A PRINCIPAL COMPONENTS ANALYSIS OF THE UK TERM STRUCTURE AND THE INFLUENCE OF FISCAL POLICY

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Dedicated to my parents for their help and encouragement over the years.

Abstract

This study examines the use of principal component techniques in analysing term structures of interest rates. It employs original methods of estimating B-Spline models with endogenous knot positions and applies the method of Dierckx (1981) to generate new data sets for the study. The variability of the knots suggests that natural market boundaries do not exist in the UK gilts market. Few, if any, of the previous studies of term structures using principal components have subjected the components to statistical testing. This is remedied in this thesis. The results suggest that only two components, a level and a slope component, are required to describe most of the variability in the term structures irrespective of the data used, but these components are not stable over time. The thesis extends the method to include partial common principal components, and using this method demonstrated the difference in the major components of selected data sets. The thesis found that changes in the principal component scores could not be accounted for by regularly published economic news, including news about the PSBR. A macromodel was estimated. This showed that the term structures in the sample were altered by changes in government spending but the movement in interest rates would depend upon how this was funded and what maturity of interest rates was studied. The model also showed that significant changes would take a long time to manifest themselves and that there was evidence that some forms of funding had unstable effects. These results provide an explanation of why news effects are difficult to discern and why there is no consensus on whether or not fiscal variables affect the term structure.

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Chapter 1

Introduction

1.1 Objectives of the Study

The term structure of interest rates is often seen as one of the main areas where the forward looking behaviour of market participants is readily observable. As long-term interest rates are believed to help determine activity then changes in expectations can have effects on the current period. Thus the term structure acts as a transmission mechanism between the future and the present. This view has received increased support since the late 1980s from analysis that suggests that the slope of the term structure or the spread between commercial and government bonds can help predict changes in economic activity.[1] Moreover, at the end of 1997 British government securities (gilts) amounted to nearly £317bn, of which £181bn was held by UK life assurance and pension funds (LAPFs), and accounts for about 15% of the LAPFs' total gross financial wealth. Hence, virtually all UK citizens are linked into the gilts market at least indirectly as investors as well as being eventually the paymasters through their taxes.

Citizens also feel the impact of changes in the term structure through its effect on government finances. For countries that have imposed limits on revenue raising, or have exhausted their tax capacity, higher interest rates mean fewer resources are available for government expenditure. Indeed, if real interest rates exceed GDP growth rates the government debt to GDP ratio can rise or fall without limit or until the financial markets are not prepared to absorb any more of the countries' debt, a situation analysed by Bispham (1987). Changes in the term structure are one of the means that financial markets may be able to discipline wayward governments. The following quote from the Financial Times makes it clear that this is more than just a theoretical possibility. ""Skandia will not buy Swedish (state) bonds until such time as the politicians, in a credible way, begin to take seriously the accelerating state of debt" said Bjorn Wolrath, Skandia's Chief Executive. The Swedish five year bond yield moved up sharply, the Swedish Krona fell and the Stockholm stock exchange fell by 2%."[2] Hence, understanding the

[1] See for example Dueker (1997) and the papers cited therein.
[2] Financial Times 2/3 July 1994 p.4.

determinants of the term structure is an important area of study for forecasting, policy setting and the welfare of the citizens as a whole.

The main aim of the thesis is to examine whether or not changes in government fiscal behaviour alters the term structure of interest rates. This is by no means settled theoretically with Ricardians, e.g. Barro (1974), arguing against any effect whilst others argue that higher deficits raise interest rates. Neither is it settled empirically with a number of studies producing results that show higher debt and deficits are negatively related to interest rates, e.g. Evans (1987), confounding both of the above positions. Again a quotation from the Financial Times illustrates the difficulty. "Sweden's National Debt Office yesterday cut its estimate of government borrowing this year from SKr40-Skr50bn (£4bn-£5bn) to Skr20-Skr30bn. It said it would scale back its auctions of nominal treasury bonds from Skr3bn to SKr2bn, to take effect from mid-November. Swedish bond yields fell sharply yesterday after the central bank said it saw continued room for interest rate reductions. Yields on long-term bonds eased 18 basis points to 7.04 per cent and one year bond rates fell 43 points to 4.66 per cent."[3] What exactly did cause the yields to fall - lower expectations of government debt issues or expectations of lower short-term interest rates? Thus an empirical examination is required to examine the interaction of debt, deficits and the term structure.

A number of areas of interaction are left unexplored in this thesis. These include questions about the optimal maturity of government debt, see Calvo et al (1991) and government reputation and debt sustainability, see, for example, Drudi and Prati (1993). Also left unexplored are fiscal theories of inflation associated with Sargent and Wallace (1981), and in depth examination of the Fisher equation, see for example Ahmed and Rogers (1996) and Gilbert and Yeoward (1994). The reason for these omissions is not that these areas are regarded as unimportant but rather that a basic question "does government debt and deficits raise interest rates" still needs to be resolved, which is the aim of this thesis.

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[3] Financial Times, 10 October 1996.

The methodology usually adopted by economists to implement empirical research of the term structure is regression analysis. The problem is that the choice of interest rates to use as the dependent variable is unlimited and, therefore, so is the potential number of regressions. Summarising the results from a large number of regressions is cumbersome and can lead to conflicts between the results through simple stochastic variation. Picking representative interest rates runs the risk of selecting atypical rates that may bias the conclusions. This thesis uses principal components analysis to produce summary indices of the term structure, the principal component scores. This significantly reduces the required number of regressions. The thesis makes a methodological contribution by subjecting the indices to a number of comparative statistical tests. Furthermore, by using partial common principal components to study term structures the method itself is extended.

1.2 Organisation of the Study

Given its importance as an area of study the term structure has generated an extensive theoretical and empirical literature. This is surveyed in chapter 2 with an emphasis on recent UK empirical work. This survey provides the theoretical underpinnings for analysis on the UK term structure undertaken in chapters 7 and 8. The term structure data for the quantitative analysis is produced in chapter 3. Methodological advances in creating term structures from bond prices and yields are also made by endogenising the setting of knot positions in spline curve estimation. The production of endogenous knot positions allows examination of "natural market boundaries" of participants within the term structure. The existence of such boundaries would cast doubt on the expectations hypotheses of the term structure. The creation of the two data sets plus two others already in existence allows comparisons of different yield curve construction methodologies to be reported in this thesis.

Chapter 4 outlines the method of principal components and provides a description of the descriptive and statistical tests that can be applied. The chapter also provides a survey of principal components work on term structures. This is a task that does not appear to have been published elsewhere. Chapter 5 examines the principal components of the data sets estimated in chapter 3. The results suggest that most of the variance in the term structure can be explained by just two components and that this result is independent of the data set or the number of maturities used. The results confirm those from the survey in chapter 4 that the first component measures the level of interest rates and the second the slope. As sensible interpretations can be placed on the components this implies that regression analysis can be performed on the principal component scores in chapters 7 and 8.

Chapter 6 further extends the use of principal components by applying partial common principal components analysis. This has never before been reported as being used on interest rate data and its potential in economics has yet to be exploited. Using this analysis and other statistical tests the relationships between principal components (over time, across different data sets and across countries) are examined in this thesis. The main results are that the components differ between data sets and are unstable over time.

Chapter 7 uses the principal component scores from two of the data sets to analyse whether news, including news about the PSBR and debt issues, causes changes in the term structure.[4] The median forecasts produced by City forecasters, which are found to be biased and inefficient are used to create the news effects. Particular attention is paid to whether the forecasts are likely to have been superseded before the Office of National Statistics (ONS) releases the actual data. Little previous work has been produced in this area but significant revisions to forecasts could bias the coefficient estimates in news regressions. The results suggest that once forecasts have been made they are unlikely to be revised. The fiscal terms are not found to produce statistically significant effects on the term structure, a result supportive of the Ricardian position. The principal component scores are used to show how news impacts on a term structure of twelve maturities despite having only estimated three equations for each of the data sets analysed.

Chapter 8 uses the dynamic IS-LM models together with a flow of funds model, surveyed in chapter 2, to estimate a model with an embedded term structure. The model is subjected to a number of simulations and the effects on the term structure are described. The results help explain those of the previous chapter and also why researchers have failed to form a consensus on the effects of fiscal policy

[4] Part of this chapter has subsequently been accepted for publication in the International Journal of Forecasting.

on interest rates. Chapter 9 provides some conclusions, describes some limitations to the research and proposes routes for further research.

Chapter 2

A Survey of Interest Rate Models

2.1 Introduction

This chapter's main aim is to examine the literature concerning the determination of interest rates from a number of theoretical perspectives with an emphasis on the implications, if any, of fiscal policy. In order to make this task manageable a number of self imposed constraints have been applied. Firstly, fiscal policy is restricted to only include the level of taxation, the level of spending, the extent to which any deficit is funded by the sale of gilt-edged securities and the maturity composition of such debt. This definition rules out examination of the taxation of gilts per se. This definition also rules out discussions of the means by which gilts are issued (for example, auctions verses tap stocks, or indexed linked against convertibles)[1]. The reason for this is simply one of confining the thesis to a manageable length. Secondly, although interest rates may be separated, using the Fisher identity, into a real and an inflation component no comprehensive attempt is made to discuss the effect of fiscal policy on the rate of inflation. The reason is again straightforward. The literature is too large to survey and it would take the research into areas, such as the labour market determination of wages, which are far from the core interest of this thesis. Thirdly, the survey is biased towards empiricism and the discussion of empirical results focuses almost entirely upon recent UK work. The reasons are that the UK is the main area of interest, US work has been thoroughly surveyed before and that recent work, through its use of extended data sets, should encompass (or more likely supersede) earlier empirical studies.

The models surveyed are: the expectations hypothesis and associated models of term premia; finance no-arbitrage models; flow of funds models and a macroeconomic perspective. Of course, each perspective borrows from the others and so these distinctions are used as a method of organising the work on the term structure rather than "water tight" theoretical compartments. In keeping with the remainder of the thesis the main emphasis is on empirical aspects of the literature.

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[1] See Breedon and Ganley (1996) for a discussion of these issues with respect to the gilt market.

2.2 The Expectations Hypothesis

The expectations hypothesis can be traced back to Fisher (1896), according to Malkiel (1989) and Shiller (1990), making it at least a century old. In their "pure" forms the hypotheses have zero term premia. However, as Ingersoll (1987) makes clear the pure expectations hypothesis is "not a single theory but a set of related (and often confused) theories".[2] Moreover, the four common versions of the pure expectations hypothesis are incompatible with each other in the sense of predicting different values for the price of a discount bond when future interest rates are uncertain. These four forms (named by Cox et al (1981)) are as follows.

1) The unbiased expectations hypothesis in which the price of a n period pure discount bond, P_t^n, is given as the inverse of the individual expectations of (1 plus) the spot rates, r_i, multiplied together over the appropriate horizon.

$$P_t^n = \frac{1}{(1+r_t)E(1+r_{t+1})\ldots E(1+r_n)} \quad \ldots(2.2.1)$$

This version of the hypothesis is derived from the no arbitrage requirement that future interest rates, f_i, are equal to the corresponding expected spot rates that is:

$$f_i = E(r_i) \ldots (2.2.2)$$

2) The return to maturity expectations hypothesis in which the price of a pure discount bond is given as the inverse of the expected value of (1 plus) the spot rates multiplied together over the appropriate horizon.

$$\frac{1}{P_t^n} = E((1+r_t)(1+r_{t+1})\ldots(1+r_n)) \quad \ldots(2.2.3)$$

This is based on the concept that holding a bond to maturity should equal the expected return on a series of one period bonds held over the same time period. The left hand side of (2.2.3) represents the total return to maturity thus the n^{th} root of this is the (geometric) average one period return, $(1+R_t^n)$.

[2] See Ingersoll (1987) p.389. Following the tradition that does not distinguish between the hypotheses we will describe this approach collectively as the expectations hypothesis.

Forms (2.2.1) and (2.2.3) will only be equivalent if spot rates are uncorrelated so that there are no cross product terms in the denominator of the return to maturity hypothesis or if there is no uncertainty about future interest rates otherwise Jensen's inequality holds.[3] In a continuous time framework (2.2.3) and (2.2.1) are tautological and so only three forms of the expectations hypothesis are recognised in the continuous time literature.[4]

3) The yield to maturity expectations hypothesis in which the price of a pure discount bond is given as the inverse of the expected value of (one plus) the spot rates multiplied together over the appropriate horizon. This is raised to the power of the inverse of the number of periods to maturity and this entire term is raised to the power of the number of periods left to maturity. For ease of exposition we follow Ingersoll (1987) and define:

$$X \equiv ((1 + r_t)(1 + r_{t+1})\ldots(1 + r_n))^{-1} \quad ...(2.2.4)$$

This enables the yield to maturity hypothesis to be expressed simply as:

$$(P_t^n)^{(\frac{-1}{(n-t)})} = E(X^{(\frac{-1}{(n-t)})}) \quad ...(2.2.5)$$

(2.2.5) captures the idea that the expected holding period return on a consecutive series of short-term bonds equals the guaranteed yield from holding a long bond until maturity. The holding period return is the change in the bond price divided by the previous period's price.

4) The local expectations hypothesis (the model usually used in the finance literature) in which the price of a pure discount bond is given as the expected value of the inverse of (1 plus) the spot rates multiplied together over the appropriate horizon. Using (2.2.4) this can be expressed as:

$$P_t^n = E(X) \quad ...(2.2.6)$$

This hypothesis is built upon the notion that the expected return of any bond over a single period is equivalent to the short rate of interest.

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3 Jensen's inequality states that E(G(X))<G(E(X)).

4 See Cox et al. (1981) p.776.

All of these four hypotheses have plausible motivations but Cox et al (1985) argue that they cannot all be correct because the term premia on each hypothesis cannot simultaneously be zero. At any one time, therefore, three of these hypotheses are incompatible with pure zero term premia. Other problems also arise. For the return to maturity and yield to maturity expectations hypotheses the price may be infinite. The yield to maturity, the return to maturity and the unbiased expectations hypotheses can all be shown to give rise to arbitrage opportunities when specified in continuous time, rational equilibrium formulations and are, therefore, invalid equilibrium specifications.[5] Provided that interest rates are always positive then the price of the discount bond is finite under the local expectations hypothesis.

Cox et al's (1985) attack on the expectations hypothesis has been challenged by Campbell (1986), who shows, using a linear approximation on monthly US data, that the differences between the hypotheses outlined above may not be significant. Moreover, McCulloch (1993) provides a counter example to Cox et al. He argues that their results are due to their assumption of a finite number of state variables to describe the state of the economy, rather than an infinite number. McCulloch (1993) conjectures that it may be possible that Cox et al's results will be resurrected for more than N maturities, if bond prices are a function of an N dimensional state vector. Fisher and Gilles (1998) show that this conjecture is false. However, in doing so, they show that the variance of the short rate is infinite, that the non-negativity of short rate cannot be guaranteed and that the forecast for the short rate path is a sine wave with a non-dampening amplitude. All of these features are highly undesirable so that, far from resurrecting the expectations hypothesis, Fisher and Gilles (1998) show the hypotheses to be implausible.

The important point to note is that the hypotheses, in all of their manifestations, have no explicit role for fiscal variables. Only if they alter the expectations of future short-term interest rates will fiscal variables matter in the pure or constant term premia versions of the expectations hypothesis. The manner by which this could occur is outside the realm of the expectations hypothesis. However, possible routes are discussed in the section on IS-LM models below. In an expectations hypothesis that

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[5] See Ingersoll (1987) pp. 399-400.

allows for time varying term premia a further avenue for the influence of fiscal variables is made available but again the mechanisms by which this could occur are outside the scope of the model. These comments may make a review of the expectations hypothesis seem pointless in the context of the aim of this thesis to analyse the effect of fiscal variables on the term structure. However, the expectations hypothesis remains the dominant model of the term structure and the thesis would be incomplete without a review of its current empirical standing. Indeed, if the expectations hypothesis had produced more supportive results it is likely that the remainder of this thesis would be concentrating on the role of fiscal variables in the formation processes of term premia.

Although from a theoretical point of view all of the expectations theories have undesirable properties this has not stopped empirical researchers from creating a voluminous literature dedicated to testing the various versions of the expectations hypothesis. In the next section we survey these empirical results but as Shiller (1990) and Melino (1988) have comprehensively covered these, with particular emphasis on US studies, we limit this survey to studies using UK data from the late 1980s onwards to avoid duplication. The choice of UK studies alone was made principally because the UK is the main focus of study in the latter chapters of this thesis.

2.3 Linear Approximations of the Expectations Hypothesis

One important point to note from the equations (2.2.1) to (2.2.6) is that they are only true for discount bonds. Allowing for coupon payments requires the use of linear approximations and these have been used in UK studies by Taylor (1992), who notes that the approximation also avoids the need to calculate spot rates, and by others (e.g. Cuthbertson and Nitzsche (1993)) for convenience. Shiller (1979) and Shiller et al (1983) derive the following approximation by writing the bond price in terms of coupon payments, c, and yields to maturity, y_t^n, [6]:

$$P_t^n = \frac{c}{y_t^n} + \frac{y_t^n - c}{y_t^n (1 + y_t^n)^n} \quad ...(2.3.1)$$

6 See Allan (1992) pp.250-251 for the derivation of (2.3.1) and for a fuller description of this relationship.

If this is substituted into the formula for the holding period return, H_t^n (capital gain plus coupon as a percentage of the price paid for the gilt), the following expression is derived:

$$H_t^n + 1 = \left(\frac{c}{y_t^{n-1}} + \frac{y_t^{n-1} - c}{y_t^{n-1}(1 + y_t^{n-1})^{n-1}} + c \right) \Bigg/ \left(\frac{c}{y_t^n} + \frac{y_t^n - c}{y_t^n(1 + y_t^n)^n} \right) \quad ...(2.3.2)$$

Using the local expectations hypothesis (that the expected holding period return, H_t^n, equals the one period spot rate plus a term premium) and (2.3.2) a first order non-linear rational expectations model relating y_t^n and y_t can be derived. To avoid the non-linearities a Taylor expansion is taken of (2.3.2) around $y_t^n = y_t^{n-1} = c = \bar{y}$ so that it is truncated after the linear term.[7] This allows a linear relationship between y_t^n and y_{t+k} to be written:

$$y_t^n = \sum_{k=0}^{n-1} g^k (1-g)/(1-g^n)\, E_t(y_{t+k}) \quad ...(2.3.3)$$

Where: g is a constant discount factor, $0 < g < 1$.

E_t is the expectation operator.

The discount factor is associated with a constant discount rate $\bar{y}$ such that $g = (1+\bar{y})^{-1}$. Using Macaulay's (1938) definition of duration, D_i.

$$D_i = (ng^n + \sum_{i=1}^{n} ig^i c)/(g^n + \sum_{i=1}^{n} g^i c) \quad ...(2.3.4)$$

Consequently, using the definition of g and D_i, (2.3.3) is the duration weighted average of future interest rates. Shiller (1979), Shiller et al (1983) and Campbell (1986) all provide correlation coefficients which suggest that the Taylor approximation error on the holding period returns does not appear to be significant.[8] However, Hall and Miles (1992) note that the correlation between the approximation and the true holding period return may be high but the approximation may still be a

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[7] Shiller (1979) pp. 1197-1199 shows that this linear form is also consistent with other forms of the expectations hypothesis.

[8] See Shiller (1979) p.1196, Shiller et al (1983) table 1 p.182, and Campbell (1986) table 1, p.191.

biased measure if the sensitivity of the approximation to changes in interest rates is too great. They tested this upon portfolios of gilts for various maturities by running regressions of the form[9]:

$$h_t^n = \alpha + \beta H_t^n \quad \text{... (2.3.5)}$$

Where: h_t^n is the true n period holding return at time t.

H_t^n is the approximation to the n period holding return at time t given by (2.3.2).

α, β are coefficients and if H_t^n is unbiased they will equal zero and unity respectively.

Hall and Miles (1992) found for UK data during the period January 1985 to March 1989 that β was statistically significantly different from unity (at the 5% level) in half of the 14 regressions undertaken whilst α was statistically significantly greater than zero in eight of the regressions. As β was generally found to be less than unity there is a tendency for the approximation to be more sensitive to changes in interest rates than are the true holding period returns. Hall and Miles results also apply to a number of other countries. Despite the relatively short time period covered, these results suggest that Shiller (1979) and Shiller et al (1983) have been too sanguine about the errors in the approximation. However, whether or not the empirical consequences of this approximation error are significant is not clear and this is an area worth further study. Bearing this caveat in mind, the next section summarises the empirical methodologies used and recent UK studies undertaken on the expectations hypothesis.

2.4 Recent UK Studies of the Expectations Hypothesis

Empirical tests of the expectations hypothesis can be split into six main methodologies of which five are discussed below.[10] These are:

1) Tests of orthogonality or market efficiency conditions. Under the assumptions that expectations are formed rationally and that term premia are constant, the excess returns should be independent of any information that is available at the time the expectations were formed. The excess returns can be

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9 The maturities of the gilts portfolios are (in years) 1-3, 3-5, 5-7, 7-10, 10-15, and 15+.

10 There does not appear to be any published work using UK data that has investigated tests derived from Euler equations and they are not considered further in this thesis.

measured as the difference between the holding period return and the spot rate; the difference between the long interest rate and the weighted average short rates; or by the difference between the forward rate and the spot rate. These measures are not the same because of Jensen's inequality.[11]. The orthogonality tests are joint hypotheses about the constancy of term premia and the rationality of forecasts. Moreover, such tests cannot prove orthogonality because they cannot rule out the possibility that another subset of variables is not orthogonal to the excess returns. The best result that can be gained is, therefore, non-rejection of the expectations hypothesis. Tests of these attributes are applied to our data sets in chapter 7.

MacDonald and Macmillan (1992) side step the joint hypotheses problem by using expectations of UK three month interbank bid rates in three month's time collected from 26 economic and financial forecasters by Consensus Economics. The time period covered is October 1987 to October 1991 and the data are collected monthly. As the frequency of data collection exceeds that of the forecast data MacDonald and Macmillan (1992) correct the coefficient covariance matrix, estimated by ordinary least squares estimates, by using Hansen's (1982) generalised method of moments (GMM). MacDonald and Macmillan (1992) also allow for heteroscedasticity in the model's errors.

MacDonald and Macmillan (1992) regress the difference between the forward rate and the appropriate spot rate (a measure of the excess return or forward premium) on the difference between the forward rate and the current spot rate. They find, under the assumption of rational expectations, that most of the variation of the forward premium can be accounted for by expected interest rate changes. On the other hand, using pooled survey data to measure expectations (i.e. using all 26 forecasters separately) the results suggest that the major (60%) source of variation in the forward premium is time varying term premia. However, these results appear to be dependent upon the use of the pooled data set that decreases the standard errors of the parameters by increasing the sample size. When rational expectations are assumed market efficiency cannot be rejected. However, when the survey expectations are summarised by using the mean or the median of the individual forecasts market efficiency is

11 See the discussion in section 2.2.

rejected by the sample. The reason is that the survey expectations are not rational because five out of the 26 individual forecasters appear to take insufficient attention to the information contained within the current forward rates. This is sufficient to make the mean of the survey forecasts irrational and accounts for the different conclusion depending upon whether rational or survey expectations are used. Overall, MacDonald and Macmillan's (1992) results reject the pure expectations hypothesis and cast doubt upon the weaker constant term premia version for the UK. However, it should be noted that these results are limited in the range of interest rates and the time period covered (just four years).

2) Variance bounds tests that were first applied by Shiller (1979). From the return to maturity form of the expectations hypothesis (2.2.3), long term interest rates are averages of expected short term interest rates. Under the assumptions that expectations are formed rationally and that term premia are constant, long term interest rates have smaller variances than the averages of the short term interest rates. This is because the expectation errors are independent of the long term interest rates and the variance of two independent variables is simply the sum of their variances. Hence, the variance of the averages of short term interest rates equals the variance of the long term interest rates plus the (non-negative) variance of the expectations errors. Similar variance bounds can be placed on the holding period returns and short term interest rates.

Variance bounds tests have found that the long rate is too variable relative to the average of the short rates.[12] However, it is not clear whether this result derives from the rational expectations versions of the expectations hypothesis being incorrect or a weakness in the testing procedure. Flavin (1983), using Monte Carlo simulations, shows that, if the short rate nearly has a unit root, in small samples the sample variance of short rates may be downward biased and this may account for long rates appearing to be too variable. The reason is that with a near unit root short term interest rates will show high persistence and, hence, deviations from the sample mean are smaller than deviations around the population mean. All that can be concluded from variance bounds test is that from the observed sample the results do not support the rational expectations theory of the term structure. Flavin's critique appears to have caused

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[12] See, for example, Shiller (1979).

researchers using UK data to abandon this line of testing, although it is still undertaken in the context of the VAR analysis outlined below. Green (1991) reports that returns on UK three month local authority rates varied between 18.5% and 4.6% whereas annualised three month returns on ten year gilts varied between -14.7% and 36.4% using monthly data over the period 1972 to 1985. This result is inconsistent with the expectations hypothesis.

3) Single equation regression tests. These use the consequence of the expectations hypothesis that the long interest rate is (up to a linear approximation) equal to the average of the expected short rates to set up testable hypotheses. Although (2.3.1) could provide the regression equation as it stands concerns that the interest rates are non-stationary cause researchers to transform the variables into weakly stationary variables so that traditional statistical tests can be applied. These stationary variables are usually the spread between a long and a short interest rate and the change in the short interest rates. As Campbell and Shiller (1991) note, the spread should predict the change in short term interest rates over the maturity of the long term bond. A positive spread implies that short term rates are expected to rise. This causes a capital loss for long term bond holders and to ensure that returns are equalised between long and short term bonds the yield on long term bonds must be higher than short term bonds. Hence the spread is positive. The term "long" bond is used for exposition purposes. In many single equation tests the maturity of the long bond is simply twice that of the short bond (see, for example, Driffill et al (1993)) so that the change in interest rates is simply the change in short term rates. Tests using long maturities that are twice the short term maturity are popular because the mathematics is conveniently simplified by this choice. The test can be performed by running a regression of the change in interest rates on a constant and the spread. In the two period case for discount bonds the regression would be:

$$r_{t+1}^m - r_t^m = \beta(2(r_t^n - r_t^m)) + \theta \quad ...(2.4.1)$$

Where r_t^n is the two period interest rate.

r_t^m is the one period rate.

6 is the constant term premia.

The β coefficient on the spread should be unity if the long bond has twice the maturity of the short bond.[13] It should be noted that this equation imposes rational expectations and replaces the expected long rate by its' ex post value. Two factors have to be taken into consideration during estimation of (2.4.1). Firstly, overlapping interest rates induce an (m-n-1) moving average process into the error term that has to be allowed for before inferences can be drawn.[14] Secondly, if the term premia, θ, is random then it and the spread are correlated and estimation of β by OLS will be biased and inconsistent. Thus instrumental variables estimation has to be used.

These procedures can produce a myriad of combinations of interest rates that can be used to test expectations models of the term structure. The results of US studies find that the regression of the change in interest rates on the spread results in parameters that are of insufficient magnitude and often the wrong sign (negative). These results are summarised in Campbell and Shiller (1991). These and the more recent results of Campbell (1995) are in accordance with the finding that longer rates tend to fall when the spread is positive. Roberds and Whiteman (1996) describe the decline in the values of the coefficients, with respect to the longer maturity, as a "smirk". This result appears to have a long pedigree with Macaulay (1938) observing a similar pattern to movements in US long rates and the spread. However, as Campbell's results show, the standard error on these estimates is often so large that the hypothesis of no relationship cannot be ruled out. This orthogonality result has received support from Ayres and Barry (1979, 1980) and Steeley (1989).

Cuthbertson and Nitzsche (1993) use regressions of the form (2.4.1) to test the implications of the rational expectations term structure using weekly UK Certificates of Deposits (CD) data for the period October 1975 to October 1992 using maturities of 4, 13, 26, 39 and 52 weeks. In contrast to the US studies, Cuthbertson and Nitzsche (1993) find that they cannot reject the null hypothesis that the change in interest rates is as predicted by the expectations hypothesis (positive) and that the parameter on the spread variable is unity. However, the standard error of the estimates is so large that neither can the

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13 In general the coefficient will be given by (m/(n-m)), where m is the maturity of the short bond and n is the maturity of the long bond.

14 This is another reason why m=2 and n=1 is an attractive combination of interest rates to study.

hypothesis that this parameter is zero be ruled out. Nevertheless, Cuthbertson and Nitzsche's results are more supportive to the expectations hypothesis than many others. They attribute this to the use of a high quality data set that avoids the problems of approximating spot yields or applying approximations on yields to maturity. The CD market is also very liquid and this, the authors claim, may make the recorded rates match the trading rates more closely than in less liquid markets.

The second type of regression test uses what Campbell and Shiller (1991) call the "perfect foresight spread". This is the spread which the expectations theory would give if there was perfect foresight, that is investors never made errors in their forecasts about future interest rates and is defined as follows:

$$S_t^{(n,m)} = (1/k)\sum_{i=1}^{k-1}(\sum_{j=1}^{i}\Delta^m r_{t+jm}^m) \quad ...(2.4.2)$$

Where: $S_t^{(n,m)} = r_t^n - r_t^m$

k=n/m

$\Delta^m r_{t+m}^m = r_{t+m}^m - r_t^m$

If short term interest rates are expected to rise over the lifetime of the long bond (i.e. the change terms are positive) then the long term interest rate has to be higher than the current short term rate. This is to equalise the returns on the long term bond and a sequence of short term bonds. This can be tested by using the perfect foresight spread as the dependent variable and a constant and the actual spread as independent variables in a regression. Again, provided that the term premia are constant, the slope coefficient should be unity.

US data also suggest that, although the expectations theory is often rejected, the perfect foresight regressions give forecasts in the correct direction for the changes in the short rate. Roberds and Whiteman (1996) describe the resulting parameter estimates as the "predictability smile". When the maturity of the long bond is three months or less short term rates generally move as predicted by the expectations hypothesis. For maturities between three months and two years short rates react insufficiently to the spread, whilst for long rates of two years or more the spread again predicts movements in short rates. Thus the parameter estimates take on a U-shape or "smile".

Cuthbertson and Nitzsche (1993), Hurn et al (1993) and Cuthbertson (1996) test the perfect foresight regressions. The former used that same data set as discussed above, whilst Hurn et al (1993) used month end one, three, six and twelve month middle Libor rates over the period January 1975 to December 1991 and Cuthbertson (1996) used weekly (Thursday) Libor rates for January 1981 to February 1992. All of these researchers find that they cannot reject the hypothesis that the parameter on the spread is unity, which is in accordance with the expectations hypothesis. Moreover, Cuthbertson and Nitzsche (1993), using four lags on the spread and the short rate of interest, and Cuthbertson (1996), using five lags on the spread and short rates, find that these variables are not statistically significant. Consequently, in these studies all the information on the perfect foresight spread appears to be captured by the actual spread.

Frachot and Lesne (1993) and Roberds and Whiteman (1996), using the Cox et al (1985) model, are able to explain the "predictability smile" and the "smirk" for the US studies. Frachot and Lesne (1993) show that if interest rates are deterministic then long term interest rates are an average of expected short rates. If interest rates are stochastic but their variances are deterministic, then long term interest rates equal the average of expected short rates plus a term premium. In both these case regressions based on (2.4.1) and (2.4.2) would support the expectations hypothesis. However, if both interest rates and their variances are stochastic, then long term interest rates are averages of expected short rates multiplied by a function, C(m-n), plus a term premium. The important point is that C(m-n) is a function of the maturity difference between the short and the long interest rates (as well as the constant mean reversion parameter and a constant that describes how stochastic the variances of interest rates actually are). Roberds and Whiteman (1996) show that, when the regression is set up in the form of (2.4.2), stochastic variances in the Cox et al model will reproduce the right hand side of the "predictability smile" as the long interest rate lengthens in maturity. They show that this is true for any parameters of the Cox et al model. However, in order to achieve parameters close to unity for short maturities the market price of risk, i.e. the covariances of changes in interest rates and the market portfolio, must be small. These same conditions will explain the "smirk" results generated from regressions of the form (2.4.1).

Frachot and Lesne (1993) applied their analysis to explaining the results of Campbell and Shiller (1991) and Fama (1984). Roberds and Whiteman (1996), on the other hand, estimate their own version of (2.4.1) and (2.4.2) and attempt to use the analysis to explain the pattern of coefficients. Both papers calibrate the model of Cox et al by searching over the parameters to find those combinations that minimise the sum of squared deviations of the actual parameters from the theoretical parameters. Thus the criterion on which to judge the success of this approach is not whether the actual and theoretical are close. Rather it is whether or not the parameters of the implied Cox et al model are plausible. Unfortunately, the calibrated coefficients are not plausible. Frachot and Lesne (1993) report a parameter that in six out of the seven calibrations is above its theoretical maximum of unity. Roberds and Whiteman (1996) themselves point out that the parameters of their calibrated Cox et al model are not close to those obtained by other researchers. Moreover, their calibrated parameters cannot explain part of the "smile" and "smirk" at longer maturities and this result also applies to a two-factor model.

Despite the indifferent empirical results, this approach offers a compelling explanation for the failure of the expectations hypothesis and further work is clearly warranted. Unfortunately, to date, the approach of Frachot and Lesne (1993) and Roberds and Whiteman (1996) has not been applied to the UK results, especially to those associated with Cuthbertson that are more supportive of the expectations hypothesis. In order to be consistent Cuthbertson's results would have to imply that the conditional variance of short term interest rates is not stochastic. Examination of this would, clearly, be an area worth further study.

4) Vector autoregression (VAR) tests were first proposed by Campbell and Shiller (1987) in response to two weaknesses with the single equation methodology discussed above. These are that the single equation models have to use GMM to correct for overlapping forecast errors that induce moving average processes. These do not work well if the degree of overlap is large relative to the sample size as the results of Monte Carlo simulations performed by Campbell and Shiller (1991) demonstrate. As an example of the size of this problem it can be noted that Cuthbertson and Nitzsche's (1993) use of

annual data (52 week) reduced their number of truly independent observations to just 18 despite using weekly data.[15]

The second problem with the single equation regression tests is that they do not provide a detailed picture of the shape of the yield curve. The VAR approach allows a forecast of the changes in short interest rates to be made over any horizon and, consequently, the behaviour of long rates can be inferred. The VAR methodology also has the advantages that alternative measures (to R^2) of the expectations theory's ability to predict the data can be derived and that a form of volatility tests can also be conducted within this framework.[16] The VAR approach assumes that the change in the short rate is a stationary process. It follows, therefore, that the spread is also a stationary process and there exists a bivariate Wold representation of these variables that may be arbitrarily approximated by a VAR. In essence the test of the expectations hypothesis involves using a Wald test on the non-linear restrictions imposed upon the VAR by equations (2.4.1) above with the unrestricted VAR.[17]

The Wald test on the VAR restrictions has had mixed results on UK data. Cuthbertson and Nitzsche (1993) reject the restrictions in four of their five VARs and Cuthbertson (1996) in four of his eight VARs, whilst Hurn et al (1993) cannot reject the restrictions in any of the six VARs they estimated. It is not clear whether the differences in the results are due to different sample frequencies, different time periods covered or the different compounding conventions used. A further possibility is that the researchers are testing different restrictions although as the Hurn et al paper is very sketchy on the restrictions that are being tested this explanation is untestable.[18] Taylor (1992), using three month UK Treasury bills and redemption yields on 5, 10, 15, and 20 year gilts, also rejects the restrictions for each of his four VARs.

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[15] It should be noted that a GMM correction on the standard errors is also used within the VAR system.

[16] See Campbell and Shiller (1987) pp.1068-1070.

[17] A simple example of the restrictions is given by Driffill et al (1993).

[18] Driffill et al (1993) show that a number of different restrictions can be applied and these can alter whether or not the restrictions are rejected by the Wald tests. See Driffill et al (1993) pp.12-14.

Using the parameters of the VAR the "theoretical spread", which is a weighted average of forecast interest rate changes, can be computed.[19] The difference between the actual spread and the "theoretical spread" is a measure of the discounted sum of expected term premia conditional on the information contained in lagged values of the spread and the change in short term interest rates.[20] The use of the present value formula adds a measure of persistence of term premia (alongside the variability of term premia) in evaluating the success of expectations hypothesis. The relationship between the spreads is often presented graphically but both Cuthbertson and Nitzsche (1993) and Hurn et al (1993) report that the correlation between the theoretical spread and the actual spread is very high (never less than 0.97). Moreover, the standard errors are small enough to suggest that this result is very robust and that the hypothesis that the correlation is unity cannot be rejected.

An alternative method of analysing the spreads is to calculate the ratio of their standard deviations. If the standard deviation of the spread is larger than that of the theoretical spread then the spread is too volatile relative to expected information about short term interest rates. Neither Cuthbertson and Nitzsche (1993) nor Hurn et al (1993) found evidence of excess volatility, although in the former paper this result is due to the standard errors being large rather than the point estimates being close to unity. On the other hand, Cuthbertson (1996) found excess volatility in three of his eight tests. MacDonald and Speight (1988) found excess volatility for 5, 10, and 20 year gilts compared with Treasury bill rates, over the period from the first quarter of 1963 to the first quarter of 1987. Their ratios of the actual innovation in the spread to the forecast innovation only indicated excess volatility for five year gilts. Over a longer data period 1952 to 1988 using quarterly data Mills (1991) found evidence of excess volatility for five and twenty year gilts and the 3 1/2% war loan, although for the five year gilt the result was marginal. Mills (1991) also found that his results were sensitive to the data period used, with the later period 1972-1988 displaying greater excess volatility than the period 1952-1971. Overall we conclude that the variance ratio tests, whilst probably of low power, are not overly supportive of the expectations hypothesis.

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[19] See Campbell and Shiller (1987) pp.1068-1069 and Campbell and Shiller (1991) p.10 for a derivation of this measure.
[20] See Campbell and Shiller (1991) p.10.

5) The expectations hypothesis can also be examined using cointegration tests associated with Engle and Granger (1987). These are based on the single equation tests discussed above and are a precursor to these tests in the sense that if long and short rates are not cointegrated then the expectations hypothesis is false. If a variable is integrated of order 1, denoted I(1), then it has to be differenced once to produce a stationary variable that is integrated of order 0, I(0). If a linear combination of two or more variables can be formed such that this combination is I(0) then the variables are described as being cointegrated. Taylor (1992) used weekly data for UK three month Treasury bills and the redemption yields on 5, 10, 15 and 20 year gilts over the period January 1985 to November 1989. He reports that all of these yields appear to be unit root series, whereas the spread between the gilts and the Treasury bill rate appears to be stationary using Phillips-Perron tests for unit roots. MacDonald and Speight (1988), Mills (1991) and Cuthbertson (1996) also find that their interest rate series are I(1) whilst the spreads are I(0). However, because their data sets overlap these two studies do not represent independent collaboration. This is a particular problem in well-researched areas such as the expectations hypothesis. Despite this quibble, the cointegration tests are consistent with the expectations hypothesis but they could, of course, be consistent with other models of interest rates.

2.5 Time Varying Term Premia

Overall, the UK results reported above are not very supportive of the pure expectations hypothesis. Given the theoretical results of Fisher and Gilles (1993), Green's (1991) comment that the expectations theory of the term structure is "an invaluable expositional tool but constituting a dead end as far as research aimed at understanding interest rates is concerned" seems correct.[21] In many cases it is not clear whether the expectations failure is due to the assumption of rational expectations being incorrect or due to non-constant term premia. If term premia are non-constant but predictable an augmented expectations hypothesis could provide an explanation of movements in interest rates and this section discusses some recent work on explaining term premia.

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21 Green (1991) pp.132-133.

Early specifications of risk premia tended to be ad hoc. Hicks (1939) postulated the liquidity premium hypothesis in which the term premia rose as the maturity of the debt increased. Thus in equilibrium the term structure would slope upwards. Market segmentation or preferred habitat theories (associated with Modigliani and Sutch (1967)) propose that investors have preferences about the length of debt they hold to ensure that maturing debt matches their own expenditure profile. The result of this is that to entice investors away from their habitats a premium has to be offered but this need not be linked in a monotonic manner to maturity length. Under this view term premia do not have a deterministic sign. Both the liquidity preference and market segmentation theories would allow the premia to be non-constant. They could depend, for example, on the terms of bank lending, short term interest rates, inflation, GDP growth and developments in the stock market. All of these could rapidly change liquidity in the market, or change relative asset supplies, which could change the viability of preferred habitats or the liquidity of segments of the term structure. More rigorous work on general equilibrium models of the term structure, e.g. Cox et al (1985), also find that the term premia can vary over time and will be influenced by a number of variables including changes in the outstanding debt maturity profile.

Yet even in these formal models there is a large element of ad hoc choices being made about the actual determinants of term premia. Sill (1993) uses the intertemporal capital asset pricing model (ICAPM) to relate the conditional (on information available up to the current period) expected excess return on US Treasury bills to the conditional covariance of the asset with the benchmark return. Sill assumes that the excess of the benchmark return over the risk free rate has a linear factor structure. Consequently, excess Treasury bill returns can be expressed as a function of the covariance of the idiosyncratic changes between the benchmark portfolio and the Treasury bill rates, which is assumed to be constant in Sill's empirical work, and the variances and covariances of the factors. The variances and covariances are estimated by a GARCH(1,1) model and are thus time varying. Despite the impressive formulation the factors, industrial production growth, inflation and a bond default premium, are arbitrarily chosen and, consequently, without these being tied down this line of research is little more than an advanced form of data mining.

Against such a background empirical researchers have a host of explanatory variables to choose from in their attempts to explain term premia. However, little progress has been made in narrowing down these variables to the core determinants of term premia. Taylor (1992) reports that the lagged variance term in a GARCH(1,1) model is not statistically significant on UK data so that there is no persistence in term premia. This result suggests that finding a stable relationship with macroeconomic variables may be difficult, as these are often highly persistent even in growth rate terms. Hall and Miles (1992) examined a portfolio of UK gilts over the period January 1985 to March 1989. Unlike Taylor (1992) they found that term premia as measured by a GARCH model was persistent in a statistical sense but that for the whole portfolio such term premia were not statistically significant. Similar conclusions could be reached when the portfolios were split into maturity bands and term premia, measured by the covariance of the innovation terms which were justified by capital asset pricing model (CAPM) considerations, were allowed to enter the GARCH model. The exception to this result was the one to three year gilt portfolio where both variance and covariance terms appear as statistically significant term premia. These results can be rationalised by noting that there is no return to investors for holding diversifiable risk and the variance of the gilt portfolios may measures only diversifiable risk. Furthermore, the risk measured by the covariance of gilt portfolios can also be diversified. Hence there is no return to this measure either. A better measure of non-diversifiable risk would contain covariances with the innovations on portfolios containing equities and foreign bonds and real capital.

There have been mixed results from using debt variables to explain term premia. Goodhart and Gowland (1977) fail to find any effects whilst Taylor (1992) does. However, Taylor's equation is misspecified as it implies that a permanent change in the proportion of asset supplies in a given maturity class will reduce/increase the excess holding period return over the short rate. This implies that for all future periods the prices of long bonds must continue to rise or fall. This opens up an arbitrage possibility and it is not clear what stops arbitrage. Alternatively, and this seems to be what Taylor had in mind, at the start of the period the gilt price falls so that the running yield rises to the required level and prices stay at this new level so that no capital gain is made. In general, however, gilt issues will not be made on the first day of each week so that this implies the price moves on the expectation of the issue of gilts. Again this means that there is an arbitrage possibility unless expectations are only formed with total certainty on the first day of each week. This seems an unappealing model. We are not therefore convinced that Taylor's results are particularly robust.

Rather like research on the pure expectations hypothesis, work on time varying term premia has thrown up conflicting results. Given the ad hoc nature of the empirical work it is not clear that this line of research will produce convincing answers to bolster the expectations hypothesis in the near term. Indeed, Frachot and Lesne (1993) and Roberds and Whiteman (1996) show that finance models can explain some of the failures of the single regression tests of the expectations hypothesis and we describe these models in the next section.

2.6 Finance Models of Interest Rate Determination

The second strand of the literature exploring the determination of interest rates may be termed the "finance or arbitrage" literature. Hull (1993) distinguishes between models in which the term structure is endogenous and models in which the term structure is exogenous. As the primary aim of the thesis is to examine the determinants of the term structure models where the term structure is exogenous, such as Ho and Lee (1986) and Hull and White (1990), are not discussed.

As Pagan et al (1995) remark there is little overlap between these models and those examined under the expectations literature described in the above section. Many of the models (but not all) within the finance literature can be nested within a relatively simple stochastic diffusion process. The variants of this process are often simply stated as the starting point for analysis. At first glance, this gives the impression that the model has been chosen simply because of its analytical and empirical tractability and its ability to meet the boundary conditions of the interest rate process.[22] This is not the case. The models are derived from a number of plausible economic assumptions, including a no-arbitrage condition, with the only arbitrary feature being the specification of the stochastic diffusion process. As such this procedure is similar to that in mainstream economics where an equation may be rigorously derived but the empirical implementation has a number of ad hoc features (linear vs. log-linear, etc.).

22 A rather dire example of this tendency is Chance (1994).

In the following paragraphs the bond pricing equation is specified, the interest rate path is specified and the implications of the model are discussed. The various different assumptions on the stochastic diffusion process are outlined and recent empirical works on these types of models are discussed. It should be emphasised that these models are designed to value contingent claims and Ho and Lee (1986) and Heath et al (1992) are central to this aim. Due to this, the ability to explain developments in the term structure is not the over riding concern of finance models, unlike the expectations hypothesis reviewed earlier and the IS-LM and flow of funds models reviewed later in this chapter. Nevertheless, the work of Frachot and Lesne (1993) and Roberds and Whiteman (1996) (as discussed in section 2.4 above) do offer an explanation of the failure of single equation regressions to conform to the expectations hypothesis. Although there are some clear empirical "winners" amongst the finance specifications, the literature is still developing rapidly so that, as Campbell et al (1994) emphasise, no one model has emerged as the consensus choice.

Finance models begin by postulating that the short term interest rate (often the instantaneous rate), r, is determined by a stochastic differential equation. We follow Brown and Dybvig (1986) in specifying a particular diffusion process, that of Cox et al (1985), but the derivation of the bond pricing equation makes it simple to substitute others.[23]

The diffusion process is given by:

$$dr = k(\theta - r)dt + \sigma\sqrt{r}dx \quad ...(2.6.1)$$

Where: dr is the change in the instantaneous interest rate.

dt is a small change in time.

σ is a standard deviation term.

$k(\theta - r)$ is a drift term.

dx is a Wiener process with a zero mean and variance dt.

23 Other derivations are possible, see, for example, Wilmott et al (1995).

The square of the diffusion process (2.6.1) can be written to leading order as:

$$(dr)^2 = \sigma^2 r dt \quad ...(2.6.2)$$

This is because for very small increments in time $(dt)^2 \to 0$, $dxdt \to 0$ and $(dx)^2 \to dt$.

Let the price, P, of a riskless, zero coupon discount bond, in period, t, maturing in period T, be designated by:

$$P(r,t,T) \quad ...(2.6.3)$$

Using Ito's Lemma the instantaneous rate of return on the bond is given by:

$$dP/P = (P_r dr + 1/2\, P_{rr}(dr)^2 + P_t dt)/P \quad ...(2.6.4)$$

Where: dp is the change in price

P_r is the partial derivative with respect to interest rates.

P_{rr} is the second partial derivative with respect to interest rates.

P_t is the partial derivative with respect to time.

By substituting (2.6.1) and (2.6.2) into (2.6.4) and rearranging the rate of return can be expressed as:

$$dP/P = \left[\left(K(\theta - r)P_r/P\right) + P_t/P + 1/2\,\sigma^2 P_{rr}/P\right]dt + \sigma\sqrt{r}\, P_r/P\, dx \quad ...(2.6.5)$$

We can summarise (2.6.5) more succinctly as:

$$dP / P = \mu(r,t,T)dt + v(r,t,T)dx \quad ...(2.6.6)$$

In an efficient market, arbitrage ensures that the expected instantaneous return on the bond is equal to the instantaneous risk free rate plus a risk premium. As this is a single factor model there is only one source of noise in the economy and this is given by $v(r,t,T)$. As noted above the expectation of dx is zero and therefore the expected return from (2.6.6) is simply given by $\mu(r,t,T)$. Hence:

$$\mu(r,t,T) = r + \lambda^*(r,t)\, v(r,t,T) \quad ...(2.6.7)$$

Where: $\lambda^{*}(r,t)$ is the premium paid to investors for accepting one unit of risk.

Brown and Dybvig (1986) assume that

$$\lambda^{*}(r,t) = \lambda\sqrt{r}/\sigma \quad ...(2.6.8)$$

By substituting (2.6.8) and the appropriate parts of (2.6.5) into (2.6.7) and rearranging we find that:

$$k(\theta - r)P_r + P_t + \frac{1}{2}\sigma^2 r P_{rr} = rP + \lambda r P_r \quad ...(2.6.9)$$

Equation (2.6.9) has the terminal condition $P(r,T,T) = 1.0$ and this allows an explicit solution to the pricing equation to be written as:

$$P(r,t,T) = A(t,T)e^{-B(t,T)r} \quad ...(2.6.10)$$

The parameters $A(t,T)$ and $B(t,T)$ are determined by the time to maturity and the coefficients $k, \lambda, 6$ and σ^2. If other functional forms for the equations (2.6.1) or (2.6.8) had been chosen then an explicit solution may not have been available and numerical solution methods would have been required.

Equations of the form of (2.6.1) have been suggested that have a range of attractive properties. For example, short term interest rates can be made to remain positive or bounded by a positive value and the short rate can be made to be mean reverting. The specification (2.6.10) results in the long term rate of interest tending towards a constant that is independent of the short rate of interest as the bond's maturity is increased. Moreover, if r is mean reverting, an increase in r will result in the slope of the term structure declining and the change in the slope and r are perfectly correlated.

From (2.6.10) the short rate determines the level of long term interest rates and the long and short rates are perfectly correlated. The reason for this is that the pricing equation is driven by a single state variable, in this case short term interest rates. Perfect correlation is clearly at odds with the empirical evidence and so the important question is not whether finance models can explain bond prices but what proportion of the variance in prices they can explain. It should also be noted that unlike the expectations literature, reviewed in the previous section, the term structure is driven only by the instantaneous rate not explicitly by changes in expectations of future rates. It could be argued, however, that time varying parameters, given a constant instantaneous interest rate, do constitute changes in expectations about the processes driving future interest rates. The difference between the models may not be as stark as it first seems. It should also be obvious that there is no role for fiscal policy in these single factor models.

Most of the empirical work has focused on single factor models with time invariant parameters. Table 2.6.1 tabulates various parameter values that have been suggested for the process given by (2.6.1).

Table 2.6.1 Parameter Values for the Model

$$dr = \delta(\theta(t) - \varpi(t)r^{\nu})dt + \sigma(t)r^{\gamma}dX$$

Authors	Parameter values	Constant parameter values
Merton (1973) and Ingersoll (1987)	$dr = \alpha dt + \sigma dX$	Yes
Vasicek (1977)	$dr = \delta(6 - r)dt + \sigma dX$	Yes
Cox et al (1985) (CIR)	$dr = \delta(\theta - r)dt + \sigma\sqrt{r}dX$	Yes
Geometric Brownian Motion	$dr = \beta rdt + \sigma rdX$	Yes
Dothan (1978)	$dr = \sigma rdX$	Yes
Brennan and Schwartz (1979)	$dr = \delta(6 - r)dt + \sigma rdX$	Yes
Constant elasticity of variance	$dr = \beta rdt + \sigma r^{\gamma}dX$	Yes
Hull and White (1990)	$dr = (\theta(t) - \varpi(t)r)dt + \sigma(t)r^{\gamma}dX$	No

Note: this table is by no means exhaustive and other models can be fitted into this framework. See Chen (1996), p.3. If ν=1 and 6=0 then $\beta = -\delta\omega$. If ν=0 then $\alpha = \delta(6 - \omega)$.

The single factor, constant parameter models have been subjected to extensive testing see, for example, Chan et al (1992) and Dahlquist (1994). The overall results indicate that none of the above models is preferred by the data. Chan et al, using US data, find little evidence of mean reversion (although this is hard to estimate from time series data) and a parameter on the levels term in volatility, γ, well in excess of unity. The former result is also found by Pagan et al (1995) and by El-Jahel et al (1996) for a

Vasicek model but not for a Cox et al square root model. For the Cox et al model the mean reversion is found to be implausibly fast as they imply that shocks are removed from the system within a couple of weeks.[24] The size of the volatility term depends upon the estimation procedure used. If GMM is used then the levels parameter is in excess of unity whilst if the simulation estimation procedure of Gourieoux et al (1993) is used they are below unity in Pagan et al's study. El-Jahel et al also found a parameter in excess of unity on US one, six and twelve month rates and for one month UK rates. Dahlquist's (1994) parameter estimates, using Swedish and Danish data, cannot rule out either the Cox et al model or the Brennan-Schwartz formulation. Campbell et al (1994) and Pagan et al (1995) both note that these parameter estimates are difficult to rationalise. Pagan et al (1995) state that "the predictions from CIR type models are therefore diametrically opposed to the data", whilst Campbell et al (1994) conclude that single factor models are "too restrictive to fit nominal interest data".[25]

Consequently, work has developed upon a further generalisation of the single factor models to encompass multiple factors. It is assumed that n state variables all follow continuous time diffusion processes and there are n+1 traded securities that are dependent on up to n of the state variables. Using a generalisation of Ito's Lemma to several stochastic variables and the same arguments as employed above, a second order differential equation can be derived which makes the price dependent upon the growth rates of the states of nature and the correlation of the volatilities of the states of nature. Two factor models include: Brennan and Schwartz (1979), who use short and long rates, Fong and Vasicek (1991) and Longstaff and Schwartz (1992), who use a short rate and its volatility and Brown and Schaefer (1994), who use the long rate and the spread between the short and the long rate. Three factor models include Dillen (1994) who uses a model including a World interest rate, a real exchange rate and the inflation rate and Chen (1996) who uses the current short rate, the mean of the short rate and the current volatility of the short rate. Duffie and Kan (1994 and 1996) define and analyse an n factor model of the term structure although they note that empirical work suggests that only two or three factors are needed in practice. There is no reason why an index of fiscal policy cannot be entered as one of the factors, however, Duffie and Kan (1994) argue that to facilitate the pricing and hedging of

24 El-Jahel (1996) p.19 and table 3, p.20.
25 Pagan et al (1995) p. 21 and Campbell et al (1994) p.31.

derivatives it is more convenient to assume that the n factors are interest rates. Despite the extra flexibility that multi-factors have over single factor models a number of empirical problems remain. For example, Brown and Schaefer (1994) report that their model of the long rate for the UK gilt market is mean averting rather than mean reverting, whilst Steeley (1989) fails to find any statistically significant parameters in his long rate process.

2.6.1 Hybrid Finance-Econometric Models

The problem of mean aversion and poorly defined parameters have led to further experimentation within the single factor framework, although it is doubtful that these extensions can be used to value contingent claims. These experiments are hybrids between the finance and the applied econometrics literature and whilst strictly in neither camp are treated as a subsections of the finance literature for expositional purposes in this thesis. The first of these experiments, due to Steeley (1990), is to generalise the mean reversion process by adding further lagged terms in the differences of short interest rates in the spirit of Hendry (see Davidson et al (1978)). Unfortunately, this approach does not improve the explanatory power of the equations, which often record R^2 of less than 10% using UK data. Furthermore, the procedure means that certain combinations of coefficients can violate the boundary condition of positive interest rates.

Another approach is to generalise the volatility process using forms of GARCH models[26]. Pagan et al (1995), Brenner et al (1994) and Koedijk et al (1993) have undertaken work of this type. All these papers found a reduction in the coefficient on volatility but, as the paper by Pagan et al shows, they do not necessarily remove the absence of mean reversion. Indeed in the Pagan et al paper interest rates appear to be mean averting using an EGARCH model although the parameters, though statistically significant, are quantitatively small[27]. Bianchi et al (1997) using a semi-parametric method found that the level of volatility was not monotonic in the level of interest rates. Essentially their model estimates a relatively general diffusion model. The model's squared errors are smoothed against the level of interest rates and are used to proxy the variance. Their procedure can be thought of as a relation to

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26 Steeley (1990) uses a GARCH model but does not allow for a level effect.
27 Pagan et al (1995) table 4, p.8.

GARCH models. They found that the volatility relationship changed after the UK's ejection from the ERM. Neither Vasicek nor Cox et al models would be able to account for this feature of the volatility.[28]

Second, the models have been applied to real as opposed to nominal interest rates. Whilst Brown and Schaefer (1994a) find evidence of a levels term in volatility, Evans et al (1992) find no evidence of heteroscedasticity. Although Evans et al find evidence of mean reversion, the mean to which the model reverts is zero. Although one of the attractive features of using real interest rates is that the boundary condition of positive interest rates need not apply, it seems implausible to believe that the mean real interest rate was zero. This is because the data used are for the UK over the period 1870 to 1975, i.e. a period which excludes much of the high inflation 1970s and 1980s when real interest rates were significantly negative. Consequently, it seems likely that the model is misspecified, possibly through Evans et al (1992) assumption that inflation expectations are simply the previous year's inflation rate. An alternative view is that what makes the finance literature a useful way of thinking about interest rates is that boundary conditions are imposed thus using real rates where one of the conditions is missing makes this approach less useful.

A third avenue of research is to note that the monetary authorities determine short term interest rates. This means that the short term interest rate does not so much diffuse as jump as the monetary authorities react to shocks. Moreover, the interest rates ratchet upwards or downwards for some periods rather than attaining a maximum (or minimum) then returning to the long run mean. This causes persistence in short term interest rates. Changes in the institutional arrangements can change the speed at which interest rates return to their long term levels. Thus taking account of monetary policy arrangements may improve the fit of these models. An example of this can be found in the estimates of Bianchi et al (1997) on UK two year bonds for periods pre and post the UK's exit from the ERM in 16 September 1992. Prior to this date the mean reverting interest rate was 10.3% and after this date it was 6.9% to 6.3% depending upon the model estimated.[29] Whilst both the GARCH, the jump processes and

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28 See Bianchi et al (1997) figure 3, p.13.

29 Bianchi et al (1997) table 2, p.10. For other work on jump processes see El-Jahel et al. (1996a).

perhaps to a lesser extent the real interest rate routes are interesting avenues of research, the little evidence available so far does not point to them as being unambiguously promising.

2.7 Flow of Funds Models

The third set of models of interest rate determination may be termed as the flow of funds models. These models range from crude, ad hoc single equation models through to multi-equation models of the entire financial system built (at least in theory if not always in practice) from optimising behaviour of participants in the markets. There is a range of models between these extremes.

Under certain assumptions the portfolio balance approach provides a straightforward expression for the yields on the assets in the investor's portfolio. Assume that the investor wishes to maximise their expected end of period utility from their wealth, U(W). This is subject to the constraint that the return on wealth is given by the return on the vector of (n x 1) assets in the portfolio, r, weighted by their share in the portfolio, the (n x 1) vector, α. Using the assumption that transaction costs are zero and the utility function exhibits constant relative risk aversion,[30] i.e.:

$$-WU''(W)/U'(W) = c \quad ...(2.7.1)$$

Where: c is a positive constant.

U'' is the second derivative.

U' is the first derivative.

The problem is to:

$$\max_{\alpha_i} \; U = \sum_{i=1}^{n} \alpha_i r_i^e - \frac{c}{2}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1} \alpha_i \alpha_j \sigma_{ij} \quad ...(2.7.2)$$

Subject to:

$$\sum_{i=1}^{n} \alpha_i = 1 \quad ...(2.7.3)$$

30 This implies that the investor is less averse to risky projects the greater the investor's wealth. See Layard and Walters (1978) pp.360-361 for a derivation of (2.7.1).

Where: σ_{ij} is the covariance of returns between the i^{th} and the j^{th} assets.

Friedman and Roley (1987) show that the single period optimal shares of each asset in total wealth, α, is given by:

$$\alpha = \frac{1}{c} B r^e + A \quad ...(2.7.4)$$

Where: r^e is the expected vector of returns, (n x 1).

B is an (n x n) matrix given by: $B = \Omega^{-1} - (d'\Omega^{-1}d)^{-1}\Omega^{-1}dd'\Omega^{-1}$

d is an (n x 1) vector of ones (unit vector).

Ω is the covariance matrix of assets returns.

A is an (n x 1) vector given by: $A = (d'\Omega^{-1}d)^{-1}\Omega^{-1}d$

(2.7.4) implies that asset demands are linear in expected returns and homogenous in wealth. If we treat the supplies of assets as exogenous and assume that expectations are rational, i.e. the actual return equals the expected return plus a mean zero, independently distributed vector of random errors, ε, then (2.7.4) can be inverted to give an expression for the endogenous asset returns:

$$r = B^{-1}c(\alpha - A) + \varepsilon \quad ...(2.7.5)$$

As (2.7.5) uses the inverse of B the response of returns to changes in asset supplies is a complicated function of the return's covariances and little in general can be said about the consequences of changes in asset supplies. In a small four asset model (money, short debt, long debt and equity) the following assumptions are required before simple analytical results can be derived. First, the return on one asset (say) money is fixed. Second, all the assets are gross substitutes in the portfolio. Third, assets closer in maturity are closer substitutes than those that are further away in maturity. It can be then shown that the effects of increasing the supply of short assets, whilst reducing long assets to keep wealth unchanged, will result in a rise in the return on the short asset and a fall in the returns on the long asset and equity. If one is not prepared to make such strong assumptions about substitutability then the signs of the effects of an open market operation described above becomes an empirical matter.

Equation (2.7.5) can be estimated by regression techniques provided that the number of observations exceeds (n-1). It is (n-1) and not n because one asset has to be dropped due to linear dependence of the asset shares through the adding up constraint. Frankel (1992) reports that such regressions "have always been very imprecise and often implausible in sign or magnitude" and Friedman (1992) describes his own results as "nonsensical" when he included non-financial wealth.[31] An alternative has been to directly parameterise the model using the observed covariance matrix of asset returns. Unfortunately, there is a wide range of estimates for the constant relative risk aversion parameter, c, so that the size of the return responses with respect to asset supplies can easily be doubled or halved. Indeed Agell and Persson (1992) more or less suggest that this constant be made up to suit the priors of the investigator.[32] Blake (1995) using data from a cross section of UK wealth holders finds that the constant relative risk aversion parameter, c, varies between 47.6 for the poorest households to 7.88 for the richest households with a weighted sample mean of 35.04. This suggests, if these results were repeated in the rest of the economy, that the effects of changing asset supplies on rates of return would be large, ceteris paribus. Green (1988), on the other hand, provides estimates using UK monthly data for July 1972 to November 1977 that are negative at -141.1 for a model with adjustment costs and -101.9 in a model without such costs. A negative finding is inconsistent with relative risk aversion and probably indicates that other restrictions in Green's models do not hold. In particular the adoption of rational expectations may be suspect as Green's data period includes the first post-war period of very high inflation rates.

There are a number of other problems with flow of funds models that also besets regression analysis. These include the choice of assets; the degree of temporal aggregation; the sample period and the method of extracting the unpredictable component of asset returns from the total return to calculate the covariance matrix. It is the unpredictable returns that are important because if all returns were predictable there would be no risk and no need to build portfolios of assets to optimise the risk-return trade-off. This leads to a further problem. If predictable returns are excluded (such as coupon payments on gilts) then the return varies mainly due to changes in prices of gilts and other instruments.

31 Frankel (1992) p.83 and Friedman (1992) p.100.

32 See Agell and Persson (1992) p.37.

In turn these price changes alter the value of wealth and hence asset shares (unless all prices move by the same amount in which case the asset shares would not alter). Consequently, the asset shares have to be treated as endogenous, as do the elements of the return covariance matrix, unless it is assumed that the changes in returns are achieved by changes in future asset prices leaving current asset prices unchanged.

These problems were confronted by Agell and Persson (1992) who demonstrate, using US data, that these considerations are empirically very important. A comparison of the return covariances and the conditional covariances found that using a four-quarter VAR resulted in a marked fall in the covariances.[33] However, the qualitative results, that an increase in long term bonds and a matching reduction in short term bonds raises the relative return on equity, remained unchanged. Agell and Persson (1992) used a moving sample vector autoregressive model to extract the unpredictable elements of real returns and to allow for these perceptions of risk to be time varying. They found that the numerical values were highly volatile.[34] Honohan (1980) in his study of UK life assurance companies also used a Bayesian approach to update his covariance matrix. When the experiment was repeated using monthly rather than quarterly data the responses, as measured by the change in returns following a change in asset supplies, are larger and have a different pattern over time.[35] If, instead of using historic asset returns to estimate the conditional covariance matrix, option data are used the results become even more volatile.[36]

The only area where changes did not seem to make much difference was when the prices of the assets were made endogenous. Agell and Persson approach the problem of endogenous prices by either allowing the current price to change or allowing the next period's price to change but not both. The results noted above all assumed that the current price remained constant and all adjustment was made

[33] See Agell and Persson (1992) table 4.1 p.40 and table 4.3 p.44. The period covered was the first quarter 1960 to the second quarter of 1988.
[34] See Agell and Persson (1992) figures 4.2a and 4.2b p.46. The period covered was the first quarter 1970 to the second quarter of 1988.
[35] See Agell and Persson (1992) figures 4.9a and 4.9b p.58.
[36] See Agell and Persson (1992) figures 5.1a and 5.1b p.62. The data period used was the final quarter of 1985 to the second quarter of 1988.

by the next period's price. With endogenous current prices and current wealth the covariance matrix of returns and the asset supplies (because all values of assets (except the numeraire) change when the volume of one asset changes) are all different. Allowing for these effects muddies the waters as to the magnitude of the effects of changing asset supplies on returns, even in simple three asset models including a risk free asset. Using a VAR model to predict future expected returns and to calculate the covariance matrix of returns Agell and Persson (1992) do not find that endogenous prices have any significant empirical effects on the model's properties.[37] However, Frankel (1992) claims that this is a simple product of the method Agell and Persson (1992) used.

The mean variance model discussed above imposes so much structure that the parameters need not be estimated statistically. The volatility of the parameter estimates suggests that the model's tractability may have been gained by the sacrifice of empirical cohesion. Yet empirical works which have allowed for richer dynamic structures have not covered themselves with glory. In a series of papers, Barr and Cuthbertson (1989, 1990, 1990a and 1990b) review previous attempts to estimate asset demand functions. They conclude that for the personal sector "such attempts have often yielded results that conflict with the chosen theoretical model or intuitive a priori views".[38] For the overseas sector "empirical results ... can only be described as 'mixed'".[39] They pass similar comments to these on asset demand studies of other financial institutions (OFIs) and banks.[40]

Honohan tests various models using a constant elasticity of substitution utility function instead of a constant relative risk aversion. Honohan (1980) notes that changing the effective sample size through the Bayesian forgetfulness parameter altered the variances in a manner not susceptible to useful summary. In general the larger the sample size the better were the parameter estimates.[41] Nevertheless, he describes his own work as unsuccessful.[42] In one of his models the parameter estimates suggest that returns do not enter the utility function only risk, and in another formulation wrong signs

37 See Agell and Persson (1992) Figures 6.1a and 6.1b p.77.

38 Barr and Cuthbertson (1989) p.2.

39 Barr and Cuthbertson (1990) p.2.

40 Barr and Cuthbertson (1990a) p.1 and Barr and Cuthbertson (1990b) p.4.

41 Honohan (1980) footnote 12 page 27.

42 Honohan (1980) p.25.

predominate.[43] Honohan (1980) concludes that these results suggest that either the mean variance approach is a severe misspecification or that errors in the data may have distorted the results.[44]

Keating (1985) describes his work on the financial side of the London Business School model. This is a monumental work covering nine sectors of the economy and thirteen assets. In fact it's sheer size and the compromises this forces upon the model both via estimation, because of software limitations, and on usability, because of the needs to run it simultaneously with the real side of the LBS model, led to numerous problems. Not least of these problems was that although Keating claimed his model was a modified version of Parkin's (1970) mean-variance framework he imposes the assumption that all the covariances of asset returns are zero thus negating a large part of the foundations upon which he claims his model is built. Moreover, as Keating assumes that banks and building societies price their deposits as fixed mark-ups on the bill-market rate the correlation between the returns on these assets should be unity.

Keating's modifications are to allow for adjustment costs of altering the portfolio, which are independent among assets because of the zero covariance assumption, and an explicit allowance for rationing of certain assets during the estimation period. Keating's claim of reinterpretation the covariance matrix to allow for the non-return utility derived from holding riskless assets seems to play no part in the estimation of his model. The model deflates the wealth variables by the GDP deflator but then omits to define the returns in real terms or allow the level and variability of inflation to enter the model. The empirical results suffer some of the problems mentioned in connection with Honohan's (1980) work. Unrestricted estimation resulted in coefficients which were theoretically unacceptable and either had values imposed, for example the adjustment cost parameters for the gilts equations for the personal sector and pension funds, whilst some parameters were set to zero without statistical testing. Courakis (1988) points out that the remaining parameters are not subjected to any extensive empirical testing and are intuitively implausible. For the personal sector time deposits are often the most costly assets to adjust, and sight deposits are regarded as the most risky asset. In the company

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43 Honohan (1980) p.28.

44 Honohan (1980) p.29.

sector equities are regarded as the least risky according to Keating's estimates. One reason for this may be that the iterative process used in estimation appears to have been stopped before the global solution was found. Thus the residual sum of squares for the personal sector equations are still falling by 6% between iterations when the search was terminated.[45] All in all, Keating's work well deserves Courakis' (1988) description of it as an "Alice in Wonderland story".[46]

Barr and Cuthbertson's work moves away from the mean variance approach and builds asset demand functions based on the "Almost-Ideal Demand System" (AIDS) of Deaton and Muellbauer (1980). Investors minimise the costs of achieving a given level of utility by altering the asset shares in their portfolios. This results in equilibrium asset shares being determined by the prices of all assets (the inverse of the expected real rate of return) and the investor's real wealth. In the short run there are costs of achieving this equilibrium. Barr and Cuthbertson (1989b, for example) use a generalisation of the standard quadratic costs function to take into account the costs of adjusting other assets in the portfolio. They are then able to write the change in the asset shares as a change in the long run equilibrium demand plus (n-1) disequilibrium terms, lagged one period, in the other asset demands. (A disequilibrium term drops out because the sum of the disequilibria must be zero.) As Barr and Cuthbertson make clear they do not believe that costs can be characterised as quadratic and they prefer to think of their specification as being a straightforward error correction mechanism. In this respect the contrast with the work of Keating (1985) could not be starker. Barr and Cuthbertson do not provide any discussion of the aggregation problems that may occur in moving from the demand functions of individual investors to estimating demand equations on a sector wide basis. They simply instrument the returns and the wealth terms to avoid this problem without any discussion of instrument suitability. Nor do they provide any evidence that their assumption of separability, made to make the estimation procedure tractable by limiting the number of alternative returns that need be considered, is correct.

Despite extensive attempts to purge data errors there are a number of empirical criticisms that can be made of this series of papers. In part these stem from the desire to test both long run restrictions on the

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[45] Keating (1985) p.100.
[46] Courakis (1988) p.625.

parameters and use some of the techniques in cointegration theory. Unfortunately, most of the series in Barr and Cuthbertson's papers are I(1) so that the standard tests of statistical significance are not valid in this framework. Nevertheless, Barr and Cuthbertson, who were aware of the problem, place considerable weight on these results.[47] One reason for this problem is that, at the time, although the problem was understood techniques were not available to surmount them.

Setting this difficulty aside and concentrating on the results for long gilt holdings, there a numerous difficulties with these equations. For the company sector Barr and Cuthbertson (1989b) find that only real wealth determines long gilt holding in the long term, relative rates of return and the own rate are statistically insignificant. In the short term only the lagged disequilibrium in gilt holdings drive changes in gilt holdings but the Box Pierce statistic suggests that the equation is misspecified.[48] A demand function driven only by wealth seems intuitively unappealing. For the overseas sector the long run equation for gilts fails the Dickey Fuller test for stationary residuals and in the long run alternative returns play no statistically significant part in determining the asset shares.[49] For the OFI sector in the long run only the own rate and the rate on hire purchase lending enter the demand for gilts equation and there is no wealth term only time trends.[50] In the short run equation there are no returns variables only changes in the time trend and the disequilibrium form the long-run equation for company securities. In particular there is no own long term disequilibrium term although the system is stable.[51] For UK banks, unless the coefficients on returns in the company securities equation are imposed, only the foreign currency lending rate and real wealth are statistically significant.[52] In the short run equation none of the terms are statistically significant.[53] For the personal sector Barr and Cuthbertson's (1989a) results suggest that when symmetry is imposed on the long run return variables the error correction

47 Barr and Cuthbertson (1990a) p.9.
48 Barr and Cuthbertson (1989) table 2, p16.
49 Barr and Cuthbertson (1990) p.11 and table 2, p13.
50 Barr and Cuthbertson (1990) table 2, p.10.
51 Barr and Cuthbertson (1990a) table 3, p.11.
52 Barr and Cuthbertson (1990b) table 3a, p.22.
53 Barr and Cuthbertson (1990b) table 3b, p.23.

terms, including the disequilbrium term from its own long-run solution, are statistically insignificant. Moreover, the equation suffers from misspecification on the basis of a Ljung-Box statistic.[54]

These results do not provide much confidence that the gilt asset demand equations have been discovered. If the statistically insignificant terms were set to zero this would imply that when a single return remained this too would have to be zero in order to maintain the adding constraint. Alternatively it might be argued that because, say, the own rate was statistically significant the other rates have to be retained otherwise a statistically significant term has to be dropped. In this respect it is disappointing that Barr and Cuthbertson did not indulge in some experimentation to see what would happen to the parameter estimates. Finally, although Barr and Cuthbertson claim that they have estimated demand equations, there is no discussion about how they have identified this from the supply of securities. Indeed, as they do not model the government sector or the supply of company sector securities the model is incomplete and one is left with the impression that these variables simply adjust passively to the demands for these assets. Whilst a vast improvement on the work of Keating (1985) there remain too many problems with the system approach to make it an attractive option for modelling interest rates.

2.8 The Static IS-LM Models

It is sometimes alleged that a close relative of the IS-LM approach is the loanable funds approach, which can be traced back to the writings of J. S. Mill, Hume and Ricardo.[55] However, as Patinkin notes "no logical significance can be attached to any distinction between these two analytical frameworks".[56] Through Walras' law they are simply manifestations of the same general equilibrium framework. The same conclusions on interest rates would be reached whether the market for bonds was substituted for the money market or, indeed, if the markets for money, short and long bonds were all included in the analysis. For this reason the loanable funds approach does not require separate analysis from that conducted in this section. This section analyses the IS-LM model including wealth effects as initially described by Christ (1968). Ricardian equivalence is ignored until a latter section.

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54 Barr and Cuthbertson (1989a) table 9, p.34.

55 See Patinkin (1965) p.366.

56 See Patinkin (1965) p.377.

In a closed economy the IS-LM model works as follows. An increase in government expenditure, funded by an increase in government debt, raises demand in the economy both directly and because the extra wealth in the form of government debt stimulates consumers' expenditure. Accelerator effects may also encourage greater investment and stock building. With the money supply assumed to be fixed, the extra demand for money to accompany extra transactions, raises the rate of interest. This effect will be increased if higher wealth raises the demand for money for portfolio investment purposes. The process will reach equilibrium when the increase in GDP is sufficient to increase taxes by an amount equal to the increase in government expenditure so that the issuance of debt ceases. Even this simple model has the potential for instability if the increase in money demand raises interest rates sufficiently to crowd out other components of expenditure so that output actually falls and the deficit and debt rise over time.[57]

Whether unstable or not, the effect of an increase in government expenditure is to raise the nominal rate of interest. The new equilibrium will see a balanced budget, higher government spending and higher debt together with higher interest rates. Thus the same level of the deficit can be associated with different levels of interest rates. Indeed, through the balanced budget multiplier an increase in government spending which is matched by a rise in taxes, so that no new debt is issued, will also raise interest rates. This is because the increase in taxes will be paid for, in part, from savings, and, therefore, the increase in government spending will be greater than the decline in consumers' expenditure. Consequently, activity will rise, pushing up interest rates provided that the money supply remains fixed. The message from the simple IS-LM model is that provided the money supply is fixed the rate of interest will rise following an increase in government expenditure. Moreover, a rise in interest rates can be accompanied by an increase in government spending whether or not the budget deficit and the level of debt rise. Thus models of interest rates require a measure of the money supply, government spending, and the level of debt (assuming that the marginal tax rate remains constant) rather than just the level of the deficit.

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57 The probability of instability would be greatly increased if the government indulged in open market operations to exchange bonds for money.

The endogenity of the budget deficit can also lead to difficulties in interpreting its effects on interest rates. Suppose that the private sector wishes to increase their capital stock and finances the increase by issuing bonds. The increase in investment expenditure raises activity and from an initial budget balance causes the government's budget to register a surplus. With the money supply assumed to be fixed, higher investment spending raises interest rates to equilibrate the money market. Hence, budget surpluses are associated with higher interest rates and, as the government retires debt and the resultant wealth effects reduce activity, interest rates will be seen to fall alongside a deterioration in the budget surplus. Neither government spending nor the marginal tax rates have changed in this example only the budget surplus/deficit. Hence, empirical models of interest rates that include budget deficits can produce misleading results. A similar result could be obtained by looking at the asset demand and supply equations. If private sector bond stocks were erroneously ignored a fall in the government bond stock could be associated with rising interest rates because of demand substitution into newly issued private debt. The issuance of private sector debt has been incorporated into the debt terms used in chapter 7 to study news effects on the term structure.

One important distinction examined by Barro (1987) is that between permanent and transitory changes in government expenditure. In a closed economy model where consumption is determined by the Euler equation, the marginal utility of consumption depends upon last period's marginal utility multiplied by the ratio of real interest rates to the rate of time preference. Assuming that there is no growth or increase in population, then consumers' expenditure is constant over time in this economy. Furthermore, the real rate of interest equals the rate of time preference. If there is an unexpected increase in government expenditure that is expected to be permanent this causes consumption to be crowded out and to remain permanently lower. As consumers' expenditure remains constant at this lower level the rate of interest remains equal to the rate of time preference so that permanent changes in government spending do not alter interest rates. If the increase in government expenditure is expected to be only temporary then consumers' expenditure is again expected to rise. As the Euler equation is only driven by unexpected events and the previous period's consumption this can only occur if real interest rates temporarily rise above the rate of time preference when government expenditure falls.

Hence, there is a need to distinguish between permanent and temporary changes in government expenditure.

Furthermore, the long term interest rate, being a weighted average of future short term rates, rises when government spending rises and remains higher throughout the period of high government spending before falling as government spending falls. Hence, the behaviour of long and short term interest rates also differ for a transitory increase in government spending. Barro (1987) showed that UK yields on consols and other perpetuities over the period 1730 to 1913 was positively and statistically significantly related to temporary government spending. He also showed that it was positively statistically significantly related to both the deficit to GDP ratio and the debt to GDP ratio. When the government spending, deficit and debt terms were all entered they became statistically insignificant because of colinearity of the terms. Consequently, there is no means of telling from this regression whether the UK did exhibit Ricardian equivalence over this extended period. Barro's work again shows the problems of omitted variable biases in single equation studies of interest rates and this problem makes many single equation models, e.g. Nunes-Correia and Stemitsiotis (1993), unreliable.

One problem with the IS-LM model is its comparative statics methodology. As the increase in government debt depends upon the time taken to move to the new equilibrium, IS-LM models are silent on the quantity of new debt eventually created. If the new equilibrium requires a large increase in GDP (because the marginal tax rate is low) which is accomplished quickly to bring the government's deficit back to equilibrium, the debt to GDP ratio could be lower in the new equilibrium than in the initial equilibrium. Consequently, higher interest rates could be associated with either higher or lower debt to GDP ratios. Thus researchers who claim that IS-LM models predict that a higher debt to GDP ratio will result in higher interest rates are mistaken. All the model predicts is that higher nominal (and real) debt will be associated with higher interest rates. The static nature of this model means that it is unsuitable for empirical implementation and indeed there are no current UK macro models that could be described in these terms.[58]

58 For a defense of IS-LM see Patinkin (1990).

Moreover, the prediction that higher government spending causes an increase in interest rates does not necessarily pass through into an open-economy IS-LM model. In an open economy there are, in principal, two regimes, a fixed exchange rate and floating exchange rate, which could be studied. However, the fixed exchange rate case implies an "equilibrium" when the government's deficit is just matched by an inflow of foreign capital. For there to be inflows the rest of the World has to run, in aggregate, a government budget surplus. Hence this "equilibrium" occurs with a continuing change in the composition of wealth holdings and this can only be sustained if government debt and foreign government debt are perfect substitutes. In this case domestic interest rates cannot deviate from the World interest rate and a fiscal expansion has no effect on interest rates or on activity. If domestic government debt and foreign government debt are not perfect substitutes then at some point the rest of the World will be no longer prepared to hold more debt without an increase in the return or they may not be prepared to hold more debt at all. Under these circumstances the "equilibrium" is just temporary and, therefore, not equilibrium at all.

If the exchange rate is floating and capital is perfectly mobile the rate of interest is determined again by the World interest rate, irrespective of changes in the government's deficit and debt. All that an increase in government spending does is crowd out an equal amount of exports, leaving activity unchanged, by appreciating the real exchange rate. However, if exchange rate expectations are formed regressively, that is that the exchange rate was expected to return to it's original level, foreign investors would require to be compensated for their expected capital loss (uncovered interest parity, UIP) and domestic interest rates would rise above the World rate.[59] Even with perfectly mobile capital it does not necessarily follow that interest rates will be invariant to changes in government spending and debt. However, regressive expectations on their own raise questions about why investors hold expectations which are consistently proved to be false and seem an unlikely expectations process for investors to hold give that an exogenous variable, government spending, has changed. Expectations that the exchange rate will depreciate along an equilibrium path after a change in government spending, i.e.

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59 For models of this form see Dornbusch (1976).

rational expectations, are more appealing and these are discussed further below. Moreover, these models retain a basic problem because if the budget is in deficit the stock of debt rises without limit and, eventually, no more debt will be accepted into the World's bond portfolio. Arguments about the country being small and hence unable to alter significantly the composition of the World's asset portfolio is simply a delaying tactic to hide the model's short term nature and its unsuitability for analysis of this form.

2.9 Dynamic IS-LM Models

There are, however, models that allow a small open economy's interest rates to deviate from the World rate without assuming regressive expectations. These models, associated with papers by Blanchard (1981), Turnovsky and Miller (1984) and Turnovsky (1986) amongst others, are relatively simple IS-LM formulations but even so they produce results about the dynamics of interest rates which were missing from the IS-LM models discussed above. In particular, they allow for both a short and a long rate of interest in their models. Allowing either the price level, GDP or government liabilities to adjust only slowly to their long-run equilibrium through the use of a continuous time framework produces the dynamics. The IS-LM models discussed above could only compare comparative static results and hence the interesting dynamics are lost or have to be put together in ad hoc stories.

Each of the models is slightly different form.[60] Blanchard (1981) includes the price of equities in a closed economy, Turnovsky and Miller (1984) have no equity effects but include the government's budget constraint in a closed economy, whilst Turnovsky (1986) has an open economy without a budget constraint and without equity prices. Despite these differences all these models produce three interesting results. First, the behaviour of long and short interest rates can differ following a change in fiscal policy and, second, that the behaviour of interest rates will differ depending upon whether or not the change in fiscal policy is anticipated. Third, to clear markets short term interest rates may be required to jump to new values.

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60 A discrete time version of these models is presented in Chapter 8 where it is estimated and so these models are not formally presented here.

Despite their simple stylised forms these models produce behaviour that is dependent upon the models' parameters. This is especially true when the price level is allowed to vary and the presence of inflation causes uncertainty about the real value of long term interest rates. In these models the real long rate is the return on a perpetuity, the price of which can be calculated from the real cash flows divided by the real discount rates. The Turnovsky (1986) model can illustrate this path indeterminacy. Although a distinguishing feature of this model is the use of long-term rates in the aggregate demand function, in the following exposition we suppress this, as it is not necessary to derive the results. It is also assumed that the transaction demand for money is related to aggregate supply rather than the actual level of aggregate demand. This enables questions of stability to be partly resolved. Furthermore, the choice of nominal as opposed to real short term interest rates in the demand for money function also alleviates further possibilities of instability.

For an unanticipated fiscal expansion the exchange rate appreciates to crowd out the extra demand. As inflation is determined by the output gap, the difference between aggregate supply (assumed to be exogenous) and aggregate demand, prices are unaffected. With no change in output and prices, there is no effect on either real or nominal (long or short) interest rates which are determined by exogenous World rates through uncovered interest rate parity and the condition that the holding period return on perpetuities has to equal the short rate.

In the case of an anticipated fiscal expansion the results are less clear cut. The anticipation of expansion causes the exchange rate to appreciate in advance of the increase in government expenditure. This is because in the long run the exchange rate must appreciate to crowd out the increase in aggregate demand so that it equals aggregate supply. It does not jump to its long run solution. If it did so the money market would be unable to clear because the short term interest rate would remain constrained at the World level. With an appreciation of the (real) exchange rate, demand falls below aggregate supply, deflation sets in and this requires that short term interest rates fall to equilibrate the increase in the real money stock with demand. Holding domestic short term instruments will, therefore, only be acceptable if the exchange rate continues to appreciate and it will only do this if it is below its long-run value. The appreciation of the currency is determined by the uncovered interest rate parity and this, in turn, is determined by the elasticity of money demand with respect to nominal interest rates. If the elasticity is large the decline in short term interest rates needs to be relatively small and, hence, the exchange rate can be close to its long-run level. However, the greater the appreciation of the exchange rate the larger will be the output gap and hence the greater the deflation. Consequently, real short term rates are likely to rise immediately following the anticipation of a fiscal expansion if the elasticity of the demand for money is relatively large (in Turnovsky's (1986) model an elasticity in excess of one is required).

Over time, the fall in output causes prices to fall further and to clear the money market nominal interest rates have to fall further. The decline in nominal short term interest rates only ceases once the expansion of fiscal policy has occurred. The path of real short run rates is less clear. If the exchange rate appreciates at a greater rate than the price level falls, activity falls further and the rate of deflation increases so that short term real rates rise during the period before the fiscal expansion commences. Alternatively, if the exchange rate appreciates at a slower rate than the price level falls, the fall in activity will be moderated and so will the deflation. Consequently, the real short rate of interest will decline until the fiscal expansion takes place both because the deflation rate moderates and because of lower nominal short term rates. Depending upon the model's parameters, the real short term interest rate just before the fiscal expansion is implemented may be higher or lower than before the fiscal expansion was anticipated.

This story is complicated by the behaviour of the real long term interest rate. As it is assumed that the holding period return on the perpetuity is equivalent to the short rate of interest the (real) long rate is simply the cash flows discounted by the (real) short term rate of interest. Consequently, the long rate reflects the short rate from both the period before and the period after the fiscal expansion. Hence to know what happens to the long rate the behaviour of the short rate after the fiscal expansion has to be understood.

Following the fiscal expansion demand is increased and this must be sufficient to cause the price level to rise reducing the real money stock. Consequently, the nominal short term interest rate begins to rise, back towards the World short term rate. If demand did not rise above aggregate supply when the fiscal expansion occurred, because there was excessive crowding out, then the price level would continue to fall, as would nominal interest rates, and to ensure uncovered interest parity the exchange rate would have to continue appreciating leading to further deflation. Hence, the system would be unstable. Ruling this out assumes that the rise in the real exchange rate, in anticipation of the fiscal expansion, only partly crowds out the increase in government expenditure. Thus, when the fiscal expansion occurs, demand exceeds aggregate supply and prices begin to rise, raising nominal interest rates.

To offset the excess demand the real exchange rate has to appreciate further and eventually all the increase in demand will be crowded out. This causes inflation to cease and, in equilibrium, the exchange rate also stops appreciating. Nominal short rates have returned to the World level of interest to ensure that uncovered interest parity is maintained. If aggregate demand and short term interest rates have returned to their original level and the money supply has remained fixed then the price level as well must have returned to its original level to ensure equilibrium in the money markets. Consequently, nominal short term interest rates fall in anticipation of a fiscal expansion and rise back towards their initial level after the fiscal expansion. Long-term rates, being a geometric average of future short term rates, follow the same pattern, but the deviation from long run equilibrium is smaller because of the forward-looking element. The actual degree of deviation will increase as the overall level of short term interest rises because future coupon payments will be discounted more heavily and hence long-term interest rates will give increased weight to the near future.

At the time of the fiscal expansion real short term interest rates jump downwards because deflation has been replaced by inflation and, consequently, real short term interest rates fall from above nominal rates to below nominal rates. As the inflation rate falls the real interest rate rises but until equilibrium is reached remains below the nominal short rate of interest. The real long rate of interest following the fiscal expansion will be above the real short rate and will rise back towards the World short term nominal rate, which is its equilibrium level. Its behaviour in the pre-expansion period, however, is dependent upon the model's parameters. If short rates are low, so that the future plays a large role in determining current rates, then upon news of the fiscal expansion the real long term rate may fall (whereas the real short rate initially rose), decline for some time, then begin to rise before the fiscal expansion occurs. At that time of the expansion real long rates will still be below their equilibrium level. An alternative, when the future is heavily discounted, is that long real rates initially rise with short rates, but not by as much, decline until the fiscal expansion at which time they begin to rise again. In this case there is no jump in interest rates at the time of the expansion because the discounting procedure smoothes this away. Finally, if real short rates rose during the period prior to the fiscal expansion, long real rates may also rise, although at some stage they will begin to decline as lower real short rates, following the fiscal expansion, will pull the long real rate below its equilibrium level. Hence, real long rates will fall below their equilibrium level for at least some of the time prior to a fiscal expansion.

From this relatively simple IS-LM model the following effects on the term structure from a fiscal expansion can be discerned. The level of the nominal term structure falls upon news of a future fiscal expansion with the curve inverting so that short rates are above long-term rates (assuming that the structure was initially flat). Depending upon how much future short term rates are discounted, nominal long rates may begin to rise prior to the fiscal expansion and at this time the inversion will have disappeared and the term structure will steepen. At that time the level of interest rates will still be below their equilibrium level but, with short rates falling and long rates rising, it is not clear how to characterise the overall level relative to a period just after the announcement was made. Once the fiscal expansion has occurred the level of interest rates begins to rise with the steepening of the term structure slowly disappearing as both long and short rates return to the World short rate of interest.

In real terms the short term rate of interest rate jumps upwards on announcement of the fiscal expansion but the long rate may either rise or fall, leaving the level of real interest rates ambiguous. Moreover, the real term structure can either steepen or invert. If it steepens short term real interest rates must rise during the period prior to the fiscal expansion and short term rates must be high to discount lower post expansion real rates. The steepening will be reduced over time and, prior to the fiscal expansion, the term structure will invert. This inversion will be eliminated at the time of the fiscal expansion by the jump downward in real short rates (caused by the change from deflation to inflation) so that the term structure slopes upwards. Over time the term structure will flatten. In these circumstances the level of interest rates initially rises upon the news, becomes ambiguous at some point prior to the expansion, falls at the time of the fiscal expansion and rises thereafter back to the World rate of interest.

If the term structure inverts upon the announcement the level of interest rates is ambiguous because the real long rate can either rise or fall. Prior to the fiscal expansion the term structure may flatten and then steepen or the steepening may have to await the fiscal expansion but at some stage, either prior to or after the fiscal expansion, the level of interest rates will be below the World equilibrium level.

Upon news of the fiscal expansion long term nominal rates initially fall, reflecting the future decline in short term rates, then rise above short term rates as the higher short term rates, after the fiscal expansion, are discounted less as time passes. The time at which long term rates rise above short term rates depends upon the level of short term rates. If these are low then the long term rate will rise above short term rates at an earlier time than if short term rates are high. Both short term and long term nominal rates will asymptotically return towards the nominal World rate of interest.

What these results highlight is that even relatively simple dynamic IS-LM models can produce complex behaviour in interest rates. Moreover, the results can be different depending upon the whether researchers focus on long or short, real or nominal rates and post-announcement or post-implementation behaviour. Only in the post-implementation period for nominal rates does this model produce the behaviour that most empirical workers seem to be testing for, i.e. an increase in fiscal policy causes interest rates to rise. Even in this case, however, fiscal policy has lowered nominal rates relative to World rates. The results also depend upon the level of interest rates themselves so that, given these change over time as World rates change, it is not perhaps surprising that research on interest rates based upon IS-LM models have produced differing results.

Furthermore, these predictions do not spill over into the closed economy IS-LM models of Blanchard (1981) and Turnovsky and Miller (1984). Without being tied in the long-run to return to the World rate of interest a fiscal expansion will result in both short and long rates being higher. In the period immediately following the expansion long rates will increase by more than short rates, so that the term structure steepens, but this steepening will moderate over time as interest rates asymptotically approach their new equilibrium. Essentially, the mechanism is that increased demand following the fiscal expansion requires a rise in short term interest rates to offset the increase in money demand and higher short term rates, both in the present and the future, raise long term rates.

In the Blanchard model (1981) there are again a multiplicity of interest rate paths from the anticipation of a fiscal expansions but these are simpler than in the Turnovsky (1986) model. The novel feature of Blanchard's model is that equity prices determine wealth, which, in turn, influences aggregate demand. The holding period return on equities and long bonds are the same as the short rate of interest through arbitrage considerations. Higher output in the future will raise profits but this will be offset by higher interest rates so that the current value of equities may rise or fall depending upon which effect is larger. If the value of equities falls this will decrease aggregate demand and short rates will also have to fall to ensure that the money market clears. If equity prices rise, short rates also rise to clear the money market. In both cases the long term rate of interest rises. This is obvious in the case where short term rates rise. However, in the case of a fall in short term rates long term rates rise. This is because the level of interest rates is such that future higher short term rates (after the fiscal expansion) dominate the effect of lower short term rates during the period between the announcement and the implementation of the fiscal expansion. Suppose that interest rates were high; so that the future was totally discounted, share prices would not fall and there would be no need for a reduction in short term interest rates. Short term interest rates only fall if the future is important but if the future is important long term rates must rise because future short term rates are expected to rise. Consequently, long term interest rates always rise in Blanchard's model. The result is that an anticipated future fiscal expansion may have ambiguous effects on the level of nominal interest rates, although eventually after implementation they will be higher. It will also cause the term structure to steepen and this steepening may either increase or decrease until the implementation of fiscal policy. After the implementation the term structure always begins to flatten out.

