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# A Game Theoretic Approach for Stable Network Topologies in Opportunistic Networks

Nils Bulling, Michael Köster, Matei Popovici

## Abstract

*Opportunistic networks* (ON) are particular types of delay-tolerant networks in which users/network entities participate in order to propagate information. Besides the advantages of these networks (e.g. decentralization, independence of communication infrastructure) they raise new problems regarding for example effectiveness, message routing, message delivery, security issues, and trust. In this paper we introduce a formal description of an ON and of optimal communication topologies, for the non-cooperative and cooperative settings. We follow a game theoretic approach and allow users to express properties about how their messages should be handled in the network by means of a logical language (for instance, message privacy may be achieved by requiring that network nodes with internet access should be avoided on the communication path). We determine the complexity of associated verification and synthesis problems of network topologies.

## 1 Introduction

The ever increasing use of online social networking services together with the popularity of new generation smart-phones and of other smart mobile devices are causing mobile networks to be overloaded. In order to solve this problem, novel communication methods and new types of network architectures, such as *delay-tolerant* [4] and *opportunistic networks* [14, 12] have emerged. Traditionally, delays are seen as networking problems caused by connectivity interruptions. In the conventional setting they are the exception. However, in *delay-tolerant networks* they are the rule: messages are (deliberately) delayed, and offloaded to alternative communication routes, in order to relieve wireless and mobile networks of data traffic [9, 11].

An *opportunistic network* is a particular type of delay-tolerant network in which participants are mobile and able to communicate at limited range (e.g. humans carrying wireless communication devices). It is assumed that: (i) global Internet access is not available, (ii) end-to-end connectivity between any two participants is not generally possible and (iii) the entire network might be disconnected, i.e. certain groups of participants might be outside the communication range of other groups from the network. In this setting, communication occurs *opportunistically*: whenever two devices are in proximity, they will consider this as an *opportunity* to exchange messages. Moreover,

the participants' mobility is exploited in order to transmit messages between disconnected groups of users. According to this *store-carry-and-forward* mechanism [1], a message is *stored* in user *A*'s buffer and is *carried* around until *A* is in communication range with another user *B*. When this happens, the message is forwarded to user *B* and the process is repeated until the final destination is reached, or the message is outdated.

One of the main advantages of opportunistic networks is the fact that they are: (i) decentralized, (ii) independent of any communication infrastructure and (iii) inexpensive. Opportunistic networks also raise new problems regarding for example effectiveness, message routing in such a dynamic environment, maximizing message delivery, security issues, and trust. Moreover, not all network topologies are desirable nor stable because (i) users may wish to avoid specific routes/users and (ii) users lack incentives to provide services (e.g. message forwarding) to other users.

In this paper we address these two problems and propose a game-theoretic communication model for opportunistic networks, in which each user (or each group of users) has certain communication preferences which express the users' goals and also restrictions on the network. Instead of using rankings or other community-dependent metrics, we use the temporal logic **CTL** for expressing these preferences. The advantage of this approach is that it allows a flexible description of various preferences like reachability or avoidance properties.

The contributions of this paper are a game-theoretic analysis of *optimal* (or *stable*) *network topologies*<sup>1</sup> for message forwarding and related complexity results. The optimal topology should minimize communication costs, while satisfying the players' goals. We model a network topology as the outcome of a strategic game in which the actions of each player consist of establishing communication channels. Then, optimal topologies correspond to game theoretic solution concepts; that is, in informal terms, a topology is stable if no agent or player has an incentive to deviate from the given topology. In this paper, we consider individual and group rationality, each leading to a different notion of optimality/stability.

We consider both a cooperative and a non-cooperative setting. Often cooperation is required as players are usually not able to achieve their goals by themselves. Apart from the game theoretic modelling approach, the complexity results regarding verification and synthesis problems of optimal topologies form the main technical results of this paper. Finally, we would like to note that a lot of work in this area has focused on game theoretic methods for package forwarding and routing strategies. Our work should not only be understood as yet another analysis but in particular as a pre-processing step. We propose a way for finding an optimal network topology and once it has been found existing methods for package routing and forwarding can be applied on top of it.

The paper is structured as follows: In Section 2 we introduce the basic ingredients of an *opportunistic network* (ON), motivate our game theoretic approach, define the *opportunistic network game*, and put game theoretic solution concepts in the context of optimal topologies. In Section 3 we propose a computational setting based on the temporal logic **CTL** (computation tree logic). In Section 4 we analyse the complexity

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<sup>1</sup>In this paper, we use the term "optimal" to refer to "stability criteria" in a game theoretic sense.

of verification and synthesis problems. Finally, in Section 5 and 6 we discuss related work and conclude, respectively.

*About the paper.* This paper is an extended version of [2]; in particular, we discuss the following shortcoming contained in the original paper:

- In the original setting, the non-cooperative solution concepts (Nash and strong Nash Equilibrium) are limited because players do usually have no incentive to set up channels for others. In non-trivial cases, actions containing *no* communication channels set for other players would always be preferred due to lower costs.

In order to address this shortcoming, we modify the *utility function* (cf. Definition 2.14) by introducing incentives for players to help other players. The *new factor*  $\frac{|\text{SAT}(\mathcal{O})|}{|N|}$  together with a *scaling factor*  $\beta$  describes the incentive for player  $i$  to help other players fulfilling their goals. The  $\beta$  value allows specifying a player's *degree of interest* in the satisfaction of the goals of other players. For  $\beta = 1$  a player is *fully interested* in helping other players whereas for  $\beta = 0$  a player is *not interested* at all whether other players' goals are satisfied or not. We note, that this new definition of utility function incorporates a cooperative component to the non-cooperative setting. It is worth pointing out that the value of  $\beta$  is defined by the opportunistic network application and not by the players themselves. Hence, the players have two choices: accept the given value of  $\beta$  and to "use" the opportunistic network or not to take part in the opportunistic network. For a fixed  $\beta$ , players still remain selfish and do only care about their utility, which now takes into account other players goals.

Moreover, in this paper we clarify that our approach is rather a communication protocol (which players cannot influence) than a decision process of each player. Thus, the system is responsible for processing user preferences and computing the appropriate actions. Also, it can be done in a decentralized way assuming that the computation is deterministic and exactly the same for all players. We also mention the use of a *conversion function* that models the correlation between costs and the value of a player's goal. Also, we correct errors and slightly modify the examples.

## 2 Optimal Opportunistic Networks

In this section we introduce an *opportunistic network* (ON), motivate our game theoretic approach, define the *opportunistic network game* which is used to determine optimal topologies. The concept of optimality depends on the specific solution concept at hand and reflects different stability conditions of a topology. An ON is defined over an *opportunistic network frame* (ONF) which models the participants of an ON (to which will henceforth also refer as *players*), the locations they can reach, the possible connections (or channels) they can establish, and a cost for each such channel. Players have the intention to send messages to one or several locations, but are interested in enforcing restrictions on the way messages are delivered. These restrictions include prohibiting specific players (or rather characteristics of players) on the message deliv-

ery path, requiring the existence of several paths towards destination, or restricting the path's length.

## 2.1 Opportunistic Networks

An *opportunistic network frame* (ONF) essentially defines a set of players  $\mathcal{P}$  and their abilities to communicate with each other. We use a neighborhood function  $N : \mathcal{P} \rightarrow 2^{\mathcal{P}}$  to model the players with whom a (communication) channel can be established; that is,  $N(i)$  is the possibly empty set of players with whom  $i$  can set up a channel. We require that  $i \notin N(i)$ . The establishment of a channel from player  $i$  to player  $j$  has cost  $c(i, j)$ . The cost function  $c$  can aggregate a number of internal and external factors related to players such as bandwidth consumption, trust level, resource usage etc. Finally, each player  $i$  attempts to satisfy a certain goal  $\phi_i$ . Goals give players the ability to impose restrictions on how messages are forwarded. One such restriction could be that any path to the destination must not include certain players.

The *value* function  $v$  quantifies the value of a player's goal. The values are subjective to the agents and can be of various origin. In this paper, we do not discuss this issue in more detail.

**Definition 2.1** (Opportunistic Network Frame). An *opportunistic network frame* (ONF) is given by  $\mathcal{F} = (P, N, Props, c, \mathcal{I}, (\phi_i)_{i \in \mathcal{P}}, v)$  where

- $\mathcal{P}$  is a finite set of players;
- $N : \mathcal{P} \rightarrow 2^{\mathcal{P}}$  is a neighborhood function.  $N(i)$  is the set of neighbors with which  $i$  can establish channels. We require that  $i \notin N(i)$ , i.e. players cannot establish channels with themselves.
- $Props$  is a set of propositional symbols that represent different user properties.
- $\mathcal{I} : \mathcal{P} \rightarrow 2^{Props}$  is a valuation function assigning, for each player  $i$ , a set of propositions which are true for player  $i$ .
- A partial cost function  $c : \mathcal{P} \times \mathcal{P} \rightarrow \mathbb{R}^{\geq 0}$ .  $c(i, j)$  is the cost for player  $i$  of establishing a channel with player  $j$ . The value  $c(i, j) \geq 0$  must be defined for any two players  $i \neq j$  provided that  $j \in N(i)$ . Moreover,  $c(i, i) = 0$  for all players  $i$ .
- $\phi_i$  is the goal for player  $i$ .
- A value function  $v : \mathcal{P} \rightarrow \mathbb{R}$  models the value of a player's goal.

