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Abstract

In this paper we deal with shift scheduling of tank trucks for a small oil company. Given are a set of tank trucks with different characteristics and a set of drivers with different skills. The objective is to assign a feasible driver to every shift of the tank trucks such that legal and safety restrictions are satisfied, the total working times of the drivers are within desired intervals, requested vacation of the drivers is respected and the trucks are assigned to the most favored drivers. We propose a two-phase solution algorithm which is based on a mixed integer linear programming formulation and an improvement procedure. Computational results are reported showing that the algorithm is able to generate good schedules in a small amount of time.

Key words: shift scheduling, tank trucks, mixed integer programming

AMS Classification: 90B35 Scheduling

1 Introduction

Staff scheduling problems deal with the issue of assigning employees to shifts (tasks) in a certain time period (usually, a week or one month) such that several regulations are satisfied, qualifications and preferences of the employees are taken into account, level of service is maximized and costs are minimized. Since business becomes more service oriented, the importance of staff scheduling has increased in recent years. Various companies and public organizations (like software companies, warehouses, airlines, railways, bus companies, hospitals, security services or call centres) want to use their staff in an efficient way covering all duties with small costs and trying to take into account preferences of their employees as much as possible. Due to a large number of constraints (e.g. different skills of the employees, different types of contracts, legal restrictions) staff scheduling is a very complicated task. Often, such schedules are still constructed manually or with the help of a spreadsheet tool. Since almost every company has its own specific requirements, it is very difficult to develop a general scheduling system which generates appropriate schedules in an automatic way.

In the literature mainly models and algorithms for specific situations are studied: see for example the survey on nurse rostering by Burke et al. [5], Kohl and Karisch [10] for airline crew scheduling, Bard et al. [1] for staff scheduling in postal service, or Mehrotra [12] for problems in call centres. In some applications schedules must be repeated according to cyclic patterns (cf. e.g. Beaumont [2] or Mason et al. [11]). More general situations are covered in Ernst et al. [9] who review applications, methods and models for staff scheduling or in Blöchliger [3] who gives a tutorial on modelling staff scheduling problems. De Causmaecker et al. [8] classified personnel scheduling problems in Belgian companies and identified four different types of scheduling problems: permanence, mobility, fluctuation and project centered planning. Meisel and Schaerf [13] considered a general employee scheduling problem and developed algorithms using constraint programming and local search. The problem of scheduling workers at a hub of a trucking system for the loading and unloading of trucks has been tackled with an integer linear programming formulation solved by column generation in Sarin and Aggarwal [14].

In this paper we deal with shift scheduling of tank trucks for a small oil company. We are not aware of any related problem in the literature. For tank trucks mainly dispatching or routing problems are studied (see e.g. Brown and Graves [4]).

In our problem we are given a set of tank trucks with different characteristics and a set of drivers with different skills. The objective is to assign a feasible driver to every shift of the tank trucks such that legal and safety restrictions are satisfied, the total working times of the drivers are within desired intervals, requested vacation of the drivers is respected and the trucks are preferably assigned to the most favored drivers.

In this paper we propose a two-phase solution algorithm which is based on a mixed integer linear programming formulation and an improvement procedure. The purpose of such an automatic system is to

- answer the strategic question of the company whether they have sufficient employees to satify their demands in each month,
- shorten the time for a manual planner in order to create a schedule for the next month,
- decrease costs by using the employees in a more efficient way (e.g. by reducing paid overtime),
- quickly modify an existing schedule due to small short-term changes (like illness of employees).

The remainder of this paper is organized as follows. After introducing the considered scheduling problem in Section 2, a mixed integer linear programming formulation is given in Section 3. Based on this model a two-phase solution algorithm is described in Section 4. After presenting computational results in Section 5 we conclude with some remarks in Section 6.

2 Problem formulation

In this section we describe the studied problem of scheduling tank trucks in more detail. Given is a planning horizon $\mathcal{T} = \{1, \dots, T\}$ of working days (Monday to Saturday), usually belonging to a single month. Let $\mathcal{T}^{MF} \subset \mathcal{T}$ be the set of all days from Monday to Friday, and $\mathcal{T}^{Sa} \subset \mathcal{T}$ be the set of all Saturdays. Furthermore, we have a set $N := \{1, \dots, n\}$ of tank drivers, divided into permanent drivers N^S (with a fixed contract, fixed salary and paid overtime) and temporary drivers (which are used when not sufficient permanent drivers are available and which are only paid for hours performed). The company specifies for each driver $j \in N$ an interval $[Z_j^{\min}, Z_j^{\max}]$ for the desired total working time in the planning horizon which is influenced by the actual working time of the previous month. Furthermore, each driver $j \in N$ specifies a subset $U_j \subseteq \mathcal{T}$ containing all days where he definitely does not want to work (vacation).

The set F contains a certain number of tank trucks with different characteristics (like length or class of risk). While for the subset $F^{1S} \subseteq F$ one day shift has to be scheduled from Monday to Friday, the trucks from the subset $F^{2S} \subseteq F$ are operated within two shifts (an early and a late shift) from Monday to Friday. Additionally, a subset $F^{Sa} \subseteq F$ contains trucks which have to be driven on Saturdays in a single shift. For each $i \in F^{1S}$ and every $t \in \mathcal{T}$ let $l_{it} \geq 0$ be the shift length of truck i on day t. Analogously, for trucks $i \in F^{2S}$ and $i \in F^{Sa}$ the corresponding shift durations are denoted by l_{it}^1 (early), l_{it}^2 (late) and l_{it}^S (Saturday).

For each truck $i \in F$ let $T_i \subseteq \mathcal{T}^{MF}$ be the subset of days (Monday to Friday) on which i has to be scheduled and $N_i \subseteq N$ be the set of drivers which are allowed to drive truck i. Similarly, for all $i \in F^{2S}$ subsets $N_i^1, N_i^2 \subseteq N$ contain all drivers which are allowed to drive i in an early and late shift, respectively. Due to personal reasons some drivers N^w should not have the same type of shifts (early or late) in two consecutive weeks, i.e. if such a driver has early shifts in one week, he must have only late shifts (or no shifts at all) the next week, etc.

