Bounds on the number and size of extra dimensions from molecular spectroscopy

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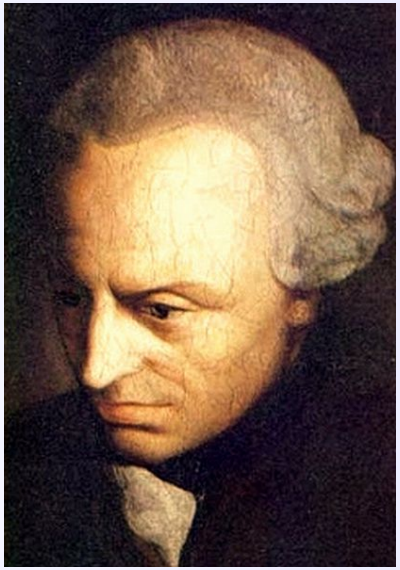
Force laws and dimensions



Immanuel Kant

Immanuel Kant

number of dimensions consequence of Newton's Universal law of gravitation

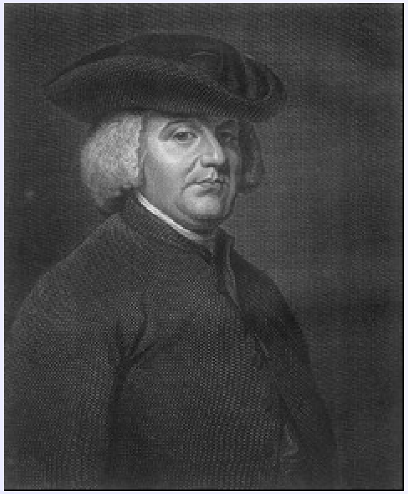
Gauss flux law:

$$\oint \vec{F} \cdot d\vec{A} = kQ_{\text{encl}}$$

3-dim : $A_{\text{V}} \propto r^2 \to F \propto \frac{1}{r^2}$

N-dim : $A_{\text{V}} \propto r^{n-1} \to F \propto \frac{1}{r^{N-1}}$

Extra spatial dimensions



William Paley

William Paley

- Stability of planetary orbits

Paul Ehrenfest

- Stability of atoms only if (N<4)

$$E = \frac{p^2}{2m} + V \sim \frac{\hbar^2}{2mr} - \frac{e^2}{r^{N-2}}$$

Paul Ehrenfest

New Physics with extra dimensions

Kaluza theory:

Einstein field equations in (4+1) dimensions

→ Maxwell equations in (3+1)-dim subspace + scalar field

Klein: Compactification: rolled-up dimensions

String theories:

Bosonic: N=26

Supersymmetric: N=10

Theodor Kaluza

Oskar Klein

ADD theory

Arkani–Hamed, Dimopoulos, Dvali

Phys. Lett. B **429**, 263–272 (1998)

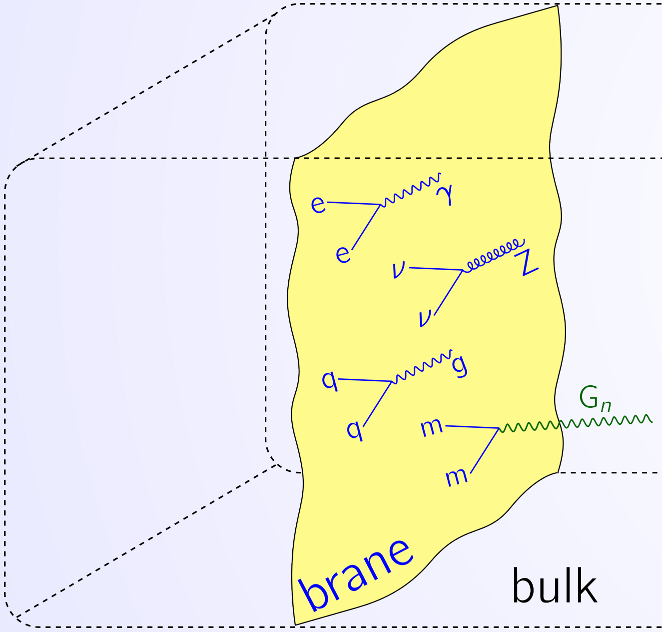
Hierarchy problem: Why is gravity so much weaker?

e.g. between two protons $\frac{V_G}{V_{em}} = 8 \times 10^{-37}$

Or equivalently: Why is the Planck mass M_{Pl} so much bigger?

$$\frac{M_{\mathrm{Pl}}}{M_Z} \sim 10^{17}$$

ADD and branes



Electromagnetism, Weak and Strong forces confined in normal (3+1)-dim space

Gravity leaks out to extra *n*-dim diluting its strength

At time of proposal, possibility of large extra dimensions up to ~mm

ADD and the hierarchy problem

ADD modified gravity with *n* extra dimensions:

$$V_{\mathrm{ADD}}(r) = -G_{(4+n)} \frac{m_1 m_2}{r^{1+n}}$$

for *r* separations larger than the compactification size R_{comp}:

$$V_{\mathrm{ADD}}(r) = -G_{(4+n)} \frac{m_1 m_2}{(R_{\mathrm{comp}})^n r} \qquad r \gg R_{\mathrm{comp}}$$

corresponds to Newtonian gravity: $V_{\mathrm{Newton}}(r) = -G \frac{m_1 m_2}{r}$

$$G_{(4+n)} = (R_{\mathrm{comp}})^n G$$

ADD and the hierarchy problem

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$$G_{(4+n)} = (R_{\mathrm{comp}})^n G$$

$$V_{\mathrm{ADD}}(r) = -G \frac{m_1 m_2}{r} \left(\frac{R_{\mathrm{comp}}}{r} \right)^n$$

enhancement factor

Constraints on ADD

Particle physics searches for new particles

(Stellar cooling from production of new particles)

