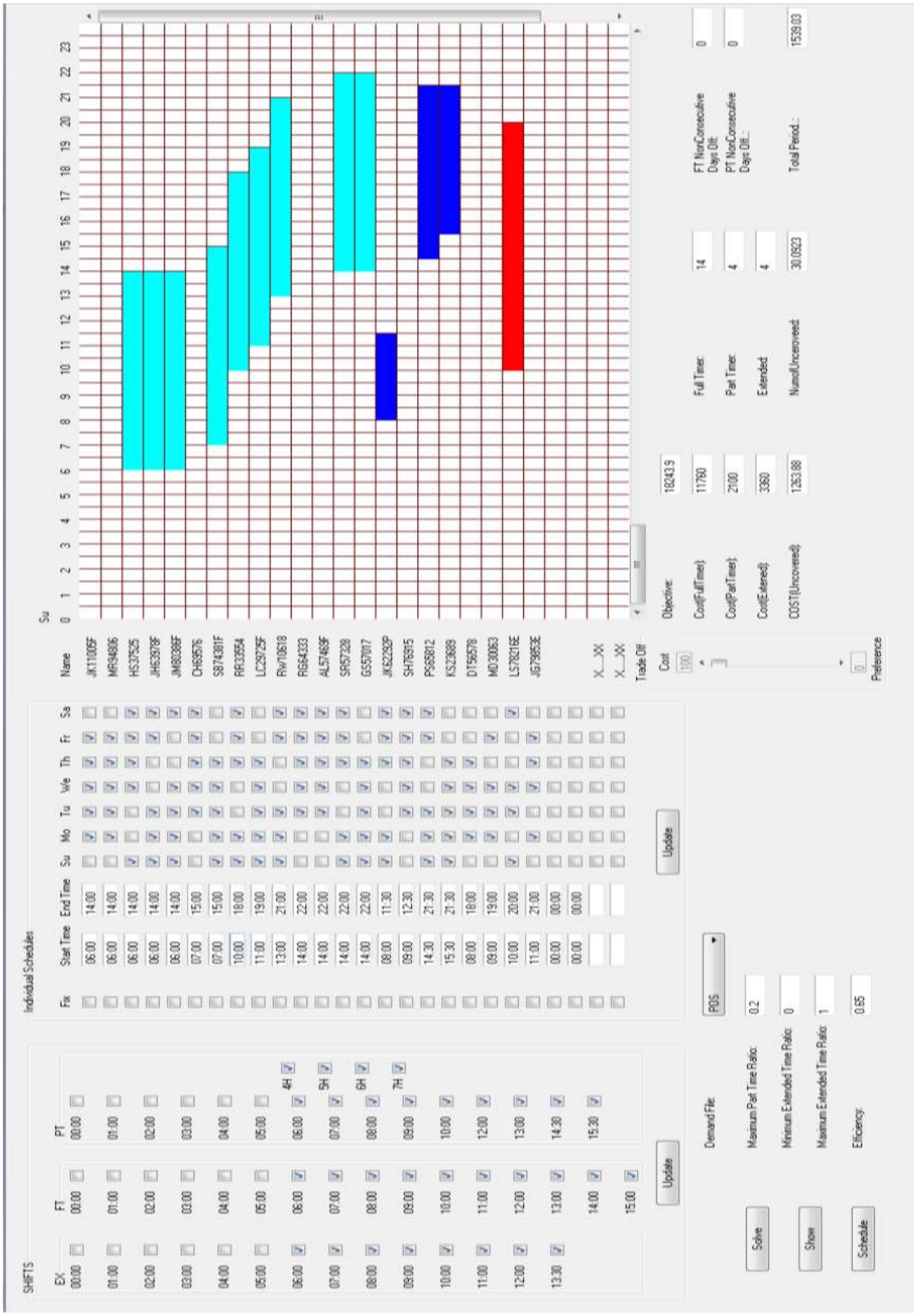

Figure 27: The Interface

APPENDIX H: Multi-Skill Cross-Training Results and Comparisons for Test Cases

Table 59: Detailed Results for Three-Skill and Four-Skill Cross-Training

Case	CT	Result	MaxC									
			10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
M1	3-Skill	Cost	58,050	56,312	55,636	55,370	56,823	55,403	55,291	54,905	55,190	55,629
		B.Bou.	53,195	52,302	52,035	52,026	52,024	52,024	52,024	52,024	52,024	52,024
		Gap	8.36%	7.12%	6.47%	6.04%	8.45%	6.10%	5.91%	5.25%	5.74%	6.48%
	4-Skill	Cost	58,141	56,312	56,085	55,005	55,383	54,742	54,628	55,292	54,796	54,957
		B.Bou.	53,207	52,301	52,028	52,024	52,024	52,025	52,024	52,024	52,024	52,024
		Gap	8.49%	7.12%	7.23%	5.42%	6.07%	4.96%	4.77%	5.91%	5.06%	5.34%
M2	3-Skill	Cost	56,692	54,538	52,801	52,824	52,446	53,601	52,603	52,565	52,831	51,755
		B.Bou.	50,754	49,979	49,720	49,659	49,671	49,651	49,650	49,645	49,641	49,640
		Gap	10.48%	8.36%	5.83%	5.99%	5.29%	7.37%	5.61%	5.56%	6.04%	4.09%
	4-Skill	Cost	54,637	53,343	53,734	51,739	52,806	51,880	52,731	52,456	52,170	51,870
		B.Bou.	50,809	49,938	49,735	49,696	49,651	49,643	49,653	49,647	49,650	49,643
		Gap	7.01%	6.38%	7.44%	3.95%	5.97%	4.31%	5.84%	5.35%	4.83%	4.29%
M3	3-Skill	Cost	76,865	74,091	73,969	74,533	73,842	74,810	74,324	73,182	72,277	73,084
		B.Bou.	70,261	69,692	69,671	69,671	69,671	69,671	69,671	69,671	69,671	69,671
		Gap	8.59%	5.94%	5.81%	6.52%	5.65%	6.87%	6.26%	4.80%	3.61%	4.67%
	4-Skill	Cost	75,167	73,734	74,072	73,123	72,694	72,580	73,319	72,356	72,277	72,384
		B.Bou.	70,217	69,701	69,671	69,672	69,672	69,671	69,671	69,671	69,671	69,671
		Gap	6.59%	5.47%	5.94%	4.72%	4.16%	4.01%	4.98%	3.71%	3.60%	3.75%
M4	3-Skill	Cost	64,336	62,205	62,295	62,639	63,063	62,475	62,763	62,600	61,818	62,526
