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ORIGINAL PAPER

# Probabilistic approach to decision-making under uncertainty during volcanic crises: retrospective application to the El Hierro (Spain) 2011 volcanic crisis

Rosa Sobradelo · Joan Martí · Christopher Kilburn · Carmen López

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Abstract Understanding the potential evolution of a volcanic crisis is crucial for designing effective mitigation strategies. This is especially the case for volcanoes close to densely populated regions, where inappropriate decisions may trigger widespread loss of life, economic disruption, and public distress. An outstanding goal for improving the management of volcanic crises, therefore, is to develop objective, real-time methodologies for evaluating how an emergency will develop and how scientists communicate with decision-makers. Here, we present a new model Bayesian Decision Model (BADEMO) that applies a general and flexible, probabilistic approach to managing volcanic crises. The model combines the hazard and risk factors that decision-makers need for a holistic analysis of a volcanic crisis. These factors include eruption scenarios and their probabilities of occurrence, the vulnerability of populations and their activities, and the costs of false alarms and failed forecasts. The model can be implemented before an emergency, to identify actions for reducing the vulnerability of a district; during an emergency, to identify the optimum mitigating actions and how these may change as new information is obtained; and after an emergency, to assess the effectiveness of a mitigating response and, from the results, to improve strategies before another crisis occurs. As illustrated by a retrospective analysis of the 2011 eruption of El Hierro, in the Canary Islands, BADEMO provides the basis for quantifying the uncertainty associated with each recommended action as an emergency evolves and serves as a mechanism for improving communications between scientists and decision-makers.

R. Sobradelo · C. Kilburn Aon Benfield UCL Hazard Centre, Department of Earth Sciences, University College London, London, UK

C. López

Observatorio Geofísico Central, Instituto Geográfico Nacional, Parque del Retiro, Madrid, Spain

R. Sobradelo (⊠) · J. Martí Institute of Earth Sciences Jaume Almera, ICTJA-CSIC, Barcelona, Spain e-mail: sobradelo@gmail.com

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#### 1 Introduction

Volcanic crises are complex and challenging to manage because of the inability to view directly the subsurface processes that operate before eruption and, during eruption, because of the time-dependent behaviour of volcanic hazards. As a result, decisions taken during emergencies are based on evaluations that contain significant uncertainty and so test the ability of communities to protect themselves and their infrastructure, as well as their ability to recover from an eruption.

A volcanic emergency is a dynamic process. It begins with unrest and continues until magma can no longer be erupted or until the magmatic system has achieved an unthreatening equilibrium such as minor degassing across a limited area around the vent. Forecasts of a volcano's behaviour change as new information is obtained and so, as a crisis proceeds, decision-makers are commonly required to keep re-evaluating the most effective response. The task is especially challenging during emergencies in densely populated regions and around volcanoes that are reawakening after an extended period of repose. In the first case, owing to the large sizes of the population and vulnerable district, the potential losses from a false alarm may be as high as those from an eruption itself. In the second, decision-makers may have little or no experience of responding to a volcanic emergency. Complexity and inexperience will both magnify the uncertainty in evaluating the consequences of alternative responses, particularly when decisions are being made under stress.

An effective method for improving how decisions are made is to prepare scenarios that describe the potential impact of an eruption. Recent procedures have focussed on scenarios for the possible eruptive behaviour of a volcano and on probabilistic criteria for evacuating populations at risk. In the first case, the Bayesian methodology proposed by Newhall and Hoblitt (2002) has been used to develop computer-assisted procedures for transforming field data into probabilities that an eruption scenario will take place (Marzocchi et al. 2008, 2010; Sobradelo et al. 2014). In the second, Woo (2008) followed the method of Katz and Murphy (1997) to develop a probabilistic criteria for evacuation decision-making within a cost-benefit analysis framework and showed how this may be quantitatively expressed in terms of the proportion of the evacuees owing their lives to the evacuation call.

The two approaches together provide a framework for assisting decision-makers during an emergency (Marzocchi and Woo 2007; Marzocchi et al. 2012). However, their evaluation of social impact has been restricted to the economic consequences of lives lost and of an evacuation. More comprehensive analyses are needed to take account also of the potential cost from injuries to people, from the loss of property and livelihoods, and from the consequences of mitigating actions beyond an evacuation. For example, even before a crisis develops, mitigating actions can be implemented that involve vulnerable infrastructure, economic and environmental interests, and the establishment of no-construction zones. Each of these actions will have an associated potential loss that will depend on a number of parameters that, in addition to the population at risk, include the vulnerability of a district and the value of its exposed economic and physical infrastructure. The availability and consequences of a mitigating action may vary as a crisis develops and so influence the final decision. Analyses must therefore be structured in a systematic, quantitative manner, so that actions already taken can be re-evaluated as necessary during an emergency. As part of Operations Research, Bayesian decision theory provides the tools to combine the philosophy, theory, methodology, and professional practice necessary to address complex decision-making problems in a formal manner. It uses procedures, methods, and tools for identifying, clearly representing, and formally assessing important aspects of a decision, for prescribing a recommended course of action, and for translating the formal representation of a decision and its corresponding recommendation into insight for the decision-maker and other stakeholders (Rice 2007; Berger 2010).

This paper presents a new, purpose-built Bayesian Decision Model, BADEMO, as a general and flexible, probabilistic model for volcano crisis management. It shows how evaluations can be improved during an emergency by applying Bayesian decision theory to a structured decision framework. It assesses the expected gains and losses from mitigating actions by integrating eruption scenarios, and their probabilities of occurrence, with data on vulnerable populations and their economic and physical infrastructure, as well as the cost of a false alarm and of a failed forecast. Most importantly, BADEMO can account for changes in the values of controlling parameters during an emergency and so enables recommended actions to be re-evaluated and modified as circumstances evolve. The general framework of the decision-making problem is first presented in terms of stages in volcanic unrest and of the associated eruption scenarios and social factors to be evaluated. Bayesian decision theory is then applied to assess the probabilities and consequences of each scenario. Finally, the application of BADEMO is illustrated by retrospectively evaluating the crisis before the 2011 eruption of El Hierro in the Canary Islands.