The model of Turnovsky and Miller (1984) emphasises the adjustments of money and bonds to ensure that the government's budget constraint is met with standard effects on the money and goods market equilibria. An anticipated increase in government expenditure causes the long rate to jump upwards because future short rates are correctly expected to be higher in the long-run. This reduces activity and to ensure equilibrium in the money market short rates have to immediately fall. Hence, the term structure steepens. Lower activity (because of higher long term rates) results in a government deficit that is financed by an increased supply of bonds. This raises the demand for money and the level of activity in the goods market. We will assume that this adjustment is stable, for expositional purposes, and that government bonds are net wealth. The increase in the demand for money, with the real money stock held constant, causes short term interest rates to rise over the pre-implementation period as do long rates (because of the increase in short rates). If bond issuance was sufficiently large, activity could be pushed back to the initial equilibrium where higher wealth would be offset by higher interest rates in both the goods and money markets. In this case short term interest rates would rise above their initial equilibrium but this is not necessarily true prior to the implementation of the fiscal expansion.

Upon implementation of the fiscal expansion in the Turnovsky and Miller model, activity increases further and to ensure equilibrium in the goods market the short rate of interest has to rise. The long rate of interest, being forward looking, has already anticipated this and does not jump upon implementation. The increase in activity is insufficient to remove the government's deficit and bond sales are used to cover the government's deficit. These cause activity to rise and increase the short rate of interest. Over time the deficit is (assumed to be) closed and activity ceases rising, with both short and long rates higher than their initial equilibrium. Consequently, the slope of the term structure is eliminated over time following a fiscal expansion.

Despite the rich variety of interest rate responses dynamic IS-LM models of the type above do not appear to have been explicitly estimated in the UK and if they have their workings have been buried within the workings of much larger macro-econometric models. Chapter 8 of this thesis reports work on the estimation of a version of these models and through the use of principal components even greater flexibility is imposed on the term structure.

2.10 Intertemporal Model of Fisher and Turnovsky

Although the IS-LM framework has continued in use to formulate interest rate models its defects are well recognised. Indeed Evans (1987), a noted user of IS-LM framework, states that the "model is neither microeconomically rigorous nor universally regarded as useful".[61] This view has led to analysis being conducted in an intertemporal optimising framework where macroeconomic behaviour is consistent with microeconomic optimisation. A noted paper in this framework is by Fisher and Turnovsky (1992). They link long and short rates through the arbitrage form of the expectations hypothesis. Consequently, with the short rate being determined by short term equilibrium in the real economy, the long rate can be calculated in a recursive manner once the path of short rates is known. Hence, there is no need to arbitrarily assign long rates to the IS function and short rates to the LM function as in the models analysed above. They also analyse a number of different types of changes in government expenditure. These involve the time horizon over which permanent changes in government expenditure is expected. These are a zero time horizon (unanticipated); an anticipated increase with a short time horizon; an anticipated increase with a medium time horizon; and an anticipated increase with a long time horizon.

In order to achieve arbitrage, Fisher and Turnovsky assume that capital markets are perfect and investors have perfect foresight with an infinite planning horizon. To ease analysis the price level is fixed and considerations of money and foreign trade are excluded from the model. These assumptions introduce some of the problems of IS-LM and rule out other areas of macroeconomic research, such as credit rationing. The ignoring of the money supply takes a step backwards from IS-LM and, therefore, the model is not an unambiguous improvement over the IS-LM model.

The model assumes that the representative agent maximises discounted welfare from consumption and leisure subject to a budget constraint that links consumption, investment in bonds and capital goods to income from production, interest income from bonds and lump sum taxes. Variations in lump sum

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61 Evans (1987) p.282.

taxes, rather than bond issuance, ensure that the government's financial constraint is always satisfied. Hence, the rate of consumer time preference and the level of short term interest rates help determine consumption and labour supply. The production function contains just capital and labour supply. Investment is determined by output from the production function minus consumption (which is determined by the budget constraint and the welfare function) minus the exogenously determined amount of government spending. In the steady state investment is zero and the marginal physical product of capital equals the (exogenous) rate of time preference that, in turn, equals the short rate of interest. Fisher and Turnovsky assume that adjustments of the capital stock between steady states is not instantaneous and thus the short rate (and the long rate) can diverge from the rate of time preference during the adjustment phase.

An unanticipated increase in government expenditure and lump sum taxes reduces consumption. This raises the marginal utility of wealth and causes an increase in the supply of labour. The extra labour supply raises the marginal product of capital which causes capital accumulation and the higher return on investment causes the short term interest rate to rise. As the extra investment raises the capital stock the marginal product of capital and short term interest rates fall back to their steady state values. Consequently, the effect of an unanticipated increase in government expenditure is to raise short term interest rates and have them decline until the transition is complete. Long term rates, being determined by future short rates, also rise but not by as much as short rates and they also decline back to their steady state level. The term structure inverts upon an unanticipated increase in government expenditure with the inversion declining over the transition. The crucial aspect of this model is that labour supply increases with an increase in government expenditure. If this did not occur a rise in government expenditure would simply crowd out the same amount of consumption and there would be no effect on interest rates (this is the result obtained by Turnovsky (1986) discussed earlier). Similarly, if adjustment of the capital stock were instantaneous there would be no effect on interest rates.

With an anticipated future increase in government expenditure the marginal utility of wealth again rises initially but by less than in the unanticipated case because the future rise in government expenditure is discounted. This sets in train the adjustments described above. However, if the time between announcement and implementation of the increase in government expenditure is long the economy can over accumulate capital. Thus the marginal physical product of capital is reduced to below its long run equilibrium and so is the short term rate of interest. Once the fiscal expansion occurs it crowds out an equivalent amount of investment so that capital falls and its marginal product rises pulling up short term interest rates towards their equilibrium level. Thus with a long announcement period short term interest rates rise, fall and then rise again. Long term rates, being a weighted average of short term rates, follow a similar path. However, if the transition period is extremely long, so that short term rates are below their long run equilibrium for a long time, then long term interest rates can fall on the announcement of a fiscal expansion. Long term rates will continue to fall until the future rises in short term rates (after the fiscal expansion) dominate the short term declines. In terms of the term structure, an anticipated expansion initially causes the term structure to invert then flatten out before it steepens and then flattens out again. Hence, Fisher and Turnovsky's model, like the three models analysed above, has some rather different implications from the standard IS-LM model. In particular, it demonstrates the importance of expectations and the horizon over which those expectations are formed.

2.11 Ricardian Equivalence

A further area that requires discussion is Ricardian equivalence and its extension to incorporate invariance propositions about open market operations as well as the more standard government borrowing equivalence proposition resurrected by Barro (1974) and surveyed, amongst others, by Seater (1993).

Ricardian Equivalence states that although government purchases affect interest rates the manner in which these expenditures are financed (i.e. through debt or taxes) is irrelevant.[62] The reason is that asset demands move one-for-one with changes in the supply of public debt because the private sector perceives the extra debt as simply delayed taxation. The private sector increases its savings to pay off the debt and by buying the newly issued debt it can match the increase in obligations exactly. Barro's (1974) contribution was to weaken the objection that debt falling due after the current generation's

62 Strictly this only refers to lump sum taxes, as taxes which change marginal tax rates can have effects on labour supply thus altering aggregate supply and thus changing interest rates.

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enforce payment of taxes whereas the private sector is unable to enforce payment of interest. Therefore the private sector restricts these individuals' access to the loan market thereby raising their discount rates. It seems equally plausible to argue that these individuals are also outside the tax net whether legally through low income or illegal through evasion. In such circumstances they will not receive tax breaks and are not required to increase their savings. Thus Ricardian Equivalence may still hold even with imperfect capital markets.

The third assumption that is violated is that of certainty of taxes. If the distribution across households of the future tax rise is known but the aggregate total is not, simply holding the appropriate proportion of the newly issued debt completely hedges exposure to future taxes. However, if the distribution across individuals is not known individuals (if they have non-increasing absolute risk aversion) react to an increase in future lump sum taxes by saving more than the current reduction in taxes.[63] Hence the results are the opposite of the standard case - a tax cut (a deficit increase) results in a fall in interest rates. On the other hand, if taxes are income based the higher future taxes, which reduce the dispersion of future disposable income, reduce uncertainty. Lower uncertainty tends to increase current consumption. If the method of financing current tax cuts raises uncertainty it will raise savings and reduce interest rates. If it reduces uncertainty interest rates will rise. Therefore, it is not clear that the use of non-lump sum taxes results in the return to the standard result of higher deficits leading to higher interest rates.

The fourth assumption that is violated is that taxes are not lump sum. With taxes imposed at a higher rate on income than on expenditure, a lower tax rate in the current period, giving rise to a greater budget deficit, could encourage greater work activity and higher savings (because after tax returns would be higher because of the lower tax rate). Higher saving rates would tend to lower pre-tax returns along side an increase in the budget deficit. In the following period when income taxes rose savings would be reduced to maintain the desired level of consumption and rising interest rates would accompany a falling deficit. These results are non-Ricardian but nor are they the standard "greater

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63 See Chan (1983) p.363.

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Ricardian equivalence can also be extended to the analysis of open market operations, the issuance of short and long term debt and the choice between the special features (index linking, or coupon paying versus zero coupon bonds) attached to gilts (see Chan (1983) and Stiglitz (1988)). Unless there is a market imperfection, a redistributive effect (perhaps through taxes being non-lump sum or through some couples being childless), imperfect private substitutes for government bonds or unless the private sector does not take into account the behaviour of the government alongside its own, debt policy will not matter. This is simply because in a fully flexible economy the private sector has already selected its optimal consumption path. Unless the government offers a new set of bonds that provide a consumption path not previously attainable there will not be any effect on activity prices and interest rates. In a sense, therefore, the equivalence propositions simply extend the theorems of Modigliani and Miller (1958) to the government sector.

The model of Chan (1983) shows why open market decisions of government may be irrelevant. At time t_0 the government issues more discount long bonds to mature at time t_L and redeems exactly the same value of discount short bonds due to mature at time t_S. As coupon bonds can be treated as a sequence of discount bonds the limitation to discount bonds has no effect on the results. The exact number of short term bonds being redeemed will depend upon the relative price of long to short bonds $p(t_0,t_L) / p(t_0,t_S)$. There is no change in the current total value of debt, the government's deficit or the money supply. Future tax liabilities have changed with less revenue being required when the remaining short bonds mature and more needed when long bond mature. These tax payments are assumed to be lump-sum and a known proportion, θ, will be required from each household.

In a model described by the above assumptions, one strategy of households is for each to sell $\theta p(t_0,t_L) / p(t_0,t_S)$ short bonds (realising $\theta p(t_0,t_L)$) and buy θ long bonds. As the supply of short bonds exactly meets the extra demand of the government prices and short term interest rates are unchanged. Similarly, the matching of demand and supply for long term bonds also results in unchanged prices and long term interest rates. Moreover, the consumption path of the household is left unchanged by this rearrangement of their portfolio. At time t_S the household receives a smaller capital repayment but this is exactly matched by a lower tax demand. At time t_L the increased tax demand can

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Chapter 3

The Estimation of Spot and Yield Curves

3.1 Introduction

This chapter provides a discussion of the means used to derive interest rates from the data used by Egginton and Hall (1994). The primary aim of the chapter is to provide interest rates at constant maturities that can be used in the principal component analysis conducted in chapter 5. If there was an agreed method by which interest rates could be estimated from the prices of coupon bonds this chapter would be rather short. There is, however, no agreed method and the chapter discusses some of the approaches that have been used in the literature. The chapter contributes to this literature by examining the use of endogenous knot positions within two estimation procedures. This is used to examine the idea of "natural market boundaries" within term structures.

For completeness the following paragraphs rehearse the definitions of terms used throughout this thesis. It should be noted that all the discrete measures discussed below have their continuous time counter parts and these are often used in theoretical models of the term structure[1].

As each gilt has a known coupon and a fixed, semi-annual, schedule for their payment dates the yield to maturity, the redemption yield, can be calculated as the solution to:

$$P_t^n = \sum_{i=t}^{t+n} \frac{c_i}{(1+R_t)^i} + \frac{v_n}{(1+R_t)^n} \quad \ldots(3.1.1)$$

where : P_t^n = the gross price at time t with n periods to maturity
C_i = the coupon made at time i
v_n = the principal repaid at time n
R_t = the yield to maturity

[1] Shiller (1990) provides an exhaustive set of definitions.

This formulation makes the yield to maturity the rate that equates the present value of the gilt and its current price. Formula (3.1.1) is an n^{th}-order polynomial in $(1+R_t)$. The polynomial has n roots but, as all the c_i and v_n are positive, the solution to (3.1.1) produces only one positive root and this is taken as the yield to maturity[2]. Under the assumption that arbitrage ensures that gilts close in maturity and coupon trade at similar prices, all that is needed to fit the yields at given maturities is an interpolation routine, for example cubic splines.

A more general formulation of (3.1.1) would allow the rate at which the coupons were discounted to vary with the maturity of the gilt. These rates are known as spot rates (or zero coupon rates). The spot rates, r_i, are linked to the price of the gilt as follows:

$$P_t^n = \sum_{i=t}^{t+n} \frac{c_i}{(1+r_i)^i} + \frac{v_n}{(1+r_n)^n} \quad ...(3.1.2)$$

An alternative method of expressing this relationship is to use discount factors that are defined as:

$$d_i = \frac{1}{(1+r_i)^i} \quad ...(3.1.3)$$

Equation (3.1.3) is often used because without the superscripts manipulation is often easier than by using spot rates directly.

However, there is no unique solution to (3.1.2) on its own for the spot rates and, in general, n-1 further gilts would be required to solve for the interest rates. The longest gilt in the data set, described below, will mature 37 years after the beginning of the data set and, with semi-annual coupons, a total of 73 gilts would be required to calculate its spot rates. This is a tall order, even with the deep nature of the UK gilt market, because the coupon dates are not synchronised between each gilt. This is compounded by the scarcity of gilts at long maturities, so that the calculation of systematic interest rates is made difficult. These problems are compounded by the presence of bonds with special characteristics that affect their prices. These include FOTRA bonds (those free of tax to residents abroad), convertibles

2 The yields to maturity used in this study were calculated using the Newton-Raphson method.

(those with options to convert into other gilts at set times), variable coupon gilts and index linked gilts[3]. Making allowances for these factors would reduce the degrees of freedom available. Consequently, unless the analysis is restricted to the very short end of the term structure, the data requirements become too great for spot rates to be calculated directly. Rather indirect methods have to be used and some of these are described in section 3.3 below.

By using spot rates implied forward rates can be calculated. Through arbitrage the current n period spot rate equals the geometric mean of the n one-period forward rates, that is the return to holding a zero coupon gilt for n periods equals the return to holding n one period zero coupon gilts sequentially. Consequently, the forward rate for the n^{th} period, f_n, can be calculated as follows:

$$(1+f_n)=\frac{(1+f_1)(1+f_2)...(1+f_{n-1})(1+f_n)}{(1+f_1)(1+f_2)...(1+f_{n-1})}=\frac{(1+R_n)^n}{(1+R_{n-1})^{n-1}} \qquad ...(3.1.4)$$

The par yield curve can be calculated from (3.1.1) by setting the current price, P_t^n, equal to the value at maturity, v_n, and it can be shown that this implies that the yield to maturity, R_t, has to equal the coupon, c_i. By substituting the par yield, y_n, for the coupon payment within (3.1.2) the par yields are simply determined by the value at maturity, v_n, and the spot rates or discount rates as given by (3.1.5).

$$y_n=\frac{v_n(1-d_n)}{\sum_{i=1}^{n} d_i} \qquad ...(3.1.5)$$

Thus the spot rates, discount rates, forward rates and par rates are all linked algebraically and knowledge of one set can be used to derive the other rates. In the analysis undertaken below we therefore concentrate on the spot rates. Analysis is also conducted on the yields to maturity, in part because it is not algebraically linked to the above measures and also because it enables comparison with earlier work undertaken by Egginton and Hall (1994). The criticisms of the yield to maturity are well known: that it is a limited measure of return and that it depends upon coupon rate so that comparing

3 Since 24 May 1995 tax reforms have meant that the distinction between FOTRA and non-FOTRA gilts disappear for practical purposes. For the sample period considered in this thesis the distinction is relevant.

gilts with different coupons is invalid (and hence there should be one redemption yield curve for each coupon rate). Despite these problems, redemption yields are widely reported in the financial press.[4]

3.2 Description of the Data

In this section a detailed description of the data is provided. The data are from two sources. The data for the UK are those used in Egginton and Hall (1994), whilst the US data are from McCulloch (1990) and McCulloch and Kwon (1993). Egginton and Hall's data were calculated from the gross prices of British government securities, gilts, for each working day between 2 January 1979 and 21 August 1990 from data published in 'Mullens Blues', which were published by Warburg Securities. Over time the maturity of a given gilt declines and, because gilts are not equally spaced through time, it is impossible to derive a consistent series for interest rates without relying on estimation or interpolation methods. In the sections below interpolation is carried out using a variant of cubic B-splines. A description of the methodology used is provided below.

Listed in Table 3.2.1 is a sample of the gilts that were traded in the secondary market between 1979 and 1990. The sample excludes those gilts that had variable interest rates, were conversion stocks, were undated or were indexed linked. In order to minimise the tax effects all gilts with coupons below 6.75% were removed from the sample. Observations on the remaining gilts were only included in the sample on a fully paid basis so that no fraction of the semi-annual coupon had to be used in the calculation of the redemption yields.

The gilt names in Table 3.2.1 have the following explanation. The first two letters indicate that the gilt is either known as a treasury (TR) or an exchequer (EX) gilt. The next two numbers and the next letter indicate the annual coupon paid on the gilt. For example 09Z indicates a coupon rate of 9.0%, Q indicates a quarter of a percentage point, H indicates a half of a percentage point and T indicates three-quarters of a percentage point is paid. The final two numbers indicate the year of redemption for the gilt.

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4 See Schaefer (1977) for details of these criticisms.

Table 3.2.1 The Gilt Edged Securities used to Construct UK Yield Curves

Bond Number	Bond Name	Redemption Date	Inclusion Date	Exclusion Date	Number of Observations	Maturity at Inclusion
1	TR09H80	14.05.80	02.01.79	17.04.79	76	1.4
2	EX13Z80	25.11.80	02.01.79	30.10.79	216	1.9
3	TR11H81	15.01.81	02.01.79	18.01.80	274	2.0
4	TR09T81	01.04.81	02.01.79	04.03.80	306	2.3
5	EX08Q81	12.06.81	02.01.79	15.05.80	358	2.4
6	EX09H81	04.08.81	02.01.79	08.07.80	396	2.6
7	EX12T81	23.11.81	02.01.79	28.10.80	476	2.9
8	TR08H82	15.01.82	02.01.79	17.06.81	642	3.0
9	TR14Z82	16.03.82	02.01.79	17.02.81	556	3.2
10	TR08Q82	05.07.82	02.01.79	07.07.81	656	3.5
11	EX09Q82	22.09.82	02.01.79	24.09.81	713	3.7
12	EX08T83	05.01.83	02.01.79	07.01.82	788	4.0
13	TR12Z83	17.03.83	02.01.79	16.03.82	836	4.2
14	TR09Q83	18.07.83	02.01.79	07.06.82	895	4.5
15	EX13H83	22.11.83	23.05.80	02.11.82	638	3.5
16	EX10Z83	12.12.83	02.01.79	02.11.82	1001	4.9
17	EX11Q84	20.02.84	12.09.79	22.02.83	900	4.4
18	EX14Z84	22.05.84	07.01.80	27.04.83	863	4.4
19	TR12Z84	26.09.84	27.09.79	15.08.83	1013	5.0
20	TR15Z85	22.02.85	19.11.79	20.01.84	1090	5.3
21	TR11H85	15.07.85	16.07.81	11.06.84	758	4.0
22	EX12Q85	22.11.85	02.01.79	12.10.84	1509	6.9
23	EX11T86	25.02.86	26.02.81	23.01.85	1016	5.0
24	TR12Z86	12.06.86	15.06.81	25.03.85	986	5.0
25	TR08H86	10.07.86	02.01.79	11.06.85	1681	7.5
26	EX14Z86	29.10.86	07.05.82	01.10.85	888	4.5
27	EX13Q87	22.01.87	23.07.79	24.01.86	1700	7.5
28	EX10H87	06.04.87	05.10.82	14.03.86	899	4.5
29	TR10Z87	12.06.87	07.08.84	15.05.86	463	2.8
30	TR12Z87	03.11.87	04.11.80	07.10.86	1546	7.0
31	TR07T88	26.01.88	02.01.79	28.01.87	2107	9.1
32	EX10H88	10.05.88	14.07.83	16.04.87	981	4.8
33	TR09H88	25.10.88	14.07.83	30.09.87	1100	5.3
34	TR11H89	22.02.89	25.02.80	26.01.88	2067	9.0
35	TR10H89	14.06.89	13.12.83	14.06.88	1156	5.5
36	EX10Z89	01.08.89	31.07.84	05.07.88	1026	5.0
37	EX11Z89	29.09.89	23.08.85	01.09.88	790	4.1
38	TR13Z90	15.01.90	02.01.79	17.01.89	2621	11.0
39	EX11Z90	12.02.90	15.08.85	14.02.89	914	4.5
40	EX12H90	22.03.90	24.09.81	27.03.89	1957	8.5
41	TR08Q90	15.06.90	02.01.79	19.06.89	2730	11.4
42	TR11T91	10.01.91	02.01.79	12.01.90	2879	12.0
43	EX11Z91	25.10.91	26.10.79	21.08.90	2823	12.0
44	TR08Z91	10.12.91	30.12.87	21.08.90	690	4.0
45	TR12T92	22.01.92	02.01.79	21.08.90	3036	13.1
46	TR10Z92	21.02.92	02.01.79	21.08.90	3036	13.1
47	TR08Z92	13.04.92	02.12.87	21.08.90	710	4.4
48	EX12Q92	25.08.92	02.01.79	21.08.90	3036	13.6
49	EX13H92	22.09.92	23.09.80	21.08.90	2586	12.0
50	TR08Q93	18.02.93	15.09.88	21.08.90	504	4.4
51	TR10Z93	15.04.93	08.01.87	21.08.90	944	6.3

Table 3.2.1 cont. The Gilt Edged Securities used to Construct UK Yield Curves

Bond Number	Bond Name	Redemption Date	Inclusion Date	Exclusion Date	Number of Observations	Maturity at Inclusion
52	TR12H93	14.07.93	02.01.79	21.08.90	3036	14.5
53	TR13T93	23.11.93	02.01.79	21.08.90	3036	14.9
54	TR08H94	03.02.94	03.01.89	21.08.90	426	5.1
55	TR14H94	01.03.94	02.01.79	21.08.90	3036	15.2
56	EX13H94	27.04.94	27.10.80	21.08.90	2562	13.5
57	TR10Z94	09.06.94	16.07.87	21.08.90	809	6.9
58	EX12H94	22.08.94	02.01.79	21.08.90	3036	15.6
59	TR09Z94	17.11.94	02.01.79	21.08.90	3036	15.9
60	TR12Z95	25.01.95	02.01.79	21.08.90	3036	16.1
61	EX10Q95	21.07.95	02.01.79	21.08.90	3036	16.5
62	TR12T95	15.11.95	02.01.79	21.08.90	3036	16.9
63	TR14Z96	22.01.96	23.07.80	21.08.90	2630	15.5
64	TR09Z96	15.03.96	02.01.79	21.08.90	3036	17.2
65	TR15Q96	03.05.96	02.01.79	21.08.90	3036	17.3
66	EX13Q96	15.05.96	02.01.79	21.08.90	3036	17.4
67	TR13Q97	22.01.97	02.01.79	21.08.90	3036	18.1
68	EX10H97	21.02.97	02.01.79	21.08.90	3036	18.1
69	TR08T97	01.09.97	02.01.79	31.12.87	2348	18.7
70	EX15Z97	27.10.97	28.04.86	21.08.90	1127	11.5
71	EX09T98	19.01.98	24.09.84	21.08.90	1542	13.3
72	TR06T98	01.05.98	02.01.79	21.08.90	3036	19.3
73	TR15H98	30.09.98	02.01.79	21.08.90	3036	19.7
74	EX12Z98	20.11.98	02.01.79	29.12.89	2869	19.9
75	TR09H99	15.01.99	02.01.79	21.08.90	3036	20.0
76	EX12Q99	26.03.99	28.08.79	21.08.90	2866	19.6
77	TR10H99	19.05.99	02.01.79	21.08.90	3036	20.4
78	TR08H00	28.01.00	14.03.88	21.08.90	637	11.9
79	TR13Z00	14.07.00	15.01.81	21.08.90	2504	19.5
80	TR10Z01	26.02.01	27.03.86	21.08.90	1149	14.9
81	TR14Z01	22.05.01	26.05.81	21.08.90	2411	20.0
82	EX12Z02	22.01.02	02.01.79	06.08.90	3025	23.1
83	TR09T02	27.08.02	19.03.86	21.08.90	1155	16.5
84	EX09Z02	19.11.02	09.12.87	21.08.90	705	14.9
85	TR13T03	24.07.03	26.07.79	21.08.90	2889	24.0
86	TR10Z03	08.09.03	18.11.86	21.08.90	981	16.8
87	TR11H04	19.03.04	23.05.79	21.08.90	2935	24.8
88	TR10Z04	18.05.04	24.12.85	21.08.90	1216	18.4
89	EX10H05	20.09.05	22.10.85	21.08.90	1261	19.9
90	TR12H05	21.11.05	22.05.79	21.08.90	2936	26.5
91	TR08Z06	05.10.06	02.01.79	31.12.86	2087	27.8
92	TR11T07	22.01.07	23.01.80	21.08.90	2760	27.0
93	TR08H07	16.07.07	15.01.87	21.08.90	939	20.5
94	TR13H08	26.03.08	29.09.80	21.08.90	2582	27.5
95	TR09Z08	13.10.08	16.09.87	25.11.87	46	21.1
96	TR08Z09	25.09.09	28.10.86	21.08.90	996	22.9
97	TR07T15	26.01.15	02.01.79	21.08.90	3036	36.1
98	EX12Z17	12.12.17	02.01.79	21.08.90	3036	38.9

The third column in Table 3.2.1 gives the redemption date of the gilt. For those gilts with multiple redemption dates, and which had not been redeemed, the latest date was chosen as the redemption date. The reason is that Egginton and Hall (1994) used the latest date to maturity and their data are used in this study. Thus for practical purposes it is easier to follow their practice. These gilts are: TR09Z96, TR06T98, TR14Z01, EX12Z02, TR13T03, TR11H04, TR12H05, TR11T07, TR13H08, TR07T15 and the EX12Z17. There are alternative assumptions that could be used, for example, that these gilts would be redeemed at the earliest opportunity. However, this assumption is equally arbitrary. An alternative assumption (used by McCulloch and Kwon (1993)) is that the gilt will be redeemed if the price is above par but purchased in the market if the price is below par - the "par rule". This would result in the expected maturity of the gilt changing endogenously. Later in the chapter we examine the fitting of endogenous knot positions and the endogenous maturities of some of the gilts would add to the complexity of interpreting the results. Effectively it would split the data into even more sub-groups than the 97 listed in the appendix to this chapter. Moreover, Bliss and Ronn (1995) show that it is not only the price that matters for the decision to call but also the volatility of interest rates. Thus concentrating on the price alone will be misleading. For these reasons the "par rule" was not used.

Given that the expectation of early redemption cannot be estimated in an error free manner this could also introduce biases into the yield estimates the consequences of which would be hard to describe a priori. With the simple assumption of late redemption the consequences are simple - the yields will be overestimated during periods when early redemption is expected with the over estimation being proportional to the trade undertaken by market participants holding such an expectation. The yields and the spot rates with longer maturities, therefore, may be biased upwards during some of the time periods analysed in this thesis.

3.3 The Estimation of Spot Curves

The seminal work on estimating yield curves is McCulloch (1971), although McCulloch himself draws attention to the earlier work of Cohen et al (1966) and Durand (1958). It is worth describing the evolution of McCulloch's work in detail because this methodology provides one of the spot estimates for the UK data (the other being drawn from data published by the Bank of England). It also provides the US spot data. However, to make it clear from the beginning, the methodologies differ in a number of respects. The UK spot data were estimated using a cubic B-spline and assumes that all tax rates are zero, whilst McCulloch (1990) used a tax adjusted cubic spline to estimate the US spot data.

McCulloch (1971) worked with a discount function that he assumed was continuously differentiable and monotonically decreasing. He also assumed that coupon payments were paid continuously, and this allowed him to use the price plus accrued interest as his price series. In McCulloch (1990) the price alone is fitted to the sum of the coupon payments, and this is the only difference in the method of constructing the US data from the methodology described below. The price is assumed to be equal to the discounted principal plus the continuously discounted coupon payments and an error term. The error term picks up a host of features including: transaction costs; tax effects; convertibility considerations; ineligibility for commercial bank purchase; the risk of default; imperfect arbitrage and errors introduced by modelling the discount function.

McCulloch (1971) chooses to model the discount function, $\delta(m)$, as a linear combination of k continuously differentiable functions $f_j(m)$, where m is the maturity of the bond and j increments from one to k.

$$\delta(m) = 1 + \sum_{j=1}^{k} \alpha_j f_j(m) \qquad \text{...(3.3.1)}$$

Each function is weighted by α_j, which was estimated by regression. When j=0 (i.e. in the current period) the discount function has to equal unity as a pound now is worth one pound. This is captured by including a constant with the value of one in the proxy for the discount function and only allowing the function $f_j(m)$ to take values for j>0. In terms of equation (3.1.3), $\delta(m)$ is the continuous time analogue of d_i. By substituting the proxy for the discount function into the expression for the discounted present value and rearranging, McCulloch (1971) derives a further expression. This links the current price (minus the principal, V_i, and a coupon payment C_i multiplied by the bond's maturity, m_i) to a linear combination of the discounted principal and coupon payments for each value of j weighted by the α_j.

$$P_i - C_i m_i - V_i = \sum_{j=1}^{k} \alpha_j \left[C_i \int_0^{m_i} f_j(\mu)\, d\mu + V_i f_j(m_i) \right] \qquad \ldots(3.3.2)$$

With the addition of an error term equation (3.3.2) can be estimated by ordinary least squares to find the values of α_j. By entering the a_j back into (3.3.1), the values of the discount function at any maturity within the data set can be calculated.

Estimation requires that the form of the functions, $f_j(m)$, must be decided, together with the value of k, the number of functions included in the discount proxy. McCulloch (1971) chose as his function a piecewise quadratic, where the knots are chosen so that there are equal numbers of bonds in each subinterval. This allows the estimated discount function to have higher resolution at maturities where there are more bonds. The choice of a quadratic function implies, however, that the forward rates estimated from the discount function have discontinuous first derivatives. This produces a "knuckle" shape to the forward rates.[5] McCulloch dismisses this by claiming that the level of the forward rate at any particular maturity has little practical significance.[6]

Two methods are proposed to set k, the number of parameters to be estimated. If k is set too low the proxy may not fit the discount function very well, whereas if k is set too high it may overfit to outliers. McCulloch suggested that either k could be set to minimise the variance of the residuals, where the denominator of the variance is given by the number of bonds minus k. Alternatively, McCulloch suggested that k should equal the nearest integer to the square root of the number of bonds. This allows the resolution of the discount function to increase with the number of bonds. It also allows the number of bonds relative to the number of parameters estimated to rise within each piecewise segment, thereby counteracting overfitting. Clearly, the square root approach is not unique, but McCulloch notes that it produces similar values for k as minimising the error variance.[7]

5 See, for example, McCulloch (1971) figure 4, p. 23.
6 McCulloch (1971) p. 31.
7 McCulloch (1971) p. 31.

The specification is completed by allowing the error term to be heteroscedastic with its variance proportional to the bid-offer spread and brokerage fees. Together these are termed e_i, where i represents each bond[8]. Consequently, weighted least squares is required for estimation, where the variables are each divided by e_i. Jordan (1984) reports that this specification reduces the number of his samples that exhibit heteroscedasticity by 60%. However, Litzenberger and Rolfo (1984) claim that, because the bid-ask spread typically increases with maturity, the correction for heteroscedasticity reduces, without justification, the weight given to longer maturity bonds.

A different assumption is used by Vasicek and Fong (1982) who assume that the error term is homoscedastic in yields. Consequently, they use generalised least squares as their estimation procedure. In particular, the error variance is assumed to be proportional to the squared derivative of prices with respect to the yield of each bond. It is also assumed that the errors are uncorrelated between bonds. This latter assumption is contentious. It is difficult to believe that a pricing error in one bond would not also be applied to close substitute bonds as well, unless the errors were derived solely from measurement errors but in this case there is no reason to believe that the errors would be heteroscedastic.

As the bid/ask spread is not available with the Egginton-Hall data, and in the light of Litzenberger and Rolfo's comment, this approach is not followed. Moreover, as the unbiasedness of the ordinary least squares (OLS) estimator only relies on the first moment properties of the model, OLS remains unbiased and consistent even in the presence of heteroscedasticity. As we are not interested in the fit of the individual parameters of the equation (because there will be 3036 equations) ignoring potential heteroscedasticity is not a problem.

By attempting to remove heteroscedasticity, McCulloch (1975) also has to use an instrumental variables (IV) approach to allow for his dependent and independent variables both being constructed from the mean of the bid/ask spread. McCulloch instruments his regression by replacing the mean of the bid/ask

8 As the estimation occurs across bonds rather than through time no mention of the autocorrelation properties of the error term need be made.

spread by the par value, as do Litzenberger and Rolfo (1984). McCulloch (1975) reports that the difference between the IV regression and ordinary least squares (OLS) is minimal.[9] However, this may be due to McCulloch constraining his capital gains tax rate to be half the income tax rate. Jordan (1984) found that the estimated tax rates were virtually unchanged between OLS and IV estimation when the capital gains rate was constrained to be 0.5, but were highly variable when the constraint was removed.[10] In fact, the use of the par value as an instrument removes much of the tax effects that McCulloch was trying to capture because the tax rates tend to be low and often zero. Langetieg and Smoot (1989), echoing the results of Jordan (1984), conclude that simply ignoring the problems that IV estimation was designed to eliminate can produce better fitting models than when IV estimation is used.

Other features of McCulloch's specification have also been challenged or alternatives tried by other researchers. Precision of the estimates at longer maturities will also be impeded, according to Litzenberger and Rolfo (1984), if the position of the knots is determined by McCulloch's equal number of bonds rule. Consequently, they do not adopt it in their empirical work preferring to set the knots exogenously at "natural market boundaries". As they themselves note, where these are believed to occur is highly subjective. Using UK data, Steeley (1990) instigated a search procedure with the number of knot and their positions with the criterion being to minimise the standard errors of the spot rates. He found that the term structure could be divided into three segments: up to five years, up to ten years and ten years and over.[11] Prior to Deacon and Derry (1994), the Bank of England set the position of its knots exogenously at zero, two, five, ten, fifteen and infinity, where time is measured in years. As the Bank's model transforms time to lie between zero and one, using an exponential function, the choice of knot points simply divides this space into nearly equal segments.[12] More recent work from the Bank of England (Deacon and Derry (1994)) notes the effects of changing the knots from being evenly spaced in transformed time to evenly spaced by the number of bonds in the sample. This results, for the

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[9] McCulloch (1975) p. 815.

[10] Jordan (1984) table II, p. 401. Although Jordan repeatedly notes that the capital gains tax rate is set at 0.5 it is more likely that it is set at half the income tax rate because in two of his regressions where the unconstrained tax rates are estimated to be zero, the constrained standard error of the regression are equivalent to the unconstrained standard error when one would expect it to be larger.

[11] Steeley (1990) p.157.

[12] See Mastronikola (1991) p.20. These knot settings were used by Egginton and Hall (1994) who used the Bank's model to interpolate their data.

forward rate curves on the 30 September 1992, of a difference of up to 13 basis points, although the effects on par and zero coupon curves are much less significant.[13]

The two further problems of McCulloch's paper were tackled in a later paper by the same author. McCulloch (1975) replaced the piecewise quadratic function with a piecewise cubic function as the proxy for the discount function. This removes the problem of the "knuckles" in the forward rates because a cubic function has a continuous second derivative. Secondly, McCulloch's 1975 paper makes allowance for tax effects on bond pricing that, as noted above, was suspected of being a major source of the error term in the regression. Essentially, however, McCulloch uses the same methodology as described above, except that a unique income tax rate and a capital gains tax rate (taken to be half the income tax rate because of the tax regime in operation in the US in the early 1970s) are supplied.

Using a discrete time period version of (3.1.1) that allows for an income tax rate of τ, the gross price of a gilt can be written in terms of discount functions as:

$$p_i + ai_i = ai_i \tau \delta(t_1) + \frac{c_i}{2}(1-\tau)\sum_{l=1}^{n} \delta(l) + V_i \delta(L) \qquad \text{...(3.3.4)}$$

Where: p_i represents the clean price of gilt i that pays a coupon of c_i annually.

ai_i represents the accrued interest on the gilt i, which since 1986 has been taxable as income.

t_1 represents the time to the payment of the tax on the accrued interest.

η represents the number of coupon payments outstanding.

$\delta()$ represents the discount function.

l represents the time of the coupon payments.

L represents the time of the redemption payment V_i.

Prior to 1986 the first term on the right hand side of (3.3.4) would be zero because accrued interest was not taxed. Currently it is also zero for charities, pension funds and individuals whose holdings within the accrued interest scheme, as defined by the Income and Corporation Taxes Act 1988, are less than

13 See Deacon and Derry (1994) p. 32, footnote 29, and p.33.

£5,000. After substituting the proxy for the discount function into equation 3.3.4, the discount function can be estimated from:

$$y_i = \sum_{l=1}^{n} \alpha_j x_{ij} \qquad ...(3.3.5)$$

Where: $$y_i = p_i + \left(ai_i - \frac{nc_i}{2} \right)(1-\tau) - V_i$$

$$x_{ij} = ai_i \tau\, f_j(t_1) + \frac{c_i}{2}(1-\tau) \sum_{l=1}^{n} f_j(l) + V_i f_j(n)$$

The tax parameter, τ, can be estimated alongside α_j by a grid search over the range zero to (almost) unity using ordinary least squares or alternatively by non-linear least squares. The tax rate does not represent the marginal rate but is an average rate of those organisations and individuals operating in the market. As, in principal, these can change from day to day then even within a given tax year the tax rate can change on a daily basis as the estimates made by Levin and Copeland (1992) demonstrate. Unfortunately, as the tax rates do not represent marginal rates, estimates derived in this manner cannot be used in modelling the determinants of interest rates.

It should also be noted that the specification implies that the participants in the market assume that the tax rate will remain fixed over the entire life of the gilt. For individual investors this seems an implausible assumption to make. Over the 1980s marginal tax rates fell sharply, and expectations that further reductions in marginal tax rates would occur could have been plausibly held. The specification (3.3.5) is, therefore, misspecified and the estimated discount factors may be biased. A further problem may arise if institutions with different tax rates operate in different segments of the yield curve (for example, individuals at the short end and pension funds at the long end) because (3.3.5) assumes that the tax rate is equivalent for all gilts. Again this may bias the discount factors.

Both Jordan (1984) and Litzenberger and Rolfo (1984) point out that McCulloch's methodology is unable to separately identify the income tax rate from the rate on capital gains with any precision. This is not a problem in the UK because gilts are free of capital gains tax, although under certain circumstances market makers have their capital gains taxed as profits at the corporation tax rate. Nevertheless, there is a "tax " problem in the UK between tax exempt institutions and those that pay income or corporation tax. The latter group would clearly prefer low coupon gilts so that their return is received in the form of (untaxed) capital gains, whilst the former, ceteris paribus, should be indifferent. This raises the possibility that there may be distinct markets (clienteles) for gilts, and estimating just one term structure may be inappropriate. Thus, because different taxpayers value gilts differently, there is an opportunity for arbitrage between investors.

The problem of tax clienteles has been tackled by the highly original work of Shaefer (1981) who first pointed out the danger of trying to estimate single term structures in the presence of these tax effects. Schaefer assumed that short selling was banned and, therefore, no arbitrage is possible. This means that the price of a gilt is determined by those who value its type of cash flow and capital gain the most. Hence the price is always more than or equal to the discounted cash flow and the redemption price. This formulation allowed him to specify a linear programming technique to select which gilts would enter the optimal portfolio for any investor with a given tax rate. He also used Bernstein polynomials to approximate the discount function because of the standard problem of the absence of cash flows in every period.

Schaefer's (1981) methodology, whilst providing a more rigorous treatment of taxation, does not provide a representative tax parameter, rather one has to be supplied by the user. Furthermore, empirical implementation often results in very few bonds being selected as being efficient. For example, Derry and Pradhan (1993) found that only six to eight gilts out of between 40 to 60 were efficient. This raises the concern that because the estimates rely on so few gilts that they are very imprecise. On the other hand, Mastronikola (1991), building upon the work of Clarkson (1978), provides an equation that determines the shape of investors' indifference curves between capital gain to redemption and (taxable) income. From this a representative tax rate can be estimated.[14] Whilst mathematically intriguing, it seems highly implausible that, other than for very short gilts, the comparison will be between the gain to redemption and the running yield. Rather the comparison will

[14] See Mastronikola (1991) p.15.

be between the expected capital gain and the running yield but this approach does not specify how such expectations could be formed. Moreover, the model assumes that coupons are paid continuously otherwise there would be no relationship between capital gains and the running yield for all but two days a year. To the extent that investors treat accrued interest as a capital gain rather than income the model will deviate significantly from the perceptions of investors. It is not, therefore, clear that the use of this additional arbitrary relationship results in an improvement over the simpler McCulloch approach. Consequently, neither Schaefer's nor Mastronikola's approaches are investigated in the remainder of this thesis.

Nor is the tax treatment outlined in (3.3.4) used. Apart from the weaknesses of the tax treatment detailed above there are four additional reasons for this. Firstly, the spot rates estimated using McCulloch's method are being compared with spot rates, estimated by the Bank of England, which assumed zero tax rates. To ensure that the spot rates only differ, in so far as is possible, because of the different estimation routines used a zero tax rate was also applied to the McCulloch type estimates of the spot rates. Secondly, the original Egginton-Hall data eliminated low coupon gilts from the sample. In terms of equation (3.3.4) this procedure, in the limit, would push the first and second terms to being simply determined by the discount function and as such whether the coupon payment is net or gross of tax is irrelevant to the estimation procedure. Thirdly, Langetieg and Smoot (1989) report that when knot positions are altered the estimates of the tax parameters also change.[15] Consequently, as the knot positions are set endogenously, the tax effects may already be captured by the model. Fourthly, the setting of a zero tax rate may be not that far from being correct, at least for some of the sample period, as Levin and Copeland's (1992) estimates of the tax parameter during 1989 to 1991 are rarely above 6% and sometimes zero.[16]

3.4 Alternative Methods of Estimating Spot Curves

In this section we outline some of the alternative methods that have been used to estimate yield curves. In some cases, Vasicek and Fong (1982) and Chambers et al (1984), the essential difference from

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15 See Langetieg and Smoot (1989) p. 206.

16 See Levin and Copeland (1992) Table 1, pp.28-31.

McCulloch's work is in the specification of the discount functions. However, in the work of Nelson and Siegel (1987), Svensson (1993) and Deacon and Derry (1994) a different approach is taken, although, as Svensson (1993) and Baum and Thies (1992) show, it can be used in conjunction with the McCulloch approach. The papers covered in this section are only some of the more well known but there exists many others potential models that could be used, for example Longstaff and Schwartz (1992) and others from the finance literature. However, given that the finance literature provides models that are to be tested on the data, constructing the data on the basis of these models is circular.

Vasicek and Fong (1982) propose that discount factors can be calculated using exponential spline fitting.[17] This amounts to applying a generalised least squares regression technique using the gross price of bonds (i.e. including accrued interest) as the dependent variable. Vasicek and Fong (1982) argue that this price is determined by the discounted values of the coupons and principal; tax effects, which are assumed to be proportional to the current yield of the bond; a dummy variable to distinguish callable from non-callable bonds and an error term, the source of which is left undetermined.

Vasicek and Fong (1982) suggest that the discount function can be approximated by a third order exponential spline of the form:

$$d(t) = \alpha_0 + \alpha_1 e^{-\alpha t} + \alpha_2 e^{-2\alpha t} + \alpha_3 e^{-3\alpha t} \quad ...(3.4.1)$$

Where: d(t) is the discount function over t periods

$\alpha_0, \alpha_1, \alpha_2, \alpha_3$ and α are parameters.

Vasicek and Fong (1982) choose to transform the argument of the discount function, t, so that only α is estimated from the above list of parameters and this is accomplished to minimises the sum of squared weighted residuals.[18] This transformation, it is claimed, avoids the use of non-linear estimation

17 Coleman et al (1992) also propose a method which is "equivalent to fitting exponential splines to estimate the discount function", p.9.

18 It can be shown that with the transformation of time that Vasicek and Fong (1982) apply, α is the limiting value of the forward rates as t approaches infinity. See Vasicek and Fong (1982) p. 346.

techniques, but Shea (1985) argues that this is incorrect and that non-linear estimation methods are still required.[19] Shea (1985) provides empirical results that suggest that Vasicek and Fong's method brings no particular advantage to the modelling of discount rates and, indeed, has a number of problems of its own. These are that the transformation of the data for low values of α can lead to bunching so that much of the estimation interval is empty of data. Shea found that in these circumstances the derived forward rates became unstable and unrealistic (negative in the example Shea provides).[20] Shea (1985) also notes that the asymptotic forward rate, α, often bear little resemblance to the sample data, so that the validity of the forward rates for maturities greater than that available from the sample is questionable.[21]

Another example of alternative discount function approximations is provided by Chambers et al (1984). They propose the use of an exponential polynomial (i.e. the exponent of an ordinary polynomial) to model spot rates. McCulloch (1971, 1975) had experimented with polynomials instead of splines but found that it over-smoothed at the short end, where there are more bonds, and under-smoothed at the long end of the yield curve, so that implausible forward rates were derived. Shea (1984) notes that over-fitting the degree of the polynomial results in estimates that pass close to all the data points so that little reliance can be placed in goodness of fit measures. The parameters of Chambers et al's (1984) exponential polynomial were estimated by regressing bond prices on the coupon (and principal) stream multiplied by the exponential polynomial. Maximum likelihood estimation was undertaken, with the variance of the error term being modelled as being dependent upon the time to maturity of the given bond. Chambers et al (1984) suggest that a sixth order exponential polynomial fitted the data best, although if homoscedasticity is assumed a third or fourth degree polynomial was preferred.[22] Chambers et al also find that heteroscedasticity is not always present in the sub-periods they analyse. Consequently, a unique choice of polynomial length cannot be made.

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19 See Shea (1985) p.321.

20 See Shea (1985) figure 2, p. 324. Note also that a B-spline estimated on the same data also exhibited negative forward rates.

21 See Shea (1985) figure 1, p. 323.

22 See Chambers et al (1984) p.243 and p.238 respectively. Note also that treasury bonds were not included in the heteroscedasticity regressions.

Moreover, Chambers et al (1984) results also indicate that the term structure and forward rates remain "erratic at the far end" although all estimated forward rates remain positive.[23] They attribute this poor result to the paucity of data at the long end of their sample, which in any case is limited to a maximum maturity of ten years. This is the same reason as given by McCulloch (1975) and it is not clear, given that Chambers et al (1984) and McCulloch used different data and polynomial lengths, whether this implies an improvement over the use of ordinary polynomials. Chambers et al (1984) also note that their maximum likelihood approach is difficult to implement and computationally burdensome. This may be one of the reasons why their approach has not been followed in the literature.

Baum and Thies (1992) have treated the problem of negative forward rates in a different manner. They estimate the yield curve using McCulloch's estimation technique, with some minor alterations, then post filter the results using Nelson and Siegel's procedure to constrain the shape of the estimated yields.[24] Nelson and Siegel (1987) believe that the yield curve has three basic shapes: monotonic, humped or S-shaped.[25] A class of functions that generate these shapes are the solutions to differential or difference equations. By setting the instantaneous forward rate equal to the solution to a second order differential equation with real and equal roots, Nelson and Siegel (1987) derive a linear equation in time adjusted maturity and four constants:

$$f(m) = \beta_0 + \beta_1(\exp(-m/\tau) + \beta_2[(m/\tau)(\exp(-m/\tau))] \quad ...(3.4.2)$$

Where: $f(m)$ = the instantaneous forward rate at maturity m

$\beta_o, \beta_1, \beta_2$ and τ are parameters.

By initiating a grid search, the three parameters and the time adjustment to maturity, τ, can be found. The selected function form ensures that as the maturity, m, of the bond increases, the limiting value of

23 See Chambers et al (1984) p.244.

24 Baum and Thies (1992) add dummy variables to their regression to allow for securities with different risk ratings in their data.

25 Nelson and Siegel (1987) p.474. Their own work on US treasury bills suggests that in only one of the 37 samples they modelled could the fit of the differential equation be described as very poor. It should be noted that one of their parameters, the time adjustment parameter, is very unstable between samples, see table 1, p. 481.

the forward rate is given by β_0 and as maturity gets shorter the forward rate tends to $(\beta_0 + \beta_1)$. Baum and Thies (1992) do not describe the consequences of imposing this filter on their own work but, using graphical methods, they compare it with the earlier work of Thies (1985). They find that it reduces the spurious volatility of the estimated yield curves.

This approach to modelling yield curves can be contrasted with the approach used below. Baum and Thies (1992) filter their zero coupon yields to produce a limited number of yield curves that can be summarised by a few parameters. In the approach used below a large range of potential yield curves are summarised in terms of principal components. Which method is best at summarising the yield curve seems to depend upon how willing one is to believe, a priori, that the yield curve can only take on a limited number of shapes. Moreover, Nelson and Siegel's filter does not necessarily produce independent summary measures of the yield curve, because there is no guarantee that the estimated parameters are uncorrelated. For these reasons the principal component method is the preferred summary measure of the yield curve.

Nelson and Siegel (1987) are able to use an integrated version of (3.4.2) to estimate spot rates because they restrict their estimation to US treasury bills. In a series of papers Svensson (1993,1994) and Dahlquist and Svensson (1994) use the Nelson and Siegel's functional form to replace cubic splines within the McCulloch estimation routine. In Svensson (1994) (3.4.2) is enhanced by a further term requiring two extra parameters to be estimated but the improvement in fit is not regarded as being significant. Dahlquist and Svensson (1994) also find that constraining the model to ensure that the sum of the parameters $(\beta_0 + \beta_1)$ equals the marginal lending rate does not lead to parameter estimates that are statistically different from the unconstrained ones.[26] The Bank of England (1995), which uses the Svensson model, does not report data for maturities of less than two years "since problems associated with estimating the curves at very short maturities may make these data unreliable". Problems in estimating data in the three month to one-year maturity range are also reported by Svensson (1993).[27] Despite this the estimated data supplied by the Bank of England (1995) will be used, in chapter 4, as a

26 See Dahlquist and Svensson (1994) table 4, p.19.
27 See Svensson (1993) p. 25.

comparison with the estimates derived from using the McCulloch methodology. It is to the practical aspects of the cubic spline routine that attention is turned in section 3.5.

3.5 Estimation of Alternative Yield Curves

Using the data described above, together with the salient points determined from the discussion of spot rate estimation methodologies, a further set of redemption yield curves and a new set of spot rates can be estimated. As these use exactly the same data as Egginton and Hall (1994) the effects of changing the estimation methodology on the estimated principal components (the subject of chapter 5) can easily be discerned. It is these comparisons, rather than the derived spot rates and yields themselves, that are of interest in the remainder of this chapter. However, it is important that the new estimates bear some relationship to accepted estimation practice, in order that the comparison of the principal component results can be regarded as valid. To this end the estimation routine makes use of cubic B-splines as recommended by Shea (1984) and Steeley (1990).[28]

Basis or B-splines can be defined as the Lagrange polynomial[29]:

$$B_p(x) = \sum_{j=p}^{p+k+1} \left[\prod_{i=p,i\neq j}^{p+k+1} 1/(e_i - e_j) \right] (x - e_j)_+^k \quad ...(3.5.1)$$

Where: $B_p(x)$ is a B-spline that is only non-zero if x is in the interval (e_p, e_{p+k+1}),

e_i and e_j are knots of which there are n,

k gives the order of the B-spline, in this case three,

x is the value at which the B-spline is to be evaluated,

$(x - e_j)_+^k$ is a truncated power function, i.e. it is the maximum of $((x-e_j,0))^k$.

The truncated power function ensures that the value of the bases (i.e. the sequence B_p, p=0,...n-k-1) underlying the B-spline are zero over a large part of the approximation space, i.e. when

28 It should be noted that the estimation procedure used by Egginton and Hall (1994) was that used by the Bank prior to the introduction of the Svensson (1994) methodology and, stripped of its tax effects, this boils down to estimation using the cubic spline routine written by Press et al (1986).

29 See Powell (1981) p.229 for details.

$x \leq e_p$ or $x \geq e_{p+k+1}$. This prevents the loss of accuracy due to cancellation that can be found in other spline representations and it also reduces the amount of calculation required. Moreover, because a recurrence relationship exists between B-splines of different degrees and different p's, calculation can be simplified and further problems due to cancellation can also be eliminated. The recurrence formula is used to estimate the B-splines in the McCulloch estimation procedure (discussed below) using a subprogram derived from De Boor (1978) and modified by Steeley and Clewlow (1993).[30]

The bases B_p, (p=0,...,n-k-1) are linearly independent, but we wish to span a space of n+k, thus 2k extra bases are required. This can be achieved by adding k knots at either end of the interval of interest, so that (p= -k,...,n-1), but the B-spline is only constructed over the interval of interest. The positioning of the extra knots is arbitrary (provided they lie outside the interval of interest) and we follow Steeley (1989) in placing them at -3, -2, -1, 45, 50, and 60 years.[31]

It can be shown that every element of the space to be spanned can be described as a weighted linear combination of the bases such as:[32]

$$S(x) = \sum_{j=-k}^{n-1} \lambda_j B_j(x) \quad ...(3.5.2)$$

The cubic B-spline representation used in the Dierckx yield program (which is also used below) is slightly different and can be written as:

$$S(M) = \sum_{i=1}^{n-4} \lambda_i N_i(M) \quad ... (3.5.3)$$

Where M is the time to maturity, n is the number of knots, $N_i(M)$ is the normalised cubic B-spline and λ_i are the coefficients that are determined by the constrained minimisation problem described below. The normalised B-spline, $N_i(M)$, is uniquely defined except for a multiplicative constant. This constant can be used to normalise the B-spline such that the value of each B-splines is of similar size

30 The original program is BSPLVB in De Boor (1978) pp. 134-135.

31 See Steeley (1989) p.157.

32 See, for example, De Boor (1978) pp.115-120.

and, in fact, they lie between 0.25 and 1.[33] This normalisation has the advantage that the loss of accuracy that bedevils ordinary cubic splines is further reduced, although it is not necessary in the B-spline application of McCulloch's procedure.

The parameters λ_i can be found, for given knot positions, by minimising the sum of the squared residuals formed by subtracting the fitted values of the B-Spline from the actual value. This is known as the l_2 norm. Other norms could be used, such as the minimisation of the sum of the absolute errors or the minimisation of the largest error (minimax). The use of the l_2 norm is retained because this allows estimation to be undertaken by using ordinary least squares. It is also consistent with the methodology used by other researchers amongst them McCulloch (1975).

3.6 Endogenous Knot Positions.

A novel aspect of the yield curve estimation routine is that the choice of the knot positions is derived endogenously rather than exogenously by the researcher. The only other researchers to have investigated this in a controlled experiment are Langetieg and Smoot (1989), although others, such as Deacon and Derry (1994), have illustrated the effect of changing the knot positions. This enables the view (see, for example, the gilt prices page of the Financial Times and Steeley (1990)) that the gilt market can be divided into three segments: "shorts", "mediums" and "longs" by the placement of two interior knots to be examined.

Deacon and Derry (1994) note that the position of the knots in spline routines has not received much attention in the literature. This, they claim, is surprising because programs already exist to optimally place knots and they cite the work of de Boor (1978).[34] However, a closer reading of de Boor (1978) shows that he is by no means convinced that variable knot positions are particularly beneficial. He argues that: only an optimal distribution of knots can be found, not their individual optimal positions; there are potentially many stationary points to the objective functions, so that the outcome is only locally optimal; and it is computationally expensive, because of the non-linear nature of the problem.

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33 See Hayes (1974) p.149 for details.

34 Deacon and Derry (1994) p.33.

Therefore it is only warranted when precise placement of the knots is essential, because, for example, the data exhibit some discontinuities or when approximations with as few as possible parameters are required.[35] For these reasons De Boor concludes that it is just as effective, and computationally easier, to choose two or three times as many knots rather than attempt to set the knot positions optimally.

Given de Boor's comments we are interested in variable knot setting for three reasons. Firstly, the use of excessive knots can cause the fitted curve to weave, potentially causing implausible estimates from the forward rate curve. Secondly, it reduces the possibility that the estimated discount factors and yields are due to an arbitrary choice of knot settings. Thirdly, the positions of the knots found by these algorithms are of interest in their own right, as they may provide evidence about the existence, location and stability of "natural market boundaries".

The idea of natural market boundaries can be linked with the market segmentation and preferred habitats theories of Culbertson (1957), Roll (1970) and Modigliani and Sutch (1966). Due to hedging behaviour, perhaps through the need to match asset and liability maturities, different segments of the term structure are dominated by different types of organisations. If the states of nature which affect the inflow or outflow of funds from these institutions are different, or they react in different manners to the same states of nature, this should manifest itself in different segments of the term structure behaving in different ways. Thus minimising the residual sum of squares in spline programs may be accomplished by setting the knot positions at these maturity boundaries so that the segments are treated as being different. Evidence against such boundaries would be found if the knot positions were unstable. If "natural market boundaries" exist and their position can be determined this would influence the choice of which debt maturity ratios should be used to examine the effects of funding on the term structure in chapters 7 and 8.

Suchomski (1991) divides the variable knot fitting routines into those that start from an initial distribution of knots and add knots to minimise a stated objective function (of which Dierckx (1981) is an example) and those which are based on the proper variation of a given number of knots (such as de

35 See De Boor (1978) p.181 and p.272.

Boor's (1978) NEWNOT program).[36] This distinction is arbitrary as the application of Dierckx's program below shows. We begin by estimating endogenous knot positions for spot curves and then apply Dierckx's method to estimate redemption yield curves.

The program to estimate spot rates uses the methodology due to McCulloch (1971,1975) as implemented by Steeley and Clewlow (1993). This is augmented by a local integer search routine that moves the two interior knots (that are initially placed at five and ten years), so that the residual sum of squares is at a local minimum. In the light of de Boor's comments noted above, we do not operate a full optimal spline interpolation algorithm but the results of this local search procedure, discussed below, suggest that implementation of a full optimal spline interpolation algorithm would not be of great value.

The procedure examines four alternative combinations of knots by holding, in turn, one of the knot positions constant and then increasing and decreasing the other knot position by one year. The residual sum of squares is recorded and the procedure repeated. The combination of knots with the lowest residual sum of squares is then chosen as the starting point for the next iteration and the procedure is repeated until the residual sum of squares cannot be reduced further. An example makes this procedure clearer. Denote the initial starting point of five years and ten years as (5,10), then its residual sum of squares will be compared with those of (4,10), (6,10), (5,9) and (5,11). If, say, (5,11) has the smallest sum of squared residuals this is compared with (4,11), (6,11), (5,10) and (5,12). The procedure continues until the residual sum of squares cannot be reduced further. Each comparison is counted as one iteration. For each day in the sample the initial starting point is always knot positions of five and ten years as this starting point was recommended by Steeley (1989).

To make the hypothesis more specific we suggest that the presence of natural market boundaries would be expected to result in the density of knot positions being greatest at, say, five and ten years. As well the iterative process begins at five and ten years if the term structure is flat, and there is no obvious advantage in positioning the knots at any particular maturity, five and ten years would be chosen as the knot locations. For this reason we would again expect bunching around the five to ten year maturities.

36 See De Boor (1978) pp. 184-186.

We would expect these knot positions to be relatively stable over time as befits an institutional factor. The null hypothesis is that the location of the knots is determined by other factors, such as the distribution of gilts in the estimation procedure or by noise in gilt prices. If either, or both, of these are important determinants of the knot positions we would not expect to record an increased density of knots at five and ten years and there is no reason to expect the knot positions to be stable over time. We do not want to be any more specific about the hypothesis because this would take us into the micro-foundations of the gilts market. Rather, the exercise is best thought of as a fishing expedition for facts that future models of boundaries might have to explain - to claim anything more without a theoretical background is simply too heroic.

The average number of iterations performed per day was 16.4, with the maximum number being 59 and the minimum being five. The search procedure led to a reduction in the sum of squared residuals by an average of 54.2%, relative to the sum of squared residuals with knots set at five and ten years, with the largest reduction being 96.4% and the smallest being 7.5%. The results suggest that a substantial gain in fit can be achieved, even when the knot positions are endogenised using a procedure that finds a local integer minimum for the residual sum of squares. These results are consistent with those of Suchomski (1991) who found that, relative to equally spaced knot positions, endogenising the knot positions resulted in a reduction in the root of the sum of the squared residuals by between 83.7% and 98.4%.[37] On the other hand, Langetieg and Smoot (1989) report that the improvement in explanatory power when the knot positions are freely estimated was not sufficient to offset the loss of degrees of freedom in the adjusted sum of squared errors.[38] Although Langetieg and Smoot (1989) do not provide a description of their algorithm, the above result suggests that it is of the knot addition form, as is Dierckx (1981), rather than having a given number of knots and varying their location. Consequently, Langetieg and Smoot's result may not necessarily contradict those reported above.

The knot locations changed between days in 383 instances (12.6% of the 3036 days) and of these 61 (2%) of the changes were accompanied by changes in the gilts underlying the analysis (see table 3.A.1

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37 See Suchomski (1991) tables 1 and 2, p.2272.

38 See Langetieg and Smoot (1989) p.201.

in the appendix). In comparison with the 96 days on which the gilts underlying the analysis change the knot positions altered in 62.9% of the cases. Statistical evidence that the knot positions are not robust to the underlying gilts within the analysis is provided by the following contingency table (table 3.6.1).

Table 3.6.1 Expected and Observed Changes in Knot Positions Controlling for Changes in Gilts

	Change in at least 1 knot position	No Change in either knot position
Change in gilts included in data	61 (12.1)	35 (83.9)
No change in gilts included in data	322 (370.9)	2618 (2569.1)

Note: The figures in parentheses are the expected frequencies per cell if the proportion were drawn from the same distribution. These are denoted by e_i. The other figures are the actual observations that are denoted as o_i.

The following statistical formula tests whether the proportions in table 3.6.1are the same[39]:

$$\sum_{i=1}^{4}(o_i - e_i)^2 / e_i$$

This statistic is approximately distributed as χ^2 with 1 degree of freedom and the value of the test is 227.1 (the critical value for 0.5% is 7.88). This confirms that changing the gilts used in the spline routine results in a statistically significantly greater proportion of changes in the location of the knot positions than do daily changes in the maturity of the gilt and/or its' price.

Of the remaining 322 days (10.9% of 2940) when the knot locations change the difference in the knot position must be due to either changes in the price of some gilts or changes in the maturity of those gilts or both. Within the data set there are 93 bank holidays in which the prices of the gilts are set at the preceding days' prices. Thus for estimation purposes the only variable that changes is that the maturity of all gilts in the sample declines by one day. Of these 93 days, five were accompanied by the exclusion of gilts. Of the remaining 88 days, there were four instances (4.5%) of knots changing positions. These effects can be analysed in the following contingency table (table 3.6.2):

39 See Spiegel (1972) pp. 202-203.

Table 3.6.2 Expected and Observed Frequencies of Knot Positions after Controlling for Changes in Prices and Gilts

	Change in at least one knot position	No change in either knot position
No change in price of gilts	4 (9.6)	84 (78.4)
Change in price of gilts	318 (312.4)	2534 (2539.6)

Note: The figures in parentheses are the expected frequencies per cell if the proportion were drawn from the same distribution. These are denoted by e_i. The other figures are the actual observations that are denoted as o_i.

The χ^2 test gives a value of 3.78, which is just below the critical value of 3.84 at the 5% level of significance. Consequently, the null hypothesis that both proportions come from the same underlying distribution cannot be rejected. Thus we cannot rule out the hypothesis that a one-day change in the maturity of all the gilts in the sample can cause the knot positions to change. This suggests that endogenising the knot positions does not contribute to our understanding of the market's perceptions of "short", "medium" and "long" gilts.

Although the effects of changes in maturity and prices cannot be separated for the remaining 2852 days it is still of interest to consider whether some feature of the pricing structure leads to a change in the knot positions. Table 3.6.3 gives the mean of the average percentage change in the prices between one day and the next day, the mean of the absolute average percentage change in the prices and the mean of the standard deviation of the price changes. All of these are distinguished by whether or not at least one knot position changed.

Table 3.6.3 Changes in Gilt Prices by Changes in the Position of the Knots Change

	Mean	Absolute mean	Standard deviation
No change in knot positions	0.0045	0.3848	0.4083
At least one change in knot positions	0.0018	0.4076	0.4634
Test of statistical difference	0.11	-1.34	-2.58

The average percentage price change is smaller for days when there is a change in knot positions but this is due to offsetting positive and negative percentage changes. When the absolute means are used the average percentage changes in prices are larger when the knot position changes than when they remain constant. Whilst this is a plausible explanation of why the knot positions change it is not unambiguous. For instance, there could be large parallel shifts in the prices that would show up as large percentage price changes but there is little reason to believe that a parallel shift in prices would alter the positions of the knots. Moreover, the difference in the average absolute prices is so small, at less than 3p on a £100 gilt, that it is difficult to believe that this could cause a change in the knot positions. Indeed, a test of the difference of the means reveals that they are not statistically significantly different as they are well below the 5% level of significance of 1.96 for a two-tailed test.[40]

Table 3.6.4 Results of the Regression of the Standard Deviation on the Knot Positions

Dependent variable	Constant	Standard deviation	Standard deviation squared	Standard deviation cubed	$\overline{R}^2$
First knot	0.03 (0.61)	-0.07 (-0.65)	-	-	0.00
First knot	0.00 (-0.05)	0.13 (0.39)	-0.15 (0.63)	-	0.00
First knot	-0.08 (-0.66)	0.69 (0.92)	-1.00 (-0.96)	0.33 (0.84)	0.00
Second knot	0.06 (0.71)	-0.11 (-0.68)	-	-	0.00
Second knot	0.05 (0.36)	-0.04 (-0.07)	-0.06 (0.16)	-	0.00
Second knot	-0.12 (-0.65)	1.05 (0.36)	-1.89 (-1.21)	0.72 (1.21)	0.00
Combined knots	0.68 (4.81)	0.39 (1.48)	-	-	0.00
Combined knots	0.31 (1.48)	2.26 (2.76)	-1.42 (-2.41)	-	0.00
Combined knots	-0.14 (-0.47)	5.46 (3.01)	-6.28 (-2.48)	1.92 (1.98)	0.00

Note: The t-statistics of the regression coefficients are in parentheses. The combined knots are the sum of the absolute changes in both knots.

40 The large sample test used is given in Spiegel (1972) pp.170-171.

On the other hand, the hypothesis that the standard deviations of the percentage price changes are drawn from different distributions cannot be rejected at the 1% level of significance.[41] This result suggests that an increase in the variability of the price between individual days may be a reason why the knot positions change. This is examined further by regressing the change in the knot positions on various combinations of the standard deviations of price changes. These combinations included a linear term, a quadratic term and a cubic term. The dependent variables were the absolute changes in the position of the first knot, the second knot and the sum of the absolute changes in both knots. The results are presented in table 3.6.4 above.

Table 3.6.4 indicates that the standard deviation of the percentage price changes between days has no explanatory power to explain movements in the first or the second knot positions or in the sum of the absolute changes in knot positions. The results in table 3.6.4 suggest that summary measures of the behaviour of the prices of gilts are unable to explain, in a simple manner, the movement in the knot positions. In turn, this suggests that the movement in prices of just a few gilts may be sufficient to alter the positions of the knots, although we cannot discount the effects of changes in maturity as it was shown above how this can change knot positions irrespective of price changes.

These findings would not be serious if the movement in the knot positions were only a year or so either way. Tables 3.6.5 and 3.6.6 show that on 2659 (87.6%) and 2731 (90.0%) days for the first and second knots respectively there were no changes in the knot positions between days[42]. Moreover, 96.2% and 95.5% of days had changes in the first and second knots respectively that were between minus one year and plus one year. For the vast bulk of the days analysed the changes in the knot positions were, consequently, relatively small. At the other extreme just 0.1% and 1.4% of the days recorded changes in the knot positions, for first and second knots respectively, that were of 20 years or more in either direction. Although the knot positions are dependent upon changes in the maturities of a given set of gilts (irrespective of price changes) and are related to the variability of the price (but not in a simple

[41] Note that because the standard deviation figures are averages of the individual standard deviations the appropriate test is one for differences of means not an F-ratio test.
[42] Tables 3.6.5 and 3.6.6 treat the knots independently. If the knots are taken together there was changes in one or both knots on 383 days.

manner), tables 3.6.5 and 3.6.6 indicate that these weaknesses of the model do not lead to a significant amount of variation in the knot positions.

Table 3.6.5 Distribution of the Changes in the First Knot Position

Change in knot position (years)	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11
Observations	2	2	1	4	3	4	5	2	1	1
Change in knot position (years)	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
Observations	3	1	4	2	6	2	0	4	9	133
Change in knot position (years)	0	1	2	3	4	5	6	7	8	9
Observations	2659	126	10	5	1	5	4	3	5	0
Change in knot position (years)	10	11	12	13	14	15	16	17	18	19
Observations	1	1	1	1	7	9	2	3	1	2

Note: Initial knot positions set at five and ten years.

Table 3.6.6 Distribution of the Changes in the Second Knot Position

Change in knot position (years)	-30	-29	-28	-27	-26	-25	-24	-23	-22	-21
Observations	1	0	5	0	5	0	3	0	1	6
Change in knot position (years)	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11
Observations	0	0	4	1	0	0	1	0	1	1
Change in knot position (years)	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
Observations	3	0	3	2	6	1	5	10	6	90
Change in knot position (years)	0	1	2	3	4	5	6	7	8	9
Observations	2731	78	8	10	6	4	4	5	3	0
Change in knot position (years)	10	11	12	13	14	15	16	17	18	19
Observations	1	1	0	1	2	1	0	1	1	0
Change in knot position (years)	20	21	22	23	24	25	26	27	28	29
Observations	0	8	3	0	3	0	3	0	6	0

Note: Initial knot positions set at five and ten years.

Table 3.6.7 The Distribution of the First Knot

year	1	2	3	4	5	6	7	8	9	10
observations	0	0	0	11	96	99	14	356	60	324
year	11	12	13	14	15	16	17	18	19	20
observations	359	232	329	445	232	26	28	8	34	1
year	21	22	23	24	25	26	27	28	29	30
observations	0	0	0	7	67	142	99	36	23	8
year	31	32	33	34	35	36	37	38		
observations	0	0	0	0	0	0	0	0		

Table 3.6.8 The Distribution of the Second Knot

year	1	2	3	4	5	6	7	8	9	10
observations	0	0	0	0	0	11	96	99	14	356
year	11	12	13	14	15	16	17	18	19	20
observations	60	324	359	232	329	445	232	2	28	8
year	21	22	23	24	25	26	27	28	29	30
observations	34	22	2	0	0	0	0	0	0	0
year	31	32	33	34	35	36	37	38		
observations	8	0	0	7	7	2	4	355		

What tables 3.6.5 and 3.6.6 do not consider, however, is the cumulative nature of these small changes. Tables 3.6.7 and 3.6.8 show that the first knot is relatively evenly distributed between 8 and 15 years, with a further cluster of observations between 24 and 30 years. The second knot is, in broad terms, evenly distributed between 10 and 17 years. However, there is a further cluster at 38 years, a greater maturity for all but nine months of the sample period (see table 3.2.1). This may be indicating that only one interior knot is required to minimise the residual sum of squares. Alternatively, it may indicate that fitting long rate data is, on occasion, rather difficult, and this requires the presence of a knot at a long-maturity. The spread of the observations makes it impossible to claim that the positions of the knots reveal anything positive about the maturities that might separate "short", "medium" and "long" term gilts.

Tables 3.6.7 and 3.6.8 illustrate that there is a close correspondence, particularly in the early years, between the first knot and the second knot two years onwards. Indeed, it is not until after the first knot exceeds 16 years and the second knot 18 years that any difference in the series occurs. Out of the 3036 days, 2637 (86.9%) have knots that are two years apart, whilst the remaining 399 (13.1%) have knots which are greater than two years apart. There are no instances in this data set of the knots being closer than two years. The average knot positions when the difference is two years are 11.4 and 13.4 years, whist the average positions are 25.8 and 36.9 when the difference between the knot positions is greater than two years. For the majority of days a difference of just two years between the knots gives no room for the distinction of a "medium" set of gilts. The spacing of the knots when the difference is greater than two years would rule out any distinction into "shorts", "medium" and "long" gilts for these days. It is again concluded that the position of knots in the spline function does not provide quantitative evidence on where market participants may perceive natural market boundaries in the term structure.

The results also indicate that for those days when the difference in the position of the knots is greater than two years the average reduction in the residual sum of squares is 91.0%, compared with a reduction of 48.6% when the difference is two years. With knot positions being set at an average of 25.8 and 36.9 years, which is beyond the maturity of most gilts within the sample, the improved fit during these days seems to imply that having two interior knots is detrimental to the fit of the spline function. Most of the large spreads between the knot positions occur in the years 1987 to 1990 and this suggests that one cause of the change in bond spacing may be that the pricing structure was different from the period 1979 to 1986.

The pricing structure was examined by comparing the standard deviation of the gilt prices on days that the knot positions increased to be more than two years apart with the previous day when the knot positions were just two years apart, provided that the gilts used in the analysis had not changed.[43] The reverse cases when the spacing of the knots was reduced to two years were also analysed. Of the 45 instances when the knot positions changed (as described above) it was found that the standard deviation of the prices of gilts was slightly larger at 12.23 when the knot spacing was greater than two years compared with a standard deviation of 12.19 when the knot spacing was two years. A test of the hypothesis that there is no difference between the average standard deviations cannot be rejected at the 32% level of significance.[44] Consequently, the variability of the gilt prices cannot explain the abrupt changes in knot spacing. It is, therefore, unsurprising that the last three regressions reported in table 3.6.4 find no relationship between the absolute size of the changes in the knot positions and the change in the standard deviations of the prices. A further explanation of the differences between days with a two-year spread and those with a greater than two-year spread is examined below.

43 It is tempting to use a principal components analysis of the prices but the changes in maturity of the gilts rule this out.

44 The test statistic was 0.133 against the standard normal curve of 1.96 at the 5% level.

On day 787 of the sample (6 January 1982) the difference between the interior knots rose to 18 years, with the first knot being at 20 years and the second knot at 38 years. Both the preceding day and the subsequent day had the same knot positions of 15 and 17 years. Days 786, 787 and 788 therefore provide a relatively controlled experiment, because the gilts included remain the same and the maturity of the gilts has changed by only 2 days, for studying the effects of prices on the knot locations.

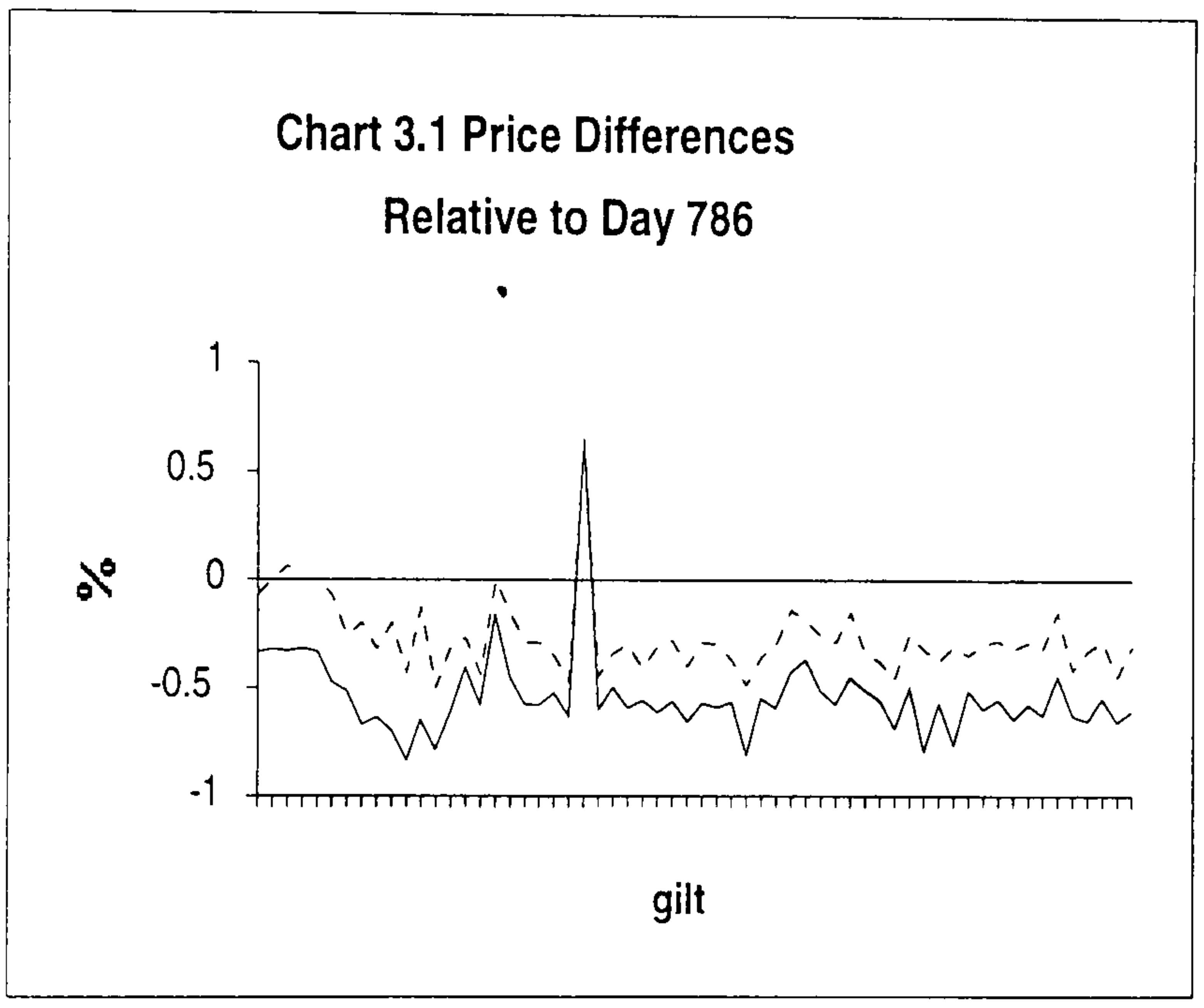


Chart 3.1 shows the percentage changes in the price of each of the 60 gilts included in the analysis relative to those on 5 January 1982. For the 6 January prices were on average 0.5% lower, ranging from +0.7% to -0.8%, and on the 7 January they were on average 0.3% lower, ranging from +0.7% to -0.5%, than on 5 January 1982. The only unusual feature of chart 3.1 is the increase in price of one gilt by 0.7% but as both days have the same increase in price, relative to 5 January, this cannot account for the change in knot positions between the 6 and 7 January 1982. The prices on the days around 6 January do not provide any indication of why the knot positions should change so significantly between these days. This tends to corroborate the view that the change in the knot positions are not informative about the market's perception of what constitutes "short", "medium" and "long" gilts.

One of the concerns of using the above method to search for knot positions is that the search ceases when a local minimum is found. Evidence that this is a problem comes from the significantly lower residual sum of squares achieved when the knot positions are widely spaced (i.e. when more iterations were performed) than when they are two years apart. Consequently, the analysis was repeated using as starting points knots at 20 years and 37 years, which were the average knot positions found when the difference between the knots was greater than two years. The objective is to find how many times the same local minimum is found as when starting knots of five and ten years where used. Following the extensive discussion above the analysis of knot positions is not believed to throw light upon the perceptions of "short", "medium" and "long" gilts and so the discussion is not repeated. Indeed with the starting knot positions being set as such high maturities it can be argued that a local search routine, such as employed here, simply cannot throw light on the "short", "medium" and "long" gilts question.

In view of these points one further alteration to the method discussed above is made. The previous results show the knot positions usually change very little between one day and the next. Consequently, the speed of the algorithm can be greatly increased by using the previous days' final solution as the initial starting point for the current day. This is provided that the difference in the knot positions is in excess of two years (i.e. that the algorithm has not become stuck with the suspected local minima). If the previous days knot positions were less than two years apart the current days' iterations begin from 20 and 37 years.

The analysis revealed that there was a large reduction in the residual sum of squares using the new search procedure described above, with the average reduction being 75.2% compared with the final solution when the search had always started from knot positions of five and ten years.[45] Clearly, local minima are present in the data and the technique must be evaluated from different starting points to achieve the best fit. This is a weakness in the method. As a consequence of the improvement in fit the location of the knots has also changed. Previously 2637 (86.9%) had a difference in knot positions of two years and this fell to just 71 (2.3%) when the higher starting positions for the knots were used. Although in the previous analysis the knot positions were never closer than two years, there were 43 (1.4%) instances of the knot positions being just one year apart with the new starting values. For the vast majority of days starting the search with higher knot positions resulted in 2922 (96.3%) having knots that were three or more years apart, compared to 13.1% when the starting points for the knots

[45] The maximum reduction in the residual sum of squares was 98.1% and the minimum was of no difference which occurred 355 (11.7%) times

were five and ten years. Searching from larger starting maturities therefore results in a greater separation of the knot positions.

These differences show up in the average knot positions for a difference of two years in the knots. The average position of the first knot is 25.3 years (see table 3.6.9) whereas in the search conducted from five and ten years it had been 11.4 years. When the knot positions are greater than two years apart the difference between the different starting points is less significant, with the higher starting knot positions averaging 26.3 years for the first knot compared with 25.8 when the starting point was five and ten years. For the second knot the corresponding knot positions were 37.8 years and 36.9 years. These knot positions are large relative to the starting positions recommended by Steeley (1990) and, as noted above, they throw no light upon the distinctions between "short", "medium" and "long" gilts.

The distribution of the second knot position (table 3.6.10) is more concentrated than when the algorithm was started at five and ten years with a range of knot positions between 20 and 38 years. The corresponding range for the knot positions calculated from a starting point of five and ten years is 6 to 38 years (see table 3.6.8). The first knot position is equally concentrated (see table 3.6.9) with a range of observations between 17 and 31 years compared with 4 to 30 years when they are determined from a starting point of five and ten years. The first knot's modal group is 26 years, although it is not as prominent as the second knot accounting for only 582 (19.2%) of the observations. The first knot position is much more stable than when the knot starting positions were five and ten years, with an absence of large jumps in the knot positions between days (except in 1979) (see table 3.6.11). The movement in the first knot is sufficient to ensure that the difference between the knots is more volatile over the middle years of the sample. However, the extreme volatility in the spacing of the knots in the latter years of the sample is absent.

The placement of the knots indicates that this method would only accept a distinction between "short" and "long" gilts because a knot of 38 years is greater than the greatest maturity used in the sample except for the first 282 days of the sample. However, with the first knot position being located between 17 and 31 years (see table 3.6.9), it also seems unlikely that this marks a "natural boundary".

Table 3.6.9 The Distribution of the First Knot

year	1	2	3	4	5	6	7	8	9	10
observations	0	0	0	0	0	0	0	0	0	0
year	11	12	13	14	15	16	17	18	19	20
observations	0	0	0	0	0	0	77	0	43	7
year	21	22	23	24	25	26	27	28	29	30
observations	80	82	132	288	517	582	438	272	227	174
year	31	32	33	34	35	36	37	38		
observations	117	0	0	0	0	0	0	0		

Note: Initial knot positions determined by the previous day's final knot positions or set at 20 years.

Table 3.6.10 The Distribution of the Second Knot

year	1	2	3	4	5	6	7	8	9	10
observations	0	0	0	0	0	0	0	0	0	0
year	11	12	13	14	15	16	17	18	19	20
observations	0	0	0	0	0	0	0	0	0	41
year	21	22	23	24	25	26	27	28	29	30
observations	15	64	4	2	10	20	9	1	1	1
year	31	32	33	34	35	36	37	38		
observations	13	10	6	1	9	23	35	2771		

Note: Initial knot positions determined by the previous day's final knot positions or set at 37 years.

Table 3.6.11 Distribution of the Changes in the First Knot Position

Change in knot position (years)	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11
Observations	0	0	0	0	0	0	0	0	1	0
Change in knot position (years)	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
Observations	0	0	0	0	0	0	0	0	3	237
Change in knot position (years)	0	1	2	3	4	5	6	7	8	9
Observations	2545	244	4	0	1	0	0	0	1	0
Change in knot position (years)	10	11	12	13	14	15	16	17	18	19
Observations	0	0	0	0	0	0	0	0	0	0

Table 3.6.12 Distribution of the Changes in the Second Knot Position

Change in knot position (years)	-30	-29	-28	-27	-26	-25	-24	-23	-22	-21
Observations	0	0	0	0	0	0	0	0	0	0
Change in knot position (years)	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11
Observations	0	0	0	0	0	1	0	1	2	1
Change in knot position (years)	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
Observations	0	0	0	1	2	1	0	3	8	27
Change in knot position (years)	0	1	2	3	4	5	6	7	8	9
Observations	2929	37	9	1	3	1	2	2	1	0
Change in knot position (years)	10	11	12	13	14	15	16	17	18	19
Observations	1	1	1	1	0	0	0	0	0	0
Change in knot position (years)	20	21	22	23	24	25	26	27	28	29
Observations	0	0	0	0	0	0	0	0	0	0

If the local minima were less stable than the global minima there would be more changes in knot positions when five and ten years were always used as the starting position than when the search begins from the previous days knot positions. However, the results point to the opposite conclusion. There are 551 changes in one or both of the knot positions when the previous day's knot positions were used as the starting values, although most are due to the first knot.[46] Of these 55 (compared to 61) were due to changes in the composition of gilts used in estimation. It has not been established that a global minimum has always been achieved by using the previous day's knot positions as the starting points (and is very unlikely to have done so). However, the large reductions in residual sum of squares suggest that the percentage of global minima will have increased. This suggests that the global minima are more unstable than local minima when looking at the number of movements. However, comparing tables 3.6.5 and 3.6.11 it can be seen that the sizes of the changes in the first knot are much smaller when the algorithm is allowed to search from the previous day's solution than when it always starts from five and ten years. This result suggests that, in terms of size of movement, global minima are more stable than local minima.

46 The corresponding number for the five and ten year starting values was 383.

The results of the use of the new starting points are straightforward. First, the model fits more closely. Second, although the knots move more often, the maturity moved is smaller. Third, the separation between the knots is greater. Fourth, the dispersion of the knot positions is smaller. Fifth the knot positions suggest that the only possible distinction is between "short" and "long" gilts. However, the dispersion of the first knot position is still large and it is centred around a high maturity so that either the distinction between "short" and "long" gilts has no behavioural content, or that the distinction is too subtle to show up in the positioning of the knots.

3.7 The Interpolation of Redemption Yields Using the Dierckx Method

In this section we use redemption yields as the data. It must be stated that the redemption yield data has no relation to spot rate curves, they have no economic foundation or interpretation, and are simply a set of data with which to test the Dierckx (1981) method. The estimates of knot positions derived from the method using redemption yields do not themselves add to the discussion of "natural market boundaries". The Dierckx method can be used to estimate spot rates, but this requires a two step procedure. The estimation is intensive in its use of computer time. Therefore, using the procedure on yields seemed a reasonable step to evaluate the usefulness of the method. It was found that the Dierckx method is unlikely to make a significant contribution to the discussion of "natural market boundaries" and so estimation on spot rates was not undertaken.

The estimation procedure is that described in Dierckx (1981) and implemented as NAG routine E02BEF in NAG (1991a). This routine minimises the sum of the squared discontinuities of the third order derivatives of the function at the interior knots, subject to the sum of the squared residuals being less than or equal to some arbitrary amount, S.[47] If S is set too low the function may contain too much noise and in an extreme case, when S=0, it will return an interpolating spline that simply passes through all the data points. Alternatively, if S is set too high the algorithm returns a weighted least squares cubic polynomial, i.e. it removes all the interior knots thereby eliminating any discontinuities in the third order derivatives. This occurred on one occasion for day 1567 (2 January 1985). The final value

[47] A related smoothing spline method has been proposed by Fisher et al (1995) and developed into a variable smoothness criterion by Waggoner (1997).

of S is an extremely close approximation to the weighted sum of the squared residuals, unless the algorithm returns a cubic polynomial. As S is set exogenously the squared residuals cannot be used as a diagnostic tool and they are not reported.

The algorithm begins by estimating a cubic spline that minimises the sum of squared residuals without any interior knots. If the sum of squared errors exceeds the criterion, S, a knot is added at the point where the fit is poorest. The procedure is repeated until the sum of squared residuals is less than S. This methodology chooses the number and position of the interior knots. In the application described below the number of interior knots is constrained to be no more than three, dividing the maturities into four sub-intervals. This is achieved by allowing S to alter so that the number of interior knots is either two or three. For 65 days (2.1%) the algorithm failed to meet this criterion after 1000 iterations and the number and positions of the knots on the last iteration were accepted as the final estimates. Once the number of knots and their positions was determined the algorithm calculates the set of coefficients of the normalised B-splines (see (3.5.3) above) which minimises the discontinuity in the third derivative.

The algorithm requires an initial value of S with which to start the process. Reinsch (1967) suggests that a good starting point can be found by using:

$$\text{var}(Y)*(M-(2*M)^{0.5}) \quad ...(3.7.1)$$

Where M is the number of gilts and var(Y) is the variance of the redemption yields. It was found, however, that (3.7.1) gave estimates of S that were too large and, therefore, constantly produced a weighted least squares cubic polynomial. Consequently, (3.7.1) was not used. Rather the value of S from the preceding day was used as the starting point for the iteration process. For the first day, 2 January 1979, a starting value of S that was half way between the values that produced a weighted least squares cubic polynomial and the value that produced an interpolating spline was used. The final value of S was determined by the iterative procedure. This involved solving the model for the given value of S, reducing that value by 1% if the number of interior knots was one or less or increasing it by 1% if the number of interior knots was greater than three. The program also produces eight exterior knots. The first four are positioned at the shortest maturity gilt in the sample and the last four are positioned at the largest maturity. All the interior knots lie between these exterior knots and their position will be determined by the highest and lowest maturities of the gilts within the sample. For all samples the highest maturity is that of EX12Z17 which begins with 38.9 years and finishes with 27.3 years. The shortest maturity varies (see the appendix to this chapter for details) but is never less than one year.

For 1657 days in the sample (54.6%) the procedure finds that two interior knots are optimal, whilst for 1314 days (43.3%) three interior knots are optimal. The remaining 65 days, 34 had one interior knot, 30 had four interior knots and one had no knots at all. On average using Dierckx's method the first interior knot was positioned at 5.9 years, the second at 10.9 years, the third at 13.8 years and the fourth at 15.5 years. However, these figures may be distorted because the number of knots changes over time and because the maturity of the largest gilt, and hence the position of four largest exterior knots, falls by 11.5 years during the sample period. The first of these distortions can be examined by looking at the knot positions for a given number of knots (see table 3.7.1)

Table 3.7.1 Knot Positions by Number of Knots

Number of Knots	Number of Observations	First Knot (years)	Second Knot (years)	Third Knot (years)	Fourth Knot (years)
0	1	-	-	-	-
1	34	9.8	-	-	-
2	1657	6.7	12.4	-	-
3	1314	4.9	9.0	13.9	-
4	30	4.5	7.5	11.8	15.5
average	3036	5.9	10.9	13.8	15.5

Note: Dierckx method used on redemption yields.

There are a number of points to note. Firstly, as more knots are added the position of the original knots declines and the extra knot is, on average, always at a greater maturity. Consequently, the average knot positions, given in the last row of table 3.7.1, are misleading, as there does not appear to be any preferred knot locations. Secondly, the spacing between the knots is relatively even and declines as the number of knots increases. Thus Dierckx's method concentrates its knot locations towards the middle and shorter maturities where more of the gilts lie. This raises the question of whether or not the use of Dierckx's method results in equal numbers of gilts between knot positions, which is the modelling strategy recommended by McCulloch. Table 3.7.2 reports the average percentage of gilts in each subsection and a χ^2 test statistic for each row that tests whether the observed frequencies are statistically different from the expected frequencies. The expected frequencies are 50%, 33.33%, 25% and 20% for each row respectively. The χ^2 statistics indicate that the hypothesis that the gilts are evenly spaced between the knots cannot be rejected at the 95% level of significance.

Table 3.7.2 Percentage of Gilts between the Interior Knots by Number of Knots

Number of Interior Knots	First Subsection	Second Subsection	Third Subsection	Fourth Subsection	Fifth Subsection	χ^2 Test Statistic
1	49.79	50.21	-	-	-	0.002 (3.84)
2	34.91	24.44	40.65	-	-	4.05 (5.99)
3	22.77	19.44	23.49	34.30	-	4.99 (7.81)
4	24.90	18.58	18.80	16.79	20.94	1.93 (9.49)

Note: The χ^2 95% critical value is given in parentheses.

To counteract the second distortion, that is the position of the knots is altered by the decline in the longest maturity in the sample, the analysis used in table 3.7.1 is repeated but each knot position is divided by the function:

$$(38.93083-0.00383*I)/(38.93083-0.00383*1518)$$

Where I is the day identifier that takes a value of 1 at the start of the sample (2 January 1979) and 3036 on the last day of the sample (21 August 1990). The value of 1518 is the value of the middle day of the sample (25 October 1984) and 38.93083 is the maturity of the longest bond in years on the first day of the sample. Consequently, the numerator is the maturity of the longest gilt at the middle point of the sample. As I increases the value of the function falls from 1.175 to 0.824. Thus, relative to the start of the sample, this function gives a 30% reduction in its size, the same as the reduction in the maturity of the largest gilt in the sample. By dividing the knots by this function the effects of the decline in the position of the upper exterior knots can be counteracted. The results are reported in table 3.7.3.

