CORE
Metadata, citation and similar papers at core.ac.uk
Provided by Digital.CSIC

Games for the Strategic Influence of Expectations

Lluís Godo
Artificial Intelligence Research Institute, IIIA
Spanish National Research Council, CSIC
Campus UAB, 08193 Bellaterra, Spain
godo@iiia.csic.es

Enrico Marchioni
Institut de Recherche en Informatique de Toulouse
Université Paul Sabatier
118 Route de Narbonne, 31062 Toulouse, France
enrico.marchioni@irit.fr

We introduce a new class of games where each player's aim is to randomise her strategic choices in order to affect the other players' expectations aside from her own. The way each player intends to exert this influence is expressed through a Boolean combination of polynomial equalities and inequalities with rational coefficients. We offer a logical representation of these games as well as a computational study of the existence of equilibria.[1]

1 Introduction

In the situations of strategic interactions modelled in Game Theory, the goal of each player is essentially the maximisation of her own expected payoff. Players, however, often care not only about maximising their own expectation, but also about influencing other players' expected outcomes. As an example, consider a number of competing investment banks selling and buying tradable assets so that the trading of financial products affects each other's profit. These banks might randomize their choices and obviously aim at maximizing their expected profit. Still, their strategy might go beyond the choice of a specific investment and they might be interested in influencing the market and the behavior of other banks possibly undermining the expected gain of their competitors.

In this work, we offer logical models to formalize these kinds of strategic interactions, called Expectation Games, where each player's aim is to randomise her strategic choices in order to affect the other players' expectations over an outcome as well as their own expectation. Expectation Games are an extension of Łukasiewicz games [9] and are based on the logics $\mathsf{E}(\mathfrak{G})$ that formalise reasoning about expected payoffs in a class of Łukasiewicz games [4]. Łukasiewicz games [9], a generalisation of Boolean games [7], involve a finite set of players P_i each controlling a finite set of propositional variables V_i, whose strategy corresponds to assigning values from the scale $L_k = \left\{0, \frac{1}{k}, \ldots, \frac{k-1}{k}, 1\right\}$ to the variables in V_i. Strategies can be interpreted as efforts or costs, and each player's strategic choice can be seen as an assignment to each controlled variable carrying an intrinsic cost. Each player is given a finitely-valued Łukasiewicz logic formula φ_i, with variables from $\bigcup_i^n \mathsf{V}_i$, whose valuation is interpreted as the payoff function for P_i and corresponds to the restriction over L_k of a continuous piecewise linear polynomial function [2].

Expectation Games expand Lukasiewicz games by assigning to each player P_i a modal formula Φ_i of the logic $\mathsf{E}(\mathfrak{G})$, whose interpretation corresponds to a piecewise rational polynomial function whose variables are interpreted as the expected values of the payoff functions φ_i. Each formula Φ_i is then meant to represent a player's goal concerning the relation between her and other players' expectations.

[1] This extended abstract is based on the article [4] and an upcoming extended version of the same work.

F. Mogavero, A. Murano, and M.Y. Vardi (Eds.):
2nd Workshop on Strategic Reasoning 2014 (SR14)
EPTCS 146, 2014, pp. 9–15, doi:10.4204/EPTCS.146.2

© L. Godo, E. Marchioni
This work is licensed under the
Creative Commons Attribution License.

2 Logical Background

The language of Łukasiewicz logic Ł (see [2]) is built from a countable set of propositional variables $\{p_1, p_2, \ldots\}$, the binary connective $\rightarrow$ and the truth constant $\overline{0}$ (for falsity). Further connectives are defined as follows:

$\neg\varphi$	is	$\varphi \rightarrow \overline{0}$,	$\varphi \wedge \psi$	is	$\varphi \& (\varphi \rightarrow \psi)$,
$\varphi \& \psi$	is	$\neg(\varphi \rightarrow \neg\psi)$,	$\varphi \vee \psi$	is	$((\varphi \rightarrow \psi) \rightarrow \psi)$,
$\varphi \oplus \psi$	is	$\neg(\neg\varphi \& \neg\psi)$,	$\varphi \leftrightarrow \psi$	is	$(\varphi \rightarrow \psi) \& (\psi \rightarrow \varphi)$,
$\varphi \ominus \psi$	is	$\varphi \& \neg\psi$,	$d(\varphi, \psi)$	is	$\neg(\varphi \leftrightarrow \psi)$.