**Remark 2.2** (Conversion function). *We note that a goal of a player is of qualitative nature; the value function, on the other hand, adds to the qualitative objective a quantitative dimension. Now, it can be the case that the value and cost functions produce values of types which cannot (easily) be compared. For instance, the cost function may measure the transmission power required to send a message or the expected number of*

lost packets, while the value function may measure the available battery on a mobile device. For this reason, we assume the existence of a conversion function  $f : \mathbb{R}^{\geq 0} \rightarrow \mathbb{R}^{\geq 0}$  which makes the values  $f(v(i))$  and  $c(j, k)$  comparable for any players  $i, j$  and  $k$ .

However, in the follow we consider, for the sake of readability, the conversion function as identity function.

A proposition  $p \in Props$  can for instance represent certain real world locations or a *communication channel* (e.g. a wifi hotspot, a mobile connection, etc.) which a player can access. A possible goal of a player could be the following: *Player 1 wants to access a specific communication channel*. In particular, goals can also be understood as restrictions on communication paths and network topologies.

**Remark 2.3** (Player goals). *In ONF each player has a single goal. It is rather straight forward to extend the setting to a set of goals, one for each player, and to assign different values to each goal. However, for a clearer presentation, we do only consider the single goal setting in this paper.*

*In our particular setting, goals are expressed using the language of computation tree logic (CTL). CTL is, in our opinion, a natural choice since model checking CTL-formulae can be done in polynomial time. Thus, it brings no significant overhead to the computational complexity of our solution concepts, described in Section 4. However, the general framework is not dependent on CTL and can be used with other suitable languages for specifying goals. For this reason, we defer the introduction of CTL and the formal definition of goal satisfaction to Section 3.*

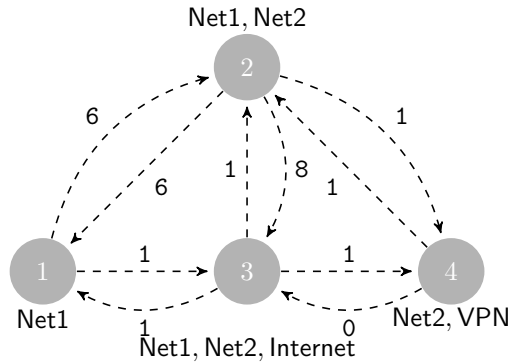


Figure 1: A simple opportunistic network frame.

**Example 2.4** (Simple ONF). Consider the ON with 4 players shown in Figure 1. The scenario describes two partially disconnected networks modeled by the propositions Net1 and Net2, respectively. Player 1, which is a member of Net1 is unable to communicate directly with player 4, which is a member of Net2. Players 2 and 3 are members of both Net1 and Net2 and thus can communicate with any other player.

Player 3 has access to Internet, and player 4 to a virtual private network, denoted as VPN. We model this scenario as a ONF where the set of propositions  $\mathcal{P}_{props} = \{\text{Net1}, \text{VPN}, \text{Net2}, \text{Internet}\}$  describes all existing networks and the valuation function defined as  $\mathcal{I}(1) = \{\text{Net1}\}$ ,  $\mathcal{I}(2) = \{\text{Net1}, \text{Net2}\}$ ,  $\mathcal{I}(3) = \{\text{Net1}, \text{Net2}, \text{Internet}\}$  and  $\mathcal{I}(4) = \{\text{Net2}, \text{VPN}\}$  describes what networks are accessible for each player.

The dashed arrows describe all possible communication channels which can be established, and they are labeled with their corresponding costs. Therefore:  $N(1) = N(4) = \{2, 3\}$ ,  $N(2) = \{1, 3, 4\}$  and  $N(3) = \{1, 2, 4\}$ . The costs are  $c(1, 2) = c(2, 1) = 6$ ,  $c(1, 3) = c(2, 4) = c(3, 1) = c(3, 4) = c(4, 2) = c(3, 2) = 1$ ,  $c(4, 3) = 0$  and  $c(2, 3) = 8$ .

Let us assume that player 1 wants to send a message to the VPN. In order to do so, one possible path is  $(1, 3, 4)$ , i.e. to send the message from player 1 over player 3 to player 4. However, let us moreover assume that player 1 requires that its message must not pass through any node which has access to Internet. We will refer to this goal as  $\phi_1$ . The single path that obeys this restriction is  $(1, 2, 4)$ .

Similarly, player 2 has the goal—to which we refer to as  $\phi_2$ —of sending a message to the Internet. Players 3 and 4 have no communication goals. The goal values for the players are,  $v(1) = 40$ ,  $v(2) = 20$  and  $v(3) = 0$ ,  $v(4) = 0$ , respectively.

**Remark 2.5.** *We formally model the absence of a communication goal, as a goal which is implicitly satisfied without taking any action (in logical terms: a tautology).*

As seen in Example 2.4, the *optimality* of a certain *network topology* depends on several issues. We may, for instance, call a topology optimal if all the players' goals are satisfied, or if the costs are minimal and the goal of a particular player is satisfied. We discuss optimality in more detail in Section 2.2. In what follows, we describe *topologies* as *structures* which depend on a certain ONF. We assume players have the intention of sending messages to one or several other players. Furthermore, we expect that destinations are not (always) directly reachable, and that messages must be routed via intermediate players. Whenever this is the case, a message will cross a sequence of players, from source to destination. A *transition*  $i \xrightarrow{k} j$  in a *topology* represents a directed communication channel in which player  $i$  forwards one (or several) messages generated by or on behalf of player  $k$ , to player  $j$ .  $j$  is the next-hop (as in relay networks) on the message delivery path. It might be the case that  $i = k$ , if  $j$  is the first hop on the message path. The cost of setting a channel is captured by  $c(i, j)$ , and it has to be put up by the sender, i.e. player  $i$  in this very case. Since we assume players are self-interested, setting up communication links and relaying messages for other players requires some kind of incentive, as we will further see.

Each player's goal expresses one (or several) destinations that should be reached, or can also enforce certain restrictions on how these destinations are reached. The expressiveness of the goals depends on the concrete goal language used. Possible restrictions/goals include, for example, *channel P should be accessible via at most two hops* or *channel P should be accessible via a path that doesn't pass through node/player i* (i.e. player  $i$  should not be able to receive the message sent), etc. We note however



that a player  $k$  can only formulate goals that involve forwarding its own messages, i.e. it can only express preferences regarding edges of the form  $i \xrightarrow{k} j$  in the topology.

In the remainder of this paper we assume that  $\mathcal{F} = (P, N, \mathcal{P}rops, c, \mathcal{I}, (\phi_i)_{i \in P}, v)$  is an opportunistic network frame, as described by Definition 2.1.

**Definition 2.6** ( $\mathcal{F}$ -topology,  $\mathcal{T}ops(\mathcal{F})$ ). An  $\mathcal{F}$ -topology is a labeled transition system  $\mathcal{T}_{\mathcal{F}} = (\mathcal{P}, \rightarrow, \mathcal{P}rops, \mathcal{I})$  where  $\mathcal{P}$  is the set of nodes and  $\rightarrow \subseteq \mathcal{P} \times \mathcal{P} \times \mathcal{P}$  where we require that  $(i, i, i) \in \rightarrow$  for all  $i \in \mathcal{P}$  and that  $c \in N(a)$  if  $(a, b, c) \in \rightarrow$  and  $a \neq c$ . The elements  $\mathcal{P}rops$  and  $\mathcal{I}$  are taken from  $\mathcal{F}$ . We write  $a \xrightarrow{b} c$  for  $(a, b, c) \in \rightarrow$ . The relation  $a \xrightarrow{b} c$  models a  $b$ -labeled transition from  $a$  to  $c$ .

The set of all  $\mathcal{F}$ -topologies is denoted  $\mathcal{T}ops(\mathcal{F})$ . We write  $\mathcal{T}$  instead of  $\mathcal{T}_{\mathcal{F}}$  if  $\mathcal{F}$  is clear from context.

The reflexive loops  $i \xrightarrow{i} i$  are due to technical reasons. They model a player's possibility to do nothing.