### 2 Decision model framework for volcanic crises management

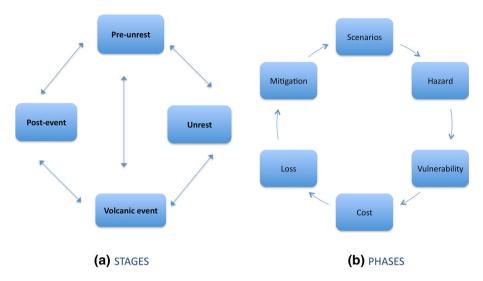
The effective management of a volcanic crisis is a cyclic process that includes a pre-unrest and a post-event stage, in addition to the unrest and volcanic event episodes (Fig. 1a). The unrest episode sets the beginning of a volcanic crisis and could evolve into a volcanic event or go back to the pre-unrest stage, with volcanic activity returning to normal background level. Sometimes a volcanic event could occur without any warning; this is, without a previous unrest episode existing, being detected or correctly interpreted. The process may evolve clockwise, as in the case of Pinatubo in 1991 (Newhall and Punongbayan 2010), starting with an unrest episode with a duration that may go from days to some years, a volcanic event episode with different eruptive phases, and a post-eruption stage where activity returns to the pre-unrest levels. A different case is offered by volcanic crises that alternate between unrest and eruption episodes for a period of time, before returning to background levels of activity. This could be the case of the ongoing crisis in Montserrat (Druitt and Kokelaar 2002). For these reasons, all stages are crucial to determine the level of preparedness and resilience of a community threatened by a volcano.

The key information required for volcanic crisis management can be represented as an event tree (Fig. 2), for which the trunk consists of a series of stages that describe the approach to eruption and subsequent recovery. Each stage acts as a node to support branches that describe the components required to evaluate the costs and the benefits of potential mitigating actions. Decisions are then made on which of the mitigation actions are most appropriate. For operational simplicity, we have divided an emergency into four stages (Fig. 1a): pre-unrest, unrest, volcanic event, and post-event. At each stage, formal

Table 1 El Hierro magmatic eruption scenarios likely to occur, and corresponding prior and posterior probabilities of occurrence	c eruption scer	larios lil	cely to occu	r, and corresponding	g prior and posterior p	robabilities of occurre	ence	
Prior				Posterior probabilities	ities			
Outcome	Scenario zone	VEI	HASSET	Phase I (7–18/7 2011)	Phase II (19/7–3/9 2011)	Phase III (4–26/9 2011)	Phase IV (27/9–7/10 2011)	Phase V (8–10/10 2011)
Eruption <200 m deep or near coast	s1 (SW- 1b)	2, 3, 4	0.23	0	0	0.04	0.09	0
	s2 (NW- 2b)	2, 3, 4	0.19	0.02	0.09	0	0	0
	s3 (NE-3b)	2, 3, 4	0.36	0	0	0	0	0
	s4 (S-4b)	2, 3, 4	0.15	0	0	0.09	0.34	0.57
Subaerial eruption	s5 (SW- 1a)	2, 3	0.23	0	0.05	0.06	0.09	0
	s6 (NW- 2a)	2, 3	0.19	0.02	0.19	0.21	0	0
	s7 (NE-3a)	2, 3	0.36	0	0	0	0	0
	s8 (S-4a)	2, 3	0.15	0	0.04	0.22	0.28	0.28
s9 Other (including submarine eruption >200 m deep)	ie eruption >2	00 m	0.08	0.95	0.63	0.38	0.21	0.15
The possible products of an eruption in scenarios 1 to 8 are lava flows, fallout, ballistic, and pyroclastic density currents. See text for detailed explanations on how these probabilities are estimated	eruption in sce	marios 1	to 8 are la	va flows, fallout, ba	ullistic, and pyroclastic	density currents. See	text for detailed explan	ations on how these

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**Fig. 1** a Volcanic crisis management is part of a longer cycle that also includes pre-unrest and post-event stages, which are crucial for determining the level of preparedness and resilience. A different degree of preparedness and reaction is required in each stage of this cycle, in order to be as effective as possible when facing a volcanic threat. b Six phases in each stage conform to a cyclic process in the decision-making problem (see text for explanations)

decision analysis is applied to physical, social, and economic data in order to decide on the preferred mitigating action.

# 2.1 Operational stages

*Pre-unrest* defines the background activity of a volcano while it does not pose a threat. This typically coincides with a state of dormancy, when magma is not exposed at the surface. At persistently active volcanoes, the background state may instead be associated with low-level activity at an open vent, such as degassing from a lava lake or minor strombolian activity. Pre-unrest provides the opportunity: (1) to acquire field data for long-term hazard assessment, including evidence for the potential styles of eruptions and the distribution of their products, as well as databases on population distributions and land use; (2) to prepare eruption scenarios and appropriate emergency responses; and (3) to implement educational programmes among decision-makers and vulnerable communities to raise awareness of volcanic hazards.

*Unrest* begins when potential precursory signals show an increase in level above background values. Such signals are normally detected by geophysical or geochemical monitoring and include an increase in local rates of micro-seismicity (or volcano-tectonic events), ground deformation, and gas release. The signals are analysed to determine whether where and when an eruption might occur. The results are regularly updated to prepare for the short-term implementation of response plans established during the stage of pre-unrest.

A volcanic event occurs when the precursory sequence culminates in a major change in the state of the volcano. The event is usually a magmatic eruption, but may in addition be characterised by a non-magmatic event, such as a phreatic eruption, intense seismic swarms, or slope failure on the volcano's flanks. At this stage, the emergency response can be focussed to accommodate a known style of event.

