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LOW ATTENUATION FREQUENCY BANDS FOR LAMB WAVES IMMERSED IN VISCOUS FLUIDS: THEORETICAL ANALYSIS AND EXPERIMENTAL VALIDATION

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ABSTRACT: The application of Lamb waves is a promising technique in ultrasonic NDE techniques for inspection and fluid characterization due to multimodal and dispersive characteristics. When a plate is in contact with a viscous fluid these waves are strongly attenuated. However, for most of the Lamb wave modes there is a low attenuation frequency band that could be used for non-destructive testing or fluid characterization. In order to explore this feature the phase velocity and attenuation curves of Lamb modes are experimentally measured in these low attenuation frequency bands, showing good agreement with theory.

INTRODUCTION

Lamb waves propagating in plate-like structures give rise to multimodal propagation which provides more information in a measurement process than simple single-mode propagation. On the other hand the theoretical analysis and physical interpretation of the experimental results become more involved. If the plate is in contact with a liquid, there is strong attenuation due to Leaky waves and viscous losses in the fluid (Dayal & Vikram 1989), nevertheless it is observed that for each mode there is a low attenuation frequency band that could be used for propagating waves through immersed plates.

For the theoretical analysis of Lamb modes the characteristic equations of wave propagation in isotropic solids were derived. The viscous fluid was modeled using the Navier-Stokes equation and the global matrix method was used to describe the multilayered media consisting of fluid-plate-fluid. This method relates the displacement and stress components at each interface resulting in a full matrix of which the solutions are obtained by applying the appropriate boundary conditions (Lowe, 1995). A spectral method was used to obtain the phase velocity and the attenuation coefficient of different Lamb wave modes at low attenuation bands. The experimental results show a good agreement with the theoretical curves.

THEORETICAL ANALYSIS

The equations that govern the elastodynamic behavior of immersed plates are expressed in terms of the potential of displacements. In this analysis an elastic isotropic solid is considered with Lamé coefficients λ and μ and mass bulk density ρ_s. The plate thickness is 2d and it is unlimited along the y axis. The Lamb waves propagate in the xz plane and are described by the displacement velocity vector (Kino, 1987).

$$\vec{U} = \nabla \times \vec{\Psi}_s + \nabla \phi_s \tag{1}$$

Considering wave propagation along the x direction and $\vec{\Psi}_s = \Psi_s \hat{y}$ and that the discontinuity occurs only in the z direction, ∂ / ∂ y = 0, the solutions of ϕ_s and Ψ_s satisfy the Helmholtz´s equations resulting in longitudinal and transversal wave equations, respectively (Kino, 1987; Rose, 1999):

$$\nabla^2 \phi_s + k_l^2 \phi_s = 0 \tag{2}$$
$$\nabla^2 \vec{\Psi}_s + k_t^2 \vec{\Psi}_s = 0 \tag{3}$$

where $k_l^2 = \omega^2 / {v_l}^2$ and $k_t^2 = \omega^2 / {v_t}^2$ are the longitudinal and transversal wavenumber, ω is the radial frequency and v_l and v_t are the longitudinal and transversal velocity, respectively. The equations (2) and (3) are harmonic time dependent and considering a medium which has a finite thickness the potential functions (ϕ_s, Ψ_s) for the solid are given by:

$$\phi_s = [A \cosh(qz) + B \sinh(qz)] e^{j\omega t} e^{-jkx} = Q(qz) e^{j\omega t} e^{-jkx} \tag{4}$$
$$\Psi_s = [C \cosh(sz) + D \sinh(sz)] e^{j\omega t} e^{-jkx} = S(sz) e^{j\omega t} e^{-jkx} \tag{5}$$

where k is the wavenumber of the guided mode, $q^2 = k^2 - k_l^2$ and $s^2 = k^2 - k_t^2$. The equations (4) and (5) correspond to the pair of partial waves propagating in the positive and negative z-direction, respectively. Coefficients A, B, C and D must be determined. Therefore, we can obtain the displacement velocity equation:

$$U_x = \frac{\partial \phi_s}{\partial x} - \frac{\partial \Psi_s}{\partial z} = -jk\phi_s - \frac{\partial \Psi_s}{\partial z} \tag{6}$$
$$U_z = \frac{\partial \phi_s}{\partial z} + \frac{\partial \Psi_s}{\partial x} = \frac{\partial \phi_s}{\partial z} - jk\Psi_s \tag{7}$$