Let *Form* denote the set of Łukasiewicz logic formulas. A valuation e from *Form* into $[0,1]$ is a mapping $e : Form \rightarrow [0,1]$ assigning to all propositional variables a value from the real unit interval (with $e(\overline{0}) = 0$) that can be extended to complex formulas as follows:

$$\begin{array}{rclrcl} e(\varphi \rightarrow \psi) & = & \min(1 - e(\varphi) + e(\psi), 1) & e(\neg\varphi) & = & 1 - e(\varphi) \\ e(\varphi \& \psi) & = & \max(0, e(\varphi) + e(\psi) - 1) & e(\varphi \oplus \psi) & = & \min(1, e(\varphi) + e(\psi)) \\ e(\varphi \ominus \psi) & = & \max(0, e(\varphi) - e(\psi)) & e(\varphi \wedge \psi) & = & \min(e(\varphi), e(\psi)) \\ e(\varphi \vee \psi) & = & \max(e(\varphi), e(\psi)) & e(d(\varphi, \psi)) & = & |e(\varphi) - e(\psi)| \\ e(\varphi \leftrightarrow \psi) & = & 1 - |e(\varphi) - e(\psi)| & & & \end{array}$$

A valuation e *satisfies* a formula φ if $e(\varphi) = 1$. As usual, a set of formulas is called a theory. A valuation e satisfies a theory T, if $e(\psi) = 1$, for every $\psi \in T$.

Infinite-valued Łukasiewicz logic has the following axiomatisation:

(Ł1) $\varphi \rightarrow (\psi \rightarrow \varphi)$, (Ł2) $(\varphi \rightarrow \psi) \rightarrow ((\psi \rightarrow \chi) \rightarrow (\varphi \rightarrow \chi))$,
(Ł3) $(\neg\varphi \rightarrow \neg\psi) \rightarrow (\psi \rightarrow \varphi)$, (Ł4) $((\varphi \rightarrow \psi) \rightarrow \psi) \rightarrow ((\psi \rightarrow \varphi) \rightarrow \varphi)$.

The only inference rule is *modus ponens*, i.e.: from $\varphi \rightarrow \psi$ and φ derive ψ.

A *proof* in Ł is a sequence $\varphi_1, \ldots, \varphi_n$ of formulas such that each φ_i either is an axiom of Ł or follows from some preceding φ_j, φ_k $(j, k < i)$ by modus ponens. We say that a formula φ can be derived from a theory T, denoted as $T \vdash \varphi$, if there is a proof of φ from a set $T' \subseteq T$. A theory T is said to be consistent if $T \not\vdash \overline{0}$.

Łukasiewicz logic is complete with respect to deductions from finite theories for the given semantics, i.e.: for every finite theory T and every formula φ, $T \vdash \varphi$ iff every valuation e that satisfies T also satisfies φ.

For each $k \in \mathbb{N}$, the finite-valued Łukasiewicz logic $Ł_k$ is the schematic extension of Ł with the axiom schemas:

(Ł5) $(n-1)\varphi \leftrightarrow n\varphi$, (Ł6) $(k\varphi^{k-1})^n \leftrightarrow n\varphi^k$,

for each integer $k = 2, \ldots, n-2$ that does not divide $n-1$, and where $n\varphi$ is an abbreviation for $\varphi \oplus \cdots \oplus \varphi$ (n times) and φ^k is an abbreviation for $\varphi \& \ldots \& \varphi$, ($k$ times). The notions of valuation and satisfiability for $Ł_k$ are defined as above just replacing $[0,1]$ by

$$L_k = \left\{0, \frac{1}{k}, \ldots, \frac{k-1}{k}, 1\right\}$$

as set of truth values. Every $Ł_k$ is complete (in the above sense) with respect to deductions from finite theories for the given semantics.

