Journal of Volcanology and Geothermal Research 290 (2015) 1–11

Contents lists available at ScienceDirect

Journal of Volcanology and Geothermal Research

journal homepage: www.elsevier.com/locate/jvolgeores

Short-term volcanic hazard assessment through Bayesian inference: retrospective application to the Pinatubo 1991 volcanic crisis

Rosa Sobradelo [a], Joan Martí [b,*]

[a] Aon Benfield UCL Hazard Centre, Department of Earth Sciences, University College London, Gower Street, London, WC1E 6BT, UK
[b] Group of Volcanology of Barcelona (GVB-SIMGEO), Earth Sciences Institute "Jaume Almera", CSIC. Lluis Sole i Sabaris s/n, 08028 Barcelona, Spain

ARTICLE INFO

Article history:
Received 2 July 2014
Accepted 26 November 2014
Available online 5 December 2014

Keywords:
Monitoring data
Unrest indicator
Bayesian inference
Volcanic hazard assessment
Short-term probabilities

ABSTRACT

One of the most challenging aspects of managing a volcanic crisis is the interpretation of the monitoring data, so as to anticipate to the evolution of the unrest and implement timely mitigation actions. An unrest episode may include different stages or time intervals of increasing activity that may or may not precede a volcanic eruption, depending on the causes of the unrest (magmatic, geothermal or tectonic). Therefore, one of the main goals in monitoring volcanic unrest is to forecast whether or not such increase of activity will end up with an eruption, and if this is the case, how, when, and where this eruption will take place. As an alternative method to expert elicitation for assessing and merging monitoring data and relevant past information, we present a probabilistic method to transform precursory activity into the probability of experiencing a significant variation by the next time interval (i.e. the next step in the unrest), given its preceding evolution, and by further estimating the probability of the occurrence of a particular eruptive scenario combining monitoring and past data. With the 1991 Pinatubo volcanic crisis as a reference, we have developed such a method to assess short-term volcanic hazard using Bayesian inference.

1. Introduction

Assessing eruption hazard scenarios in probabilistic ways has become a main challenge in modern volcanology (Newhall and Hoblitt, 2002; Marzocchi et al., 2004, 2006; Aspinall, 2006; Martí et al., 2008; Neri et al., 2008; Sobradelo and Martí, 2010). An effective method for assessing volcanic hazard is to prepare scenarios that describe the potential impact of an eruption. The most commonly used procedures have focused on scenarios for the possible eruptive behavior of a volcano. The event tree based Bayesian methodology proposed by Newhall and Hoblitt (2002) has been used to develop computer-assisted procedures for transforming field data into the probability that a volcanic scenario will take place (Marzocchi et al., 2007, 2010; Sobradelo et al., 2013).

Limitations in the Bayesian methodology to incorporate and evaluate monitoring data using event tree structures has meant that these methods are mostly used for long-term hazard assessment. Elicitation of expert judgment has been widely used to complement the Bayesian event tree with monitoring information (Aspinall, 2006; Martí et al., 2008; Neri et al., 2008) when dealing with short-term hazard assessment. However, this method has a human decision component which adds an additional source of bias to the model, which requires the event tree structure to be as simple as possible, does not allow for epistemic and aleatoric uncertainties to be dealt with in a formal probabilistic way, unlike the Bayesian methods, and require the elicitation team to meet in order to update the probabilities each time new data arrives. Alternatively (Marzocchi et al., 2007, 2010) fuzzy logic has been used to incorporate monitoring data into the first three nodes of the event tree structure, and to quantify the probability of a magmatic eruption in the next time window. This approach requires a specific set of unrest indicators to be defined at each step of the event tree (branch and node), and a threshold value to be set a priori for each one. However, this reduces its applicability to volcanoes with long repose periods and without monitoring information from previous unrest/eruptive episodes.

To apply the Bayesian event tree methodology for the evaluation of different eruptive scenarios when conducting short-term hazard assessment, an automated and systematic approach is needed to complement past information with monitoring data, and to estimate the probability of a volcanic event in the next time interval. This method should incorporate information on monitoring data using flexible criteria to determine the relevant parameters and corresponding critical levels, as these may vary across volcanic systems. Although there is a group of precursory signals commonly used in most volcanoes (seismicity, ground deformation, gas, etc), the absolute values and trigger threshold (that is, values that set off the event), may significantly differ from one volcano to another.

* Corresponding author.
E-mail address: joan.marti@ictja.csic.es (J. Martí).

http://dx.doi.org/10.1016/j.jvolgeores.2014.11.011

In order to provide a simple and automated way of assessing the evolution of the volcanic system from looking at the monitoring signals, we propose a flexible probabilistic approach to incorporate monitoring information for the quantification of short-term volcanic hazard that looks for significant changes in the values of the measured unrest indicators, across consecutive time intervals. Rather than focusing on the absolute value of each variable, this method compares its degree of change with respect to the previous time interval. In each case, a variation which is considered significant can be defined in advance given the specific characteristics of the volcano being studied.

The activity of the volcano is analyzed through a series of reports or bulletins generated at each time interval with updated precursory data. The probabilistic method presented here uses Bayesian inference to estimate for each variable (unrest indicator) the probability of experiencing a significant variation between now and the next report. This information can be interpreted on its own or combined into a set of "precursory signals" that can be linked to a particular evolution of the unrest episode and, if this predicts an eruption, to a particular eruptive scenario. The short-term probability of an occurrence for that particular unrest and potential eruptive scenario can be estimated using Bayesian inference, complementing the monitoring data with past records. The final probabilities can then be used to assess the different mitigation actions associated with each scenario and estimate the corresponding potential risk (Sobradelo et al., submitted for publication). The application of the method is illustrated by retrospectively evaluating the volcanic crisis before the 1991 eruption of Pinatubo for stratovolcano cases. The same approach can be used for other volcano types, and it could be easily adapted to study similar natural hazards.

2. The Pinatubo 1991 volcanic crisis

Mount Pinatubo, in the Philippines, erupted in June 1991 after more than 500 years of repose and two months of monitored unrest (Punongbayan and Newhall, 1996). Before that, there was no scientific monitoring on the volcano, so the actual beginning of the unrest is unknown. The main eruption is assumed to have started on June 12 with the formation of vertical eruption columns, and generated the climatic explosive event (VEI 6) on June 15, (Hoblit et al., 1996; Wolfe and Hoblit, 1996). However, several minor eruptive episodes had already occurred in the previous days with some steam explosion and vigorous fumaroles in early April, some phreatic explosions on mid to late May, and the extrusion of a dacite spine on June 7 (Hoblit et al., 1996). Unrest was marked mainly by a significant increase in seismicity, ground deformation and gas emissions. Seismicity was characterized by the presence of volcano-tectonic, LP, tremor, and hybrid events, located at two distinct source regions, one near the summit of the volcano at depths of 0 to 3 km and another approximately 5 km to the Northwest at depths of 2–6 km (Cornelius and Voight, 1996; Harlow et al., 1996). Ground deformation marked a significant inflation of the upper part of the volcano between June 4 and June 7 (Ewert et al., 1996), coinciding with an increase in shallow seismicity and a decrease in the SO2 emissions (Daag et al., 1996) which taken together were interpreted as indications that magma was ascending from the chamber to the surface.

Very little was known about the geology and past volcanology of Mount Pinatubo before the beginning of the unrest, and most of this knowledge was related to mineral and geothermal exploration around the volcano (Newhall et al., 1996; Punongbayan et al., 1996). When unrest began, and the potential for a volcanic eruption was obvious, extra field work was undertaken which established that Pinatubo was the site of large explosive eruptions in the relatively recent past. This explosive activity was responsible for the deposition of numerous pyroclastic flow deposits and fallout layers, and associated lahars, on the lower flanks of the volcano (Newhall et al., 1996).

A preliminary hazard assessment was conducted, including an event tree and a hazard map with the three major volcanic hazards (pyroclastic flows, air fall, and lahars), in order to forecast the possible outcome and products of a future eruption (Punongbayan et al., 1996; Newhall and Hoblitt, 2002), and. This was used as the basis on which to update the probabilities of each potential scenario, according to the monitoring information which was periodically sent by the parties responsible for volcano surveillance in the form of different bulletins. This was the result of a joint effort by the Philippine Institute of Volcanology and Seismology (PHIVOLCS) and the U.S. Geological Survey Volcano Crisis Assistance Team (USGS VCAT), which allowed decision-makers to manage massive evacuations, resulting in a significant reduction of the potential impact the eruption could have caused on the local population (Punongbayan et al., 1996).