For every truck $i \in F$ a preference list of acceptable drivers is given, where the drivers are ordered w.r.t. decreasing preferences. From these lists coefficients $\alpha_{ij} \in [0, 1]$ for $i \in F, j \in N$ are derived measuring how suitable driver i is for truck j. We set $\alpha_{ij} = 0$ for the most favored driver, $\alpha_{ij} = 1$ for infeasible drivers, and $\alpha_{ij} \in]0, 1[$ for acceptable, but not most preferred drivers.

The objective is to find a feasible schedule which satisfies the following hard constraints:

(H1) a feasible driver is assigned to every truck on all days in the required shifts,

(H2) each driver is assigned to at most one shift per day,

(H3) each driver is only assigned to a truck if he is available on the corresponding day,

(H4) each driver drives at most 55 hours per week,

(H5) no driver has an early or day shift on the day directly after a late shift,

(H6) no driver has both early/day and late shifts in a week, (H7) each driver from the set N^w does not have the same type of shifts (early or late) in two consecutive weeks,

(H8) all permanent drivers have 'compact' working and non-working periods, i.e.

- they do not work only one day per week,
- if they have free days in a week (which are not explicitly specified as vacation days), these days must be enlargements of the weekend (i.e. the drivers have some consecutive free days before or after Sunday).

Note that constraints (H5) are automatically fulfilled if condition (H6) is satisfied. However, we include it as a separate condition in order to be independent from other constraints. Additionally, the following soft constraints should be fulfilled as much as possible:

(S1) every driver has an actual working time which is contained in his desired total working time interval,

(S2) the trucks are assigned to the most preferred drivers,

(S3) the drivers are not assigned to a large number of different trucks in a week and do not often change the assigned trucks.

In order to ensure that always a feasible complete schedule exists, we introduce a dummy driver d who is allowed to drive any truck in every shift. Furthermore, no constraints are imposed on him (i.e. he can also drive several trucks on the same day simultaneously). Since it is not desired that he is used in a schedule, we set $Z_d^{\min} = Z_d^{\max} = 0$ and penalize positive working time accordingly.

3 A MIP formulation

In this section we give an exact mixed integer linear programming formulation for the described scheduling problem including all hard constraints (H1)-(H8) and the soft constraints (S1),(S2). Since it is difficult to integrate condition (S3) into the MIP model, it is not included here, but taken into account in a second step after solving the MIP (cf. Section 4).

We assume that the planning horizon $\mathcal{T}$ equals one month and denote by $\omega \in \{4,5\}$ the number of weeks in $\mathcal{T}$, by $\mathcal{T}_\tau \subset \mathcal{T}$ for $\tau = 1,\ldots,\omega$ all days of week τ and by t_τ^{Sa} the Saturday in week τ. Furthermore, let $\mathcal{T}^{Tu}, \mathcal{T}^{We}, \mathcal{T}^{Th}, \mathcal{T}^{Fr} \in \mathcal{T}$ be the sets of Tuesdays, Wednesdays, Thursdays, and Fridays in the planning horizon, respectively, We introduce binary decision variables

- $x_{ijt} \in \{0,1\}$ for $i \in F^{1S}, j \in N \cup \{d\}, t \in T_i$ with

$$x_{ijt} = \begin{cases} 1, & \text{if truck } i \text{ is assigned to driver } j \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

- $x^S_{ijt} \in \{0,1\}$ for $i \in F^{Sa}, j \in N \cup \{d\}, t \in \mathcal{T}^{Sa}$ with

$$x^S_{ijt} = \begin{cases} 1, & \text{if truck } i \text{ is assigned to driver } j \text{ at Saturday } t \\ 0, & \text{otherwise} \end{cases}$$

- $x^1_{ijt} \in \{0,1\}$ for $i \in F^{2S}, j \in N \cup \{d\}, t \in T_i$, and $x^2_{ijt} \in \{0,1\}$ for $i \in F^{2S}, j \in N \cup \{d\}, t \in T_i$ with

$$\begin{aligned} x^1_{ijt} &= \begin{cases} 1, & \text{if truck } i \text{ is assigned to driver } j \text{ on day } t \text{ in the early shift} \\ 0, & \text{otherwise} \end{cases} \\ x^2_{ijt} &= \begin{cases} 1, & \text{if truck } i \text{ is assigned to driver } j \text{ on day } t \text{ in the late shift} \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

Additionally, we have the following variables:

- $\Delta^+_j, \Delta^-_j \geq 0$ for $j \in N \cup \{d\}$ which measure for the drivers the deviations from their desired total working times (Δ^+_j for overtime, Δ^-_j for undertime)
- binary variables $v^1_{j\tau}, v^2_{j\tau} \in \{0,1\}$ for $j \in N, \tau = 1, \ldots, \omega$ with

$$\begin{aligned} v^1_{j\tau} &= \begin{cases} 1, & \text{if driver } j \text{ is assigned to day or early shifts in week } \tau \\ 0, & \text{otherwise} \end{cases} \\ v^2_{j\tau} &= \begin{cases} 1, & \text{if driver } j \text{ is assigned to late shifts in week } \tau \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

In the objective function the following weighting factors are used:

- $w^+_j, w^-_j \geq 0$ for all $j \in N \cup \{d\}$ are penalty factors for a deviation from the desired total working times (w^+_j for times larger than $Z^{\max}_j$, w^-_j for times smaller than $Z^{\min}_j$). Permanent drivers get larger weights than temporary drivers, the dummy driver has a very large weight w^+_d.
- $\hat{w}_j \geq 0$ for all $j \in N$ penalizes situations when a driver is assigned to a truck for which he is not a favored driver (again temporary drivers get smaller weights).