It is sometimes useful to introduce constants in addition to $\overline{0}$ that will denote values in the domain L_k. Specifically, we will denote by $Ł_k^c$ the Łukasiewicz logic obtained by adding constants $\overline{c}$ for every value $c \in L_k$. We assume that valuation functions e interpret such constants in the natural way: $e(\overline{c}) = c$.

A McNaughton function [2] is a continuous piecewise linear polynomial functions with integer coefficients over the *n*th-cube $[0,1]^n$. To each Łukasiewicz formula $\varphi(p_1,\ldots,p_n)$ we can associate a McNaughton function f_φ so that, for every valuation e

$$f_\varphi(e(p_1),\ldots,e(p_n)) = e(\varphi(p_1,\ldots,p_n)).$$

Every Ł-formula is then said to define a McNaughton function. The converse is also true, i.e. every continuous piecewise linear polynomial function with integer coefficients over $[0,1]^n$ is definable by a formula in Łukasiewicz logic. In the case of finite-valued Łukasiewicz logics, the functions defined by formulas are just the restrictions of McNaughton functions over $(L_k)^n$. In this sense, we can associate to every formula $\varphi(p_1,\ldots,p_n)$ from $Ł_k$ a function $f_\varphi : (L_k)^n \to L_k$. As for each $Ł_k^c$, the functions defined by a formula are combinations of restrictions of McNaughton functions and, in addition, the constant functions for each $c \in L_k$. The class of functions definable by $Ł_k^c$-formulas exactly coincides with the class of all functions $f : (L_k)^n \to L_k$, for every $n \geq 0$.

The expressive power of infinite-valued Łukasiewicz logic lies in, and is limited to, the definability of piecewise linear polynomial functions. Expanding Ł with the connectives $\odot, \to_\Pi$ of Product logic [6], interpreted as the product of reals and as the truncated division, respectively, significantly augments the expressive power of the logic. The $Ł\Pi\frac{1}{2}$ logic [3] is the result of this expansion, obtained by adding the connectives $\odot, \to_\Pi, \overline{\frac{1}{2}}$, whose valuations e extend the valuations for Ł as follows:

$$e(\varphi \odot \psi) \;=\; e(\varphi)\cdot e(\psi), \qquad e(\varphi \to_\Pi \psi) \;=\; \begin{cases} 1 & e(\varphi) \leq e(\psi) \\ \frac{e(\psi)}{e\varphi} & \text{otherwise} \end{cases}, \qquad e\left(\overline{\tfrac{1}{2}}\right) \;=\; \tfrac{1}{2}.$$

Notice that the presence of the constant $\overline{\frac{1}{2}}$ makes it possible to define constants for all rationals in $[0,1]$ (see [3]). $Ł\Pi\frac{1}{2}$'s axioms include the axioms of Łukasiewicz and Product logics (see [6]) as well as the following additional axioms, where $\Delta\varphi$ is $\neg\varphi \to_\Pi \overline{0}$:

- (ŁΠ1) $(\varphi \odot \psi) \ominus (\varphi \odot \chi) \leftrightarrow \varphi \odot (\psi \ominus \chi)$,
- (ŁΠ2) $\Delta(\varphi \to \psi) \to (\varphi \to_\Pi \psi)$,
- (ŁΠ3) $\Delta(\varphi \to_\Pi \psi) \to (\varphi \to \psi)$,
- (ŁΠ4) $\overline{\frac{1}{2}} \leftrightarrow \neg\overline{\frac{1}{2}}$.

The deduction rules are modus ponens for & and $\to$, and the necessitation rule for Δ, i.e.: from φ derive $\Delta\varphi$. $Ł\Pi\frac{1}{2}$ is complete with respect to deductions from finite theories for the given semantics [3].