The type of volcanic hazard assessment done at the time of the crisis was based on expert elicitation, using as a starting point the major volcanic hazards identified in a previous long-term volcanic hazard assessment, complemented with short-term monitoring data (Punongbayan et al., 1996).

3. Short-term volcanic hazard assessment

In volcanic hazard assessment, long-term hazard assessment is based on historical and geological data and theoretical and physical models, and is basically used for territorial planning and the definition of emergency plans. It is conducted during a quiet phase of the volcano (eg. Marzocchi et al., 2010; Sobradelo et al., 2013). Short-term hazard assessment is done when the volcano goes into unrest and consists in complementing the long-term hazard assessment with continuous monitoring data. Therefore, the main objective in monitoring volcanic unrest is to forecast whether or not such an increase in activity will end up with an eruption, and if so, which eruptive scenario is the most likely.

Continuous monitoring should identify the different stages in the evolution of an unrest episode by detecting any increase in activity that may be indicated by a change in the monitored geophysical and geochemical parameters. However, determining when these parameters will reach a maximum value or will pass a threshold after which the eruption will occur is at present a nearly impossible task.

Each volcano has different characteristics (internal structure, rock rheology, magma composition, etc) which may result in different maximum values or thresholds for the monitored parameters before reaching the eruption conditions. It is a big challenge to estimate short-term probabilities at each stage of a volcanic unrest episode, as new data arrives. The method proposed here is a simple approach, through Bayesian inference, to incorporate monitoring data and automatically update short-term probability estimates, which does not require a previously set threshold value for the unrest indicators.

3.1. Monitoring variables

3.1.1. Definition of unrest indicators

Volcano monitoring includes several ground-based and remote techniques which are able to detect any change (i.e.: departure from the background) in the geophysical and geochemical variables that characterize the state of activity of a volcanic system (Scarpa and Tilling, 1996). Most of these variables are associated with changes produced by the movement of fresh magma and associated fluids inside the volcano's plumbing system (Sparks, 2003; Cañón Tapia, 2014). Therefore, these changes will be noticed in the form of variations in the amount of seismicity and of its characteristics (frequency, etc), ground deformation, variations in the gravity, magmatic and/or electric fields, and changes in the amount and nature of volcanic gases released to the atmosphere, etc. Some of these variations, or rather some combinations of them, have been used as unrest indicators of eruptive activity (Voight, 1988; Cornelius and Voight, 1994; De-La-Cruz-Reyna and

Reyes-Davila, 2001; Kilburn, 2003; Bell and Kilburn, 2012; Segall, 2013, 2013), although the exact amount and type of change may significantly vary for each volcano (Chouet and Matoza, 2013). Therefore, as indicated before, the identification and use of geophysical and geochemical information should not be linked to standardized values, as they require careful examination in each particular situation.

3.1.2. Monitoring variables commonly used as unrest indicators

In this paper we prefer to use the term unrest "indicator" rather than "precursor" as we will consider observables and also some interpretations in the group of variables that we will take into account to apply our methodology. Based on previous studies, we include a list of the most representative monitoring variables (observables) and interpretations that have regularly been used to forecast potential eruptive activity (Sparks, 2003; Sandri et al., 2004; Chouet and Matoza, 2013; Phillipson et al., 2013; Segall, 2013): 1) seismicity increase (i.e.: number of seismic events); 2) real-time seismic-amplitude measurement (RSAM) acceleration; 3) accumulated energy released rate increase; 4) lateral migration of seismicity; 5) vertical ascent of seismicity; 6) deep seismicity; 7) shallow seismicity; 8) volcano tectonic events; 9) long period events; 10) tremor events; 11) hybrid events; 12) gas flux increase; 13) H2O increase; 14) SO2 increase; 15) CO2 increase; 16) presence of other gases; 17) fluids temperature increase; 18) total strain increase; 19) inflation rate (i.e.: ground velocity) increase; 20) lateral migration of the source of ground deformation; 21) vertical migration of the source of ground deformation; 22) delta g/delta h anomaly; 23) presence of new fractures; 24) phreatic explosions; and 25) appearance of fresh magma at surface. These parameters may be grouped into four categories: seismicity, gases, ground deformation, and others, for which general changes may be noted by observers, even without operating a monitoring network near the volcano. For the purpose of this study if the unrest indicator has experienced a significant variation with respect to previous measurements taken, the corresponding variable will simply take value "Y" (yes, 1) or "N" (no, 0) otherwise.

3.1.3. Unrest outcomes and unrest indicators

As each volcanic system may behave in a different way, it will require a specific combination of precursory signals to identify unrest and raise the alarm. At present there is no general definition to clearly identify the stages of unrest. This is particularly obvious if we compare volcanoes characterized by long quiescent periods with volcanoes with long on-going eruption episodes (such as Etna, Stromboli, Montserrat, etc), where identification of new unrest episodes may be more challenging than in well known and regularly erupting volcanoes, with clear quiescence periods between eruptions (e.g.: Piton de la Fournaise).

The evolution of an unrest episode will depend on the causes of the unrest (magmatic, tectonic, or geothermal), which can lead to different outcomes (magmatic eruption, phreatic explosion, sector failure, or other), in different locations, and with different possibilities of eruption magnitude, products, extent, etc. We define an *unrest outcome* as one particular combination of these different possibilities. Each particular scenario is expected to result from a particular pattern in precursory activity. However, there are factors in each scenario that cannot be anticipated by looking at monitoring data, but which can be anticipated by looking at the products from past events, so this is why we must also consider past data when doing short-term hazard assessment. Table 1 shows three simple sources of unrest (unrest scenarios), and the corresponding values needed for each to occur.

3.2. The monitoring data for the 1991 Pinatubo volcanic crisis

Systematic volcano monitoring at Mount Pinatubo started in early April 1991 (see Punongbayan and Newhall (1996) references herein), with the deployment of different portable and permanent equipment at different dates, so the information recorded up to mid May, when a full monitoring network was operating, may not be complete. Table 2 illustrates the parameters monitored that served as precursory signals for the duration of the unrest period previous to the 1991 Pinatubo eruption.

Using the summary information from Table 1 we build our Table 2 with information on the state of each unrest indicator at each time bulletin, based on the particular information provided for each variable (Cornelius and Voight, 1996; Daag et al., 1996; Ewert et al., 1996; Harlow et al., 1996).

3.3. The impact of past events on short-term volcanic hazard assessment

To best assess how the volcanic unrest will evolve, it is also important to know how the volcano has acted in the past. For this reason, it is relevant to incorporate historical and geological information or any previous long-term hazard assessment. In the case of Pinatubo there were no previous studies done at the time of the unrest preceding the 1991 eruption, except for the logic tree mentioned earlier (Newhall et al., 1996; Punongbayan et al., 1996), based on an aerial exploration of the area when unrest started and which allowed scientists to detect the hazardous nature of the volcano. The logic tree presented to civil defense and military leaders on 17 May highlighted a number of potentially hazardous scenarios.

Based on this preliminary long-term volcanic hazard assessment, combined with existing monitoring information, the probability that a pyroclastic flow would reach into the populated areas - 15 km from the summit - within one year was estimated to be about 1% on 17 May, and about 8% on 10 June (Newhall, 2000). This estimate would most probably have risen significantly if the event tree had been updated after the explosive phase on June 7–14, 1991.

Table 1
Combination of unrest indicators needed to be present for a particular type of unrest to develop. The value 1 means *Yes*, a value of 0 means *No* and a blank space means it can take either value.

Unrest indicators	Unrest type: Magmatic	Geothermal	Tectonic
Overall seismicity increase			
Seismicity increase	1	1	1
RSAM acceleration	1	1	
Accumulated energy released rate increase	1	1	1
Lateral migration of seismicity			
Vertical migration of seismicity	1	0	0
Deep seismicity		0	
Shallow seismicity	1	1	
VT events	1	1	0
LP events	1	1	0
Tremor events	1	1	0
Hybrid events	1	1	0
Overall gas increase			
Gas flux increase	1	1	0
H2O increase		1	0
CO2 increase			0
SO2 increase		0	0
Others	1		0
Fluids temperature increase	1		0
Overall ground deformation increase			
Strain increase	1	1	1
Inflation rate increase			0
Lateral migration		0	0
Vertical migration		0	0
$\Delta g/\Delta h$ anomaly			0
Other changes			
Fractures			
Phreatic explosions			0
Fresh magma		0	0

4. Probability model: short-term volcanic hazard assessment through Bayesian inference

At this point we introduce Bayesian inference as a way to merge and quantify the information from precursory signals, and to detect possible patterns across different unrest indicators. Furthermore, the method will provide a way to quantify the uncertainty surrounding the monitoring variables, and estimate the probability that an unrest indicator will experience a significant variation in the next time interval compared to the previous one.