		B.Bou.	59,974	59,206	59,118	59,097	59,092	59,089	59,096	59,089	59,089	59,089
		Gap	6.78%	4.82%	5.10%	5.66%	6.30%	5.42%	5.84%	5.61%	4.42%	5.50%
	4-Skill	Cost	64,488	63,289	62,318	62,531	61,898	62,118	61,929	61,804	61,862	61,716
		B.Bou.	59,934	59,234	59,104	59,091	59,091	59,091	59,095	59,089	59,091	59,089
		Gap	7.06%	6.41%	5.16%	5.50%	4.53%	4.87%	4.58%	4.39%	4.48%	4.26%
L1	3-Skill	Cost	92,982	91,688	91,093	89,548	91,958	91,945	90,380	90,299	90,260	89,535
		B.Bou.	86,893	85,813	85,446	85,457	85,437	85,434	85,434	85,434	85,435	85,434
		Gap	6.55%	6.41%	6.20%	4.57%	7.09%	7.08%	5.47%	5.39%	5.35%	4.58%
	4-Skill	Cost	93,343	92,815	91,952	90,211	90,407	89,618	90,478	90,810	90,241	89,300
		B.Bou.	86,352	85,511	85,434	85,434	85,434	85,434	85,435	85,434	8,5434	85,434
		Gap	7.49%	7.87%	7.09%	5.30%	5.50%	4.67%	5.57%	5.92%	5.33%	4.33%
L2	3-Skill	Cost	102,258	100,433	98,975	97,299	96,989	96,897	95,935	96,374	96,464	95,980
		B.Bou.	93,567	92,025	91,780	91,777	91,775	91,775	91,774	91,774	91,775	91,775
		Gap	8.50%	8.37%	7.27%	5.68%	5.38%	5.29%	4.34%	4.77%	4.86%	4.38%
	4-Skill	Cost	100,985	96,671	98,025	96,851	96,428	96,878	95,952	95,720	95,018	95,873
		B.Bou.	93,046	91,888	91,775	91,776	91,774	91,774	91,774	91,775	91,774	91,774
		Gap	7.86%	4.95%	6.38%	5.24%	4.83%	5.27%	4.35%	4.12%	3.41%	4.27%
L3	3-Skill	Cost	116,212	112,239	111,768	112,002	112,066	111,920	111,367	111,658	111,252	110,365
		B.Bou.	108,264	106,548	106,250	106,243	106,241	106,242	106,241	106,246	106,241	106,246
		Gap	6.84%	5.07%	4.94%	5.14%	5.20%	5.07%	4.60%	4.85%	4.50%	3.73%
	4-Skill	Cost	114,762	111,564	111,736	113,009	111,266	110,778	111,358	111,584	111,833	110,115
		B.Bou.	107,784	106,360	106,241	106,242	106,241	106,241	106,241	106,241	106,241	106,241
		Gap	6.08%	4.66%	4.92%	5.99%	4.52%	4.10%	4.60%	4.79%	5.00%	3.52%