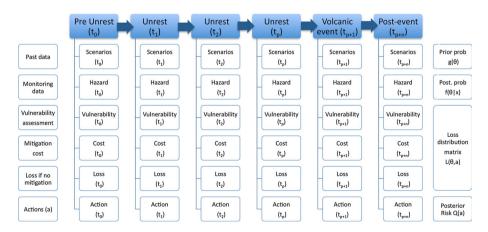
The *post-event* stage describes the interval in which the volcano returns to background conditions. The interval may continue from days to years, during which time the emergency response is directed towards rescue and recovery.

# 2.2 Decision analysis

The process of reaching a decision during an emergency can formally be divided into deterministic, probabilistic, and informational phases (Kaufman and Thomas 1977; Thomas and Samson 1986). The deterministic phase integrates fundamental information on the threatening process (e.g. the type of volcanic phenomenon); the probabilistic phase determines the probability that a vulnerable component (e.g. the exposed populations and infrastructure) will be threatened by one or more processes; and the informational phase evaluates the outcome of each threat, including the consequences of pursuing a specified mitigating action. All phases must be completed at each stage of a crisis (Fig. 2). They are addressed in a specific order, progressing from general evaluations of a volcano's potential behaviour to more specific evaluations of the consequences of a given mitigating response.

# 2.2.1 Deterministic phase: scenarios

The deterministic phase assesses the potential range of behaviour of a volcano and its likelihood of occurrence. Event *scenarios* are identified by taking account not only of how a particular volcano has behaved in the past, but, by comparing the behaviour of similar types of volcanos elsewhere, also of additional behaviour patterns that could occur. Input data are obtained from geological and volcanological field observations, monitoring records, and the results from theoretical models and from expert elicitation.



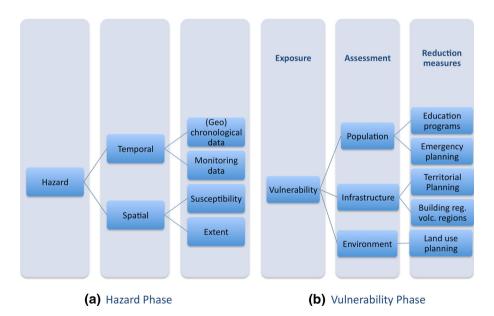
**Fig. 2** Decision model structure. General form of the decision problem as a hierarchical event tree structure made of four stages (nodes), pre-unrest, unrest, volcanic event, and post-event and six phases (branches) per stage, scenarios, hazard, vulnerability, cost, loss and mitigation

# 2.2.2 Probabilistic phase: volcanic hazard

The probabilistic phase evaluates the *volcanic hazard*, defined as the probability that a destructive phenomenon may affect a particular area in a particular time interval. The probability that a phenomenon will affect an area of a given size is estimated from the event scenarios. For long-term evaluations, especially during pre-unrest, the probability that the phenomenon will occur within a specified time interval is estimated from its past frequency of occurrence, based on historical observations and on geological data. For short-term evaluations, once unrest has begun, the probability is evaluated from monitoring data and from analyses of previous crises (Fig. 3a). As defined, the volcanic hazard has a space component related to how likely (susceptible) a new vent will open in a particular area opening (Martí and Felpeto 2010). To do a complete volcanic hazard assessment, we need to assess the volcanic susceptibility and the potential extent of each product. The relative likelihood of each scenario is then determined using various methods (Newhall and Hoblitt 2002; Aspinall 2006; Felpeto et al. 2007; Marzocchi et al. 2008, 2010; Sobradelo and Martí 2010; Sobradelo et al. 2014).

# 2.2.3 Informational phase: vulnerability, cost of mitigation actions, loss if no mitigation actions

The informational phase evaluates the potential costs of an eruption and of mitigating procedures. It combines the results from the probabilistic phase with assessments of vulnerability and its associated economic impact. Figure 3b shows the information required in this phase and the corresponding mitigation actions.



**Fig. 3** a The information required in the hazard phase is (geo)chronological and/or monitoring data for the temporal assessment, and susceptibility and extent for the spatial hazard. **b** Educational programs, emergency, territorial and land use planning, as well as building regulations, will all play a vital role in the volcanic crisis management and will determine the resilience of a community to face a volcanic event

*Vulnerability* is defined as the loss that is expected from a particular hazard. It is a dynamic quantity that changes with time. During pre-unrest, for example, the vulnerability of a district may increase as its population becomes larger, but decrease if suitable engineering defences are implemented or the number of evacuation routes is increased; it will also vary with changes in land use and in economic activity. During unrest and eruption, the vulnerability will evolve to reflect short-term changes in circumstance, including a change in eruptive behaviour, the movement of people to different districts, and the disruption of mitigation actions (e.g. caused by the blockage of escape routes). Vulnerability must therefore be reassessed repeatedly as an emergency develops.

Vulnerability measures the potential cost of a hazard before implementing additional mitigating procedures. For a full economic assessment, account must also be taken of the *cost* of the mitigation actions themselves, as well as of the *loss* due to the failure to follow those actions. The type of actions necessary will depend on a community's existing level of preparedness, as reflected by its vulnerability, and on its ability to respond to new mitigating procedures during an emergency. In addition, even under ideal conditions, when appropriate mitigation strategies have been implemented, losses will occur owing to unexpected irregularities and to the stochastic nature of human behaviour. Thus, the recommended design of engineering defenses may have underestimated the potential magnitude of a hazard; structures may have been constructed with defective material; and individuals or communities may refuse to evacuate a threatened district.

#### 2.2.4 Mitigation strategies

Merging the information from the previous phases, the decision-maker should have by now sufficient information to evaluate the various alternatives, establish a course of action, and perform sensitivity analysis in relation to the optimal strategy, which may lead to further information gathering. The objective here is for the decision-maker to develop insight into the decision and determine a clear course of action. Much of the insight developed in this phase results from exploring the implications of the formal decision model developed during the previous phases. Central to these implications is the formal recommendation for action.