The Bayesian method applied here follows the same structure as the one applied in Sobradelo and Martí (2010) for long-term hazard assessment to estimate the likelihood at each node of the event tree. For simplicity, the method presented here only considers two possible values for the random (dummy) variables (Yes, No).

4.1. Probability estimate for each individual unrest indicator

The probabilistic evaluation of each unrest indicator is done at each time interval or unrest stage, when the parties responsible for the volcano monitoring network report on the current state of activity of the volcano. This information is then assessed by comparison with the previous report. The evaluation will focus on any significant variation in the monitoring activity with respect to the last report, and will then look at the overall evolution. We consider a significant variation to be any change (increase, decrease, migration) compared with the previous value for the same indicator. In the case of an escalation towards the onset of an eruptive event, most indicators tend to show a significant increase in their values (Chouet and Matoza, 2013; Cañón Tapia, 2014), but a significant decrease (or just a significant variation) could also occur, as in gas composition. These conditions should be addressed in advance by the corresponding monitoring experts.

Adopting this approach, the monitoring information is used to observe the relative evolution of the precursory data and to determine the tendency of the activity to increase or decrease. Rather than waiting for a particular precursory signal to reach a specific value (threshold), we assess its variation with respect to previous measurements, as a result of the variation of the system, by observing the evolution of the corresponding probabilities of occurrence, looking for significant or sudden changes. This can further allow comparison of the unrest episodes of two volcanoes with different starting points and different behaviours, by just looking at the relative changes of the system as the monitoring signals evolve. There may be situations where we want to know if the unrest indicator reaches a particular critical value. In that case the range of all possible values of the variable can be segmented into continuous, non-overlaping intervals and each interval modelled as a dummy variable. This will be set to Y or N depending on the observed value falling in the same interval as the critical value or not.

Defining each unrest indicator R_i, $i = 1, \ldots, n$, as a dummy variable, where n is the number of unrest indicators, means each can have only two outcomes, either *success*, Y, with probability θ_i, or *failure*, N, with probability $(1–\theta_i)$. Table 2 shows the R_i unrest indicators and corresponding values across different time bulletins. We understand by *success* the occurrence of that particular unrest indicator, that is to say the value of that particular monitoring variable has experienced a significant variation with respect to its previously measured stage, and *failure* otherwise. Let T_i represent the number of equal length time intervals with *success* for unrest indicator i. Both T_i and θ_i are random variables. T_i takes values in the interval $[0, \infty]$ and θ_i takes values in $[0,1]$.

θ_i is unknown and all its possible values are deemed equally likely a priori. This uncertainty can be described by assigning to θ_i a uniform distribution on the interval [0,1]. This is appropriate because θ_i being a probability, can take only values between 0 and 1; furthermore, the uniform distribution assigns equal probability density to all points in the interval, which reflects the fact that no possible value of θ_i is, a priori,

Table 2
Information on unrest indicators at different stages of the Pinatubo unrest period before the climatic eruption on June 15th, 1991. Aside from the first column that refers to July–August 1990 the rest of the columns refer to year 1991. The value 1 means *Yes*, the value 0 means *No*, and a blank space means *No information*.

Unrest indicators evolution	7-8 90	15/3-2/4	2/4-26/5	26-27/5	2/6	3/6	5/6	7/6	8/6	9/6	10/6	12/6	13/6	14/6	15/6
Overall seismicity increase	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Seismicity increase			0	0	1	1	1	1	1	1	1	1	1	1	1
RSAM acceleration			0	0	1	1	1	1	1	1	1	1	1	1	1
Accum. energy released rate increase			0	0	1	1	1	1	1	1	1	1	1	1	1
Lateral migration of seismicity			0	0	1	0	0	0	0	0	0	0	0	0	0
Vertical migration of seismicity		1	1	1	1	0	0	0	0	0	0	0	0	0	0
Deep seismicity			0	1	1	1	1	1	1						
Shallow seismicity			1	1	1	1	1	1	1	1	1	1	1	1	1
VT events		1	1	1	1	1	1	1	1	1	1	1	1	1	1
LP events			1	1	1	1	1	1	1	1	1	1	1	1	1
Tremor events			1	1	1	1	0	0	0	1	1	1	1	1	1
Hybrid events															
Overall gas increase	1	1	1	1				1	1	1	1				
Gas flux increase			1	1	0	0	0	1	1	1	1				
H_2O increase															
CO_2 increase															
SO_2 increase			1	1	0	0	0	1	1	1	1				
Others															
Fluids temperature increase															
Overall ground deform. increase			0	0	0	1	1	1	0	0	0	0			
Strain increase			0	0	0	1	1	1	0	0	0	0			
Inflation rate increase			0	0	0	1	1	1	0	0	0	0			
Lateral migration			0	0	0	0	0	0	0	0	0	0			
Vertical migration			0	0	0	0	0	0	0	0	0	0			
$\Delta g/\Delta h$ anomaly															
Other changes	1		0	0	1	1	1	1	1	1	1	1	1	1	1
Fractures	1		1												
Phreatic explosions	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0
Fresh magma	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

deemed more likely than any of the others. In the case that a priori beliefs, physical models or past data allow us to assign different initial values for the θ_i parameters, then, as the uniform distribution is a particular case of the beta distribution with parameters $\alpha = \beta = 1$, we would simply model this random variable with a Beta distribution of parameters (α, β). The Beta distribution is a continuous probability distribution having two parameters (when there are more than two parameters it is called a Dirichlet distribution). One of its most common uses is to model one's uncertainty about the probability of success of an experiment.

The only requirement for the prior distribution is that it should represent the knowledge about θ_i, before observing the current data. The prior distribution can be specified to be entirely subjective, or to depend on past data or be weak or non-informative (uniform prior). The prior distribution represents our "best guess". A priori, we know that the event R_i happens with probability θ_i, and we are interested in knowing what is the posterior value of θ_i after observing the data, that is, we want to know $p(\theta_i|T_i = y_i)$ (also written as $p(\theta_i|y_i)$), where y_i is the number of time windows that had a success, for unrest indicator R_i, out of n_i equal length non overlaping time windows (eg. the time from one monitoring bulletin to next).

Using Bayesian inference this is computed as:

$$p(\theta_i|y_i) \propto p(\theta_i) \times p(y_i|\theta_i) \tag{1}$$

Where θ_i is a random variable that follows a beta distribution of parameters (α_i, β_i) (uniform distribution if $\alpha_i = \beta_i = 1$) and y_i is a random variable that follows a binomial distribution of parameters (n_i, θ_i). By the convenient property that the beta distribution is a conjugate prior of the binomial distribution, we find that the right hand side product of Eq. (1), that is, the posterior distribution of θ_i, follows a beta distribution of parameters $(\alpha_i + y_i, \beta_i + (n_i - y_i))$. And so, we estimate $p(\theta_i|y_i)$ with the expected value E and corresponding variance V of a random variable that follows a beta distribution with parameters $(\alpha_i + y_i, \beta_i + (n_i - y_i))$, that has the form:

$$E[\theta_i|y_i] = \frac{\alpha_i^*}{\alpha_i^* + \beta_i^*} \tag{2}$$

$$V[\theta_i|y_i] = \frac{\alpha_i^* \beta_i^*}{(\alpha_i^* + \beta_i^*)^2 (\alpha_i^* + \beta_i^* + 1)} \tag{3}$$

where $\alpha_i^* = \alpha_i + y_i, \beta_i^* = \beta_i + (n_i - y_i)$ and the parameters α_i, β_i will be determined by:

$$\alpha_i = E[\theta_i](\lambda_i + J - 1); \beta_i = (1 - E[\theta_i])(\lambda_i + J - 1) \tag{4}$$

where $E[\theta_i]$ is the central value inferred by a priori models and/or of the theoretical beliefs, and will account for the aleatoric uncertainty, while λ_i controls the confidence at which $E[\theta_i]$ is considered a reliable estimate and will account for the epistemic uncertainty (Sobradelo and Martí, 2010). Both these parameters will be inputs to the model. J is the number of possible mutually exclusive and exhaustive values that the variable can take, which in our case is 2 as we use binary variables.