Cavendish-type experiments and tests of inverse square law for gravity

Propose tests based spectroscopy

Our system: two-proton H_2

Few-body system, simple nuclei

neutrals: H_2, D_2, HD (also ions: H_2^+, D_2^+, HD^+)

Tractable system from *ab initio* theory

H_2: K. Pachucki, J. Komasa: J. Komasa *et al.*, *J. Chem. Theory Comput.* **7**, 3105 (2011)

Effect of weak, strong, and gravitational forces negligible

Any other effect may be new physics

ADD effect in molecules

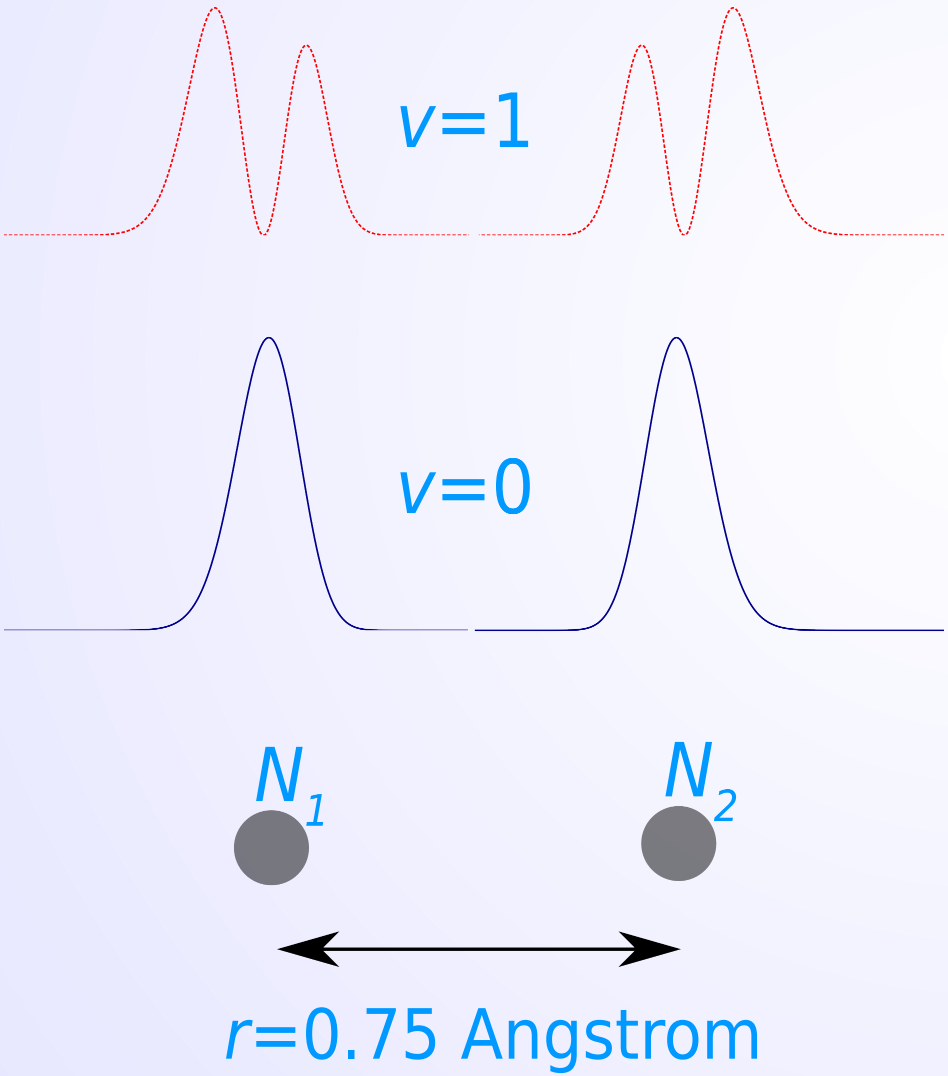
$$V_{\mathrm{Newton}}(r) = G\frac{m_1 m_2}{r} = N_1 N_2 \frac{\alpha_G}{r}\hbar c \qquad \alpha_G = \frac{G m_{\mathrm{p}}^2}{\hbar c}$$

$$V_{\mathrm{ADD}}(r) = \left(\frac{R_{\mathrm{comp}}}{r}\right)^n N_1 N_2 \frac{\alpha_G}{r}\hbar c \qquad \text{for } r \ll R_n$$

Perturbation causing level shift:

$$\langle V_{\mathrm{ADD}} \rangle = \alpha_G \hbar c N_1 N_2 (R_{\mathrm{comp}})^n \int_0^{R_{\mathrm{comp}}} \Psi^*(r) \frac{1}{r^{n+1}} \Psi(r) r^2 \mathrm{d}r$$

Effect on transitions



Level shifts:

$$\langle V_{\mathrm{ADD}} \rangle_1 = \langle \Psi_1 | V_{\mathrm{ADD}} | \Psi_1 \rangle$$