**Remark 2.7** ( $\mathcal{F}$ -topologies and labeled transition systems). *Labeled transition systems are often used to describe the states of a certain system, and labeled transitions represent possible actions which can be taken, in order to reach some state from another. We would like to emphasize that in our setting, nodes in a  $\mathcal{F}$ -topology represent players and their properties. Edges correspond to the possible communication channels they can establish.*

**Proposition 2.8.** *We have  $|\mathcal{T}ops(\mathcal{F})| \leq 2^{|\mathcal{P}|^3}$ .*

**Definition 2.9** (Opportunistic Network). An *opportunistic network* (ON) is a tuple  $\mathcal{O} = (\mathcal{F}, \mathcal{T})$  consisting of an opportunistic network frame  $\mathcal{F}$  and an  $\mathcal{F}$ -topology  $\mathcal{T} \in \mathcal{T}ops(\mathcal{F})$ .

Intuitively, an opportunistic network  $\mathcal{O}$  is obtained by taking one possible instantiation of the opportunistic frame  $\mathcal{F}$ . Such an instantiation consists of labeled transitions between players. The label stands for the player on whose behalf the message is forwarded.

**Example 2.10.** (Simple ON) We continue Example 2.4. Figure 2 shows an ON in which the goal  $\phi_1$  (we recall that  $\phi_1$  expresses the goal of reaching VPN without ever visiting a node in which Internet holds) of player 1 is violated. The violation is caused by the labeled transition  $1 \xrightarrow{1} 3$  which causes player 1's messages to be forwarded to player 3 who has access to Internet. Also, we would like to emphasize that player 1 sets two channels to player 3: one for itself and one on behalf of player 2.

Our ultimate goal is to use our approach for real-world communication. As communication is costly (e.g. because of limited battery power or network usage fees) the *costs* of a network have to be taken into account as well. For this purpose, we introduce the *cost of a network* which is defined as the sum of all established channel costs. This allows to compare networks to each other.

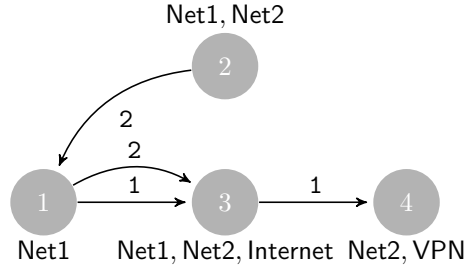


Figure 2: A simple opportunistic network. The reflexive edges (i.e.  $i \xrightarrow{i} i$  for all players  $i$ ) are omitted to improve the readability.

**Definition 2.11** (Cost). Let  $\mathcal{O} = (\mathcal{F}, \mathcal{T})$  be an ON. The cost of player  $i$  in  $\mathcal{O}$  is defined as the costs of all outgoing edges from player  $i$ , i.e.  $cost_i(\mathcal{T}) = \sum_{(i,k,j) \in \mathcal{E}} c(i,j)$  where  $i$  is fixed. The *total cost* of ON  $\mathcal{O}$  is defined as  $cost(\mathcal{O}) = \sum_{i \in \mathcal{P}} cost_i(\mathcal{T})$ .

**Remark 2.12** (Summing up Costs). Notice that according to Definition 2.11, whenever two or more communication channels are established to the same player, the cost of each such channel is added. Clearly, there are also settings in which the other extreme, that the same channel can be used by arbitrarily many users, makes good sense. Currently, we are working on a more realistic setting which takes into account a more sophisticated cost scheme.

Given this definition we can ask what is the best ON? Is it the one with the minimal costs? This depends on the behavior of the players, e.g. whether they are social, strictly self-interested, cooperative, or non-cooperative. Notice that a player may not unconditionally follow the satisfaction of its goal. If the player's costs exceed the value of its goal the player may be better off not establishing a link at all. Players often reason strategically. In particular, this is the case if costs are involved and network nodes belong to different organizations, which often is a realistic scenario in opportunistic nets and relay networks.

**Example 2.13** (Cost of an ON). In our example (Figure 2), the individual costs  $cost_i(\mathcal{O})$  and the overall costs  $cost(\mathcal{O})$  are given as follows:  $cost_1(\mathcal{O}) = 2$ ,  $cost_2(\mathcal{O}) = 6$ ,  $cost_3(\mathcal{O}) = 1$ ,  $cost_4(\mathcal{O}) = 0$  and  $cost(\mathcal{O}) = 9$ . The channel costs are included in the definition of the ONF shown in Figure 1.

## 2.2 What does an optimal solution look like?

In this section we informally discuss properties of optimal ONs. It is straightforward that optimality is highly affected by: (i) the goal fulfillment of a player, (ii) the goal value, (iii) the players' costs, and (iv) other players' goal fulfillment (depending on the parameter  $\beta$ , cf. Definition 2.14). Given a player  $i$  who participates in creating a

topology  $\mathcal{T}$  by setting up channels, we require a measure of its profit in  $\mathcal{T}$ , i.e. a *utility value*, which takes into account the properties (i-iv).

Rational players usually prefer topologies in which they get higher utility over those in which they get less utility. However, they can only set up a channel with their (directly accessible) neighbours and thus depend on the other players' actions in most of the cases. Therefore, it might be the case that achieving an optimal topology requires the contribution of other players and a certain deal of *compromise*. For instance, players might be required to accept higher costs, in order for their goals to become satisfied at all. As a result, finding optimal topologies amounts to finding the optimal compromises, the ones that are acceptable by each player. In other words, players act in a *highly strategic* manner.

*Game theory* offers a framework for analysing such strategic interactions. Therefore, one natural solution is to make use of game theoretic solution concepts for non-cooperative and cooperative games to analyse and to determine optimal topologies. This approach has already been followed by many researchers. We discuss related work in Section 5.

In the non-cooperative setting we model players with different degrees of selfishness, which always prefer the topology awarding them the greatest utility. The first solution concept we study, the *Nash equilibrium*, describes ON optimality in terms of *individual* stability. The question we ask is whether a player will accept setting (some of) its channels given the set of channels set by all the other players.

We easily notice that individual deviation is a limited means for a player to achieve a desirable outcome, therefore we turn to a stronger solution concept, the *strong Nash equilibrium*, to address *group deviations*. In contrast to the Nash equilibrium solution concept, we ask whether a *group* can deviate to increase its payoff. The intuition is that players can partly communicate to find solutions in which *each* of the deviating group members is better off. This is quite a strong assumption and is relaxed by the last optimality concept we consider: the *core*.

The *core*, which is rather a cooperative concept, is used to examine group deviations which allows for the *transfer of utility* between players. In our opinion, this concept is more sensible than the strong Nash equilibrium: if players are already assumed to jointly deviate, why should they not be able to agree on a payoff division which is beneficial for all members? We would like to note that we are not concerned with the actual payoff distribution here. An ON which does not satisfy the requirements of the *core* describes an unstable network. The instability is caused by a group of (deviating) players which can achieve a strictly higher group utility than the sum of the utilities of each individual member in the ON. In this paper we are only considering whether such a group utility can be achieved and do not discuss the construction of a fair payoff distribution.

We would like to note that there is a strong relation between these solution concepts. Indeed, we claim that a topology in the core is also strong Nash optimal and that every strong Nash optimal topology is also Nash optimal. A formal treatment is left for future research.

### 2.3 The Opportunistic Network Game

In Section 2.2 we motivated the use of game theory to give a characterisation of optimal ONs, that is, optimal topologies for an ONF. If we assume no cooperation between players, in particular no payoff distribution, the utility of a player is given by the value of its goal, if satisfied, weighted with a factor describing the incentive to contribute to the goals of other players minus the costs of the channels established by this very player.

**Definition 2.14** (Utility  $u_i(\mathcal{O})$ ). The utility of player  $i$  in the ON  $\mathcal{O}$  is defined as

$$u_i(\mathcal{O}) = \begin{cases} \left( \beta \frac{|\text{SAT}(\mathcal{O})|}{|\mathcal{P}|} + (1 - \beta) \right) v(i) - \text{cost}_i(\mathcal{O}) & \text{if } \phi_i \text{ is satisfied in } \mathcal{O}, \\ -\text{cost}_i(\mathcal{O}) & \text{otherwise.} \end{cases}$$

where  $\text{SAT}(\mathcal{O}) = \{j : \phi_j \text{ is satisfied in } \mathcal{O}\}$ , and  $\beta \in [0, 1]$  is a scaling factor.

We say that a player  $i$  (strictly) prefers  $\mathcal{O}$  over  $\mathcal{O}'$  iff  $u_i(\mathcal{O}) > u_i(\mathcal{O}')$ ; then, we write  $\mathcal{O} \succ_i \mathcal{O}'$ . Analogously, we define  $\succeq_i$  with respect to  $\geq$ . (We would like to note that in Section 4 we will be more concrete about what ‘‘satisfaction’’ means wrt. to a specific language to express goals.)