On the other hand, for small displacement velocities in isotropic materials the relationship for stresses components are given by the generalized Hooke's law and with the aid of (4) and (5) and $\mu / j\omega = -j\rho_s \omega / k_t^2$ e $\lambda + 2\mu / j\omega = -j\rho_s \omega / k_l^2$, which leads to:

$$T_z = -\frac{j\omega\rho_s}{k_l^2}(k^2 + Q^2)\phi_s - \frac{2j\omega\rho_s}{k_t^2} k \frac{\partial \Psi_s}{\partial z} \tag{8}$$
$$T_{xz} = -\frac{2j\omega\rho_s}{k_t^2} k \frac{\partial \phi_s}{\partial z} + \frac{j\omega\rho_s}{k_t^2}[k^2 + S^2]\Psi_s \tag{9}$$

Fluid modeling

For the viscous fluid modeling, the equation of motion and the stress tensor are obtained through the Navier-Stokes equation, which relates the continuity and the state equations (Landau & Lifshitz, 1966). The displacement velocity and the components of the stress tensor can be written in the general form as established:

$$U = -\left[\frac{1}{k_f^2} + \frac{1}{\omega\rho_f}\left(\zeta + \frac{2}{3}\right)\right]\nabla(\nabla U) - \frac{j\eta}{\omega\rho_f}\Delta U \tag{10}$$

$$T_z = -P + \eta\left[2\frac{\partial U_z}{\partial z} - \frac{2}{3}\left(\frac{\partial U_x}{\partial x} + \frac{\partial U_z}{\partial z}\right)\right] + \zeta\left(\frac{\partial U_x}{\partial x} + \frac{\partial U_z}{\partial z}\right) \tag{11}$$

$$T_{xz} = \eta\left(\frac{\partial U_x}{\partial z} + \frac{\partial U_z}{\partial x}\right) \tag{12}$$

Where ρ_f is the density, c_f is the propagation velocity, η and ζ are the shear and bulk viscosity coefficients, $k_f = \omega/c_f$ is the wavenumber of the guided mode in the fluid and P is the force acting on a point on the surface (Landau & Lifshitz, 1966). Remembering that the displacement velocity vector can be written in terms of a vector potential, $\vec{\Psi}$, and a scalar potential, ϕ, given in (2) for the solid, the longitudinal and transversal wave equations can be obtained for the fluid like it was done for the solid (Nayfeh & Nagy, 1997):

$$\nabla^2\phi_f + k_{ef}^2\phi_f = 0 \tag{13}$$

$$\nabla \times (\nabla \times \Psi_f) - \frac{\omega\rho_f}{j\eta}\Psi_f = 0 \tag{14}$$

where the effective wavenumber is defined as:

$$k_{ef}^2 = \frac{1}{\frac{1}{k_f^2} + \frac{1}{\omega\rho_f}\left(\zeta + \frac{2}{3}\right)} \tag{15}$$

From the equations (13) and (14) we can obtain the longitudinal and transversal wave equations like for a solid by multiplying the potentials with the coefficients $m_1^2 = k^2 - k_{ef}^2$ and $m_2^2 = k^2 - \omega\rho_f/j\eta$. Using the decomposition in partial waves for the fluid and assuming that the displacement velocity vector is given by (1), the displacement equation for fluid is:

$$U_x = -jk\phi_f - \frac{\partial\Psi_f}{\partial z} \tag{16}$$

$$U_z = \frac{\partial\phi_f}{\partial z} - jk\Psi_f \tag{17}$$

Assuming that $\lambda = \rho_f\omega^2/k_{ef}^2$ and $\mu = j\eta\omega$, it can be proved that (11) and (12) coincide with the expressions of the stress tensor for the solid. Developing the equation tensor for the viscous fluid analogously to the solid:

$$T_z = \eta[m_1^2 + m_2^2]\phi_f - 2j\omega\rho_f\eta k\frac{\partial\Psi_f}{\partial z}, \tag{18}$$

$$T_{xz} = -2j\eta\omega\rho_f\eta k\frac{\partial\phi_f}{\partial z} - \eta[m_1^2 + m_2^2]\Psi_f. \tag{19}$$