Table 3.7.3 Knot Positions Relative to that of the Largest Maturity by Number of Knots

Number of Knots	Number of Observations	First Knot (years)	Second Knot (years)	Third Knot (years)	Fourth Knot (years)
0	1	-	-	-	-
1	34	10.2	-	-	-
2	1657	6.9	12.6	-	-
3	1314	4.9	8.9	13.7	-
4	30	5.0	8.3	12.7	16.7
average	3036	6.0	10.9	13.7	16.7

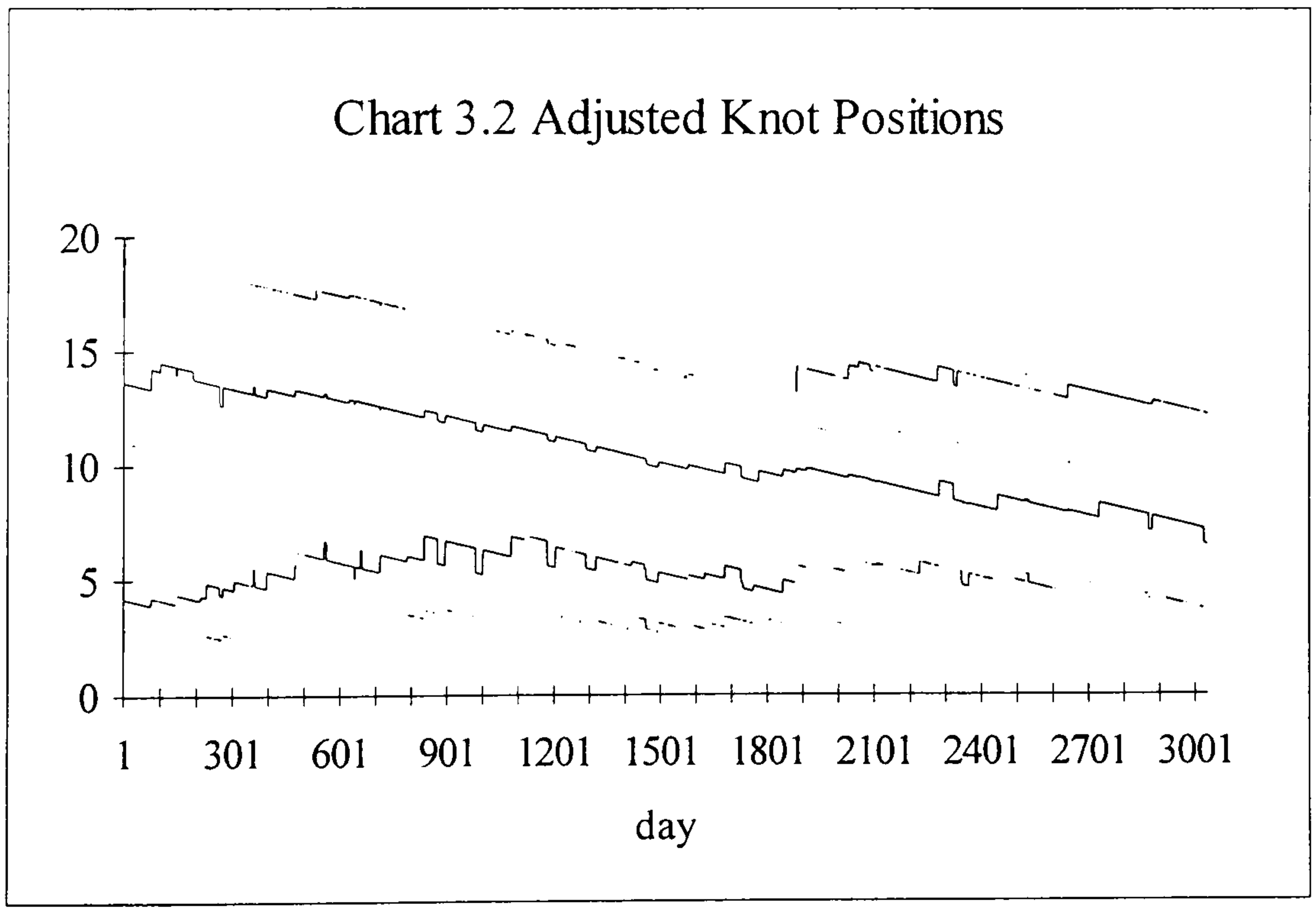
Note: Dierckx method used on redemption yields.

The effects of this weighting scheme are to leave the conclusions drawn from the unweighted data virtually unchallenged. Although, relative to the three interior knot case, the first knot position does slightly increase when a fourth knot is added. Nevertheless, the overall conclusion, that there does not appear to be any preferred knot locations, remains intact.

Comparing tables 3.7.1 and 3.7.3 when the number of interior knots is three, the knot positions are smaller when the data are weighted than unweighted, indicating that the three knot yield curves tend to come from earlier in the sample, whereas the opposite is true of the other yield curves. This was confirmed by calculating the average day, using one at the start of the sample and 3036 on the last day of the sample, for the yield curves by the number of interior knots. The average day length for yield curves with three interior knots is 1401.8 (i.e. under half way through the sample) whereas for two interior knots it is 1591.2 (i.e. above half way through the sample). A t-test gave a value of 5.92, indicating that the means are statistically different at the 1% level of significance. Moreover, the yield curves with one knot and four knots have average sample positions of 1971.9 and 2101.4 respectively (i.e. much higher than the average sample position of 1518.5). Thus, the latter half of the sample shows less stability than the first half with a greater variability in the number of knots selected. This result is consistent with the estimation of the spot rates when the starting positions were always five and ten years, which also found increased instability in the second half of the sample.

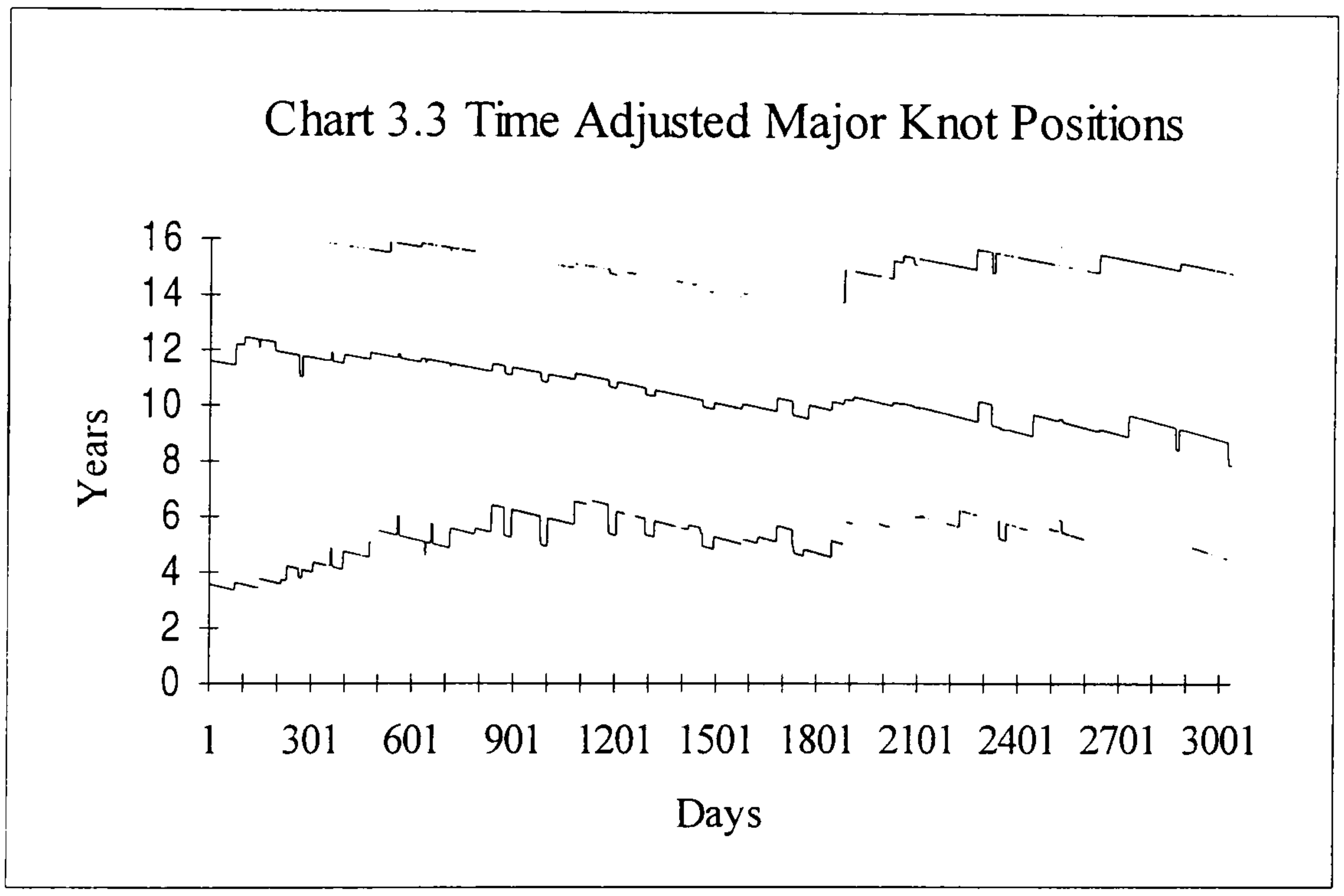
Another way of looking at instability is to find how many times the number of estimated knot positions changed between one day and the next. Of the 3035 days on which the knots could change there were 969 (31.9%) changes in the number of knots. Of these 39 (40.6%) were on one of the 96 days on which the data changed because of the addition or exclusion of gilts in the sample. Ignoring these changes the number of knots still changed in 31.6% of the observations. The immediate conclusion to be drawn is that the number of knot positions is not stable over time.

Furthermore, even if the number of knots is constant it does not necessarily mean that they are in the same position. Allowance has to be made for the fact that all of the gilts mature by one day between observations and so changes of 0.004 of a year are ignored. It was found that, of the 2066 observations when the number of knots remained constant, there were 173 (8.5%) changes in their positions. Of these changes, 90 were greater than one year, whilst 40 were just two or three days. The rest of the changes appeared to be evenly spread between these extremes. This indicates that the major source of instability comes from changes in the number of knots, and not from the position of the knots once the number has been decided.



This is illustrated by chart 3.2. The data used to form this chart are the knot positions described in the above section reordered so that they came from six different knot positions. This was accomplished by hand with the aim being to produce the most stable knot positions possible. The results as shown in chart 3.2 are rather striking. Adjusted in this manner there does appear to be two relatively stable knot positions at six years and between 14 to 12 years, until April 1986 and at eight to ten years and 17 years in the second half of the sample. Chart 3.2 suggests that there may be a further two knot positions that are intermittent giving six segments to the term structure. The regularity of their position seems to argue against them being simply noise.

Removing the distortion caused by the shortening of the longest maturity gilt during the sample period, by dividing by the expression (3.7.2), the central knot positions still show a downward trend over time (see chart 3.3). The downward trends in the shortest and the longest maturity knots have been removed by the adjustment. Nevertheless, the adjustment clearly, leaves some problems to be resolved. Does the term structure have four segments or does it just have three? If the latter, why does the position appear to change in April 1986?



The major change in the knots being in April 1986 rules out one possible explanation for the shift the introduction of dual capacity dealing and other changes in the UK gilt market (known as Big Bang) on 26 October 1986. We suspect that the results portrayed in charts 3.2 and 3.3 are due to a combination of factors. Firstly, there is an element of them being "a trick of the eye", whereby the ability of the brain to construct patterns has constructed regular patterns where none actually exist. Secondly, they may be artefacts of the method used to construct the knot position. As the target residual sum of squares, S, is set at the previous day's value for the first iteration, this may result in the method locating the knot positions at the same maturity for a period of days. The method itself may therefore produce stable knot locations. For these reasons we remain unconvinced that this technique would provide any evidence of natural market boundaries in the UK gilt market if applied to spot rates. Consequently this application was not undertaken.

3.8 Summary and Conclusion

This chapter has reviewed a number of approaches to modelling yield curves. It has applied one of the better known methods, devised by McCulloch, to British Government gilts using daily data for the period 1979 to 1990. The chapter has contributed to the literature by enhancing the McCulloch approach to allow the knot positions to be determined endogenously. It has examined the use of this approach and described the problems, the finding of local minima, and the advantages, which are much improved fits. This was used to analyse whether or not there are natural market boundaries present in the gilts market. The results suggested that the knot positions were highly volatile and that, therefore, the distinction between "short", "medium" and "long" gilts was of little value. Overall the results do not suggest that endogenising the knot positions provides any strong evidence that there exist natural boundaries in the yield curve. Hence, the knot positions are of no use in setting the maturity bands for the study of funding effects in chapters 7 and 8. However, just because the method failed to find evidence of market boundaries this does not mean that they are no present. It may be the case that a more subtle technique than knot location in spline curves is required to find them.

An alternative method of endogenising the knot positions, due to Dierckx (1981), was used on redemption yields. The results suggested that an application to spot rates would not be worthwhile as spacing the knots so that equal numbers of gilts lie in each segment seems acceptable. As this has been suggested and implemented by McCulloch repeating the exercise to calculate spot rates seems unjustified. The work reported in this chapter has also succeeded in its aim of providing spot and yield data at constant maturities. These data will be used in the principal component analysis conducted in chapters 5 and 6. Before this is undertaken chapter 4 surveys previous work using principal components. It describes the theory behind the method, some of its extensions, such as common principal components, and some of the statistical tests and descriptive statistics that will be used in chapters 5 and 6.

Appendix 3.1 The Gilts used to Estimate the Yield Curves

Table 3.A.1 The Gilts used to Estimate the Yield Curves

Dates	Number of Days	Number of Gilts	Gilts Included in the Yield Curves by Numbers
02.01.79 to 17.04.79	76	47	1-14, 16, 22, 25, 31, 38, 41, 42, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-75, 77, 82, 91, 97, 98.
18.04.79 to 21.05.79	24	46	2-14, 16, 22, 25, 31, 38, 41, 42, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-75, 77, 82, 91, 97, 98.
22.05.79	1	47	2-14, 16, 22, 25, 31, 38, 41, 42, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-75, 77, 82, 90, 91, 97, 98.
23.05.79 to 20.07.79	43	48	2-14, 16, 22, 25, 31, 38, 41, 42, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-75, 77, 82, 87, 90, 91, 97, 98.
23.07.79 to 25.07.79	3	49	2-14, 16, 22, 25, 27, 31, 38, 41, 42, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-75, 77, 82, 87, 90, 91, 97, 98.
26.07.79 to 27.08.79	23	50	2-14, 16, 22, 25, 27, 31, 38, 41, 42, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-75, 77, 82, 85, 87, 90, 91, 97, 98.
28.08.79 to 11.09.79	11	51	2-14, 16, 22, 25, 27, 31, 38, 41, 42, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-77, 82, 85, 87, 90, 91, 97, 98.
12.09.79 to 26.09.79	11	52	2-14, 16, 17, 22, 25, 27, 31, 38, 41, 42, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-77, 82, 85, 87, 90, 91, 97, 98.
27.09.79 to 25.10.79	21	53	2-14, 16, 17, 19, 22, 25, 27, 31, 38, 41, 42, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-77, 82, 85, 87, 90, 91, 97, 98.
26.10.79 to 30.10.79	3	54	2-14, 16, 17, 19, 22, 25, 27, 31, 38, 41, 42, 43, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-77, 82, 85, 87, 90, 91, 97, 98.
31.10.79 to 16.11.79	13	53	3-14, 16, 17, 19, 22, 25, 27, 31, 38, 41, 42, 43, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-77, 82, 85, 87, 90, 91, 97, 98.
19.11.79 to 04.01.80	35	54	3-14, 16, 17, 19, 20, 22, 25, 27, 31, 38, 41, 42, 43, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-77, 82, 85, 87, 90, 91, 97, 98.
07.01.80 to 18.01.80	10	55	3-14, 16-20, 22, 25, 27, 31, 38, 41, 42, 43, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-77, 82, 85, 87, 90, 91, 97, 98.
21.01.80 to 22.01.80	2	54	4-14, 16-20, 22, 25, 27, 31, 38, 41, 42, 43, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-77, 82, 85, 87, 90, 91, 97, 98
23.01.80 to 22.02.80	23	55	4-14, 16-20, 22, 25, 27, 31, 38, 41, 42, 43, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-77, 82, 85, 87, 90-92, 97, 98
25.02.80 to 04.03.80	7	56	4-14, 16-20, 22, 25, 27, 31, 34, 38, 41, 42, 43, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-77, 82, 85, 87, 90-92, 97, 98
05.03.80 to 15.05.80	52	55	5-14, 16-20, 22, 25, 27, 31, 34, 38, 41, 42, 43, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-77, 82, 85, 87, 90-92, 97, 98
16.05.80 to 22.05.80	5	54	6-14, 16-20, 22, 25, 27, 31, 34, 38, 41, 42, 43, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-77, 82, 85, 87, 90-92, 97, 98
23.05.80 to 08.07.80	33	55	6-20, 22, 25, 27, 31, 34, 38, 41, 42, 43, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-77, 82, 85, 87, 90-92, 97, 98
09.07.80 to 22.07.80	10	54	7-20, 22, 25, 27, 31, 34, 38, 41, 42, 43, 45, 46, 48, 52, 53, 55, 58-62, 64-69, 72-77, 82, 85, 87, 90-92, 97, 98
23.07.80 to 22.09.80	44	55	7-20, 22, 25, 27, 31, 34, 38, 41, 42, 43, 45, 46, 48, 52, 53, 55, 58-69, 72-77, 82, 85, 87, 90-92, 97, 98
23.09.80 to 26.09.80	4	56	7-20, 22, 25, 27, 31, 34, 38, 41, 42, 43, 45, 46, 48, 49, 52, 53, 55, 58-69, 72-77, 82, 85, 87, 90-92, 97, 98
29.09.80 to 24.10.80	20	57	7-20, 22, 25, 27, 31, 34, 38, 41, 42, 43, 45, 46, 48, 49, 52, 53, 55, 58-69, 72-77, 82, 85, 87, 90-92, 94, 97, 98
27.10.80 to 28.10.80	2	58	7-20, 22, 25, 27, 31, 34, 38, 41, 42, 43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 82, 85, 87, 90-92, 94, 97, 98
29.10.80 to 03.11.80	4	57	8-20, 22, 25, 27, 31, 34, 38, 41, 42, 43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 82, 85, 87, 90-92, 94, 97, 98

04.11.80 to 14.01.81	52	58	8-20, 22, 25, 27, 30, 31, 34, 38, 41, 42, 43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 82, 85, 87, 90-92, 94, 97, 98
15.01.81 to 17.02.81	24	59	8-20, 22, 25, 27, 30, 31, 34, 38, 41, 42, 43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 82, 85, 87, 90-92, 94, 97, 98
18.02.81 to 25.02.81	6	58	8, 10-20, 22, 25, 27, 30, 31, 34, 38, 41, 42, 43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 82, 85, 87, 90-92, 94, 97, 98
26.02.81 to 25.05.81	63	59	8, 10-20, 22, 23, 25, 27, 30, 31, 34, 38, 41, 42, 43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 82, 85, 87, 90-92, 94, 97, 98
26.05.81 to 12.06.81	14	60	8, 10-20, 22, 23, 25, 27, 30, 31, 34, 38, 41, 42, 43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
15.06.81 to 17.06.81	3	61	8, 10-20, 22-25, 27, 30, 31, 34, 38, 41, 42, 43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
18.06.81 to 07.07.81	14	60	10-20, 22-25, 27, 30, 31, 34, 38, 41, 42, 43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
08.07.81 to 15.07.81	6	59	11-20, 22-25, 27, 30, 31, 34, 38, 41, 42, 43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
16.07.81 to 23.09.81	50	60	11-25, 27, 30, 31, 34, 38, 41, 42, 43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
24.09.81	1	61	11-25, 27, 30, 31, 34, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
25.09.81 to 07.01.82	75	60	12-25, 27, 30, 31, 34, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
08.01.82 to 16.03.82	48	59	13-25, 27, 30, 31, 34, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
17.03.82 to 06.05.82	37	58	14-25, 27, 30, 31, 34, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
07.05.82 to 07.06.82	22	59	14-27, 30, 31, 34, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
08.06.82 to 04.10.82	85	58	15-27, 30, 31, 34, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
05.10.82 to 02.11.82	21	59	15-28, 30, 31, 34, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
03.11.82 to 22.02.83	80	57	17-28, 30, 31, 34, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
23.02.83 to 27.04.83	46	56	18-28, 30, 31, 34, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
28.04.83 to 13.07.83	55	55	19-28, 30, 31, 34, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
14.07.83 to 15.08.83	23	57	19-28, 30-34, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
16.08.83 to 12.12.83	85	56	20-28, 30-34, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
13.12.83 to 20.01.84	29	57	20-28, 30-35, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
23.01.84 to 11.06.84	101	56	21-28, 30-35, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
12.06.84 to 30.07.84	35	55	22-28, 30-35, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
31.07.84 to 06.08.84	5	56	22-28, 30-36, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
07.08.84 to 21.09.84	34	57	22-36, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 72-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
24.09.84 to 12.10.84	15	58	22-36, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 71-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98

15.10.84 to 17.01.85	69	57	23-36, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 71-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
18.01.85 to 25.03.85	47	56	24-36, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 71-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
26.03.85 to 11.06.85	56	55	25-36, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 71-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
12.06.85 to 14.08.85	46	54	26-36, 38, 40-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 71-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
15.08.85 to 22.08.85	6	55	26-36, 38-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 71-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
23.08.85 to 01.10.85	28	56	26-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 71-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
02.10.85 to 21.10.85	14	55	27-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 71-77, 79, 81, 82, 85, 87, 90-92, 94, 97, 98
22.10.85 to 23.12.85	45	56	27-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 71-77, 79, 81, 82, 85, 87, 89-92, 94, 97, 98
24.12.85 to 24.01.86	24	57	27-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 71-77, 79, 81, 82, 85, 87-92, 94, 97, 98
27.01.86 to 14.03.86	35	56	28-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 71-77, 79, 81, 82, 85, 87-92, 94, 97, 98
17.03.86 to 18.03.86	2	55	29-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 71-77, 79, 81, 82, 85, 87-92, 94, 97, 98
19.03.86 to 26.03.86	6	56	29-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 71-77, 79, 81-83, 85, 87-92, 94, 97, 98
27.03.86 to 25.04.86	22	57	29-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-69, 71-77, 79-83, 85, 87-92, 94, 97, 98
28.04.86 to 15.05.86	14	58	29-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-77, 79-83, 85, 87-92, 94, 97, 98
16.05.86 to 07.10.86	103	57	30-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-77, 79-83, 85, 87-92, 94, 97, 98
08.10.86 to 27.10.86	14	56	31-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-77, 79-83, 85, 87-92, 94, 97, 98
28.10.86 to 17.10.86	15	57	31-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-77, 79-83, 85, 87-92, 94, 96-98
18.10.86 to 31.12.86	32	58	31-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-77, 79-83, 85-92, 94, 96-98
01.01.87 to 07.01.87	5	57	31-43, 45, 46, 48, 49, 52, 53, 55, 56, 58-77, 79-83, 85-90, 92, 94, 96-98
08.01.87 to 14.01.87	5	58	31-43, 45, 46, 48, 49, 51-53, 55, 56, 58-77, 79-83, 85-90, 92, 94, 96-98
15.01.87 to 28.01.87	10	59	31-43, 45, 46, 48, 49, 51-53, 55, 56, 58-77, 79-83, 85-90, 92-94, 96-98
29.01.87 to 16.04.87	56	58	32-43, 45, 46, 48, 49, 51-53, 55, 56, 58-77, 79-83, 85-90, 92-94, 96-98
17.04.87 to 15.07.87	64	57	33-43, 45, 46, 48, 49, 51-53, 55, 56, 58-77, 79-83, 85-90, 92-94, 96-98
16.07.87 to 22.09.87	49	58	33-43, 45, 46, 48, 49, 51-53, 55-77, 79-83, 85-90, 92-94, 96-98
23.09.87 to 30.09.87	6	59	33-43, 45, 46, 48, 49, 51-53, 55-77, 79-83, 85-90, 92-98
01.10.87 to 25.11.87	40	58	34-43, 45, 46, 48, 49, 51-53, 55-77, 79-83, 85-90, 92-98
26.11.87 to 01.12.87	4	57	34-43, 45, 46, 48, 49, 51-53, 55-77, 79-83, 85-90, 92-94, 96-98
02.12.87 to 08.12.87	5	58	34-43, 45-49, 51-53, 55-77, 79-83, 85-90, 92-94, 96-98

09.12.87 to 29.12.87	15	59	34-43, 45-49, 51-53, 55-77, 79-90, 92-94, 96-98
30.12.87 to 31.12.87	2	60	34-49, 51-53, 55-77, 79-90, 92-94, 96-98
01.01.88 to 26.01.88	18	59	34-49, 51-53, 55-68, 70-77, 79-90, 92-94, 96-98
27.01.88 to 11.03.88	33	58	35-49, 51-53, 55-68, 70-77, 79-90, 92-94, 96-98
14.03.88 to 17.05.88	47	59	35-49, 51-53, 55-68, 70-90, 92-94, 96-98
18.05.88 to 05.07.88	35	58	36-49, 51-53, 55-68, 70-90, 92-94, 96-98
06.07.88 to 01.09.88	42	57	37-49, 51-53, 55-68, 70-90, 92-94, 96-98
02.09.88 to 14.09.88	9	56	38-49, 51-53, 55-68, 70-90, 92-94, 96-98
15.09.88 to 02.01.89	78	57	38-53, 55-68, 70-90, 92-94, 96-98
03.01.89 to 17.01.89	11	58	38-68, 70-90, 92-94, 96-98
18.01.89 to 14.02.89	20	57	39-68, 70-90, 92-94, 96-98
15.02.89 to 24.03.89	28	56	40-68, 70-90, 92-94, 96-98
27.03.89 to 19.06.89	61	55	41-68, 70-90, 92-94, 96-98
20.06.89 to 29.12.89	139	54	42-68, 70-90, 92-94, 96-98
01.01.90 to 12.01.90	10	53	42-68, 70-73, 75-90, 92-94, 96-98
15.01.90 to 06.08.90	146	52	43-68, 70-73, 75-90, 92-94, 96-98
07.08.90 to 21.08.90	10	51	43-68, 70-73, 75-81, 83-90, 92-94, 96-98

Note: The gilt numbers in column four of the table refer to those assigned in table 3.2.1.

Chapter 4

Theoretical Considerations of Principal Components

4.1 Survey of Previous Principal Components Decompositions of Term Structure Data

This chapter has three main aims. First, to critically review previous work using principal components on term structure data. Second, to describe the method of principal components. Third, to describe a series of statistical tests that can be applied to principal component estimates. The original contribution to the literature from this chapter is derived from the survey, which, to my knowledge, has never previously been undertaken. The motivation for discussing the test procedures is that previous studies using principal components analyses of interest rates have completely ignored the information that such tests can provide. A number of the tests are applied in chapters 5 and 6 to the data sets constructed in chapter 3. The appendix to this chapter discusses some alternative methods to principal components and why they have not been used in this thesis.

One area not covered by this survey is the use of principal components to analyse the extent of financial integration between countries or blocs of countries. Examples of papers from this area of research include White and Woodbury (1980), Nellis (1982), Helbling and Wescott (1995) and Pentecost and Holmes (1995a, 1995b). What distinguishes the above papers from those surveyed in table 4.1.1 is that they select a single maturity to compare across countries, or at best a short and a long maturity, rather than exploiting the whole term structure. As this thesis is concerned with the term structure the financial integration papers are not discussed in this chapter. However, a test of financial integration that exploits the whole term structure, Partial Common Principal Components (PCPC), is used to examine financial integration between the US and UK in chapter 6.

The motivation behind a chapter devoted almost entirely to the theoretical and statistical considerations of principal components analysis is simply that, as the survey below reveals, such considerations, in particular the statistical tests appear to be completely absent in previous work. It is to the survey of previous work that we now turn.

Despite the age of principal components analysis, and its attractiveness as a method of reducing the dimensions associated with movements in yields, discount rates, interest rates and bond prices, there have been relatively few studies that have used this technique. In this section these studies are reviewed with the aim of elucidating the various methodologies employed so that these can be subjected to statistical testing in chapters 5 and 6.

Table 4.1.1 sets out the main background features of the principal components studies. That a table can be used to summarise the information is a clear indication that this form of analysis is in its infancy. There are two main results that are claimed from these studies. These are:

First, that for the UK, Canada and USA at most three principal components are sufficient to describe virtually all the variation in term structure data. The smallest percentage (where a percentage, or the means of calculating one, are reported in the papers listed in table 4.1.1) that the first three components account for is 93% in the paper by Egginton and Hall (1994). The implication of this finding is that only three variables have to be explained before a comprehensive explanation of the movements in the term structure is available. Indeed, even if only the first component were understood this would account for an impressive level of explanatory power for the term structure, 85% in the case of Egginton and Hall (1994).

On the other hand, Garbade and Urich (1988) suggest that the explanatory power of the first two components using West German and Japanese data is rather lower than in the other countries they analysed, at 91% and 83% respectively. They conclude that this "implies the presence of quantitatively important higher order modes of fluctuation".[1] Garbade and Urich (1988) offer the conjecture that because the short end of the yield curves in these countries are thinly traded and short sales are more expensive, "bumps and wiggles" in the yield curve can persist through time and these are picked up as higher order components.[2] However, they provide no evidence to support these conjectures. Indeed, the explanatory power for West Germany is not much lower than that reported by Egginton and Hall for

[1] Garbade and Urich (1988) p.6.
[2] Garbade and Urich (1988) p.4.

the UK. The results in chapter 5 suggest that the data used by Egginton and Hall (1994) are relatively noisy and this may explain the similarity in explanatory powers.

The second main finding is that the first principal component represents parallel shifts in the term structure, the second principal component represents changes in the slope of the term structure, whilst the third component measures what are termed "curvature" by Litterman and Scheinkman (1991) and Garbade (1986), a "twist" by Steeley (1990), a "butterfly" by Beckers (1993) and a "kink" by Egginton and Hall (1994)[3]. That the principal components have interpretable properties is a crucial reason for the continuing interest in this methodology. However, the diversity of names for the third principal component gives a clear indication that there is little agreement about how it affects the term structure. The similarity of names given to the first and second principal components also tends to disguise differences in findings. Garbade and Urich (1988) results suggest that the first and second components cannot be interpreted as the level and the slope of the yield curve using German data.

The most worrying aspect of these studies is that the conclusions reported above all rely on subjective interpretation. Not one of the studies summarised in table 4.1.1 gave any formal statistical tests of either the proportion of the variance explained or the variability of the coefficients.[4] To make matters comparatively worse, the literature that studies interest rate convergence across countries has reported the results of statistical tests.[5] Yet formal testing is certainly called for because as such a high proportion of variability is explained by the first component, the remaining components may be indistinguishable from each other. Consequently, there may not be a reason to pay any more attention to the second and third components than to subsequent components. Similarly, the interpretation of the results relies solely on visual inspection but it is the ability to label the components with simple names that makes the results so interesting. To be fair, Krzanowski (1984) notes that the lack of formal analysis is endemic with the use of principal components because, he believes, that the "optimal" nature

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[3] Litterman and Scheinkman (1991) p. 54, Garbade (1986) p. 1, Steeley (1990) p. 346, Beckers (1993) p. 11, Egginton and Hall (1994) p. 151.

[4] Although Litterman and Scheinkman (1991) conducted a likelihood ratio test it is not reported in their paper.

[5] See, for example, Pentecost and Holmes (1995a, 1995b).

of the results causes them to be accepted uncritically. Flury (1988) puts the blame, less desirably, on the absence of tests within easily available statistical packages.

Table 4.1.1 Main Background Features of Principal Component Studies

Paper	Data	Frequency	Longest Period	Maximum Maturity	Country	Method
Hester (1969)	Money market interest rates and capital market yields	Monthly	January 1958 to January 1969	Long term rates	USA	not known
Vercruysse et al. (1971)	Money market interest rates and capital market yields	Monthly	1957 to 1969	Long term rates	Belgium	not clear
Fase (1973)	Money market interest rates and capital market yields	Monthly	January 1960 to December 1970	Perpetual	Holland	Correlation matrix
Logue and Sweeney (1984)	Change in eurocurrency CDs	Monthly	January 1977 to February 1982	12 months	USA, Japan, Germany Switzerland, UK, France	Covariance matrix
Garbade (1986)	Change in spot rates & yields	Weekly	June 1983 to December 1985	30 years	USA	Covariance matrix
Boothe & Glassman (1988)	First differences of yields	Monthly	January 1972 to December 1984	31 years	USA and Canada	Cross product
Garbade & Urich (1988)	Change in yields on par bonds	Weekly	February 1987 to April 1988	10 years	USA, Canada, UK, Japan, Germany	Covariance matrix
Dybvig (1989)	Innovations in log discount factors	Monthly and annual	December 1952 to December 1987	5 years	USA	Covariance matrix
Steeley (1990)	Spot rates	Weekly	October 1985 to October 1987	18 Years	UK	Covariance matrix
Litterman & Scheinkman (1991)	Excess returns on zeros and coupon bonds	Weekly	January 1984 to August 1988	28 years	USA	Covariance matrix
Beckers (1993)	Normalised spot rate changes	Monthly	1948 to 1992 (USA only)	30 years	USA, UK & 12 others	Not clear
Strickland (1993)	Forward rates	Daily	November 1987 to August 1990	10 years	USA, UK	Not clear
Fraser (1993)	Interest rates on certificates of deposits	Monthly	January 1970 to March 1992	1 year	UK	Covariance matrix
Egginton & Hall (1994)	Redemption yields	Daily	January 1979 to August 1990	24 years	UK	Correlation matrix
Wilson (1994)	Percentage changes in zero coupon yields	Daily	January 1980 to September 1993	5 years	Switzerland	Correlation matrix
Pagan et al (1995)	Spreads over 1 month rate	Monthly	December 1946 to February 1991	10 years	US	Covariance

Sources: From the papers referenced except Hester for (1969), which is cited in Fase (1973).

Few of the papers cited in table 4.1.1 provide a discussion of the choice of dependent variable. In a number of cases this is acceptable because this choice is determined by the aims of the paper and the use of principal components is merely means to further analysis. However, for Garbade (1986), Garbade and Urich (1988), Litterman and Scheinkman (1991), and Beckers (1993) the use of principal components analysis is the core of their papers and, consequently, the omission of a justification for the choice of dependent variable is serious. Even when the choice of data is discussed, as in Dybvig (1989), it is unconvincing. Dybvig (1989) argues that using log discounts is less sensitive to errors-in-variables problem. He claims that "using forward rates puts lots of weight in little bumps in the yield curve and using yields puts too much weight on the shortest maturity whose yield is most affected by the bid-ask spread or not having the right quote date or maturity date on which funds are received".[6] For the UK gilts market it is hard to believe that finding the right quote or maturity dates is likely to be a major difficulty; and for those markets where it is more difficult, short maturity yields can be excluded from the analysis. For the forward rates the question is simply whether or not the "little bumps" in the yield curve are a true manifestation of forward rates or an artefact of the yield curve estimation procedure. If it is the latter then the estimation procedure needs to be changed, rather than discarding the use of forward rates. The choice of dependent variable is evaluated in chapter 5.

The use of discount factors, as preferred by Dybvig (1989), has its own problems not least of which is finding a set of discount bonds with a broad range of maturities. Moreover, the differences in maturities between discount bonds may change over time and, to hold this constant, some interpolation method may be required, which introduces similar problems to those faced when using redemption yields. The main criterion by which the dependent variables should be chosen is the use that the principal components are to be put.

Boothe and Glassman's (1988) work has particular data peculiarities. They aggregate their data into eight maturity classes that were found by coalescing 31 one-year maturity classes until each class had at least one bond present for each month between 1972 and 1984.[7] The yields were constructed as the unweighted average yield for each class. This means that the data are not standardised over time so that, in principal, the maturity of the longest bond analysed could have fallen by 13 years during the

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[6] Dybvig (1989) footnote 5, p. 12.

[7] The eight maturity classes were 1, 2, 3, 4, 5, 6-9, 10-13 and 14-31 years. Boothe and Glassman (1988), p.9.

period analysed. It would have been preferable to use the bond data to estimate yields at standardised points in time. Although this might have meant that some longer maturity data would have to be ignored because extrapolation to these longer-term maturities when the data is absent is very imprecise.

Egginton and Hall (1994) and Wilson (1994) normalise the data to ensure that they have means of zero and variances of unity but there is no discussion of whether this is actually necessary. The papers by Garbade (1986) and Garbade and Urich (1988) simply assert that using changes in interest rates results in the series having zero means and they make no attempt to standardise the variances. On the other hand, the normalisation applied by Beckers (1993) seems unlikely to achieve data with either zero means or variances of unity.[8] The work of Steeley (1990) can throw some light on this matter. In his PhD thesis Steeley (1989) used changes in spot rates whereas his 1990 paper used the spot rates themselves. The broad conclusions remain the same, although there have been some changes in the eigenvectors as would be expected.[9] This may indicate that normalisation has little effect on the results but a formal demonstration of this has not yet been provided.

The choice of variables from which the principal components are formed will, however, determines the form the principal components take. This can be illustrated by the following examples.[10] Suppose first that all the bonds are discount bonds and that a parallel shift occurs in all their redemption yields. Under these circumstances the spot yields and the forward rates will also shift in a parallel fashion and with the same magnitude and direction as the change in the redemption yields. The price of the discount bonds will move in the opposite direction and the change will be larger the longer the maturity of the bond. If, on the other hand, the redemption yields of the discount bonds change so that the slope of the yield curve changes then the slope of the spot curve changes, by the same magnitude and the prices of the bonds move in the reverse direction. In both these instances the maturity at which the spot curve and prices pivot is identical to that for the redemption yields. This is not the case for the forward

[8] The change in the spot rate is multiplied by the square root of 8% divided by the spot rate at five-year maturity. It seems possible that Beckers intends it to induce homoscedastic variances to the spot rates but this seems unlikely given the findings on the Cox, Ingersol and Ross model discussed in chapter 2.
[9] Compare Steeley (1990) table 2, p. 345 with Steeley (1989) table 8.1, p. 218.
[10] The choice of variable has been investigated more formally by Lekkos (1999) for use in a study of factor analysis.

rate curve where the pivot maturity is shorter than that of the redemption yields. The forward rate also exhibits a much more volatile response at longer maturities and less response at short maturities than the redemption yield curve. For discount bonds, therefore, movements in the redemption yields will only be matched exactly by movements in spot rates.

For coupon bonds the same methodology can be applied.[11] For a shift in all redemption yields, spot rates can be regarded as also moving in a parallel fashion and again the price shows larger variation at greater maturities. For a change in the slope of the yields, the price of the bond again shows more volatility as the maturity of the bond increases, but the maturity at which the price pivots about is the same as that around which the yields pivot. For the spot rates the magnitude of the change in the rates no longer matches the change in the yields. The discrepancy increases with the maturity and the pivot point has a shorter maturity than that of the yield changes.

The above comments suggest that comparisons between yield curve studies using principal components will not be fruitful unless the type of data used in each of the studies matches. This problem is compounded by whether or not the data are standardised as discussed above. Furthermore, the criticisms of redemption yields, noted in section 3.1 of chapter 3, mean that comparisons of principal components using redemption yields and spot rates only provide information about the technique of principal components, not about the structure of interest rates that can only be deduced from examining spot rates.

Whilst there are a number of similarities between the studies outlined in table 4.1.1 there is insufficient overlap by which the consequences of specific changes in data or methodology can be ascertained with any certainty. The papers by Garbade (1986) and Garbade and Urich (1988), for example, cover the same country, have the same frequencies of observations and use the same methodology. However, they differ in the data used, time periods covered and the maximum maturities analysed. The closest matching studies are, on the basis of table 4.1.1, Garbade (1986) and Litterman and Scheinkman

[11] An analysis of par bonds cannot be undertaken because for any shock to the redemption yields, such that the coupon no longer equals the yield, the bond is no longer a par bond.

(1991). The main differences between these papers are the dependent variables used and the sample period, which is slightly longer in the latter study.

One important finding from the above papers is that although the interpretation of the second principal component as a measure of the slope is well established, the manner in which the yield curve reacts to changes in the second principal component have not been established. In Garbade (1986) an increase in the second principal component reduces yields at the short end and increases them at the long end. The reverse is reported in the paper by Garbade and Urich (1988). However, is as noted below, the eigenvectors are only identified up to an arbitrary scaling of ± 1 and, consequently, the reversal of signs may not be particularly meaningful. Nevertheless, both of these papers seek to interpret the eigenvectors of the second component by reference to their signs and this is clearly unwarranted.[12] Of more concern, because it is less likely to be due to the arbitrary scaling of the principal components, Dybvig (1989) using monthly data finds that the signs of the eigenvectors at the short and long ends reverse themselves when the frequency of the data is changed. Dybvig does not comment upon this result. If the slope measure is not robust over time or estimation procedure then the usefulness of the principal components methodology is reduced. This feature is examined in chapter 5.

These studies are open to the criticism that their results are simply products of a specific historical period and, are therefore, not general.[13] Evidence to refute this might have been available from Beckers (1993) who runs principal components on a number of sub-periods for his data. His evidence suggests that there may be some instability in the response of changes in spot yields to changes in the principal components. However, it is by no means conclusive because the evidence is graphical, which makes it difficult to interpret and unavailable for statistical testing. Beckers' own interpretation is that the three main principal components are 'stable and persistent through time'.[14] Although Dybvig (1989) also analyses subsets of his data, there are other changes as well, in particular to the maturity range

12 See Garbade (1986) p. 3 and Garbade and Urich (1988) p. 5.

13 As noted in chapter 1 this argument implies that either the institutional arrangements matter more than underlying structure (tastes and technology) or that the structure is time varying. Only in the latter sense is this a valid criticism, as we would expect the structure to become apparent irrespective of the institutional arrangements. If the structure is time varying then the criticism is valid and the "deeper parameters" that guide the change in the structure still need to be found.

14 Beckers (1993) p. 11.

covered. Consequently, his comment that the first principal component in the longer, annual, series does not appear to reflect parallel shifts in interest rates, unlike the monthly case, may not be due to changes in the time period alone.[15] Yet when Dybvig used both the monthly and annual data sets he concluded that the first principal component was the same in both data sets. Wilson (1994) splits his sample into ten non-overlapping sub-periods and finds that the eigenvectors, especially those of the third principal component, which he describes as resembling spaghetti, are unstable. There, consequently, appears to be ambiguity in whether or not the principal components are stable over time. This suggests that further tests of the stability of the principal components decomposition of the yield curve are required and these are performed in chapter 6.

4.2 The Method of Principal Components

The technique of principal components can be traced back to a paper by Karl Pearson (1901), although a practical method of computation was not suggested until Hotelling (1933). Principal components analysis can be derived as follows. Let X be a (p x n) matrix of p observed variables over n time periods or n experiments. The covariances of the elements of X are given by the (p x p) matrix denoted by S. Let z_1 be a (1 x n) vector formed from:

$$z_1 = a_1^{'}X \quad ...(4.2.1)$$

Where: $a_1^{'}$ is a (1 x p) vector.

z_1 is known as the first principal component scores.

The sample variance of z_1 is given by $a_1^{'}Sa_1$ and the first principal component is defined as being the linear combination given by (4.2.1) that maximises the variance of the first principal component, subject to the constraint that $a_1^{'}a_1=1$ (which makes a_1 unique except for its sign). This can be written as:

$$V_1 = a_1^{'}Sa_1 - l_1\left(a_1^{'}a_1 - 1\right) \quad ...(4.2.2)$$

Where: l_1 is a Lagrange multiplier.

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15 Dybvig (1989) table 6 p. 18.

Maximising V_1 with respect to the vector a_1' gives:

$$Sa_1 = l_1 a_1 \quad ...(4.2.3)$$

This is a homogeneous set of p equations in p unknowns and a non-trivial solution is:

$$|S - l_1 I| = 0 \quad ...(4.2.4)$$

Where: I is a (p x p) identity matrix.

Consequently, from (4.2.4), l_1 is an eigenvalue of S and the solution a_1 is its corresponding eigenvector. There are p eigenvalues of S but if (4.2.3) is pre-multiplied by a_1' and the constraint $a_1'a_1 = 1$ is imposed then:

$$a_1'Sa_1 = l_1 \quad ...(4.2.5)$$

The eigenvalue l_1 equals the variance of z_1 and, as this is being maximised, l_1 is chosen to be the largest eigenvalue. The coefficients a_1 therefore correspond to the largest eigenvalue of S and using (4.2.1) they define z_1 as the first principal component scores.

To find the second principal component, z_2, an analogous route is used with the addition of a further constraint that z_2 be orthogonal to z_1, i.e. $a_2'a_1 = a_1'a_2 = 0$. Maximising the variance of the second principal component, V_2, subject to both constraints is achieved using two Lagrange multipliers, l_2 and m so that:

$$V_2 = a_2'Sa_2 - l_2(a_2'a_2 - 1) - m(a_2'a\ ) \quad ...(4.2.6)$$

Differentiating (4.2.6) with respect to a_2' and setting the results equal to 0 for all p equations leads to the solution:

$$(S - l_2 I)a_2 = \frac{1}{2} m a_1 \quad ...(4.2.7)$$

Pre-multiplying (4.2.7) by a_1' and recalling that $a_1'a_1 = 1$ and $a_1'a_2 = 0$, we find that:

$$a_1'Sa_2 = \frac{1}{2} m \quad ...(4.2.8)$$

However, pre-multiplying (4.2.3) by a_2' and imposing the orthogonality constraint implies that $a_2'Sa_1 = 0$ and, since this is a scalar and S is symmetric, its transpose also equals zero. This implies, from (4.2.8), that m is zero. The solution to (4.2.7) is, therefore, similar to that of (4.2.3) and the coefficients, a_2, of the second principal component scores, $z_2 = a_2' X$, are the eigenvectors of the second largest eigenvalue, l_2.

This process can be repeated up to p times, so that the ith principal component is formed from the eigenvector corresponding to the ith largest eigenvalue of S. The ith principal component has the greatest variance for all a_i, subject to $a_i' a_i = 1$ and $a_j' a_i = 0 \, (j < i)$. Principal component analysis, therefore, constructs a set of p variables, the principal component scores, which are orthogonal to each other and that have variances equivalent to their eigenvalues in descending order of magnitude.

One property of eigenvalues is that their sum is equivalent to the trace of S and the proportion of the variance that each principal component accounts for can, therefore, be calculated from $l_i / tr(S)$. If S is a symmetric, positive semi-definite matrix all the eigenvalues are real and positive. Consequently, the proportions $l_i / tr(S)$ are always positive. If only a few principal components, say q, account for most of the variation then the use of this method allows a reduction in the dimensions of the problem from p to q. It is the ability to reduce the dimensions of the problem that makes principal components attractive to use on yield curve data.[16]

This raises the question will the eigenvalues be different so that at least one eigenvector can be said to contain the most information about the data (corresponding to the dominant eigenvalue). Fortunately the Perron Frobenius theorem answers this question. This theorem shows that for a non-negative primitive matrix at least one of its eigenvalues is positive and greater in absolute value than all the other eigenvalues. The matrix S is said to be primitive if and only if $S^t > 0$ for some t, i.e. all (p x p) elements of S^t are positive for some t (t will be less than or equal to p^2-2p+2). A primitive matrix is always irreducible and irreducible matrices cannot have 2 independent eigenvectors that are both non-negative. Moreover, the Perron-Frobenius theorem also guarantees that for this dominant eigenvalue the corresponding eigenvector will be positive.

16 A comprehensive list of the uses to which principal components analysis can be put can be found in Karson (1982) p. 210.

It should also be noted that the principal components are a simple linear combination of the data and that more general non-linear techniques are available. These are discussed in, for example, Gnanadesikan (1977), Hastie and Stuetzle (1989) and Gifi (1990). Gnanadesikan (1977) proposes that the original variables be augmented by adding quadratic terms and higher order terms to the original variables and then undertaking ordinary principal components analysis. There are a number of problems with this approach. One problem is that the number of variables, p, may now exceed the number of observation periods, n.[17] Secondly, the scales of the variables are different between the linear and quadratic terms and, as principal components are not invariant to scale, this can cause difficulties (see below). Finally, Flury (1994) found that the example used by Gnanadesikan (1977) to demonstrate his method was very sensitive to small data errors. Hastie and Stuetzle's (1989) work involves fitting a curve through the data that minimises the sum of the squared orthogonal distances between the curve and the data, whereas principal components analysis simply fits a straight line. Krzanowski and Marriott (1994) dismiss Hastie and Stuetzle's approach as simply another method for non-linear curve fitting.[18] Gifi (1990), on the other hand, is mainly concerned with extending the quantitative approach described above to deal with qualitative variables. For these reasons non-linear approaches are not followed in the remainder of this thesis.

Linear principal components itself has three main difficulties. These are: components are not independent of the scales in which X is measured; some means of deciding how many principal components capture all of the significant variation in the data is required; as are means of interpreting the components themselves. These difficulties are discussed in turn below.

If principal components analysis was performed, then one of the variables was to have its scale of measurement changed (from, say, millions to billions) the principal components would not be equivalent to those calculated initially. Moreover, if the variables contained within X are of different scales then a linear combination may not provide a meaningful interpretation. With yield curve data this is not a problem. However, if the variability of the yields is sufficiently different, then in

[17] Krzanowski (1987b) has devised a method of reducing p to n-1 by selecting the best n-1 variables in the sense of preserving the data structure.
[18] Krzanowski and Marriott (1994) p.198.

calculating the first principal component the largest coefficients will be given to the variables with the greatest variability. In the extreme the principal component will correspond to the variable with the largest variance and no new information will be derived from the analysis.

These difficulties can be overcome by standardising the variables within X, but no general theory exists to provide a basis for the choice of one method over any other. The most common method is to subtract from the variables their means and then divide by their standard deviations to produce a new set of variables with mean zero and unit variance.[19] Principal component analysis is carried out on the correlation matrix of X after standardisation by the above method because the covariance matrix of the standardised variables equals the correlation matrix. This simple standardisation has a number of consequences.

First, the eigenvectors can change not only their magnitude but also their signs and, consequently, interpretation of the resulting principal components will be different. It has to be recalled, however, that the normalisation a'a=1 is not unique in sign. Second, the amount of variation explained by each principal component can change markedly. An example of both these factors is provided by Kendal (1980) using soil sample data.[20]

Thirdly, Jobson (1992) claims that the use of standardised variables with equal variance will cause the coefficients of the first principal component to tend to be equal.[21] In such cases the finding that the first component is associated with the level of the yield curve is not particularly informative. However, Jobson (1992) offers no proof of his assertion and it can be noted that Kendal's soil sample data produces an eigenvector that has the coefficient on the first variable used is over 28 times as large as that on the third variable.[22] As is discussed below, nearly equal off-diagonals in a correlation matrix will also tend to produce coefficients on the first principal component that are nearly equivalent to each other. Moreover, because the succeeding coefficients are orthogonal, they must contain a mixture of

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19 An alternative standardisation method is to use logarithms. See Morrison (1976) p.286.

20 Kendal (1980) pp. 20-23.

21 Jobson (1992) p.371.

22 Kendal (1980) table 2.6, p. 23.

positive and negative coefficients. This does not mean that the positive and negative coefficients have to be consecutive and, hence, there is no reason to suppose that the second coefficient is necessarily the slope of the yield curve. It should also be noted that by the Perron-Frobenius theorem if each element of the covariance or correlation matrix is positive (which it usually will be in the case of yields) then all of the coefficients of the first eigenvalue will have the same sign.[23]

Fourthly, it can be shown that there is no method by which a simple transformation will allow one set of principal components to be derived from knowledge of another set.[24] Unfortunately, there does not appear to be any accepted method within principal components that will provide a test of whether or not standardisation is required. Consequently, in chapter 5 principal components analysis will be carried out on both standardised and unstandardised data to compare the results and test Jobson's assertion discussed above.

4.3 Testing Inferences on Eigenvalues

Once principal components have been calculated, the second problem is deciding how many of the components contain all the relevant information. As the analysis can generate up to p components, unless the system is less than full rank, ignoring any of the components will leave some of the variation in X unexplained. There are various methods for deciding how many components capture all the salient information. These are outlined below.

1) A "goodness of fit" criterion by which an arbitrarily large percentage, G, of the variance is selected, and the number of components, j, required so that $\sum_{i=1}^{j} l_i / tr(S) > G$. In the case of a correlation matrix the trace of S is replaced by p, the number of variables used in the analysis. This method is highly subjective and ignores any information that might be present in the pattern of the eigenvalues. An approximate test of whether or not the first k components account for G percent of the variance is available but it is only an approximation and the arbitrary nature of the choice of G remains.[25]

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23 See, for example, Gifi (1990) p.304.
24 See, for example, Krzanowski (1988) pp. 65-66.
25 See Mardia et al. (1979) for details.

2) The "scree diagram" plots the eigenvalues against the number of their components, j. The only components that are regarded as significant are those for which the rate of change of the plot is noticeably negative. An alternative, suggested by Jolliffe (1986), is to use the log of the eigenvalues. Again this procedure is highly subjective.

3) The "average eigenvalue" criterion by which only those components with eigenvalues greater than the average of the eigenvalues are deemed to hold significant information. When the analysis is conducted on a correlation matrix the criterion is that the retained eigenvalues are all greater than or equal to 1 (the "eigenvalue-one" or Kaiser criterion). Jolliffe (1986) suggests that the critical value should be 0.7 on the basis of simulation work. Kendal (1980) dismisses this criterion as "a very rough-and-ready procedure for which it is difficult to advance a convincing theoretical justification".[26]

4) A variant on 3 above is the "average generalised variance", which retains the component if its eigenvalue exceeds the geometric mean of S.

5) A "cross-validation" approach can be used which exploits the relationship between the eigenvalue decomposition, on which principal components analysis is based, and the singular value decomposition of the data matrix:

$$\underset{(nxp)}{X} = \underset{(nxp)(pxp)(pxp)}{UWV'} \qquad ...(4.3.1)$$

Where: the columns of U (n x p) are formed from the p orthonormalized eigenvectors of XX'

the columns of V (p x p) are formed from the p orthonormalized eigenvectors of X'X

$$\mathrm{W} = \mathrm{diag}(\mathrm{w}_1, w_2, \ldots, w_p)$$
$$where\ w_1 \geq w_2 \geq \ldots \geq w_p$$

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26 Kendal (1980) p.27.

The w_i are the non-negative square roots of the eigenvalues of X'X or XX' and the normalization is such that UU', U'U, V'V, VV'=I, where I is the appropriate identity matrix.[27]

The $(i,j)^{th}$ element of X can be written as:

$$x_{ij} = \sum_{t=1}^{p} u_{it} w_t v_{tj} \quad ...(4.3.2)$$

If m was chosen such that m<p then:

$$x_{ij} = \sum_{t=1}^{m} u_{it} w_{tt} v_{tj} + \varepsilon_{ij}^{m} \quad ...(4.3.3)$$

Each choice of m will be associated with an error ε_{ij}^{m} and these errors can be summarised for each m as:

$$PRESS(m) = (1/np)(\sum_{i=1}^{n} \sum_{j=1}^{p} (\varepsilon_{ij}^{m})^2) \quad ...(4.3.4)$$

The ideal measure of PRESS(m) would be to have all information about the n^{th} period deleted and then make the prediction. This is not possible because the derived X is of insufficient size. As an alternative, U is formed when the j^{th} column of X has been deleted and V is formed when the i^{th} row of X has been deleted. W is constructed by multiply together the square roots of the two W matrices formed when the j^{th} column and the i^{th} row of X have been deleted. Thus to describe cross-validation techniques as predictive is not strictly correct.

Eastment and Krzanowski (1982) suggest that the optimum choice of m can be made from the following function:

$$W = \left(\left(\frac{PRESS(m-1) - PRESS(m)}{n+p-2m} \right) \Bigg/ \frac{PRESS(m)}{np+(2(m-p)-n} \right) \quad ...(4.3.5)$$

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27 See Krzanowski (1988) p.126 for an overview of singular value decomposition. Note that if X is square and symmetric then the singular value decomposition is known as a spectral decomposition of X.

The numerator represents the marginal predictive power of the mth component, whilst the denominator represents the average predictive power of m. Consequently, the mth component should only be retained if W>1, i.e. the marginal gain in predictive power is such that it will increase the average predictive power. If all the values of W are below unity then all the components have to be retained. The criterion of W>1 remains, however, subjective. Eastment and Krzanowski (1982) present evidence that the number of components retained when using the W statistic can be either more or less than other methods might suggest. They conclude that the choice between the W statistic and other methods depends upon whether the estimated components are to be used on future data or simply as a method of describing the data.[28] In this thesis the large data set implies that day by day elimination of observations is impractical and the W statistic is not calculated.

Each of the above methods rely upon a subjective analysis of what constitutes a significant result and, whilst they may aid comparison between studies on different data sets, they do not provide, other than by tradition, a method of calibrating principal components analysis. For this we have to turn to statistical tests. It is assumed for each of these tests that the distribution of the population X is multivariate normal. The violation of this assumption does not appear to have been the subject of empirical or theoretical studies and the consequences of its violation are unclear.

It can be shown that if the same algebra as used above is applied to the maximum likelihood estimator of the covariance matrix, [(n-1)/n]S, where S is the sample covariance matrix, the sample principal components are the maximum likelihood estimator of the population principal components as are the sample eigenvectors and eigenvalues.[29] Similar results apply to the sample correlation matrix.

Letting λ_i be the ith eigenvalue from the population corresponding to the sample eigenvalue l_i, Girshick (1939) has shown that the distribution of:

$$(l_i - \lambda_i)/(2\lambda_i^2 / n)^{0.5} \qquad ...(4.3.6)$$

28 Eastment and Krzanowski (1982) p.75, table 1 and p.76.
29 See Anderson (1958).

(4.3.6) approaches the standard normal distribution as n increases[30]. From this a large sample confidence interval can be calculated as:

$$l_i / (1 \pm (2/n)\chi^2(1))^{0.5} \qquad ...(4.3.7)$$

Where: $\chi^2(1)$ is the appropriate value from the χ^2 distribution with one degree of freedom.

These confidence intervals are reported in the analysis undertaken in chapter 5. Note that both $\chi^2(1)$ and n need to be chosen such that the denominator of (4.3.7) is non-negative. Moreover, (4.3.7) cannot be used to identify eigenvalues that are insignificantly different from zero. Small values of eigenvalues cannot come from a population that has a corresponding eigenvalue of zero because such samples simply cannot be generated unless the sampling procedure induces measurement errors that mimic an extra dimension to the data. Thus, at best (4.3.7) can help indicate the range of the variances of each principal component.

As we cannot test for an eigenvalue being zero a different approach is required. The approach adopted is to test for the equality of the eigenvalues. If all the eigenvalues are found to be equal then no transformation of the data is required. If the last k eigenvalues are found to be equal, then the choice becomes either to concentrate on the p-k different eigenvalues or on all p eigenvalues because there is no justification in deciding one or more of the k eigenvalues are more significant than the rest. Schott (1988) goes so far as suggesting that under these circumstances the last k principal components are simply picking up noise in the data.[31]

The test (which is also known as a test of sphericity) takes the form of a likelihood ratio, where the ratio is formed from the arithmetic mean of the last k eigenvalues divided by the geometric mean of those eigenvalues as follows:

$$(n)(p-k)(\log_e(\sum_{i=k+1}^{p} l_i /(p-k)) - \log_e((\prod_{i=k+1}^{p} l_i)^{-(p-k)})) \qquad ...(4.3.8)$$

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30 Where the covariance as opposed to the correlation matrix is being used to calculate principal components.

31 See Schott (1988) p.794.

(4.3.8) is distributed as a χ^2 distribution with (p-k-1)(p-k+2)/2 degrees of freedom for a covariance matrix. Lawley (1956) notes that for a correlation matrix the criterion does not even asymptotically follow a χ^2 distribution, although it will approximately do so if the first k eigenvalues are large relative to the eigenvalues of the k+1 onwards.[32] For a correlation matrix Bartlett (1954) has shown that the degrees of freedom depend on the variances in the first k components and (p-k-1)(p-k+2)/2 is an upper limit to the degrees of freedom. Schott (1988), building upon the work of Lawley (1956), notes that (4.3.8) is asymptotically distributed as a linear combination of χ^2 variables. Consequently the distribution of the statistic could be proxied by:

$$c\chi_d^2 \qquad \ldots(4.3.9)$$
$$where: c = \sigma^2 / 2\mu$$
$$d = 2\mu^2 / \sigma^2$$

The asymptotic mean of (4.3.8), μ, and its variance, σ^2, are derived from the last p-k eigenvectors, the elements of the correlation matrix itself and the average of the last p-k eigenvalues.[33] The degrees of freedom, d, of the statistic are evaluated to the nearest integer. Schott (1988) presents simulation results that suggest that his statistic is much less erratic than using an unadjusted version of (4.3.8).[34] However, Schott's approach uses Kronecker products to form matrices of size p^2 x p^2. In some samples used in chapter 5, p is in excess of 45 and this implies matrices with over 4.1 million elements. The computational burden of calculating Schott's approximation (4.3.9) for all of the correlation samples is therefore too great it will not be used in the remainder of this thesis.

For the eigenvalues derived from a covariance matrix Bartlett (1954) suggests that (n) in (4.3.8) is replaced by:

$$n-(2p+11)/6 \quad \ldots(4.3.10)$$

32 See Lawley (1956) p.134.
33 See Schott (1988) pp.794-795 for details.
34 See Schott (1988) Table 1, p.796 for details.

This is known as Bartlett's test of isotropy and is distributed as a χ^2 distribution with p(p-1)/2 degrees of freedom. This test is reported in the analysis undertaken in chapter 5. Lawley (1956), on the other hand, suggests that (n) in (4.3.8) be replaced by:

$$n-k-\{(2(p-k)+1+2/(p-k))/6\}+l^2\sum_{i=1}^{k}(l_i-l)^{-2} \quad ...(4.3.11)$$

Where: l is the mean of $l_{k+1}.....l_p$.

(4.3.11) is approximately distributed as χ^2 with (p-k-1)(p-k+2)/2 degrees of freedom. The approximation error is given by n^{-2} which rapidly diminishes with an increase in the sample size.

These tests of eigenvalue equality can be further modified to test whether or not the smallest k eigenvalues equal a given value (but not zero) and whether or not the intermediate r eigenvalues are equivalent.[35] The usual procedure is to let k=0 and perform the test. If the hypothesis that all p eigenvalues are equal can be rejected (a test statistic above the critical χ^2 level) the test is repeated until k=p or a k<p is found for which the hypothesis is rejected.

It should be noted that these are large sample tests but it has not been established how large the sample has to be before they are valid. Krzanowski (1983) reports the results of Monte Carlo simulations that indicate that Bartlett's test of isotropy tends to lead to values of k that are higher than those found using the subjective tests (such as the scree test). The conclusion to be drawn from this survey is that for small sample sizes the choice of the number of significant principal components remains highly subjective. The survey also suggests that, because the tests that can be applied to a correlation matrix are more approximate, the choice between a correlation matrix, which does eliminate scale effects, and a covariance matrix, which does not eliminate scale effects, is not clear cut.[36]

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35 See Karson (1982) p.207 and Morrison (1976) p.294 respectively.

36 Schott's (1988) test (4.3.9) is only applicable to small numbers of attributes because of the computational burden it imposes.

4.4 Testing Inferences about Eigenvectors

The third problem referred to in section 4.2 is the interpretation of the principal components once they have been calculated. Such interpretation is aided by having variables in the same dimension but there is no guarantee that any sensible interpretation can be placed on the individual components. Moreover, there is a lack of informative tests with which to analyse the eigenvectors. Anderson (1963) has proposed the following asymptotic test of the equality of the first eigenvector with a pre-specified eigenvector, α_{ip}.[37]

$$n(l_i\alpha_{ip}'S^{-1}\alpha_{ip} + (1/l_i)(\alpha_{ip}'S\ \alpha_{ip}) - 2) \quad ...(4.4.1)$$

$$\text{Where } S = \sum_{i=1}^{p} l_i a_{ip} a_{ip}'$$

$$S^{-1} = \sum_{i=1}^{p} l_i^{-1} a_{ip} a_{ip}'$$

Where: n is the sample size, l_i, is the i^{th} eigenvalue and a_{ip} is the p^{th} element of the estimated eigenvector.[38] This statistic is asymptotically distributed as χ^2 with (p-1) degrees of freedom and assumes that the variables are distributed normally.

The problem with this test is that there are infinite numbers of repetitions that could be conducted.[39] It may be objected that there exists an infinite number of null hypotheses for virtually all statistical tests. However, economic theory often provides the null hypothesis in the form that the parameter is zero or one. Hypotheses about the eigenvectors are much less forthcoming. Anderson's test can be used to test whether the data came from an equicorrelation matrix (where the diagonal values are unity, the off-diagonals are all equal and greater than zero but less than or equal to unity). In this situation all the coefficients of the first principal component are equal to the inverse of the square root of the number of variables, p, and so the pre-specified vector is easy to construct. Unfortunately, other matrices, for example an equipredictability covariance matrix, also produce similar answers, and so the usefulness of Anderson's test remains doubtful.[40]

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[37] Testing subsequent eigenvectors can only be performed as a joint test and requires a more general form of 4.4.1 and so is not undertaken in this thesis.
[38] See Flury (1988) p.34.
[39] The problem of having a given eigenvector or set of eigenvectors as the null hypothesis also applies to, for example, the test of Mallows (1961).
[40] See Morrison (1976) pp. 290-291 for an example of this form of matrix.

The choice of the null hypothesis is, of course, far more limited when the comparison of eigenvectors is between different sub-samples. An obvious test would be test whether or not the sub-sample eigenvectors equal the whole sample eigenvectors. However, because the hypothesis is constructed from an analysis of the data actually contained in the sub-sample, the statistical validity of Anderson's test can be questioned. Moreover, Schott (1987) reports that there is a large probability of a type 1 error using Anderson's test if the sample size is small. This is because the test depends on the eigenvalues being distinct and if they are close in size the associated eigenvectors will be unstable with large variances.[41]

In the analysis conducted in chapter 5 we test whether the first eigenvector takes the following form:

$$\alpha_{1,p,12} = 1.3844 * \alpha_{1,p,23} = 1.9365 * \alpha_{1,p,45} \quad ...(4.4.2)$$

That is the first eigenvector derived from 12 observations is equivalent to those derived from 23 and 45 observations except for the presence of scaling factors. These factors are derived from the fact that the sum of the squared coefficients equals one, so that the greater the number of observations the smaller the coefficients are on average. This has the advantage that the hypothesis is supplied by the other data sets, but again this opens some questions about the statistical basis of the test. As subsequent eigenvectors would have to be jointly tested with the first we only use Anderson's test on the first eigenvector.

Girshick (1939) derived some large sample results for the variance and covariance of the coefficients. These are:

$$\operatorname{cov}(a_{ji}, a_{jm}) = \frac{l_j}{n}\left(\sum_{u=1,u\neq j}^{p} \frac{l_u}{(l_u - l_j)^2}(a_{ui}a_{um})\right) \quad ...(4.4.3)$$

41 Schott (1987) p.106 provides a two-stage test that controls for type 1 error, regardless of sample size. A type 1 error is the rejection of the null hypothesis when the null hypothesis is true.

$$\text{var}(a_{ji}) = \frac{l_j}{n}\left(\sum_{u=1,u\neq j}^{p} \frac{l_u}{(l_u - l_j)^2} a_{ui}^2\right) \quad ...(4.4.4)$$

$$\text{cov}(a_{ji}, a_{lk}) = \frac{-l_j l_l}{n(l_j - l_l)^2} a_{jk} a_{li} \quad ...(4.4.5)$$

(4.4.3) gives the covariance between the i^{th} and the m^{th} coefficients in the j^{th} eigenvector, (4.4.4) gives the variance of the i^{th} coefficient in the j^{th} eigenvector. (4.4.5) gives the covariance of the i^{th} coefficient in the j^{th} eigenvector against the k^{th} coefficient in the l^{th} eigenvector. These formulae can be used as subjective indicators of the robustness of the eigenvector. Large variances and covariances indicate that the estimates of the eigenvalue may be unreliable between samples. One disadvantage of using variances and covariances is that they generate a vast amount of data, and to circumvent this only those combinations of coefficients that have a correlation coefficient in excess of (an arbitrary) 95% are reported in chapters 5 and 6.

An alternative to calculating the variances of the eigenvalues and the covariances of the eigenvector coefficients analytically is to use bootstrap or jackknife methods. These statistics can be obtained by taking sub-samples of the data, estimating principal components and storing the eigenvalues and eigenvectors. A distribution of the eigenvectors and eigenvalues can be calculated around their mean values. For example, the standard error of the i^{th} eigenvalue is given by:

$$\left\{(n-1)\left[\sum_{j=1}^{n}\left(l_{ij} - \hat{l}_i\right)^2\right]/n\right\}^{0.5} \quad ...(4.4.6)$$

Where: n = number of samples

l_{ij} = i^{th} eigenvalue when row j of the data is deleted

$\hat{l}_i$ = the mean of the i^{th} eigenvalue.

A similar formula can be used to calculate the standard error of the eigenvector coefficients. The problem remains how to select the sub-samples. With the large data set of 3036 observations omitting one day at a time would be impractical, as would random sampling arrangements. Moreover, as the analytical variances are available it is not clear why the computational burden of jackknife methods need be accepted. Consequently, jackknife standard errors are not calculated.

Krzanowski (1984) has derived a formula that estimates the effect on the first $(p-1)$ coefficients of small changes in the value of the eigenvalue, the Krzanowski Tolerance. This is gives the new vector of coefficients as:

$$a_{i,n} = (a_i \pm a_{i+1}(T)^{0.5})/(1+T)^{0.5} \quad ...(4.4.7)$$

Where: T is given by: $\varepsilon/(l_i - l_{i+1})$

ε represents an arbitrary change in l_i.

Krzanowski (1984) suggests that ε be taken as a fixed percentage of l_i and in the analysis in chapter 5 10% is used. As the signs of the coefficients are arbitrary it is the magnitude of the change rather than its direction that is of interest. It is also important to note that it is the difference in the sequential eigenvalues, not their absolute levels, that determines the robustness of the coefficients. Unfortunately, there is no guide to how large (4.4.3), (4.4.4), (4.4.5) and (4.4.7) have to be to indicate unreliability. Many authors endorse Kendal's view that it would "be unwise to lean heavily on the numerical value of any particular coefficient in the eigenvector".[42] The view endorsed in this thesis is that it is unwise to lean heavily on the value of eigenvectors until enough tests have been carried out to establish the eigenvectors' fragility. Nevertheless, interpretation of the principal components therefore remains something of an art rather than a science.

4.5 Stability of Principal Components Between Samples

There are also tests of whether or not the principal component vectors from any sub-samples are the same as that from any other group. This clearly has applications to the stability over time of the principal components analysis of interest rates. If the eigenvectors are not statistically equivalent between sub-periods then the whole-sample eigenvectors are at best an amalgam of various sub-sample eigenvectors. At worst they may be a combination of the eigenvectors and random effects, if the

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42 Kendal (1980) p.28.

stochastic variance is large relative to the deterministic variance in some sub-periods. If these cases do occur, then failure to determine the variables that drive the principal components, which is the concern of chapter 8, may be because the eigenvectors are unstable rather than because the explanatory variables themselves are unimportant.

An alternative question is to ask whether or not different samples for the same time periods result in the same principal components being derived. This can be used as a measure of whether or not the different methodologies used to construct yield curves do result in significant differences. Tests could also be performed as to whether different interest rates provide much the same information, i.e. does it matter if a two year or a 3 year maturity gilt is used? A fourth application is to enquire whether or not similar principal components exist across countries. If there are common principal components in this case then the factors that drive the yield curve are likely to be international.

Analysis of these questions involves finding whether or not a simultaneous reduction of dimensionality in several groups that preserve the variances of the data is possible. This will be possible if the subspace spanned by the most important principal component vectors is the same for all groups.[43] Krzanowski (1979) provides a method of calculating the critical angles between the subspaces spanned by the first m principal components. The distribution of this angle is not known, although Krzanowski (1982) provides some simulation results with which to calibrate the calculated angles. A further problem is that if the q^{th} and the $(q+1)^{th}$ eigenvalues are close then the eigenvectors will have large variances (see the discussion below). This may lead to large angles between the subspaces leading to erroneous rejection of the hypothesis of a common subspace. A graphical presentation has been developed by Keramidas et al (1987), whilst Schott (1991) provides a χ^2 test and Flury (1987) a likelihood ratio test. Schott bases his analysis upon the common principal components (CPC) model that generalises the single population principal components to that of several populations (or potential populations). Flury (1987) provides a further generalisation in that his test is based upon partial common principal components (PCPC). Unlike CPC, PCPC only assumes that some of the eigenvectors are equivalent. Moreover, there is no restriction that these need be associated with the

43 See Schott (1991) p.771.

largest eigenvalues. An asymptotic version of PCPC is used in Chapter 6 and a discussion of this technique is presented below.

In section 4.2 one method of deriving principal components using a Lagrange multiplier process was described. It is, however, easier to explain common principal components by using the spectral decomposition theorem.[44] This may be stated as follows. Let X be a real symmetric (p x p) matrix, then there exists an orthogonal (p x p) matrix, A, and a diagonal (p x p) matrix, Λ, such that:

$$A'XA = \Lambda \quad ...(4.5.1)$$

If the eigenvalues of X are all distinct, the matrix A is uniquely defined up to multiplication by -1 and the re-ordering of the columns, with the columns of A being the eigenvectors of X. The diagonal elements of Λ are the associated eigenvalues.

Recall from section 4.2 that:

$$XA = \lambda A \quad ...(4.5.2)$$

Where: λ is the vector of eigenvalues.

Let a_1 and a_2 be two associated eigenvectors with λ_1 and λ_2 as the corresponding eigenvalues. Then pre-multiplying (4.5.2) for the first eigenvector, a_1, and eigenvalue, λ , combination by a transposed second eigenvector, a_2, and vice versa results in the following two equations:

$$a_2'Xa_1 = \lambda_1 a_2' a_1 \quad ...(4.5.3)$$

$$a_1'Xa_2 = \lambda_2 a_1' a_2 \quad ...(4.5.4)$$

As X is symmetrical and λ and λ_2 are distinct (by the Perron Frobenius theorem) taking the transpose of (4.5.4) and subtracting from (4.5.3) means that both equations can only hold if $a_1' a_2 = 0$, i.e. they are orthogonal. Thus by putting the eigenvectors, a, as the columns of A in (4.5.1) ensures that it is orthogonal. Using this orthogonality condition, together with (4.5.3) imply that $A'XA$ is diagonal and,

44 See Krzanowski (1988) p.126.

hence, so is Λ. Moreover, because we impose $a_j^{'}a_j = 1$ this implies that the eigenvectors are unique (up to their sign) and thus so is Λ.

Pre-multiplying (4.5.1) by A and then post-multiplying the result by A' we can write:

$$X_{cpc}^{(i)} = A\Lambda A^{'} = \sum_{j=1}^{p} \lambda_j a_j a_j^{'} \quad ...(4.5.5)$$

This is called the spectral decomposition of X and by using (4.5.5) the statement of PCPCs becomes straightforward.

Let X_i be the covariance matrix from observations on the i^{th} sample then:

$$X_i = A^{(i)}\Lambda_i A^{(i)'} \quad ...(4.5.6)$$

The only difference from equation (4.5.1) above is that in (4.5.6) the matrices of eigenvectors are partitioned:

$$A^{(i)} = (A_c, A_s^{(i)}) \quad ...(4.5.7)$$

Where A_c is a (p x q) matrix of the q eigenvectors that are common to all the i samples, whilst $A_s^{(i)}$ is a (p(p-q)) matrix of the (p-q) eigenvectors that are specific to each of the i samples. Clearly if q=p then the model is of common principal components. Using the spectral decomposition theorem (4.5.5) this can be written as:

$$X_i = \sum_{j=1}^{q} \lambda_{ij} A_j A_j^{'} + \sum_{j=q+1}^{p} \lambda_{ij} A_j^{(i)} A_j^{(i)'} \quad ...(4.5.8)$$

From (4.5.8) it is clear that PCPCs do not impose conditions upon the eigenvalues only upon some of the eigenvectors. This means that there is no reason why, in principal, the common eigenvectors should explain the same percentage of the total variation in a sample or, indeed, that the common eigenvectors have associated eigenvalues that are numerically significant.

Although there exists an algorithm to calculate common principal component models (the FG algorithm) no commercially available algorithm exists for PCPCs. Flury (1988) outlines a maximum likelihood method for estimating PCPCs but, as Flury notes, it is extremely cumbersome with, for example, a small model of twelve variables, with two common principal components and two data sets requiring the evaluation of 153 Lagrange multipliers in the likelihood function. Fortunately, Flury (1988) outlines an approximate maximum likelihood method that has the attractive property that testing down from a full common principal component model to PCPC models with q tending to p can be performed.

The methodology is straightforward. Partition the eigenvector matrix as calculated by the maximum likelihood estimates of the common principal components model as in (4.5.7) above. Then form the matrix:

$$A_s^{(i)'} X_i A_s^{(i)} \quad ...(4.5.9)$$

The ((p-q) x (p-q)) matrix (4.5.9) can be rotated such that:

$$Q_i' A_s^{(i)'} X_i A_s^{(i)} Q_i \quad ...(4.5.10)$$

(4.5.10) is diagonal (i.e. Q_i' forms the eigenvector of (4.5.9)) and the matrix $A_s^{(i)}Q$ replaces A_s^i in the partitioned eigenvector matrix (4.5.7). Using the new eigenvectors, estimates of the eigenvalues can be obtained and so can estimates of the covariance matrices under the hypothesis that the partial components model is correct using the sample version of (4.5.5). Using the estimates of the samples' covariance matrices under either common or partial common principal components a log-likelihood statistic can be constructed as:

$$\chi_{cpc}^2 = \sum_{i=1}^{k} n_i \ln\left(\frac{\det X_{cpc}^{(i)}}{\det X_i}\right) \quad ...(4.5.11)$$

For the partial model, $X_{cpc}^{(i)}$ is replaced by $X_{pcpc}^{(i)}$ and the common principal component model statistic is asymptotically distributed as χ^2 with ((k-1)p(p-1)/2) degrees of freedom. The partial model is asymptotically distributed as χ^2 with ((k-1)q(2p-q-1)/2) degrees of freedom. As the common model forces more structure on the data than the partial common principal component model, it would be expected that common principal component models would have larger test statistics and that these would decline (in the limit to zero) as the number of specific components was allowed to rise. Using this test we can analyse whether moving from a "general" formulation with only one common partial component to a full common principal component model is valid.[45] This procedure is used in chapter 6 to test the stability of eigenvectors between samples, across techniques and across countries.

4.6 Finding Influential and Untypical Observations using Principal Components

As Krzanowski (1987a) makes clear there are further questions that can be asked about particular observations (in the case of yield curves particular days) by distinguishing "influential" and "untypical" observations. An influential observation is one whose omission causes large changes in the results of the analysis but that shows no obvious distinguishing features in terms of its measured values. An untypical observation, on the other hand, can be distinguished as an outlier in the measured data.

The influence or importance of the variables themselves can be descriptively determined by the use of procrustes analysis. This has an advantage over alternative methods proposed by Jolliffe (1972,1973) (see below) and McCabe (1984) in that the retained data will reproduce as closely as possible the general features of the entire data set.[46] Jolliffe (1987), however, argues that procrustes analysis should be used in conjunction with other techniques because procrustes analysis concentrates on finding group structures and discards other information. Krzanowski (1987b), however, believes this to be an advantage. He points out that the methods proposed by Jolliffe (1972,1973) and McCabe (1984) often fail to find the same variables when applied to the same data set.

Procrustes analysis can be defined as follows. Let Y be an (n x k) matrix of principal component scores, where k is the number of dimensions that provides the best approximation to the true data set. Z

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[45] Although some alternative tests exist, as the results are subjective in the case of Keramidas et al (1987); lack critical values in the case of Krzanowski (1979); and are less general than that described in the text, Schott (1991), we have no hesitation in restricting our analysis to Flury's log likelihood test.
[46] See Krzanowski (1987b) p.23.

is the (n x k) matrix of principal component scores when the number of variables in the data set has been reduced by at least one. Procrustes analysis seeks to find the minimum sum of squared differences between the elements of Y and Z when the elements of Z have been matched under translation (i.e. mean centred) and rotation. These two processes are undertaken because the analysis is interested in relative positions rather than absolute positions. Rotation is achieved by multiplying Z by an orthogonal matrix, Q, where QQ'=I. Thus the sum of squared deviations can be written as:

$$M^2 = \text{trace}\,(YY' + ZQQ'Z' - 2YQ'Z') = \text{trace}\,(YY' + ZZ' - 2YQ'Z) \qquad ...(4.6.1)$$

To minimise M^2, 2YQ'Z has to be maximised. This can be achieved by the use Lagrange multipliers and it can be shown that the required rotation is given by Q=VU', where U and V' are the matrices defined in (4.3.1) above by the singular value decomposition of Z'Y. The M^2 statistic can be used in a backward elimination procedure to find a pre-determined number of variables that best retain the features of the overall data set. Backward elimination removes each variable in turn and calculates the value of M^2. The variable that causes the maximum increase in M^2 is permanently removed from the data set. This procedure is repeated until only the desired number of variables is retained. This procedure has been implemented by Krzanowski (1987b) using an algorithm to speed up calculation of the singular value decomposition devised by Bunch and Nielsen (1978)

Procrustes analysis is used in chapter 6 to analyse a problem not often faced by statisticians. That is, because the term structure is constructed, the number of variables that could be sampled is unlimited; i.e. the choice of maturities is arbitrary. One method by which the appropriate number of maturities to be used could be ascertained would be to estimate principal components on various sub-samples and compare the results. However, analysis would depend to a large extent on judgement and, especially when the test statistics are included, would require a vast amount of tables and space. For these reasons selecting the spacing of a given number of maturities included in the principal components analysis is conducted using procrustes analysis.

Jolliffe (1972) presents eight methods of discarding variables of which four involve the use of principal components, although two are rejected because they are either computationally too slow or because they give unsatisfactory results.[47] Jolliffe's two remaining principal component methods are known as B2 and B4. B2 requires the estimation of principal components and the eigenvector associated with the smallest eigenvalue is examined to find the variables with the largest parameters. These variables are then deleted from the data set. Jolliffe's method B4 repeats the above procedure in reverse by studying the eigenvectors of the largest eigenvalues and retaining those variables that have attracted large values.[48] Although Jolliffe (1972,1973) shows that his methods work well on both real and artificial data, the use of visual inspection introduces a significant element of arbitrariness into the procedure.

Similar problems arise if Flury's (1988) concept of redundancy is implemented. A variable is redundant for one or more principal components if its corresponding coefficients in the eigenvectors are zero. However, it is unlikely that an element in an eigenvector will be exactly zero so that an arbitrary decision on how small the element has to be needs to be made. For this reason tests of redundancy and the B1 and B2 tests are not used in this thesis. Instead a procrustes analysis (as described above) is used on a reduced data set to examine which subset of variables contains the most information thereby implicitly setting the omitted variables' coefficients to zero in the eigenvectors.

It is also possible to estimate descriptive statistics to illustrate the importance of each day's spot rates on the estimated principal component coefficients. The procedure, due to Krzanowski (1979), is straightforward. After deciding the minimum number of principal components that describe the data, say m; a matrix of the first m principal component coefficients can be constructed, say V_m. Similarly a matrix, $V_{m,i}$, can be constructed by recalculating the principal component coefficients when the i^{th} day has been deleted. An (m x m) matrix can be formed from $V_m^{'}V_{m\,i}$ that, when it has undergone singular value decomposition, can supply the diagonal matrix D. From D the smallest element, d, can be chosen and used to calculate the maximum critical angle, θ , from the formula:

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47 See Jolliffe (1972) p.164, table 1.

48 This procedure is clearly easier if a correlation matrix rather than a covariance matrix is used in the principal components analysis.

$$6 = \cos^{-1}(d) \quad ...(4.6.2)$$

However, as there are no distributional results available to suggest when 6 is large, this procedure is not followed. Krzanowski (1987) suggests that if 6 is 10 degrees and above, then the omitted day is influential in determining the overall pattern of the principal component coefficients.

As has been shown there are a large number of tests to which a principal components analysis of interest rates can be subjected. Not all of them will be used in the following chapters because of the absence of distributional results, and, in some, cases the computational burden that would be imposed. Nevertheless, the selection used still represents a significant improvement on previous work where no descriptive or statistical tests are reported.

4.7 Summary and Conclusions

The chapter has provided a summary of the method of principal components and drawn together descriptions of a series of descriptive and statistical tests that the principal components of the interest rate data can be subjected. A novel aspect of the chapter is that it has drawn together a survey of previous work on interest rates using principal components, and this has not previously been attempted. The work seems to suggest that there is a superficial similarity in the results but on closer examination there are a number of weaknesses in interpretation by a number of authors. There is disagreement on whether or not the results are stable, how to characterise the third principal component and, indeed, whether or not the third component is simply noise. However, the single weakness of previous studies is the complete absence of statistical testing, and without this as a basis for discussion there is little head way that can be made in answering questions about the nature and stability of the components.

Appendix 4.1 Alternative Methods to Principal Components

One area of potential confusion is the relationship between principal components and other multivariate forms of analysis, in particular, factor analysis. What principal components analysis achieves is that each constructed component maximises the explained part of the variances of the observations. In contrast, classical factor analysis is designed to parsimoniously reproduce the correlations between the observations using as few factors as possible. Moreover, factor analysis imposes restrictions upon the elements of the covariance matrix of the observations whereas principal components does not.[49] As the primary aim of using principal components is as a data reduction technique its use is preferred in this thesis over factor analysis.

Another approach is to analyse the data using cointegration techniques to find the common trends that drive interest rates. The intuition behind this approach is straightforward. If each of the interest rates contains a stochastic trend, there is no reason why this trend may not be shared between a number of the variables. More formally, if each interest rate is integrated of order 1, I(1), then Stock and Watson (1988) (building upon the work of Beveridge and Nelson (1981)) show that the vector y (n x 1) of variables of interest can be expressed as:

$$y_t = y_o + A\tau_t + a_t \quad ...(4A.1)$$

Where: y_t is the vector of variables.

y_0 is the initial value of the y vector, i.e. at time zero.

τ_t is the stochastic trend vector.

a_t are the transitory components that are stationary moving averages.

A is a matrix of coefficients.

The stochastic trend evolves as follows:

$$\tau_t = \pi + \tau_{t-1} + v_t \quad ...(4A.2)$$

Where: π is a vector of constants.

v_t is a vector of random errors.

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[49] See Kloek (1990) p.205 and Adelman (1990) p.91.

Equations (4A.1) and (4A.2) imply that the vector y can be separated into a permanent stochastic trend component and a transitory cyclical component, where the cycle feature is driven by the serial correlation of the series. When the variables in y are I(1) and they do not form a cointegrating vector (i.e. they have no tendency to move together in the long run) then each of their trends is independent (i.e. there are n common trends). If, however, there are r cointegrating vectors then there are n-r common stochastic trends.[50] A similar procedure can isolate common stochastic cycles (as described by a_t in equation (4A.1)) by analysing the short run dynamic behaviour as embodied in the first differences of y. If a vector can be found (the codependence vector) such that it can be used to produce a linear combination of the first differences of y that are unforecastable then the vector, y, may share cycle features between its elements. Thus if there are only n-s stationary moving average processes there are (n-s) common stochastic trends.

The presence and the number of common trends can be tested for using standard tests for cointegration, such as the trace test and eigenvalue test, whilst common cycles can be tested for using a test based on canonical correlations or by χ^2 likelihood test restrictions on the VAR. Vahid and Engle (1993) have shown that if the number of cointegrating vectors and the cofeature vectors sum to the number of variables, then the vector y can be decomposed into its trend and cycle features. This uses the estimated cointegration and codependence vectors without having to invert the vector error correction model that underlies the estimation procedure.

The common trend approach has been used on short term US interest rates by Stock and Watson (1988) (albeit mainly as an illustration of the technique) and by Pagan et al (1995) also on US rates. Both find just one common trend. Stock and Watson (1988) argue that this finding is consistent with the expectations hypothesis of the yield curve. Other applications have analysed the existence of a World interest rate, for example Pain and Thomas (1997) and Helbling and Wescott (1995). The common trend approach, by using a stochastic trend, is much more flexible than the first principal component in determining the level of interest rates. This is its primary advantage over principal components but one

[50] The terminology is the reverse of what might be expected. It is not the common trends that are shared but rather linear combinations of these common trends.

that could be matched by using recursive estimates of principal components. However, as Helbling and Wescott (1995) point out, a stochastic trend can be too flexible so that too much variation in interest rates can be assigned to changes in the trend.

Using a common trend approach a measure of a trend interest rate may be constructed which is the counterpart (although not numerically) to the first principal component. The remaining principal components are forced into the cyclical component. To the extent that there are more than two factors that describe interest rates this is a weakness of common trends. There are also a number of different decompositions that can be used to divide interest rates into trend and cyclical components. This is because of the difficulty in identifying the parameters of the trend and cycles and the loading matrices that determine how these feed into a given variable. Thus Stock and Watson (1988) propose one specification, whereas 'Kasa (1992) proposes another. Escribano and Pena (1994) show that the decomposition proposed by Gonzalo and Granger (1995) is equivalent to that of Stock and Watson(1988).

Moreover, the common trends approach suffers from the same difficulties as other cointegrating studies. The choice of lag length to be used in the VAR can alter the number of cointegrating vectors found and there can be disagreements between the tests as to the number of cointegrating vectors. Even when cointegrating and codependence vectors are found their interpretation may be difficult. (This is also a problem with the principal components approach as discussed above.) All of these problems are to be found in Pain and Thomas' (1997) paper. For these reasons the common trends approach is not followed in this thesis.

Chapter 5

The Robustness of Principal Components Analysis to Changes in Data.

5.1 Introduction

The previous chapter surveyed the method of principal components analysis (PCA) and outlined some tests and descriptive statistics that can be used to reach conclusions about the value of principal components in analysing the term structure. In this chapter we apply principal component analysis to the data bases that were estimated in chapter 3. We also use the cubic spline yield curve from Egginton and Hall (1994), spot and forward data, estimated by the Svensson method, from the Bank of England (1995) and spot, par and forward data from McCulloch and Kwon (1993).

It is important to re-emphasise that redemption yields are not measures of interest rates. Therefore, using redemption yields cannot throw any light on questions such as whether the first principal component can be described as the level of interest rates. However, what the use of redemption yields can do is throw light upon the properties of principal components analysis. As spot and redemption yields are measuring different things (indeed it is not clear what, if anything, the redemption yields measure) then the principal component analysis of these series should produce different results. The comparison of redemption yields and spot rates therefore only reveals information about the usefulness of principal components. It does not, and is not intended to, reveal anything about the term structure of interest rates.

Each data set is given a three-letter identifier. The first letter describes the method of estimation and is one of five types: a simple cubic, C, the Dierckx method, D, the Svensson method, S, the B-spline method, B, and McCulloch method, M.[1] The second letter represents one of four data types:

[1] It should be recalled that the B-spline method uses the McCulloch method as part of its estimation procedure.

redemption yields, Y, spot rates, S, forward rates, F and par rates, P. The final letter identifies either UK data, £, or US data, $. The data sets under consideration are as follows:

1) Cubic spline redemption yields calculated by Egginton and Hall (1994), henceforth CY£.

2) Variable knot redemption yields calculated using Dierckx's method as described in chapter 3, henceforth DY£.

3) Spot rates, estimated using the Svensson method, from the Bank of England (1995), henceforth SS£.

4) Spot rates using a B-spline endogenous knot position estimation procedure as described in chapter 3, henceforth BS£.

These four series represent the core data sets on which most of the comparisons will be carried out. The four remaining series, listed below, allow two further comparisons to be carried out between spot and forward rates and spot and par rates.[2]

5) Forward rates, estimated by the Svensson method, from the Bank of England (1995), henceforth SF£.

6) US spot rates from McCulloch and Kwon (1993), henceforth MS$.[3]

7) US forward rates from McCulloch and Kwon (1993), henceforth MF$.

8) US par rates from McCulloch and Kwon (1993), henceforth MP$.

The combinations of five estimation methods, four data types and two countries potentially give 40 different time series that could be constructed. However, in practice this is not the case. I do not have access to the data which McCulloch and Kwon used to generate MS$, MP$ and MF$ and hence it is not possible to construct US data using any of the other four methods. This reduces the potential number of combinations to 20. Furthermore, the Svensson method, the B-spline and McCulloch methods are all designed to calculate spot rates. To go back and calculate yields, which are theoretically inferior to spot rates, is a dubious practice. Secondly, arguments put forward in chapter 3 suggest that the B-spline is a superior estimation technique to the cubic spline used in the NAG routine. Therefore to estimate

2 The results of the par/spot comparison show that there is little point in undertaking a par/forward comparison.

3 Real spot rates and spot inflation curves estimated by the Bank of England are studied in chapter 8.

spot rates using the cubic spline routine would be a retrograde step.[4] Thirdly, the Dierckx method was designed to be applicable to yield data. To adapt it to calculate spot, par or forward rates would require a two step estimation routine from an initial estimate of the spot, par or forward rates. However, the results in chapter 3 do not suggest that that this route of research is worth pursuing.

This leaves nine potential data sets spot, forward and par rates estimated by Svensson, B-spline and McCulloch methods. However, the B-spline method uses the McCulloch approach as part of its estimation procedure and, given that we prefer the B-spline method to the simple cubic used in the McCulloch method, the B-spline method can be regarded as encompassing the McCulloch method. Finally, a comparison of the par and spot rate eigenvalues and eigenvectors for the US data, MP$ and MS$, shows that the data are extremely close, and it was concluded that little is lost by omitting a fuller analysis of par rates for UK data. Thus, instead of appearing to ignore 32 potential combinations, in fact only one useful category is ignored, the B-spline estimates of forward rates. However, as we can compare the forward rates for both the US and the UK data and as forward rates are not the central concern of this thesis, we feel justified in ignoring this category. The eight data sets therefore give a better coverage of the plausible permutations than their number suggests. Yet this raises the question why not just concentrate on four series, the B-spline and the Svensson methods for spot and forward rates? The answer is that whilst this is possible, comparing and contrasting the results from the eight series actually used can potentially provide strong support for the use of principal components analysis at less analytical cost than by analysing data sets which only differ in one dimension. Thus it can be argued that the variety of methods and term structure data produced is a strength not a weakness of this chapter.

Both DY£ and BS£ data, which were estimated in chapter 4, can be calculated at three-month intervals for maturities between two and 24 years[5]. This enables a direct comparison to be undertaken with the CY£ and SS£ data that were obtained from other sources. The interpolated data is used to calculate the

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[4] It should be noted that the CY£ data set was available at low cost from the work of Egginton and Hall (1994). They used the cubic spline routine because this calculation method was the one used at the time at the Bank of England when the work was carried out and not to have used it would have caused tensions between various departments of the Bank.

