An Investigation in the Theory of Voluntary Provision of Public Goods and Income Tax Evasion under the Hypothesis of Ethical Behaviour on the Part of Economic Agents

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Summary

In this work we discuss a number of issues in the theory of voluntary provision of public goods and income tax evasion under the assumption that individuals are ruled by a notion of Kantian morality. Our justification for imposing such an assumption is that models incorporating the traditional assumption of rational egoism are unable to explain the many real world examples of successful private provision of public goods, of which compliance to tax rules can be taken as an example. In the first part of this work, after having reviewed the literature on private provision of public goods and justified our alternative approach, we introduce and formalize the notion of Kantian behaviour. We investigate efficiency of private provision of public goods under Kantian behaviour and we also compare Kantian provision with alternative models of public goods supply. Precise conditions on the structure of individual preferences which would ensure efficiency of private provision of a public good under Kantian behaviour are derived. It is also shown that while Kantian supply of a public good is in general still characterized by underprovision it tends to be more efficient than public good provision under a democratic system as represented by the Median voter theorem. Finally, using the notion of Lindahl equilibrium, a different way of assessing under/over provision of a public good under Kantian behaviour is derived. In the second part of this work, building upon the analysis on Kantian behaviour developed in the previous sections, we address the phenomenon of income tax evasion as an example of voluntary (non) provision of a public good. We present a model where the amount of tax that a taxpayer wishes to evade is determined on the basis of his perception of the fairness of his fiscal treatment, with respect to both governmental supply of public goods and the perceived behaviour of the other taxpayers. The coercive powers of the state, as well as the taxpayer's attitude toward risk, determine only the extent to which this desired level of tax evasion is reached in practice. It is shown that this approach is able to produce implications for the relationship between the characteristics of public expenditure, the tax rates and tax evasion which are more consistent with both intuition and empirical evidence than the results of the conventional model of income tax evasion. Furthermore, it also allows one to address other important questions such as the effect of government X-inefficiency on tax evasion.

Chapter I Introduction and Overview

I.1 Selfishness, Fairness and Rationality

Economic theory relies heavily upon the assumption that economic agents behave as "rational egoists", where rationality refers to the way in which individuals organize their actions and selfishness to the content of these actions. Such an assumption has received severe criticisms in the past and it is still submitted to unsympathetic scrutiny by scholars inside and outside the realm of economics. Nevertheless, the concept of rational egoism characterizes most of the current work in economics. This widespread acceptance of the notion is due to a number of reasons. First, history; that is, the long tradition that the notion of "economic man" has in the discipline. Second, usefulness: that is, the fact that the assumption of rational egoism is a particularly good assumption to employ, especially in formal work, since it allows one to reduce dramatically the complexity of human behaviour whilst still retaining important features of it in the market place. The third is that the notion of rational egoism is associated with some fundamental results of economic theory --such as the Pareto optimality of competitive equilibria (Sen, 1987).

Given this general consensus on the notion of rational egoism, a work which purposes to address some selected economic issues by relaxing this assumption requires at least two justifications. First, it must show that the assumption of rational egoism is particularly unsuitable for the issue at hand, that is, that it misses some fundamental features of human behaviour in the field under analysis. Second, it must show that the analytic complexity added to models by relaxing this assumption pays off in terms of results which are more in line with observed data than those provided by traditional models.

In this work we try to show that, in the context of models of voluntary provision of public goods and tax evasion, abandoning the notion of rational egoism in favour of a notion of ethical behaviour can be justified in the double sense explained above. Our basic argument will run as follows. In situations characterized by voluntary provision of public goods, the assumption of rational egoism implies that private provision of public goods is largely suboptimal or even non-existent. This strongly clashes with the many real world examples of successful private contribution schemes, such as charities or collective organizations (see chapter II). It is also inconsistent with the fact that most people pay their taxes even when it would be in their best interest not to do so, a fact which can also be thought of as an example of voluntary contribution to a public good (see chapter V). A possible explanation for these phenomena, as suggested by Sen (1977) and Johansen (1977), is that human beings are sophisticated enough to understand that in a situation of voluntary provision of public goods, the adoption of a strictly selfish strategy by everybody may be detrimental in terms of everybody's own selfish objectives. As a consequence, they might be stimulated to follow those rules of behaviour that they perceive might produce a better outcome if followed by everybody.

Building upon the seminal work of Laffont (1975) and, especially, of Sugden (1984), we will model these rules as ethical rules of behaviour. The main idea is simply that there are some basic norms of social behaviour ("practical morality") that, if followed by individuals, will tend to promote, in public goods situations, a higher level of economic efficiency than would result if everybody behaved selfishly (Phelps, 1975). We will assume that people are clever enough to understand this and that, as a consequence, they may be willing to follow, in some cases at least, these social norms. Thus, differently from the more sociologically oriented work by Akerlof (1980), it is here assumed that compliance to social rules is an autonomous choice rather than the result of social pressure. This of course does not mean to deny that external forces such as group pressures, reputation, etc. may also have a role in enforcing individuals' obedience to rules.

Following a quite established tradition in economics, we will introduce these ethical rules as a constraint imposed on the maximization of a selfish utility function. Less conventionally, and following the seminal work of Laffont (1975) and Sugden (1984), we will model ethical behaviour by means of a notion of Kantian morality: that is, by the idea that an individual would consider it morally right to act as he would wish that everybody else should act in the same circumstances (see chapter III). We adopt this rule of behaviour for a number of reasons. First, the Kantian rule is a very simple and well known ethical rule of behaviour, so much so that it can be found in slightly different forms in several ethical systems (see chapter III). Thus, it is not too unrealistic to assume that people may know and be influenced in their behaviour by this rule. Second, in order to implement the Kantian rule an individual need only know his own preferences. Thus, Kantian behaviour is realistic even in the sense that it does not require individuals to have information which goes beyond the amount that we usually assume economic agents possess. Third, since Kantian behaviour is based only on each individual's preferences, some fundamental notions of economic theory, such as Pareto efficiency, can be still used in assessing the results of individual behaviour under Kantian rules[1].

In the following we will also consider several different ways of modeling the notion of Kantian morality. In chapters III and IV, for reasons which will be explained below, we will discuss and formalize the notion of "pure" Kantian behaviour in the context of a model of private provision of a public good. In contrast, in the chapters on tax evasion (chapters VI and VII), we will consider a model where Kantian behaviour is weakened by reciprocity considerations: that is, where an individual wishes to behave as a

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[1]This would not have been the case if we had followed Sen's (1977) suggestion to model morality as the result of an ordering of orderings.

Kantian if and only if he perceives that everybody else does likewise. The latter interpretation follows more closely the approach advocated by Sugden (1984) and by the present author elsewhere (Bordignon, 1987). In order to distinguish between these two approaches, I shall refer to "pure" Kantian behaviour simply as Kantian behaviour, and to Kantian behaviour weakened by reciprocity rules as "fair" behaviour.

I.2 An Overview of the work

As argued in the previous section, abandoning the notion of rational egoism requires some justifications. The first and most important justification for considering an alternative approach to individual behaviour in the context of private provision of public goods is simply that the conventional theory does not work. This is shown in the first part of chapter II, where, building upon some previous work by Andreoni (1988a) and Sugden (1982), the traditional model of voluntary provision of public goods is spelled out in detail, its main implications derived analytically, and the predictions of the model contrasted with a large amount of empirical and experimental evidence. The results of this exercise show that the traditional model is falsified by empirical data. We then conclude that the traditional model of private provision of public goods is in need of deep revision.

The most recent literature on private provision of public goods has shown an increasing awareness of the many shortcomings of the traditional model and several different approaches have been proposed to amend the conventional theory. The second part of chapter II reviews in detail this literature. The conclusions which can be drawn by this survey are quite disappointing. An alternative theory to the Nash contribution model is hardly forthcoming and the attempts which have been made so far to build up a different theory by relaxing alternatively one of the three founding assumptions of the model --Nash conjectures, purity of the public good and individualistic maximizing behaviour-- all suffer from severe drawbacks. However, among these last attempts, the research strategy which looks more promising is the abandonment of the assumption of rational egoism in favour of a notion of Kantian behaviour, as discussed by Laffont (1975) and Sugden (1984). This approach too is however open to some criticisms, which are presented and discussed in the last section of chapter II.

Some of the points made at the end of chapter II are taken up again in chapter III. In this chapter we address a basic issue: suppose that individuals are subjected to such a strong ethical rule as Kantian behaviour; would then private provision of a public good be efficient? To answer this question, we consider two different ways of formalizing Kantian behaviour, the traditional one introduced by Laffont and a second one which takes into account "the ability to pay" of the different individuals. We derive precise conditions on the structure of individual preferences and on income distribution which would ensure efficiency of private provision of a public good under Kantian behaviour. The performance of Kantian behaviour in efficiency terms is also studied for some important classes of individual preferences. The results of these exercises show that, in general, Kantian provision of a public good is characterized by Pareto inefficiency and that furthermore there is some tendency toward underprovision. Thus, Kantian behaviour is consistent with the empirical evidence which suggests that private provision of public goods, while strictly greater than that predicted by the conventional model, is still characterized by underprovision (see chapter II).

In the tradition of welfare economics, the fact that Kantian behaviour is characterized by inefficiency would support government intervention in order to reestablish conditions of Pareto optimality. But, in the spirit of the new-new welfare economics, a market failure is not a sufficient condition for invoking public intervention. It has still to be shown that government can in some sense do better than private citizens. In order to cast some light on this issue, in chapter IV private provision of a public good under Kantian behaviour is compared with public provision under a political process where the results of the median voter theorem apply. It is shown that for some important classes of preferences Kantian behaviour is always strictly more efficient than public provision under a political process. This result then suggests that Kantian behaviour, while in general Pareto inefficient, may be characterized by a higher level of efficiency than alternative public good provision schemes.

In chapter IV we also discuss the links between Kantian provision of a public good and Lindahl equilibria. Using the notion of Lindahl equilibrium, a different way of assessing under/over provision of a public good is derived. It is shown that whenever Kantian behaviour is inefficient it would always be possible to find a Lindahl equilibrium which would maintain each individual on the same indifference surface but using less total income. Furthermore, the extent of the "waste" introduced by Kantian behaviour is an increasing function of the difference, in absolute value, between the summation of the marginal rates of substitution and the marginal rate of transformation. These results might also be used for measuring the amount of inefficiency generated by alternative methods of public good provision. Finally, in the last section of chapter IV, we offer some suggestions for further work on Kantian behaviour and, more generally, for private provision of public goods under unselfish behaviour.

In the second part of this work we try to add some flesh to the notion of private provision of public goods under ethical behaviour by showing that this approach is suitable for application to a larger class of economic phenomena. The example chosen is income tax evasion. The theoretical literature on income tax evasion is surveyed in chapter V, which also introduces intuitively our alternative approach to the problem. The basic argument of chapter V is as follows. The conventional literature on tax evasion, in the tradition of "selfish" economics, looks at the phenomenon of non-compliance to tax rules simply as the result of an individualistic calculus of expected costs and benefits. Taxpayers wish to evade entirely their income tax, and the only reason why they might not do so is because there is some non-zero probability of being caught evading. Detection usually leads to punishment by government in the form of pecuniary penalties (surcharge rates) imposed on detected evaded tax or detected evaded income. The relationship between government and taxpayers is, in this approach, simply one of coercion.

The main problem with this literature is that it produces unpalatable results in terms of both intuition and empirical evidence. First, estimated compliance to tax rules in the real world is much larger than is predicted by this literature (see chapter V). Second, the portfolio choice approach to income tax evasion predicts, under the conventional assumption of decreasing absolute risk aversion, a negative relationship between evaded tax and the tax rate. This implication too is clearly rejected by both empirical and experimental literature.

Building upon some previous sociological work on tax evasion, we suggest an alternative approach to the phenomenon. In this approach, taxpayers see their relationship with government also as a relationship of exchange, where they exchange purchasing power in return for governmental goods and services. It is assumed that taxpayers have a perception of what should be "fair" terms of trade between their private consumption and government provision of public goods. If taxpayers perceive their tax burden as "unfair", both in terms of governmental supply of public goods and in terms of the perceived tax treatment of the "other" taxpayers, we assume that they will wish to evade taxes in order to reestablish fairness in their relationship with the other actors of the fiscal system. Thus, the decision to evade by a single taxpayer is taken on the basis of an ethical judgment on the fairness of the fiscal system. The coercive powers of the state and the individual attitude toward risk determine only the extent to which this desired level of tax evasion is reached in practice.

Chapter VI formalizes these ideas in the context of a model with two types of consumers and two goods, private consumption and a public good supplied by the state. We use the work on Kantian behaviour developed in chapter III, weakened by reciprocity rules, to determine the amount of tax that an individual would consider it fair to evade. In chapter VI we analyze the model in a short run perspective, where tax revenue and public expenditure can be considered as independent phenomena. In chapter VII we extend the model to the case where government budget always balances so that, given the tax rate and the tax behaviour of taxpayers, public expenditure is also determined.

The comparative statics results of chapter VI imply zero tax evasion for low values of the tax rate and tax evasion increasing in the tax rate for at least some medium range of values of the tax rate. Thus, our approach is consistent with the empirical literature on this issue. Furthermore, a number of interesting and testable implications are derived for the relationship between the amount of tax evaded by the different social classes, the distributional characteristics of public expenditure and income distribution.

Chapter VII reproduces the comparative statics exercises of chapter VI for the case of endogenous public expenditure. A larger amount of ambiguity characterizes this case but it is shown that whenever the comparative statics derivatives can be signed, the same results of the previous chapter emerge. In the last section of chapter VII, in the context of a simpler economy with identical individuals, we address another important issue which has, so far, escaped scrutiny in the literature: the effect on tax evasion of X-inefficiency in public production of public goods. It is shown that, for any level of public good supply, tax evasion unambiguously falls as the amount of X-inefficiency in the public sector decreases. The result is instead ambiguous and depends on a crucial parameter if it is the tax rate to be kept constant during the exercise. Finally, in the conclusions to chapter VII we advance some suggestions for further work which could be done, in the area of tax evasion, by using our model.

Chapter VIII concludes this work by pointing out a main caveat of the analysis of the previous chapters and by proposing a tentative list of other related fields in economics where the ethical approach to voluntary provision of public goods could be successfully applied. It is in fact a conviction of the writer that, upon reformulation, the voluntary contribution model could offer interesting insights in other fields beyond those in which it is usually applied.

Part I: Voluntary Contribution of Public Goods

Chapter II The Nash Contribution Model: Theoretical Results, Empirical Tests and Proposed Solutions

II.1 Introduction

Economists, political scientists, sociologists, scholars of industrial relations etc., all found their intuition of individual behaviour in collective organizations on the basis of the results of the standard Nash contribution model. In this chapter we argue that this model is in need of deep revision. None of the predictions of the model finds more than weak support in the empirical literature and most of the results of the model are clearly falsified by observed data. This does not imply that the main result of the traditional model, free-riding behaviour and the consequent undersupply of public goods by the private sector, is not a real phenomenon. But it does imply that this phenomenon requires a different explanation from that provided by the Nash contribution model. In this chapter we also argue that among the many alternatives proposed in the literature the one which looks more promising is the abandonment of the assumption of individualistic maximizing behaviour in favour of a notion of ethical behaviour.

In order to make our point clear we will proceed as follows.

In sections II.2 to II.4 the basic Nash contribution model is spelled out in detail and its implications for the inefficiency of private sector supply of a public good and its main comparative statics results are derived analytically. Thus, the effects on private provision of a public good of changes in the size of the economy, in governmental provision and in income distribution are considered. In section II.5 these predictions of the Nash contribution model are contrasted against a large amount of empirical and experimental evidence. It will be shown that the Nash contribution model does not survive the test. Consequently, the rest of the chapter is dedicated to an examination of the main alternatives to the traditional model proposed in the literature.

Section II.7 discusses miscellaneous different analysis which share the characteristics of implying a quite clear departure from the structure of the Nash contribution model. While most of this work provides interesting insights for the working of private contribution schemes in particular cases, it will be argued that none of it is general enough to be applied to the many real world instances of private provision of public goods. In sections II.8.1 to II.8.3 we then turn to the analysis of the literature which has attempted to supply a general theory of voluntary provision of public goods by questioning the three basic assumptions on which the Nash contribution model is built: Nash conjectures, purity of the public good, and individualistic maximizing behaviour. In section II.8.1 it is shown that non-Nash behaviour cannot solve the problems of the traditional model and indeed it can only make them worse. In section II.8.2 it is shown that models which consider impure public goods can only work at the price of unpalatable assumptions on individual behaviour. We are then left with the assumption of individualistic maximizing behaviour. In section II.8.3 we discuss three different models of "unselfish" behaviour: altruism, social norms of behaviour and ethical behaviour. It is argued that only the latter is able to provide a satisfactory theory of voluntary provision of public goods and we offer a number of suggestions for further research in this area, some of which are taken up in the next two chapters. The conclusions close the chapter.

II.2 The Nash Contribution Model

In order to aid understanding, let us start by describing the economy where we will place our discussion. In the following we will consider a simple two-good economy, with a pure public good Q, and a pure private good y_i , $i=1,..,n$, where the suffix indicates the individual who consumes it. We will further assume that the production function of the public good is simply additive in individual contributions and we will set the unit of measure of private contributions so as to make the (constant) marginal rate of transformation between the private and the public good equal to one: that is, we assume $\Sigma q_i = Q$ where q_i is individual i's contribution. Neither the assumption on the number of goods nor the assumption on the production function for the public good is essential for the results to follow and we will later indicate where they may be of some importance. The three assumptions which instead lie at the heart of the traditional model of voluntary provision of public goods[1] (Sugden, 1982) are as follows: 1) individualistic maximizing behaviour; 2) "purity" of public good; 3) Nash conjectures on the part of the individual agents.

The assumption of individualistic maximizing behaviour is imposed without justification since, as discussed in the previous chapter, it is the usual hypothesis employed in economics. In a formal model, such an hypothesis is introduced by assuming that each individual i maximizes an utility function, $U^i(.)$, defined on the two goods. Conventionally, we will assume such a function to be strictly quasi-concave.

The second hypothesis, the "purity" assumption, requires the public good to be characterized by complete non-rivalness and complete non-excludability in consumption across individuals -- or inside the particular group of individuals under consideration. In contrast with the previous one, this second hypothesis is instead explicitly introduced as a simplifying assumption. In our context, as we shall see in more detail below (see II.8.2), the assumption of purity hinges especially on the fact that each contributor gains utility only from the total supply of the public good and not from his own individual contribution. This will be captured in

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[1]Also known as "equilibrium with subscriptions" (Malinvaud, 1969) and "independent adjustment model" (Buchanan, 1968).

our model by writing the utility function of each individual as depending, in addition to private consumption, only on the total amount of public good supply; i.e. $U^i=U^i(y_i,Q)$.

The third assumption, the assumption of Nash conjectures, implies that each individual, when making his own decision, takes the total contribution of the other individuals as given. This is usually justified by referring to the relative "smallness" of the single individual with respect to the number of individuals composing the group. In large economies, where the model is supposed to apply, each individual is too small to appreciate the influence that his contribution may exert on other individuals' behaviour. Thus, he takes the latter as given. In our model, this will be captured by assuming that for each individual i the total provision of the public good by the other individuals, that we will indicate with Q_{-i} , is a known, given amount.

These three assumptions, together with the hypothesis on the production function for the public good, allow us to write individual i's problem as :

$$(1) \qquad \max_{y_i,q_i} \; U^i(y_i,q_i+Q_{-i})$$

$$\text{s.t.}\; y_i+q_i=I_i; \qquad q_i\geq 0$$

where I_i is individual i's lump sum income. Following Bergstrom et al. (1986) and Andreoni (1988a), let us now note that the Nash assumption also implies that when an individual chooses

q_i he is really choosing Q; we can stress this fact by adding and subtracting Q_{-i} from the LHS of the budget constraint in eq.(1) and by rewriting individual i's problem as:

$$(1_b) \qquad \max_{y_i, Q} \; U^i(y_i, Q)$$

$$\text{s.t. } y_i + Q = I_i + Q_{-i} = W_i \, ; \qquad Q \geq Q_{-i}$$

Note that the purity assumption implies that other individuals' behaviour affects i's choice of Q also through an income effect on the budget constraint. This will be of the greatest importance in the results to follow. Leaving aside the inequality constraint, the problem in eq.(1_b) is akin to a standard consumer choice problem. The assumption of strict convexity of preferences implies that problem (1_b) has a unique solution which in turn is a function of individual i's income; let us then write the demand function for the public good of individual i as $f^i(I)$. For simplicity we will assume $f^i(.)$ to be a differentiable function. Taking into account the inequality constraint total provision of the public good will then be equal to $Q = \max\{f^i(I_i + Q_{-i}), Q_{-i}\}$. We can then write individual i's contribution as:

$$(2) \qquad q_i = \max\{f^i(I_i + Q_{-i}) - Q_{-i}, 0\}$$

The three assumptions together, and of course especially the latter, lead the theorist to employ in the context of the model above a concept of Nash equilibrium. That is, an equilibrium is a vector of individual contributions $\{q_i^*\}$ such that $\{q_i^*, y_i^*\}$ solves problem (1) for all i. The standard assumption of convexity of preferences, by implying continuity of the individual reaction function in eq.(2) to other individuals' contributions, guarantees the existence of a Nash equilibrium in this model. Of course, such an equilibrium may be neither unique nor stable. But, as can be easily shown, if the income derivative of each individual demand function is everywhere positive but smaller than unity --i.e. if $0<f_I^i<1$ $\forall i$-- the Nash equilibrium in the model above is both unique (Bergstrom et al., 1986) and, under a simple adjustment mechanism, locally stable (Cornes and Sandler, 1986)[2]. $0<f_I^i<1$ simply implies that the marginal propensity to consume out of income for both the private and the public good is strictly positive; that is, that both goods are normal goods. As the assumption of normality is entirely intuitive in this context and as, furthermore, it is largely confirmed by available empirical studies[3], the model works very neatly and allows one to perform with ease the usual

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[2] To get an intuition for this result, simply note that from eq.(2), $0<f_I<1$ implies that each individual reaction function is a contraction in Q_{-i}. As is well known from game theory, this condition is sufficient to ensure both uniqueness and stability of Nash equilibria in non cooperative games (see Friedman, 1986).

[3] Empirical studies on charitable giving indicate the income effect alone to be around 0.03-0.04, while most studies indicate that the income elasticity lies between 0.4 and 0.8. Data quoted by Sugden (1982) and Andreoni (1988a).

comparative statics exercises.

Given its simplicity, the subscription model above has been applied, with only slight modifications, to the positive analysis of several economic phenomena. For instance, charities and philanthropic institutions (Becker, 1974, Young, 1982, Collard, 1978), voting behavior (Margolis, 1982), saving behaviour across generations (Sen, 1967, Marglin, 1963), collective organizations and pressure groups (Olson, 1965), military expenditure in alliances (Olson and Zeckhauser, 1966), strike behaviour (Naylor, 1989), federal government expenditures and many others. In normative terms, by exploiting its main qualitative result, free-riding behaviour (see below), the model has been applied to a much larger class of economic situations, becoming one of the main arguments in support of government intervention in such fields as redistributive transfer policy (Hochman and Rodgers, 1970), health care (Arrow, 1963), education (Stiglitz, 1974), national defense (Sandler and Forbes, 1980), saving (Dasgupta et al., 1972), licenses, collective bargaining (Tarantelli, 1986) and so on. Indeed, in a Hobbesian tradition, the same existence of the State has been justified as the result of an attempt by rational agents to cope with the free-riding problem (Orbell and Wilson, 1977). Given the central role that free-riding behaviour exerts in the literature on public goods, it deserves special attention.

II.3 Free-riding Behavior

Loosely speaking, with the term "free-riding" the literature indicates a number of reasons why we should expect the equilibrium allocation in the model above to be Pareto inefficient and furthermore, to be characterized by underprovision of the public good. This definition is deliberately loose, because the free-riding problem is really characterized by three different and quite separate phenomena, sometimes confused in the literature (McMillan, 1979).

The first feature, that we could call here, following Cornes and Sandler (1986:80), <u>systemic</u> free-riding, depends on the fact that each individual fails to appreciate the effect that his contribution exerts on the utilities of his fellows. Since each utility function is dependent on the same quantity of the public good (purity assumption), each individual contribution affects simultaneously the level of welfare of everybody else. There are therefore spillover effects which are not internalized by the private sector and as usual, this leads to inefficient equilibria. Thus, in this respect, the free-riding problem is nothing but a special case of the larger class of market failures associated with externalities.

The result of Pareto inefficiency of the Nash equilibrium can be immediately derived by simply noting, from the first-order conditions for the solution of problem (1), that in a Nash equilibrium $U^h_Q/U^h_y(y^*_h,Q^*)=1$ for all individuals h such that $q^*_h>0$ and $0<U^k_Q/U^k_y(I_k,Q^*)\leq 1$ for all individuals k such that $q^*_k=0$. Then, summing over the two groups of individuals:

$$(3)\quad \Sigma U^i_Q/U^i_y(y^*_i,Q^*) = \Sigma_h U^h_Q/U^h_y(y^*_h,Q^*) + \Sigma_k U^k_Q/U^k_y(I_k,Q^*) \geq H \geq 1$$

where H is the subset of individuals whose contributions are strictly positive and where the two inequalities in eq.(3) cannot both hold as equalities at the same time. Hence, by recalling the Samuelson condition for optimality in public good production, we conclude that the public good, in a Nash equilibrium, is not supplied at the optimal level.

Showing that the Nash equilibrium is not only inefficient but also characterized by underprovision of the public good, or even more, quantifying the extent of systemic free-riding is instead more tricky. This is so because, unless individual preferences have some very special structure (Bergstrom and Cornes, 1983), the optimal level of public good supply is not independent of the distribution of income. It then follows that by changing the contribution of the single individual, one would also change the optimal level itself. Thus, in this literature, scholars usually content themselves with showing that the Nash equilibrium is characterized by <u>local</u> underprovision of a public good. To see this, suppose that we increase by ε the contribution of each individual h such that $q^*_h>0$, while keeping the allocation of each non-contributor constant. Evaluating the utility function of an individual $j \in H$ at the new allocation, we get $U^j = U^j(y^*_j - \varepsilon, \Sigma[q^*_h + \varepsilon])$. By differentiating the utility function so obtained with respect to ε and evaluating it at $\varepsilon=0$, we get:

(3b) $\partial U^j/\partial\varepsilon\big|_{\varepsilon=0} = U^j_Q[H-1]\geq 0 \qquad \forall j,\ j\in H$

Since the utility of each non contributor is increasing in Q at unchanged private good consumption, we then conclude that in a Nash equilibrium, if there are at least two contributors, the utility of everybody would increase following a small increase in the contribution of all contributors: hence, in a Nash equilibrium, the public good is locally underprovided.

Similar results of undersupply of the public good in a Nash equilibrium, but which hold globally and not only locally, can be reached by using other devices --as for example by comparing the optimal and the actual contribution at unchanged individual shares (Cornes and Sandler, 1986). We will use some of these devices in the next two chapters when we evaluate the efficiency of Kantian provision[4].

The second feature of the free-riding problem, that again following Cornes and Sandler (1986) we can term microlevel free-riding, has to do with the slope of each individual reaction function with respect to other individual contributions. If both the public good and the private good are normal goods, an increase in the contribution to the public good by another individual (or

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[4]See chapter III section 4, and chapter IV section 3.

public agency) will certainly reduce each individual contribution. Moreover, this reduction will be less than the increase in the contribution of the other individual. To see this, let us differentiate q_i in eq.(2) above with respect to Q_{-i}:

(4) $$\partial q_i/\partial Q_{-i} = -1 + f_I^i$$

Clearly, the normality assumption implies $-1<\partial q_i/\partial Q_{-i}<0$. The intuition behind this result is quite straightforward. A change in the contribution of another individual has both an income and a substitution effect. The first element on the RHS of eq.(4) represents the pure substitution effect: since the public good enters in the same amount in the utility function of each individual an increase in the contribution of an other individual leads each individual to reduce his contribution to the public good by exactly the same amount of the perceived increase in public good supply. However there is also an income effect which is captured by the second element in the RHS of eq.(4). An exogenous increase in public good supply is equivalent for the individual under consideration to an exogenous increase in his income endowment. Such an increase, in turn, induces the individual to buy more of the public good if the latter is a normal good thus partially offsetting the substitution effect above.

Note that, while the exact magnitude of the income effect on the demand for the public good is clearly an empirical issue, the reaction function of an individual could become positively sloped in other contributions only if we assumed that the public good is a superior good. In our two good economy this would have the unpalatable effect of making the private good, representing private consumption, an inferior good. The latter is clearly an implication that most economists would find hard to accept. Therefore, if the assumption of non-superiority of the public good is taken as reasonable, the predictions of the standard model of voluntary provision of a public good are clear cut. Each individual should reduce his contribution --i.e. should take a free-ride-- in the face of a perceived increase in the contributions of other individuals, or more generally, in the face of an exogenous increase in public good supply. The extent of this "crowding out" phenomenon in turn is negatively related to the size of the income effects on the demand for the public good. In the next paragraph, we will discuss in detail the striking implications that this argument imposes on the comparative statics properties of the model, in terms of 1) changes in the number of contributors; 2) changes in the exogenous supply of public good by government; and 3) changes in the distribution of income across contributors.

Finally, the last but probably best known feature of the free-rider problem is represented by the lack of incentives for each individual to reveal his true preferences for the public good. In private good economies, the larger the number of individuals the lower are the incentives for each individual to misrepresent his preferences (Hurwicz, 1972). In public good economies quite the opposite is true. Not only does it pay for each individual to dissemble his preferences but the scope for profitable dissembling increases with the size of the economy (Cornes and Sandler, 1986: 102- 103). For obvious reason this last feature of free-rider behaviour has been termed <u>informational</u> free-riding (McMillan,1979).

This last feature of free riding behaviour, however important and widely discussed in the literature, is really extraneous to the present discussion and has been quoted here only for the sake of completeness. This is so, because informational free-riding raises problems of eliciting truthful revelation of preferences for <u>public</u> provision of the public good while we concentrate here only on voluntary provision from the private sector. Furthermore, informational free-riding is better addressed in models which consider a type of strategic behaviour on the part of the individual agents which goes beyond the simple Nash conjectures that we assumed from the outset (see Laffont, 1987). For a discussion of the problem and some solutions proposed in the literature see McMillan (1979) and the excellent survey by Laffont (1987). In the following, we will concentrate only upon the two previous features of free-rider behaviour, systemic and microlevel free-riding.

II.4 The traditional model: other implications.

The basic predictions of the model discussed so far are, of course, part of the standard background of any economist. However, there are other important implications of the traditional subscription model which are less well known, but worth considering in order to assess the validity of the model. All of them crucially depend on the magnitude of the income effects and, at a deeper level, on the assumed purity of the public good. All of them, in different ways, cast doubts on the empirical relevance of the model, at least for "reasonable" assumptions on the magnitude of the income effects.

II.4.1: Changes in the number of contributors

Since the seminal work by Olson (1965), there has been a presumption in public good theory that increasing the number of contributors would reduce the level of efficiency reached by the simple contribution model discussed above. Olson's argument was threefold. First, he argued that moving from small to large groups would induce individuals to shift away from some more complex strategic behaviour to simple Nash conjectures, thus reducing the incentive to individually contribute to the public good (see also Buchanan, 1967 and II.8.1 below). Second, he argued that in a Nash equilibrium, increasing the number of the individuals would increase the optimal level of public good supply and third, that, on the contrary, the actual level of public good provision would fall as the size of the group of contributors increases.

While the first argument is usually accepted in the literature and the second is true if the public good is a normal good (Cornes and Sandler, 1986:83-85), the third argument has not gone unchallenged. Both McGuire (1974) and Chamberlin (1974) showed that in a two good economy with identical individuals, as long as the public good is a normal good, increasing the number of individuals would increase the private supply of the public good. To see this, let us rewrite eq.(2) for an economy with identical individuals where everybody is a contributor. Evaluating eq.(2) at the equilibrium allocation we get:

$$(5) \qquad q_i^* = f(I+[n-1]q_i^*) - [n-1]q_i^*$$

Totally differentiating eq.(5) with respect to n:

$$(6) \qquad \partial q_i/\partial n = -[1-f_I]q_i^*/\{[1-f_I]n+f_I\}$$

which is unambiguously negative. From (6) it follows:

$$(7) \qquad dQ/dn = q_i^* + n\partial q_i/\partial n = f_I q_i^*/\{[1-f_I]n+f_I\}$$

which is instead unambiguously positive as long as the public good is a normal good. The rationale for this result is straightforward. If the public good is a normal good, adding an extra individual who contributes an extra unit to the public good would induce the other individuals, in equilibrium, to reduce their total contribution by less than unity. Hence, actual total contribution must increase. However, while total contribution increases, each individual contribution must fall since, as we saw in the previous paragraph, each reaction function is negatively sloped in the contributions of the other individuals. Also note that the RHS of eq.(7) is decreasing in n and for $n \Rightarrow \infty$, $dQ/dn \Rightarrow 0$; that is, in large economies the total supply of the public good would tend to converge to a finite amount.

Chamberlin and McGuire 's results were extended by Andreoni (1988a) to an economy with different types of individuals. In doing this Andreoni (1988a) also proved another important result (see also Bergstrom et al., 1986), that I quote here for future reference. Suppose that individuals are characterized by identical tastes and different incomes. Then: (1) only individuals with income above a given threshold will contribute in equilibrium and (2) each contributor will consume exactly the same quantity of the private good in equilibrium. To see this, recall that in equilibrium, for each i such that $q_i>0$, $Q=f(I_i+Q_{-i})$. Since by assumption $f_I>0$ everywhere, f(.) can be inverted. It then follows:

(8) $$f^{-1}(Q)= I_i+Q_{-i}$$

Adding Q to both sides of eq.(8) and rearranging we get:

(9) $$q_i= I_i+ Q - f^{-1}(Q)$$

Let now I^* be the level of I_i such that $I^*=f^{-1}(Q)-Q$. It is then clear that $q_i>0$ as $I_i>I^*$ and $q_i=0$ if $I_i\leq I^*$. This proves (1). Second note that $y_i=I_i-q_i$. Hence, from (9), for $\forall i$ such that $q_i>0$:

$$(10) \qquad y_i= f^{-1}(Q) - Q =I^*$$

which proves (2). Extending Chamberlin and McGuire 's result, Andreoni (1988a) proved that for $n\Rightarrow\infty$, $I^*\Rightarrow I^{max}$, where I^{max} is the income of the richest individual in the society. From this it follows that as the number of contributors tend to infinity 1) average contribution tends to zero; 2) only the very richest members of the community contribute; 3) total contribution converges to a finite value. By considering different "types" of individuals Andreoni (1988a) managed to extend the result above even to an economy with heterogeneous preferences. The result still holds, with the only difference that as $n\Rightarrow\infty$ it is now the most "generous" type -in terms of both income and preferences- to pay for the public good.

Summing up, the results reached in this section imply that in a large economy, total private provision of the public good does not depend on the size of the population and that we should observe only the rich contributing to the public good.

II.4.2 Changes in government supply of public goods

The Nash contribution model can be easily extended to include a public sector as well. It is sufficient to assume that a share of the public good is exogenously supplied by government and compute the equilibrium allocation of private contributions under the assumption that each individual observes government provision. Of interest here is the question: what would happen to private provision if government exogenously increased its supply of the public good ?

Suppose first that government increases its contribution without raising extra revenue from the set of contributors. For example, government may run a public deficit or it may raise extra revenue from non contributors (see below). We can study this effect in our simple model by assuming that a share of the public good, say ζ, is supplied exogenously by the state. Assuming an economy with identical individuals for simplicity, we can write the individual contribution in equilibrium as:

$$(11) \qquad q_i^* = f(I+[n-1]q_i^* + \zeta) - [n-1]q_i^* - \zeta$$

Then by differentiating with respect to ζ and setting $d\zeta>0$ we get:

$$(12) \qquad dq_i = -[1-f_I]d\zeta/\{f_I + n[1-f_I]\} <0$$

Since $dQ = d\zeta + ndq_i$, eq.(12) implies:

$$(13) \qquad dQ = d\zeta f_I/\{f_I + n[1-f_I]\} >0$$

The rationale for the results in eq.(12) and (13) is of course the same that we saw above. An exogenous increase in public good provision has both a substitution and an income effect: and government provision of the public good does not crowd out completely private provision due to the presence of positive income effects on the demand for the public good. However, note from eq.(13) above that if the economy is "large" --i.e. if n tends to infinity-- the crowding out effect will be approximately complete even if the income effects are very strong[5]. Needless to say the results above can be easily extended to an economy with different individuals (Andreoni, 1988a). Therefore, if the economy is large the predictions of the model are clear cut: an exogenous increase in public good supply by government should reduce private supply to the public good dollar for dollar, or approximately so.

Let us then ask what would happen if government increased public good supply with a balanced budget, that is by raising extra revenue. Suppose first that each individual is a contributor to the public good (i.e. each individual contribution is strictly positive) and that the extra revenue is raised through a system of lump sum taxes. Also assume that for each individual his lump sum tax is smaller in absolute value than his contribution to the

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[5]This was originally argued by Margolis (1982) and subsequently proved formally by Sugden (1982) and Andreoni (1988a).

public good. Then, as Warr (1982) and Roberts (1984) showed the crowding out effect is again complete. Each individual simply reduces his contribution by the amount of the increased lump sum tax, thus completely offsetting government behaviour.

In order to see this result let Q^* be the private supply of the public good in the original Nash equilibrium, before government intervention, and let q_i^* and Q_{-i}^* be the individual contributions in this equilibrium, so that $Q^*=q_i^*+Q_{-i}^*$. Let τ_i be the lump sum tax imposed on each individual i, where $|\tau_i|<q_i^*$, and let q_i^{**} and Q_{-i}^{**} be the equilibrium contributions after government intervention. By definition of Nash equilibrium and from problem (1_b) above it follows that $\{y_i^*,Q^*\}$ are the equilibrium choices of individual i when his total exogenous income is $W_i^*=[I_i+Q_{-i}^*]$. After government intervention, individual i's exogenous income shifts to $W_i^{**}=[I_i-\tau_i+Q_{-i}^{**}+\Sigma\tau_i]$. Suppose now that, after government intervention, each individual other than individual i changes his contribution in the opposite direction and by the same amount as the change in his lump sum tax: that is, suppose $Q_{-i}^{**}=Q_{-i}^*-\Sigma_{j\neq i}\tau_j=\Sigma_{j\neq i}(q_j^*-\tau_j)$. Then by substituting in the expression above we obtain $W_i^*=W_i^{**}$. The budget constraint of consumer i is therefore unchanged: it then follows that $\{y_i^*,Q^*\}$ is still optimal after government intervention. To achieve $\{y_i^*,Q^*\}$ consumer i must simply set his contribution so as to offset his lump tax --i.e. he must set $q_i^{**}=q_i^*-\tau_i$-- which is possible since by assumption $|\tau_i|<q_i^*$. Therefore if q_i^* ,∀i are the equilibrium choices before government intervention $q_i^{**}=q_i^*-\tau_i$ ∀i are the equilibrium choices

after government intervention.

Note that, differently from the previous case, where individuals reversed their increased endowment on the purchase of private goods, in the latter case not only the total quantity of the public good supplied but also the allocation of private goods across individuals remains unchanged. That is, government action is completely neutral to private sector allocations. Note that this remarkable result, while strictly depending on the assumption of purity of the public good, does not require either an additive production function for the public good or for that matter, the Nash assumption itself (see Bergstrom et al., 1986 and II.8.1 below).

It could be thought that the neutrality result above however depends on the existence of a single public good and on the fact that government uses lump sum taxes. But, in a path breaking paper Bernheim (1986) claimed a much stronger result. He showed that in a model with many public goods and different individuals, as long as each individual contributes to at least a public good, any distortional tax and transfer (for example, labour taxes) leaves the total supply of public goods and each individual demand for the private goods completely unaltered. Crowding out is now complete in the even stronger sense that any government action and not only lump sum financed increases in public good supply is completely neutral to private sector allocations. However, differently from Warr's results, as was later clarified by Andreoni (1988a) and Boadway et al. (1987), Bernheim's findings require stronger informational assumptions than usually made in the Nash contribution model. Bernheim requires each individual to "see through" the government budget (i.e. realizing that the government budget is binding) and also to correctly conjecture the behaviour of the other agents in the economy. Whether these assumptions are realistic enough, or if myopic Nash behaviour is a more convincing hypothesis is still an open question in the literature, as we shall see below (see II.8.1).

The above criticisms do not apply of course to Warr's results. The latter can instead be rightly criticized on the ground that they require interior solutions (i.e. each individual is a contributor) and that Warr considers only marginal changes in taxes and public good supply. In the "real world", contributors to a single public good are only a subset of the collectivity of the taxpayers and huge changes in taxes and public expenditure are also sometimes made. If, as an effect of government changes, the set of contributors also changes government policy can still be effective. Building upon such intuition, Bergstrom et al. (1986) presented a number of comparative statics results which somewhat weaken Warr's neutrality results. If, for example, as we saw previously, government finances its increased supply of the public good by collecting extra revenue from the set of non contributors, or, alternatively if some contributor is taxed by more than his original contribution then total public good supply will increase. Thus, in these cases the neutrality results are correspondingly weakened and government policy is still effective

- at least to a degree.

I used the word "weakened" above on purpose. In fact, I would like to argue, Bergstrom et al.'s findings qualify but do not drastically alter the above crowding out results. The force of their argument lies in the fact that the set of contributors to a single public good is usually much smaller than the overall set of taxpayers. However, as their own theorem 7 shows (Bergstrom et al., 1986:44-47), if there are many public goods and each individual is a contributor to at least one public good --which is of course a much weaker assumption to impose: see Bernheim (1986)-- the same neutrality results emerge, under some qualifications[6]. Furthermore, and this is the real issue here, if attention is restricted to an economy where everybody contributes, it is clear that government can increase total supply of the public good only by completely crowding out some contributors. That is, there cannot be real joint provision, public and private, of the public good. In Andreoni's (1988a) own words: "joint provision is a veil". I believe that it is the intuitive and empirical relevance of this extremely strong result which is at issue here, rather than the problem of the adjustment of the "extensive" versus the "intensive" margin, as in Bergstrom et

[6] Bergstrom et al (1986), in a model with two public goods, show that if the sets of contributors are disjoint, any income redistribution inside each group of donors would leave the total supply of each public good unchanged. If the two groups are not disjoint, it is required that the income distribution leaves the aggregate income of each group of contributors unchanged for the Warr's neutrality results to go through.

al.'s paper.

II.4.3 Changes in income distribution

In a paper related to the one quoted above, Warr (1983) showed that total provision of a public good is also, in a Nash equilibrium, independent of income distribution across contributors. As a moment's thought can show, this is really a simple extension of the previous argument. Suppose that in a Nash equilibrium where everybody strictly contributes to a public good, government redistributes income across contributors through a series of neutral tax increases and tax decreases. Again, as in the previous case, if the extra income added or subtracted from each individual is less than, in absolute value, his original contribution to the public good, each individual can simply offset this change by varying his contribution in the opposite direction and by the same amount as the change in his income endowment. In our model, this can be immediately seen by writing $\tau_i=\Delta I_i$, where $\Sigma\tau_i=0$, in section II.4.2 above and by repeating our previous argument.

Therefore, in a Nash equilibrium private provision of a public good (and private good allocation) is unaffected by income (re)distribution. As in the previous case, this result must be qualified. If the set of contributors does not coincide with the whole collectivity and/or if the change in the income endowment of an individual exceeds in absolute value his original contribution, the neutrality results may not wholly occur (see Bergstrom et al., 1986). However, as in the previous case, this qualification must also be qualified in the sense that: 1) the neutrality results will still in general occur, at least as an approximation (Andreoni, 1988a); and 2) leaving aside the adjustment on the "extensive" margin, it is the independence of private provision from income distribution inside the group of contributors which is the real result to be checked for intuitive and empirical relevance.

In conclusion, we can summarize the results of the traditional model of voluntary provision of a public good as follows. Because of the existence of systemic free-riding behaviour, private provision will in general be characterized by a) inefficiency and b) underprovision of the public good. Because of the existence of microlevel free-riding behaviour, c) the reaction function of each individual will be negatively sloped in the contribution of everybody else (if the public good is not a superior good). Because of (c), the equilibrium allocation of the subscription model will be characterized by a general invariance property (Andreoni, 1988a). In equilibrium, private supply of a public good will be invariant, or approximately so, with respect to changes in *a*) government supply of the public good, *b*) income distribution, *c*) subsidies to giving, and *d*) size of the population.