Dispersion equations

The characteristic dispersion relations for systems with different layers can be found applying the method of global matrix. This technique relates displacement and stress components of the lower surface of the last layer with the upper surface of the first layer into a single matrix. In this method the matrix is composed of 4(n-1) equations, where n is the number of layers in the system. These equations satisfies the boundary conditions of each layer and are influenced, in a particular interface, by the waves that come from the lower interface (Lowe, 1995). In this work the system considered consists of a solid isotropic plate of thickness 2d completely immersed in a viscous fluid. For modeling the plate, equations (6), (7), (8) and (9) are used. For the upper layer the stress and displacement velocity are defined by (16), (17), (18) and (19). The solutions are separated into symmetric and antisymmetric modes. For symmetrical modes the displacement velocity, U_z, and stress, T_{xz}, components vanish when z = 0, then with the boundary conditions imposed at the interface z = d, the characteristic dispersion relation is given by:

$$\begin{vmatrix} (k^2+s^2)\cosh qd & 2jks\cosh sd & k_t^2/\rho_s\left(j\rho_f+2\mu k^2/\omega\right) & 2\mu k m_2 k_t^2/\omega\rho_s \\ 2jkq\sinh qd & -(k^2+s^2)\sinh sd & 2km_1\mu k_t^2/\omega\rho_s & -k_t^2/\rho_s\left(j\rho_f+2\mu k^2/\omega\right) \\ q\sinh qd & jk\sinh sd & -m_1 & k \\ jk\cosh qd & -s\cosh sd & k & m_2 \end{vmatrix} = 0 \tag{20}$$

For antisymmetric modes the displacement components, U_x, and stress, T_{zz}, vanish when z = 0, then it follows that the characteristic dispersion relation for these modes is given by:

$$\begin{vmatrix} (k^2+s^2)\sinh qd & 2jks\sinh sd & k_t^2/\rho_s\left(j\rho_f+2\mu k^2/\omega\right) & 2\mu k m_2 k_t^2/\omega\rho_s \\ 2jkq\cosh qd & -(k^2+s^2)\cosh sd & 2km_1\mu k_t^2/\omega\rho_s & -k_t^2/\rho_s\left(j\rho_f+2\mu k^2/\omega\right) \\ q\cosh qd & jk\cosh \mathrm{sd} & -m_1 & k \\ jk\sinh qd & -s\sinh sd & k & m_2 \end{vmatrix} = 0 \tag{21}$$

EXPERIMENTAL SETUP AND RESULTS

An experimental verification was conducted, regarding the measurement of the low attenuation values for some symmetric (S0, S1 and S2) and antisymmetric (A1 and A2) modes in several frequency bands. Aluminum plates of 1, 2 and 3 mm-thickness immersed in water were tested. The Lamb waves are generated and received by two wideband 5 MHz transducers (Panametrics) coupled to wedges and operated in pitch-catch. One of the transducers is excited with Gaussian-envelope tone bursts generated by a function generator (Tektronix AFG 3101). The pulse central frequencies are chosen according to the low attenuation frequency bands obtained theoretically.

These bursts are amplified by using a 204L Broadband Power Amplifier (Electronics & Innovation) before feeding the emitting transducer. The waves received by the other transducer are digitalized by a digital oscilloscope (Agilent MSO7014B) and downloaded to a computer for signal processing. The distance between the transducers was varied from 15 to 85 mm, in 1 mm steps. The experimental configuration is shown in Figure 1. Symmetric modes are obtained using a wedge made of methacrylate with an angle $\alpha = 30°$ and for antisymmetric modes, α was chosen to be 25°.