[5] Using the subroutine E02BBF, see NAG (1991a) for details.

covariances and correlations that are used as the basis for the principal components analysis. One difficulty in reporting the principal components of the McCulloch and Kwon (1993) data sets is that the spacing of the data is irregular. Their data is monthly between the months 0 to 18, then quarterly to two years, then semi-annually to three years, then annually to 35 years with a final jump to 40 years. Data on all 56 maturities is not available for the entire data period from December 1947 to February 1991. The longest complete data set has 33 observations, with the longest maturity being 13 years. Over 39% of the observations are below one year in maturity and over 60% are below two years in maturity. This is a very different maturity structure from the one studied using the UK data, whatever its source, and, consequently, inter-country comparisons are not made in this chapter. Such comparisons would, of course, be complicated by the fact that the US data is month-end, rather than daily, and covers the period 1951 to 1991, rather than just 1979 to 1990. A more consistent data set using month end data from the SS£, a time period of March 1982 to February 1991 and maturities of two years to 24 years (23 observations) is analysed in chapter 6. In this chapter the main objectives are to compare and contrast the US results for different types of term structures (spot, forward and par) as a crosscheck on the comparisons made on spot and forward rates using the Bank of England data.

Egginton and Hall (1994) used the TSP package to derive their principal components. However, this package automatically standardises the mean and variances of the yields so that many of the test procedures outlined in chapter 3 are no longer valid.[6] Consequently, a new program was written using NAG library subroutine G03AAF to calculate the principal components. G03AAF automatically calculates Bartlett's test of isotropy. To this was added Krzanowski's (1984) test of the variability of the eigenvectors and Girshick's (1939) variance and covariances for the eigenvectors. This latter test is also transformed to give correlation coefficients but, to save space, the results are reported only for the first 3 eigenvectors (this restriction does not alter the conclusions) and then only in a form of a cumulative frequency table. The results for Girshick's variances and covariances were validated against the results reported in Jackson and Hearne (1973) and those for Krzanowski's (1984) tests from results reported in that paper itself. With the exception of the comparison of the spot rates with the par and forward rates these tests are not deployed until chapter 6. Krzanowski's (1987b) variable selection procedure is,

[6] See Hall and Cummins (1993) for details of the TSP principal components analysis.

however, used to throw further light upon the selection of the number of observations and their maturities.

The following questions need to be examined using the principal component program:

A) Does altering the data matrix from a covariance to a correlation matrix make any significant differences to the results of a principal components analysis?

B) Do the results alter if the sampling of the data across the term structure is altered by entering more data?

C) Does changing to different representations of the term structure, par, spot or forward, alter the results in any significant manner?

D) Do the results differ if the data is changed from yields to spot rates?

E) Are the first three components of the spot rates consistent with the interpretation from chapter 4 of them being measures of the level, slope and twist of the term structures?

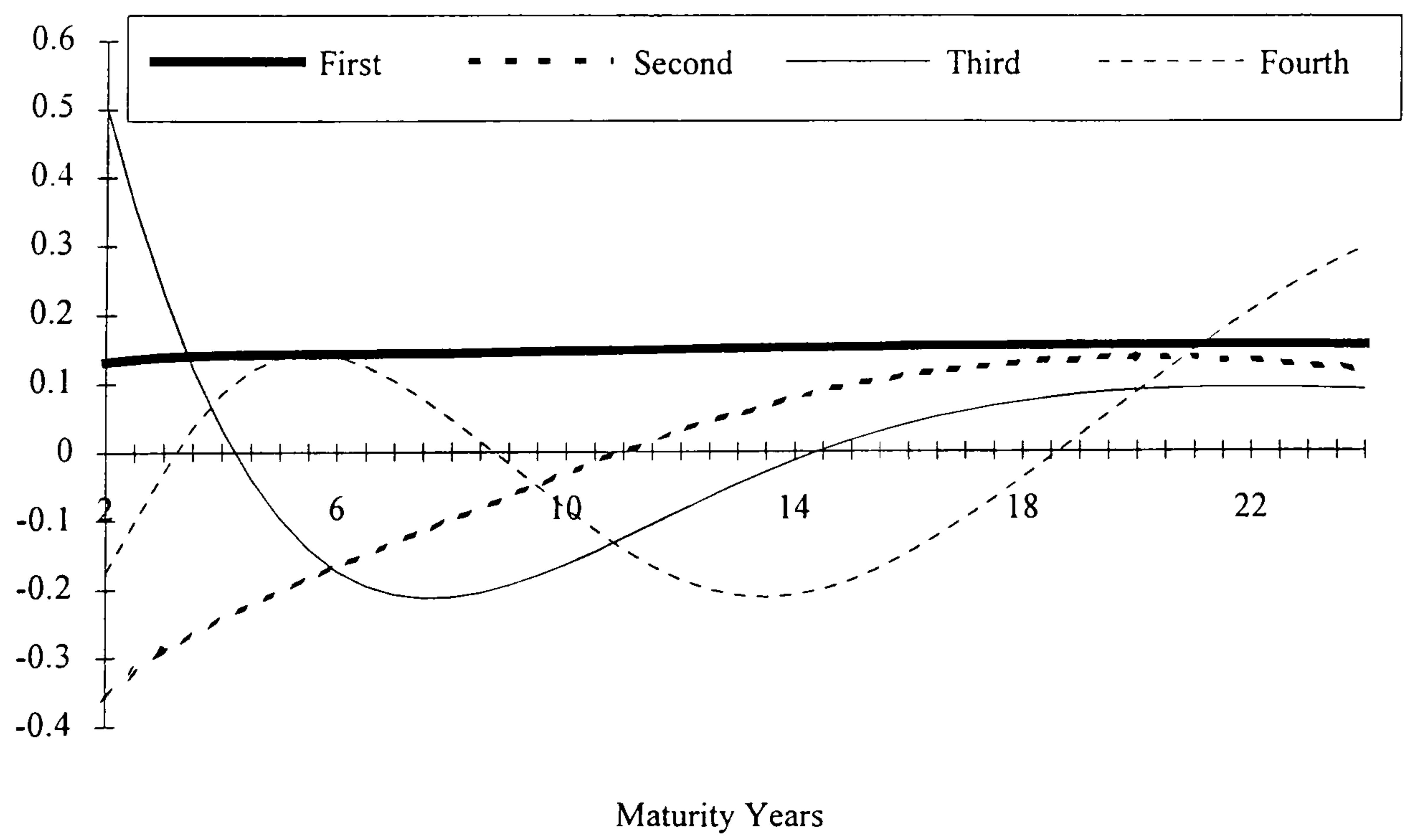
If the answers to questions A), B) and C) are, on balance, "no" then it can be concluded that the principal components decomposition of the term structure is a valid method with which to analyse the term structure. If the answer to D) is "no" then principal components analysis cannot distinguish between data on interest rates (spot rates) and meaningless data (redemption yields). This would be a weakness in the approach. If the answer to E) is "yes" then the principal component scores also have a simple interpretation that increases their attractiveness as means of summarising term structures.

The plan of the chapter is as follows in section 5.2 we present the results for a single data set, DY£, to provide an overview (or benchmark) for the following results. In section 5.3 we compare and contrast the results of PCA using covariances as opposed to correlation matrices for both the eigenvectors and eigenvalues. Changes to the eigenvalues and eigenvectors when the number of observations are altered is examined in section 5.4. Section 5.5 comparisons are made across the different types of series: spot, par and forward. Section 5.6 compares and contrasts the spot and the yield components to see if differences between them can be discerned. Section 5.7 provides a summary and conclusions.

5.2 The General Properties of the Principal Components

In chapter 4 the survey of papers using PCA suggested that yield data could be summarised by a few components and that these corresponded to the level, the slope and a twist in the term structure. In this section we investigate how well the data sets listed above conform to this general description. To give a feel for the results using the levels of the DY£ data are shown in chart 5.1. As can be seen all the eigenvectors adjust smoothly across maturities and thus little information is lost by presenting the eigenvectors at two-year intervals in the tables that follow. The first eigenvector of DY£ is, for all intent and purposes, flat with values ranging from 0.132 to 0.155. This eigenvector is clearly dominant explaining 97% of the variance in the data. The second eigenvector measures the slope with the coefficients changing sign at 11 years maturity and ranging from -0.358 to 0.093. The second principal component explained 2.3% of the variance. The third eigenvector explains a twist in the data being positive for maturities at and below 4 years and above 14 years. It explains 0.5% of the variance of the data. Finally, the fourth eigenvector has a complicated twist shape, which explains a minuscule 0.1% of the data.

Chart 5.1 Eigenvectors from DY£



Similar results could be presented for other data sets. However, the CY£ data set, although it has the same qualitative interpretation, differs in quantitative terms from the other data sets and the UK forward rate data, SF£, also differs in qualitative terms. These data sets are discussed in more detail in sections 5.5 and 5.6.

5.3 The Effects of Changing Matrix Types on Principal Components

There are a number of potential reasons why the explanatory power and the eigenvectors should differ between the US and the UK data. These are that: the data is from a different country; it has a much longer time span; the sampling frequency is monthly, rather than daily, and the spot rates sampled differ both in the maximum maturity considered and the time interval between maturities. Given these differences the US data is set aside until section 5.5 and in the following paragraphs only the UK data are studied.

Table 5.3.1 Eigenvalues, Percentage of Variance Explained, Bartlett's Test of Isotropy and the Lawley Correction for DY£.

First Eigenvalue			Second Eigenvalue			Third Eigenvalue			Fourth Eigenvalue		
value	%	chisq	value	%	chisq	value	%	chisq	value	%	chisq
11.56	96.3	-	0.33	2.8	-	0.09	0.8	-	0.01	0.1	-
34.81	96.4	269136	0.99	2.7	168834	0.27	0.7	135677	0.04	0.1	97489
		269223			168852			135662			97456
22.25	96.8	-	0.57	2.5	-	0.15	0.6	-	0.01	0.0	-
66.97	96.8	773981	1.65	2.4	556519	0.43	0.6	477484	0.07	0.1	389823
		774233			556578			477430			389692
43.64	97.0	-	1.05	2.3	-	0.26	0.6	-	0.04	0.1	-
131.26	97.0	1865286	3.08	2.3	1412040	0.74	0.5	1235407	0.13	0.1	1047534
		1865899			1412193			1235268			1047184

Note Calculated from the level of data over the period 2 January 1979 to 21 August 1990.
row 3 uses 12 observations and a correlation matrix
row 4 uses 12 observations and a covariance matrix
row 6 uses 24 observations and a correlation matrix
row 7 uses 24 observations and a covariance matrix
row 9 uses 45 observations and a correlation matrix
row 10 uses 45 observations and a covariance matrix
% = percentage of total variance explained
Chisq rows 4, 7 and 10 is Bartlett's test of Isotropy
Chisq rows 5, 8 and 11 is the Lawley correction.

Table 5.3.2 Eigenvalues, Percentage of Variance Explained, Bartlett's Test of Isotropy and the Lawley Correction for CY£.

First Eigenvalue			Second Eigenvalue			Third Eigenvalue			Fourth Eigenvalue		
value	%	chisq	value	%	chisq	value	%	chisq	value	%	chisq
10.05	83.7	-	0.72	6.0	-	0.28	2.3	-	0.24	2.0	-
34.93	83.6	67124 67146	2.68	6.4	16667 16669	1.13	2.7	8912 8912	0.72	1.7	6167 6166
19.38	84.3	-	1.07	4.7	-	0.50	2.2	-	0.37	1.6	-
68.79	84.6	159248 159300	4.00	4.9	46842 46847	1.89	2.3	31319 31316	1.16	1.4	23807 23800
37.84	84.1	-	1.92	4.3	-	0.98	2.2	-	0.56	1.2	-
135.50	84.4	407052 407186	7.20	4.5	173852 173871	3.56	2.2	140330 140315	1.85	1.2	121474 121434

Note: See table 5.3.1.

Table 5.3.3 Eigenvalues, Percentage of Variance Explained, Bartlett's Test of Isotropy and the Lawley Correction for BS£.

First Eigenvalue			Second Eigenvalue			Third Eigenvalue			Fourth Eigenvalue		
value	%	chisq	value	%	chisq	value	%	chisq	value	%	chisq
10.81	90.1	-	0.84	7.0	-	0.26	2.2	-	0.09	0.7	-
41.35	90.7	255832 255915	3.04	6.7	186871 186891	0.94	2.1	157906 157888	0.25	0.5	125781 125738
20.99	91.3	-	1.48	6.4	-	0.41	1.8	-	0.11	0.5	-
80.55	91.8	662817 663033	5.32	6.1	508650 508704	1.57	1.8	435118 435068	0.29	0.3	341012 340897
41.35	91.9	-	2.77	6.1	-	0.74	1.6	-	0.13	0.3	-
158.93	92.3	1501731 1502225	9.93	5.8	1174660 1174787	2.85	1.7	1006439 1006325	0.33	0.2	754383 754131

Note: See table 5.3.1.

Table 5.3.4 Eigenvalues, Percentage of Variance Explained, Bartlett's Test of Isotropy and the Lawley Correction for SS£.

First Eigenvalue			Second Eigenvalue			Third Eigenvalue			Fourth Eigenvalue		
value	%	chisq	value	%	chisq	value	%	chisq	value	%	chisq
9.55	79.6	-	1.94	16.2	-	0.46	3.9	-	0.04	0.3	-
10.49	79.3	233488 233593	2.19	16.5	202684 202713	0.49	3.7	174550 174522	0.05	0.4	137890 137824
18.69	81.2	-	3.47	15.1	-	0.77	3.4	-	0.06	0.3	-
20.52	81.3	608899 609175	3.80	15.1	536941 537020	0.82	3.2	470090 470015	0.08	0.3	379591 379414
36.94	82.1	-	6.54	14.5	-	1.40	3.1	-	0.10	0.2	-
40.57	82.3	1376903 1377532	7.07	14.4	1221128 1221312	1.48	3.0	1074536 1074368	0.13	0.3	869744 869340

Note: See table 5.3.1 except that the statistics are calculated from the level of data over the period 31 March 1982 to 21 August 1990.

From tables 5.3.1 to 5.3.4 it can be seen that the first principal component (PC1) explains between 79.3% and 97.0% of the variation in the UK spot and yield data. The lower estimate is from the SS£ data (table 5.3.4) and the higher estimate is from the DY£ data (table 5.3.1). Thus, although PC1 explains a substantial proportion of the variance, this proportion is not always as large as had been indicated by the papers surveyed in chapter 4. Bartlett's test of isotropy (and the Lawley correction) indicates that the hypothesis that the eigenvalues are the same between components can be easily rejected, indeed the computerised χ^2 function was unable to calculate a probability for any of the tests of isotropy because the statistic was so large. This result was true for all components analysed. The lack of statistical testing in previous work in this area, noted in chapter 4, is not, therefore, as serious for practical purposes as that chapter suggested.

The yield and spot data, the second principal component (PC2) explains between 2.3%, for the DY£ data (table 5.3.1) and 16.5% using the Bank of England spot data, SS£, (table 5.3.4) of the variation. Even for the SS£ data there is a substantial difference between the explanatory powers of the first and second components. Cumulatively PC1 and PC2 explain between 88.4% using CY£ data (table 5.3.2) and 99.3% using the DY£ data (table 5.3.1). Thus for almost all purposes only two principal components are required to explain the data. The results for the spot rates reconfirm those cited in chapter 4.

For the third principal component (PC3) the percentage of variation explained is at most 3.9% using the SS£ data (table 5.3.4) and as low as 0.5% using the DY£ data (table 5.3.1). The range of percentages for the fourth eigenvalue range from virtually 0%, using the DY£ data (table 5.3.1), to 2%, using the CY£ data (table 5.3.2). Although the third and fourth eigenvalues explanatory powers overlap in percentage terms, Bartletts' tests of isotropy and the Lawley correction suggest that the eigenvalues are distinct in all cases.

In general, there are no systematic changes in explanatory power between the covariance and correlation matrices, ceteris paribus. In terms of the percentage of the variance explained the measures are always close for each of the data sets. The largest difference is just 0.6% for PC1 for the BS£ data with 12 observations, table 5.3.3. The conclusion that the greatest proportion of the variances can be accounted for by just two or three eigenvalues is, therefore, independent of the matrix used to form the eigenvectors.

Table 5.3.5 compares the differences that changes between the covariance and correlation matrices make for the eigenvectors. As can be seen the absolute differences between the eigenvectors are small. For the first principal component the largest average difference is just 0.0473 for the SS£ data set.

There is little overall pattern to the differences between the first eigenvectors when moving from a covariance to a correlation matrix. For the SS£ data set the two year coefficients are larger for a covariance matrix than a correlation matrix, whilst at 24 years maturity they are larger for the correlation matrices. For the remaining three data sets the reverse is true. The reason for this is because the pattern of variances differs between the data sets with the variance falling with maturity for SS£ but tending to rise for the other data sets. Using a correlation matrix standardises the diagonal at one. If Jobson's assertion discussed in chapter 4 is correct, this will raise the eigenvector coefficients at longer maturities for SS£ but reduces it for the other data sets for a correlation matrix as opposed to a correlation matrix. It should be emphasised that the changes in the coefficients remain small when moving from a correlation to a covariance matrix for the first principal component.

Table 5.3.5 Mean Absolute Differences in Eigenvector Coefficients between Covariance and Correlation Matrices

	First Principal Component		
	12 observation	24 observation	45 observations
DY£	0.0074	0.0053	0.0038
CY£	0.0288	0.0199	0.0143
SS£	0.0473	0.0327	0.0229
BS£	0.0237	0.0167	0.0118
	Second Principal Component		
	12 observation	24 observation	45 observations
DY£	0.0059	0.0046	0.0033
CY£	0.0367	0.0227	0.0141
SS£	0.0725	0.0482	0.0332
BS£	0.0301	0.0191	0.0127
	Third Principal Component		
	12 observation	24 observation	45 observations
DY£	0.0059	0.0042	0.0033
CY£	0.0542	0.0227	0.0158
SS£	0.0693	0.0437	0.0287
BS£	0.0518	0.0260	0.0148

For the second principal component table 5.3.5 indicates that the differences between the coefficients are small in absolute terms when moving from a covariance to a correlation matrix. There is little pattern in the size of the differences relative to the differences from the first principal component. SS£ and BS£ always have larger differences, whereas DY£ has smaller differences and CY£ has larger differences at 12 and 24 observations but smaller at 45 observations. Similarly, a random pattern seems to operate for the third principal component. The absolute difference between covariance and correlation coefficients for CY£ being larger for the third principal component than for the second component, whilst the reverse is true for SS£. It remains true that the absolute differences between the coefficients from covariance and correlation matrices are small for the third component, as they are for the first two components.

Overall, we conclude that there are virtually no differences in the percentage of the variance explained by each component and only small differences in the eigenvectors when the matrices is changed from a covariance to a correlation matrix. Consequently, in the following sections we use covariance matrices because this allows a greater range of statistical tests to be conducted than for correlation matrices. In the next section we consider whether differences in the number of observations used has any significant and systematic consequences for the eigenvalues and eigenvectors.

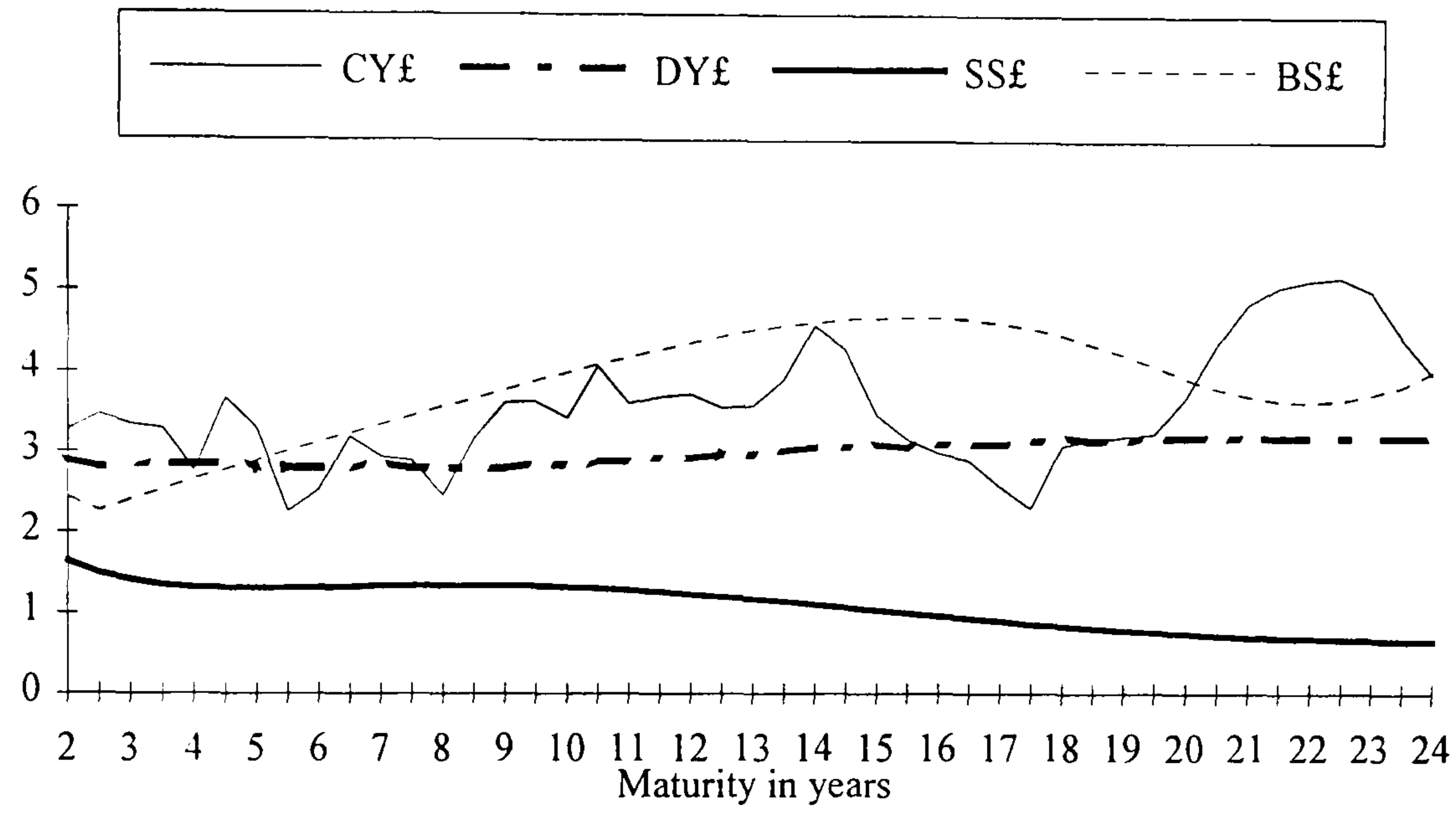
5.4 The Effects of Changing the Number of Observations on the Principal Components

In the previous section it was established that the type of matrix used in the analysis was virtually irrelevant for principal components analysis. This section enquires whether or not the number of observations used in the analysis systematically alters the eigenvalues and eigenvectors.

To begin, do the eigenvalues move systematically with the number of observations used in estimation? The evidence from the DY£ (table 5.3.1) and the spot rates, BS£ and SS£, (tables 5.3.3 and 5.3.4) suggest that as the number of observations rises the explanatory power of PC1 rises and that of PC2 falls. The increase in the percentage of the variation explained by PC1 rises by between 0.6% for the DY£ data (table 5.3.1) and 3.0% for the SS£ data (table 5.3.4) as the number of observations is increased from 12 to 45. Thus the rise is not large. For the spot rates the third principal component (PC3) also falls as the sample size increases. However, the total explanatory power of the first three eigenvalues is left unchanged by altering the number of observations.

For the CY£ data (table 5.3.2) the position is ambiguous with the maximum explanatory power of PC1 being achieved with 24 observations. Both PC2 and PC3 record falls in explanatory power as the observations are increased. Indeed, the drop in explanatory power for PC2 is sufficient to reduce the total explanatory power of all three eigenvalues by between 1.4% to 1.6%, the opposite result for the other three data sets.

Chart 5.2 Variance of Interest Rates



Except for CY£, the results above imply that by adding more observations the dominance of the level of the data becomes even greater. However, relative to the data already present, the new observations give little extra information about the slope. This occurs because each time new observations are added they are within the maturity range of two to 24 years. Consequently, once the effect of general changes in the level of interest rates is removed the (conditional) variation in the new observations will, on average, be less than the variation in the data already present. Conversely, in terms of levels the new observations will have broadly the same variation. Thus, the second principal component will explain less and the first principal component more of the total variance. However, it should be stressed that this result is an artefact of the sampling regime by which observations are added rather than a general result. If the extra yields had been all added at long (or all at short) maturities the variation of the second principal component would have risen, whilst that of the first principal component would have fallen as a percentage of total variance. Nor would this result hold if the main factors were not measures of the level and the slope of the data. Thus it is not changes in the number of observations that causes the systematic change, but the systematic entry of the data by maturity into the data set.

Adding more observations using the data set CY£ does not result in a consistent picture. This is because, as can be seen from chart 5.2, unlike the other data sets there is no consistent pattern to the variances of the CY£ data set. Hence, by adding more data, there is little reason to suspect that the matrices being entered will move in a consistent manner.

We use charts 5.3 to 5.7 to study the first principal components' eigenvectors. In these charts the factor loadings have been standardised by dividing through by the coefficient on the respective two-year yields. As the sum of the squared coefficients for each component is constrained to equal unity by design, the relative size of the coefficients as the number of observations changes is irrelevant. Consequently, standardisation does not cause a loss of information. The second point to note is that only 12 maturities' coefficients are shown, even though for the 23 and 45 observation cases more coefficients were estimated. This restriction is for ease of comparison across estimation techniques and none of the conclusions drawn in this section are substantially altered by this restriction.

Chart 5.3 First Eigenvector by Number of Observations, DY£

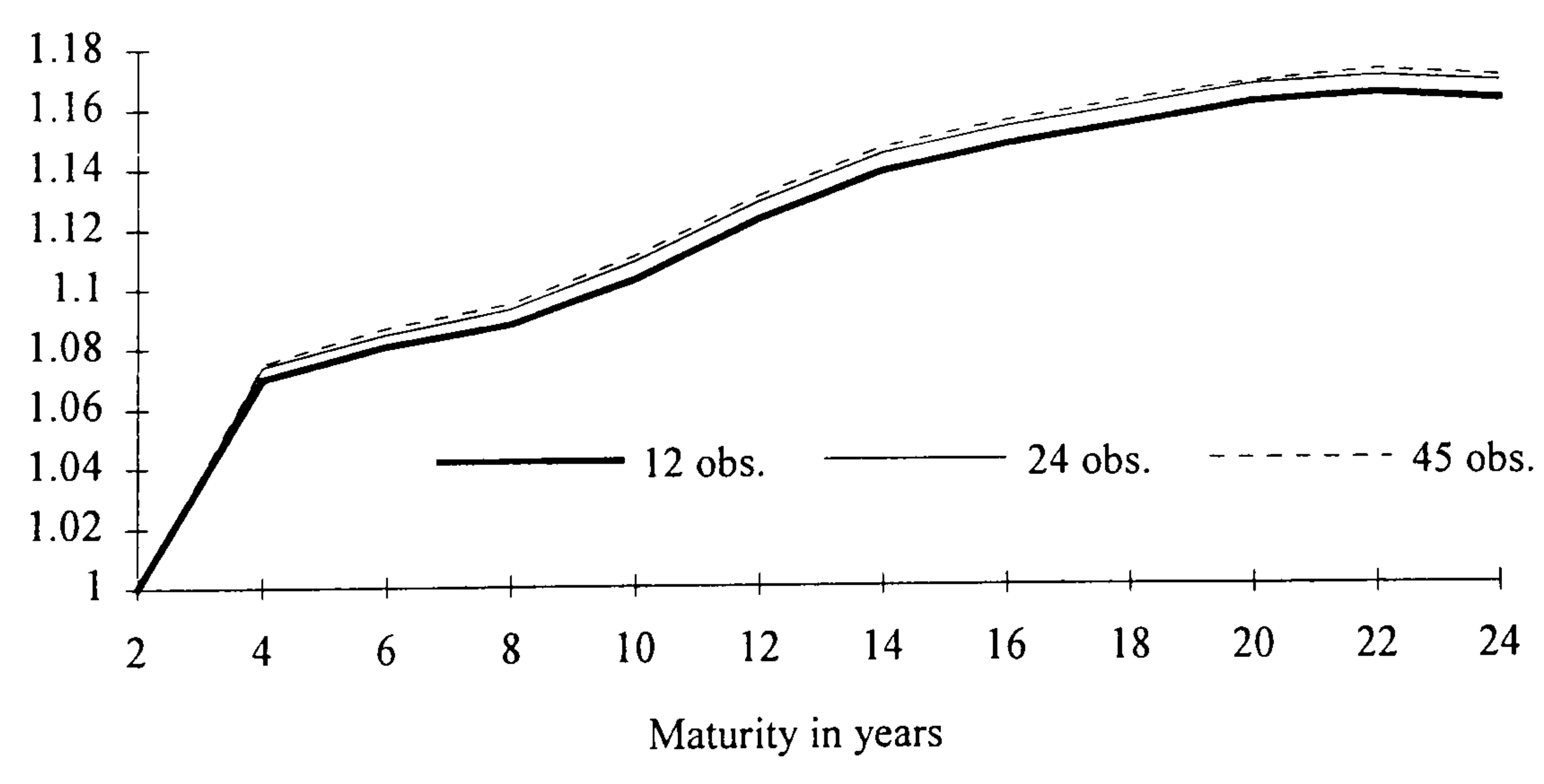


Chart 5.4 First Eigenvector by Number of Observations, SS£

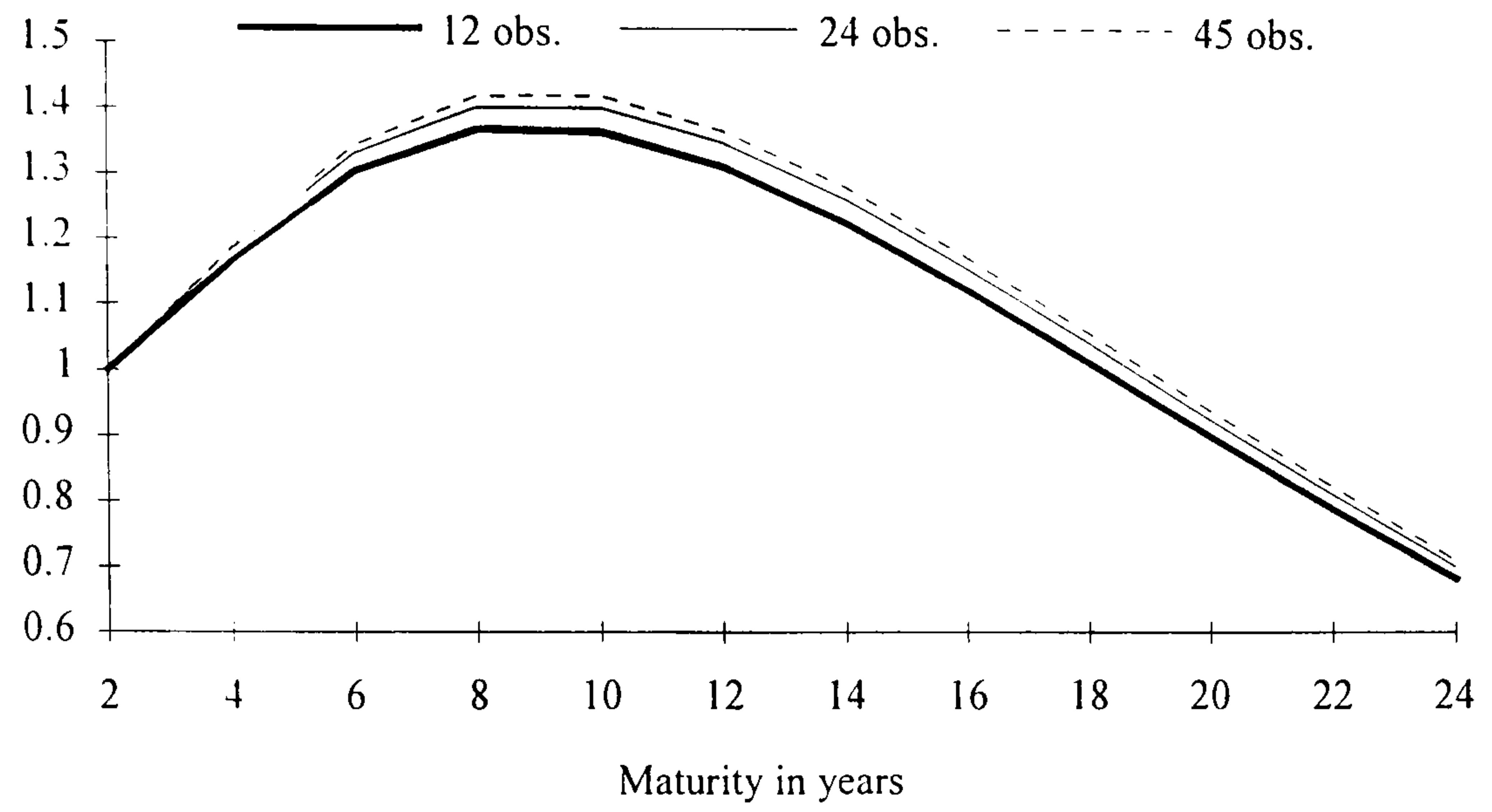


Chart 5.5 First Eigenvector by Number of Observations, BS£

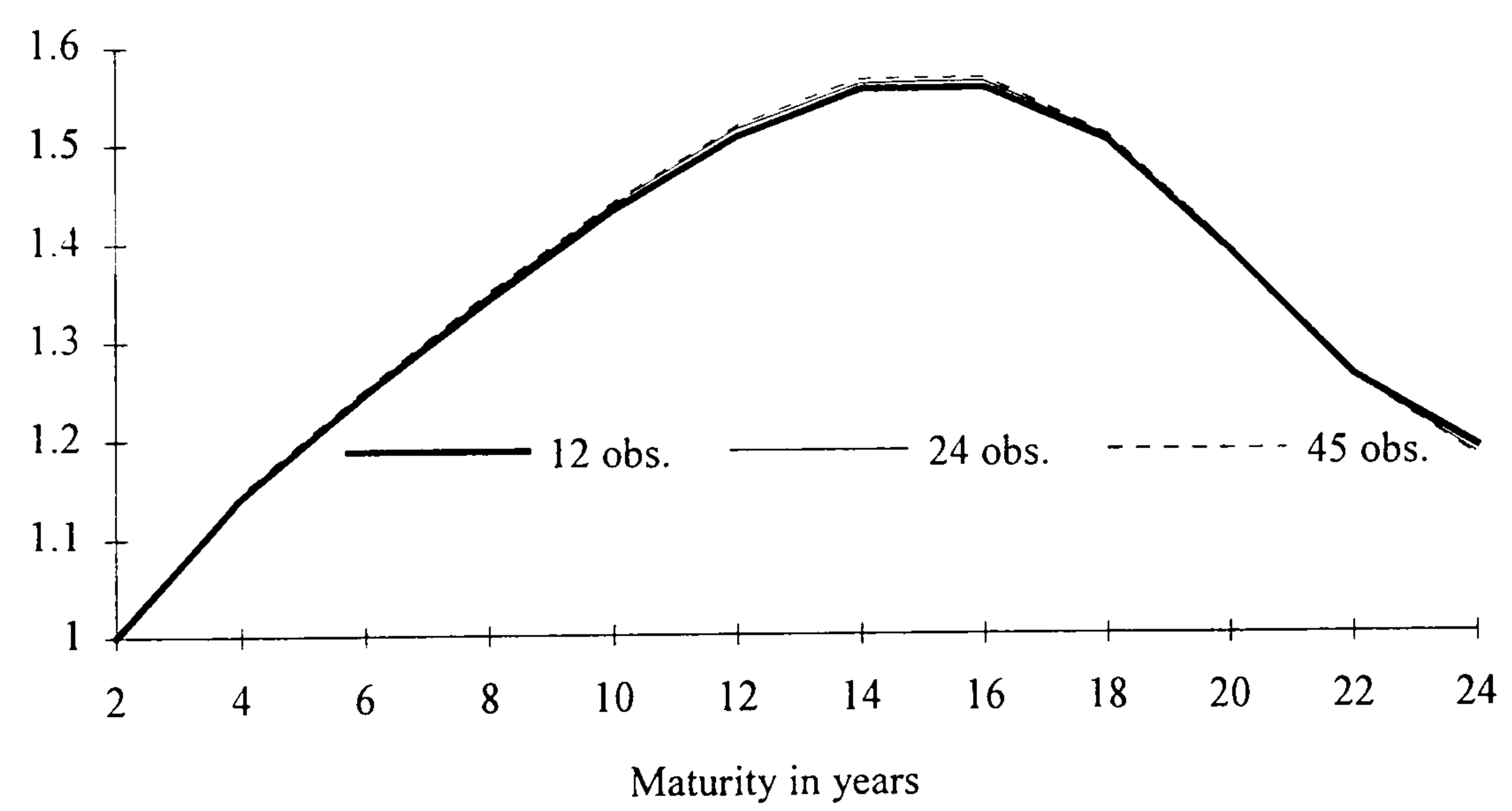
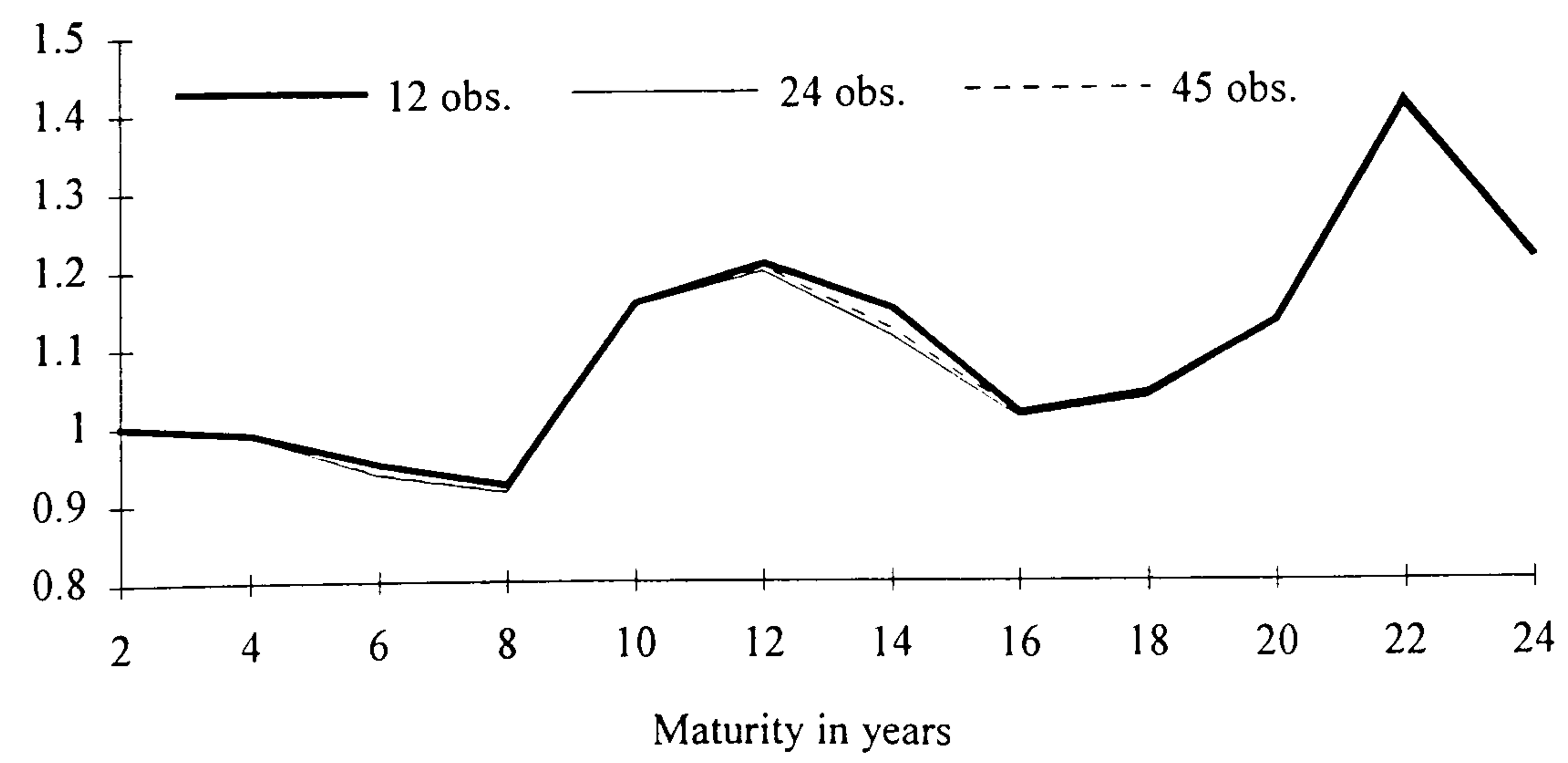


Chart 5.6 First Eigenvector by Number of Observations, CY£



It is clear from these charts that the number of observations makes very little difference to the pattern of the first eigenvector coefficients, ceteris paribus. Once the changes in the level of the coefficients on the two year interest rates are allowed for, there is relatively little difference in relative coefficient sizes as the number of interest rates sampled increases for a given matrix type and maturity. The largest absolute difference between the standardised coefficients is 5.5% for the SS£ spot rates at a ten year maturity (chart 5.4) comparing the 12 and 45 observation coefficients. For the DY£ and the spot rates (charts 5.3, 5.4 and 5.5) there is a tendency for the size of the coefficients to marginally rise (even after standardisation) as the number of rates used increases. For the CY£ data, chart 5.6, there is, if anything, a U-shaped pattern but a firm conclusion is difficult. Given that the average absolute difference in relative coefficients for all data types is just 1.2%, the overall conclusion is that changing the number of rates used in the analysis has little influence on the relative size of the first principal component coefficients.[7]

Charts 5.7 to 5.10 plot the second eigenvector by the number of observations. The immediate conclusion is again that changing the number of observations has little influence on the size of the coefficient after they have been standardised. The average absolute difference for all coefficients from any data type is 4.7%. However, the largest absolute difference is 17.8% at the 22 year of maturity for the CY£ data comparing the 12 and the 45 observation components. Again there is evidence that, even after standardisation, the coefficients are greater in absolute size the greater the number of observations. However, the CY£ data produced four (out of 11) observations where the 45 observation coefficients were not larger in absolute terms than the 23 observation coefficients. Whilst the divergence between the coefficients is larger in size than for the first principal component, each data set show the same pattern of coefficients as the number of observations increases. Changing the number of observations does not, therefore, have any significant consequences for the interpretation of the eigenvectors of the second principal component.

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[7] This result was confirmed by using Anderson's (1963) test discussed in chapter 4. For example, the DY£ data a comparison of the 12 maturites eigenvector with the 23 maturities produced a chi squared statistic of 11.32 against a 5% rejection criterion of 19.68. However, whilst never rejecting the hypothesis that the eigenvectors were equivalent, the exact values of the test were found to be highly sensitive to rounding errors and are not therefore reported.

Chart 5.7 Second Eigenvector by Number of Observations, DY£

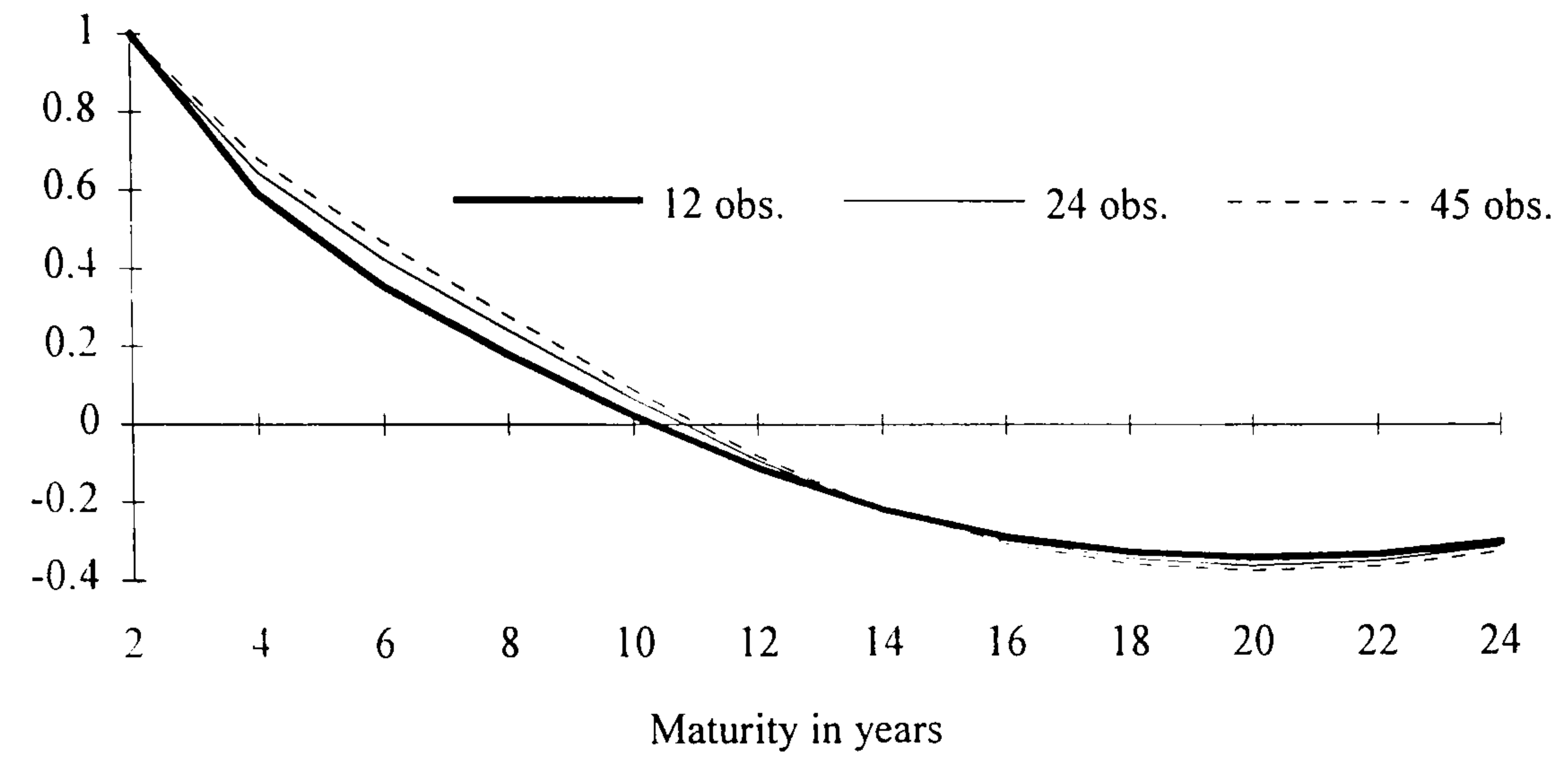


Chart 5.8 Second Eigenvector by Number of Observations, SS£

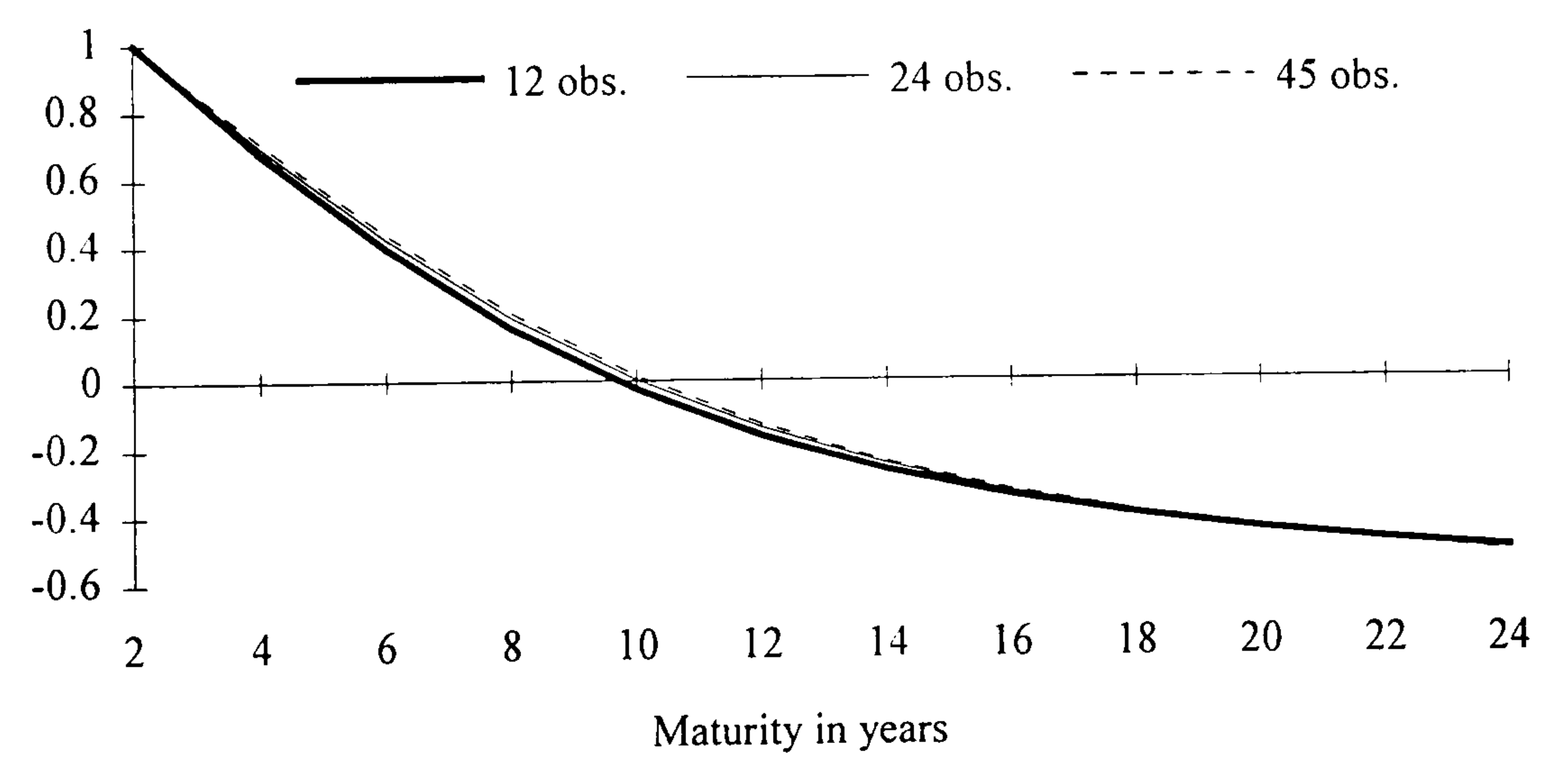


Chart 5.9 Second Eigenvector by Number of Observations, BS£

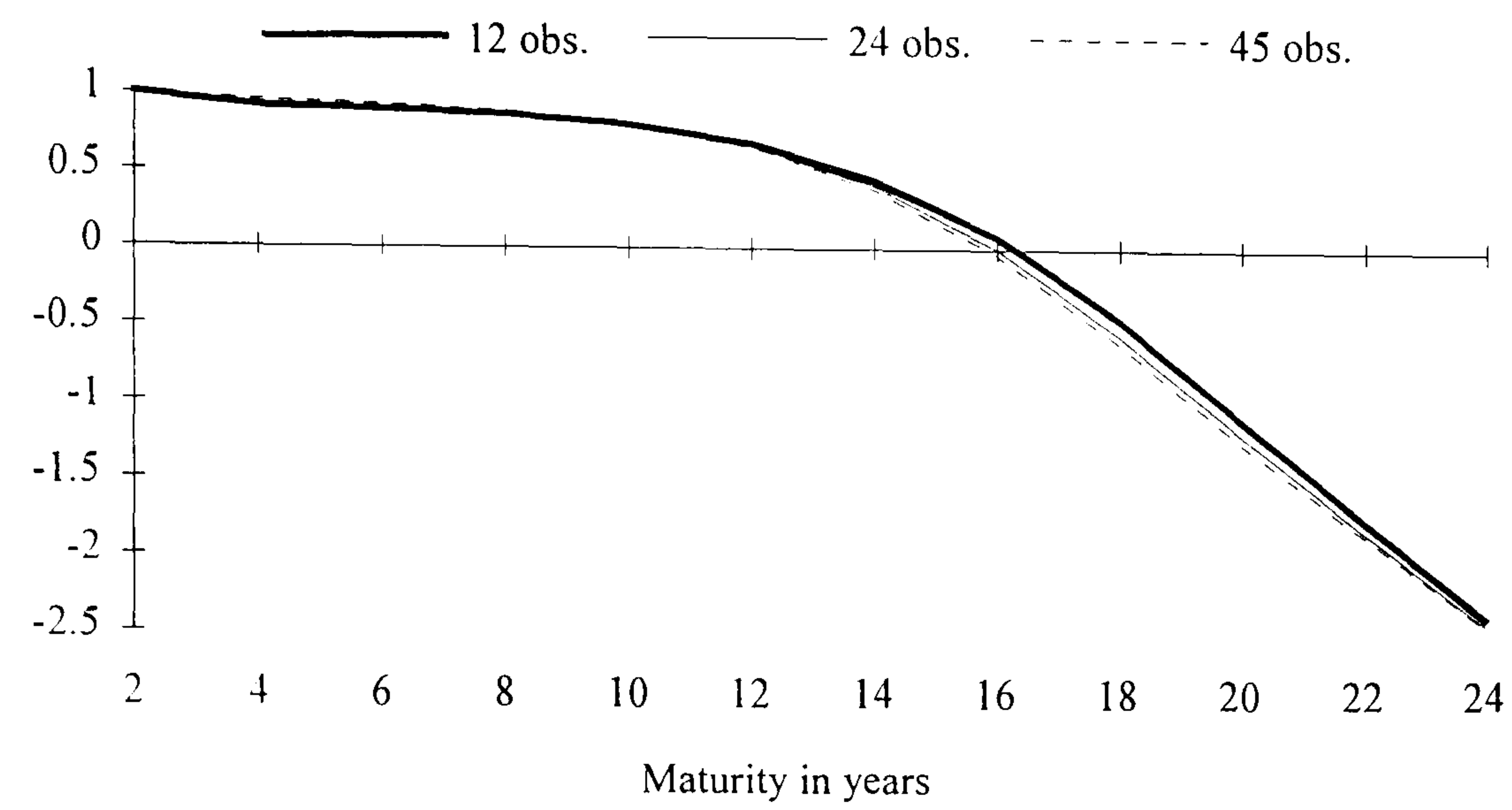
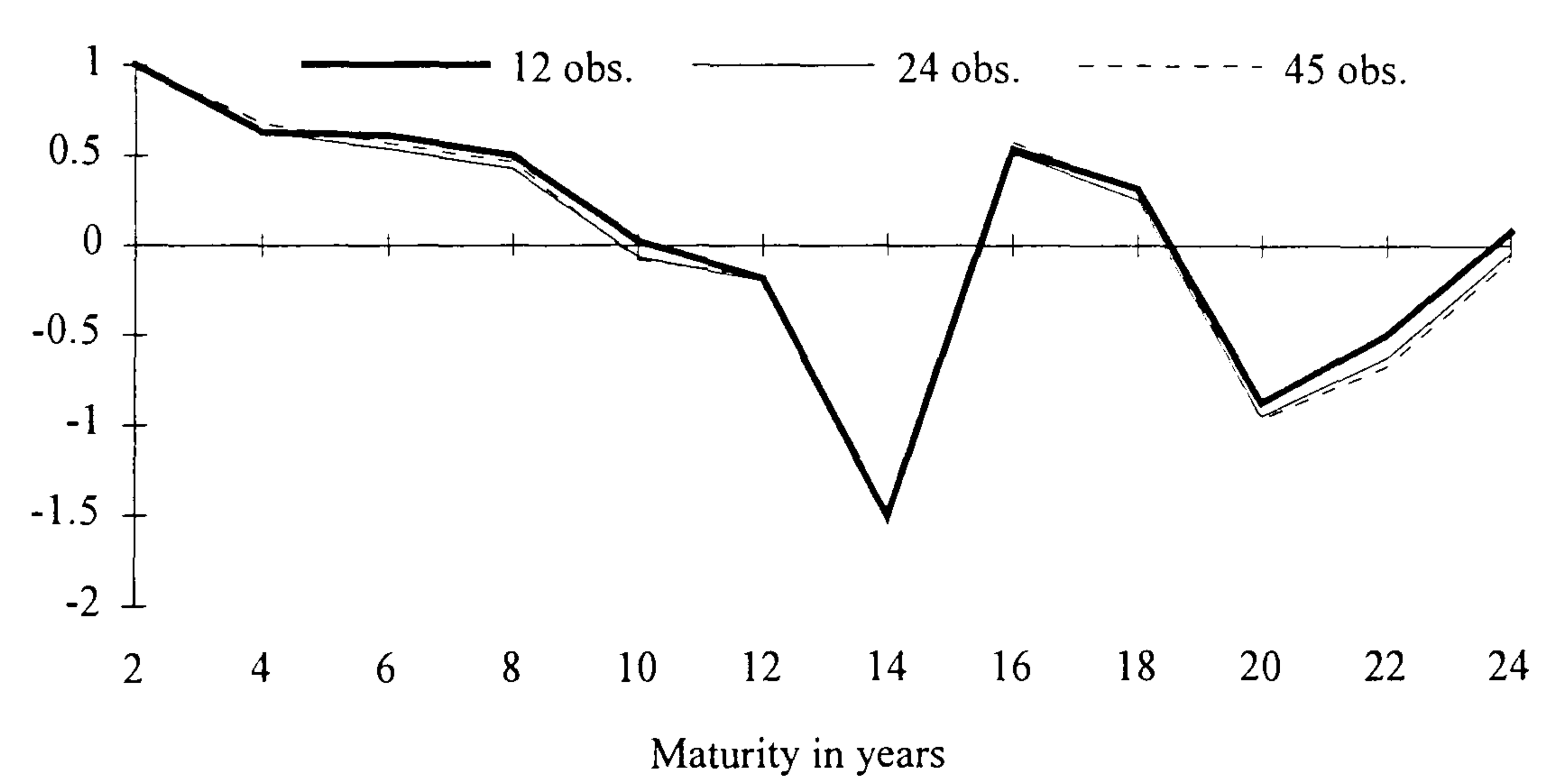


Chart 5.10 Second Eigenvector by Number of Observations, CY£



Charts 5.11 to 5.14 show that whilst the interpretation of each of the third principal components is unchanged by adding more observations the size of the coefficients is significantly altered, even after standardisation. For the CY£ data (chart 5.14) the difference between the 12 and 45 observation at 14 years maturity for the eigenvector coefficients is 100.9% and the average difference for all data sets is 17.2%. As it clear from the charts, it is not just a few outliers that cause the high percentage differences but is present throughout the maturity range. There is no systematic pattern to this for DY£ and SS£ (charts 5.11 and 5.12) the move from 12 to 45 observations tends to diminish the differences in the coefficients compared to those at two-years. Whereas for BS£ and CY£ (charts 5.13 and 5.14) the

move from 12 to 45 observations enhances the differences from the two-year parameter. The results suggest that, unlike the previous two components, care should be taken in deciding how many observations are used in the analysis because the results are likely to be sensitive to this choice. However, as these components explain at most 3.7% of the variation in the data (see table 5.3.4), unless the researcher is particularly interested in the third component this sensitivity will not matter much.

Chart 5.11 Third Eigenvector by Number of Observations,
DY£

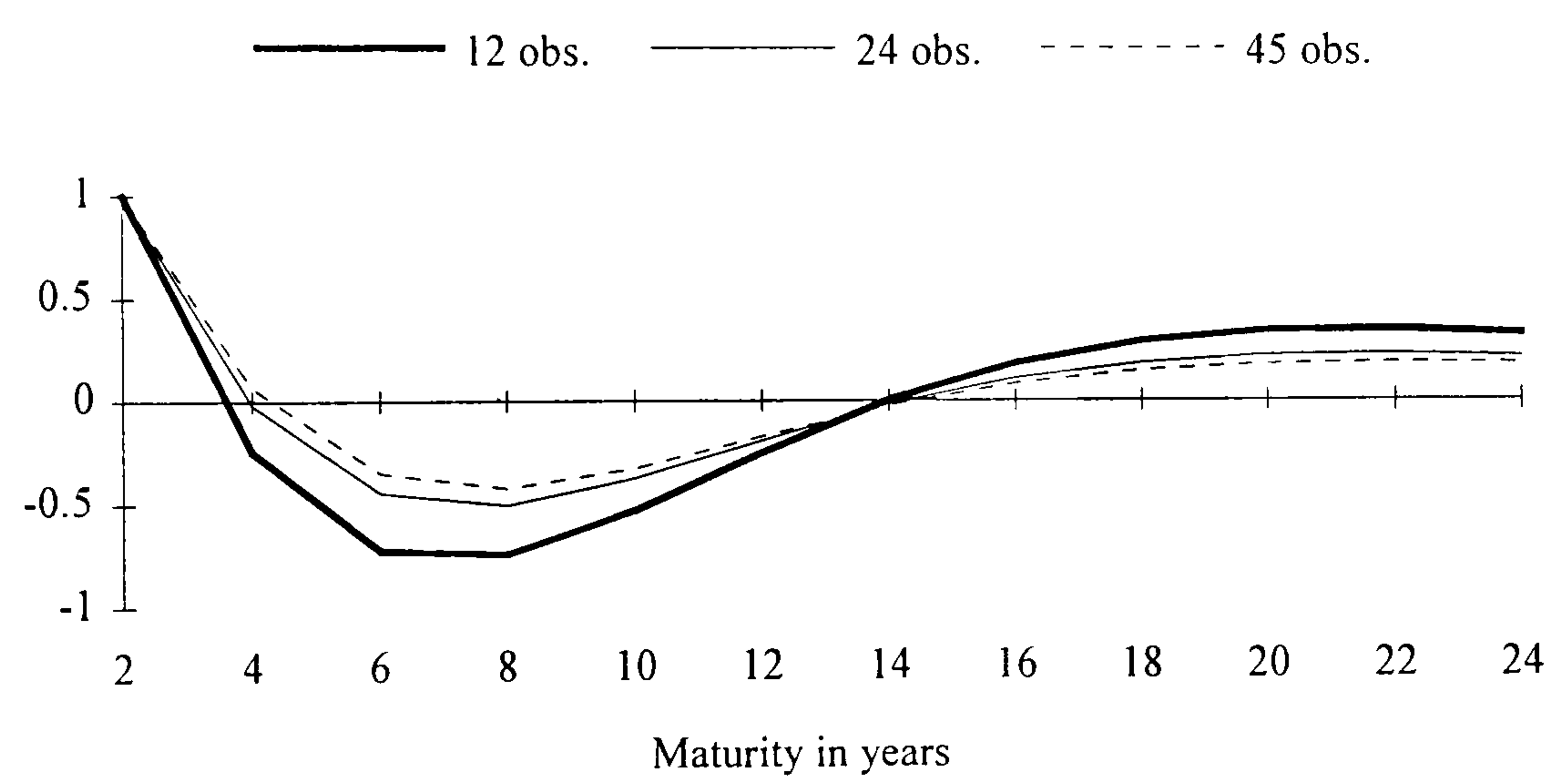


Chart 5.12 Third Eigenvector by Number of Observations,
SS£

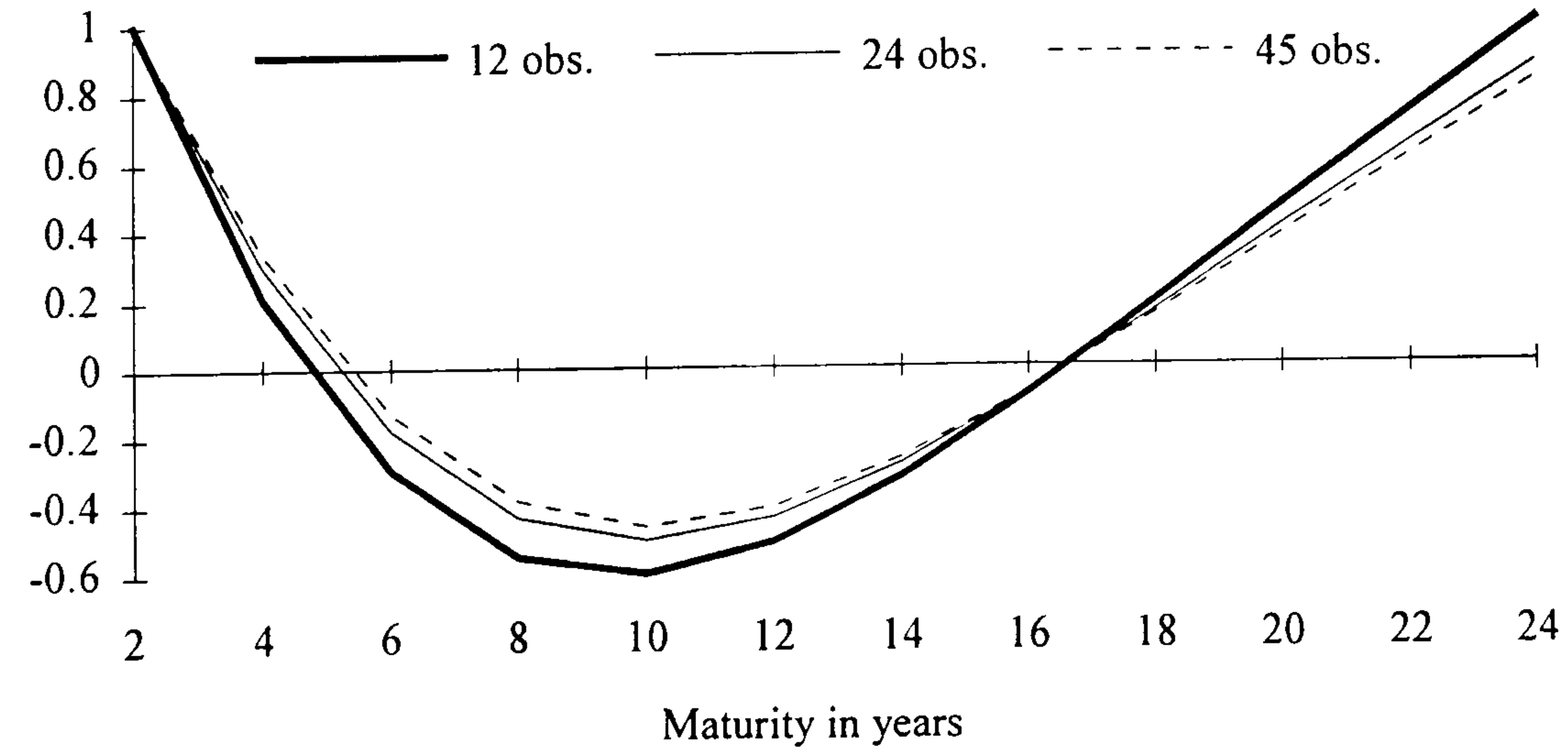


Chart 5.13 Third Eigenvector by Number of Observations, BS£

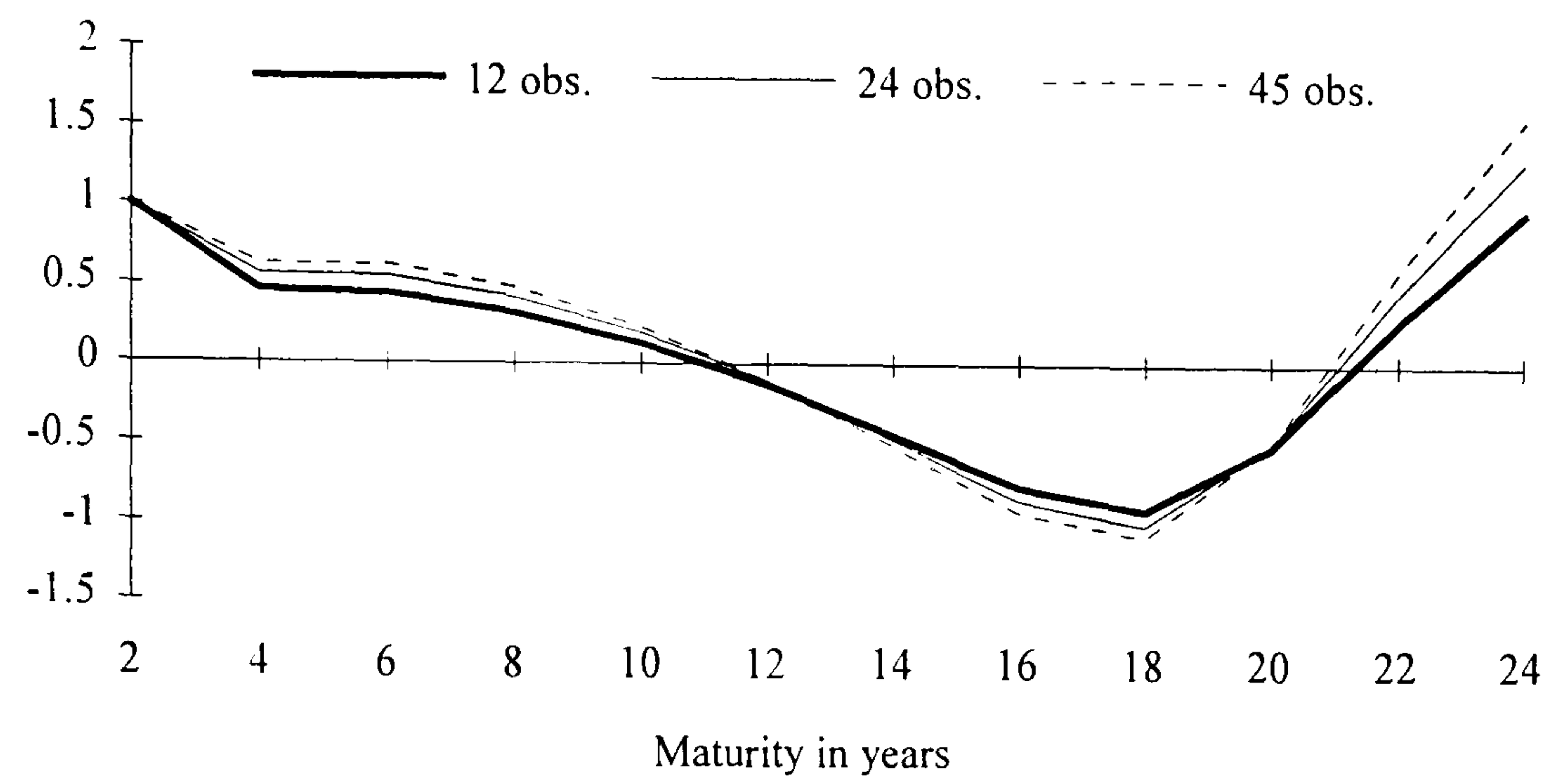
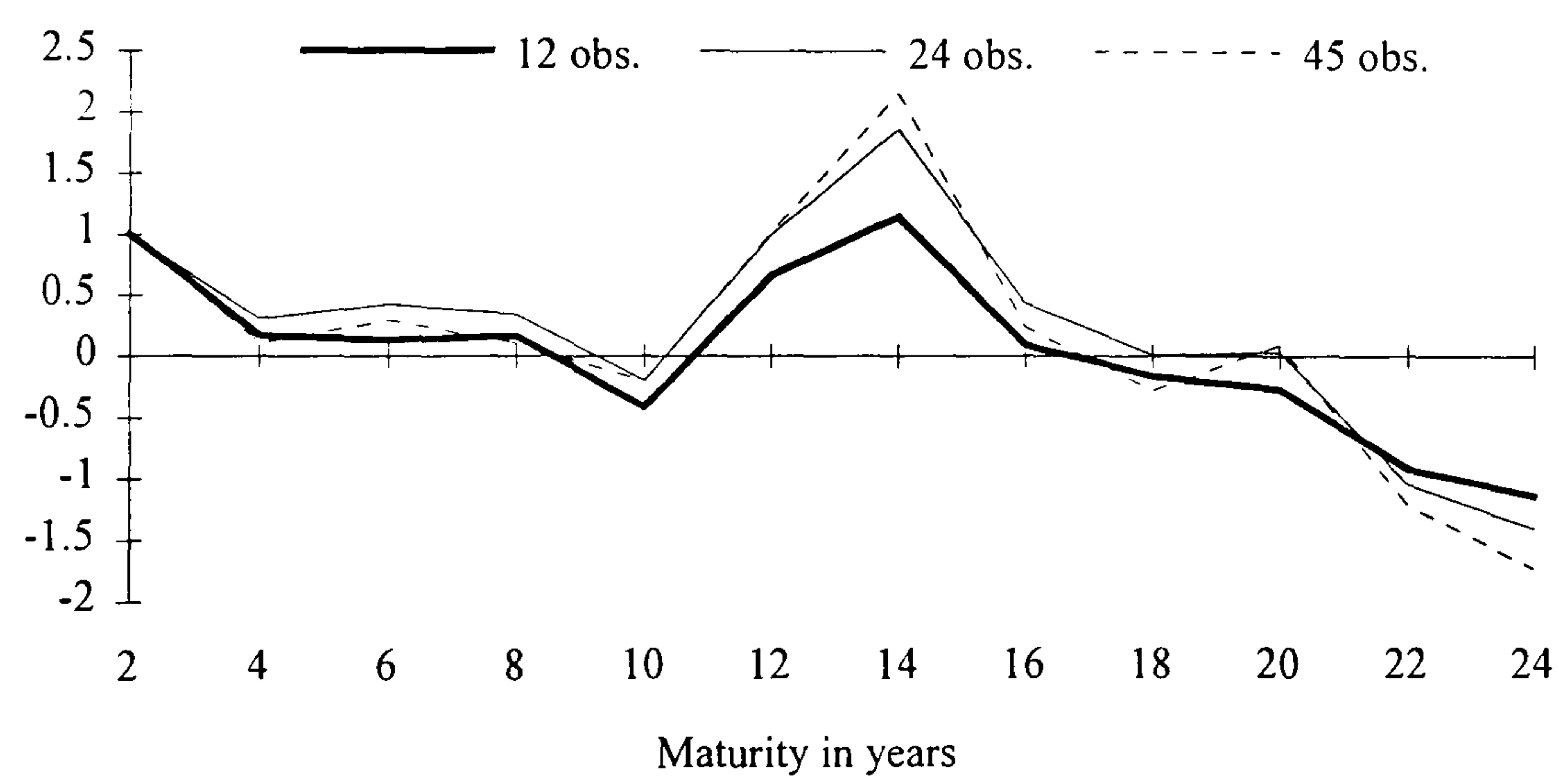


Chart 5.14 Third Eigenvector by Number of Observations, CY£



So far we have compared the eigenvectors from different numbers of yields and spot rates that were equally spaced between two and 24 years in maturity. There is, however, no reason why the information contained in term structures should be equally spaced. We examine this by using Krzanowski's (1987b) variable selection procedure, which was discussed in chapter 4. The variable selection was carried out on a sub-sample of the data that comprised month end data for the 23 observation spot curves constructed using both the BS£ and the SS£ data. A reduced sub-set was required simply because the full data set of 45 variables each with 3036 observations was too large for

The University of Warwick's UNIX computer to solve within four hours. By restricting the problem manageable solution times could be achieved.

The problem was to find 12 spot rates from the original 23 that best describe the data. The results are reported in table 5.4.1 below:

Table 5.4.1 The 12 Spot Maturities Selected in Years.

BS£	9	10	11	12	13	14	15	16	17	18	19	24
SS£	2	3	6	7	8	9	10	11	13	14	15	16

The data sets do not correspond exactly, the SS£ data starts at the end of March 1982, giving 101 observations, whilst the BS£ data begins on 2 January 1979 and finishes on 21 August 1990 giving 139 observations. Even bearing this in mind the differences are considerable. The BS£ data selects medium to long spot rates, whilst the SS£ data selects short to medium spot rates. However, the two curves have seven out of 12 maturities in common and the probability of doing this is only 0.3%. There could be two explanations for this difference. Either the spot data are very different or the deletion is nearly random, because the data contained within the spot rates are very similar. The fact that the two data sets seem to have information contained in separate ends of the spot curves does not suggest random deletion. Table 5.4.2 examines this in detail, by comparing the M^2 (the sum of squared residuals from the procrustes analysis) of those variables that are deleted with those that remain.

Table 5.4.2 Percentage M^2 from SS£

Iteration	1	2	3	4	5	6	7	8	9	10	11
Lowest M^2 as a % of twelfth M^2	57.1	87.8	88.0	91.8	99.4	98.9	96.7	99.3	99.4	99.3	99.6
Lowest M^2 as a % of next lowest M^2	98.9	99.6	98.4	99.6	99.9	100.0	97.9	99.7	100.0	99.6	99.6
Lowest M^2 as a % of lowest M^2 in final selection	82.5	95.5	98.1	95.2	99.9	99.9	97.1	99.4	100.0	99.3	99.6

Note: M^2 is calculated using equation 4.6.1 of chapter 4.

With the SS£ data, table 5.4.2, it is noticeable that after the fourth variable has been deleted the percentage difference between the deleted variable and the twelfth variable becomes very small, never being less than 3.3%. Similarly, the percentage difference between the eliminated variable and the next lowest score is never greater than 2.1%. The third row shows that, except for the first variable to be deleted, the percentage difference is never less than 4.8%. All of these results suggest that the choice between which variable is eliminated and which one stays is quite fine. Moreover, the percentage in the third row is always larger (except for the 11^{th} iteration by construction) than the percentage in the first row. This implies that at least one variable that was eventually retained was in the rejection region (i.e. had an M^2 that was ranked below the twelfth in order of magnitude) for each of the iterations. However, examination of the first elimination run finds that of the 11 with the smallest M^2, seven are eliminated by the final elimination run.

Table 5.4.3 Percentage M^2 from BS£

Iteration	1	2	3	4	5	6	7	8	9	10	11
Lowest M^2 as a % of twelfth M^2	38.3	71.2	81.2	80.7	89.1	92.7	91.5	94.8	96.4	98.3	98.2
Lowest M^2 as a % of next lowest M^2	84.7	93.1	100.0	96.8	99.5	98.8	98.6	97.8	99.1	98.6	98.2
Lowest M^2 as a % of lowest M^2 in final selection	38.7	71.9	81.3	80.9	89.1	93.6	91.7	94.8	96.5	98.6	98.2

Note: M^2 is calculated using equation 4.6.1 of chapter 4.

With the BS£ data the separation between rejected variables and retained variables seems much clearer, table 5.4.3. The lowest M^2 as a percentage of the twelfth M^2 is always lower and only by the eleventh iteration has the percentage achieved the magnitude that the SS£ data recorded. Similarly, a comparison of the third row in table 5.4.3 shows that the excluded spot rate was always a lower percentage of the M^2 statistic of the smallest variable that is eventually retained in the eleventh iteration. Comparing the first and third rows shows that in eight iterations at least one variable was in the rejection region but was eventually retained following the eleventh iteration. Moreover, the percentage differences between the first and third rows are much smaller with the BS£ data than with the SS£ data. It can also be noted that there were nine variables in the rejection region on the first iteration that had been excluded by the end of the eleventh iteration. These results suggest that with the BS£ data it is easier to separate various spot rates than with the SS£ data.

These results have been inconclusive. The SS£ data could involve more random deletion and the BS£ data less, but the evidence is not clear cut. However, it can be concluded that the equal spacing of spot rates along term structures does not appear to be necessary to capture the maximum amount of information contained within the data. Exactly what that spacing should be though remains a topic for future research. It is interesting to note that the spacing of the maturities in table 5.4.1 broadly corresponds to the maturities whose variances are the highest (see chart 5.2). This observation may provide an interesting avenue for further research.

5.5 Comparing and Contrasting Spot, Par and Forward Rates[8]

In chapter 4 it was outlined that the principal component results may be sensitive to the type of interest rate data used in the form of spot, par and forward data. In this section we analyse this contention by comparing the principal components of the 12 observation covariance matrices from the Bank of England spot (SS£) and forward rate (SF£) data and US par (MP$), spot (MS$) and forward (MF$) data from McCulloch and Kwon (1993).

As noted in the introduction, the US data is irregularly spaced and 33 observations are too many to be reported succinctly in table form.[9] Consequently, the results are reported using just 12 of the 33 estimated parameters from the eigenvectors. This choice enabled almost equal spacing between each of the 33 maturities used in the analysis. The maturities range from zero to 13 years and the full list is given in table 5.5.3 below.

As table 5.5.1 shows, the first component dominates the US interest rate data, accounting for a minimum of 96.6% of the data variance in the case of the forward rates. The second component

[8] More details of the correlation statistics for the US spot, par and forward rates and the UK forward rates can be found in the appendix to this chapter in tables 5.A.9 to 5.A.32

[9] It is also unnecessary, as the conclusions drawn are equally applicable to all 33 observations.

accounts for between 1.6% and 2.4% and the remaining components have a negligible amount of explanatory power. Thus all the series appear to conform to the results reported in section 5.3. Furthermore, there is no appreciable difference in explanatory power between the various interest rate series.

Table 5.5.1 Eigenvalues, Percentage of Variance Explained and Bartlett's Test of Isotropy for the US Data by Type of Term Structure

	First Eigenvalue			Second Eigenvalue			Third Eigenvalue		
Type	value	%	chisq	value	%	chisq	value	%	chisq
Spot	326.01	97.9	189170	5.90	1.8	131955	0.68	0.2	105870
Forward	323.60	96.6	130622	8.00	2.4	80904	1.54	0.5	64845
Par	325.44	98.1	189437	5.33	1.6	130801	0.65	0.2	105083

Note: % = percentage of total variance explained, chisq= Bartlett's test of isotropy.
Estimated using month end data 1951 to 1991, using 33 observations along the term structure.

Completely the opposite result is found for a comparison of the Bank of England's spot and forward rate data. What can be immediately comprehended from Bartlett's test of isotropy (see table 5.5.2) is that, like the spot data, the principal components are distinct. However, the percentage of the variance accounted for by the first principal component is much lower, and the percentage for the second component much higher, than in the corresponding spot rate data. Together the first two principal components also account for a slightly lower percentage of the variance, 91.6%, than do the spot data, 95.8%, giving a slightly increased role to the subsequent components.

Table 5.5.2 Eigenvalues, Percentage of Variance Explained and Bartlett's Test of Isotropy for the Bank of England Spot and Forward Rates.

	First Eigenvalue			Second Eigenvalue			Third Eigenvalue		
	value	%	chisq	value	%	chisq	value	%	chisq
Spot	10.49	79.6	233488	2.19	16.5	202684	0.49	3.7	174550
Forward	11.0	51.2	212064	8.76	40.4	200894	1.47	6.8	169492

Note : % = percentage of total variance explained, chisq= Bartlett's test of isotropy. Estimated using daily data 31 March 1982 to 21 August 1990 using 12 observations along the term structure.

Tables 5.5.3 and 5.5.4 indicate there is little difference between the eigenvectors for each of the US interest series, either in terms of pattern or numerically, for the first two principal components. In each case the first eigenvector represents the level of interest rates (although each series has slightly lower coefficients at the short and long ends than in the middle maturities) and the second represents a measure of the slope. For the par and the spot data the coefficients are also numerically similar for the third principal component, table 5.5.5. Although, the eigenvector for the third principal component of forward rates is numerically different from the spot and par rates eigenvectors, its pattern is very similar. Thus all the third principal components for the US data can be interpreted as a twist in the interest rate curves.

Table 5.5.3 Eigenvectors and Krzanowski Tolerances (KT) for the First Principal Component Using US Data.

Yield Maturity (years)	Spot PC	Spot KT (10%)	Forward PC	Forward KT (10%)	Par PC	Par KT (10%)
0	0.163	0.0671	0.161	0.0744	0.164	0.0689
0.25	0.174	0.0635	0.176	0.0705	0.174	0.0649
0.5	0.177	0.0532	0.181	0.0691	0.177	0.0536
0.75	0.179	0.0427	0.179	0.0401	0.178	0.0424
1.0	0.179	0.0282	0.180	0.0125	0.178	0.0275
1.25	0.179	0.0148	0.181	0.0021	0.179	0.0137
1.5	0.179	0.0043	0.178	0.0080	0.178	0.0029
2.5	0.176	0.0223	0.173	0.0319	0.176	0.0239
5.0	0.172	0.0559	0.171	0.0612	0.172	0.0565
8.0	0.169	0.0741	0.164	0.0609	0.169	0.0731
11.0	0.166	0.0802	0.166	0.0693	0.167	0.0789
13.0	0.166	0.0838	0.172	0.0784	0.166	0.0817

Note: The Krzanowski tolerance is constructed from a 10% change in the value of the component's eigenvalue. By adding or subtracting the tolerance from the elements of the eigenvector, a band can be constructed within which the element would vary if the eigenvalue was incorrectly measured by upto 10%. The principal components were estimated using a Covariance Matrix of 33 Observations from the month end sample 1951-1991

Table 5.5.4 Eigenvectors and Krzanowski Tolerances (KT) for the Second Principal Component Using US Data.

Yield Maturity (years)	Spot PC	Spot KT (10%)	Forward PC	Forward KT (10%)	Par PC	Par KT (10%)
0	-0.221	0.0468	-0.244	0.0510	-0.227	0.0468
0.25	-0.209	0.0331	-0.231	0.0200	-0.214	0.0325
0.5	-0.175	0.0190	-0.227	0.0016	-0.177	0.0180
0.75	-0.140	0.0075	-0.132	0.0065	-0.140	0.0062
1.0	-0.093	0.0106	-0.041	0.0176	-0.090	0.0122
1.25	-0.049	0.0457	0.007	0.0461	-0.045	0.0468
1.5	-0.014	0.0704	0.026	0.0622	-0.010	0.0697
2.5	0.073	0.0785	0.105	0.0650	0.079	0.0773
5.0	0.184	0.0838	0.201	0.0784	0.186	0.0817
8.0	0.244	0.0244	0.200	0.0274	0.241	0.0262
11.0	0.264	0.0443	0.227	0.0869	0.260	0.0421
13.0	0.276	0.0569	0.257	0.1044	0.269	0.0515

Note: See table 5.5.3

Table 5.5.5 Eigenvectors and Krzanowski Tolerances (KT) for the Third Principal Component Using US Data.

Yield Maturity (years)	Spot PC	Spot KT (10%)	Forward PC	Forward KT (10%)	Par PC	Par KT (10%)
0	0.613	0.0110	0.331	0.0047	0.638	0.0098
0.25	0.144	0.0039	0.167	0.0117	0.127	0.0054
0.5	-0.035	0.0322	0.007	0.0398	-0.048	0.0337
0.75	-0.119	0.0646	-0.103	0.0659	-0.127	0.0644
1.0	-0.148	0.0766	-0.181	0.0624	-0.150	0.0755
1.25	-0.153	0.0820	-0.173	0.0741	-0.149	0.0803
1.5	-0.146	0.0169	-0.154	0.0076	-0.140	0.0197
2.5	-0.112	0.0377	-0.147	0.0720	-0.098	0.0371
5.0	-0.008	0.0569	-0.041	0.1044	0.006	0.0515
8.0	0.076	0.0107	0.083	0.0001	0.082	0.0171
11.0	0.139	0.0490	0.262	0.0537	0.132	0.0464
13.0	0.179	0.0711	0.315	0.0563	0.161	0.0624

Note: See table 5.5.3.

For the US data the Krzanowski tolerances indicate that there is a tendency for the short and long maturities to vary more than the middle maturities for all of the first eigenvectors (table 5.5.3). This exaggerates the slight bend noted above. This implies that the coefficients would be humped or U-shaped under erroneous measurement of the first eigenvalues. However, given the relative size of Bartlett's test of Isotropy in table 5.5.1, such erroneous measurement seems unlikely. The second eigenvectors would retain their shape as a measure of the slope of the interest rate curves according to the Krzanowoski tolerances (table 5.5.4). The third principal components' eigenvectors would retain their interpretation as a twist in the interest rate curves (table 5.5.5) if their respective eigenvalues were incorrectly measured by 10%.

Table 5.5.6 Eigenvectors and Krzanowski Tolerances (KT) for the First Four Principal Components Using SF£.

Yield Maturity (years)	First PC	First KT (10%)	Second PC	Second KT (10%)	Third PC	Third KT (10%)	Fourth PC	Fourth KT (10%)
2	0.176	0.0709	-0.125	0.1120	0.763	0.0071	-0.471	0.0622
4	0.239	0.1656	-0.292	0.0861	0.429	0.0320	0.283	0.0066
6	0.246	0.2288	-0.404	0.0642	0.061	0.0534	0.412	0.0610
8	0.191	0.2475	-0.437	0.0458	-0.155	0.0712	0.201	0.1352
10	0.097	0.2355	-0.415	0.0751	-0.229	0.0210	-0.063	0.0924
12	-0.012	0.2078	-0.367	0.0691	-0.211	0.0836	-0.249	0.0251
14	-0.122	0.1746	-0.308	0.0476	-0.145	0.1087	-0.324	0.0440
16	-0.225	0.1418	-0.250	0.0203	-0.062	0.0990	-0.295	0.0846
18	-0.316	0.1120	-0.198	0.0071	0.022	0.0622	-0.186	0.0859
20	-0.395	0.0861	-0.152	0.0320	0.098	0.0066	-0.020	0.0484
22	-0.463	0.0642	-0.113	0.0534	0.163	0.0610	0.182	0.0220
24	-0.520	0.0458	-0.081	0.0712	0.218	0.1352	0.403	0.1178

Note: Principal components were estimated using a covariance matrix of 12 observations from the daily sample 31 March 1982 to 21 August-1990

The Bank of England forward rate data, on the other hand, differs from its US counterpart in that the eigenvector of the first principal component is more easily interpreted as the slope of the forward rate curve, rather than the level (see table 5.5.6). Whilst the second component is always the same sign, the sizes of the parameters are noticeably lower at the short and longer maturities than in the middle of the forward rate curve. Thus, unlike the US data, the interpretation of the forward rate curves is different from the other spot data.

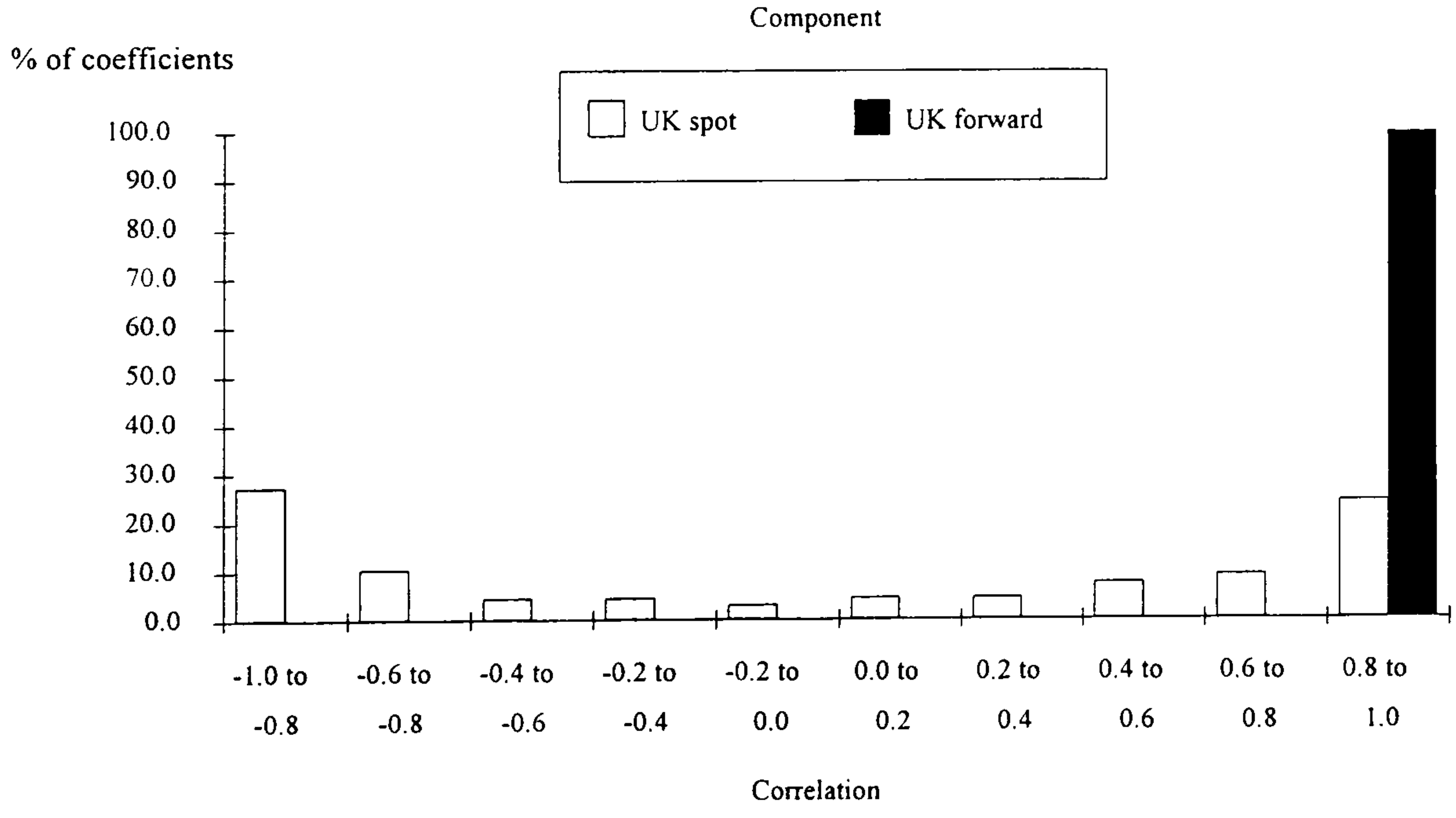
Table 5.5.7 Standard Deviations (SD) and Coefficients of Variation (CV) for the First Four Eigenvectors Using SF£.