How do these results fare with respect to empirical and experimental evidence ?

II.5.1 Checking the traditional model: empirical evidence

The first fact which casts some doubts on the general philosophy of the model above is the casual observation that we are surrounded by quite a large number of privately supplied public goods. Large organizations such as trade unions, political parties, cooperatives, pressure groups, each of which produce an output which can be thought of as a public good for their members, seem to be able to cope quite well with the free-rider problem. In most countries, philanthropic organizations, churches and religious foundations, theaters, concerts, health research campaigns, natural parks, political movements, etc. rely heavily and happily enough on the voluntary contribution of private citizens. Large sums of money are freely donated in many countries to charities and philanthropic institutions. Just to give an idea of the order of magnitude of these donations, in 1981 total donations to charity amounted to 2-3% of the GNP in USA (Kurz, 1984, Andreoni, 1988a), and in the United Kingdom private gifts to the biggest five charities exceeded £60 m in 1980[7].

And money is not of course the only thing to be donated. In most countries, private organizations, religious or not, supplementing governmental agencies in such fields as education,

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[7]Quoted by Sugden (1982): data from Charities Aid Foundation, 1981.

health, social services, general charities, are organized only on the bases of volunteer labor supply. Again just to offer some quantitative indications: in USA , 5% of the entire stock of USA labor force is donated freely each year (Menchik and Weisbrod, 1987); in Italy, a citizen over eight spends over one third of his spare time working for some voluntary organization.

All the above makes one feel a bit uneasy with a theory which predicts that an individual will always take a free-ride when he can. But stronger doubts arise when one looks more carefully at the comparative static results (*a*) to (*d*) above. First, data on charitable giving show that most organizations are "large" in the double sense that they collect very large sums of money (see above) from the small contributions of a very large number of individuals (more than 85% of all households make donations to charities in the USA; see Andreoni, 1988a). Furthermore, the total sum of money collected is usually much larger than the income of each individual contributor taken alone. These three facts together clash with result (*d*) above which, on the contrary, would require that as the number of givers becomes very large only very few individuals, the richest, would contribute a practically unchanged amount of public good[8].

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[8]The incapacity of the traditional model to account for some basic facts of the empirical literature on charities is also confirmed by a simulation experiment attempted by Andreoni (1988a). The latter simulates the working of the subscription model, using actual data from USA income distribution and actual estimates of the income elasticity of giving. The purpose of the exercise is to calculate the expected value of free-riding, total giving, and

Second, as for result (α) above, several authors have tried to detect and to estimate, with reference to charitable giving, the extent of the "crowding out" phenomenon discussed in the previous paragraph. Strictly speaking, the results of the different papers are not comparable because they use different sets of data. Nevertheless, the insights offered are remarkable. First, there is no evidence of the complete (or approximately so) crowding out phenomenon predicted by theory. Second, evidence is contradictory even on the issue of the existence of a crowding out phenomenon. For example, Abrams and Schitz (1978) find a crowding out effect of the order of 28% and similar indications are also reported by Russel Roberts (1984). In contrast, Schwartz (1970) finds a <u>positive</u>, even if weak, correlation between private and public giving. Reece (1979) finds no evidence of the crowding out effect. Schiff (1985) finds mixed evidence. While public expenditure of the type "cash spending" on the needy is shown to reduce private giving, public expenditure on "social measures" increases private provision. Menchik and Weisbrod (1987), in a study which analyses the voluntary supply of labour, also find a

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average giving as the number of contributors increases. The results are, to say the least, impressive. In a community of 25 members 99% of the individuals should take a free-ride; if the number of contributors goes up to two hundred, estimated free-riding goes up to 99.998 % ; beyond five hundred free-riding is virtually complete. Total giving converges very soon to the very small figure of 2,300 dollars. As a consequence, there is practically no difference between the total amount of public good supplied by a community of 50 members and a community with a million members. None of these results is of course in line with the empirical facts quoted above.

clear positive relationship between local government spending and volunteering. A clear indication of complementarity rather than substitutability between private and public expenditure on public goods also emerges from the more institutional work of Salamon (1986) and Deans and Ware (1987).

In conclusion, while the presence of some amount of crowding out for some items of public expenditure may be an issue, the approximately complete crowding out of the type predicted by theory is not an issue at all. None of the studies quoted above gets even close to confirming this prediction, while there are clear indications in the literature of a relationship between private and public giving which goes in the opposite direction than that predicted by theory. Clearly, all these results strongly clash with predictions (*a*) and (*c*) above.

As for result (*b*) above, I am not aware of any empirical study which has tried to test if private provision is independent of income distribution. However the implausibility of the result speaks for itself.

<u>II.5.2 Checking the traditional model: experimental studies</u>

Another piece of empirical evidence on voluntary provision of public goods is represented by the experimental literature[9]. This

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[9] A third source of evidence on the free-rider issue is surveys. This technique is still in its infancy, but it has already been able to offer considerable insights. Thorsby and Withers (1986),

type of empirical analysis suffers of course from obvious problems concerning the interpretation and the reliability of the results. This is particularly true in our case, because experimenters do not know subjects' preferences for the public good. Thus the best they can do is to induce such preferences on the individuals. This remark is of course rather obvious but it must be realized that this fact alone may introduce severe biases in the results, particularly when one is trying to measure the extent of free-riding behaviour. Furthermore, most of the studies that we are about to discuss, did not carefully set up the theory that they were trying to test (see below), thus adding ambiguity to the interpretation of their results.

With these caveats in mind, we can begin to examine the literature. At an early stage, most works were only interested in detecting free-riding behaviour, defined as the percentage of the actual level to the (experimenter induced) "optimal" level of contribution to the public good. Early studies of this type include Bohm (1972), Sweeney (1973) and Scherr and Babb (1975). Bohm finds no evidence of free-riding behaviour while the other two studies detect some amount of "weak" free-riding. None of them

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for example, by exploiting the interval technique developed by Bohm (1979) find little evidence of strategic misrepresentation of preferences in a study on the demand for Arts in Australia. On a sample of 625 subjects, 65% turn out to be honest respondents and 35% free-riders. Of the latter, only one-third are "strong" free-riders. For a survey of the method employed and of the results obtained in previous attempts, see the authors quoted above.

find support for the "strong" amount of free-riding predicted by theory[10]. These results are however somewhat ill defined due both to the simplicity of the experiment (very little variance in the laboratory conditions was imposed) and the lack of an explicit discussion of the underlying theory to be tested.

Smith's (1980) experiments are better conceived. However, he attempts to estimate the amount of free-riding in a very peculiar voluntary contribution scheme. In such a scheme, the public good is provided if and only if 1) the actual amount of voluntary contributions exceeds the cost of producing the public good and 2) each individual agrees, ex-post, on the cost share implicit in the voluntary contribution scheme above. Smith finds no support for the hypothesis of free-riding behaviour.

Weaker results, but in the same direction are presented by Marwell and Ames (1980) who report on several experiments attempted by Marwell and his associates at the University of Madison. The setting up of the empirical test is fairly robust. Twelve different experiments are attempted and the theory underlying the experiments is checked by confrontation with experts in the field[11]. All the results, with only a significant

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[10]The distinction between "strong" and "weak" free-riding was introduced by Marwell and Ames (1980) and it has been subsequently adopted in the literature. There is, of course, some ambiguity in determining the extent of free-riding which falls in the two categories. See below for some indications of the range of values which are commonly accepted in the literature.

[11]The twelve different experiments intended to test the effects on private provision of changes in the laboratory conditions. The following variations were considered: discrete public good, skewed

exception[12], point toward a clear refutation of the "strong" version of free-rider behaviour. While theory would predict a zero (or approximately zero) provision, individuals voluntarily contribute on average between 40% and 60% of their resources to the public good (a 100% contribution to the public good would represent the "optimal" amount of contribution). Marwell and Ames conclude that while a "weak" version of the free-rider hypothesis can be supported, the "strong" version is rejected by the experiment. Similarly, Schneider and Pommerehme (1981) find very little support for the hypothesis of free-riding behaviour. Up to 96% of the "true" marginal willingness to pay was declared on average by the individuals.

All the studies reported so far thus seem to reject the hypothesis of strong free-riding behaviour predicted by theory while there is support for some weaker version of the same hypothesis. However, as argued above, most of these studies fail to explicitly define the theory they are trying to test, thus introducing some ambiguity in the interpretation of their findings. Building up on such an observation, Kim and Walker

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resources and individual returns from public good investment, small and large groups, non divisibility, high stakes, experienced subjects, experiment with graduate economics students and different levels of information about other behaviour. As is reported in the text, with one exception, no significant variation in the results emerged.

[12]The exception is constituted by the group formed by graduate students of Economics. This group showed a much larger amount of free-riding than average amount. Marwell and Ames interpret this result as an effect of the teaching of Economics on students' perception of the fairness of free-riding.

(1984) claim that the previous studies are not really tests for the results of the Nash contribution model and that as a consequence their findings are invalidated. Their argument is that any scientific theory has a domain of applicability which is determined by a set of side assumptions and that a theory cannot be tested unless it is made sure that these side assumptions are also respected. With reference to the studies quoted above Kim and Walker (1984) identify nine invalidating factors[13] that seem to have been present, at least partially, in all the experiments. In contrast, Kim and Walker set up an experiment of their own where they attempt to control for all these factors. Their results show a larger (but note, not complete) amount of free-riding behaviour than that detected by the previous studies.

While the attempt to introduce carefully the theory to be tested is surely praiseworthy, it should however be noted that the study by Kim and Walker can be criticized on their own terms. In fact, while these authors identify several invalidating factors (see note 13), their main point surely hinges on the invalidating factor number 5: "uncertainty and disequilibrium". The idea is simply that public good theory refers to an equilibrium situation where there is not uncertainty about other individual contributions and where each individual has adjusted his behaviour

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[13]These factors are as follows : impure public good, discrete public good, group optimum unknown, misunderstanding and vagueness, uncertainty and disequilibrium, insufficient economic motivation, small groups, transitory endowment income, lack of anonymity.

to other individuals' behaviour. In practical terms, this induces Kim and Walker to repeat their experiment several times (eleven) in order to "give subjects time to reflect upon their own and the others' previous pledges". Since voluntary contributions show a clear tendency to fall as the experiment is repeated, and since most previous studies (except Smith and Sweeney) did not allow for repetition, Kim and Walker conclude that the results of the previous studies are invalidated because they were really referring to a disequilibrium situation.

It is not entirely clear if this conclusion is correct. As a matter of fact, there is something of a paradox here: Public good theory refers to, and makes strong predictions only for, situations characterized by single period choices (i.e. public good theory is static) and for groups where the identity of the participants is not important. Yet, Kim and Walker's experiment refers to a model which is essentially dynamic (the experiment is repeated several times) and where furthermore participants are always the same. It is not altogether clear what should be the predictions of public good theory in a dynamic context where individuals may play intertemporal strategies[14].

This suggests that we should attempt to allow for learning by

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[14]These issues are discussed in Andreoni (1988b), who reports of an experiment where he attempts to test for the two main alternative hypotheses which have been put forward to explain the reduction in individuals' contributions as the experiment is repeated: learning and strategies. Both hypotheses are rejected by the experiment.

using some different device from that used by Kim and Walker[15]. It also shows that Kim and Walker's findings might also be invalidated on the ground that public good theory is static and not dynamic and that their interpretation of the effects of repetition (in terms of learning) is only one of many possible interpretations which could be given. Similar remarks could also be applied to the interesting and more sophisticated work by Isaac and his associates (1984, 1985, 1988), who also consider repeated experiments. Roughly speaking, their results are consistent with all previous findings, in the sense that in particular contexts they replicate most of the previous results. In general, none of the three competing theories[16] that they consider is consistent with all results. Weak free-riding, however, seems to prevail in the sense of being consistent with a larger set of results. Repetition reduces average contribution. Interestingly enough, contrary to prediction (*d*) above, increasing the number of

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[15]For example, instead of repeating the game, an alternative strategy might be to use experienced subjects; i.e. subjects who have already participated in the same type of experiment in different groups. As we report in a previous note Marwell and Ames (1981) do make a similar attempt but their results do not show any significant difference with respect to unexperienced subjects. For further discussion on this issue see Andreoni (1988b).

[16]Their model is built up in such a way that the dominant strategy for each individual is to provide nothing for the public good while the optimal level of contribution would require strictly positive individual contributions. Zero contribution is then defined as strong free-riding, optimal contribution as Lindahl behaviour and any intermediate situation as weak free-riding behaviour.

individuals involved in the experiment does not reduce average giving as long as each individual's marginal per capita benefit from the public good does not fall following the increase in the number of contributors (i.e. as long as the public good is pure).

Summing up, even taking into account all the caveats we mentioned above, a clear indication seems to emerge from the experimental literature quoted so far: free-riding behaviour is a real phenomenon but its importance seems to have been greatly overemphasized by theory[17]. Let us also note, for future reference, that several researchers have suggested, as a possible explanation of their findings, that subjects were not only driven by mere selfishness in deciding their contribution. For example, most of the subjects in Marwell and Ames's experiments declared that they were "concerned with fairness" when deciding upon their own level of provision to the public good. In line with this hypothesis of fair behaviour, Smith (1980) notes that in his contribution scheme "rich people tend to give more and poor people to give less than their Lindahl shares to the public good". Other researchers, such as Schneider and Pommerehme (1981), speak generally of altruistic preferences as a motivation of subjects' behaviour. Finally, Mestelman and Feerny (1987) in an experimental study, find that

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[17]Our survey of the experimental literature thus confirms Margolis' statement (1982:6): "the conventional economic model not only predicts (correctly) the existence of problems of free-riding, but also predicts (incorrectly) such severe problems that no society we know could function if its members actually behaved as the conventional model implies they will".

"ideology" (i.e. people's perception of the fairness of free-riding) matters in determining the extent of free-riding behaviour. Participants expected to be biased against free-riding, did actually free ride less than average people[18].

II.6 A first summing up

All the empirical literature quoted in the previous two paragraphs indicates that the traditional model of voluntary provision of public goods is in need of drastic revision. None of the predictions of the model is really supported by empirical evidence. Casual observation and all the bulk of experimental analysis tend to suggest that, to some extent at least, people manage to avoid the problem of (systemic) free-riding. This casts serious doubts on predictions (a) and (b) above, that is on the presumed suboptimality of private contribution of public goods[19]. Similarly, none of the results summarized by the "invariance"

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[18] The subjects of the experiments were participants in a congress against earth pollution.

[19] This of course does not imply that one could not argue for suboptimality of private provision of public goods by using other arguments. For example, if the public good must be produced and individuals cannot be excluded from its consumption, no private firm would ever produce the public good because it would be unable to charge a price for it. Furthermore, even if some excluding mechanism could be found, the latter would never be optimal if the public good is pure. In fact, in the latter case, the marginal cost to associate another individual to the consumption of the public good is zero and therefore it is never optimal to exclude an individual from the consumption of the public good. For a good discussion of this and related themes see Oakland (1987).

proposition above finds support in the empirical analysis. The comparative static results on the size of the group of contributors are decisively falsified by the empirical data on large organizations (prediction *d*); there is weak, if any, evidence of a crowding out phenomenon at work between private and public supply of public goods (prediction *a*); and the general invariance property of private allocations to any governmental distortionary and redistributive policy is simply too strong to be taken seriously (predictions *b* and *c*).

The recent literature on public goods has shown an increasing awareness of the many shortcomings of the traditional model. Several attempts have been made either to abandon altogether or to change the traditional contribution model. In the following, without pretense of completeness, we will review and discuss the most interesting attempts made in the literature to date. In so doing, we will also offer arguments in support of the alternative approach that we will adopt in the following chapters.

II.7 Alternative models of voluntary provision of public good

A first attempt which comes naturally to mind is to exploit more fully the game theoretical structure implicit in public good situations. Several attempts of this sort have been made in the literature. Attention has been almost exclusively restricted to noncooperative games, because in large economies the ability of agents to communicate and make binding agreements is dubious (see however Foley, 1970). The main idea of this literature is that concentrating upon simple games and allowing the agents to play public good games several times may encourage cooperation, thus explaining the observed low level of free-riding behaviour. To date, however, these attempts do not seem to be very satisfactory.

For one thing, introducing a more abstract game theoretical structure usually imposes a cost in terms of further simplifying assumptions to be imposed on the structure of the problem. For example, usually very simple games are considered, with binary choices (i.e. contribute or not contribute to a public good) and a limited set of players. This is of course pretty far from the kind of problems usually addressed by public goods theory. Second, a game theoretical structure requires one to explicitly define the payoff structure of the game and the strategy and information sets of the players. As for the former, modeling a public good situation as a "Prisoner's Dilemma" game or as a "Chicken" game -- both roughly compatible with the implicit payoffs of public good situations -- may entail rather different solution concepts and therefore entirely different results[20].

In general, it is however difficult to find strong results in this literature. It is now known for example that in a Prisoner's

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[20]As is well known, in fact, the former game has a dominant strategy, the latter has not and different equilibrium concepts may be proposed. As an example of the "Chicken" game analyses of public goods situation see Bliss and Nalebuff (1984) and Lipnowsky and Maital (1983). For a discussion of several public good games with different incentive structures, see Palfrey and Rosenthal (1988).

Dilemma game, repetition and incomplete information may entail greater cooperation from the individuals (Taylor, 1976; Friedman, 1986; Kreps et al., 1982). However, results are in general ambiguous and difficult to extend to economies with several agents (Taylor, 1976). In contrast, it has instead proved more fruitful to consider less abstract and simpler games but which allow one to introduce more structure into the problem.

Examples of this strategy are offered by Hirshleifer (1983), Bagnoli and Lipman (1986, 1987) and Guttman (1978, 1984, 1987). Hirshleifer (1983) considers cases where the total supply of the public good is not additive in the individual contributions but depends on either the smallest (i.e. $Q=\min\{q_1, \quad ,q_n\}$) or alternatively, the largest individual contribution to the public good (i.e. $Q=\max\{q_1, \quad ,q_n\}$). Bagnoli and Lipman (1986), building upon Hirshleifer, consider a simple contribution game with complete information where the public good is a discrete variable --which is provided if and only if the amount of resources collected reaches a given threshold-- and where if total private provision fails to reach the provision point level, individual contributions are returned to contributors. Guttman considers a sequential two stage Nash contribution model, where in the first stage individuals choose a "matching rate" and in the second a "flat contribution", where the matching rate represents a commitment by individuals to contribute a certain portion of the aggregate flat contribution of everybody else and the flat contribution is instead an independent decision to contribute to the public good.

All these models share the characteristics of implying, at least in some special cases, efficiency in private supply of the public good and in any case a lower amount of inefficiency than that implied by the traditional contribution model discussed above[21]. All these papers provide therefore interesting insights into the working of the contribution model under different assumptions on the production function of public goods and individual behaviour. Nevertheless, their results sound somewhat artificial. In most of the real world examples of private contributions of public goods, none of the distinctive features of the papers above is really present.

The simple summation rule is likely to be a better approximation to the "real" production function of public goods than either the smallest or the largest contribution rule, when attention is restricted to the main real world instances of private provision of public goods: collective organizations and

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[21] In Hirshleifer's model the smallest contribution rule is certainly more efficient than either the traditional summation rule or the greatest contribution rule. Bagnoli and Lipman, using as their concept of equilibrium a refinement of the usual Nash equilibrium concept, the notion of proper equilibria, reach the remarkable result that all proper equilibria in pure strategies of their contribution game are Pareto efficient allocations and all Pareto efficient allocations can be sustained as proper equilibria in pure strategies (see also Bagnoli and Lipman, 1987 for an empirical test of their model strongly supportive of their theoretical findings). Guttman's model implies efficiency in private provision to the public good for identical individuals and a higher level of efficiency than in the traditional model for heterogeneous individuals.

charities. Similarly, the fact that in Bagnoli and Lipman private contributions are returned to individual contributors if resources collected are not sufficient to pay for the public good surely biases the result toward an higher provision than in the normal case. It must be realized, however, that very few, if any, real world examples of private contribution schemes are organized along Bagnoli and Lipman lines.

Guttman's model is open to both logical and empirical criticisms. On logical grounds, while even selfish individuals have an incentive to select in the first game positive matching rates --because it encourages higher flat contributions by others-- they also have an incentive not to fulfill their commitment to match other contributions in the second game. It is then not clear what keeps individuals in Guttman's model from reverting to the usual Nash behaviour of the traditional model thus undermining the entire contribution scheme. On empirical grounds, Guttman can quote only a single and very particular example of a real world contribution scheme which resembles his model[22].

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[22]The empirical evidence that Guttman quotes in support for his model is the system of matching grants by US Federal government to single states' expenditure on public goods. Note that the former looks more similar to an external agent with coercive powers than to a typical agent of a contribution scheme. This takes us back to the logical criticism to Guttman's model advanced in the text, since it seems that the latter to work requires some external agent endowed with coercive powers over players.

None of the previous attempts seems therefore to be able to offer a satisfactory solution to the many shortcomings of the traditional model[23]. Leaving aside their normative implications --which may turn out to be important in other contexts-- the main result which emerges from the literature reviewed in this section is a general indication to look more carefully at the institutional features of the particular private contribution scheme under analysis, before concluding dogmatically that theory predicts an inefficient level of private provision of public goods. However what these models fail to do is to offer a general theory of voluntary contributions to public goods which may be applied successfully to the many real world instances of private

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[23] Another avenue of research which looks promising is represented by the analysis of private provision of public goods in conditions of uncertainty. Austen-Smith (1980, 1981) and Shogren (1987), for example, have shown that if an individual has only a probabilistic knowledge of the total contributions to the public good by the other individuals and if he is characterized by decreasing absolute risk aversion he will tend to supply more to the public good relative to the case where he knows exactly the total provision to the public good by the other individuals. The comparison is performed by employing an hypothesis of mean preserving spread: that is, by assuming that the expected value of other individuals' contributions in the case of uncertainty is equal to the actual value of other individuals' contribution in the case of certainty. Here however lies also the limit of the approach. In fact, if each individual increases his contribution to the public good because of uncertainty as argued in this approach, we should also expect, at least in the long run, that the expected value of other individuals' contribution would also increase relative to the case of certainty. But this in turn, would also imply, under the assumption of income normality of the private good, a reduction in each individual contribution. Considering both the effects it is therefore unclear if uncertainty should increase <u>total</u> private provision of the public good with respect to the certainty case analyzed in the previous sections.

provision. It is then time to consider the literature which has attempted to build such a theory by questioning the basic tenets of the traditional model: that is, by questioning the three fundamental assumptions over which the model is built.

II.8.1 Modifying the traditional model: non-Nash behaviour

The first assumption that one would wish to relax in the context of the contribution model above is certainly the assumption of Nash behaviour. This is so for a number of reasons. First, Nash behaviour is myopic: as we will see below, Nash conjectures on the slope of the reaction function of other individuals turn out to be generally wrong in equilibrium. Second, it seems intuitively clear that if an individual believes that by raising his contribution he can induce other individuals to increase theirs he will be stimulated to do so. If many individuals behave likewise, a higher provision of the public good may result than in the traditional model. Third, there is an established tradition in the public good literature which attributes a higher efficiency to private provision of public goods in small groups on the basis of the argument that in such groups it is easier to maintain expectations of matching behaviour (Olson, 1965; Buchanan, 1967).

This verbal tradition was explicitly modeled by Cornes and Sandler (1984b, 1986). To see how their argument works, let us consider an economy with identical individuals and suppose that each individual i believes that the other individuals will react to changes in i's own contribution by modifying their choices. Following Cornes and Sandler (1984b) we can model this expectation by writing $Q^{e}_{-i}=Q^{e}_{-i}(q_i,Q_{-i})$, where Q^{e}_{-i} is i's expectation of the total contribution of the other individuals expressed as a function of their actual or initial choice, Q_{-i}, and as a function of i's own contribution, q_i. For simplicity, let us linearize i's expectations on other behaviour by writing $\partial Q^{e}_{-i}/\partial q_i=\sigma$, where σ is some constant. Using these assumptions we can rewrite problem (1) as:

$$(14) \qquad \max_{q_i} \quad U(I-q_i;q_i+Q^{e}_{-i})$$

$$\text{s.t. } q_i\geq 0$$

Taking into account the inequality constraint, the first order condition for problem (14), including the complementary slackness condition, can be written as:

$$(15) \qquad \{-U_y+U_a[1+\sigma]\}q_i=0 \qquad q_i\geq 0 \qquad -U_y+U_a[1+\sigma]\leq 0$$

Assuming first $q_i>0$, eq.(15) can be rearranged to give:

$$(16) \qquad U_a/U_y=1/[1+\sigma]$$

where we assume the corresponding second order condition to hold whenever eq.(16) holds. Eq.(16) illustrates Cornes and Sandler's argument: if $\sigma>0$ (expectations of matching behaviour) the implicit price of the public good that an individual faces is smaller than one and, under the normality assumption, this will certainly stimulate a higher provision to the public good than under Nash behaviour (i.e. for $\sigma=0$). The intuition is also straightforward: if individual i expects other individuals to match at least partially his contribution, by giving up an unit of his private consumption he will gain more than an unit of the public good; hence the price that individual i perceives he must pay for an unit of the public good is effectively smaller than one.

It would then seem that if expectations of matching behaviour can be taken as reasonable, non-Nash behaviour, by implying a larger private provision of the public good in equilibrium, could represent a possible escape route to the difficulties of the traditional Nash contribution model. An obvious way to check the reasonableness of expectations of matching behaviour is to verify if the latter are <u>consistent</u> in equilibrium, where, following the oligopoly literature (Perry, 1982; Bresnahan, 1981), an expectation (conjectural variation) is defined as consistent if it is identical to the optimal response of the other agents at the equilibrium based upon that expectation. Unfortunately, as Sugden (1985) has shown, if the public good is not a superior good expectations of matching behaviour cannot be consistent in equilibrium. To see this, let us define a <u>consistent equilibrium</u>

(Perry, 1982) as a vector of private contributions $\{q^*\}$ such that:

$$(17a)\quad U(I-q_i^*, q_i^*+Q^e_{-i}(q_i^*, Q_{-i}^*)) \geq U(I-\hat{q}_i, \hat{q}_i+Q^e_{-i}(\hat{q}_i, Q_{-i}^*)) \quad \forall i,\ \forall \hat{q}_i$$

$$(17b)\quad Q^e_{-i}(q_i^*, Q_{-i}^*)=Q_{-i}^* \quad \forall i$$

$$(17c)\quad \partial Q_{-i}(q^*)/\partial q_i=\sigma \quad \forall i.$$

Condition (17a) imposes that at the equilibrium no individual has an incentive to unilaterally change his choice; condition (17b) requires each individual to be correct, at the equilibrium, about his expectations on the <u>level</u> of other individuals contributions, and finally condition (17c) requires each individual to be correct, at the equilibrium, also about the <u>slope</u> of other individuals' reaction-function. As should be clear, the concept of consistent equilibrium is just a refinement of the concept of Nash equilibrium that we used so far, a refinement which is obtained by making condition (17b) explicit and by adding a condition on the (local) rightness of conjectural variations in equilibrium, i.e. by adding condition (17c).

In order to get an explicit solution for condition (17c) above, let us evaluate eq.(16) at a symmetric equilibrium allocation $\{q^*\}$:

$$(18)\quad -U_y(I-q_j^*;(n-1)q_j^*+q_i^*) + U_Q(I-q_j^*;(n-1)q_j^*+q_i^*)[1+\sigma]=0 \quad \forall j$$

where $q_j^*=q_i^*>0$. By differentiating eq.(18) with respect to q_i and q_j and solving for dq_j/dq_i we get the actual response of j's contribution to an exogenous change in i's contribution: and since all individuals are identical $\partial Q_{-i}(q^*)/\partial q_i=\{n-1\}dq_j/dq_i$. Condition (17c) then imposes[24]:

(19) $D^{-1}[n-1]\{-U_{aa}[1+\sigma]+U_{ya}\}=\sigma$

where $D=\{U_{yy}+U_{aa}[n-1][1+\sigma]-U_{ya}[n+\sigma]\}$ and where all derivatives are evaluated at the equilibrium allocation $\{q^*\}$. Solving and rearranging, (19) can be rewritten as:

(20) $[n-1][1+\sigma]\{-U_{aa}[1+\sigma]+U_{ya}\} + \sigma\{-U_{yy}+U_{ya}[1+\sigma]\}=0$

To interpret this expression note that the terms in the curly parentheses of eq.(20) capture the effects of income changes on an individual demand for the private and the public good. This can be easily seen by differentiating the first order condition in eq.(18) with respect to I and q_j and by keeping the contributions of everybody else constant. Doing this and solving for dq_j/dI we get $dq_j/dI=\{-U_{yy}+U_{ya}[1+\sigma]\}/J$, where $J=-\{U_{yy}+U_{aa}(1+\sigma)-U_{ya}(2+\sigma)\}>0$

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[24] I read but I am unconvinced by the alternative condition for consistency of conjectures derived by Scafuri (1988). Note that, differently from what argued by Scafuri, our condition does take into account the interaction among the contributions of the remaining (n-1) agents following from the changes in i's contribution.

from quasi-concavity of the utility function. Furthermore, as $dq_j+dy_j=dI$ from the budget constraint, $dy_j/dI=\{-U_{\alpha\alpha}[1+\sigma]+U_{y\alpha}\}/J$. Then, by dividing both sides of eq.(20) by J and rearranging we get:

(21) $\sigma = -[n-1]\alpha/\{[n-1]\alpha+[1-\alpha]\}$

where I have simplified the notation by writing $dy_j/dI=dy/dI=\alpha$. Clearly, the sign of σ depends on the sign of the income effect. First note that, as also argued by Cornes and Sandler (1984b), Nash conjectures can be consistent only if there are no income effects on the demand for the private good (i.e. $\alpha=0$). Second note that, in line with our previous analysis, if there are no income effects on the demand for the public good (i.e. $\alpha=1$), each individual should predict a complete offsetting behaviour by part of the other agents (i.e.$\sigma=-1$). Third, note that as Sugden (1985) rightly argued, if both the private and the public good are normal goods (i.e. $0<\alpha<1$) $\sigma<0$ certainly; and as also argued by Sugden $\sigma>0$ requires $\alpha<0$; that is, it requires private consumption to be an inferior good. The rationale for these results is of course the same as we saw in the previous sections: if the public good is a normal good, each individual reaction function is negatively sloped in other individuals' contributions. Therefore, if an individual correctly predicts the reactions of other individuals, he must predict offsetting behaviour. But this, of course, going back to eq.(16), would also induce each individual to contribute even less to the public good than in the Nash contribution model, thus making the free-rider problem more severe.

While the above is sufficient to show that abandoning the assumption of Nash behaviour is unlikely to offer a solution to the problems of the traditional model, other striking implications of eq.(21) should be noted. First, note that even if $\alpha<0$, for $n>[2\alpha-1]/\alpha$, $\sigma<0$ certainly; that is, in large economies, even if private consumption is an inferior good, consistent conjectures are negative: i.e. individuals must predict offsetting behaviour on the part of the other agents. This strengthens Sugden's argument even further. Second note that, if $\alpha\neq0$, as n tends to infinity σ tends to -1; that is, in very large economies, irrespective of the sign of the income effects, each individual should predict complete offsetting behaviour on the part of the other individuals. Sugden (1985) noted this and argued that in very large economies private provision of the public good should approach zero; but our previous formulation of the concept of consistent equilibrium suggests even more striking conclusions.

To see this, suppose that Sugden is right and that the consistent symmetric equilibrium is characterized by zero private provision of the public good: i.e. let $q_i=0$, $\forall i$. But if $Q_{-i}=0$, it would seem reasonable to argue that individual i's consistent conjectures about the slope of the reaction function of the other individuals can only be non-negative since no individual can supply a negative contribution to the public good: that is, whatever the process by which an individual i comes to have expectations about other behaviour, for $Q_{-i}=0$, he cannot expect consistently offsetting reactions by the other individuals.

More generally, even if $Q_{-i}\neq 0$, it would seem a natural requirement of consistency of conjectures to impose that, if individual i rightly conjectures that the level of the other individuals' contribution is Q_{-i}, he cannot consistently expect the fall in other individuals' contributions to be larger than this initial value. It then follows:

$$(21a) \qquad Q^{\circ}_{-i}(q_i', Q_{-i}) - Q^{\circ}_{-i}(q_i, Q_{-i}) \geq -Q_{-i}$$

whenever $Q^{\circ}_{-i}(q_i, Q_{-i})=Q_{-i}$. We can then state:

<u>Proposition 1</u> "If $U_a/U_y(I,0)>1$, a consistent symmetric equilibrium where $q_i=0$, $\forall i$, does not exist."

<u>Proof</u> "Suppose $q_i=Q_{-i}=0$ is a consistent equilibrium. Then from condition (17b) above, $Q^{\circ}_{-i}(0,0)=Q_{-i}=0$. Consider now a small increase in i's contribution to $q_i'>0$. The corresponding change in the expected contribution of everybody else is then:

$$\Delta Q^{\circ}_{-i} = Q^{\circ}_{-i}(q_i',0) - Q^{\circ}_{-i}(0,0) \geq 0$$

by equation (21a) above. Invoking the mean value theorem we can rewrite the above expression as

$$\Delta Q^e_{-i} = \{\partial Q^e_{-i}(\bar{q}_i, 0)/\partial q_i\} q_i' \geq 0$$

for some $\bar{q}_i \in (0, q_i')$. Consider now the expected utility differential resulting from the small increase in i's contribution:

$$\Delta U = U(I - q_i', \; q_i' + Q^e_{-i}(q_i', 0)) - U(I, 0)$$

Letting $q_i' \Rightarrow 0$, the previous expression can be written as

(21b) $$dU = \{-U_y(I,0) + U_\alpha(I,0)[1 + \partial Q^e_{-i}(\bar{q}_i, 0)/\partial q_i]\} q_i' > 0$$

since $[-U_y(I,0) + U_\alpha(I,0)] > 0$ by assumption. Hence if $Q_{-i} = 0$ and individual i rightly conjectures both the level of contribution and the slope of the reaction function of other individuals, $q_i = 0$ cannot be the optimal choice of individual i. This shows that $q_i = 0$, $\forall i$, cannot be an equilibrium. QED"

The condition $U_\alpha/U_y(I,0) > 1$ simply implies that at zero provision of the public good, if the individual faced a price for the public good equal to one, he would wish to spend some of his income on the public good. It is therefore an entirely innocuous assumption which we have been implicitly imposing in all the chapter. The rationale for proposition 1 is simple: if $Q_{-i} = 0$ and each individual i rightly conjectures that $\partial Q_{-i}/\partial q_i \geq 0$, each i would have an incentive to deviate and to contribute a positive quantity to the public good. Condition (17a) is therefore not satisfied which implies that a symmetric consistent equilibrium at $q_i=0$, $\forall i$ cannot exist. Note that this result does not depend on either the number of individuals or the income effects on the demand for the public good; it is only a consequence of our hypothesis that at $Q_{-i}=0$ each individual i rightly conjectures that $\partial Q_{-i}/\partial q_i \geq 0$.

Thus, proposition 1 shows that a consistent equilibrium characterized by a zero contribution from everybody is impossible. But would a symmetric consistent equilibrium where everybody provides a positive quantity be possible? Let us start with the simpler case of an economy where there are no income effects on the demand for the public good. We can state:

Proposition 2 "Suppose that in an economy with identical individuals the demand for the public good by each individual is everywhere characterized by zero income effects: then a consistent symmetric equilibrium with a positive contribution from everybody does not exist"

Proof "Suppose that $q_i=q>0$, $\forall i$, is an equilibrium. From eq.(21) above, if individual preferences are such that $\alpha=1$ everywhere, consistent conjectures must entail $\sigma=-1$. Evaluating eq.(15) at $Q_{-i}=[n-1]q>0$ and $\sigma=-1$ we get

(21c) $$[-U_y(I-q_i, q_i+[n-1]q)]<0 \quad \text{for any } q_i \geq 0;$$

which implies that the optimal choice of individual i is to set $q_i=0$. This shows that $q_i=q>0, \forall i$ is not an equilibrium. QED"

The rationale for proposition 2 is straightforward. Going back to eq.(16) if $\sigma=-1$ it is as if each individual i faced an infinitely large price for the public good. In fact, any positive contribution by individual i brings about a corresponding reduction in the contributions of the other individuals; therefore if individual i reduced his private consumption he would not get any extra amount of the public good in exchange. Thus, i's best choice is to set his contribution equal to zero.

Putting together proposition 1 and proposition 2 we find that in an economy with zero income effects on the demand for the public good a symmetric consistent equilibrium does not exist. But as we saw above, the demand for the public good is usually characterized by positive income effects. Thus, in this case, consistent symmetric equilibria with positive contributions may exist. On the other hand, as we saw in eq.(21) above, if $n \Rightarrow \infty$, $\sigma \Rightarrow -1$, even if there are positive income effects on the demand for the public good. Would then a consistent symmetric equilibrium exist in a large economy with positive income effect on the demand for the public good?

Proposition 3 "Suppose that in an economy with identical individuals the public good is everywhere a normal good and suppose further that $\exists\ \delta$, $\delta>1$, such that $1<U_a/U_y(I,0)<\delta$. Then there is an n^* such that for $n>n^*$ a consistent symmetric equilibrium must fail to exist"

Proof "If the public good is a normal good the private good must also be a normal good; it follows:

(21d) $\quad U_a/U_y(I,[n-1]q) \leq U_a/U_y(I,0) < \delta$

for any $q\geq0$ and any $n>1$. From eq.(21) above, as n increases σ falls; let then n^* be such that $[1-\sigma]^{-1}=\delta$. Hence, for any $n>n^*$, $[1-\sigma]^{-1}>\delta$. Consider now an economy with n^{**} identical individuals, where $n^{**}>n^*$. From proposition 1 above, $q_i=0$, $\forall i$ cannot be an equilibrium. Suppose then that $q_i=q>0$, $\forall i$, is an equilibrium. Evaluating eq.(15) at $Q_{-i}=[n^{**}-1]q>0$ and using again the assumption of normality of the two goods, we get:

(21e) $\quad U_a/U_y(I-q_i,q_i+[n^{**}-1]q) < [1+\sigma]^{-1} \qquad \forall q_i\geq0$

which implies that the optimal choice of individual i is to set $q_i=0$. Hence $q_i=q$,$\forall i$, cannot be a consistent equilibrium. QED"

The condition $U_a/U_y(I,0)<\delta$ implies that there exists a price for the public good so high that individuals would prefer not to buy any of the public good and spend all their income in the purchase of the private good. The public good is not, in other words, an essential good[25]. I would like to argue that for many public goods the condition of non-essentiality may be considered reasonable enough. If this conjecture is correct, proposition 1 to 3 would then imply that symmetric consistent equilibria in large economies may fail to exist. This of course does not imply that asymmetrical equilibria cannot exist either as the next proposition shows.

<u>Proposition 4</u> "Consider an economy with two identical individuals, $i=1,2$, and suppose that each individual demand for the public good is characterized by zero income effects. Also suppose that $\partial q_j^{\circ}(q_i,0)/\partial q_i=0$, $i,j=1,2$ and $i\neq j$. Then, an allocation where $q_j^*=0$ and $q_i^*=\{q_i|\max U(I-q_i,q_i)\}$ $i,j=1,2,$ $i\neq j$ is a consistent equilibrium."

<u>Proof</u> "To prove the proposition, it is enough to check that conditions (17a) to (17c) are satisfied at the allocation above. q_i^* satisfies (17a) to (17c) if $q_j^{\circ}(q_i^*,0)-q_j(q_i^*)=0$ and $\partial q_j^{\circ}(q_i^*,0)/\partial q_i=\partial q_j(q_i^*)/\partial q_i=0$, where $q_j(q_i)$ indicates the reaction function of individual j. Suppose now that individual j holds consistent conjectural variations about i's behaviour at q_i^*; that

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[25]For a precise definition of essential goods see chapter IV, section 3.

is, since there are no income effects on the demand for the public good, suppose $\partial q_i^{\circ}(0,q_i^*)/\partial q_j = \partial q_i(0)/\partial q_j = -1$. By the previous argument, the optimal response of individual j is then to set $q_j(q_i^*)=0$; furthermore, since $q_j=0$ is the optimal response of individual j for any $q_i>0$, $q_j=0$ must also be the optimal response of individual j in a small neighborhood of q_i^*. This implies $\partial q_j(q_i^*)/\partial q_i=0$. Therefore at $q_j^*=0$ and $q_i=q_i^*$, each individual correctly conjectures both the level and the slope of the reaction function of the other individual at that allocation; and each individual maximizes his utility with respect to the correct conjecture about the other individual's behaviour. Hence $q_j^*=0$ and $q_i=q_i^*$ i,j=1,2 i≠j is a consistent equilibrium.QED"

Summing up, unless private consumption is an inferior good, consistent conjectures must necessarily imply expectations of offsetting behaviour by the other individuals; and if the economy is large, consistent conjectures must be negative even if the private good is an inferior good. Furthermore, the larger the number of individuals the smaller the private provision of the public good; and if the economy is very large a symmetric equilibrium may fail to exist. Going back to sections II.5 and II.6, it is then clear that consistent non-Nash behaviour can only make the explanation of the empirical evidence on private provision of public goods more difficult. A solution could be to impose matching behaviour without requiring it to be consistent; but in the selfish framework adopted in the literature this would be difficult to justify. Furthermore, note that even with arbitrary conjectures (i.e non-consistent conjectures) the basic invariance results that we indicated above would remain unchanged. This is so because even in an arbitrary conjectured equilibrium the reaction function of each individual would still be negatively sloped in the contribution of the other individuals if both goods are normal goods, thus reproducing all the invariance results presented above. This can easily be seen by working on the first order conditions in eq.(18).

II.8.2 Modifying the traditional model: impure public goods

A second attempt is to drop the assumption of "purity" of the public good by introducing elements of "privateness" in an individual decision to provide the public good. This solution has also a long tradition in the history of the discipline[26] and it has by now been proposed by so many writers that it is threatening to become a new orthodoxy[27]. Indeed, the same literature on club goods

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[26]Olson (1965), for example, suggested that large organizations manage to exist despite the free-rider problem because they offer "selective incentives" --i.e. private goods and services -- to their members. As an example, think of a Union which offers together with its membership card price reductions for travel expenses. However, as Stigler (1974) was quick to remark, Olson's solution is open to the logical objection that in a competitive market we should expect private firms to emerge able to offer the same services at a lower price than the collective organizations. This is so because the private firms would not have to pay for the production of the public good itself.

[27]For a recent survey of the literature on mixed goods see Else (1988).

and local public goods has sprung from the simple observation that many so-called public goods have private goods characteristics. As is by now well known, this simple fact is able to change considerably the traditional results of inefficiency of private provision of these goods (see Sandler and Tschirhart, 1980).

Club goods theory cannot be applied straightforwardly to the empirical facts that we are discussing here because the distinctive features of clubs are not present in our case (see Cornes and Sandler,1986: 159-160). There are however several alternative ways of introducing elements of privateness even in the context of large organizations and philanthropic institutions. For instance, in the case of philanthropic giving, a simple way is to deny altogether the public good nature of the charity and to model individual decisions to donate simply as the purchase of a pure private good. This solution characterizes, for example, the work of Weisbrod and his associates (Weisbrod, 1975; 1977; Weisbrod and Dominguez,1980, Menchik and Weisbrod 1987). In their formulation, the donor gets utility only from his own contribution to the charity and not from the total contribution by all other individuals. Therefore, to all intents and purposes, individual contribution is a pure private good.

Clearly, such a solution would eliminate at once all the problems related to private provision of public goods and indeed the same notion of free-riding behaviour would become meaningless. However, one cannot help feeling that such a solution is equivalent to throwing out the baby with the bath water. On intuitive grounds, it would seem very strange to argue that the characteristics of the people benefiting from the donations do not affect the decision of an individual to donate. Yet, if an individual gets utility only from his own donation, this is what has been implicitly assumed. Moreover, Weisbrod's solution is also open to a logical objection. If two (rich) individuals get utility only from the act of donating, they would surely be better off by donating to each other part of their income rather than by giving it to a third poor person who cannot reciprocate. Yet, as a matter of fact, rich people donate to poor people and voluntary organizations are characterized by the fact that they attempt to help the sick, the poor and the elderly.