Using these variables the problem can be formulated by (1)-(33). The objective in (1) is to minimize a weighted sum of violations of the soft constraints (S1) and (S2). Constraints (2)-(5) ensure that condition (H1) is respected, i.e. that to every truck in the required shifts (day, early, late, Saturday) a feasible driver is assigned. Conditions (6)-(7) deal with (H2) and ensure that each driver is assigned to at most one shift per day. Equalities (8)-(9) take care of the non-availabilities of the drivers from condition (H3). Condition (H4) is regarded in (10) by requiring that the sum of all shift lengths for a driver is at most 55. Inequalities (11)-(12) forbid that a driver has an early or day shift directly after a late shift (condition (H5)) and (13)-(14) ensure that each driver from the set N^w does not have the same type of shifts in two consecutive weeks (H7). Constraints (15)-(16) guarantee that for each driver j the variables Δ^+_j, Δ^-_j are equal to the deviations from the desired total working time interval $[Z^{\min}_j, Z^{\max}_j]$. Furthermore, inequalities (17)-(18) ensure that the binary variables $v^1_{j\tau}, v^2_{j\tau}$ are set to one if driver j is assigned to an early/day or late shift in week τ. Conditions (19) forbid that a driver has both early/day and late shifts in a week (condition (H6)).

The first part of (H8) is taken into account in inequalities (20) by requiring that each permanent driver either drives on at least two days or on no day in a week. Constraints (21)-(24) deal with the second part of (H8) requiring that for permanent drivers free days are enlargements of the weekend. For example, condition (21) states that if a permanent driver has a free Tuesday (not vacation), he is not allowed to work on the Monday before. Similarly, (22) implies that if a driver has a free Wednesday, he is not allowed to work on the Tuesday and Monday before. Analogously, due to (23) a driver with a free Thursday should not drive on the next Friday and Saturday. Finally, constraints (25)-(28) ensure that trucks are not assigned to infeasible drivers.

$$\begin{aligned}\min \sum_{j\in N\cup\{d\}} (w_j^+\Delta_j^+ + w_j^-\Delta_j^-) &+ \sum_{i\in F^{1S}}\sum_{j\in N_i}\sum_{t\in T_i} \hat{w}_j\alpha_{ij}x_{ijt} + \sum_{i\in F^{Sa}}\sum_{j\in N_i}\sum_{t\in \mathcal{T}^{Sa}} \hat{w}_j\alpha_{ij}x^S_{ijt} \\ &+ \sum_{i\in F^{2S}}\sum_{t\in T_i}\Big(\sum_{j\in N_i^1} \hat{w}_j\alpha_{ij}x^1_{ijt} + \sum_{j\in N_i^2}\hat{w}_j\alpha_{ij}x^2_{ijt}\Big) \qquad (1)\end{aligned}$$

$$\sum_{j\in N_i\cup\{d\}} x_{ijt} = 1 \qquad (i\in F^{1S}, t\in T_i) \qquad (2)$$

$$\sum_{j\in N_i^1\cup\{d\}} x^1_{ijt} = 1 \qquad (i\in F^{2S}, t\in T_i) \qquad (3)$$

$$\sum_{j\in N_i^2\cup\{d\}} x^2_{ijt} = 1 \qquad (i\in F^{2S}, t\in T_i) \qquad (4)$$

$$\sum_{j\in N_i\cup\{d\}} x^S_{ijt} = 1 \qquad (i\in F^{Sa}, t\in \mathcal{T}^{Sa}) \qquad (5)$$

$$\sum_{i\in F^{1S}} x_{ijt} + \sum_{i\in F^{2S}} (x^1_{ijt}+x^2_{ijt}) \le 1 \qquad (j\in N, t\in \mathcal{T}^{MF}) \qquad (6)$$

$$\sum_{i\in F^{Sa}} x^S_{ijt} \le 1 \qquad (j\in N, t\in \mathcal{T}^{Sa}) \qquad (7)$$

$$\sum_{i\in F^{1S}} x_{ijt} + \sum_{i\in F^{2S}} (x^1_{ijt}+x^2_{ijt}) = 0 \qquad (j\in N, t\in U_j\cap \mathcal{T}^{MF}) \qquad (8)$$

$$\sum_{i\in F^{Sa}} x^S_{ijt} = 0 \qquad (j\in N, t\in U_j\cap \mathcal{T}^{Sa}) \qquad (9)$$

$$\begin{aligned}&\sum_{i\in F^{1S}}\sum_{t\in \mathcal{T}_\tau\cap T_i} l_{it}x_{ijt} + \sum_{i\in F^{Sa}}\sum_{t\in \mathcal{T}_\tau\cap \mathcal{T}^{Sa}} l^S_{it}x^S_{ijt} \\ &+ \sum_{i\in F^{2S}}\sum_{t\in \mathcal{T}_\tau\cap T_i} l^1_{it}x^1_{ijt} + \sum_{i\in F^{2S}}\sum_{t\in \mathcal{T}_\tau\cap T_i} l^2_{it}x^2_{ijt} \le 55 \qquad (j\in N, \forall\tau) \qquad (10)\end{aligned}$$

$$\sum_{i\in F^{2S}} (x^2_{ijt} + x^1_{ij,t+1}) + \sum_{i\in F^{1S}} x_{ij,t+1} \le 1 \qquad (j\in \bigcup_{i\in F^{2S}} N_i^2, t\in \mathcal{T}^{MD}) \qquad (11)$$

$$\sum_{i\in F^{2S}} x^2_{ij,t-1} + \sum_{i\in F^{Sa}} x^S_{ijt} \le 1 \qquad (j\in \bigcup_{i\in F^{2S}} N_i^2, t\in \mathcal{T}^{Sa}) \qquad (12)$$

$$\sum_{i\in F^{2S}} (x^1_{ijt} + x^1_{ijt'}) \le 1 \qquad (j\in N^w, t\in \mathcal{T}_\tau, t'\in \mathcal{T}_{\tau+1},\ \forall\tau) \qquad (13)$$

$$\sum_{i\in F^{2S}} (x^2_{ijt} + x^2_{ijt'}) \le 1 \qquad (j\in N^w, t\in \mathcal{T}_\tau, t'\in \mathcal{T}_{\tau+1},\ \forall\tau) \qquad (14)$$

