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Finite-Difference Approximation of Wave Equation: a Study Case of the SIMA Velocity

Model

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Abstract

Synthetic seismograms enable to model the theoretical seismic response of the Earth interior due to different structural features and changes in the physical properties of crust and mantle. This approximation provides a best understanding of the real seismic data recorded in field experiments. In this paper, we are showing the development and application of a new scheme based on a multi-order explicit finite-difference algorithm for acoustic waves in a 2D heterogeneous media. The results of the modeling are compared with the seismic data acquired within the SIMA project providing new insight about the internal structure of the subsurface allowing improving the velocity model obtained in previous works.

Keywords: Synthetic seismograms, finite-difference, seismic modeling

1. Introduction

The approximation of wave propagation has been introduced and widely used in forward and inverse problems of acoustic and elastic waves e.g., [1], [2], [3], [4], [5], [6]. Seismic forward modeling has been an asset to aid the interpretation of controlled source seismic data. Refraction and wide-angle seismic reflection data still makes broad use of ray tracing schemes that allow the estimation of relatively simple velocity models, usually flat layer cake models. This type of interpretation does not take into account the fine details of the full wave field which undoubtedly provides new aspects which need to be understood to fully appreciate the complexities of the subsurface. In conventional reflection interpretation scattering energy, or even S-wave arrivals are not commonly used.

The synthetics simulation of the full wave-field is therefore very useful once the fine details of the recorded data can be introduced in the interpretation. Ray tracing is a solution of the wave equation which when used in seismic refraction seismic data interpretation only uses the first arrival of the more energetic P-wave phases. The full solution of the wave equation can be achieved by several approaches being one of the most used, the finite differences. There are different developments which can reproduce, simulate and solve the wave equation using finite differences with different degrees of exactness (see [7] for more details). There are second, fourth order developments [8], staggered and/or coincident grid approaches, as well as mixed spectral and finite difference solutions [9]. Early developments [10] of the finite difference solution to the wave equation are mostly

oriented to seismic exploration where the data is characterized by limited offsets and the models are relatively small when compared with academic studies that address the structure of the lithosphere and/or the characteristic structures of the deep crust. The increase in the size of the model increases the difficulties in the computation of the wave-field and the requirements on the software and on the hardware. Current technical developments make it feasible to run relatively large models (crustal or even lithospheric scale) in low cost computer systems and relatively simple codes are available.

In the current case the simplification is achieved by using second order finite difference solution of the wave equation. However the complexity is defined by the scale of the model which we aim to simulate. This is the case of the relatively recently acquired controlled source wide-angle seismic reflection data [11] acquired across the ATLAS orogen in Morocco. The wide-angle seismic transect covers a distance of over 700 km. Therefore, the crustal model consists of 700 km long by an 80 km thick.

2. Mathematical formulations

The 2D acoustic equation

The second-order partial differential equation describing acoustic-wave propagation in a two dimensional heterogeneous medium in the rectangular coordinates x and z [12]; [13] can be written as

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x} \left[\mu \frac{\partial u}{\partial x} \right] + \frac{\partial}{\partial z} \left[\mu \frac{\partial u}{\partial z} \right] + F(x, z, t), \quad (1)$$

where u is the wave displacement, $\rho(x, z)$ is the density, t the time, $\mu(x, z)$ is the elasticity parameter, F is the local source.

The term F corresponds to displacement due directly to the source of energy (explosion) from the total displacement in a rectangular region surrounding the source point.

The equation in (1) will be approximated using the finite-difference method. The centered differences can be used to represent the time differentials of the first terms, in the case of the second term, we will need a more elaborated calculations.

We consider the term

$$\frac{\partial}{\partial x} \left[c^2(x, z) \frac{\partial u}{\partial x} \right] \quad (2)$$

as a generalization of the terms of the second-hand side of equation (1) having partial derivatives with respect to one spatial coordinate only. Where ρ and μ were

replaced by the expression $c(x, z)$, with $c(x, z)$ is the speed of sound in the medium, and $= \sqrt{\frac{\mu}{\rho}}$.

In the following, we replace $c^2(x, z)$ by its discrete value $c^2(i, j)$ at the grid point (i, j) . We define $c^2(i, j)$ to be the average value of $c^2(x, z)$ over a rectangle, of sides Δx and Δz , centered at the grid point $(i\Delta x, j\Delta z)$. i, j and k are defined to be integer.