Similarly, for the posterior probability of *failure* (N), the expected value

$$E[(1 - \theta_i)|(n_i - y_i)] = \frac{\beta_i^*}{\alpha_i^* + \beta_i^*} \tag{5}$$

and the variance remains the same as in Eq. (3)

Table 3 shows the posterior probabilities for all the unrest indicators computed at each time bulletin using Eq. (2). In the first bulletin we do not have any information on the initial probabilities, so our "best guess" for the probability of "success" of each unrest indicator is non-

Table 3
Evolution of the unrest indicators at different stages of the Pinatubo unrest period, before the climatic eruption on June 15th, 1991, with corresponding probability of "success" for each individual indicator. All columns refer to year 1991.

	4-5 91	26-27/5	2/6	3/6	5/6	7/6	8/6	9/6	10/6	12/6	13/6	14/6	15/6
Overall seismicity increase	.8	.83	.86	.88	.89	.9	.91	.92	.92	.93	.93	.94	.94
Seismicity increase	.27	.22	.33	.42	.48	.53	.58	.61	.64	.67	.69	.71	.73
RSAM acceleration	.27	.22	.33	.42	.48	.53	.58	.61	.64	.67	.69	.71	.73
Accum. energy released rate increase	.27	.22	.33	.42	.48	.53	.59	.63	.67	.71	.74	.77	.78
Lateral migration of seismicity	.27	.22	.33	.29	.26	.23	.21	.19	.18	.17	.16	.15	.14
Vertical migration of seismicity	.6	.67	.71	.63	.56	.5	.45	.42	.38	.36	.33	.31	.29
Deep seismicity	.27	.39	.48	.54	.59	.63	.67						
Shallow seismicity	.47	.56	.62	.67	.7	.73	.76	.78	.79	.81	.82	.83	.84
VT events	.6	.67	.71	.75	.78	.8	.82	.83	.85	.86	.87	.88	.88
LP events	.47	.56	.62	.67	.7	.73	.76	.78	.79	.81	.82	.83	.84
Tremor events	.47	.56	.62	.67	.59	.53	.48	.53	.56	.6	.62	.65	.67
Hybrid events													
Overall gas increase	.8	.83				.85	.86	.88	.88				
Gas flux increase	.47	.56	.48	.42	.37	.43	.48	.53	.56				
H_2O increase													
CO_2 increase													
SO_2 increase	.47	.56	.48	.42	.37	.43	.48	.53	.56				
Others													
Fluids temperature increase													
Overall ground deformation increase	.27	.22	.19	.29	.37	.43	.39	.36	.33	.31			
Strain increase	.27	.22	.19	.29	.37	.43	.39	.36	.33	.31			
Inflation rate increase	.27	.22	.19	.29	.37	.43	.39	.36	.33	.31			
Lateral migration	.27	.22	.19	.17	.15	.13	.12	.11	.1	.1			
Vertical migration	.27	.22	.19	.17	.15	.13	.12	.11	.1	.1			
$\Delta g/\Delta h$ anomaly													
Other changes	.53	.44	.52	.58	.63	.67	.7	.72	.74	.76	.78	.79	.8
Fractures	.73												
Phreatic explosions	.4	.33	.43	.5	.56	.5	.45	.42	.38	.36	.33	.31	.29
Fresh magma	.2	.17	.14	.13	.11	.2	.27	.33	.38	.43	.47	.5	.53

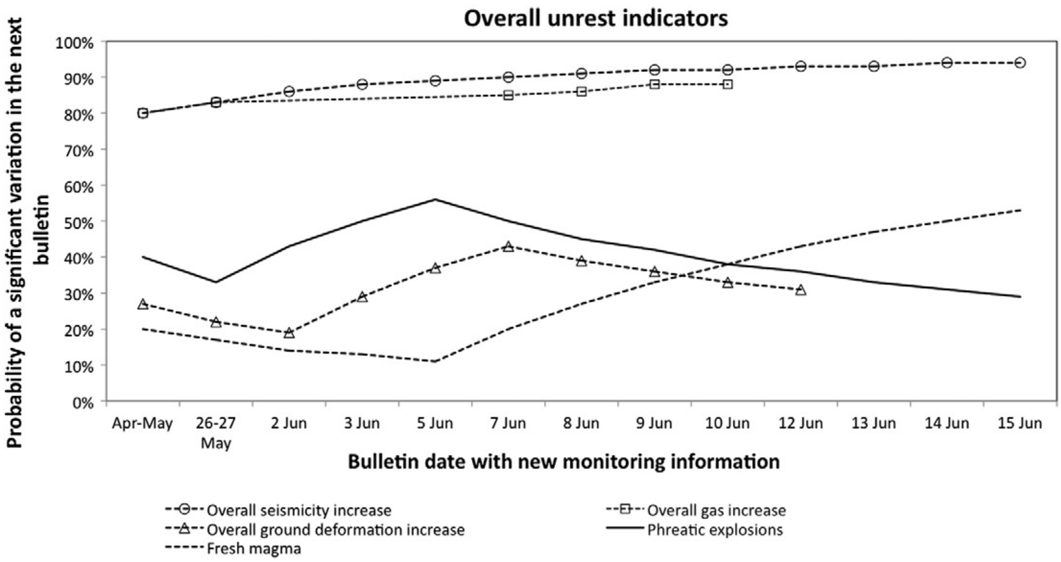


Fig. 1. Evolution of probabilities for the overall unrest indicators as the unrest episode evolves.

informative, and we assign 0.5 to each option, with a maximum epistemic uncertainty value of 1. In this way, if new data arrives it will contribute significantly to update our prior probabilities. We set the length of the studied time window to be the time from the last report. We are looking at only one report since the last so the time interval investigated is 1. Using this information on Eqs. (2) and (4) we obtain the updated (posterior) probability of any particular unrest indicator for the next time interval (bulletin).

For the following bulletins, as the time window for the study is the interval in between reports, we use as our prior probability the posterior probability from the previous one, and so the posterior probability in the last report now becomes the prior weigh input for our $E[\theta_i]$ parameter. With this assumption, we ensure the same data is not used twice in the probability estimation and remove the bias that different lengths between reports would add to the estimations. If there is no information for a particular unrest indicator, we can either assume the same probability as in the last bulletin for which data existed, implying a constant rate until new data arrives, or we can do a linear interpolation between reports for which data is reported. In this case, as it is done retrospectively, we opted for the second option.

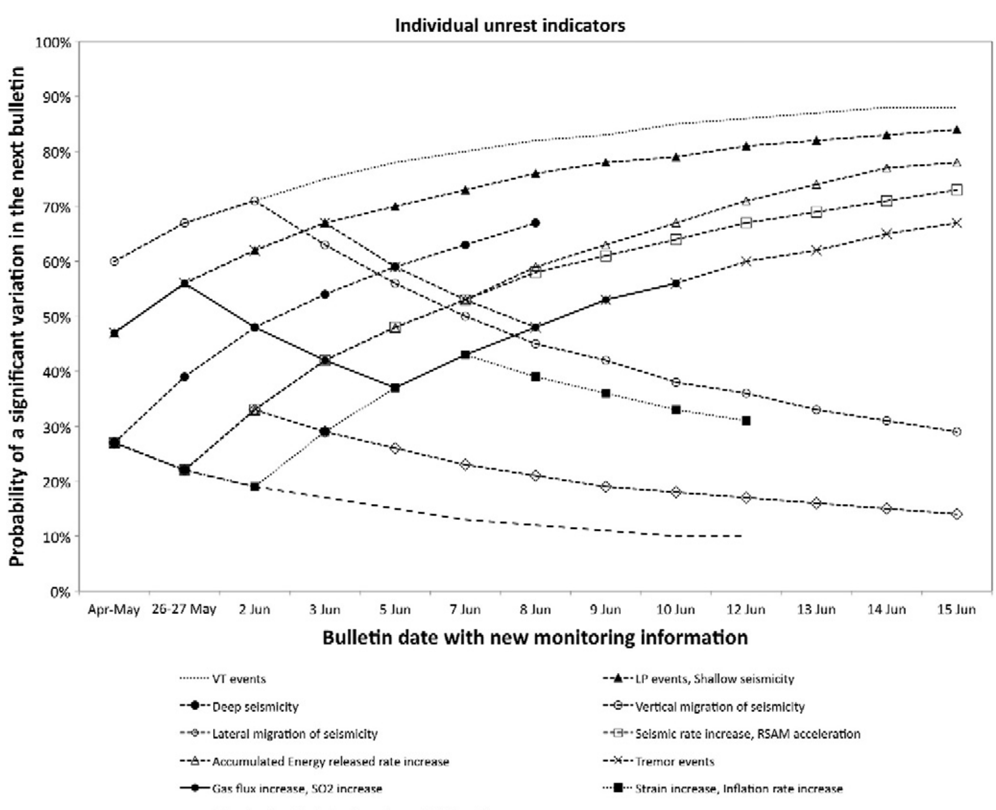


Fig. 2. Evolution of individual probabilities for each unrest indicator as the unrest episode evolves.