Therefore, assuming that the utility of the donors depends on the level of welfare of the people benefiting from the charity seems to be unavoidable both on intuitive and on theoretical grounds. But this, of course, compels one to introduce an element of publicness in the analysis of charitable giving. Similar remarks apply of course even more strongly to large organizations other than philanthropic organizations, such as trade unions.

Probably in order to avoid the theoretical and intuitive pitfalls of eliminating tout court the public good features of charity donations, several authors have proposed a sort of mid-way solution. Bernheim (1986), Andreoni (1988a), Cornes and Sandler (1984a), Posnett and Sandler (1986), all argue that private provision of public goods should be studied in models where individuals gain utility from both their contribution to the public good <u>and</u> the total level of public good supply[28]. If an individual gets utility from both his own contribution and the total supply of the public good, each dollar spent on public good provision can be thought of as producing jointly two types of goods: a pure "private" good (the individual satisfaction deriving from his own donation) and the public good itself. Clearly, the traditional results of the Nash contribution model may change in this case. To see this, let us rewrite the utility function of the representative consumer i as depending upon three goods or "characteristics" (Cornes and Sandler, 1984a, 1986):

$$(22) \qquad U^i = U^i(y_i, Q, z_i)$$

where y_i and Q have the interpretation offered above. The novelty of the approach comes from the third good or characteristic: z_i. The latter is supposed to be a pure private good which is produced by purchasing an unit of the public good: it is, in the interpretation usually offered, the private satisfaction accruing to individual i as a result of the feeling of "having done his own duty". For simplicity let us suppose that z's production function is a linear function of individual i's contribution:

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[28]In a recent study on collective action, Hirschman (1982) also suggests a similar explanation for citizens' involvement in political activity.

(23) $z_i = \bar{z} + \gamma q_i$

where γ is some positive constant, $\bar{z}$ is some exogenous value (it might also be set equal to zero) and q_i is, as above, individual i's contribution. The first order condition for the maximization of eq.(22) subject to eq.(23) and the budget constraint $I_i = q_i + y_i$ yields:

(24) $U^i_Q/U^i_y(I_i - q^*_i, q^*_i + Q_{-i}, \gamma q^*_i + \bar{z}) + \gamma U^i_z/U^i_y(I_i - q^*_i, q^*_i + Q_{-i}, \gamma q^*_i + \bar{z}) = 1$

where we assume an interior solution for q_i and the related second order condition to hold at q^*_i. In general, at least for $\bar{z}=0$, eq.(24) entails a larger individual provision to the public good than in the normal case[29]. The real novelty of the approach

[29]This can be easily seen by differentiating eq.(24) with respect to q_i and γ and evaluating the resulting derivative $dq_i \backslash d\gamma$ at $\gamma=0$:

$$dq_i/d\gamma|_{\gamma=0} = \{q^*_i[U^i_{Qz} - U^i_{yz}] + U^i_z\}/D$$

where $D = -[U^i_{yy} + U^i_{QQ} - 2U^i_{yQ}] > 0$ from the second order condition and where each derivative is evaluated at the optimal choice for q^*_i. The equation above shows the effect of introducing a small jointness in production on individual contribution, using as starting point the contribution in the normal case. Clearly, if we evaluate the expression above at $\bar{z}=0$, U^i_z is likely to be very large thus implying $dq_i/d\gamma > 0$, whatever the sign of the first two effects. Hence, jointness in production is likely to produce, _ceteris paribus_, a larger individual provision to the public good. Note however that the conditions for efficiency in public good provision also change in the joint production model, implying a larger individual contribution with respect to the normal case. It is therefore unclear if the joint production model implies a however comes from its implications for the slope of the individual reaction function with respect to changes in other contributions. To see this let us differentiate eq.(24) with respect to q_i and Q_{-i}; solving for dq_i/dQ_{-i} we get:

$$(25) \qquad dq_i/dQ_{-i} = [-U^i_{ya} + U^i_{aa} + \gamma U^i_{za}]/D$$

where $D=-[U^i_{yy}+U^i_{aa}+\gamma^2 U^i_{zz}-2U^i_{ya}-\gamma 2U^i_{yz}+\gamma 2U^i_{az}]>0$ from the second order condition. The first two terms on the RHS of eq.(25) can be shown to capture the usual substitution and income effects that we derived earlier (see below) and are therefore together negative if both the private good y and the public good are normal goods. The last term is however uncertain; U^i_{za} measures the effect of a small change in Q on the marginal utility for the private good z_i. If such effect is positive, i.e. if z_i and Q are "complementary" goods (Cornes and Sandler, 1984a), the sign of dq_i/dQ_{-i} becomes uncertain and it might also turn out to be positive. The intuition is straightforward. If Q and z_i are complementary, an exogenous increase in Q will push the individual to "buy" more of the private good z_i, thus inducing him to increase his contribution to the public good. Clearly, if such a complementarity effect is very strong, it might overcome the substitution effect induced by an increase in the contribution of the other individuals thus

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higher efficiency in private provision to the public good.

generating a positively sloped reaction function.

The point is of course whether such a complementarity effect is likely to arise in practice and to be strong enough to overcome the substitution effect. Intuitively speaking, it is difficult to see why this should be the case. If the private benefits of donating depend on the satisfaction of having "done one's bit", it is not clear why such a feeling should deepen following an increase in other contributions. Rather, if a sense of ethical duty lies behind this feeling, one would expect it to remain basically unchanged. In terms of our formalization above this would imply $U^i_{z\alpha}=0$, thus reproducing the typical reaction function associated with Nash behaviour.

On this issue we are also able to offer a different interpretation for eq.(25) above. To see this let us differentiate the first order condition in eq.(24) alternatively with respect to q_i and I_i, and with respect to q_i and to $\bar{z}$; doing this and solving for dq_i/dI_i and $dq_i/d\bar{z}$ we get:

$$dq_i/dI_i = [-U^i_{yy} + U^i_{\alpha y} + \gamma U^i_{zy}]/D$$

(26)

$$dq_i/d\bar{z} = [-U^i_{yz} + U^i_{\alpha z} + \gamma U^i_{zz}]/D$$

where D has the interpretation given above. Then by summing and subtracting 1 from the RHS of eq.(25) and rearranging, dq_i/dQ_{-i} can be rewritten as:

(27) $dq_i/dQ_{-i} = -1 + dq_i/dI_i - \gamma dq_i/d\bar{z}$

The first two terms on the RHS of eq.(27) are the substitution and income effect discussed in the previous paragraphs. It can now be noted that a necessary but not sufficient condition for a positively sloped reaction function is $dq_i/d\bar{z} < 0$; that is, from eq.(23), $dz_i/d\bar{z} < 1$. This in turn has a nice interpretation. A small increase in $\bar{z}$ will produce both a substitution and an "income" effect due to the fact that an exogenous increase in $\bar{z}$ increases utility and small increases in utility are equivalent to small increases in income. Therefore $dz_i/d\bar{z} < 1$ if the private good z_i is not a superior good; that is, if the increase in utility generated by the exogenous increase in $\bar{z}$ does not induce the individual demand for z_i to increase so much as to make $dq_i/d\bar{z}$ positive.

If this interpretation is correct, it is then clear that the sign of the reaction function in eq.(27) above depends upon the assumptions made on the income effects on the demand for the private good z_i. For example, if the private characteristic z_i behaves as a normal good --i.e. if $0 < dz_i/d\bar{z} < 1$-- and if the income effects on the demand for the public good have the small magnitude detected by the empirical studies (see section II.3), the individual reaction function should still be non-positively sloped in other individuals' contributions even in a joint production model. As should be clear, a positively sloped reaction function really requires the private characteristic generated by the individual contribution to be an inferior good: that is, it requires $dz_i/d\bar{z}<0$.

These considerations show that while the joint production formulation is able to reduce some of the problems associated with the traditional contribution model[30], it also requires very peculiar assumptions to work. Thus, without wishing to deny the important insights offered by the impure public good approach, there seems to be room enough for considering alternative explanations. On this ground, note that the same justification offered in the literature for the notion of private benefits accruing from public good contributions seems to point more toward an explanation of giving as motivated by some feeling of ethical "duty" rather than by a preference for donating[31].

II.8.3 Modifying the traditional model: unselfish behaviour

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[30]In particular the assumption of impure public goods, by breaking the link between lump-sum income and exogenous changes in public good supply, would also eliminate, at least to an extent, the invariance results summarized in section II.3 (see Andreoni,1988a).

[31]Before leaving the section on impure public good, we should mention the important paper by Palfrey and Rosenthal (1988). These authors call their model an "altruistic" model, but what they really do is to introduce private benefits from contributing, since "altruism" is modeled as a component of utility which depends solely on how much an individual contributes. The interesting feature of their model is that while the pecuniary pay-offs of the game are common knowledge, "altruism" is assumed to be private information. They then get a game of imperfect information which is solved by using the notion of Bayesian Nash equilibrium, due to Harsanyi. Palfrey and Rosenthal obtain strong results which are then tested in several different types of binary contribution games.

We are then left with the last assumption of the traditional contribution model: the assumption of selfish maximizing behaviour. To be sure, there has always been debate in the literature on the relevance of this assumption for public good situations. For example, both Olson and Buchanan, two founding fathers of the discipline, have always been very careful in limiting the range of applicability of the assumption of selfishness in the context of the analysis of private provision of public goods[32]. Both authors are however deeply convinced that public good theory is perfectly adequate for the analysis of large economic organizations and the subsequent literature has tended to forget even the mild caveats that the earlier authors had imposed on the theory. On the other hand, as we discussed at length in the previous chapter, public good theory is not consistent with the empirical facts that we know about private provision of public goods in large groups. Moreover, as we showed in the previous paragraphs, the attempts proposed so far in the literature to amend the traditional model are far from being satisfactory.

There seems therefore to be room enough to explore the

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[32]Olson (1965) deliberately excludes "religious and philanthropic" organizations from his analysis on the ground that people's behaviour in these organizations may not be motivated by strictly defined self-interest. Buchanan (1978) restricts the domain of applicability of public good theory to "large organizations" exactly because, he argues, ethical and social norms of behaviour are likely to prevail in smaller groups.

implications of dropping the assumption of selfishness even in the context of large groups and organizations. Why should then people behave unselfishly in these situations? First, because there is no reason why they should not do so. The assumption of selfishness is only a useful simplifying hypothesis on human behaviour and not a literal description of people's attitudes and behaviour. Thus, considering alternative hypotheses to selfishness is an entirely legitimate research strategy to follow. The point is of course if such a strategy is fruitful. A hint that this could be the case is provided by the experimental evidence on public goods quoted above. As we pointed out there, several authors offered explanations for their findings in terms of "feelings of fairness", altruism or ethical norms. On intuitive grounds this sounds realistic. Basic norms of social behaviour such as "be honest", "tell the truth", etc. if followed by individuals would certainly reduce free-riding behaviour.

There is moreover another reason why we should expect unselfish behaviour to be particularly relevant in public good situations. As we argued in chapter I, in such situations, unselfish behaviour may be perceived by the individuals themselves as instrumental[33] to the attainment of selfish objectives. As

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[33]There is a growing economic literature that, in the spirit of evolutionary biology, attempts to explain unselfish behaviour as instrumental to achieve selfish objectives. An up-to-date bibliography is in Frank (1987), who also offers an excellent example of this approach by explaining feelings of guilt and concern for fairness as pre-commitment devices in Prisoners' Dilemma type of situations. For a first attempt to verify

Johansen (1977:148) has put it

"...everybody would understand that the system would produce very bad results and possibly come to a breakdown if everybody concealed their preferences for the public goods. Although concealment of preferences corresponds to a sort of non cooperative equilibrium, and thus to individual rationality in a narrow sense, everybody would realize that it generates a very inferior solution. The collective might be able to break out of this equilibrium and instead establish something more like a cooperative equilibrium, based on a more true representation of preferences, out of the joint understanding of the necessity of this for the long-run workability of the system".

The problem is, of course, how to model satisfactorily these ideas in the context of a model of voluntary provision of public goods. Early writers, such as Becker (1974) and Collard (1978) proposed to use the notion of altruistic preferences. But as our previous discussion has shown, altruism alone cannot be a solution. In fact, if two individuals are altruistic toward a third the welfare of the latter represents, in effect, a pure public good for the former: and why should then individuals contribute toward this public good?[34]

This remark holds also for the interesting extension of the usual altruistic model offered by Margolis (1982). The latter assumes that each individual's behaviour can be characterized by two utility functions: a first which represents his own

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empirically the importance of fair rules for price and wage setting behaviour of firms see Kahneman et al. (1986).

[34] For a similar argument and related criticisms of the notion of altruistic preferences see Sugden (1982, 1984).

self-interested preferences and a second which represents the preferences of the reference group of the individual-- as perceived by the individual himself. Postulating a form of "participation altruism" --individuals gain utility from donating resources to the group-- and an "allocation rule" which establishes how the resources of the individual come to be allocated between the selfish and the group oriented utility function, Margolis's model explains the larger private provision of the public good observed in reality. However, as in the traditional model discussed above, Margolis's theory still predicts negatively sloped individual reaction functions with respect to others' contributions[35], thus reproducing the basic invariance properties of the traditional model.

An alternative line of research, originated by Akerlof (1980), explains unselfish behaviour in terms of "social customs". The idea is that there are some customs (i.e. rules of behaviour) which are followed by individuals in spite of the fact that it would be in their self-interest to disobey them. The binding force which causes individual obedience to the custom is the social

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[35]The reason is simple: an increase in the contribution to the public good by another individual reduces the marginal utility of contributing of the group-oriented preferences. Thus the individual should divert some of his resources to other activities. Another problem with Margolis's theory is that, as group-preferences are completely unaffected by selfish preferences, the arguments of the group oriented utility function are imprecise. Thus, the model is unable to predict to which public good an individual should contribute. For these and related criticisms see Sugden (1984).

sanction imposed by the loss of reputation from breaking the rule[73]. Akerlof uses his model to explain job discrimination and the persistence of higher wages than market-clearing wages in labor markets, but it is clear that if we identify the social custom as saying "pay for the public good" his model can be used successfully even in the context of the empirical cases we are concerned with[74].

But, however suggestive, Akerlof's theory meets a fundamental objection in the context of our problem: the fundamental ad-hoc nature of the social custom. Why, in fact, should the social custom say "pay for the public good" and not, for example, "pay for the public good only in such and such situation"? Indeed, as the same analysis of Akerlof suggests, any possible equilibrium could be sustained as the result of the imposition of some set of "social customs". Without a theory which would allow one to select among these rules, the results would remain basically indeterminate. Now, while there is a growing literature (Schotter, 1981, Sugden, 1986) which is trying to explain the emergence of

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[73] Akerlof's model is dynamic and provides an intuition on how social customs may evolve. The idea is that in any single period there will always be some deviant people who nevertheless decide to follow their own interest and break the code. If the number of deviants overcomes a given threshold, in the subsequent periods there will be fewer and fewer people following the social custom until the latter disappears.

[74] For an example of authors who use Akerlof's model to solve the free-rider problem in public good situations see Naylor (1989) and Booth (1985).

norms and "institutions" as the result of repeated simple games, the results obtained to date are far too weak and ambiguous to allow for a theory to be built upon them. Without denying the important insights offered by Akerlof's analysis, it does seem therefore that a satisfactory theory of private provision of public goods must be looked for elsewhere.

The hypothesis of ethical behaviour may offer a more satisfactory solution. In a sense, we could think of morality as selecting among all the set of possible social customs or "rules of behaviour" which could be followed by the individuals. Furthermore, if attention is restricted to simple basic universal moral codes it sounds quite realistic to assume that people may know and be influenced by these rules in their actual behaviour. Of course, introducing an hypothesis of ethically oriented behaviour does not mean to deny that social sanctions have a role in enforcing the obedience to such rules.

The work of Laffont (1975) and Sugden (1984) represents a first step in this direction. Laffont introduces and formalizes the notion of Kantian behaviour in a model with identical individuals (see next chapter). He shows that in an economy with externalities and public goods efficiency will result if everybody acts as a Kantian -- i.e. if everybody takes his decisions on the basis of the hypothesis that everybody else will do as he decides to do. Sugden (1984), building upon Laffont, presents a more satisfactory theory based on a notion of reciprocity and Kantian behaviour. Sugden assumes that each individual in the economy selects, for each group he belongs to, his Kantian contribution to the public good, where the latter is computed as in Laffont (that is, on the basis of the hypothesis that everybody else in the group would contribute the same absolute amount that the Kantian individual decides to contribute). The Kantian contribution represents the maximum amount that each individual would perceive as morally right to contribute in each group he belongs to. How much of this maximum amount an individual will actually provide depends on the behaviour of the other individuals composing the group through mechanisms of reciprocity. Sugden assumes that in each group each individual will feel obligated to contribute --i.e. to "reciprocate"-- at least the smallest contribution provided by any other individual in the group. In turn, a "group" is defined as any possible coalition which can be formed from the original group of contributors, including the singleton groups formed by each individual alone.

The argument offered by Sugden in support of his formalization is based on an hypothesis of "practical morality". Assuming that an individual always acts as a Kantian would seem to require too much for the morality of the typical individual and would probably be also ethically disputable. However, assuming that an individual would feel obliged to pay for the public good if everybody else in the group of contributors did the same is certainly a much less stringent requirement. And this is particularly true if one assumes, as Sugden does, that each individual feels obliged to provide, in each single group he belongs to, only the smallest contribution provided in that group.

The results that Sugden obtains in his model are indeed remarkable. In general, the model will be characterized by multiple Nash equilibria. All of them but one would entail a larger provision of the public good than selfishness. On the other hand, unless individuals are identical, each equilibrium will be characterized by undersupply of the public good. Thus, Sugden's model is consistent with the empirical evidence which shows a positive (even if smaller than predicted by traditional theory) amount of free-rider behaviour (see above). Second, his model predicts matching behaviour on the part of the individuals. Thus Sugden's model is not affected by most of the invariance properties of the Nash contribution model summarized in the previous sections.

Sugden's model thus constitutes a very important starting point for the elaboration of an alternative theory of voluntary provision of public goods. Despite this, several criticisms could be raised against his formalization of ethical behaviour. First, his interpretation of the Kantian rule, as Laffont's, does not take into account the differences in "ability to pay" of the different individuals. This is a serious weakness in a theory based on the hypothesis that individuals are ruled by notions of fairness. Second, his definition of group is also not convincing in the context of his analysis. In the real world the groups toward which an individual may have feelings of belonging or reciprocity are likely to be exogenous to the analysis and surely do not coincide with the notion of coalition in game theory. Third, his way of modeling reciprocity rules is also open to question. In his model, each individual is obliged to provide only the smallest contribution provided by everybody else in the group. But this would imply that if in a group to which individual i belongs, one individual provides zero while everybody else provides large positive quantities to the public good, individual i would feel obligated only to provide zero. This sounds strange; it looks more reasonable to assume that the contribution that each individual would feel obligated to offer is some function of the total contribution of everybody else in the group[75]. These and other related issues will be taken up in the following chapters.

Summing up, among the many alternatives that we discussed in this chapter, the abandonment of the assumption of individualistic maximizing behaviour in favor of a notion of ethical behaviour seems to be the one which offers the most promising results in terms of both intuition and empirical evidence. Most work remains however to be done in this exciting but largely unexplored area. In the next two chapters we will attempt to offer a contribution to this line of inquiry, building on the work by Laffont (1975) and Sugden (1984).

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[75] I have considered this issue elsewhere; see Bordignon (1987).

II.9 Conclusions

In this chapter we reviewed the basic Nash contribution model for private provision of public goods. It was shown that this model produces results which are largely at odds with the empirical and experimental evidence on voluntary provision of public goods. We then discussed in detail several attempts made in the literature to either abandon altogether or to modify substantially the traditional model. We concluded that the attempt of relaxing the assumption of selfishness in favour of a notion of ethical behaviour appears to be the most promising research strategy. In the next two chapters we will offer a contribution to this line of inquiry.

Chapter III

Was Kant Right? The Voluntary Provision of Public Goods under the Principle of Unconditional Commitment

Table III.1 List of Symbols used in Chapter III and Chapter IV

n = number of individuals in the economy indexed by i, $i = 1...n$;

Q = total quantity of public good;

y_i = private good consumed by individual i;

I_i = exogenous income of individual i;

q_i = individual i's contribution to the public good;

U^i = utility function for individual i;

$P^i = U^i_\alpha/U^i_y$ = marginal rate of substitution between private and public good for individual i;

Q^*_i/n = individual i's contribution under the FKR;

$y^*_i = I_i - Q^*_i/n$ = i's private good consumption under the FKR;

$Q^* = \Sigma\, Q^*_i/n$ = total contribution under the FKR;

$\bar{I} = \Sigma I_i/n$ = average income;

α_i = i's preference for the public good;

$\bar{\alpha} = \Sigma\alpha_i/n$ = average preference;

q^i_j = optimal contribution selected by individual i for individual j when i behaves according to the SKR;

$Q^i = \Sigma_j q^i_j$ = total public good supply selected by individual i when he behaves according to the SKR;

$y^i = I_i - q^i_i$ = optimal amount of private good consumption selected by individual i when he behaves according to the SKR;

$\hat{q}_i = \max(0, q^i_i)$ actual contribution of individual i under the SKR;

$\hat{y}_i = I_i - \hat{q}_i$ = actual consumption of the private good by individual i when he behaves according to the SKR;

$\hat{Q} = \Sigma\, \hat{q}_i$ = total contribution under the SKR;

Q^o = efficient level of public good supply.

Chapter III Was Kant Right?

III.1 Introduction

In the previous chapter we critically reviewed the recent literature on private provision of public goods. We ended up suggesting that abandoning the hypothesis of selfish behaviour in favour of a notion of ethical behaviour may offer an important starting point for a more adequate formulation of the theory. In this and in the next chapter we will attempt to offer a contribution to this line of research.

In this chapter, in particular, we address a question which, surprisingly enough, has never been addressed directly in the literature. Suppose that people do behave in a context of voluntary provision of a public good according to some notion of ethical behaviour: would then the public good be provided at an efficient level? And, if this is not the case, would it tend to be over or under provided? And how would this compare with the traditional theories put forward to explain public good provision such as Cournot-Nash behaviour, Median voter theory, and Lindahl equilibrium ?

As the results could differ under different ethical rules we preferred to reformulate the question as follows. Suppose that among the set of all the possible rules of morality that individuals could follow in providing a public good we consider only the subset of "realistic" rules of behaviour, where realism is referred to the amount of information that an individual could reasonably possess. Suppose further that among the last subset we select the rule which most strongly constrains behaviour in a group-concerned way: is this rule able to guarantee an efficient provision of the public good?

The main result which emerges from the analytic work developed in this chapter is that private provision of the public good is in general inefficient even if individuals are constrained by such a strong ethical rule as Kantian behaviour. Only if individual preferences have some very peculiar structure and/or if income is equally distributed across the population is Kantian behaviour efficient. This result casts serious doubts on the possibility of private provision of public goods to be efficient without public intervention. However, market failures are not a sufficient condition for public provision unless it is shown that governments can do better. The next chapter addresses this last issue more in depth comparing Kantian behaviour with alternative models of public good supply.

The present chapter is organized as follows. In section III.2, we present and try to justify our choice of Kantian behaviour as one of the strongest rules of fairness individuals could reasonably follow in providing a public good. In section III.3, we offer two different interpretations of the Kantian rule, according to whether such a rule is interpreted in a "absolute" or in a "relative" sense. In section III.4, the likelihood of the first Kantian rule to produce the public good efficiently is studied in the general case and by means of particular parameterizations of individual preferences. In section III.5, we perform the same analysis for the second Kantian rule and we also investigate the links between the performance of the two rules. A number of interesting results are obtained which are summarized and discussed further in section III.6. Finally, in the Appendix we compare Kantian and selfish contribution by an individual by means of a simple parameterization.

III.2 The Principle of Unconditional Commitment

As stressed in the previous chapter, casual observation and empirical results suggest that people do not behave, in a context of voluntary provision of public goods (henceforth VPPG), according to the naive conception of rational egoism that economists employ. This observation, however, begs the question: if people do not behave according to selfishness how do they behave? In the final section of the previous chapter, commenting on Sugden (1984), we suggested that a notion of Kantian behaviour, weakened by reciprocity rules, may represent an useful approximation to individual behaviour. In this chapter, however, we want to restrict our attention to a rule of behaviour which is, on one hand, realistic enough to be actually followed by individuals, at least in some (admittedly extreme) circumstances, and, on the other hand, strong enough to be considered a benchmark of group oriented behaviour. We wish to suggest that the Kantian rule, or categorical imperative, has much to be said for it as such a rule.

In his "Fundamental Principles of the Methaphysic of Morals", Kant states the "supreme" principle of morality as follows: "Act as if the maxim of thy action were to become by thy will a Universal Law of Nature". In modern words, an act is morally good if and only if the agent who performs it could will its maxim to have the same necessity of a Law of Nature i.e. if the agent could will that everybody else in the same circumstances would do the same thing on the basis of the same principle.

For our aim and following the literature quoted in the last chapter, we will vulgarize this principle restating it as saying "Act always in such a way as you would <u>like</u> everybody else to do in the same circumstances". This is a vulgarization, because what an agent "could will" in Kant is just what the "rational man could will", while in our (and others') formulation an agent would always act as he would <u>prefer</u> everybody else acted in the same circumstances. While this formulation is open to Sen's criticism on the distinction between choices and preferences (Sen, 1973), we believe that it is closer to actual behaviour than the original Kantian rule would be, especially when one consider the difficulty of assessing what the "rational man could will". Furthermore, this vulgarization has the additional advantage in limiting the level of information needed to implement the Kantian rule for each individual to the knowledge of his own preferences (see below).

Thus, in our formulation, the Kantian rule is a very simple and general rule of behaviour, so much that it can be found, in slightly different forms, in many different ethical systems[1]. But it is also a very strong rule and it is hard to believe that people actually follow it in day-by-day life. However, as suggested in chapter I, in the context of VPPG, where the interaction among agents is public information, people may at least <u>think</u> according to it --in the sense that this is how they feel they <u>ought to</u> behave-- and they may also <u>act</u> according to it, if they perceive that everybody else is doing likewise[2]. In what follows we will instead assume that people <u>always</u> act according to the Kantian principle, <u>regardless</u> what everybody else is doing. In this sense, the Kantian principle imposes more constraint on individual behaviour than other notions of "fairness" with a more distinct sociological flavour (such as for example concepts of fairness based on notions of "reciprocity" or "reputation"; see the references quoted in the previous chapter) in so much as its validity is independent (unconditional) of other behaviour. This is unlikely to be realistic but it strengthens our previous point that Kantian behaviour represents a benchmark of group oriented behaviour. This is also confirmed by the fact that, as is well known in the literature (see Collard, 1978), and as we

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1 Consider, for example, the Bible: "All things whatsoever ye would that men should do to you, do ye even so to them: for this is the law and the prophets"; Matthew,7:12 and the more prudential Confucius:"What you do not want done to yourself, do not do to others" <u>Anaclets</u> .

2 This is indeed the interpretation suggested by Sugden (1984). See previous chapter.

will also show in the next section, in the interpretation given above Kantian behaviour is <u>formally</u>, if not logically, equivalent to an extreme form of altruism.

Thus, the Kantian rule seems to have the characteristics that we indicated as desirable for our exercise. It is simple and realistic enough to be considered as actually influencing behaviour, at least in some circumstances, and it is strong enough to subsume other conceivable rules of fairness and altruism. Therefore, we feel authorized to perform our exercise on its basis.

The problem is how to translate it into a norm of action for individuals sharing the benefits of a public good. Clearly, everything depends on how we interpret the word "action" in our previous statement of the rule. If we take it as meaning the "absolute" level of contribution, the Kantian rule simply becomes "provide as much as you would like everybody else to provide to the public good". This interpretation, relatively straightforward, is indeed the one chosen by Laffont (1975) in his seminal contribution on Kantian behaviour. However, as we argued in chapter II, this interpretation overlooks the differences in the ability to pay of the different individuals. It seems rather unlikely to suppose that in performing the mental exercise that he needs in order to compute his Kantian contribution, an individual would not take in account his relative position in the income distribution. Thus, a second interpretation of the word "action" could be in terms of "the <u>relative</u> amount of contribution" and the rule could become "provide always as much as you would like everybody else to do relative to their income capabilities".

Note, however, that an analysis of the second interpretation of the rule does not rule out the interest of analyzing the first rule as well, since there may be cases where income differences are either not relevant or are not considered as such by individuals in determining their level of contribution. Examples of the latter case may be in terms of voluntary provision of time, or blood, in contrast with money, to charities (see Titmuss, 1971).

III.3 A formal approach to the principle of unconditional commitment: first and second Kantian rule

In order to formalize our Kantian rules we have first to describe the economy where we wish to carry on our analysis. For simplicity, we will work with the simplified economy which is customary to analyze in the public good literature and that we already discussed in the previous chapter. This will also allow us to compare our results with the traditional results presented in the literature.

Thus, consider an economy with only two goods, y_i, a private good, where i indicates the individual who consumes it, and a (pure) public good, Q. Let n be the number of individuals composing the society, where $i=1...n$. Again only for simplicity suppose further that the public good is only privately supplied and let the production function of the public good be linear and additive in individual contributions:

$$(1) \qquad Q = \Sigma q_i \qquad\qquad i = 1....n$$

where q_i is the contribution of individual i. We impose the obvious constraint that $q_i \geq 0$, $\forall i$. Also, let the preferences of individual i be represented by a strictly quasi-concave and three times differentiable utility function, U^i.

$$(2) \qquad U^i = U^i(y_i, Q)$$

and let the budget constraint for each individual i be given by

$$(3) \qquad I_i = y_i + q_i$$

where I_i, the income, is exogenously given and where the unit of measurement of the individual contribution is selected so as to make the (constant) marginal rate of transformation between the two goods equal to unity. We are then ready to introduce our two Kantian rules.

III.3.1 First Kantian Rule

The first Kantian rule (henceforth FKR) is easily formalized once it is recognized that the statement "provide the absolute amount of contribution that you would like everybody else to provide" is equivalent to the statement "select an amount of contribution on the basis of the hypothesis that everybody else will provide what you decide to provide". In the context of our model this would imply that individual i solves his optimization problem by assuming that $Q=nq_i$. Substituting the latter expression in (3), substituting for the budget constraint in the utility function, and indicating with Q_i the <u>total</u> amount of the public good that individual i would like everybody to enjoy, i's optimization problem can be written as

$$(4) \qquad \max_{Q_i} \; U^i(I_i - 1/nQ_i \, , \; Q_i)$$

Equation (4) then shows that under the FKR each individual behaves as if he faced an implicit price for the public good equal to $1/n$. This is so because when acting as a Kantian each individual acts as if he expected everybody else to match completely his contribution. As a consequence the amount of private good that each individual must give up for an extra unit of the public good is simply $1/n$.

The assumption of strict quasi-concavity of the utility function entails that Q_i^*, the optimal choice of Q_i in (4), is unique. The principle of unconditional commitment then imposes upon individual i the obligation to supply Q_i^*/n, <u>regardless</u> of what everybody else is doing. If everybody acts according to the

FKR, total provision of the public good is then $Q^* = \Sigma Q_i^*/n$. For future reference (see chapter IV) note that the FKR introduces a wedge between the price used by individuals to reach their decision and the price that they actually pay for the public good. While ex-ante each individual selects his contribution as if he faced a price of $1/n$, ex-post the price that he pays is $Q_i^*/\Sigma Q_i^*$.

III.3.2 Second Kantian Rule

According to the second interpretation of the principle of unconditional commitment, individual i selects his contribution on the basis of the hypothesis that everybody else provides what i would like them to provide, taking into account the differences in income across individuals. The most natural way to formalize this idea is by assuming that individual i maximizes a function of utility functions by selecting a different amount of contribution for each individual, according to the income endowment of the latter. The fact that individual i assumes that everybody else provides what i himself would like them to provide can be formalized by supposing that individual i lends his preferences to everybody else: that is, by assuming that in performing his optimizing exercise i acts as if everybody else had i's own preferences but their true income. This also answers our requirement of informational realistic rules of behaviour. In fact, while income differences can be observed or more or less correctly inferred, the preferences of the other individuals are in general unknown and it is hard to see how our individual, however ethically motivated, could come to get this information. In the following we assume instead that income distribution is costlessly and correctly observable by each individual composing the society.

Let us therefore model individual i's behaviour under the second Kantian rule (henceforth SKR), as if he maximized a sort of personal "welfare function" where each individual has i's own preferences but their true income; selecting the utilitarian formulation for simplicity[3], individual i solves

$$(5) \qquad \max_{q_1 \ldots q_n} \quad \Sigma_j \; U^i(I_j - q_j \; , \; \Sigma q_j)$$

Let us call q_j^i the amount of contribution by individual j which solves (5), where j = 1...n.

Note that we have not allowed in (5) individual i to consider the non-negativity constraint imposed on each individual contribution. This is so because in (5) individual i is involved in an ethical choice and it seems natural to allow him the maximum freedom (i.e. the possibility of income transfers across individuals) in deciding what ought to be a fair contribution from

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[3]As is easy to check any other welfare function would produce the same result as long as the social weight imposed on each individual were the same. This last condition is clearly implicit in the symmetry of the principle of the categorical imperative. Also note that, as we argued above, in the formalization of eq.5 Kantian behaviour coincides with an extreme form of paternalistic altruism, where each individual treats everybody else as he treats himself.

each individual. As a result it may happen that $q_j^i \leq 0$, for some j (see below). However, even if in solving (5) i selects a contribution for each individual composing the community, it should be recalled that this is just a mental exercise used by i to compute his Kantian contribution, q_i^i, and the latter is the only one which is relevant for our discourse. Thus we may have a problem only if q_i^i is negative. Since individual i has no way, in a market economy, to enforce a transfer in his favor we assume that in this case his actual contribution to the public good would be simply zero. We can then write the actual contribution of individual i to the public good under the SKR, let us call it $\hat{q}_i$, as $\hat{q}_i = \max(q_i^i, 0)$. Clearly, if everybody behaves according to the second Kantian rule, total public good supply will be $\hat{Q} = \Sigma \hat{q}_i$.

Since our rationale for considering two different Kantian rules is only based on the existence of income differences across individuals it would be very convenient if the two rules coincided in absence of income differences. This is indeed the case.

Result 1 "Let $I_i = I$, $\forall$ i . Then $q_i^i = Q_i^*/n$ $\forall$ i ."

Proof. See below.

Result 1 then shows that the SKR is really a generalization of the FKR. If income were equally distributed, each individual would provide exactly the same amount under both the rules.

III.4 Voluntary provision of public goods under the principle of unconditional commitment

Suppose now that in an economy like the one described above all individuals acted according to the first or alternatively the second Kantian rule. Would public good provision be efficient? Let us consider the two rules in turn. Commence with the first rule.

III.4.1 FKR and public good provision

According to the FKR, each individual i selects Q_i on the basis of the following formula:

$$(5) \qquad P^i(I_i - Q_i^*/n \, , \, Q_i^*) = 1/n$$

where $P^i = U^i_Q/U^i_y$ is the marginal rate of substitution. Assessment of efficiency or inefficiency of public good provision requires one to evaluate the summation of the marginal rates of substitution across all individuals, each evaluated at $(I - Q_i^*/n \, ; \, Q^*)$ for each i, and compare it with the marginal rate of transformation between the two goods. Calling the Pareto efficient level of public good provision Q^o and using (5) we can then write the condition for efficiency as follows :

$$(6) \qquad Q^o = Q^* \text{ iff } \Sigma P^i(I_i - Q_i^*/n \, , \, Q_i^*) = \Sigma P^i(I_i - Q_i^*/n \, , \, Q^*)$$

There are two trivial cases where (6) must necessarily hold. The first is when each individual is characterized by the same income and preferences. In this case, as it is easy to verify $Q_i^* = Q^*$ $\forall i$ and the RHS and the LHS of (6) coincide for each element of the summation. Of course, this case is of no interest because in general individuals differ both in terms of income and preferences. The second case where (6) must necessarily hold is when each individual's demand for the private good is characterized by zero income elasticity. In this case in fact, each marginal rate of substitution is independent of the amount of public good supplied and again the RHS and the LHS of (6) coincide for each element in the summation. Clearly, the empirical relevance of this second case depends on the interpretation that we give to the private good in our model. If we attribute it the natural interpretation of representing private consumption, the case of zero income elasticity for the private good is clearly very unrealistic (see chapter II).

Beyond the special cases above it is clearly unlikely that condition (6) holds for unspecified utility functions. This fact alone shows that Kantian behaviour will in general be inefficient. However, we would like to be able to say something more about Kantian provision. In particular, we would like to know if Kantian behaviour will tend to bring about over or under provision of the public good and to identify the conditions under which this will occur. This would require us to be able to infer something about over or under provision of the public good when the summation of the marginal rates of substitution differs from the marginal rate of transformation. As we noted in chapter II, strictly speaking, this is not generally possible because Q^{o} itself is in general a function of the income distribution across individuals. Therefore if we move from Q^{*}, Q^{o} will also change in a direction which will depend on how we share the extra burden of the public good across individuals.

There are however special senses in which we can dare express assessments on optimal quantities by examination of the optimality conditions. In the next chapter (see section 3) we offer a novel way to infer over/under provision of the public good by examination of the sum of the marginal rates of substitution. In this chapter we content ourselves with noting the following. Suppose that each $P^{i}(.)$ is characterized by the propriety that a small increase in the contribution to the public good by any individual --including individual i-- would make each $P^{i}(.)$ fall. Then if $\Sigma P^{i}(I_{i}- Q_{i}^{*}/n,Q^{*})>1$ (<1) and if we increase (decrease) slightly the contribution of each individual the sum of the marginal rates of substitution will get closer to the marginal rate of transformation thus reducing the extent of inefficiency. Indeed, if each marginal rate of substitution is everywhere characterized by the propriety above and we keep increasing (decreasing) the contribution of each individual we must eventually end up with an efficient level of public good supply, Q^{o}, where $Q^{o} > Q^{*}$ ($Q^{o}<Q^{*}$). Therefore, if each $P^{i}(.)$ has the property discussed above, we can talk of Q^{*} as over provided if

$\Sigma P^i(I_i - Q_i^*/n, Q^*)<1$ and of Q^* as under provided if $\Sigma P^i(I_i - Q_i^*/n, Q^*)>1$[4].

As can be easily verified by considering a small change in the contribution of an individual i, a sufficient but not necessary condition for $P^i(.)$ to be decreasing in q_i is that $P^i_a<0$ and $P^i_y>0$, $\forall i$. This condition is strictly tied up with the income derivative of the demands for the two goods; in particular, if $P^i_y>0$ everywhere the demand for the public good by individual i will be everywhere an increasing function of income and similarly, if $P^i_a<0$ the demand for the private good will be everywhere an increasing function of income. Thus our condition of decreasing marginal rates of substitution really requires only income normality of both goods and for the reasons stressed in the chapter II this is quite a mild assumption to impose. Unless otherwise stated, we will then always assume in what follows that both goods are normal goods.

Let us now enquire into the issue of over/under provision of the public good under the FKR. We begin with a very general case and subsequently we will consider some simple parameterizations of the utility functions.

III.4.2: a general case

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[4]It must be noted however that we are here talking of over or under provision only in a very special sense. In particular if we actually performed such an increase the change would not be in general Pareto improving: some individuals would be made worse off as a consequence of the change.

In this section we will address the problem of efficiency/inefficiency of Kantian provision in a very general setting. This will allow us to identify the factors which are likely to be important in determining the direction and the extent of inefficiency in public good provision under the FKR. In later sections we will provide some economic intuition for the results obtained in this section. To this purpose suppose that each individual utility function can be written as

(7) $U^i(y_i, Q) = U(\alpha_i, y_i, Q)$

where α_i is a parameter which captures the difference in preferences for the public good across the individuals. We assume that the parameter α is distributed continuously and that the utility function is differentiable at any required order in α. As in the previous section, income is given exogenously and is allowed to vary across individuals. In general terms, we can then write the marginal rate of substitution as a function of the three parameters α, I, Q :

(8) $P = P(\alpha, I, Q)$

We assume that $P_\alpha > 0$ -- i.e. that the larger is α the stronger are the preferences for the public good-- and following our previous discussion that $P_\alpha < 0$. Note that if we keep the total supply of the public good constant a small increase in income is equivalent to a small increase in the private good. Thus, by assuming normality of the public good we can also sign the last partial derivative in (8), as $P_I>0$.

If each individual i behaves according to the FKR he will select his optimal contribution Q_i^*/n by equalizing P^i to $1/n$. It must therefore be identically true that

$$(9) \quad P(\alpha_i, I_i - 1/nQ(I_i,\alpha_i),\ Q(I_i,\alpha_i)) \equiv 1/n$$

where $Q_i^*=Q(I_i,\alpha_i)$. Consider now the individual characterized by average income $\bar{I}$ and average preference $\bar{\alpha}$ and let $\bar{Q}$ represent his optimal choice. Using (9) and (6) we can then state : $Q^o \gtrless Q^*$ as $X \gtrless 0$ where

$$(10) \quad X = \Sigma P(\alpha_i, I_i, Q^*) - nP(\bar{\alpha},\bar{I},\bar{Q})$$

Suppose now that the differences $(\alpha_i-\bar{\alpha})$ and $(I_i-\bar{I})$ are for each i sufficiently small. We can then approximate the differences $P(\alpha_i,I_i,Q^*)-P(\bar{\alpha},\bar{I},\bar{Q})$ as a second order Taylor expansion around the point $(\bar{\alpha},\bar{I},\bar{Q})$. Doing this and using the fact that $\Sigma(\alpha_i-\bar{\alpha})=\Sigma(I_i-\bar{I})=0$ we can rewrite (10) as

$$(11) \quad X \cong nP_\alpha(Q^*-\bar{Q})+1/2P_{\alpha\alpha}\Sigma(\alpha_i-\bar{\alpha})^2+1/2P_{II}\Sigma(I_i-\bar{I})^2+1/2P_{\alpha\alpha}(Q^*-\bar{Q})^2+ P_{I\alpha}\Sigma(\alpha_i-\bar{\alpha})(I_i-\bar{I})$$

Equation (11) then indicates --to the second order of approximation-- the factors which are likely to matter in determining over/under provision of the public good. The sign of the first term in the approximation depends on the concavity/convexity of the demand for the public good in income and preferences which in turn depends on the concavity/convexity of the marginal rate of substitution in the three parameters above (see below). The second term in the expansion captures both the concavity/convexity of the marginal rate of substitution and the variance and the covariance of the distribution of income and preferences across the population. For example, if richer people have also stronger preferences for the public good the term $\Sigma(\alpha_i-\bar{\alpha})(I_i-\bar{I})$ will be positive; negative in the opposite case. Some important cases can be obtained as special cases of (11). For example, the case of identical preferences can be easily obtained by imposing $(\alpha_i-\bar{\alpha})=0$ in (11).

However, (11) also shows that very little can be said on public good provision under the FKR in the general case. Some extra structure on preferences must be imposed if we want to derive interesting results. This is what we are going to do in the next sections. For future reference let us however state:

<u>Result 2</u> "At the first order of approximation the public good under the FKR will be under provided (over provided) if P(.) is concave (convex) in (α,I,Q)"

<u>Proof</u> "Suppose P is concave in (α, I, Q): then

$P(\bar{\alpha},\bar{I},Q^*) \geq 1/n\Sigma P(\alpha_i, I_i, Q_i^*) = P(\bar{\alpha},\bar{I},\bar{Q})$

which implies by normality of the private good $Q^* \leq \bar{Q}$. QED"

III.4.3 Additive Utility Function

In this section we assume that each individual utility function has the form:

$$(12) \qquad U^i = f^i(y_i) + \alpha_i \frac{Q_i^{1-\beta}}{1-\beta}$$

where I only impose the conditions that $f_y^i > 0$ and $f_{yy}^i < 0$, $\forall i$ and $\beta > 0$. These conditions guarantee normality of both goods for each individual. The parameterization in (12) implies that each individual has the <u>same</u> constant elasticity of marginal utility of the public good. However, note that the presence of a parameter α and the possibility allowed to the function $f()$ to vary across individuals, allows quite a large amount of differences in individual preferences.