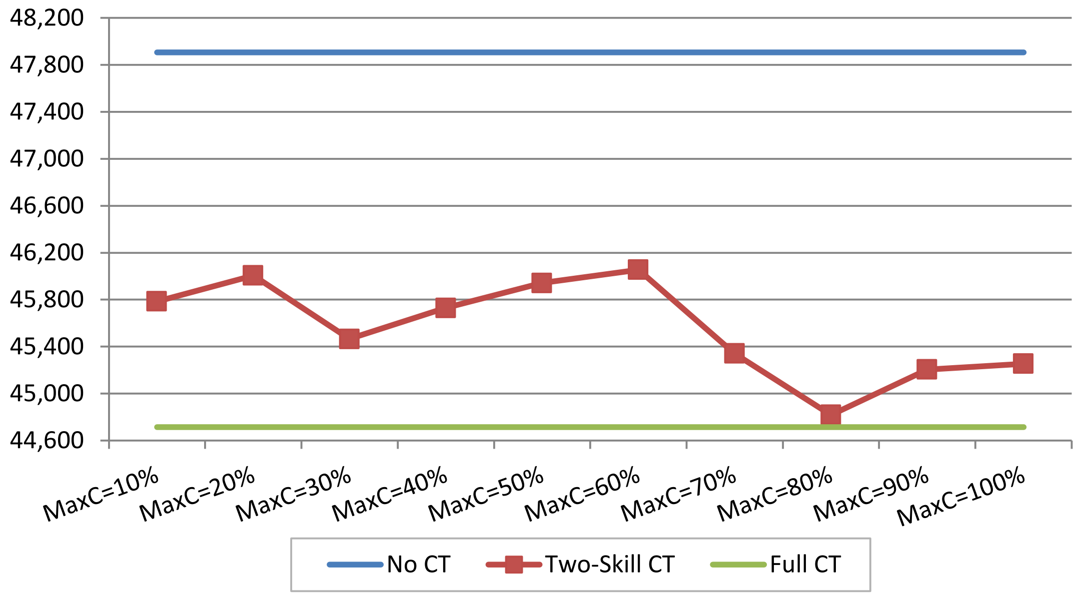

Figure 28: Limited Cross-Training Results for Case S1

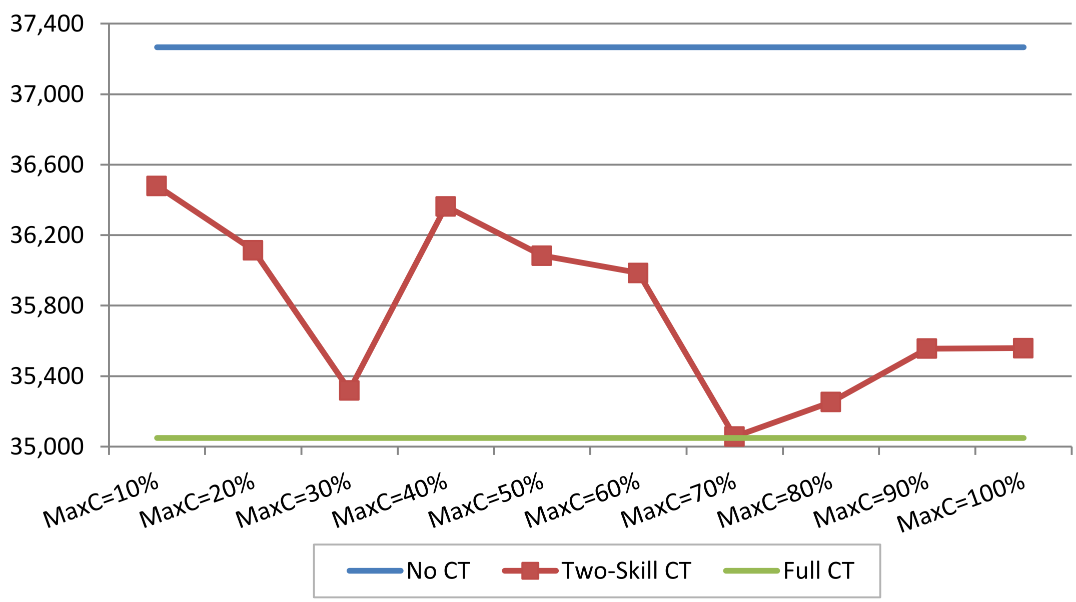

Figure 29: Limited Cross-Training Results for Case S2

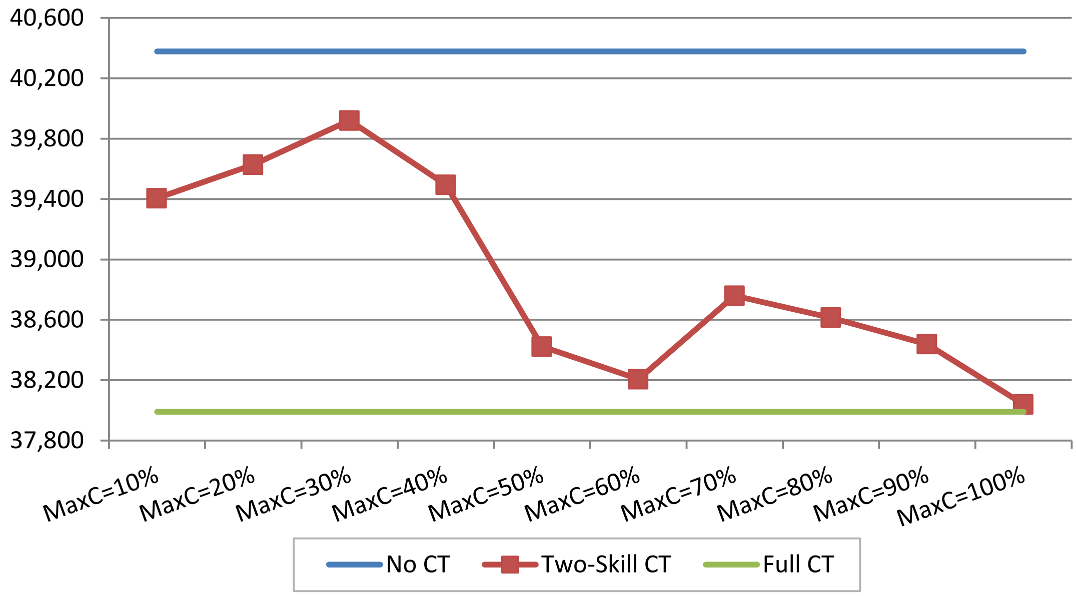

Figure 30: Limited Cross-Training Results for Case S3

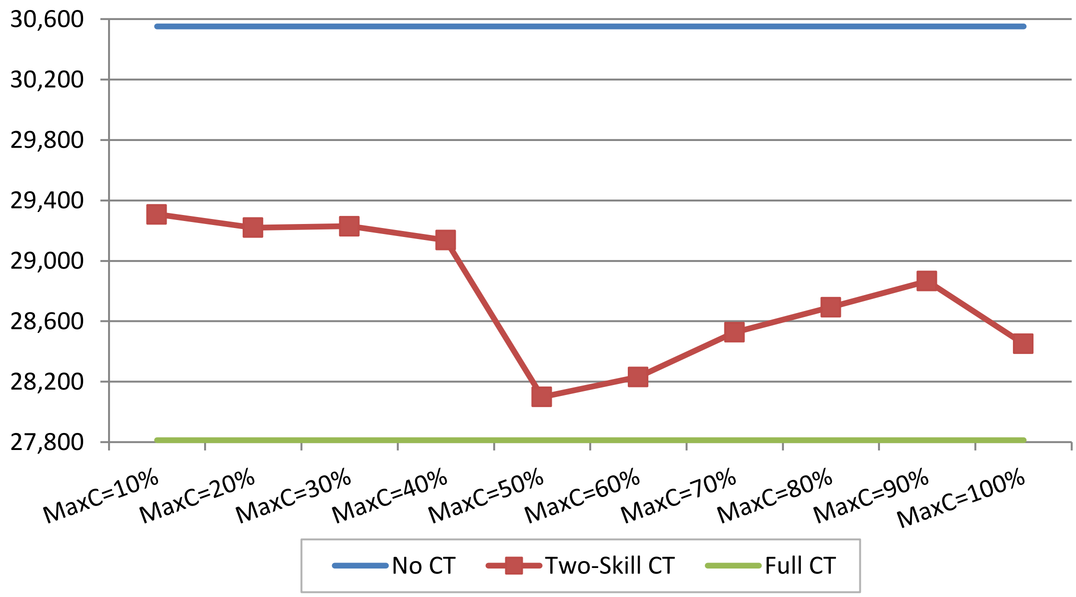

Figure 31: Limited Cross-Training Results for Case S4

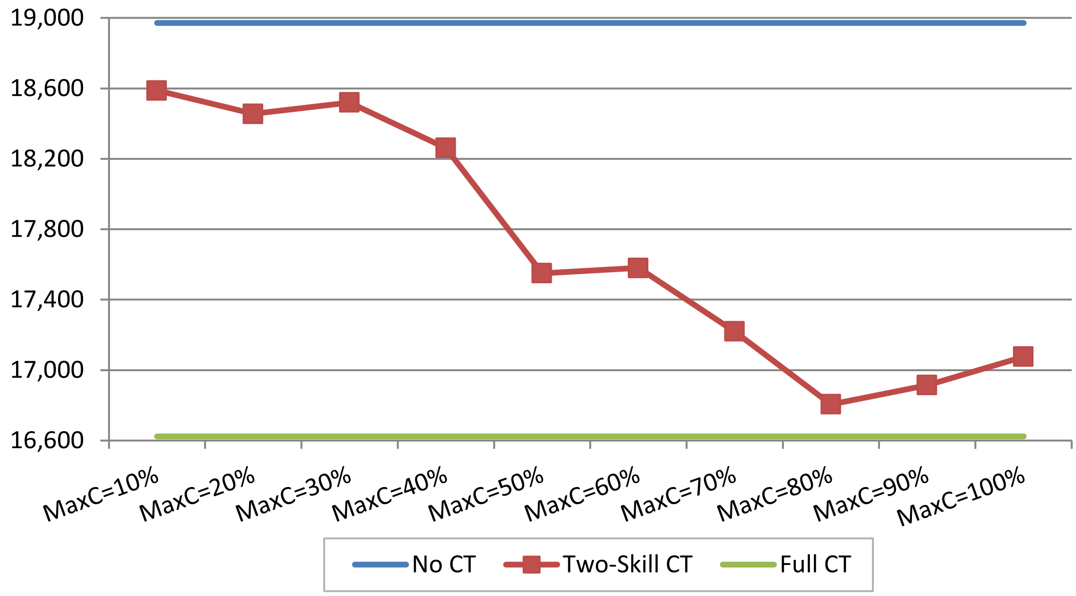

Figure 32: Limited Cross-Training Results for Case S5

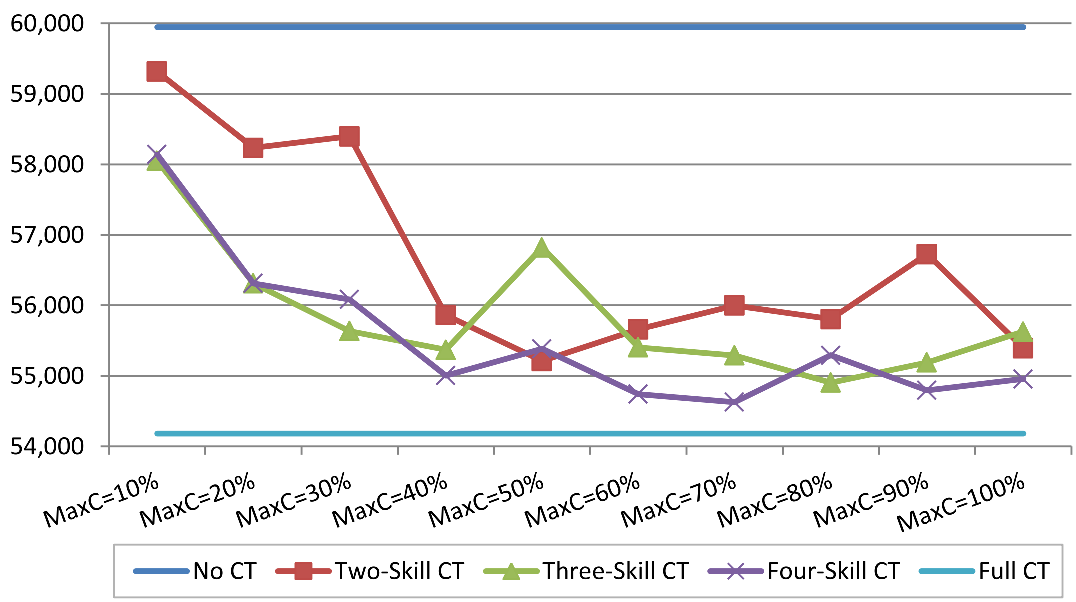

Figure 33: Limited Cross-Training Results for Case M1

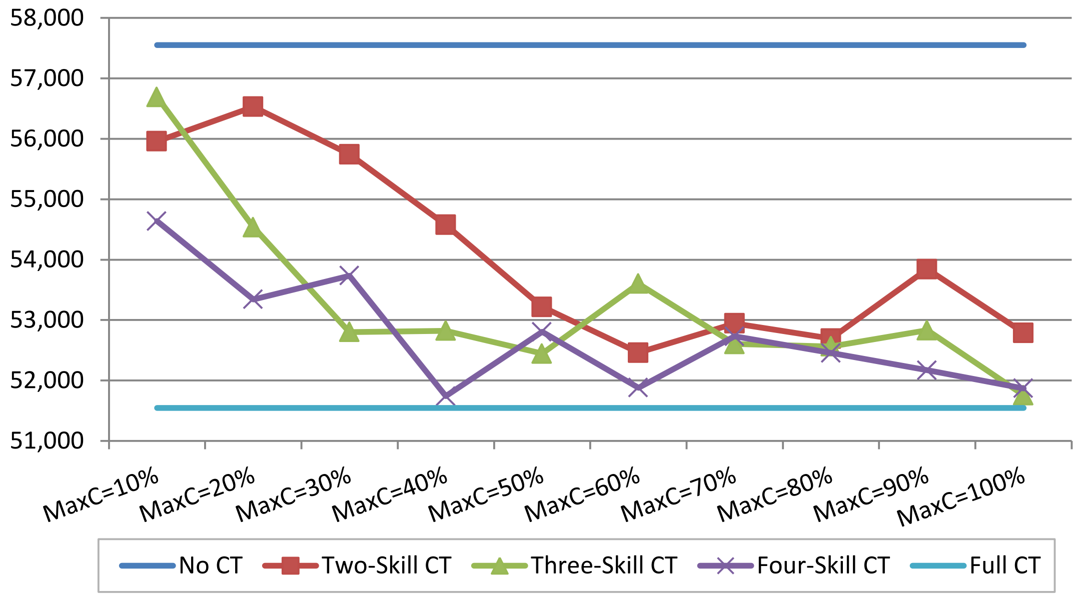

Figure 34: Limited Cross-Training Results for Case M2

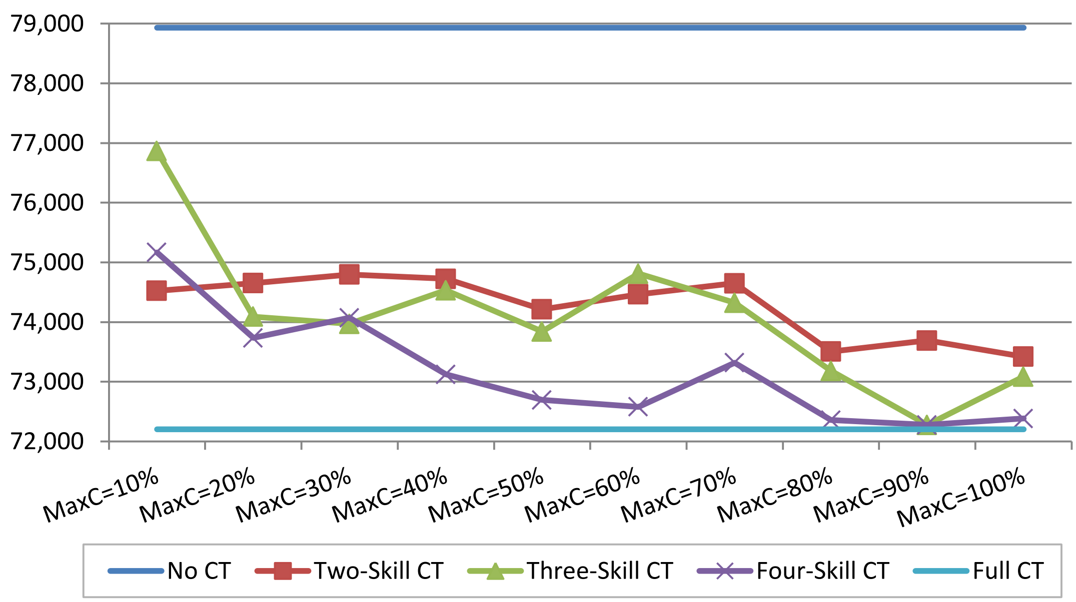