Using the input information from Table 2, we compute the probability estimates at each stage. Figs. 1 and 2 show how the probability evolves in time for each unrest indicator. Unrest indicators for which there is very little or no information (increase in number of seismic events, hybrid events, fumarole temperature increase, etc) are not considered in the analysis.

As the monitoring reports continue to arrive, and the probability estimates are updated for each unrest indicator, our level of confidence in the assigned prior weights increases, and so we reflect this in the epistemic uncertainties by increasing the value of λ_i by 1 at each time bulletin, so by 15 June, 1991 the value used would be 15. This means that as the crisis progresses and we receive more information, we are incorporating the previous knowledge and gradually increasing our confidence in the prior probabilities.

4.2. Probability estimate for a particular eruptive scenario

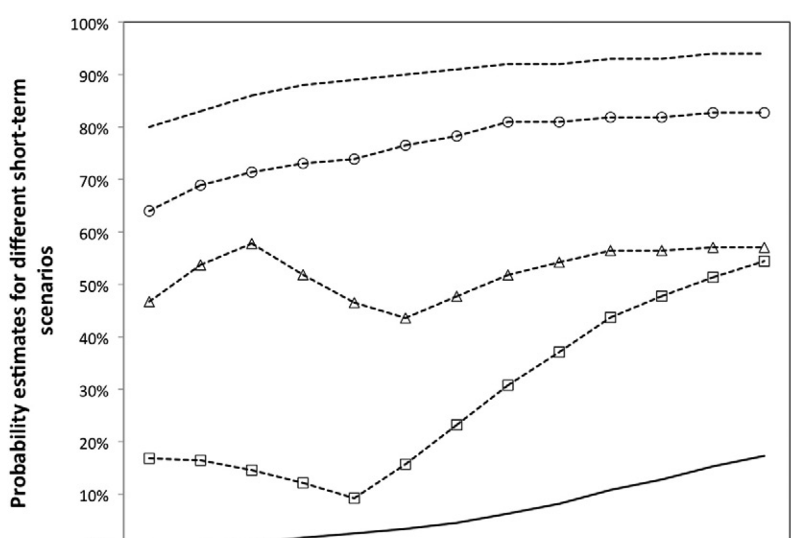
In this section we show how the monitoring information can be combined to assess the short-term probability of a possible eruptive scenario. After extra field work was carried out, it was determined that Pinatubo had originated large explosive eruptions in the relatively recent past, responsible for the numerous pyroclastic flow deposits and fallout layers, with associated lahars, exposed on the lower flank of the volcano. The logic tree presented to civil defense and military leaders on May 17, 1991 (Newhall et al., 1996; Punongbayan et al., 1996), during what was then still low-level, steady-state unrest, showed three possible eruptive scenarios associated with the major volcanic hazards: pyroclastic flows, air fall and lahars, which could reach Clark Air Base and adjoining cities of Angeles and Mabalacat. The first version of this tree was the basis to update the probabilities of occurrence as monitoring information arrived.

Formally speaking, let s be a particular volcanic scenario whose probability we want to estimate $p(s)$. Define s as a combination of n independent precursory signals, $m_1, m_2, \ldots, m_n$, then $p(s) = p(m_1, m_2, \ldots, m_n) = \prod_{i=1}^{n} p(m_i)$, where $p(m_i)$ is the probability of *success* of unrest indicator m_i computed as in Eq. (2). So we get:

$$p(s) = E(s) = \prod_{i=1}^{n} E(\theta_i | y_i) \tag{6}$$

This step is equivalent to computing the long-term probability for each eruptive scenario as in Eq. (11) of Sobradelo and Martí (2010). Therefore, once we have computed the corresponding posterior probabilities for each unrest indicator, and know their probability density functions, we can compute the probability of a particular scenario by multiplying their individual probabilities. As with the assumption of independence of the nodes when building event trees for volcanic hazard assessment, addressed in Sobradelo et al. (submitted for publication), the assumption of independence of the unrest indicators made here has an impact on the final results. This initial condition is set for simplicity and practical application of the Bayesian inference methodology. In general, some unrest indicators need not be independent of each other, in which case a different mathematical approach may be more suitable. Future work is needed to address this. However, it remains to be proven whether the presumed accuracy increase in the probability estimates would justify the additional complexity that dependency would introduce to the model settings and calculations. The list of unrest indicators in Section 3.1.2 was defined to include only those that would contribute with new information to the process. However, caution should be taken when selecting a particular combination of unrest indicators to describe an unrest outcome and a possible eruptive scenario.

As the product of the variance is not the same as the variance of the product, to compute the corresponding variance of the estimate we use the approximation technique based on the Delta method (Rice, 2007) to determine the asymptotic distribution of the variance $\hat{\sigma}^2$ and



	Apr-May	26-27 May	2 Jun	3 Jun	5 Jun	7 Jun	8 Jun	9 Jun	10 Jun	12 Jun	13 Jun	14 Jun	15 Jun
Overall seismicity increase only	80%	83%	86%	88%	89%	90%	91%	92%	92%	93%	93%	94%	94%
Overall seismicity and gas increase	64%	69%	71%	73%	74%	77%	78%	81%	81%	82%	82%	83%	83%
Overall seismicity, gas and deformation	47%	54%	58%	52%	47%	44%	48%	52%	54%	56%	56%	57%	57%
Overall seismicity, gas, deformation and fresh magma	17%	16%	15%	12%	9%	16%	23%	31%	37%	44%	48%	51%	54%
Seismicity, RSAM, energy release, shallow seismicity, VT, LP, tremor	0.12%	0.13%	1%	2%	2%	3%	5%	6%	8%	11%	13%	15%	17%

Fig. 3. Evolution of the probability of a magmatic eruption based on different combinations of unrest indicators.

corresponding standard deviation $\hat{\sigma} = \sqrt{\hat{\sigma}^2}$ for each scenario s, as follows:

$$\hat{\sigma}^2 = E(s)^2 \sum_{i=1}^{n} \frac{V(\theta_i|y_i)}{[E(\theta_i|y_i)]^2} \tag{7}$$

where, $E(s)$ is from Eq. (6), $E(\theta_i|y_i)$ from Eq. (2), and $V(\theta_i|y_i)$ from Eq. (3). See Sobradelo et al. (2013), for further details on how to derive Eq. (7).

Once the source of the unrest has been established as magmatic, the next step is to assess the type of outcome from this unrest. Suppose that for a magmatic eruption to occur, a particular combination of unrest indicators needs to happen. Let us use the seismicity indicators for example, and say that a magmatic eruption is more likely to occur if there is seismicity increase, RSAM acceleration, accumulated energy release rate increase, shallow seismicity, VT and LP events and tremor. Using Eq. (6) we compute these probabilities. Fig. 3 shows how using different combinations of unrest indicators, the total probability of an eruption varies. The challenge is to select the combination of unrest indicators that best describes this particular volcanic system.