While Ł is the logic of McNaughton functions, $Ł\Pi\frac{1}{2}$ is the logic of piecewise rational functions over $[0,1]^n$, for all n (see [10]). In fact, the function defined by each $Ł\Pi\frac{1}{2}$-formula with n variables corresponds to a supremum of rational fractions

$$\frac{P(x_1,\ldots,x_n)}{Q(x_1,\ldots,x_n)}$$

over $[0,1]^n$, where $P(x_1,\ldots,x_n), Q(x_1,\ldots,x_n)$ are polynomials with rational coefficients. Conversely, every piecewise rational function with over the unit cube $[0,1]^n$ can be defined by an $Ł\Pi\frac{1}{2}$-formula.

3 Logics for Łukasiewicz Games with Expectations

In this section we briefly introduce Łukasiewicz games on $Ł_k^c$ along with the logics $\mathsf{E}(\mathfrak{G})$ to represent expected payoffs in classes of games. $\mathsf{E}(\mathfrak{G})$ will be the basis upon which Expectation Games are defined.

3.1 Łukasiewicz Games

Definition 3.1 ([9]) *A Łukasiewicz game $\mathscr{G}$ on $Ł_k^c$ is a tuple $\mathscr{G} = \langle \mathsf{P}, \mathsf{V}, \{\mathsf{V}_i\}, \{\mathsf{S}_i\}, \{\varphi_i\}\rangle$ where:*

1. $\mathsf{P} = \{P_1, \ldots, P_n\}$ *is a set of* players*;*
2. $\mathsf{V} = \{p_1, \ldots, p_m\}$ *is a finite set of propositional variables;*
3. *For each $i \in \{1, \ldots, n\}$, $\mathsf{V}_i \subseteq \mathsf{V}$ is the set of propositional variables under control of player P_i, so that the sets V_i form a partition of V, with $|\mathsf{V}_i| = m_i$, and $\sum_{i=1}^n m_i = m$.*
4. *For each $i \in \{1, \ldots, n\}$, S_i is the strategy set for player P_i that consists of all valuations $s : \mathsf{V}_i \to L_k$ of the propositional variables in V_i, i.e. $\mathsf{S}_i = \{s \mid s : \mathsf{V}_i \to L_k\}$.*
5. *For each $i \in \{1, \ldots, n\}$, $\varphi_i(p_1, \ldots, p_t)$ is an $Ł_k^c$-formula, built from variables in V, whose associated function $f_{\varphi_i} : (L_k)^t \to L_k$ corresponds to the* payoff function *of P_i, and whose value is determined by the valuations in $\{\mathsf{S}_1, \ldots, \mathsf{S}_n\}$.*

We denote by $\mathsf{S} = \mathsf{S}_1 \times \cdots \times \mathsf{S}_n$ the product of the strategy spaces. A tuple $\vec{s} = (s_1, \ldots, s_n) \in \mathsf{S}$ of strategies is called a *strategy combination*. With an abuse of notation, we denote by $f_{\varphi_i}(\vec{s})$ the value of the payoff function f_{φ_i} under the valuation corresponding to the strategy combination $\vec{s}$.

Given a game $\mathscr{G}$, let $\delta : \mathsf{P} \to \{1, \ldots, m\}$ be a function assigning to each player P_i an integer from $\{1, \ldots, m\}$ that corresponds to the number of variables in V_i: i.e.: $\delta(P_i) = m_i$. δ is called a *variable distribution function*. Given a game $\mathscr{G}$, the *type* of $\mathscr{G}$ is the triple $\langle n, m, \delta\rangle$, where n is the number of players, m is the number of variables in V, and δ is the variable distribution function for $\mathscr{G}$.