**Remark 2.15** (The scaling factor  $\beta$  and non-cooperative players). *The scaling factor  $\beta$  describes the player’s attitude concerning the satisfaction of other players’ goals. In the examples that follow, we consider  $\beta$  to be 0 (completely indifferent to whether or not the goals of other players are satisfied), 1 (completely motivated to help other players satisfying their goals) and 0.5 (in-between). Thus, an agent becomes more indifferent (resp. more interested) to satisfying other players’ goals, if  $\beta \rightarrow 0$  (resp.  $\beta \rightarrow 1$ ). However, even if players are fully interested, they only help others if their goal is satisfied and has a strictly positive value. Note that for  $\beta = 0$  we get the setting described in [2].*

Finally, we note that for  $\beta > 0$  the definition of the utility function contains a cooperative flavor. Again, we would like emphasize that the value of  $\beta$  is defined by the opportunistic network application and not by the players themselves.

Moreover, players are also selfish in the sense that if their individual goal is not satisfied they have no incentive to set up any communication channel.

**Remark 2.16** (Utility-Value Dependency). *The overall Utility  $\sum u_i(\mathcal{O})$  is always smaller than or equal to the sum of all values the players get when their goals are achieved, i.e.,  $\sum u_i(\mathcal{O}) \leq \sum v(i)$ . This can easily be seen as follows:*

$$\begin{aligned} \sum_{i \in \mathcal{P}} u_i(\mathcal{O}) &\leq \sum_{i \in \mathcal{P}} \left( \beta \frac{|\text{SAT}(\mathcal{O})|}{|\mathcal{P}|} + (1 - \beta) \right) v(i) - \text{cost}_i(\mathcal{O}) \\ &\leq \sum_{i \in \mathcal{P}} (\beta + (1 - \beta)) v(i) - \text{cost}_i(\mathcal{O}) \\ &= \sum_{i \in \mathcal{P}} v(i) - \text{cost}_i(\mathcal{O}) \\ &\leq \sum_{i \in \mathcal{P}} v(i). \end{aligned}$$

The costs of a player depend on the channels it creates. From a game theoretic perspective these are the actions of the player, i.e. an action of a player is a set of transition/channels. But when it comes to an implementation of our approach, the system

is responsible for processing user preferences, and computing the appropriate actions for each player (in a centralized or distributed fashion). Therefore, setting up channels is the task of the system, which users cannot directly control. However, they can indirectly modify the system behaviour, by modifying their preferences. Thus, the system can be rather seen as a communication protocol each player has to follow.

**Definition 2.17** (Actions). The  $\mathcal{F}$ -actions of player  $i$  in an ONF  $\mathcal{F}$  are given by:  $Actions_i = 2^{\{i \xrightarrow{j} k \mid j \in \mathcal{P} \text{ and } k \in N(i)\}}$ . We define  $Actions = \times_{i \in \mathcal{P}} Actions_i$ . An element  $a \in Actions$  is called  $\mathcal{F}$ -action profile. We omit  $\mathcal{F}$  if clear from context.

It is easy to see that each action tuple gives rise to an  $\mathcal{F}$ -topology and thus to an ON.

**Definition 2.18** ( $\mathcal{F}(a)$ ). Given an  $\mathcal{F}$ -action profile  $a = (a_1, \dots, a_{|\mathcal{P}|})$  we use  $\mathcal{F}(a)$  to refer to the ON  $(\mathcal{F}, \mathcal{T})$  where  $\mathcal{T} = (\mathcal{P}, (\bigcup_{i \in \mathcal{P}} a_i) \cup \{i \xrightarrow{i} i \mid i \in \mathcal{P}\}, Props, \mathcal{I})$ .

Now it is easy to see that for each ON  $\mathcal{O} = (\mathcal{F}, \mathcal{T})$  there is an  $\mathcal{F}$ -action  $a$  such that  $\mathcal{O} = \mathcal{F}(a)$  and vice versa. Hence, we define the utility for player  $i$  for an  $\mathcal{F}$ -action profile  $a$  as the utility of  $i$  in  $\mathcal{F}(a)$ :

$$(\star) \quad u_i(a) := u_i(\mathcal{F}(a)).$$

We are ready to associate an ONF with a strategic game:

**Definition 2.19** (Opportunistic Network Game). Let  $\mathcal{F}$  be an ONF. The  $\mathcal{F}$ -opportunistic network game (ONG), is given by the tuple  $\mathcal{G}_{\mathcal{F}} = (\mathcal{F}, Actions, u)$  where:

- $Actions$  is the set of  $\mathcal{F}$ -action profiles defined in Def. 2.17 and
- $u : \mathcal{P} \times Actions \rightarrow \mathbb{R}$  is the payoff function defined in  $(\star)$ .

We do also lift the preference relations  $\succ$  from Definition 2.14 to action profiles:  $a \succ_i^{\mathcal{F}} a'$  iff  $\mathcal{F}(a) \succ_i \mathcal{F}(a')$ . Relation  $\succeq_i^{\mathcal{F}}$  is defined analogously.

**Example 2.20** (Simple ONG). The scenario from Example 1 formulated as an ONG  $\mathcal{G}_{\mathcal{F}} = (\mathcal{F}, Actions, u)$  looks as follows:

- $\mathcal{F}$  is defined as in Example 2.4,
- $Actions_1 = 2^{\{1 \xrightarrow{j} 2, 1 \xrightarrow{j} 3 \mid j \in \mathcal{P}\}}$ ,  $Actions_2 = 2^{\{2 \xrightarrow{j} 1, 2 \xrightarrow{j} 3, 2 \xrightarrow{j} 4 \mid j \in \mathcal{P}\}}$ ,  $Actions_3 = 2^{\{3 \xrightarrow{j} 1, 3 \xrightarrow{j} 2, 3 \xrightarrow{j} 4 \mid j \in \mathcal{P}\}}$ ,  $Actions_4 = 2^{\{4 \xrightarrow{j} 2, 4 \xrightarrow{j} 3 \mid j \in \mathcal{P}\}}$ .
- The following action profile  $a \in Actions$  results in the ON from Fig. 2:  
 $a = (\{1 \xrightarrow{1} 3, 1 \xrightarrow{2} 3\}, \{2 \xrightarrow{2} 1\}, \{3 \xrightarrow{1} 4\})$ . Let  $\beta = 0$ . The utility of this action profile is given by:

$$\begin{aligned} u_1(a) &= -2, \\ u_2(a) &= 20 - 6 = 14, \\ u_3(a) &= 0 - 1 = -1, \\ u_4(a) &= 0 - 0 = 0. \end{aligned}$$

If the agents are more cooperative ( $\beta = \frac{1}{2}$ ) it changes to:

$$\begin{aligned} u_1(a) &= -2, \\ u_2(a) &= \frac{1}{2} \cdot \frac{3}{4} \cdot 20 + (1 - \frac{1}{2}) \cdot 20 - 6 = 11.5, \\ u_3(a) &= \frac{1}{2} \cdot \frac{3}{4} \cdot 0 + (1 - \frac{1}{2}) \cdot 0 - 1 = -1, \\ u_4(a) &= \frac{1}{2} \cdot \frac{3}{4} \cdot 0 + (1 - \frac{1}{2}) \cdot 0 - 0 = 0. \end{aligned}$$

Finally, if the agents are fully cooperative ( $\beta = 1$ ) it becomes:

$$\begin{aligned} u_1(a) &= -2, \\ u_2(a) &= 1 \cdot \frac{3}{4} \cdot 20 + (1 - 1) \cdot 20 - 6 = 9, \\ u_3(a) &= 1 \cdot \frac{3}{4} \cdot 0 + (1 - 1) \cdot 0 - 1 = -1, \\ u_4(a) &= 1 \cdot \frac{3}{4} \cdot 0 + (1 - 1) \cdot 0 - 0 = 0. \end{aligned}$$

In the following section we discuss how we can use ONGs to find optimal ONs.

## 2.4 Optimal Solutions for Non-Cooperative Players

In this section we discuss classic solution concepts of non-cooperative game theory. That is, players choose their actions independently from each other and no payoff division takes place.

**Definition 2.21** (Nash equilibrium). A *Nash equilibrium* of an ONG  $\mathcal{G}$  is an action profile  $a^* \in \mathcal{A}$  such that, for each player  $i \in \mathcal{P}$  and all actions  $a_i \in \mathcal{A}_i$  we have that  $(a_{-i}^*, a_i^*) \succeq_i (a_{-i}^*, a_i)$ .

Intuitively, a Nash equilibrium is a stable action profile, i.e., given the actions of the other players, no player can individually deviate and increase its payoff.

**Example 2.22** (Nash solution). We continue Example 2.20. First of all, notice that player 4's and player 3's goal value are zero, therefore player 3 will not set up any channel since the costs would decrease his utility. A channel from player 4 to player 3, however, costs nothing, thus player 4 has no incentive to deviate from the choice of setting (or not setting) such a channel, no matter what other players do. Additionally, player 4 will not establish a channel to player 2 because it would decrease its utility. Thus, we have two distinct scenarios: (i) player 4's action contains  $4 \xrightarrow{2} 3$  (which helps satisfying player 2's goal) or (ii) player 4's action is an arbitrary member of  $O = 2^{\{4 \xrightarrow{i} 3 | i \in \{1,3,4\}\}}$ .