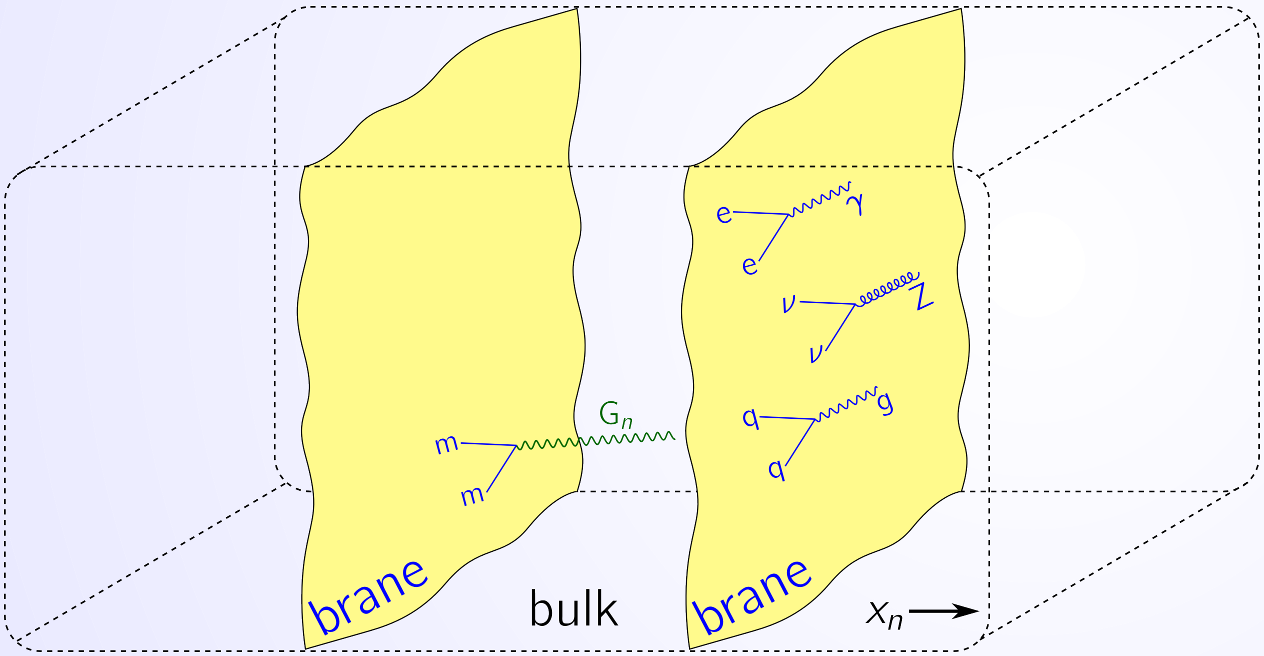
$$\langle V_{\mathrm{ADD}} \rangle_0 = \langle \Psi_0 | V_{\mathrm{ADD}} | \Psi_0 \rangle$$

Transition shift:

$$\Delta \langle V_{\mathrm{ADD}} \rangle = \langle V_{\mathrm{ADD}} \rangle_1 - \langle V_{\mathrm{ADD}} \rangle_0$$

Differential effect

Randall-Sundrum theory



Gravity confined in different brane and SM interactions in another

Gravity leaks out to extra dim with strength exponentially weakening

Extra dimension need not be compactified

Level energies in molecules

$$\Delta E \equiv E_{\rm exp} - E_{\rm calc}$$

$$\Delta E \rightarrow 0 + \delta E$$

$$\delta E = \sqrt{\delta E_{\rm exp}^2 + \delta E_{\rm calc}^2}$$

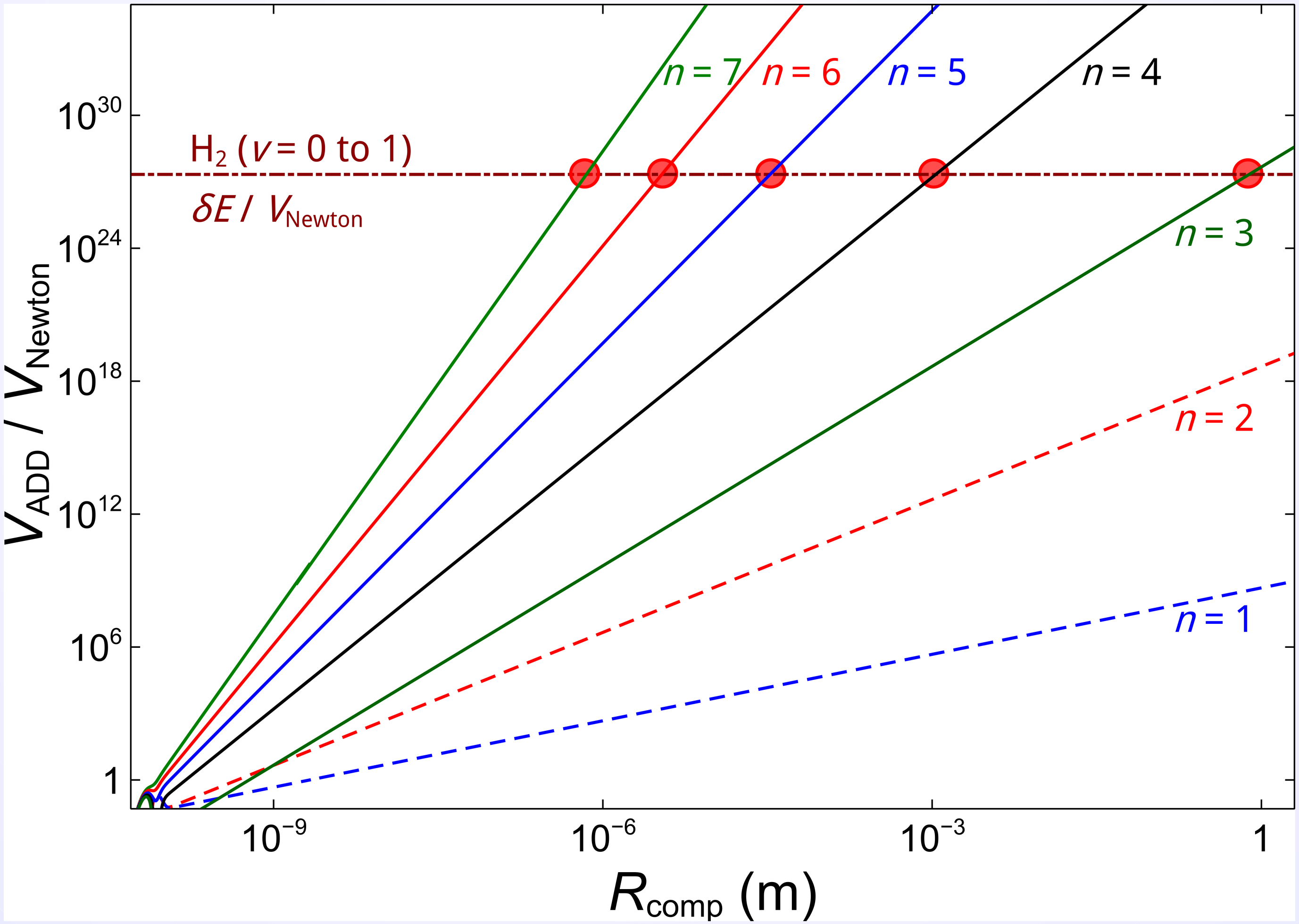
species	transition	δE (cm^{-1})
H_2	$v = 0 \rightarrow 1$	0.00020
	$v = 0 \rightarrow 2$	0.004
	$v = 0 \rightarrow 3$	0.004
	D_0	0.0012
HD	$v = 0 \rightarrow 1$	0.00025
	D_0	0.0012
D_2	$v = 0 \rightarrow 1$	0.00018
	$v = 0 \rightarrow 2$	0.001
	D_0	0.0011
HD^+	$v = 0 \rightarrow 1$	0.000005
	$v = 0 \rightarrow 4$	0.000017