# 3 Bayesian decision model (BADEMO) for volcanic crises management

Decision theory quantifies the process of identifying optimal decisions in the presence of uncertainty (Berger 2010). Using Bayesian analysis, it combines data from the stages described above to identify a range of actions and their associated losses and uncertainties. The aim is to choose the action that minimises the expected loss (Rice 2007).

The decision-maker chooses an action *a* from a set *A* of all possible actions based on the observation of a random variable, or data, *X*, with a probability distribution that depends on a parameter  $\theta$ , called the *state of nature*. A statistical decision function, *d*, depends on the available data *X* and is used to choose a particular action, a = d(X). By taking action *a*, the decision-maker incurs a loss,  $L(\theta, a)$ , which depends on both the state of nature  $\theta$  and the action *a*. The loss function represents the loss suffered by taking action *a* when the true parameter value (state of nature) is  $\theta$ ,  $(L(\theta, a) \ge 0)$ .

Using a Bayesian approach, the Expected Loss (also formally labelled the Posterior Risk) for action a is

$$Q(a) = \int L(\theta, a)h(\theta|x)d\theta$$
(1)

where

$$h(\theta|x) = \frac{f(x|\theta)g(\theta)}{\int f(x|\theta)g(\theta)d\theta}$$
(2)

is the so-called posterior distribution of  $\theta$  given that the variable X takes the observed value x, and so Q(a) is the sum of the losses from different states of nature weighted by their probabilities of occurrence. The optimal decision rule is the function d(x) that minimises the posterior risk for each value of x.

Since we are making decisions in the presence of uncertainty, the actual incurred loss,  $L(\theta, a)$ , will never be known with certainty (at least at the time of the decision-making), so a natural method of proceeding in the face of this uncertainty is to consider instead the *expected* loss of making a decision and then choose an *optimal* decision with respect to this expected loss. The use of Bayesian expected loss directly incorporates the consequences of the uncertainty.

The posterior probability  $h(\theta|x)$  describes the hazard and is proportional to the product of the probability of occurrence of a particular state of nature  $\theta$ , i.e. the distribution function  $g(\theta)$  prior to observing the data, multiplied by the probability of observing these values of x if the true state of nature was indeed  $\theta$ ,  $f(x|\theta)$ , i.e. the conditional distribution of x given  $\theta$ . During unrest, x would be the monitoring data,  $\theta$  would be the possible eruptive scenarios, and  $g(\theta)$  would be the probability of occurrence of each scenario before unrest.

Figure 2 represents the decision framework of BADEMO with the corresponding parameter matrix in each phase and stage. As an example, a simple algorithm for finding the optimal decision is as follows:

- 1. Identify the states of nature (scenarios),  $\theta$ , to be evaluated and corresponding prior probabilities,  $g(\theta)$ .
- 2. Using available data, expert elicitation, existing models, monitoring data, etc. define the conditional probability function,  $f(x|\theta)$  and compute the posterior distribution  $h(\theta|x)$ .
- 3. Define the set of actions (choices) *a* to be evaluated.
- 4. Define the loss distribution associated with each action and state of nature,  $L(\theta, a)$ . The loss includes both damage and indirect economic losses, and it is a function of the vulnerability, this is, the population at risk, the infrastructures and the environment.
- 5. For each action, *a* calculates the posterior risk or expected loss Q(a), with respect to the different states of nature  $\theta$ .

Apart from deciding on the optimal mitigation strategy, another important aspect is the time of the implementation (Marzocchi and Woo 2007), to ensure the available safety estimated time (ASET) before onset is larger than the required safety estimated time (RSET) to implement an action. The longer we wait, the more we know about the evolution of the process and the lower the uncertainty, but it decreases our ASET before the volcanic event. ASET will depend on the escalation rate to the event, whereas RSET will depend mainly on the type and length of the mitigation action, including the population at risk and road infrastructures. So the Posterior Risk Q is also a function of time, Q(a, t), and it will be determined by the point in time during the volcanic crises when the decision is called. Further work is needed to define strategies that ensure  $Q(a, RSET) \leq Q(a, ASET)$  for each volcanic system.

# **4** BADEMO applied retrospectively to the El Hierro volcanic eruption (2011–2012)

El Hierro is the youngest and most westerly of the Atlantic Canary Islands, about 470 km from the northwest coast of Africa. After at least 200 years of repose, a submarine eruption began on 10 October 2011, about 2.0–2.5 km from the southern coastal town of La Restinga (López et al. 2012). The eruption occurred at 300 m below sea level (Fig. 4) and was preceded by nearly three months of unrest, during which more than 11,000 seismic events, 4 cm of surface deformation, and anomalous gas emissions were recorded by the monitoring networks of Spain's National Geographical Institute (IGN; López et al. (2012); Martí et al. (2013)) and the Volcanological Institute of the Canary Islands (INVOLCAN; Ibánez et al. (2012)). The eruptive activity decreased on 27 February 2012 and, since then until the time of writing (December 2014), only residual gas emissions have been registered from the main vent.

Although the eruption was offshore, it had a devastating effect on tourism and fishing, which are the mainstays of the island's economy. The occupancy of hotel rooms declined from 50 to 4 %. Marine life was depleted within a radius of 3 km from the vent, forcing the migration of larger sea creatures to other locations. Within just 2 weeks from the start of eruption, the cost to the island's population of 10,960 was estimated to have been  $\in$ 26 million (*Diario Sesiones Parlamento Canario, Num. 13. 26/Oct/2011*, www.gobcan.es).