Depending on  $\beta$  we can compute different Nash equilibria. For example, for  $\beta = 1$  the following set of strategy profiles contains Nash equilibria:  $S_1 \cup S_2 \cup S_3 \cup S_4$  where

$$\begin{aligned} S_1 &= \{(\emptyset, \{2 \xrightarrow{2} 3\}, \emptyset, X) \mid X \in O\}, \\ S_2 &= \{(\emptyset, \{2 \xrightarrow{2} 4\}, \emptyset, \{4 \xrightarrow{2} 3\}, X) \mid X \in O\}, \text{ and} \\ S_3 &= \{(\{1 \xrightarrow{1} 2, 1 \xrightarrow{2} 3\}, \{2 \xrightarrow{2} 1, 2 \xrightarrow{1} 4\}, \emptyset, X) \mid X \in O\} \\ S_4 &= \{(\{1 \xrightarrow{1} 2\}, \{2 \xrightarrow{1} 4, 2 \xrightarrow{2} 4\}, X \cup \{4 \xrightarrow{2} 3\}) \mid X \in O\}. \end{aligned}$$

Let us refer to the following action profiles as follows:

$$\begin{aligned}
 \text{NE}_1 &= (\emptyset, \{2 \xrightarrow{2} 3\}, \emptyset, \emptyset) \in S_1 \\
 \text{NE}_2 &= (\emptyset, \{2 \xrightarrow{2} 4\}, \emptyset, \{4 \xrightarrow{2} 3\}, \emptyset) \in S_2 \\
 \text{NE}_3 &= (\{1 \xrightarrow{1} 2, 1 \xrightarrow{2} 3\}, \{2 \xrightarrow{2} 1, 2 \xrightarrow{1} 4\}, \emptyset, \emptyset) \in S_3 \\
 \text{NE}_4 &= (\{1 \xrightarrow{1} 2\}, \{2 \xrightarrow{1} 4, 2 \xrightarrow{2} 4\}, \{4 \xrightarrow{2} 3\}) \in S_4
 \end{aligned}$$

Figure 3 shows the ONs associated with  $\text{NE}_2$  and  $\text{NE}_3$  and Figure 4(a) shows  $\text{NE}_4$ . To increase readability, we have omitted all propositions except Internet and VPN.

Note that in the case of  $\text{NE}_2$ , player 2 cannot deviate from  $2 \xrightarrow{2} 4$  to obtain a better payoff as player 4 has set the channel  $4 \xrightarrow{2} 3$ . This profile give player 2 utility 14. This scenario emerging from  $\text{NE}_2$  leaves player 1 with its goal unsatisfied, however the player has no better alternative, given the actions of the other players. The utility values for  $\beta = 1$  are given in the following: For  $\beta = 1$  (fully interested):

$$\begin{aligned}
 a \in S_1 : \quad & u_1(a) = u_3(a) = u_4(a) = 0 \\
 & u_2(a) = \frac{3}{4} \cdot 20 + 0 \cdot 20 - 8 = 7 \\
 a \in S_2 : \quad & u_1(a) = u_3(a) = u_4(a) = 0 \\
 & u_2(a) = \frac{3}{4} \cdot 20 + 0 \cdot 20 - 1 = 14 \\
 a \in S_3 : \quad & u_1(a) = \frac{4}{4} \cdot 40 + 0 \cdot 40 - 6 - 1 = 33 \\
 & u_2(a) = \frac{4}{4} \cdot 20 + 0 \cdot 20 - 6 - 1 = 13 \\
 & u_3(a) = u_4(a) = 0 \\
 a \in S_4 : \quad & u_1(a) = \frac{4}{4} \cdot 40 - 6 = 34 \\
 & u_2(a) = \frac{4}{4} \cdot 20 - 2 = 18 \\
 & u_3(a) = u_4(a) = 0
 \end{aligned}$$

Finally, Figure 4(b) shows a network which does not correspond to a Nash equilibrium because player 2 would be better off removing the channel to player 3.

Also for other values of  $\beta$  some Nash equilibria disappear. For example, for  $\beta = 0$ ,  $\text{NE}_2$  remains a Nash equilibrium whereas  $\text{NE}_4$  is no longer a Nash equilibrium although it offers a higher utility for player 2. In Example 2.32 we will see that the profile  $\text{NE}_4$  is in the core of the same game.

A nice property of the Nash Equilibrium is that it ensures efficiency concerning the ON, i.e., it removes channels that are unnecessary and/or channels that are set to fulfil a goal, which is already fulfilled by a better path. In Fig. 4(b) the channel  $2 \xrightarrow{2} 3$  is redundant since the goal of player 2 is already fulfilled by  $a = (\{2 \xrightarrow{2} 1\}, \{1 \xrightarrow{2} 3\})$ .

**Remark 2.23** (Limitations of Nash Equilibrium). *We can easily notice that, as  $\beta$  approaches 0, we obtain fewer Nash Equilibria. In Example 2.22, for values of  $\beta$  which are strictly lower than  $\frac{1}{10}$ , the action profiles from  $S_3$  are no longer Nash Equilibria,*

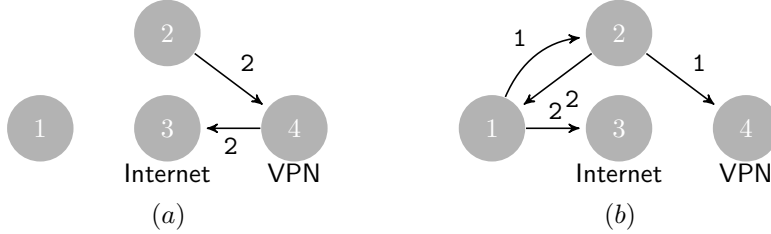


Figure 3: For  $\beta = 1$ , Figure (a) shows Nash Equilibrium  $NE_2$ ; and Figure (b) the Nash equilibrium  $NE_3$ . Again, we omit the reflexive edges in the interpretation as ON, cf. Definition 2.18.

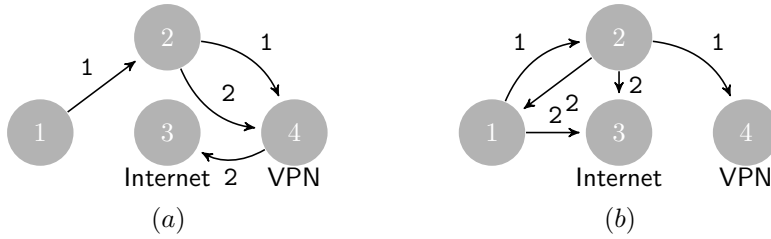


Figure 4: For  $\beta = 1$ , Figure (a) shows the Nash Equilibrium  $NE_4$ ; and Figure (b) a slightly modified ON that is not a Nash Equilibrium. Again, we omit the reflexive edges in the interpretation as ON, cf. Definition 2.18.

since Player 1 may now achieve a higher utility by not setting the channel  $1 \xrightarrow{2} 3$ , even though the goal of Player 2 is now not satisfied. The situation is the same from Player 2's perspective (and regarding the channel  $2 \xrightarrow{1} 4$ ), but for values of  $\beta$  lower than  $\frac{1}{5}$ .

When  $\beta$  is 0, no player will set channels for others, given that the channel costs and goal values are non-zero. In these situations, the only Nash Equilibria that exist are those associated to players which satisfy their goals by themselves, or in which cost-free channels are involved. In Example 2.22 this is the case with the Nash Equilibria from  $S_1$  and  $NE_2$ , respectively.

It is easy to see that players may not behave very cooperative in the case of Nash equilibria. If a player's goal cannot be satisfied, the player has no incentive to establish any non cost-free channels:

**Proposition 2.24.** *Suppose  $a^*$  is a Nash equilibrium such that player  $i$ 's goal is not satisfied or  $v(i) = 0$  in  $\mathcal{F}(a^*)$ ; then for all communication channels  $i \xrightarrow{j} k \in a_i^*$  the following is true:  $c(i, k) = 0$ . This holds for any value of  $\beta$ .*



Proposition 2.24 captures the intuition that players are not expected to exhibit altruism, in the absence of any payoff incentive. If it was the case that  $c(i, k) > 0$ , for some channel  $i \xrightarrow{j} k$ , then player  $i$  would be better off by not setting such a channel, and therefore  $a^*$  is not a Nash equilibrium.