Yield Maturity (years)	First PC SD	First PC CV (%)	Second PC SD	Second PC CV (%)	Third PC SD	Third PC CV (%)	Fourth PC SD	Fourth PC CV (%)
2	0.0133	7.57	0.0178	-14.26	0.0064	0.84	0.0102	-2.18
4	0.0266	11.11	0.0220	-7.54	0.0057	1.33	0.0083	2.94
6	0.0363	14.76	0.0222	-5.50	0.0070	11.46	0.0023	0.57
8	0.0393	20.59	0.0173	-3.96	0.0057	-3.64	0.0042	2.11
10	0.0374	38.42	0.0091	-2.19	0.0047	-2.04	0.0045	-7.11
12	0.0330	-275.72	0.0027	-0.73	0.0050	-2.36	0.0032	-1.29
14	0.0278	-22.73	0.0112	-3.63	0.0053	-3.66	0.0027	-0.83
16	0.0225	-10.03	0.0203	-8.10	0.0051	-8.15	0.0031	-1.04
18	0.0178	-5.63	0.0284	-14.39	0.0043	20.00	0.0031	-1.68
20	0.0137	-3.47	0.0355	-23.40	0.0040	4.04	0.0026	-13.24
22	0.0103	-2.23	0.0416	-36.78	0.0049	2.99	0.0028	1.53
24	0.0077	-1.48	0.0468	-58.03	0.0071	3.25	0.0050	1.25

Note: See table 5.5.6

However, the estimates of the first principal component for the UK forward rates are less securely estimated than the corresponding spot coefficients. The Krzanowski tolerances (table 5.5.6) are larger for all the maturities between four and 18 years. This is a consequence of the lower percentage of the variance of the data being explained by the first principal component. The standard deviation of the coefficient estimates (table 5.5.7) and the coefficients of variation are always larger than the corresponding estimates of the spot rates. Indeed, the coefficients between four and 14 years could conceivably be zero if the first eigenvalue was incorrectly measured by 10%. Despite these findings, sampling error alone could not make the first principal component into a measure of the level of forward rates. On the other hand, chart 5.15 suggests that the coefficients are highly correlated with each other, with none of the correlations being less than 0.80. In contrast, only 24.25% of the correlations are over 0.80 for the spot rate data (see chart 5.15). Thus the forward rate eigenvectors will tend to keep its shape when subjected to measurement error.

Chart 5.15 Within Component Coefficient Correlations for the First Component



For the second principal component of the UK forward rates the Krzanowski tolerances (table 5.5.6) are, with the exception of the two-year coefficient, smaller than the corresponding estimates for the spot rate. This is due to the greater relative size of the second eigenvalue of the forward data compared to the other eigenvalues. For each coefficient the standard deviation (table 5.5.7) is larger for the forward rates and the same is true, with the exception of the ten-year and twelve-year estimates, for the coefficients of variation. The Krzanowski tolerances (table 5.5.6) and the standard deviations of the coefficients (table 5.5.7) also suggest that sampling errors would tend to move the coefficients on the shorter and longer maturities more than the medium term maturities. There is no evidence from these statistics that measurement error could account for the second principal component not being the slope of the term structure.

Table 5.5.8 Standard Deviations (SD) and Coefficients of Variation (CV) for the First Principal Component's Eigenvectors Using US Data.

Yield Maturity (years)	Spot SD	Spot CV	Forward SD	Forward CV	Par SD	Par CV
0	0.0020	1.23	0.0025	1.57	0.0020	1.21
0.25	0.0014	0.80	0.0019	1.05	0.0014	0.78
0.5	0.0011	0.63	0.0018	0.97	0.0011	0.60
0.75	0.0009	0.51	0.0011	0.63	0.0009	0.49
1.0	0.0007	0.37	0.0008	0.43	0.0006	0.35
1.25	0.0005	0.26	0.0007	0.38	0.0004	0.24
1.5	0.0004	0.21	0.0007	0.40	0.0003	0.19
2.5	0.0006	0.33	0.0011	0.62	0.0006	0.32
5.0	0.0012	0.67	0.0016	0.92	0.0011	0.64
8.0	0.0015	0.90	0.0016	0.95	0.0014	0.84
11.0	0.0017	1.00	0.0019	1.14	0.0016	0.93
13.0	0.0018	1.07	0.0023	1.31	0.0016	0.98

Note: Principal components were estimated using a covariance matrix of 33 month end observations from 1951-1991

Table 5.5.9 Standard Deviations (SD) and Coefficients of Variation (CV) for the Second Principal Component's Eigenvectors Using US Data.

Yield Maturity (years)	Spot SD	Spot CV	Forward SD	Forward CV	Par SD	Par CV
0	0.0121	-5.50	0.0128	-5.24	0.0128	-5.63
0.25	0.0043	-2.06	0.0057	-2.47	0.0043	-2.01
0.5	0.0023	-1.30	0.0043	-1.89	0.0023	-1.31
0.75	0.0026	-1.88	0.0044	-3.34	0.0028	-1.99
1.0	0.0030	-3.21	0.0054	-13.19	0.0031	-3.42
1.25	0.0031	-6.43	0.0052	76.45	0.0032	-7.06
1.5	0.0031	-22.31	0.0052	19.70	0.0031	-32.61
2.5	0.0030	4.05	0.0055	5.23	0.0028	3.58
5.0	0.0020	1.08	0.0040	1.97	0.0018	0.95
8.0	0.0019	0.78	0.0040	2.02	0.0019	0.81
11.0	0.0031	1.16	0.0070	3.10	0.0030	1.15
13.0	0.0042	1.51	0.0094	3.64	0.0038	1.42

Note: See table 5.5.8

Table 5.5.10 Standard Deviations (SD) and Coefficients of Variation (CV) for the Third Principal Component's Eigenvectors Using US Data.

Yield Maturity (years)	Spot SD	Spot CV	Forward SD	Forward CV	Par SD	Par CV
0	0.0256	4.17	0.0309	9.34	0.0241	3.77
0.25	0.0145	10.09	0.0113	6.81	0.0148	11.68
0.5	0.0082	-23.71	0.0130	177.55	0.0079	-16.46
0.75	0.0048	-4.05	0.0099	-9.61	0.0045	-3.53
1.0	0.0041	-2.75	0.0077	-4.25	0.0044	-2.90
1.25	0.0053	-3.45	0.0072	-4.16	0.0056	-3.72
1.5	0.0060	-4.09	0.0087	-5.70	0.0061	-4.39
2.5	0.0077	-6.85	0.0105	-7.13	0.0073	-7.42
5.0	0.0065	-85.73	0.0107	-26.07	0.0057	88.34
8.0	0.0051	6.65	0.0098	11.86	0.0051	6.19
11.0	0.0079	5.67	0.0093	3.57	0.0075	5.70
13.0	0.0110	6.18	0.0154	4.89	0.0097	6.04

Note: See table 5.5.8

The standard deviations of the US coefficients are small with the maximum coefficient of variation being 1.6% at the instantaneous maturity for the forward rate data on the first principal component's eigenvector (table 5.5.8). The standard deviations of the coefficients rise for each of the data sets comparing the first and the second and the second and the third eigenvectors. This results, on occasion, in some large coefficients of variation for the third principal component when the coefficient is small at the six-months and five-year maturities. Nevertheless, on the whole the coefficients of variation are relatively small, usually less than 5% for the second principal components' eigenvectors (table 5.5.9) and 10% for the third components (table 5.5.10).

Girshick's (1939) results for the covariances of the eigenvector coefficients (equation (4.4.4) of chapter 4) can be used to investigate the interpretation of principal components. The covariances for the first three eigenvectors of the US data are depicted in charts 5.16 to 5.18. The correlation coefficients of the US par rates within the various components show a similar pattern to those of the spot rates (charts 5.16 to 5.18) and, hence, are not shown separately. These charts record the correlations between the coefficients within a given eigenvector. Chart 5.16 shows the correlations of the coefficients of the first eigenvectors for spot and forward rates have a large range of correlations, both positive and negative. Moreover, notable proportions, especially for the spot rates, have large correlations in absolute size. Thus measurement error in the coefficients would lead to deformation in the patterns of the first eigenvectors. Hence, its interpretation is not quite as secure as might have been suspected from the coefficients of variation alone.

Interpretations of the correlations for the second and third eigenvectors of the US data are more difficult because the eigenvectors contain both positive and negative coefficients. Thus, movements of the coefficients in opposite directions under measurement error may actually enhance the interpretation of the coefficients as the slopes and twists of the term structures. Nevertheless, such a fortuitous combination of correlations cannot be guaranteed. Consequently, the spread of the correlation estimates must raise a slight concern that the components would deform under measurement error. These concerns have to be set against the strong evidence from other statistics that the results are robust to measurement error.

Chart 5.16 Within Component Coefficient Correlations for the First Component

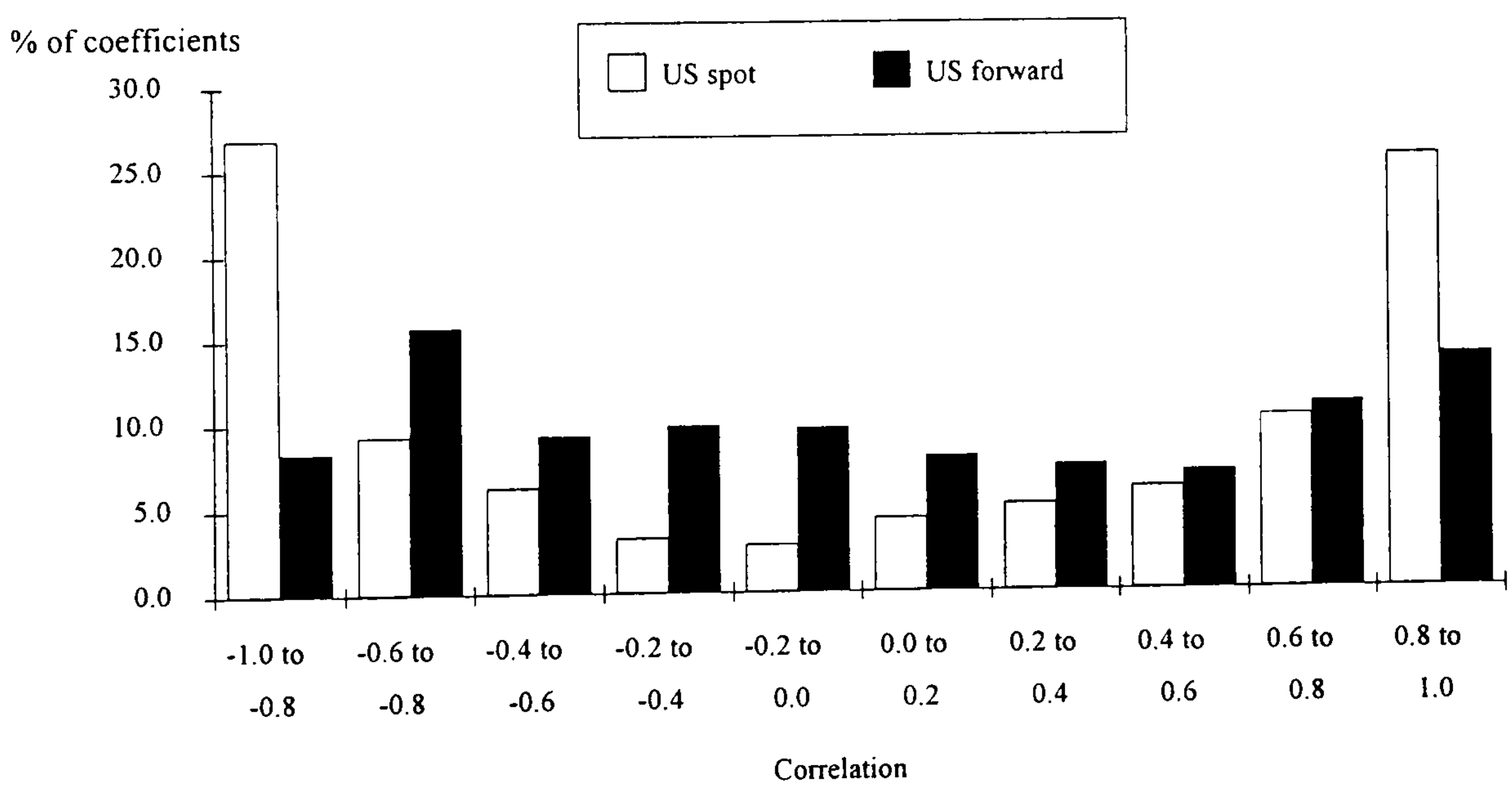


Chart 5.17 Within Component Coefficient Correlations for the Second Component

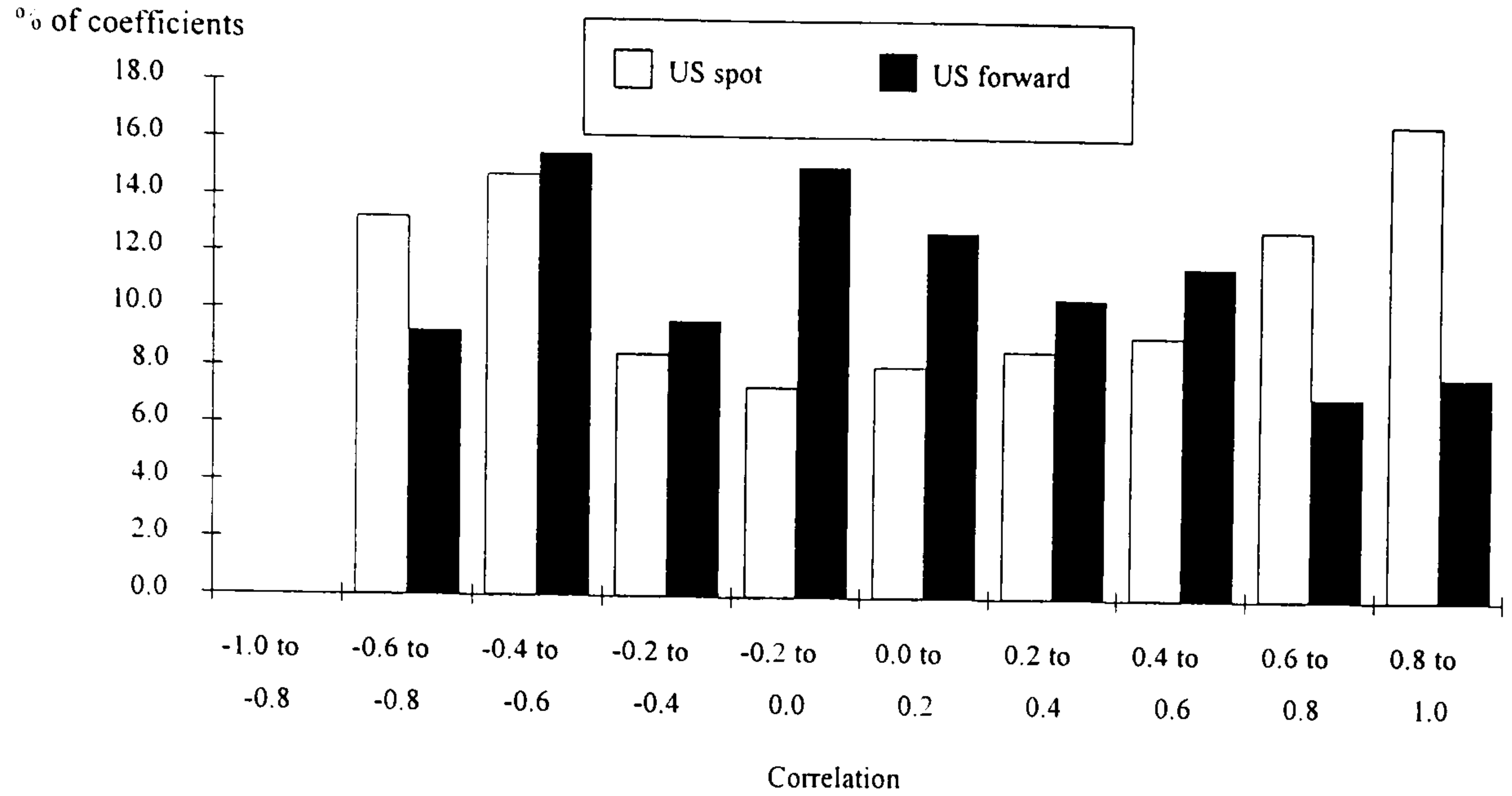
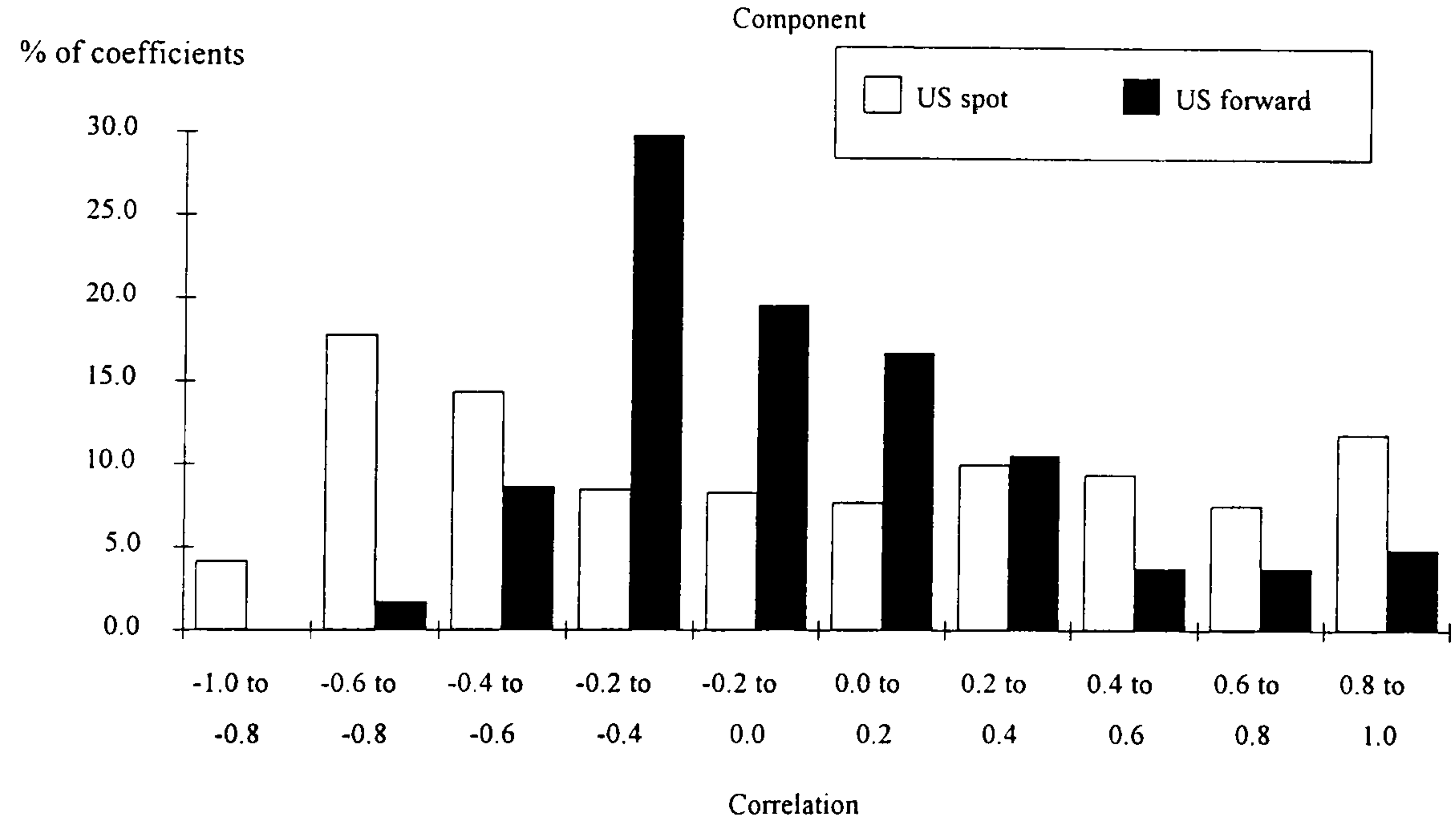


Chart 5.18 Within Component Coefficient Correlations for the Third Component



Girshick's (1939) measure of the covariances of the coefficients across eigenvectors (equation 4.4.5 of chapter 4), i.e. the covariance of the i^{th} coefficient in eigenvector j with the k^{th} coefficient in eigenvector l, are depicted in charts 5.19 to 5.21. The interpretation of these results for the US spot and forward rates is straightforward. Measurement errors from other eigenvectors do not spill over into the measurement of the first three eigenvectors because the correlations between the coefficients are very low. The vast majority (never less than 93.9%) of the correlations are between –0.1 and 0.1. Thus, for example, measurement errors in the second eigenvectors' coefficients will not have any material impact upon the estimates of the first eigenvectors' coefficients.

Chart 5.19 Across Component Coefficient Correlations for the First Component

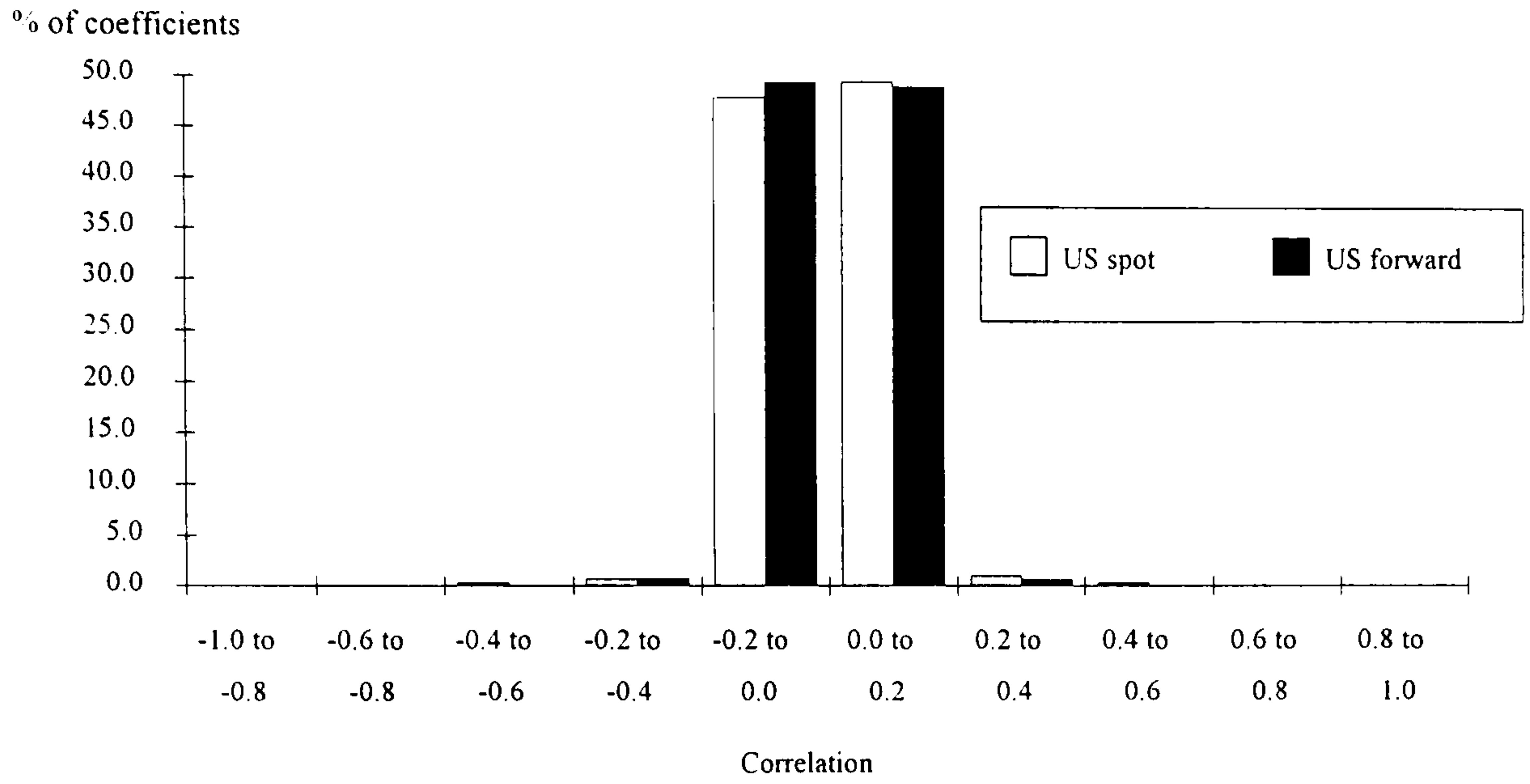


Chart 5.20 Across Component Coefficient Correlations for the Second Component

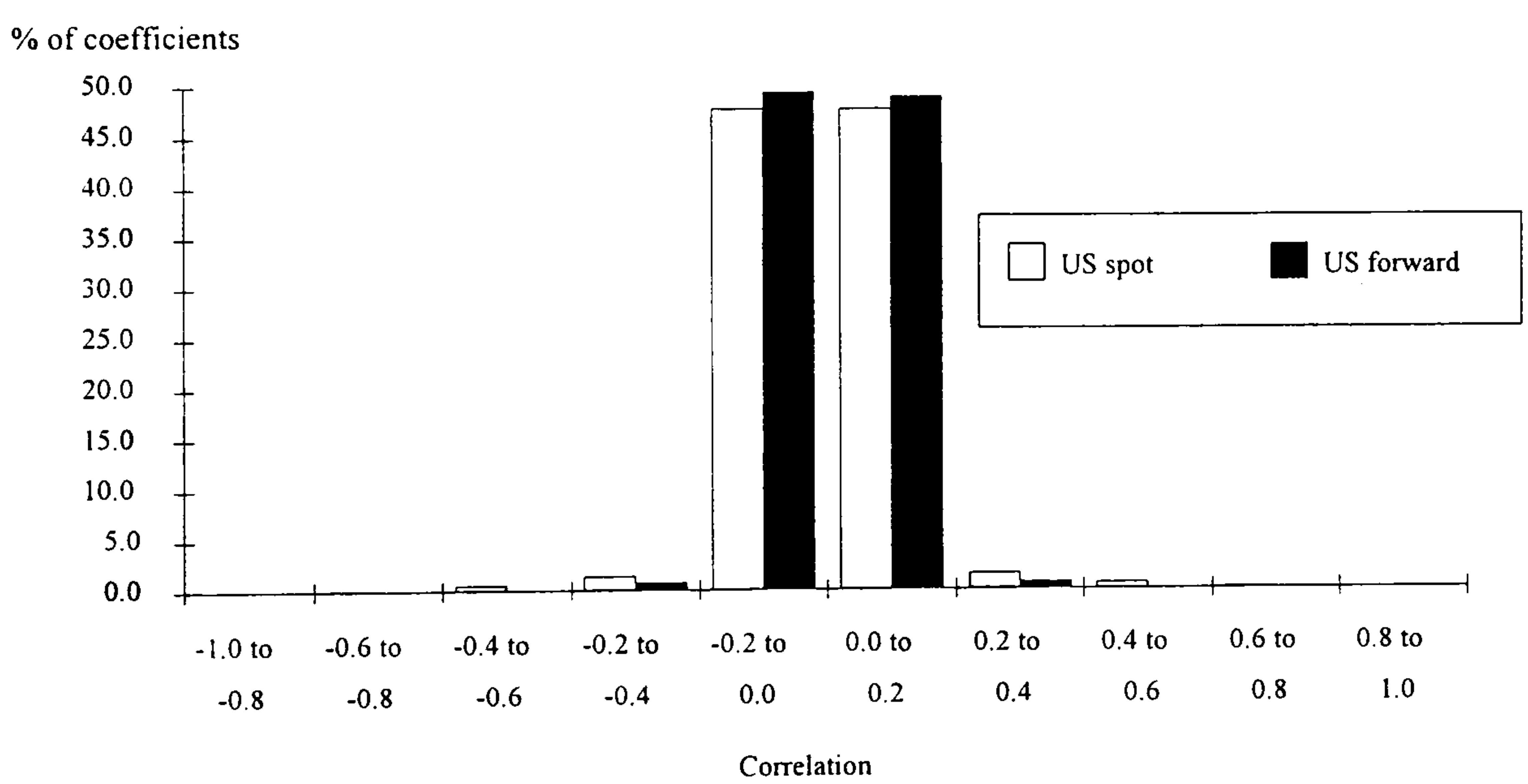
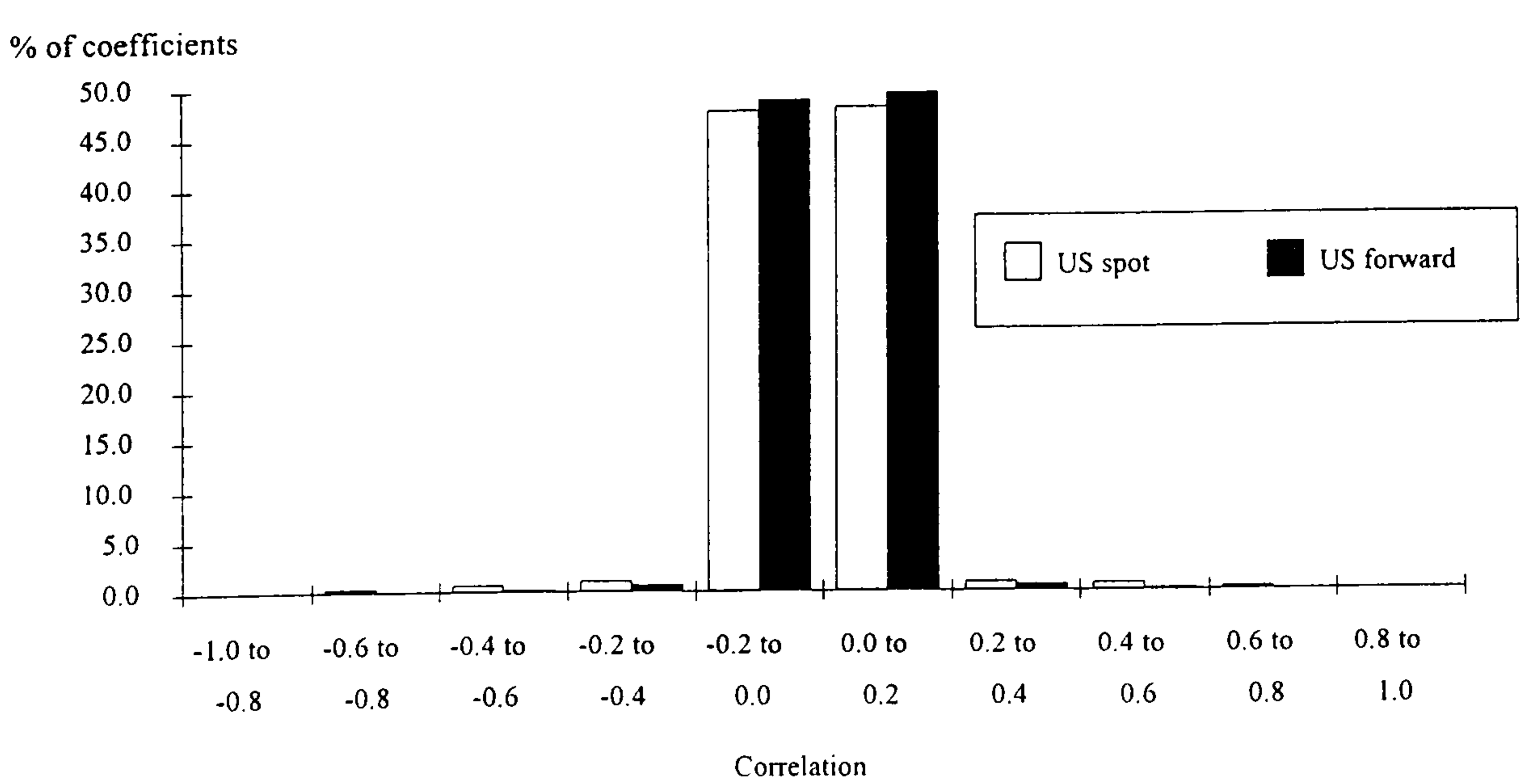


Chart 5.21 Across Component Coefficient Correlations for the Third Component



As has already been noted, chart 5.15 indicates that the interpretation of the first component of the UK forward rates is secure, because all the coefficients are highly correlated. This strong claim cannot be made for the Bank of England spot rates for the first component, where, like the US data, the chart indicates a wide range of correlations. Hence, the spot rate coefficients are likely to deform under measurement error and its interpretation as the level of interest rates is not quite as secure as might have been hoped.

For the second and third components of both the UK spot and forward rates, charts 5.22 and 5.23, the correlations reveal that measurement errors will cause the eigenvectors to deform to some extent. Hence the interpretations of these components are, again, not quite as secure as might be hoped. However, charts 5.24, 5.25 and 5.26 reveal that measurement errors from coefficients in one eigenvector do not spill over into the coefficient estimates in the other eigenvectors. In this sense the estimated eigenvectors are secure.

Chart 5.22 Within Component Coefficient Correlations for the Second Component

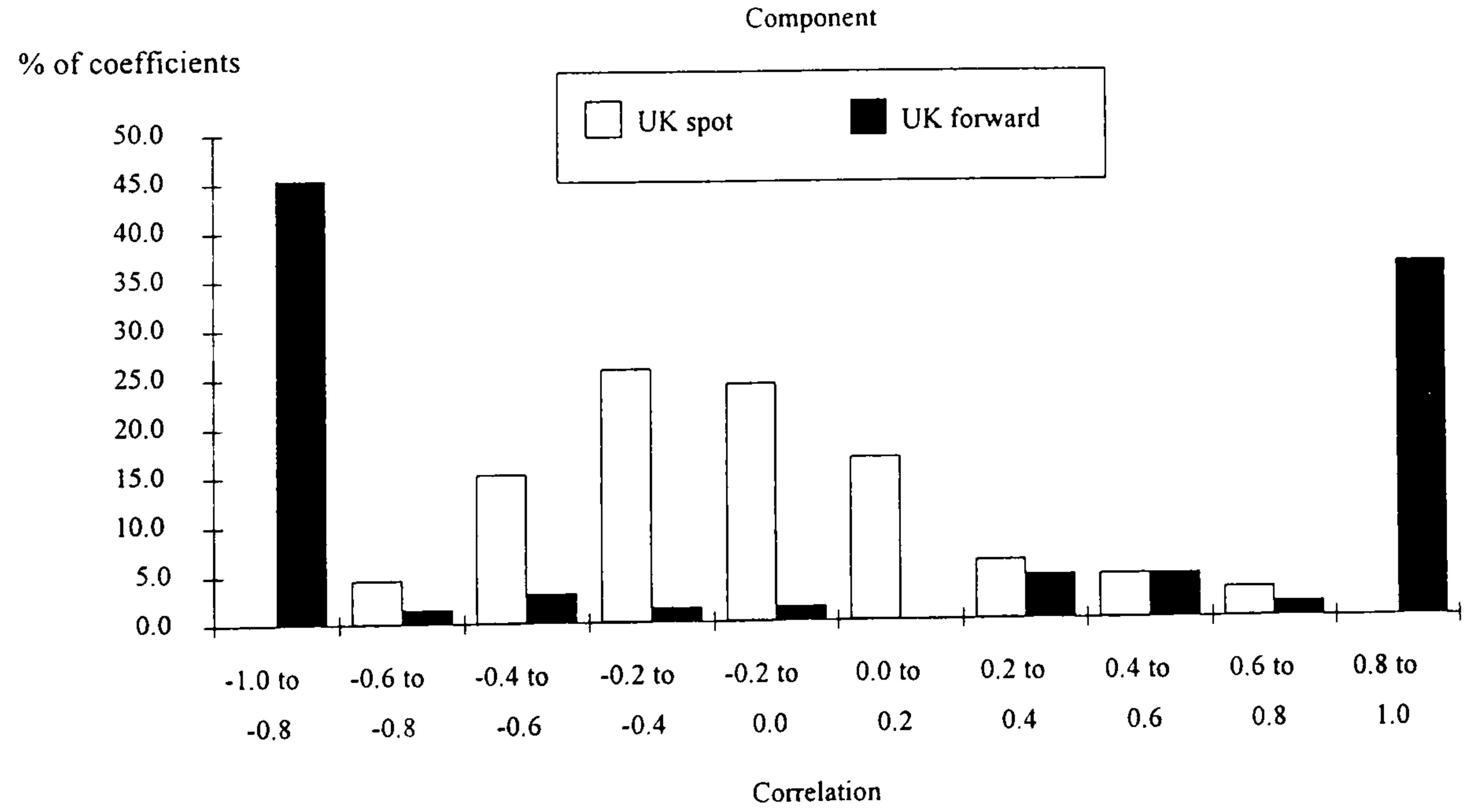


Chart 5.23 Within Component Coefficient Correlations for the Third Component

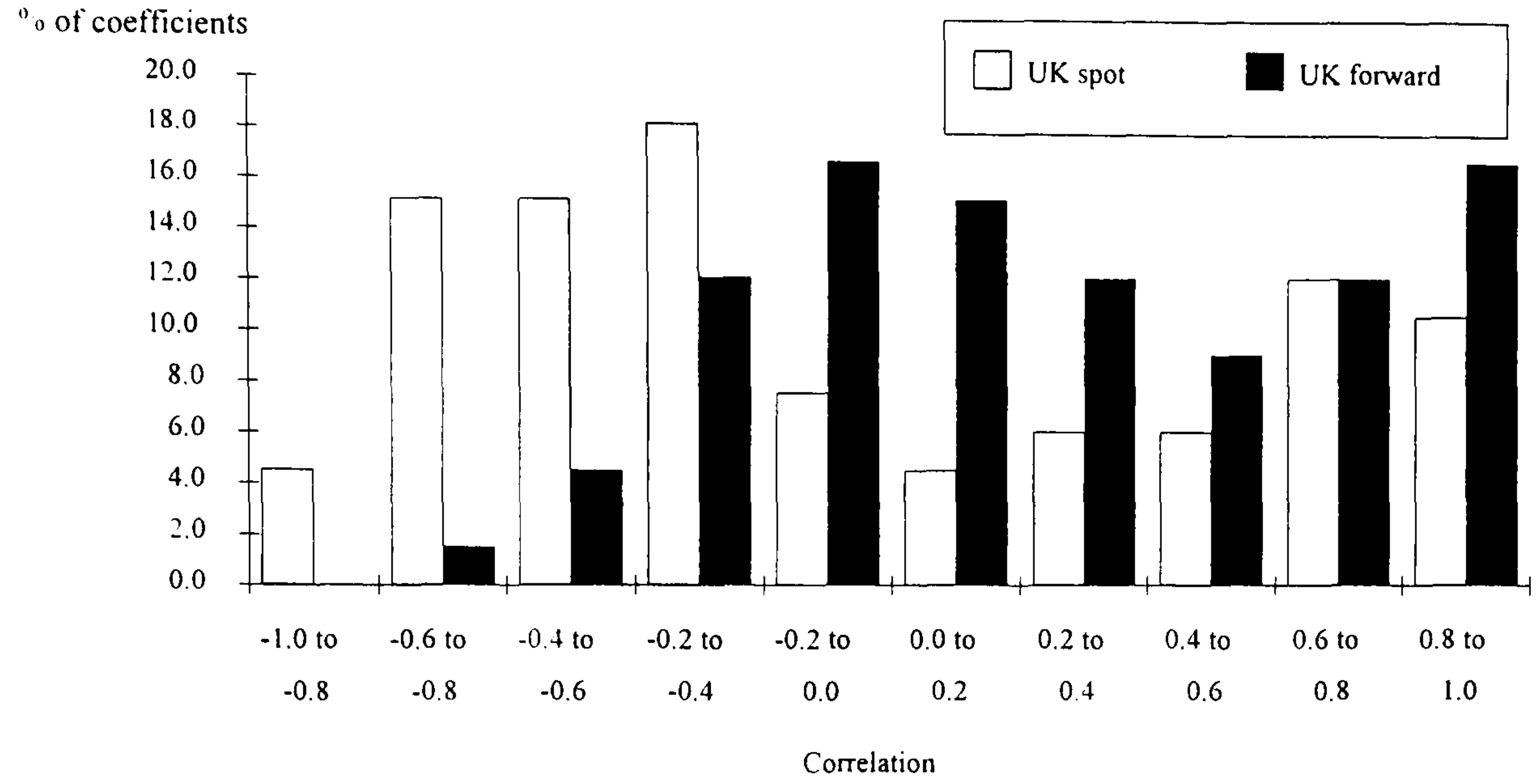


Chart 5.24 Across Component Coefficient Correlations for the First Component

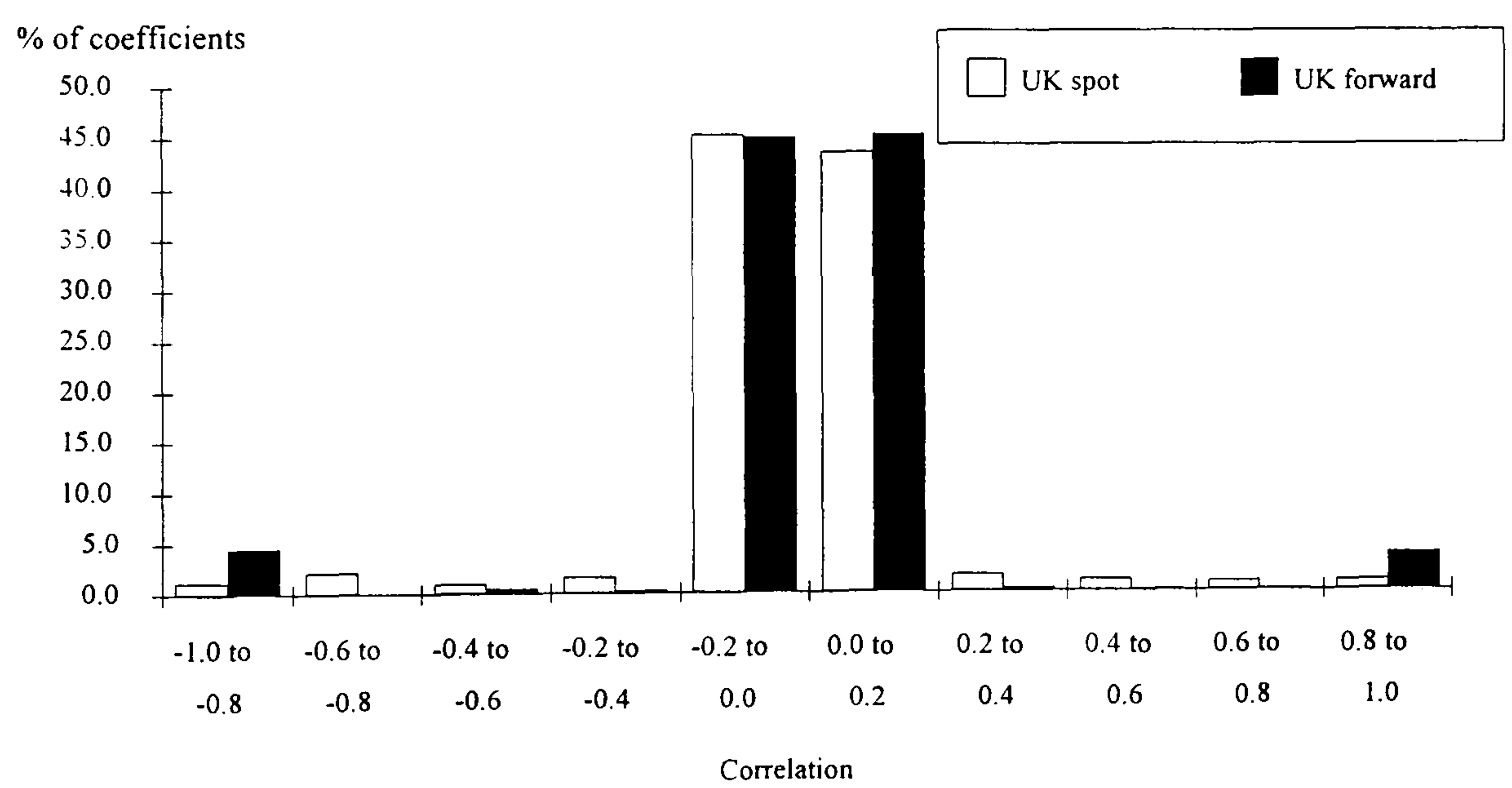


Chart 5.25 Across Component Coefficient Correlations for the Second Component

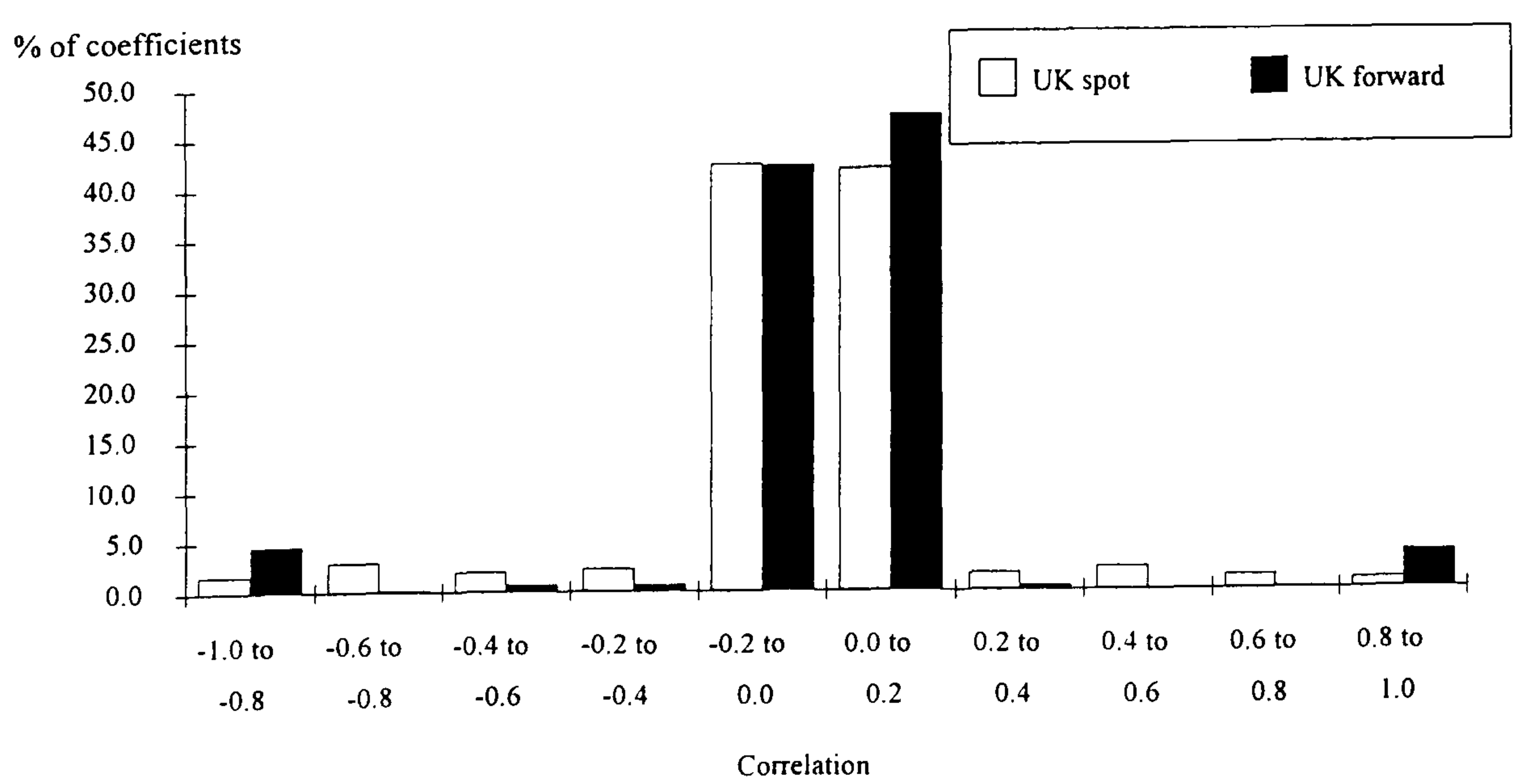
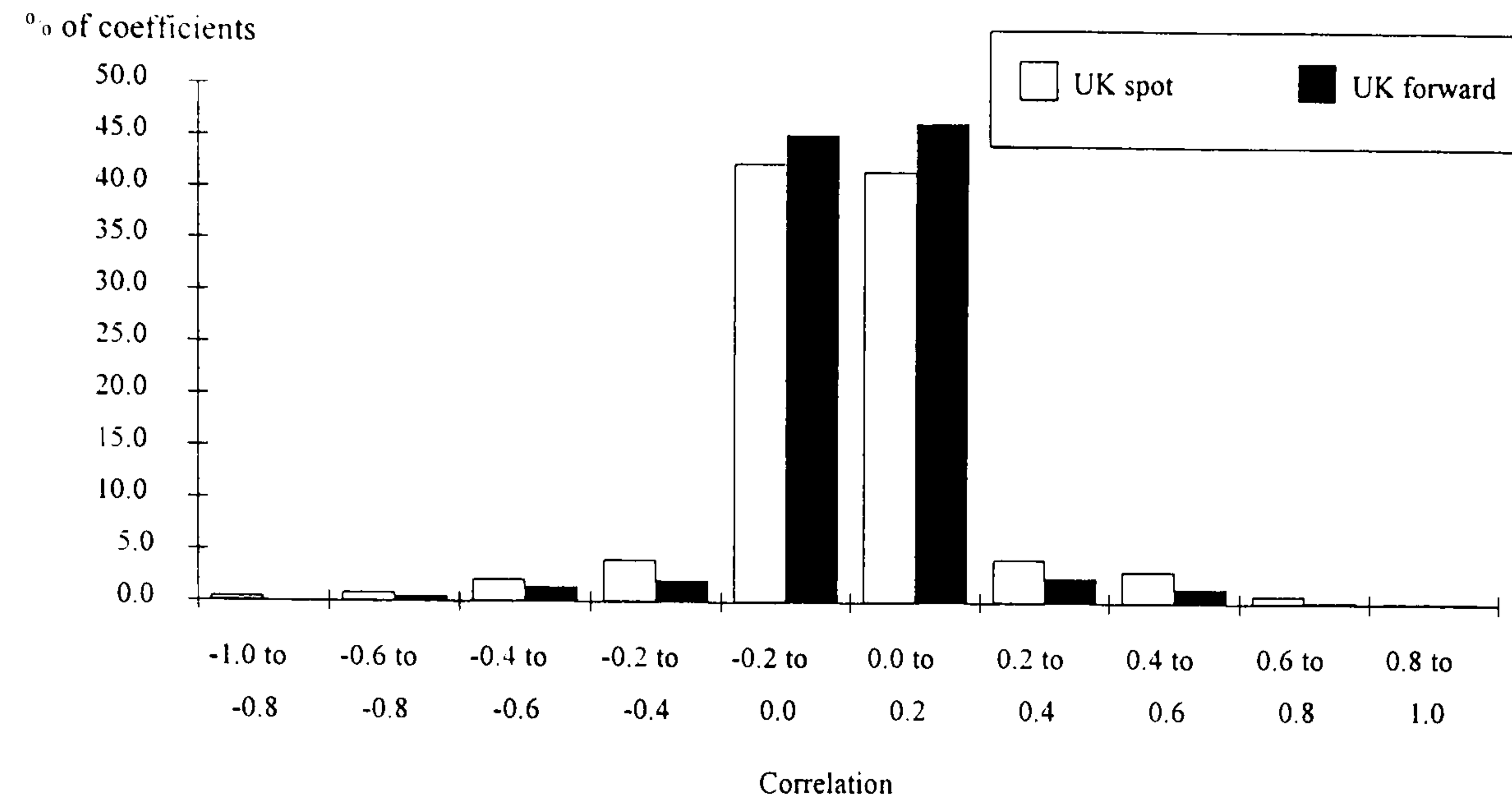


Chart 5.26 Across Component Coefficient Correlations for the Third Component



Are the different patterns of eigenvectors for the spot and forward rates of the Bank of England and the US data consistent? Take first the Bank of England results. Assume that all interest rates rise and fall together, giving a positive set of coefficients for the first principal component, but that longer rates rise and fall by less than short rates. Consequently, a rise in current interest rates implies a smaller rise in forward rates and, depending on the covariances between the short and long rates, may imply a fall in long term forward rates. Therefore, to explain the maximum amount of variation in the data, long term forward rates need to be loaded into the principal component with different signed coefficients from those of short forward rates. Thus the results using the Bank of England spot and forward rates are consistent with each other.

The above result is data set specific. A fall in the covariances between the long and short spot rates does not imply that the covariances between short and long forward rates are necessarily negative. Hence, as in the case of the US data, a positive set of eigenvectors can be found for the first principal component of both the forward and spot rates. What causes the negative result is a large reduction in the correlation between consecutive interest rates. If, say, the eight year spot rate rises from 10% to 11% but the nine year rate only rises from 10% to 10.4%, this implies that the one year forward rate at nine years will fall to 9.8%. If, on the other hand, the nine-year rate had risen to 10.6% the forward rate would also have risen to 10.2%. It is also noticeable from this example that the variability of longer-term forward rates is reduced relative to that of longer-term spot rates. Consequently, this component explains less of the relative variation in the data and hence the relative eigenvalue is lower than for the spot data.

An explanation of the different results for PC2 can be provided by noting that the Bank of England and the US data differ in terms of the maturities used. The Bank of England data uses spot and forward rates of two-year maturity upwards (because data at the short end of the spot curve is believed to be unreliable by the Bank itself). The US data includes the instantaneous spot and forward rates. This difference is crucial in explaining the different results from the second principal component of forward rates.

Assume that a shock to the spot rates occurs that alters the slope of the spot curve, say by reducing short rates and raising long rates, so steepening the curve. However, as only the slope and not the level of spot rates has been altered, the post-shock spot rate and the pre-shock spot rate curves intersect (at the pivot maturity). These changes translate into the following changes in the forward rate curve. The change in the instantaneous forward rate is, in this case, negative and equal to the decline in the instantaneous spot rate. As the instantaneous forward rate is lower, but the two spot rate curves intersect at some given maturity, this implies that at least one of the post-shock forward rates between zero and the pivot maturity must be higher than the corresponding pre-shock rate. Moreover, at maturities higher than the pivot maturity, the spot rates are higher than their pre-shock counterparts by an increasing amount. Therefore, above the pivot maturity the forward rates will be increased by the shock.

Consequently, the instantaneous forward rate and a number of other rates will be negatively correlated. In order to explain the maximum amount of variability from this slope shock the weights on the forward rates will be of different signs between the short and long ends of the forward rate curve. The actual size of these weights will depend upon the relationship between the spot rates, but the above paragraph provides a general description. Further, the pivot maturity for the forward rate curve will be shorter than the pivot maturity for the spot rate. This is because, as noted above, at least one post-shock forward rate has to be higher to ensure that the spot rates intersect at the pivot point, but this, in turn, implies that the forward pivot point is at a lower maturity. Both of these conditions are met in the eigenvectors of the second principal component for US rates. The eigenvectors of both the spot and the forward second components change signs (see table 5.5.4) and the pivot maturity for the forward rates, at between 1.0 and 1.25 years, is lower than the spot rate pivot maturity of between 1.5 to 2.5 years. As the Bank of England data excludes all the forward rates with maturities of less than two years, the change in sign is effectively excluded. Consequently, the second principal component's eigenvectors all having the same sign is an artefact of the data. Due to this, and the difficulty placing an interpretation on the eigenvectors, the Bank of England forward rate data is not modelled in latter chapters.

To summarise the results found so far. We have found that the number of observations used to construct the matrices makes little difference to the results. Altering the matrix between covariance and correlation matrices makes only slight differences for the results. Using US data the principal components for the par and spot rates were found to be virtually indistinguishable. However, whereas the principal components from the US spot and forward rate curves were very close, a comparison of the Bank of England spot and forward rate components revealed very different results. We argued that at least part of the difference in results can be accounted for by the different maturities used in the US and UK forward rate data. In the next section we analyse the consequences of noise in the data on principal components by comparing the results for UK spot and redemption yield data. However, statistical questions about the stability of the results are postponed until chapter 6, in part to keep the size of this chapter to manageable proportions.

5.6 The Effect of Noise on Principal Components

It should be noted that by comparing spot rates and redemption yields we are comparing the incomparable. The former is a measure of interest rates, whilst it is unclear what, if anything, the latter measures. The justification for the comparison is as follows. Our estimates of the spot rates undoubtedly contain noise so that they do not equal the true spot rates. The redemption yields too can be thought of as estimates of the true spot rates but with a very high noise to information ratio. The noise element takes a form that cannot be described by a simple function, it may be time varying and subject to jumps and rapid shifts in its distribution. Indeed, the noise is likely to be correlated, for some time periods, with the yield estimates themselves.[10] The presence of the noise makes it ideal to examine the properties of principal components. Furthermore, because redemption yields and spot rates move in the same domain, problems about the scale of the variables, which were identified in chapter 4 as a weakness of the principal component method, are avoided.

The results of principal component analysis on the four measures of spot and redemption yield data can throw up a vast range of results. However, they can be characterised by three cases. First, the principal components of the spot and redemption data can be distinguished on the basis of their eigenvalues or eigenvectors. Provided that the eigenvalues and eigenvectors for the two estimates of the spot rates cannot be distinguished, then it can be concluded, with a fair degree of certainty, that the interpretations of the eigenvectors (possibly as the level, slope and twist of interest rates) is valid. The second case is that the spot and redemption yield components cannot be distinguished. As they are entirely different data the inability to distinguish between them is a weakness of principal components analysis. In the extreme, it would suggest that many undesirable estimation routines for spot rates (such as redemption yields) could be employed because principal components will still translate them into a small number of factors purporting to measure the level and slope of the term structure. The third case is that all of the principal components can be distinguished from each other. In this situation drawing conclusions on spot rates say across countries would only be valid if the spot rates had been calculated using the same methodology. It is, of course, unlikely that the results of the comparisons of spot and yield principal components will fall neatly into one of these categories. It must be emphasised, yet again, that the use of spot and yield data is designed to throw light upon the use of principal components not upon the term structure of interest rates, which can only be described by spot rate data.

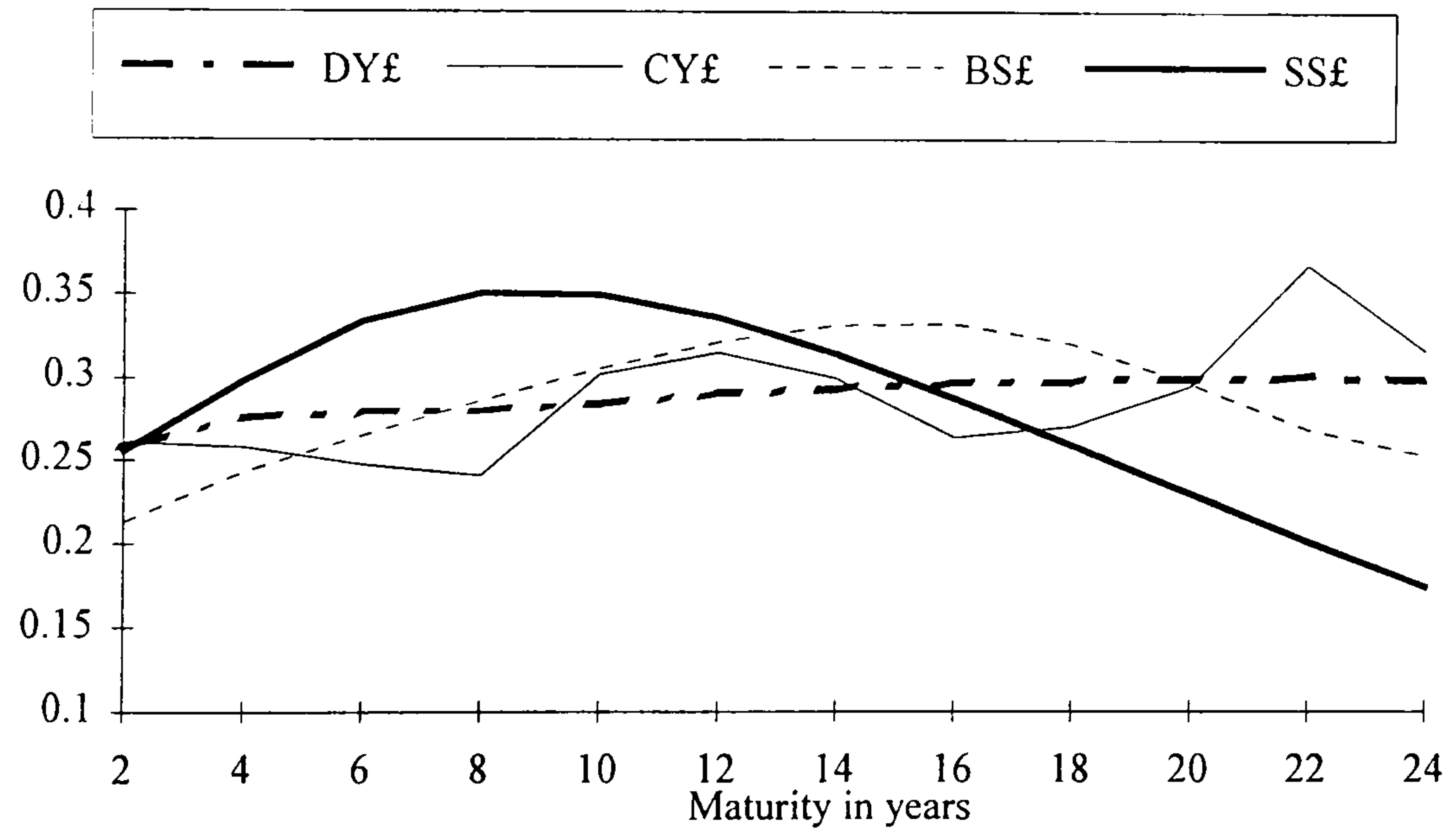
We begin our analysis by asking is there any evidence that the explanatory powers of the eigenvalues change systematically between spot and yield data other things remaining equal? Comparing tables 5.3.1, 5.3.2 and 5.3.3, which all use the same gilt data base, the percentage of the variance explained by the first eigenvalues of the spot rates of table 5.3.3 always lie in between the corresponding yield

[10] It should be noted that subtracting the estimated spot rates from the redemption yields gives the difference of the measurement errors so that the measurement error on redemption yields is not identified.

estimates of tables 5.3.1 and 5.3.2 for the first component. In table 5.3.4 the explanatory power of the first principal component is much lower than the corresponding estimates from the other data sets, although, as the Bank of England data period is shorter, the comparison is not completely watertight. For subsequent components the results are less clear cut. With the second component the explanatory power is always higher for the spot rates, BS£ and SS£, than for the yields, DY£ and CY£. Whereas BS£ eigenvalues always lie in between those of the yield data and SS£ eigenvalues always lie above the other estimates for the third and fourth components.

Thus there appears to be no systematic pattern to the eigenvalues when comparing spot and yield data. The fact remains that for each of the data sets the first eigenvalue easily explains most of the variation in the data and that this explanatory power declines rapidly (even for SS£) for the other data sets.

Chart 5.27 First Eigenvector Coefficents



Moving now to the first principal components eigenvectors, which are portrayed in chart 5.27. The coefficients are all of the same sign and of approximately similar magnitudes within each data set. However, the pattern is noticeably different between the data sets. CY£ is volatile and has a pattern reminiscent of its variances shown in chart 5.2. Both CY£ and DY£ have a tendency for the coefficients to rise with maturity. However, the spot rates, BS£ and SS£, have a slight humped shape, although, as the maturities at which the peaks of the hump occurs ranges from eight to 16 years, this similarity should not be taken too far. What chart 5.27 displays is that the coefficients of the yield to redemption data and the spot rates from BS£ are much closer in size and pattern that the first eigenvector coefficients of the spot data from the Bank of England, SS£.

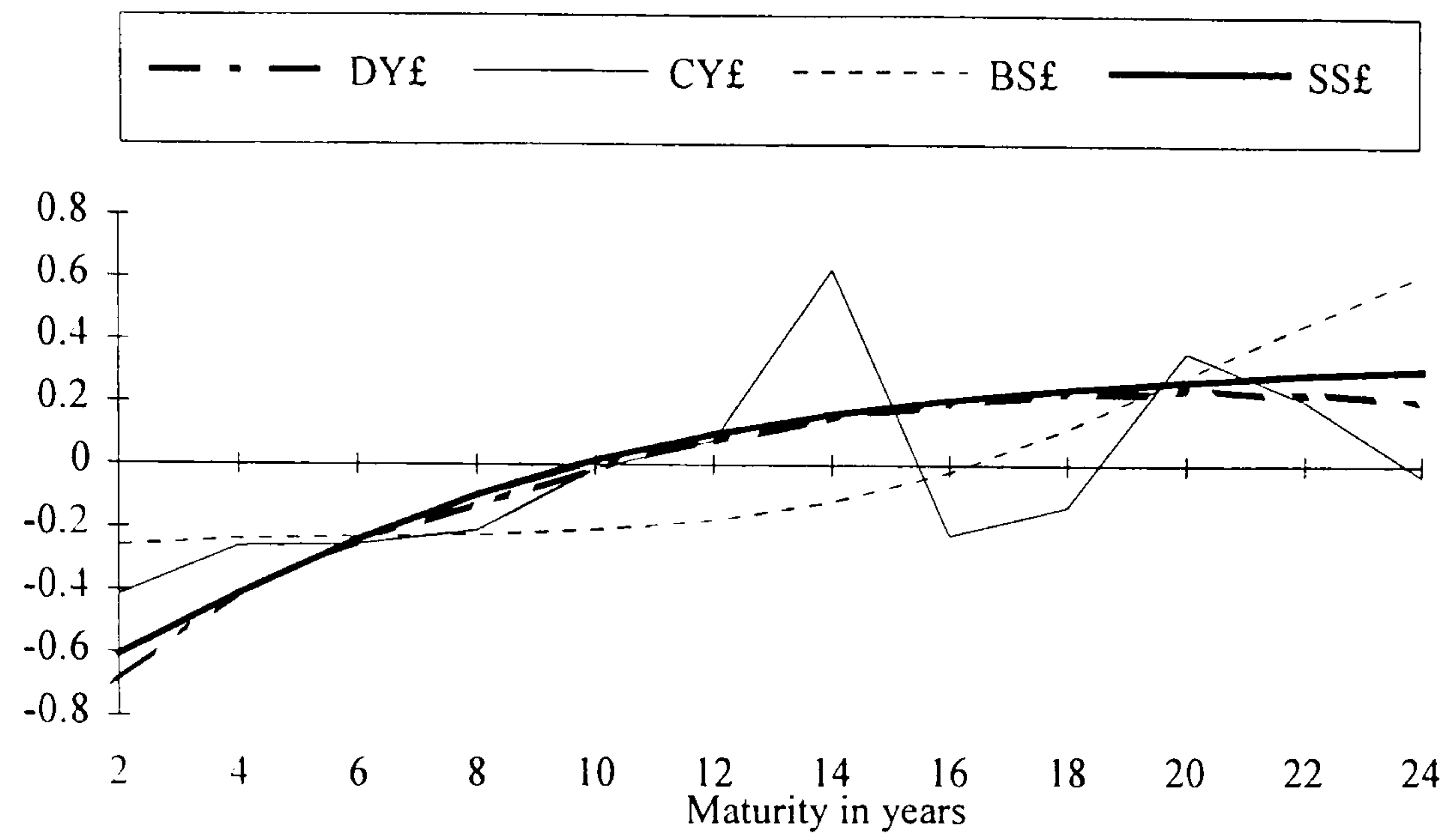
Table 5.6.1 Maximum and Minimum Standardised Eigenvector Coefficients of PC1, % Difference from the Two-Year Rate.

Matrix type	Covariance	Covariance	Covariance	Correlation	Correlation	Correlation
Observations	12	23	45	12	23	45
DY£						
Maximum	16.5	17.1	17.3	10.9	11.4	11.6
Minimum	0.0	0.0	0.0	0.0	0.0	0.0
Difference	16.5	17.1	17.3	10.9	11.4	11.6
CY£						
Maximum	40.9	40.2	40.6	10.8	11.1	11.3
Minimum	-7.6	-8.5	-8.4	-7.0	-8.5	-7.6
Difference	48.5	48.7	49.0	17.8	19.6	18.9
BS£						
Maximum	55.9	56.6	57.0	11.5	12.4	12.9
Minimum	0.0	0.0	0.0	-9.2	-9.3	-9.4
Difference	55.9	56.6	57.0	20.7	21.7	22.3
SS£						
Maximum	36.9	40.2	42.0	73.3	74.9	75.7
Minimum	-31.7	-29.8	-28.9	0.0	0.0	0.0
Difference	68.6	70.0	70.9	73.3	74.9	75.7

Note: Calculated from daily data 2 January 1979 to 21 August 1990 except for SS£, which starts on 31 March 1982.

However, represented in a different manner we are able to separate the spot and the redemption yields Table 5.6.1 portrays the maximum and minimum values of the first principal components as a percentage of the coefficient on the two year rate. As can be seen the spot data, BS£ and SS£, are more variable than the yield data, DY£ and CY£. The difference between the maximum and minimum coefficients is nearly 76% for the Bank of England spot rates. Of course, large percentage variations on small coefficients result in small changes in the absolute value of the coefficients. Furthermore, in half the cases the minimum value is at the two-year maturity and this will over emphasise the variability of the other coefficients. However, the SS£ data have a shorter time period than the other three data sets and this could account for part of its larger difference. Nevertheless, portrayed in this manner, the initial conclusion that the first principal component of the spot rates represents the level of interest rates needs to be treated with a little bit of caution.

Chart 5.28 Second Eigenvector Coefficents



The eigenvectors of the second principal component are presented in chart 5.28[11]. Leaving aside the CY£ data, the chart clearly shows that the second component is a measure of the slope, irrespective of the number of observations or matrix used. The only slight blemish to this description is for the DY£ data, where the coefficients on both the 22 and 24 year maturities decrease in size.

The CY£ coefficients are highly erratic and cannot be construed as being similar to the other data. This result conflicts with that reported in Egginton and Hall (1994) who found that the second principal component did measure the slope. Even their (unpublished) results for the 12 observation contain two anomalous coefficients but, nonetheless, their second component does look like a measure of the slope. The reason for the difference in these results is that Egginton and Hall (1994) used a program (TSP) which standardised the yields by subtracting the means and dividing by the standard deviation. As this adjustment is applied before the correlation matrix is calculated it is hardly surprising that Egginton and Hall's coefficients are different from those shown in chart 5.28. What appears to happen is that standardising removes some of the noise from the series, so that the underlying patterns are more easily discerned, hence the results show a more clear cut slope measure than shown in chart 5.28.

Leaving aside the CY£ results there is little agreement on the maturity at which the coefficients change sign (the pivot maturity). For the DY£ coefficients the pivot maturity is always between 10 and 12

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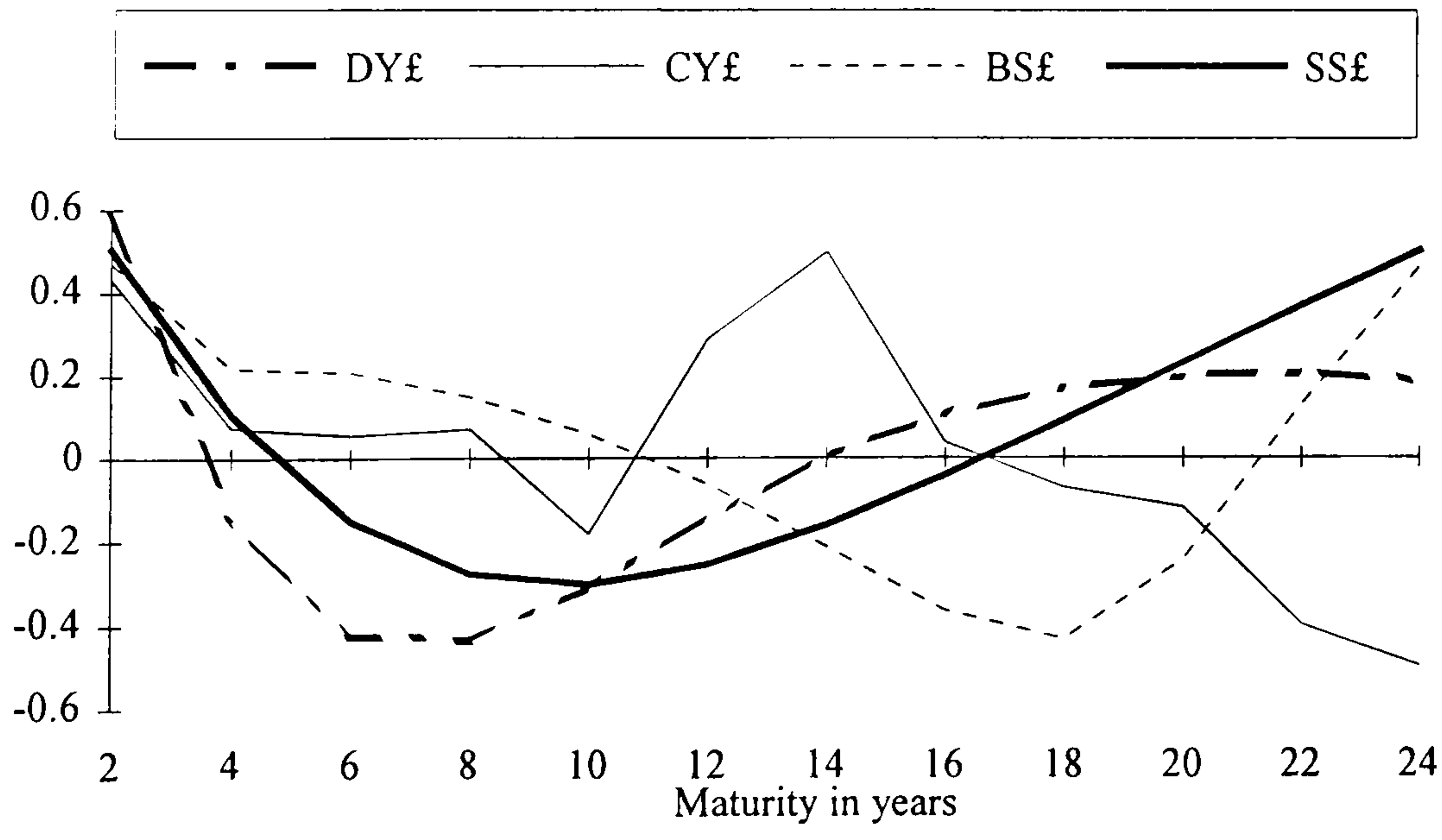
11 Further information about the second and third principal component eigenvectors is contained in the appendix in tables 5.A.1 to 5.A.8.

years regardless of the number of observations or matrix type used. For the BS£ data it is between 14 and 18 years, whilst for the SS£ data it is between 8 and 14 years. For these latter two data sets there is no consistent pattern except to note that the pivot maturities are constant for the correlation matrix results but not the covariance matrices. Although in chart 5.28 DY£ and SS£ pivot at virtually the same maturity this is a "fluke", other combinations of observations and matrix types do not show this consistency.

It is also noticeable that the patterns of coefficients are similar for DY£ and SS£ but very different from the pattern shown for BS£. Although all three data sets measure the slope of the data BS£ would show greater changes at long maturities than at short maturities whilst DY£ and SS£ show the reverse. The coefficients of the second principal component again demonstrate that there are no systematic differences between the yield and the spot data but rather that differences in estimation technique matter more.

Chart 5.29 Third Eigenvector Coefficents



The work surveyed in chapter 4 had suggested that the third principal component could be described as a "twist" or "kink". For each of the data sets analysed in this chapter, with the exception of the CY£ data, this appears to be an adequate description. The coefficients of the third eigenvectors (see chart 5.29) initially decline, then start to increase. For the BS£, DY£ and the SS£ data this is true irrespective of the number of observations used and the type of matrix employed. The only slight blemish to this description are the coefficients on the 24 year gilts for the DY£ data, which all fall marginally compared with the 22 year gilt parameter. However, the maturity at which the parameter is smallest is unstable across the number of observations used and differences in matrix types. For the DY£ data the minimum parameter is achieved at a maturity of eight-years; for the SS£ spot rates at 10 and 12-years and by the BS£ rates at 18-years. This large variation suggests that, for the third component, estimation techniques may be playing a large part in determining the minimum maturity. These differences mean that there is no evidence of systematic differences in the coefficients between the spot and yield data.

The CY£ data, on the other hand, defies simple description. Whilst the parameters may appear to be almost random there are some common elements. The parameters tend to fall between two and 10 years maturity, then rise between 10 and 14 years, before falling between 16 and 24 years. Of course, there are a number of deviations from this description, but as a generalised description it is fair. It seems highly likely that the reason that the third principal component of the cubic spline data differs from those in the other data sets is due to the much greater variability of the data, as indicated by the variances shown in chart 5.2.

Three conclusions can be drawn from this section. Firstly, there is no evidence that spot eigenvectors differ in a systematic manner from yield eigenvectors of DY£. Secondly, the CY£ eigenvectors are notably different from the corresponding eigenvectors of the other data sets. Thirdly, the spot rate principal components have a number of differences that suggest that the technique of estimation may matter more than the use of data. If these preliminary findings are confirmed, then they suggest that the interpretation of principal components as measuring features of the term structure should be treated with more caution than is usually applied. In chapter 6 we investigate whether or not these conclusions can be sustained using a range of statistical techniques.

5.7 Conclusions

In the introduction five broad questions were posed which can now be answered. The results suggest that changing the data matrix from a covariance to a correlation matrix does not make any significant difference to the eigenvalues or to the eigenvectors. Secondly, entering observations at more maturities makes only a slight difference to the results. Results from a data selection procedure suggest that the maximum amount of variation in the data will not necessarily be achieved by the equal spacing of the maturities between different data sets. Thirdly, the results using US data suggest that par data and spot data can be interchanged without significant effect on the eigenvectors and eigenvalues. This result also stands for the US spot and forward rate analysis. However, it does not hold for the UK spot and forward rates, which are very different both in terms of eigenvalues and in the interpretation that can be placed on the main eigenvectors. An explanation of this difference was provided in terms of the different maturities sampled in the two databases. Comparisons of the spot and yield principal components indicate that the CY£ components could be distinguished from the other series. However, the components from the other series were for certain features less easy to distinguish and there was little evidence of consistent features for the two spot rate series. This preliminary finding is examined in more detail in chapter 6, where a statistical analysis, including partial common principal components, is carried out. Finally, the spot rate data for the UK does conform to being measures of the level, slope and a twist in the term structure. This finding must be slightly tempered by the results described in section 5.6 and the statistical findings using Girshick's (1939) covariances of coefficients.

Appendix 5.1 Summary Statistics on the UK and US Data

Table 5.A.1 Comparison of the Loadings for the Second Principal Component by Sample Size and Type of Data Matrix using DY£

Matrix type	Covariance	Covariance	Covariance	Correlation	Correlation	Correlation
Observations	12	23	45	12	23	45
Eigenvalue (%)	0.99 (2.73)	1.67 (2.41)	3.08 (2.28)	0.33 (2.78)	0.57 (2.46)	1.05 (2.32)
Maturity						
2 (actual)	-0.693	-0.505	-0.358	-0.693	-0.505	-0.358
2	1.000	1.000	1.000	1.000	1.000	1.000
4	0.590	0.649	0.681	0.584	0.643	0.676
6	0.354	0.427	0.467	0.342	0.415	0.456
8	0.179	0.243	0.278	0.163	0.227	0.262
10	0.023	0.067	0.089	0.004	0.047	0.069
12	-0.113	-0.092	-0.082	-0.131	-0.112	-0.103
14	-0.219	-0.217	-0.218	-0.233	-0.232	-0.234
16	-0.289	-0.300	-0.309	-0.298	-0.310	-0.320
18	-0.330	-0.348	-0.361	-0.334	-0.353	-0.366
20	-0.345	-0.365	-0.379	-0.346	-0.367	-0.382
22	-0.336	-0.355	-0.368	-0.336	-0.356	-0.369
24	-0.303	-0.318	-0.330	-0.306	-0.321	-0.333

Note: Calculated from daily data from 2 January 1979 to 21 August 1990.

Table 5.A.2 Comparison of the Loadings for the Second Principal Component by Sample Size and Type of Data Matrix using CY£

Matrix type	Covariance	Covariance	Covariance	Correlation	Correlation	Correlation
Observations	12	23	45	12	23	45
Eigenvalue (%)	2.68 (6.41)	4.00 (4.92)	7.20 (4.50)	0.72 (6.03)	1.07 (4.65)	1.92 (4.30)
2 (actual)	-0.416	-0.333	-0.240	-0.407	-0.337	-0.238
2	1.000	1.000	1.000	1.000	1.000	1.000
4	0.626	0.625	0.680	0.639	0.675	0.744
6	0.613	0.537	0.567	0.651	0.605	0.639
8	0.503	0.428	0.464	0.532	0.468	0.516
10	0.024	-0.065	-0.083	-0.148	-0.170	-0.175
12	-0.187	-0.203	-0.177	-0.302	-0.259	-0.259
14	-1.514	-1.502	-1.475	-1.424	-1.321	-1.387
16	0.538	0.516	0.577	0.500	0.544	0.601
18	0.316	0.256	0.310	0.189	0.209	0.275
20	-0.881	-0.956	-0.972	-1.061	-1.054	-1.080
22	-0.500	-0.635	-0.678	-0.595	-0.614	-0.637
24	0.080	-0.047	-0.083	-0.101	-0.139	-0.137

Note: Calculated from daily data from 2 January 1979 to 21 August 1990.

Table 5.A.3 Comparison of the Loadings for the Second Principal Component by Sample Size and Type of Data Matrix using BS£.

Matrix type	Covariance	Covariance	Covariance	Correlation	Correlation	Correlation
Observations	12	23	45	12	23	45
Eigenvalue (%)	3.04 (6.7)	5.32 (6.1)	9.93 (5.8)	0.84 (7.0)	1.48 (6.4)	2.765 (6.1)
Maturity						
2 (Actual)	-0.259	-0.190	-0.137	-0.326	-0.233	-0.165
2	1.000	1.000	1.000	1.000	1.000	1.000
4	0.915	0.939	0.955	0.803	0.850	0.878
6	0.892	0.915	0.931	0.713	0.756	0.782
8	0.864	0.879	0.889	0.632	0.668	0.689
10	0.802	0.804	0.806	0.536	0.561	0.575
12	0.678	0.660	0.651	0.406	0.417	0.421
14	0.449	0.406	0.381	0.218	0.208	0.200
16	0.077	0.002	-0.039	-0.058	-0.092	-0.117
18	-0.458	-0.558	-0.614	-0.443	-0.506	-0.547
20	-1.109	-1.205	-1.256	-0.937	-1.019	-1.070
22	-1.761	-1.829	-1.860	-1.436	-1.525	-1.577
24	-2.393	-2.441	-2.457	-1.797	-1.888	-1.939

Note: Calculated from daily data from 2 January 1979 to 21 August 1990.

Table 5.A.4 Comparison of the Loadings for the Second Principal Component by Sample Size and Type of Data Matrix using SS£

Matrix type	Covariance	Covariance	Covariance	Correlation	Correlation	Correlation
Observations	12	23	45	12	23	45
Eigenvalue (%)	2.19 (16.5)	3.80 (15.1)	7.07 (14.4)	1.94 (16.2)	3.47 (15.1)	6.54 (14.5)
Maturity						
2 (actual)	-0.606	-0.456	-0.332	-0.518	-0.381	-0.275
2	1.000	1.000	1.000	1.000	1.000	1.000
4	0.677	0.699	0.710	0.848	0.859	0.865
6	0.393	0.423	0.437	0.604	0.617	0.624
8	0.157	0.188	0.203	0.377	0.388	0.393
10	-0.026	0.002	0.015	0.191	0.197	0.201
12	-0.165	-0.141	-0.131	0.032	0.035	0.036
14	-0.269	-0.250	-0.242	-0.111	-0.113	-0.114
16	-0.346	-0.333	-0.328	-0.250	-0.256	-0.259
18	-0.405	-0.397	-0.395	-0.387	-0.398	-0.403
20	-0.449	-0.448	-0.448	-0.521	-0.536	-0.543
22	-0.484	-0.488	-0.491	-0.642	-0.662	-0.671
24	-0.512	-0.521	-0.527	-0.743	-0.765	-0.777

Note: Calculated from daily data from 31 March 1982 to 21 August 1990.

Table 5.A.5 Comparison of the Loadings for the Third Principal Component by Sample Size and Type of Data Matrix using DY£

Matrix type	Covariance	Covariance	Covariance	Correlation	Correlation	Correlation
Observations	12	23	45	12	23	45
Eigenvalue (%)	0.27 (0.74)	0.43 (0.61)	0.74 (0.54)	0.09 (0.76)	0.15 (0.64)	0.26 (0.57)
Maturity						
2 (actual)	0.583	0.597	0.502	0.579	0.593	0.498
2	1.000	1.000	1.000	1.000	1.000	1.000
4	-0.244	-0.016	0.067	-0.248	-0.020	0.064
6	-0.726	-0.447	-0.348	-0.735	-0.455	-0.356
8	-0.746	-0.508	-0.424	-0.751	-0.513	-0.431
10	-0.532	-0.379	-0.327	-0.522	-0.375	-0.324
12	-0.250	-0.191	-0.171	-0.228	-0.177	-0.160
14	0.004	-0.017	-0.025	0.028	-0.001	-0.011
16	0.184	0.107	0.080	0.204	0.120	0.092
18	0.290	0.182	0.145	0.305	0.192	0.154
20	0.340	0.219	0.177	0.350	0.225	0.183
22	0.347	0.226	0.186	0.353	0.230	0.189
24	0.321	0.214	0.179	0.328	0.218	0.181

Note: Calculated from daily data from 2 January 1979 to 21 August 1990.

Table 5.A.6 Comparison of the Loadings for the Third Principal Component by Sample Size and Type of Data Matrix using CY£

Matrix type	Covariance	Covariance	Covariance	Correlation	Correlation	Correlation
Observations	12	23	45	12	23	45
Eigenvalue (%)	1.13 (2.7)	1.89 (2.3)	3.56 (2.2)	0.28 (2.3)	0.50 (2.2)	0.98 (2.2)
2 (actual)	0.434	0.244	0.154	0.463	0.196	0.159
2	1.000	1.000	1.000	1.000	1.000	1.000
4	0.170	0.304	0.107	0.091	0.309	0.111
6	0.131	0.424	0.290	0.074	0.640	0.312
8	0.164	0.342	0.108	0.250	0.504	0.063
10	-0.416	-0.202	-0.209	-0.390	-0.233	-0.227
12	0.665	0.996	1.018	0.694	1.165	0.855
14	1.150	1.869	2.159	1.078	2.221	1.733
16	0.090	0.439	0.238	-0.265	0.636	0.228
18	-0.166	0.011	-0.281	-0.498	0.083	-0.410
20	-0.272	0.030	0.091	-0.159	0.135	-0.059
22	-0.924	-1.050	-1.222	-0.679	-0.991	-1.006
24	-1.151	-1.416	-1.742	-0.996	-1.651	-1.586

Note: Calculated from daily data from 2 January 1979 to 21 August 1990.

Table 5.A.7 Comparison of the Loadings for the Third Principal Component by Sample Size and Type of Data Matrix using BS£.

Matrix type	Covariance	Covariance	Covariance	Correlation	Correlation	Correlation
Observations	12	23	45	12	23	45
Eigenvalue (%)	0.94 (2.1)	1.57 (1.8)	2.85 (1.7)	0.26 (2.2)	0.41 (1.8)	0.74 (1.6)
Maturity						
2 (Actual)	0.471	0.315	0.209	0.550	0.379	0.246
2	1.000	1.000	1.000	1.000	1.000	1.000
4	0.463	0.563	0.626	0.279	0.415	0.504
6	0.441	0.553	0.624	0.225	0.362	0.453
8	0.320	0.417	0.476	0.104	0.219	0.289
10	0.128	0.195	0.231	-0.052	0.028	0.068
12	-0.132	-0.107	-0.105	-0.236	-0.199	-0.198
14	-0.450	-0.480	-0.518	-0.442	-0.457	-0.500
16	-0.776	-0.860	-0.935	-0.642	-0.711	-0.797
18	-0.920	-1.017	-1.097	-0.718	-0.812	-0.914
20	-0.517	-0.510	-0.506	-0.398	-0.420	-0.446
22	0.278	0.465	0.612	0.282	0.411	0.537
24	0.982	1.315	1.577	0.865	1.102	1.338

Note Calculated from daily data from 2 January 1979 to 21 August 1990.

Table 5.A.8 Comparison of the Loadings for the Third Principal Component by Sample Size and Type of Data Matrix using SS£

Matrix type	Covariance	Covariance	Covariance	Correlation	Correlation	Correlation
Observations	12	23	45	12	23	45
Eigenvalue (%)	0.49 (3.7)	0.82 (3.2)	1.48 (3.0)	0.46 (3.9)	0.77 (3.4)	1.40 (3.1)
Maturity						
2 (actual)	0.503	0.436	0.339	0.555	0.448	0.338
2	1.000	1.000	1.000	1.000	1.000	1.000
4	0.209	0.300	0.341	0.393	0.447	0.473
6	-0.299	-0.181	-0.130	-0.118	-0.050	-0.018
8	-0.551	-0.435	-0.387	-0.410	-0.343	-0.314
10	-0.600	-0.502	-0.462	-0.527	-0.467	-0.440
12	-0.506	-0.435	-0.407	-0.524	-0.472	-0.449
14	-0.320	-0.280	-0.265	-0.432	-0.390	-0.371
16	-0.080	-0.074	-0.072	-0.266	-0.235	-0.222
18	0.186	0.159	0.148	-0.037	-0.019	-0.011
20	0.462	0.401	0.377	0.243	0.245	0.247
22	0.735	0.642	0.605	0.552	0.537	0.531
24	0.998	0.874	0.826	0.859	0.827	0.815

Note: Calculated from daily data from 31 March 1982 to 21 August 1990.

Table 5.A.9 Within Component Correlations for the First Component Using MS$.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	16.67	10.23	4.73	4.55	3.41	2.84	1.89	1.33	1.33	1.52
Number	176	108	50	48	36	30	20	14	14	16
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	2.46	1.89	2.08	3.03	2.65	3.41	4.55	5.68	7.01	18.75
Number	26	20	22	32	28	36	48	60	74	198

Note: Correlations calculated using Girshick's (1939) equation. See equation (4.4.3) of chapter 4. The underlying eigenvectors were calculated from a covariance matrix of 33 maturities using month-end data from 1951 to 1991. Total number of correlations = 1056

Table 5.A.10 Within Component Correlations for the Second Component Using MS$.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	3.22	10.04	7.77	7.01	2.46	6.06	2.84	4.55
Number	0	0	34	106	82	74	26	64	30	48
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	5.11	3.03	4.36	4.36	3.79	5.49	6.06	7.01	5.87	10.98
Number	54	32	46	46	40	58	64	74	62	116

Note: See table 5.A.9

Table 5.A.11 Within Component Correlations for the Third Component Using MS$.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	4.17	7.01	10.8	9.09	5.3	4.36	4.17	5.3	3.03
Number	0	44	74	114	96	56	46	44	56	32
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	4.92	2.84	4.92	5.11	6.63	2.84	3.22	4.36	4.36	7.58
Number	52	30	52	54	70	30	34	46	46	80

Note: See table 5.A.9

Table 5.A.12 Within Component Correlations for the First Component Using MP$.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	16.67	10.04	5.11	4.17	3.6	2.27	2.27	1.7	1.52	1.7
Number	176	106	54	44	38	24	24	18	16	18
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	1.52	2.65	1.89	2.65	3.79	3.03	3.98	5.3	7.39	18.75
Number	16	28	20	28	40	32	42	56	78	198

Note: See table 5.A.9

Table 5.A.13 Within Component Correlations for the Second Component Using MP$.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0.57	4.36	11.74	6.63	6.44	3.41	4.92	3.79	2.84
Number	0	6	46	124	70	68	36	52	40	30
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	4.17	4.73	2.65	3.41	4.55	5.11	5.3	7.39	7.39	10.61
Number	44	50	28	36	48	54	56	78	78	112

Note: See table 5.A.9

Table 5.A.14 Within Component Correlations for the Third Component Using MP$

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	3.79	6.82	11.55	9.09	5.87	3.79	4.55	2.84	4.36
Number	0	40	72	122	96	62	40	48	30	46
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	4.73	4.92	5.49	4.55	4.55	2.84	3.22	4.17	4.36	8.52
Number	50	52	58	48	48	30	34	44	46	90

Note: See table 5.A.9

Table 5.A.15 Within Component Correlations for the First Component Using MF$

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	8.33	10.98	4.73	5.3	3.98	5.11	4.73	4.73	4.92
Number	0	88	116	50	56	42	54	50	50	52
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	3.98	3.98	3.79	3.6	3.41	3.6	4.92	6.06	6.44	7.39
Number	42	42	40	38	36	38	52	64	68	78

Note: See table 5.A.9

Table 5.A.16 Within Component Correlations for the Second Component Using MF$.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0.95	8.33	6.82	8.71	5.49	4.17	6.44	8.71
Number	0	0	10	88	72	92	58	44	68	92
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	6.25	6.63	5.3	5.3	8.33	3.41	5.49	1.7	3.79	4.17
Number	66	70	56	56	88	36	58	18	40	44

Note: See table 5.A.9

Table 5.A.17 Within Component Correlations for the Third Component Using MF$.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0	1.7	1.89	6.82	13.64	16.29	10.61	9.09
Number	0	0	0	18	20	72	144	172	112	96
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	9.66	7.2	6.25	4.36	2.08	1.7	1.7	2.08	2.08	2.84
Number	102	76	66	46	22	18	18	22	22	30

Note: See table 5.A.9

Table 5.A.18 Across Component Correlations for the First Component for MS$.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0	0.01	0.11	0.22	0.55	0.19	0.12	47.84
Number	0	0	0	4	38	78	192	67	43	16672
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	49.28	0.26	0.37	0.69	0.24	0.11	0	0	0	0
Number	17174	90	128	240	84	38	0	0	0	0

Note: Correlations calculated using Girshick's (1939) equation. See equation (4.4.5) of chapter 4. The underlying eigenvectors were calculated from a covariance matrix of 33 maturities using month-end data from 1951 to 1991. Total number of correlations = 34848

Table 5.A.19 Across Component Correlations for the Second Component Using MS$.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0.01	0.06	0.2	0.37	0.84	0.54	0.69	47.16
Number	0	1	3	21	71	129	291	189	241	16433
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	46.97	0.89	0.65	0.91	0.4	0.22	0.08	0.01	0	0
Number	16369	309	226	317	138	77	29	4	0	0

Note: See table 5.A.18

Table 5.A.20 Across Component Correlations for the Third Component Using MS$.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0.03	0.12	0.21	0.29	0.37	0.5	0.55	0.86	46.93
Number	0	11	41	74	100	130	175	193	301	16354
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	47.25	0.89	0.48	0.44	0.38	0.36	0.21	0.09	0.03	0
Number	16464	310	169	152	132	124	74	32	11	1

Note: See table 5.A.18

Table 5.A.21 Across Component Correlations for the First Component Using MF$

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0	0	0	0	0	0.79	0.41	49.08
Number	0	0	0	0	0	0	0	275	143	17105
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	48.61	0.4	0.7	0	0	0	0	0	0	0
Number	16939	141	245	0	0	0	0	0	0	0

Note: See table 5.A.18

Table 5.A.22 Across Component Correlations for the Second Component Using MF$.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0	0	0.02	0.13	0.29	1.06	0.68	47.97
Number	0	0	0	0	7	45	102	370	237	16717
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	47.74	0.66	0.94	0.33	0.16	0	0	0	0	0
Number	16638	231	329	115	57	0	0	0	0	0

Note: See table 5.A.18

Table 5.A.23 Across Component Correlations for the Third Component Using MF$

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0	0	0.02	0.13	0.32	0.36	0.74	48.12
Number	0	0	0	0	7	45	110	125	258	16769
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	48.65	0.87	0.29	0.33	0.17	0	0	0	0	0
Number	16954	303	102	116	59	0	0	0	0	0

Note: See table 5.A.18

Table 5.A.24 Across Component Correlations for the First Component Using MP$.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0	0.03	0.09	0.16	0.62	0.19	0.16	47.91
Number	0	0	0	9	31	55	216	65	57	16694
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	49.23	0.26	0.46	0.64	0.17	0.09	0	0	0	0
Number	17154	91	160	223	60	32	1	0	0	0

Note: See table 5.A.18

Table 5.A.25 Across Component Correlations for the Second Component Using MP$.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0.01	0.01	0.11	0.17	0.37	0.87	0.55	0.73	47.04
Number	0	2	5	39	60	128	304	190	253	16393
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	46.92	0.91	0.75	0.84	0.37	0.22	0.11	0.03	0	0
Number	16349	318	261	291	128	75	40	12	0	0

Note: See table 5.A.18

Table 5.A.26 Across Component Correlations for the Third Component Using MP$.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0.03	0.15	0.23	0.28	0.43	0.45	0.54	0.72	47.09
Number	0	11	51	81	98	150	156	187	252	16411
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	47.3	0.71	0.47	0.4	0.43	0.38	0.22	0.11	0.04	0
Number	16484	249	163	141	149	134	77	40	13	1

Note: See table 5.A.18

Table 5.A.27 Within Component Correlations for the First Component Using SF£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0	0	0	0	0	0	0	0
Number	0	0	0	0	0	0	0	0	0	0
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	0	0	0	0	0	0	0	0	12.12	87.88
Number	0	0	0	0	0	0	0	0	16	116

Note: Correlations calculated using Girshick's (1939) equation. See equation (4.4.3) of chapter 4. The underlying eigenvectors were calculated from a covariance matrix of 12 observations from the daily sample 31 March 1982 to 21 August 1990. Total number of correlations = 132

Table 5.A.28 Within Component Correlations for the Second Component Using SF£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	37.88	7.58	0	1.52	1.52	1.52	1.52	0	1.52	0
Number	50	10	0	2	2	2	2	0	2	0
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	0	0	0	4.55	3.03	1.52	0	1.52	4.55	31.82
Number	0	0	0	6	4	2	0	2	6	42

Note:see table 5.A.27

Table 5.A.29 Within Component Correlations for the Third Component Using SF£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	1.52	0	1.52	3.03	4.55	7.58	4.55	12.12
Number	0	0	2	0	2	4	6	10	6	16
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	9.09	6.06	3.03	9.09	7.58	1.52	6.06	6.06	9.09	7.58
Number	12	8	4	12	10	2	8	8	12	10

Note:see table 5.A.27

Table 5.A.30 Across Component Correlations for the First Component Using SF£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	4.17	0.38	0	0	0	0.51	0.25	0	0.25	44.82
Number	66	6	0	0	0	8	4	0	4	710
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	44.76	0.63	0.19	0.13	0.13	0	0	0.06	0.95	2.78
Number	709	10	3	2	2	0	0	1	15	44

Note: Correlations calculated using Girshick's (1939) equation. See equation (4.4.5) of chapter 4. The underlying eigenvectors were calculated from a covariance matrix of 12 observations from the daily sample 31 March 1982 to 21 August 1990. Total number of correlations = 1584.

