Body rocking or lift off in flow

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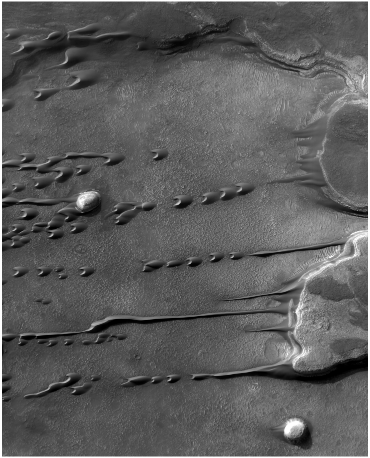
Te Whare Wānanga o Waitaha
CHRISTCHURCH NEW ZEALAND

Phillip L Wilson

30 January 2014

Rock 'n' roll on Mars



HiRISE/MRO/LPL/NASA
(Barchans $\sim 200m$)

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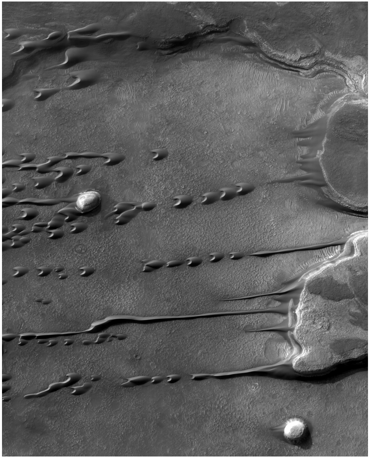

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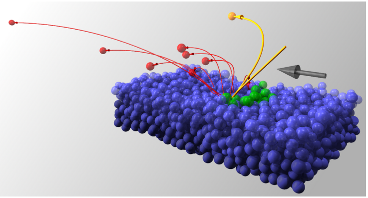
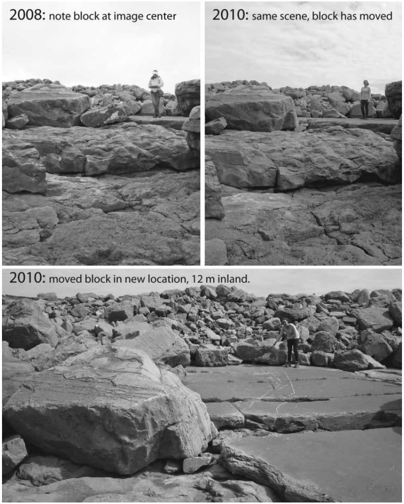


Carneiro *et al.*, Phys. Rev. Lett. (2013)

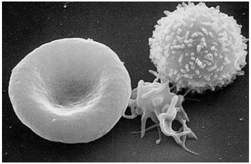
Other motivations



Cox *et al.*, J. Geol. (2012)

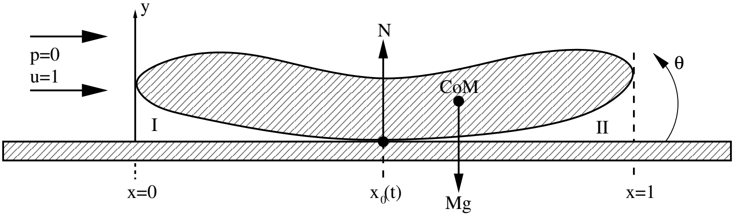
Bernews

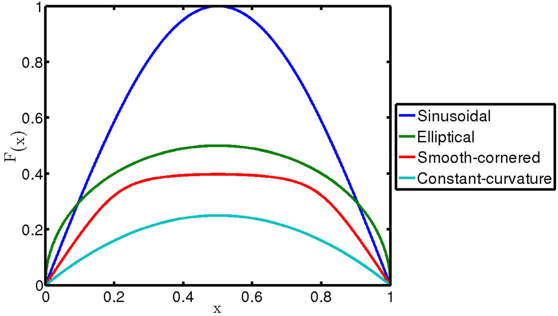
Team Solutions



public domain

Configuration





Governing equations

$$H(x,t) = h(t) + (x - x_c)\theta(t) - F(x);$$
$$H = H_x = 0 \text{ at } x = x_0(t).$$