**Definition 2.25** (Strong Nash equilibrium). A *strong Nash equilibrium* of an ONG  $\mathcal{G}$  is an action profile  $a^* \in \text{Actions}$  such that, there is no coalition  $C \subseteq \mathcal{P}$  and no joint action  $a_C \in \times_{i \in C} \text{Actions}_i$  such that we have that:

$$\begin{aligned} (a^*_{-C}, a_C) \succeq_i (a^*_{-C}, a^*_C) & \text{ for all } i \in C \text{ and} \\ (a^*_{-C}, a_C) \succ_i (a^*_{-C}, a^*_C) & \text{ for some } i \in C. \end{aligned}$$

The strong Nash equilibrium  $a^*$  is a Nash equilibrium where no coalition can be formed that can cooperatively deviate from the action profile such that *all* of the members of the coalition get a payoff at least as high as in  $a^*$  and at least one deviating player gets a strictly better payoff. The difference between a Nash and a strong Nash equilibrium is that in order to ensure the property, in the latter case we have to take all possible coalition deviations into account while for the former only single player deviations have to be considered. This means that strong Nash equilibria are more stable than Nash equilibria but also more restrictive. Concerning the ONG a drawback of the (strong) Nash equilibrium is that as soon as a player's goal is not satisfied it only executes actions without negative costs, i.e., the player becomes (more or less) passive. This is a direct consequence of Proposition 2.24.

**Remark 2.26** (Strong Nash and group deviations). *Notice that, given  $\beta = 1$  (fully interested in other players goals), the Nash Equilibrium of  $NE_3$  does not ensure the highest possible utility for Player 2, despite having his goal fulfilled. But in order to achieve a better utility, the actions of Player 2 are not sufficient. However, players 2 and 4 can jointly achieve such an outcome.*

**Example 2.27** (Strong Nash Equilibrium). For  $\beta = 1$  there is a unique strong Nash Equilibrium in the previous example, which is shown in Figure 5:  $(\{1 \xrightarrow{1} 2\}, \{2 \xrightarrow{1} 4, 2 \xrightarrow{2} 4\}, \{4 \xrightarrow{2} 3\})$  (indeed this is  $NE_4$  from Figure 4). The Nash Equilibria from  $S_1$  and  $S_3$  as well as  $NE_2$  are not strong, since the coalition  $C = \{1, 2, 4\}$  will always prefer the action profile  $(\{1 \xrightarrow{1} 2\}, \{2 \xrightarrow{1} 4, 2 \xrightarrow{2} 4\}, \{4 \xrightarrow{2} 3\})$ .

## 2.5 Optimal Solutions for Cooperative Players

In the previous solution concepts a player is not willing to deviate if the deviation would not increase its individual utility. This holds, even if the overall sum of the utilities of the deviating group would be much higher. So, what if players cooperate and are allowed to transfer utility, to set up side payments? We define the utility of a group as follows (without explaining *how* the payoff is actually divided).

**Definition 2.28** (Group utility). Let  $X \subseteq \mathcal{P}$  be a group of players and  $\mathcal{O}$  be an ON. We define  $u_X(\mathcal{O}) = \sum_{i \in X} u_i(\mathcal{O})$ . As before we define the group utility of action profiles  $a \in \text{Actions}$  as  $u_X(a) = u_X(\mathcal{F}(a))$ . Similarly, we also lift the preference relations  $\succ_i$   $\succeq_i$  to groups of players,  $\succ_X$  and  $\succeq_X$ , respectively.

**Example 2.29** (Group utility). The group utility of the ON shown in Fig. 3(b) is:  $u(\mathcal{O}) = u(1, a) + u(2, a) + u(3, a) + u(4, a)$  where  $a = (\{1 \xrightarrow{1} 2, 1 \xrightarrow{2} 3\}, \{2 \xrightarrow{2} 1, 2 \xrightarrow{1} 4\}, \emptyset, \emptyset)$ . For  $\beta = 0 : u(\mathcal{O}) = 46$ , for  $\beta = \frac{1}{2} : u(\mathcal{O}) = 52$  and for  $\beta = 1 : u(\mathcal{O}) = 46$ . Similarly, the group utility of the ON shown in Fig. 3(a) is  $u(\mathcal{O}) = 19$ ,  $u(\mathcal{O}) = 16, 5$  and  $u(\mathcal{O}) = 14$ , respectively.

Finally, we lift the strong Nash equilibrium concept to the setting in which players can transfer payoff. Now, a player may establish channels even if its goal is *not* satisfied.

**Definition 2.30** (Core). The *core* of an ONG consists of the set of all action profiles  $a$  such that there is no coalition  $X \subseteq \mathcal{P}$  and no action profile  $a'$  which agrees with  $a$  for all players  $\mathcal{P} \setminus X$  such that  $a' \succ_X a$ .

**Remark 2.31** (Core). *We would like to note that our notion of core is somewhat different from the standard game theoretic notion: We assume that the players who are not deviating stick to their actions. Currently, we are also working on a slightly different definition of the core which is more in line with its game theoretic counterpart: the deviating coalition on its own (without using channels from players outside the coalition) must be better off.*

**Example 2.32** (Core). We continue Example 2.29. The action profile  $(\{1 \xrightarrow{1} 2\}, \{2 \xrightarrow{1} 4, 2 \xrightarrow{2} 4\}, \{4 \xrightarrow{2} 3\})$  yields a group utility of  $u(\mathcal{O}) = 40 + 20 - (6 + 1 + 1) = 52$  under  $\beta = 1$  and is the only member of the core.

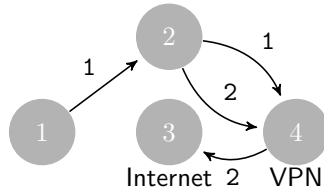


Figure 5: The core.

Intuitively, the core provides the best topology  $\mathcal{T}$  in  $\text{Tops}(\mathcal{G})$  regarding the (social) payoff of all players and adds some stability assumptions.

**Remark 2.33** (Limitations of the core). *Note that the core will not contain ONs in which players abandon their own goal for helping others (apart for trivial cases in which helping comes for free).*

### 3 Computational Setting

In the following, we introduce the temporal logic **CTL** (Computation Tree Logic) for expressing players' goals in an ON.

#### 3.1 Preferences as Temporal Formulae

In this section we define a goal of an player to be expressed as a **CTL**-formula. In the following, we review the syntax and semantics of the logic.

The *language of CTL* [3] is given by all formulae generated by the grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid E(\varphi \mathcal{U} \varphi) \mid E\bigcirc\varphi \mid E\Box\varphi.$$

where  $p \in \mathcal{Props}$  is a proposition. The Boolean connectives are defined by their usual abbreviations. The basic temporal operators are  $\mathcal{U}$  (*until*) and  $\bigcirc$  (*in the next state*). The path quantifier  $E$  (*there is a path*) allows to existentially quantify over possible system behaviors; that is, in our case, over communication paths. The dual universal path quantifier  $A$  (*for all paths*) and the additional temporal operators  $\diamond$  (*eventually*) and  $\Box$  (*always from now on*) can be defined as macros:  $\diamond\varphi \equiv \top \mathcal{U} \varphi$ ,  $A\bigcirc\varphi \equiv \neg E\bigcirc\neg\varphi$ ,  $A\Box\varphi \equiv \neg E\diamond\neg\varphi$ , and  $A\varphi \mathcal{U} \psi \equiv \neg E((\neg\psi) \mathcal{U} (\neg\varphi \wedge \neg\psi)) \wedge \neg E\Box\neg\psi$ .

**Example 3.1** (Goals). The goals of Example 2.4 can be expressed as **CTL**-formulae as follows:  $\phi_1 = E(\neg\text{Internet} \mathcal{U} (\text{VPN} \wedge A\Box\neg\text{Internet}))$ ,  $\phi_2 = E\diamond\text{Internet}$ , and  $\phi_3 = \phi_4 = \top$ . We would like to note that one could imagine other formalizations capturing the informal (and ambiguous) description of goal  $\phi_1$ . Here, we actually express that no node with internet access is visited until VPN is true and then, that on all possible extensions it is not possible to visit a node with internet access via a channel established on behalf of player 1.

The standard semantics of **CTL** is defined over Kripke structures. Given a  $\mathcal{F}$ -topology  $\mathcal{T}$  we simply ignore the labels and interpret the resulting structure as Kripke structure. This is done by adjusting the definition of a path.

**Definition 3.2** (Communication path). A *communication path*  $\lambda = i_0 i_1 \dots \in \mathcal{P}^\omega$  in  $\mathcal{T}$  is an infinite sequence of players/nodes that are interconnected by channels; that is, for all  $j = 0, 1, 2, \dots$ , there exists some  $k \in \mathcal{P}$  (not necessary the same, for all  $j$ ) such that  $i_j \xrightarrow{k} i_{j+1}$ . We use  $\lambda[j]$  to denote the  $j$ th player ( $i_j$ ) on path  $\lambda$  (starting from  $j = 0$ ) and  $\lambda[j, \infty]$  to denote the subpath of  $\lambda$  starting from  $j$  (i.e.  $\lambda[j, \infty] = \lambda[j] \lambda[j+1] \dots$ ). We write  $\Lambda(i)$  to refer to the set of all paths that start with player  $i$ .