Computing the first order conditions for choice of Q_i^* by an individual i we get

$$(13) \qquad \frac{\alpha_i \, Q_i^{*-\beta}}{f_y^i(y_i^*)} = \frac{1}{n} \qquad \forall i$$

where $y_i^* = I_i - 1/nQ_i^*$. We are interested in the value of

$$(14) \qquad \Sigma P^{i}(y_i^*, Q^*) = \Sigma \frac{\alpha_i \; Q^{*-\beta}}{f_y^i(y_i^*)}$$

Solving (13) for $f_y^i(y_i^*)$, substituting in (14) and rearranging we get

$$(15) \qquad \Sigma P^{i}(y_i^*, Q^*) \gtrless 1 \quad \text{as} \quad 1/n \; \Sigma(Q_i^{*\beta}) \gtrless (Q^*)^{\beta}$$

Thus $Q^o > Q^*$ if $\beta > 1$ and $Q^o < Q^*$ if $\beta < 1$. Usual estimations for the parameter β (for private goods) gives values between 1 and 2 (see Stern, 1973). If these results could be extended to public goods they would then imply a clear tendency toward under provision of the public good.

III.4.4 Identical and Homothetic Preferences

Assume that each individual i is characterized by identical and homothetic preferences. Then it has to be true that for each i

$$(16) \qquad \frac{I_i - 1/nQ_i^*}{Q_i^*} = k$$

i.e. the ratio of the two goods selected by each individual must be equal to the same constant k. Hence substituting for the budget constraint and solving for $1/nQ_i^*$

9 "with" from before $\bar{Q}$

$$(17) \qquad \frac{Q_i^*}{n} = \frac{I_i}{nk + 1}$$

Summing over the n individuals

$$(18) \qquad Q^* = \frac{\Sigma I_i}{nk + 1}$$

Now, consider the individual with the <u>average</u> income and call with $\bar{Q}$ the <u>total</u> amount of public good provision selected by this individual according to the FKR. From (16)

$$(19) \qquad \bar{Q} = \frac{n}{nk + 1} \; \frac{\Sigma I_i}{n} = \frac{\Sigma I_i}{nk + 1} = Q^*$$

i.e. the average individual is <u>decisive</u> in the sense that the amount of public good that he would choose for the collectivity is the amount which is actually supplied by the collectivity. For future reference we can then state

<u>Result 3</u> "Suppose that each individual is characterized by the same homothetic preferences and that everybody acts according to the FKR. Then the total amount of public good supplied equals the amount of public good selected by the individual endowed with average income"

Using (19) and the budget constraint for each individual, we can write the first order condition for the average individual as

$$(20) \qquad P\left(\frac{1}{n}\Sigma\left(I_i - \frac{1}{n}Q_i^*\right), \bar{Q}\right) = \frac{1}{n}$$

Efficiency in public good provision would then require

$$(21) \qquad P\left(\frac{1}{n}\Sigma\left(I_i - \frac{1}{n}Q_i^*\right), \bar{Q}\right) = \frac{1}{n}\Sigma P\left(I_i - \frac{1}{n}Q_i^*, \bar{Q}\right)$$

i.e. if the marginal rate of substitution of the average individual, evaluated at $\bar{Q}$, is equal to the average of the marginal rates of substitution, evaluated at $\bar{Q}$, public good provision under the FKR is efficient. As the sharp reader will have surely recognized, this is just a special case of Bowen's theorem where the "average voter" takes the place of the "median voter" (see Bergstrom, 1979). In the next chapter we will discuss in details the links between Kantian provision and the median voter theorem. For the moment note that from (21) we can write

$$(22) \quad \bar{Q} \gtrless Q^o \quad \text{as } P\left(\frac{1}{n}\Sigma y_i^*, \bar{Q}\right) \gtrless \Sigma P(y_i^*, \bar{Q})/n$$

where as above $y_i^* = (I_i - \frac{1}{n}Q_i^*)$. Then if the marginal rate of substitution is concave/convex in the private good, Kantian behaviour will be characterized by over/under provision of the public good. As is easy to check this is exactly the result that we should have expected from (11), evaluating it at $Q^* = \bar{Q}$ and at

$\alpha_i = \bar{\alpha}$. However, there are important differences between (11) and (22). First, (11) is only valid for very small differences in income distribution while (22) holds everywhere. Second, differently from (11), (22) does not require the marginal rate of substitution to be differentiable. Third, and more importantly, (22) allows us an easy interpretation of the economic conditions lying behind over/under provision of the public good.

In fact, if we assume a constant elasticity of substitution between the two goods, the marginal rate of substitution, for some admissible transformation of the utility function, must take the form $(Q/y)^{p-1}$, where p is the parameter in the utility function. It is then clear that $\bar{Q} \gtrless Q^{o}$ as $p \gtrless 0$ or equivalently $\bar{Q} \gtrless Q^{o}$ as $\varepsilon \gtrless 1$, where ε indicates the elasticity of substitution between the two goods. This result would also suggest, as the previous one, that the public good would tend to be under provided under the FKR. This is so because we would expect quite a low elasticity of substitution between the public good and private goods taken as a whole, since they are typically quite different types of goods.

This concludes our analysis of the FKR. Summarizing we saw that Kantian behaviour will in general tend to produce the public good at an inefficient level, at least as long as both goods are normal goods. It is unfortunately difficult to obtain general results on the direction of the inefficiency but the examples that we considered seem to suggest that the public good will tend to be under provided under the FKR. Let us then move to the SKR.

III.5 Public Good Provision under the Second Kantian Rule

Suppose that in an economy as the one described above each individual behaves according to the second Kantian rule. Would then the public good be provided at an efficient level? The key result needed to answer this question is summarized in the next theorem.

Result 4. "Let $Q^i = \sum_j q_j^i$ represent the total amount of public good that individual i would select for the community if he acted according to the SKR. Then:

4.a $\quad Q^i = (Q : \max_Q U^i(\bar{I} - Q/n, Q))$

4.b $\quad q_j^i = Q^i/n + (I_j - \bar{I})$ "

Proof "Consider the first order conditions for the solution of problem (5). Written in full and evaluated at the optimal choices,

$$(23) \quad \begin{array}{l} \sum_j U_Q^i(Q^i, I_j - q_j^i) = U_y^i(Q^i, I_1 - q_1^i) \\ \vdots \\ \sum_j U_Q^i(Q^i, I_j - q_j^i) = U_y^i(Q^i, I_n - q_n^i) \end{array}$$

Since the LHS in (23) is the same for each of the n rows it follows that

$$U_y^i(Q^i, I_s - q_s^i) = U_y^i(Q^i, I_t - q_t^i) \quad \forall\ s,t \in n, \text{ which entails}$$

$$(I_s - q_s^i) = (I_t - q_t^i) = y^i \qquad \forall s,t \in n$$

where y^i represents the common level of the private good selected by individual i for each member of the community. Using (1) and (3),

(24) $Q^i = \Sigma I_i - n y^i$

dividing (24) by n and rearranging

(25) $\bar{I} = y^i + 1/nQ^i$

Selecting the i^{th} row in (23) and evaluating it at the optimal choices

$$nU_Q^i(Q^i,\ \bar{I} - 1/nQ^i) = U_y^i(Q^i,\ \bar{I} - 1/nQ^i)$$

which are the first order conditions for the problem

$$\max_Q U^i(\bar{I} - 1/nQ,\ Q)$$

This proves 4.a. Now note that each individual contribution must be such that

(26) $y^i = I_j - q^i_j \quad \forall j,\ j=1\ldots n$

Substituting (26) in (25)and solving for q^i_j

$$q^i_j = 1/nQ^i + (I_j - \bar{I})."\text{QED}.$$

Result (4) then shows that Kantian behaviour under the second Kantian rule can really be thought as a two stage maximizing process. At the first stage each individual i selects an optimal level of public good supply and a single level of private good consumption as if he were endowed with average income and as if he were facing a price for the public good equal to 1/n. In the second step individual i selects a different contribution for each individual, including himself, so as to equalize the consumption of the private good across the individuals.

The extreme form of "egalitarism" implied by the SKR may seem excessive even for an ethical rule of behaviour. Few of us, I suppose, however ethically motivated, would consider as "just" a resource distribution perfectly egalitarian across individuals. It should however be realized that we are here considering a very special type of economy, a timeless economy where income is not produced but is, so to say, obtained as a gift from heaven. Without of course willing to defend the realism of the Kantian rules, it is my opinion that in such an economy a perfect egalitarian distribution of resources has some claim for being considered as the just distribution.

From a normative viewpoint, the only serious objections to income equalization derive from those conceptions of distributive justice which insist on the fact that income is produced by individuals through effort and pain and that an individual should have a special right to command power over those resources that he himself has contributed to creating. Similarly, in other conceptions, such as the procedural approach to justice (see Nozick, 1973), the ethical claim to complete resource equalization is rejected because it does not take in account the fact that the present income distribution is the result of an historical process where resources have been exchanged and transferred among individuals following commonly agreed rules of property and exchange. This is not of course the place to discuss these concepts of distributive justice. I do however believe that similar arguments, more or less consciously formulated, lie at the bottom of the instinctive repulse that many people feel for a complete equalization of resources. This raises the issue of a convincing formulation of ethical rules of behaviour for more realistic economies (see next chapter) but it also strengthens our point that for the economy that we are considering in this paper our way of formalizing Kantian behaviour is quite adequate[5].

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[5] Also note that the SKR produces from this point of view results similar to the traditional Nash contribution model. As is shown in the previous chapter, individuals characterized by identical preferences and different incomes would contribute to the public good so as to equalize private consumption across individuals.

Result 4 allows us to clarify the links existing between the first and the second Kantian rule. For example, the economic rationale behind result 1 becomes immediately obvious. If income is equally distributed, each individual really faces the same problem when acting according to the first or the second rule and this explains the identity of choices in the two cases. However, when income is not equally distributed there are important differences between the two rules in terms of efficiency in public good provision as the next result illustrates.

<u>Result 5</u> "Suppose that each individual is characterized by the same preferences. Then if $q_i^i \geq 0$ $\forall i$, public good provision under the SKR is efficient. If $q_i^i < 0$ some i, public good under the SKR is over provided."

<u>Proof</u> "If everybody has the same preferences the problem in (23) is the same for each individual. This entails $q_i^i = q_i^j$ $\forall i,j$ which also implies $Q^i = Q^j$ and $y^i = y^j$ $\forall i,j$. Then if $q_i^i \geq 0$ $\forall i$, $\hat{q}_i = \max(0, q_i^i) = q_i^i$ which entails $\hat{Q} = \Sigma\hat{q}_i = Q^i$ and $\hat{y}_i = I_i - \hat{q}_i = y^i = \hat{y}$ $\forall i$. Selecting any row in (23) and evaluating it at the optimal choices $(\hat{Q}, \hat{y})$:

$$\Sigma P(\hat{Q}, \hat{y}) = nP(\hat{Q}, \hat{y}) = 1$$

and public good provision is efficient. If $q_i^i < 0$ some i, $\hat{q}_i = 0$; i.e. individual i will provide more than it would be optimal for him to provide while everybody else will provide exactly the optimal quantity; then public good is over provided."QED

Thus, in contrast with the FKR, identity of preferences is sufficient to guarantee efficiency under the SKR if the non-negativity constraint on individual contributions is not binding. The result is in a way obvious. Under the SKR each individual behaves as if he had average income and, if the non-negativity constraint does not bind, with identical preferences we really have a community of identical individuals each of whom faces a price of $1/n$ for the public good. The efficiency result is then immediate.

The crucial condition is of course that the non-negativity constraint must be non-binding for each individual. The likelihood of this condition to occur depends mainly on the extent of the inequality in income distribution across individuals. If such an inequality is very small we should expect it to hold while it will surely not hold if this inequality is large. It would be interesting to enquire into the size of inequality in income distribution which is just compatible with a $q_i^i \geq 0$ for each i. But it is also clear that we should not expect general results on this ground since q_i^i is also a function of the preferences for the public good. In order to get an intuition about the size of the inequality in income distribution which is compatible with a $q_i^i \geq 0$ for each i we have then preferred to address this question by means of a simple parameterization of the utility function. The results are presented in the appendix to this chapter. Anticipating the latter here, our simple parameterization suggests that for realistic values in income inequality the poorest members in the society would surely not pay voluntarily for the public good. This is in a way interesting because it shows that the decision not to pay for the public good may well be the result of an ethical choice rather than the result of mere selfish preferences. Indeed, as we also show in the appendix, a poor individual who acted on the basis of the SKR might well decide to provide <u>less</u> towards the public good than he would do if he acted on the basis of mere selfishness. This simple result, that I believe to be interesting in its own, is at the basis of the fairness approach to income tax evasion which is presented in the second part of the present work.

Let us consider result 5 again. It seems to suggest a better performance of the SKR in efficiency terms than the FKR, exactly because the former introduces a redistribution of the burden of the public good across individuals in proportion to their ability to pay. This is clearly illustrated by the next result.

<u>Result 6</u> "Suppose each individual is characterized by the same homothetic preferences and suppose that $q_i^i \geq 0$. Then the total provision of the public good under the two rules is the same"

<u>Proof</u> "Let q^a be the optimal provision for the individual with average income under the SKR and let $\bar{Q}/n$ be the optimal provision for the same individual under the FKR. Then by result 1, $q^a = \bar{Q}/n$. By result 3, $Q^* = \bar{Q}$. By result 5, identity of preferences and $q_i^i \geq 0$

$\forall i$ implies $Q^i=\hat{Q}$ each i. Then for the individual with average income $q^a=\hat{Q}/n + (\bar{I}-\bar{I})=\hat{Q}/n$. It then follows $Q^*=\bar{Q}=nq^a=\hat{Q}=\Sigma q^i_i$."QED

Then, if individuals are characterized by the same homothetic preferences they would provide the same total amount of public good under both rules. Yet, as we saw above, public good provision is in general inefficient under the FKR, even if individuals have the same homothetic indifference map, while it is always efficient under the SKR if preferences are identical and $q^i_i \geq 0$. We are therefore compelled to conclude that the same amount of public good, supplied by the same community, is efficient under one rule and inefficient under the other! This (apparent) paradox is easily solved once it is realized that the efficient level of public good provision is not independent of income distribution. What the SKR does, in the presence of identical and homothetic preferences, is to redistribute income across the population (trough the equality of private good consumption) so as to make the chosen level of public good efficient. In some loose sense, we could therefore say that, in the case of the FKR, it is not the provision of the public good which is inefficient but is the distribution of income which is non optimal.

So far we have only considered the case where preferences are identical and we have seen that the SKR is undoubtedly better than the FKR, at least as long as the non-negativity constraint is not binding. How would the two rules fare when preferences are not identical?

Result 7 "Suppose that the structure of preferences is such that the public good is under/over provided under the FKR. Then providing that $q_i^i \geq 0$ $\forall i$ the public good is also under/over provided under the SKR"

Proof "We only prove the result for underprovision because exactly the same argument can be used for overprovision. For the FKR to be always characterized by under provision it must be true that for any income distribution,

(27) $P^i(y_i^*, Q_i^*) = \frac{1}{n}$ entails $\Sigma P^i(y_i^*, Q^*) > 1$

Consider now the SKR. Selecting the i^{th} row in (23) and evaluating it at the optimal choices,

(28) $P^i(y^i, Q^i) = \frac{1}{n}$

if $q_i^i \geq 0$ $\forall i$, $\hat{q}_i = q_i^i = \frac{Q^i}{n} + (I_i - \bar{I})$. It then follows $I_i - \hat{q}_i = y^i$ and $\hat{Q} = \Sigma \hat{q}_i = \Sigma Q^i/n$. Assessment of efficiency or inefficiency of public good provision under the SKR requires us to evaluate

(29) $\Sigma P^i(y^i, \Sigma Q^i/n) = x$

But from (27) and (28), $x > 1$.QED"

Again the rationale behind this result is obvious. If the structure of preferences is such that, for <u>any</u> income distribution, the FKR is characterized by under/over provision of the public good this must also hold if income is equally distributed. Now, if the non-negativity constraint is not binding, each individual in acting according to the SKR, behaves as if he had average income and he faced an implicit price for the public good equal to $1/n$; that is, he behaves as if he were subjected to the FKR but he had average income. The result then follows naturally.

An implication of result 7, as would be very easy to check, is that if we chose the same parameterizations of preferences for the SKR that we used for the FKR in the previous sections the same results in terms of over/under provision of the public good would emerge. Then we can conclude that the same tendency toward under provision that we saw emerging for the FKR would characterize the SKR as well. Of course, this conclusion must be qualified in the sense that it only holds if the non negativity constraint is not binding for each individual. If this is not the case, as we saw in result 5 , the fact that some individual with income below average may decide to supply zero to the public good under the SKR may offer a corrective to the tendency toward under provision.

Also note that result 7 holds even if public good provision under the FKR is efficient. That is, if Kantian behaviour under the FKR is efficient so must be Kantian behaviour under the SKR. This indeed shows that the SKR is in a precise sense more efficient than the FKR. In fact, while the SKR is always efficient when the FKR is efficient the opposite, as result 6 shows, does not hold.

This general remark on the greater efficiency of the SKR should not however be overstressed. Results 5 to 7 were obtained on the basis of the hypothesis that the non-negativity condition on each individual contribution was not binding. But as we remarked above, this is unlikely to hold unless income distribution is close to egalitarian. On the other hand, if the income distribution is close to egalitarian the comparative advantage of the SKR over the FKR is lost.

So far we have compared the two rules only in terms of their relative performance in approximating efficiency in public good provision. But it may also be interesting to ask about the difference in quantity provision under the two rules, both individually and at a social level. The next result summarizes what can be said.

<u>Result 8</u> "Suppose that the public good is everywhere a normal good for each individual. Then people with income below (above) average will certainly provide less (more) under the SKR than under the FKR"

<u>Proof</u> "From eq.4 and result 4 we can write

$$Q_i^* = \phi^i(I_i, 1/n) \quad \text{and} \quad Q^i = \phi^i(\bar{I}, 1/n)$$

where $\phi^i(.)$ is individual i's demand function for the public good. Then,

$$Q_i^*/n = \phi^i(I_i, 1/n)/n \quad \text{and} \quad \hat{q}_i = \max(0,\ \phi^i(\bar{I}, 1/n)/n + (I_i - \bar{I}))$$

Clearly if the non-negativity constraint is binding (which requires $\bar{I} > I_i$) $\hat{q}_i = 0$ and $Q_i^*/n > 0 = \hat{q}_i$. If the non-negativity constraint is not binding we must compare

$$(30) \qquad Q_i^*/n - \hat{q}_i = 1/n(\phi^i(I_i, 1/n) - \phi^i(\bar{I}, 1/n)) - (I_i - \bar{I})$$

Applying a first order Taylor approximation at the first term in the RHS of (30) and rearranging :

$$(31) \qquad Q_i^*/n - \hat{q}_i \cong (\phi_I^i 1/n - 1)(I_i - \bar{I})$$

where ϕ_I^i represent the partial income derivative of the demand function for the public good, evaluated at $(\bar{I}, 1/n)$. Normality of the public good requires $(\phi_I^i 1/n) < 1$ which implies $Q_i^*/n \gtrless \hat{q}_i$ as $I_i \gtrless \bar{I}$." QED.

Summing (31) over the individuals we can also get --at the first order of approximation and assuming that the non-negativity constraint is not binding for each individual-- the difference in total provision under the two rules as follows:

(32) $Q^{*} - \hat{Q} \cong \Sigma\phi_{I}^{i} 1/n(I_i - \bar{I})$

Quite intuitively (32) implies that total public good provision under the FKR will tend to be larger (smaller) than total provision under the SKR if poor people are characterized by lower (higher) preferences for the public good.

III.6 Conclusions

In this chapter we addressed the question: is ethical behaviour able to enforce an efficient provision of a public good? The answer is in general negative. Even if all individuals were subjected to as extreme a form of morality as the categorical imperative they would not in general supply the public good efficiently. Furthermore, the examples that we considered in the paper would also suggest that both the Kantian rules tend toward an under provision of the public good-- even if such a tendency is partially counteracted for the SKR by the presence of a non-negative constraint on individual contributions. Thus, it would seem a reasonable conjecture to suppose that under some more realistic behavioural rules --rules that so to say lie in between the extreme form of morality embodied in the categorical imperative and the pure selfishness of the traditional model-- the public good would tend to be seriously under provided. This of course opens the way to state provision of the public good as supplementing private provision.

This conclusion is however open to many caveats. First, we have not modeled explicitly state provision of the public good in our model. How a community of Kantian individuals would or should react to increased government provision is an open issue. This is of course even more true if we considered some more realistic rule of behaviour, where we could not escape the issue --that we avoided in this paper by invoking the principle of the unconditional commitment implicit in the categorical imperative-- of the interdependence among individual behaviour.

Second, there is a subtle issue which has not been considered in this paper. In all our discussion so far we have implicitly assumed that efficiency in public good provision is a desirable outcome. On the other hand, even if efficiency is desirable it is not in general a paramount consideration. In particular, with different individuals one may well wish other ethical principles beyond Pareto optimally to guide one's assessment of the different outcomes. Then, if the categorical imperative is considered as a reasonable ethical principle, why should the state wish to intervene in the economy even if Kantian behaviour is inefficient? There is in other words a normative issue that we have avoided in the analysis of this paper. We will come back to this in chapter IV and chapter VIII.

Finally, in the spirit of the "new-new" welfare economics, indicating the presence of a market failure is not a sufficient condition for invoking public intervention. In a second best world it has still to be shown that the second best provision of the state is in some sense better than the second best provision by the market. This issue will also be taken up in the next chapter.

Appendix: Ethical versus selfish behaviour.

As we discussed above, ethical behaviour as represented by the SKR may dictate to an individual with income below average a zero, or even a negative contribution to the public good. As we also saw above the likelihood of this to happen will depend both on the inequality in income distribution across individuals and on the distribution of preferences for the public good in the population. In order to get an idea of the income differential needed to bring about a strictly positive contribution to the public good by all individuals we will consider in this appendix a simple example. Thus, suppose that the preferences of each individual i can be represented by the utility function :

A.1 $$U^i = y_i Q^{\alpha_i}$$

where α_i is a parameter which captures the differences in preferences for the public good across the population. Using result 4 we can identify the optimal contribution by individual i under the SKR by maximizing A.1 under the constraint $y_i + Q/n = \bar{I}$ and by redistributing the burden for the public good across individuals so as to equalize private consumption. Doing this and rearranging individual i's desired contribution can be written as:

$$\text{A.2} \quad q_i^i = I_i - \frac{\bar{I}}{1+\alpha_i}$$

where as above I_i is individual income and $\bar{I}$ is average income. Clearly $q_i^i \geq 0$ as $(1+\alpha_i)I_i \geq \bar{I}$. To insure strict concavity of the utility function α_i must lie in the interval (0,1). If the preferences for the public good are very high $(\alpha_i \to 1)$ $q_i^i \geq 0$ as $I_i \geq \bar{I}/2$; if the preferences for the public good are low $(\alpha_i \to 0)$ $q_i^i \geq 0$ as $I_i \geq \bar{I}$. Clearly, this latter condition is bound to fail for some i unless income is equally distributed. However, note that even in the former case $(\alpha_i \to 1)$ the amount of income inequality allowed to keep $q_i^i \geq 0$ $\forall i$ is very limited indeed; for example, in the case of a two person economy, the income of the poorest could not be smaller than 1/3 of the income of the richest. To the extent that these results could be generalized to other cases, we should therefore expect quite a large number of individuals at the bottom of the income distribution to provide zero to the public good under the SKR.

Chapter IV: Was Kant Right? Kantian Provision of Public Goods and Alternative Models of Public Good Supply

IV.1 Introduction

In the previous chapter we inquired into the efficiency of public good provision under Kantian behaviour. In this chapter we take a more normative view by discussing the relative performance of Kantian behaviour *vis a vis* other rules of public good provision. This will allow us both to clarify the nature of Kantian behaviour and to discuss its desirability.

In chapter III we saw that public good provision under Kantian behaviour is in general characterized by inefficiency and that, in particular, there is some tendency toward underprovision. As we remarked above this may offer some support for public supply of public goods. On the other hand, even if we concentrate only on the efficiency issue, for this conclusion to be warranted it should still be shown that, under the same informational constraints imposed on the Kantian individuals of the previous section, government is in some sense able to do better. In order to approximate government's behaviour in "realistic" market economies characterized by a democratic political system we exploit in section IV.2 a simple model of voting over public good levels where the results of the median voter's theorem apply. General results are, of course, difficult to reach but the indications that we get on the basis of a simple parameterization of preferences clearly point toward a better performance, in efficiency terms, of Kantian behaviour with respect to public good supply under a political process.

This result should of course not be taken as meaning that actual private supply of public goods is more efficient than public provision. For one thing there are serious doubts that the median voter theory is really able to explain actual public supply of public goods. Recent empirical evidence suggests that this is not the case even where the institutional environment more closely approximates the conditions under which the median voter's model applies (Aragon et al., 1988). Besides, for the conclusion above to be warranted one should at least require further research on more realistic rules than Kantian behaviour in the context of models of voluntary supply of a public good.

In spite of these limitations, the result above is of some interest because it suggests that in a second best world Kantian behaviour may approximate efficiency better than alternative rules of public good provision. This is also confirmed by the results in section IV.3. In this section the issue of the relative performance of Kantian behaviour in efficiency terms is addressed from a different point of view, by analyzing the links between Kantian provision and Lindahl equilibria. It is shown that whenever Kantian provision (under the FKR) is efficient it could be sustained as a Lindahl equilibrium for some lump-sum redistribution of income. Similarly, it is also shown that whenever Kantian behaviour is inefficient it would always be possible, if lump sum transfers were available, to find a Lindahl equilibrium which would guarantee to each individual the same level of utility but with less total income. These results suggest that if we take Pareto-efficiency as our reference standard the case for Kantian behaviour must lie in the inability of government to reach a first-best allocation.

In section IV.4 we address a side issue which was left unasked in the previous chapter, that is the effect of income redistribution on Kantian provision. The results obtained as well as some limitations of the present work are further commented upon in section IV.5 which closes the paper. In this final section we also offer some suggestions for further research.

IV.2 Kantian Behaviour versus public good provision under a Democratic System

In the literature on public supply of public goods it is customary to model government behaviour as determined by the result of a political process, where the median voter's choice is decisive in setting the amount of the public good supplied by the state[1]. The median voter theory seems therefore a natural candidate to employ to characterize public supply in our

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[1]The theoretical limitations of such an approach are of course well known (see Atkinson and Stiglitz, 1980 and Mueller, 1979) and by now there is also some empirical evidence which suggests that the median voter's theorem is a poor predictor even where the conditions required for its validity are roughly respected (see Aragon et al., 1988). This should induce the reader to take the results of the next sections with a grain of salt.

comparison between private provision of the public good under Kantian-behaviour and public good provision by the state.

A second reason to consider the results of such a model is that there seem to be strong similarities between the results that we reached in the previous chapter and the results which are usually obtained in this literature. For example, Stiglitz (1974) and Bergstrom (1979) by using parameterizations of preferences similar to the ones that we used in the previous chapter obtain results very close to ours in terms of efficiency of public good provision under the median voter theory. As we will see such a resemblance becomes obvious once the substantial similarity between the two approaches is correctly understood.

Let us then consider in some detail the model by Stiglitz (1974) who presents the results that most closely resemble ours[2]. Stiglitz discusses a model of public good provision through the political process where the public good is financed by means of a proportional tax on the income of individuals, and where individuals spend on a unique private good what is left of their income after having paid the tax. The demand for the public good by an individual i is then given by the solution to the following problem:

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[2]In Stiglitz's model, under the assumption of identical preferences, homothetic preferences show the same phenomena of over/under provision of the public good for the same ranges of values that we saw determining over/under provision under Kantian behaviour. See below.

$$(1) \quad \max_{Q_i} U^i(Q_i, (1 - Q_i/n\bar{I})I_i)$$

where we have reformulated Stiglitz's model using the notation introduced in the previous chapter. By comparing eq.(1) in the present chapter with eq.(4) in the previous chapter we note immediately the similarity with the problem that an individual faces when acting according to the FKR. The main difference lies in the fact that the price of the public good under a proportional tax system is not, as in eq.(4) of the previous chapter, just $1/n$ but it is $1/n$ times an augmenting factor given by the ratio between individual income and average income. This might suggest that we could think of Kantian behaviour as if individuals were faced with a "head" tax on public good provision, like the one considered for example by Bowen (1943) in his seminal paper on public goods. But as we argued above this is true only ex-ante; ex-post an individual subjected to the FKR supplies only his selected contribution. Therefore, the share of the public good that i has to bear, or alternatively the effective price that he pays is just Q_i^*/nQ^* which, unless individuals are identical, is in general different from $1/n$. In other words, as noted above, Kantian behaviour drives a wedge between the decision price, the price on which basis individuals reach their decision, and the effective price that they pay.

The distinction is of great importance. For example, assume that as in our example in section III.4.4 of the previous chapter, individuals are characterized by identical and homothetic preferences, which is also one of the examples considered by Stiglitz (1974). Then, by simply dividing eq.(16) by eq.(17) of the previous chapter, we obtain:

$$(2) \qquad \frac{Q_i^*}{nQ^*} = \frac{I_i}{n\bar{I}}$$

that is, the effective price (i.e. share of total expenditure on public good) that individuals characterized by homothetic and identical preferences pay for the public good when acting according to the FKR is identical to the price that they would face if the public good was financed through a proportional wealth tax. However, even if the mechanism for sharing the burden for the public good is the same under Kantian behaviour and a proportional tax system, the total supply of the public good under the two mechanisms will in general differ. In fact, as we saw above, under the FKR it is the individual with average income who is decisive if preferences are identical and homothetic, while Stiglitz appeals to the median voter theorem to establish that it is the amount selected by the individual with median wealth which is supplied in a political system based on majority voting. In general, there is no reason to suppose that average and median wealth coincide and in particular Stiglitz works under the hypothesis -- which is also a well known empirical fact of most market economies -- that median income is less than average income. Hence we should not expect total provision of the public good to be the same under the two processes.

Suppose however that the elasticity of substitution is constant and equal to unity: then the amount of public good selected by each individual under a proportional tax system is the same for each individual. Hence, the amount selected by the average individual is equal to the amount selected by the median individual. Furthermore, since the price that the average individual pays under a proportional tax system is just $1/n$ (see eq.(1) above) it follows that this individual will select the same amount of public good provision under both a proportional tax system and under Kantian behaviour. It then follows that with identical and homothetic preferences and unitary elasticity of substitution total public good supply will be the same under both Kantian behaviour and a democratic process characterized by majority voting. More than that, as we saw in the previous chapter and as Stiglitz also illustrates, it will also be efficient under both processes due to the identity of choices across individuals. What if the elasticity of substitution is different from one?

Result 1 "Assume that individuals are characterized by the same homothetic preferences, with constant elasticity of substitution between the private and the public good. Also assume that average income is greater than median income. Then if the elasticity of substitution is greater/smaller than one the amount of public good which would be supplied if everybody acted according to the FKR is smaller/larger than the amount which would be provided in a democratic system under majority voting and a proportional wealth tax system."

<u>Proof</u> "If preferences are characterized by a constant elasticity of substitution between the two goods, the utility function can be written, for some admissible transformation, as

$$U = (y_i^p + Q_i^p)^{1/p}$$

By eq.(4) and by result 3 in the previous chapter total provision of the public good under the FKR is then given by:

$$Q^* = \frac{n\bar{I}}{1 + n^{p/p-1}}$$

From eq.(1) the total amount provided under a political process, Q^m, is instead given by:

$$Q^m = n\bar{I}/[1 + (n\bar{I}/I^m)^{p/p-1}]$$

where I^m represents median income. Then,

$$Q^* \gtrless Q^m \quad \text{as} \quad (\bar{I}/I^m)^{p/p-1} \gtrless 1$$

but $I^m < \bar{I}$ by assumption: hence $0<p<1$ implies $Q^m > Q^*$ and $p<0$ implies $Q^m < Q^*$. QED"

Stiglitz (1974) also shows that $Q^o \lessgtr Q^m$ as $\varepsilon \gtrless 1$ for identical homothetic utility functions with constant elasticity of substitution, where as in the previous chapter, Q^o represents the Pareto efficient level of public good supply and ε is the (constant) elasticity of substitution between the public and the private good. Putting together these results with the ones obtained in the previous chapter, section III.4.4, we can state:

<u>Result 2</u> "Suppose that individuals are characterized by identical homothetic preferences with constant elasticity of substitution. Also assume that average income is greater than median income. Then public good provision under the FKR is always weakly more efficient than government provision under a democratic process with majority voting and a proportional wealth tax system".

<u>Proof</u> "By result 1 above and eq.(22) in the previous chapter it follows that :

$$\text{if } \varepsilon>1 \text{ , } \quad Q^m>Q^*>Q^o;$$

$$\text{if } \varepsilon<1 \text{ , } \quad Q^m<Q^*<Q^o. \text{ QED"}$$

We say weakly in the statement of result 2 because as we saw above if $\varepsilon=1$, Kantian provision and median voter choice coincide and they are both efficient (see Stiglitz, 1974). Result 2 then shows that for an important class of preferences Kantian behaviour, if in general still inefficient, is strictly more efficient than public good provision under a political process, at least to the extent that the latter can be represented by the median voter literature. This does not allow us, of course, to reach general results on the relative performance of the two mechanisms. But it seems to suggest that in many cases of importance Kantian provision of the public good will tend to be closer to efficiency than public good provision under a political system.

This conjecture finds some support in the attempts made in the literature to extend Stiglitz's results to more general cases. Bergstrom (1979: Theorem 2, pg.221) for example shows that if individuals are characterized by utility functions of the form

$$(3)\quad U^i = \ln y_i + \alpha_i \ln Q$$

(in our notation), two assumptions are needed to guarantee efficiency of public good provision under a democratic system with majority voting and a proportional wealth tax system: 1) a symmetric distribution of the α_i's across the population; and 2) that the distribution of the α_i's is not correlated with the I_i's distribution. In contrast, it is immediately apparent by writing $f^i(y_i) = \ln y_i$ in eq.(12) of the previous chapter, Kantian provision is always efficient if preferences are represented by a Cobb-Douglas utility function. Finally note that the similarity of results between the median voter literature and Kantian behaviour should not be over stressed. Both Stiglitz and Bergstrom have to impose very strong restrictions on individual preferences to get meaningful results, while we were able to characterize Kantian behaviour for a much larger class of preferences.

IV.3 Kantian Behaviour and Lindahl Equilibria

The optimality of Kantian behaviour in the case of Cobb-Douglas utility functions would lead one to suspect that, somehow, individuals acting according to the FKR turn out by selecting the correct tax shares that would enforce a Lindahl equilibrium. This is indeed the case. Before proving it formally let us define a Lindahl equilibrium in our framework.

Definition "A Lindahl equilibrium is a vector of tax shares $(t_1^+ \ldots t_n^+) \geq 0$ where $\Sigma t_i^+ = 1$ and an allocation vector $(y_1^+ \ldots y_n^+, Q^+)$ such that for each i, (y_i^+, Q^+) maximizes $U^i(y_i, Q)$ subject to $y_i + t_i^+ Q = I_i$"

We can then state:

Result 3 "Suppose that individuals characterized by the utility function in eq.(3) above were asked to pay a tax, t_i^*, on the public good and let $t_i^* = Q_i^* / nQ^*$, where Q_i^* is the total amount of public good selected by individual i when he acts according to the FKR and $Q^* = \Sigma Q_i^* / n$. Then, the resulting allocation is a Lindahl equilibrium. Furthermore, the public good demanded by each individual in the Lindahl equilibrium above coincides with Q^*."

Proof "From eq.(3) above and eq.(4) of the previous chapter, $Q_i^*/n = \alpha_i I_i/(1+\alpha_i)$ which implies $t_i^* = \dfrac{\alpha_i I_i/(1+\alpha_i)}{\Sigma\alpha_i I_i/(1+\alpha_i)}$

First note, $\Sigma t_i^* = 1$. Now suppose that each individual were charged with a tax on the purchase of the public good given by t_i^*; then, individual i would solve:

$$\max_Q \quad \ln(I_i - t_i^* Q) + \alpha_i \ln Q$$

Let Q_i^L be the solution to the problem above; then from first order conditions on utility maximization:

$$Q_i^L = \frac{\alpha_i I_i}{1+\alpha_i} \cdot \frac{1}{t_i^*} = \Sigma\alpha_i I_i/(1+\alpha_i) \text{ QED"}$$

Analyzing Kantian behaviour we have then implicitly found a way for generating Lindahl tax shares if individuals are characterized by log-linear utility functions[3]. That is, if individuals were asked to decide their demand for the public good on the basis of an head tax - equal contribution--and after were instead asked to pay $1/n$ of their chosen amount, the resulting provision of public good would be Pareto optimal.

To result 3 can also be given a different interpretation. Let

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[3]In reality for a larger class of preferences since as is apparent from eq.(12) of the previous chapter, $f^i(y_i)$ may differ across individuals and need not take the logarithmic form.

$P^{i*}=P^i(I_i-Q_i^*/n,Q^*)$ indicate the value of the marginal rate of substitution of individual i evaluated at the final allocation of goods that individual i receives as a consequence of Kantian behaviour (see previous chapter). Let us call m_i^* the minimum amount of income that should be given to individual i so as to support the allocation $\{I_i-Q_i^*/n,Q^*\}$ at price P^{i*} as an utility maximizing choice.[4] Then it is clear that with log-linear utility functions $m_i^*=I_i$. This is so because with these utility functions $P^{i*}=Q_i^*/nQ^*$; it then follows:

$$(4) \qquad m_i^* = I_i - \frac{Q_i^*}{n} + \frac{Q_i^*}{nQ^*}Q^* = I_i$$

Eq.(4) clarifies the economic rationale behind Result 3. It shows that in the case of log-linear utility functions government could enforce the allocation $\{I_i-Q_i^*/n,Q^*\}$ as a Lindahl equilibrium by simply facing each individual i with the personalized price P^{i*}. Or to put it differently, only correct information about individual preferences would be needed while lump sum transfers of income across individuals would not be necessary. However with other classes of preferences efficiency of Kantian behaviour does not imply that $m_i^*=I_i \ \forall i$, and lump sum income redistribution might be needed. The following result illustrates.

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[4]If preferences are strictly convex we are guaranteed that such an income exists; see Neary and Roberts (1980).

Result4 "Assume that individual preferences are strictly convex and that lump sum transfers are available. Then if Q^* is Pareto efficient it could be sustained as a Lindahl equilibrium for some lump sum redistribution of income."

Proof "By definition: $m_i^* = I_i - Q_i^*/n + P^{i*}Q^*$; summing over the n individuals:

$$\begin{aligned} \Sigma m_i^* &= \Sigma(I_i - Q_i^*/n + P^{i*}Q^*) \\ &= \Sigma I_i - Q^* + Q^*\Sigma P^{i*} \\ &= \Sigma I_i \end{aligned}$$

since by assumption $\Sigma P^{i*} = 1$. QED"

That is, when Kantian behaviour is efficient, the resulting allocation is a Lindahl equilibrium for some lump sum redistribution of income. This is of course not surprising since as is well known (see Cornes and Sandler, 1986: 95-98) if preferences are strictly convex and lump sum transfers are available each Pareto efficient allocation can be supported as a Lindahl equilibrium. The point is however that, when efficient, Kantian behaviour can achieve what a government could achieve only if it is endowed with perfect information about individual preferences and it can use lump sum transfers of income across individuals. Since in most cases governments lack both, result 4 gives in a sense an idea of the best that Kantian behaviour could achieve in comparison with public provision, at least as long as the discussion is limited to the particular ethical principle embodied in the notion of Pareto efficiency[5].

While this is an interesting result, it should not be forgotten that as we saw previously Kantian behaviour is in general inefficient. Therefore, by definition, by using the same amount of total resources(=income) in the economy it should be possible to find a different allocation of goods across individuals which would guarantee to everybody a higher level of welfare than in the Kantian equilibrium. Or, alternatively (see Dierker and Lenninghaus, 1988), it should be possible to find a different allocation of goods which would guarantee to everybody the same level of welfare as in the Kantian equilibrium but using less total resources. In this latter case the difference between the total resources used in the Kantian allocation and the total resources which would be needed to reach the same level of welfare for each individual if such resources were employed efficiently can be thought as giving a measure of the waste introduced by Kantian behaviour.

By using the notion of Lindahl equilibrium the next result tries to make this last concept more precise. It also allows one to see in a different light the relationship, that we discussed in the previous chapter (section III.4.1), between the value of the sum of the marginal rates of substitution and the issue of

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[5]As we remarked in the conclusion of the previous chapter Kantian behavior in itself is an ethical principle which could conflict with Pareto efficiency. Moreover since Lindahl equilibria are in general not unique, one could still inquire on the desirability of the particular Lindahl equilibrium that Kantian behaviour selects when efficient.

over/under provision of the public good. In order to prove this result we need, in addition to strict convexity of preferences, to impose the assumption that for each individual the private good is an essential good (see Willig, 1978)[6]. This assumption implies that the indifference curves of each individual i never intersect the $y_i=0$ axis. Since here y_i represents the total amount of private consumption for individual i, the assumption of essentiality of the private good seems quite reasonable.

<u>Result 5</u> "Suppose that preferences are strictly convex and that the private good is an essential good for each individual. Then, if $\Sigma P^i(I_i-Q_i^*/n,Q^*)\neq 1$, there exists an allocation (y_i',Q') such that $Q'\gtrless Q^*$ as $\Sigma P^i(I_i-Q_i^*/n,Q^*)\gtrless 1$, $\Sigma P^i(y',Q')=1$ and $U^i(y',Q')=U^i(I_i-Q_i^*/n,Q^*)$. Furthermore, the total amount of income needed to support the allocation (y',Q') as a Lindahl equilibrium is less than ΣI_i."

<u>Proof</u> "Let $U^{i*}=U^i(y_i^*,Q^*)$, where $y_i^*= I_i-Q_i^*/n$, and let us rewrite the marginal rate of substitution as $P^i=P^i(U^{i*},Q)$, where the private good y_i is set so as to reach utility level U^{i*} when

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[6]The definition of essentiality of a good is as follows. Consider an economy with n goods and I consumers, indexed by $i = 1,2,....I$, who have preferences defined on the space of goods. Then good x_1 is said to be essential to consumer i if for any bundle $x = x_1,x_2,..x_n$ with $x_1>0$ there does not exist a bundle $x_1'=0,x_2',..x_n'$ such that $U^i(x)=U^i(x')$; i.e. a good is essential if any bundle excluding it can not match a bundle including it. See Willig (1978) for a more complete discussion.

public good consumption is Q. Then, strict convexity of preferences and essentiality of the private good imply that $P^i(U^{i*},Q)$ is a monotonically decreasing function of Q; i.e. $P^i_Q(U^{i*},Q)<0$ everywhere. Hence, if $\Sigma P^i(U^{i*},Q^*)>1$ (<1) there must exist a $Q'>Q^*$ ($Q'<Q^*$) such that $\Sigma P^i(U^{i*},Q')=1$. This proves the first part of the theorem. Now, let $P^{i}{}'=P^i(U^{i*},Q')$ and let y_i' be such that $U^i(y_i',Q')=U^{i*}$. Let $E^i(P,U)$ indicate the expenditure function for individual i; i.e. $E^i(P,U)$ is the minimum level of income needed to reach utility U when the price of the public good is P. Then, by construction, $E^i(P^{i}{}',U^{i*})$ is the minimum level of income needed to support the allocation (y_i',Q') as an utility maximizing choice. By definition of expenditure function it then follows:

$$E^i(P^{i}{}',U^{i*}) < y_i^* + P^{i}{}'Q^*$$

Summing over the individuals:

$$\Sigma E^i(P^{i}{}',U^{i*}) < \Sigma y_i^* + Q^*\Sigma P^{i}{}'$$

$$< \Sigma I_i - Q^* + Q^*$$

$$< \Sigma I_i .\ \text{QED".}$$

Result 5 shows that if lump sum transfers were available and individual preferences known, a benevolent government could arrange things so as to ensure to each individual the same level of utility reached as a consequence of Kantian behaviour; furthermore since $\Sigma E^i(P^{i}{}',U^{i*}) < \Sigma I_i$, government could redistribute the extra resources to the individuals so as to reach an allocation which would certainly dominate in the Pareto sense the Kantian allocation. Thus, as we argued in the introduction, if we stick to Pareto efficiency as our only standard of reference, Kantian behaviour can only be defended in a second best world, where individual preferences are unknown and lump sum transfers not available.