Definition 3.2 (Class) *Let $\mathscr{G}$ and $\mathscr{G}'$ be two Łukasiewicz games $\mathscr{G}$ and $\mathscr{G}'$ on $Ł_k^c$ of type $\langle n, m, \delta\rangle$ and $\langle n, m, \delta'\rangle$, respectively. We say that $\mathscr{G}$ and $\mathscr{G}'$ belong to the same class $\mathfrak{G}$ if there exists a permutation $\mathfrak{j}$ of the indices $\{1, \ldots, n\}$ such that, for all P_i, $\delta(P_{\mathfrak{j}(i)}) = \delta'(P_i)$.*

Notice that what matters in the definition of a type is not which players are assigned certain variables, but rather their distribution.

Let $\mathscr{G}$ be a Łukasiewicz game on $Ł_k^c$. A *mixed strategy* π_i for player P_i is a probability distribution on the strategy space S_i. By π_{-i}, we denote the tuple of mixed strategies $(\pi_1, \ldots, \pi_{i-1}, \pi_{i+1}, \ldots, \pi_n)$. P_{-i} denotes the tuple of players $(P_1, \ldots, P_{i-1}, P_{i+1}, \ldots, P_n)$. Given the mixed strategies $(\pi_1, \ldots, \pi_n)$, the *expected payoff* for P_i of playing π_i, when P_{-i} play π_{-i}, is given by

$$exp_{\varphi_i}(\pi_i, \pi_{-i}) = \sum_{\vec{s}=(s_1,\ldots,s_n)\in\mathsf{S}} \left(\left(\prod_{j=1}^{n} \pi_j(s_j) \right) \cdot f_{\varphi_i}(\vec{s}) \right)$$

3.2 The Logics $\mathsf{E}(\mathfrak{G})$

Given a class of games $\mathfrak{G}$ on $Ł_k^c$, the language of $\mathsf{E}(\mathfrak{G})$ is defined as follows: (1) The set NModF of non-modal formulas corresponds to the set of $Ł_k^c$-formulas built from the propositional variables $p_1, \ldots, p_m$. (2) The set ModF of modal formulas is built from the atomic modal formulas $\mathsf{E}\varphi$, with $\varphi \in$ NModF, using the connectives of the $Ł\Pi\frac{1}{2}$ logic. $\mathsf{E}\varphi$ is meant to encode a player's expected payoff of playing a mixed strategy, given the payoff function associated to φ. Nested modalities are not allowed.

A model **M** for $\mathsf{E}(\mathfrak{G})$ is a tuple $\langle \mathsf{S}, e, \{\pi_i\}\rangle$, such that:

1. $\mathsf{S} = \mathsf{S}_1 \times \cdots \times \mathsf{S}_n$ is the set of all strategy combinations, i.e.

$$\{\vec{s} = (s_1, \ldots, s_n) \mid (s_1, \ldots, s_n) \in \mathsf{S}_1 \times \cdots \times \mathsf{S}_n\}.$$

2. $e : (\text{NModF} \times \mathsf{S}) \to L_k$ is a valuation of non-modal formulas, such that, for each $\varphi \in \text{NModF}$ $e(\varphi, \vec{s}) = f_\varphi(\vec{s})$, where f_φ is the function associated to φ and $\vec{s} = (s_1, \ldots, s_n)$.
3. $\pi_i : \mathsf{S}_i \to [0,1]$ is a probability distribution, for each P_i.

The truth value of a formula Φ in $\mathbf{M}$ at $\vec{s}$, denoted $\|\Phi\|_{\mathbf{M},\vec{s}}$, is inductively defined as follows:

1. If Φ is a non-modal formula $\varphi \in \text{NModF}$, then $\|\varphi\|_{\mathbf{M},\vec{s}} = e(\varphi, \vec{s})$,
2. If Φ is an atomic modal formula $\mathsf{E}\varphi$, then $\|\mathsf{E}\varphi\|_{\mathbf{M},\vec{s}} = exp_\varphi(\pi_1, \ldots, \pi_n)$.
3. If Φ is a non-atomic modal formula, its truth value is computed by evaluating its atomic modal subformulas and then by using the truth functions associated to the $\text{Ł}\Pi\frac{1}{2}$-connectives occurring in Φ.