Figure 35: Limited Cross-Training Results for Case M3

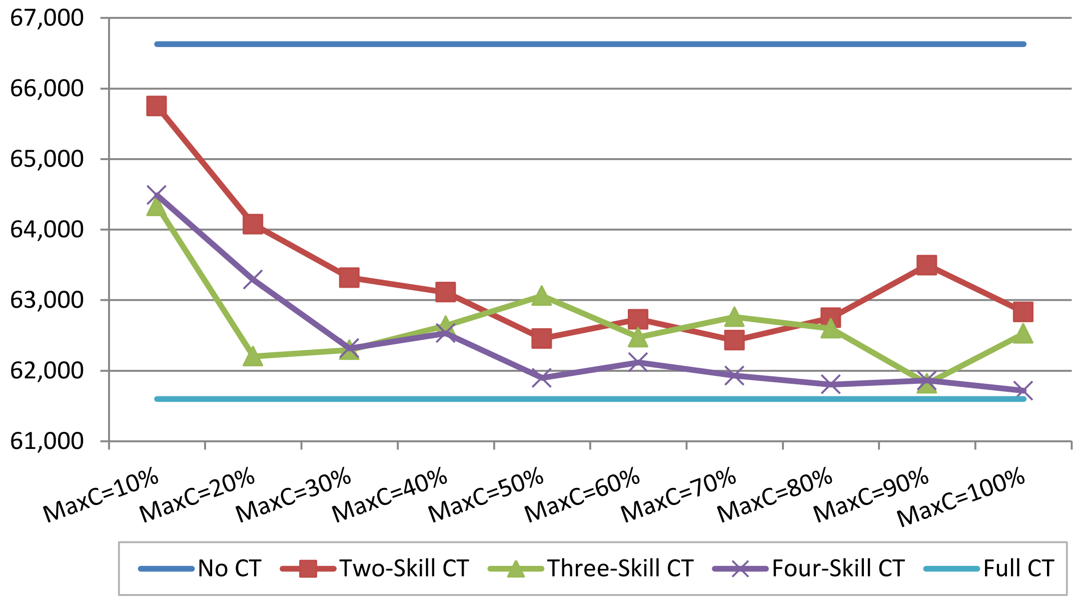

Figure 36: Limited Cross-Training Results for Case M4

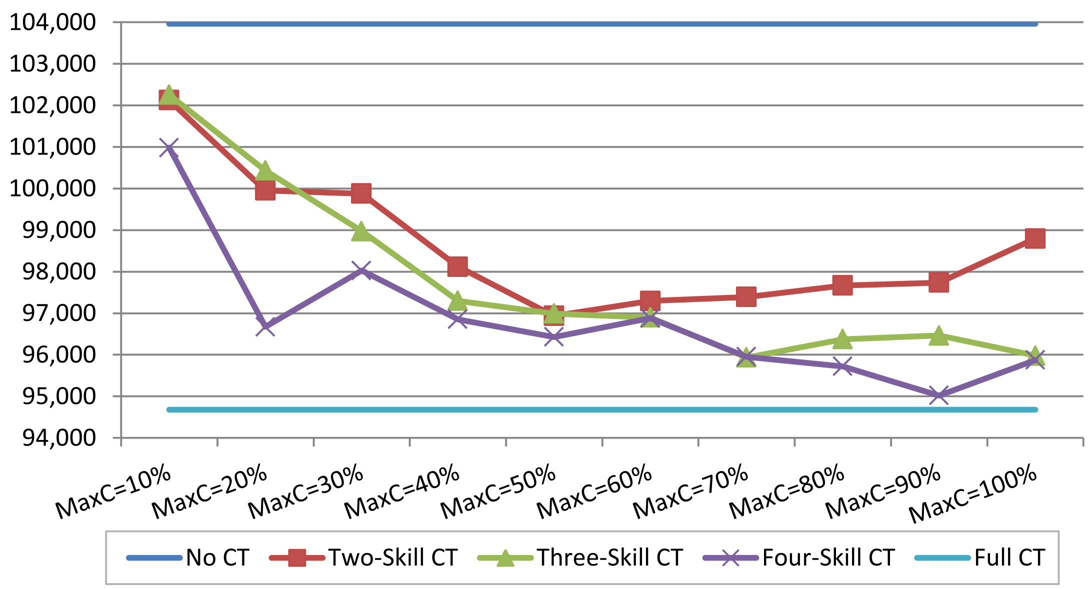

Figure 37: Limited Cross-Training Results for Case L2

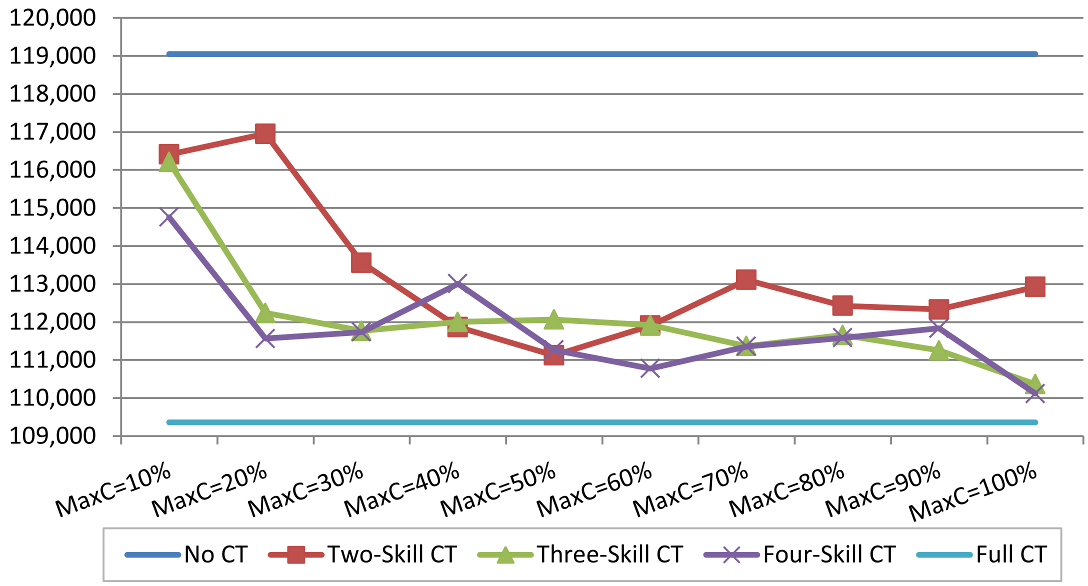

Figure 38: Limited Cross-Training Results for Case L3

APPENDIX I: Comparisons of No CT, Partial Limited CT, and Full CT

Table 60: Comparison of No CT, Partial Limited CT, and Full CT for Case S1

Comparison of		MaxC										Avg.
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	
No CT	2-Skill CT	4.4%	4.0%	5.1%	4.5%	4.1%	3.9%	5.4%	6.4%	5.6%	5.5%	**4.9%**
No CT	Full CT	6.7%										**6.7%**