As one would have expected, it is also easy to show that the extent of "waste" introduced by Kantian behaviour, as measured by the expression $\Sigma I_i - \Sigma E^i(P^{i\prime}, U^{i*})$, is a positive function of the difference between the sum of the marginal rates of substitution, evaluated at the Kantian allocation, and the marginal rate of transformation; that is, it is a positive function of $|\Sigma P^i(U^{i*}, Q^*) - 1|$. To see this, let us define a constrained expenditure function[7] $\hat{E}^i = \hat{E}^i(U, P, Q)$ as the minimum level of income needed to reach utility level U when the price of the public good is P and public good supply is fixed at Q. That is,

$$\hat{E}^i(U,P,Q) = \min_{y_i} \; y_i + PQ \quad \text{s.t.} \quad U^i(y_i, Q) = U$$

Evaluating $\hat{E}^i(U,P,Q)$ at $U=U^{i*}$, $Q=Q^*$ and $P=\phi^i=Q_i^*/\Sigma Q_i^*$ we get:

$$\hat{E}^i(U^{i*}, \phi^i, Q^*) = y_i(U^{i*}, Q^*) + \phi^i Q^* = y_i^* + Q_i^*/n = I_i \tag{5}$$

Consider now the partial derivative of $\hat{E}^i(U^{i*}, \phi^i, Q^*)$ with respect to Q:

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[7]See Deaton and Muellabauer (1980) and Neary and Roberts (1980) for a discussion of the properties of this function.

(6) $$\hat{E}^i_Q(U^{i*},\phi^i,Q^*)= \frac{\partial y_i}{\partial Q}(U^{i*},Q^*) + \phi^i = \phi^i - P^{i*}$$

since $\partial y_i/\partial Q$, evaluated at (U^{i*},Q^*) is simply the negative of the marginal rate of substitution evaluated at the same allocation of goods. Let us now evaluate the constrained expenditure function $\hat{E}^i$ at (U^{i*},ϕ^i,Q') where, as above, Q' is such that $\Sigma P^i(U^{i*},Q')=1$:

(7) $$\hat{E}^i(U^{i*},\phi^i,Q')= y_i' + \phi^i Q'$$

Summing over the individuals we get:

(8) $$\Sigma\hat{E}^i(U^{i*},\phi^i,Q')= \Sigma y_i' +Q'= \Sigma E^i(P^{i}{}',U^{i*})$$

that is, the total income needed to reach the allocation (y_i',Q') as an utility maximization choice for each i when the individual price for the public good is $P^{i}{}'$ is the same total income needed to reach the same allocation when each individual i is constrained to consume Q' at price ϕ^i and to reach utility level U^{i*}-- which is obvious since $\Sigma\phi^i=\Sigma P^{i}{}'=1$. From eq.5 and eq.8 it follows that we can write the waste introduced by Kantian behaviour as

(9) $$\Sigma I_i - \Sigma E^i(P^{i}{}',U^{i*}) = \Sigma\hat{E}^i(U^{i*},\phi^i,Q^*) - \Sigma\hat{E}^i(U^{i*},\phi^i,Q')$$

Using eq.6, the change in income associated with the <u>discrete</u>

change in public good provision from Q^* to Q' in the RHS of eq. (9) can be represented as a line integral

$$(10) \quad \Sigma I_i - \Sigma E^i(P^{i}{}',U^{i*}) = \Sigma\int_{Q'}^{Q^*}(\phi^i - P^i(U^{i*},Q))dQ$$
$$= \int_{Q'}^{Q^*}(1 - \Sigma P^i(U^{i*},Q))dQ$$

which is what we wanted to show in the first place. Note that from result 5 $dQ \gtrless 0$ as $\Sigma P^i(U^{i*},Q^*) \lessgtr 1$; thus the expression on the LHS of eq.10 is always positive as long as Kantian behaviour is inefficient.

IV.4 Income Distribution and Kantian Supply of a Public Good

In chapter II we insisted on the general invariance properties of the Nash contribution model. In particular we saw that in a Nash equilibrium where everybody is a contributor marginal transfers of income across contributors would leave the total supply of the public good and each individual consumption of the private good completely unchanged. It may then be interesting to ask if Kantian allocations are characterized by the same properties. Let us consider the two Kantian rules in turn.

As for the FKR, in general, income redistribution should affect the total provision of public good as well as the private consumption of each individual. Quite trivially, a redistribution of income from the rich to the poor will increase/decrease the total provision of the public good according to whether poor people have a higher/lower marginal propensity to consume the public good than rich individuals. There is however a special case where this does not happen.[8]

<u>Proposition 1</u> "Suppose that individuals acting according to the FKR are characterized by the same homothetic preferences. Then any redistribution of income across individuals would leave the total supply of public good unchanged"

The result is obvious once it is recalled that with homothetic preferences it is the average individual who is "decisive" (see result 3 of chapter III) and any income redistribution would leave this average income unchanged. Note, however, that both the level of private consumption and the contribution of each individual will in general change following the redistribution, which will therefore have welfare effects. In particular any income redistribution in an egalitarian direction should increase the efficiency of Kantian behaviour under the FKR by closing the gap between the LHS and the RHS of eq.(22) in the previous chapter. Indeed, as the inequality in income distribution tends to zero, the FKR tends to coincide with the SKR which is always efficient with identical preferences as we saw in the previous chapter.

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[8]Another special case is, of course, the case of quasi linear utility functions with zero income effects on the demand for the public good.

The story changes radically if we consider the SKR. Much stronger neutrality results can be obtained if the inequality constraint is not binding. The next result illustrates.

<u>Result 6</u> " Let $\hat{q}_i$ indicate individual i's contribution under the SKR. Assume that $\hat{q}_i>0$, $\forall i$ and let ΔI_i indicate the transfer of income (positive or negative) accruing to individual i. Then if $|\Delta I_i| < \hat{q}_i$, $\forall i$ the total supply of the public good and each individual consumption of private good will remain completely unchanged following the income redistribution."

<u>Proof</u> "By result 4 of the previous chapter, if $\hat{q}_i>0$ $\forall i$, $\hat{q}_i=q_i^i=Q^i/n + (I_i-\bar{I})$. Total provision of the public good is then $\Sigma\hat{q}_i=\Sigma Q^i/n$. Consider now a transfer of income to individual i. Let $q_i^i{}'$ represent i's desired contribution after the transfer and $\hat{q}_i{}'= \max (0,q_i^i{}')$ his actual contribution after the transfer. Since Q^i/n depends only on average income it will not change following the transfer; it then follows from result 4 of the previous chapter:

$$q_i^i{}' = Q^i/n + (I_i-\bar{I}) + \Delta I_i = \hat{q}_i + \Delta I_i >0$$

by assumption. Then $\hat{q}_i{}'=q_i^i{}'$ and $\Sigma\hat{q}_i{}'=\Sigma(\hat{q}_i + \Delta I_i)=\Sigma\hat{q}_i$. QED"

Thus in an economy where everybody acts according to the SKR a marginal income redistribution would have the same effects as in the Nash contribution model (see chapter II). That is, if each individual contribution is strictly positive and the income transfer is marginal the effect on the Kantian allocation of a slight income redistribution is nil. Impressively enough, the effects of a slight income redistribution on the Kantian allocation when the inequality constraint is binding also tend to replicate the comparative static results of the Nash contribution model in the same case (see chapter II and Bergstrom et al, 1986).

Result 7 "Let q_i^i indicate the desired contribution of individual i under the SKR. Suppose that $q_i^i<0$, some i. Then a redistribution of income toward greater\smaller equality will reduce\increase total public good provision"

Proof "For simplicity suppose one redistributes income from individual j to individual i while keeping the contribution of everybody else constant. By the non-negativity constraint on individual contribution $q_i^i<0$ implies $\hat{q}_i=0$. Let Q_k be the contribution of all individuals except i and j, before the redistribution. Then the total provision of the public good before the transfer, Q_b is

(11) $$Q_b = Q_k + \hat{q}_j$$

By result 6, after the redistribution, individual j will provide $\hat{q}_j - \Delta I_i$, where ΔI_i is the income transfer, and individual i will provide $\max(0,\ q_i^i + \Delta I_i)$ while everybody else will keep his contribution unchanged. Summing over the individuals we get:

(12) $$Q_a = Q_k + \hat{q}_j - \Delta I_i + \max(0,\ q_i^i + \Delta I_i)$$

where Q_a represents total public good supply after the income transfer. Subtracting (11) from (12):

(13) $$Q_a - Q_b = -\Delta I_i + \max(0,\ q_i^i + \Delta I_i)$$

then $Q_a \lessgtr Q_b$ as $\Delta I_i \gtrless 0$. QED"

Summing up, we can then say that Kantian behaviour, at least under the SKR formulation, is characterized by the same invariance properties of the Nash contribution model (for income redistribution). If we take these results as unrealistic, results 6 and 7 would then suggest that (pure) Kantian behaviour does not represent a convincing alternative model of actual individual behaviour with respect to the traditional Nash contribution model. On the other hand, as we remarked in the previous chapter, this was not the reason for introducing and discussing Kantian behaviour in the first place.

IV.5 Kantian behaviour: limitations and suggestions for further research

In the previous chapter we argued that the way we chose to formalize Kantian behaviour was quite adequate for the type of economy we were considering, a static economy where individuals do not earn their income. However, if we move to a more realistic economy, a different approach to Kantian behaviour should be chosen if we are to avoid serious logical pitfalls. On the one hand, as we already remarked in the previous chapter, in an economy where individuals earn their income we should consider the fact that individuals may feel they are entitled to a larger share of resources if they have worked harder or risked more than other people. This would suggest that we should reformulate our Kantian rules in terms of individual "effort" rather than individual contribution. This would probably weaken the extreme egalitarianism of the SKR by imposing a sort of horizontal equity constraint on the maximization problem in eq.(5) of the previous chapter.

Similarly, if we introduced a time dimension in our analysis, we should also consider the fact that people may learn about other individuals' preferences by observing their behaviour. This is a crucial point for our discourse because we justified our Kantian rules in the first place on the basis of informational constraints on other individuals' preferences.

As an example of the type of issue which could be addressed in a dynamic economy, suppose that individuals can observe the total supply of public good by the other individuals composing the society. Then, if individual i has decided, say by following the FKR, to contribute Q^*_{io}/n at time t_o, by simply observing at time t_1 that $Q^*_{io} \neq Q^*_1$ (where the second suffix indicates the time period) he would realize that not everybody is like him or alternatively that not everybody acts according to the Kantian rules. Indeed, a number of different possibilities open up according to how we interpret the notion of Kantian behaviour and individual i's reaction. For example, if i takes the observation above as an indication that the other individuals do not follow the Kantian rules he may either decide to stick to his previous behaviour (thus strictly following the principle of unconditional commitment) or, probably more realistically, to change his contribution. This last case would lead one to introduce some sort of reciprocity rules of the type considered by Sugden (1984) (see chapter II) and that we also discuss in the next chapters.

Alternatively, if individual i interprets the observation above as an indication that the other Kantian individuals have different preferences for the public good he may decide to change his behaviour so as to take this into account. It would seem reasonable to argue, for instance, that without contradicting the spirit of the principle of the categorical imperative, one individual may feel obliged to offer more towards the public good if he perceives that his preferences for the latter are greater than the preferences of his fellows. For example, we might imagine that individual i will revise upward (downward) his contribution as long as his marginal willingness to pay for the public good is larger (smaller) than the share that he actually pays for it; that is, we could write individual i's contribution at time t, q_{it}, as

$$(14) \qquad q_{it} = q_{it-1} + \alpha(P^i(I_i - q_{it-1}, \Sigma q_{it-1}) - q_{it-1}/\Sigma q_{it-1})$$

where $0 \leq \alpha \leq 1$ is some constant which measures the speed of adjustment, $t=0,1,2,\ldots$ and where $q_{io} = Q_i^*/n$, $\forall i$. Note that an equilibrium in the system of n non linear difference equations above is a Lindahl equilibrium. It would then be interesting to analyze the conditions under which such an equilibrium exists, is unique and stable. However, this will have to wait for further research.

IV.6 Conclusions

In this chapter we discussed and clarified the links between Kantian behaviour and alternative methods of public good provision such as provision through a political system (exemplified by the median voter theorem) and Lindahl equilibria. Finally we commented upon the main limitations of the present work and we offered some suggestions for further research. The main result which emerges from this chapter is as follows. If we take as our only ethical principle the Pareto principle, Kantian behaviour rates quite badly. As we illustrated at length in the previous chapter public good provision under Kantian behaviour is typically suboptimal. Then as is shown in section IV.4, a perfectly informed government which uses lump sum transfers could certainly do better -in terms of individuals' own preferences- than a society of Kantian individuals. On the other hand, in a second best world, where government lacks both correct information and lump sum tools the superiority of government intervention is more dubious. In order to get some insights on this last issue we compared in section IV.2 public good provision under a political system with Kantian provision. It was shown that for an important class of preferences Kantian provision is strictly more efficient than public good provision under a political system. While this may not say much on the issue of the relative performance between public and actual private provision of public goods --since both Kantian behaviour and Median voter theory are unlikely to be realistic description of private and public provision of public goods-- it suggests that in a number of cases Kantian behaviour may be preferable, in a second best world, to alternative methods of public good provision.

An Investigation in the Theory of Voluntary Provision of Public Goods and Income Tax Evasion under the Hypothesis of Ethical Behaviour on the Part of Economic Agents

A two volume Ph.D dissertation: volume 2

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Introduction

In the first part of the present work we considered some issues relative to the problem of voluntary provision of public goods. In chapter II we surveyed the relevant literature, both theoretical and empirical, and we argued that a more general approach to human motivations and behaviour may be needed in order to gain a more complete understanding of the phenomenon. In chapters III and IV we provided an example of this different approach by comparing, in an highly abstract setting, private and public good provision under the extreme hypothesis that individual behaviour was ruled by the Kantian categorical imperative. We showed that interesting insights can be gained by following this approach and that subtle issues, both theoretical and analytical, arise which deserve further attention.

The case of voluntary provision of public goods is not however the only situation where considering alternative approaches to human behaviour may turn out to be fruitful. Indeed, it will be our point in this part of the work to argue that, by enlarging the range of phenomena which are traditionally addressed by the contribution model outlined above, a number of other important economic problems may receive interesting and more satisfactory explanations than those provided by selfish models.

The example that we will be using to illustrate this point is represented by the phenomenon of (income) tax evasion. Here we are confronted with a well-formulated, elegant economic theory, based on the assumption of rational egoism, which nevertheless produces results which are both counter-intuitive and in sharp contrast with available evidence. It will be our task in the following chapters to show that integrating the traditional model with an approach based on "fairness" allows one to get both more convincing explanations of the phenomenon and analytic results which are not contradicted by empirical evidence. Furthermore, it also allows one to consider questions which, in spite of their intuitive relevance, cannot be addressed in the traditional model -- as for example the effect of public sector inefficiency on tax compliance. In order to model "fairness" we will make use of the analytic work on the Kantian rules developed in chapters III and IV. In contrast with the strategy followed in those chapters, however, we will explicitly consider here rules of reciprocity, thus following more closely the approach advocated by Sugden (1984) and by the present author elsewhere (Bordignon, 1987).

Our line of attack is as follows. In chapter V we review the most significant theoretical literature on the argument, drawing attention to the inconsistencies with empirical evidence generated by the conventional model, and we introduce intuitively our fairness approach to tax evasion. In chapter VI we set up the basic structure of the model and, after having discussed the characteristics of the equilibrium, we perform a series of comparative static exercises in the case of wholly exogenous public expenditure. In the same chapter we also try to solve some of the ambiguities emerged by imposing a parameterization on the preferences of the individuals. In chapter VII we analyze the case of wholly endogenous public expenditure. We first replicate the comparative static exercises attempted in the previous chapter. Subsequently we consider a case where fairness is violated by the State through inefficiencies in the production of a public good. Finally, in the conclusions to this chapter the main findings of the analysis are summarized, the principal caveats discussed and some avenues for further research indicated.

Chapter V: A Fairness Approach to Income Tax Evasion: a Survey of the Literature and an Intuitive Presentation of the Approach

V.1 Tax Evasion: A Quick Review of the Literature

The conventional economic model of income tax evasion, as presented in the seminal paper by Allingham and Sandmo (1972)[1] --henceforth, AS -- and subsequently greatly extended in the literature views the decision of evading simply as the result of a portfolio choice. In the original AS paper, the typical taxpayer decides by maximization of an expected utility function how to distribute a fixed amount of resources (his pre-tax income) between a safe asset (his post tax income if he fully pays the tax) and a risky asset (evaded income). Evaded income represents a risky asset because it may induce both positive and negative returns depending on whether the individual is caught cheating the tax system. For interior solutions, as one would expect, the amount of evaded income turns out to be influenced -- in addition to the other parameters of the problem-- negatively for risk averse individuals by the parameters set up by government to control tax behaviour: the expected probability of detection, supposed to be given and known to the taxpayer, and the penalty

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[1] Another important seminal paper is Srinivasan (1973).

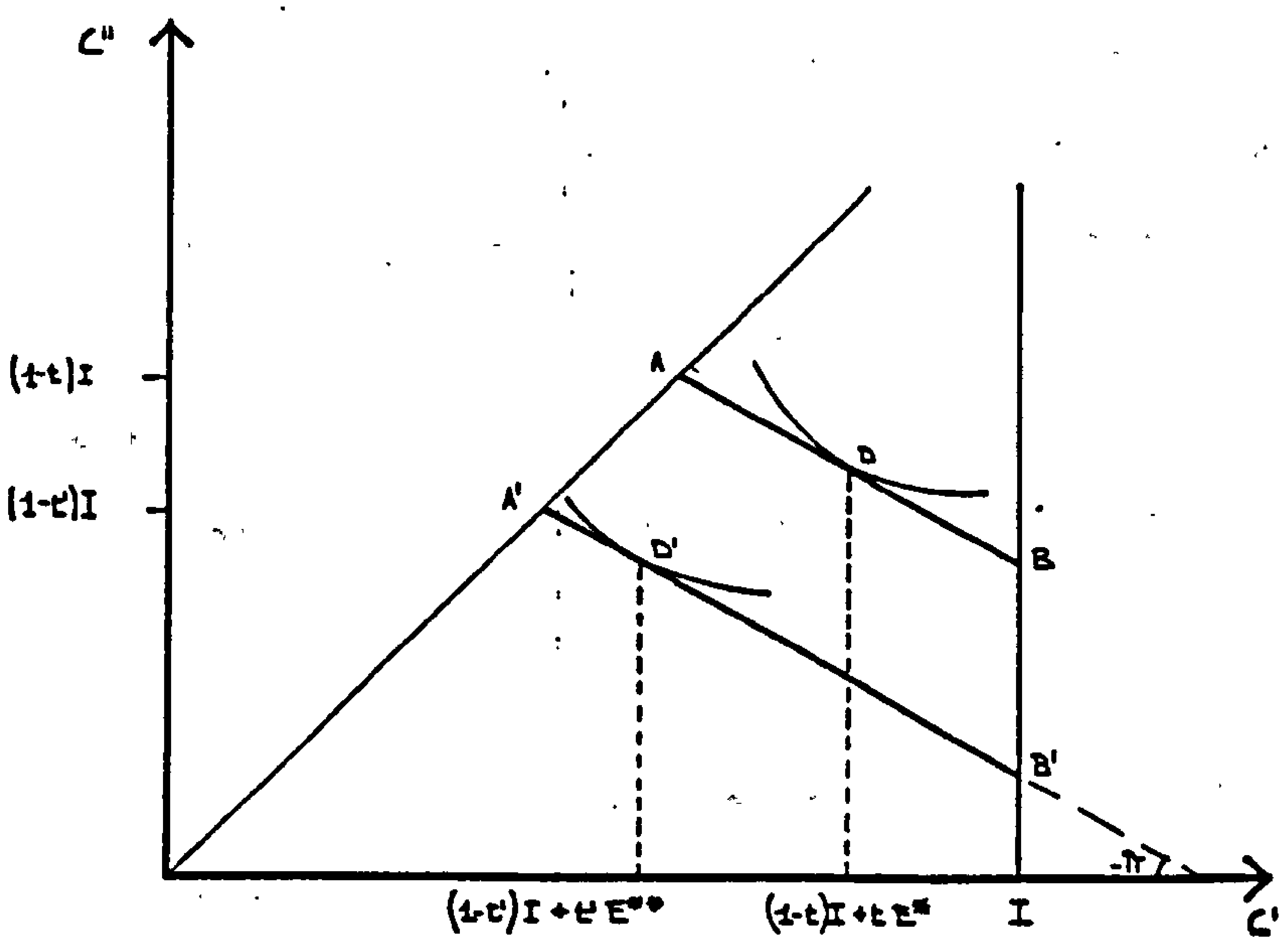
rate applied in the case of detection.

In contrast, the effect of a tax increase on evaded income turns out to be, in the AS model, ambiguous. An increase in the tax rate has in fact both a substitution and an income effect and under the conventional assumption of decreasing absolute risk aversion (henceforth DARA) the net effect on evaded income is undecided. The substitution effect derives from the fact that increasing the tax rate reduces the relative price of evaded income thus inducing more evasion, while the income effect results from the fact that increasing the tax rate makes the taxpayer, in expected terms, poorer in both states of the world thus triggering, under the assumption of DARA, a reduction in evaded income. Subsequently, Yitzhaki (1974) and Christiansen (1980) showed that if the surcharge rate is imposed on <u>evaded tax</u>, rather than on evaded income, (an institutional feature which is common to many countries, including the USA, Israel and Italy), this ambiguity disappears. The substitution effect cancels out, and under DARA, the effect of an increase in the tax rate on evaded tax is certainly negative.

Since the latter comparative static result may not be immediately apparent and it is quite central to the ensuing discussion, let us illustrate it by means of a simple graphical argument, borrowed from by Cowell (1985a). Let I represent the pre-tax income of the representative taxpayer, t the proportional rate of tax, E the income undeclared and π the surcharge rate on evaded tax. The consumption of the taxpayer in the favorable state of the world, when he is not caught, is then $C'=(1-t)I + tE$ and the consumption when he is caught is $C''=(1-t)I - \pi tE$. The taxpayer's budget constraint is represented in fig. V.1 by the segment AB, where point A is located on the certainty line (i.e. E=0) and the point B lies on the locus of points where the taxpayer evades the tax completely (i.e. E=I). Note that the slope of the budget line is simply $-\pi$. Assuming an interior solution, the amount of evaded income will be determined by the point of tangency between an indifference curve and the budget line ; say, at point D in the picture. An increase in the tax rate from t to t', where t'>t, will then move the budget line parallel downward, say to A'B' in the picture, and the new point of tangency will be D'. As drawn in the picture, the assumption of DARA implies that D' must lie to the left of D, and that further the distance between the two optimal points on the C' axis must be larger than the distance between the two optimal points on the C'' axis. This unambiguously implies a lower level of evaded tax (i.e. tE) corresponding to a higher rate of tax.

This central prediction of the conventional model is not only counter-intuitive but it is also contradicted by both empirical (Clotfelter, 1983; Slemrod, 1985; Crane and Nourzad, 1986) and experimental evidence (Friedland et al., 1978). It thus represents the most serious inconsistency between the conventional model and reality. Not surprisingly therefore, the subsequent literature has dedicated a lot of effort in trying to overcome it.[2] This is not

Figure V.1



[2]Most work has moved in the direction of relaxing the assumptions imposed by AS in their original paper. Thus, Koskela (1983) considers a progressive tax system rather than a proportional one; Benjamini and Maital (1985) study the case where individuals do not behave according to expected utility theory but are characterized by the errors in perception experimentally singled out by Tversky and Kahneman; Pencavel (1979), Watson (1985), and Cowell (1985b) introduce a labor-leisure choice in the model; Allingham and Sandmo (1972) themselves and Russell and Rickard (1987) introduce an element of dynamics considering more periods and retroactive penalties; Sproule (1985) analyzes the case where individuals are characterized by imperfect information about the parameters of the fiscal system. In general the results of these papers are disappointing for the issue at hand. They show either the persistence of a negative relationship between the tax rate and the amount of evaded tax or, even worse, a complete ambiguity in the comparative static properties of the model. This is, for example, the case of introducing a labor supply choice in the model. For a more complete discussion of these issues see Cowell (1985a).

the place to review this ever-growing literature (see note 2) but it is fair to say that the attempts made so far to extend the AS framework have shown that the result above is surprisingly robust and that it survives in much more complex models than that considered by AS.[3] Below, we will comment in detail on one of the most serious attempts made in the literature to overcome it. For the time being, let us note other inconsistencies of the conventional model.

In particular, by modeling the decision to evade simply as the result of an individual calculus of expected costs and benefits, conventional theory would predict that, if the expected net benefits from evading became positive and large enough people would certainly evade and presumably even large amounts of taxes. Yet, despite theory, this is not what seems to be happening in reality. For example, Hansson (1985:286) notes that in Sweden, where pecuniary penalties for tax evasion amount to only 1% - 1.4% of estimated tax evasion (this is to be compared with marginal tax rates in the range of 50% - 90%) "only the assumption of an extreme degree of risk aversion would keep tax evasion within the range of estimated tax evasion". In a similar vein are the results of an experimental study by Baldry (1986). The latter reports that

[3]The robustness of the result of a negative relationship between the tax rate and the amount of evaded tax in analytic models is such as to have convinced some economists that that relationship must represent a feature of reality rather that an insufficiency of theory: see Dubin et al. (1987). It is maybe superfluous to add that this is neither my conviction nor the conviction of most of the economists working in the field.

in a tax evasion experiment a substantial portion of the subjects simply refused to evade taxes, despite the fact that expected returns from evasion were largely positive. In contrast, and quite interestingly, in a second experiment, formally equivalent to the tax evasion experiment, but presented to people as a gambling experiment, people did lay positive bets whenever the expected returns from the bet were positive. As Baldry concludes: "whatever tax evasion is, it is not a gamble".

Clearly, similar results suggest that other considerations, beyond the individualistic calculus of expected costs and benefits, are crucial to the decision to evade. In discussing the results of his experiment, Baldry suggests that moral considerations are likely to play a role in determining people's tax behaviour. Other experimental and empirical evidence strongly supports this conclusion. For example, Friedland et al. (1978), Spicer and Becker (1980), Spicer and Thomas (1982), all find that the decision to evade is strongly influenced by the perception that people have of the tax burden facing them as being "fair" or "unfair". We will come back to this in the next section.

The two above mentioned features -- the wrong expected sign for tax changes on tax evasion and the inability of the model to take into account Baldry's type of evidence -- surely represent the most serious weaknesses of the conventional model. However there are at least other two aspects of the phenomenon of tax evasion which are not adequately treated in the AS framework.

The first concerns the interdependence among taxpayers. It is commonly held, by practitioners in the field and economists alike, that people tend to evade more the more they perceive other taxpayers to evade. Here evidence is poor and contradictory (contrast Spicer and Lundstedt, 1976 with Spicer and Hero, 1985) but no serious research has been undertaken on this issue, at least as far as I am aware of. Attempts have been made to introduce this feature in formal models (see Benjamini and Maital, 1985 and Gordon, 1987a, 1987b). But this has been done, at least so far, by postulating ad hoc the existence of "social stigma" on tax evasion and by making the latter negatively dependent on the amount of total tax evasion. Clearly, a better explanation of the existing links between taxpayers would be welcome.

A more serious criticism of models of the AS type concerns the so-called "public aspect" of tax evasion. "Government not only takes away but it also gives back", in form of goods and services. Unless it is believed that the utility function of the taxpayers is separable in private goods and publicly provided private and public goods, government expenditure would presumably exert an influence on the tax behaviour of the taxpayer. Curiously enough, although this public feature of tax behaviour was noted relatively early in the literature (Kolm, 1973) very little theoretical and empirical work has been done on it until very recently. Only Hansson (1985) and Cowell and Gordon (1988) -- henceforth CG -- deal explicitly with it on a theoretical ground. The latter paper in particular presents a number of interesting characteristics. However, since I am going to use CG's model in building my own model in chapter VII, let me postpone the discussion of their paper to that chapter. Here, I want only to comment upon their main result. By introducing (endogenously[4]) public expenditure in a model of tax evasion CG manage to offer an explanation for the the observed positive relationship between tax evasion and the tax rate. In particular, by exploiting some parameterizations of the utility functions of taxpayers, CG show that the effect of a tax increase on evaded tax tends to be positive if public goods are "scarce", while it tends to be negative if public goods are "abundant". The rationale behind this result is as follows. If public goods are strongly underprovided (overprovided) an increase in the rate of tax and therefore in public goods supply would tend to make individuals better off (worse off) on average in both states of the world, thus triggering, under the hypothesis of DARA, an increase (decrease) in tax evasion.

In a framework where selfishness is imposed from the outset, the logic of the result is indisputable. Indeed, as the sharp reader will surely have realized, it just represents an extension to an uncertain world of the traditional free-rider argument (see chapters II), with the assumption of DARA which takes the place of the traditional income normality assumption on the demand for public goods. However, one can not help feeling that there is something odd going on here. For, underprovision (overprovision)

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[4]With endogenous public expenditure, I just mean that government's budget always balances, so that once the rate of tax is determined so is public expenditure. See chapter VII.

of the public good implies that an individual[5] would be willing to pay a higher (lower) tax in exchange for a higher amount of public good. Yet, schizophrenically, the selfish individual in the CG model resists a tax increase, evading more --thus undermining the expected increase in public good supply-- when he would welcome an increase in tax, while he reduces tax evasion --thus stimulating a further increase in public good supply-- when he would contrast an increase in tax. Not only this, but given the positive relationship between the rate of tax and public expenditure --due to the assumption of a balanced government budget-- the above mentioned result would also imply that people tend to evade more in response to a tax increase if the initial rate of tax is low and to evade less in the opposite case. Or, to put it differently, a 1% tax increase from an initial rate of tax of, say, 1% should increase tax evasion, while the same 1% increase in the tax rate from an original average tax rate of, say, 90% should reduce tax evasion! Any non-economist --and a few economists for that matter-- would find the argument strange indeed.[6] Rather, intuition would suggest the opposite to be more plausible. And indeed there is at least an experimental study (Friedland et al.,

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[5]In the CG model all individuals are identical. See chapter VII, section VII.2 for a proof and a discussion of the result referred to in the text.

[6]In fairness, it must be said that CG have become aware of the counter-intuitiveness of their argument. In the paper published in the Journal of Public Economics, in contrast with the original Working Paper, the emphasis on the result is much reduced: compare Cowell and Gordon, 1986 with Cowell and Gordon, 1988.

1978) which shows evaded tax to be an increasing convex function of the tax rate rather than a concave one as implied by the CG result.

Hence, the introduction of public expenditure in a model of tax evasion by itself does not seem to be able to remove the counter-intuitiveness of the results of the portfolio model. In contrast, as we will show in the next chapters, the introduction of public expenditure coupled with fairness rules does indeed allow one to make some steps toward a better reconciliation of theoretical results with both reality and common sense.

Finally, let me note a last feature of the AS type of model which requires further work. The critical note by Kolm (1973) on the AS paper has generated a vast literature on the issue of "optimal" tax evasion: i.e. the selection by government of optimal values for the penalty rates, the tax rate and the expenditure on detection. As is by now well known this problem has no easy solutions because the results turn out to be very sensitive to the assumption made about the maximand of government (for example, expected tax revenue versus expected utilitarian welfare function: see Benjamini and Maital, 1985 and Sandmo, 1981). Moreover, there are still unsolved logical and ethical problems which make the proposed solutions even more questionable.[7] However, there is

[7] I am referring here to the issue on whether the utility functions of evaders should be considered in the welfare function of government. See the references quoted in the text and Cowell (1985a) for a discussion of this problem.

another fundamental issue which has been so far overlooked in the literature. If, as I am going to argue in the next section and as our previous references seem to suggest, the decision to evade is strongly influenced by fairness considerations, do we not run the risk, by assuming selfish utility functions as in the AS framework, of producing policy suggestions which are qualitatively wrong and perhaps counter-effective? To take an extreme example: suppose government intends to maximize an expected utilitarian welfare function. As is well known from the work of Stiglitz (1982) the solution to this problem can clash with some ethical principles of public finance, such as the principle of horizontal equity. But, even if we leave aside the ethical problem, if people have a strong feeling for equal treatment of equals the perceived injustice of the fiscal system may actually trigger an explosion of anti-social behaviour of which tax evasion would be only an element. In this case, the cure could actually worsen the illness. It is therefore not surprising that, in the best example so far advanced in the literature of optimal taxation with tax evasion (Sandmo, 1981), the author concludes by raising the question of an "appropriate treatment of notions of justice and morality". We will come back to this in chapter VII.

Summing up, there seems to be a need for a theory which, by integrating the conventional portfolio choice model of income tax evasion, makes it able to explain, or at least to be consistent with, the following four stylized facts:

1) evaded tax is positively linked with tax changes, at least

over some ranges;

2) a consistent fraction of taxpayers choose not to evade even when it would be in their selfish interest to do so;

3) the behaviour of a taxpayer is influenced by the behaviour of other taxpayers in the same reference group;

4) the behaviour of a taxpayer is influenced by government expenditure on goods and services.

In the following chapters we will show how is possible, by exploiting the framework developed in the previous chapters on voluntary provision of public goods, to build up such a theory.

V.2 A Fairness Approach to Income Tax Evasion: An Intuitive Explanation

In the previous section we saw that the conventional model of income tax evasion, based on the notion of rational egoism, meets serious problems in explaining the empirical facts known about the phenomenom. It seems then worth asking which consequences the adoption of a different approach, based on a notion of practical morality, would have for the analysis of the problem at hand. First, it would suggest that one looked at the decision to evade by a single taxpayer as the result of a process of social interactions rather than as the choice of an isolated individual. Second, it would require one to identify the "actors" involved in the process and to establish the particular "rules of fairness" that these actors are supposed to follow.

Fortunately, there already exists a literature, founded on the psychological theory of social exchange, which has attempted to look at the phenomenon of tax evasion from this perspective (see especially Spicer and Lundstedt, 1976). In this approach the decision to evade by a single taxpayer can be understood as the result of a social process involving (at least) three different actors: the single taxpayer, the government, and the "other" taxpayers. Each of these actors is tied up to the others by a different channel of social interactions. The relationship between the individual taxpayer and the government involves at least two elements. On the one hand, there is certainly the element of coercion typically studied in the AS type of models. On the other hand, there is also an element of exchange. From the latter point of view, the taxpayer can be seen as exchanging purchasing power in return for governmental goods and services. Here, the rule of fairness is also clear. For the consumer to be satisfied by this exchange, he must perceive the "terms of trade" --implicitly determined in the exchange with government-- as being "fair". That is, there has to be a link, perceived as fair, between the amount of purchasing power transferred to government and the quantity and quality of goods and services transferred back from government to the consumer[8]. It seems also reasonable to argue that if the

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[8]The governmental goods and services supplied to taxpayers, which I refer to in the text, must be interpreted in a wide sense. If taxpayers are altruistic, for example, and the welfare of the poor members of the society enters in their utility functions, an efficient redistribution policy by government may well be

taxpayer perceives his terms of trade with government as being "unfair" he will try to adjust his behaviour so as to correct this perceived unjustice. As Spicer and Becker (1980:173) put it "tax evasion may be seen partly as a means by which taxpayers attempt to restore equity in their terms of trade with the government".

Another feature of the same relationship involves the other taxpayers as well. Even if the taxpayer is satisfied, as a whole, by the terms of trade between private consumption of the collectivity and the amount of goods and services supplied by the state, he may still perceive an inequity between his fiscal treatment and the tax treatment of the other taxpayers. According to Homans (1961) perceived inequity in exchange relationships creates a sense of distress in the participants. And according to Adams (1965), this sense of distress may induce participants to reduce inequity by adjusting their contributions to the exchange relationship. Again, tax evasion can be seen as an attempt by the single taxpayer to restore fairness at the level of exchange with government.

Finally, there is a third element which involves more directly the relationship between the individual taxpayer and the "other" taxpayers. Even if the individual taxpayer perceives the "institutionalized" terms of trade between the government and the collectivity of taxpayers, including himself, as being "fair"

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perceived by consumers as part of the exchange. In the same direction moves Thurow's interpretation of income distribution as a publicly produced public good (Thurow, 1971).

--that is, if the taxpayer considers the fiscal structure as being fair and the pattern of governmental expenditure on goods and services as being adequate-- what really matters from the point of view of the individual taxpayer is the actual behaviour of the other taxpayers. If the latter evade taxes, the individual taxpayer may still feel as "unfair" the tax burden lying on himself. This, even if the individual taxpayer would be willing to pay the taxes --i.e. he would be willing to accept the terms of trade offered by government-- provided that the other taxpayers did the same. Again, tax evasion can be seen as an attempt to restore "fairness", this time in the relationship with the other taxpayers.

Summing up, this approach sees the decision to evade by a single taxpayer as largely determined by the perceived fairness of the existing relationships between him, the state and the "other" taxpayers. The coercive powers of the state represent just but one, and not necessarily the most important, of the determinants of tax evasion. If these are the ideas, intuitively quite satisfactory and with some experimental evidence to support them[9], there remains the problem of how to formalize them. In order to understand the actual formalization attempted in the next chapter, consider for a moment a world where the state supplies public goods but it does not have coercive powers on individuals. Should

[9]See the works by Spicer and his collaborators quoted above in the text.

we expect individuals to pay the taxes? As we know from chapter II, in a model where selfishness is assumed from the outset and where each taxpayer is too small to affect in any way the supply of public goods by government, the answer must necessarily be negative. Each individual would take a free-ride and if government's powers of coercion are nil each individual would evade completely the taxes. This is, of course, the conventional argument in support of the view that government must have in the first place coercive powers.

But suppose now, following our previous discussion, that individuals are ruled by considerations of fairness. Then, even in the absence of coercion by government, each individual would adjust his tax payment so as to reach fair terms of trade with respect to the government and the other taxpayers. Let us call q_i^F the amount of his income that an individual i would perceive as fair to give to the state in exchange for the received goods. On the basis of our previous discussion q_i^F will be a function of several elements: the amount of goods and services provided by the state, the tax structure, and the actual level of tax evasion by the other taxpayers. (For the sake of the presentation we assume here that all these elements are known and given to individual i: see next chapter for a discussion on the informational requirements). Having established a value for q_i^F, individual i will instead be confronted by the state with an income demand of t_iI_i, where t_i is the average rate of tax that individual i is asked to pay on his income and I_i represents his given income.

Then, we can determine the <u>desired</u> level of tax evasion by individual i simply as the difference between the amount of taxes he is asked to pay and the amount of taxes that he would wish to pay:

$$z_i = t_i I_i - q_i^F .$$

where z_i is the desired level of evaded tax. Then, leaving aside the problem of corner solutions (see below), in a world where governments do not have coercive powers but individuals are ruled by norms of fairness each taxpayer i would evade exactly z_i. But of course in the real world governments do have coercive powers and they usually try to control tax evasion by imposing penalties and controls on tax declarations. Therefore, in such a world, even the fairness ruled taxpayer must realize that there is a risk involved in evading. Hence, even if his desired level of evasion is still z_i, the taxpayer may nevertheless decide to evade less than z_i if he perceives that doing otherwise he could put in danger the very goal that he is pursuing through tax evasion: the reestablishment of fair terms of trade. Therefore how much of z_i a taxpayer i will actually manage to realize in practice will depend on the usual parameters of the portfolio choice model: individual i's attitude toward risk, the probability of being caught if evading and the penalty rate on evaded tax.

Several elements of this model should be noted. First, note that we split the actual decision of evading by a single taxpayer in two parts. On the one hand, and previously on logical grounds, the individual taxpayer decides which part of his tax, if any, he wishes to evade. According to our fairness approach, this decision is taken on the basis of the perceived equity of the tax system in the three dimensions that we indicated previously. Subsequently, on the basis of the parameters by which government controls tax evasion the individual decides how much of this desired tax evasion he can reach in practice. Morality, in other words, constrains tax behaviour in advance of any other considerations on the riskiness to evade. This seems reasonable. Also, there is some experimental evidence which supports the view that the decision to evade can be split in the two stages indicated above (Friedland et al., 1978).

Second, there is no guarantee in this approach that $t_i I_i \geq z_i \geq 0$. For example, $z_i > t_i I_i$ entails that an individual feels so disadvantaged by the fiscal system that he would consider it fair not to pay a tax but to receive a subsidy from government. On the other hand, $z_i < 0$ implies that individual i would wish to pay a higher amount of taxes than he is asked to pay. Since no individual can either evade a negative amount of income or force the state to pay him a subsidy, in the next chapter I will model these two cases as corner solutions. This does not mean however that I consider them as unrealistic or economically irrelevant; more simply, they are likely to affect other phenomena beyond tax evasion. For example, if a relevant share of the population feels so disadvantaged by the fiscal system as to consider "fair" a negative tax contribution we should expect as a consequence social conflict to arise elsewhere in the economy, for instance in the labor market and\or in the political arena[10]. Similarly, if an individual would consider it fair to be taxed more heavily than he is, we may expect that he will try to reestablish a fair equilibrium by giving part of his extra income away, for example by contributing to a charity.[11]

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[10]The recent labor conflicts in Italy for a "just" fiscal system seem to represent a good example of this situation.

[11]This point deserves further attention since it could help to cast some light on the motivations for giving. At the very end, when an individual decides to donate one dollar to a charity, does not this mean that he would be willing to pay a dollar more of taxes provided that he were sure that this extra dollar was spent on that charitable activity ? Note also that altruistic behaviour is not inconsistent with evading taxes; in terms of our approach this could be interpreted in the sense that the evader either does not trust the state as supplier of public goods or he does not agree with the way in which the state decides to spend the money collected through taxes (i.e. the composition of public expenditure). In other words, an evader may not necessarily object to the magnitude of his tax burden but he may object to the way in which his money is spent by the state. I think that at least in Italy --where we observe both a very high level of tax evasion and a large amount of voluntary provision to charities and where there is a widespread legitimate distrust among citizens in the ability of the state to spend efficiently tax revenue-- this interpretation could have some appeal.

Finally, the equation above allows us to appreciate the difference between my fairness approach and the portfolio choice model of income tax evasion. As should be clear, the latter represents but a special case of the former, a special case where q_i^F is always identically equal to zero for all individuals. In this case, in fact, as in the AS type of model, each individual would wish to evade entirely the taxes and only the use of coercive powers by government could restrain him from doing so. In my approach, however, there may be cases where an individual is willing to pay, at least to some extent, the tax independently of any coercion by government; and in any case the maximum amount of evaded tax is bounded above by the desired level of tax evasion. As I am going to show in the next two chapters, this difference is enough to introduce drastic changes in the comparative statics properties of the model.

If the above discussion provides an intuitive explanation of the formalization of tax behaviour attempted in the next chapter, there still remains the problem of how to model satisfactorily the process by which an individual comes to select a value for q_i^F, the fair tax. As anticipated above, I will make use here of my Kantian rules coupled with reciprocity considerations. The main idea is as follows. I will suppose that the typical taxpayer, faced with a given supply of public goods by the state, will first express a moral judgment of the Kantian type by asking himself "assuming that I can select a different tax payment for each individual composing the society, how much do I think that I and my fellow taxpayers should pay for the public goods supplied by the state?". In line with the analysis developed in chapters III and IV the answer to this question will be based on the idea that a Kantian individual would consider it fair to pay to the state, in exchange for the given supply of public goods, as much as he would wish other individuals in similar conditions to pay. As a result of this ethical judgment, the typical taxpayer will then determine a vector of "just" taxes for himself and the other individuals composing the society; for obvious reasons, in the next chapter I will be calling this vector of taxes the "Kantian taxes". If the individual taxpayer acted according to the principle of unconditional commitment this would be the end of the story because he would then wish to pay his Kantian tax regardless of the tax behaviour of the other taxpayers.

But in the context of tax compliance, it seems to be both more realistic and more in line with the literature quoted above, to assume that the typical taxpayer will also be influenced by considerations of reciprocity. That is, it seems more reasonable to suppose that the tax a taxpayer wishes to pay will also be influenced by the perceived tax behaviour of the other taxpayers. We will then model this feature by assuming that a taxpayer will wish to pay his Kantian tax if and only if he perceives that the other taxpayers do the same. If the other taxpayers do not "keep the deal" and pay to the state an amount which is different from what the Kantian individual would like them to pay he will also adjust his desired tax contribution by revising upward or downward his "Kantian tax". Therefore the "fair tax" that at the end of this process an individual desires to pay turns out to be a function of both his selected "Kantian tax" and the difference between how much he would wish the other taxpayers to pay and how much he perceives that they actually pay.