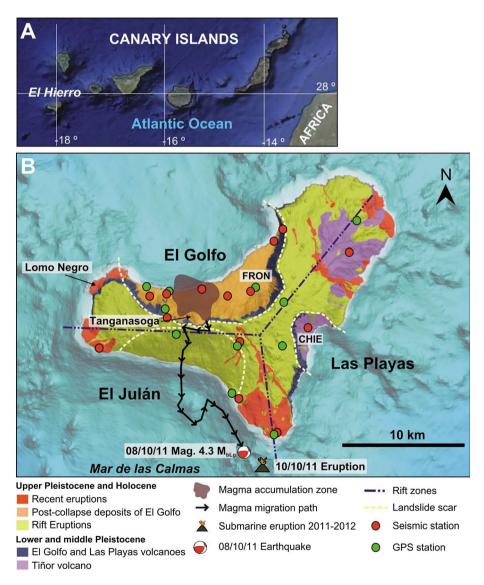
Owing to the long previous repose interval, local decision-makers on El Hierro were not familiar with responding to a volcanic emergency. The crisis thus provides a good test of the effectiveness of BADEMO to combine scientific, social, and economic information in a manner that is of practical assistance to decision-makers.

4.1 Pre-unrest stage: scenarios and prior probability of occurrence, vulnerability assessment, actions and cost-loss distribution matrix

The 2011 eruption was the first recorded on El Hierro. Analyses during pre-unrest must therefore rely on geological information to constrain eruption scenarios. The island has an approximately triangular outline with sides 27, 23, and 20 km long (Fig. 4). Eruptions with an estimated Volcano Explosivity Index (VEI) of 2 or less have produced monogenetic cinder cones and lava flows primarily, but not exclusively, along three rift zones that radiate from the centre of the island (Guillou et al. 1996; Carracedo et al. 2001; Becerril et al. 2014; Martí et al. 2013). The activity has continued offshore, and bathymetric studies have revealed a significant number of well-preserved, and possibly recent, submarine volcanic cones, particularly along the southern rift (Gee et al. 2001). Between the rift zones, in contrast, episodes of flank collapse have left the major scars of El Golfo, Las Playas, and El Julán (Fig. 4; Day et al. (1997); Martí et al. (2013)). Although volcanism across the island has been dominated by mafic eruptions of lavas and localised explosive activity (Stroncik et al. 2009), hydromagmatic eruptions from coastal and near-offshore vents have generated pyroclastic density currents (PDCs), while Holocene eruptions of trachy-phonolite have emplaced pyroclastic fall and flow deposits from a location close to the present Tanganasoga volcano on the western rift zone (Pedrazzi et al. 2013).

The chances of a zone being affected by an eruption are determined by the product of the susceptibility, or relative probability, that an eruption will occur in that zone and the probability that it will be affected by the products from an eruption anywhere on the island. The susceptibility was evaluated following the method of Becerril et al. (2013, 2014), who used the distribution of volcanic centres to divide El Hierro into five primary zones (Fig. 5): the three rift zones (1, 3, and 4), The El Golfo scar (2), and far-offshore (5).

volcanic complexes of Tiñor and El Golfo-Las Playas (Fig. 5). The type and distribution of products are governed by the style of eruption. As summarised in Table 1, we considered the following scenarios (based on Becerril et al. 2014):



**Fig. 4** a Location map of the Canary Islands. **b** Simplified geologic map of El Hierro showing the main morphological and structural features, and the epicentral migration of seismicity with time (see Martí et al. 2013 and references within for additional information). Locations and focal mechanisms of the earthquake preceding the onset of the eruption and location of the vent are also shown. *Dark blue dashed lines* are traces of the rift zones. *White dashed lines* are traces of landslides scars. CHIE, Seismic station; FRON, Frontera GPS station

(1) in Subzones a and b, a mafic eruption with a VEI of 2, comparable to the most common type on the island, as well as a less-common felsic eruption with a VEI of 3, and (2) additionally in Subzones b and c, a hydromagmatic eruption with a VEI of 3 or 4. Eruptions with a VEI of 2 or less were associated with the production of lava flows of short extent and cinder cones and those with a VEI of 3 and 4 with cinder cones, pyroclastic fall and density currents of medium extent. We also considered the possibility of a far-offshore eruption in Zone 5.

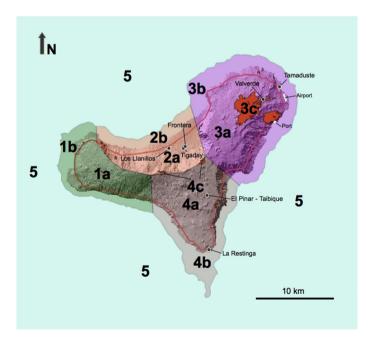
The probability of occurrence of each scenario was estimated using HASSET, the Bayesian event tree approach as described in Sobradelo et al. (2014), and the input data shown in Table 2. We assume that renewed unrest is of magmatic origin and so assign a weight of 1 to the corresponding branches in node *Unrest* and *Origin*. Based on an analysis of global volcanic unrest since 2000 (Phillipson et al. 2013), we assume also that the unrest has a probability of 64 % of culminating in an eruption and assign this value to the prior weight of the corresponding branch of the node *Outcome*. From the susceptibility analysis of Becerril et al. (2013), the prior weights for eruptions in Zones 1 to 5 are 0.2175, 0.1725, 0.2825, 0.11, and 0.2175, respectively. Although no eruptions have been documented in Zone 5, the susceptibility analysis assigns a weight of 0.2175 to this scenario to account for the likelihood of near-offshore eruptions that have not been documented. We assume total epistemic uncertainty and assign the value 1 to all data weight parameters.

Column HASSET in Table 1 shows the probability of a magmatic eruption in each zone. We consider subaerial and submarine areas in the same zone for the purpose of estimating the prior probability of a magmatic eruption, so the initial probabilities are the same for scenarios in subaerial (subzones "a") and coastal (subzones "b") locations. From this preliminary analysis, Zone 3 emerges with the largest volcanic hazard, followed by Zones 1, 2, and 4.

