Essays in Financial Forecasting

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Declaration

I declare that any material contained in this thesis has not been submitted for a degree to any other university. I further declare that one paper titled "The out-of-sample success of term structure models as exchange rate predictors: a step beyond" drawn from Chapter 2 of this thesis has been published in the *Journal of International Economics*, 2002, 60, pp. 61-83. The paper "Modelling and forecasting stock returns: Exploiting the futures market, regime shifts and international spillovers" drawn from Chapter 3 of this thesis is under review with the *Journal of Applied Econometrics* (revised and resubmitted) while the paper "Exchange Rates and Fundamentals: Evidence on the Economic Value of Predictability", drawn from Chapter 4 of this thesis, has been submitted to a refereed journal.

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Abstract

Forecasting is central to economic and financial decision-making. Government institutions and agents in the private sector often base their decisions on forecasts of financial and economic variables. Forecasting has therefore been a primary concern for practitioners and financial econometricians alike, and the relevant literature has witnessed a renaissance in recent years. This thesis contributes to this literature by investigating three topical issues related to financial and economic forecasting.

The first chapter finds its rationale in the large literature suggesting that standard exchange rate models cannot outperform a random walk forecast and that the forward rate is not an optimal predictor of the spot rate. However, there is some evidence that the term structure of forward premia contains valuable information for forecasting future spot exchange rates and that exchange rate dynamics display nonlinearities. This chapter proposes a term-structure forecasting model of exchange rates based on a regime-switching vector equilibrium correction model which is novel in this context. Our model significantly outperforms both a random walk and, to a lesser extent, a linear term-structure vector equilibrium correction model for four major dollar exchange rates across a range of horizons.

The second chapter proposes a vector equilibrium correction model of stock returns that exploits the information in the futures market, while also allowing for regime-switching behavior and international spillovers across stock market indices. Using data for three major stock market indices since 1989, we find that: (i) in sample, the model outperforms several alternative models on the basis of standard statistical criteria; (ii) in out-of-sample forecasting, the model does not produce significant gains in terms of point forecasts relative to more parsimonious alternative specifications, but it does so both in terms of market timing ability and in density forecasting performance. The importance of these gains is illustrated with a simple application to a risk management problem.

The third chapter re-examines a major puzzle in international finance that is the inability of exchange rate models based on monetary fundamentals to produce better out-of-sample forecasts of the nominal exchange rate than a naive random walk. While prior research has generally evaluated exchange rate forecasts using conventional statistical measures of forecast accuracy, this chapter investigates whether there is any economic value to the predictive power of monetary fundamentals for the exchange rate. We estimate, using a framework that allows for parameter uncertainty, the economic and utility gains to an investor who manages her portfolio based on exchange rate forecasts from a monetary fundamentals model. In contrast to much previous research, we find that the economic value of the exchange rate forecasts implied by monetary fundamentals can be substantially greater than the economic value of forecasts obtained using a random walk across a range of horizons.

In sum this thesis adds to the relevant literature on forecasting financial variables by providing insights and evidence to researchers and indicating potential avenues for futures research.

Chapter 1

Introduction

Forecasting is central to economic and financial decision-making. Government institutions and agents in the private sector often base their decisions on forecasts of financial and economic variables. Forecasting has therefore been a primary concern for practitioners and financial econometricians alike, and the relevant literature has witnessed a renaissance in recent years. This thesis contributes to this literature by investigating three topical issues related to financial and economic forecasting which particularly attracted our personal attention. These issues are mainly concerned with the so-called 'exchange rate disconnnect puzzle', which highlights just how weak is the relationship between the exchange rate and virtually any macroeconomic aggregate or fundamental suggested by open-economy macroeconomic theory, and the prediction of internationally integrated stock market indices.