4.2.1. Probability estimate incorporating past data

So far we have estimated the probability of the occurrence of a particular scenario using monitoring data only. As mentioned in Section 3.3, in addition to the monitoring data, it is also important to look at the past behavior of the volcano, since this may define the potential outcome of the unrest. In the case of Pinatubo, we want to use our Bayesian approach to assess the short-term probability of having a magmatic eruption that originates pyroclastic flows and/or fallout. The same eruptive scenarios that were evaluated in the logic tree presented on 17 May, as described in Sections 3.3 and 4.2. Call this scenario u. We consider lahar as a secondary hazard, for which an eruption with pyroclastic flows and/or fall out needs to occur first. Using monitoring data, we can approximate the odds of having a volcanic eruption with the approach described in Section 4.2, as this scenario is directly related to the evolution of the precursory signals. However, to assess the eruption hazards we need to use information on past volcanic events, as precursory signals provide little or no information on the outcome of a potential eruption. This can be done using the Bayesian approach, also used in Sobradelo et al. (submitted for publication), as follows:

$$h(u|v) = \frac{f(v|u)g(u)}{\int f(v|u)g(u)du} \tag{8}$$

Where $h(u|v)$ is the posterior distribution of scenario u after we observed monitoring data v. The posterior probability $h(u|v)$ describes the hazard, where $g(u)$ is the probability of occurrence of scenario u before we observe the monitoring data v, this is, the long-term probability of scenario u. The term $f(v|u)$ is the conditional distribution of monitoring data v given scenario u, which can be interpreted as the probability of observing these values of the unrest indicators if scenario u was to happen.

As the denominator in Eq. (8) is a constant, let $c = (\int f(v|u)g(u)du)^{-1}$, and we get:

$$h(u|v) = cf(v|u)g(u) \tag{9}$$

The probability function h is estimated with the expected value. For simplicity in the notation, denote $E(h)$ and $V(h)$ as the expected value and variance associated with $h(u|v)$, $E(f)$ and $V(f)$ as the expected value and variance associated with $f(v|u)$, computed with Eqs. (6) and (7) respectively, and $E(g)$ and $V(g)$ as the expected value and variance to describe $g(u)$, which can be estimated using the Bayesian event tree methodology described in (Sobradelo and Martí, 2010) and automated in HASSET (Sobradelo et al., 2013), (other methods may also be used (Aspinall, 2006; Marzocchi et al., 2010)), or with expert elicitation as in the original logic tree of Pinatubo. Then:

$$E(h) = cE(fg) = cE(f)E(g) \tag{10}$$

$$V(h) = c^2V(fg) = c^2\left[(E(f))^2V(g) + (E(g))^2V(f) + V(f)V(g)\right] \tag{11}$$

And so we have written the expected value and variance of a particular scenario as a function of the expected value and variances of the variables that measure the uncertainty associated with monitoring data, weighted by the uncertainty associated with past events.

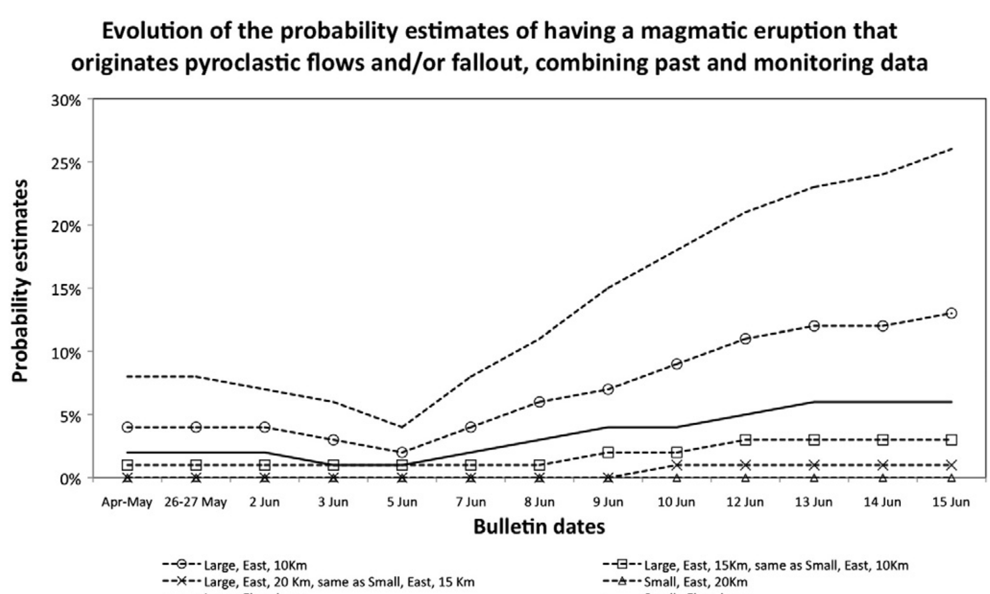


Fig. 4. Evolution of the probability of a magmatic eruption for various scenarios, merging past information on size, location and extent with monitoring data on overall seismicity increase, gas increase, deformation and fresh magma.

Fig. 4 shows the evolution of the short-term probabilities in the Pinatubo example, computed with Eq. (9), merging past data and monitoring information on overall seismicity, overall gas increase, deformation and presence of fresh magma. The long-term probabilities for each scenario are from the logic tree presented in 17 May.

5. Discussion and results

Fig. 3 shows the evolution of the probability estimates of an eruption based on different combinations of unrest indicators. The last scenario, formed from a combination of monitoring information on total seismicity, RSAM, energy release, shallow seismicity, VT, LP events, and tremor, estimated the probability of an imminent eruption to be less than 1% as of May 1991, and 8% by 10 June. These are consistent with the estimations reported initially in the Pinatubo crisis Punongbayan et al. (1996); Newhall et al. (1996). However, if we look at the second scenario, formed from combining information on overall seismicity increase and gas increase only, the probability estimates jump to 64% and 81% respectively for the same dates. The results in the first example suggest that the probabilities could have been underestimated, causing delays in the implementation of adequate mitigation actions. On the contrary, the resulting probabilities from the second case could have been overestimated, incurring in unnecessary economic costs if mitigation actions are implemented too soon.

The challenge is to find an adequate combination of unrest indicators (observations and interpretations) that best describe this particular volcano, weighting down those parameters that may be less relevant to it. In this respect, there seems to be a good compromise between the third scenario (based on overall seismicity, gas and deformation) and the fourth scenario in Fig. 3 (based on overall seismicity, gas, deformation, and fresh magma). In our case, we gave more importance to the existence of fresh magma towards the total probability of an eruption. The non existence of fresh magma in the earlier stages of the unrest brings the total probability of eruption down to 9% on 5 June, as opposed to an 89% if we look at the overall seismicity increase only. The same effect is observed to a lesser extent when combining indicators on seismicity and gas.

Assume that we wanted to assess the type of unrest observed as of April 1991, given that this could be either magmatic, geothermal or tectonic. Table 1 shows the expected behavior of each indicator for a particular type of unrest to occur. Comparing those values with the observed precursory behavior in April 1991 shown in Table 2, we can say from looking at the seismicity indicators that the unrest is likely to be magmatic, as some of the observed unrest indicators clearly rule out geothermal and tectonic unrest.

So far we have shown in Fig. 3 the evolution of the probability estimates based on monitoring information alone. Let us take the fourth scenario from Fig. 3, then, when the short-term probability estimates based on overall seismicity, overall gas increase, deformation and presence of fresh magma are updated with information on past events (Section 4.2.1), we obtain a better assessment on the potential size and extent of the volcanic hazards. Table 4 and Fig. 4 show for the same monitoring information how the evolution of the probability of a volcanic event varies significantly across different scenarios. The probability of having a large magnitude event increases drastically from June 5th onwards, driven by the significant increase in having a large magnitude event directed to the East and affecting distances up to 10 km. This could be understood as an indication that mitigation actions should be considered to protect the East side up to a distance of 10 Km.

These probability results need to be used together with the cost and losses associated to those areas. This should be the next step of the analysis, where the output probabilities would serve as input parameters in a probabilistic decision model. The cost of implementing actions, and associated losses if the event occurs before the area is protected, are incorporated to assess each mitigation action Marzocchi and Woo, 2007; Sobradelo et al. (submitted for publication).

5.1. The unrest indicators

With the methodology explained in previous sections, we compute the probability estimates for each unrest indicator at each time interval. Figs. 1 and 2 show for each unrest indicator the probability that the activity will increase significantly by the next time interval (assumed to be of the same length as the interval since last) if the pattern observed in the past is to be continued. As an example, figure Fig. 1 shows the 0.83 overall seismicity increase on 26–27 May 1991 reported in Table 3. This is computed using Eq. (2) as $(4+1)/(4+1+1)$, where the value $4 = 0.80(4+1)$ is computed using Eq. (4).