EJS, Koelemeij, Komasa, Pachucki, Eikema, Ubachs, *Phys Rev D* **87**, 112008 (2013).

Constraints from H_2 X(v=0,1)

$$\delta E > \langle V_{\mathrm{ADD}} \rangle$$

$$(R_{\mathrm{comp}})^n < \frac{\delta E}{V_{\mathrm{Newton}}\left(\Delta\left\langle r^{-(n+1)}\right\rangle\right)}$$



Constraints

n	R_{comp}(m)			
	H_2 (1-0)	H_2 D_0	D_2 D_0	HD^+ (4-0)
2	2.2×10^{4}	1.0×10^{4}	4.8×10^{3}	2.8×10^{3}
3	7.7×10^{-1}	1.9×10^{-1}	1.2×10^{-1}	1.0×10^{-1}
4	1.1×10^{-3}	8.5×10^{-4}	5.9×10^{-4}	7.0×10^{-4}
5	3.3×10^{-5}	3.2×10^{-5}	2.4×10^{-5}	3.1×10^{-5}
6	3.4×10^{-6}	3.7×10^{-6}	2.9×10^{-6}	3.0×10^{-6}
7	6.9×10^{-7}	7.8×10^{-7}	6.4×10^{-7}	6.3×10^{-7}

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	H_2 (1-0)	H_2 D_0	D_2 D_0	HD^+ (4-0)
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7	6.9×10^{-7}	7.8×10^{-7}	6.4×10^{-7}	6.3×10^{-7}

LHC (Atlas): $R_3 < 3.7 \times 10^{-10}$ m

Supernova cooling: $R_3 < 4 \times 10^{-7}$ m

Cavendish-type: $R_2 < 130 \times 10^{-6}$ m

QCD (nucleon masses): $R_7 < 2.4 \times 10^{-10}$ m

Conclusions

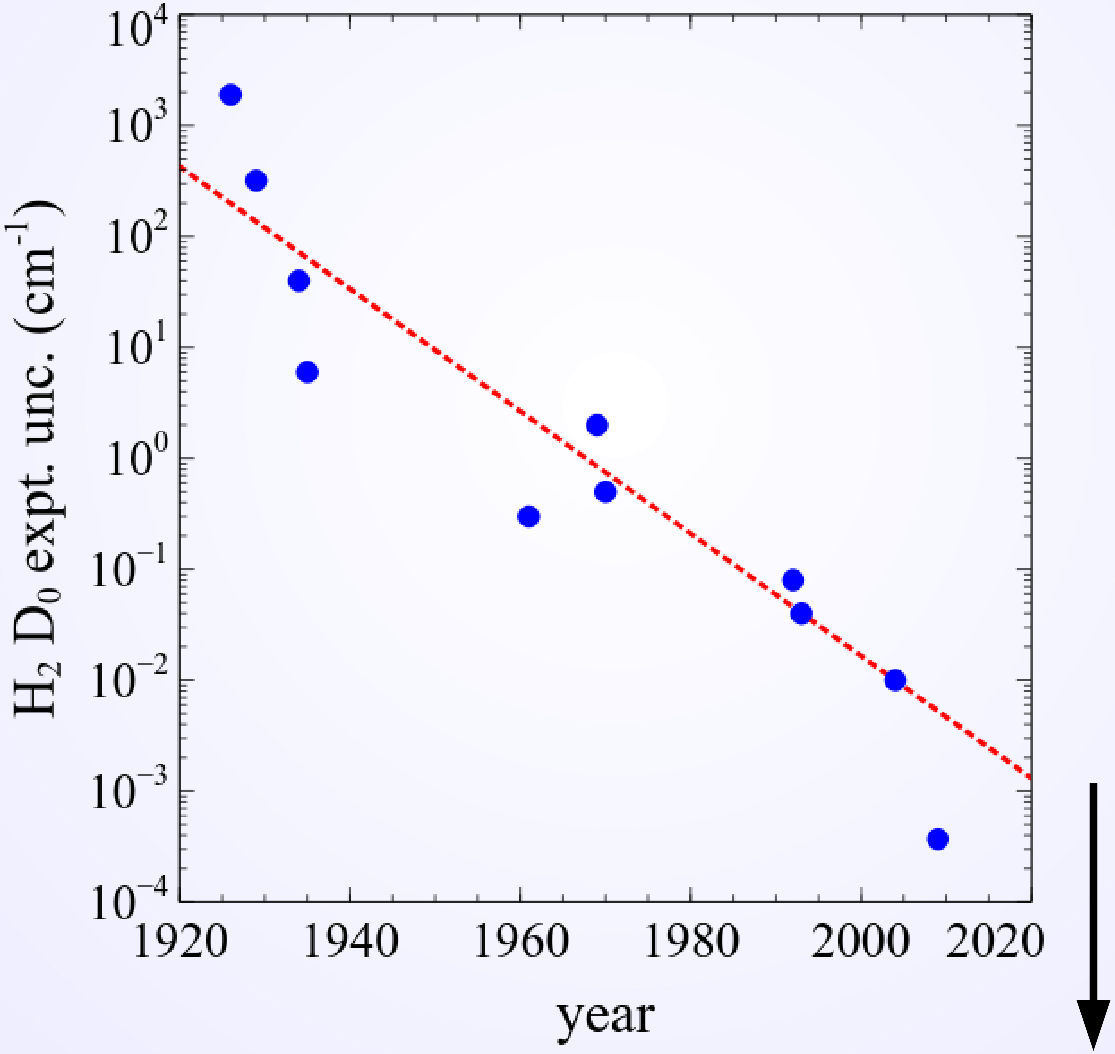
High-precision molecular spectroscopic results and accurate *ab initio* theory are used to constrain new physics