$$
\begin{aligned}
&\sum_{i\in F^{1S}}\sum_{t\in T_i} l_{it}x_{ijt} + \sum_{i\in F^{Sa}}\sum_{t\in \mathcal{T}^{Sa}} l^S_{it}x^S_{ijt} \\
&+\sum_{i\in F^{2S}}\sum_{t\in T_i} l^1_{it}x^1_{ijt} + \sum_{i\in F^{2S}}\sum_{t\in T_i} l^2_{it}x^2_{ijt} - Z^{max}_j - \Delta^+_j \le 0 \quad (j\in N\cup\{d\})
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
&Z^{min}_j - \sum_{i\in F^{1S}}\sum_{t\in T_i} l_{it}x_{ijt} - \sum_{i\in F^{Sa}}\sum_{t\in \mathcal{T}^{Sa}} l^S_{it}x^S_{ijt} \\
&-\sum_{i\in F^{2S}}\sum_{t\in T_i} l^1_{it}x^1_{ijt} - \sum_{i\in F^{2S}}\sum_{t\in T_i} l^2_{it}x^2_{ijt} - \Delta^-_j \le 0 \quad (j\in N\cup\{d\})
\end{aligned} \tag{16}
$$

$$
\left(\sum_{t\in\mathcal{T}_\tau}\left(\sum_{i\in F^{2S}} x^1_{ijt} + \sum_{i\in F^{1S}} x_{ijt}\right) + \sum_{i\in F^{Sa}} x^S_{ijt^{Sa}_\tau}\right)/|\mathcal{T}_\tau| - v^1_{j\tau} \le 0 \quad (j\in N,\ \forall\tau) \tag{17}
$$

$$
\sum_{t\in\mathcal{T}_\tau}\sum_{i\in F^{2S}} x^2_{ijt}/|\mathcal{T}_\tau| - v^2_{j\tau} \le 0 \quad (j\in N,\ \forall\tau) \tag{18}
$$

$$
v^1_{j\tau} + v^2_{j\tau} \le 1 \quad (j\in N,\ \forall\tau) \tag{19}
$$

$$
\sum_{t\in\mathcal{T}_\tau}\left(\sum_{i\in F^{2S}}\left(x^1_{ijt}+x^2_{ijt}\right) + \sum_{i\in F^{1S}} x_{ijt} + \sum_{i\in F^{Sa}} x^S_{ijt}\right) - 2\left(v^1_{j\tau}+v^2_{j\tau}\right) \ge 0 \quad (j\in N^S,\ \forall\tau) \tag{20}
$$

$$
\begin{aligned}
&\sum_{i\in F^{1S}} x_{ij,t-1} + \sum_{i\in F^{2S}}\left(x^1_{ij,t-1}+x^2_{ij,t-1}\right) \\
&-\left(\sum_{i\in F^{1S}} x_{ijt} + \sum_{i\in F^{2S}}\left(x^1_{ijt}+x^2_{ijt}\right)\right) \le 0 \quad (j\in N^S, t\in\mathcal{T}^{Tu}\setminus U_j)
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
&\sum_{i\in F^{1S}}\left(x_{ij,t-2}+x_{ij,t-1}\right) + \sum_{i\in F^{2S}}\left(x^1_{ij,t-2}+x^2_{ij,t-2}+x^1_{ij,t-1}+x^2_{ij,t-1}\right) \\
&-2\left(\sum_{i\in F^{1S}} x_{ijt} + \sum_{i\in F^{2S}}\left(x^1_{ijt}+x^2_{ijt}\right)\right) \le 0 \quad (j\in N^S, t\in\mathcal{T}^{We}\setminus U_j)
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
&\sum_{i\in F^{1S}} x_{ij,t+1} + \sum_{i\in F^{2S}}\left(x^1_{ij,t+1}+x^2_{ij,t+1}\right) + \sum_{i\in F^{Sa}} x^S_{ij,t+2} \\
&-2\left(\sum_{i\in F^{1S}} x_{ijt} + \sum_{i\in F^{2S}}\left(x^1_{ijt}+x^2_{ijt}\right)\right) \le 0 \quad (j\in N^S, t\in\mathcal{T}^{Th}\setminus U_j)
\end{aligned} \tag{23}
$$

$$
\sum_{i\in F^{Sa}} x^S_{ij,t+1} - \left(\sum_{i\in F^{1S}} x_{ijt} + \sum_{i\in F^{2S}}\left(x^1_{ijt}+x^2_{ijt}\right)\right) \le 0 \quad (j\in N^S, t\in\mathcal{T}^{Fr}\setminus U_j) \tag{24}
$$

$$
\begin{aligned}
&x_{ijt} = 0 && (i\in F^{1S}, j\in N\setminus N_i, t\in T_i) && (25)\\
&x^S_{ijt} = 0 && (i\in F^{Sa}, j\in N\setminus N_i, t\in \mathcal{T}^{Sa}) && (26)\\
&x^1_{ijt} = 0 && (i\in F^{2S}, j\in N\setminus N^1_i, t\in T_i) && (27)\\
&x^2_{ijt} = 0 && (i\in F^{2S}, j\in N\setminus N^2_i, t\in T_i) && (28)\\
&x_{ijt} \in \{0,1\} && (i\in F^{1S}, j\in N\cup\{d\}, t\in T_i) && (29)\\
&x^S_{ijt} \in \{0,1\} && (i\in F^{Sa}, j\in N\cup\{d\}, t\in \mathcal{T}^{Sa}) && (30)\\
&x^1_{ijt}, x^2_{ijt} \in \{0,1\} && (i\in F^{2S}, j\in N\cup\{d\}, t\in T_i) && (31)\\
&v^1_{j\tau}, v^2_{j\tau} \in \{0,1\} && (j\in N, \tau=1,\ldots,\omega) && (32)\\
&\Delta^+_j, \Delta^-_j \ge 0 && (j\in N\cup\{d\}) && (33)
\end{aligned}
$$

4 A solution algorithm

In this section we present a two-phase solution algorithm for the considered problem which is based on the MIP model stated in the previous section. As mentioned before, the soft constraint (S3) ensuring that the drivers do not often change the assigned trucks, is not taken into account in the MIP model. For this reason, in the following we proceed in two phases: in the first phase we solve the MIP (ignoring (S3)), in the second phase we try to improve the resulting solution with respect to (S3).