Overall, the total seismicity experienced a gradual increase from 2 April 1991 to 15 June, whereas the ground deformation showed an increase between 3 and 7 June, and a decrease after that. The gas parameters (based on overall information detected and on gas flux and SO2 measurements) experienced an overall gradual increase up to the last measurements on 10 June. On 5 June, 2 days before the ground deformation stops increasing, the presence of fresh magma begins to increase and the number of phreatic explosions stops.

Fig. 2 shows the evolution of the unrest indicators studied individually. Mapping them into a common probability scale makes it easier to detect abnormal trends, possible patterns, and compare the behavior of different indicators. In our example, there seems to be a clear division between those unrest indicators that increase as the crisis evolves towards an eruption and those that decrease. It is also important to detect the inflexion point in some key unrest indicators, as in the SO2 increase on June 5, where the inflation rate increase stops on June 7th, and the tremor increases on June 8th. These inflexion points between June 5 and June 7th could be an indication that the system was experiencing a shift into a new phase.

Some of the unrest indicators have similar probability estimate curves because they show the same pattern of successive *Y* and *N* as the crisis evolves. As this behavior is expected for certain groups of unrest indicators in specific scenarios, in particular seismicity indicators, it serves as a way to spot abnormal behavior. For example, if we see a

Table 4
Short-term probabilities merging past data and monitoring information on *overall seismicity increase, overall gas increase, ground deformation increase* and presence of *fresh magma*.

Size		Location		Extent		LT pr.	4-5 91	26-27/5	2/6	3/6	5/6	7/6	8/6	9/6	10/6	12/6	13/6	14/6	15/6
Large	.8	East	.4	10Km	.75	.24	.04	.04	.04	.03	.02	.04	.06	.07	.09	.11	.12	.12	.13
				15Km	.2	.064	.01	.01	.01	.01	.01	.01	.01	.02	.02	.03	.03	.03	.03
				20Km	.05	.016	0	0	0	0	0	0	0	0	.01	.01	.01	.01	.01
Small	.2	East	.4	10Km	.75	.06	.01	.01	.01	.01	.01	.01	.01	.02	.02	.03	.03	.03	.0
				15Km	.2	.016	0	0	0	0	0	0	0	0	.01	.01	.01	.01	.01
				20Km	.05	.004	0	0	0	0	0	0	0	0	0	0	0	0	0
Large	.8	Else	.6			.48	.08	.08	.07	.06	.04	.08	.11	.15	.18	.21	.23	.24	.26
Small	.2	Else	.6			.12	.02	.02	.02	.01	.01	.02	.03	.04	.04	.05	.06	.06	.06
ST probability estimates							.17	.16	.15	.12	.09	.16	.23	.31	.37	.44	.48	.51	.54

significant increase in seismicity but no VT events, it could indicate a tectonic rather than a magmatic unrest. It could also suggest inconsistencies in the measurement devices.

Alternatively, if the increase in activity across different reports varies substantially for a particular time interval with respect to a previous one, the absolute value of the increase can be reflected in the probability curve by adjusting the corresponding prior information for that particular bulletin. In our example, the accumulated energy release rate increase and the RSAM acceleration show the same pattern in Table 2, but we see in Fig. 2 that from 7 June the probability curve for the accumulated energy release rate increase is above the RSAM trend. This is because the absolute value of the energy rate increase from 8 June onwards was substantially larger than for the significant increases experienced in the period leading up to 7 June, and so we reflected this significantly larger increase in the prior weights assigned, putting more emphasis on the later changes. With this approach we are able to capture and highlight relevant shifts in the precursory activity.

6. Conclusion

Understanding the evolution of volcanic unrest is one of the fundamental tasks to forecast volcanic activity. The time variation of the different geophysical and geochemical parameters monitored by the surveillance networks, or simply the variation in the normal behavior directly noticed by an observer located close to the volcano, may indicate that an unrest episode is evolving towards a volcanic eruption. However, quantifying this degree of evolution and the time to the onset of the eruption is not an easy task, due to the fact that forecasting volcanic eruptions is still a young science. Even more important is the fact that each volcano or volcanic system responds in a different way when an over-pressurized batch of magma tries to reach the surface. In addition to this, the possibility of having unrest triggered by geothermal or tectonic causes (i.e.: without movement of fresh magma) may complicate even more the understanding of this complex problem.

In this study we present a quantitative approach through Bayesian inference, consisting of an open structure that allows mapping individual unrest indicators into a common probability scale, which can then be combined to asses the hazard of a particular scenario. This approach estimates (for each unrest indicator) the probability of experiencing a significant variation between two consecutive stages or time intervals during the unrest episode, which can be interpreted as an evolution towards the onset of a particular eruptive scenario. This means that we are not putting restrictions on any particular value of the unrest indicator, but rather emphasizing the relative change with respect to a previous stage. As we update the probability estimates at each point, using only the new information since the last bulletin, we also make the model independent of the length of the interval between reports. The short-term probability of occurrence for an eruptive scenario takes into account monitoring data, as well as any relevant past history of the volcano (long-term hazard assessment). As defined, the method can be applied to any volcanic system, regardless of their characteristics, allowing comparisons between them.

Most important, the method provides a rapid and formal way of transforming precursory information into a common probabilistic scale for comparison, useful to assist decision makers and experts during an evolving crisis, and detect sudden changes or shifts in the activity of the volcano that may require immediate attention. The method also allows identifying alternative unrest indicators for those volcanic systems that are less monitored than others. For instance, if there is no monitoring system in place but there is overall seismicity felt by the population, or overall deformation, these can be taken into account to assess the short-term probabilities. The fact that we consider observables (i.e. precursors) and interpretations as unrest indicators does not challenge the validity of our approach. Basically, it claims to identify changes in apparent tendencies or patterns, rather than absolute values that could be interpreted as fixed trigger indicators. This methodology could be applied to the study of all volcano types and is equally applicable to monogenetic fields and collapse calderas. For example, in monogenetic volcanism, lateral migration of seismicity and deformation may be crucial to understand the evolution of the unrest (e.g.: Martí et al., 2013). The methodology can also be used to compare the behavior of the same parameters for different eruptions from the same volcano and try to look for a pattern. The epistemic and aleatoric uncertainties can be formally adjusted at each step of the unrest and for each unrest indicator via the model input parameters (prior and data weights), as explained in previous sections. The selection of the prior probabilities in a Bayesian model has a subjective component, which in some occasions has been object of criticism. The tool presented here does not claim to replace existing approaches to the management of a volcanic crisis, but tries to assist in the interpretation and analysis of relevant data. Caution should always be executed regarding the interpretation and applicability of any approach to an evolving crisis. Although this approach has been developed for volcanic hazard, it could be adapted to assess similar natural hazards.

Acknowledgments

This research has been partially funded by the European Commission (FP7 Theme: ENV.2011.1.3.3-1; Grant 282759: VUELCO) and by the Aon Benfield UCL Hazard Centre. All data and information used to reproduce the results in the case study have been extracted from the public source Punongbayan and Newhall (1996) edited monograph on the Mt Pinatubo eruption. Comments from the JVGR editor Lionel Wilson and the reviews of Olivier Jacquet and an anonymous reviewer are greatly appreciated.

References

Aspinall, W.P., 2006. Structured elicitation of expert judgment for probabilistic hazard and risk assessment in volcanic eruptions. In: Mader, H.M., et al. (Eds.), Statistics in volcanology. Special Publication of IAVCEI, Geological Society of London.

Bell, A., Kilburn, C., 2012. Precursors to dyke-fed eruptions at basaltic volcanoes: insights from patterns of volcano-tectonic seismicity at Kilauea volcano, Hawaii. Bull. Volcanol. 74, 325–339.

Cañón Tapia, E., 2014. Volcanic eruption triggers: A hierarchical classification. Earth Sci. Rev. 129, 100–119.

Chouet, B., Matoza, R., 2013. A multi-decadal view of seismic methods for detecting precursors of magma movement and eruption. J. Volcanol. Geotherm. Res. 252, 108–175.

Cornelius, R., Voight, B., 1994. Seismological aspects of the 1989–1990 eruption at Redoubt Volcano, Alaska – the materials failure forecast method (FFM) with RSAM and SSAM seismic data. J. Volcanol. Geotherm. Res. 62, 469–498.