Table 5.A.31 Across Component Correlations for the Second Component Using SF£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	4.17	0.38	0.06	0.13	0.13	0.63	0.38	0.32	1.01	41.29
Number	66	6	1	2	2	10	6	5	16	654
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	46.65	0.57	0.32	0.13	0.06	0	0	0.06	0.95	2.78
Number	739	9	5	2	1	0	0	1	15	44

Note: see table 5.A.30

Table 5.A.32 Across Component Correlations for the Third Component Using SF£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0.13	0.38	0.57	0.82	0.95	1.14	2.02	43.37
Number	0	0	2	6	9	13	15	18	32	687
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	44.32	2.21	1.39	1.07	0.95	0.44	0.13	0.06	0.06	0
Number	702	35	22	17	15	7	2	1	1	0

Note: see table 5.A.30

Chapter 6

Testing the Stability and Consistency of the Term Structure using Partial Common Principal Components

6.1 Introduction

There are two central questions posed in this chapter. Firstly, how robust are the descriptions of the UK principal components given in chapter 5? Secondly, how alike are the term structures derived from different methods of estimation, over different time periods and from different countries, in this case the UK and the US? These questions can be regarded as a continuation of the analysis undertaken in chapter 5 and the analysis proceeds on three levels. The first is relatively straightforward. In the preceding chapters the principal component scores have been ignored, with analysis concentrating entirely upon the eigenvectors and eigenvalues. Here we utilise a simple idea that if the principal components behave in a similar manner, then the principal component scores from different samples should be highly correlated. Moreover, if the same underlying forces drive the component scores, the orthogonality of each of the scores from within a sample should also be repeated across samples. This idea can be analysed using correlation techniques, which is applied in section 6.2. This appears to be a novel approach that has not previously been used in the literature on principal components analysis of term structures. One reason is that principal components studies often lack a time dimension in their data, as they are usually concerned with cross sections of characteristics. The cross sectional nature of the data means that the principal component scores only contain data for the set pattern in which they were analysed and, hence, cross-comparisons are meaningless, unlike time series data. This does not, however, explain why correlations of principal component scores have not been used in the term structure studies.

Second, the eigenvectors estimated in chapter 5 of the UK spot and yield data are subjected to a number of statistical tests in section 6.3. The objective is to study whether or not the differences noted in chapter 5 between the eigenvectors are statistically significant. Again, as noted in chapter 4, the use of statistical tests is a novel approach at least as far as principal components analysis of term structure data is concerned. These tests include the Krzanowski tolerances, measures of the variances of the eigenvectors and an examination of the within and across component correlations of the coefficients.

The third approach utilised in this chapter is partial common principal components (PCPCs) due to Flury (1988). Essentially, Flury outlines a hierarchy of relationships between sample covariance matrices. At the highest level is the proposition that all the covariance matrices under consideration are equal. For this relationship the appropriate manner of estimating principal components is to pool the data and this is the method used in chapter 5 when the full sample is estimated from a series of sub-samples across time. The second level assumes that each covariance matrix is proportional to each of the others, but the coefficient of proportionality may change between each of the matrices. The third level is to assume that each of the covariance matrices can be decomposed into a common set of eigenvectors but the eigenvalues differs for each sample. This is known as common principal components. The fourth level is to assume that only some of the eigenvectors are common and that some are specific as are all the eigenvalues from each sample. This is known as partial common principal components analysis. It is with these latter two types of analysis that the second part of this chapter is concerned. Section 6.4 applies PCPC, which was described in chapter 4, across time. Section 6.5 applies it across techniques and in section 6.6 it is applied across data from different countries. Section 6.7 provides some conclusions.

6.2 Correlation of Principal Component Scores

The principal component scores are defined by equation (4.2.1) in chapter 4. The i^{th} vector of principal component scores, Z_i, is defined as:

$$Z_i = a_i^{'} X \quad ...(6.2.1)$$

Where: a_i is the i^{th} eigenvector

X is the data matrix

As the eigenvectors are orthogonal this implies that the correlation between scores within each data set is zero. However, there is no reason to suppose that the correlations across data sets need be zero. Table 6.2.1 records the correlation coefficients between the principal component scores. All the columns and rows except for 4, 8 and 12 correspond to the full sample period, 2 January 1979 to 21 August 1990 (i.e. 3036 observations). Columns and rows 4, 8 and 12, which corresponds to the Bank of England spot curves, are calculated over the period 31 March 1982 to 21 August 1990 (i.e. 2190 observations). Columns 1 to 4 are the correlations of the first principal component scores, 5 to 8 are the correlations of the second principal component scores and columns 9 to 12 are the correlations of the third principal component scores. It should be recalled that the signs of the eigenvectors are arbitrary so that the sign of the correlation does not convey any information. It should be noted that yields and spot rates are inherently different and, therefore, table 6.2.1 should not be interpreted as a comparison of yield and spot rates. Rather the table analyses the ability of principal components analysis to separate yields from spot rate data.

Table 6.2.1 Correlation of Principal Component Scores

		PC1				PC2				PC3			
		1	2	3	4	5	6	7	8	9	10	11	12
PC1	1	1.00	0.99	0.99	0.98	0.00	-0.09	-0.09	-0.05	0.00	-0.01	0.01	-0.07
	2	0.99	1.00	0.98	0.95	0.08	0.00	-0.09	0.09	0.00	0.00	-0.04	-0.15
	3	0.99	0.98	1.00	0.97	0.08	-0.02	0.00	0.12	0.04	0.02	0.00	-0.09
	4	0.98	0.95	0.97	1.00	-0.17	-0.38	-0.29	0.00	-0.47	0.17	0.42	0.00
PC2	5	0.00	0.08	0.08	-0.17	1.00	0.79	-0.04	0.87	0.00	0.39	-0.58	-0.32
	6	-0.09	0.00	-0.02	-0.38	0.79	1.00	-0.12	0.61	-0.01	0.00	-0.47	-0.30
	7	-0.09	-0.09	0.00	-0.29	-0.04	-0.12	1.00	0.01	0.57	0.23	0.00	0.27
	8	-0.05	0.09	0.12	0.00	0.87	0.61	0.01	1.00	0.00	0.46	-0.78	0.00
PC3	9	0.00	0.00	0.04	-0.47	0.00	-0.01	0.57	0.00	1.00	0.19	-0.29	0.46
	10	-0.01	0.00	0.02	0.17	0.39	0.00	0.23	0.46	0.19	1.00	-0.20	-0.03
	11	0.01	-0.04	0.00	0.42	-0.58	-0.47	0.00	-0.78	-0.29	-0.20	1.00	0.23
	12	-0.07	-0.15	-0.09	0.00	-0.32	-0.30	0.27	0.00	0.46	-0.03	0.23	1.00

Note: Column and rows 1, 5 and 9 are calculated using DY£.
Column and rows 2, 6 and 10 are calculated using CY£.
Column and rows 3, 7 and 11 are calculated using BS£.
Column and rows 4, 8 and 12 are calculated using SS£.

As can be seen from table 6.2.1, all the first principal component scores in the sub-matrix defined by rows and columns 1 to 4 are highly correlated with each other, the lowest correlation being 0.95. For the second principal component scores, the matrix of rows and columns 5 to 8, the results are less clear cut. There is a positive correlation between rows 5, 6 and 8 but these scores are not at all correlated with those of row 7 (using BS£) where the highest (in absolute terms) correlation is -0.12. For the third principal component scores the results are even more mixed. Column 9 (using DY£) is correlated with column 12 (using SS£) but all the other rows and columns show little correlation.

These results can be partly explained by examining the "off-diagonal" sub-matrices. If these eigenvectors are similar these sub-matrices should contain (absolutely) small or zero correlations. Yet examination of column 7 with rows 9, 10 and 12 and column 11 with rows 5, 6 and 8 does not find small correlation coefficients. This suggests that the second principal component scores from the BS£ are more closely associated with the third principal component scores of the other data sets than it is with the second principal component scores. Likewise, the third principal component scores of the BS£ data is more closely associated with the second principal component scores of the other term structures. This was not obvious from the comparisons of the 12 observation eigenvectors conducted in the last chapter. This illustrates the point that although the eigenvectors appear the same movements in the variables underlying the eigenvectors are also important for summary measures such as the principal component scores. For the DY£, CY£ and SS£ data sets the correlations between the first principal component scores and the second and third principal component scores are all low. This is not true of the SS£ data in column 4, nor is it true for many of the second and third principal component scores when they are compared together.

Overall these results suggest that if the principal component decompositions of the term structure are similar then this similarity is more likely to be found in the level of interest rates than in the slope or twist of the term structures. This finding would be consistent with the higher principal components containing a higher noise to signal ratio than the lower principal components, although the method by which principal components achieves this is unclear. The result also suggests that the search for common components is more likely to be fruitful with the first eigenvector than with the other eigenvectors and the analysis, in section 6.4, is conducted with this in mind.

6.3 Consistency Across Techniques

The comparison of eigenvectors when different estimation techniques were used in chapter 5 revealed some notable differences, in particular for CY£. This section presents statistical evidence that suggests that these differences are statistically significant. It begins by using standard deviations and the Krzanowski tolerances to gauge the sampling variability of the eigenvectors. In the next section PCPC analysis is used to ascertain whether or not the results are statistically different. In the previous chapter the cross-matrix comparisons and the comparisons by the number of observations indicated that the coefficients were not sensitive to these choices. Thus restricting the analysis to results from the 12 variable covariance matrices can easily be justified. The statistical procedures used are, in any case, not applicable to correlation matrices.

Tables 6.3.1 and 6.3.2 give the first eigenvectors' coefficients, their standard deviations, coefficients of variation and the associated Krzanowski tolerances for each of the four UK data sets, DY£, CY£, BS£ and SS£. As can be seen from the third and seventh columns of tables 6.3.1 and 6.3.2, the standard deviation of the coefficient estimates tend to be small. The coefficients of variation (columns four and eight of the tables) indicate that at most the standard deviation is 3.1% of the coefficient estimate and is usually (in 38/48 of the comparisons) below 1%. What is noticeable, however, is that the standard deviation on the two-year gilt tends to be relatively large for each of the four data sets. Nevertheless, it appears highly unlikely that the variation in parameter sizes of the first principal components is due to sampling. The standard deviations are simply too small to allow more than a few coefficients to intersect and thus we conclude that the first eigenvectors are distinct.

Table 6.3.1 Standard Deviations (SD), Coefficients of Variation (CV) and Krzanowski Tolerances (KT) for the First Eigenvectors using DY£ and CY£.

Maturity (years)	DY£ eigen-vector	DY£ SD	DY£ CV (%)	DY£ KT (10%)	CY£ eigen-vector	CY£ SD	CY£ CV (%)	CY£ KT (10%)
2	0.2583	0.0024	0.92	0.2116	0.261	0.0031	1.18	0.130
4	0.2765	0.0013	0.48	0.1249	0.258	0.0022	0.84	0.082
6	0.2792	0.0010	0.37	0.0749	0.248	0.0020	0.81	0.080
8	0.2810	0.0008	0.29	0.0379	0.241	0.0021	0.87	0.066
10	0.2850	0.0005	0.19	0.0049	0.302	0.0016	0.53	0.003
12	0.2902	0.0004	0.14	0.0239	0.315	0.0016	0.52	0.024
14	0.2942	0.0005	0.18	0.0463	0.300	0.0039	1.32	0.197
16	0.2966	0.0007	0.23	0.0612	0.265	0.0024	0.89	0.070
18	0.2984	0.0008	0.26	0.0699	0.271	0.0023	0.85	0.041
20	0.3001	0.0008	0.27	0.0730	0.295	0.0026	0.89	0.115
22	0.3010	0.0008	0.27	0.0711	0.368	0.0021	0.57	0.065
24	0.3005	0.0008	0.27	0.0642	0.317	0.0022	0.70	0.010

Note: The Krzanowski tolerances are constructed from a 10% change in the value of the component's eigenvalue. By adding or subtracting the tolerance from the elements of the eigenvector, a band can be constructed within which the elements would vary if the eigenvalue were erroneously measured by up to 10%.

Table 6.3.2 Standard Deviations (SD), Coefficients of Variation (CV) and Krzanowski Tolerances (KT) for the First Eigenvectors using BS£ and SS£.

Maturity (years)	BS£ eigen-vector	BS£ SD	BS£ CV (%)	BS£ KT (10%)	SS£ eigen-vector	SS£ SD	SS£ CV (%)	SS£ KT (10%)
2	0.2129	0.0022	1.04	0.0807	0.256	0.0079	3.09	0.2031
4	0.2431	0.0014	0.59	0.0738	0.299	0.0051	1.72	0.1375
6	0.2654	0.0014	0.53	0.0720	0.334	0.0031	0.93	0.0798
8	0.2860	0.0013	0.46	0.0697	0.351	0.0018	0.52	0.0320
10	0.3052	0.0012	0.38	0.0647	0.350	0.0015	0.43	0.0053
12	0.3213	0.0010	0.30	0.0547	0.336	0.0018	0.53	0.0335
14	0.3314	0.0009	0.26	0.0363	0.315	0.0022	0.70	0.0546
16	0.3319	0.0011	0.32	0.0062	0.288	0.0026	0.91	0.0703
18	0.3201	0.0014	0.44	0.0370	0.260	0.0031	1.18	0.0822
20	0.2963	0.0017	0.57	0.0895	0.231	0.0035	1.54	0.0913
22	0.2692	0.0025	0.91	0.1421	0.202	0.0041	2.00	0.0984
24	0.2535	0.0035	1.40	0.1932	0.175	0.0046	2.61	0.1040

Note: See table 6.3.1

A further way of looking at the sample variability of the coefficients is to calculate the Krzanowski tolerances, as discussed in chapter 4. These are tabulated in columns five and nine of tables 6.3.1 and 6.3.2. As the tolerance is calculated from an arbitrary percentage, confidence bands are not particularly meaningful. Rather the use of the tolerance is to work out whether or not the coefficients become more similar in size if the eigenvalue is varied. Given the results of Bartlett's test of isotropy and the Lawley correction discussed in chapter 5, the eigenvalues are very securely estimated and thus a tolerance of 10% is, in all probability, excessive. The Krzanowski tolerances tend to confirm the results of the standard deviations, in particular, that the change in the parameter on two-year gilts would be relatively larger than the change on other parameters. It should also be noted that there is a tendency for the middle maturities to be more robust to shocks to the eigenvalues than the parameters of the short and long maturities. Nevertheless, the Krzanowski tolerances confirm that the parameters of the first principal components are robustly estimated and therefore distinct.

Another check on the robustness of the interpretation of the first component is to calculate the degree to which the parameters co-vary using Girshick's (1939) measures. These correlations are reported in the appendix in tables 6.A.1 to 6.A.4. The tables reveal that, except for the CY£ data, there is a tendency for the parameters to be highly correlated with other parameters in the PC1. There are between 21% and 33% pairs of coefficients that have absolute correlation coefficients in excess of 0.9 and there are approximately the same numbers of pairs with positive and negative correlation coefficients. In contrast, there are relatively few parameter pairs with low correlations. Whereas the CY£ data set has no absolute correlation coefficients that are larger than 0.6. These results suggest that the first eigenvectors would deform under measurement errors. The evidence is not strong enough to out weigh the evidence from the coefficients of variation and the Krzanowski tolerances that the eigenvectors are distinct.

Not only are the coefficients potentially correlated within their own component but also they are also potentially correlated with coefficients from other eigenvectors (see Girshick (1939)). These are reported in tables 6.A.5 to 6.A.8, 6.A.13 to 6.A.19 and 6.A.21 to 6.A.24 in the appendix to this chapter. As these correlations involve all 12 of the principal components it is convenient to discuss all of the results together. The overwhelming finding is that the coefficients are uncorrelated for each of the 12 comparisons (4 data sets by 3 components). Only the SS£ spot data set has more than 205 of the comparisons being absolutely greater than -0.1 or 0.1. Just 0.6% of the comparisons for the first 3 eigenvectors result in correlations in excess of 0.9 in absolute terms.[1] These results strongly suggest that the eigenvectors' coefficients are uncorrelated and even if doubts were cast about the numerical values of coefficients and their interpretation for one eigenvector this would not cause the interpretations of the other eigenvectors to be changed.

[1] Even this figure is an overestimate because it double counts the correlations within each of the first three eigenvectors.

Table 6.3.3 Standard Deviations (SD), Coefficients of Variation (CV) and Krzanowski Tolerances (KT) for the Second Eigenvectors using DY£ and CY£.

Maturity (years)	DY£ eigen-vector	DY£ SD	DY£ CV (%)	DY£ KT (10%)	CY£ eigen-vector	CY£ SD	CY£ CV (%)	CY£ KT (10%)
2	-0.6927	0.0076	-1.1	0.0699	-0.416	0.0110	-2.65	0.041
4	-0.4090	0.0027	-0.65	0.0730	-0.261	0.0072	-2.76	0.115
6	-0.2454	0.0056	-2.3	0.0711	-0.255	0.0062	-2.43	0.065
8	-0.1241	0.0057	-4.62	0.0642	-0.209	0.0078	-3.73	0.010
10	-0.0159	0.0042	-26.72	0.1075	-0.010	0.0074	-75.54	0.070
12	0.0781	0.0026	3.27	0.0505	0.078	0.0080	10.24	0.111
14	0.1516	0.0018	1.21	0.0008	0.630	0.0112	1.77	0.192
16	0.2005	0.0021	1.04	0.0371	-0.224	0.0095	-4.23	0.015
18	0.2287	0.0025	1.09	0.0587	-0.131	0.0102	-7.77	0.028
20	0.2391	0.0028	1.17	0.0689	0.367	0.0078	2.13	0.045
22	0.2327	0.0031	1.32	0.0701	0.208	0.0096	4.59	0.154
24	0.2101	0.0033	1.59	0.0649	-0.033	0.0121	-36.37	0.192

Note: see table 6.3.1

Table 6.3.4 Standard Deviations (SD), Coefficients of Variation (CV) and Krzanowski Tolerances (KT) for the Second Eigenvectors using BS£ and SS£.

Maturity (years)	BS£ eigen-vector	BS£ SD	BS£ CV (%)	BS£ KT (10%)	SS£ eigen-vector	SS£ SD	SS£ CV (%)	SS£ KT (10%)
2	-0.2586	0.0084	-3.23	0.0370	-0.606	0.0075	-1.24	0.0822
4	-0.2365	0.0036	-1.54	0.0895	-0.411	0.0043	-1.05	0.0913
6	-0.2306	0.0037	-1.59	0.1421	-0.238	0.0048	-2.02	0.0984
8	-0.2233	0.0030	-1.36	0.1932	-0.096	0.0057	-5.96	0.1040
10	-0.2074	0.0022	-1.07	0.0214	0.016	0.0059	36.97	0.1021
12	-0.1752	0.0021	-1.20	0.0221	0.100	0.0054	5.38	0.0861
14	-0.1162	0.0036	-3.09	0.0754	0.163	0.0045	2.77	0.0545
16	-0.0199	0.0057	-28.57	0.1301	0.210	0.0037	1.76	0.0137
18	0.1185	0.0067	5.66	0.1542	0.245	0.0035	1.42	0.0317
20	0.2868	0.0040	1.41	0.0866	0.273	0.0042	1.53	0.0786
22	0.4553	0.0025	0.54	0.0466	0.294	0.0055	1.86	0.1251
24	0.6188	0.0070	1.13	0.1645	0.311	0.0070	2.26	0.1699

Note: See table 6.3.1

Tables 6.3.3 and 6.3.4 tabulate the standard deviations, coefficients of variation and Krzanowski tolerances for the second principal components. They indicate that, although the standard deviations of the coefficient estimates tend to be larger than those of the corresponding first principal component coefficient are, the coefficients are still precisely estimated. Indeed, the coefficient of variation only becomes large when the coefficient itself becomes small at the central maturities of the term structures, although the CY£ data, table 6.3.3, does provided a counter example to this. The Krzanowski tolerances also suggest that, on the whole, a shock to the measurement of the eigenvalues would not change the interpretation of the second principal components. It is noticeable from table 6.3.4 that the shocks applied to the six and eight-year coefficients of the BS£ rates are relatively large. A relatively small shock would result in some deterioration of the acceptability of the second coefficient being the slope interpretation. However, as noted above, it is highly unlikely that there has been a significant mismeasurement of the value of the eigenvalues so the interpretation that the second components, except in the case of the CY£ data, are measures of the slope of the data is very secure. Moreover, as the numerical values are also securely estimated the results strongly suggest that the technique used to estimate spot and yield curves are important in determining the pattern of the eigenvectors.

A further examination of the robustness of the coefficients can be carried out by examination of the coefficient correlations both within and between eigenvectors. These are given in tables 6.A.9 to 6.A.16 in the appendix. In general the coefficients are correlated within the same eigenvectors but not across eigenvectors. This is the same result as found for the first principal component but the results are slightly less decisive. Within components the number of extreme coefficients, greater than 0.9 in absolute terms, has fallen relative to the first principal component coefficients from 19.7% to 14.0%, whilst that of small correlations, between -0.1 to 0.1, has increased from 8.0% to 12.1%. The across coefficient correlations find a very slight opposite movement with small correlations declining as a percentage from 89.5% to 84%, whilst large correlations increased from 0.1% to 0.3%. Overall the conclusion is that the second eigenvectors are likely to deform under measurement error because the coefficients are correlated. However, they will not deform when the measurement error occurs in eigenvectors other than the second ones. Like the results for the first eigenvectors, these results are not strong enough to suggest that the second components may be insecurely estimated.

Table 6.3.5 Standard Deviations (SD), Coefficients of Variation (CV) and Krzanowski Tolerances (KT) for the Third Eigenvectors using DY£ and CY£.

Maturity (years)	DY£ eigen-vector	DY£ SD	DY£ CV (%)	DY£ KT (10%)	CY£ eigen-vector	CY£ SD	CY£ CV (%)	CY£ KT (10%)
2	0.5827	0.0091	1.57	0.0587	-0.434	0.0170	-3.91	0.028
4	-0.1419	0.0063	-4.47	0.0689	-0.074	0.0164	-22.19	0.045
6	-0.4231	0.0039	-0.93	0.0701	-0.057	0.0135	-23.78	0.154
8	-0.4346	0.0023	-0.54	0.0649	-0.071	0.0192	-26.92	0.192
10	-0.3099	0.0023	-0.75	0.0613	0.181	0.0142	7.86	0.080
12	-0.1454	0.0033	-2.30	0.1200	-0.289	0.0110	-3.8	0.080
14	0.0024	0.0040	165.31	0.1328	-0.500	0.0168	-3.36	0.048
16	0.1070	0.0038	3.52	0.0975	-0.039	0.0270	-68.83	0.280
18	0.1692	0.0033	1.97	0.0343	0.072	0.0282	39.2	0.281
20	0.1984	0.0034	1.70	0.0399	0.118	0.0192	16.3	0.041
22	0.2020	0.0042	2.06	0.1121	0.401	0.0100	2.5	0.006
24	0.1871	0.0053	2.82	0.1700	0.500	0.0161	3.22	0.087

Note: See table 6.3.1

Table 6.3.6 Standard Deviations (SD), Coefficients of Variation (CV) and Krzanowski Tolerances (KT) for the Third Eigenvectors using BS£ and SS£.

Maturity (years)	BS£ eigen-vector	BS£ SD	BS£ CV (%)	BS£ KT (10%)	SS£ eigen-vector	SS£ SD	SS£ CV (%)	SS£ KT (10%)
2	0.4713	0.0110	2.32	0.1542	0.503	0.0091	1.80	0.0317
4	0.2180	0.0044	2.01	0.0866	0.105	0.0068	6.48	0.0786
6	0.2080	0.0047	2.28	0.0466	-0.151	0.0049	-3.27	0.1251
8	0.1506	0.0046	3.08	0.1645	-0.277	0.0026	-0.95	0.1699
10	0.0601	0.0042	6.91	0.0732	-0.302	0.0020	-0.66	0.0279
12	-0.0620	0.0033	-5.28	0.0466	-0.255	0.0028	-1.08	0.0716
14	-0.2120	0.0021	-0.99	0.0062	-0.161	0.0034	-2.09	0.0829
16	-0.3657	0.0020	-0.54	0.0440	-0.040	0.0036	-8.95	0.0695
18	-0.4334	0.0036	-0.82	0.0810	0.094	0.0037	3.92	0.0395
20	-0.2436	0.0049	-2.00	0.0568	0.233	0.0038	1.62	0.0005
22	0.1310	0.0068	5.19	0.0100	0.370	0.0041	1.12	0.0457
24	0.4626	0.0093	2.01	0.0437	0.503	0.0049	0.97	0.0928

Note: See table 6.3.1

The third principal components' eigenvectors are, like the first two eigenvectors, securely estimated with small standard deviations (see tables 6.3.5 and 6.3.6) and little evidence of a pattern, except to note that the CY£'s standard deviations are larger than the other data sets. The standard deviations are, in general, larger than for the preceding components and this may be an indication that there is an increase in the noise associated with the estimates, but the deterioration is by no means numerically significant. Analysis of the Krzanowski tolerances do not provide any evidence against the view that the third principal component measures twists in the data. However, the large value for the tolerance on the 24 year maturity of the DY£ data (table 6.3.5, column five), coupled with a decline in the coefficient value, may indicate that the form of this twist might be very slightly different for this data set when compared with the others.

There is some evidence that the correlations between the coefficient estimates are more diffuse than for the first two components. In particular, the number of large coefficients on the within eigenvector comparisons has declined to an average of 9.1% compared with 19.7% for PC1 and 14.0% for PC2. There has been a decline in the percentage of small correlations across components to 82.8% from 89.5% and 84.0% for the first and second components respectively. As in other cases these correlations give little cause to believe that either the numerical sizes or the interpretation of the eigenvectors needs to be reconsidered. Again the results suggest that the differences of the eigenvectors are features of the techniques underlying the estimation of the data and not sampling variability.

In chapter 5 we showed that it is difficult to distinguish between the eigenvectors of spot and yield data. In section 6.2 it was also showed that the principal components scores were, for the first principal components, highly correlated. Consequently, using movements in the principal components scores are unlikely to distinguish between the data. Our statistical analysis, so far in this section, has also shown that the estimates of the eigenvectors are robust. These findings are a concern because the spot rates measure interest rates whilst the yield data does not. If principal components analysis produces similar "reduced" data from initial data known to be different for the first eigenvectors then the information that distinguishes between the data must be contained in the lower order principal components. Consequently, discarding lower order components loses important information on interest rates. Fortunately, by calculating a PCPC model (discussed in section five of chapter 4) we can show that this conclusion is incorrect and that we can distinguish between eigenvectors.

The PCPC analysis uses the log-likelihood statistic to test for the similarity of the eigenvectors and the analysis takes two forms. Firstly, all four data sets are tested for the presence of common eigenvectors over the period 1983 to 1989. Secondly, the data set is split into two groups, the two yield data sets and the two spot rate data sets, and the CPC and PCPC models are re-estimated.

Table 6.3.7 All Four Techniques Compared, 1983 to 1989.

Partials	χ^2	Degrees of freedom	χ^2 5%	χ^2 1%
0 (CPC)	23396.8	198	231.8	247.2
2	23385.9	195	228.6	243.9
3	23125.0	189	222.1	237.2
4	22496.4	180	212.3	227.1
5	21839.2	168	199.2	213.6
6	21371.3	153	182.9	196.6
7	20494.4	135	163.1	176.2
8	19695.9	114	139.9	152.1
9	16445.5	90	113.1	124.1
10	10733.8	63	82.5	92.0
11	5684.6	33	47.4	54.8

Note: χ^2 statistics are calculated from $\chi^2 = \sum_{i=1}^{k} n_i \ln(\frac{\det X_i^{(j)}}{\det X_i})$

Where: k =number of groups, n_i =number of observations per group, $X_i^{(j)}$ is the covariance matrix with j partials and X_i is the covariance matrix of the ith group data.

As table 6.3.7 shows the technique used to construct the yields and spot rates do matter. There are highly statistically significant rejections of the hypothesis that all the eigenvectors are the same (the CPC model) and all of the PCPC models. Again the rejection of the model springs from constraining the eigenvectors with the higher eigenvalues but, unlike the comparison across time, the number of eigenvectors that are important in determining the rejection is about four (i.e. eight partials) compared with two to three across time. One reason for the resounding rejection of common eigenvectors could be that the table is comparing spot and yield variances whereas, by their very nature, these are different. However, table 6.3.8 shows that this is not the case. When the two spot data sets are compared with each other and the two yield data sets are compared, there is still complete rejection of all the common restrictions on the eigenvectors. The test values are much greater for the spot values, but this may be due to the data set underlying the Bank of England's spot curves being different from the spot rates underlying the data set constructed in chapter 3.[2] These results strongly support the conclusion that the eigenvectors from different data sets can be distinguished. The apparently small differences in the eigenvectors' coefficients are, therefore, important.

[2] The Bank's data was collected from its dealers whereas the data used in estimating the models in chapter 3 came from the Broker Mullens.

Table 6.3.8 Spot and Yield Techniques Compared, 1983 to 1989.

Partials	Spot rates	Yields	Degrees of freedom	χ^2 5%	χ^2 1%
0 (CPC)	8913.3	6674.2	66	86.0	95.6
2	8834.6	6643.0	65	84.8	94.4
3	8832.7	6540.2	63	82.5	92.0
4	8795.6	6311.8	60	79.1	88.4
5	8678.3	5769.5	56	74.5	83.5
6	8620.5	5658.0	51	68.7	77.4
7	8438.1	5615.8	45	61.7	70.0
8	7524.4	4760.9	38	53.4	61.2
9	7255.6	3660.1	30	43.8	50.9
10	4282.7	2368.1	21	32.7	38.9
11	2028.5	730.8	11	19.7	24.7

Notes: See table 6.3.7.

6.4 Stability over Time

In chapter 5 no examination was carried out on the stability of the eigenvectors over time. This is rectified in this section. The analysis is in two stages. Firstly, a descriptive approach is adopted that compares and contrasts the eigenvectors for a twelve observation covariance matrix by year. As an analysis of this using the covariances and correlations statistics would generate an inordinate amount of data this is omitted. Secondly, statistical testing of the hypothesis that eigenvectors are not stable over time is carried out by use of PCPC models and log likelihood statistics. Clearly, if the eigenvectors were unstable over time this would mean that care would have to be attached to the interpretation of eigenvectors and their use in forming principal component scores. In particular, using scores that are calculated in one period to analyse data from another period could be misleading.

We begin by splitting the sample into each of its constituent 11 years.[3] For PC1 only the DY£ data produces eigenvectors with the same sign for all the parameters for all 11 yearly sub-samples (see table 6.4.1). For the other three data sets, nine out of 29 sub-samples have at least one change of sign (see tables 6.4.2 to 6.4.4) for the first principal component. Moreover, even for the sub-samples where the eigenvectors are all the same sign, the range of parameter estimates suggests that in many cases the first component can be thought of as the level of interest rates only with difficulty. For example, in the year 1988 the DY£ data produces a parameter on the two-year maturity that is 345 times larger than that on

[3] Ignoring the year 1990 for which only partial data is available for the CY£ data set.

the 20-year maturity (see table 6.4.1). On the whole, however, the DY£ data puts forward the strongest case for PC1 measuring the level of the data. Five out of the seven sub-samples for the SS£ data (table 6.4.4) and six out of eleven sub-samples for the BS£ data (table 6.4.3) can be thought of as measures of the slope of the data. Unfortunately, these same sub-samples usually produce PC2s (tables 6.4.8 and 6.4.7) that can also be interpreted as slope coefficients. Consequently, four of the SS£ sub-samples and five of the BS£ sub-samples appear to have two slope measures. The CY£ data, on the other hand, (table 6.4.2) produces highly variable parameter estimates for PC1, none of which can be interpreted as a measure of the slope whilst the sample for 1979 to 1983 and 1984 could be loosely interpreted as a measure of the level.

Table 6.4.1 Loadings on the First Principal Components by Year Using DY£

year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
maturity											
2	0.337	0.413	0.340	0.360	0.240	0.362	0.171	0.223	0.301	0.690	0.540
4	0.363	0.372	0.331	0.325	0.176	0.292	0.327	0.393	0.464	0.463	0.386
6	0.336	0.332	0.324	0.303	0.192	0.323	0.387	0.447	0.456	0.376	0.323
8	0.311	0.297	0.314	0.289	0.234	0.332	0.381	0.413	0.414	0.306	0.294
10	0.291	0.268	0.301	0.283	0.279	0.319	0.347	0.339	0.353	0.224	0.256
12	0.275	0.247	0.287	0.280	0.311	0.301	0.314	0.266	0.275	0.138	0.210
14	0.263	0.238	0.275	0.277	0.326	0.282	0.286	0.214	0.198	0.067	0.177
16	0.256	0.240	0.266	0.274	0.331	0.266	0.263	0.186	0.143	0.024	0.170
18	0.251	0.244	0.260	0.270	0.331	0.252	0.244	0.178	0.111	0.004	0.177
20	0.249	0.247	0.254	0.266	0.330	0.241	0.228	0.186	0.101	0.002	0.201
22	0.249	0.249	0.249	0.262	0.327	0.233	0.216	0.205	0.112	0.011	0.233
24	0.250	0.250	0.243	0.257	0.323	0.226	0.205	0.229	0.142	0.023	0.270

Note: Estimated using covariance matrices of 12 observations, using daily data. Maturity is in years.

Table 6.4.2 Loadings on the First Principal Components by Year Using CY£

year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
maturity											
2	0.390	0.357	0.370	0.385	0.265	0.437	0.186	0.201	0.170	0.763	0.287
4	0.320	0.352	0.123	0.252	-0.086	0.358	-0.363	0.325	0.182	0.092	0.158
6	0.302	0.257	0.314	0.268	0.390	0.359	-0.368	0.404	0.094	0.223	0.133
8	0.301	0.097	0.023	0.254	0.179	0.255	-0.278	0.328	0.209	0.293	-0.166
10	0.248	0.224	0.295	0.311	0.203	0.350	-0.425	0.320	-0.222	0.067	0.359
12	0.317	0.315	0.330	0.276	0.417	0.227	-0.249	0.313	0.384	0.320	-0.106
14	0.274	0.181	0.189	0.329	0.034	0.201	-0.078	0.243	0.065	0.168	-0.425
16	0.225	0.428	0.392	0.350	0.016	0.243	-0.242	0.137	0.134	-0.298	0.533
18	0.349	0.374	0.438	0.287	0.222	0.210	-0.281	-0.412	0.218	-0.079	0.396
20	0.069	0.155	0.318	0.264	0.299	0.229	-0.342	0.280	0.103	-0.034	-0.136
22	0.304	0.295	0.265	0.268	0.481	0.294	-0.283	0.181	-0.494	-0.193	0.094
24	0.239	0.241	0.033	0.158	0.389	0.175	-0.196	0.156	-0.604	-0.078	0.243

Note: See table 6.4.1

Table 6.4.3 Loadings on the First Principal Components by Year using BS£

year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
maturity											
2	0.165	0.177	0.317	0.290	0.178	0.353	0.355	0.486	0.565	0.750	0.637
4	0.193	0.105	0.322	0.288	0.254	0.289	0.369	0.359	0.341	0.226	0.184
6	0.215	0.119	0.346	0.307	0.273	0.297	0.367	0.346	0.335	0.194	0.191
8	0.213	0.136	0.367	0.324	0.290	0.306	0.363	0.336	0.326	0.181	0.200
10	0.199	0.157	0.380	0.338	0.306	0.313	0.354	0.323	0.312	0.177	0.209
12	0.175	0.183	0.378	0.344	0.318	0.316	0.338	0.305	0.292	0.176	0.219
14	0.146	0.216	0.350	0.338	0.327	0.314	0.310	0.280	0.262	0.179	0.229
16	0.126	0.256	0.286	0.315	0.330	0.303	0.267	0.245	0.223	0.185	0.239
18	0.150	0.306	0.194	0.275	0.324	0.283	0.207	0.199	0.172	0.193	0.248
20	0.276	0.365	0.101	0.227	0.306	0.253	0.129	0.144	0.111	0.204	0.258
22	0.467	0.442	0.045	0.186	0.277	0.216	0.036	0.081	0.043	0.221	0.270
24	0.648	0.577	0.037	0.165	0.241	0.180	-0.060	0.017	-0.024	0.244	0.287

Note: See table 6.4.1

Table 6.4.4 Loadings on the First Principal Components by Year using SS£

year	1983	1984	1985	1986	1987	1988	1989
maturity							
2	0.110	0.460	0.389	0.450	0.457	0.662	0.592
4	0.082	0.451	0.447	0.445	0.423	0.504	0.319
6	0.103	0.414	0.436	0.411	0.375	0.383	0.254
8	0.146	0.366	0.392	0.364	0.331	0.269	0.245
10	0.195	0.314	0.339	0.315	0.293	0.169	0.246
12	0.245	0.263	0.284	0.267	0.261	0.086	0.246
14	0.290	0.217	0.230	0.224	0.234	0.020	0.243
16	0.330	0.174	0.179	0.183	0.210	-0.032	0.237
18	0.365	0.135	0.130	0.147	0.190	-0.072	0.229
20	0.395	0.101	0.084	0.114	0.171	-0.102	0.220
22	0.420	0.071	0.039	0.083	0.155	-0.125	0.212
24	0.440	0.045	-0.003	0.055	0.139	-0.142	0.203

Note: See table 6.4.1

The PC2 and PC3 from the CY£ data appear to be almost random rather than representing a slope or a "twist" in the data (see tables 6.4.6 and 6.4.10). For the DY£ data (table 6.4.5) PC2 seems to represent the slope for the years 1979 to 1984, but to represent a twist for the years 1985 to 1989. For these years neither PC1 nor PC3 seem to represent the slope.

Table 6.4.5 Loadings on the Second Principal Components by Year using DY£

year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
maturity											
2	-0.767	-0.649	-0.491	-0.833	-0.595	-0.738	-0.928	-0.888	-0.502	-0.558	-0.217
4	-0.336	-0.377	-0.462	-0.272	-0.484	-0.381	0.279	-0.069	0.127	0.080	0.090
6	-0.086	-0.139	-0.280	-0.005	-0.336	-0.116	0.212	0.247	0.313	0.280	0.326
8	0.063	0.033	-0.115	0.109	-0.204	0.050	0.091	0.271	0.209	0.342	0.430
10	0.140	0.151	0.009	0.149	-0.093	0.134	0.016	0.175	0.088	0.322	0.377
12	0.175	0.222	0.106	0.155	-0.005	0.174	-0.026	0.080	0.050	0.263	0.237
14	0.188	0.255	0.188	0.159	0.063	0.194	-0.047	-0.011	0.013	0.203	0.075
16	0.192	0.264	0.257	0.166	0.119	0.204	-0.051	-0.075	-0.077	0.173	-0.089
18	0.195	0.258	0.298	0.171	0.170	0.209	-0.043	-0.106	-0.204	0.175	-0.226
20	0.199	0.241	0.308	0.174	0.217	0.208	-0.028	-0.107	-0.335	0.204	-0.332
22	0.205	0.218	0.295	0.175	0.259	0.204	-0.013	-0.079	-0.438	0.255	-0.385
24	0.214	0.193	0.265	0.175	0.296	0.197	-0.001	-0.027	-0.478	0.325	-0.363

Note: See table 6.4.1

Table 6.4.6 Loadings on the Second Principal Components by Year using CY£

year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
maturity											
2	-0.182	-0.287	-0.042	-0.506	0.037	-0.678	0.917	-0.499	-0.236	-0.072	-0.035
4	-0.215	0.033	-0.222	0.570	0.746	0.040	-0.041	0.024	-0.291	0.610	0.006
6	0.378	0.156	0.456	0.081	-0.282	0.128	-0.092	-0.096	-0.250	-0.009	0.144
8	-0.203	0.597	-0.040	0.537	-0.050	0.295	0.117	0.005	-0.263	-0.005	-0.513
10	0.321	0.175	0.171	-0.208	0.181	0.066	0.256	0.215	-0.461	-0.215	0.302
12	0.142	0.045	-0.257	0.107	0.021	0.128	-0.149	-0.159	-0.123	0.282	0.085
14	-0.095	-0.018	-0.574	0.121	0.402	0.462	0.024	0.600	-0.292	0.458	0.715
16	0.181	0.027	0.491	-0.108	0.335	0.087	0.092	0.188	-0.425	0.176	0.190
18	-0.112	-0.451	-0.141	-0.087	0.194	0.036	0.151	0.038	-0.267	0.032	0.086
20	0.716	0.545	-0.165	-0.066	0.120	0.070	0.100	0.235	-0.204	-0.137	-0.089
22	-0.041	0.019	-0.119	-0.163	0.037	-0.289	0.068	-0.324	-0.244	0.413	-0.223
24	-0.212	0.041	-0.108	0.072	0.029	0.323	-0.027	-0.335	-0.251	0.259	-0.070

Note: See table 6.4.1

Table 6.4.7 Loadings on the Second Principal Components by Year using BS£

year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
maturity											
2	-0.300	-0.372	-0.295	-0.012	-0.318	-0.314	-0.118	-0.293	-0.044	-0.309	-0.267
4	-0.212	-0.295	-0.150	-0.277	-0.333	-0.289	-0.164	-0.167	-0.190	0.399	-0.344
6	-0.204	-0.306	-0.134	-0.266	-0.291	-0.243	-0.114	-0.114	-0.139	0.390	-0.286
8	-0.222	-0.312	-0.110	-0.247	-0.249	-0.197	-0.071	-0.066	-0.092	0.366	-0.236
10	-0.254	-0.310	-0.068	-0.211	-0.198	-0.141	-0.024	-0.012	-0.039	0.331	-0.182
12	-0.294	-0.295	-0.001	-0.145	-0.134	-0.072	0.031	0.050	0.025	0.277	-0.117
14	-0.334	-0.259	0.100	-0.037	-0.050	0.014	0.098	0.123	0.101	0.209	-0.040
16	-0.349	-0.192	0.229	0.113	0.054	0.118	0.182	0.209	0.191	0.119	0.054
18	-0.286	-0.084	0.359	0.286	0.175	0.236	0.280	0.303	0.293	0.011	0.164
20	-0.061	0.069	0.453	0.431	0.306	0.357	0.392	0.400	0.403	-0.114	0.290
22	0.249	0.256	0.484	0.491	0.430	0.464	0.513	0.491	0.513	-0.248	0.428
24	0.493	0.469	0.474	0.451	0.520	0.532	0.634	0.562	0.612	-0.374	0.573

Note: See table 6.4.1

Table 6.4.8 Loadings on the Second Principal Components by Year using SS£

year	1983	1984	1985	1986	1987	1988	1989
maturity							
2	-0.326	-0.578	-0.099	-0.260	-0.056	-0.024	-0.214
4	-0.307	-0.257	-0.051	-0.230	-0.149	0.044	-0.190
6	-0.353	-0.032	0.066	-0.157	-0.186	0.111	-0.214
8	-0.373	0.118	0.138	-0.061	-0.196	0.187	-0.227
10	-0.350	0.213	0.145	0.038	-0.166	0.253	-0.194
12	-0.289	0.268	0.097	0.131	-0.099	0.303	-0.119
14	-0.199	0.294	0.009	0.213	-0.003	0.337	-0.015
16	-0.090	0.302	-0.107	0.284	0.111	0.358	0.105
18	0.032	0.296	-0.239	0.345	0.238	0.370	0.231
20	0.159	0.281	-0.380	0.398	0.370	0.374	0.357
22	0.288	0.261	-0.524	0.443	0.504	0.373	0.477
24	0.416	0.236	-0.668	0.483	0.636	0.370	0.591

Note: See table 6.4.1

For the third principal component (with the exception of the CY£ data) the remaining data sets (tables 6.4.9, 6.4.11 and 6.4.12) can usually be thought of as supporting twists but the nature of these is highly variable. The maturity of the minimum value of the parameter for the DY£ is between four and 18 years depending upon the data set. For 1988 the third principal component of the BS£ data (tables 6.4.11) seems to represent the slope of the term structure rather than a twist. Hence, the second and the third principal components seem to have exchanged meaning in this year and for this particular data set.

Table 6.4.9 Loadings on the Third Principal Components by Year using DY£

year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
maturity											
2	0.463	0.451	0.689	0.363	0.734	0.478	-0.329	0.196	0.522	0.318	0.683
4	-0.403	-0.178	-0.039	-0.554	-0.236	-0.295	-0.567	0.451	0.109	-0.110	0.195
6	-0.400	-0.420	-0.361	-0.512	-0.443	-0.554	-0.384	0.318	-0.094	-0.122	-0.140
8	-0.330	-0.442	-0.426	-0.292	-0.371	-0.413	-0.127	0.139	-0.253	-0.193	-0.277
10	-0.221	-0.335	-0.323	-0.079	-0.210	-0.141	0.109	-0.064	-0.209	-0.227	-0.216
12	-0.083	-0.159	-0.146	0.089	-0.063	0.065	0.251	-0.262	0.031	-0.192	-0.053
14	0.056	0.023	0.012	0.186	0.011	0.173	0.312	-0.375	0.273	-0.094	0.038
16	0.160	0.169	0.107	0.224	0.043	0.217	0.315	-0.405	0.321	0.063	0.000
18	0.227	0.253	0.151	0.222	0.062	0.218	0.274	-0.370	0.191	0.241	-0.130
20	0.263	0.270	0.156	0.192	0.074	0.187	0.206	-0.290	-0.045	0.402	-0.267
22	0.275	0.240	0.135	0.145	0.081	0.135	0.125	-0.185	-0.311	0.505	-0.361
24	0.272	0.181	0.099	0.091	0.083	0.073	0.046	-0.073	-0.534	0.509	-0.359

Note: See table 6.4.1

Table 6.4.10 Loadings on the Third Principal Components by Year using CY£

year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
maturity											
2	0.166	0.242	0.169	-0.573	-0.186	0.554	-0.197	-0.340	0.006	-0.208	0.307
4	-0.637	0.741	0.399	-0.106	-0.302	-0.365	0.292	0.033	0.070	0.084	0.325
6	0.260	0.020	0.177	-0.166	-0.427	-0.193	-0.436	-0.071	-0.216	0.066	0.005
8	0.065	0.203	0.541	-0.039	0.459	0.175	-0.187	0.127	0.041	-0.090	0.511
10	0.255	-0.276	0.096	0.161	-0.185	-0.137	0.509	-0.604	0.186	0.219	-0.142
12	0.285	0.053	-0.079	-0.012	-0.384	0.218	-0.449	0.041	0.235	0.182	0.456
14	-0.085	-0.130	0.010	-0.223	0.347	0.244	-0.337	-0.285	-0.280	-0.136	0.172
16	-0.340	-0.282	-0.017	0.671	-0.065	-0.094	-0.221	0.026	-0.587	-0.718	0.198
18	0.122	-0.287	-0.496	0.219	0.107	-0.010	0.068	-0.441	0.611	0.403	0.013
20	-0.402	-0.280	-0.079	0.052	0.179	-0.059	-0.047	0.457	0.160	0.273	-0.294
22	0.075	-0.011	0.192	0.161	0.328	-0.515	0.054	-0.101	-0.023	0.254	-0.394
24	-0.210	-0.120	0.429	-0.170	0.154	0.296	0.137	-0.006	0.190	0.146	0.014

Note: See table 6.4.1

Table 6.4.11 Loadings on the Third Principal Components by Year using BS£

year	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
maturity											
2	0.303	0.855	0.891	0.950	0.918	0.874	0.927	0.822	0.824	0.561	0.722
4	0.328	-0.022	-0.135	-0.048	-0.099	-0.189	-0.160	-0.228	-0.248	0.114	-0.242
6	0.298	-0.050	-0.142	-0.062	-0.108	-0.178	-0.165	-0.233	-0.240	0.019	-0.260
8	0.242	-0.077	-0.141	-0.080	-0.115	-0.171	-0.157	-0.225	-0.232	-0.044	-0.267
10	0.163	-0.104	-0.134	-0.098	-0.118	-0.165	-0.148	-0.208	-0.219	-0.097	-0.264
12	0.052	-0.136	-0.122	-0.120	-0.118	-0.155	-0.128	-0.177	-0.198	-0.149	-0.253
14	-0.106	-0.169	-0.106	-0.140	-0.113	-0.136	-0.105	-0.135	-0.173	-0.202	-0.236
16	-0.317	-0.200	-0.081	-0.142	-0.098	-0.114	-0.069	-0.083	-0.137	-0.256	-0.207
18	-0.521	-0.215	-0.026	-0.123	-0.061	-0.077	-0.028	-0.019	-0.095	-0.310	-0.172
20	-0.465	-0.187	0.073	-0.073	0.005	-0.015	0.017	0.054	-0.048	-0.355	-0.126
22	-0.137	-0.067	0.193	-0.007	0.099	0.077	0.057	0.133	-0.004	-0.389	-0.072
24	0.086	0.277	0.235	0.010	0.242	0.209	0.076	0.215	0.039	-0.399	-0.014

Note: See table 6.4.1

Table 6.4.12 Loadings on the Third Principal Components by Year using SS£

year	1983	1984	1985	1986	1987	1988	1989
maturity							
2	0.612	0.128	0.835	0.660	0.636	0.403	0.266
4	0.507	0.198	0.158	0.176	0.299	0.158	-0.569
6	0.171	0.046	-0.180	-0.159	0.008	-0.117	-0.495
8	-0.099	-0.130	-0.274	-0.301	-0.191	-0.293	-0.159
10	-0.246	-0.243	-0.255	-0.327	-0.304	-0.357	0.117
12	-0.290	-0.270	-0.197	-0.288	-0.347	-0.332	0.264
14	-0.259	-0.221	-0.137	-0.212	-0.334	-0.247	0.297
16	-0.183	-0.108	-0.093	-0.115	-0.282	-0.122	0.249
18	-0.084	0.052	-0.071	-0.008	-0.203	0.025	0.151
20	0.026	0.243	-0.072	0.104	-0.105	0.182	0.025
22	0.136	0.456	-0.096	0.219	0.004	0.341	-0.114
24	0.241	0.681	-0.140	0.333	0.118	0.499	-0.255

Note: See table 6.4.1

We continue this section by using PCPC to study the stability of principal components between the sub-periods to gain a statistical answer to complement the observations derived from tables 6.4.1 to 6.4.12. A CPC model is estimated and the eigenvectors are ordered such that they correspond with the size of the eigenvalues from the first data set analysed. The log-likelihood statistic is calculated and the procedure is repeated dropping, in turn, the restriction that the eigenvectors are equivalent on the eigenvectors with the smallest eigenvalues until only one common eigenvector remains.

The results of this procedure, reported in table 6.4.13, are clear cut. In no instance can the hypothesis that the model contains no (large eigenvalue) common eigenvectors be rejected. The evidence confirms the impression from tables 6.4.1 to 6.4.12 that the eigenvectors are not stable over time. Even with only one common eigenvector the smallest χ^2 is greater than the 1% critical value by a factor of almost 11.7 times for BS£ and by 45.3 times for CY£. Of course, this does not necessarily mean there are no common eigenvectors some of those associated with low eigenvalues may be common but this has been disguised by the imposition of common eigenvectors for the larger eigenvalues. This has not been tested for, however, because eigenvectors associated with the small eigenvalues have little influence in explaining the covariances of the samples.

Table 6.4.13 Log Likelihood Statistics for CPC and PCPC Models, 1979-1989

Partials	CY£	BS£	DY£	Degrees of freedom	χ^2 5%	χ^2 1%
0 (CPC)	32920.9	6487.5	11271.2	660	720.9	747.5
2	32624.8	6482.6	11115.8	650	710.4	736.8
3	31780.0	6478.3	10926.3	630	689.5	715.5
4	30396.7	6459.5	10684.6	600	658.1	683.5
5	29076.1	6435.9	10292.3	560	616.2	640.8
6	26988.6	6398.1	9451.8	510	563.6	587.2
7	24964.6	6343.1	9024.0	450	500.5	522.7
8	21280.5	6111.1	7085.5	380	426.5	447.1
9	17402.3	5702.5	5518.4	300	341.4	359.9
10	11993.8	4995.7	4632.2	210	244.8	260.6
11	6681.2	1723.4	3347.3	110	135.5	147.4

Note: See table 6.3.7

Table 6.4.14 Log Likelihood Statistics for CPC and PCPC Models for SS£ Data, 1983-1989

Partials	SS£	Degrees of freedom	χ^2 5%	χ^2 1%
0 (CPC)	2175.9	396	464.4	443.4
2	2119.4	390	457.9	437.1
3	2106.3	378	444.9	424.3
4	2096.7	360	425.4	405.2
5	2087.6	336	399.2	379.8
6	2070.0	306	336.5	347.8
7	2057.0	270	327.0	309.3
8	2020.4	228	280.6	244.2
9	1921.5	180	227.1	212.3
10	1776.1	126	165.9	153.2
11	1078.0	66	95.6	86.0

Note: See table 6.3.7

As the Bank of England data is only collected from 1982, the first full year of data is from 1983. This means that the degrees of freedom are different from the other three data sets and this is set out in a separate table, (table 6.4.14). The results are similar to those noted before; there is strong rejection of the CPC and all of the PCPC models. Hence, it is concluded that the covariance matrices of the Bank of England's spot curves are not stable over time. What is noticeable is that the rate of increase in the χ^2 statistic is greatest when the models have common eigenvectors imposed on the eigenvectors with the largest eigenvalues. Moving from a CPC model to a model with two or three specific eigenvectors results in only a small change in the χ^2 statistics but moving from ten to eleven specific eigenvectors (two to one common eigenvectors) produces much greater change in the χ^2 statistics. Clearly, tying down the eigenvectors that have small eigenvalues has little affect on the rejection of the CPC model.

These results suggest that the eigenvectors are not stable over time and that the decomposition of the term structure into identifiable level, slope and twist terms is heavily dependent upon the period chosen. For small sub-samples of one year this decomposition often breaks down. This result is consistent with that reported by Wilson (1994). One reason for this result could be that the factors driving the level of interest rates tend to be relatively slow moving so that over the short time period of one year there is a greater tendency for stochastic influences to dominate the variance of interest rates. Evidence could be derived for this by noting that for protracted periods monetary authorities leave short-term interest rates unchanged. For example, in 1980 there were two changes in base rates, whereas in 1988 there were 13. Thus the volatility of the driving factors may change and this, in turn, would influence whether the level or the slope contributed the most of the variation to term structures. Fuller examinations of the variables that may influence the principal components are given in chapters 7 and 8.

6.5 An Application Across Countries

As a final analysis using common components we apply the technique to the Bank of England's spot data and the US data as estimated by McCulloch and Kwon (1993). The US was chosen as it has been found to be the dominant country in interest rate setting in a number of studies, for example Pain and Thomas (1997). As McCulloch and Kwon's data set is only month end this reduces the number of observations. Although both data sets would allow spot rates at all annual maturities between two years and 24 years to be investigated, in order to maintain consistency with analysis in the previous sections, only the data sets using 12 maturities between two and 24 years have been used. The data period used is March 1982 to February 1991. The start date was determined by the start of the Bank of England series and the end date by the finish of the McCulloch and Kwon data. To test whether the results are sensitive to the data period the samples are split at August 1986, which was chosen as the simple mid-point of the sample.

Table 6.5.1 Comparison of US and UK Spot Rates

Partials	1982-3 to 1991-2	1982-3 to 1986-8	1986-9 to 1991-2	Degrees of freedom	χ^2 5%	χ^2 1%
0 (CPC)	376.5	241.6	182.6	66	86.0	95.6
2	376.4	241.3	172.7	65	84.8	94.4
3	370.4	239.6	172.3	63	82.5	92.0
4	362.3	238.9	171.0	60	79.1	88.4
5	359.9	231.0	166.9	56	74.5	83.5
6	331.6	221.6	165.2	51	68.7	77.4
7	315.1	210.9	161.0	45	61.7	70.0
8	283.9	186.3	153.5	38	53.4	61.2
9	198.6	173.8	142.5	30	43.8	50.9
10	168.0	105.8	115.2	21	32.7	38.9
11	161.0	57.9	43.3	11	19.7	24.7

Notes: See table 6.3.7

As can be seen from table 6.5.1, the data rejects the hypotheses that the eigenvectors of the US and UK spot rates are the same. Moreover, this is true for both the earlier period and the later period of the sample. We conclude that there is little indication from these results that the variances underlying the

US and UK government term structures are becoming more similar. The only glimmer of evidence for this is that the size of the χ^2 falls, with one exception, for each of the comparisons between the earlier and later sub-periods so that the degree of rejection is less. It would be useful to extend this analysis to the later part of the 1990s to see whether or not the reduction in the statistics has continued. Of course, these results make no allowances for changes in exchange rates that may make the covariances of the term structures between the US and the UK more similar. Alternatively, analysis of real spot rates, if exchange rates move in line with expected relative prices, might also over turn the results found above. Such adaptations would be another avenue for future research. Finally, it is worth noting that although the degrees of freedom are the same between tables 6.5.1 and 6.5.2, the χ^2 statistics are much larger for data that used only UK data. Undoubtedly, the different sample sizes (2190 against 89) play a large role in this finding but the different techniques used to estimate the US and UK data may account for part of the rejection of the common eigenvectors hypothesis. Further work should also attempt to construct term structures using the same techniques before applying tests for common eigenvectors.

6.6 Conclusions

This chapter has examined the robustness of principal components to changes in the technique used to estimate the term structure, changes in the sample period and by comparing term structures between the US and the UK. This chapter extends the work set out in chapter 5 in which the qualitative similarities of the eigenvector representation of the term structure covariances were established. Analyses of the correlations of the principal component scores suggested that the first principal components were similar across all four UK spot and yield data sets. The other components had low correlations, and the analysis suggested that the movement in the second principal component for the BS£ scores were more closely associated with movements in the third principal component scores than it was with the scores of the other second principal components.

The calculated standard deviations of the eigenvectors and the Krzanowski tolerances, for plausible variability in the eigenvalues, also suggest that the variations in the eigenvectors' coefficients are not due to sampling errors. Fortunately, the use of partial common principal components analysis casts considerable doubt upon the similarity of the major eigenvectors, and this appears to be the first time that this technique has been used on term structure data. The results show that the choice of estimation technique, the sample used (a result supported by examination of the eigenvectors calculated for each year) and the country studied can all result in significant differences in the eigenvectors. The results also suggested that there had been a slight reduction in the degree of rejection of the PCPC models between the early 1980s and the later 1980s for a comparison of the UK and US term structures. Further results to see whether this has continued in the 1990s would be of interest. However, the results indicate that definitive results are unlikely to be attained until the technique used to estimate the term structures of different countries has been standardised.

Appendix 6.1 Across and Within Component Correlations for UK Term Structures

Table 6.A.1 Within Component Correlations for the First Component using DY£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	7.58	9.09	12.12	7.58	1.52	7.58	0.00	3.03	1.52	1.52
Number	10	12	16	10	2	10	0	4	2	2
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	1.52	6.06	1.52	1.52	1.52	4.5	4.55	4.55	9.09	13.64
Number	2	8	2	2	2	6	6	6	12	18

Note: Correlations calculated using Girshick's (1939) equation. See equation (4.4.3) of chapter 4. The underlying eigenvectors were calculated from a covariance matrix of 12 observations from the daily sample 2 January 1979 to 21 August 1990. Total number of correlations = 132.

Table 6.A.2 Within Component Correlations for the First Component using CY£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0	3.03	3.03	12.12	12.12	7.58	13.64	9.09
Number	0	0	0	4	4	16	16	10	18	12
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	10.61	4.55	4.55	12.12	6.06	1.52	0	0	0	0
Number	14	6	6	16	8	2	0	0	0	0

Note: See table 6.A.1.

Table 6.A.3 Within Component Correlations for the First Component using BS£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	12.12	9.09	7.58	6.06	1.52	3.03	7.58	3.03	0	1.52
Number	16	12	10	8	2	4	10	4	0	2
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	3.03	0	3.03	7.58	1.52	3.03	6.06	4.55	7.58	12.12
Number	4	0	4	10	2	4	8	6	10	16

Note: See table 6.A.1.

Table 6.A.4 Within Component Correlations for the First Component using SS£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	16.67	10.61	7.58	3.03	1.52	3.03	3.03	1.52	1.52	1.52
Number	22	14	10	4	2	4	4	2	2	2
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	3.03	1.52	1.52	3.03	3.03	4.55	3.03	6.06	7.58	16.67
Number	4	2	2	4	4	6	4	8	10	22

Note: Correlations calculated using Girshick's (1939) equation. See equation (4.4.3) of chapter 4. The underlying eigenvectors were calculated from a covariance matrix of 12 observations from the daily sample 31 March 1982 to 21 August 1990. Total number of correlations = 132.

Table 6.A.5 Across Component Correlations for the First Component using DY£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0.00	0.00	0.00	0.00	0.00	0.51	0.69	2.02	1.52	48.30
Number	0	0	0	0	0	8	11	32	24	765
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	43.12	2.21	0.88	0.51	0.25	0.00	0.00	0.00	0.00	0.00
Number	683	35	14	8	4	0	0	0	0	0

Note: Correlations calculated using Girshick's (1939) equation. See equation (4.4.5) of chapter 4. The underlying eigenvectors were calculated from a covariance matrix of 12 observations from the daily sample 2 January 1979 to 21 August 1990. Total number of correlations = 1584

Table 6.A.6 Across Component Correlations for the First Component using CY£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0	0	0	0	0	0	1.83	46.65
Number	0	0	0	0	0	0	0	0	29	739
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	49.05	2.46	0	0	0	0	0	0	0	0
Number	777	39	0	0	0	0	0	0	0	0

Note: See table 6.A.5.

Table 6.A.7 Across Component Correlations for the First Component using BS£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0.25	0.13	0.19	0.38	0.76	0.69	1.77	48.86
Number	0	0	4	2	3	6	12	11	28	774
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	39.65	2.4	1.39	1.45	1.01	0.44	0.25	0.38	0	0
Number	628	38	22	23	16	7	4	6	0	0

Note: See table 6.A.5.

Table 6.A.8 Across Component Correlations for the First Component using SS£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0.32	0.95	1.2	1.01	0.44	0.57	0.69	0.95	1.7	43.69
Number	5	15	19	16	7	9	11	15	27	692
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	42.49	1.14	1.14	0.57	0.82	0.38	0.51	0.44	0.76	0.25
Number	673	18	18	9	13	6	8	7	12	4

Note: Correlations calculated using Girshick's (1939) equation. See equation (4.4.5) of chapter 4. The underlying eigenvectors were calculated from a covariance matrix of 12 observations from the daily sample 31 March 1982 to 21 August 1990. Total number of correlations = 1584

Table 6.A.9 Within Component Correlations for the Second Component using DY£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	3.03	7.58	12.12	9.09	1.52	6.06	4.55	1.52	1.52	4.55
Number	4	10	16	12	2	8	6	2	2	6
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	3.03	3.03	1.52	1.52	3.03	3.03	6.06	9.09	6.06	12.12
Number	4	4	2	2	4	4	8	12	8	16

Note: See table 6.A.1.

Table 6.A.10 Within Component Correlations for the Second Component using CY£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0	4.55	7.58	1.52	6.06	13.64	12.12	12.12
Number	0	0	0	6	10	2	8	18	16	16
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	12.12	12.12	4.55	6.06	0	3.03	1.52	3.03	0	0
Number	16	16	6	8	0	4	2	4	0	0

Note: See table 6.A.1.

Table 6.A.11 Within Component Correlations for the Second Component using BS£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	4.55	7.58	10.61	9.09	4.55	1.52	4.55	1.52	1.52
Number	0	6	10	14	12	6	2	6	2	2
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	3.03	3.03	1.52	0	1.52	6.06	6.06	6.06	6.06	21.21
Number	4	4	2	0	2	8	8	8	8	28

Note: See table 6.A.1.

Table 6.A.12 Within Component Correlations for the Second Component using SS£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0	0	0	1.52	3.03	9.09	1.52	6.06
Number	0	0	0	0	0	2	4	12	2	8
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	6.06	1.52	4.55	3.03	6.06	7.58	4.55	9.09	16.67	19.7
Number	8	2	6	4	8	10	6	12	22	26

Note: See table 6.A.4.

Table 6.A.13 Across Component Correlations for the Second Component using DY£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0.19	0.19	0.44	0.57	0.57	1.52	1.26	2.34	1.96	42.11
Number	3	3	7	9	9	24	20	37	31	667
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	41.92	1.52	1.26	0.88	1.20	0.57	0.88	0.19	0.38	0.06
Number	664	24	20	14	19	9	14	3	6	1

Note: See table 6.A.5.

Table 6.A.14 Across Component Correlations for the Second Component using CY£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0	0.13	0	0.13	0.51	0.51	3.16	44.07
Number	0	0	0	2	0	2	8	8	50	698
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	46.59	3.85	0.32	0.44	0.13	0.06	0.13	0	0	0
Number	738	61	5	7	2	1	2	0	0	0

Note: See table 6.A.5.

Table 6.A.15 Across Component Correlations for the Second Component using BS£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0.13	0.44	1.14	1.33	0.19	0.63	1.83	1.14	2.08	38.07
Number	2	7	18	21	3	10	29	18	33	603
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	40.97	2.59	2.34	1.83	1.2	1.26	1.39	0.82	0.38	0.25
Number	649	41	37	29	19	20	22	13	6	4

Note: See table 6.A.5.

Table 6.A.16 Across Component Correlations for the Second Component using SS£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0.32	1.39	1.58	1.39	1.01	1.01	1.2	1.14	1.14	41.35
Number	5	22	25	22	16	16	19	18	18	655
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	41.1	0.88	0.82	0.95	1.2	1.14	0.76	0.63	0.76	0.25
Number	651	14	13	15	19	18	12	10	12	4

Note: See table 6.A.8.

Table 6.A.17 Within Component Correlations for the Third Component using DY£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	1.52	3.03	10.61	4.55	9.09	6.06	10.61	7.58	3.03	4.5
Number	2	4	14	6	12	8	14	10	4	6
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	1.52	3.03	3.03	3.03	6.06	0.00	4.55	7.58	6.06	4.55
Number	2	4	4	4	8	0	6	10	8	6

Note: See table 6.A.1.

Table 6.A.18 Within Component Correlations for the Third Component using CY£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	1.52	1.52	7.58	12.12	10.61	9.09	4.55	13.64
Number	0	0	2	2	10	16	14	12	6	18
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	4.55	10.61	9.09	7.58	0	3.03	4.55	0	0	0
Number	6	14	12	10	0	4	6	0	0	0

Note: See table 6.A.1.

Table 6.A.19 Within Component Correlations for the Third Component using BS£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	12.12	9.09	9.09	4.55	1.52	12.12	6.06	3.03	0
Number	0	16	12	12	6	2	16	8	4	0
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	0	1.52	3.03	3.03	1.52	0	4.55	6.06	6.06	16.67
Number	0	2	4	4	2	0	6	8	8	22

Note: See table 6.A.1.

Table 6.A.20 Within Component Correlations for the Third Component using SS£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	3.03	0	6.06	9.09	4.55	4.55	4.55	3.03	6.06	3.03
Number	4	0	8	12	6	6	6	4	8	4
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	1.52	1.52	7.58	4.55	7.58	1.52	7.58	3.03	10.61	10.61
Number	2	2	10	6	10	2	10	4	14	14

Note: See table 6.A.4.

Table 6.A.21 Across Component Correlations for the Third Component using DY£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0.19	0.19	0.44	0.57	0.57	1.26	0.95	1.01	1.83	42.99
Number	3	3	7	9	9	20	15	16	29	681
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	42.30	2.59	1.07	0.82	1.14	0.57	0.88	0.19	0.38	0.06
Number	670	41	17	13	18	9	14	3	6	1

Note: See table 6.A.5.

Table 6.A.22 Across Component Correlations for the Third Component using CY£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0	0	0.13	0.19	0.63	0.82	1.14	3.41	43.06
Number	0	0	0	2	3	10	13	18	54	682
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	44.19	3.16	1.33	1.14	0.51	0.19	0.13	0	0	0
Number	700	50	21	18	8	3	2	0	0	0

Note: See table 6.A.5.

Table 6.A.23 Across Component Correlations for the Third Component using BS£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0.19	0.57	1.2	1.64	0.82	0.88	1.45	0.69	1.96	41.73
Number	3	9	19	26	13	14	23	11	31	661
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	38.32	2.34	1.14	1.14	1.07	1.33	2.08	0.69	0.44	0.32
Number	607	37	18	18	17	21	33	11	7	5

Note: See table 6.A.5.

Table 6.A.24 Across Component Correlations for the Third Component using SS£.

Correlation	-1.0 to -0.9	-0.9 to -0.8	-0.8 to -0.7	-0.7 to -0.6	-0.6 to -0.5	-0.5 to -0.4	-0.4 to -0.3	-0.3 to -0.2	-0.2 to -0.1	-0.1 to 0.0
%	0	0.44	0.38	0.44	0.88	1.26	1.83	2.21	2.9	39.65
Number	0	7	6	7	14	20	29	35	46	628
Correlation	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
%	39.02	2.78	2.15	2.08	1.83	1.26	0.57	0.19	0.13	0
Number	618	44	34	33	29	20	9	3	2	0

Note: See table 6.A.8.

Chapter 7

ONS News and Movements in the Principal Components of the Term Structure

7.1 Introduction

In this chapter we investigate the hypothesis that the UK gilts market is efficient in that it only reacts to new information, "news". Clearly knowledge about how the term structure reacts to news could be highly profitable to market makers and in itself justifies continued research into this area. Three further justifications can also be offered. Firstly, by using principal components we can potentially analyse the whole yield curve from two years up to 24 years of maturity in a compact manner. Consequently, broad conclusions on the level and the slope of the term structure can be drawn. Secondly, we have available two term structures for the UK. By testing for news effects on these different databases, concerns that news effects depend upon the data used can potentially be lessened. Thirdly, we have collected expectations data for UK data releases that have been insufficiently analysed previously. The combination of principal components and the relatively untried expectation data has the potential to provide significant results in an area where past research has been unsuccessful.[1] For these reasons we are more than justified in examining news effects in the gilt market.

The plan of the chapter is as follows. In sections 7.2 to 7.9 the expectations data are analysed. Section 7.2 acts as an introduction to these sections. Section 7.3 of this chapter briefly reviews alternative methods of constructing expectations and compares the results of the MMS survey with that conducted by Dow Jones. In sections 7.4 and 7.5 the tests for rationality and revisions of expectations are discussed. After a description of the data used, the results for the tests of rationality and timeliness are presented in the sections 7.7 and 7.8. In section 7.9 the numerical significance of the forecasts' biases and inefficiencies are assessed, and this part of the chapter concludes with a summary of the main findings. The results suggest that the expectations are not rational but that the inefficiency and bias are

[1] To jump ahead, this potential is not fulfilled by the results reported in this chapter.

small and that the surveys are timely. Establishing this is important in justifying why the data from MMS International can be used in testing the efficiency of the gilts market.

In section 7.10 tests of efficient markets are introduced, and section 7.11 discuses some previous work using UK data. Section 7.12 compares and contrasts the means and variances of changes in the principal component scores on days when ONS data is released and on days when it is not. The next section creates the non-ONS data that are used to ensure that movements in the principal component scores are not erroneously assigned to movements in ONS data when they are due to other factors. This section also tests for orders of integration. Sections 7.14, 7.15 and 7.16 use regression analysis to test the effects of news (both ONS and non-ONS) on the first three principal components. The first component regressions are subjected to stability testing in section 7.17. Section 7.18 numerically examines the effects of news on the term structure in both the short and the long run, whilst section 7.19 provides some conclusions.

7.2 Measuring News Effects

An important area of research is how "news" is reflected in changes in financial variables. By "news" we mean unexpected extra information, and a major difficulty is separating data into expected and unexpected components. Indeed some items simply cannot be distinguished in this manner, for example the membership of the UK's Monetary Policy Committee, because it is impossible to quantify the news element. Consequently, economic analysis of news has focused almost entirely upon the effects of economic data releases that have the dual benefit of being quantified and having a preannounced release date. The actual data release is relatively straightforward to find (although care has to taken to ensure that the data is free of subsequent revisions that have in some instances substantially altered the figures), but defining what constitutes the expected data is more difficult. This requires some form of forecast to be available; and, although there are numerous ways in which economic forecasts can be constructed, the basic choice has been to either use published forecasts or to build specific forecasting models for the problem at hand. This chapter examines the usefulness of the forecast survey published by Standard and Poor's MMS on nine monthly UK data series. In particular it examines whether the survey is consistent with other published surveys; whether the survey departs significantly from rationality and whether there is any evidence that expectations change between the construction of the forecast and the release of the data. This latter aspect, the timeliness of the forecast, has received little coverage in the literature but could potentially invalidate many of the studies of news effects on asset prices because the news would be measured with error.

7.3 Alternative Methods of Constructing Expectations

One method of constructing expectations is to use the forecasts from econometric models as published by, for example, the London Business School or the National Institute of Economic and Social Research. However, although these forecasts are well publicised they are for quarterly data, so that they lack the precision required to estimate high frequency news effects. An alternative is for the researcher to build forecasting models such as single equation regressions and vector autoregressions (VARs). A problem with this is that when the data is subject to revisions (as most of the ONS's economic data are) the model has to be re-estimated after every data release as otherwise the model would be making use of information that was not available at that time. This can become very time consuming, particularly if the functional form has to be rechecked each time the model is re-estimated. VARs can easily be overfitted, giving overdue weight to spurious relationships and they often have no background in economic theory to justify their parameter estimates so that, a priori, interpretations of the forecasts are again difficult. Moreover, it is not clear that the estimated equations will necessarily represent the views of market participants.

This chapter uses an alternative approach to those outlined above by utilizing the median forecasts produced by between 15 to 20 City forecasters (usually economists) as collected by Standard & Poor's MMS, an economic information company. The survey is anonymous but for the UK most of the large investment banks resident in the City are contacted. For other European countries, investment banks resident in that country will be surveyed as well as City institutions to ensure a large enough sample. For the UK the survey covers most of the major ONS data releases currently about nine monthly releases plus two quarterly series GDP and the current account, although it usually restricts itself to the "headline" figures rather than the minutiae of the data releases. MMS faxes respondents at the beginning of the week asking for their forecasts of ONS data that are to be published in the following week. Non-respondents are contacted by phone on Thursday and early Friday to increase the response rate. The median forecast is calculated and it is made available to MMS's clients by both fax and Bloomberg terminals on Friday, and the surveys are passed to Reuters and Bridge News and other news services. The results of the survey are also published in the Financial Times' Diary Section, usually on a Monday. The forecast horizon for the median forecast therefore ranges from a minimum of three days (Friday to Monday) to a maximum of seven days (Friday to the Friday of the following week).

Although MMS data has been used to analyse money supply announcements by MacDonald and Torrance (1987), to my knowledge UK non-monetary MMS data has not been used previously, and this provides another motivation for the analysis conducted below. The median is used because this, rather than the mean forecast, is the measure provided by MMS. The individual forecasts are not provided with the survey data so that calculation of the means is not possible. The advantage of using these data set is that they provide a relatively up to date view of the market's forecasts that reflects a number of forecasting methods and may, consequently, be less susceptible to forecast errors from inappropriate model specifications.

There are, however, a number of problems with using survey data to measure expectations. These are that: there is no reason why the most influential forecast should necessarily be represented by the median; the median forecasts constructed by other institutions, such as Dow Jones, sometimes differ from MMS's and the final problem is that expectations may have changed since the survey was undertaken. All of these imply that any forecast as a measure of expectations means could be measured with error and, consequently, that estimates of news effects on asset prices could be biased. The first problem is common to the use of other techniques to estimate expectations, and there is no reason to believe that, say, a particular VAR is the most influential forecast. Nevertheless, because the forecasts are collected from institutions whose businesses are actively involved in transactions of financial assets, there is a higher probability, compared with other methods, of the most influential forecasts being captured in the MMS survey. It can also be argued that the surveys are unlikely to be influential if there are significant departures from rationality. This is tested for in the following sections.

A comprehensive comparison of alternative surveys is impossible simply because the surveying organisations rarely keep a record of the data. However, the correlation between the MMS and Dow Jones survey using data from the beginning of 1995 is high, never being less than 0.73 in the case of producer input prices and six out of the nine economic series used below all have correlations in excess of 0.9.[2] This suggests that both of these surveys will tend to capture similar movements in the market's perception. Moreover, it should be noted that some organisations undertaking the surveys are principally interested in generating news stories and, as such, the statistical methodology, let alone sample sizes, are not as strict as one would hope. For these reasons and because it has a longer coverage the MMS survey is the preferred choice.

7.4 Testing for Bias and Efficiency

Although there is no method that can calibrate the extent to which the market's true perception and the MMS median forecast diverge, one method of assessing the usefulness of MMS survey data involves using the rational expectations hypothesis (REH), associated with Muth (1960). This states that market participants use all cost efficient knowledge to forecast economic variables so that their forecasts are unbiased and efficient.

This can be expressed as follows:

$$Y_{it} = \alpha_i + \beta_i Y_{it}^e + \varepsilon_{it} \quad \dots (7.4.1)$$

Where: Y_{it} is the i[th] economic variable, e.g. producer prices, retail sales, M4, unemployment, underlying average earnings, PSBR, industrial production and RPI.

Y_{it}^e is the median forecasts of the i[th] economic variable for time t made at time k.

α_i, β_i are coefficients.

ε_{it} is a random variable that is normally distributed with a mean of zero and a constant variance.

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2 Unfortunately some data are missing even over this short period and as the majority of the series are believed to be I(1) this rules out the use of regressions to test equality.

If the forecast is rational the coefficients α_i will be zero and the β_i will be one. The error term, ε_{it}, will be uncorrelated with itself (no serial correlation). It must also be uncorrelated with any economic variable that was available at time k and that was costless to acquire and process (the orthogonality condition), i.e. there is no information available at time k (k<t) that could be used to improve the forecast.

Equations such as (7.4.1) can be used to assess whether or not the MMS median forecasts are rational. If the MMS forecasts diverge substantially from rationality it is unlikely that innovations in Y_{it} will provide an unbiased estimate of the news in data releases because market participants are unlikely to pay much attention to such forecasts. However, evaluating what constitutes a substantial divergence is problematic. Past research suggests that the orthogonality conditions are usually violated. For example, using UK data both Evans and Gulamani (1984) and Pesaran (1985) reject the orthogonality conditions for inflation forecasts. Taylor (1988) found evidence of "irrational" expectations by just over 50 leading City of London investment managers on wage and price inflation, the FT All Share index and the US S&P composite index. Moreover, Evans and Gulamani (1984) find that by adding past inflation, the money supply, and unemployment, to equations like (7.4.1) they can improve the predictive accuracy of it's forecasts (measured by the standard deviation of the forecast error) by 30%. MacDonald and Torrance (1987) cannot reject unbiasedness but can reject orthogonality at the fourth lag of the forecast error for £M3, although overall F-tests cannot reject the hypothesis of orthogonality.

The second stage of analysis is, therefore, to examine whether the orthogonality condition holds by testing whether the error term, ε_{it}, is independent of available information. In order to study this, equation (7.4.2) is estimated:

$$Y_{it} = \alpha_i + \beta_i Y_{it}^e + \gamma_{it} Z_{tl} + \delta_t \quad ...(7.4.2)$$

Where: Z_{tl} is a vector of other variables that are available at time k.

δ_{it} is an error term that is normally distributed with a mean of zero and a constant variance.

γ_{il} is a vector of parameters.

If the γ_{il} parameters are statistically different from zero, then market forecasters are not using all available information and hence are inefficient. By using equation (7.4.2) we can potentially improve upon the published forecasts. In this chapter Z_{tl} is limited to the 12 lags (l=1.... 12) of the uncorrected forecast errors ($Y_{it} - Y_{it}^e$) because we would expect these to capture systematic effects from a variety of sources. Twelve lags were chosen, as this should be sufficient to capture any possible seasonal effects in the data.

7.5 Timeliness of the Forecasts

One remaining problem of using survey data is that of the survey's forecasts being out of date by the time the actual data is released. For example, the MMS expectation data can be up to eight days old by the time the data is released and so it may no longer reflect the expectations held in the market. Consequently, estimates of the news element in data releases, and their subsequent effects on financial prices, can be misleading. There are three ways of capturing timeliness effects.

The first is to argue that forecasts are made with a "ragged edge" when not all data are available. Hence when new data are published this allows another part of the ragged edge to be filled; and, if published data contain news, this causes other simultaneously determined concurrently dated forecasts to be revised. Thus a third set of terms can be added to equation (7.4.2) that comprises the information that becomes available between the time the forecast is made and the publication of the data. This includes all the forecast errors, ($Y_{jt} - Y_{jt}^e$) for all the other j ($j \neq i$) variables.

Secondly, not all news between the survey and publication dates will be captured in the forecast error terms ($Y_{jt} - Y_{jt}^e$). To allow for this a second set of financial variables can be added to equation (7.4.2). The reasoning for this is as follows. Suppose some news arrives that causes expectations to be revised but that it is not directly observable to the researcher. The change in expectations causes market participants to transact and change the prices of financial variables. Hence the change in financial prices, such as the trade weighted exchange rate, three-month LIBOR and the FT-SE 100, are a measure of the revision of expectations. These are referred to as ΔX_t.

The third justification for the presence of the $(Y_{jt} - Y^e_{jt})$ and ΔX_t terms is as follows. Suppose, having made their forecast the forecasters wish to improve upon it until the time of publication. Having used all previously published data to form their initial expectation the main sources of news will be from $(Y_{jt} - Y^e_{jt})$ and ΔX_t. Thus forecasters include these variables in their models when trying to form predictions. This can be thought of as a hybrid VAR approach with the initial forecast, Y_{it}, acting as a proxy for the lagged terms in traditional VAR models. Thus the distinction between survey based and VAR or model generated expectations is less clear cut than the distinction drawn in section 7.2. This justification for the inclusion of $(Y_{jt} - Y^e_{jt})$ and ΔX_t is that together with Y_{it} this forms the expectations generating process. It remains of interest whether or not the initial forecast, Y_{it}, s unbiased and efficient because if it is not market markers will be systematically trading, initially at least, on incorrect forecasts and these trades may have to be unwound as the forecast is updated.

The evidence for these effects is, however, far from conclusive. For US money supply announcements Grossman (1981) found that the change in the Treasury bill rate contained information that improved forecasts but Liu (1994) found that the S&P-500 share price index did not. In this chapter the daily change since the forecast was made of the exchange rate, three month LIBOR rates and equity prices as measured by the FT-SE 100 are included in the regressions. As the levels of these variables were already known at the time the forecasts were made it is assumed only the change in these financial prices reflects news. Thus the difference between the Z_{tl} and the ΔX_t is that the former are available before the forecast is made whilst the latter is not. This gives the following equation:

$$Y_{it} = \alpha_i + \beta_i Y^e_{it} + \gamma_{il} Z_{tl} + \eta_i (Y_{jt} - Y^e_{jt}) + \kappa_i \Delta X_t + v_{it} \quad ...(7.5.1)$$

Where: $(Y_{jt} - Y^e_{jt})$ is a vector of forecast errors on the economic variables for all j except i.

ΔX_t is a vector of changes in financial variables available after the construction of the forecast.

η_i and κ_i are vectors of parameters.

v_{it} is a random error of mean zero and constant variance.

The interpretation of forecast errors $(Y_{jt} - Y^{e}_{jt})$ and the change in the financial variables, ΔX_t, is that they appear in the equation not because they are thought to determine Y_{it}, but because they would have influenced the formation of expectations, Y^{e}_{it}, had they been known at time k. Alternatively, ΔX_t may appear in the equation because as expectations are revised market participants trade in financial markets thereby altering prices. The change in prices is an indication that expectations have been revised and that Y^{e}_{t} is an out of date measure of expectations. The parameters η_i and κ_i therefore represent a combination of the effects of the economic variables on the formation of expectations as well as the relationship between expectations and the outturn as given by the parameter β_i. As equation (7.5.1) encompasses equations (7.4.1) and (7.4.2) it is used as the starting point for the analysis. It should be noted that the presence or otherwise of the terms $(Y_{it} - Y^{e}_{it})$ and ΔX_t is not a test of rational expectations as neither vectors were known at time k. Their presence is a test of whether or not the efficiency of the initial forecast could be improved upon using these variables.

7.6 The Data

Expectations data were obtained from MMS on nine monthly economic series. These are tabulated in table 7.6.1. The choice of variables was constrained by the need to obtain sufficient degrees of freedom. For example, expectations of quarterly GDP growth for which only 20 observations were available have not been used. The majority of data is for the month on month percentage change but the PSBR and unemployment data are for changes in the level and the underlying earnings series is for annual percentage change. In this latter case it is important to note that this series is only a one-step ahead forecast because the forecasts do not overlap. Indeed, although the MMS surveys are published weekly, they only contain forecasts of these variables once a month in the week of their publication. This has some important consequences: one is that moving average error processes are not induced so there is no need to correct the variance-covariance matrix of the parameter estimates using the methods proposed by White (1980) and Hansen and Hodrick (1980). Furthermore, because the data is not categorical ("up, stay the same, down" as used in some CBI surveys for example), the problems of measurement errors are much reduced (see Pesaran (1985)), obviating the need for an instrumental variables (IV) estimation procedure.

The corresponding outturns were compiled from various copies of the ONS publications Economic Trends and Financial Statistics. Except for the RPI data, which is never revised, care was taken to use the initial estimates, as the current levels of variables reflect not only rebasing over time but also the affects of revisions. The financial data on the trade weighted exchange rate, three month LIBOR and the FT-SE 100 were derived from Datastream as none of these variables are subject to revisions. As table 7.6.1 shows the data contains variables that are integrated at both I(1) and I(0), but both the forecast and the outturn on each variable were always of the same order of integration. All the financial variables and the forecast errors were found to be I(0).

Equation (7.5.1) was estimated by ordinary least squares with 12 lags on the dependent variable's forecast errors. The forecast errors made on M4 were, however, excluded from the other equations because by including them the estimation period would be unduly constrained. A number of other forecast errors were excluded because of simultaneous publications (such as the publication of the producer price input and output series in the same press release). The sample size of each equation was chosen to ensure that the maximum number of observations was used. The equation for industrial production dropped the first 25 observations because no data is available for that period on the forecast errors made on the PSBR. For the other equations the elimination of other forecast errors has resulted in the sample sizes being determined by the lag on the forecast errors given in table 7.6.2, row four. A number of the equations also contained dummy variables (see table 7.6.2, row 5) whose presence was designed to ensure that the residuals from the equations were normally distributed so that inferences could be drawn from the test statistics also presented in table 7.6.2. Omitting the dummy variables does not change the statistical conclusions drawn. Table 7.6.2 records the final form and the parameter values of the variables of interest.

Table 7.6.1 Orders of Integration of the Forecast and Actual Data

Series	Data period	Number of observations	Order of integration of expectations	Order of integration of outturn
Producer input prices (month on month % change).	November 1982 - March 1995	149	I(0) ADF(12)= -3.16 ADF(0)= -8.04	I(0) ADF(0)= -8.04
Retail sales volume (month on month % change).	December 1981 - March 1995	160	I(0) ADF(10)= -4.15 ADF(8)= -3.26	I(0) ADF(8)= -3.26
M4 (month on month % change).	March 1988 - March 1995	85	I(1) ADF(4)= -7.02	I(1) ADF(4)= -7.24
Unemployment (month on month change).	August 1982 - March 1995	152	I(1) ADF(2)=-9.75	I(1) ADF(0)=-12.53
Underlying average earnings (% change on same month of previous year).	November 1982 - December 1994	146	I (1) ADF(2)= -4.63	I (1) ADF(6)= -2.29
PSBR (£mn).	December 1983 - March 1995	136	I(1) ADF(12)= -4.37	I(1) ADF(12)=-4.25
Producer output prices (month on month % change).	November 1982 - March 1995	149	I(1) ADF(10)= -12.65	I(1) ADF(11)= -6.81
Industrial production (month on month % change).	November 1981 - February 1995	160	I(0) ADF(11)=-3.03	I(0) ADF(0)=-9.91
RPI (month on month % change).	December 1981 - March 1995	160	I(1) ADF(10)= -8.80	I(1) ADF(11)= -6.62

Note: The orders of integration were calculated using an augmented Dickey Fuller test, ADF(I), of lag length i. The appropriate lag length was set by inspection of the ADF equations using the lag criterion methodology suggested by Hendry and Doornik (1996).[3]

[3] Hendry and Doornik (1996) pp.41-42.

Table 7.6.2 Efficiency, Orthogonality and Regression Diagnostic Tests

Series	Producer input prices	Retail sales volume	M4	Un-employ-ment	Average earnings	PSBR	Producer output prices.	Industrial output	RPI
α_i (t-statistic)	-0.05 (-0.71)	.02 (0.28)	-0.06 (-0.72)	-1.23 (-0.72)	0.03 (0.48)	-218.6 (-2.25)	0.017 (0.6)	-0.18* (-2.32)	-0.04 (2.27)
β_i (t-statistic) (t-statistic)[a]	0.90 (11.57) (-1.29)	1.08 (8.12) (0.60)	1.10 (11.12) (1.01)	1.04 (20.87) (0.80)	1.00 (108.61) (0.00)	1.00 (30.07) (0.00)	0.95 (13.60) (-0.72)	1.18 (8.22) (1.25)	1.09 (31.77) (2.62)
Lagged forecast errors included (lags)	None	(-1), (-2) (-3), (-6) (-10)	(-1), (-2)	None	(-1)	(-6)	(-12)	(-1), (-12)	(-2), (-5), (-6)
Forecast errors for other variables (param-eter, t-statistic)	None	None	Un-employ-ment (-.01, -2.15)	Retail sales (5.10, 2.09) PSBR (-.02, -3.86)	None	None	None	PSBR (0.0003, 2.97)	Ex-change rate (-0.05, -2.46)
Dummy variables included	None	February 1984	None	None	March 1984 August 1989	January 1987 March 1992	May 1989 June 1991	February 1988 June 1991	March, April, September 1987 November 1988
Johansen	n/a	n/a	18.84* (0.00) 18.69* (0.00)	1.51 (0.47)	0.99 (0.61)	7.76* (0.02) 1.00 (0.61)	0.02 (0.99)	n/a	21.06* (0.00) 25.75* (0.00)
Corrected R^2	0.49	0.49	0.62	0.77	0.99	0.89	0.67	0.48	0.89
Residual LM(12)	10.77 (0.55)	16.73 (0.16)	19.57 (0.08)	7.68 (0.81)	13.43 (0.34)	11.38 (0.50)	9.04 (0.70)	9.92 (0.62)	12.48 (0.41)
ARCH (12)	8.53 (0.74)	11.35 (0.50)	18.33 (0.11)	11.81 (0.46)	13.25 (0.35)	16.33 (0.18)	7.58 (0.82)	10.44 (0.58)	6.08 (0.91)
Normality	1.44 (0.49)	2.34 (0.31)	1.40 (0.84)	0.95 (0.62)	6.98 (0.03)	1.67 (0.43)	1.83 (0.40)	1.81 (0.41)	0.89 (0.64)

Note: Variables defined as in table 7.6.1. Johansen LR test first number is for the joint hypothesis and the second figure refers to the hypothesis that β_i is unity. The marginal significance levels for the Johansen tests and the diagnostic tests are in parentheses. A * indicates rejections of unbiasedness using the appropriate t-statistic or LR test. (t-statistic)[a] indicates a test from the coefficient being unity.