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$$M\ddot{h}(t) = \int_0^1 p(x,t)\,\mathrm{d}x + N(t) - Mg^+,$$

$$I\ddot{\theta}(t) = \int_0^1 (x - x_c)p(x,t)\,\mathrm{d}x + (x_0 - x_c)N(t).$$

Numerical solutions of small-t equations

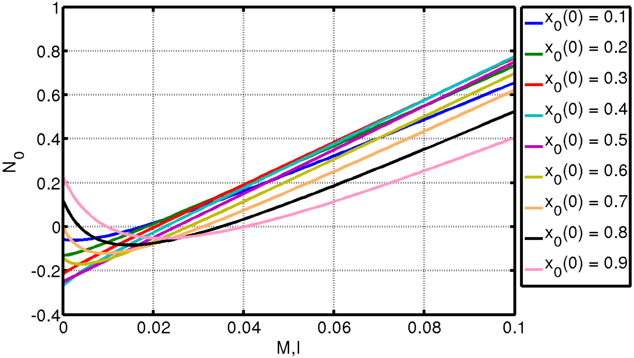

Figure: Sinusoidal body $F(x) = \sin(\pi x)$.

Numerical solutions of small-t equations

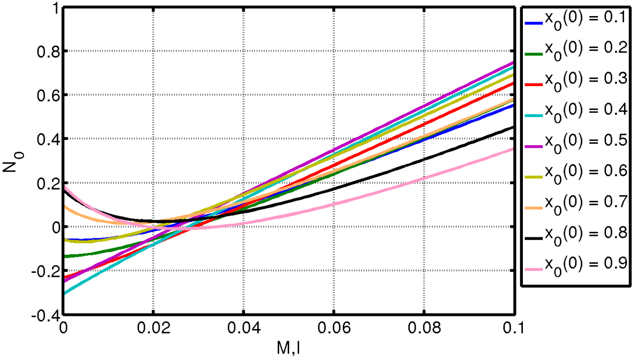

Figure: Elliptical body $F(x) = \sqrt{x - x^2}$.

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Numerical solutions of small-t equations

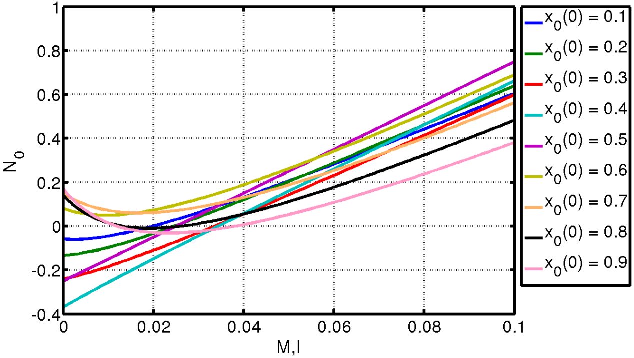

Figure: Smooth-cornered body.

Numerical solutions of small-t equations

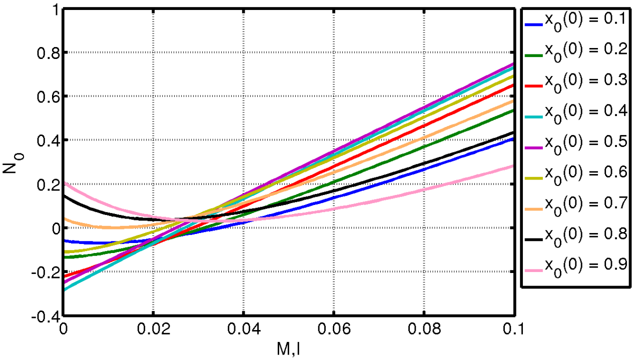

Figure: Constant-curvature body $F(x) = x(1 - x)$.