Cornelius, R., Voight, B., 1996. Real-time seismic amplitude meaurement (RSAM) and seismic espectral amplitude measurement (SSAM) analysis with the Materials Failure Forecast Method (FFM), June 1991 explosive eruption at Mount Pinatubo. In: Punongbayan, R., Newhall, C. (Eds.), Fire and mud: eruptions and lahars of Mount Pinatubo, Philippines. PHIVOLCS, University of Washington Press, Quezon City, Seattle, pp. 249–268.

Daag, A., Tubionosa, B., Newhall, C., ngo, N.T., Javier, D., Dolan, M., De-Los-Reyes, P., Arboleda, R., Martinez, M., Regalado, M., 1996. Monitoring sulfur dioxide emissions at Mount Pinatubo. In: Punongbayan, R., Newhall, C. (Eds.), Fire and mud: eruptions and lahars of Mount Pinatubo, Philippines. PHIVOLCS, University of Washington Press, Quezon City, Seattle, pp. 409–414.

De-La-Cruz-Reyna, S., Reyes-Davila, G., 2001. A model to describe precursory material-failure phenomena: applications to short-term forecasting at Colima volcano, Mexico. Bull. Volcanol. 63, 297–308.

Ewert, J., Lockhart, A., Marcial, S., Ambubuyog, G., 1996. Ground deformation prior to the 1991 eruption s of Mount Pinatubo. In: Punongbayan, R., Newhall, C. (Eds.), Fire and mud: eruptions and lahars of Mount Pinatubo, Philippines. PHIVOLCS, University of Washington Press, Quezon City, Seattle, pp. 329–338.

Harlow, D., Power, J., Laguerta, E., Ambubuyog, G., White, R., Hoblit, R., 1996. Precursory seismicity and forecasting of the June 1991, eruption of Mount Pinatubo. In: Punongbayan, R., Newhall, C. (Eds.), Fire and mud: eruptions and lahars of Mount Pinatubo, Philippines. PHIVOLCS, University of Washington Press, Quezon City, Seattle, pp. 285–306.

Hoblit, R., Wolfe, E., Scott, W., Couchman, M., Pallister, J., Javier, D., 1996. The preclimatic eruptions of Mount Pinatubo, June 1991. In: Punongbayan, R., Newhall, C. (Eds.), Fire and mud: eruptions and lahars of Mount Pinatubo, Philippines. PHIVOLCS, University of Washington Press, Quezon City, Seattle, pp. 457–512.

Kilburn, C., 2003. Multiscale fracturing as a key to forecasting volcanic eruptions. J. Volcanol. Geotherm. Res. 125 (3–4), 271–289.

Martí, J., Aspinall, W., Sobradelo, R., Felpeto, A., Geyer, A., Ortiz, R., Baxter, P., Cole, P., Pacheco, J., Blanco, M., Lopez, C., 2008. A long-term volcanic hazard event tree for Teide-Pico Viejo stratovolcanoes (Tenerife, Canary Islands). J. Volcanol. Geotherm. Res. 178 (3), 543–552.
Martí, J., Pinel, V., López, C., Geyer, A., Blanco, M., Abella, R., Tárraga, M., Castro, A., Rodrguez, C., 2013. Causes and mechanisms of the 2011-2012 El Hierro (Canary Islands) submarine eruption. J. Geophys. Res. 118, 823–839.
Marzocchi, W., Woo, G., 2007. Probabilistic eruption forecasting and the call for an evacuation. Geophys. Res. Lett. 34.
Marzocchi, W., Sandri, L., Gasparini, P., Newhall, C., Boschi, E., 2004. Quantifying probabilities of volcanic events: The example of volcanic hazard at Mount Vesuvius. J. Geophys. Res. 109.
Marzocchi, W., Sandri, L., Furlan, C., 2006. A quantitative model for volcanic hazard assessment. In: Mader, H.M., et al. (Eds.), Statistics in volcanology. Special Publication of IAVCEI, Geological Society of London.
Marzocchi, W., Sandri, L., Selva, J., 2007. BET_EF: a probabilistic tool for long- and short-term eruption forecasting. Bull. Volcanol. 70 (5), 623–632.
Marzocchi, W., Sandri, L., Selva, J., 2010. BET_VH: a probabilistic tool for long-term volcanic hazard assessment. Bull. Volcanol. 72, 705–716.
Neri, A., Aspinall, W.P., Cioni, R., Bertagnini, A., Baxter, P.J., Zuccaro, G., Andronico, D., Barsotti, S., Cole, P., Esposti-Ongaro, T., Hincks, T.K., Macedonio, G., Papale, P., Rosi, M., Santacroce, R., Woo, G., 2008. Developing an event tree for probabilistic hazard and risk assessment at Vesuvius. J. Volcanol. Geotherm. Res. 178 (3), 397–415.
Newhall, C., 2000. Volcano warnings. In: Sigurdsson, H., et al. (Eds.), Encyclopedia of volcanoes. Academic Press, San Diego, California, pp. 1185–11198.
Newhall, C.G., Hoblitt, R.P., 2002. Constructing event trees for volcanic crisis. Bull. Volcanol. 64, 3–20.
Newhall, C., Daag, A., Delfin, F., Hoblit, R., McGeehin, J., Pallister, J., Regalado, W., Rubin, M., Tubionosa, B., Tamayo, R., Umbal, J., 1996. Eruptive history of mount pinatubo. In: Punongbayan, R., Newhall, C. (Eds.), Fire and mud: eruptions and lahars of Mount Pinatubo, Philippines. PHIVOLCS, University of Washington Press, Quezon City, Seattle, pp. 165–196.
Phillipson, G., Sobradelo, R., Gottsmann, J., 2013. Global volcanic unrest in the 21st century: An analysis of the first decade. J. Volcanol. Geotherm. Res. 264, 183–196.
Punongbayan, R., Newhall, C., 1996. Fire and mud: eruptions and lahars of Mount Pinatubo, Philippines. PHIVOLCS, University of Washington Press, Quezon City, Seattle.
Punongbayan, R., Newhall, C., Butista, M., García, D., Harlow, D., Hoblit, R., Sabit, J., Solidum, R., 1996. Eruption hazard assessment and warnings. In: Punongbayan, R., Newhall, C. (Eds.), Fire and mud: eruptions and lahars of Mount Pinatubo, Philippines. PHIVOLCS, University of Washington Press, Quezon City, Seattle, pp. 67–86.
Rice, J.A., 2007. Mathematical statistics and data analysis. Duxbury Press, Belmont, California.
Sandri, L., Marzocchi, W., Zaccarelli, L., 2004. A new perspective in identifying the precursory patterns of eruptions. Bull. Volcanol. 66, 263–275.
Scarpa, R., Tilling, R. (Eds.), 1996. Monitoring and mitigation of volcano hazards. Springer-Verlag, Berlin Heidelberg.
Segall, P., 2013. Volcano deformation and eruption forecasting. In: Pyle, D., Mather, T., Biggs, J. (Eds.), Remote sensing of volcanoes and volcanic processes: Integrating observation and modelling. Geological Society, London, Special Publications, London.
Sobradelo, R., Martí, J., 2010. Bayesian event tree for long-term volcanic hazard assessment: Application to Teide-Pico Viejo stratovolcanoes, Tenerife, Canary Islands. J. Geophys. Res. Solid Earth 115 (B5).
Sobradelo, R., Bartolini, S., Martí, J., 2013. HASSET: a probability event tree tool to evaluate future volcanic scenarios using Bayesian inference. Bull. Volcanol. 76 (1), 1–15.
Sobradelo, R., Martí, J., Kilburn, C., López, C., 2014. Probabilistic approach to decision making under uncertainty during volcanic crises. Nat. Hazards http://dx.doi.org/10.1007/s11069-014-1530-8.
Sparks, R., 2003. Forecasting volcanic eruptions. Earth Planet. Sci. Lett. 210, 1–15.
Voight, B., 1988. A method for prediction of volcanic-eruptions. Nature 332, 125–130.
Wolfe, E., Hoblit, R., 1996. Overview of the eruptions. In: Punongbayan, R., Newhall, C. (Eds.), Fire and mud: eruptions and lahars of Mount Pinatubo, Philippines. PHIVOLCS, University of Washington Press, Quezon City, Seattle, pp. 3–20.