The potential cost to and loss associated with each scenario were estimated using economic and social data from Spain's National Civil Protection and Emergencies Agency and the IGN. El Hierro's population is concentrated within several villages grouped into three municipalities. The villages in this study are as follows: Frontera, Los Llanillos, and Tigaday in the municipality of La Frontera (population 4124); Tamaduste and Valverde in the municipality of Valverde (5035), with the only commercial airport and port on the island; and La Restinga, Taibique and El Pinar in the municipality of El Pinar de El Hierro (1.801). For the simulation, we have assumed that it is necessary to relocate for 30 days those in the village nearest to the new vent. Following Blerald (1986), Spain's National Civil Protection have estimated the cost of evacuation to be equivalent to 60 % of the monthly Gross Domestic Product (GDP; using 2011 values for the Canary Islands, www. ine.es). This cost includes any health and safety issues resulting from the relocation and maintenance of the evacuees, as well as logistic costs directly or indirectly related to the evacuation process. The cost of not evacuating was also estimated from the expected loss of income due to the interruption of work, that is, the estimated impact on the economy based on the total annual per capita Gross Domestic Product (GDP) of the Canary Islands in 2011 (www.ine.es), due either to permanent factors (death, serious injury, etc.) or to temporary ones (minor injuries, business closure, etc). For the purpose of this study, no casualties were considered, as the 30-day impact on the economy of losing the work force, whether it is due to death or not, is already accounted for. This assumption would have a different impact on the losses, reflected in the recovery cost, if we were considering a longterm effect (years to decades) on the economy. In that case, to quantify more accurately the economic losses, we would also have to account for the economic compensation received from government and insurance companies. However, as these are input parameters to our model given by those responsible for managing the volcanic crisis, deciding their value is neither within the scope of this paper nor the responsibility of the scientists. The resulting actions to be costed were thus (Table 3): a1, evacuate Los Llanillos; a2, evacuate Tamaduste and close the port and airport; a3, evacuate La Restinga and halt fishing activities; a4, evacuate Frontera and Tigaday; a5, evacuate Valverde village and close the port and airport; a6, evacuate Taibique; and a7, do nothing.

4.2 The unrest stage: posterior probabilities and posterior risk for each action

The first sign of unrest at El Hierro was recognised on 7 July 2011 from an anomalous north-eastward displacement of the GPS station FRON on the north side of the island (Fig. 4). The monitoring system was subsequently enhanced, and, by the time of the eruption on 10 October 2011, the additional precursory signals being recorded included changes in local seismicity, gravity, magnetism, and gas emissions (López et al. 2012; Martí et al. 2013). In response to the increasing unrest, the emergency services had by late September 2011 evacuated several families from La Restinga and had made plans to evacuate the whole island if necessary. On Tuesday 11 October, the regional government of the Canary Islands congregated at a local sports field approximately 600 inhabitants of La Restinga for evacuation. According to the Canary Islands government, the decision was made "due to the possibility that the centre of the eruption might move closer to the coast".



**Fig. 5** To perform a volcanic hazard assessment of the island from the geological setting described above, we divide the island into five zones according to the susceptibility study of Becerril et al. (2013), based on the distribution of volcanic lineations, eruption fissures, and location of vents: the three rift zones (1, 3, and 4), the El Golfo scar (2), and far-offshore (5). The first four zones were further subdivided into subzones "a" along a rift or collapse scar, "b" along the coast, including offshore to a depth of 200 m, and "c" around the volcanic complexes of Tiñor and El Golfo-Las Playas

Based on the behaviour of local seismicity and deformation, the unrest can be divided into five distinct phases (López et al. 2012): Phase I (7–18 July 2011), during which ground movement at station FRON continued to the NE at less than 1 mm d<sup>-1</sup>; Phase II (19 Jul–3 Sep 2011), when the rate of occurrence of local earthquakes increased to peak values of hundreds per day, the mean rate of seismic energy release was about 2 GJ d<sup>-1</sup> and most of the events were located 10–15 km beneath the El Golfo district; Phase III (4–26 Sep 2011), when the mean rates of ground movement at FRON increased to 1– 2 mm d<sup>-1</sup> and changed direction to the north; Phase IV (27 Sep–7 Oct 2011), when the mean rate of seismic energy release increased by an order of magnitude to about 70 GJ d<sup>-1</sup> and the epicentres shifted to off the southwest coast of the island; and Phase V (8–10 Oct 2011) when, following a magnitude 4.3 earthquake on 8 October (the largest-magnitude event up to that moment), a seismic swarm occurred at depths of less than 5 km and included epicentres less than 5 km offshore from La Restinga.

Although defined retrospectively, the phases coincide with significant changes in precursory behaviour and, consequently, changes in the level of concern as the crisis evolved. Data from each stage can thus be used to evaluate how BADEMO would have performed during the crisis. Based on the five phases, we assigned the conditional probabilities for each scenario to compute the posterior probabilities using Eq. (2), as shown in Table 1. Figure 6 shows the posterior risk for the response actions listed above, computed using Eq. (1), and its variation with time. The loss due to an action was estimated from the sum of the weighted losses for an event occurring in scenarios s1 to s8. In the case of a far-offshore eruption (scenario s9), the loss was estimated from the cost of implementing any of the actions a1 to a6, weighted by the posterior probability of scenario 9.

Node	Event	Past data	Prior weight	Data weight
Unrest	Yes	29	1	1
Unrest	No	0	0	1
Origin	Magmatic	29	1	1
Origin	Geothermal	0	0	1
Origin	Seismic	0	0	1
Origin	Other	0	0	1
Outcome	Magmatic eruption	29	0.64	1
Outcome	Sector failure	0	0.18	1
Outcome	Phreatic eruption	0	0	1
Outcome	No eruption	0	0.18	1
Location	Zone 1	6	0.2175	1
Location	Zone 2	5	0.1725	1
Location	Zone 3	10	0.2825	1
Location	Zone 4	4	0.11	1
Location	Zone 5	0	0.2175	1

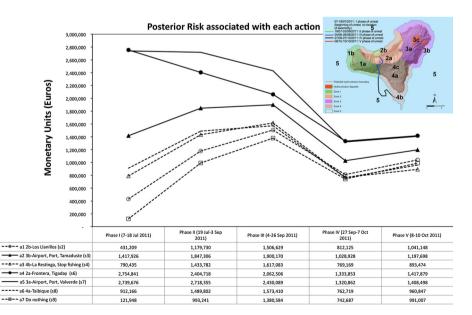
**Table 2** Input data for HASSET used to evaluate the probability of each scenario at time  $t_0$  given unrest, without taking into account monitoring data and using, only information on past events