Numerical solutions of small-t equations

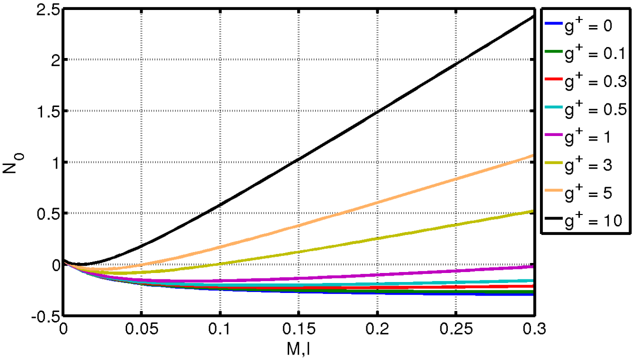

Figure: Constant-curvature body with $x_0(0) = 0.7$.

Numerical solutions of small-t equations

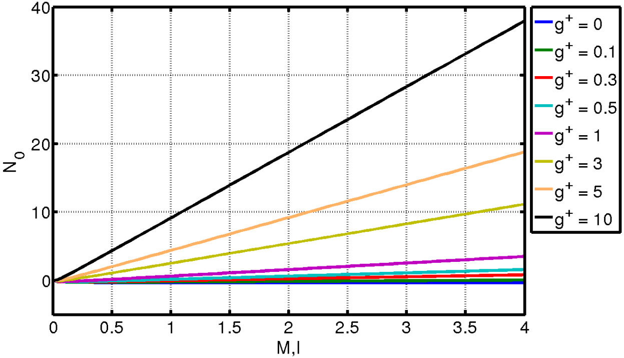

Figure: Constant-curvature body with $x_0(0) = 0.7$.

$O(1)$ time, negligible fluid effects

Mass- and moment of inertia-dominated.

$$\begin{aligned} M\ddot{h}(t) &= N(t) - Mg^+ , \\ I\ddot{\theta}(t) &= (x_0 - x_c)N(t) , \\ H(x,t) &= h(t) + (x - x_c)\theta(t) - F(x) , \\ H &= H_x = 0 \text{ at } x = x_0(t) . \end{aligned}$$

In subsequent analysis, a key equation:

$$\alpha\ddot{x}_0 + \beta\dot{x}_0^2 = (x_0 - x_c)g^+.$$

Here, α, β depend on I, M, and body shape.

Evolution of contact position

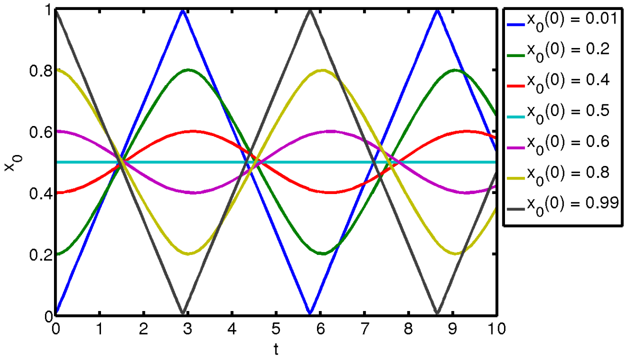

Figure: Sinusoidal body, with $g^+ = 10$, $x_c = 0.5$, varying initial contact point location, and zero initial contact point velocity.

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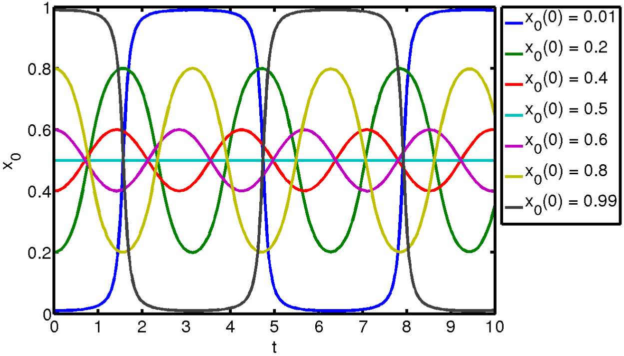

Figure: Elliptical body, with $g^+ = 10$, $x_c = 0.5$, varying initial contact point location, and zero initial contact point velocity.