Since the valuation of a modal formula Φ does not depend on a specific strategy combination but only on the model $\mathbf{M}$, we will often simply write $\|\Phi\|_{\mathbf{M}}$ to denote the valuation of Φ in $\mathbf{M}$.

Theorem 3.3 (Completeness) *Let Γ and Φ be a finite modal theory and a modal formula in $\mathsf{E}(\mathfrak{G})$. Then, $\Gamma \vdash_{\mathsf{E}(\mathfrak{G})} \Phi$ if and only if for every model $\mathbf{M}$ such that, for each $\Psi \in \Gamma$, $\|\Psi\|_{\mathbf{M}} = 1$, also $\|\Phi\|_{\mathbf{M}} = 1$.*

4 Expectation Games

In this section we introduce a class of games with polynomial constraints over expectations. These games expand Lukasiewicz games by assigning to each player a formula Φ_i of $\mathsf{E}(\mathfrak{G})$, whose interpretation corresponds to a piecewise rational polynomial function whose variables are expected values. The formula Φ_i is meant to represent a player's goal concerning the relation between her and other players' expectations.

Definition 4.1 *An Expectation Game $\mathscr{E}_\mathscr{G}$ on $\mathsf{E}(\mathfrak{G})$ is a tuple $\mathscr{E}_\mathscr{G} = \langle \mathscr{G}, \{\mathsf{M}_i\}, \{\Phi_i\} \rangle$, where:*

1. *$\mathscr{G}$ is a Łukasiewicz game on Ł_k^c, with $\mathscr{G} \in \mathfrak{G}$,*
2. *for each $i \in \{1, \ldots, n\}$, M_i is the set of all mixed strategies on S_i of player P_i,*
3. *for each $i \in \{1, \ldots, n\}$, Φ_i is an $\mathsf{E}(\mathfrak{G})$-formula such that every atomic modal formula occurring in Φ_i has the form $\mathsf{E}\psi$, with $\psi \in \{\varphi_1, \ldots, \varphi_n\}$, i.e. the payoff formulas in $\mathscr{G}$.*

A model $\mathbf{M} = \langle \mathsf{S}, e, \{\pi_i\} \rangle$ of $\mathsf{E}(\mathfrak{G})$ for a game $\mathscr{E}_\mathscr{G}$ is called a *best response model* for a player P_i whenever, for all models $\mathbf{M}' = \langle \mathsf{S}, e, \{\pi_i'\} \rangle$ with $\pi_{-i}' = \pi_{-i}$,

$$\|\Phi_i\|_{\mathbf{M}'} \leq \|\Phi_i\|_{\mathbf{M}}.$$

An expectation game $\mathscr{E}_\mathscr{G}$ on $\mathsf{E}(\mathfrak{G})$ is said to have a *Nash Equilibrium*, whenever there exists a model $\mathbf{M}^*$ that is a best response model for each player P_i. In that case $\mathbf{M}^*$ is called an *equilibrium model*.

Example 1. Let $\mathscr{E}_\mathscr{G}$ be any expectation game where each P_i is simply assigned the formula $\Phi_i := \mathsf{E}\varphi_i$. This game corresponds to the the situation where each player cares only about her own expectation and whose goal is its maximisation. Clearly, by Nash's Theorem [11], every $\mathscr{E}_\mathscr{G}$ of this form admits an Equilibrium, since it offers a formalisation of the classical case where equilibria are given by tuples of mixed strategies over valuations in a Łukasiewicz game.