Condition (S3) consists of two parts:

- If a driver works in some week, the total number of different trucks he is assigned to in this week should be as small as possible.
- If a driver is assigned to more than one truck within a week, the number of changes from one truck at one day to another truck on the next day should be as small as possible.

For a given solution let $u_{j\tau} \in \mathbb{N}$ be the number of different tank trucks driver $j \in N$ is assigned to in week τ. Furthermore, define

$$u'_{j\tau} = \begin{cases} u_{j\tau} - 1, & \text{if } u_{j\tau} > 1, \\ 0, & \text{otherwise} \end{cases}$$

and let $\tilde{u}_{j\tau}$ be the number of changes for driver j from one truck at one day to another truck on the next day in week τ if $u_{j\tau} > 1$ and zero otherwise. If a driver has a free day in a week, the resulting day is considered as a "joker", i.e. no changes (before and afterwards) are counted. For example, if in one week a driver j is assigned to the truck sequence (1, 1, free, 2, 2), we have $\tilde{u}_{j\tau} = 0$.

In order to take into account the changes of trucks within a week we modify the objective function (1) by adding the two additional terms

$$\sum_{\tau=1}^{\omega} \sum_{j \in N} w'_j u'_{j\tau} + \sum_{\tau=1}^{\omega} \sum_{j \in N} \tilde{w}_j \tilde{u}_{j\tau} \tag{34}$$

with weighting factors $w'_j, \tilde{w}_j \geq 0$ for all drivers $j \in N$.

After finding a feasible solution to the MIP model (with objective function (1)), we evaluate this solution with respect to the enlarged objective function (1)+(34). Then we try to improve this value by changing some shift assignments (i.e. by assigning tank trucks to other drivers on certain days). Such changes are only performed if the resulting solution is feasible with respect to the hard constraints (H1)-(H8).

Our improvement procedure consists of two stages, which will be described in the following. Since condition (S3) refers to one week, the procedure considers every week in the planning horizon separately.

Stage 1: For the first stage the solution of the MIP is used as initial solution. The idea of this stage is to directly swap assigned tank trucks for pairs of drivers (if possible) on certain days in the considered week. A detailed description of this procedure for a fixed week τ can be found in Figure 1.

```
 1.  Calculate a list $L$ containing all drivers $j \in N$ with $u_{j\tau} > 1$ ordered
     according to non-decreasing $u_{j\tau}$-values;
 2.  Take the next driver $j \in L$;
 3.  IF $u_{j\tau} > 1$ in the current (possibly modified) solution holds THEN
 4.     Determine a most driven tank truck $i \in F$ of driver $j$ in week $\tau$;
 5.     FOR every day $t \in \mathcal{T}_\tau$ where $j$ does not drive $i$ DO
 6.         Determine the driver $j'$ who drives $i$ on day $t$;
 7.         If feasible, change the tank trucks for $j$ and $j'$ on day $t$;
 8.     ENDFOR
 9.     IF the new solution does not have a better objective value THEN
10.         Discard all changes involving driver $j$;
11.  ENDIF
12.  GOTO Step 2;
```

Figure 1: Stage 1 of the improvement procedure

In Step 1 all drivers who drive more than one tank truck in the considered week τ are sorted in ascending order with respect to the number of different trucks driven in that week. In the following steps all these drivers are considered for possible changes by trying to assign each driver his most driven truck on more days. If in Step 4 the most driven tank truck is not unique, possible changes are made for every most driven tank truck and the truck leading to the best new solution value is taken. In order to avoid cycles during the changes of tank trucks, the following checks were integrated before performing a change in Step 7: If in Step 7 truck i is the most driven truck for both drivers j and j', the driver who drives the truck on more days in the week is preferred. If the number of days is the same for both drivers and one of the drivers is a permanent and the other a temporary driver, the permanent driver is preferred. Otherwise, the currently assigned driver is preferred.

If after one pass through the list L (Steps 2-12 for all $j \in L$) a better solution is obtained, this solution is accepted, and the whole procedure is repeated with a new list L. The procedure stops if during one scan through the current list no improved solution is found.

Example: Let us consider a small example for one week with $n = 9$ drivers and 8 tank trucks which have to be all driven in a single day shift from Monday to Friday. All feasible trucks for each driver are listed in the second column of the table in Figure 2. In this figure also a feasible solution is shown, where $\sum_{j=1}^{9} u'_{j\tau} + \sum_{j=1}^{9} \tilde{u}_{j\tau} = 10 + 17 = 27$.

The improvement procedure now performs the following steps:

1. The list L is calculated containing the drivers Brown, Jones, Roberts, Taylor, Evans, Thomas, and Wilson in this order.

	feasible trucks	Mo 1	Tu 2	We 3	Th 4	Fr 5	$u_{j\tau}$	$u'_{j\tau}$	$\tilde{u}_{j\tau}$
Brown	1,7	1	1	7	7	1	2	1	2
Evans	1,3,7	3	3	1	1	7	3	2	2
Jackson	8	8	8	8	-	-	1	0	0
Jones	2,3	2	2	3	2	2	2	1	2
Roberts	5,6	6	6	5	6	5	2	1	3
Smith	7,8	-	-	-	8	8	1	0	0
Taylor	4,5	5	5	4	5	4	2	1	3
Thomas	4,6,7	7	7	6	4	6	3	2	3
Wilson	2,3,4	4	4	2	3	3	3	2	2

Figure 2: A feasible solution

2. Brown does not drive his most driven tank truck 1 on days 3 and 4, where truck 1 is assigned to Evans. Since it is feasible to change tank trucks 1 and 7 on these days and the objective value is improved, this change is accepted.