The first issue finds its rationale in the large literature starting with the Meese and Rogoff (1983a,b) studies. Their robust finding that standard empirical exchange rate models could not outperform a simple random walk forecast has been devastating and after twenty years of research models which consistently and significantly outperform a naive random walk are still elusive. A parallel finding in the exchange rate literature, also dating from the early 1980s, was that the forward rate is not an optimal predictor of the future spot exchange rate (see e.g. Hansen and Hodrick, 1980; Frankel, 1980;

Bilson,1981), or equivalently that the forward premium is not an optimal predictor of the rate of depreciation, as the efficient markets hypothesis, at least in its risk-neutral formulation, would suggest (see Frankel and Rose, 1995; Taylor, 1995). Attempts to locate the source of this failure of the risk-neutral efficient markets hypothesis either in the presence of stable, significant and plausible risk premia, or in some sense in the failure of rational expectations when applied to the foreign exchange market as a whole, have also met with mixed and very limited success (see Peel and Pope, 1991; Lewis, 1995; Taylor, 1995; Coakley and Fuertes, 2001b).

More recently Clarida and Taylor (1997) argued that the failure of the forward rate optimally to predict the future spot rate did not necessarily imply that forward rates did not contain valuable information for forecasting future spot exchange rates. Clarida and Taylor develop what they term an 'agnostic' framework for linking spot rate and forward rate movements without assuming anything at all specific about risk premia or expectations formation except that departures from the risk-neutral efficient markets hypothesis (RNEMH) drive at most a stationary wedge between forward and expected future spot rates. Using this framework, they have been able to extract sufficient information from the term structure of forward premia to outperform the random walk forecast - and a range of alternative forecasts - for several exchange rates in out-of-sample forecasting.

Alongside the work on exchange rate forecasting, another strand of the literature has developed in which increasingly strong evidence of nonlinearities of one sort or another in exchange rate movements has been reported. One element of this, dating at least to Booth and Glassman (1987), has been the mounting evidence that the conditional distribution of nominal exchange rate changes is well described by a mixture of normal distributions and that, consequently, a Markov switching model may be a logical characterization of exchange rate behavior (e.g. see Engel and Hamilton, 1990; LeBaron, 1992; Engel, 1994; Engel and Hakkio, 1996; Engel and Kim, 1999). However, although Markov-switching models fit nominal exchange rate data very well, in general they do not produce superior forecasts to a random walk or the forward rate on the basis of conventional forecasting criteria (see e.g. Engel, 1994). Overall the literature on nonlinear modelling of exchange rates has produced models that fit satisfactorily and forecast well in sample but that in general fail to beat simple random walk models or linear specifications in out-of-sample forecasting (e.g. see Diebold and Nason, 1990; Engel, 1994; Meese and Rose, 1990, 1991).

In this thesis we investigate whether allowing for nonlinearities in the underlying data-generating process for the term structure yields superior exchange rate forecasts. This is done through estimating a fairly general three-regime Markov-switching vector equilibrium correction model (MS-VECM) for spot rates and the term structure of forward rates which is essentially based on an extension of Markovian regime shifts to a nonstationary framework, for which the underlying econometric theory has recently been developed[1]. Given the evidence of significant regime-switching behavior in exchange rate movements discussed above, this seems a natural way to extend the Clarida-Taylor analysis, even though this involves estimating and forecasting from a sophisticated multivariate nonlinear model. Indeed, to the best of our knowledge, the research reported in this chapter represents the first application of Markov-switching in a multivariate cointegrated framework to exchange rate modelling and forecasting.

The second issue concerns the behavior of international stock market indices. A large body of research on modeling and forecasting stock returns has investigated the relationship between spot and futures prices in stock index futures markets. In particular, a number of empirical studies have focused on the persistence of deviations from the cost

[1] Several authors have recently begun to use the term 'equilibrium correction' instead of the traditional 'error correction' as the latter term now seems to have a different meaning in some recent theories of economic forecasting (e.g. see Clements and Hendry, 1998, p. 18). Since the term 'equilibrium correction' conveys the idea of the adjustment considered in the present context quite well, we use this term below.

of carry and have investigated the relationship between spot and futures prices in the context of vector autoregressions using cointegration or equilibrium correction models (see Dwyer, Locke and Yu, 1996; Neely and Weller, 2000, and the references therein). The rationale underlying this line of research is that the cost of carry model and variants of it predict that spot and futures prices cointegrate and their long-run relationship is characterized by a long-run equilibrium defined by the futures basis, implying both mean reversion in the basis and the existence of a vector equilibrium correction model (VECM) for spot and futures prices. This literature has generally reported evidence that the futures market contains valuable information for modeling and/or forecasting stock returns.