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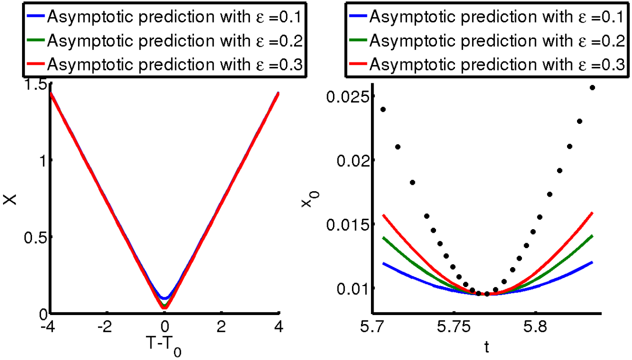

Figure: Analytical prediction of rocking behaviour obtained by asymptotic analysis. Sinusoidal body, conditions as before. Unscaled on the right. Dots are numerical solutions.

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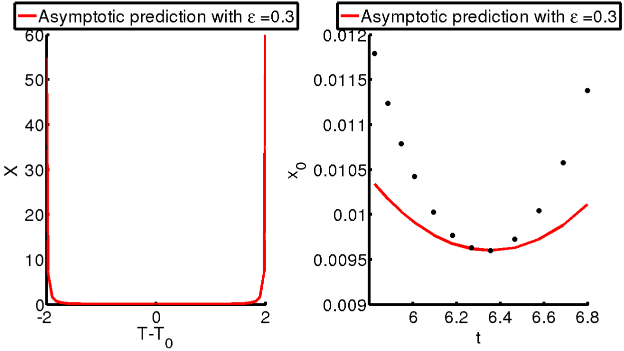

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$O(1)$ time, fluid effects

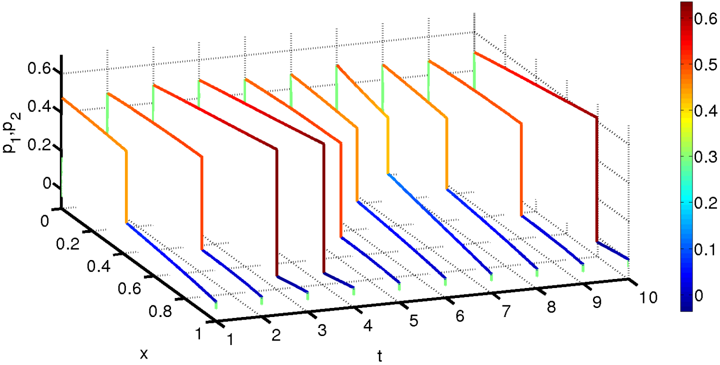
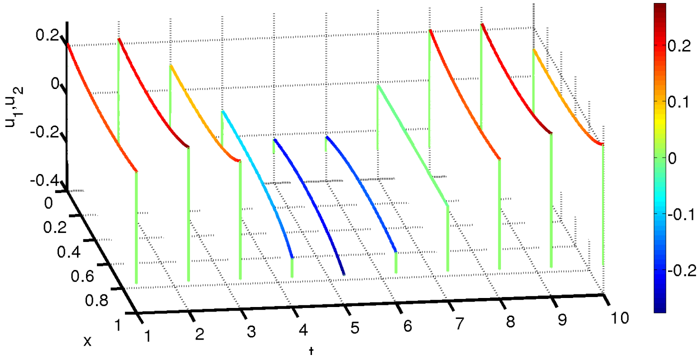


Figure: Sinusoidal body, $M = I = 0.125$, $x_0(0) = 0.33, \dot{x}_0(0) = 0$.