2-Skill CT	Full CT	2.3%	2.8%	1.6%	2.2%	2.7%	2.9%	1.4%	0.2%	1.1%	1.2%	**1.8%**

Table 61: Comparison of No CT, Partial Limited CT, and Full CT for Case S2

Comparison of		MaxC										Avg.
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	
No CT	2-Skill CT	2.1%	3.1%	5.2%	2.4%	3.2%	3.4%	5.9%	5.4%	4.6%	4.6%	**4.0**%
No CT	Full CT	6.0%										**6.0**%
2-Skill CT	Full CT	3.9%	3.0%	0.8%	3.6%	2.9%	2.6%	0.0%	0.6%	1.4%	1.4%	**2.0**%

Table 62: Comparison of No CT, Partial Limited CT, and Full CT for Case S3

Comparison of		MaxC										Avg.
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	
No CT	2-Skill CT	2.4%	1.9%	1.1%	2.2%	4.8%	5.4%	4.0%	4.4%	4.8%	5.8%	**3.7**%
No CT	Full CT	5.9%										**5.9**%
2-Skill CT	Full CT	3.6%	4.1%	4.8%	3.8%	1.1%	0.6%	2.0%	1.6%	1.2%	0.1%	**2.3**%

Table 63: Comparison of No CT, Partial Limited CT, and Full CT for Case S4

Comparison of		MaxC										Avg.
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	
No CT	2-Skill CT	4.1%	4.4%	4.3%	4.6%	8.0%	7.6%	6.6%	6.1%	5.5%	6.9%	**5.8**%
No CT	Full CT	9.0%										**9.0**%
2-Skill CT	Full CT	5.1%	4.8%	4.9%	4.6%	1.0%	1.5%	2.5%	3.1%	3.7%	2.2%	**3.3**%

Table 64: Comparison of No CT, Partial Limited CT, and Full CT for Case S5

Comparison of		MaxC										Avg.
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	
No CT	2-Skill CT	2.0%	2.7%	2.4%	3.7%	7.5%	7.3%	9.2%	11.4%	10.8%	10.0%	**6.7**%
No CT	Full CT	12.4%										**12.4**%