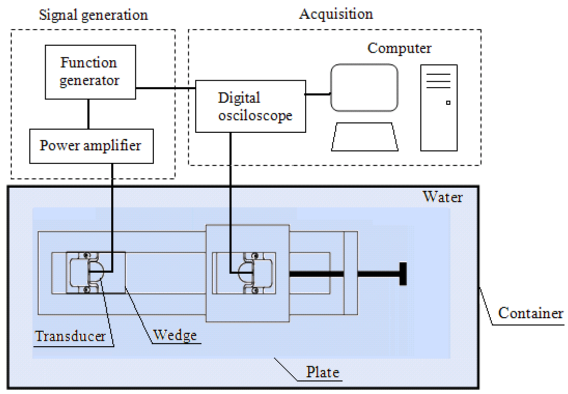
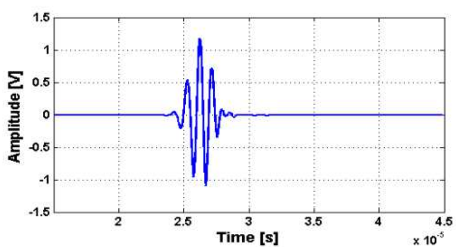
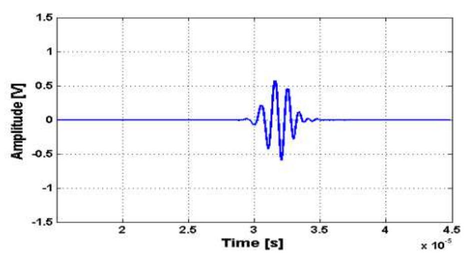


Figure 1 – Experimental setup for measurement of Lamb wave modes in a plate immersed in a viscous fluid.

Dispersion and attenuation curves are obtained using a time-frequency treatment. The S0 symmetric mode at different positions of the receiver transducer is plotted on Figure 2. It was obtained using a central frequency of 1 MHz and the 1 mm-thickness immersed plate. The amplitude of the detected signal decays exponentially with the distance.



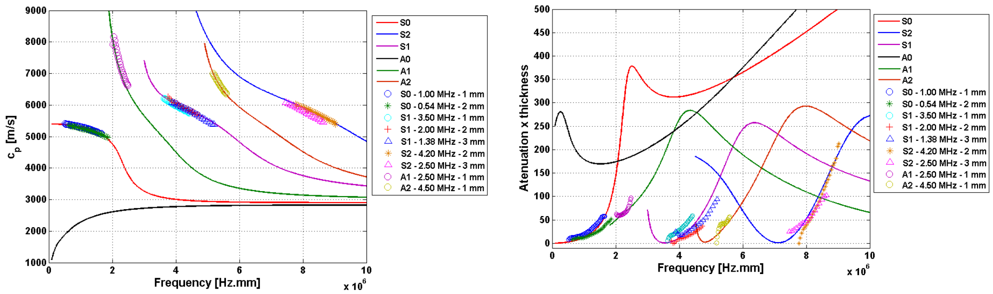
(a)



(b)

Figure 2 - Propagating pulses at 1 MHz for 1 mm-thickness immersed plate corresponding to the S0 mode. a) For 15 mm distance through the plate between both transducers and b) for 45 mm distance.

To analyze the signals, a sliding Gaussian window is translated according to the maximum amplitude of the signal in order to eliminate undesired frequencies. The spectral components of the signals are obtained with successive Fourier Transforms performed with 2048 points. From the magnitude and phase of the FFT phase velocity and attenuation curves are calculated. The low attenuation frequency ranges studied were around 1 MHz (S0 mode), 4,2 MHz (S1 mode), 8 MHz (S2 mode), 2,5 MHz (A1 mode) and 5,5 MHz (A2 mode). Both theoretical and experimental results are presented together on Figure 3 for different plate thicknesses and excitation frequency. The experimental results are in accordance with the theoretical model. Small discrepancies found for attenuation data may be originated on differences between real and theoretical values of the materials used in modeling and imperfections and fluid motion in the plate surfaces.



(a) (b)

Figure 3 – Theoretical and experimental results for the Lamb wave propagation in plate immersed in water. a) Phase velocity versus frequency. b) Attenuation versus frequency.

CONCLUSIONS

In this paper an analytical technique to model the propagation of mechanical waves in submerged plates in a viscous fluid is described. This technique, can also be used for fluid-coated plates. The viscous fluid was modeled using the Navier-Stokes equation and the global matrix method was used to model the multilayered media consisting of fluid-plate-fluid. Theoretical calculus was carried out to obtain the phase velocity and the attenuation curves as a function of frequency. Frequency bands of low attenuation were observed and an experimental verification was conducted regarding the measurement of these low attenuation values for some symmetric and antisymmetric modes (S0, S1, S2, A1 and A2) in these selected frequency bands. From the acquired waveforms the attenuation and velocities were calculated as a function of frequency for several modes. The experimental results show a good agreement with the theoretical curves.

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