Having introduced intuitively my fairness approach to income tax evasion let us now turn to the model itself.

Chapter VI: A Fairness Approach to Income Tax Evasion: the Model and the Analysis with Exogenous Public Expenditure

Table VI.1: List of the symbols used in Chapters VI and VII

i = 1,2 types of individuals

h = 1,...,N/2 number of individuals of each type

C_{ih} = private consumption of an individual h belonging to group i

G = public good

U^i = utility function of individuals of type i

I_i = lump sum income of individuals of type i

q_{ih} = tax payment of an individual h belonging to group i

$1/\psi(N)$ = marginal rate of transformation between the private good and the public good

$1/\Psi = N/\psi(N)$ average marginal rate of transformation of the private good in the public good for large economies

$w^i = U^i_G/U^i_C$ = the fair price of G for an individual of type i

$\bar{I}$ = average income

q^i_j = the Kantian contribution selected by an individual of type i for an individual of type j

$t^i_j = q^i_j/I_j$ = the Kantian tax selected by an individual of type i for an individual belonging to group j

q^F_{ih} = the fair tax of an individual h belonging to group i

t_i = average rate of tax imposed by the state on individual of type i

x_j = average amount of evaded tax by individuals of type j

x_i^* = average amount of evaded tax by individuals of type i except individual h belonging to group i

β_1^i = reciprocity weight imposed by an individual of type i on the perceived average tax payment by individuals of the same type

β_2^i = reciprocity weight imposed by an individual of type i on the perceived average tax payment by individuals of the other type

z_{ih} = desired level of evaded tax by an individual h belonging to group i

$\bar{x}_{ih}$ = fairness constraint (possible desired evaded tax) by an individual h belonging to group i

p_i = probability of being detected if evading for an individual of type i

π = surcharge rate imposed on detected evaded tax

x_{ih}^o = the expected utility choice of evaded tax for an individual h belonging to group i

$\hat{x}_{ih}$ = actual evaded tax for an individual h belonging to group i

s_i = $(1-p_i)-\pi p_i$ =expected price on unit of evaded tax for an individual of type i

ϑ^i = shorthand for $\beta_2^i/(1-\beta_1^i)$

ε^i =quantity elasticity of the fair price for an individual of type i

δ =parameter which express government's beliefs about expected tax evasion

k = I_2/I_1 = index of income inequality

D = shorthand for $(s+\pi)/\pi$

VI.1: The Model: Preliminaries

In order to keep the discussion as simple as possible I will consider an economy populated by only two types of identical individuals, individuals of type 1 and individuals of type 2, indexed by $i=1,2$. These two types should be thought of as representing in the simplest possible way the different social groups in which a society can be meaningfully divided for our aims: employees versus professional workers, working class versus middle class, rentiers versus workers. From our point of view, the important feature which distinguishes these different social groups is that they are usually characterized by different income endowments and by different opportunities for evading. This is due both to the different way in which these different social groups raise their income and to the fact that government expenditure for detecting tax evasion is usually different across these groups. The assumption of identity inside each group entails that each individual of a given type is characterized by the same preferences and endowed by the same income as any other individual of the same type. This is equivalent to assuming that differences in income and opportunities for evading inside a social group are less relevant than differences across social groups. I take this as being quite reasonable.

In the paper I will also assume that there is the same number of individuals in each group: i.e. I will postulate that there are

N/2 individuals in both group 1 and group 2. This is just to avoid having to distinguish in the paper between the different shares of the two groups in the total population and in no way will affect the results. When needed, I shall distinguish an individual from the group he belongs to by adding a suffix h to the variable under consideration. For example, C_{ih} is the private consumption of an individual h belonging to group i, where h=1....N/2 and i=1,2.

Each type's preferences can be represented by a strictly concave, twice differentiable utility function which obeys the axioms of expected utility theory. Such a utility function, denoted $U^i(.)$, i=1,2, is defined on two types of goods, a pure private good (private consumption), C_{ih}, and a public good G. This is only for simplicity and the analysis could be easily extended to many goods. We can then write the utility function of a representative consumer h belonging to group i as:

$$(1) \qquad U^i = U^i(C_{ih}, G) \qquad i=1,2 \;,\; h=1....N/2$$

I will also assume that each type of individual is endowed with an exogenously given lump-sum income that we indicate by I_i, i=1,2. This assumption implies that we are considering here an economy where the decision to evade and the fundamental leisure-work decision are completely separated choices. In this extreme form this assumption is surely unrealistic; but the price of adding a leisure-work choice to the model would be very high indeed. First, as I recalled in note 2 of the previous chapter, tax evasion models with labor supply choice are usually characterized by a complete ambiguity in the signs of the comparative statics derivatives. Second, as argued in chapters III and IV, my formalization of Kantian rules can be defended only in an economy where income is not earned. I am therefore obliged to impose such an assumption: it will be the task of further research to verify if the results go through even in an economy with labor supply decisions.[1]

Turning now to the production function of the public good, we assume, for simplicity, that the supply of the public good is proportional to the amount of the private good used as input in its production. Letting q_{ih} represent individual h's "contribution" to the public good --i.e. q_{ih} is the amount of his private consumption which individual h gives up for the production of the public good -- we write the production function as:

(2) $$G=1/\psi(N)\ \Sigma_i \Sigma_h q_{ih}$$

where $\psi(.)$ represents the marginal rate of transformation (henceforth MRT). Note that in equation (2) we write the MRT as a function of the number of individuals composing the society and we further assume that $d\psi(N)/dN > 0$. This can be thought of as a crowding assumption: the public good is not pure and some amount

[1] It should be noted that most of the literature on tax evasion is couched in terms of economies with lump sum incomes. This is so to avoid the ambiguity noted in the text.

of rivalry is present in consumption. For the public goods that we will be considering in this paper this seems to be perfectly adequate (see below). Following Gordon and Cowell (1988) we further assume the two following limiting assumptions on the production function of the public good:

$$\lim_{N\to\infty} 1/\psi(N) = 0$$
(3)
$$\lim_{N\to\infty} N/\psi(N) = 1/\Psi > 0$$

These assumptions imply that for large economies, where $N\to\infty$, for each individual h, $\partial G/\partial q_{ih} = 1/\psi(N) = 0$; that is, for each individual h the amount of public good supplied in the economy is fixed and cannot be modified by unilaterally changing his contribution. This assumption, in addition of being entirely reasonable for the kind of problems that we address, will allow us to simplify greatly the analysis when we consider the case of endogenous public expenditure. In what follows, we will always assume an economy where N is "large".

VI.2 The Fair Tax and the Desired Level of Tax Evasion

As anticipated in the previous chapter, in order to compute the fair tax we have first to determine the rate of tax that a typical individual would wish to pay when acting as a Kantian. In chapter III we saw that an individual behaving according to the second Kantian rule would select fair contributions to a public good by following a two-step process (see result 4, chapter III). First, he would select an optimal amount of public good supply and a unique level of private good consumption for all the members of the community as if he were endowed with average income and faced a price of the public good equal to the MRT divided by N. Second, he would distribute the burden of paying for the selected amount of public good among individuals so as to equalize private consumption.

It then seems natural to assume that a Kantian individual would follow exactly the same two-step process in deciding a fair contribution to the state when it is the latter which will supply the public good. In this case however it is not the fair quantity of public good supply which has to be chosen, since that is unilaterally set up by the state, but rather the <u>fair price</u> to be paid for the given amount of public good. We will then model the process of reaching a Kantian contribution by assuming that first, a Kantian individual selects a fair price to pay for the given amount of public good supply as if he were endowed with average income and he had to pay a "price" for the given amount of public good equal to the MRT divided by N. By multiplying this fair price for the given amount of public good supplied we get the amount of income that, on average, the Kantian individual would wish that all the individuals composing the economy paid to the state. Finally, we redistribute this average amount across individuals so as to equalize private consumption.

As for the fair price itself we assume that it is equal to the marginal evaluation or marginal willingness to pay --in terms of the numeraire, private consumption-- that a Kantian individual would impose on the the given amount of public good supplied. Using equations (1) to (3) we then write the fair price w^i for an individual belonging to group i as:

$$(4) \qquad w^i(G,\bar{I}) = U^i_G/U^i_C(G;\ \bar{I}-\Psi G)$$

That is, the price that a Kantian individual belonging to group i would perceive as fair is equal to his marginal rate of substitution evaluated at the given amount of public good and at the level of private good which would result if the Kantian individual were endowed with average income ($\bar{I}$) and he had to pay for the public good a "price" equal to the MRT divided by N. By multiplying this fair price for the given amount of public good supply and by redistributing the resulting amount across individuals so as to equalize private consumption, as indicated above, we get the amount of money that an individual belonging to group i would wish that individuals belonging to his group and to the other group paid to the state:

$$(5) \qquad \begin{aligned} q^i_j &= (I_j - \bar{I}) + w^i(G,\bar{I})G \\ q^i_i &= (I_i - \bar{I}) + w^i(G,\bar{I})G \end{aligned} \qquad i=1,2\ ;\ j=1,2\ ;\ i\neq j$$

where q_j^i is the Kantian contribution that an individual belonging to group i would wish that an individual belonging to group j paid to the state --and similarly for q_i^i. Equation (5) derives strictly from the analysis of the second Kantian rule performed in chapter III. Note however that in getting through to equation (5) we had implicitly to assume that individual i not only knows the distribution of income in the population --which in terms of large aggregates may be not too unrealistic-- but he also has accurate estimates of the production function for the public good. This certainly requires much more information than that a taxpayer typically possesses. Note however that it would impossible to talk meaningfully of perceived fairness of the terms of trade between citizens and the state if we did not assume that taxpayers have at least a rough idea of the existing trade off between purchasing power (private consumption) and supply of public goods. And, as many survey researches show (see for example, Likert, 1966) people do make judgments regarding the fairness of the relationship between what they pay to the state and what they receive back[2]. Furthermore, most of the so-called public goods supplied by the state are really private goods with

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[2]As Spicer and Lundstedt (1976:296) put it "[while] it can be argued that many taxpayers are not able to assess the exact value of what they pay or what they receive from government in return...it seems reasonable to suggest that taxpayers have general impressions and attitudes concerning their and other terms of trade with government"

public good characteristics (for example, both education and health enter into this category). For this type of good our assumption above is not too unrealistic, because typically such goods are also privately supplied. Then, simply by looking at the existing terms of trade in the private market, an individual is likely to obtain a rough knowledge of the production function for these goods.[3]

Turning again to equation (5) it is now easy to obtain the Kantian taxes for each type of individuals. We just define them as the average rates resulting by dividing q_j^i for the income of the corresponding type of individual; that is:

$$
(6) \qquad \begin{aligned} t_i^i &= q_i^i/I_i \\ t_j^i &= q_j^i/I_j \end{aligned} \qquad i=1,2;\ j=1,2;\ i\neq j
$$

t_j^i (t_i^i) is the Kantian rate of tax: that is, it is the average rate that a Kantian individual of type i would wish an individual of type j (i) to pay on his income. Note, from equation (5), that the Kantian tax is a function of the total provision of

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[3]In a previous version of this paper, I studied the case where individuals had only a probabilistic knowledge of the production function for the public good. However, in addition to complicating the model quite seriously, this hypothesis produced different results from the ones presented here only if it was assumed that the perception individuals had of the production function was seriously biased either in one direction or another. Since there is no reason to suppose that this occurs in the real world I decided to stick to the simpler version.

the public good by the state as well as of the income distribution. This is what one would have expected.

Turning now to the fair tax, we argued in the previous chapter that this should be a function of both the Kantian tax selected by an individual and of his perception of the actual tax behaviour of the other taxpayers. We assume here that each individual knows the <u>average</u> tax payment of each group composing the society. This does not sound too unrealistic. In fact, such a contribution is just given by the difference between the income tax raised on each individual of each group and the average level of tax evasion inside that group. At least for large aggregates it is plausible to suppose that an individual may come to get some idea about these two variables. For example, at least in Italy, both pieces of information are available in the press.[4] Let us then write the fair tax as a function of the Kantian tax and of the average level of tax payment inside each group:

$$(7,8) \qquad q^{F}_{ih} = f(q^{i}_{i};\ t_i I_i - x^{*}_{i};\ t_j I_j - x_j)$$

where t_i (t_j) is the average rate of tax imposed by the state

[4]In Italy, in May, the Newspapers usually publish the main tables of the Annual Report of the Minister of Finance on tax collection. These tables contain the average income declared by each single category. Together with the new Coefficienti Presuntivi di Reddito (Presumed Income Coefficients) these data would allow one to compute quite easily the estimated tax evasion for each single category. Some economic newspapers have already attempted this exercise: see for example Il Sole 24 Ore, 7\8\1989.

on the income of individuals of type i (j)[5], x^*_i is the average level of tax evaded by all the individuals in group i except h and x_j is the average level of tax evaded by all individuals belonging to group j (that is, $x_j=2\Sigma x_{jh}/N$ and $x^*_i=2\Sigma_{k\neq h}x_{ik}/[N-2]$). Even in this simpler formulation many specifications of eq.(7,8) would be still possible. Mainly for simplicity we choose a linear approximation of eq.(7,8), where the fair tax of an individual h belonging to group i depends, in addition to the Kantian tax, on the difference between the tax that the individual would like the other taxpayers to pay and what they actually pay on average:

$$(9)\qquad q^F_{ih} = q^i_i - \beta^i_1(t^i_i I_i - (t_i I_i - x^*_i)) - \beta^i_2(t^i_j I_j - (t_j I_j - x_j))$$

where $0\leq\beta^i_t\leq 1$, $t=1,2$, $\beta^i_1+\beta^i_2\leq 1$, represent the <u>reciprocity weights</u> on the other contributions. Note that in line with our previous discussion, eq.(9) implies that individual h would wish to pay exactly the chosen Kantian rate of tax if everybody else did the same and would revise upward or downward his wished contribution following other behaviour. From eq.(9), by subtracting q^F_{ih} from $t_i I_i$, we can finally get the desired level of

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[5]As yet, I have not really introduced a tax system in the model and the formulation adopted in the text is intended to maintain the maximum of generality. What I am implicitly assuming is that given a particular fiscal system (proportional or progressive) individuals are able to compute the average rate of tax on their and other type's income. However, in the rest of the paper, as it is customary in most of the literature on tax evasion I will assume a proportional tax system. See chapter VII for some comments on this issue.

tax evasion; doing this and rearranging terms:

$$(10) \quad z_{ih} = (1 - \beta_1^i) I_i (t_i - t_i^i) + \beta_1^i x_i^* + \beta_2^i x_j + \beta_2^i I_j (t_j^i - t_j)$$

where as above z_{ih} represents the desired level of tax evasion. Eq.(10) offers a nice decomposition of the desired level of tax evasion into its components. As argued in the previous chapter, the desired level of tax evasion turns out to be a function of the amount of public good supplied by the state (through the Kantian taxes), the fiscal structure (through the actual rates of taxes), and the level of tax evasion by the other individuals composing the society. It does seem therefore that our formulation of the desired level of tax evasion is able to capture the essence of the fairness approach to income tax evasion outlined in the previous chapter.

The signs of the partial derivatives of z_{ih} also seem to be in line with both intuition and the sociological literature quoted above. Ceteris paribus, the desired level of tax evasion is in fact an increasing function of the difference between the actual tax that an individual is asked to pay and the Kantian tax; an increasing function of the level of perceived tax evasion by the other individuals in the society; and a decreasing function of the difference between the actual tax imposed on the individuals of the other type and the tax that individual h would wish them to pay. All these comparative static results respect the flavour of the sociological literature quoted above.

Also note that the linear specification of the desired level of tax evasion obtained in eq.(10) offers a further advantage: depending on the values attributed to the reciprocity weights, it allows one to get as special cases many important alternative models of tax evasion. For example, for $\beta_1^i=\beta_2^i=0$, we return to the case of pure Kantian behaviour studied in the chapters III and IV; for $\beta_1^i=1$ and $\beta_2^i=0$ we get a pure reciprocity model; and so on. For future reference, note that if we assume the limiting case that $\beta_1^i+\beta_2^i=1$ the Kantian taxes disappear and we get a model of reciprocity among social groups corrected by differences in income distribution (this can be easily verified by substituting eq.(5) and (6) in eq.(10)).

Finally note that there is no a-priori reason why z_{ih} should lie inside the range of possible tax evasion. For the reasons stressed in the previous chapter we avoid this problem by modeling the extreme cases as corner solutions; that is, we define a new variable $\bar{x}_{ih}$ such that:

$$
(11) \quad
\begin{aligned}
\bar{x}_{ih} &= z_{ih} && \text{if} \quad 0 \le z_{ih} \le t_i I_i \\
\bar{x}_{ih} &= 0 && \text{if} \quad z_{ih} < 0 \\
\bar{x}_{ih} &= t_i I_i && \text{if} \quad z_{ih} > t_i I_i
\end{aligned}
$$

$\bar{x}_{ih}$ could be termed the amount of <u>feasible</u> desired tax evasion. But, for reasons which will become immediately apparent and to avoid confusion with the desired level of tax evasion, I will instead call $\bar{x}_{ih}$ the <u>fairness constraint</u> on income tax evasion.

VI.3 The Tax Evasion Choice

$\bar{x}_{ih}$ then represents the (possible) amount of tax that an individual h, taking into account the behaviour of the other individuals in the society and the amount of public good supplied by the state, would evade if he did not face any risk in doing so. But, as we argued in the previous chapter, there is usually a risk in evading and an individual must take it into account when deciding how much of his desired tax evasion he can achieve in practice. It then seems natural to model taxpayer behaviour as the result of a constrained maximization process, where the taxpayer maximizes an expected utility function --which takes into account the riskiness of evading-- subject to the constraint represented by the amount of possible desired tax evasion --which takes into account the fairness of evading. Calling p_i the probability of detection for an individual of type i, and π ($\pi>0$) the penalty imposed on tax evaded[6], both supposed known and given to the taxpayer, we can express the individual taxpayer's problem as follows:

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[6]Note that, for obvious reasons, while we distinguish between the probability of detection for two groups, we assume an unique penalty rate.

$$(12) \quad \max_{x_{ih}} (1-p_i)U^i((1-t)I_i+x_{ih};G) + p_iU^i((1-t)I_i-\pi x_{ih};G)$$

$$\text{s.t. } 0 \le x_{ih} \le \bar{x}_{ih}$$

where in eq.(12) I have assumed, for simplicity, a proportional tax system (i.e. $t_1=t_2=t$). Let x^o_{ih} be the amount of evaded tax that the individual h would choose by maximization of the expected utility function in (12) if he were <u>not</u> subject to the fairness constraint. Then we can more easily express the solution to problem (12), let us call it $\hat{x}_{ih}$, as $\hat{x}_{ih}=\min(x^o_{ih};\bar{x}_{ih})$.

It is easy to check that the last equation does indeed reflect the intuitive argument for a fairness approach to income tax evasion that we presented in the previous chapter. Suppose first that $x^o_{ih}>\bar{x}_{ih}$; then selfishness --i.e. the individual calculus of expected costs and benefits-- dictates an higher level of tax evasion that the amount that the taxpayer would consider it fair to evade. But, according to our fairness approach, no individual would ever evade more than $\bar{x}_{ih}$, simply because he would consider it "unfair" to do so. Therefore, $\hat{x}_{ih}=\bar{x}_{ih}$. Suppose next that $x^o_{ih}<\bar{x}_{ih}$. Then the individual would wish to evade up to $\bar{x}_{ih}$; but evading any amount above x^o_{ih} could be self-defeating in terms of h's own perceptions of the risk involved, thus undermining the reestablishment of fair terms of trade that the individual is trying to achieve, in the first place, by evading. Then, $\hat{x}_{ih}=x^o_{ih}$.

As should also be clear, x^o_{ih} represents the solution to the problem of income tax evasion as presented in the usual portfolio choice model. Not surprisingly therefore, the comparative statics derivatives of x^o_{ih} are entirely in line with the results typically reached in this literature. In order to simplify the presentation, I will here limit myself to stating these results, for future reference, without providing a formal proof[7]. The references given next to each result indicate to the interested reader where he can find the formal proofs.

The existence of an unique solution to the maximization of the expected utility function in (12) and the second order conditions are guaranteed by the assumption of strict concavity of the utility function that we imposed at the beginning of the paper and by the convexity of the constraint. The assumption of concavity implies also risk-averse behaviour on the part of the typical taxpayer. An interior solution for x^o_{ih} (i.e. a solution where $0 \leq x^o_{ih} \leq tI_i$) is guaranteed by the following two conditions (Allingham and Sandmo, 1972):

$$(1 - p_i) - \pi p_i = s_i > 0$$

(13)

$$(1 - p_i)U^i_c(I_i;G) + p_i U^i_c((1 - \pi t)I_i;G) < 0$$

where the suffix in the utility function indicates the

---

[7]Most of these results are however proved, in passing, in chapter VII.

partial derivative with respect to private consumption. The first condition above has a ready interpretation. For the risk-averse taxpayer to evade, the expected gains from the evasion must be greater than the expected costs; or to put it differently, the expected "price" of tax evasion, s_i, must be positive. In what follows, unless otherwise stated, I will always assume the conditions stated in eq.(13) to hold.

The assumption of risk-averse behaviour is enough to sign the partial derivatives of x^o_{ih} with respect to p_i and π, which are both negative (Allingham and Sandmo, 1972). The comparative statics derivatives with respect to the income and the tax rate are instead in the general case ambiguous; however, as recalled in the previous chapter, the assumption of DARA allows one to sign both of them as follows: $\partial x^o_{ih}/\partial I_i>0$, $\partial x^o_{ih}/\partial t<0$[8] (see Yitzhaki, 1974). The derivative with respect to G is instead ambiguous even in this latter case; however Cowell and Gordon (1988) have shown that if the demand for the public good is characterized by zero income elasticity and assuming both risk aversion and DARA, then it follows that $\partial x^o_{ih}/\partial G>0$ (see chapter VII). Another case where the latter derivative can be signed is given by separability of the utility function in private and public good. In this case trivially, $\partial x^o_{ih}/\partial G=0$. I shall exploit both of these special cases in the next sections.

It is perhaps worth remembering that the derivatives above

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[8]Indeed, as is easy to verify, $\partial x^o_{ih}/\partial I_i = -\partial x^o_{ih}/\partial t[1- t]/I_i$.

are not the equilibrium derivatives. So far, neither have we taken in account the interdependence among the agents nor have we defined government behaviour. This will be our task in the next section.

VI.4 Equilibrium and Comparative Statics Properties with Exogenous Public Expenditure

VI.4.1 The Equilibrium

Before analyzing the equilibrium in my model we have first to discuss government behaviour. In fact, as I am going to show, the properties of the equilibrium turn out to be strictly dependent on the assumption made about the way in which government is supposed to finance public expenditure. If we adopt a short run perspective public expenditure and tax revenue are likely to be independent. In the real world different governmental agencies are in charge of decisions concerning taxation and expenditure; and the possible discrepancies between the two are covered by public budget surplus or deficit. In the short run then, public expenditure can be considered "exogenous" in the sense that it can be determined, at least to an extent, independently of revenue considerations. Therefore, in the short run, we can think of government as disposing of two separate decision variables, the tax structure and public expenditure. In the long run, on the contrary, the only appropriate concept of equilibrium is one where the public budget is fully balanced, that is, where all public expenditure is covered by tax revenue. In this case, public expenditure becomes "endogenous" in the sense that once government has selected the rate of tax, tax revenue and therefore public expenditure is, given the tax behaviour of taxpayers, wholly determined. Hence in the long run, government can only control a choice variable, the tax structure and it loses the control of total public expenditure (but not necessarily of its composition: see chapter VII, note 8).

In the rest of this chapter we will analyze the extreme case of wholly exogenous public expenditure leaving the analysis of endogenous public expenditure to the next chapter. To put it a bit more formally, we assume that, in the short run, government can freely choose among the following set of parameters $A=\{p_1,p_2,t,\pi,G\}$. We can then state formally our definition of equilibrium as follows:

Equilibrium "Given A, an equilibrium is a vector of evaded taxes, x_{ih}, such that $x_{ih}=\hat{x}_{ih}, \forall i, \forall h$."

That is, by recalling the definition of $\hat{x}_{ih}$ given in the previous section, an equilibrium is a vector of evaded taxes such that each individual, taking the behaviour of any other agent in the economy and the parameters set up by government as given, maximizes his (expected) selfish utility function subject to his fairness constraint. Note that, given A, our model represents just a simple example of a non-cooperative game under Nash behaviour, where the players are the taxpayers and where the strategy set of each player is constrained by the amount of (possible) tax evasion. Then in order to prove the existence of an equilibrium in my model I will just need to use the standard tools of game theory for this type of game. To this end note that with exogenous public expenditure an individual taxpayer's choice is influenced by the other taxpayers' choices only through the fairness constraint; we stress this fact by writing $\hat{x}_{ih}=\min(x^o_{ih};\bar{x}_{ih}(x_{i-h}))=\hat{x}_{ih}(x_{i-h})$ where x_{i-h} indicates the vector of choices of tax evasion of all the individuals composing the society except the individual h belonging to group i. Second, note that $\hat{x}_{ih}(x_{i-h})$ --the best reply function of individual h in the terminology of game theory-- is a continuous function of x_{i-h}. We can then state:

<u>Proposition 1</u> "A Nash equilibrium exists"

<u>Proof</u> "Let $X=\{x \text{ in } R^N: 0\leq x_{ih}\leq tI_i$ for $i=1,2$ and $h=1,\ldots,N/2\}$ and let $F(X)=\{\hat{x}_{11}(x_{1-1}),\ldots,\hat{x}_{1N/2}(x_{1-N/2}),\hat{x}_{21}(x_{2-1}),\ldots,\hat{x}_{2N/2}(x_{2-N/2})\}$. X is clearly a compact and convex set. From continuity of each individual best reply function, the function F(X) defines a continuous function from the set X in itself. Hence, by Brouwer's Fixed Point Theorem it must exist a fixed point. That point is an Nash equilibrium in our game"

Hence an equilibrium exists. But we can prove under reasonable assumptions a much stronger result:

Proposition 2 "Suppose $\beta_1^i+\beta_2^i<1$, i=1,2. Then equilibrium is unique"

Proof "By using the definition of a contraction[9], it is easy to verify that $\hat{x}_{ih}=\min(x_{ih}^o;\bar{x}_{ih}(x_{i-h}))=\hat{x}_{ih}(x_{i-h})$ is a contraction if both x_{ih}^o and $\bar{x}_{ih}(x_{i-h})$ are contractions in x_{i-h}. But x_{ih}^o is trivially a contraction and $\bar{x}_{ih}(x_{i-h})$ is a contraction if $\beta_1^i+\beta_2^i<1$. Then, provided that the last condition holds for i=1,2, each individual best reply function is a contraction which implies that F(X) is also a contraction: then equilibrium is unique. (See Friedman, 1986:44)."

The intuition behind this last result is pretty clear. As we noted at the end of section VI.2 above, if $\beta_1^i+\beta_2^i=1$ the Kantian taxes disappear from the computation of desired tax evasion and we are left with a simple model of reciprocity across social groups. It is then obvious that in such model we may have multiple equilibria. In what follows we will stick to the reasonable case where $\beta_1^i+\beta_2^i<1$. This allows us to prove another important result:

---

[9]Let f(x) be a function with domain $A \subset R^m$ and range $B \subset R^n$. If there is a positive scalar $\lambda<1$ such that for any x and $x' \in A$, $d(f(x),f(x')) \leq \lambda d(x,x')$, where d(.) indicates the distance between the two points, then f(x) is a contraction. If f(x) is a differentiable function, the definition above is equivalent to $\Sigma_i |\partial f_j/\partial x_i| \leq \lambda$ for each component $f_j(x)$ of $f(x)=\{f_1(x),\ldots,f_n(x)\}$.

Proposition 3 "If $\beta_1^i+\beta_2^i<1$, $i=1,2$, equilibrium is symmetric: i.e. $\hat{x}_{ih}=\hat{x}_i$, $\forall h$, $i=1,2$."

Proof "Suppose not. Then there exist at least two individuals A and B belonging to the same group i such that $\hat{x}_{iA}\neq\hat{x}_{iB}$ in equilibrium. But, by assumption of identical individuals inside each group, if $\hat{x}_{iA}$ and $\hat{x}_{iB}$ are equilibrium choices it is possible to swap the choices between the two individuals still obtaining an equilibrium: that is, an allocation where A evades $\hat{x}_{iB}$, B evades $\hat{x}_{iA}$, and everybody else keeps his choice unchanged is still an equilibrium. But then we would have two equilibria, in contradiction with Proposition 2 above. Hence, $\hat{x}_{iA}=\hat{x}_{iB}$ for all A and B belonging to group i."

Then, in equilibrium, each individual of the same group must evade the same amount of tax. This will allow us in the next sub-section to consider the comparative static results of the model as resulting from the interactions of only two individuals. Finally note that $\beta_1^i+\beta_2^i<1$, $i=1,2$ also implies, under a simple adjustment mechanism, that the unique symmetric equilibrium is also locally stable (see Cornes and Sandler, 1986:92 and chapter VII, section VII.1).

VI.4.2 Comparative statics results

Assuming $\beta_1^i+\beta_2^i<1$, $i=1,2$, equilibrium is then unique and symmetric. This implies that such equilibrium can only be characterized by one of the following four cases:

(1) both social groups are constrained in their tax behaviour: i.e. $\hat{x}_1=\bar{x}_1$ and $\hat{x}_2=\bar{x}_2$;

(2) both social groups are unconstrained in their tax behaviour: i.e. $\hat{x}_1=x_1^o$ and $\hat{x}_2=x_2^o$;

(3)-(4) One social group is constrained and the other is unconstrained in their tax behaviour: i.e. $\hat{x}_i=\bar{x}_i$ and $\hat{x}_j=x_j^o$ $i=1,2$, $j=1,2$, $i\neq j$.

The next table summarizes the four possible cases:

Table VI.2: equilibria with exogenous public expenditure

		type 2	
		constrained	unconstrained
type 1	constrained	I $\bar{x}_1$; $\bar{x}_2$	II $\bar{x}_1$; x_2^o
	unconstrained	III x_1^o ; $\bar{x}_2$	IV x_1^o ; x_2^o

Without imposing additional structure on the model it is impossible to tell in which region the equilibrium lies (see below). Note however that in line with Baldry's results we now have three regions (from I to III) where at least one group of individuals evades less than would be in its selfish interest to do. Also note that, as we are about to show, the comparative statics results of the model will depend on the region where the equilibrium happens to lie. In the following exercises we will always assume interior solutions for both x_i^0 and $\bar{x}_i$ $i=1,2$ (i.e. $0<x_i^0<tI_i$ and $0<\bar{x}_i=z_i<tI_i$).

VI.4.2.1: Changes in t

Let us start by considering a change in the rate of tax. In region IV, under the assumption of exogenous public expenditure, the results will just replicate those of the traditional portfolio model (see section VI.3): both x_2^0 and x_1^0 will fall as t increases. In region I, however, where both groups are constrained in their tax behaviour we obtain different results. By considering eq.(10) and using the fact that in the symmetric equilibrium $x_i^*=\hat{x}_i=\bar{x}_i$, $i=1,2$ we can write the fairness constraint in equilibrium as:

$$(14)\quad \bar{x}_i = I_i(t - t_i^i) + \vartheta^i\bar{x}_j + \vartheta^i I_j(t_j^i - t) \qquad i=1,2\ ,\ j=1,2\ i\neq j$$

where $\vartheta^i=\beta_2^i/(1-\beta_1^i)$. Then, by totally differentiating $\bar{x}_1$ and $\bar{x}_2$, as expressed in (14), with respect to t and solving the system we get:

$$\partial \bar{x}_1/\partial t = I_1 > 0$$

(15)

$$\partial \bar{x}_2/\partial t = I_2 > 0$$

that is, individuals of both types would resist an increase in the rate of tax by attempting to evade completely the augmented tax[10]. It is important to stress the intuition behind this result: at unchanged public good supply the taxpayers, who were already constrained in their behaviour by considerations of fairness, perceive the increase in their tax rate as "unfair": thus, since an increased amount of tax evasion pays off in terms of expected costs and benefits, they react by increasing tax evasion. To this author the above sounds a quite convincing explanation of the reason why taxpayers --or at least a reasonable amount of "honest" taxpayers-- should wish to increase their level of tax evasion

[10]Note however that tax revenue would in any case increase following the increase in the tax rate since part of the evaded taxes would be detected and taxed at the surcharge rate. To see this note that expected tax revenue (≅actual tax revenue since in large economies the probability of being detected will coincide with the proportions of individuals actually detected) can be written as:

$$R \cong N/2\{t[I_1+I_2] - s_1\bar{x}_1 - s_2\bar{x}_2 - 2c(p_1,p_2)\}$$

where c(.) is the average cost needed to enforce the probabilities of detection p_1 and p_2. By differentiating the equation above with respect to t we get $\partial R/\partial t = N/2\Sigma I_i(1-s_i) > 0$ since $s_i < 1$.

following an increase in the tax rate.

The effect of an increase in the proportional rate of tax in regions II and III is illustrated by the following derivative:

$$(16)\quad \partial \bar{x}_i/\partial t = I_i + \vartheta^i \partial x^o_j/\partial t - \vartheta^i I_j \quad i=1,2 \;,\; j=1,2 \;,\; i\neq j$$

As can be seen from eq.(16) there are two tendencies at work here. On one hand there is the positive effect due to the increase of the tax rate above the Kantian rate of tax; this is captured by the first element in the equation above. On the other hand there are the two negative effects due to the increase in the tax rate faced by the individuals in the other group and by the decrease in the amount of tax evaded by the latter: these are captured by the third and the second element of eq.(16). The net effect is ambiguous: hence the question marks in table VI.3.

Table VI.3: effects of changes in t

		type 2	
		constrained	unconstrained
		I	II
type 1	constrained	$\partial \bar{x}_1/\partial t > 0$	$\partial \bar{x}_1/\partial t$?
		$\partial \bar{x}_2/\partial t > 0$	$\partial x^o_2/\partial t < 0$
		III	IV
	unconstrained	$\partial x^o_1/\partial t < 0$	$\partial x^o_1/\partial t < 0$
		$\partial \bar{x}_2/\partial t$?	$\partial x^o_2/\partial t < 0$

VI.4.2.2: Changes in G

As we recalled in section VI.3 the effect of changes in public expenditure on evaded taxes is ambiguous in the standard portfolio choice model. So, in order to make our results comparable, we will consider in this section only the main special case where this ambiguity disappears: the case of zero income effects on the demand for the public good. Going back to eq.(4), note that this assumption implies that the marginal rate of substitution (i.e. the fair price) of the representative taxpayer depends only upon the given supply of the public good.

Let us start with region I. Not surprisingly, since we modeled the fair price as a marginal rate of substitution, the effects of changes in G on tax evasion turn out to depend on a quantity elasticity. This can be seen by substituting eq.(5) and eq.(6) in eq.(14) and totally differentiating with respect to G. Forming the system and solving we get:

$$(17)\quad \partial\bar{x}_i/\partial G = -1/V[(1-\theta^i)w^i(\varepsilon^i+1)+\theta^i(1-\theta^j)w^j(\varepsilon^j+1)]$$

$$i=1,2\ ,\ j=1,2\ ,\ i\neq j$$

where $V=(1-\theta^i\theta^j)>0$ by assumption and where $\varepsilon^i=(w^i_G G/w^i)$, $i=1,2$, is the elasticity of the marginal willingness to pay with respect to changes in public good supply. Then $\partial\bar{x}_1/\partial G<0$ (>0) as both $|\varepsilon^1|<1$ <u>and</u> $|\varepsilon^2|<1$ ($|\varepsilon^1|>1$ <u>and</u> $|\varepsilon^2|>1$) and similarly for

$\partial\bar{x}_2/\partial G$. Different types of elasticities for the two types of individuals (i.e., for example, $|\varepsilon^1|>1$ and $|\varepsilon^2|<1$) would then produce ambiguous results, while elasticities which are both greater or smaller than unity allow one to sign unambiguously the effects of a change in public good supply on tax evasion. Also note that if we assume that the two types of individuals have identical preferences and identical reciprocity weights (i.e. if $w^i=w^j=w$ and $\vartheta^i=\vartheta^j$) eq.(17) reduces to $\partial\bar{x}_1/\partial G=\partial\bar{x}_2/\partial G=-w(\varepsilon+1)$. The two types of individuals would then react to the change in G by shifting their evaded tax in the same direction and by the same amount.

The rationale for the result above is straightforward. If the marginal willingness to pay is quantity inelastic (i.e.$|\varepsilon^i|<1$ i=1,2) a 1% increase in public good supply would reduce the price that an individual regards as fair to pay for the public good by less than 1% : then the total amount of income that the individual considers fair to pay to the state in exchange for the public good supplied must increase following the increase in G and consequently tax evasion must fall. Vice versa for the opposite case of quantity elastic w^i.

The intermediate cases of regions II and III are less clear-cut. By differentiating eq.(14) with respect to G we get:

$$(18)\quad \partial\bar{x}_i/\partial G= -(1-\vartheta^i)w^i(\varepsilon^i+1)+\vartheta^i\partial x_j^o/\partial G \qquad i=1,2\ ,\ j=1,2\ ,\ i\neq j$$

Since $\partial x_j^o/\partial G>0$ for a public good with zero income effects (see section VI.3) the sign of $\partial \bar{x}_i/\partial G$ is certainly positive for $|\varepsilon^i|>1$, while is ambiguous in the opposite case. Finally, under the assumption of exogenous public good supply, the signs of the derivatives in region IV are both positive ($\partial x_i^o/\partial G>0$, i=1,2). The next table summarizes the results with respect to changes in G:

Table VI.4: effects of changes in G

		type 2	
		constrained	unconstrained
		I	II
type 1	constrained	$\partial \bar{x}_1/\partial G \gtrless 0$, $\partial \bar{x}_2/\partial G \lessgtr 0$ $\quad |\varepsilon^i| \gtrless 1$	symmetric to III
		III	IV
	unconstrained	$\partial x_1^o/\partial G >0$	$\partial x_1^o/\partial G>0$
		$\partial \bar{x}_2/\partial G?$ $\quad |\varepsilon^2|<1$	
		$\partial \bar{x}_2/\partial G>0$ $\quad |\varepsilon^2|>1$	$\partial x_2^o/\partial G>0$

VI.4.3 A First Summing Up

Summing up, the comparative statics results for the case of exogenously given public expenditure show that:

(1) The signs of the comparative statics derivatives depend on the region where the equilibrium lies;

(2) In some regions individuals evade less than would be suggested

by a pure calculus of expected costs and benefits;

(3) At least in some regions, individual and total tax evasion turn out to be a positive function of the tax rate;

(4) Public expenditure is not neutral to the decision to evade by individuals and its effects on tax evasion are determined (in the constrained regions) by the quantity elasticity of the "fair price".

As argued in chapter VI, results (1) to (4) are more in line with both intuition and empirical evidence than the traditional results of the portfolio choice approach to tax evasion. The main limit of the approach illustrated in the previous sections is that, at the level of extreme generality that we considered above, we were unable to predict where the equilibrium tends to lie with respect to different values of the parameters selected by government. In order to solve some of these ambiguities and to address some other important questions we will consider in the next section a simple example of the model outlined above. Here we limit ourselves to stating the following result:

<u>Proposition 4</u> "Suppose that (1) individual preferences and reciprocity weights are the same across social groups (i.e $U^1=U^2=U$ and $\vartheta^1=\vartheta^2=\vartheta$), (2) the expected price for evaded tax is the same across social groups ($s_1=s_2=s>0$): then, under DARA, a regime where rich people are unconstrained and poor people are constrained by fairness considerations is impossible. That is, either region II or region III must disappear."

<u>Proof</u> "Without loss of generality let $I_2>I_1$. Note from eq.(4) that $U^1=U^2$ implies $t^i_j=t^i_i$, i=1,2 , j=1,2 , i≠j. Suppose that $\hat{x}_2=x^o_2$ and $\hat{x}_1=\bar{x}_1$ is an equilibrium. Then, from DARA and assumptions (1) and (2) above, in equilibrium $\bar{x}_2\geq x^o_2>x^o_1\geq\bar{x}_1$, where $x^o_1>0$ by assumption (2). Substituting from eq.(10) and eq.(11) in the inequalities above and solving for x^o_2 in equilibrium we get:

(19) $z_2=I_2(t - t^2_2) + \vartheta(\max(0,z_1)- I_1(t - t^1_1))\geq\bar{x}_2\geq x^o_2$

(20) $x^o_2>x^o_1\geq\max(0,z_1=I_1(t - t^1_1) + \vartheta(x^o_2 - I_2(t - t^2_2))$

Suppose first that $z_1>0$. Then, by substituting for z_1 in eq.(19) and by solving both eq.(19) and eq.(20) for x^o_2 we get:

(21/2) $I_2(t - t^2_2) > I_1(t - t^1_1)$

By substituting from eq.(5) and eq.(6) in t^i_i, i=1,2, the inequality in (21/2) requires $I_2<I_1$ which is impossible by assumption. Next, suppose $z_1\leq 0$: then substituting in eq.(19) and (20) and repeating the steps above we get:

(23/4) $I_2(t - t^2_2) > I_1(t - t^1_1)[1 + \vartheta - \vartheta^2] > I_1(t - t^1_1)$

since $\vartheta<1$ by assumption. This is again impossible; we then conclude that $\hat{x}_2=x^o_2$ and $\hat{x}_1=\bar{x}_1$ cannot be an equilibrium.QED"

Hence, if differences in preferences and in opportunities for evading across social groups are not relevant, poor people can never be constrained if rich people are not constrained by fairness considerations. This sounds reasonable enough. Proposition 4 then suggests that, if differences in preferences and in opportunities for evading are small, equilibrium will shift from one region to another, following a change in the rate of tax and/or in public good supply, without ever entering region II (for $I_z>I_1$). The example discussed in the next section confirms this conjecture.

Section VI.5 An Example: Cobb-Douglas Utility Functions

As we saw in the previous section, it is in general impossible to establish which particular regime will prevail in equilibrium. This is of course not surprising. In the general case, the two types of individuals differ both in terms of their personal characteristics and in terms of the probability of detection that they face. We are therefore unable to decide, for example, who, between poor or rich people, will tend to evade more in equilibrium. This substantial ambiguity of the general model, if expected, is however unfortunate for two basic reasons. First, because as we saw above, the results of the model depend upon the region where the equilibrium lies. Second, because it does not allow one to address some of the most interesting questions which could be asked in the model. For example, in which region the equilibrium tends to lie for different values of the tax rate or the effects of public expenditure on tax evasion. In particular, one would wish to investigate not only the effects of total expenditure on tax evasion --i.e. the issue of over/under provision of public goods-- but also the effects that the <u>composition</u> of public expenditure --i.e. its being more or less in favor of poor versus rich people-- may have on tax evasion.

In order to gain some insights on the issues above it is then necessary to lose some generality by reducing our general model to simpler and more tractable terms. We choose to do so by imposing a simple parameterization on the preferences of individuals and by introducing other simplifying assumptions. In the following we will indicate which assumptions are crucial and which assumptions could be eliminated without harm.

Let us start by assuming that both types of individuals are characterized by the same type of preferences, representable by a Cobb Douglas utility function in logarithmic form:

$$(25) \qquad U^i = \log C_{ih} + \alpha_i \log G \qquad h=1,\dots,N/2;\ i=1,2$$

The parameters α_i, $0<\alpha_i<1$, capture in a very simple way the differences in preferences for the public good across social groups. Note that the parameterization in eq.(25) implies separability of preferences in public and private good. This is crucial because, as explained in section VI.3, separability entails that the unconstrained solution x_i^o is independent of public good supply. In the present context this is acceptable because we are here mainly interested in the effects of government expenditure on the perceived fairness of the terms of trade and the latter affects only $\bar{x}_i$.

We also assume that the two types of individuals face the same probability of being caught if evading, p, which implies $s_1=s_2=s$. This assumption, together with the utility specification in (25), implies that, if unconstrained by fairness considerations, rich people will evade in absolute terms more than poor people (that is, $x^o_1 \gtrless x^o_2$ as $I_1 \gtrless I_2$). This sounds intuitively realistic. Thus, the assumption above on p, if clearly not necessary, represents a simple way to introduce this feature of the real world in our model.