2-Skill CT	Full CT	10.6%	9.9%	10.2%	9.0%	5.3%	5.4%	3.5%	1.1%	1.7%	2.7%	**5.9**%

Table 65: Comparison of No CT, Partial Limited CT, and Full CT for Case M1

Comparison of		MaxC										Avg.
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	
No CT	2-Skill CT	1.1%	2.9%	2.6%	6.8%	7.9%	7.2%	6.6%	6.9%	5.4%	7.6%	**5.5**%
	3-Skill CT	3.2%	6.1%	7.2%	7.6%	5.2%	7.6%	7.8%	8.4%	7.9%	7.2%	**6.8**%
	4-Skill CT	3.0%	6.1%	6.4%	8.2%	7.6%	8.7%	8.9%	7.8%	8.6%	8.3%	**7.4**%
No CT	Full CT	9.6%										**9.6**%
2-Skill CT	Full CT	8.7%	7.0%	7.2%	3.0%	1.9%	2.7%	3.2%	2.9%	4.5%	2.2%	**4.3**%
3-Skill CT		6.7%	3.8%	2.6%	2.1%	4.6%	2.2%	2.0%	1.3%	1.8%	2.6%	**3.0**%
4-Skill CT		6.8%	3.8%	3.4%	1.5%	2.2%	1.0%	0.8%	2.0%	1.1%	1.4%	**2.4**%