3. Jones does not drive his most driven tank truck 2 on day 3, where truck 2 is assigned to Wilson. Since it is feasible to change tank trucks 2 and 3 on this day and the objective value is improved, this change is also accepted.

4. Roberts does not drive his most driven tank truck 6 on days 3 and 5, where truck 6 is assigned to Thomas. Since Thomas is not allowed to drive truck 5 of Roberts, no change is performed.

5. Taylor does not drive his most driven tank truck 5 on days 3 and 5, where truck 5 is assigned to Roberts. Since Roberts is not allowed to drive truck 4 of Taylor, no change is performed.

6. In the following iterations also for Evans, Thomas and Wilson no changes are possible.

7. A new list L is calculated containing Evans, Roberts, Taylor, Wilson, and Thomas. Since for none of them a pairwise interchange is feasible, the procedure stops with the solution shown in Figure 3, where $\sum_{j=1}^{9} u'_{j\tau} + \sum_{j=1}^{9} \tilde{u}_{j\tau} = 6 + 11 = 17$.

Stage 2: For the second stage the (possibly changed) solution after applying the first stage is used as initial solution. The idea of this stage is also based on swapping tank trucks, but instead of only swapping pairs of trucks, longer sequences of different changes are allowed.

During the procedure some assignments of drivers to tank trucks are temporarily removed. If in a step not immediately a new driver is assigned to a truck, this truck is put into a list of "free" tank trucks. For this purpose, the set F_{free} contains pairs (i,t) of tank trucks $i \in F$ and days $t \in \mathcal{T}_\tau$ with corresponding shift types (early/late, day) for which new drivers have to be found in later steps. A detailed description of this procedure can be found in Figure 4.

In Step 2 again all drivers who drive more than one tank truck in the considered week τ are sorted in ascending order with respect to the number of different trucks driven in

	feasible trucks	Mo 1	Tu 2	We 3	Th 4	Fr 5	$u_{j\tau}$	$u'_{j\tau}$	$\tilde{u}_{j\tau}$
Brown	1,7	1	1	1	1	1	1	0	0
Evans	1,3,7	3	3	7	7	7	2	1	1
Jackson	8	8	8	8	-	-	1	0	0
Jones	2,3	2	2	2	2	2	1	0	0
Roberts	5,6	6	6	5	6	5	2	1	3
Smith	7,8	-	-	-	8	8	1	0	0
Taylor	4,5	5	5	4	5	4	2	1	3
Thomas	4,6,7	7	7	6	4	6	3	2	3
Wilson	2,3,4	4	4	3	3	3	2	1	1

Figure 3: Solution after applying stage 1 of the improvement procedure

that week. In the following steps these drivers are considered iteratively and it is tried to assign them their most driven tank truck on more days of the week. Instead of swapping trucks for pairs of drivers as in stage 1, we directly assign the most driven tank truck to the current driver on a day when he does not drive this truck. This assignment causes that the previously assigned tank truck has no driver anymore and that a driver changes from a working to a free day on the considered day. This infeasible situation has to be repaired in the following steps. In order to avoid cycles during the assignment of tank trucks, similar checks as in stage 1 are integrated before performing an assignment in Steps 10 and 17.

When the complete list L is scanned, we have a list of tank trucks with no driver on certain days. In Steps 27 to 30 we try to find drivers for these tank trucks. Therefore, we determine all drivers who are allowed to drive the free tank trucks. For every acceptable driver we check whether it is feasible that he drives an additional tank truck (according to his vacation, total working time and 'compact' working period). Because of the 'compact' working period the order of the considered days for a free tank truck is important. For example, consider the situation of one week (days 1 to 5) where a tank truck is free on the days 1,2 and 3 and only one feasible driver for this truck exists. This driver has free on the days 1,2 and 3 and drives a tank truck on the days 4 and 5. If we first consider day 1 we could not assign him the tank truck because this contradicts with the 'compact' working period. The same holds for day 2. For day 3 it is possible to assign him the tank truck. After this assignment on day 3 we can also assign him the tank truck on day 2 and after that also on day 1. So we consider the days in an arbitrary order but whenever we have made an assignment we consider the remaining days again.

If at the end the list of free tank trucks is empty and the objective value has been improved, we accept the new improved solution. Otherwise, all changes are discarded. If after one pass through the list L and the list of free tank trucks a better solution is accepted, the whole procedure is repeated with a new list L. The procedure stops if during one pass of stage 2 no improved solution is found.

```
 1.  F_free := ∅;
 2.  Calculate a list L containing all drivers j ∈ N with u_jτ > 1 ordered
     according to non-decreasing u_jτ-values;
 3.  Take the next driver j ∈ L;
 4.  IF u_jτ > 1 in the current (possibly modified) solution holds THEN
 5.     Determine a most driven tank truck i ∈ F of driver j in week τ;
 6.     FOR every day t ∈ T_τ where j does not drive i DO
 7.         IF the pair (i,t) is in F_free for the corresponding shift THEN
 8.            Assign driver j to truck i on day t in the corresponding
               shift and remove the pair (i,t) from F_free;
 9.         ELSE
10.            Assign driver j to truck i on day t in the corresponding
               shift and put the pair (î,t) into the list F_free where î is
               the previously assigned truck of driver j in that shift;
11.            Determine the driver j' who drives i on day t in that shift;
12.            IF it is infeasible that driver j' has free on day t THEN
13.               Determine a most driven tank truck i' ∈ F of j' in week τ;
14.               IF (i',t) ∈ F_free THEN
15.                  Assign driver j' to truck i' on day t in the corres-
                     ponding shift and remove the pair (i',t) from F_free;
16.               ELSE
17.                  Assign driver j' to truck i' on day t;
18.                  Determine the driver j'' who drives i' on day t;
19.                  IF it is infeasible that j'' has free on day t THEN
20.                     Set j' := j'' and GOTO Step 15;
21.               ENDIF
22.            ENDIF
23.         ENDIF
24.     ENDFOR
25.  ENDIF
26.  GOTO Step 3;
27.  FOR every pair (i,t) ∈ F_free DO
28.     Try to find a feasible driver for truck i in the corresponding
        shift on day t;
29.  IF F_free ≠ ∅ OR the new solution does not have a better value THEN
30.     Discard all changes;
```

Figure 4: Stage 2 of the improvement procedure

Example: We start the second stage with the solution from Figure 3. Then in the second stage the following steps are performed:

1. Evans gets his most driven tank truck 7 on days 1 and 2 and truck 3 is put into the list F_{free} for these days. For driver Thomas it is acceptable that he has free on days 1 and 2.