	Zone	Vulnerability			Actions		
Erupuon < 200 In ueep of near coast 10 2b 3b	Submarine 1b SW 2b NW 3b NE 4b S	None Orchilla Lighthouse Frontera coastal population Airport, Port and coastal po Coastal nopulation El Pinar.	None Orchilla Lighthouse Frontera coastal population Airport, Port and coastal population of Valverde municipality Coastal booulation El Pinar. La Restinga port	de municipality	None None al. Evacuation Los Llanillos (333 hab) a2. Close Airport, Port Evacuate Tama a3. Stor fishing activities and evacuate	None None al. Evacuation Los Llanillos (333 hab) a.2. Close Airport, Port Evacuate Tamaduste (286 hab) a.3. Stoo fishing activities and evacuate La Restinea (547 hab)	(c 547 hab)
Subaerial eruption 1a 2a 3a 3a 4a	1a SW 2a NW 3a NE 4a S	Nuestra Sra de Los Reyes Frontera inland population Airport, Port and coastal p El Pinar inland population	Nuestra Sra de Los Reyes Sanctuary Frontera inland population Airport, Port and coastal population of Frontera municipality El Pinar inland population		None A. Evacuate Frontera and Tigad a5. Close Airport, Port and Evac a6. Evacuate Taibique (837 hab)	None A. Evacuate Frontera and Tigaday (2835 hab) a5. Close Airport, Port and Evacuate Valverde village (1686 hab) a6. Evacuate Taibique (837 hab)	e (1686 hab)
Any of the above					a7. Do nothing		
Event Zone		Scenario Actions	30d evacuation cost <sup>a</sup>	30d tourism cost <sup>b</sup>	Fishing sector <sup>c</sup>	30d Implementation cost <sup>d</sup>	30d Estimated loss <sup>e</sup>
Eruption <200 m deep or near coast 2b NV 3b NF 4b S	2b NW s2 3b NE s3 4b S s4	a1 a2 a3	329,770 283,226 541,694	1,076,160	159,500	329,770 1,359,386 701,194	549,617 1,548,203 1,062,324
Subaerial eruption 2a NW 3a NE 4a S	2a NW 86 3a NE 87 4a S 88	a4 a5 a6	2,807,500 1,669,646 828,881	1,076,160		2,807,500 2,745,806 828,881	4,679,168 3,858,903 1,381,469
Quantities are in euros <sup>a</sup> Estimated by Spain's National Civil Protection as 60 % of total annual per capita GDP of €19,806 for the Canary Islands in 2011 (www.inc.es) <sup>b</sup> Of closing airport and port. On average, there are 304 tourists per day. Tourists in the Canary Islands stay on average 9.2 days and spend €118 <sup>c</sup> Compensation for stopping activities for 30 days <sup>d</sup> Sum of cost of evacuation, tourism-related losses, and compensation to fishing sector ( <i>BOC 127 de 29 de junio de 2012</i> ) <sup>e</sup> If no mitigation action and event happens. Potential loss of workforce for 30 days and corresponding impact on the economy computed as the estimated losses per person based on €19,806 annual GDP times the number of people in the area, plus losses from tourism and the fishing sector	1 Protect rage, the s for 30 -related 1 ppens. P4	ion as 60 % of tot: re are 304 tourists days losses, and compen otential loss of woi ber of people in thu	al annual per capita GD per day. Tourists in the nation to fishing sector rkforce for 30 days and e area, plus losses from	DP of €19,806 for the Canary Islands st e Canary Islands st • ( <i>BOC 127 de 29 d</i> • corresponding imp • tourism and the fi	he Canary Islands i ay on average 9.2 ( <i>e junio de 2012</i> ) act on the economy shing sector	n 2011 (www.ine.es) days and spend €118 < computed as the estimated 1	losses per person

#### 4.3 Evaluation of results and posterior risk of mitigation actions

To illustrate how the model is applied, we here derive the posterior risk of  $\notin$ 431,209 associated with action  $a_1$ , evacuation of Los Llanillos, as early as in Phase I (Fig. 6). This is computed using Eq. (1) for  $Q(a_1)$  as the sum of all the cost and losses that we would incur by evacuating Los Llanillos, and the event affects scenarios s1 to s9, weighted by the probability of these scenarios happening during Phase I. The fifth column in Table 1 gives the posterior probabilities of occurrence of each scenario in Phase I. The last column in the bottom part of Table 3 provides the 30-day estimated losses associated with each action. That is, if we take action  $a_1$  and the event in scenario  $s_2$  occurs, we would have taken the correct action and we would only incur the  $\in$  329,770 evacuation cost; if we take action a1 but the event in scenario s3 takes place, we would incur a loss of €1,548,203, which would be interpreted as the penalty for taking action  $a_1$  instead of action  $a_2$ . Similarly, for the remaining actions, if the event happens in scenario s4 but we took action a1 instead of action a3, the 30-day estimated losses are  $\in 1,062,324$ . The same applies to scenarios s6 to s8 associated with actions a4 to a6, respectively. In the event that nothing happens, that is, scenario s9, but we took action a1, we assume no losses but only the cost of implementing the action. As mentioned above, these costs and losses for each action have to be weighted by the probability of occurrence of their associated scenario, and so combining all, we get:  $Q(a_1) = 329,770 \times 0.023 + 1,548,203 \times 0 + 1,062,324 \times 0$  $+4,679,168 \times 0.023 +$  $3,858,903 \times 0 + 1,381,469 \times 0 + 329,770 \times 0.953$  which is approximately 431,209.