Let  $\mathcal{T}$  be a  $\mathcal{F}$ -topology and  $i \in \mathcal{P}$  be a player/node in  $\mathcal{T}$ . The semantics of **CTL**-formulae is given by the satisfaction relation  $\models^{\text{CTL}}$  defined below:

$$\begin{aligned} \mathcal{T}, i \models^{\text{CTL}} p & \quad \text{iff } p \in \mathcal{I}(i) \text{ and } p \in \mathcal{Props}; \\ \mathcal{T}, i \models^{\text{CTL}} \neg\varphi & \quad \text{iff } \mathcal{T}, i \not\models^{\text{CTL}} \varphi; \\ \mathcal{T}, i \models^{\text{CTL}} \varphi \wedge \psi & \quad \text{iff } \mathcal{T}, i \models^{\text{CTL}} \varphi \text{ and } \mathcal{T}, i \models^{\text{CTL}} \psi; \end{aligned}$$

$\mathcal{T}, i \models^{\text{CTL}} \text{E}\bigcirc\varphi$  iff there is a path  $\lambda \in \Lambda(i)$  such that  $\mathcal{T}, \lambda[1] \models^{\text{CTL}} \varphi$ ;

$\mathcal{T}, i \models^{\text{CTL}} \text{E}\Box\varphi$  iff there is a path  $\lambda \in \Lambda(i)$  such that  $\mathcal{T}, \lambda[j] \models^{\text{CTL}} \varphi$  for every  $j \geq 0$ ;

$\mathcal{T}, i \models^{\text{CTL}} \text{E}\varphi \mathcal{U} \psi$  iff there is a path  $\lambda \in \Lambda(i)$  such that  $\mathcal{T}, \lambda[j] \models^{\text{CTL}} \psi$  for some  $j \geq 0$ , and  $\mathcal{T}, \lambda[k, \infty] \models^{\text{CTL}} \varphi$  for all  $0 \leq k < j$ .

Finally, we define the satisfaction of goals in an ON. The idea is that player  $i$ 's goal is satisfied if the underlying topology in which only channels intended for  $i$  are considered satisfies its goal. We also note that the goal formula is interpreted in a communication path starting from player  $i$ . (Note that the definition of satisfaction is also used within an ONG.)

**Definition 3.3** (Satisfaction in ON). Let  $\mathcal{O} = (\mathcal{F}, \mathcal{T})$  be an ON. For a player  $i \in \mathcal{P}$  we write  $\mathcal{T}|_i$  to refer to the  $\mathcal{F}$ -topology in which each transition  $(k, l, m) \in \rightarrow$  with  $l \neq i$  is removed. The goal  $\phi_i$  of player  $i$  is satisfied in  $\mathcal{O}$ , denoted by  $\mathcal{O} \models \phi_i$ , iff  $\mathcal{T}|_i, i \models^{\text{CTL}} \phi_i$ .

In the following we give some examples to illustrate the usefulness of **CTL** for expressing goals. Subsequently, in Section 4 we show that **CTL** has good computational properties regarding ONs, cf. Proposition 4.3.

**Example 3.4.** Notice that goals  $\phi_1$  and  $\phi_2$  described in Example 3.1 are both satisfied in the topology  $\mathcal{T}$  from Fig. 3 (b).  $\mathcal{T}|_1, 1 \models^{\text{CTL}} \phi_1$  since there exists the communication path  $\lambda = 1, 2, 4, 4, \dots$  on which Internet is not true until (i) VPN is true and (ii) Internet can never be true (on any path) further on. Similarly, the communication path  $\lambda = 2, 1, 3, 3, \dots$  is a witness for  $\mathcal{T}|_2, 2 \models^{\text{CTL}} \phi_2$ . For illustration we consider a few other goals:

- $\text{A}\Diamond\text{VPN}$  requires that on all paths (set for player 1) VPN must be accessible. This is not true in Fig. 3(b), but is true if the channel  $3 \xrightarrow{1} 4$  would have been established.
- $\text{E}\Box(\text{VPN} \vee \text{E}\bigcirc\text{Internet})$  expresses that on all paths from player 1 and at each hop-node, VPN is true or Internet is accessible via a direct neighbour. This goal is satisfied in  $\mathcal{T}|_1$  if the path  $\lambda = 1, 2, \dots$  would exist in  $\mathcal{T}|_1$ .

## 4 Complexity of Finding Optimal Solutions

In the following we analyze the complexity of finding optimal opportunistic networks. Throughout this section we assume that we are given the ONF  $\mathcal{F} = (\mathcal{P}, N, \text{Props}, c, \mathcal{I}, (\phi_i)_{i \in \mathcal{P}}, v)$  with goals given as **CTL**-formulae, the  $\mathcal{F}$ -topology  $\mathcal{T}$  and that  $\mathcal{O} = (\mathcal{F}, \mathcal{T})$  is an ON. Moreover, we use  $\mathcal{G}_{\mathcal{F}}$  to refer to the  $\mathcal{F}$ -opportunistic network game.

Complexity results are always with respect to the size of the input. As input we take an ON or an ONF. We measure the size of both objects in terms of the number of players and the sum of the lengths of the goal formulae.

**Definition 4.1** (Size). We use  $|\phi_i|$  to denote the length of the formula. The *size* of  $\mathcal{O}$  and of  $\mathcal{F}$  is defined as  $|\mathcal{P}| + \sum_{i \in \mathcal{P}} |\phi_i|$  (i.e. the sizes are given by the number of players and the sum of the lengths of all goal formulae).

We note that the number of transitions in  $\mathcal{T}$  is polynomial in the number of players (more precisely,  $\leq |\mathcal{P}|^3$ , also cf. Proposition 2.8). This justifies that we base the size of the input solely on the number of players and lengths of the formulae.

**Definition 4.2** (Optimal opportunistic networks). Given an ON  $\mathcal{O} = (\mathcal{F}, \mathcal{T})$  we say that  $\mathcal{O}$  is Nash-optimal (resp. strong Nash-optimal, core-optimal) if the action profile  $a \in \text{Actions}$  with  $\mathcal{F}(a) = \mathcal{T}$  is a Nash equilibrium (resp. a strong Nash equilibrium, in the core) of the  $\mathcal{F}$ -opportunistic network game  $\mathcal{G}_{\mathcal{F}}$ .

## 4.1 Verification of Optimal Solutions

The following result follows immediately from [3]:

**Proposition 4.3** ([3]). *For any player  $i \in \mathcal{P}$ , checking whether  $\mathcal{O} \models \phi_i$  is P-complete with respect to the size of  $\mathcal{O}$ .*

Now we turn to checking whether a given opportunistic network is Nash-optimal.

**Proposition 4.4** (Checking Nash optimality). *Checking whether  $\mathcal{O}$  is Nash-optimal is coNP-complete for all values of  $\beta$ .*

*Proof. Membership:* We show that the complement is in NP. Let  $\mathcal{O}$  be the given ON and let  $a$  be the action profile with  $\mathcal{F}(a) = \mathcal{O}$ . We guess a player and an action  $a_i$  of  $i$ , i.e. a set of channels. Let  $a'$  be the action profile  $a$  with  $i$ th action replaced by  $a_i$ . We then check whether  $\mathcal{F}(a')$  is preferred by the current player to  $\mathcal{O}$ . If so,  $\mathcal{O}$  is not Nash-optimal. By Proposition 4.3 the latter can be done in deterministic polynomial time.

*Hardness:* We reduce the *Minimum Cover Problem*<sup>2</sup> to the complement of our problem. Given a set  $S$ , the subsets  $S_1, \dots, S_n \subseteq S$ , and a value  $m \leq n$ , we introduce a propositional symbol  $p_u \in \text{Props}$  exactly for each element  $u \in S$ . We note that  $\text{Props}$  is finite as  $S$  is finite. For each subset  $S_k$ , we introduce a player  $i_k$  such that  $p_u \in \mathcal{I}(i_k)$  iff  $u \in S_k$ . These players all have the same goal:  $\top$ . We define a special player  $i^*$  having as goal  $\phi^* = \bigwedge_{p \in \text{Props}} \text{E} \circ p$  (this is a finite conjunction because  $\text{Props}$  is finite). The player can establish a channel with all other players.  $v(i^*)$  has value  $m + 1$  and  $c(i^*, i) = 1$ , for all  $i \neq i^*$ . All other costs and values are set to zero. We denote by  $a$  the action profile in which each player sets no edge, and by  $\mathcal{O}$  the resulting ON. Then, there is a covering of the universe  $U$  with  $m$  subsets from  $S_1, \dots, S_n$  iff  $\mathcal{O}$  is not Nash-optimal. Now we have: there is a covering of  $S$  iff  $i^*$  can satisfy its goal with positive utility (by setting channels to all players/elements in the covering) iff  $\mathcal{O}$  is not

<sup>2</sup>Minimum cover [7] takes as input a finite set  $S$  and subsets  $S_1, \dots, S_n$  of  $S$  as well as an integer  $m \leq n$ . The question is whether there are  $S_{i_1}, \dots, S_{i_m}$  such that  $\bigcup_{j=1, \dots, m} S_{i_j} = S$ . This is an NP-complete problem. Note that  $S_{i_j}$  and  $S_{i_k}$  may be equal for some  $1 \leq j, k \leq m$ .