2. Roberts gets his most driven tank truck 6 on days 3 and 5 and truck 5 is put into the list F_{free} for these days. For driver Thomas it is acceptable that he has free on day 3. But it is not acceptable that driver Thomas also has free on day 5 because in this case he only would drive one day in the week (i.e. (H8) is violated). Thus, in the next step Thomas gets his most driven truck 4 on day 5. For driver Taylor it is acceptable that he has free on day 5.

3. Taylor gets his most driven tank truck 5 on day 3 (from the list F_{free}) and tank truck 4 is put into the list F_{free} for day 3.

4. Wilson gets his most driven tank truck 3 on days 1 and 2 (from the list F_{free}) and truck 4 is put into the list F_{free} for these days.

5. Thomas drives only one tank truck, i.e. no changes are tried for him.

6. After one scan through the list L the list F_{free} contains the pairs $(5,5)$, $(4,1)$, $(4,2)$, $(4,3)$. Since it is possible to assign tank truck 5 to driver Taylor on day 5, tank truck 4 to Thomas on days 1,2, and 3, and the resulting solution is better, all changes are accepted.

7. Since in the next iteration the list L is empty, the procedure is stopped. We get the solution shown in Figure 5 with the better solution value $\sum_{j=1}^{9} u'_{j\tau} + \sum_{j=1}^{9} \tilde{u}_{j\tau} = 0$.

	Mo 1	Tu 2	We 3	Th 4	Fr 5	$u_{j\tau}$	$u'_{j\tau}$	$\tilde{u}_{j\tau}$
Brown	1	1	1	1	1	1	0	0
Evans	7	7	7	7	7	1	0	0
Jackson	8	8	8	-	-	1	0	0
Jones	2	2	2	2	2	1	0	0
Roberts	6	6	6	6	6	1	0	0
Smith	-	-	-	8	8	1	0	0
Taylor	5	5	5	5	5	1	0	0
Thomas	4	4	4	4	4	1	0	0
Wilson	3	3	3	3	3	1	0	0

Figure 5: Solution after applying stage 2 of the improvement procedure

5 Computational results

In this section we present some computational results for real-world data provided by the oil company.

In the considered test instances $n = 30$ tank drivers exist, which are divided into 25 permanent (numbers 1,. . . ,25) and 5 temporary drivers (numbers 26,. . . ,30). Furthermore, two drivers are contained in the set N^w. Additionally, there are 17 tank trucks $i = 1, \dots, 17$. The first three tank trucks have to be driven in two shifts (an early and a late shift), all other tank trucks are only operated within a single shift from Monday to Friday. Moreover, the first four tank trucks have also to be scheduled for a single shift on Saturdays. The shift lengths are $l_{it} = l_{it}^1 = l_{it}^2 = 10$ hours for all tank trucks from Monday to Friday, and $l_{it}^S = 5$ on Saturdays. In Table 1 all tank trucks with their associated drivers can be found. Each tank truck has four preferred drivers (only for truck 7 no most favored driver exists, indicated by '-' in the table). For some trucks additional acceptable drivers are given in the column "miscellaneous drivers". The orders of the drivers correspond to the orders in the preference lists of acceptable drivers for the tank trucks.

Tank truck	Early shift drivers	Late shift drivers	Miscellaneous drivers
1	13,8,9,26	16,8,9,26	27,28,29,19
2	19,9,8,26	14,9,8,26	27,28,13,16,29,21
3	21,12,8,9	12,21,8,9	27,28,29
4	23,8,9,26		3,27,28,15,29
5	20,5,15,26		3,27,6
6	4,7,22,10		
7	-,15,9,20		
8	11,5,15,26		
9	3,15,5,26		
10	17,15,27,1		
11	2,7,10,26		
12	25,15,17,20		1,27,9
13	10,7,26,2		
14	6,15,5,26		27
15	22,7,4,10		
16	1,15,25,20		
17	18,24,26,30		

Table 1: Tank trucks and associated drivers

The default total working time intervals $[Z_j^{\min}, Z_j^{\max}]$ are set to $[122, 180]$ (hours) for the permanent drivers and to $[0, 50]$ for the temporary drivers. If a driver has requested a large number of vacation days, his interval may be reduced accordingly. Furthermore, one of the permanent drivers (number 24) has a smaller total working time interval of $[0, 50]$ because he is only a "jumper" for one tank truck. Additionally, since two of the temporary drivers are only available on Saturdays (drivers 28 and 29), these drivers have a reduced total working time interval of $[0, 20]$.

After some computational experiments and discussions with the company the weighting factors were chosen as follows. The coefficients $\alpha_{ij} \in [0,1]$ for $i \in F, j \in N$ (measuring how suitable driver i is for truck j) are set to $\alpha_{ij} = 0$ for the most favored driver and $\alpha_{ij} \in \{0.1, 0.2, 0.3\}$ for the other acceptable drivers (ordered according to the preference lists). The miscellanous drivers get coefficients $\alpha_{ij} = 0.5$.

The weighting factors in the objective function are set to $w_j^+ = w_j^- = 10$, $\hat{w}_j = 3$, $w'_j = \tilde{w}_j = 2$ for permanent drivers $j \in N^S$, $w_j^+ = w_j^- = \hat{w}_j = w'_j = \tilde{w}_j = 1$ for temporary drivers, and $w_d^+ = 10000$ for the dummy driver. The temporary drivers get much smaller weighting factors since it is common that in some months they drive more and in other months they drive less hours. Furthermore, in general they are not the most favored drivers for a tank truck and may act as a "jumper" being assigned to different trucks in a week.