Going back to eq.(12), we can then write the unconstrained solutions to eq.(12) for the two types of individuals as:

$$(26) \qquad x^o_i = [sI_i(1-t)]/\pi \qquad i=1,2$$

where, in line with eq.(13), $0<x^o_i<tI_i$ if $s>0$ and $t>s/(s+\pi)$, $i=1,2$. In the following we assume both these conditions to hold. Note that x^o_i is a decreasing function of t, with $x^o_i=0$ for $t=1$.

Turning now to the fairness constraint we assume first, for simplicity, that the reciprocity weights are the same across social groups (i.e. $\beta^1_s=\beta^2_s=\beta_s$, $s=1,2$ which implies $\vartheta^1=\vartheta^2=\vartheta$). Second, following the analysis of the previous section, we impose the condition that $\beta_1+\beta_2<1$ so as to guarantee us an unique symmetric equilibrium. Using eq.(25) and eq.(4) the price that an individual h belonging to group i would consider fair to pay to the state can then be readily calculated as:

$$(27)\quad w^i(\bar{I},G)=[\alpha_i(\bar{I}-\Psi G)]/G \qquad i=1,2$$

where as above $\bar{I}$ is average income and Ψ is the marginal rate of transformation for large economies. Then, by using eq.(10), eq.(11), eq.(12), eq.(26), eq.(27) and the symmetry of equilibrium we can express the individual choice as:

$$(28)\quad \hat{x}_i=\min[s(1-t)I_i/\pi;\ \max(0;\ (1-t)(\vartheta I_j-I_i)+\ (1-\vartheta)((1-\alpha_i)\bar{I}\ +\ \alpha_i\Psi G)\ +\ \vartheta x_j)]$$

for $i=1,2$ and $i\neq j$. In order to close the model and to make the individual choice dependent only on the rate of tax selected by government we need a further step. We need some way of linking, even with exogenous public expenditure, revenue to public expenditure. We do this, by assuming that government selects the rate of tax according to the rule $t=\delta\Psi G/\bar{I}$, where $1\leq\delta\leq1/(1-s)$. G is the amount of public good that government decides, ex-ante, to provide and δ is a parameter which express government's beliefs about expected tax evasion. If $\delta=1$, government expects everybody to pay the tax and therefore the rate of tax is selected so as to enforce a balanced budget if everybody does turn out to pay all the tax. If government expects some tax evasion, δ will be raised proportionally and finally if government expects everybody to evade completely the taxes $\delta=1/(1-s)$ so as to enforce even in this case a balanced budget.[11] In passing, it should be noted that this process is likely to be, in the short run, closer to actual government behaviour than assuming always a balanced public budget.

Without loss of generality, let $I_2 \geq I_1$ and let us simplify the notation by writing $I_1 = I$ and $I_2 = kI$, with $k \geq 1$. Then by substituting for $G = \bar{I}t/\delta\Psi$ in eq.(28) above and using eq.(26), we can write the choices of the two types of individuals as:

(29)

$$\hat{x}_1 = I\min[s(1-t)/\pi;\max\{0;(1-t)(\vartheta k-1)+(1-\vartheta)(1-\alpha_1(1-t/\delta))(1+k)/2+\vartheta x_2\}]$$

$$\hat{x}_2 = I\min[s(1-t)k/\pi;\max\{0;(1-t)(\vartheta-k)+(1-\vartheta)(1-\alpha_2(1-t/\delta))(1+k)/2+\vartheta x_1\}]$$

In this simplified version of the model the decision to evade by the two types of individual can then be seen to depend on four parameters in addition to s: the tax rate (t), the distribution of preferences for the public good (α_1, α_2), the income distribution (k) and the reciprocity weights (ϑ). By working out the conditions

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[11] In the text we are assuming for simplicity that the cost to enforce the probability of detection p is negligible with respect to the entire tax revenue and it therefore does not appear in the public budget. Also note that, assuming rational expectations on the part of government, δ would be implicitly determined by the equation:

$$\delta = \bar{I}/\{\bar{I} - s/2[\hat{x}_1(\delta\Psi G/\bar{I}) + \hat{x}_2(\delta\Psi G/\bar{I})]\}$$

where $\hat{x}_i(.)$ indicates that $\hat{x}_i$ is a function of $\delta\Psi G/\bar{I}$.

which establish an equilibrium in the different regions we can get a glimpse of the influence of the different parameters on tax evasion. Thus, suppose that the equilibrium lies in the unconstrained region IV; for this to be the case the two following inequalities must hold:

$$(30a) \qquad (1-\vartheta)(1-\alpha_1(1-t/\delta))(1+k)/2 \geq (1-t)(1-k\vartheta)D$$

$$(30b) \qquad (1-\vartheta)(1-\alpha_2(1-t/\delta))(1+k)/2 \geq (1-t)(k-\vartheta)D$$

where $D=(s+\pi)/\pi$. Note that the RHS of eq.(30b) is larger, for $k>1$, than the RHS of eq.(30a): thus, if (30a) holds as an equality, (30b) cannot hold unless $\alpha_2>\alpha_1$; that is, unless public expenditure is "regressive". Also note that while the LHS of (30a) is certainly positive, the RHS of the same equation is likely to become negative for large amounts of inequality in income distribution. Thus, irrespective of the characteristics of public expenditure, poor people are likely to be unconstrained if rich people are unconstrained by fairness considerations. Finally note that the LHS of both eq.(30a) and eq.(30b) are increasing in t while the RHS of both equations are decreasing in t: this suggests that the unconstrained equilibrium can only be reached at relatively high rates of tax. And in fact by summing the corresponding sides of eq.(30a) and eq.(30b) and solving for t, we can get, as a necessary but not sufficient condition, the minimum value of t needed to enforce an equilibrium in the unconstrained region:

$$(31) \qquad t \geq \delta(D-1+\bar{\alpha})/(D\delta+\bar{\alpha})$$

where $\bar{\alpha}=(\alpha_1+\alpha_2)/2$. Note that t in eq.(31) is increasing in the average preferences for the public good ($\bar{\alpha}$): that is, the more individuals prefer on average the public good the higher must be the rate of tax faced by individuals for them to become unconstrained in their tax behaviour.

The analysis of the conditions which would enforce a constrained equilibrium (i.e. $\hat{x}_i=\bar{x}_i$, i=1,2) shows that this requires, in contrast to the previous case, a relatively low rate of tax, and in particular, symmetrically to the case above, a tax rate less than the RHS of eq.(31). The intuition behind this result is clear: as we saw in the previous section evaded tax is increasing in the tax rate in the constrained region and is decreasing in the unconstrained region. Thus, at low levels of the rate of tax (i.e for $t \Rightarrow 0$) the fairness constraint is likely to be much lower than the amount of evaded tax suggested by the expected utility calculus and individuals will tend to be constrained in their tax behaviour. By increasing the tax rate the fairness constraint will tend to increase continuously while the selfish amount of evaded tax will fall continuously. Thus, there has to be a level of the rate of tax where selfishness dictates a higher level of tax evasion than that desired by individuals. From that rate of tax onwards the taxpayers will start to be unconstrained in their tax behaviour. This suggests that we should observe constrained tax behaviour at low levels of the tax rate and unconstrained behaviour at high level of the tax rate. The special case considered below confirms this conjecture.

Turning now to the intermediate cases of regions II and III, using eq.(29), one immediately gets the conditions which would enforce an equilibrium where poor people are unconstrained and rich people are constrained in their tax behaviour (i.e. $\hat{x}_1 = x_1^o$ and $\hat{x}_2 = \bar{x}_2$):

$$(32) \quad (k-\vartheta)(1-\alpha_1(1-t/\delta)) \geq (1-k\vartheta)(1-\alpha_2(1-t/\delta))$$

For k>1 this condition is likely to hold while if k=1 it can only hold for $\alpha_1 > \alpha_2$. The condition for the reverse case of $\hat{x}_1 = \bar{x}_1$ and $\hat{x}_2 = x_2^o$ is of course identical to eq.(32) with the reverse sign. It is then clear that if there is a strong inequality in income distribution the latter equilibrium cannot exist even if public expenditure is strongly progressive (i.e. $\alpha_1 > \alpha_2$). This is of course in line with proposition 4 above.

Summing up, for each level of t, it will be the more likely that rich individuals are constrained in their tax behaviour and the poor unconstrained the higher is income inequality and the higher are the preferences of rich people for the public good relative to the preferences of poor people. Furthermore, keeping everything else constant, individuals of both types will tend to be constrained at low tax rates and unconstrained at high tax rates.

The effect of the reciprocity weights on the equilibria is more ambiguous because they tend to keep the behaviour of the two types of individuals closely related. In order to cast further light on characteristics of the model let us then consider two extreme values for the reciprocity weights: $\vartheta=0$ (i.e. $\beta_2^i=0$ $i=1,2$) and $\vartheta=1$ (i.e. $\beta_1^i+\beta_2^i=1$ $i=1,2$). The former case refers to a situation where reciprocity effects across social groups are absent or very small; the latter to a situation where the Kantian taxes (and therefore public expenditure) do not play any role in determining individual behaviour. Beginning with the former case and substituting for $\vartheta=0$ in eq.(29) we get:

$$(33) \qquad \hat{x}_1=\min[s(1-t)I/\pi;\max\{0;(-(1-t)+(1-\alpha_1(1-t/\delta))(1+k)/2)I\}]$$

$$\hat{x}_2=\min[s(1-t)Ik/\pi;\max\{0;(-(1-t)k+(1-\alpha_2(1-t/\delta))(1+k)/2)I\}]$$

First note that in eq.(33) $z_2<z_1$ if

$$(34) \qquad \alpha_1<\alpha_2+2(k-1)(1-t)/[(1+k)(1-t/\delta)]$$

Condition (34) must hold if $k\geq 1+(2(\delta-t)/(\delta+t(1-2\delta))$. Thus for example, if $\delta=1$, $k=3$ is enough to guarantee condition (34) to hold for any admissible value of α_1 and α_2. Second note that, as is easy to verify, for $t\Rightarrow 0$ and $k>1$, $z_2<0$ (i.e. $\bar{x}_2=0$) while z_1 can be still positive if $\alpha_1<(k-1)/(k+1)$; that is, for large amounts of inequality in income distribution individuals of type 1 may still wish to "evade" --i.e. to receive a subsidy-- even if the tax rate is zero. On the other hand, for $t \Rightarrow 1$, both z_1 and z_2 are positive. By comparing these results with the values of x_i^o in eq.(26) for the extreme values of t, it is clear that at low levels of t individuals will be constrained and at high levels of t unconstrained. We can check this by computing the value of t which makes z_i just equal to zero and by computing next the value of t which sets z_i equal to x_i^o. Doing this for the two types we get:

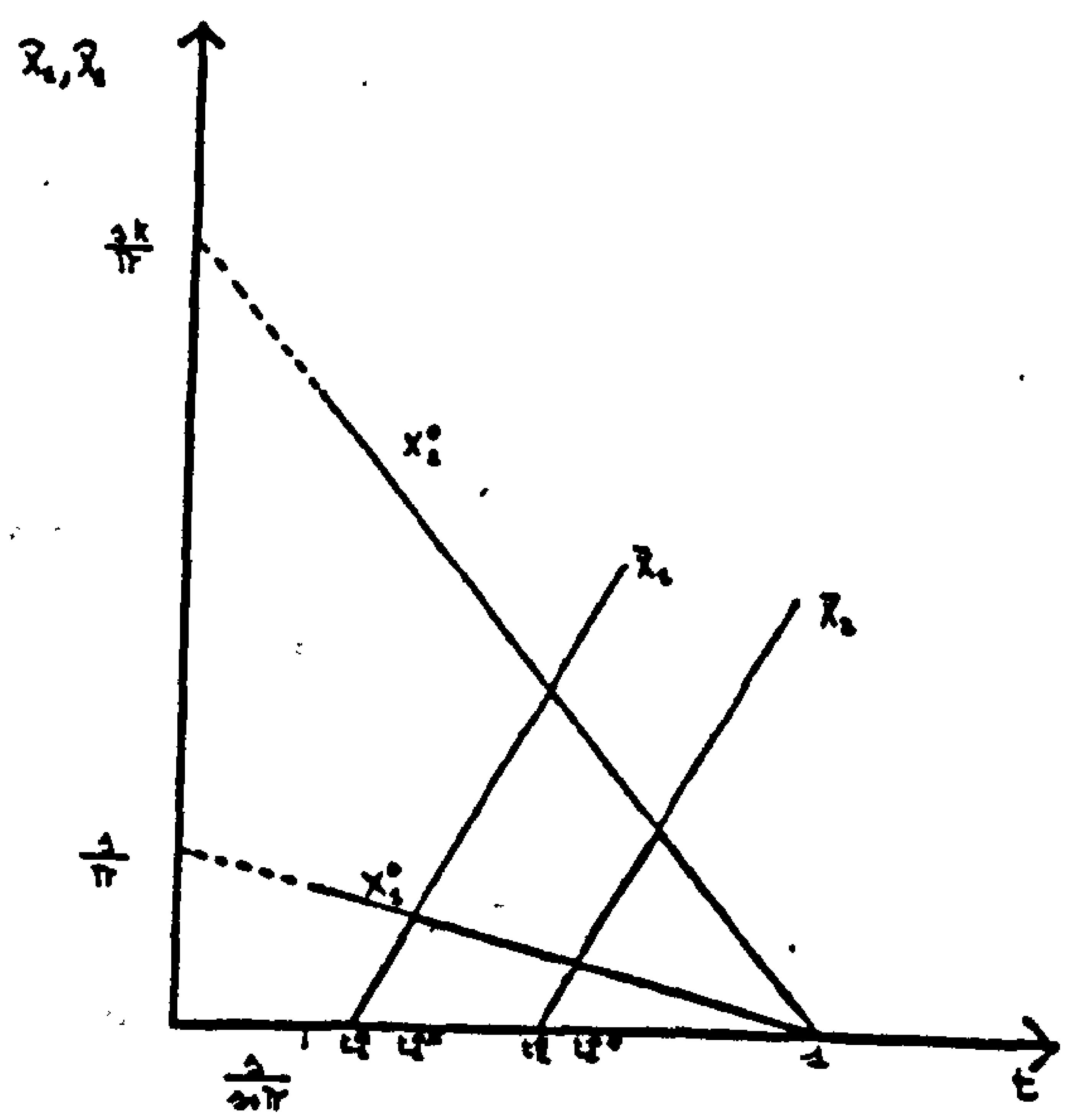
$$(35) \qquad \begin{aligned} t_2^* &= 1 - F_2/(2k\delta + \alpha_2(1+k)) \\ t_2^{**} &= 1 - F_2/(2k\delta D + \alpha_2(1+k)) \\ t_1^* &= 1 - F_1/(2\delta + \alpha_1(1+k)) \\ t_1^{**} &= 1 - F_1/(2\delta D + \alpha_1(1+k)) \end{aligned}$$

where t_i^* is the rate of tax which makes $z_i=0$, t_i^{**} is the rate of tax such that $z_i=x_i^o$ and $F_i=(1+k)[\alpha_i(1-\delta)+\delta]$. Note that $0<t_2^*<t_2^{**}<1$ certainly and $0<t_1^*<t_1^{**}<1$ if $\alpha_1>(k-1)/(k+1)$. It is also clear that if preferences for the public good across social groups do not differ too much $t_2^*>t_1^*$ and $t_2^{**}>t_1^{**}$. In any case, even if public expenditure is strongly in favor of poor people it can be shown that, for $\delta \Rightarrow 1$, condition (34) is enough to guarantee $t_2^*>t_1^*$ and $t_2^{**}>t_1^{**}$. The sign of $(t_1^{**}-t_2^*)$ is instead uncertain: however, for identical preferences $k \geq 2$ is enough to guarantee $t_1^{**}<t_2^*$ and for $\delta \Rightarrow 1$ the same inequality can be obtained for $k \geq 3$. For high values of δ $(t_1^{**}-t_2^*)$ becomes ambiguous and we can even get an

inequality of the opposite sign.

The next picture summarizes the results reached so far for selected values of the parameters k, α_i, δ and D[12].

Figure VI.1



As figure VI.1 shows, at very low levels of the tax rate there is no tax evasion; at higher levels of the rate of tax poor individuals start evading --and their evaded tax is an increasing function of the tax rate-- while rich individuals evaded tax presents a flat range. By increasing further the rate of tax poor people's evaded tax becomes negatively sloped in the tax rate

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[12] In the picture we assume $\delta=1$, $k\geq3$ and $\alpha_i>(D-2/(1+k))$. As is easy to verify these assumptions imply $t_2^{**}>t_2^{*}>t_1^{**}>t_1^{*}>s/(s+\pi)$. For different sets of assumptions see the next pictures.

while rich people's tax evasion is positive and increasing in the tax rate. Finally, only at relatively high levels of the tax rate does tax evasion present the traditional downward-sloping feature of the portfolio-choice model. Note that at low levels of the tax rate --or public good supply since for fixed δ the two variables are linked by a constant-- the tax evasion of the poor is greater than the tax evasion of the rich while the opposite holds at high levels of the tax rate. Also note that in figure VI.1 there is never a case where the poor are constrained and the rich are not.

Using figure VI.1, we can easily illustrate the effects of changes in income inequality and in the progressiveness of public expenditure on the tax behaviour of the two types of individuals. A reduction in k, for example, would reduce x_2^o, increase $\bar{x}_2$ and reduce $\bar{x}_1$ for any level of t, while x_1^o would remain unchanged. Similarly, an increase in the "progressiveness" of public expenditure (maybe a change in the composition of public expenditure so as to make it more attractive to the poor rather than the rich) would increase $\bar{x}_2$ and decrease $\bar{x}_1$ for any level of t.[13] The next figures illustrate.

In figure VI.3 we have imagined a sufficiently big change in the composition of public expenditure as to make $t_1^* > t_2^*$ (i.e. condition (34) does not hold). Note that in this case, with public expenditure strongly progressive and limited inequality in income

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[13]These results are in line with the empirical evidence on this issue: see Becker et al. (1987).

distribution, the rich evade more than the poor for any level of the tax rate.

Figure VI.2: effects of a decrease in income inequality

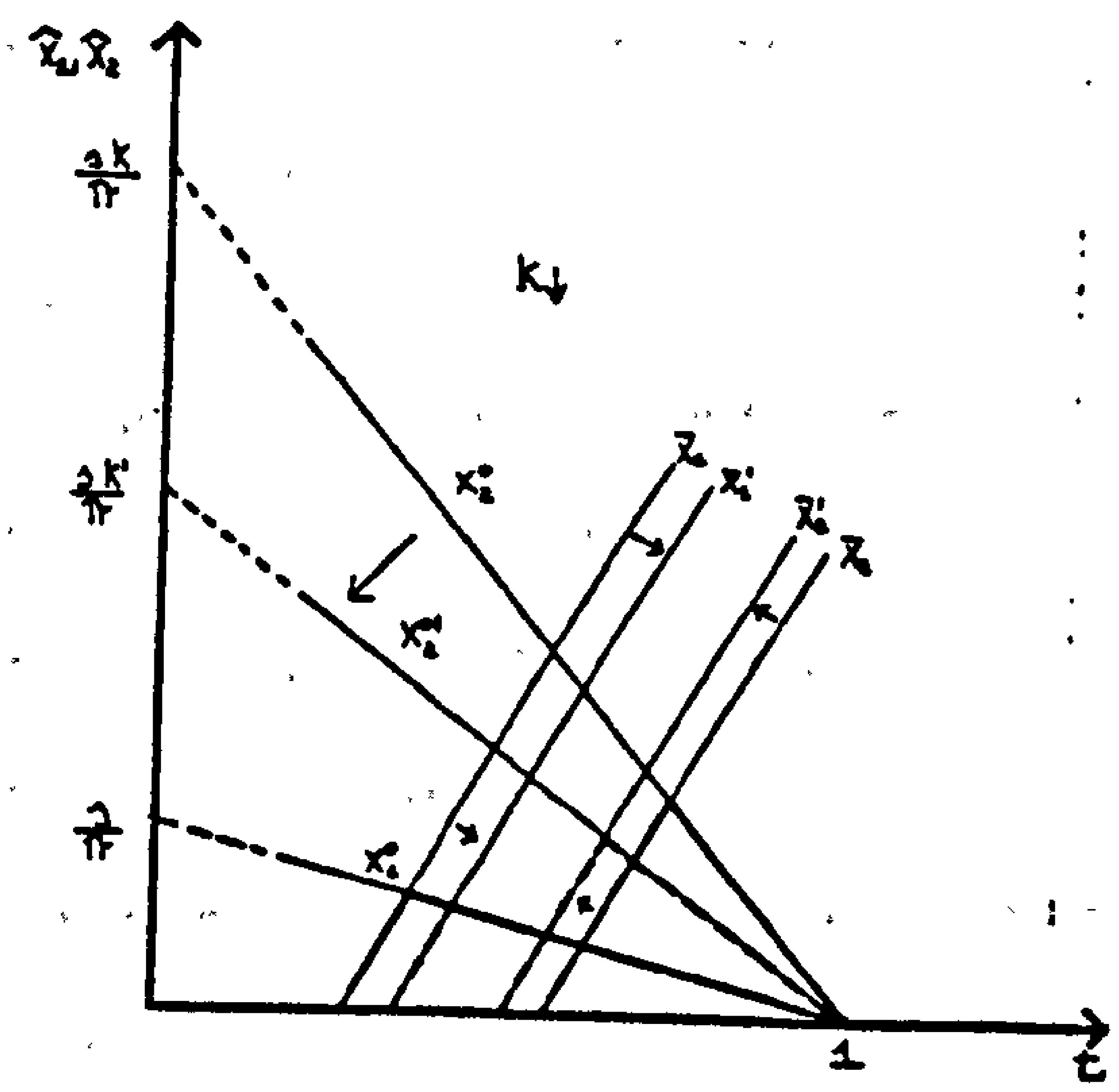
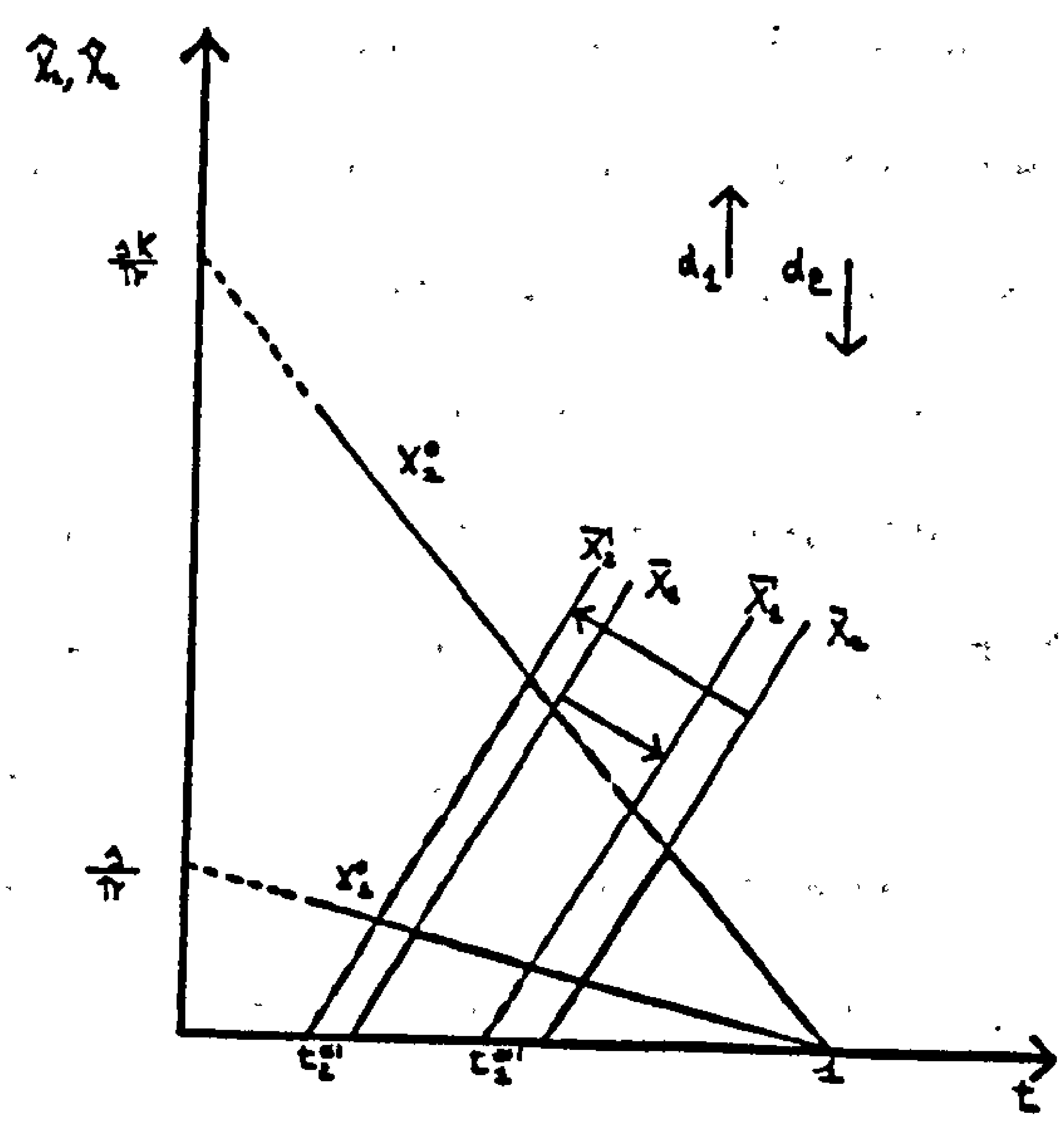


Figure VI.3: effects of an increase in the progressiveness of public expenditure



Summing up, the most interesting feature of the exercise attempted in this section is that it seems to predict a zero or very low level of tax evasion for low levels of the tax rate: an intermediate range of values for the tax rate where tax evasion is roughly increasing in the tax rate and a final range for high levels of the tax rate where tax evasion is decreasing in the tax rate --if such levels would ever be reached. If not entirely satisfactory, such results are undoubtedly more in line with observed facts than the results of the traditional portfolio choice model of tax evasion. Furthermore the effects of changes in income inequality and in the progressiveness of public expenditure on tax evasion are also intuitively appealing and in line with experimental evidence (see Becker et al., 1987).

Let us then consider the opposite case of $\vartheta=1$. As we recalled above, in this case public expenditure --and therefore differences in preferences for the public good across social groups-- does not play any role in determining the position of the fairness constraints. This is a general result (see eq.(10)) but in the present framework this can be very easily verified by substituting for $\vartheta=1$ in eq.(29). In contrast, x_i^o would be still influenced by public expenditure and it is only the particular parameterization chosen in this section --i.e. the separability of preferences in public and private good-- which makes x_i^o unaffected by G.

Having made this point clear, let me remind the reader that in section VI.4.2 the assumption $\vartheta<1$ was used to prove uniqueness of equilibrium. However, that condition was only a sufficient but not necessary condition for uniqueness of equilibrium, as the next proposition illustrates:

<u>Proposition 5</u> "Suppose $\theta=1$. Then, if $k>D$, the only symmetric equilibrium for any t such that $0\leq t\leq \bar{1}$ is one where $\hat{x}_1=x_1^0$ and $\hat{x}_2=\bar{x}_2=0$".

<u>Proof</u> "Substituting for $\theta=1$ in eq.(29) above we get:

(36a) $$z_1=(1-t)(k-1)I + x_2$$

(36b) $$z_2=-(1-t)(k-1)I + x_1$$

By definition of $\hat{x}_i$ and from eq.(36b) it follows that:

(37) $$z_2(x_1=\hat{x}_1)\leq z_2(x_1=x_1^0)=I[s(1-t)/\pi - (1-t)(k-1)]$$

Differentiating $z_2(x_1^0)$ with respect to t:

(38) $$\partial z_2(x_1^0)/\partial t= I(k-D)$$

Then, if $k>D$, $z_2(x_1^0)$ is everywhere an increasing function of t. Hence it must reach a constrained maximum at the highest admissible value for t, t=1. At t=1, $z_2(x_1^0)=0$. Hence, $z_2(\hat{x}_1)\leq 0$ for $0\leq t\leq 1$. Then, for all $0\leq t\leq 1$, $\hat{x}_2=\bar{x}_2=0$. Back substituting in eq.(35a), we then get that $\hat{x}_1=x_1^0$ if $z_1(x_2=0)\geq x_1^0$; that is $\hat{x}_1=x_1^0$ if

(39) $\quad z_1(x_2=0)-x_1^o=(1-t)I(k-D)\geq 0$

which certainly holds for k>D.QED"

Note that the condition k>D is a very mild condition indeed. In fact, since s<1 and π is in general larger than 1[14], D is smaller than 2 and for reasonable values of s and π, D is quite likely to approach unity. Then if $\theta \Rightarrow 1$ a very small amount of income inequality should be enough to enforce an equilibrium where rich people never evade and poor people are never constrained in their tax behaviour. Note however that in proving proposition 5 we assumed a symmetric equilibrium: proposition 5, in other words, does not rule out the possibility that, even with k>D, there may be multiple asymmetrical equilibria. For 1<k<D and $0\leq t<1$ we obtain multiple symmetric equilibria; an equilibrium where $\hat{x}_1=x_1^o$ and $\hat{x}_2=\bar{x}_2=0$ is still possible but we can also obtain equilibria for any value of $0\leq x_i<x_i^o$, i=1,2. An equilibrium in the unconstrained region with $\hat{x}_1=x_1^o$ and $\hat{x}_2=x_2^o$ is instead impossible. Finally, for t=1 and k<D there is an unique equilibrium for $\hat{x}_1=0$ and $\hat{x}_2=0$.

The analysis of the case for $\theta=1$ then illustrates the role played by the Kantian tax in my model: it is as if the Kantian tax

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[14] On logical grounds, for the surcharge rate to be a penalty, it would be enough that $\pi>0$. On the other hand, in most countries, π is larger than one. For example in Italy, π is approximately 3.

selected a particular equilibrium among the many offered by a pure reciprocity model.

VI.6 Conclusions

In this chapter we set up the basic structure of our model and analyzed it in the case of fully exogenous public expenditure. A number of interesting results were obtained. The behaviour of the taxpayers turned out to depend on the region where the equilibrium lies. There are regions where individuals are constrained in their tax behaviour and therefore evade less than predicted by selfishness and other regions where they behave as predicted by the traditional portfolio choice model. In the constrained regions tax evasion tends to be an increasing function of the tax rate. Also, our parameterization of the model in section 5 suggests that constrained tax behaviour tends to occur at low levels of the tax rate and unconstrained behaviour at high levels of the rate of tax. Thus, we should observe a zero or very low level of tax evasion associated with very low tax rates and positive tax evasion increasing in the tax rate associated with higher rates of tax. The portfolio choice effect of decreasing tax evasion in the tax rate can be obtained only for relatively high rates of tax and, consequently, it can also never be observed if the tax rate is below a given threshold.

In the chapter we also analyzed the effects of public expenditure, income inequality, and the reciprocity weights on the tax behaviour of the different social groups. Public expenditure was shown to affect tax evasion in the constrained region according to a quantity elasticity, while the distribution of preferences for the public good across social groups affects the tax behaviour of the different groups of individuals in the expected direction. The analysis of the effects of changing the level of inequality in income distribution on the equilibrium also produces intuitive results. Finally, we considered, in the example of section 5, two extreme values for the reciprocity weights. The results showed the crucial role played by the Kantian tax in selecting an equilibrium among the many made possible by a simple reciprocity model. Let us then turn to the case of fully endogenous public expenditure.

Chapter VII: A Fairness Approach to Income Tax Evasion: the Analysis with Endogenous Public Expenditure

VII.1 The equilibrium

As argued in the previous chapter, in the long run, the only appropriate concept of equilibrium is one where the public budget always balances; that is, where public expenditure is endogenously determined by tax revenue. We have then to reformulate the model presented in the previous chapter so as to allow for endogenous public expenditure. This can be easily done as follows.[1]

Let R be the total tax revenue collected by government. In order to get an expression for R, recall that in our model each individual h belonging to group i knows the total amount of tax evaded by the other individuals in the two groups. Let us then call R_{i-h} the total expected revenue that government would collect if individual h belonging to group i fully paid his tax. We can express R_{i-h} as follows:

$$(1)\ R_{i-h} = N/2[t(I_1+I_2)-2c(p_1,p_2)]-\Sigma_{k\neq h}E(x_{ik})-\Sigma_h E(x_{jh})$$

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[1]In this section I follow strictly the analysis of Cowell and Gordon (1988). See their paper for a more extended discussion of the model and for the analysis of a number of cases which are not considered here.

where E(.) is the expected value operator and c(.) is the average cost needed to enforce probabilities of detection p_1 and p_2. Of course, $R=R_{i-h}-E(x_{ih})$. Using eq(2) of the previous chapter, we can then express the total supply of the public good as:

$$(2) \qquad G=N/\psi(N)[1/2(t(I_1+I_2)-2c(p_1,p_2)] \\ -1/\psi(N)[\Sigma_{k\neq h}E(x_{ik})+\Sigma_h E(x_{jh})]-1/\psi(N)[E(x_{ih})]$$

Taking the limit of eq(2) for $N\Rightarrow\infty$, using the assumptions imposed in the previous chapter on the production function of the public good and the fact that for a large number of individuals the probability of being detected in each group will tend to coincide with the actual share of taxpayers caught evading[2] we can then rewrite eq.(2) as

$$(3) \qquad G=[t\bar{I}-c(p_1,p_2)-1/2(s_i x_i^*-s_j x_j)]/\Psi=\underline{R}/\Psi$$

where $\underline{R}$ is just a shorthand for the expression in parentheses above and the other symbols have the meaning specified in the previous chapter. Note that $\underline{R}$ is the expected average tax revenue for a large economy. Eq.(3) then implies that for each individual

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[2]That is, for $N\Rightarrow\infty$, $\Sigma_{k\neq h}E(x_{ik})\cong((N/2)-1)s_i x_i^*\cong(N/2)s_i x_i^*$ and $\Sigma_h E(x_{jh})\cong(N/2)s_j x_j$.

h the amount of public good supplied is a known, given amount. Using eq(3) above and the analysis developed in the previous chapter we can express the individual optimizing problem in the case of endogenous public expenditure as:

$$(4) \qquad \max_{x_{ih}} \; (1-p_i)U^i((1-t)I_i + x_{ih}, \underline{R}/\Psi) + p_i U^i((1-t)I_i - \pi x_{ih}, \underline{R}/\Psi)$$

$$\text{s.t. } 0 \le x_{ih} \le \bar{x}_{ih}$$

As in the previous chapter, let us express the individual choice in eq(4) as $\hat{x}_{ih} = \min(x^o_{ih}; \bar{x}_{ih})$, where x^o_{ih} is the solution to problem (4) without considering the fairness constraint. Note that for given $\underline{R}$ the first and second order conditions for the unconstrained solution of eq.(4) are the same conditions analyzed for exogenous public expenditure.

While the above follows strictly the analysis of the previous chapter, note however that there are now two important differences. First, unless each type's preferences are separable in public and private good, the unconstrained solution x^o_{ih} is now a function of other individual behaviour as well. This is so because the choices of the other individuals in terms of tax evasion affect total tax revenue and therefore, through eq(3), public expenditure. Second, the fairness constraint is now a function of other individuals' behaviour through a double channel. On the one hand there is still the direct or reciprocity effect which we studied previously. On the other hand, there is now an indirect effect which operates through the effect of other people choices on public good supply and therefore on the marginal willingness to pay and the Kantian taxes.

In spite of these differences it is still very easy to prove that an equilibrium exists in my model. Again, as in the previous chapter, let us define an equilibrium as a vector of evaded taxes x_{ih}, such that $x_{ih}=\hat{x}_{ih}$ for $\forall i$ and $\forall h$. Without repeating all the steps followed in the previous chapter, section VI.4.1, to prove the existence of an equilibrium, simply note that even in the case of endogenous public expenditure the individual best reply function $\hat{x}_{ih}=\min(x^{o}_{ih};\bar{x}_{ih})$ is still a continuous function of x_{i-h}. This is so because the utility function is assumed to be differentiable in G and this guarantees that both $x^{o}_{ih}(.)$ and $\bar{x}_{ih}(.)$ (for interior solutions) are differentiable in x_{i-h}[3].

[3]To be more precise, if we assume the conditions expressed in eq.(13) of the previous chapter to hold, $0<x^{o}_{ih}<t_i I_i$. Then, x^{o}_{ih} is certainly continuous in x_{i-h} due to the fact that the utility function is assumed to be differentiable in G and that G(.) is differentiable in x_{i-h} from eq.(3) of the previous chapter. Hence x^{o}_{ih} is differentiable in x_{i-h} and therefore it must also be continuous in it. Similarly, $\bar{x}_{ih}$ is also continuous in x_{i-h}. In fact, if $0<z_{ih}<t_i I_i$, $\bar{x}_{ih}=z_{ih}$ and $\bar{x}_{ih}$ is certainly continuous in x_{i-h} because the marginal willingness to pay is differentiable in x_{i-h} and therefore $\bar{x}_{ih}$ is differentiable in x_{i-h}; if either $z_{ih}\leq 0$ or $z_{ih}\geq t_i I_i$, $\bar{x}_{ih}$ is trivially continuous in x_{i-h} because it does not depend on it. Therefore both x^{o}_{ih} and $\bar{x}_{ih}$ are continuous functions of x_{i-h}.

Therefore they must be continuous function of x_{i-h} which in turn implies that $\hat{x}_{ih}(.)$ is also continuous in x_{i-h}. Therefore, as in the previous chapter, we obtain individual best reply functions which are continuous functions of other behaviour and the proof of the existence of equilibrium follows naturally.

Hence, at least an equilibrium exists. Unfortunately, while it is not difficult to find sufficient conditions which would ensure a unique equilibrium even in the case of endogenous public expenditure[4], such conditions are not as easily interpretable as in the case of exogenous public expenditure. In particular, there seems to be no reason why they should hold in the general case. We then conclude that with endogenous public expenditure multiple equilibria cannot be ruled out. Unfortunately, this also implies that we cannot rule out the possibility that some of the resulting equilibria may be asymmetric: that is, that identical individuals may be characterized by different choices in an equilibrium. In

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[4]Sufficient conditions for uniqueness of equilibrium can be easily found by determining the conditions which would guarantee both $x^{o}_{ih}(.)$ and $\bar{x}_{ih}(.)$ to be contractions in x_{i-h}. In fact, as it can be easily shown, if both $x^{o}_{ih}(.)$ and $\bar{x}_{ih}(.)$ are contractions in x_{i-h}, $\hat{x}_{ih}(.)=\min(x^{o}_{ih},\bar{x}_{ih})$ must also be a contraction in x_{i-h}. We did this, but as stated in the text, the derived conditions did not allow for a clear interpretation and therefore they have not been reported here.

the following we will concentrate only upon symmetrical equilibria, that is, equilibria where $\hat{x}_{ih}=\hat{x}_i$, $\forall h$, $i=1,2$, and we further impose a condition which would guarantee such equilibria to be locally stable. The idea is that asymmetrical equilibria, even if they can still exist, may be unstable and therefore they would tend to disappear in the long run. Following Cornes and Sandler (1986:92), under a simple adjustment mechanism, local stability of a symmetric Nash equilibrium with two types of individuals is ensured by the following condition[5]:

(5) $$(\partial\hat{x}_1/\partial x_2)(\partial\hat{x}_2/\partial x_1)<1$$

where the derivatives in eq.(5) are evaluated at the equilibrium choices for $\hat{x}_i$, $i=1,2$. As we will see in the next

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[5]In order to get eq.(5) we postulate an adjustment mechanism of the following type:

$$dx_i/d\tau = \upsilon_i\{\hat{x}_i(x_j)-x_i[\tau]\} \quad i=1,2 \;\; j=1,2 \;\; i\neq j$$

where υ_i is a positive constant, $x_i[\tau]$ is the actual value of x_i at time τ and $\hat{x}_i(x_j)$ is the optimal value of evaded tax, according to eq.(4) above. That is, each individual adjusts to his optimal value with some delay. Linearizing the equation above around the equilibrium and computing the conditions for local stability we obtain eq.(5) in the text. Note that in the equation above we are assuming that identical individuals will tend to deviate from the equilibrium simultaneously, in the same direction and by the same amount (see Seade, 1980 for a discussion of the validity of this hypothesis in Cournot equilibria). Also note that in the case of corner solutions (i.e. $x_i^o=\bar{x}_i$) the derivatives above must be interpreted as directional derivatives. For a more satisfactory discussion of the stability conditions in the case of an income tax evasion model with endogenous public expenditure see Cowell and Gordon (1988).

sections condition (5) is also helpful to sign the comparative static derivatives.

VII.2 Comparative static analysis

Unfortunately, differently from the case studied in the previous chapter, the comparative statics analysis with endogenous public expenditure turns out to be characterized by a much larger amount of ambiguity. Practically, where there are two types of individuals differing both in terms of personal characteristics and in terms of opportunities for evading, the effect of a change in the tax rate on evaded tax can not be signed in any region where an equilibrium may lie. It is then worthless to concentrate on the general case and it is instead convenient to focus on some particular cases where we can get at least a feeling for the effects of changes in the parameters on equilibria.

Let us then introduce some simplifications in the general model above. In the following we will always assume: 1) the demand for the public good is characterized by zero income effects; 2) the probability of detection is the same for each type (i.e $s_1=s_2=s$). The first condition is crucial since it allows us to sign the effects of a change in the tax rate on x_i^o ; the second is not strictly speaking necessary but it allows one to simplify considerably the treatment and it seems quite innocuous. As we are going to show, the two assumptions above are not enough to allow one to sign the comparative statics derivatives in the different regions and extra assumptions will have to be imposed.

Let us then start by supposing that the equilibrium under consideration lies in the unconstrained region IV; that is, $\hat{x}_i = x_i^o$ i=1,2. Assuming an interior solution (i.e the conditions expressed in eq.(13) of the previous chapter hold), we can analyze the effects of changes in the rate of tax by differentiating the first order conditions and by invoking the implicit function theorem. The first order conditions for the unconstrained solution to problem (4) can be written as:

$$(6) \quad (1-p)U_c^i(C_{ih}^1, G(x_i^*, x_j)) - \pi p U_c^i(C_{ih}^2, G(x_i^*, x_j)) = 0 \qquad i,j=1,2 \ i \neq j$$

where $C_{ih}^1 = (1-t)I_i + x_{ih}^o$, $C_{ih}^2 = (1-t)I_i - \pi x_{ih}^o$ and G has been written as above to remind the reader that G is now, through eq(3), a function of the behaviour of other individuals. Assuming a symmetric equilibrium in region IV, we get $x_{ih}^o = x_i^* = x_i^o$ and $x_j = x_j^o$. By differentiating (6) with respect to t:

(7)

$$\partial x_1^o/\partial t = 1/DET\{[\Delta^2 - sw^2\Gamma^2/2\Psi][\Gamma^1(I_1 - \bar{I}w^1/\Psi)] + [sw^1\Gamma^1/2\Psi][\Gamma^2(I_2 - \bar{I}w^2/\Psi)]\}$$

$$\partial x_2^o/\partial t = 1/DET\{[\Delta^1 - sw^1\Gamma^1/2\Psi][\Gamma^2(I_2 - \bar{I}w^2/\Psi)] + [sw^2\Gamma^2/2\Psi][\Gamma^1(I_1 - \bar{I}w^1/\Psi)]\}$$

where $\Delta^i = (1-p)U_{cc}^i(C_i^1, G) + \pi^2 p U_{cc}^i(C_i^2, G) < 0$ by concavity of the utility function, $\Gamma^i = (1-p)U_{cc}^i(C_i^1, G) - \pi p U_{cc}^i(C_i^2, G) > 0$ by assumption of DARA, $w^i = U_G^i/U_C^i(G)$ is what we called in the previous chapter the fair price and $DET=\Delta^1\Delta^2-s(w^1\Gamma^1\Delta^2+w^2\Gamma^2\Delta^1)/2\Psi >0$ by condition (5) above[6]. As stated eq.(7) is difficult to interpret; but note that if we assume identical incomes and identical preferences across social groups eq.(7) reduces to:

$$(8)\ \partial x^O/\partial t=1/DET\{\Delta[\Gamma\bar{I}(1-w/\Psi)]\}$$

It can now be seen that the sign of eq.(8) depends on the sign of $(1-w/\Psi)$. To interpret this expression, note that $w=\Psi$ identifies a Pareto-optimal level of public good supply. In fact, by eq.(3) of the previous chapter $\Psi=\psi(N)/N$; therefore $w=\Psi$ can be rewritten as $Nw=\psi(N)$, which is the Samuelson condition for efficiency in public good provision. Eq.(8) then shows that, with identical individuals, evaded tax is increasing (decreasing) in the tax rate if public good is underprovided (overprovided), that

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[6]Recall that we are assuming zero income effects on the demand for the public good. This assumption implies that $U_G/U_C(.)$ does not depend on C. Therefore, $U_{CG}=(U_G/U_C)U_{CC}$. By differentiating eq.(6) we get an expression in U_{CG} which, using the above, can be written as :

$$(1-p)U^i_{CG}(C^1_i,G)-\pi pU^i_{CG}(C^2_i,G)=(U^i_G/U^i_C)[(1-p)U^i_{CC}(C^1_i,G)-\pi pU^i_{CC}(C^2_i,G)]$$
$$= w^i\Gamma^i$$

using the shorthand introduced in the text and the fact that the fair price, with zero income effects on the demand of the public good, depends only upon G (see eq(4) of the previous chapter). Finally, note that $\partial x^O_i/\partial I_i=-(1-t_i)\Gamma^i/\Delta^i$. Therefore, if following the hypothesis of DARA we assume $\partial x^O_i/\partial I_i>0$, $\Gamma^i>0$ certainly. For further details, see Cowell and Gordon (1988).

is if $w>\Psi$ (if $w<\Psi$). This is of course the Cowell and Gordon result reported earlier; but note now that eq.(7) shows that this result is not very robust. In particular, even if we assumed identical preferences across social groups, the sign of the derivatives in eq.($\bar{7}$) would still be totally ambiguous. We have better luck if we assume identical incomes and different preferences across social groups. In this case, as is easy to verify, $w^i<\Psi<w^j$ implies $\partial x_i^o/\partial t<0$ and $\partial x_j^o/\partial t>0$, $i,j=1,2$ $i\neq j$. However this condition has nothing to do with the issue of over/under provision of the public good and the nice link derived by Cowell and Gordon between tax behaviour and over/under provision of the public good is lost in an economy with different agents.