Nash-optimal. Finally, we note that the goals of all players apart from  $i^*$  are trivially true. Hence, the value of  $\beta$  does not matter.  $\square$

The proof for the next proposition is done in the very same way with the only difference that one guesses a set of players and their actions (instead of a player and an action).

**Proposition 4.5** (Checking strong Nash optimality). *Checking whether  $\mathcal{O}$  is strong Nash-optimal is coNP-complete for all values of  $\beta$ .*

**Proposition 4.6** (Checking core optimality). *Checking whether  $\mathcal{O}$  is core-optimal is coNP-complete for all values of  $\beta$ .*

*Proof. Membership:* We show that non-membership is in NP. We guess a tuple  $(C, a'_C)$  where  $C \subseteq \mathcal{P}$  is a set of players and  $a'_C \in \times_{i \in C} \text{Actions}_i$  an action profile of  $C$ . Let  $a$  be the action profile with  $\mathcal{F}(a) = \mathcal{O}$  and let  $a''$  be  $a$  with  $C$ 's actions replaced by  $a'_C$ , i.e.  $a'' = (a_{-C}, a'_C)$ . Now, we can construct  $\mathcal{F}(a'')$  and check whether  $a'' \succ_C^{\mathcal{F}} a$  in deterministic polynomial time. So, we have shown that the problem of core-optimality is in coNP. *Hardness:* The same construction as in the proof of Proposition 4.4 works.  $\square$

## 4.2 Synthesis of Optimal Solutions

In the last section, we have shown that the *verification problems* are all coNP-complete. The *synthesis problem* refers to the problem of *constructing* an optimal solution and not to just checking whether we are given one. Formally, we are given an ONF  $\mathcal{F}$ , one of the three optimality concepts  $C$  (i.e. Nash, strong Nash, core) and would like to construct a  $\mathcal{F}$ -topology  $\mathcal{T}$  such that  $\mathcal{O} = (\mathcal{F}, \mathcal{T})$  is  $C$ -optimal.

Firstly, we introduce the associated *decision problem* to each of the three synthesis/function problems: Does there exist a  $\mathcal{F}$ -topology such that  $\mathcal{O} = (\mathcal{F}, \mathcal{T})$  is  $C$ -optimal?

It is easy to see that the synthesis problem is at least as hard as the associated decision problem. Formally, we have the following result:

**Theorem 4.7** (Synthesis problems). *Let  $\mathcal{F}$  be an ONF. The decision problem whether there is a  $\mathcal{F}$ -topology  $\mathcal{T}$  such that  $(\mathcal{F}, \mathcal{T})$  is Nash-optimal (resp. strong Nash-optimal, core-optimal) is in  $\Sigma_2^P$ .*

*Moreover, if such a  $\mathcal{F}$ -topology  $\mathcal{T}$  exists it can be computed by a non-deterministic Turing machine which runs in polynomial time and which has access to a NP-oracle.*

*Proof.* We show that all decision problems are in  $\Sigma_2^P$ : We guess a  $\mathcal{F}$ -topology in non-deterministic polynomial time and check whether it is optimal wrt. one of the three optimality notions (cf. Propositions 3-5). This shows that the problem is in  $\text{NP}^{\text{coNP}} = \text{NP}^{\text{NP}} = \Sigma_2^P$ . Now it is also obvious that the synthesis problem can be implemented by a non-deterministic Turing machine which runs in polynomial time and which has access to an NP-oracle.  $\square$

Finally, we also claim hardness for the decision problems:

**Claim 4.8.** : *The decision problem whether there is a  $\mathcal{F}$ -topology  $\mathcal{T}$  such that  $(\mathcal{F}, \mathcal{T})$  is Nash-optimal (resp. strong Nash-optimal, core-optimal) is  $\Sigma_2^P$ -complete.*

**Remark 4.9** (Discussion on the complexity). *The complexity results justify our choice of CTL to express goals. The model checking/verification problem of CTL is only P-complete and the expressiveness of the language is still sufficient to express interesting properties. Richer languages like CTL\* and LTL do already have a PSPACE-complete model checking problem. Also the complexities of the optimality checks and decision problems are in line with complexity results for finding Nash equilibria etc. in strategic games and one cannot hope for better worst-case complexity results [8]. (Note that in our setting the number of actions is exponential in the size of an opportunistic network frame  $\mathcal{F}$ .)*

## 5 Related Work

In this section we discuss related work. In [15] routing and forwarding protocols in wireless ad-hoc networks are considered. Similar to our approach, strategic games are used as models. The authors show that there is no forwarding-dominant protocol and propose cooperation-optimal protocols as solution for non-cooperative selfish players. Similarly, the authors in [6] analyze Nash equilibria of packet forwarding strategies in a fixed network topology. Unlike other existing game-theoretic approaches, which are aimed at describing how communication can be established in ad-hoc (or opportunistic) networks, our work is not focused on defining routing schemes or forwarding strategies. We consider such information to be known: the neighbourhood function  $N$  describes all possible channels, and the cost function may encapsulate measurable channel parameters such as throughput, required emitting power, etc.

Our method is focused on taking player preferences into account, when establishing routes in any particular type of ad-hoc network. As already seen, using languages such as CTL, players have the ability to enforce certain restrictions on how messages should be forwarded. Depending on the particular network at hand, our framework can be used as a standalone tool for establishing a communication network, or as a complement to existing routing and forwarding strategies which can now be defined on top of our established (optimal) topologies.

The impact of offloading mobile data traffic from 3G networks is discussed in [9] and efficient algorithms are proposed. Next, in [11] the routing problem in a delay-tolerant network with path failures is considered. The authors introduce a framework for studying the effectiveness of sending the same message over different paths in order to maximise the delivery rate. In this approach, the expected failure of a path is governed by a certain probability distribution. The work from [10] extends this setting. Path failures are replaced by a mobility model, which takes into account the players' social relationships, and assigns metrics such as *popularity ranking* or *centrality* (the importance of a node in a metric). Using this mobility model, routing performance and

efficiency are improved [10]. An alternative approach, based on a Markovian model of mobility, is discussed in [1].

Quality of service (QoS) is considered in [13], from the perspective of individual selfishness: players may often be reluctant to participate in an opportunistic network, either for personal security issues, or for avoiding the consumption of battery power or computational resources. The model from [13] uses competitive markets in order to enforce optimal player QoS in an opportunistic network.

The results from [10, 13, 9] show that the behaviour of an opportunistic network is heavily dependent on the human factor. The algorithms from [10] exploit social structures within a community, whereas [13] exploits the idea of a self-interested player which is willing to compromise, in order to achieve some benefit (QoS). It is expected that humans' involvement in the way communication occurs in an opportunistic network will increase, and the player's choice in the way messages are routed will be equally important as other performance factors.

In contrast to [10, 1] our modeling approach does not attempt to build a mobility model. We rather assume that the set of locations which are reachable by a player are known. As a result, our setting relies on existing methods such as [1, 11], for detecting these locations (or selecting the most likely reachable ones).

## 6 Conclusions

As opportunistic networks become more popular, players would like to have more control over the way their messages are delivered in the network. As a result, routing methods that maximise delivery should also be complemented by preference-based routing mechanisms. Our approach exploits the *expressiveness of CTL* for formulating routing preferences, and uses standard game-theoretic tools in order to characterize *stable/optimal topologies*. The solution concepts we study explore different sides of stability: against individual deviation and group deviation. For the latter, we also consider the case when groups might decide to exchange payoff.

*Future work:* A limitation of our setting, is that it captures a snapshot of the evolution of an opportunistic network: the number of players, and the way they can communicate is fixed. It would be interesting to see how a dynamically changing set of players affect the stability, and also whether computing new equilibria/network topologies can make use of previously computed ones. Currently, goals are evaluated with respect to the player's position and can only express properties of the player's message delivery path. It would be interesting to explore games in which this restriction is removed, and thus giving players the ability to specify properties of other player's message delivery paths; that is, a player's preferences can take into account other players' communication. Currently, we are also working on a prototypical implementation of our approach to compare it with existing routing mechanisms. For practical use it is also important to assess the computational complexity and run-time behavior as well as the *usability*. The latter involves issues such as: how can users not familiar with logics like CTL specify properties and how can this be done on pocket devices such as mobile phones? To



address these issues one can for example use graphical notation for CTL-formulae [5] for describing basic goals such as:  $p$  must be accessible on some path of  $q$  must not be accessible on any path.

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