Table 66: Comparison of No CT, Partial Limited CT, and Full CT for Case M2

Comparison of		MaxC										Avg.
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	
No CT	2-Skill CT	2.8%	1.8%	3.1%	5.2%	7.5%	8.9%	8.0%	8.4%	6.4%	8.3%	**6.0**%
	3-Skill CT	1.5%	5.2%	8.3%	8.2%	8.9%	6.9%	8.6%	8.7%	8.2%	10.1%	**7.5**%
	4-Skill CT	5.1%	7.3%	6.6%	10.1%	8.3%	9.9%	8.4%	8.9%	9.4%	9.9%	**8.4**%
No CT	Full CT	10.4%										**10.4**%
2-Skill CT	Full CT	7.9%	8.8%	7.5%	5.6%	3.1%	1.7%	2.6%	2.2%	4.3%	2.4%	**4.6**%
3-Skill CT		9.1%	5.5%	2.4%	2.4%	1.7%	3.8%	2.0%	1.9%	2.4%	0.4%	**3.2**%
4-Skill CT		5.7%	3.4%	4.1%	0.4%	2.4%	0.6%	2.3%	1.7%	1.2%	0.6%	**2.2**%

Table 67: Comparison of No CT, Partial Limited CT, and Full CT for Case M3

Comparison of		MaxC										Avg.
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	
No CT	2-Skill CT	5.6%	5.4%	5.2%	5.3%	6.0%	5.7%	5.4%	6.9%	6.6%	7.0%	**5.9**%
	3-Skill CT	2.6%	6.1%	6.3%	5.6%	6.5%	5.2%	5.8%	7.3%	8.4%	7.4%	**6.1**%
	4-Skill CT	4.8%	6.6%	6.2%	7.4%	7.9%	8.0%	7.1%	8.3%	8.4%	8.3%	**7.3**%
No CT	Full CT	8.5%										**8.5**%
2-Skill CT	Full CT	3.1%	3.3%	3.5%	3.4%	2.7%	3.0%	3.3%	1.8%	2.0%	1.7%	**2.8**%
3-Skill CT		6.1%	2.5%	2.4%	3.1%	2.2%	3.5%	2.9%	1.3%	0.1%	1.2%	**2.5**%
4-Skill CT		3.9%	2.1%	2.5%	1.3%	0.7%	0.5%	1.5%	0.2%	0.1%	0.2%	**1.3**%

Table 68: Comparison of No CT, Partial Limited CT, and Full CT for Case M4

Comparison of		MaxC										Avg.
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	
No CT	2-Skill CT	1.3%	3.8%	5.0%	5.3%	6.3%	5.9%	6.3%	5.8%	4.7%	5.7%	**5.0**%
	3-Skill CT	3.4%	6.6%	6.5%	6.0%	5.4%	6.2%	5.8%	6.0%	7.2%	6.2%	**5.9**%
	4-Skill CT	3.2%	5.0%	6.5%	6.2%	7.1%	6.8%	7.1%	7.2%	7.2%	7.4%	**6.4**%
No CT	Full CT	7.6%										**7.6**%
2-Skill CT	Full CT	6.3%	3.9%	2.7%	2.4%	1.4%	1.8%	1.3%	1.8%	3.0%	2.0%	**2.7**%
3-Skill CT		4.3%	1.0%	1.1%	1.7%	2.3%	1.4%	1.9%	1.6%	0.4%	1.5%	**1.7**%
4-Skill CT		4.5%	2.7%	1.2%	1.5%	0.5%	0.8%	0.5%	0.3%	0.4%	0.2%	**1.3**%

Table 69: Comparison of No CT, Partial Limited CT, and Full CT for Case L2

Comparison of		MaxC										Avg.
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	
No CT	2-Skill CT	1.8%	3.9%	3.9%	5.6%	6.8%	6.4%	6.3%	6.1%	6.0%	5.0%	**5.2**%
	3-Skill CT	1.6%	3.4%	4.8%	6.4%	6.7%	6.8%	7.7%	7.3%	7.2%	7.7%	**6.0**%
	4-Skill CT	2.9%	7.0%	5.7%	6.8%	7.2%	6.8%	7.7%	7.9%	8.6%	7.8%	**6.9**%
No CT	Full CT	8.9%										**8.9**%
2-Skill CT	Full CT	7.3%	5.3%	5.2%	3.5%	2.3%	2.7%	2.8%	3.1%	3.1%	4.2%	**3.9**%
3-Skill CT		7.4%	5.7%	4.3%	2.7%	2.4%	2.3%	1.3%	1.8%	1.8%	1.4%	**3.1**%
4-Skill CT		6.2%	2.1%	3.4%	2.2%	1.8%	2.3%	1.3%	1.1%	0.4%	1.2%	**2.2**%

Table 70: Comparison of No CT, Partial Limited CT, and Full CT for Case L3

Comparison of		MaxC										Avg.
		10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	
No CT	2-Skill CT	2.2%	1.8%	4.6%	6.0%	6.6%	6.0%	5.0%	5.6%	5.6%	5.1%	**4.9**%
	3-Skill CT	2.4%	5.7%	6.1%	5.9%	5.9%	6.0%	6.5%	6.2%	6.6%	7.3%	**5.9**%
	4-Skill CT	3.6%	6.3%	6.1%	5.1%	6.5%	6.9%	6.5%	6.3%	6.1%	7.5%	**6.1**%
No CT	Full CT	8.1%										**8.1**%
2-Skill CT	Full CT	6.1%	6.5%	3.7%	2.2%	1.7%	2.3%	3.3%	2.7%	2.6%	3.2%	**3.4**%
3-Skill CT		5.9%	2.6%	2.2%	2.4%	2.4%	2.3%	1.8%	2.1%	1.7%	0.9%	**2.4**%
4-Skill CT		4.7%	2.0%	2.1%	3.2%	1.7%	1.3%	1.8%	2.0%	2.2%	0.7%	**2.2**%

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