$O(1)$ time, fluid effects



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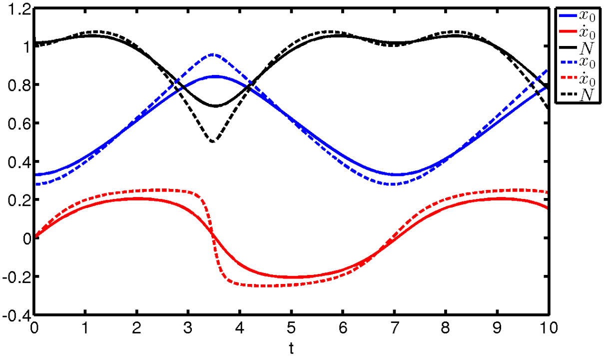

Figure: Sinusoidal body, $M = I = 0.125$, $\dot{x}_0(0) = 0$, and $x_0(0) = 0.33$ (solid lines) or $x_0(0) = 0.28$ (dashed lines).

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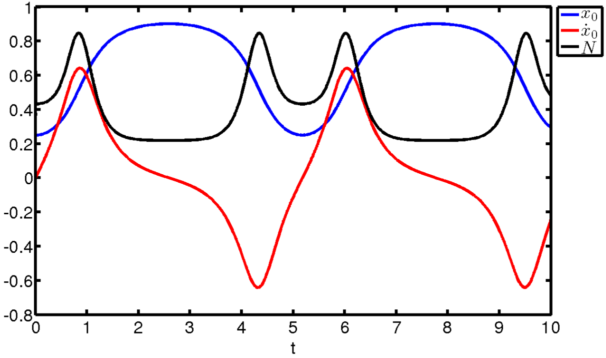

Figure: Elliptical body, $M = I = 0.08$, $x_0(0) = 0.25, \dot{x}_0(0) = 0$.

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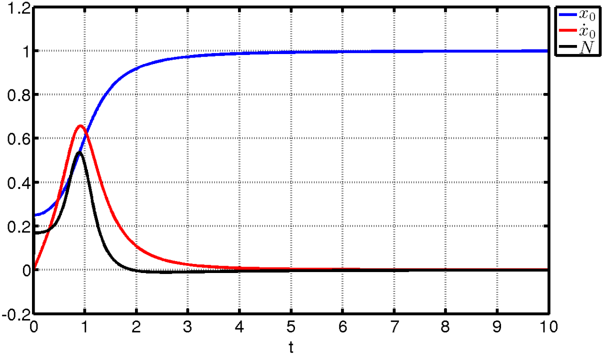

Figure: Elliptical body, $M = I = 0.05$, $x_0(0) = 0.25, \dot{x}_0(0) = 0$. Lift off occurs at $t \approx 2$ (subsequent results not physically meaningful).

$O(1)$ time, fluid effects

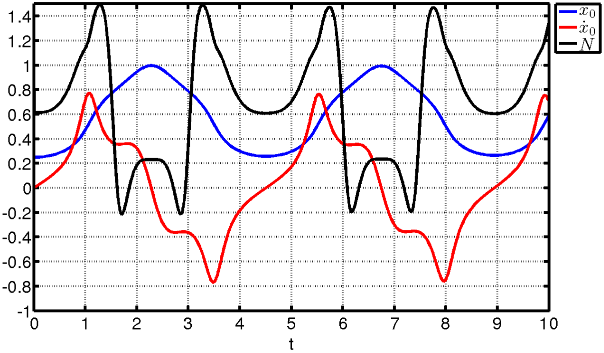

Figure: Smooth body, $M = I = 0.1$, $x_0(0) = 0.25, \dot{x}_0(0) = 0$. Lift off occurs at $t \approx 1.6$ (subsequent results not physically meaningful).