7.7 Tests for Bias and Efficiency

For the I(0) variables, input prices and retail sales, the hypothesis that the forecast is an unbiased predictor of the actual outturn cannot be rejected at the 5% level of significance (table 7.6.2, rows 2 and 3). It can be rejected for industrial production because the constant is statistically different from zero. In the remaining equations the presence of forecasts that are I(1) means that the standard t-statistics are inappropriate tests of hypotheses that β_i is unity. For these equations the Johansen (1988) likelihood ratio test was applied (table 7.6.2, row 6). These tested the joint hypothesis that α_i and β_i were zero and unity respectively. If the joint test was failed the hypotheses that the value of α_i was as given in table 7.6.2, row two and β_i was unity was also tested. The Johansen tests on the equations for unemployment, average earnings and output prices all suggest that the joint hypothesis cannot be rejected. However, the PSBR, M4 and RPI equations all reject the joint hypothesis and the forecasts are, consequently, biased. For the PSBR the reason for this bias is that the constant term is statistically significantly different from zero, i.e. the forecasts tend to overpredict this variable. For the RPI and M4 equations the hypothesis that β_i is unity can also be rejected with Johansen test scores of 25.75 and 18.19.

Only the equations for producer input prices and unemployment do not contain statistically significant lags of past forecast errors and, consequently, all the remaining forecasts are inefficient in that they have failed to use all relevant information at the time the forecast was made. Moreover, it cannot be ruled out that a more extensive search of the available information set would also reveal further inefficiencies for the input prices and unemployment forecasts. Thus only two out of the nine forecasts are both unbiased and efficient.

Table 7.6.2 also provides diagnostic analysis of the equations. The residuals, v_{it}, are not serially correlated on the basis of a Lagrange Multiplier test (row eight), nor is there evidence of heteroscedastic errors according to an ARCH test (row nine). With the exception of the underlying earnings equation, all the v_{it} are normally distributed on the basis of a Bera/Jarque test (row ten) although this is due in a number of cases to the introduction of dummy variables to remove outliers. The seventh row of table 7.6.2 gives the corrected R^2 which indicates that the equations explain between 48% and 99% of the variation in the actual data, although in a number of instances this performance is flattered by the inclusion of dummy variables. In statistical terms the results reported above are consistent with the findings of bias and inefficiency of forecasts found by other researchers.

7.8 Timeliness Results

Row four of table 7.6.2 reports the results of whether the forecast performance can be enhanced by the inclusion of forecast errors that have become available in the period between the forecast being produced and the publication of the data by the ONS. The forecast equations for M4, unemployment and industrial production all find statistically significant forecast errors from other variables. With the exception of the M4 forecast, an explanation of the signs of the parameters is difficult. For example, it is not clear why an under-forecast of the PSBR, i.e. the economy is weaker than expected, should be associated with an under-forecast of industrial production, i.e. the economy is stronger than expected, unless there is some mechanism linking public sector non-cyclical expenditure and output. Similar reasoning applies to the unemployment-forecast equation. Moreover, recursive regressions reveal that the unemployment term in the M4 equation is unstable and is often close to zero.

Only the RPI equation contains a statistically significant financial variable. The model suggests that a depreciation of the trade-weighted exchange rate is associated with the actual inflation rate being higher than the forecast rate. This is consistent with the behaviour of the financial markets, which revise their inflation forecast upwards leading to some selling of sterling causing the depreciation of the exchange rate. However, recursive regressions again suggest that for much of this period the parameter is not statistically different from zero and this effect should not be relied upon.

The use of a further data set of markets' expectations can be used to examine the updating of economic forecasts. The Dow Jones Telerate survey is conducted both one and two weeks in advance of the publication of the actual data so that the updating of expectations can be observed directly. Unfortunately there are two caveats to this data set. The results of the survey have only been stored since 1995 and, even so, not all data is available for this short time period so that it is of insufficient size to conduct elaborate statistical analysis. Moreover, the number of respondents in each survey is not constant. There is no guarantee that all respondents forecast all of the variables in the two-week period and there is no method of telling whether the individual company respondents change between different survey dates. Consequently, changes in the median expectations may simply reflect changes in the respondents to the survey rather than the updating of forecasts as new data becomes available. This caveat needs to be borne in mind when reviewing the results.

Table 7.8.1 Expectations Changes Two and One Week Before Publication of the Dow Jones Telerate Survey.

Variable	Number of Observations	Number which did not change between surveys	Number of changes	Number of changes less than ± 0.1%
RPI	16	12	4	4
Industrial production	15	5	10	7
Underlying average earnings	14	10	4	4
Retail sales	14	7	7	4
PSBR	15	3	12	n/a
M4	13	3	10	9
Unemployment	15	6	9	n/a
Producer prices input	15	4	11	4
Producer prices output	15	11	4	3

Notes: Variables are defined as in table 7.6.1.

The results reported in table 7.8.1 show that for 61 out of 132 of the surveys there was no change in the respondent's expectations between the first and the second survey. Of the remaining 71, the majority, 37, change by only the minimum amount 0.1% between surveys.[4] Thus 98 of the 132 surveys report no or only small changes in the forecast. Thus the Dow Jones Telerate survey tends to support the econometric results reported above that new information (apart from new information on the variable itself that we cannot, of course, test for) does not cause expectations to be updated to any large extent.

[4] This criterion is not used for unemployment and the PSBR, which are measured in thousands and £mn respectively, and the percentage change in underlying average earnings where the minimum change in the data is 0.25%.

Thus the chapter presents two strands of evidence on whether MMS surveys can be used to calculate forecast errors that provide the news elements in studies of asset pricing. Both of these strands point in the same direction. The regression results show that for a few variables the release of ONS data and changes in financial variables between the date the forecast is made and the date the ONS data is published may cause the forecasts to be updated. However, these effects often appear to be statistically unstable and hard to explain from economic theory. Consequently, the forecast findings are regarded as statistical aberrations. Hence, there is no evidence of forecasts being updating between the date the forecast is made and the date at which the ONS publishes the data. This is supported by the evidence from the Dow Jones survey that forecasts once made are not significantly updated. As there is no reason to suspect that other market participants react differently from those surveyed by MMS or Dow Jones these results are likely to hold for other surveys.

The implication of the timeliness finding is that studies of asset prices that use news components calculated from surveys of market participants are not introducing biases into their regressions simply because their forecasts were collected in advance of the data being published. Thus, for example, the results of Becker et al (1992), which find that news calculated from MMS surveys has low explanatory power for the long gilt future traded on LIFFE and that the news effects are unstable, is not due to the MMS surveys being out of date by the time of the publication of the data. Failures of this study and others like it to explain more than a tiny fraction of the movements in asset prices must come from other sources, perhaps for example, the forecast errors of traders are more important in determining asset prices than the errors of economists.

7.9 Numerical Significance

Before we dismiss the abilities of City forecasters, the numerical as opposed to the statistical performance of the equations needs to be checked against the forecast errors made by assuming that the forecasts are efficient and orthogonal. Table 7.9.1 shows that for the majority of equations there are no appreciable reduction in the average absolute errors. For example, for M4 forecasts are improved by 6 percentage points, by 800 persons per month for the unemployment figures and £195mn per month for the PSBR equations. The reason for this lack of substantial improvement is simply that, although a number of terms are statistically significant, the sizes of these parameters are small as are the deviations from efficiency. Consequently, the reduction in the forecasting error is also small. The absolute mean forecasting errors can also be contrasted with the revisions to the published data series between their first release and their values as published currently. Only for the PSBR is forecast error appreciably different from the errors arising from data revisions.

Table 7.9.1 Comparison of Model Residuals and Forecast Errors

	Absolute mean of model residuals	Absolute mean of unadjusted forecast errors	Absolute mean of data revisions	Standard deviation of absolute model residuals	Standard deviation of absolute unadjusted forecast errors	Standard deviation of absolute data revisions
Producer Input Prices	0.576	0.571	0.706	0.471	0.490	0.594
Retail Sales Volume	0.765	0.885	0.805	0.600	0.770	0.718
M4	0.361	0.423	0.416	0.276	0.295	0.295
Unemployment	13.313	14.117	8.020	10.823	12.394	8.695
Underlying average earnings	0.142	0.146	0.055	0.150	0.190	0.112
PSBR	719.0	914.0	200.9	615.5	1342.5	349.6
Producer Output Prices	0.118	0.126	0.180	0.119	0.100	0.187
Industrial Production	0.616	0.751	0.763	0.580	0.483	0.677
RPI	0.124	0.163	n/a	0.099	0.186	n/a

Note: Variables defined as in table 7.6.1.

Similar conclusions can be reached about the variability of forecast errors. Although for the majority of variables the variability is reduced by adjusting the median forecasts, this gain is not large except for the PSBR. Relative to the revisions to the data series the raw and adjusted forecasts are not appreciably worse except in the case of the PSBR. Given that modelling of this sort is not costless to market participants, the small gains in forecasting accuracy may not be worth the effort in eliminating them.

This is one reason why forecasting inefficiency persists over time despite numerous papers demonstrating its presence.

The main conclusion reached is that despite failing to be rational, the MMS median forecasts are likely to be accepted as accurate forecasts by market participants and can, therefore, be used to derive news elements. The second conclusion is that the majority of these series do not contain statistically significant currently dated variables; and when they do, the parameter estimates are small and/or unstable. Hence, although forecasts may be revised between their publication date and the release of the data, the forecasts cannot be adjusted in a straightforward manner for this effect. However, what these regressions do show is that if, say, interest rates react to the release of data on a certain variable, it is likely that it is due to news on this variable itself rather than due to the revisions to forecasts of as yet unpublished data that is driving the price changes of the financial asset.

7.10 Tests of Efficient Markets

The view that financial markets are efficient is the dominant view of capital markets in the developed world whether these markets be for equities, debt or foreign exchange. By efficient is simply meant that market participants use all relevant information that is sufficiently cheaply available to price assets.[5] As a consequence the market price of assets will always equal their equilibrium value in the absence of transaction costs. Blake (1990) describes this as a continuous stochastic equilibrium. The reason for the term "stochastic" is because of news. If assets trade at their equilibrium value and this reflects all relevant information then the expectation of the n-period ahead price is simply the price in the current period. Consequently, if markets are efficient then only new information, "news", will move prices. As news is random, otherwise it would not be news, this means that movements in prices will also be random and hence unpredictable at the current time. Thus asset prices are martingales with the expected price of the asset being its current price plus a random element. If prices drift upwards over time, because the expected return is positive, asset prices are submartingales.[6]

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[5] See Fama (1991) p.1575.

[6] See Blake (1990) p.245.

Roberts (1967) suggested three general forms of efficiency that have increasing informational requirements attached to them. Weak form efficiency implies that price movements cannot be predicted from past movements in prices, semi-strong efficiency asserts that prices contain all the relevant information that is publicly available. Strong form efficiency implies that not only is publicly available information but also private information is reflected in prices. In the remaining sections of this chapter we are interested in testing weak and semi-strong forms of efficiency because we have no access to the private information sets that would allow testing of strong form efficiency. Violations of either of these forms would violate strong form efficiency.

The primary reason for the interest in efficiency is that violations of it may open up profitable investment strategies. However, not all apparent violations indicate unexploited profit opportunities especially if no consideration is made of transaction costs, funding costs or risk adjustment. Nevertheless, testing efficiency is a necessary first step and this justifies examination in this chapter. A second reason for interest is that studies on the UK gilts markets have not found any consistent pattern in which news items moves prices. Consequently, if the gilt market were found to be efficient, greater information on which economic series potentially carry news and those that do not would be of interest. Such information might help the design of further academic studies about what fundamental information was carried in news items which moved asset prices but was absent in other forms of news.

In the section 7.11 we review UK previous studies. Section 7.12 studies the variability of the principal component scores on days when ONS News is released and days when it is not. The finding that the Bank of England data (SS£) and the B-spline variable knot data (BS£) appear to behave differently is the justification for examining both data sets in the sections that follow. The next section describes the creation of news data and outlines some of the problems that may arise in using regression analysis on new effects. Sections 7.14 to 7.16 describe the regression results for the first, second and third principal component scores. The stability of the first component equations is examined in section 7.17. Section 7.18 examines the numerical size of the news effects on the term structure of interest rates and section 7.19 offers some conclusions.

7.11 Previous Analysis of News Effects on UK Interest Rates

Goodhart and Smith (1985) report disappointing results for news effects. Using 20 year gilt prices and the three month interbank rate for the period January 1977 to December 1983, they find that neither news about the percentage change in the RPI, the visible trade balance nor the central government borrowing requirement had any effect on these interest rates. Only the percentage change in £M3 had any effect on bond prices, a one percentage point forecasting error reducing the price by £0.45, and no effect on the three month interest rate. Moreover, there is some indication that the sizes of the news effects increase as the event window is increased, and this points to the market being inefficient.

Becker et al (1992) also report few significant findings using the long gilt future traded on LIFFE for the period 2 January 1986 to 28 December 1990. This work is important because it uses as the independent variables the same MMS data as used below and, hence, it provides a comparison with which the use of principal component scores as the dependent variables can be made. By taking the log of the ratio of prices at 11.30 and 11.45 a.m., i.e. just before and just after the announcement of data by the ONS, Becker et al (1992) find that news on the current account, the visible trade balance, the PSBR and retail sales all had statistically significant effects on the long bond return. On the other hand, industrial production, M0, producer output prices, the RPI, and unemployment did not have statistically significant parameters. Overall, the equation explained 18.6% of the variation in prices. However, using opening to closing prices the explanatory power falls to 1.9% and only visible trade remains statistically significant. For the 15-minute event window the parameters are unstable. If the equation is split by using dummy variables, the visible trade deficit only records a statistically significant parameter after the 1987 Louvre Accord. Apart from the intercept no other variable is statistically significant at the 5% level or above.

Using a slightly different methodology, Elmendorf et al (1992) study the variance of the holding period return on consols between 1900 and 1920. They conclude that news is unable to explain more than a small part in the variation of UK bond prices. Thus despite evidence that news does stimulate trading activity, none of the three studies discussed above provides convincing evidence that ONS data announcements provide anything more than a tiny percentage of the news that drives markets. A less charitable explanation would be to suggest that the gilt edged market is inefficient or driven by non-fundamental factors including fads and noise.

7.12 Comparison of Means and Variances on ONS and Non-ONS News Days

We begin by using F-tests to examine whether or not the variance of the change in the principal component scores are greater on days that UK news is released than on non-release days. As there is no systematic component to the arrival of other news its arrival on news days and non-news days is equally likely and, hence, should cancel out. Table 7.12.1 sets out the F-tests for the principal component scores from the BS£ data. The data begins on 6 January 1984, a date determined by the first publication of monthly PSBR figures and ends on 21 August 1990, a date determined by the end of the spot data. During this period there were 420 days with news and 1302 days without ONS news.

Table 7.12.1 F-Tests on Principal Component Scores using BS£

First difference of	Variance ONS*	Variance non-ONS*	F-test	Mean ONS (x1,000)	Mean non-ONS (x1,000)	Z-test on absolute data means
First Component	11.40	6.07	1.88	3.37	2.46	5.11
Second Component	38.58	7.65	5.04	-6.20	2.77	-28.69
Third Component	2.54	3.16	0.81	-1.59	1.77	-36.58

* multiplied by 1 million

For the degrees of freedom available (419,1301) the 95% critical value is 1.14 and the 99% value is 1.20. Consequently, the F-tests reject the hypothesis that the data has the same variance for changes in the first and second principal component scores. The arrival of news from the ONS seems to induce statistically significantly more variability in the changes in the level and slope of the term structure than the arrival of other news. This result does not hold for the third principal component that appears to be less variable on days when ONS news is released. Moreover, the arrival of ONS news during this sample period produces a greater mean change (in absolute terms) in both the first and second principal components for these days than on other days in the sample. The mean change is smaller (in absolute terms) for the third principal component. Moreover, as can be seen from the Z-test of the equality of the means, the difference is statistically significant at the 1% level.

Table 7.12.2 F-Tests on Principal Component Scores using SS£

First difference of	Variance ONS*	Variance non-ONS*	F-test	Mean ONS (x1,000)	Mean non-ONS (x1,000)	Z-test on absolute data means
First Component	55.00	1.22	44.72	7.41	1.11	17.34
Second Component	0.25	1.10	0.23	0.50	-1.05	40.79
Third Component	2.72	0.22	12.50	1.65	0.47	14.49

* multiplied by 1 million

These results are confirmed for the change in the first principal component score by using the SS£ data (see table 7.12.2). Both the variances and (in absolute terms) the mean change of the components are larger on ONS-news days than on other days. For the other two components the results differ from the BS£ data, the variance of the second component is not statistically different between ONS news days and other days, whilst the third component's variance is statistically different. The mean of the second component is smaller in absolute terms on ONS news days than on other days, whilst that of the third component is larger. The results presented in tables 7.12.1 and 7.12.2 suggest that there may be significant effects of ONS news releases on some aspects of the term structure, and in sections 7.14 to 7.17 this is examined using regression techniques. However, these results also suggest that there is little coherence between these descriptions of the data sets (a result confirmed in chapter 6); and, consequently, it is important that both data sets are used in the regression analysis.

7.13 Creating Non-ONS News and Testing for Integration

Tests of efficient markets using interest rate data are often set up in terms of the equality of holding period returns and the short rate of interest, that is the local expectations version of the expectations hypothesis.[7] It is usually assumed that the term premia are constant and therefore, aside from a constant term, only the unexpected components of variables that affect bond prices will cause the excess holding period return to change. The excess holding period return is defined as the difference between the holding period return and the short rate. However, not all surprises can be identified, and there will remain a random error term. A major weakness with this approach is that if the money or debt surprises

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7 See, for example, Plosser (1987).

are due to endogenous changes in the economy, they are likely to be correlated with the error term. Hence, the parameter estimates will be inconsistent if estimated by OLS, but finding instruments with significant explanatory power for the expectational error will be difficult. Indeed, Thorbecke (1993) blames the inability of single equation estimates to find conclusive evidence of the effects of deficits on interest rates as being due to simultaneity and temporal aggregation problems.[8] Thomas and Abderrezak (1988) suggest that the use of interest rate differentials will remove the simultaneity problem. Although they supply evidence from Granger causality tests to support this view, they offer no theoretical underpinning. As they use the gap between trend GDP and actual GDP as a measure of cyclical conditions in their equations, and there is a large amount of evidence to suggest that the slope of the term structure can predict changes in economic activity, their results may be a statistical fluke. Indeed, using interest differentials (excess returns) may not be the best means of reducing aggregation and simultaneity problems and may introduce problems of its own.

To remove simultaneity problems we adopt a different approach. We select a number of general financial prices that have in common that their markets or constituent markets are deep and liquid. Consequently, these variables (the FT-SE 100, the effective trade weighted exchange rate (EER), the number of Deutsche marks per pound (DM), the number of US dollar per pound, and the three month Libor rate) are all likely to move in response to changes in the economy. Hence, by entering these variables into the regression the error term should be purged of effects emanating from the general economy. The right hand side variables and the error term will be uncorrelated and ordinary least squares estimators of the parameters will be consistent.

Even if this process does not work fully, there is a second reason for expecting the ONS news and the residual to be uncorrelated, and this is due to the timing of the publication of the data. If the shock occurs on a given date, say 1 June, then because of the publication lag of one month or more, the expectational error on a variable refers to the month of May, that is before the shock took place. Hence the expectational error and the June shock cannot be correlated and the OLS estimates are consistent. By the time the June figure is published in July the shock is fully incorporated in expectations and again

[8] Thorbecke (1993) p.1 and p.10.

there should be no correlation between the expectational error and the lagged shocked term. Consequently, daily data is preferable to monthly and quarterly data

However, by including the financial variables (FT-SE 100, EER, DM, US dollar, and Libor) in the regressions will lead to multicolinearity amongst the parameter estimates. To circumvent this problem the change in the financial variables are purged of the effects of ONS related news by running regressions[9]. However, if regressions are being run these can also be designed to extract expected changes in the financial variables to leave only the non-ONS related news. This is performed by including five lagged dependent variables in the regression as a measure of how markets form their expectations. The expectational formation could, of course, have included other variables as regressors and longer lag lengths. However, the usual reason for including longer lag lengths is to cover all the seasonal factors; but as this would require over 250 lagged dependent variables, this is impossible for practical purposes. Therefore, the regressions are limited to the form of:

$$Y_{k,t} = \delta + \sum_{i=1}^{5} \alpha_i Y_{k,t-i} + \sum_{j=1}^{8} \beta_j Z_{j,t} \quad \ldots(7.13.1)$$

Where: $Y_{k,t}$ is the dependent variable and is the daily percentage change in the FT-SE 100 index, the daily percentage change in the sterling trade weighted (effective) exchange rate, the daily percentage change in the DM per sterling exchange rate, the daily percentage change in the US dollar per pound exchange rate and the daily difference in the mid-rate three month Libor.[10]

$\delta, \alpha_i, \beta_j$ are vectors of constants to be estimated by the regressions.

Z_j are the ONS news for average earnings, industrial production, producer input prices, producer output prices, retail sales, RPI, unemployment and the PSBR all as defined as in the previous sections on bias and efficiency of the MMS forecasts.

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[9] Changes in base rates are still treated as being entirely unexpected and so no regressions are undertaken. There were no Chancellor/Governor meetings or MPC during the sample period, these being latter innovations.

[10] Percentage changes are used in a number of these variables because they are nominal values and would be expected to be I(1) but the news elements are I(0). Thus ONS news would not be expected to explain the level of these variables. Hence taking percentage changes ensures that as much as possible of the ONS news effect is purged.

The results are relatively easy to describe. Invariably the first lagged dependent variable is always statistically significant at the 5% level using standard t-tests. Except for the US dollar regression one other of the lags t-2 to t-5 are also statistically significant but exactly which of these lags is statistically significant varies between the regressions. The magnitude of the estimated parameters on the lagged dependent variables is usually small (with the exception of the Libor equation). At most a 100 basis points increase in yesterdays financial prices would increase today's financial prices by 8 basis points. For Libor a 100 basis points increase would result in an increase in Libor of 20 basis points the next day but this would be reduced to just eight basis points two days after the initial increase. For the ONS news effects statistically significant parameters at the 5% are usually absent except for retail sales news on the effective exchange rate and the DM rate. Again the numerical effect is small with a 100 basis point forecasting error being associated with a fall of nine basis points for both exchange rates. The overall explanatory powers of the equations are very small in terms of R^2. With the exception of the Libor regression, all the other regressions explain less than 2% of the variation in the dependent variables whilst the Libor equation does scarcely better, explaining just 5.5% of the daily change in Libor. These results suggest that movements in financial variables are not strongly related to either their recent behaviour or the arrival of news. Nevertheless, we use the residuals from these regressions as regressors in our analyses of the principal component scores. They represent non-ONS related news so that problems of multicolinearity are minimised. Their presence also ensures that the parameter estimates are consistent because the correlation between the error term and the ONS news terms has been removed.

Apart from the economic news from the ONS described in the previous section, we also analyse the effects of the bond issues in the form of gilts and private bonds. In the former the series are those used by Egginton and Hall (1994), whilst the latter are a new series collected from Euromoney Publication's Bondware software. The inclusion of private data is important because it is believed that the UK authorities deliberately shortened the debt maturity to encourage the issuance of corporate debt. The authorities believed that gilts and corporate bonds were close substitutes and, therefore, as discussed in chapter 2, issuance of corporate bonds would be expected to impact upon the gilt term structure in a similar manner to the issuance of gilts. The private sector data included all markets and offer types in sterling, excluding shares and warrants. The institutions covered were all private banks, private corporations, private financial companies, private utilities, any other private institutions and supranational institutions. Only bonds rated AA+ and above by Standard and Poor's or Aa1 by Moodys were included in the data set. This latter restriction reduced the number of bonds from 1667 issuances to just 110. Moreover, this means that prior to 13 February 1986 no bonds were issued that fitted into the above credit rating despite the data being collected from January 1980. However, including data for issuances of lower investment grades would reduce the expected substitutability between the bonds and gilts and thereby diminish the effect on the gilt term structure.

There are, however, a number of difficulties in using the bond and gilt stocks. These take the form of dating when all the information was available. For gilts during the 1980s and 1990s for about 20% of issues there has been a difference of, on average, five working days between the announcement of the issue and its actual issue. For reverse auctions the difference between announcement data and auction date was one calendar month. Thus it is possible that the term structure adjusted prior to the issue of the gilts. However, the timing of issue still potentially contains a significant amount of information the importance of which may be linked to the size of the reverse auction or issue. In the case of the reverse auctions the degree of coverage, the number of times the offers to sell exceeded the Bank's offers to buy, and the extent that expectations of a sale were falsified for many potential sellers may have both influenced prices. The reverse happens for pre-announced sales. However, as the degree of cover varies between auctions this effect may not be very stable, but this does not mean it will be absent. These factors offer significant scope for further work. In this thesis they are noted as a means of justifying the use of issue dates in our examination of news effects.

Table 7.13.1 Unit-Root Tests, 13 January 1984 to 21 August 1990

Variable	Test statistic	Test Type	Variable	Test statistic	Test Type
average earnings	-41.410	DF	pc3 BS£	-33.093	ADF(1)
industrial production	-41.447	DF	pc3 SS£	-26.755	ADF(5)
M4	-41.415	DF	bond issue 1	-41.422	DF
producer input prices	-41.434	DF	bond issue 2	-41.420	DF
producer output prices	-41.402	DF	bond issue 3	-41.416	DF
retail sales	-41.402	DF	bond issue 4	-41.405	DF
RPI	-41.449	DF	bond issue 5	-31.410	ADF(1)
unemployment	-41.466	DF	total bond issues	-41.603	DF
PSBR	-41.624	DF	FT residuals	-41.397	DF
FT	-19.213	ADF(3)	EER residuals	-41.416	DF
EER	-17.085	ADF(4)	US dollar residuals	-41.457	DF
US dollar	-38.214	DF	DM residuals	-41.427	DF
DM	-16.705	ADF(4)	Libor residuals	-41.433	DF
Libor	-18.002	ADF(4)	RPI squared	-41.590	DF
Base rate	-22.184	ADF(2)	FT residuals squared	-13.940	ADF(5)
pc1 BS£	-16.956	ADF(4)	Libor residuals squared	-21.395	ADF(1)
pc1 SS£	-42.267	DF	FT residuals cubed	-16.459	ADF(4)
pc2 BS£	-27.247	ADF(2)	Libor residuals cubed	-33.044	ADF(1)
pc2 SS£	-25.507	ADF(4)			

Note: Critical values: 5%=-2.864 1%=-3.437; Constant included. Where: pci is the change in the i^{th} principal component score. Bond issue 1...5 represents the daily percentage change in bond stocks of varying maturities when a new issue is made. The maturity bands are: bond issue 1 is less than one year to maturity; bond issue 2 is one to five-years to maturity; bond issue 3 is five to ten-years to maturity; bond issue 4 is 10 to 15 years to maturity; bond issue 5 is over 15 years to maturity. The first nine variables are the news elements calculated by subtracting the MMS median forecast from the actual data release. Base rate is the daily change in the base rate. FT is the daily change in the FT-SE 100, EER is the daily change in the Bank of England Effective Exchange Rate, US dollar, DM and Libor are the daily changes in these exchange rates. When residuals are used these are calculated from the respective regression equation.

Before the regression analysis is performed it is necessary to establish the order of integration of the variables. This is done using Dickey Fuller (DF) and Augmented Dickey Fuller (ADF) tests with lags of up to five days. The results are straightforward - all the variables are integrated of order zero, i.e. they are stationary, at the 1% level of significance and above. As any linear combination of stationary variables is itself stationary, regression residuals using these variables will also be stationary. Weak stationarity implies that both the mean and the variance of a variable are independent of time and that the variance is a finite number. Unless the residuals exhibit these features the model cannot be regarded as a plausible explanation of the data generating process. Thus the variables we have chosen can, in principle, form an explanation of the principal components of the spot rates. It should be noted that the unit root tests for the news variables differ from those reported in table 7.6.1 of this chapter because these news variables are monthly whereas those in table 7.13.1 are daily.

7.14 News Effects on the First Principal Component Scores

One problem of using excess holding period returns is that if news raises the level of all interest rates there may be only a small effect on excess returns. However, some forms of news will affect the term structure, as was discussed in chapter 2 for the Turnovsky and Miller (1984) model and Turnovsky (1986), and only these will be identified by regressions of the news on excess holding period returns. This suggests that using principal component scores, which have already been shown to measure the level of interest rates and the slope of the term structure, can help isolate these effects, and it is to these regressions that we now turn.

The news regression was initially specified with eight ONS news releases variables, changes in base rates that are always taken to be a surprise, six bond issue news effects, and the non-ONS news effects on five financial variables, as measured by the residuals from the regressions. To these were added five terms in the squares and cubes of the RPI news, FT-SE 100 news or Libor news. These squared and cubed terms were suggested by an analysis of the residuals, which suggested that the heterogeneity of the error variance might be due to a functional form misspecification. Such analyses also suggested that there may be some role for cross products in the functional form but as many of the variables are predominantly zero, cross product terms are often little more than dummy variables because of the reduced likelihood of both variables being non-zero on a given day. Consequently, cross product terms were not added to the regressions. In order to retain consistency across the equations, whether from the BS£ or SS£ data and across the first, second and third principal components, these square and cubic terms were added to all the regressions.

Table 7.14.1 Regression Results for the Change in the First Principal Component using BS£

	Regression 1		Regression 2		Regression 3	
	coefficient	t-statistic	coefficient	t-statistic	coefficient	t-statistic
constant	0.008292	2.08	0.006347	1.59	0.003513	0.90
average earnings	0.142160	1.73	-	-	-	-
industrial production	0.013227	0.86	-	-	-	-
producer input prices	0.011817	0.41	-	-	-	-
producer output prices	-0.228070	-2.02	-0.244210	-2.18	-0.225080	-2.01
retail sales	0.028259	2.12	0.031904	2.36	0.031864	2.24
RPI	-0.019524	-0.13	-	-	-	-
unemployment	-0.000948	-0.95	-	-	-	-
PSBR	0.000015	1.13	-	-	-	-
Base rate	0.099088	2.38	0.118490	2.51	0.079576	1.77
bond issue 1	0.019551	2.16	0.004667	1.17	-	-
bond issue 2	0.057108	1.39	-	-	-	-
bond issue 3	0.035916	1.10	-	-	-	-
bond issue 4	0.061177	1.48	-	-	-	-
bond issue 5	0.023475	0.55	-	-	-	-
bond issue total	-0.237290	-1.65	-	-	-	-
FT residual	-0.056792	-8.58	-0.056613	-8.48	-0.056389	-8.30
EER residual	-0.083095	-3.95	-0.096522	-7.06	-0.096007	-6.98
US dollar residual	-0.036575	-4.52	-0.033885	-4.37	-0.035231	-4.55
DM residual	-0.013402	-0.80	-	-	-	-
Libor residual	0.511150	9.06	0.557180	10.39	0.535040	9.62
RPI squared	0.110140	0.75	-	-	-	-
FT residuals squared	-0.000316	-0.19	-	-	-	-
Libor residuals squared	-0.329980	-2.76	-0.192380	-1.57	-	-
FT residuals cubed	0.000640	2.58	0.000661	2.07	0.000665	2.06
Libor residuals cubed	0.222230	1.84	-	-	-	-
R^2	0.37		0.36		0.36	
RSS	44.97		45.53		45.83	
AR 1-2	3.84 (0.022) F(2,1694)		3.60 (0.028) F(2,1709)		2.39 (0.092) F(2,1711)	
ARCH 1	35.19 (0.000) F(1,1694)		36.37(0.000) F(1,1709)		36.36 (0.000) F(1,1711)	
Normality $\chi^2(2)$	160.57 (0.000)		162.98 (0.000)		171.21 (0.000)	
X_i^2	8.95 (0.000) F(47,1648)		21.02 (0.000) F(19,1691)		25.77 (0.000) F(16,1696)	
$X_i * X_j$	3.63 (0.000) F(252,1443)		10.12 (0.000) F(63,1647)		13.553 (0.000) F(44,1668)	
RESET	0.08 (0.782) F(1,1695)		2.41 (0.000) F(1,1710)		8.50 (0.004) F(1,1712)	
Variance Instability	1.098		1.051		1.087	
Joint Instability	7.395		4.220		4.168	

Note: AR 1-2 is a test of first or second order autocorrelation in the residuals. ARCH 1 is a test of autoregressive conditional heteroscedasticity in the residuals. Normality is a test for the normal distribution of the residuals. X^2 , X_i*X_j and RESET are all tests of functional form misspecification. The variance instability and joint instability test for non-constancy of the residual variance and of the residual variance and the coefficient estimates. Variables as defined in table 7.13.1, significance levels in parentheses. ONS news variables are in bold type.

As table 7.14.1 shows few of the news effects were statistically significant and for the sake of parsimony those with t-statistics less than the 5% critical value of 1.96 were dropped. The t-statistics are calculated on the basis of White's (1980) heteroscedastic-consistent standard errors (as are all the OLS regressions reported in this section). The search for a parsimonious model resulted in just eleven coefficients being retained out of an original 26 (table 7.14.1, regression 2), and this reduction resulted in a further two parameters on the remaining bond issue term and the squared Libor news term also becoming statistically insignificant. Removing these terms and re-estimating (table 7.14.1, regression 3) resulted in the change in base rates being statistically insignificant. The presence of the cubed term in the FT residual means that the response of the level of the term structure is non-linear for equity news, being negative for news effects smaller than minus 9.2% and also negative for news about the stock market of between 0.0% and 9.208%.

The second part of the table shows that this parsimonious specification has some statistical difficulties. In particular, the residuals seems to exhibit autoregressive conditional heteroscedasticity of order one, ARCH(1), and the residuals are not normally distributed. The regression also fails all of the tests for functional form X^2, X_i*X_j (both tests due to White (1980)) and the Ramsey (1969) RESET test for the omission of the squared predicted values from the model. The instability tests (due to Hansen (1992)) suggest that the model coefficients are not jointly constant. For the parsimonious model this is because the residual variance is non-constant as the individual coefficients for the variables do not exhibit statistically significant instability[11]. The error term does not appear to exhibit first or second order autocorrelation.

The non-normality of the errors reported above may be due to specific effects to the term structure, which result in outliers to the residuals, or the residual distribution may be inherently non-normal. In the former case the removal of a small number of large residuals should result in the variables being normally distributed. To test this the largest absolute residuals are removed from the data set and the test for normality is repeated. As can be seen from table 7.14.2 even removing the 100 largest residuals

[11] The individual coefficient tests for stability are not reported in order to conserve space. Statistically significant deviations from the most parsimonious form of the OLS models are noted in the text.

results in all but one of residual series (that for the BS£ second principal component) remaining non-normal using the normality test implemented by Hendry and Doornik (1996). Thus even if 5.8% of the sample is eliminated the non-normality of the residuals usually remains.

Although this is far from a formal test it does suggests that the view that a few aberrant observations is the cause of the non-normality of the residuals is not tenable. There is no simple correction for non-normality short of finding the source of the news that generates the non-normal residuals; and, as we have used most of the regular quantitative news items as regressors, this would seem to be an impossible task. Consequently, the test statistics reported below may be biased, and this needs to be borne in mind whilst the results are being analysed.

Table 7.14.2 Tests of Normality Omitting Outliers

Number of outliers omitted	BS£ PC1	BS£ PC2	BS£ PC3	SS£ PC1	SS£ PC2	SS£ PC3
0	173.0	12787.7	1158.0	1593.8	2922.0	328.0
10	126.2	1134.8	1034.8	1394.6	2025.0	265.8
20	116.3	1093.3	964.2	1420.9	1571.1	254.1
100	20.5	5.3	12.2	1053.2	656.4	127.0

Note: PCi is the i^{th} principal component. The test is distributed as $\chi^2(2)$ and the 1% critical value is 9.21.

The results of the regression using the first principal component from the SS£ data are reported in table 7.14.3. The same procedure is used as for the BS£ data. The diagnostic statistics show that the regression exhibits residual autocorrelation, heteroscedasticity and non-normal errors. The only test that the model passes is the RESET test for model specification against an alternative that the squared predicted dependent variable has been omitted. It should be noted that the inclusion of up to five lags of the dependent variable in the regression did not remove the residual autocorrelation. The parsimonious model, regression 5, exhibits within sample instability both due to variance non-constancy and because of the coefficients on the effective exchange rate and Libor residuals being statistically non-constant.

Regressions 3 and 5 both find statistically significant news effects from retail sales with the same sign and similar (but not identical) coefficient values. Although for the SS£ data the base rate is statistically significant, it is not for the BS£ data. Both data sets find roles for the residuals from the FT-SE 100, the effective exchange rate, the US dollar and Libor terms. Again these terms each have the same signs and similar magnitude between the two data sets. The models differ in that the BS£ data contains no role for Libor residual squared but does contain a role for the FT-SE 100 residual cubed. Whereas the reverse is true of the SS£ data. These results suggest that ONS data releases do not provide news that systematically alters the level of interest rates in the term structure apart from news about retail sales. In particular, news about fiscal developments, whether in the form of unexpected changes in the PSBR or changes in the stock of gilts and its maturity composition, do not alter the level of interest rates. However, news emanating from unknown (i.e. non-ONS) sources that affects other financial markets does appear to have effects on the term structure of interest rates.

Table 7.14.4 below completes the analysis of the first principal components by estimating ARCH(1) models for both the BS£ data (regressions 6 to 8) and SS£ data (regression 9). In each case a statistically significant parameter could be found on the lagged squared residuals. However, attempts to explain the variance of the residuals by using news effects for retail sales and producer prices were less successful. The models usually failed to converge, and on the one occasion when they did the parameter was statistically insignificant (regression 8). Moreover, the ARCH test for heteroscedasticity is now passed for each of these equations, although the residuals remain non-normally distributed. The results also confirm that for the BS£ data changes in the base rate are statistically insignificant but by comparing regressions 7 and 8 it can be seen that no significant changes to the model occurs by dropping this variable.

Table 7.14.3 Regression Results for the Change in the First Principal Component using SS£

	Regression 4		Regression 5	
	coefficient	t-statistic	coefficient	t-statistic
constant	0.009448	1.80	0.006574	1.26
average earnings	0.197030	1.93	-	-
industrial production	0.009842	0.52	-	-
producer input prices	0.011758	0.36	-	-
producer output prices	-0.219340	-1.70	-	-
retail sales	0.043029	2.75	0.052667	3.35
RPI	-0.109010	-0.74	-	-
unemployment	-0.000889	-0.65	-	-
PSBR	0.000017	0.94	-	-
Base rate	0.101600	2.04	0.127120	2.39
bond issue 1	0.014927	1.28	-	-
bond issue 2	0.055708	1.15	-	-
bond issue 3	0.047306	1.17	-	-
bond issue 4	0.075466	1.47	-	-
bond issue 5	0.017173	0.33	-	-
bond issue total	-0.239430	-1.30	-	-
FT residual	-0.064279	-7.48	-0.045045	-3.20
EER residual	-0.083943	-3.20	-0.124440	-6.79
US dollar residual	-0.037539	-3.67	-0.026749	-2.27
DM residual	-0.033487	-1.61	-	-
Libor residual	0.608660	8.76	0.659410	9.46
RPI squared	0.190150	1.35	-	-
FT squared	-0.001096	-0.46	-	-
Libor squared	-0.334610	-2.45	-0.267060	-2.26
FT cubed	0.000674	1.80	-	-
Libor cubed	0.116180	0.91	-	-
R^2	0.30		0.28	
RSS	76.285		78.963	
AR 1-2	23.85 (0.000) F(2,1694)		23.04 (0.000) F(2,1712)	
ARCH 1	375.53 (0.000) F(1,1694)		264.01(0.000) F(1,1712)	
Normality $\chi^2(2)$	1227.3 (0.000)		1593.8 (0.000)	
X_i^2	3.700 (0.000) F(47,1648)		19.483 (0.000) F(13,1700)	
$X_i * X_j$	1.20 (0.000) F(252,1443)		11.502 (0.000) F(34,1679)	
Reset	0.19 (0.67) F(1,1695)		0.542 (0.462) F(1,1713)	
Variance Instability	0.892		0.878	
Joint Instability	6.482		3.733	

Note: See 7.14.1

Table 7.14.4 ARCH Models of the First Principal Components

	Regression 6		Regression 7		Regression 8		Regression 9	
	BS£		BS£		BS£		SS£	
Variable	Coefficient	t-value	Coefficient	t-value	Coefficient	t-value	Coefficient	t-value
constant	0.005583	1.49	0.005883	1.57	0.005931	1.59	0.009164	1.91
producer output prices	-0.24848	-2.45	-0.265380	-2.62	-0.265870	-2.67	-	-
retail sales	0.033601	2.60	0.032063	2.48	0.032569	2.50	0.051185	3.30
base rate	0.067298	1.93	-	-	-	-	0.092944	2.10
FT residual	-0.05926	-12.65	-0.059535	-12.71	-0.059687	-12.71	-0.044173	-10.96
EER residuals	-0.08984	-7.51	-0.087995	-7.37	-0.087688	-7.33	-0.120870	-8.23
US Dollar residual	-0.03982	-5.71	-0.039857	-5.70	-0.039926	-5.71	-0.034971	-4.08
Libor residuals	0.51044	14.64	0.534710	16.33	0.533500	16.27	0.645000	14.10
FT residuals cubed	0.000498	6.71	0.000499	6.73	0.000504	6.70	-	-
Libor residuals squared	-	-	-	-	-	-	-0.234370	-3.22
ARCH constant	-0.02208	-22.31	-0.022063	-22.23	-0.022056	-22.24	0.033707	23.44
ARCH lagged squared residuals	-0.17584	-4.89	-0.179200	-4.93	-0.179430	-4.93	0.233170	6.58
ARCH retail sales residuals	-	-	-	-	0.001839	0.56	-	-
Log Likelihood	2285.14		2283.27		2283.42		1887.97	
ARCH F(1,1709)	0.032 (0.86)		0.058 (0.81)		0.068 (0.79)		0.179 (0.67)	
Normality $\chi^2(2)$	138.20 (0.00)		139.42 (0.00)		139.31 (0.00)		1090.4 (0.00)	

Note: Variables defined as in table 7.13.1, significance levels in parentheses. ONS news variables in bold type.

7.15 News and the Second Principal Components

Tables 7.15.1 and 7.15.2 summarise the equations for the second principal components of BS£ and SS£ respectively. The final models (regressions 12 and 15) eliminate all the ONS news variables and the changes in the structure of gilts and corporate bonds maturities. Both models contain lagged dependent variables with parameters such that adjustment to news is almost complete within 20 days. The SS£ regression (regression 15, table 7.15.2) also includes the base rate whilst the BS£ regression (regression 12, table 7.15.1) includes the FT residual, Libor and Libor Squared residuals. As the BS£ equation does not exhibit heteroscedasticity in an autoregressive first order form, no ARCH models are estimated. The SS£ data does exhibit first order autoregressive heteroscedasticity; but when an ARCH(1) model is estimated (regression 16), the base rate term becomes statistically insignificant. The second principal component equations also share very low R^2 and the conclusion drawn is that ONS news effects, bonds and gilts issues and changes in financial variables cannot explain changes in the slope of the UK term structure.

Table 7.15.1 Models of the Second Principal Component using BS£ Data

	Regression 10		Regression 11		Regression 12		Regression 13	
Variable	Coefficient	t-value	Coefficient	t-value	Coefficient	t-value	Coefficient	t-value
Constant	-0.000954	-0.48	-0.00069	-0.36	-0.00056	-0.29	-0.00066	-0.34
pc2 lag 1	-0.119810	-4.87	-0.11692	-4.89	-0.11338	-4.76	-0.11498	-4.79
pc2 lag 2	-0.042695	-1.73	-	-	-	-	-0.03802	-1.55
pc2 lag 3	-0.090349	-3.72	-0.08168	-3.40	-0.07624	-3.21	-0.07994	-3.34
pc2 lag 4	-0.049481	-1.90	-0.04116	-1.63	-	-	-	-
pc2 lag 5	-0.016559	-0.65	-	-	-	-	-	-
average earnings	-0.024306	-0.79	-	-	-	-	-	-
industrial production	0.001912	0.21	-	-	-	-	-	-
input prices	-0.016456	-1.75	-	-	-	-	-	-
output prices	0.035671	0.73	-	-	-	-	-	-
retail sales	0.001885	0.34	-	-	-	-	-	-
RPI	0.006354	0.17	-	-	-	-	-	-
unemployment	0.000815	1.99	-	-	-	-	-	-
PSBR	-0.000009	-1.55	-	-	-	-	-	-
Base rate	0.014221	0.80						
bond issue 1	-0.010726	-2.71	-0.00337	-2.43	-	-	-	-
bond issue 2	-0.031672	-1.66	-	-	-	-	-	-
bond issue 3	-0.027183	-1.63	-	-	-	-	-	-
bond issue 4	-0.031230	-1.49	-	-	-	-	-	-
bond issue 5	-0.036506	-1.51	-	-	-	-	-	-
bond issue total	0.160810	2.19	0.039244	1.54	-	-	-	-
FT residual	0.006974	3.04	0.006292	3.67	0.006288	3.66	0.005971	3.44
EER residual	-0.002643	-0.26	-	-	-	-	-	-
US dollar residual	0.007625	1.78	0.00779	2.08	0.007783	2.08	-	-
DM residual	0.006189	0.87	-	-	-	-	-	-
Libor residual	0.066665	2.81	0.052599	2.67	0.052823	2.68	0.041188	2.27
RPI squared	-0.010563	-0.33	-	-	-	-	-	-
FT squared	-0.000181	-0.72	-	-	-	-	-	-
Libor squared	0.120530	3.01	0.078273	2.67	0.075614	2.63	0.083394	2.83
FT cubed	-0.000032	-1.06	-	-	-	-	-	-
Libor cubed	-0.069624	-1.85	-	-	-	-	-	-
R^2	0.05		0.04		0.04		0.03	
RSS	10.74		10.85		10.89		10.92	
AR 1-2	1.33 (0.27) F(2,1689)		1.97 (0.14) F(2,1710)		3.12 (0.04) F(2,1713)		2.37 (0.09) F(2,1713)	
ARCH 1	0.006 (0.94) F(1,1690)		0.001 (0.97) F(1,1710)		0.002 (0.96) F(1,1713)		0.007 (0.93) F(1,1713)	
Normality $\chi^2(2)$	1229.6 (0.00)		1237.6 (0.00)		1231.5 (0.00)		1287.7 (0.00)	
X_i^2	0.60 (0.99) F(57,1633)		1.44 (0.11) F(17,1694)		2.23 (0.01) (F11,1703)		0.70 (0.73) F(11,1703)	
$X_i * X_j$	0.57 (1.00) F(368,1322)		1.508 (0.01) F(42,1659)		2.74 (0.00) F(26,1688)		0.86 (0.67) F(26,1688)	
Reset	0.19 (0.66) F(1,1690)		1.00 (0.32) F(1,1711)		0.76 (0.38) F(1,1714)		1.65 (0.20) F(1,1714)	
Variance Instability	0.60		0.58		0.59		0.63	
Joint Instability	7.82		5.11		4.77		4.34	

Note: See table 7.14.1

Table 7.15.2 Regressions of the Second Principal Component using SS£ Data

	Regression 14		Regression 15		Regression 16	
Variable	Coefficient	t-value	Coefficient	t-value	Coefficient	t-value
Constant	-0.00242	-0.42	-0.00151	-0.29	0.00037	0.09
pc2 lag 1	-0.37819	-6.40	-0.37442	-6.44	-0.25679	-8.10
pc2 lag 2	-0.21062	-4.00	-0.21357	-4.07	-0.15370	-7.34
pc2 lag 3	-0.19572	-3.59	-0.19414	-3.62	-0.16641	-8.40
pc2 lag 4	-0.10437	-2.26	-0.10438	-2.31	-0.15791	-7.12
pc2 lag 5	-0.11968	-3.20	-0.12000	-3.19	-0.090517	-4.26
average earnings	0.08507	1.00	-	-	-	-
industrial production	0.01778	1.15	-	-	-	-
producer input prices	0.02439	0.94	-	-	-	-
producer output prices	0.20396	1.24	-	-	-	-
retail sales	-0.00707	-0.34	-	-	-	-
RPI	0.11492	1.53	-	-	-	-
unemployment	-0.00017	-0.16	-	-	-	-
PSBR	0.00001	0.43	-	-	-	-
Base rate	0.04517	0.93	0.075307	2.05	0.055398	1.85
bond issue 1	-0.00125	-0.08	-	-	-	-
bond issue 2	-0.03034	-0.77	-	-	-	-
bond issue 3	-0.03646	-1.08	-	-	-	-
bond issue 4	-0.01276	-0.30	-	-	-	-
bond issue 5	-0.02999	-0.63	-	-	-	-
bond issue total	0.08189	0.51	-	-	-	-
FT residual	-0.00032	-0.05	-	-	-	-
EER residual	-0.01065	-0.40	-	-	-	-
US dollar residual	0.00345	0.33	-	-	-	-
DM residual	0.01005	0.43	-	-	-	-
Libor residual	0.08264	1.36	-	-	-	-
RPI squared	-0.12723	-1.86	-	-	-	-
FT squared	0.00168	1.36	-	-	-	-
Libor squared	0.06699	0.44	-	-	-	-
FT cubed	0.00042	2.28	-	-	-	-
Libor cubed	-0.02816	-0.20	-	-	-	-
ARCH Constant	-	-	-	-	0.025466	21.02
ARCH Lagged residuals squared	-	-	-	-	0.61645	9.38

	Regression 14	Regression 15
R^2	0.15	0.14
RSS	81.40	82.48
AR 1-2	0.250 (0.78) F(2,1689)	1.356 (0.26) F(2,1713)
ARCH 1	82.09 (0.00) F(1,1689)	79.11 (0.00) F(1,1713)
Normality $\chi^2(2)$	2945.2 (0.00)	2922.0(0.00)
X_i^2	3.2841 (0.00) F(57,1633)	13.13 (0.00) F(12,1702)
$X_i * X_j$	1.222 (0.01) F(367,1323)	8.38 (0.00) F(27,1687)
Reset	1.273 (0.26) F(1,1690)	0.80 (0.37) F(1,1714)
Variance Instability	0.738	0.739
Joint Instability	4.152	1.587

Notes: See table 7.14.1

7.16 The Third Principal Component Regressions

Table 7.16.1 presents the regressions for the third principal components. The results are quite dissimilar between the BS£ data and the SS£ data. The former, regression 18, passes all the diagnostic tests (table 7.16.2), except for residual normality, whilst the latter, regression 20, fails all the tests except for Reset and the joint variance and coefficient instability tests. This latter result is due to the constancy of the individual coefficients rather than the constancy of the error variance, which fails both the variance test and the ARCH(1) test. The third principal component from the SS£ data fails the test for residual autocorrelation despite the presence of five lagged dependent variables.

With one exception, bond issuance at the longest maturity, the news variables that are statistically significant differ between the two data series. The BS£ data have the RPI residual and its square significant, whereas the SS£ data only have the residual on Libor significant, as the coefficient on the industrial production residual becomes statistically insignificant when first order ARCH effects are accounted for in regression 21.

The different structures and parameters of the lagged dependent variables also mean that the proportion of a shock that occurs to the model differs. Following a one-unit shock the BS£ model eventually results in the third component being 0.833 higher, whereas for the SS£ data the corresponding figure is 0.399. Both models complete almost all of the adjustment within 20 days. The presence of the squared RPI residual means that the response of the BS£ third component is non-linear. It is only positive over the range 0.0 to 0.711. As this upper limit is well above the absolute mean of the forecast errors recorded in table 7.9.1 of 0.163, the non-linear effects may not be often present in the data. Finally, it should be noted that the explanatory power of the BS£ model is woefully low at just 3% compared with around 23% for the SS£ data.

Table 7.16.1 Regressions of the Third Principal Component Scores.

	BS£				SS£					
	Regression 17		Regression 18		Regression 19		Regression 20		Regression 21	
Variable	Coefficient	t-value	Coefficient	t-value	Coefficient	t-value	Coefficient	t-value	Coefficient	t-value
Constant	0.0006	0.35	0.0012	0.73	0.0008	0.14	0.0014	0.27	-0.0021	-0.49
pc3 lag 1	-0.1508	-6.14	-0.1460	-5.95	-0.5165	-14.82	-0.5168	-14.95	-0.5583	-17.36
pc3 lag 2	-0.0573	-2.19	-0.0546	-2.07	-0.3293	-8.85	-0.3271	-8.94	-0.3359	-11.33
pc3 lag 3	-0.0226	-0.75	-	-	-0.2146	-5.71	-0.2136	-5.73	-0.2506	-8.53
pc3 lag 4	-0.0257	-1.00	-	-	-0.1830	-4.98	-0.1849	-5.08	-0.2066	-7.71
pc3 lag 5	0.0043	0.13	-	-	-0.1411	-4.42	-0.1385	-4.31	-0.1558	-6.93
average earnings	-0.0194	-0.62	-	-	-0.0595	-0.91	-	-	-	-
industrial production	-0.0026	-0.36	-	-	-0.0416	-2.20	-0.0412	-2.19	-0.0410	-1.93
producer input prices	0.0040	0.53	-	-	-0.0387	-1.39	-	-	-	-
producer output prices	-0.0524	-1.51	-	-	-0.1800	-1.22	-	-	-	-
retail sales	0.0043	0.95	-	-	0.0043	0.23	-	-	-	-
RPI	0.0505	2.13	0.04780	1.98	0.0467	0.70	-	-	-	-
unemploy-ment	-0.0006	-1.52	-	-	0.0006	0.49	-	-	-	-
PSBR	-0.0000	-1.02	-	-	-0.0000	-1.53	-	-	-	-
Base rate	-0.0248	-1.86	-	-	0.0457	1.01	-	-	-	-
bond issue 1	0.0079	1.73	-	-	0.0104	0.75	-	-	-	-
bond issue 2	0.0180	1.13	-	-	0.0556	1.15	-	-	-	-
bond issue 3	0.0169	1.18	-	-	0.0527	1.24	-	-	-	-
bond issue 4	0.0251	1.41	-	-	0.0420	0.80	-	-	-	-
bond issue 5	0.0464	2.31	0.0206	2.42	0.0989	1.51	0.0593	2.10	0.0703	2.49
bond issue total	-0.1203	-1.73	-	-	-0.2001	-1.01	-	-	-	-
FT residual	0.0011	0.57	-	-	0.0118	2.05	-	-	-	-
EER residual	-0.0005	-0.06	-	-	0.0139	0.53	-	-	-	-
US dollar residual	0.0020	0.56	-	-	-0.0031	-0.30	-	-	-	-
DM residual	0.0011	0.19	-	-	0.0053	0.26	-	-	-	-
Libor residual	-0.0192	-0.98	-	-	0.0995	1.61	0.1260	2.46	0.1066	3.01
RPI squared	-0.0690	-3.02	-0.0672	-2.96	-0.0476	-0.60	-	-	-	-
FT squared	-0.0005	-1.51	-	-	-0.0023	-2.22	-	-	-	-
Libor squared	0.0595	1.90	-	-	0.0505	0.51	-	-	-	-
FT cubed	-0.0001	-1.57	-	-	-0.00050	-3.30	-	-	-	-
Libor cubed	0.0051	0.16	-	-	0.0738	0.64	-	-	-	-
ARCH constant	-	-	-	-	-	-	-	-	0.0283	19.30
ARCH lagged squared residuals	-	-	-	-	-	-	-	-	0.4106	7.12

Note: Variables defined as in table 7.13.1. ONS news variables are in bold type.

Table 7.16.2 Regression Diagnostic Tests For the Third Principal Component

	Regression 17	Regression 18	Regression 19	Regression 20	Regression 21
R^2	0.04	0.03	0.24	0.23	-
RSS	7.80	7.93	73.49	74.66	-
AR 1-2	0.181 (0.83) F(2,1689)	0.884 (0.41) F(2,1714)	20.884 (0.00) F(2,1689)	26.489 (0.00) F(2,1711)	-
ARCH 1	0.028 (0.87) F(1,1689)	0.011 (0.91) F(1,1714)	82.472 (0.00) F(1,1689)	83.341 (0.00) F(1,1711)	2.282 (0.13) F(1,1708)
Normality $\chi^2(2)$	1158.0 (0.00)	1158.0 (0.00)	333.91 (0.00)	327.97 (0.00)	387.33 (0.00)
X_i^2	0.877 (0.73) F(57,1633)	0.190 1.00) F(9,1706)	3.524 0.00) F(57,1633)	10.946 (0.00) F(16,1696)	-
$X_i * X_j$	1.055 (0.25) F(368,1322)	0.205 (1.00) F(18,1697)	1.545 (0.00) F(368,1322)	5.848 (0.00) F(44,1668)	-
Reset	0.156 (0.69) F(1,1690)	0.021 (0.89) F(1,1715)	1.448 (0.23) F(1,1690)	1.300 (0.25) F(1,1712)	-
Variance Instability	0.313	0.302	1.161	1.228	-
Joint Instability	6.223	1.056	5.420	2.450	-

Note: AR 1-2 is a test of first or second order autocorrelation in the residuals. ARCH 1 is a test of autoregressive conditional heteroscedasticity in the residuals. Normality is a test for the normal distribution of the residuals. X^2 , X_i*X_j and Reset are all tests of functional form misspecification. The variance instability and joint instability test for non-constancy of the residual variance and of the residual variance and the coefficient estimates.

What conclusions can be drawn from these results? The non-normality of the residuals makes interpretation of the t-statistics problematical, but the evidence such as it is suggests that of the ONS data releases only news about producer output prices, retail sales and the RPI effect the term structure. The first two data releases affect the level of interest rates, and the RPI affects the kink in the term structure in a non-linear manner. No ONS data releases were found to have a significant effect on the slope of the term structure. For fiscal variables, the PSBR was always statistically insignificant, and only one bond issue term was found to be statistically significant for the third principal component of the SS£ data. The results are, therefore, not supportive of the view that fiscal news effects are important determinants of the shape of the term structure. The equations also find a role for news that emanates from other financial variables that have been purged of ONS news effects. These undoubtedly contribute to the greater explanatory power of the regressions reported above than was found by Becker et al (1992).

Nevertheless, the results are far from perfect as there are a number of instances of equation misspecification either through explicit tests such as the RESET and X^2 tests and through the presence of autocorrelated residuals. Given the range of the data included, it is difficult to believe that adding further variables, such as monthly trade statistics or consumer credit figures, or by entering overseas data would cure these problems. Rather the solution to residual autocorrelation probably lies in using a much more extensive set of lagged dependent variables and lags on the independent variables as well. The problem of misspecification may be rectified by a much more aggressive use of squared and cubed variables, plus some use of cross product terms and non-linear filters on the variables. Taken to an extreme such models would contain thousands of variables that would exhaust the degrees of freedom of even the relatively large data bases used in this study. Unfortunately, therefore, a more limited approach has to be used in examining non-linearity.

The disappointing results for the second and third principal components in terms of explanatory power prompt the question whether the regression approach adopted in this study a good strategy. There are a number of ways of looking at this. Firstly, it should be recalled that the second and third components explain at most 17.5% of the variation in the data (see table 7.A.1 in the appendix). Thus being unable to explain movements in these variables does not mean that we cannot explain changes in the levels of interest rates where most of the variability occurs. Secondly, it may be the case that ONS data contains little that is relevant to the determination of prices in the gilt market thus finding that interest rates do not react to ONS news would be unsurprising.

The above points raise the question what form of news would move the markets? Does the news have to be totally unexpected as opposed to ONS news where the timetable of publication is known in advance or is the important news a series of one-off events that could not be captured except via dummy variables in a regression analysis? If one-off news is the main cause of changes in gilt prices, then the alternative strategy used in section 7.12 may be the best approach of discerning news effects. However, such an approach risks becoming circular - prices moved significantly on a given day, and this must have been driven by a piece of news irrespective of whether or not such news seems trivial. Another alternative would be to only perform the regression on days when ONS news was released thus allocating all of the variation in gilt prices to ONS news. The problem with this is that any systematic movement between omitted variables and the ONS news will result in the parameters being inconsistent. Furthermore, such a cross section approach invalidates almost all of the diagnostic statistics that can be used to test the appropriateness of the specification. This is not very appealing.

Rather than dropping the regression approach, I expect that refining the measurement of news will allow progress to be made in determining which ONS news items move the markets. One avenue of research would be to drop the assumption that the news in the data release is instantaneously incorporated into the market's expectations. By allowing it to take time for market participants to fully evaluate or learn the importance of data the market will continue to react to past news for some time. For example, it may take two or three months of biased predictions before the market is fully convinced that a new trend in a variable has been established. This does not mean that the market is inefficient: it just means that interpretation of stochastic data is difficult. This learning approach moves analysis of news away from a simple expected/unexpected dichotomy and puts it into the context of how the news changes the market's perception of the direction and speed that the economy is developing. As such it has the potential to explain non-linear effects, why the market's reaction to news may vary over time (thus building on the results of section 7.17) and why markets may appear to react with a lag to some news items. The drawback to this approach is that separating data releases into an expected and unexpected component is no longer enough, what would need to be known is how the news item has changed the market's forward expectations for all relevant ONS variables. Whilst Kalman filtering models would be useful in this role such an exercise must await further research.

7.17 Stability of News Effects

There are two main reasons to expect that the parameter estimates of news effects will not be constant over time. Firstly, as discussed in chapter 2, whether the change in the PSBR was due to cyclical or structural influences and the reaction of the authorities will determine the sign of the response to changes in fiscal stance. This latter factor means that news effects estimated over long periods of time are likely to be unstable unless changes in the authorities' reaction function are accounted for. This section briefly studies the stability of the estimated parameters. Secondly, market participant's perceptions of what constitutes significant news may change over time, and this may be related to the cyclical state of the economy. Both of these reasons suggest that better results might be obtained if the regressions were conducted over shorter periods in which the state of the economy was, therefore, more constant.

To do this the monthly GDP series calculated by the National Institute is used to distinguish three phases of the economic cycle between January 1984 and August 1990. Between January 1984 and May 1985 the economy operated at below trend capacity with the output gap being 0.8% in January 1984 and 0.3% in May 1984. From June 1985 to November 1988 activity exceeded trend capacity by an increasing percentage from 0.3% to 6.4%, although during the first year of this period the excess of output over trend was disguised by the effects of the year long miners' strike. The third period, December 1988 to August 1989 activity began to slow so that the excess of output above trend declined from 6.0% to 2.9%.

Tables 7.17.1 and 7.17.2 give the parameter estimates and t-statistics for the regressions over the three sub-periods for the first principal component scores. A few observations will make it clear that the model is not stable over time. For example, the base rate coefficient estimated using SS£ data (table 7.17.1) is statistically significant in both the earliest and the latest periods, but the sign of the parameter has changed between the periods. The RPI terms change signs between the middle and later periods. Retail sales gain significance in the later period and producer output prices gain statistical significance in the middle period but not at other times. The coefficient on the Libor residual is over twice as large in the middle period as in the earliest period. There is evidence of bond supply effects in the middle period but not in the earliest or later periods. As can be seen from table 7.17.2 almost exactly the same comments can be made about the split sample regressions using the BS£ data. As the behaviour of the ONS news parameters is similar over the two models this suggests that the instability of the parameters is not simply noise, rather they are measuring the differing emphasis put on news over the cycle and changes in the reaction function of the authorities. These results are not definitive and further work on this topic seems to be warranted

Table 17.7.1 Regression Stability using SS£ Data

observations	7 to 365		366 to 1278		1279 to 1727	
Variable	Coefficient	t-value	Coefficient	t-value	Coefficient	t-value
Constant	0.022625	2.13	0.005128	0.62	0.025547	2.17
average earnings	0.0834	1.01	0.032087	0.16	0.39078	1.63
industrial production	-0.011389	-0.27	0.015117	0.62	-0.034749	-0.91
producer input prices	-0.056183	-0.70	0.029234	0.70	0.01032	0.19
producer output prices	0.10648	0.58	-0.639970	-2.39	-0.003150	-0.02
retail sales	0.011484	0.39	0.033762	1.39	0.081635	2.63
RPI	0.26838	1.56	-0.399390	-2.89	0.78309	3.26
unemployment	-0.005069	-1.91	-0.001663	-0.93	0.0015682	0.63
PSBR	-4.11E-05	1.15	0.000029	1.27	1.83E-05	0.00
Base rate	0.19231	2.61	0.034358	0.47	-0.1503	-2.79
bond issue 1	0.18399	1.17	0.045124	2.86	0.30167	1.03
bond issue 2	0.93322	1.30	0.184880	2.02	0.87379	1.00
bond issue 3	0.60936	1.26	0.128170	1.70	1.1575	1.03
bond issue 4	0.7782	1.24	0.192440	2.35	0.73541	1.34
bond issue 5	0.83398	1.50	0.141100	1.46	0.65118	0.86
bond issue total	-3.368	-1.34	-0.686320	-2.92	-3.1525	-0.87
FT residual	-0.09905	-6.42	-0.054534	-4.63	-0.0799	-3.59
EER residual	-0.090897	-1.94	-0.071455	-1.94	-0.10535	-1.82
US dollar residual	-0.03175	-1.82	-0.041209	-2.66	-0.034804	-2.01
DM residual	0.024554	0.71	-0.047177	-1.55	-0.030051	-0.70
Libor residual	0.35062	3.55	0.825250	6.04	0.584000	3.44
RPI squared	-0.14104	-0.30	0.440470	3.92	-0.93474	-3.54
FT squared	-0.005227	-0.81	-0.000307	-0.13	-0.019114	-1.25
Libor squared	-0.45932	-1.92	-0.423320	-0.95	-0.19075	-0.36
FT cubed	0.0045044	2.27	0.000616	1.68	0.000714	0.06
Libor cubed	0.3318	1.70	-0.122560	-0.09	-0.37277	-0.31

Note: Variables defined as in table 7.13.1

Table 7.17.2 Regression Stability using BS£ Data

observations	7 to 365		366 to 1278		1279 to 1727	
Variable	Coefficient	t-value	Coefficient	t-value	Coefficient	t-value
Constant	0.022636	2.12	0.002169	0.37	0.022856	2.46
average earnings	0.006443	0.09	-0.109280	-0.68	0.438010	2.24
industrial production	0.009154	0.27	0.008492	0.38	-0.019885	-0.69
producer input prices	-0.028780	-0.43	0.033417	0.92	-0.020406	-0.39
producer output prices	0.061585	0.38	-0.608970	-2.97	-0.093458	-0.56
retail sales	0.014318	0.53	0.023895	1.20	0.046863	1.70
RPI	0.315770	1.95	-0.291490	-2.65	0.630890	3.74
unemployment	-0.006923	-2.82	-0.001182	-1.00	0.000723	0.36
PSBR	-0.000023	-1.00	0.000018	1.11	0.000043	0.01
Base rate	0.173120	2.25	0.053487	0.90	-0.106380	-1.94
bond issue 1	0.097192	0.59	0.041777	3.18	0.010621	0.04
bond issue 2	0.470270	0.62	0.157090	2.04	-0.004824	-0.01
bond issue 3	0.288030	0.57	0.105040	1.66	-0.000848	0.00
bond issue 4	0.371890	0.56	0.125920	1.92	0.187550	0.41
bond issue 5	0.494240	0.84	0.080251	1.05	0.024394	0.04
bond issue total	-1.774300	-0.67	-0.526300	-2.54	-0.026741	-0.01
FT residual	-0.108120	-7.16	-0.043099	-5.16	-0.065126	-3.80
EER residual	-0.079031	-2.00	-0.087973	-3.10	-0.088399	-1.92
US dollar residual	-0.036893	-2.14	-0.042512	-3.82	-0.020211	-1.50
DM residual	0.021517	0.64	-0.012102	-0.53	-0.027169	-0.84
Libor residual	0.371970	3.98	0.652970	7.20	0.427930	3.08
RPI squared	-0.448660	-1.05	0.342040	3.72	-0.708730	-3.90
FT squared	-0.004506	-0.70	0.000242	0.15	-0.013252	-1.12
Libor squared	-0.390940	-1.57	-0.277720	-0.86	-0.093241	-0.23
FT cubed	0.005871	2.63	0.000538	2.14	0.000457	0.05
Libor cubed	0.301150	1.46	-0.071310	-0.07	-0.693190	-0.73

Note: Variables defined as in table 7.13.1

7.18 Numerical Significance of News Effects

In this section the numerical significance of the parameter values is examined by calculating how news affects the term structure of interest rates. The first three components explain between 99.5% and 99.2% of the total variance of the data, and these high percentages justify terminating the study at the first three principal components.[12] The eigenvectors are reported below, and for each data set the first corresponds to the level, the second the slope and the third a kink in the term structure. Letting A represent the eigenvector matrix, and through the orthogonality condition it's inverse is A'. P is the vector of principal component scores and X the vector of interest rates. Then it is simple to show that:

[12] The eigenvalues and eigenvectors are reported in the appendix.

$$A' \Delta P = \Delta X \ ...(7.18.1)$$

Where: Δ is the change operator.

As ΔP is formed from the left hand variables in the regressions numbered 9, 16, and 21 for SS£ and 8, 12, and 18 for BS£ above it can be easily substituted out. Hence it is straightforward to calculate the change in the term structure from economic shocks. It is assumed that the fourth to the twelfth principal components are all unaffected by the shocks and hence their changes are set to zero. Using either the ARCH models or the most parsimonious representation of each of the components, tables 7.18.1 to 7.18.4 show the effects of shocks on the term structure. The size of the each of the shocks is taken as the average absolute forecast error given in table 7.9.1 and by the average absolute residual or change for the financial and debt variables. Each of these averages is only calculated for the days that the data changed so that the absolute change for all days would be much smaller.

There are three main points to be made. Firstly, the changes are small when compared with the both the average interest rates, and their variances. It would take a considerable number of shocks all in the same direction before there was a significant change in the term structure. This means that for an individual ONS data release there is little incentive for market makers to trade on the basis of their expectations because the systematic return is low. For a £1mn position on a two-year gilt trading at par, the average shock to the retail sales figures would return just £200 using SS£ data. As there is little incentive to trade on expectations of ONS data releases there is little incentive to produce accurate forecasts. Consequently, this may well be one reason why the MMS survey forecasts, analysed in the first part of this paper, are biased and inconsistent. Of course, as forecast performance deteriorates then the returns to trading on ONS data releases would increase and this may encourage renewed efforts to improve forecasting ability. Thus market forces hold the forecasters in a form of stasis, not bad but certainly not as good as it easily could be.

Secondly, the response of the term structure to shocks is not always of the same sign, e.g. the long maturity debt change and the RPI data for the BS£ data and base rates for the SS£ data. Hence, the choice of maturity in single regression studies may well produce conflicting results between studies. This result also means that it should be possible to construct portfolios of gilts that are immune to certain types of news. A simple example would be buying a mixed portfolio of short and medium term bonds. This will be immune to changes in long gilt issuance in certain proportions of holdings because the spot rates, and hence prices, move in opposite directions.

Finally, not all of the shocks take time to reach their final value. If anything the majority overreact and then decline back to their long run level, although the adjustment, as noted above is completed within four weeks. This latter result implies that the gilts market is not strictly weak-form efficient. A simple strategy will result in profits above those attainable from a simple buy and hold strategy. Following positive Libor news the trader should go long on gilts because over the next 19 days or so the price will rise, as revealed by the autocorrelated nature of the price movements; but this autocorrelation of price changes invalidates weak form efficiency. However, the gains to such strategies will be small and may be outweighed by transaction costs so that in practice the gilts market may be weak-form efficient.

Table 7.18.1 SS£ Data Short Term Responses to News

Maturity	retail sales news	long gilt funding news	base rate news	FT-SE 100 news	effective exchange rate news	US dollar news	Libor news
2	0.0201	0.0330	0.0453	-0.0139	-0.0158	-0.0079	0.0256
4	0.0193	0.0021	0.0347	-0.0134	-0.0152	-0.0076	0.0205
6	0.0180	-0.0119	0.0276	-0.0124	-0.0141	-0.0071	0.0172
8	0.0163	-0.0169	0.0213	-0.0112	-0.0128	-0.0064	0.0146
10	0.0144	-0.0167	0.0155	-0.0099	-0.0113	-0.0057	0.0127
12	0.0125	-0.0134	0.0103	-0.0086	-0.0098	-0.0049	0.0112
14	0.0107	-0.0081	0.0058	-0.0074	-0.0084	-0.0042	0.0101
16	0.0091	-0.0016	0.0019	-0.0063	-0.0071	-0.0036	0.0093
18	0.0075	0.0053	-0.0015	-0.0052	-0.0059	-0.0030	0.0086
20	0.0060	0.0123	-0.0044	-0.0042	-0.0047	-0.0024	0.0080
22	0.0047	0.0194	-0.0069	-0.0032	-0.0037	-0.0018	0.0076
24	0.0034	0.0263	-0.0091	-0.0024	-0.0027	-0.0014	0.0073

Note: Measured as changes with 1%=1. Calculated from the regressions numbered 9, 16, and 21. The producer output and RPI news effects are zero.

Table 7.18.2 SS£ Data Long Term Responses to News

Maturity	retail sales news	long gilt funding news	base rate news	FT-SE 100 news	effective exchange rate news	US dollar news	Libor news
2	0.0201	0.0132	0.0361	-0.0139	-0.0158	-0.0079	0.0229
4	0.0193	0.0008	0.0303	-0.0134	-0.0152	-0.0076	0.0204
6	0.0180	-0.0047	0.0258	-0.0124	-0.0141	-0.0071	0.0182
8	0.0163	-0.0068	0.0215	-0.0112	-0.0128	-0.0064	0.0161
10	0.0144	-0.0067	0.0174	-0.0099	-0.0113	-0.0057	0.0141
12	0.0125	-0.0054	0.0135	-0.0086	-0.0098	-0.0049	0.0123
14	0.0107	-0.0032	0.0101	-0.0074	-0.0084	-0.0042	0.0108
16	0.0091	-0.0006	0.0070	-0.0063	-0.0071	-0.0036	0.0094
18	0.0075	0.0021	0.0043	-0.0052	-0.0059	-0.0030	0.0081
20	0.0060	0.0049	0.0019	-0.0042	-0.0047	-0.0024	0.0070
22	0.0047	0.0077	-0.0003	-0.0032	-0.0037	-0.0018	0.0060
24	0.0034	0.0105	-0.0022	-0.0024	-0.0027	-0.0014	0.0051

Note: Measured as changes with 1%=1. Calculated from the regressions numbered 9, 16, and 21. The producer output and RPI news effects zero at all maturities. If the news item does not enter the equations for the second or third principal component scores its short run and long run responses are equivalent.

Table 7.18.3 BS£ Data Short Term Responses to News

Maturity	producer output price news	retail sales news	RPI news	long gilt funding news	FT-SE 100 news	effective exchange rate news	US dollar news	Libor news
2	-0.0123	0.0106	0.0044	0.0078	-0.0124	-0.0095	-0.0075	0.0171
4	-0.0125	0.0108	-0.0020	-0.0035	-0.0146	-0.0097	-0.0076	0.0160
6	-0.0122	0.0105	-0.0039	-0.0069	-0.0151	-0.0094	-0.0074	0.0149
8	-0.0112	0.0096	-0.0037	-0.0065	-0.0143	-0.0086	-0.0068	0.0134
10	-0.0099	0.0085	-0.0025	-0.0044	-0.0129	-0.0076	-0.0061	0.0116
12	-0.0088	0.0076	-0.0012	-0.0020	-0.0117	-0.0068	-0.0054	0.0101
14	-0.0080	0.0069	0.0002	0.0003	-0.0109	-0.0061	-0.0049	0.0090
16	-0.0076	0.0065	0.0013	0.0023	-0.0105	-0.0058	-0.0046	0.0084
18	-0.0075	0.0064	0.0022	0.0039	-0.0105	-0.0058	-0.0046	0.0082
20	-0.0076	0.0066	0.0023	0.0040	-0.0107	-0.0059	-0.0047	0.0084
22	-0.0079	0.0068	0.0030	0.0053	-0.0110	-0.0061	-0.0048	0.0087
24	-0.0082	0.0070	0.0028	0.0049	-0.0113	-0.0063	-0.0050	0.0091

Note: Measured as changes with 1%=1. Calculated from the regressions numbered 8, 12, and 18. The base rate news effect is zero at all maturities.

Table 7.18.4 BS£ Data Long Term Responses to News

Maturity	producer output price news	retail sales news	RPI news	long gilt funding news	FT-SE 100 news	effective exchange rate news	US dollar news	Libor news
2	-0.0123	0.0106	0.003677	0.0065	-0.0130	-0.0095	-0.0075	0.0167
4	-0.0125	0.0108	-0.00164	-0.0029	-0.0148	-0.0097	-0.0076	0.0158
6	-0.0122	0.0105	-0.00329	-0.0058	-0.0151	-0.0094	-0.0074	0.0149
8	-0.0112	0.0096	-0.00309	-0.0054	-0.0142	-0.0086	-0.0068	0.0134
10	-0.0099	0.0085	-0.00211	-0.0037	-0.0128	-0.0076	-0.0061	0.0117
12	-0.0088	0.0076	-0.00097	-0.0017	-0.0116	-0.0068	-0.0054	0.0102
14	-0.0080	0.0069	0.000136	0.0002	-0.0107	-0.0061	-0.0049	0.0091
16	-0.0076	0.0065	0.001101	0.0019	-0.0103	-0.0058	-0.0046	0.0085
18	-0.0075	0.0064	0.001827	0.0032	-0.0103	-0.0058	-0.0046	0.0084
20	-0.0076	0.0066	0.001914	0.0034	-0.0105	-0.0059	-0.0047	0.0085
22	-0.0079	0.0068	0.00252	0.0044	-0.0108	-0.0061	-0.0048	0.0089
24	-0.0082	0.0070	0.002321	0.0041	-0.0111	-0.0063	-0.0050	0.0093

Note: Measured as changes with 1%=1. Calculated from the regressions numbered 8, 12, and 18. The base rate news effect is zero at all maturities. If the news item does not enter the equations for the second or third principal component scores its short run and long run responses are equivalent.

Can any economic interpretation be put on these numerical findings? For the FT-SE 100 a positive shock lowers the level of the term structure because in order to maintain proportionality of holding period returns between equities and gilts the price of gilts has to rise and hence interest rates have to fall. For the exchange rate variables a shock appreciation implies that interest rates must fall otherwise uncovered interest parity will fail. An increase in Libor feeds through the term structure on the basis of the expectations hypothesis but with some belief that it will be partly reversed so that longer rates do not rise as much. For a base rate shock the rise is expected to be more than offset at the long end of the maturity range. For the long funding variable a positive shock increases the supply and hence reduces the price of long bonds raising long interest rates relative to shorter-term rates. The rise in rates of two to four years' maturity can be interpreted as due to higher expected inflation. This is either because the funds raised by the gilt issue are used to fund projects that raise activity or because the government will choose to fund its borrowing by a higher inflation rate so that short rates rise. Higher than expected RPI inflation raises two-year rates as higher interest rates are expected, but because these usually have effects spread over a number of years inflation, will be lower over the medium term and so will interest rates. Hence the term structure rates decline in the medium term. The argument then reverses itself for longer maturites and long-term interest rates rise. A positive retail sales shock indicates a faster growing economy than expected and hence the authorities will raise interest rates to compensate. Thus there are explanations, although they may not always be mutually compatible, for each of the variables with the exception of producer output price shocks. It is not clear why higher inflation from this source would lead to a fall in the level of the term structure. That interpretation can be made of the changes in interest rates resulting from shocks provides comfort that the procedures are finding results that cannot be dismissed as statistical aberrations.

7.19 Conclusions

This chapter has shown one possible reason why economists' forecasts are biased and inconsistent - because the returns to accurate forecasting of monthly ONS data series is low as the effect ONS news releases have on gilts prices is small and unstable. This chapter has provided some innovative results. Particularly, it showed that there is little reason to believe that survey data as used in news studies will become out of date within a one to two week time horizon. The potential for the degradation of the forecast survey has been ignored in almost all of the literature on news effects and left open the possibility that the estimated parameters were inconsistent. As the returns to being correct are small there is little point in revising a forecast once it is made. Consequently, the results are internally consistent. Rather than criticising the forecasting accuracy of City forecasters, perhaps the appropriate question is to ask how they do so well given that they are unlikely to devote many resources to producing the forecast.

The chapter also showed that it is possible to use principal components to analyse news effects. The estimated models explained more of the variation in the data than does the single equation approach of, for example, Becker et al (1992), although part of this may be due to using a wider data set. The results are found to be unstable, and there is some evidence of non-linear responses, although these are unlikely to be significant for the majority of news releases. As in previous work not all ONS data releases are statistically significant. In particular, there is little evidence that news about the PSBR or gilt issues or redemptions, other than at a long maturity, have any effect on the term structure. Moreover, the low explanatory power of the equations for the second and third principal component scores means that the presence of lagged dependent variables does not imply that weak form efficiency of the gilts market should be rejected. This conclusion is strengthened when it is recalled that no allowance for transaction costs has been made.

The instability of the results also implies that the government cannot use its informational advantage to maximise the price received from gilt tap sales. The government knows the figures the ONS will publish a few days in advance. As the market's forecast is also known, the government can calculate the news element If the responses to news were stable the government could take advantage by issuing tap stock on the days when the price would rise secure in the knowledge that debt issuance would not depress prices. However, because the response to news is unstable, the government cannot use this informational advantage. The presence of debt change variables in the regressions rule out the possibility that the responses to ONS news are unstable because of decisions by the government as to whether or not to issue or redeem debt.

Clearly more work can be done in this area with the simultaneous modelling of the changes in principal component scores and the news models of the other financial variables would be worth investigating. In view of the results recording the high correspondence between the largest residuals across some components it would also be worth finding out whether this also applied to other financial variables and indeed what actually happened on these days when large residuals were recorded. In the next chapter the simultaneous determination of the first two principal component scores within a small macro model is examined, and the event study approach is left to future researchers.

Appendix 7.1 Eigenvalues and Eigenvectors of the First Three Components

Table 7.A.1 Eigenvalues and Eigenvectors of the First Three Components

	SS£			BS£		
	First principal component	Second principal component	Third principal component	First principal component	Second principal component	Third principal component
Eigenvalue	6.954	1.141	0.348	6.81	0.814	0.19
% of total variance	82.0	13.4	4.1	86.5	10.3	2.4
Spot rate maturity (years)						
2	0.443	0.508	0.575	0.367	0.742	0.461
4	0.427	0.241	0.037	0.374	0.286	-0.206
6	0.397	0.095	-0.207	0.364	0.068	-0.412
8	0.359	-0.015	-0.295	0.334	-0.033	-0.388
10	0.318	-0.106	-0.291	0.296	-0.097	-0.264
12	0.276	-0.179	-0.234	0.262	-0.148	-0.121
14	0.237	-0.237	-0.141	0.238	-0.191	0.017
16	0.200	-0.283	-0.028	0.226	-0.226	0.138
18	0.165	-0.318	0.092	0.223	-0.250	0.229
20	0.133	-0.344	0.215	0.228	-0.260	0.240
22	0.103	-0.364	0.338	0.236	-0.252	0.316
24	0.076	-0.378	0.458	0.244	-0.223	0.291

Chapter 8

A Small Stylised Macro Model with an Embedded Term Structure

8.1 Introduction.

This chapter analyses the dynamic behaviour of the term structure using a small stylised macro model of the form suggested by Turnovsky (1986), Turnovsky and Miller (1984), Blanchard (1981) and, more recently, Webber (1997). The central feature of these models is an IS function that contains long term interest rates and an LM curve that is driven by short term interest rates. However, empirical researchers in the UK have found it difficult to distinguish a role for long term interest rates in the UK. One reason for this may be multicolinearity that makes separate identification of short and long term rates difficult. Britton and Whitley (1997) could find no statistically significant role for long term rates, even in the form of the spread, in equations for domestic demand nor the money demand equation for the UK. However, they could find a role for the spread in money demand equations for Germany and France.[1] This result suggests that the models mentioned above may be too specific in their specification of long rates in the IS function and short rates in the LM function. A second criticism of these models is that real activity is generally regarded as being driven by real and not nominal interest rates. On the other hand, money demand is driven by nominal rates because the alternative investments (on bonds) are also deflated by the same inflation rate (and thus cancels out leaving the nominal terms). Thirdly, these models use the expectations hypothesis to tie long term rates and short term rates together over time. However, as surveyed in chapter 2, the expectations hypothesis has not been successful in empirical studies and it may be that other models of interest rates can provide a better empirical explanation of movements in the term structure. The model presented below rectifies these criticisms.

Although the model is highly stylised, it contains elements of two of the four main approaches to interest rate determination: the flow of funds approach and the macro (IS-LM) approach. Moreover, the equilibrium correction mechanism used in a number of the equations is consistent with the empirical (if

[1] See Britton and Whitley (1997) table 2, p161.

not the theoretical) representations of the mean reversion models of the finance literature. The model also includes two other interest rate relationships, a Fisher identity, which is imposed and, potentially, uncovered interest rate parity. Thus the model draws extensively upon the models which have been surveyed in chapter 2 whilst at the same time being innovative in its use of principal components to summarise the term structure. The model is also innovative in that it uses monthly data, rather than quarterly or annual data, to estimate the model.

The model is designed to allow analysis of the effects of fiscal policy changes upon the term structure. These include changes in government spending, changes in the average tax rate, balanced budget changes, changes in the stock of government debt and composition effects of government debt. For any simulation all but one of these fiscal variables are exogenous, but one has to be endogenous to ensure that the government's budget identity is observed. Moreover, the effects can be separated into real and nominal interest rate changes. Thus the model allows an analysis of many of the fiscal policy issues discussed in chapter 2. The simulations at the end of this chapter focus on the model's properties with respect to these fiscal variables and suggests that single equation models relating interest rates and fiscal changes are likely to provide highly misleading results.

The second section of this chapter continues by describing the model. The third section describes the data and provides some prerequisite testing of the level of integration of the variables. Section 4 reports the estimation methodology and the estimation results. Section 5 describes the results of simulation of the model and the final section provides some conclusions and pointers towards further research that could be carried out on the model.

8.2 The Model

The model, excluding parameters and lagged terms, can be described in the following set of equations.

$$Y = Y^* + f(\Delta RR1, \Delta RR2, \Delta(\frac{E * P}{P_w}), GBL^*, M^*, \Delta G, INF^*) \quad ...(8.2.1)$$

Equation 8.2.1, the IS function, states that expenditure, Y, will deviate from its trend, Y^*, because of changes in real interest rates as measured by the first two principal components scores of the real term structure, RR1 and RR2; changes in the real exchange rate, $(E*P/P_W)$, where E is the nominal exchange rate measured so that an increase is an appreciation of the domestic currency in terms of foreign currency, P is the domestic price level discussed latter, and P_W is the "world" price level; deviations in financial wealth from equilibrium holdings, $GBL^*$and M^*, where GBL represents the government's long gilt obligations and M represents other non-gilt liabilities of the government; and changes in exogenous real central government expenditure, G.[2] INF^* is the disequilibrium from the long-run inflation equation, which was included as a measure of possible relative price misinterpretation as suggested by Lucas (1972).

As deviations of expenditure from trend are assumed to be a temporary phenomenon this implies that in the long run a permanent rise in government expenditure crowds out an equivalent amount of private sector expenditure plus net exports (exports minus imports). The speed at which full crowding out occurs is determined by the lags in the model and is an empirical matter. The model also allows for disequilibrium in asset holdings to spill over into the determination of private sector expenditure. A small parameter on the disequilibrium wealth terms does not, however, imply that Ricardian Equivalence holds. It may just imply that financial wealth is not an important determinant of expenditure.

$$\frac{M}{GB + M} = f(Y, ZEROPC1, ZEROPC2, R) \quad ...(8.2.2)$$

Equation 8.2.2 is a standard asset demand function written with non-gilt assets as a proportion of total wealth on the left hand side, being determined by real GDP, the first two principal components of the nominal term structure, ZEROPC1 and ZEROPC2; and the level of the base rate, R. Non-gilt assets include all of the central government's own financing requirement except for the issue of gilts. The level of the nominal term structure ZEROPC1 will be negatively associated with the demand for non-gilt balances because it represents the opportunity cost of holding non-gilt assets. The slope parameter,

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2 Clearly the sum of all disequilibria is zero and, therefore, disequilibrium in short gilt holdings is not included in the model.

ZEROPC2, will be positively related if non-gilt assets are closer substitutes to short term rather than long term gilts. The greater the slope the lower the level of short term bond rates and, hence, the greater the demand for non-gilt assets. The base rate term, R, can be justified in a number of ways. It can be thought of as the measure of short term instruments not covered by the term structure and so will attract a negative coefficient. Alternatively, and more likely, it could measure the return on non-gilt balances themselves (the "own return"), and would, therefore, attract a positive coefficient.[3] This is to be determined by estimation. As R is set exogenously by the monetary authorities it can be seen as a means of monetary control that allows the government to alter the demand for other assets and thus gives it greater freedom in the financing of its budget deficits.

$$\frac{GBL}{GB + M} = f(Y, ZEROPC1, ZEROPC2, R) \quad ...(8.2.3)$$

Equation (8.2.3) is the demand for long bonds, GBL, as a share of total wealth. This may be positively or negatively related to the level of expenditure, Y, positively related to the level of nominal interest rates, ZEROPC1, positively related to the slope of the term structure, ZEROPC2, (the sign being determined by the sign pattern of the second eigenvector in table 8.2.1), and negatively related to the level of base rates, R. Equations (8.2.2) and (8.2.3) imply a further equation for the demand for short gilts through the adding up constraints. As Y is determined by (8.2.3), and GB and M are determined by the government's budget constraint, (8.2.4), and R is treated as being exogenous, equations (8.2.3) and (8.2.4) determine ZEROPC1 and ZEROPC2 and, hence, from these the term structure can be deduced. Due to this (8.2.3) and the equivalent for short gilts are inverted to give interest rate adjustment equations that are then estimated.

The supplies of non-gilt liabilities and gilts is governed by the central government's budget identity (8.2.4) that is:

$$\Delta M + \Delta GBL + \Delta GBS \equiv (G - TY) * P + IP \quad ...(8.2.4)$$

3 This is why it enters the interest payments equation 8.2.5 below.

Where: T is the exogenously set net average tax rate

ΔGBS is the change in the stock of short gilts

IP are interest payments on debt, which are assumed to be free of tax.

Two of the terms on the left-hand side of (8.2.4) are exogenous with the supply of the remaining asset being determined by the identity. Alternatively, the government could choose to supply a given amount of financial assets and it could either alter its real expenditure or the tax rate to maintain the identity. The identity, therefore, imposes no behavioural assumptions and, in particular, it does not ensure that the government is solvent, so this is not a budget constraint as it is sometimes described. The tax term, T, is the net average tax rate because it also picks up the effects of cyclical changes in the economy on expenditures such as unemployment benefit. The tax parameter changes each month to match the seasonal pattern of tax payments to validate the identity without the need of seasonal dummies. Central government expenditure, NOMG (=G*P), includes a measure of negative taxation in the form of income tax allowances.

The interest payments of the government, IP, are simply the product of the coupons paid per bond multiplied by the corresponding number of bonds. The model, however, does not predict coupons but spot rates. If it is assumed that the bonds are issued at par, so that the coupon rate equals the spot rate, then the change in interest payments equals the current spot rate on new issues minus the redeemed (if any) gilts times the spot rate when issued. However, this implies knowledge of the redemption schedule, which is a degree of detail that the model does not possess. All the model gives information on is the net change in gilts, not whether this simultaneously comprises redemptions and issues. Hence, far from being a simple identity, IP has to be modelled, and the following form is chosen.

$$\begin{aligned} IP_t - IP_{t-1} = f(&(ZEROPC1_t * GB_t - ZEROPC1_{t-6} * GB_{t-6}), (ZEROPC2_t * GBS_t - ZEROPC2_{t-6} * GBS_{t-6}), \\ &(ZEROPC2_t * GBL_t - ZEROPC2_{t-6} * GBL_{t-6}), (R_t * M_t - R_{t-6} * M_{t-6})) \quad \ldots(8.2.5) \end{aligned}$$

Equation (8.2.5) implicitly assumes that there is a constant flow of redemptions and new issues within the same maturity class so that interest payments reflect changes in the term structure even if the net issue of gilts is zero and the maturity structure is unchanging. Other equations are equally plausible. However, some restrictions have to be placed on the equation. In particular multiplicative formulations have to be avoided because there is no guarantee that the spot rate measure, ZEROPC1 and ZEROPC2, will always remain positive and this could lead to indeterminacy and perverse results. The lag of six months reflects the semi-annual nature of coupon payments.

Equation (8.2.6) determines the expected level of inflation, INFPC1, over the future two to 24 years. The expected inflation rate rises if GDP exceeds the trend level, if sterling denominated import price inflation rises and if non-gilt assets rise as a percentage of total wealth. If the economy is growing at its trend rate, world inflation is constant and the asset ratio is constant expected UK inflation will also be constant.

$$\Delta INFPC1 = f((Y - Y^*), \Delta(E * P_W), M/(GB + M)) \quad ...(8.2.6)$$

$$INFPC2 = f(R, (G/Y), t, (GBL/GBS), \Delta M/(\Delta M + \Delta GBS + \Delta GBL)) \quad ...(8.2.7)$$

The second component of inflation, INFPC2, measures the change in inflationary expectations along the term structure. As in the long run the level of inflation is determined by world inflation, the slope of inflationary expectations is postulated to be determined by the government's policy instruments in equation (8.2.7). These take some time to work through into actual inflation. Monetary policy is hardly ever abruptly altered and fiscal policy is usually only changed once a year. Consequently, the setting of these instruments gives the private sector clues about the longer-term intentions of the government. Determining what sign the variables should take is dependent upon the sign pattern of the second eigenvector given in table 8.2.2. This shows that as long-term inflation expectations rise relative to short term expectations INFPC2 falls. A relaxed current policy would indicate that inflation was likely to be higher in the future than it is currently and, hence, inflationary expectations would slope upwards (INFPC2 falls). Thus, INFPC2 is positively related to the level of base rates and the marginal tax rate and is negatively related to the ratio of government expenditure to trend GDP, the ratio of the long bonds to short bonds and the proportion of the current deficit that is being financed by non-gilt liabilities. The rationale for the latter two terms is as follows. If the government increases the long bond stock, financial markets perceive that this is because the authorities believe that long term rates are currently cheap. One possible reason for this is that they intent to let the inflation rate rise in the future, thus reducing the value of the real debt and interest repayments. Hence, financial markets increase their inflationary expectations, long bond spot rates rise and this justifies the authorities' view that current long-term spot rates were cheap. Similarly, the rise in the proportion of the deficit that is financed by the issue of non-gilt liabilities is an indication that future inflation is likely to rise relative to current inflation. This occurs as excess non-gilt balances are, in part, spent raising activity above trend and eventually raising the inflation rate.