We generated 15 test instances for the planning horizon July 2009. In the first instance no vacation for the drivers has to be considered. In the other 14 instances vacation is randomly distributed to the drivers. The main characteristics of the 15 instances can be found in Table 2. For each instance the total number of vacation days as well as the total number of drivers with vacation days are specified. If a driver could not reach his maximal working time $Z_j^{\max}$ (since he requested too much vacation), the corresponding value was reduced.

Instance	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
# vacation days	0	5	10	15	15	20	20	25	25	25	30	35	35	40	40
# vacation drivers	0	1	2	3	2	2	4	3	4	5	6	5	2	4	5

Table 2: Test instances with varying vacation

In the following we present some computational results for the two-phase algorithm. The algorithm was implemented in Java and tested on a PC with a 1.6 GHz-processor and 512 MB main storage under the system software Windows XP Professional. In order to solve the MIP we used the non-commercial solver CBC [6] with a time limit of 10 minutes. For comparison we also solved each MIP with the commercial product CPLEX 11.0 [7] on an Intel Core 2 Duo E8400 processor with 3 GHz and 2 GB main storage and a time limit of 2 hours.

Our results for the 15 test instances can be found in Table 3. For each instance the solution value (1) calculated by CPLEX and the corresponding computation time (in seconds) are presented. Instances 1,7 and 9 could not be completely solved to optimality with CPLEX within 2 hours, but very good solutions were found (for these three instances the maximal primal-dual gap after 2 hours was 0.05 %). The next columns present the objective value from formula (1) after solving the MIP with CBC in 10 minutes and the enlarged objective value (1)+(34) for the MIP solution. The last column contains the enlarged objective value after applying the improvement procedure to the solution of the MIP. The computation times for the improvement procedure were in the range of some seconds.

For example, let us have a closer look at the results for the fourth instance. After solving the MIP with CBC and a time limit of 10 minutes, the objective value (1) is given by $219.9 = 170 + 49.9$, where $170 = 10 \cdot (3 + 14)$ is the sum of the deviations from the desired total working times of the drivers (here only overtime occurs for two drivers) and 49.9 is the sum of the penalties for tank trucks not assigned to their most preferred drivers.

Instance	CPLEX		CBC		improved
	(1)	time	(1)	(1)+(34)	(1)+(34)
1	219.0	7200	219.1	259.1	257.7
2	219.0	13	219.1	261.1	256.9
3	219.0	8	219.0	261.0	257.4
4	219.9	4813	219.9	289.9	285.3
5	249.0	1989	249.0	301.0	298.2
6	299.6	1159	299.7	355.7	342.9
7	219.5	7200	219.5	241.5	236.1
8	292.0	12	292.0	328.0	325.2
9	229.3	7200	229.5	257.5	257.5
10	219.5	14	219.5	267.5	256.1
11	280.8	10	280.8	326.8	321.8
12	229.9	4	229.9	277.9	250.6
13	253.0	12	253.0	303.0	300.2
14	343.8	3	343.8	387.8	377.8
15	287.7	6	287.7	375.7	362.0

Table 3: Computational results for working time intervals $[122, 180]$

Instance	CPLEX		CBC		improved
	(1)	time	(1)	(1)+(34)	(1)+(34)
1	35.2	7200	35.2	87.2	79.6
2	35.8	7200	35.8	77.8	72.8
3	36.0	2820	36.1	80.1	77.5
4	36.5	7200	36.9	70.9	63.1
5	36.8	492	36.8	82.8	74.4
6	36.3	80	36.3	70.3	66.6
7	38.3	64	38.3	64.3	57.4
8	41.5	11	41.5	79.5	73.9
9	37.1	189	37.4	81.4	73.7
10	38.5	13	38.5	70.5	70.5
11	41.3	16	41.3	75.3	56.0
12	37.8	17	40.8	118.8	84.7
13	39.8	28	39.8	67.8	60.4
14	49.7	21	49.7	107.7	83.4
15	46.5	50	48.4	94.4	90.7

Table 4: Computational results for working time intervals $[122, 200]$

The enlarged objective value (1)+(34) is given by 289.9 where (34) equals $34 + 36$. After applying the improvement procedure we get the value $285.3 = 170 + 55.3 + 29 + 31$. This means that the improvement procedure has decreased the number of changes of tank trucks to $29 + 31$, but worsened the term for non-favored drivers from 49.9 to 55.3 (however, in total this is a better solution).

The results show that the oil company does not have sufficient employees to satify their demands in any month because already for the first instance without any vacation overtime cannot be avoided. Thus, it would be advisable to employ at least one additional driver (or increase the maximal working times of some drivers) because in every instance there are at least two temporary drivers with overtime.

If we increase the default total working time intervals to $[122, 200]$ for the permanent drivers, we get the results from Table 4. Instances 1,2 and 4 could not be solved to optimality with CPLEX within 2 hours. For these instances the maximal primal-dual gap was 0.42 %. It can be seen that with the increased intervals no driver deviates from his given total working time interval (i.e. the sum of all Δ^+- and Δ^--values is equal to zero). For this reason the objective values are much smaller than for the intervals $[122, 180]$.

The computational results show that (at least for the considered instances) it is sufficient to use the non-commercial mixed integer programming solver CBC. The largest absolute deviation between the solution values calculated by CPLEX and CBC is only 0.2 for the instances with the total working time intervals $[122, 180]$ and 3.0 for the instances with intervals $[122, 200]$.

6 Concluding remarks

In this paper we proposed a two-phase solution algorithm for the shift scheduling problem of a small oil company. The computational results show that the algorithm using a non-commercial MIP-solver is able to generate good schedules satisfying all hard constraints in a small amount of time. The company was very satisfied with the results.

For further research it would be interesting to study more intensively the rescheduling problem which arises if short-term changes have to be made for an existing schedule. With our current approach the MIP can be resolved after fixing all variables of days in the past to the values of the previously calculated schedule. Besides the used objective function an additional term could be integrated trying to minimize the deviations from the previous schedule.

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