By assuming that the media are locally homogeneous in the immediate vicinity of each grid point, we get:

The approximation to (2)

$$\frac{c^2\left(i + \frac{1}{2}, j\right) [u(i + 1, j, k) - u(i, j, k)] - c^2\left(i - \frac{1}{2}, j\right) [u(i, j, k) - u(i - 1, j, k)]}{(\Delta x)^2} \quad (3)$$

where the averages $c^2\left(i + \frac{1}{2}, j\right)$ and $c^2\left(i - \frac{1}{2}, j\right)$ are defined as

$$c^2\left(i \pm \frac{1}{2}, j\right) = \frac{c^2(i \pm 1, j) + c^2(i, j)}{2} \quad (4)$$

The same approximation is used for the other term.

The final equation from (1) and the previous approximations is

$$\begin{aligned} \frac{u(i, j, k+1) - 2u(i, j, k) + u(i, j, k-1)}{(\Delta t)^2} &= \frac{1}{\Delta x} \left\{ \left[\frac{c^2(i+1, j) + c^2(i, j)}{2} \right] \left[\frac{u(i+1, j, k) - u(i, j, k)}{\Delta x} \right] - \right. \\ &\left. \left[\frac{c^2(i, j) + c^2(i-1, j)}{2} \right] \left[\frac{u(i, j, k) - u(i-1, j, k)}{\Delta x} \right] \right\} + \\ \frac{1}{\Delta z} &\left\{ \left[\frac{c^2(i, j+1) + c^2(i, j)}{2} \right] \left[\frac{u(i, j+1, k) - u(i, j, k)}{\Delta z} \right] - \right. \\ &\left. \left[\frac{c^2(i, j) + c^2(i, j-1)}{2} \right] \left[\frac{u(i, j, k) - u(i, j-1, k)}{\Delta z} \right] \right\} + F(i, j, k) \end{aligned} \quad (5)$$

Ideally, the wanted source is an impulse of zero width, but practically, the best we can attain is a narrow wavelet with minimum sidelobes. As example of practical wavelets (Fig. 1), for different seismic sources:

- Ricker zero-phase wavelet, which approximates dynamite source.
- Klaunder zero-phase wavelet, which approximates Vibroseis source.

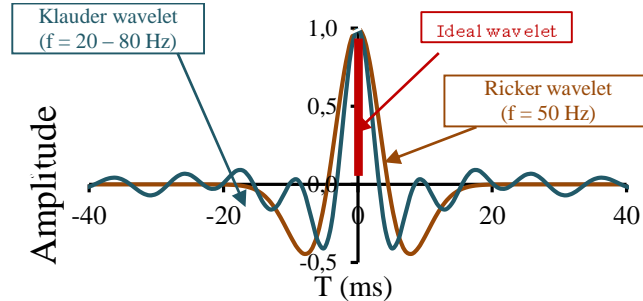


Figure 1 : zero-phase wavelets

The source in the example presented in this paper is a dynamite explosion, so in the simulation, it corresponds to a vertical body force, whose time history is the first derivative of the zero-phase Ricker wavelet:

$$f(t) = -\exp(b) f_{max} [f_{max} \cos(c) (t - t_0) + \pi \sin(c)],$$

$$b = -\frac{1}{2} f_{max}^2 (t - t_0)^2,$$

$$c = \pi f_{max} (t - t_0),$$

The finite-difference algorithm needs to be stable to get a physically meaningful numerical calculation, i.e., the difference between the exact and the numerical solutions of a finite-difference equation must remain bounded as the time index k increases, Δt remaining fixed for all i and j . The stability is maintained in this case if the von Neumann stability criterion ($c_{max} \frac{\Delta t}{\Delta x} \leq \sqrt{\frac{1}{2}}$; for $\Delta x = \Delta z$, see [14]), where c is the largest sound speed of the model, is maintained.

The error in the finite-difference solution is due to the conditions' approximation at the free surface boundary and the interfaces. In the free surface, the stress needs to be eliminated, and in the interfaces between two different acoustic materials, boundary conditions require both the stress and the displacement to be continuous [12].

3. Results

The finite difference algorithm has been tested using the velocity model obtained for the SIMA (Seismic Imaging of the Moroccan Atlas) velocity model [11]. The model consists on a layered 2D velocity scheme, 460 km long and 50 km depth. The wavefields are recorded by 890 receivers spaced by ~ 800 m following the geometry of the field acquisition. The source is located at the depth of $z=50$ m and

500m from the limit of the model, that corresponds approximately to the sixth shot near to Merzouga (Sahara domain).