The first two principal components of the nominal term structure are determined by the interaction of the asset demand equations and the government's budget constraint. The corresponding inflation components are determined by the output gap equation and by government policy. This means that real rates, which help determine expenditure, can be derived from the principal component version of the Fisher identity, which is described below.

Letting NR represent the vector of nominal term structure principal component scores for all n interest rates, REAL being a vector of n real interest rates and IR be the corresponding vector of inflation rate expectations, the vector of the real rate principal component scores RR can be derived as follows:

$$REAL \equiv NOM - INF \text{ (Fisher identity)} \quad ...(8.2.8)$$

$$NR = NOM * A \quad ...(8.2.9)$$

$$IR = INF * B \quad ...(8.2.10)$$

$$RR = REAL * C \quad ...(8.2.11)$$

$$RR \equiv NR * A'C - IR * B'C \quad ...(8.2.12)$$

A, B, and C are the eigenvectors of the nominal, inflation and real interest rate series respectively. As the eigenvectors are constructed under the assumption that they are orthogonal, the inverse of each of these matrices is simply its transpose. Hence, once the principal component scores for nominal rates and inflation expectations are known, the principal component scores for real rates are known as well using (8.2.12). Tables 8.2.1 to 8.2.5 record the eigenvectors that allow (8.2.12) to be calculated. As can be seen from the tables 8.2.1 to 8.2.3 the percentage of the variance explained by the first two eigenvalues is never less than 97.6% (table 8.2.2). This high percentage justifies the decision to model only the principal component scores for the first two eigenvectors.

Table 8.2.1 The Transposed Eigenvector Matrix of the Nominal Interest Rates (A')

maturity (years)	2	4	6	8	10	12	14	16	18	20	22	24
eigen-vector												
1	0.460	0.405	0.371	0.340	0.308	0.276	0.245	0.215	0.187	0.161	0.137	0.115
2	-0.430	-0.264	-0.148	-0.049	0.041	0.123	0.196	0.260	0.317	0.366	0.408	0.445
3	0.633	0.051	-0.214	-0.316	-0.321	-0.269	-0.182	-0.080	0.032	0.150	0.264	0.378
4	-0.403	0.468	0.411	0.123	-0.098	-0.261	-0.302	-0.268	-0.156	-0.015	0.179	0.370
5	-0.175	0.536	0.017	-0.534	-0.222	-0.109	0.240	0.313	0.313	0.022	-0.214	-0.187
6	0.045	-0.175	0.114	0.142	-0.160	-0.225	0.553	-0.178	0.183	-0.400	-0.350	0.452
7	0.013	-0.136	0.308	-0.020	-0.295	-0.182	0.512	-0.189	-0.356	0.436	0.225	-0.317
8	0.026	-0.085	0.194	-0.557	0.588	0.030	0.093	-0.499	0.142	-0.058	0.129	-0.006
9	0.001	-0.004	-0.033	0.163	0.000	-0.352	0.071	0.100	0.271	-0.504	0.626	-0.338
10	0.051	-0.279	0.346	0.149	-0.071	-0.303	-0.326	-0.072	0.604	0.332	-0.241	-0.190
11	0.066	-0.350	0.591	-0.319	-0.103	0.116	-0.168	0.487	-0.209	-0.272	0.050	0.111
12	-0.013	0.013	0.101	-0.022	-0.516	0.665	-0.050	-0.377	0.279	-0.158	0.159	-0.082

Note: First eigenvalue explains 90.8% of the variance and the second eigenvalue explains 6.9%.

Table 8.2.2 The Transposed Eigenvector Matrix of the Inflation Expectations (B')

maturity (years)	2	4	6	8	10	12	14	16	18	20	22	24
eigen-vector												
1	0.408	0.374	0.358	0.340	0.318	0.293	0.267	0.241	0.216	0.192	0.169	0.148
2	0.683	0.285	0.083	-0.040	-0.122	-0.177	-0.215	-0.241	-0.259	-0.269	-0.275	-0.278
3	0.474	-0.120	-0.309	-0.330	-0.271	-0.175	-0.066	0.049	0.161	0.269	0.368	0.461
4	-0.324	0.479	0.375	0.088	-0.164	-0.286	-0.307	-0.267	-0.143	0.018	0.217	0.418
5	-0.174	0.590	-0.148	-0.331	-0.414	0.089	0.138	0.286	0.310	-0.034	0.030	-0.335
6	-0.008	0.024	0.117	-0.428	0.261	0.074	0.318	-0.215	-0.395	-0.106	0.597	-0.240
7	0.019	0.012	-0.294	0.464	-0.424	0.533	-0.027	-0.366	-0.132	0.014	0.276	-0.077
8	-0.021	0.047	0.056	-0.190	0.196	0.146	-0.407	-0.257	0.091	0.738	-0.070	-0.330
9	0.032	-0.176	0.203	0.181	-0.136	-0.416	0.326	-0.451	0.553	0.012	0.135	-0.263
10	-0.025	0.192	-0.354	-0.106	0.485	0.162	-0.278	-0.304	0.442	-0.419	0.066	0.139
11	0.027	-0.109	-0.019	0.299	0.061	-0.249	-0.490	0.431	0.043	-0.125	0.501	-0.372
12	0.056	-0.323	0.580	-0.298	-0.263	0.437	-0.273	0.026	0.243	-0.264	0.033	0.045

Note: First eigenvalue explains 85.7% of the variance and the second eigenvalue explains 11.9%.

Table 8.2.3 The Transposed Eigenvector Matrix of Real Interest Rates (C')

maturity (years)	2	4	6	8	10	12	14	16	18	20	22	24
eigen-vector												
1	0.544	0.398	0.331	0.291	0.263	0.242	0.224	0.209	0.196	0.184	0.174	0.165
2	0.585	0.229	0.057	-0.048	-0.121	-0.177	-0.222	-0.259	-0.292	-0.319	-0.343	-0.365
3	0.42	-0.073	-0.241	-0.332	-0.338	-0.29	-0.19	-0.073	0.066	0.208	0.347	0.492
4	-0.409	0.563	0.422	0.122	-0.133	-0.303	-0.267	-0.215	-0.126	-0.011	0.114	0.258
5	0.065	-0.336	0.346	0.184	-0.094	-0.377	-0.087	0.463	-0.373	0.437	-0.094	-0.133
6	0.087	-0.341	-0.111	0.648	0.182	-0.164	0.069	-0.466	-0.196	-0.124	0.313	0.102
7	0.006	-0.081	0.285	-0.047	-0.358	-0.285	0.518	-0.342	0.449	0.164	-0.013	-0.295
8	0.023	-0.138	0.207	-0.016	0.019	-0.074	-0.414	0.235	0.354	-0.355	0.549	-0.39
9	0.038	-0.272	0.34	0.103	-0.423	0.642	-0.329	-0.251	0.004	0.147	-0.083	0.084
10	0.026	-0.100	-0.003	0.005	0.483	-0.184	-0.451	-0.247	0.455	0.354	-0.353	0.015
11	-0.030	0.226	-0.452	0.562	-0.447	-0.054	-0.155	0.259	0.302	0.048	-0.188	-0.072
12	-0.053	0.277	-0.274	-0.072	0.041	0.169	-0.08	-0.209	-0.232	0.559	0.377	-0.501

Note: First eigenvalue explains 69.2% of the variance and the second eigenvalue explains 28.9%.

Table 8.2.4 The Transposed Eigenvector Matrix of Nominal Interest Rates Multiplied by the Eigenvector Matrix of Real Interest Rates, (A'C)

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.990	-0.025	-0.135	0.022	0.002	0.007	-0.001	0.000	0.004	0.002	-0.004	-0.003
2	0.010	-0.967	0.242	-0.075	-0.002	-0.018	0.008	0.001	-0.009	-0.004	0.009	0.008
3	0.137	0.247	0.956	-0.047	-0.019	-0.037	0.002	-0.006	-0.020	-0.013	0.017	0.021
4	-0.015	-0.063	0.073	0.983	-0.023	0.095	-0.068	-0.007	0.064	0.011	-0.072	-0.029
5	0.000	0.002	-0.042	0.134	-0.160	-0.848	0.315	-0.030	-0.329	-0.111	0.117	0.032
6	0.000	-0.002	0.014	-0.013	-0.174	0.222	0.397	-0.372	-0.102	-0.150	0.009	-0.768
7	0.000	0.002	0.006	0.028	0.458	0.154	0.589	-0.199	0.044	-0.372	-0.274	0.404
8	-0.001	0.001	0.006	-0.054	-0.401	0.009	0.143	0.059	-0.121	0.364	-0.813	0.074
9	0.001	0.001	0.000	0.025	-0.142	0.298	0.217	0.762	-0.421	-0.272	0.104	-0.029
10	0.001	0.002	0.020	0.034	0.328	-0.049	0.467	0.286	0.227	0.690	0.200	-0.149
11	0.001	0.001	0.016	-0.031	0.413	-0.322	-0.209	0.357	0.281	-0.290	-0.429	-0.453
12	0.000	0.000	-0.005	-0.022	-0.524	-0.056	0.248	0.166	0.742	-0.244	0.103	0.100

Table 8.2.5 The Transposed Eigenvector Matrix of Expected Inflation Multiplied by the Eigenvector Matrix of Real Interest Rates, (B'C)

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.984	-0.120	-0.130	0.010	0.003	0.006	-0.001	0.000	0.003	0.002	-0.003	-0.003
2	0.133	0.985	0.105	0.023	-0.003	0.001	-0.003	-0.001	0.001	-0.001	-0.001	0.001
3	0.115	-0.114	0.964	-0.198	-0.015	-0.048	0.012	-0.001	-0.028	-0.012	0.026	0.025
4	0.009	-0.046	0.201	0.969	-0.025	0.081	-0.053	-0.016	0.067	-0.008	-0.048	-0.018
5	0.003	0.003	-0.045	0.109	-0.285	-0.709	0.253	0.154	-0.171	-0.301	0.363	0.253
6	0.000	0.000	-0.005	-0.025	-0.150	0.111	0.044	0.164	-0.212	-0.415	-0.733	0.426
7	0.000	0.000	-0.003	-0.060	-0.308	0.438	-0.033	-0.062	0.540	-0.352	0.380	0.382
8	-0.001	0.001	-0.002	0.016	0.149	-0.209	0.046	-0.031	0.282	0.631	-0.120	0.662
9	0.000	0.000	0.011	0.027	-0.082	0.339	0.880	0.220	-0.096	0.198	0.046	-0.066
10	0.000	-0.001	-0.007	-0.026	-0.804	0.087	-0.284	0.230	-0.172	0.417	-0.026	-0.089
11	-0.001	0.001	0.006	0.019	0.349	0.179	-0.261	0.810	-0.169	-0.004	0.265	0.157
12	0.000	0.000	0.013	-0.039	-0.061	-0.293	0.077	0.433	0.695	-0.054	-0.310	-0.365

The third to the twelfth principal component scores of both inflation expectations and nominal interest rates are assumed to be exogenous. At first glance this may not seem to matter because these components explain little of the variation in inflation and nominal interest rates. However, a priori, there is no reason to suspect that the combination of variables that contains the largest variance of these variables will contain the greatest explanatory power for, say, activity. Rather the argument for exclusion of the higher order principal components rests upon the fact that these components have no simple interpretation. Explaining a variable by another that cannot be given a simple description, except mathematically, and hence is unlikely to have an easily identifiable explanation is not productive. Consequently, higher-order principal components are treated as exogenous to other variables in the model, and they play no role in the simulations performed later in this chapter.

A standard macro-model usually imposes the result that in the long-run inflation (INF) and inflationary expectations are equivalent. As two measures of inflationary expectations are identified in this model an alternative condition is imposed. In the long run inflation (as measured by the change in the price level over the same month of the previous year) is a linear combination of the first and second principal components scores of the Bank of England's inflation expectations series.

In the dynamic equation, changes in both of the inflationary expectations terms plus changes in the effective exchange rate and world prices drive changes in the inflation rate. Once the inflation rate is known it is simple to calculate the price level and from that calculate real from nominal variables.

$$INF = f(INFPC1, INFPC2, \Delta E, \Delta P_W) \quad ...(8.2.13)$$

The effective exchange rate is postulated to be determined by two main factors: relative interest rates between the UK, US and Germany, with higher UK rates leading to an appreciation, and the ratio of nominal GDP to trend nominal GDP. This latter term is proxied by trend real GDP multiplied by world prices. The reason for this latter term is that if nominal demand exceeds its trend level, the exchange rate would be expected to appreciate to reallocate output away from exports and towards domestic demand. This would simultaneously making imports cheaper to purchase (assuming that the UK is a small nation with no influence on world prices).

$$E = f((Y * P)/(Y^* * P_w), R, GER3MON, US3MON) \quad ...(8.2.14)$$

The trend rate of expenditure, Y^*, is derived from the following equation:

$$Y_t^* = (1 + g) * Y_{t-1}^* \quad ...(8.2.15)$$

(8.2.15) requires that the monthly underlying growth rate be known and that a point at that either activity was at trend or the deviation from trend was known. There is a wide range of estimates of the size of the output gap and the underlying growth rate, but fortunately these have been summarised by a series of papers from the Economic advisers to the Chancellor ("the Wise Men"). These estimate that the output gap in June 1996 ranged from -0.25% to 3% of GDP and that the long term underlying growth rate ranged from 2% to 3%. Taking a simple average of these figures suggests that the underlying growth rate is 2.4% (in month-on-month terms, g is 0.002%) and that the output gap was 1.7% in June 1996. Using (8.2.15) and the above estimates an exogenous estimate of trend GDP can be calculated.

For completeness it should be noted that:

$$GB = GBL + GBS \quad ...(8.2.16)$$

A list of the data sources is given in appendix 8.1.

8.3 Estimation

A prerequisite to modelling is an analysis of the order of integration of the variables to be used. These are recorded in table 8.3.1. Using the ADF test, all of the variables with the exception of the average tax rate and the first principal component of the real rates are integrated of order one, I(1). The second principal component of the real interest rates is borderline I(0) and is treated as I(0) for consistency with the first component. The implication of this finding is that unless the vectors of variables cointegrate, the residual terms from these equations will not be stationary; and the model will not constitute an acceptable representation of the data generating process. The estimation period for the ADF statistics is November 1984 to April 1996. Unless otherwise stated all dynamic equations are estimated over this period and the equilibrium specifications are estimated over the period April 1984 to April 1996. The difference in the estimation period allows for the inclusion of six lagged differences in the dynamic equation (this requires seven months of data to be excluded from the estimation).

Table 8.3.1 Orders of Integration.

Variable and mnemonic	Level	First Difference
Tax receipts (T*P*Y)	ADF(5)= -1.024	ADF(4)= -10.945**
Nominal government expenditure (NOMG)	ADF(5)= 0.602	ADF(4)= -9.426**
Interest payments (IP)	ADF(5)= 1.001	ADF(4)= -30.360**
Prices (P)	ADF(0)= -0.682	ADF(6)= -5.425**
Effective Exchange Rate (E)	ADF(1)= -1.545	ADF(0)= -8.168**
World prices (P_W)	ADF(5)= 0.266	ADF(4)= -5.789**
UK Libor (R)	ADF(2)= -0.875	ADF(3)= -4.951**
US Treasury Bill rate (US3MON)	ADF(6)= -2.155	ADF(5)= -3.605**
German 3 month rate (GER3MON)	ADF(5)= -0.798	ADF(0)= -14.165**
GDP (Y)	ADF(5)= -1.404	ADF(4)= -2.913*
Short gilt stock (GBS)	ADF(0)= 4.1467	ADF(3)= -3.212*
Long gilt stock (GBL)	ADF(2)= 1.0169	ADF(1)= -6.157**
Other government debt (M)	ADF(6)=0.471	ADF(5)= -3.806**
Nominal GDP (Y*P)	ADF(2)= -0.444	ADF(6)= -4.889**
Average marginal tax rate (t)	ADF(6)= -6.945**	ADF(4)= -11.099**
First principal component on the spot rates (ZEROPC1)	ADF(1)= -2.069	ADF(6)= -6.167**
Second principal component on the spot rates (ZEROPC2)	ADF(0)= -2.618	ADF(6)= -9.733**
First principal component on inflation expectations (INFPC1)	ADF(0)= -1.761	ADF(0)= -11.154**
Second principal component on inflation expectations (INFPC2)	ADF(0)= -2.678	ADF(0)= -12.963**
First principal component on the real rates (RR1)	ADF(4)= -3.274*	ADF(6)= -5.884**
Second principal component on the real rates (RR2)	ADF(0)= -2.770	ADF(0)= -12.396**

Note: Critical values are 1% = -3.481, 5% = -2.883. ** = 1% significance, * =5% significance. Lag on ADF determined by the maximum (up to six) statistically significant lagged dependent variable in the regression.

Estimation of the bond equations of the form of (8.2.3) and, through the adding up constraints, the calculation of the parameters on the non-gilt liabilities of the government is achieved by using the Johansen (1988) technique. As the supply of both short and long term gilts is treated as exogenous, these equations are inverted and are used to determine the first and second principal components of nominal spot rates. However, as the cointegrating vectors are easy to re-parameterise we describe them as asset demand equations of the form of (2.7.4) from chapter 2. The only significant difference from (2.7.4) is the presence of a real GDP term, which helps determine the asset shares. This is justified by noting that as the private sector becomes more liquid (in the sense of having higher income) it can alter its portfolios more easily by diverting that month's cash flow to or from an asset class. Hence, when income is high, investors may be prepared to invest a greater share in less-liquid investments than when income is low. As GDP is an endogenous variable within the system, together with the two interest rate terms, this means that the maximum number of cointegrating vectors can be three. One theoretical problem with this specification is that the asset shares can theoretically be in excess of 100%, and whilst it is possible that some asset share is negative (i.e. the government is lending) this is not plausible for the demand for gilt equations. However, this effect of GDP changes on asset shares would not manifest itself for many decades so for practical purposes does not matter.

Table 8.3.2 Tests for Cointegrating Vectors between the First Principal Components and GDP.

Hypothesis number of cointegrating vectors	Maximum eigenvalue statistic	95% critical value	Trace Statistic	95% critical value
p=0	37.73	32.85	88.99	34.9
p≤1	30.09	26.19	51.26	20.0
p≤2	21.17	18.43	21.17	9.2

Table 8.3.2 reports the maximum eigenvalue tests and the trace statistic tests for the number of cointegrating vectors. The results support the hypothesis that there are three cointegrating vectors. In each case the hypothesis given in the first column can be rejected at the 1% level of significance using critical values from Osterwald-Lenum (1992). However, none of the cointegrating vectors correspond to our priors about the determinants of GDP. Consequently, in the analysis that follows the number of cointegrating vectors is restricted to two.

Table 8.3.3 Long Run Parameters on the Asset Demand Equations

	long gilt to wealth ratio	short gilt to wealth ratio	Non-gilt liabilities to wealth ratio	first principal component	second principal component	real GDP 1990 £bn	UK Libor	Constant
Parameter	-1.0	0.0	-	-.008196	-.006283	-.003513	0.0	0.64121
SE	-	-	-	0.003183	0.003274	0.001848	-	0.13058
RSE	-	-	-	2.57	1.92	1.90	-	4.91
Parameter	0.0	-1.0	-	0.010698	0.00	0.018276	0.0	-0.76766
SE	-	-	-	0.001884	-	0.002504	-	0.14881
RSE	-	-	-	5.68	-	7.30	-	5.16
Parameter	-	-	-1.0	-0.0025	0.006283	-0.01476	0.0	1.12645

Note: SE=standard error. RSE=ratio of parameter to standard error.

The equations imply that the long gilt holdings (the first row of table 8.3.3) are negatively related to both the first and second components of the principal components of the spot data whilst short debt (the fourth row of table 8.3.3) is only related to the first component. Thus a rise in the level of interest rates increases the demand for short gilts at the expense of both long gilts and other government liabilities. A rise in GDP also has a similar qualitative effect. Thus over time a rise in income would raise the asset share of short gilts. The mechanism by which the principal components are determined in the long-run is straightforward from these equations. In equilibrium, GDP equals its trend level; and, as the supply of short gilts to wealth is exogenous, the short term gilts equation determines the first principal component, the level of interest rates with a greater supply of gilts raising spot rates. With the first component determined by the supply of short gilts, the long gilt equation determines the second principal component, the slope of the term structure. The greater the supply of long gilts the lower the second principal component and, from the eigenvector in table 8.2.1 row two, this implies that either long spot rates are higher or short term rates are lower. Non-gilt liabilities passively adjust to the first and second components to ensure that the adding up constraints are met. Of course, these results only derive from the long-run, partial sub-system of asset demands, and they may not necessarily hold in the full model simulations. Indeed they need not hold in the short term for the sub-system itself.

The flow of funds models were reported with the wealth ratios as the dependent variables in order to facilitate identification of the underlying functions. However, the supply of gilts is determined exogenously and, hence, what the estimated functions are actually determining are the first and second principal components of nominal interest rates. The models can be inverted to describe the long run levels of these variables and the differences between these long run equilibria and the actual levels can be calculated as below:

Residual for the first principal component of nominal interest rates (Resz1),

$$Resz1=Zeropc1-93.47541597*(\text{short gilts/ total debt of the central government})+1.708356702*(\text{real GDP}/1000)-71.75733782 \quad \ldots(8.3.1)$$

Residual for the second principal component of nominal interest rates (Resz2),

$$Resz2=Zeropc2+159.1596371*(\text{long gilts/ total debt of the central government})+1.304472386*Zeropc1+0.559127805*(\text{real GDP}/1000)-102.0547509 \quad \ldots(8.3.2)$$

It should be noted that although the first (ZEROPC1) and second (ZEROPC2) principal components scores are uncorrelated, the first component enters the equilibrium correction term for the second component because both components determine the demand for long gilts. These residuals form the equilibrium correction terms used in the dynamic regressions of the changes in the first and second principal components of nominal interest rates, which are described below. These two residuals (lagged one month) form the equilibrium correction mechanisms reported in tables 8.3.4 and 8.3.5.

The dynamic model for the first principal component of nominal interest rates is reported in table 8.3.4.

Table 8.3.4 Dynamic Equation for the Change in ZEROPC1

Variable	Coefficient	t-statistic
Constant	-0.15535	-1.483
Change in real GDP at lag 6	-0.0011837	-2.276
Change in UK Libor	1.0619	6.374
Change in UK Libor at lag 1	-0.69619	-4.122
Change in the ratio of short gilts to total government debt at lag 5	-45.667	-3.012
Change in the ratio of long gilts to total government debt at lag 5	-43.905	-2.802
Change in the ratio of long gilts to total government debt at lag 6	-39.650	-2.689
Dummy variable, 1 in September 1986	4.3730	4.721
Equilibrium correction mechanism at lag 1 as defined in text, Resz1 (equation 8.3.1).	-0.077968	-2.979
Diagnostic statistics		
R^2	0.426	
RSS	106.317	
AR 1-2 F(2,127)	1.4967 (0.23)	
ARCH 1 F(1,127)	3.4519 (0.07)	
Normality Chi^2 (2)	0.038 (0.98)	
X^2 F(15,113)	0.69227 (0.79)	
X*X F(36,92)	0.72383 (0.86)	
RESET F(1,128)	1.8716 (0.17)	

Notes: AR 1-2 is a test of first or second order autocorrelation in the residuals. ARCH 1 is a test of autoregressive conditional heteroscedasticity in the residuals. Normality is a test for the normal distribution of the residuals. X^2 , X_i*X_j and RESET are all tests of functional form misspecification. The variance instability and joint instability test for non-constancy of the residual variance and of the residual variance and the coefficient estimates. Figures in parentheses are the statistical significance of the diagnostic statistics.

The equation finds, unlike the long-run equations, a role for UK Libor so that a rise in Libor initially raises the level of interest rates. A role is also found for the change in the ratio of long gilts to total government debt. The presence of both short and long gilt ratios to total government debt with almost equal parameters implies that an increase in the ratio of non-gilt debt of the government would temporarily raise the level of interest rates. This effect would then be modified by the equilibrium correction term and amplified by the change in the long gilt ratio with a lag of six months. Despite the absence of lagged dependent variables the dynamics will be extended by the presence of the level of interest rates in the equilibrium correction term, which has attracted a correctly signed and statistically significant coefficient. A role for the change in GDP was also found which was consistent with the sign in the long run equation. A dummy variable was required to ensure that the residuals were normally distributed in order to allow valid diagnostic tests - all of which the equation passed.

Table 8.3.5 Dynamic Equation for the Change in ZEROPC2

Variable	Coefficient	t-statistic
Constant	-0.021017	-0.660
Change in the ratio of long gilts to total government debt at lag 5	-16.187	-2.656
Change in UK Libor	-0.09998	-1.516
Change in the second principal component at lag 2	-0.18007	-2.862
Dummy +1 September 1992	2.8922	7.47
Dummy +1 March 1986 and -1 August 1993	1.4408	5.421
Dummy +1 October 1986, -1 November 1988, +1 August 1990	1.1597	5.377
Equilibrium correction mechanism at lag 1, Resz2 (equation 8.3.2).	-0.030776	-0.466
Diagnostic statistics		
R^2	0.497	
RSS	17.9374	
AR 1-2 F(2,128)	1.3034 (0.28)	
ARCH 1 F(1,128)	0.20236 (0.65)	
Normality Chi^2 (2)	3.3372 (0.19)	
X^2 F(13,116)	0.51995 (0.91)	
X*X F(20,109)	0.44479 (0.98)	
RESET F(1,129)	0.19769 (0.66)	

Note: See table 8.3.4

The model for the change in the second principal component of nominal interest rates (table 8.3.5) is less satisfactory although the model, after the addition of three dummies to ensure normally distributed errors, passes all the diagnostic tests. The model contains one lagged dependent variable at two lags and the change in the ratio of long gilts to total debt of the government at lag five. The change in UK Libor is retained, although statistically insignificant, because without its presence the equilibrium correction term is incorrectly signed (positive). The equilibrium correction term is small in absolute terms indicating that adjustment to equilibrium is slow.

After some experimentation it was found that a plausible long-run solution for the effective exchange rate equation could be found only for the period October 1992 to April 1996. This is hardly surprising, as during the 1980s and early 1990s the exchange rate was a member of two exchange rate regimes, the unofficial Lawson tracking of the DM (see Bowen (1995)) and the ERM period. To expect a single equation to cope with these regime changes would be asking too much. Instead of applying regime swapping approaches of, for example, Hamilton (1990), a simpler approach of limiting the data sample to the period October 1992 to April 1996 has been adopted. The main advantage of this is that it is simple to implement.

We do not use the Johansen procedure for the exchange rate model as interpretation of the structural cointegrating vectors was found to be highly subjective. The absence of Johansen estimation is shared with all the large UK macro econometric models. Sorting out which eigenvector corresponds with which structural model is simply too subjective for large numbers of equations. Moreover, as this model contains a number of distinct features, for example the modelling of inflationary expectations, there are no clear guides to the parameter values that can be expected on the basis of past research. Its unique nature also means that some of the identifying restrictions, for example that in the long-run inflation equals inflation expectations, although present have different forms from that usually encountered in macro models. For these reasons the single equation approach is preferred. The remainder of the model was estimated using a two-step Engle and Granger (1987) approach supplemented, on occasion, by the Engle and Yoo (1989) three-stage approach.

After some experimentation the following long-run form for the effective exchange rate was found to produce cointegrating residuals:

$$E=\exp(4.5630+0.62002*\ln((Y*P)/(Y^{*}*P_{W}))+$$
$$0.21296*\ln((1+UKLIBOR/100)/(1+GER3MON/100))) \quad ...(8.3.3)$$

ADF(12)=-3.099 critical value 5%=-2.963, 1%=-3.666.

This equation implies that for every 1% nominal GDP (Y*P) exceeds trend GDP ($Y^{*}*P_{W}$) the exchange rate is 0.62% higher. Thus excess activity is partly offset by having a higher exchange rate. In the long run real GDP would be expected to be equivalent to trend GDP, Y^{*}, so that this term is a relative price term. This might appear to contradict the purchasing power parity (PPP) hypothesis that relatively high UK prices would be associated with a lower exchange rate. However, the rejection of PPP is not convincing because the hypothesis refers to internationally tradable goods and services whilst P is the price deflator of all UK GDP. Secondly, the effect of prices on the exchange rate is a property of the system not a single equation and therefore changes in relative prices on the long-run exchange rate have to await simulations of the whole model. The second term from the exchange rate equation is that the relative interest rates, and this implies that an increase of 1% in UK short term interest rates relative to three-month German rates will cause the exchange rate to appreciate in the long-run by 0.2%.

To check that these effects are statistically significant the Engle and Yoo (1989) procedure was used. This found that the parameter on relative nominal incomes had a coefficient of 0.661251 with a t-statistic of 23.7 and the relative interest rate term had a parameter of 0.26798 and a t-statistic of 3.01. Hence, both terms are statistically different from zero. As can be seen from the ADF(12) statistic the equation cointegrates at the 5% level.

Using the long-run equation to form the equilibrium correction mechanism (ECM) the second stage equation can be formed from the first difference of these variables. The lags have been restricted to six on each variable to avoid the models becoming too cumbersome. The model is estimated using ordinary least squares. The variable with the lowest statistically significant parameters is removed and the procedure is repeated until a parsimonious model is achieved that passes the diagnostic tests. The result of this procedure reported in table 8.3.6 is as follows.

Table 8.3.6 Dynamic equation for the Change in the Effective Exchange Rate.

Variable	Coefficient	t-statistic
Constant	-0.00457	-0.647
Change in the log of the exchange rate at lag 1.	0.41712	3.679
Change in the log of the exchange rate at lag 6.	0.24367	2.142
Change in the log of the ratio of nominal GDP to trend GDP	0.73680	18.618
Change in the log of the ratio of nominal GDP to trend GDP at lag 1	-0.37101	-4.241
Change in the log of the ratio of nominal GDP to trend GDP at lag 6	-0.24499	-2.524
Change in relative short term interest rates at lag 5	0.40924	1.833
Equilibrium correction term (the residual of equation 8.3.3) at lag 1	-0.54012	-4.638
Diagnostic statistics		
R^2	0.944	
RSS	0.00042	
AR 1-2 F(2,32)	0.130 (0.88)	
ARCH 1 F(1,32)	0.237 (0.63)	
Normality Chi^2 (2)	0.197 (0.91)	
X^2 F(14,19)	0.586 (0.84)	
RESET F(1,33)	3.385 (0.08)	

Note: See table 8.3.4

The equilibrium correction term has the correct sign indicating that if the effective exchange rate is above its equilibrium value, about half of this disequilibrium will be eliminated in the next month. The coefficients of the dynamic equation indicate that there is some overshooting of the exchange rate in response to a change in nominal income relative to trend. The adjustment is relatively protracted with the presence of two lagged dependent variables. The only surprise is the absence of a statistically significant relative interest rate term. The interest rate at lag five is only retained because without it the RESET test is failed, recording a value of 5.81 at a significance level of 0.02. All the other tests, for autocorrelation, heteroscedasticity, normality of the errors, and functional form, are easily passed.

The first principal component of inflation expectations was found to have a cointegrating vector (significant at the 1% level and above on the ADF(10) statistic) and this is reported in table 8.3.7:

Table 8.3.7 First Principal Component of Inflation Expectations

Variable	Coefficient
Constant	-14.100
Difference between real GDP and trend GDP as a percentage of trend GDP	0.55239
Non-gilt assets as a percentage of total government financial assets	0.63404
ADF(10) critical values 5% =-2.885, 1%=-3.486	-5.8686

In the long run inflationary expectations are determined by the percentage deviation of output from trend (which would be expected to be zero) and by the percentage of non-gilt to total government liabilities. This latter term may indicate that the greater the liquidity of assets held by the non-government sector, the greater is the threat of higher spending and higher inflation. Consequently, expectations of inflation are also higher.

The dynamic equation for the first principal component of inflationary expectations contained the first differences of the terms in the long-run equation plus the percentage change over the previous month of the effective exchange rate and world prices. These variables were all entered with up to six lags and the residual from table 8.3.7 was added at the first lag as the equilibrium correction mechanism. This equation was substantially simplified by sequentially removing the variable with the lowest t-statistic until the parsimonious form given in table 8.3.8 was arrived at.

Table 8.3.8 Equation for the First Difference of the First Principal Component of Inflation Expectations

Variable	Coefficient	t-statistic
Constant	-0.0510	-0.70
Change in first principal component of inflation expectations at lag 1	0.231	2.93
Change in the percentage difference between output and trend output at lag 5	0.643	3.77
Change in the percentage of non-gilt government assets as a percentage of all government assets	0.302	2.729
Change in the percentage of non-gilt government assets as a percentage of all government assets at lag 2 minus the change in the percentage of non-gilt government assets as a percentage of all government assets at lag 5	-.242	-2.992
Change in the percentage change in world prices at lag 3	0.035	1.083
The equilibrium correction term (the residuals from the equation in Table 8.3.7) at lag 1	-0.480	-7.12
Diagnostic statistics		
R^2	0.35	
RSS	84.484	
AR 1-2 F(2,123)	2.764(0.07)	
ARCH 1 F(1,123)	3.687(0.06)	
Normality Chi^2 (2)	11.726(0.00)	
X^2 F(12,112)	1.183(0.30)	
X*X F(27,97)	1.246(0.22)	
RESET F(1,124)	0.158 (0.69)	

Note: See table 8.3.4

The simplification of the equation led to the removal of all the difference terms in the effective exchange rate. The change in the percentage change in world prices is retained, despite its statistical insignificance at the 5% level, because its removal leads to numerous rejections of the diagnostic statistics. Even its presence cannot remove the non-normalcy of the residuals, and experiments with dummy variables also failed remove this problem. Examination of a histogram of the residuals suggests that the non-normality has to do with a platykurtic and skewed distribution and not to the presence of a few outliers. There is little that can be done to rectify this problem accept to bear in mind that the test statistics probably underestimate the degree of fragility of the equation, but the significance of this is unknown.

A cointegrating vector for the second principal component of inflationary expectations, INFPC2, is recorded in table 8.3.9.

Table 8.3.9 Second Principal Component of Inflation Expectations

Variable	Coefficient
Constant	13.894
UK Libor	0.20374
Real government expenditure to real GDP ratio	-9.5575
Ratio of the long gilt debt to short gilt debt	-4.5901
Ratio of non-gilt debt to total government debt	-25.773
ADF(10)=-3.2185 Critical values 5%=-2.883, 1% =-3.481	

The vector cointegrates at the 5% level of significance. It should be recalled that as INFPC2 decreases, long-term inflationary expectations rise relative to the short-term. Thus, higher UK Libor, which increases INFPC2, is associated with a relative fall in long-term inflationary expectations. Increases in all of the other terms are associated with a rise in relative long-term inflationary expectations. The equation confirms that government policy settings are important for determining changes in inflationary expectations at a given point in time and, through the Fisher identity, in determining the slope of the real term structure. This is important because the equation to determine interest payments by the central government could not find any role for Libor or for non-gilt liabilities. In such circumstances it might appear that funding government expenditure via non-gilt liabilities would carry no financial cost. However, because it raises inflation expectations and real rates have to remain constant (otherwise GDP would deviate from trend in the long run), nominal interest rates on gilts will have to rise. Hence, the equation in table 8.3.9 implies that there is a financial penalty on extra funding by non-gilt sources.

The dynamic equation for INFPC2, found using the variable deletion method explained above, is given in table 8.3.10.

Table 8.3.10 Dynamic Equation for the Change in the Second Principal Component of Inflationary Expectations

Variable	Coefficient	t-statistic
Constant	0.0050	0.12
Change in the ratio of long gilt assets to short gilt assets	-1.8490	-1.61
Change in the ratio of non-gilt assets to total government debt at lag 6.	-19.575	-2.966
Change in 3 month Libor minus the change in 3 month Libor at lag 6	0.19015	3.221
Equilibrium correction term (the residual from the equation from table 8.3.9) at lag 1	-0.22342	-4.211
Dummy for September 1992	-2.3769	-4.655
Diagnostic statistics		
R^2	0.343	
RSS	32.02	
AR 1-2 F(2,130)	0.624(0.54)	
ARCH 1 F(1,130)	0.001(0.97)	
Normality Chi^2 (2)	5.810(0.06)	
X^2 F(10,122)	0.436(0.91)	
X*X F(20,116)	0.533(0.92)	
RESET F(1,131)	0.005(0.94)	

Note: See table 8.3.4

The equation in Table 8.3.10 has the correct sign on the equilibrium correction mechanism, and the signs of the other variables are consistent with their long run coefficients. The change in the ratio of long gilt assets to short gilt assets is retained in the specification, even though it is statistically insignificant at the 5% level, because without it being present the residuals are non-normal failing the χ^2 test at the 5% level. The addition of a dummy variable for September 1992, when it takes the value of one and is zero for all other months, was also required to ensure that the residuals are normally distributed.

In the long run inflation and inflation expectations would be expected to be equal. To be consistent with the expectation series inflation is defined as the change in the GDP deflator for the current month and the same month of the previous year, divided by the price deflator for the month in the previous year multiplied by 100. In this model, which uses two measures of inflationary expectations from the principal components analysis, inflation is found to form the cointegrating relationship given in table 8.3.11.

Table 8.3.11 Cointegrating Vector for Inflation

Variable	Coefficient
Constant	4.5664
First principal component of inflationary expectations	0.14918
Second principal component of inflationary expectations	0.78306
ADF(10) critical values 5% =-2.885, 1%=-3.486	-3.8456

The equation shows that in the long-run, inflation is higher when the first component of inflationary expectation rises and also increases as the second component rises (i.e. short-term expected inflation increases relative to long-term inflation).

The dynamic inflation equation, again simplifying from a specification containing six lags of the differences, is reported in table 8.3.12. The estimation period for this equation was April 1985 to April 1996.

Table 8.3.12 Dynamic Equation for Inflation

Variables	coefficient	t-statistic
Constant	-0.42187	-0.888
Change in the second principal component of inflation expectations at lag 4	0.16263	2.030
Change in inflation at lag 1 minus the change in inflation at lag 2	0.18744	4.002
The change in the first component of inflation expectations at lag 1 plus the change in the first component of inflation expectations at lag 3	-0.11721	-2.991
Equilibrium correction mechanism (residuals from equation given in table 8.3.11) at lag 1	-0.16686	-5.083
Dummy, –1 for February 1990 and +1 for February 1991	3.4472	8.785
Diagnostic statistics		
R^2	0.58	
RSS	33.6205	
AR 1-2 F(2,118)	2.445 (0.09)	
ARCH 1 F(1,118)	1.227 (0.27)	
Normality Chi^2 (2)	5.8071 (0.06)	
X^2 F(10,109)	0.352 (0.96)	
X*X F(16,103)	0.567 (0.90)	
RESET F(1,119)	0.003 (0.96)	

Note: See table 8.3.4

After searching down from six lags, the final dynamic model found no role for the rate of change of the effective exchange rate or in world prices. The inclusion of the dummy variable was required to ensure that the residuals were normally distributed. Once this is included, the model passes all the diagnostic tests. The model contains some further terms in the changes of the first and second components of the inflation term structure and the lag of the change in inflation, the parameters of which are all statistically different from zero. The equilibrium correction term is correctly signed and statistically different from zero.

In the long-run the model imposes the restriction that GDP must equal its trend level. Unfortunately, over the short time period chosen for the analysis (November 1984 to April 1996) this is not the case, and the deviations are so protracted that it is not surprising that GDP and its trend do not form a cointegrating vector. Nonetheless, the theory is imposed on the dynamic equation so that the equilibrium correction mechanism ensures that in the long-run GDP equals its trend value. The dependent variable is the change in the deviation of GDP from its trend, and this is included on the right hand side with up to six lags. Also included in the equation are the differences in the first and second components of the real interest rate term, the change in real government spending and the change in the real exchange rate. All of these variables were lagged up to six times. In addition, the residuals from the long-run short gilts equation and from the (implied) long run other government liabilities equation were also included at lag one to proxy disequilibrium wealth effects. Finally, the residuals from the long-run inflation equation (table 8.3.11) at lag one were also included as a measure of a "Lucas-type" inflation surprises. Such surprises may cause agents to misinterpret changes in inflation as a change in relative prices. Hence they change their behaviour thereby causing a deviation of output from its trend level.

This equation was sequentially simplified until the parsimonious relationship given in table 8.3.13 was found.

Table 8.3.13 Dynamic Equation for the Change in the Deviation of GDP from its Trend

Variable	Coefficient	t-statistic
Constant	-6.4058	-0.55
Change in the deviation of GDP from its trend at lag 5	0.4266	5.68
Change in the deviation of GDP from its trend at lag 6	0.2199	2.90
Change in the first principal component of the real interest rate at lag 4	-37.565	-2.39
Change in the real exchange rate	-6.3407	-2.14
Deviation of GDP from its trend at lag 1	-0.0106	-1.163
Diagnostic statistics		
R^2	0.26	
RSS	2260599.74	
AR 1-2 F(2,124)	0.431(0.65)	
ARCH 1 F(1,124)	0.0877(0.77)	
Normality Chi^2 (2)	0.0394 (0.98)	
X^2 F(10,115)	1.207(0.29)	
X*X F(20,105)	1.244(0.23)	
RESET F(1,125)	1.546(0.22)	

Note: See table 8.3.4

The equation passes all the diagnostic tests for the normality of errors, heteroscedasticity, and functional form. The lagged dependent variables are signed so that shocks from the level of real interest rates and the real exchange rate take some time to completely feed through onto the deviation of GDP from its trend. Higher real interest rates and an appreciation of the real exchange rate both temporarily depress activity relative to trend. No roles were found for inflation surprises, disequilibrium wealth terms nor for changes in real government expenditure. Although correctly signed, the equilibrium correction term attracts a small coefficient that is not statistically significant at the 5% level. It is retained to ensure coherence with the theory, but the calculation of trend GDP is, clearly, an area requiring further work.

Many attempts were made to model interest payments. However, the results were generally unsatisfactory, either because of incorrectly signed parameters, statistically insignificant parameters or failures to pass diagnostic tests. As the interest payments equation is little more than a linking equation, a decision was taken to use a relatively simple equation in the model that would have clear properties and would not therefore interfere with the interpretation of the model's overall properties. The equation reported in table 8.3.14 was adopted.

Table 8.3.14 Dynamic Equation for the Change in Interest Payments as a Ratio of Total Gilts Outstanding Minus in Interest Payments as a Ratio of Total Gilts Outstanding at Lag 6.

	Coefficient *1000	t-statistics
Constant	-.007499	-0.736
First principal component of nominal interest rates minus first principal component of interest rates at lag 6	0.034661	0.958
Second principal component of nominal interest rates multiplied by the ratio of short gilts to total gilts minus the second principal component of nominal interest rates multiplied by the ratio of short gilts to total gilts at lag 6	0.12209	0.867
Diagnostic statistics		
R^2	0.02	
RSS	0.0001719	
AR 1-2 F(2,127)	1.052 (0.35)	
ARCH 1 F(1,127)	0.2556 (0.31)	
Normality Chi^2 (2)	0.759 (0.68)	
X^2 F(4,124)	0.047 (0.99)	
X*X F(5,123)	0.145 (0.98)	
RESET F(1,128)	0.380 (0.54)	

Note: See table 8.3.4

This equation models the change in interest payments, as a percentage of gilts outstanding, between the current month and six months earlier (coupons on gilts are paid at six month intervals). This is determined by the change in the level of interest rates, as measured by the first principal component, and by the change in the slope of the term structure weighted by the proportion of gilts that are short term. It should again be recalled that the sign of the principal component scores are arbitrary. Hence, determining what happens to interest payments as maturity of the gilt stock changes is determined by the sign of the second principal component.

In the model (see table 8.2.1, row 2) a rise in the second component implies a steepening of the term structure. Thus, for a given maturity structure, a greater second component implies a rise in interest payments. This is consistent with longer debt being more expensive to service because the government is effectively buying extended freedom from making repayments of principal.

8.4 Simulations

In the descriptions of the model's equations no analysis of the equations' dynamic properties was undertaken. The reason is straightforward - the dynamic properties of each single equation are irrelevant. What matters are how the equations interact as a system. Moreover, no dynamic tracking exercise is conducted because, as Hendry and Doornik (1996) argue, all this shows is whether or not "difficult to model" variables have been exogenised, not whether the model is valid. To illustrate the system's properties a number of simulations are reported in this section. These are:

1) a permanent 1% increase in real government expenditure funded by an increase in non-gilt liabilities.

2) a permanent 1% increase in real government expenditure funded by an increase in short gilt liabilities.

3) a permanent 1% increase in real government expenditure funded by an increase in long gilt liabilities.

4) a permanent 1% increase in real government expenditure funded by an increase in taxes.

5) a temporary decrease in the tax rate of 1% funded by an increase in non-gilt liabilities.

6) a temporary decrease in the tax rate of 1% funded by an increase in short gilts.

7) a temporary decrease in the tax rate of 1% funded by an increase in long gilts

8) a permanent 1% increase in the short gilt stock funded by a matching decrease in the long gilt stock with all subsequent adjustments made to the long gilt stock.

9) a permanent 1% increase in the long gilt stock funded by a matching decrease in the short gilt stock with all subsequent adjustments made to the short gilt stock.

The model was initially solved so that the residuals set tracked the actual monthly data and, hence, the base case is the observed data.[4] All the simulations are solved over 120 months. Few macro models are regularly solved over such a large number of periods; in Wallis et al (1985) for example, the government expenditure simulation is solved over a maximum of 20 periods (five years times four quarters in the case of the National Institute).[5] The extended simulation period has two consequences. Firstly, instability in the model is more apparent. If instability is not reported in other macro-model simulations it may simply be because they have not been given sufficient time to run. Secondly, after 120 periods the standard error bands around the simulations could be expected to be very large, and this

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[4] The model is solved using the LBS Modeler software that was kindly supplied by Brian Henry and James Nixon.

[5] See Wallis et al (1985) table 2.2, p.36.

should be remembered when drawing conclusions from the simulation results reported in the tables below.

It should be noted that although the simulations use permanent changes in real government expenditure temporary changes would also be expected to result in permanent changes in the endogenous variables of the models.[6] The reason is that temporary changes result in permanent changes in the liabilities of the central government. As the ratio of the individual liabilities to total liabilities are also permanently altered, the determinants of demand for liabilities of the central government must also be permanently altered.

The nine simulations are by no means an exhaustive list of the simulations that could be performed using this model. Even so, a complete description of all the endogenous variables within the model over all of the ten year simulation period for just the nine simulations listed above would produce a significant amount of output. In order to avoid this, the descriptions of the behaviour of the non-term structure variables are given in overview in table 8.4.1. The behaviour of the nominal and real term structure structures at one year, two, three, four, five and ten years after the start of the simulations are recorded in the nine tables 8.4.2 to 8.4.4 and 8.4.6 to 8.4.11. A description of why the behaviour occurs should flesh out the overview provided by table 8.4.1.

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[6] Indeed they do in simulations on temporary increases in government spending that have been undertaken but not reported.

Table 8.4.1 Overview of Percentage Differences from Base in the Final Four Months of the Nine Simulations

	Y	P	EER	IP	ZERO PC1	ZERO PC2	INF PC1	INF PC2	RR1	RR2	M	GBS	GBL
1	0.14	-1.53	-0.86	1.03	-4.22	16.60	9.42	11.23	-17.55	22.67	8.75	0.00	0.00
2	-1.01	6.26	3.17	30.52	29.79	-23.07	-24.76	-49.45	86.67	6.23	0.00	43.44	0.00
3	0.06	-2.25	-1.36	9.41	-7.58	-34.22	-12.86	5.72	-4.25	-77.33	0.00	0.00	27.44
4	0.00	-0.24	-0.15	0.00	-0.01	0.03	0.00	0.79	0.02	-0.67	0.00	0.00	0.00
5	-0.04	-0.57	-0.38	0.43	-0.93	6.02	1.94	2.17	-3.47	10.51	1.73	0.00	0.00
6	-0.39	4.45	2.47	13.89	15.54	-15.89	-12.48	-26.75	44.26	-4.49	0.00	20.36	0.00
7	0.01	-1.16	-0.71	2.29	-2.04	-14.79	-4.03	1.37	-1.05	-32.73	0.00	0.00	8.08
8	0.00	0.47	0.29	0.74	0.73	-0.93	-0.71	-1.48	2.19	-0.37	0.00	1.00	0.11
9	0.05	-0.65	-0.37	-1.22	-2.02	0.01	0.93	3.45	-5.16	-3.68	0.00	-2.20	1.00

Note: The numbers in the first column refer to the simulations listed above. The final four months rather than the last month is used to average out any slight anomalies in the monthly data. The mnemonics are explained in the appendix to this chapter.

The overview table highlights a number of points. Firstly, the effect on the economy of an increase in government spending or reduced taxes depends upon the means used to finance these fiscal changes. With the exception of short gilt financing (simulations 2 and 6) the changes in activity by the final four months of the simulations are small. This result is not surprising as, in the long-run, GDP is tied to its trend level. In simulations 2 and 6, GDP is depressed relative to the base because of an appreciation in the real exchange rate and a rise in real interest rates. The exchange rate appreciates because prices have risen relative to world prices; and, in order to reduce the differential inflation rates between the UK and the rest of the world, the exchange rate appreciates to depress activity and subdue inflation. The increased supply of short gilts raises the level of nominal interest rates. The lower level of activity and the lower ratio of non-gilt liabilities to total liabilities (due to the higher stock of short gilts) reduce inflation expectations. Consequently, real interest rates have risen above their base level further depressing activity. Lower activity reduces tax revenues whilst higher interest rates and the higher debt stock increase interest payments. To finance this more short gilts are issued, and the process cumulates so that short gilt financing of increased real government spending and tax cuts is unstable. It should be noted that this instability would be curtailed if the government set nominal, instead of real, expenditure plans. In these circumstances the rise in prices would raise tax revenue but not nominal government expenditure so that the deficit would not cumulate to such an extent.

For money financed and long gilt financed government expenditure increases and tax decreases (simulations 1, 3, 5 and 7) the real exchange rate depreciates and real interest rates fall (subject to a caveat discussed below) hardly changing activity. Without the fall in activity tax revenues are unchanged and lower interest rates means that the rise in interest payments caused by higher debt issuance is much reduced relative to simulations 2 and 6. Consequently, the level of debt issuance is much lower (compare simulations 1 and 3 against 2 and 5, and 7 against 6 in table 8.4.1) although as the stocks are different sizes this accounts for some of the differences in table 8.4.1. The exchange rates depreciated because the price level is lower. The price level is lower because the higher stock of non-gilt liabilities and long gilt liabilities raise long-term inflationary expectations relative to the two-year horizon. One means that such expectations can be fulfilled is by having lower current inflation and, hence, lower current prices. The effects on the level of real interest rates (RR1) are muted because the slope of the real term structure changes, with some rates falling and some rising relative to the base. Thus most of the stimulatory effects on GDP flow from changes in the exchange rate and, as this is relatively small, the effects on GDP are small as well. For simulation 4, the tax financed government expenditure increase, the absence of changes in government liabilities means that the effects on the economy are small.

The final two simulations (simulations 8 and 9) are asset swaps. In simulation 8 the short gilt stock is permanently increased by 1% and any changes to the deficit is financed by long gilts. Whereas in simulation 9 the long gilt stock is increased by 1% and the short gilt stock absorbs the changes in the government's deficit. The results are not mirror images of each other, although the signs of the percentage changes reported in table 8.4.1 are usually opposite. It is noticeable that whereas simulation 8 results in an increase in gilt liabilities of 0.6% by the end of the simulation, simulation 9 results in a reduction of 0.8% in gilt liabilities. With lower liabilities and nominal interest rates, interest payments are lower, and this is the reason that short gilt stocks are lower in simulation 9. Swapping long gilts for short seems to provide an opportunity for the government to reduce the total debt, lengthen the debt maturity, and reduce interest payments whilst lowering inflation and having little effect on activity. Unfortunately, as discussed below, the model is likely to be unstable so that applying this on a large scale could have disastrous consequences. The main point to note from simulations 5 to 9 is that the manner of financing a given level of real government expenditure matters for the development of macroeconomic variables. The estimated model is not Ricardian.

So far we have not described the effects of these simulations on the term structure, in part, because interpreting the principal component scores that the model determines is cumbersome. However, by using the first two rows of tables 8.2.1 and 8.2.3 and the principal component scores from the simulations and base runs, the nominal and real term structures can be calculated, and these are reported in tables 8.4.2 to 8.4.4 and 8.4.6 to 8.4.11. In order to save space the term structures are only calculated at the end of one, two, three, four, five and ten years from the beginning of the simulation. As inflationary expectations can be worked out from these tables they are not reported to save space. In tables 8.4.2 to 8.4.4 and 8.4.6 to 8.4.11 a change of 1.00 equals an increase in interest rates of 100 basis points.

Table 8.4.2 Effects of a Non-Gilt Financed Government Expenditure Increase on the Term Structure

maturity	2	4	6	8	10	12	14	16	18	20	22	24
end of year						nominal						
1	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
2	-0.04	-0.03	-0.02	-0.01	-0.01	0.00	0.01	0.01	0.02	0.02	0.03	0.03
3	-0.15	-0.11	-0.09	-0.06	-0.04	-0.02	-0.01	0.01	0.03	0.04	0.05	0.06
4	-0.32	-0.24	-0.19	-0.14	-0.10	-0.06	-0.02	0.01	0.04	0.07	0.09	0.11
5	-0.49	-0.37	-0.28	-0.21	-0.14	-0.08	-0.02	0.03	0.08	0.12	0.15	0.19
10	-1.31	-0.93	-0.67	-0.44	-0.23	-0.04	0.14	0.30	0.44	0.56	0.67	0.76
end of year						real						
1	0.00	-0.02	-0.02	-0.03	-0.03	-0.03	-0.03	-0.04	-0.04	-0.04	-0.04	-0.04
2	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08
3	-0.25	-0.20	-0.18	-0.17	-0.16	-0.15	-0.15	-0.14	-0.14	-0.13	-0.13	-0.13
4	-0.47	-0.34	-0.29	-0.25	-0.23	-0.21	-0.20	-0.18	-0.17	-0.16	-0.16	-0.15
5	-0.70	-0.49	-0.39	-0.34	-0.30	-0.26	-0.24	-0.22	-0.20	-0.18	-0.17	-0.15
10	-1.77	-1.06	-0.72	-0.52	-0.38	-0.27	-0.18	-0.11	-0.04	0.01	0.06	0.11

Note: Change in interest rates from base. 1.0 equals an increase in rates of 100 basis points. Calculated from simulation 1.

Table 8.4.2 reports the results of the non-gilt financed government expenditure increase on the term structure of interest rates reported at twelve maturities between 2 years and 24 years. The initial effects on nominal interest rates are negligible but over time the increase in non-gilt liabilities causes the level of short term interest rates to fall relative to the base whilst raising longer term interest rates. By year ten the two year spot rate has fallen by 1.3% and the 24 year spot rates have risen by nearly 0.8%. Real interest rates are reduced relative to the base, except for rates of 20 years or more maturity in year ten of the simulation. Whilst the change in real rates is broadly flat in the first two years of the simulation the real term structure steepens from year three onwards. By year ten the slope is nearly 1.9% with real two year rates being nearly 1.8% below the base. The reason is that the extra issuance of non-gilt liabilities to total liabilities and the rise in the ratio of real government expenditure to real GDP both directly affect inflationary expectations. Thus a non-gilt financed increase in government expenditure raises inflationary expectations so that real rates are reduced relative to the base. As non-gilt liabilities include notes and coins and other liquid assets the rise in inflationary expectations can be given a monetarist interpretation.

Table 8.4.3 Effects of a Short Gilt Financed Government Expenditure Increase on the Term Structure

maturity	2	4	6	8	10	12	14	16	18	20	22	24
end of year						nominal						
1	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
2	0.14	0.13	0.12	0.11	0.10	0.10	0.09	0.08	0.07	0.07	0.06	0.05
3	0.34	0.30	0.27	0.25	0.23	0.20	0.18	0.15	0.13	0.11	0.10	0.08
4	0.64	0.55	0.49	0.43	0.38	0.33	0.28	0.23	0.19	0.15	0.11	0.08
5	1.11	0.93	0.82	0.72	0.62	0.52	0.43	0.34	0.26	0.19	0.12	0.06
10	4.62	3.75	3.17	2.67	2.18	1.72	1.29	0.88	0.52	0.19	-0.11	-0.37
end of year						real						
1	-0.12	-0.01	0.04	0.08	0.10	0.12	0.13	0.14	0.15	0.16	0.17	0.18
2	0.17	0.18	0.19	0.19	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.21
3	0.43	0.40	0.39	0.38	0.38	0.37	0.37	0.37	0.36	0.36	0.36	0.36
4	0.82	0.71	0.67	0.64	0.63	0.61	0.60	0.59	0.58	0.57	0.56	0.56
5	1.39	1.17	1.07	1.01	0.97	0.94	0.91	0.89	0.87	0.85	0.84	0.82
10	5.36	3.97	3.34	2.96	2.70	2.50	2.33	2.19	2.06	1.95	1.85	1.77

Note: Change in interest rates from base. 1.0 equals an increase in rates of 100 basis points. Calculated from simulation 2.

Table 8.4.4 Effects of a Long Gilt Financed Government Expenditure Increase on the Term Structure

maturity	2	4	6	8	10	12	14	16	18	20	22	24
end of year						nominal						
1	-0.03	-0.03	-0.04	-0.05	-0.05	-0.05	-0.05	-0.06	-0.06	-0.06	-0.06	-0.06
2	-0.05	-0.07	-0.10	-0.11	-0.13	-0.14	-0.15	-0.15	-0.16	-0.16	-0.17	-0.17
3	-0.04	-0.11	-0.15	-0.19	-0.22	-0.25	-0.27	-0.29	-0.30	-0.31	-0.32	-0.33
4	-0.02	-0.13	-0.22	-0.29	-0.35	-0.40	-0.44	-0.47	-0.50	-0.53	-0.55	-0.56
5	0.08	-0.11	-0.25	-0.37	-0.47	-0.55	-0.63	-0.69	-0.74	-0.78	-0.82	-0.85
10	0.89	0.32	-0.09	-0.43	-0.74	-1.01	-1.25	-1.45	-1.63	-1.78	-1.91	-2.02
end of year						real						
1	0.06	0.02	0.00	-0.01	-0.02	-0.03	-0.03	-0.04	-0.04	-0.04	-0.04	-0.05
2	0.12	0.03	-0.02	-0.04	-0.06	-0.08	-0.09	-0.10	-0.11	-0.11	-0.12	-0.12
3	0.23	0.06	-0.03	-0.08	-0.11	-0.14	-0.16	-0.18	-0.20	-0.21	-0.22	-0.23
4	0.42	0.12	-0.03	-0.12	-0.18	-0.23	-0.27	-0.30	-0.33	-0.35	-0.37	-0.39
5	0.74	0.24	0.00	-0.14	-0.25	-0.32	-0.39	-0.44	-0.48	-0.52	-0.55	-0.59
10	2.08	0.72	0.06	-0.34	-0.62	-0.83	-1.00	-1.15	-1.27	-1.37	-1.47	-1.55

Note: Change in interest rates from base. 1.0 equals an increase in rates of 100 basis points. Calculated from simulation 3.

Tables 8.4.3 and 8.4.4 record the effects on the nominal and real term structures for simulations 2 and 3. The changes to the term structure of short gilt financed government spending conform to expectations. The extra gilt supply raises nominal interest rates and the rise would be greatest for short maturities. Three years after the start of the simulation this is also true for real interest rates. The long gilt simulation, however, fails to conform to this expectation. The increased supply of long gilts reduces interest rates and causes long rates to fall more than short rates although after five years short rates begin to rise. The long-run effects of changes in the short and long gilts on the first principal component score (reported in table 8.4.5) are of different signs, as are the effects on the second principal component score. However, it can be seen from table 8.4.1 that the percentage changes in the second principal components for simulations 2 and 3 are of the same sign. The reason is that the effects of the changes in the short gilt stock in simulation 2 are being masked by the effects of changes in other variables, in this case the change in the first principal component, which rises by nearly 29.8% by the end of the simulation.

Table 8.4.5 Direct Effects of Funding on the Long-Run Component Scores

Method of financing deficits	First Principal Component Score	Second Principal Component Score	Second Principal Component Score including effect of change in first component
Non-gilt financed (M)	-93.475*GBS/W2	159.159*GBL/W2	(159.159*GBL+121.868*GBS) /W2
Short gilt financed (GBS)	93.475*(GBL+M)/W2	159.159*GBL/W2	(37.291*GBL-121.868*M)/W2
Long gilt financed (GBL)	-93.475*GBS/W2	-159.159*(GBS+M)/W2	-(159.159*M+ 37.291*GBS)/W2

Where W2=(GBS+GBL+M)*(GBS+GBL+M) and the constants come from equations 8.3.1 and 8.3.2. The constant 37.291=159.159-1.304*93.475 and 121.868=1.304*93.475.

As can be seen from table 8.4.5 unless the value of short gilts exceeds that of long gilts plus non-gilt liabilities the change in the first principal component of a given change in gilts, ceteris paribus, will be smaller in absolute terms for long gilts. Moreover, the direct change in the second principal component will always be positive on non-gilts and short gilts but negative on long gilts (column three of table 8.4.5). However, when the effects of the change in the first component are added (column four of table 8.4.5) it can be seen that the consequences of changes in short gilts on the second principal component are ambiguous. Unless long gilts are more than 3.268 times as large as non-gilt liabilities the effect of changes in short gilts will be to reduce the second principal component. As non-gilts exceed long gilts throughout the base simulation (at most long gilts are 74% of non-gilt liabilities) changes in short and long gilt stocks will have the same sign (negative) in the long-run. Table 8.4.5 highlights that the results reported in these simulations are base dependent; but, as the changes in the asset stocks have tended to be relatively small, base dependency would be unlikely to change the qualitative nature of the results reported in this section. Table 8.4.5 also demonstrates that if, as is likely, single instrument financing is unstable, then in the long run the increase in the instrument will eventually push the parameter on the principal components to zero because W2 becomes very large; and, hence, the coefficients fall towards zero. Thus there is a limit to the change in the term structures as the government's liabilities increase.

Table 8.4.5 also shows that, unless changes to the debt structure are all the same percentage of their outstanding stocks, financing the deficit by the issue of debt will always change nominal interest rates in the long run. The percentage value itself does not matter because only the relative sizes of the debt instruments matter. In the long-run it is the changes in the asset stocks and not the changes in government expenditure that induces changes in the term structures because in the long-run equilibrium GDP is unaffected by changes in government spending. This can be seen from simulation 4 where the increase in government expenditure is financed by taxation and the effects on the term structures are negligible (see table 8.4.6).

Table 8.4.6 Effects of a Tax Financed Government Expenditure Increase on the Term Structure

maturity	2	4	6	8	10	12	14	16	18	20	22	24
end of year						nominal						
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
end of year						real						
1	0.02	0.01	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
2	0.02	0.01	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
3	0.02	0.01	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
4	0.02	0.01	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
5	0.02	0.01	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
10	0.02	0.01	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01

Note: Change in interest rates from base. 1.0 equals an increase in rates of 100 basis points. Calculated from simulation 4.

In both the short and the long gilt simulations inflationary expectations are reduced (unlike for the non-gilt simulation). For the short gilt simulation the change in real rates are more pronounced than for nominal rates (table 8.4.3). For long gilt financing (table 8.4.4) the change in inflationary expectations is smaller in absolute terms, and this moderates the change in real interest rates for maturities in excess of six years. Whilst gilt financing reduces inflationary expectations the slopes of the expectations term structures for short and long gilts move in opposite directions in simulations 2 and 3. Short gilt financing leads to lower expectations of inflation in the distant future because the short term nature of the extra gilts means that investors can penalise the government for inflation by charging them higher spot rates in the future if need be. Alternatively, long gilt finance tends to raise future inflation expectations, relative to the short term, because investors will find it harder to penalise the government because the longer nature of the new debt means, ceteris paribus, that new issues of debt are less frequent. This difference arises from the presence of the ratio of long to short gilts in the second principal component score of inflationary expectations. The different value of the first component of inflationary expectations, reported in table 8.4.1, is due almost entirely to the different sizes of the change in the gilt stocks (and to a lesser extent the change in GDP in the case of short gilt financing).

Although it is less clear in the overview table 8.4.1, table 8.4.4 shows that long gilt financing also appears to be unstable, although the instability is less apparent by year ten of the simulation than for the short gilt financed simulation reported in table 8.4.3. To an even smaller extent there may be evidence from table 8.4.2 that non-gilt financed government spending is also unstable. It should be pointed out that this instability is a product of the parameter estimates and is not inherent in the structure of the model per se.

The instability of the model does not arise from an unstable relationship between temporary changes in the liabilities of the government and interest rates. If each of the liabilities in turn is raised by 1% and in the next month returned to its base level for the rest of the simulation, with all extra deficits or surpluses funded by changes in taxes, then the effects on nominal and real interest rates die away to zero quite rapidly. The instability lies not in the interest rate equations but rather in how changes in the rest of the economy affect the liability stocks of the government. In turn this can be summarised by two factors. Does the increase in liabilities increase GDP by enough to ensure that extra tax revenues are greater than the increase in interest payments derived from higher liabilities, and do changes in nominal interest rates raise interest rates by more than any increase in GDP raises tax revenues?

In order to investigate these factors further the effects of a temporary decrease in the tax rate of 1% for twelve months are reported in tables 8.4.7 to 8.4.9. As was shown in table 8.4.6 the consequences of government spending funded by tax changes is negligible, and by using tax changes even this residual element is removed. By making the tax change only temporary it was hoped that the dynamics would become easier to describe as the model returns to its base path.

Table 8.4.7 Effects of a Temporary Decrease in the Tax Rate of 1% Funded by an Increase in Non-Gilt Liabilities.

maturity	2	4	6	8	10	12	14	16	18	20	22	24
end of year						nominal						
1	0.00	0.01	0.01	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03
2	-0.13	-0.10	-0.07	-0.05	-0.04	-0.02	0.00	0.01	0.03	0.04	0.05	0.06
3	-0.30	-0.23	-0.18	-0.14	-0.10	-0.06	-0.03	0.01	0.03	0.06	0.08	0.10
4	-0.26	-0.17	-0.12	-0.07	-0.02	0.02	0.06	0.10	0.13	0.15	0.17	0.19
5	-0.48	-0.36	-0.27	-0.20	-0.13	-0.06	0.00	0.05	0.10	0.14	0.18	0.21
10	-0.40	-0.27	-0.18	-0.11	-0.04	0.03	0.08	0.14	0.18	0.22	0.26	0.29
end of year						real						
1	-0.06	-0.07	-0.08	-0.08	-0.08	-0.08	-0.08	-0.09	-0.09	-0.09	-0.09	-0.09
2	-0.24	-0.19	-0.17	-0.16	-0.15	-0.14	-0.14	-0.13	-0.13	-0.12	-0.12	-0.12
3	-0.44	-0.31	-0.25	-0.21	-0.19	-0.17	-0.15	-0.14	-0.13	-0.12	-0.11	-0.10
4	-0.58	-0.38	-0.28	-0.23	-0.19	-0.16	-0.13	-0.11	-0.09	-0.08	-0.06	-0.05
5	-0.65	-0.40	-0.28	-0.21	-0.16	-0.12	-0.09	-0.07	-0.04	-0.02	-0.01	0.01
10	-0.52	-0.28	-0.16	-0.09	-0.04	0.00	0.03	0.06	0.08	0.10	0.12	0.13

Note: Change in interest rates from base. 1.0 equals an increase in rates of 100 basis points. Calculated from simulation 5.

In simulation 5, the temporary cut in the tax rate funded by non-gilt liabilities, leaves non-gilt liabilities 1.73% higher in the final four months of the simulation although they had been 3.0% higher two years after the beginning of the simulation. This can be taken as evidence that non-gilt finance is stable for temporary changes in tax rates (the deviation of GDP from base in table 8.4.1 being regarded as being minor). The extra issuance of non-gilts in this simulation stabilises at a value equivalent to about 1.7% of the base value in the long-run. Likewise, the funding of the temporary tax cut by long gilts also appears stable as the deviation of long gilts from their base value had risen to 16% after five years and three-months but in the final years of the simulation has levelled out at 8%. On the other hand, short gilt financing does appear to be unstable. GDP in the final four-months of the simulation is nearly 0.4% lower and this causes extra short term gilts to be continued to be issued ten years after the start of the simulation. Consequently, in the final four-months of the simulation short term gilts are nearly 20.4% greater than their base value.

It is noticeable that simulation 5 (funding by non-gilt liabilities) and simulation 7 (funding by long gilts) match their counter parts for a permanent change in government expenditure (simulations 1 and 3) in qualitative terms. They are smaller in quantitative terms apart from GDP for simulation 5, which we have claimed above is trivially different from its base value. For the short gilts, however, the signs of the percentage changes in the second component scores of real interest rates is negative for the temporary tax change but positive for the permanent government expenditure increase. However, comparison of tables 8.4.3 and 8.4.8 do not reveal any substantive differences in the qualitative behaviour of the term structures.

Table 8.4.8 Effects of a Temporary Decrease in the Tax Rate of 1% Funded by an Increase in Short Gilt Liabilities.

maturity	2	4	6	8	10	12	14	16	18	20	22	24
end of year	nominal											
1	0.08	0.08	0.07	0.07	0.07	0.07	0.07	0.07	0.06	0.06	0.06	0.06
2	0.34	0.30	0.28	0.26	0.24	0.22	0.20	0.18	0.16	0.14	0.12	0.11
3	0.60	0.51	0.46	0.41	0.36	0.31	0.26	0.22	0.18	0.14	0.11	0.07
4	0.85	0.71	0.61	0.53	0.45	0.37	0.29	0.23	0.16	0.10	0.05	0.01
5	1.16	0.95	0.81	0.68	0.57	0.45	0.35	0.25	0.16	0.08	0.00	-0.06
10	2.58	2.06	1.71	1.40	1.11	0.83	0.58	0.34	0.13	-0.07	-0.24	-0.39
end of year	real											
1	0.05	0.13	0.16	0.19	0.20	0.22	0.23	0.23	0.24	0.25	0.25	0.26
2	0.38	0.38	0.39	0.39	0.39	0.40	0.40	0.40	0.40	0.40	0.40	0.40
3	0.72	0.61	0.56	0.53	0.52	0.50	0.49	0.48	0.47	0.46	0.45	0.44
4	1.04	0.83	0.73	0.68	0.64	0.61	0.58	0.56	0.54	0.53	0.51	0.50
5	1.41	1.08	0.94	0.85	0.79	0.75	0.71	0.67	0.65	0.62	0.60	0.58
10	2.95	2.11	1.72	1.48	1.32	1.20	1.09	1.01	0.93	0.86	0.81	0.75

Note: Change in interest rates from base. 1.0 equals an increase in rates of 100 basis points. Calculated from simulation 6.

Table 8.4.9 Effects of a Temporary Decrease in the Tax Rate of 1% Funded by an Increase in Long Gilt Liabilities.

maturity	2	4	6	8	10	12	14	16	18	20	22	24
end of year	nominal											
1	-0.08	-0.10	-0.12	-0.13	-0.14	-0.15	-0.16	-0.17	-0.17	-0.17	-0.17	-0.17
2	-0.06	-0.12	-0.16	-0.19	-0.22	-0.25	-0.27	-0.28	-0.30	-0.31	-0.31	-0.32
3	0.04	-0.06	-0.13	-0.18	-0.23	-0.28	-0.32	-0.35	-0.38	-0.40	-0.41	-0.43
4	0.12	-0.02	-0.12	-0.20	-0.28	-0.34	-0.40	-0.44	-0.48	-0.52	-0.54	-0.57
5	0.23	0.05	-0.08	-0.19	-0.29	-0.37	-0.45	-0.51	-0.57	-0.62	-0.65	-0.69
10	0.51	0.26	0.07	-0.08	-0.22	-0.35	-0.46	-0.56	-0.64	-0.71	-0.77	-0.82
end of year	real											
1	0.12	0.03	-0.01	-0.03	-0.05	-0.07	-0.08	-0.08	-0.09	-0.10	-0.11	-0.11
2	0.20	0.04	-0.04	-0.09	-0.12	-0.14	-0.16	-0.18	-0.20	-0.21	-0.22	-0.23
3	0.35	0.11	-0.01	-0.08	-0.13	-0.17	-0.20	-0.23	-0.25	-0.27	-0.29	-0.30
4	0.50	0.17	0.00	-0.09	-0.16	-0.22	-0.26	-0.29	-0.32	-0.35	-0.37	-0.39
5	0.69	0.25	0.04	-0.09	-0.18	-0.25	-0.30	-0.35	-0.39	-0.42	-0.45	-0.48
10	0.91	0.33	0.05	-0.12	-0.24	-0.33	-0.40	-0.46	-0.51	-0.56	-0.60	-0.63

Note: Change in interest rates from base. 1.0 equals an increase in rates of 100 basis points. Calculated from simulation 7.

The final two simulations, numbered 8 and 9, examine the effects of shortening and lengthening the maturity of gilts. It has been argued (see, for example, Friedman (1992)) that shortening the maturity of gilts would result in longer rates falling, thereby stimulating investment in physical capital and improving the supply side potential of the economy. Given the inability of previous researchers to discern a role for long-term interest rates this result seems unlikely for the UK. Moreover, in our model (see table 8.4.10) the extra issuance of short term gilts initially causes the level of nominal interest rates to rise. It is only after two years that the rise is greater at the short end than the long end of the term structure and only after ten years that long interest rates are lower than in the base case. The shortening of the debt maturity also raises the level of real interest rates (because inflation expectations fall) and at long maturities these stay above the base simulation even after ten years. It seems unlikely, therefore, that the capital stock would have been increased by the shortening of the government's debt maturity.

Table 8.4.10 Effects of a Permanent 1% Increase in Short Gilts Funded by a Change in Long Gilts.

maturity	2	4	6	8	10	12	14	16	18	20	22	24
end of year						nominal						
1	0.06	0.06	0.06	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
2	0.08	0.08	0.08	0.09	0.09	0.09	0.09	0.08	0.08	0.08	0.08	0.08
3	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.08	0.08	0.08	0.07
4	0.10	0.09	0.09	0.09	0.09	0.08	0.08	0.07	0.07	0.07	0.06	0.06
5	0.10	0.09	0.09	0.08	0.07	0.07	0.06	0.06	0.05	0.05	0.04	0.04
10	0.13	0.10	0.08	0.07	0.05	0.04	0.02	0.01	0.00	-0.01	-0.02	-0.03
end of year						real						
1	0.01	0.04	0.06	0.07	0.08	0.08	0.09	0.09	0.09	0.10	0.10	0.10
2	0.04	0.07	0.08	0.09	0.09	0.10	0.10	0.11	0.11	0.11	0.11	0.11
3	0.07	0.09	0.09	0.10	0.10	0.10	0.11	0.11	0.11	0.11	0.11	0.11
4	0.09	0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
5	0.10	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.08	0.08	0.08	0.08
10	0.15	0.11	0.08	0.07	0.06	0.06	0.05	0.05	0.04	0.04	0.04	0.03

Note: Change in interest rates from base. 1.0 equals an increase in rates of 100 basis points. Calculated from simulation 8.

As table 8.4.11 shows lengthening the maturity of gilt liabilities conversely reduces nominal interest rates with the largest reduction being at the short end of the term structure. Again the differences from base are numerically small until the end of the simulation and thus it is unsurprising that the effect on GDP is very slight, raising it by less than 0.1% in the final four months of the simulation. Nevertheless, higher activity contributes to reduced short term gilts issuance, and this in turn reduces interest payments as do lower nominal interest rates. It is the reduction in the short gilts as a percentage of all government debt that results in the lower interest rates seen in table 8.4.11. As table 8.4.1 illustrates, by the end of the simulation the effect of this funding change on the nominal slope term, ZEROPC2, is negligible. For real interest rates the effects of a lengthening in maturity reduces interest rates (because inflation expectations rise) with the term structure shifting in an almost parallel fashion.

Simulation 9 again emphasises the importance of allowing for the interactions between the real economy and the liability stocks in the determination of term structures. Thus flow of funds models, which were surveyed in chapter 2, section 2.7, are likely to be misleading about the effects on interest rates of changes in government securities unless they are embedded within an macroeconomic model. In particular they are likely to miss instability in the system, which simulation 9 suggests will occur if the simulation period was to be extended further.

Table 8.4.11 Effects of a Permanent 1% Increase in Long Gilts Funded by a Change in Short Gilts.

maturity	2	4	6	8	10	12	14	16	18	20	22	24
end of year						nominal						
1	-0.05	-0.05	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06
2	-0.07	-0.07	-0.07	-0.07	-0.07	-0.07	-0.07	-0.07	-0.07	-0.07	-0.06	-0.06
3	-0.08	-0.08	-0.08	-0.08	-0.07	-0.07	-0.07	-0.07	-0.07	-0.06	-0.06	-0.06
4	-0.08	-0.08	-0.08	-0.07	-0.07	-0.07	-0.06	-0.06	-0.06	-0.05	-0.05	-0.05
5	-0.09	-0.09	-0.08	-0.08	-0.07	-0.07	-0.06	-0.06	-0.05	-0.05	-0.04	-0.04
10	-0.23	-0.20	-0.19	-0.17	-0.15	-0.14	-0.12	-0.11	-0.09	-0.08	-0.07	-0.06
end of year						real						
1	-0.01	-0.04	-0.05	-0.06	-0.07	-0.07	-0.07	-0.08	-0.08	-0.08	-0.08	-0.09
2	-0.03	-0.06	-0.07	-0.07	-0.08	-0.08	-0.08	-0.09	-0.09	-0.09	-0.09	-0.09
3	-0.06	-0.07	-0.07	-0.08	-0.08	-0.08	-0.09	-0.09	-0.09	-0.09	-0.09	-0.09
4	-0.07	-0.07	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08
5	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08
10	-0.22	-0.20	-0.19	-0.18	-0.18	-0.18	-0.18	-0.17	-0.17	-0.17	-0.17	-0.17

Note: Change in interest rates from base. 1.0 equals an increase in rates of 100 basis points. Calculated from simulation 9.

8.5 Conclusions

This chapter has shown is that it is possible to model movements in real and nominal term structures encompassing 24 interest rates with a macro model that is compact and easy to understand. This is an extension of principal components analysis that has not been implemented before. The model has a number of potential uses. For the UK Debt Management Office (DMO) knowledge of how the term structure evolves over time as a consequence of changes of current changes in debt would clearly be useful in achieving the DMO's aim of minimising the cost of government borrowing. Further, as the term structures can be calculated in both real and nominal terms, the model could be adapted to allow the DMO to minimise costs from issuing index linked as well as nominal coupon gilts.

For investors the model could be used in Monte-Carlo simulations to ascertain those gilt maturities that would enter a portfolio so that the portfolio was least susceptible to economic shocks. In turn gilt maturities that were most volatile to economic shocks would be expected to attract the largest risk premia to compensate for their riskiness. By performing Monte-Carlo simulations but dropping one source of variation at a time the macro-economic sources of the term premia can be more closely identified. The identification of the important macroeconomic variables that change term premia offers the possibility of profitable arbitrage between different gilt maturities. This provides a clear agenda for future research.

Although the model has clear potential in these finance-related areas it would not be the preferred choice for forecasting macroeconomic developments. It is not designed to do this and there are a number of macroeconometric models that would clearly out perform it in this task. There is no reason why the equations from this model could not be integrated into a larger model, for example the LBS's, so that this weakness would be easy to overcome with the consent of the LBS to integrate the two models together.

The simulations highlight a number of points on the interactions of fiscal policy and term structures. Firstly, for an increase in government spending or a decrease in tax rates the method used to finance the extra deficit is important, indeed the financing rule determines the outcome. Thus the question "does government spending affect interest rates?" is incomplete unless the question also specifies how any extra deficits are to be financed. If government spending is financed by extra tax revenue it has little effect on real or nominal interest rates. Conversely, if government spending is financed by short gilt sales then the effects on the term structure are (absolutely) much larger. Secondly, it matters which interest rate is examined as the effects of changes in interest rates are usually larger at the ends of the term structures than in the middle maturity interest rates. Thus if, say, a ten year gilt is chosen to represent interest rates the conclusions drawn would differ from those drawn if a two year or a 24 year gilt were chosen. Thirdly, the effects of changes in spending take some time to manifest themselves. If the question was posed "do government spending changes significantly effect the term structure in the short term, say, up to two years?" the answer would be no. Alternatively, the long-run (ten or more years) effects can be large for some financing modes. Fourthly, it was shown that the long-run effects could change depending upon the relative sizes of the outstanding debts of the government. Given these findings, it is not hard to understand why the work using single equation estimates of government spending on interest rates has resulted in mixed and inconclusive results.

The model provides evidence that gilt financed deficits may be unstable in the long-run. This is consistent with the findings of the simple wealth enhanced IS-LM models surveyed in chapter 2, section 2.8. This suggests that policies to shorten the debt such as Operation Twist in the USA (see Modigliani and Sutch (1966)) should not be undertaken lightly. Constructing a simple constant funding policy that had no effects on interest rates in the long-run was shown to be possible only if the relative debt stocks to total government debt was kept constant and there were no other shocks. The model is therefore non-Ricardian in that the financing of deficits matters for the evolution of interest rates. The model also exhibits the property that as a liability increases as a percentage of all government liabilities the effect of increasing that liability by an even greater amount is reduced. This is akin to the Keynesian liquidity trap but, unlike the liquidity trap, this can occur at any level of interest rates, for any of the government's liabilities.

There are numerous changes that could be investigated within the framework of the macro-model; far too many to undertake even if the whole of this thesis was devoted to the subject. A few avenues of further research that could be applied to this model are suggested below. One avenue would be to see if the model's statistical properties could be improved by using alternative measures of trend output or to check if the model was robust to alternative estimation procedures such as FIML now that the specification has been established. It would be relatively straightforward to increase the number of variables to endogenise trend output by including labour market and capital stock variables. In turn, this would require the asset share equations to be altered. With the largest changes in interest rates occurring after a protracted length of time, it would be an advantage to allow aggregate supply, which is also believed to change relatively slowly, to be endogenous. Clearly other changes are possible by distinguishing more non-government sectors. Other assets and liabilities (in particular index linked gilts) could also be identified. This would again alter the asset share equations. Work could also be undertaken on investigating simple rules for minimising interest payments that would allow deficits to be gilt financed without the risk of the economy becoming uncontrollable.

Appendix 8.1 Data Sources

The following are endogenous variables in the model:

Y, monthly GDP index measure supplied by Martin Weale of the National Institute (see Salazar et al (1997)), monthly average, seasonally adjusted. Converted to an expenditure base in £billions by multiplying the average 1990 level of GDP at factor cost at 1990 prices (ONS code CAOP).

RR1, first principal component score of the Bank of England's real term structure with rates at two year intervals between 2 years and 24 years maturity, not seasonally adjusted.

RR2, second principal component score of the Bank of England's real term structure with rates at two year intervals between 2 years and 24 years maturity, not seasonally adjusted.

E, Bank of England's trade weighted effective exchange rate index (1990=100), monthly average, not seasonally adjusted, (ONS code AJHX).

INFPC1, first principal component score of the Bank of England's inflation term structure with rates at two year intervals between 2 years and 24 years maturity, not seasonally adjusted.

INFPC2, second principal component score of the Bank of England's inflation term structure with rates at two year intervals between 2 years and 24 years maturity, not seasonally adjusted.

M, non-gilt financial liabilities of the central government calculated as total financial liabilities minus gilt liabilities.

T, tax receipts by central government are calculated as the sum of inland revenue receipts, (ONS code ACAB) plus Customs and Excise receipts (ONS code ACAC) plus social security receipts (ONS code ABIA) plus other receipts (ONS code ABIC).

IP, interest payments on government debt (ONS code ABIE).

P, interpolated series using the quarterly GDP price deflator at factor cost (ONS code DJCM).

ZEROPC1, first principal component score of the Bank of England's nominal term structure with rates at two year intervals between 2 years and 24 years maturity, not seasonally adjusted.

ZEROPC2, second principal component score of the Bank of England's nominal term structure with rates at two year intervals between 2 years and 24 years maturity, not seasonally adjusted.

The following variables are treated as exogenous:

Y^*, trend real GDP calculated as described in text.

GB, face value of outstanding government bonds. Source Bank of England Quarterly Bulletin various issues Operation of Monetary Policy Chapter, from tables and text, not seasonally adjusted, £billions.

G*P, nominal government spending is measured as net departmental outlays of the central government (ONS code ABIG) minus privatisation receipts (ONS code ABIF) minus receipts of interest and dividends (ONS code ABIB)

P_W the world price level as measured by the sterling based import unit value index (ONS code DJBC\DJDJ) divided by the trade weighted exchange rate index (E).

GER3MON, calculated as three-month Sterling inter bank offered rate (ONS code AJWR) minus the differential with German three-month rates (ONS code AJHZ).

US3MON, calculated as three-month Sterling inter bank offered rate (ONS code AJWR) minus the differential with US three-month rates (ONS code AFBI).

R, Sterling three-month inter bank offered rate in London (ONS code AJWR).

Chapter 9

Conclusions

9.1 Summary and Implications

The motivation for studying the term structure of interest rates is straightforward. It is believed to provide a link between how expectations of the future are transmitted into activity today. At a more basic level virtually all UK adults are investors, either directly or indirectly via pension funds, in the UK gilts market. Yet, despite its importance, there remains widespread disagreement about the effects of government fiscal behaviour on the term structure. As the government is the supplier of gilts this is tantamount to claiming that the effects emanating from changes in the supply side of the gilts market are not understood - a situation that requires resolution.

In this thesis we have thoroughly examined principal component decompositions of the UK term structure of interest rates and used this approach to throw light on this area of research. Outside the finance field most research on interest rates concentrates on at most two interest rates a short and a long rate. The great attraction of principal components analysis is its ability to reduce the dimensionality of the data, which allows tractable solutions. Thus the research can analyse movements in the whole of the term structure, and a key concern of previous work - "are these results specific to the maturities chosen?" can be bypassed. However, previous research in this area, which was surveyed in chapter 3, has entirely ignored the statistical basis of principal components analysis. This omission is rectified in this thesis by subjecting the principal components to a battery of statistical tests, many of which have not been applied to term structure data before.

There are two major new results that can be referenced by future researchers. Firstly, the components are invariant to the type of matrix used. This is an important result because it enables the use of covariance matrices, which in turn allow a far greater range of statistical tests. The second result is that the principal components are invariant, apart from scaling, to the number of maturities used. This means that researchers do not have to worry about getting the maximum number of maturities, just that they are spread sufficiently over the term structure. However, the results also showed that the principal components from term structures estimated by alternative methods are different. This result was demonstrated by the use of partial common principal components, a technique never before used on term structure data. This means that inter-country comparisons where the data have not been estimated by the same technique bias the results against finding similar decompositions of the term structure. This point has not been sufficiently emphasised in previous inter-country comparisons. The partial common principal components analysis also indicted that it was possible to distinguish between the various measure of spot rates and redemption yields. This is a reassuring result given that only spot rates measure movements in interest rates.

The results also support results that were previously known. It found that most of the variability in term structure data could be explained by two components, a level term and a slope term, and statistical tests confirm that the eigenvectors are not stable across time. An implication of this is that care needs to beexercised when using principal components scores to ensure that they are calculated from the appropriate sample. The results also showed that if the sample period was too short the characterisation of the first principal component, as the level, and the second, as the slope, need not hold. Researchers need to bear this in mind when selecting data periods.

In reaching these conclusions a number of UK term structures were analysed. Spot and redemption yields were constructed using new methods of estimation that endogenised the knot positions. One has been previously published, although not used to my knowledge on UK term structure data while one was an entirely new method within a B-spline procedure. The knot positions were intensively studied to see whether they could indicate the presence of natural market boundaries. Such an approach does not seem to have been applied to UK term structures in previous work. However, the knot positions were highly variable, and it seems unlikely that market boundaries or preferred habitats of investors play any role in determining the shape of the UK term structure.

Chapter 7 investigates how "news" may affect the term structure of interest rates using the first two principal component scores. Using regression and Johansen estimation techniques, this chapter shows that although the MMS survey of City forecasters is biased and inefficient, the numerical significance of this is small. Moreover, evidence suggests that the survey is not out of date by the time that the ONS publishes its data. The analysis of the timeliness of forecasts appears to be new to the UK literature, and this result removes one of the arguments against using surveys of forecasts to derive news. Regressions using the principal component scores as the dependent variables found that the government's fiscal policy, as measured by gilt issues or the unexpected changes in the PSBR, have no affect on the term structure of interest rates. The regressions also have two other implications. Firstly, because there is no statistically significant relationship between news and term structure movements, there is little incentive for City forecasters to improve their forecasting performances at least as far as the forecasts given to MMS, as opposed to private forecasts, are concerned. Secondly, knowledge of the news element cannot help predict movement in the term structure. Consequently, advance knowledge of data releases possessed by the Bank of England (or the Debt Management Office) cannot help it plan to issue debt into markets that are rising (thereby getting a better price) or purchase debt to stabilise interest rates when news suggests the market may fall.

Chapter 8 explores the use of principal component scores in a small, stylised macroeconomic model focusing on the role of fiscal policy. The model has an exogenous supply side, an aggregate demand function, a three-asset flow of funds model, which is inverted to model the first two components scores of the Bank of England's spot-rate term structure, and equations for the first two component scores of the Bank of England's inflation expectations term structure. Using a vector version of the Fisher identity, a real interest rate term structure can be derived. Changes in government spending and taxes are linked to the flow of funds model via the government's budget identity. The model has a number of novel features. Firstly, it is estimated using monthly rather than quarterly or annual data. Secondly, it estimates equations for inflation expectations using Bank of England data, which has never been used before in this fashion. Thirdly, the use of principal components can allow a richer set of effects on the asset demand and real side of the economy. Finally, the use of principal components allows the effects of numerous (in this model 12) interest rates to be analysed.

It was found that changes in the assets supplied and not the change in spending had the major effects on the term structures. The size of the effect depended upon the type of debt being issued and the sign and size were dependent upon the magnitude of the asset stocks. There was evidence that some forms of gilt financing were unstable, a possibility noted in chapter 2, but the consequences of this would take a long time to manifest themselves. The main implication from this model is that it allows a path to reconcile the results of previous studies of the term structure and fiscal variables. In this thesis chapter 7 showed that it was difficult to find any statistically significant effects from fiscal variables on the term structure. This model suggests that the reason for this was that the effects require a protracted period of time before they become apparent. Thus news studies that focus on daily or hourly movements in the term structure are unlikely to detect such effects. Thus, the finding of limited news effects from fiscal variables in chapter 7 does not contradict the finding in chapter 8. Secondly, because the slope of the term structure changes over time, by selecting different maturities different results can be found. This can help explain some of the mixed empirical results described in chapter 2. Finally, as the response of the term structure also depends upon the relative proportions of liabilities and as this changes over time, it is possible that the effects of fiscal policy on interest rates may appear unstable even though the underlying parameters are constant. Thus the model can provide a range of arguments to explain why empirical testing for the effects of fiscal policy on interest rates has produced such mixed results.

All research has limitations and this thesis is no exception. With hindsight it would have been preferable to have applied the Dierckx (1981) method of endogenising knot positions to spot rates rather than just to the redemption yields. This would have allowed direct comparison of the results of using principal components techniques in chapters 5 and 6. It is likely that this would strengthen the result that different techniques produce term structures that can be distinguished by using partial common principal components. The second limitation is that Monte-Carlo studies were not performed to ascertain the distributions of the test and descriptive statistics used in the principal components analyses. Without this the introduction of test statistics, whilst an improvement on other work in this area, is incomplete. Knowledge of the distribution of the test statistics would clearly be of use in the statistical analysis of data from outside the finance field. Thirdly, no attempt was made to exploit the information contained in the autocorrelated nature of interest rate data. The final limitation is that no attempt was made in chapter 8 to distinguish between expected and surprise changes in government spending despite the importance attached to this distinction in the dynamic IS-LM models reviewed in chapter 2.

9.2 Avenues for Future Research

The work in this thesis has suggested a number of extensions and further avenues for research that may be profitable to explore. To my knowledge there appears to be no analysis of the effect of non-normalcy of the data on principal component test statistics. Given that financial data is almost invariably non-normal this is an area that needs to be examined further. One method of doing this would be to conduct Monte-Carlo experiments on artificial data sets designed to mimic spot rates.

A second avenue of research would be to repeat the estimation of endogenous knot spline curves on US data. This would eliminate the problem of estimation differences hiding similarities between the US and UK data and it would facilitate tests on the hypothesis that the world's capital markets are becoming more integrated. This would allow a re-run of the tests on partial common principal components between the US and UK data to be much more specific.

The third avenue of potential research would extend the use of partial common principal components to European countries and Japan to test for increasing capital market integration. The method could also be applied to money market rates and equity markets to identify common components across markets.

Fourth, further examination of the lower order principal components could be conducted with the aim of seeing if they are systematic or just noise. The estimated news elements could then be applied to the lower order principal components to test whether news account for the movements in lower order principal components or are just noise.

Research in the thesis showed that knowledge about ONS data releases could not be used to predict movements in the principal component scores. It would be useful if portfolios of hypothetical gilts could be constructed to test further the hypothesis that extra knowledge of ONS releases, i.e. a better forecasting performance than those produced by the MMS surveys, are valueless. It would also be of value to see whether portfolios of gilts can be constructed that are immune to the noise generated by forecasting errors of ONS data releases. If such portfolios can be constructed this is a further reason why biased and inefficient forecasting errors are not eliminated by City forecasters.

Much further research could be carried out refining the macro model reported in chapter 8. One area of interest would be to analyse, using both simulations and optimal control, a debt issuance policy that minimised interest payments when the model is subjected to shocks. Is this policy robust or does it lead to instrument instability? How does the policy compare with what the Bank of England actually did? Does this policy conflict with the other aim of monetary policy, which is to maintain the value of the currency? On a slightly different tack, is it possible to build portfolios of gilts that are dynamically immune in the short and long term to macro-economic shocks that such a model can mimic? If certain maturities have risks that are non-diversifiable and thus earn term premia what are the shocks that generate this result? Answers to these questions would clearly be of interest to the investors.

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