Example 2. Not every expectation game has an equilibrium. In fact, consider the following game $\mathscr{E}_\mathscr{G} = \langle \mathsf{P}, \mathsf{V}, \{\mathsf{V}_i\}, \{\mathsf{S}_i\}, \{\varphi_i\}, \{\mathsf{M}_i\}, \{\Phi_i\} \rangle$, with $i \in \{1, 2\}$, where:

(1) $\varphi_1 := p_1$ and $\varphi_2 := p_2$, and (2) $\Phi_1 := \neg d(\mathsf{E}(p_1), \mathsf{E}(p_2))$ and $\Phi_2 := d(\mathsf{E}(p_1), \mathsf{E}(p_2))$.[2]

The above game can be regarded as a particular version of Matching Pennies with expectations. In fact, while P_1 aims at matching P_2's expectation, P_2 wants their expectations to be as far as possible. It is easy to see that there is no model **M** that gives an equilibrium for $\mathscr{E}_\mathscr{G}$. Therefore:

Proposition 4.2 *There exist Expectation Games on* $\mathsf{E}(\mathfrak{G})$ *that do not admit a Nash Equilibrium.*

5 Complexity

Definition 5.1 *For a given game $\mathscr{E}_\mathscr{G}$, the* MEMBERSHIP *problem is the problem of determining whether there exists an equilibrium model* **M**. *For a given game $\mathscr{E}_\mathscr{G}$ and model* **M** *with with rational mixed strategies $(\pi_1, \dots, \pi_n)$, the* NON-EMPTINESS *problem is the problem of determining whether* **M** *belongs to the set of Nash Equilibria.*

Recall that the first-order theory $\mathsf{Th}(\mathbb{R})$ of real closed fields is the set of sentences in the language of ordered rings $\langle +, -, \cdot, 0, 1, < \rangle$ that are valid over the field of reals [8]. The existence of an equilibrium in a game $\mathscr{E}_\mathscr{G}$ can be expressed through a first-order sentence ξ of $\mathsf{Th}(\mathbb{R})$:

Proposition 5.2 *For each Expectation Game $\mathscr{E}_\mathscr{G}$ there exists a first-order sentence ξ of the theory $\mathsf{Th}(\mathbb{R})$ of real closed fields so that $\mathscr{E}_\mathscr{G}$ admits a Nash Equilibrium if and only if ξ holds in $\mathsf{Th}(\mathbb{R})$.*

As a consequence of the above, it is easy to see that a game $\mathscr{E}_\mathscr{G}$ admits an equilibrium if and only if there exists a quantifier-free formula in the language of ordered rings that defines a non-empty semialgebraic set over the reals [8].

We exploit the connection with $\mathsf{Th}(\mathbb{R})$ to determine the computational complexity of both the MEMBERSHIP and the NON-EMPTINESS problem. In fact, given a game $\mathscr{E}_\mathscr{G}$, it can be shown that the sentence ξ can be computed from $\mathscr{E}_\mathscr{G}$ but its length is exponential in the number of propositional variables of the payoff formulas ϕ_i. Deciding the validity of a sentence in $\mathsf{Th}(\mathbb{R})$ is singly exponential in the number of variables and doubly exponential in the number of alternations of quantifier blocks [5]. It can be shown that for every game the alternation of quantifiers in ξ is always fixed. As a consequence, we obtain:

Theorem 5.3 *Given an Expectation Game $\mathscr{E}_\mathscr{G}$ the* NON-EMPTINESS *problem can be decided in* 2-EXPTIME.

Deciding the validity of a sentence with only existential quantifiers in $\mathsf{Th}(\mathbb{R})$ can be solved in PSPACE [1]. We can show that, given a game $\mathscr{E}_\mathscr{G}$ and model **M** with rational mixed strategies $(\pi_1, \dots, \pi_n)$, we can compute in polynomial time an existential sentence of $\mathsf{Th}(\mathbb{R})$ whose validity is equivalent to the fact that **M** is an equilibrium model.

Theorem 5.4 *Given an Expectation Game $\mathscr{E}_\mathscr{G}$ and a model* **M** *with rational mixed strategies $(\pi_1, \dots, \pi_n)$, the* MEMBERSHIP *problem can be decided in* PSPACE.