The code enabled us to simulate the wave propagation through the model; in Fig. 2 we can visualize the wave propagation and reflections on the models' layers, respectively at 4.5s, 7.5s, 9s and 10s after the shooting time. The synthetic seismic record obtained shows the seismograms for every receiver deployed along the profile, being the horizontal axis distance to the source and the vertical time and the amplitude variations in function of time correspond to the arrival of a seismic wave.

The comparison with the real seismic record Fig. 3 presents similarities, especially in term of first arrivals (first energy arriving at every receiver) and the reflected energy. Because of this modeling is acoustic, no refractions are observed in the synthetic model; another relevant difference is the presence of clear arrivals in the synthetic seismogram in far offset from the shot point, which is not the case in the real shot gather, this is the result of the decay of the seismic amplitude as a function of time due to the spherical spreading and elastic attenuation that takes place in the earth interior.

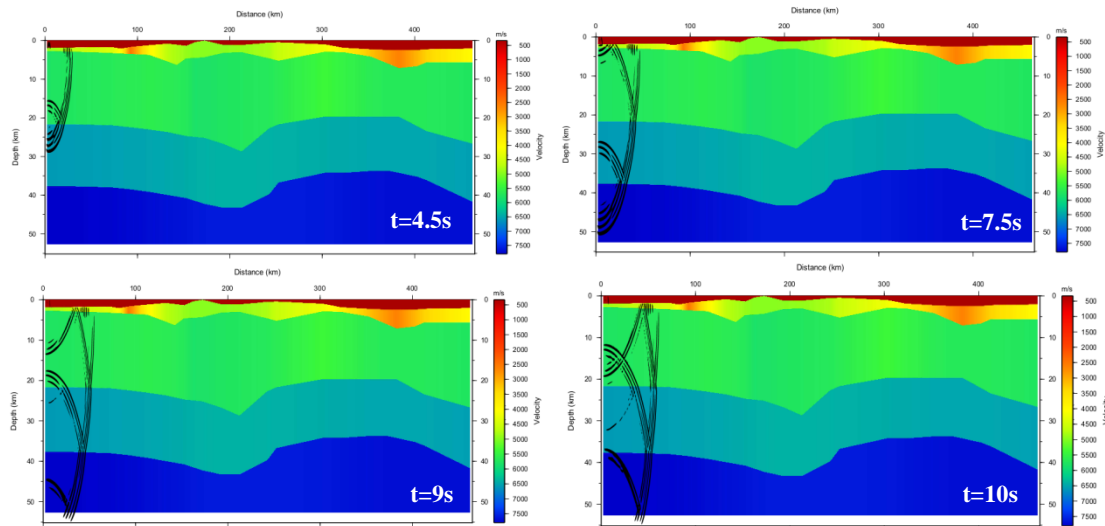


Figure 2 : Snapshots of the wave propagation simulation in time

The major difficulties in the solution of differential equations by Finite difference schemes and in particular the wave equation include: 1) the numerical dispersion, 2) numerical artifacts due to sharp contrasts in physical properties and, 3) the absorbing boundary conditions.

Numerical dispersion is caused when the grid (model grid) sampling interval is not small enough [15] when compared with the dominant frequency of the signal perturbation. In order to overcome this difficulty the grid interval has to be less than $\frac{1}{4}$ of the wavelength of the source signal. Nevertheless, even in this case the high frequency tails of the source signal generates high frequency very low amplitude noise which is recorded in the seismogram.

Sharp contrasts of physical properties especially when a very small number of grids are involved (very small size localized heterogeneities) generate numerical singularities during the simulation.

This is solved again by decreasing the grid size so that the velocity anomalies are mapped by a large number of grids and also by smoothing the contrasts in physical properties at the boundaries of the structures (for example, boundaries are thicker than a single grid point). Other approaches include integration schemes around the grid points [16]. The latter is used in the current scheme.

Finally to avoid reflections from the sides (limits) of the model a relatively thick band of the models around the sides needs to behave like a sponge so that the sides will not generate a reflection. There are several ways to account for this one is the well known absorbing boundary conditions [17], [18] a second way is based on the recent advances on perfect matching boundaries [19].