Comparisons set constraints for number and size (volume) of extra spatial dimensions

Spectroscopic method in Angstrom-separation range independent of and complement other methods

Talk based on: EJS, A. N. Schellekens, B. Gato-Rivera and W. Ubachs, *New J. Phys* **17**, 033015 (2015).

Prospects



Natural linewidth:

~10^{-16} cm^{-1}

Thank you for your attention.

ADD effect in molecules

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$$V_{\text{ADD}}(r) = \left(\frac{R_{\text{comp}}}{r}\right)^n N_1 N_2 \frac{\alpha_G}{r}\hbar c \qquad \text{for } r \ll R_n$$

Perturbation causing level shift:

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Differential effect in transitions:

$$\Delta V_{\text{ADD}} = V_{\text{Newton}}(R_{\text{comp}})^n \Delta\left(\left\langle r^{-(n+1)}\right\rangle\right)_{\Psi_1,\Psi_2}$$

ADD and the hierarchy problem

ADD modified gravity with *n* extra dimensions:

$$V_{\mathrm{ADD}}(r) = \frac{m_1 m_2}{M_{(4+n)}^2 M_{(4+n)}^n} \frac{1}{r^{n+1}}$$

$$V_{\mathrm{ADD}}(r) = \frac{m_1 m_2}{M_{(4+n)}^2 (M_{(4+n)} R_n)^n} \frac{1}{r} \qquad \text{for } r \gg R_n$$

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Compared to Newtonian gravity:

$$V_{\mathrm{N}}(r) = G \frac{m_1 m_2}{r} = \frac{m_1 m_2}{M_{\mathrm{Pl}}^2} \frac{1}{r}$$

$$M_{\mathrm{Pl}}^2 = M_{(4+n)}^2 (M_{(4+n)} R_n)^n$$

Fundamental mass $M_{(4+n)}$ may still be small, while observed M_{Pl} becomes large due to extra dimensions.

ADD in molecular transition

$$V_{\mathrm{N}}(r) = G\frac{m_1 m_2}{r} = N_1 N_2 \frac{\alpha_G}{r}\hbar c \qquad \alpha_G = \frac{G m_{\mathrm{p}}^2}{\hbar c}$$

$$V_{\mathrm{ADD}}(r) = \left(\frac{R_n}{r}\right)^n N_1 N_2 \frac{\alpha_G}{r}\hbar c \qquad \text{for } r \ll R_n$$

Perturbation causing level shift:

$$V_{\mathrm{ADD}} = \alpha_G \hbar c N_1 N_2 R_n^n \int_0^{R_n} \Psi^*(r)\frac{1}{r^{n+1}}\Psi(r) r^2 \mathrm{d}r$$

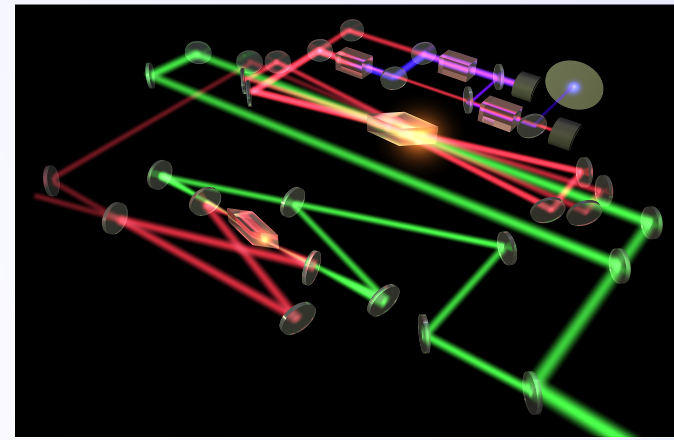
Differential effect in transitions:

