On paraconsistency and fuzzy logic

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In this communication we will report on the research done on paraconsistency and fuzzy logics as a result of the joint collaboration of two groups of the MatoMUVI project, the Brasilian group of the CLE at the University of Campinas and the Spanish group of IIIA-CSIC from Barcelona.

On the one hand, paraconsistency is the study of logics (as deductive systems) having a negation operator $\neg$ such that not every contradiction $\{\varphi, \neg\varphi\}$ trivializes or explotes. In other words, a paraconsistent logic is a logic having at least a contradictory, non-trivial theory. On the other hand, systems of mathematical fuzzy logic (MFL) study the question of vagueness from a foundational point of view based on many-valued logics. In this sense MFL can be considered as a degree-based approach to vagueness.

Among the plethora of paraconsistent logics proposed in the literature, the so-called *Logics of Formal Inconsistency* (LFIs), proposed in [3] (see also [2]), play an important role, since they internalize in the object language the very notions of consistency and inconsistency by means of specific connectives (either primitive or not).

Systems of MFL, understood as truth-preserving many-valued logics in the sense of [8, 4], are not paraconsistent. Indeed, in these systems, $\varphi \& \neg\varphi$ is always evaluated to 0, and hence any formula can be deduced from the set of premises $\{\varphi, \neg\varphi\}$. However, the situation is different if one considers, for each truth-preserving logic L, its companion $L^{\leq}$ that preserves degrees of truth as studied in [1]. In fact, in these systems $L^{\leq}$, a formula φ follows from a (finite) set of premises Γ when, for all evaluations e on a corresponding class of L-chains, $e(\varphi) \geq \min\{e(\psi) \mid \psi \in \Gamma\}$. Obviously, if L is not pseudo-complemented, there is always some evaluation e such that $e(\varphi \wedge \neg\varphi) > 0$. This says that $\{\varphi, \neg\varphi\}$ is not explosive in $L^{\leq}$ and thus, there are degree-preserving fuzzy logics that are paraconsistent.

Therefore the aim of the joint research programe has been to find and study logics that can handle both inconsistency and graded truth at once. The main outcomes are summarized next.

In [7] we have explored the notion of paraconsistent fuzzy logic in the degree-preserving paradigm, studying their paraconsistency features as logics of formal inconsistency. We have also considered their expansions with additional negation connectives, and correpond-ing first-order formalisms. Finally, we have compared our approach to other paraconsistent logics in the literature.

In [5], given an axiomatic extension L of MTL (that is not SMTL), we have first studied natural conditions (in the sense used in LFIs) to require to a consistency operator on L-chains. These conditions are then used to define both a semilinear truth-preserving logic over the language of L expanded with a consistency operator, as well as its paraconsistent degree preserving companion. Finally we have considered several extensions capturing several further properties one can ask to the consistency operator.

Finally, in [6] we have started the study of intermediate logics between the usual truth-preserving Łukasiewicz logic Ł, that is explosive, and its degree preserving compan-ion $Ł^{\leq}$, that is paraconsistent. We have shown there are infinitely-many explosive and paraconsistent logics in between and we provide some general results about these logics. A more detailed description of the family of intermediate logics is presented in the particular case of finitely-valued Łukasiewicz logics $Ł_n$.

References

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