In this paper, we are only presenting the acoustic approximation for this velocity model and only one shot gather is modeled. The objective is to apply both approximations, acoustic and elastic, using moreover S-wave arrivals, and do the modeling of all the existing shot gathers, to obtain information about the velocity model that can provide a more detailed knowledge of the Atlas Mountains.

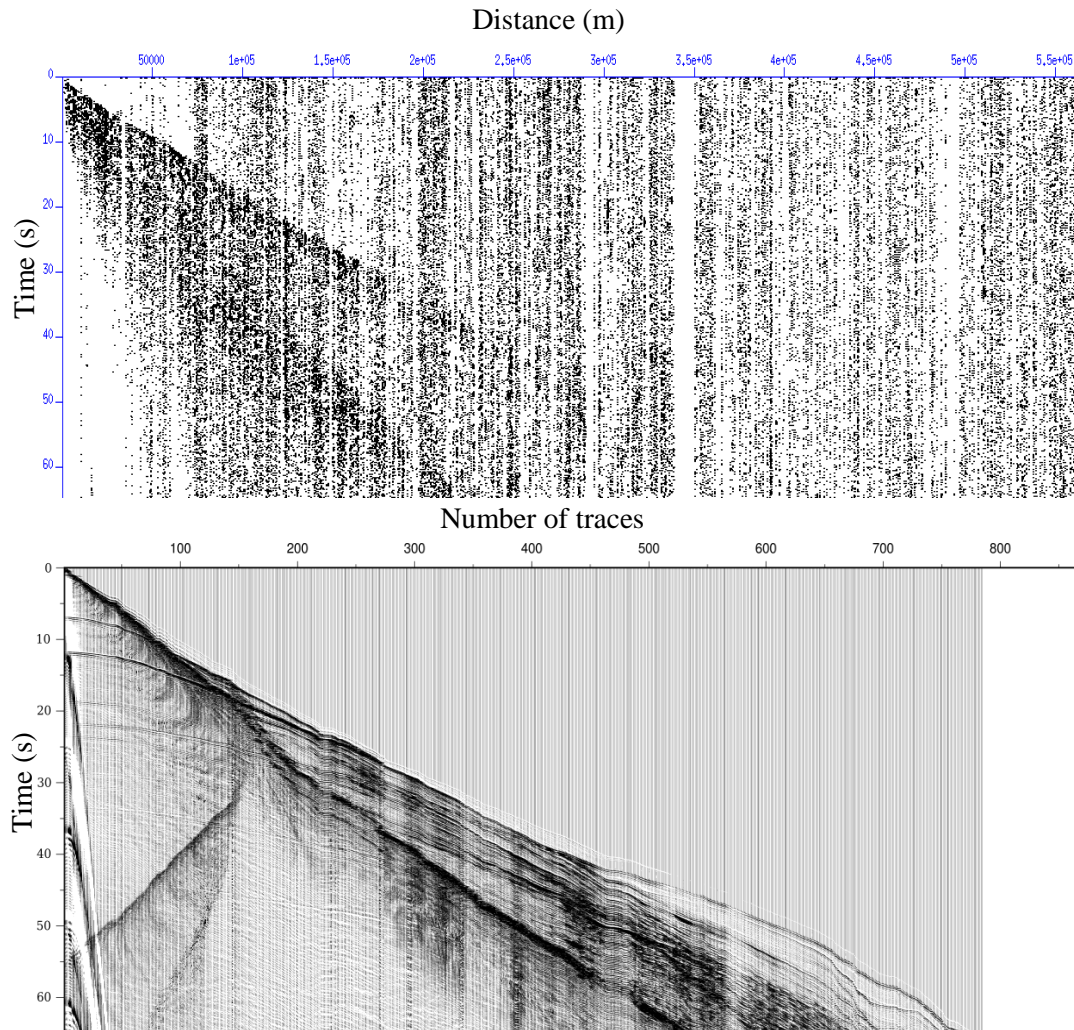


Figure 3 : Sixth Shot gather from SIMA experiment and the corresponding synthetic seismogram

4. Conclusion

In this paper, we consider a finite-difference wave equation solver to create synthetic seismograms that simulate the earth response to an excitation by dynamite explosion, we also used as input to the code a realistic velocity model from the SIMA project and we noticed that the fitting between real and calculated time-distance graphs is not perfect, to minimize those differences, modifying the velocity model and using also the elastic approximation to include refraction and head waves, which lead us to obtain a more accurate velocity model.

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