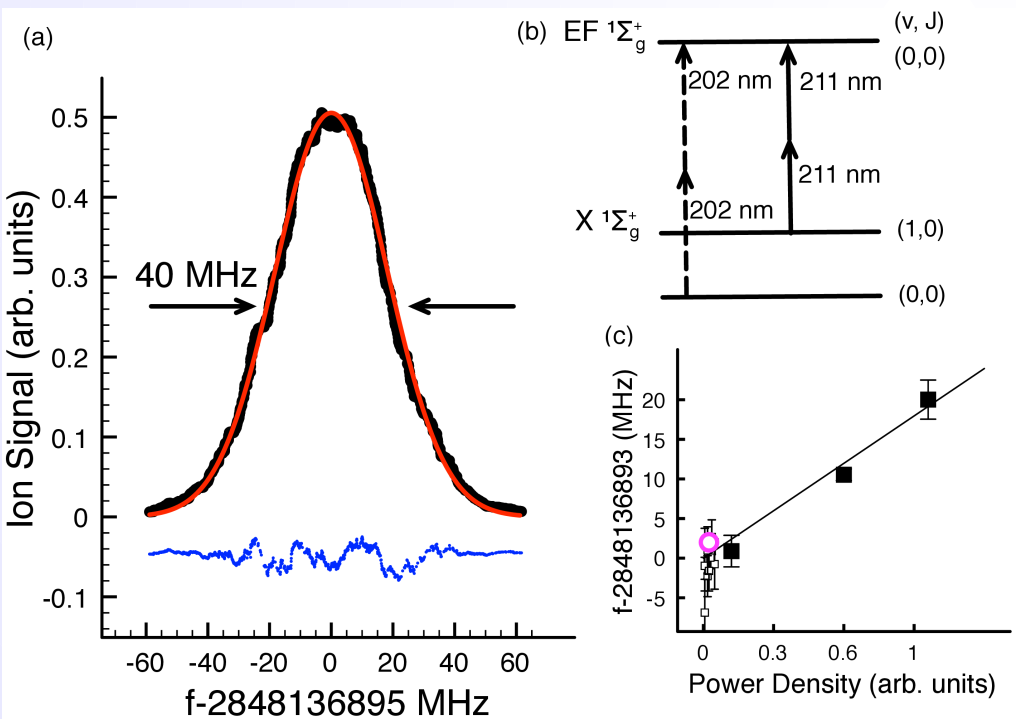
$$\Delta V_{\mathrm{ADD}} = \alpha_G \hbar c N_1 N_2 R_n^n \left(\left\langle r^{-(n+1)} \right\rangle_{\Psi_1} - \left\langle r^{-(n+1)} \right\rangle_{\Psi_2} \right)$$

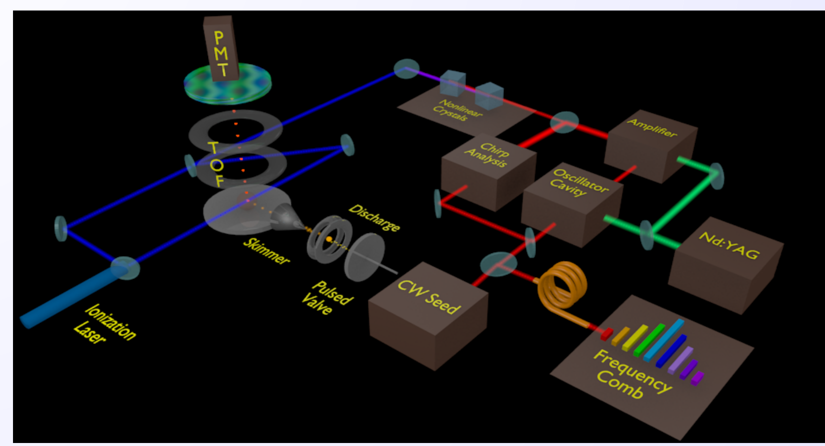
H_2 spectroscopy

Features

- Narrowband UV sources
- Absolute frequency calibration
- 2-photon Doppler-free REMPI
- Sagnac alignment
- Delayed ionisation
- ac-Stark extrapolation