Transition from rocking to lift off is smooth

With negligible flow, a key equation was:

$$\alpha \ddot{x}_0 + \beta \dot{x}_0^2 = (x_0 - x_c) g^+,$$

where α, β depended on M, I, and body shape.

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With fluid effects, the corresponding equation becomes:

$$\alpha \ddot{x}_0 + \beta \dot{x}_0^2 = (x_0 - x_c) g^+ + \frac{i_2 - (x_0 - x_c) i_1}{M},$$

where

$$i_1 = \int_0^1 p(x,t)\,\mathrm{d}x \quad , \quad i_2 = \int_0^1 (x - x_c) p(x,t)\,\mathrm{d}x.$$

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- Thus the lift-off generating mechanism of i_1, i_2 — an effect of “added mass” and evolution in the fluid-body interaction — is still dominated by the $O(1)$ global contributions.

Discussion

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  - Reptation and clashes, multiple bodies — see also Smith & Wilson, Proc. Roy. Soc. A (2011).

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- Dimensional threshold wind speed for lift-off on Mars is 2–3 times that on Earth.
- Threshold speed $\propto \sqrt{\text{particle size}}$.
- Both results consistent with Martian observations in Wang, Int. J. Land Processes Arid Enviro. (2012).

$O(1)$ time, fluid effects

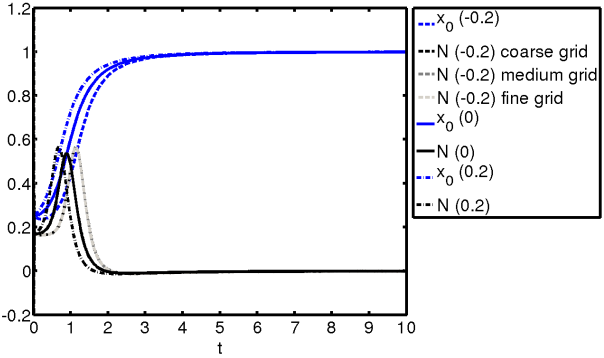
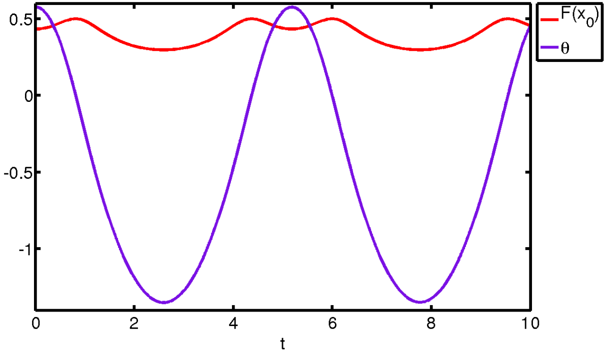


Figure: Elliptical body, $M = I = 0.05$, initially $x_0(0) = 0.25$ and $\dot{x}_0(0)$ is either $-0.2, 0$, or 0.2. Also shown: grid-independence.

$O(1)$ time, fluid effects



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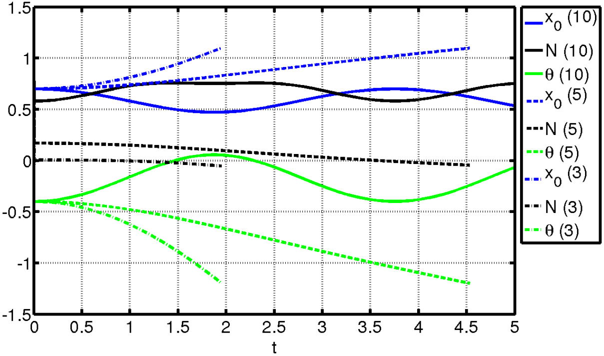

Figure: The influence of the gravity parameter g^+ (value in parentheses in legend) on the behaviour of the constant-curvature body. Here, $M = I = 0.1$, $x_0(0) = 0.7, \dot{x}_0(0) = 0$. Results corresponding to values of x_0 greater than unity are not physical, but are still shown.