Table 1 and Fig. 6 show the results from the BADEMO analysis. In Phase I, the set of actions with the minimum cost is *Evacuate Los Llanillos* and *Do nothing*, since the final (posterior) probabilities are significant only for the El Golfo district (Zones 2a and 2b) in



**Fig. 6** Evolution of the posterior risk associated with each action across the different phases of the volcanic crisis (and corresponding scenario from Table 1). The *inset map* shows the hazard zones as indicated in text and Fig. 5, and the position of seismicity during each phase of unrest. See text for detailed explanations

which anomalous ground deformation was first recognised. However, the action with the minimum (posterior) risk is *Do nothing*. The risk of taking actions *a2*, *a4*, and *a5* is very large, owing to the cost of closing the airport and the port, and of evacuating Valverde, Tamaduste and Frontera, when the chances of an eruption near by are non-existent. Moreover, the final (posterior) probability of no eruption is very large at this stage, indicating that the actions would carry large economic penalties.

From July 19th to September 26th (Phases II and III), the most probable scenarios are still *s*<sup>2</sup> and *s*<sup>6</sup> in Zone 2, followed by *s*<sup>9</sup>. The updated probability of no eruption has decreased significantly from 0.95 in Phase I to 0.38 in Phase III. By September 4th (Phase III), scenarios in Zone 4 are most likely to occur, reflecting the southward migration of the earthquake epicentres. The probability of Scenario *s*<sup>8</sup>, in particular, has increased from 0.04 in Phase II to 0.22 in Phase III. The set of actions with the lowest posterior risk, apart from action *Do nothing* (€1,380,584), is now *a*1, *a*3, and *a*6 (€1,506,629, €1,617,083, and €1,573,410, respectively), related to a halting of fishing activities and to the evacuation of Zones 4 and 2b. Actions *a*2, *a*4, and *a*5 have decreased their posterior risk with respect to Phase I. The penalty of implementing these actions is now lower since the probability of no eruption has decreased significantly from 0.95 to 0.38.

The trend towards an increasing risk in Zone 4 continued throughout Phases IV and V. By the time the eruption began, the action with lowest risk (of  $\in$ 893,474) was to evacuate La Restinga and to stop fishing activities (*a3*). This was the action actually taken during the emergency. However, it is notable that the action to do nothing (*a7*) had only a slightly larger associated risk ( $\in$ 991,007), so that the penalty for doing nothing would have been similar to the estimated losses for evacuating La Restinga and to halting fishing activities.

#### 5 Discussion

Using information available during the El Hierro crisis, BADEMO identified the preferred responses of "no action" and "evacuation of La Restinga and cessation of fishing". The second option coincided with what actually occurred. The eruption on 10 October 2011 finally began in Zone 5 two kilometres offshore from La Restinga, on the southern tip of the island, at a depth of 300 m below sea level (Fig. 4). It had no direct impact on the settlement and so the option of "no action" would have emerged as the more cost-effective. However, no action may have been interpreted at the time as a measure of indecision and so, from a political viewpoint, evacuation of La Restinga may still be considered as the best response.

The posterior probabilities computed by BADEMO are used to weigh the potential losses of each mitigating action. The probabilities are not to be interpreted in absolute terms but rather to be used as a measure for the relative importance of each action as a crisis evolves. At El Hierro, for example, Zone 3 is most vulnerable to a magmatic eruption, because it contains the largest population, as well as the commercial port and airport. Actions associated with this zone have a large estimated risk of incurring a cost without the zone actually being affected by an eruption. Action *a5* has the highest estimated risk of incurring a cost without the zone actually being affected by an eruption, as a volcanic event affecting Zone 3a in the North East is not likely. The penalty for taking this action, and not actions *a3*, *a4*, or *a6* associated with the more likely scenarios in Zones 2 and 4, is large.

The agreement between the retrospective analysis and actual outcome confirms the applicability of BADEMO to a real emergency. It also demonstrates how the objective

procedure identified an alternative option that could have been considered had BADEMO been available at the time. The results are thus encouraging for BADEMO to be applied and tested during emergencies that involve larger populations and greater economic consequences.

The advantage of BADEMO is that it combines the specific costs and potential losses with corresponding distributions of local susceptibility and vulnerability to eruptions and, from these, ranks the consequences of a range of mitigating actions. Indeed, BADEMO can be viewed as a tool for improved communication between monitoring scientists, who provide volcanological information, and those responsible for deciding the appropriate action plans and mitigating strategies.

### 6 Conclusion

Decisions made during volcanic emergencies require numerous factors to be evaluated simultaneously. The relative importance of each factor may vary from one emergency to the next and so decisions are commonly subjective and strongly influenced by the experience of the decision-maker. Subjectivity can be reduced by acquiring and analysing key data in a systematic and transparent manner and by presenting the results in a clear and simple form that can be readily understood under the conditions of high stress during an emergency.

Based on Bayesian Decision Theory, the new model BADEMO offers a flexible, probabilistic model for managing volcanic emergencies. As illustrated for the 2011 eruption of El Hierro, the model provides integrated scenarios for evaluating the potential physical and economic impact of an eruption. Unlike previous models that focussed on the consequences of evacuations, BADEMO identifies a spectrum of mitigating actions that take account also of the potential cost from injuries to people, from the loss of property and livelihoods, and from the consequences of the mitigating actions themselves. It can be run iteratively during an emergency, so that decisions can be revised as new information is obtained; such information includes new interpretations of eruption precursors and updated evaluations of vulnerability after a mitigating action has been implemented.

Even before an emergency begins, BADEMO can improve a community's resilience and preparedness, by identifying mitigating actions to decrease the vulnerability of a district and by raising awareness among decision-makers of the impact of eruptions. For example, the model can be used to select the most effective locations for constructing new evacuation routes, or to highlight areas in which economic development would be least affected by an eruption. It may also be operated as a training simulator to illustrate the consequences of implementing a particular mitigating action. This application is especially useful for decision-makers who are unfamiliar with volcanic activity. Finally, after an emergency, the model can be run retrospectively to assess the effectiveness of a mitigating response and, from the results, to improve strategies before another emergency occurs.

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