M. L. Niu *et al.*, J. Mol. Spectrosc. **300**, 44 (2014)

Example: H_2 spectroscopy

Features

- Narrowband UV sources
- Absolute frequency calibration
- 2-photon Doppler-free REMPI
- Sagnac alignment
- Delayed ionisation
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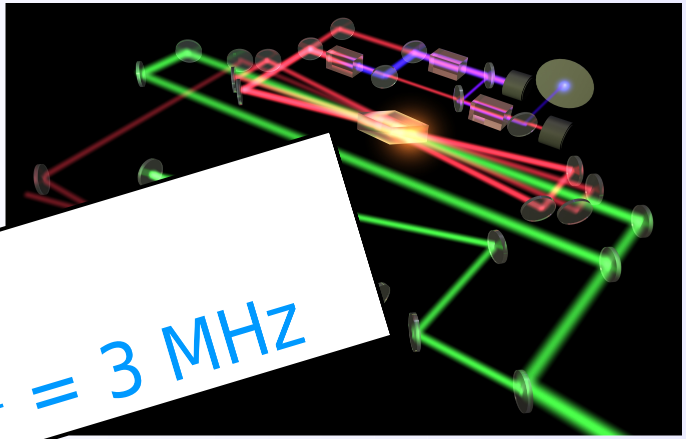
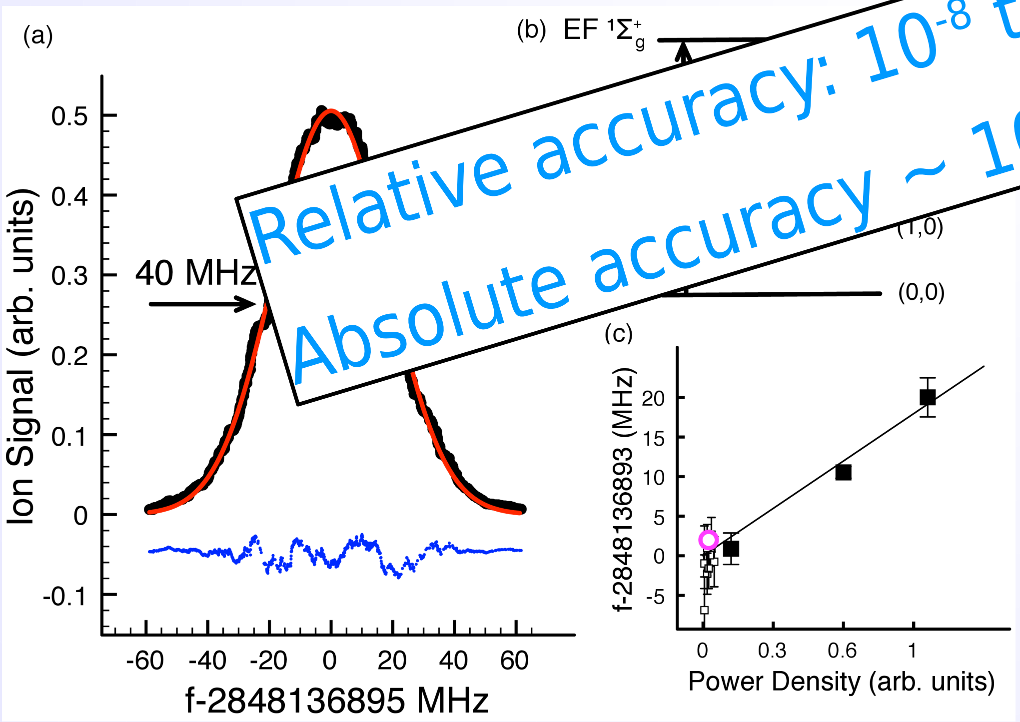
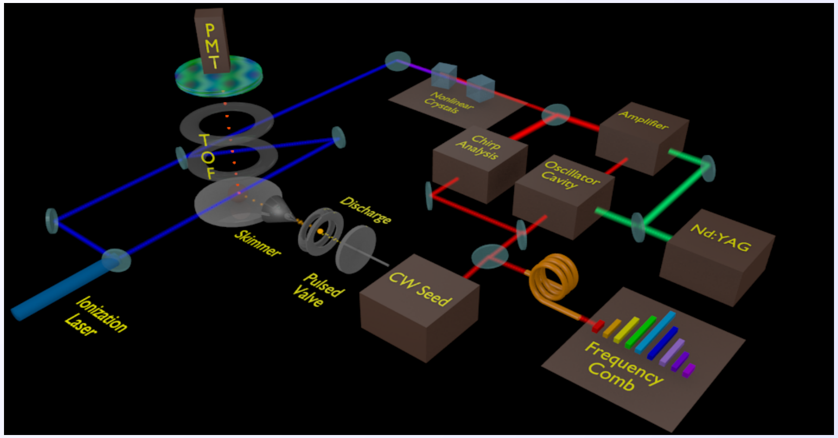


Relative accuracy: 10^{-8} to 10^{-10}

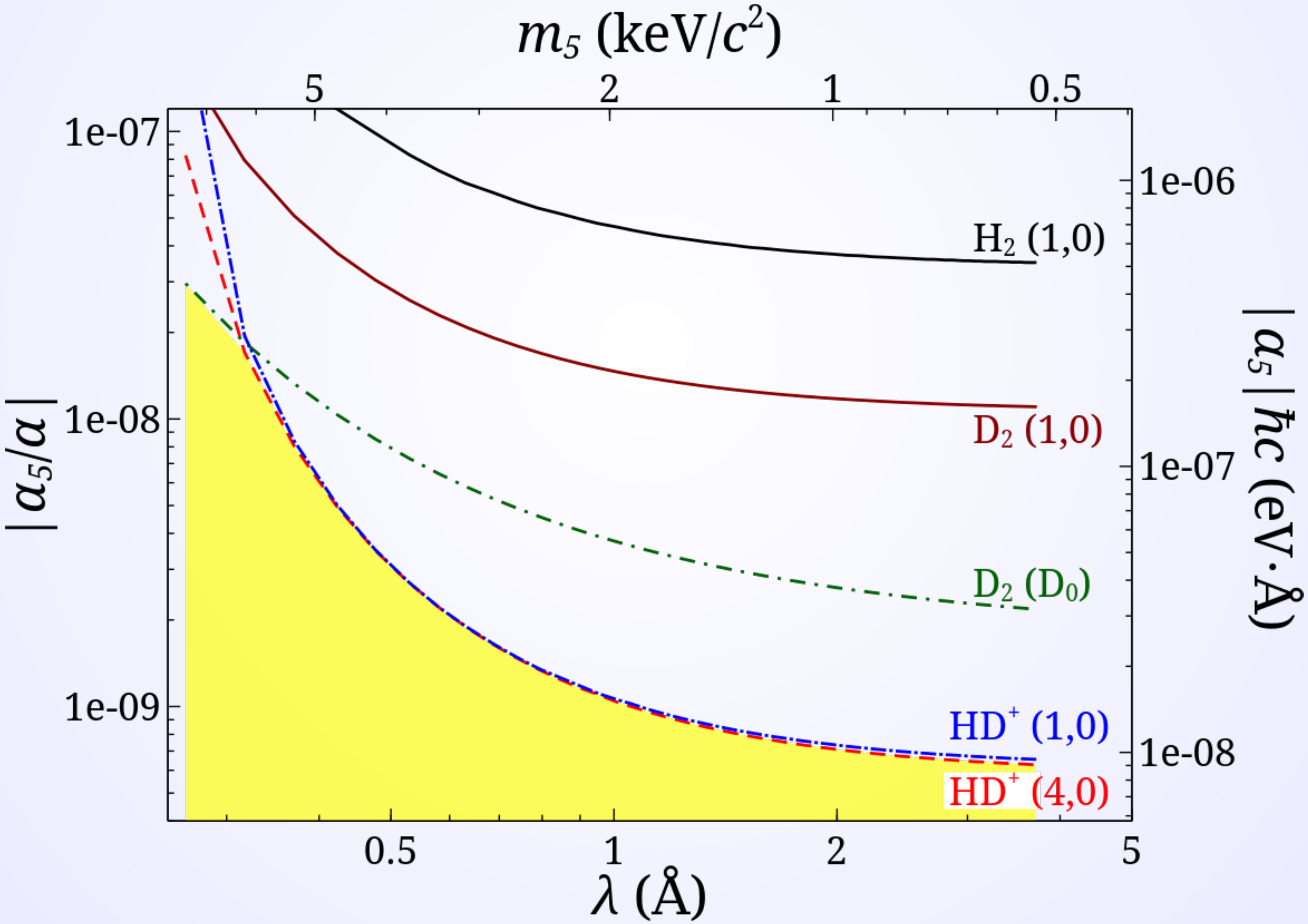
Absolute accuracy ~ 10^{-4} cm^{-1} = 3 MHz