Also note that the counter-intuitive results of the portfolio choice model survive even in our extension of the model to different agents: in eq.(7) above, in the special cases derived for egalitarian income distribution, the individuals who have to gain (to lose), in expected terms, from the increase in the tax rate increase (decrease) their evaded tax following an increase in the tax rate. The driving force behind these results is of course, as explained in chapter VI, the assumption of DARA.

Let us then consider the opposite case of an equilibrium lying in region I, where both types of individuals are constrained in their tax behaviour. Assuming an interior solution for $\bar{x}_i$ (i.e. $0<z_i<tI_i$) and a symmetric equilibrium we can write $\bar{x}_i$ as:

(9) $\bar{x}_i = I_i[t - t_i^i(\bar{x}_i, \bar{x}_j)] + \vartheta^i \bar{x}_j + \vartheta^i I_j[t_j^i(\bar{x}_i, \bar{x}_j) - t]$

where I have already substituted for the symmetric equilibrium and where the Kantian tax has been written as a function of the amount of evaded taxes in order to remind the reader of the indirect influence that now other individual behaviour exerts, through eq.(3), on the fairness constraint. Unfortunately, comparative statics analysis with the general formulation in eq.(10) above produces ambiguous results. Let us then simplify greatly eq.(10) by assuming identical preferences and identical reciprocity weights ($U^1 = U^2 = U$ and $\vartheta^1 = \vartheta^2 = \vartheta$). Then, by differentiating eq.(10) with respect to t we get:

(10) $\partial\bar{x}_i/\partial t = I_i - T[(1-s)\bar{I}]/(1-sT)$ $\qquad i=1,2$

where $T = w(\varepsilon+1)/\Psi$, and where, as in the previous chapter, ε indicates the quantity elasticity of the marginal willingness to pay. To sign (10) note first that the stability condition (5) requires $1/s > T$[7]; therefore the denominator of the second term in equation (10) is certainly positive. Hence if $|\varepsilon| > 1$, $T < 0$ and both individuals will certainly increase their evaded tax following an increase in the rate of tax. If $|\varepsilon| < 1$ we get ambiguous signs and

[7] With identical preferences and reciprocity weights condition (5) reduces to $(1-\vartheta^2)(1-sT) > 0$; hence, for $\vartheta < 1$, it requires $1/s > T$.

$\partial \bar{x}_i/\partial t$ may turn out to be negative. Note however that if $I_i = \bar{I}$ (identical incomes), $\partial \bar{x}/\partial t<0$ would require $|\varepsilon|<(w-\Psi)/w$; this can only be possible if the public good is strongly underprovided. Thus, in an economy where income differences are limited, tax evasion in the constrained region can become decreasing in the tax rate only if the public good is strongly underprovided and the fair price is inelastic in the public good.

The intuition behind these results is quite straightforward. With endogenous public expenditure an increase in the tax rate has two effects: a direct one through the tax rate and an indirect one through the effect of the change of the tax rate on public good supply. By comparing eq.(10) above with eq.(15) and eq.(17) of the previous chapter, it is clear that the first element in eq.(10) captures the direct effect and the second element the indirect one. Thus, if the fair price is elastic in public good supply the two effects act in the same direction (see chapter VI, section VI.2) and we get unambiguous results, while if the fair price is inelastic the two effects affect the fairness constraint in opposite directions thus producing ambiguous results. Also note that in performing the exercise above we assumed $z_i>0$; but, as the case analyzed in the next section illustrates, this cannot be the case if individuals are identical and G is underprovided. Therefore, at least for the case of identical individuals, $\bar{x}_i$ is positively sloped in the tax rate for all the range where $\bar{x}_i>0$.

Finally, let us consider the mixed regions II and III. Here ambiguity is even larger than in the two previous cases. In order to make some progress let us then assume identical preferences, identical reciprocity weights and, without loss of generality, $I_2>I_1$. Then, from proposition 4 of the previous chapter, if an equilibrium lies in the mixed regions, it must be in region III, that is where $\hat{x}_1=x_1^0$ and $\hat{x}_2=\bar{x}_2$. Assuming a symmetric equilibrium and differentiating with respect to t:

$$(11a) \qquad \partial x_1^0/\partial t=1/H\{\Gamma^1[A+(\bar{I}w/\Psi-I_1)]\}$$

$$(11b) \qquad \partial \bar{x}_2/\partial t=1/H\{-\Gamma^1\vartheta[A+(\bar{I}w/\Psi-I_1)]-\Delta^1[I_2-\vartheta I_1+(1-\vartheta)\bar{I}T]\}$$

$A=s/2[(\vartheta I_1-I_2)w/\Psi+I_1(1-\vartheta)T]$, $\quad H=[s\Gamma^1(1+\vartheta)/2\Psi-\Delta^1(1-s(1-\vartheta)T/2]$. Needless to say the sign of (11) is in general ambiguous. Note however that if $|\varepsilon|>1$ (i.e $T<0$), $H>0$ and $A<0$. Then, if inequality in income distribution is not too large and/or the public good is overprovided (i.e if $\bar{I}w/\Psi<I_1$) $\partial x_1^0/\partial t<0$ and $\partial \bar{x}_2/\partial t$ is ambiguous, while if the public good is strongly underprovided but inequality in income distribution is limited (i.e if $\bar{I}w/\Psi>I_1$ but the second term in (11b) is positive) $\partial x_1^0/\partial t$ is ambiguous and $\partial \bar{x}_2/\partial t>0$. Roughly speaking, these results are in line with the analysis performed above for the more clear-cut cases of region I and IV.

Summing up, comparative statics analysis with endogenous public expenditure[8] shows a much greater amount of ambiguity than

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[8]In a previous version of the paper I attempted a comparative statics exercise with respect to changes in the composition of public expenditure. The simultaneity problem that arises with

the analysis with exogenous public expenditure. However, when they can be signed, the comparative static results are roughly in line with the analysis of the previous chapter. Tax evasion is likely to be increasing in the constrained region and it was shown that the key parameter in this region is represented by the quantity elasticity of the fair price. Tax evasion may instead be increasing or decreasing in the unconstrained region depending on over/under provision of the public good. In the mixed regions results are even more ambiguous but the key parameters are still the ones indicated above. The analysis above also indicates that clear-cut results with endogenous public expenditure can only be reached by assuming identical individuals. In order to get a more precise understanding of the implications of the model and to address some other important questions, it is then worthwhile to

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endogenous public expenditure, can be easily solved by assuming that government decides, ex-ante, to devote a share of total tax revenue to a particular public good. For example, if there are two public goods G_1 and G_2, government may decide ex-ante to spend λ of total tax revenue on public good G_1 and $(1-\lambda)$ on public good G_2, where $0\leq\lambda\leq1$. Thus, eq.(3) above could be written as $G_1=\lambda\underline{R}/\Psi_1$ and $G_2=(1-\lambda)\underline{R}/\Psi_2$ and eq.(4) and the equations for the fair price could be similarly changed. The goal of the exercise was to investigate the effects that changes in the composition of public expenditure (i.e. shifts in λ) may have on tax evasion in the different regions and to relate these effects to some characteristics of the public goods. Unfortunately, the results did not repay the analytic effort and for this reason they have not been presented in the text. The results turn out to be ambiguous in all regions and I only get the weak result that in the constrained region I, with identical preferences, complementary public goods implies $\partial\bar{x}/\partial\lambda|_{\lambda=0}<0$.

consider the case of identical individuals in somewhat more detail.

VII.3 Identical Individuals and Government X-Inefficiency

In the previous chapter and in section VII.2 above, we considered cases where the decision to evade by a single taxpayer was motivated by a perception of inequity in his fiscal treatment, both in relationship with other taxpayers and in relationship with the given supply of public goods by government. However, these are not the only cases where an individual may perceive himself to be involved in an "unfair" deal with government. Another case which is often quoted in the press as a major determinant of tax evasion, at least in Italy, is that where fairness is violated by the state itself through "inefficiency" in the production of public goods. The basic argument is that government is unable to spend taxpayers' money efficiently and that, consequently, the quantity and quality of public goods supplied is not adequate with respect to total tax revenue collected.

In this section we study the implications of this argument, being mainly interested in the effects that shifts in the efficiency of the state as a producer of public goods may have on income tax evasion and on the relationship between the tax rate and evaded tax. Since we are not interested in this section in motivations for evading arising from perceived inequity in the tax treatment of the different types of taxpayers, we simplify the problem by assuming identical individuals. Moreover, for the reasons stressed above we simplify further the problem by assuming zero income effects on the demand for the public good. Implicitly, the case studied in this section will also allow us to get a more precise understanding of the implications of the model for the relationship between the tax rate and evaded tax in the case of identical individuals.

Let us then start by assuming that, in line with our previous analysis, the "true" production function for the public good, in a large economy, is given by:

(12) $G=\underline{R}/\Psi$

We assume that individuals know the "true" production function of the public good and that their computation of the fair tax is based on this production function. We can then introduce government X-inefficiency by assuming that the latter produces the public good according to the formula:

(13) $G=\alpha\underline{R}/\Psi$

where $0<\alpha\leq 1$ is our "efficiency" index of public good production. If $\alpha=1$, governmental production is X-efficient, while if $\alpha<1$ governmental production is X-inefficient in the sense that a higher amount of the public good could be produced with the same revenue. Eq.(13) is probably the simplest possible way of capturing analytically the complex issue of the low quality of government supply. From eq.(3) above, in an economy with identical individuals, average expected revenue for an individual h can be simply written as:

$$(14) \qquad \underline{R} = tI - sx^{*} - c(p)$$

where as above $x^{*}=\Sigma_{k\neq h}x_k/(N-1)$. In the following, we will assume that the average cost for enforcing probability of detection p is negligible with respect to average revenue and we will therefore set $c(p)=0$ in eq.(14). This is just a simplification which will make the graphical illustration of the results of the model easier. We will indicate however where it is likely to be of some importance.

In an economy with identical individuals, the desired level of tax evasion (see eq.(10), eq.(5) and eq.(6) of chapter VI) can be simply expressed as:

$$(15) \qquad z_h = [1-\beta][tI-w(G)G] + \beta x^{*}$$

Let us consider first the constrained region I. Assuming a symmetric equilibrium in this region (i.e. $x^{*}=\bar{x}$), substituting in (15) and recalling our definition of $\bar{x}$ in chapter VI, we can write:

$$(16) \qquad \bar{x}(t,\alpha) \geq [tI - w(G)G]$$

Clearly, (16) will hold as an equality if the RHS of (16) is positive or equal to zero (i.e. if $tI>z_h-z\geq 0$) and as an inequality if the RHS of (16) is negative (i.e. if $z_h-z<0$). In the latter case $\bar{x}=0$. Note that in (16) we wrote $\bar{x}$ as a function of t and, through (14), of α. Let us assume first that the RHS of (16) is greater than zero; then (16) holds as an equality and $\bar{x}>0$. By substituting for $x=\bar{x}$ in eq.(14) and in eq.(13):

(17) $$G(t,\alpha) = \alpha[tI - s\bar{x}]/\Psi$$

Eq.(16) and eq.(17) form a system of simultaneous equations in α and t; solving the system and differentiating respectively for α and t, we get:

(18a) $$\partial G/\partial t = \alpha I[1-s]/[1-\alpha sT]\Psi$$

(18b) $$\partial G/\partial\alpha = [tI-s\bar{x}]/[1-\alpha sT]\Psi$$

(19a) $$\partial\bar{x}/\partial t = I[1-\alpha T]/[1-\alpha sT]$$

(19b) $$\partial\bar{x}/\partial\alpha = -T[tI-s\bar{x}]/[1-\alpha sT]$$

where, as in the previous section, $T=w[\varepsilon+1]/\Psi$. The stability condition (5) above[9] imposes $[1-\alpha sT]>0$; therefore $\partial G/\partial t>0$ and $\partial G/\partial\alpha>0$ certainly, while $\partial\bar{x}/\partial\alpha>0$ [<0] as $|\varepsilon|>1$ [$|\varepsilon|<1$]. The sign of $\partial\bar{x}/\partial t$ is instead less certain; as we saw in the previous section, it is certainly positive if $|\varepsilon|>1$ but it may become negative if $|\varepsilon|<1$. Note however, as in eq.(10) above, that $\partial\bar{x}/\partial t<0$ would require $|\varepsilon|<[\alpha w-\Psi]/\alpha w$. Let us then call G^{*} that level of G such that $w(G^{*})=\Psi$ and G^{**} that level of G such that $w(G^{**})=\Psi/\alpha$. Inspection of eq.(12) shows that G^{*} is the Pareto optimal level of public good supply with respect to the "true" production function. Also note that $G^{*}\geq G^{**}$ for $\alpha\leq1$. It is then clear that for $G\geq G^{**}$ $\partial\bar{x}/\partial t>0$ certainly. This will turn out to be useful below. The comparative statics results of eq.(18) and eq.(19) above are of course in line with our previous analysis and do not require further comments[10].

In the exercise above we assumed $z=\bar{x}>0$: It is now time to ask at which level of G this is possible. To see this, fix G at $\bar{G}$, assume that $\bar{x}>0$, solve (16) for $w(\bar{G})\bar{G}$, and subtract (17) from the expression thus obtained. Elementary algebra then shows:

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[9] Since in this section we consider only identical individuals, we change slightly the adjustment mechanism by assuming now a small deviation from the symmetric equilibrium of all individuals simultaneously. As can be easily verified, for $\bar{x}>0$, local stability imposes $[1-N]<\alpha sT<1$. See Cornes and Sandler (1986:93-4) for further details.

[10] Simply note that with $\alpha<1$, it is as if public expenditure were "exogenous", in the sense that by shifting α we can change G without having to change public revenue. It is then clear why our results for $\partial\bar{x}/\partial\alpha$ are exactly the same as the results achieved in the previous chapter for changes in G: see chapter VI, section VI.4.2.

$$(20) \qquad \bar{G}[w(\bar{G})-\Psi] = [1-\alpha]tI - [1-\alpha s]\bar{x}$$

Next suppose that $\alpha=1$ and $\bar{G}\leq G^{*}$; that is, assume that the public good is underprovided. Then, $w(\bar{G})\geq\Psi$ certainly and the LHS of (20) is greater than or equal to zero. In contrast, if $\alpha=1$, the RHS of (20) is certainly negative for $\bar{x}>0$. Therefore eq.(20) cannot hold as an equality. We then conclude that for $\bar{G}\leq G^{*}$ and $\alpha=1$, $\bar{x}=0$ and (16) holds as an inequality. We have established:

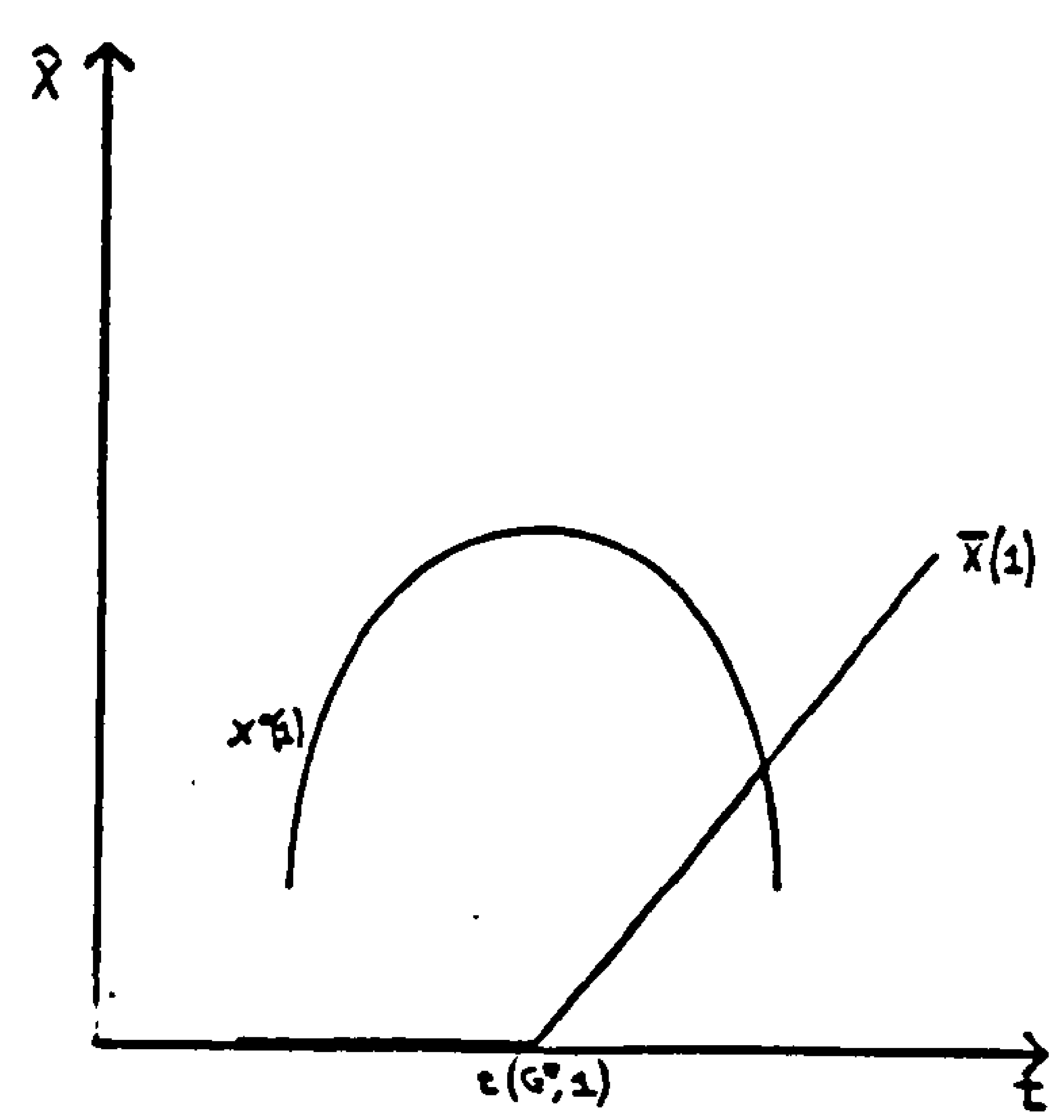
<u>Proposition 1</u> "If government is an efficient producer of the public good, individuals will never evade for G less than or equal to the optimal level"

The intuition is of course straightforward: if $G\leq G^{*}$ the fair price that an individual is ready to pay for the given supply of the public good is larger than the cost of providing that amount of public good; therefore if government simply sets $tI=\Psi G$ individuals would never evade. This result may look very strong, but it should be noted that we have assumed away here any problem concerning the distribution of the tax burden across social groups. Therefore taxpayers actually act as Kantians. Moreover, as can be easily verified, if we introduce back in eq.(14) the cost of enforcing a given probability of detection, tax evasion turns out to be positive, if small, at $G=G^{*}$ for $\alpha=1$.

Going back to eq.(19a) above, we now see that the ambiguity about the sign of $\partial\bar{x}/\partial t$ disappears: whenever $\bar{x}>0$, for $\alpha=1$, the fairness constraint is certainly increasing in the tax rate. In order to get a complete picture of individual behaviour let us finally consider the unconstrained solution x^o. By invoking eq.(8), we can immediately establish that the unconstrained solution x^o is, for $\alpha=1$, a concave function of the tax rate with an unique maximum associated with $G=G^*$. Putting together the two pieces of analysis, it is then clear that, in an economy with identical individuals and an efficient government, tax evasion is zero for all $G\leq G^*$, is positive and increasing in t for a range of values of t such that $G>G^*$, and might become positive and decreasing in the tax rate for high values of t, if such high values of t were ever to be reached. Figure VII.1 illustrates:

Figure VII.1

Tax evasion with identical individuals and an efficient government



If still not entirely satisfactory, because we cannot rule out the possibility that at high tax rates evaded tax may become a decreasing function of the rate of tax, it is clear that the relationship between tax evasion and the tax rate presented in the picture above is far more in agreement with intuition and observed behaviour than the implications of the traditional portfolio model of income tax evasion. It is maybe worthwhile to stress again the intuition behind the picture above: for $G \leq G^{*}$ an increase in the tax rate would increase the welfare of the taxpayers: therefore, the latter do not resist the increase in the tax rate and they keep evaded tax at zero. If $G^{*} \geq G$ an increase in the tax rate would instead reduce individual welfare and therefore individuals attempt to resist this increase by increasing tax evasion. With all the caveats that the analysis still presents (see next section) the above seems to the present author a much more reasonable explanation of individual behaviour than that offered by the conventional models of income tax evasion.

Let us now turn to the case where $\alpha<1$. In order to identify the range of values of G such that $\bar{x}(t,\alpha)>0$, assume $\bar{x}>0$, fix $G=\bar{G}$, and let t adjust so as to keep $G=\bar{G}$ when varying α. Solving eq.(17) for t at $G=\bar{G}$, and substituting in eq.(16), we get:

$$(21) \quad \bar{x}(\bar{t},\alpha) = \bar{G}[\Psi-\alpha w(\bar{G})]/[1-s]\alpha$$

where $\bar{t}$ is the rate of tax needed to keep $G=\bar{G}$ at any level of

α. Clearly, $\bar{x}>0$ for $\bar{G}>G^{**}$ and $\bar{x}=0$ for $\bar{G}\leq G^{**}$. Going back to eq.(19a), this implies that whenever $\bar{x}>0$, the fairness constraint is an increasing function of the tax rate. This of course simply repeats our previous argument. Note however than now $\bar{x}>0$ for $G=G^{*}$: that is, individuals wish to evade at the Pareto optimal level of public good supply.

In order to see the effect of changes in α on tax evasion let us first ask how the rate of tax would change so as to keep $G=\bar{G}$ following a change in α. Assuming $\hat{x}=\bar{x}>0$ (i.e. $\bar{G}>G^{**}$), differentiating totally eq.(16) and eq.(17) at $G=\bar{G}$ and solving for $dt/d\alpha$ we get:

$$(22) \qquad dt/d\alpha|_{G=\bar{G}} = -[tI-s\bar{x}]/[1-s]\alpha I$$

which is unambiguously negative. Since this holds for any G it must also hold for $G=G^{*}$; that is, a fall in efficiency by government increases the rate of tax needed to enforce the Pareto optimal level of public good supply. Also note that from eq.(22) and eq.(16) $d\bar{x}/d\alpha|_{G=\bar{G}}<0$: that is, a fall in α must unambiguously increase the fairness constraint for any level of G. To get a complete picture of individual behaviour let us finally examine the effects of changes in α in the unconstrained solution x^{0}. By substituting for G from eq.(13) in eq.(6) above and by differentiating with respect to t and α we get:

(23) $\partial x^{o}/\partial t = \Gamma I[1-\alpha w/\Psi]/DE$

(24) $\partial x^{o}/\partial \alpha = -[w\underline{R}\Gamma/\Psi]/DE$

where $DE=[\Delta - s\alpha w\Gamma/\Psi]$. $DE<0$ and therefore $\partial x^{o}/\partial \alpha>0$[11] and $\partial x^{o}/\partial t \gtrless 0$ as $\Psi \lessgtr \alpha w$. These are of course the results already reached above: but note that now at $G=G^{*}$, $\partial x^{o}/\partial t<0$ for $\alpha<1$. Putting together these results with the ones reached before, it is easy to characterize the effects of a fall in government efficiency on tax evasion. Let us start from $\alpha=1$ and consider a small fall in α: then, from eq.(19a) and eq.(24) above, if $|\varepsilon|<1$ ($|\varepsilon|>1$) $\bar{x}$ will increase (decrease) and x^{o} decrease for any level of t. Furthermore, following the decrease in α, the level of t needed to enforce the Pareto optimal level of public good supply [$t^{*}=t(G^{*},\alpha)$] must increase (eq.(22)), the fairness constraint at t^{*} must be positive (eq.(21)), and $\partial x^{o}/\partial t$ must be negative at $t=t^{*}$ (eq.(24)). Figure VII.2 illustrates for the two cases of $|\varepsilon|<1$ and $|\varepsilon|>1$.

As the figure indicates, for $|\varepsilon|<1$, a fall in α increases evaded tax for any level of t up to the point where $x^{o}(\alpha)=\bar{x}(1)$, and it reduces it afterwards. In contrast, if $|\varepsilon|>1$, the fall in α reduces evaded tax for any level of t. Hence, depending on the

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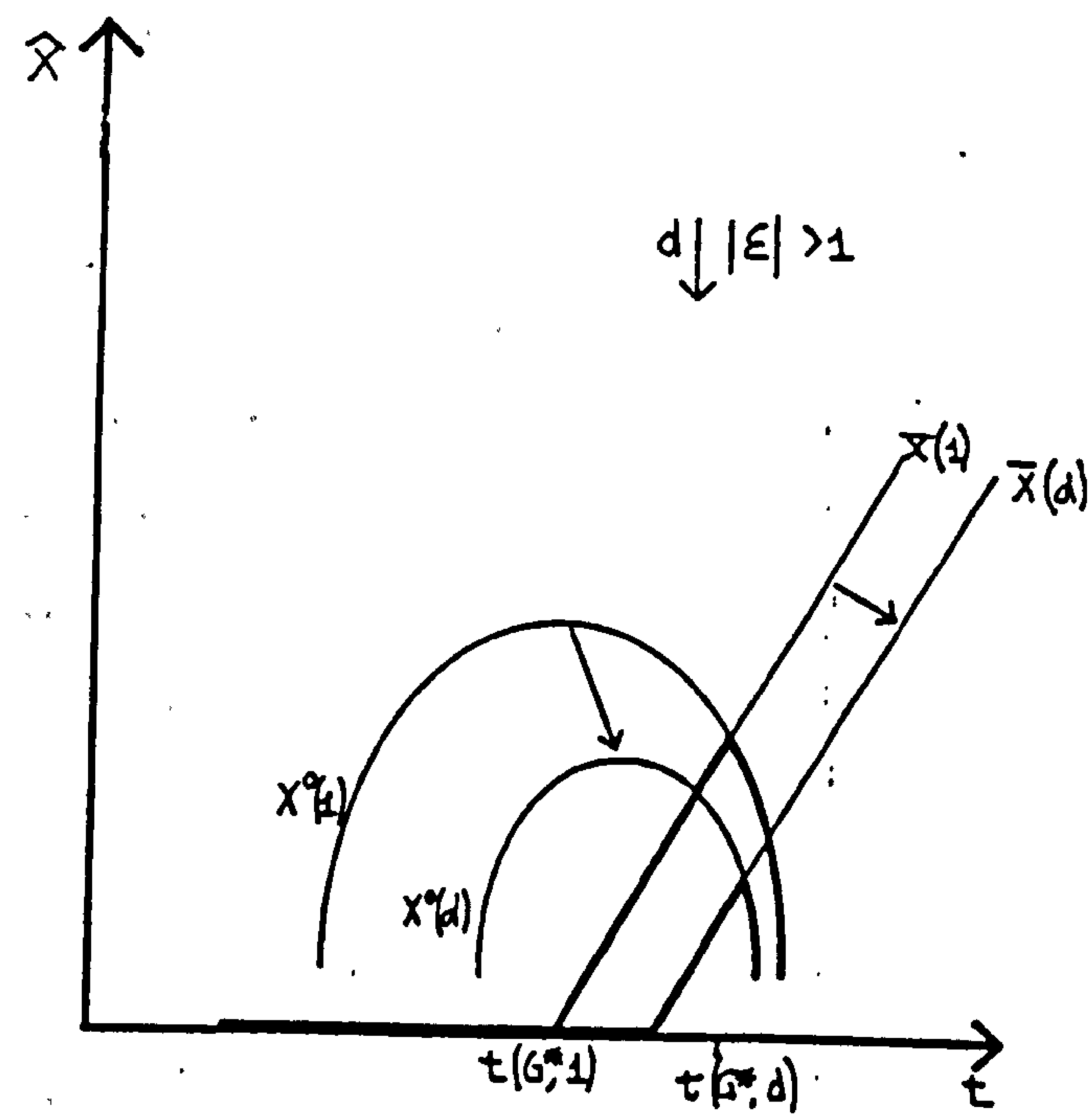
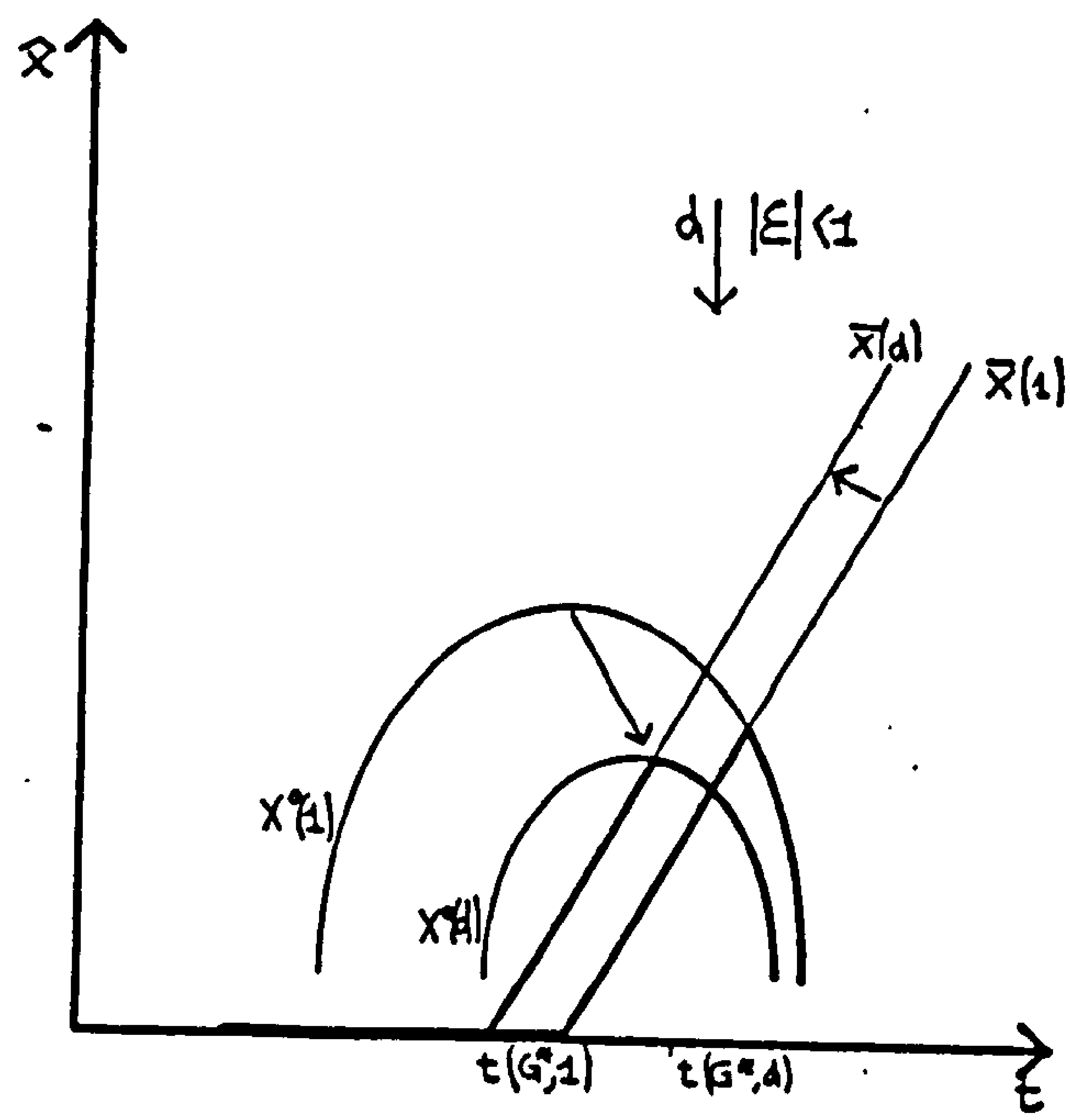
[11]For the reason explained above, $\partial x^{o}/\partial \alpha$ has of course the same sign as $\partial x^{o}/\partial G$ in the case of exogenous public expenditure.

Figure VII.2





value of $|\varepsilon|$ an increase in government efficiency may actually increase tax evasion. This conclusion runs against the very argument which stimulated the present analysis. It should be noted however that in the picture above we plotted tax evasion against the rate of tax. If we repeated the same exercise by keeping G constant the results would change: as we noted above, in the region where $\hat{x}-\bar{x}>0$, tax evasion would certainly increase, for any level of G, as α decreases.

Finally, note that we kept α constant for all the exercise. But it is often suggested that the public sector tends to be less efficient the larger it is (because of increasing bureaucracy, more complex management problems, increasing administrative costs, etc.). In our simple model this could be captured by assuming that α is a decreasing function of t, since the latter variable represents a proxy for the amount of resources allocated to the public sector. Assuming $\hat{x}-\bar{x}>0$ and $|\varepsilon|<1$ it is then easy to establish from eq.(16) and eq.(17) above that if $|\alpha_t \alpha/t|>tI(1-s)/GTs$ total revenue will fall as t increases. That is, if individuals are ruled by notions of fairness we may obtain a Laffer curve even in an economy without labor supply.

VII.4 Conclusions

In this chapter we extended the analysis to the case of endogenous public expenditure (balanced public budget). In section VII.1 we modified the model so as to allow for endogenous public expenditure, in section VII.2 we repeated the comparative statics exercises of the previous chapter and in section VII.3 we analyzed the model in the case of identical individuals. In the latter section we also investigated the effects of government X-inefficiency on tax evasion. The comparative statics results are in general ambiguous and can be signed only in some special cases. However, when they can be signed, they are roughly in line with the results achieved in the previous chapter. Evaded tax turns out to be increasing in the tax rate in the constrained region and increasing or decreasing in the unconstrained region according to the over/under supply of the public good. A special case where ambiguity disappears completely is the case of identical individuals and zero income effects on the demand for the public good. Accordingly, we studied this case in detail in section VII.3. It was shown that in this case tax evasion is zero for low levels of the tax rate (up to the point where public good supply is Pareto optimal if the administrative costs for detecting tax evasion are small), is increasing in the tax rate for higher rates of tax and may decrease in the tax rate at very high rates of tax, if such levels would ever be reached. This of course confirms our results of the previous chapter and the comments advanced there apply here as well. Finally, we investigated the effects of shifts in government efficiency as a producer of public goods on tax evasion. In contrast to what one could have expected, an increase in government X-efficiency can increase or decrease tax evasion for any level of the tax rate depending on the quantity elasticity of the fair price. However, at least in the constrained region, tax evasion would certainly fall for any level of public good supply following an increase in government X-efficiency.

Let me now end up this section with some general remarks on the analysis developed in this and in the previous chapter. The model presented in chapters VI and VII is surely primitive. The informational requirements needed by taxpayers to compute the fair tax are likely to be too strong to be realistic and we limited the analysis to an economy without a labor choice decision. These caveats of the model indicate that the latter must be taken simply as a first attempt to consider in a formal analysis the complex issue of the fairness of the relationship between taxpayers and the state. In the present context I wish only to stress that, in spite of its limitations, the model presented in the chapters above represents an improvement upon the traditional model of income tax evasion. The theoretical foundations of the model are deeply rooted in a sociological literature which is supported by a substantial amount of empirical evidence. The results of the analysis are far more in line with intuition and the empirical evidence than the results of the portfolio-choice model. Going back to section V.1, we managed to achieve all the targets that we imposed on our research agenda. Tax evasion was shown to be zero at low levels of the tax rate and increasing in the tax rate for at least some range of the rates of tax; there are equilibria where individuals choose not to evade even if it would be in their selfish interest to do so; and public expenditure affects tax behaviour without producing counter-intuitive results. We further showed in the previous chapter that income distribution and the distributional characteristics of public expenditure are also important factors in determining an individual's attitude toward tax evasion. Needless to say, all these predictions could be tested against empirical and experimental evidence.

Also note that a number of extensions of the analysis could be easily performed. For example, in the previous chapter we set up the model so as to allow for a general fiscal structure but we subsequently performed the analysis only for a proportional tax system. It would be interesting to enlarge the model to consider a progressive tax system; given the strong egalitarian bias of the Kantian individuals, a reasonable conjecture would suggest that evaded tax in the constrained region should fall for poor and rise for rich individuals. Similarly, we did not consider in the model that tax revenue is in part spent on redistributive transfers to the private sector. It might be worthwhile to try to introduce this feature in the model as well: again, intuition would suggest that desired tax evasion should increase for rich and fall for poor people, but this should be verified. Also in the model we assumed that everybody is ruled by fairness considerations: but it may be more realistic to imagine that there is a proportion of taxpayers who are completely selfish. Given the reciprocity weights and the indirect effects on public expenditure it is likely that even a small share of taxpayers who are completely selfish could raise tax evasion considerably. But again this needs to be verified more carefully in a dynamic model. Finally, maybe the most important problem which could be addressed in the model above concerns the issue of the selection of optimal rates of tax, penalty rates and expenditure on detection by government in an economy where individuals are ruled by fairness considerations. I reported in section VI.1 on the difficulties within the traditional model of addressing these questions; and my feeling is that the results of an optimal tax evasion exercise in my model could be very different from the traditional ones. Indeed, if there is a policy suggestion which could be drawn by the present analysis it is that the reestablishment of "fairness" in the relationship between taxpayers and the state might be a more effective tool for reducing tax evasion than increasing the coercion on taxpayers.

Chapter VIII: Conclusions

VIII.1 Concluding Remarks

In the first part of this work we justified and formalized the notion of Kantian behaviour in a model of private supply of a public good. In the second part of this work we applied the same approach to income tax evasion, seen as an example of voluntary (non) provision of public goods. We argued that this approach is able to offer interesting insights for the analysis of the phenomena at hand and some results were reached which seem to support our claim. In the concluding section of each of the two parts we discussed the caveats of the analysis performed in that part and we also offered some suggestions for further research in the two fields considered. Rather than repeating again the arguments discussed at the end of each section, let me instead conclude this work by pointing out a general caveat pertaining to the approach followed in the previous chapters and by indicating some other economic fields where the adoption of an ethical approach to human behaviour may turn out to be fruitful.

Beginning with the former, I believe that the foundations of Kantian morality would repay a more detailed analysis than that performed in this work. In modeling Kantian behaviour we basically followed the literature on this issue and argued that in the context of the simple economy that we were considering, a timeless economy where income is not earned, that way of formalizing

Kantian behaviour could be considered adequate. In the final section of chapter IV we also offered some suggestions on how the notion of Kantian behaviour could or should be reformulated in the context of more complex economies. However, the same notion of Kantian behaviour was introduced simply on the basis of a verbal argument and some extra work on this issue would be highly desirable.

In particular, it might be of interest to determine whether Kantian behaviour can be derived formally from a set of axioms reflecting more fundamental ethical principles, such as the notion of symmetry of treatment among agents. This would allow one to clarify the structure of Kantian behaviour, thus making it easier to extend the notion to different contexts, and would also allow one to understand better the links between Kantian morality and the alternative interpretations of fairness which have been proposed in the literature in different contexts --such as the notion of fair allocations popularized by Varian (1974) and Baumol (1986), the fair distribution in terms of needs and deserts discussed by Sen (1984) in several works, and the many notions of fair behaviour which are commonly used in labour economics (see, for example, the the work of Akerlof, 1984 and Frank, 1985). This enterprise, if successful, might also allow one to use the notion of Kantian morality as a different way to assess the desirability of economic processes and results, together with, or alternative to, the many tools that economists already employ to this end.

While the above shows that much work on Kantian morality has still to be done and that our formalization represents only a first attempt to address this issue. I would nevertheless like to argue that, in spite of their limitations, the models introduced in chapter III might be used, upon reformulation, to cast light on several economic phenomena. A first field where they could be applied is labour economics. I am referring here not only to the many examples of collective action which characterize this field --as for example joining a union, worker participation in a strike and so on-- but also to more substantial issues such as the determination of wages in collective bargaining and the issue of why firms pay wages above the market-clearing level.

For example, the literature on efficiency wages (Akerlof and Yellen, 1986) is based on the hypothesis that workers reduce their labor effort if their real wage is cut, so that it can be optimal for a firm to pay a wage which is higher than the market clearing wage. In this literature however it has never been explained clearly why workers' effort should be positively correlated with the real wage. In order to solve this problem, in a recent paper Akerlof and Yellen (1988) suggest that the effort produced by a worker should be modeled as an increasing function of the wage paid by the firm less a subjectively determined "fair" wage. However, this fair wage is not explained in the theory and it is introduced ad hoc as the wage of some reference group or a weighted average of the wages paid in the relevant industry. One could perhaps attempt to use our Kantian rules weakened by reciprocity considerations to model the process by which a worker comes to select the wage that he would consider it fair to receive in exchange for the effort produced. Again, as in our model on income tax evasion, reduction of effort by the worker could then be interpreted as an example of (non) voluntary contribution. It might also be interesting to see if this approach is able to offer a rationale for some empirical regularities in the wage setting behaviour of firms (see Akerlof and Yellen, 1988:44) which the efficiency wage literature fails to explain.

Another important field where the model above could be applied is in the case of cooperative enterprises. Since the seminal paper by Sen (1966) a growing literature has developed discussing the way in which labour-managed firms select the output to produce and how the resulting profits are distributed among workers. This is clearly an issue where our formalization of Kantian behaviour weakened by reciprocity considerations might offer some insights. Here again, as in the original Sugden paper (Sugden, 1984), the voluntary contribution of an individual should be modeled in terms of the work effort employed in the production of the output of the labour-managed firm. The problem for the labour-managed firm would then become that of determining simultaneously the output of the firm and the allocative rule which would maximize the welfare of its members. It could be interesting to check if this approach would be able either to solve or to offer an alternative rationale for some of the most striking results of this literature, such as the negatively sloped supply function of the labour-managed firm.

It would be very easy to offer many other examples of economic fields where the models discussed in the previous chapters might be applied, upon reformulation. It of course remains to be seen whether their application in the indicated fields would turn out to be useful, in the double sense of offering interesting insights and producing analytic results in line with observation. It is hoped however that the indications offered above are enough to suggest that the approach followed in the previous chapters is rich in potential applications. The writer's hope is that it will also turn out to be fruitful.

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