[2] Where $\neg d(\mathsf{E}(p_1), \mathsf{E}(p_2))$ is interpreted as $1 - |exp_{p_1}(\pi_1, \pi_2) - exp_{p_2}(\pi_1, \pi_2)|$ and $d(\mathsf{E}(p_1), \mathsf{E}(p_2))$ as $|exp_{p_1}(\pi_1, \pi_2) - exp_{p_2}(\pi_1, \pi_2)|$ (see [4]).

6 Extensions and Future Work

This work lends itself to several extensions and generalizations. On the one hand we plan to study the notion of correlated equilibria for Expectation Games as well as to determine the complexity of checking their existence. In addition, we are interested in studying games where an external agent can exert influence on the game by imposing constraints on the payoffs and the expectations. This agent would then play the role of an enforcer by pushing the players to make choices that agree with her dispositions. Also, we plan to investigate games based on infinite-valued Łukasiewicz logic [2] where players have infinite strategy spaces. Finally, we intend to explore possible relations with stochastic games and whether our framework can be adapted to formalize those kinds of strategic interactions.

Acknowledgements

Godo acknowledges support from the Spanish projects EdeTRI (TIN2012-39348-C02-01) and AT (CONSOLIDER CSD 2007-0022). Marchioni acknowledges support from the Marie Curie Intra-European Fellowship NAAMSI (FP7-PEOPLE-2011-IEF).

References

[1] J.F. Canny. Some algebraic and geometric computations in PSPACE. In *Proc. of the 20th ACM Symposium on Theory of Computing*, 460–467, 1988, doi:10.1145/62212.62257.

[2] R. Cignoli, I. M. L. D'Ottaviano, D. Mundici. *Algebraic Foundations of Many-Valued Reasoning*, Volume 7 of *Trends in Logic*, Kluwer Academic Publishers, Dordrecht, 2000, doi:10.1007/978-94-015-9480-6.

[3] F. Esteva, L. Godo, F. Montagna. The ŁΠ and ŁΠ$\frac{1}{2}$ logics: two complete fuzzy systems joining Łukasiewicz and product logic. *Archive for Mathematical Logic*, 40: 39–67, 2001, doi:10.1007/s001530050173.

[4] L. Godo, E. Marchioni. Logics for Non-Cooperative Games with Expectations. In *Proc. of the* 11th *European Workshop on Multi-Agent Systems*, Toulouse, France, 2013, available at http://ceur-ws.org/Vol-1113/paper7.pdf.

[5] D. Y. Grigor'ev. Complexity of deciding Tarski algebra. *Journal of Symbolic Computation*, 5: 65–108, 1988, doi:10.1016/S0747-7171(88)80006-3.

[6] P. Hájek. *Metamathematics of Fuzzy Logic*. Volume 4 of *Trends in Logic*, Kluwer Academic Publishers, Dordrecht, 1998, doi:10.1007/978-94-011-5300-3.

[7] P. Harrenstein, W. van der Hoek, J.J.Ch. Meyer, C. Witteveen. Boolean games. In *Proc. of the* 8th *Conference on Theoretical Aspects of Rationality and Knowledge*, J. van Benthem (Ed.), Siena, Italy, 287–298, 2001, available at http://www.tark.org/proceedings/tark_jul8_01/p287-harrenstein.pdf.

[8] W. Hodges. *Model theory*. Cambridge University Press, Cambridge, 1993, doi:10.1017/CBO9780511551574.

[9] E. Marchioni, M. Wooldridge. Łukasiewicz Games. In *Proceedings of the* 13th *International Conferece on Autonomous Agents and Multiagent Systems*, Paris, France, 2014, to appear.

[10] F. Montagna, G. Panti. Adding structures to MV-algebras. *Journal of Pure and Applied Algebra*, 164: 365–387, 2001, doi:10.1016/S0022-4049(00)00169-9.

[11] J. Nash. Non-cooperative games. *The Annals of Mathematics*, Second Series, 54(2): 286–295, 1951, available at http://www.jstor.org/stable/1969529.