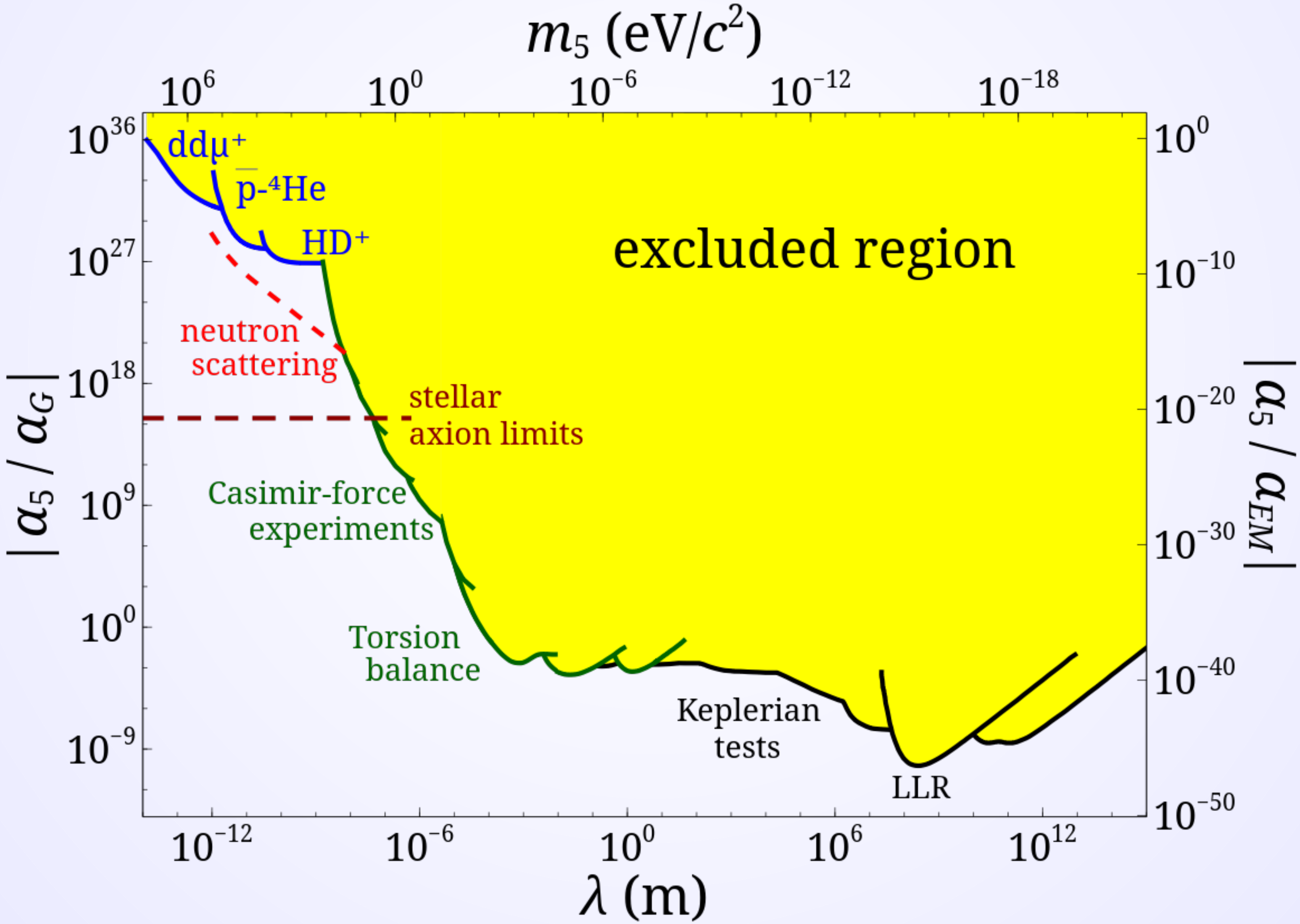
M. L. Niu *et al.*, J. Mol. Spectrosc. **300**, 44 (2014)

Fifth force constraints: $\alpha_5 < \frac{\delta E}{N_1 N_2 \Delta Y_\lambda}$



EJS, Koelemeij, Komasa, Pachucki, Eikema, Ubachs, *Phys Rev D* **87**, 112008 (2013).

Fifth-force constraints



EJS, W. Ubachs, V.I. Korobov, J. Mol. Spectrosc. **300**, 65 (2014)

The SM interactions

In molecules (and atoms): $r \sim a_0$ (Bohr radius)

- Electromagnetic (QED): $D_0 \sim 4.5$ eV
- Weak < 10^{-12} eV
- Strong < 10^{-400} eV
- Gravity ~ 10^{-37} eV