A missing link which has not been addressed in the above-mentioned literature is that trading activity does not take place for one index per unit of time. Indeed, it is more likely that traders place orders and take positions simultaneously using different indices given that stock and futures markets for different indices are closely linked by both hedging activities and cross-market arbitrage. This may generate comovements across stock market indices and, in turn, the cross-correlation between different indices may be potentially very useful in improving empirical models of stock returns (e.g. see Eun and Shin, 1989; Engle and Susmel, 1994; Koutmos and Booth, 1995; Lee, 1995; Karoly and Stulz, 1996).

Alongside the work on modeling and forecasting stock prices and returns, another strand of the literature has developed where increasingly strong evidence of nonlinearities in stock price movements has been documented. One element of this has been the mounting evidence that the conditional distribution of stock returns is well described by a mixture of normal distributions (e.g. see Rydén, Teräsvirta and Åsbrink, 1998, and the references therein) and that, consequently, a Markov switching model may be a logical characterization of stock returns behavior (e.g. see, *inter alia*, LeBaron, 1992;

Hamilton and Susmel, 1994; Hamilton and Lin, 1996; Ramchand and Susmel, 1998a,b; Rydén, Teräsvirta and Åsbrink, 1998; Susmel, 1999; Perez-Quiros and Timmermann, 2001). Also, not only Markov-switching models fit stock returns data well, but they have often been proved to produce superior forecasts to several alternative conventional models of stock returns (e.g. see Hamilton and Susmel, 1994; Hamilton and Lin, 1996).

In this thesis we tie together these somewhat different, albeit related, strands of research on modelling and forecasting stock returns. In particular, we investigate whether allowing for nonlinearities and international spillovers in the underlying data-generating process for a VECM that links spot and futures prices yields an improvement, in terms of both in-sample fit and out-of-sample forecasting, over models of stock returns that do not allow for nonlinearities and/or international spillovers. The evaluation of the relative performance is based on conventional statistical criteria for point forecasting performance as well as on the ability of the models to forecast the true predictive density of stock returns out of sample. In fact, we argue and provide evidence that density forecast accuracy is more appropriate for evaluating our competing models since stock returns are non-normally distributed and we are considering nonlinear models consistent with non-normal densities (see, *inter alia*, Diebold, Gunther and Tay, 1998; Granger and Pesaran, 1999; Tay and Wallis, 2000; Timmermann, 2000). Further, we illustrate the practical importance of our results on density forecasting with an application to financial risk management. Financial risk is generally measured by means of Value-at-Risk (VaR), which can be defined as the expected maximum loss over a target horizon within a given confidence interval. Users of the VaR methodology generally assume that expected returns are normally or t-distributed. However, this assumption contrasts with the large amount of empirical evidence suggesting that the distribution of financial asset returns is not normal. Point forecast analysis and testing procedures based upon it do not take into account these features, so that VaR analysis often relies on dubious parametric distributional assumptions. In our simple application we analyze the out-of-sample forecasting performance of our proposed empirical models of stock returns and we investigate the implications of these forecasts for a risk manager who has to quantify the risk associated with holding the stock indices in question over a one-week horizon.

The third issue investigated in this thesis concerns with the re-examination of the 'exchange rate disconnect puzzle' under a new perspective. In fact, prior research on the ability of monetary-fundamentals models to forecast exchange rates relies on *statistical* measures of forecast accuracy, like mean squared errors. Surprisingly little attention has been directed, however, to assessing whether there is any *economic* value to exchange rate predictability. Our contribution fills this gap in that we investigate the ability of a monetary-fundamentals model to predict exchange rates by measuring the economic or utility-based value to an investor who relies on this model to allocate her wealth between two assets that are identical in all respects except the currency of denomination. We do so by focusing on two key questions. First, as a preliminary to the forecasting exercise, we ask how exchange rate predictability and parameter uncertainty affect optimal portfolio choice for investors with a range of horizons up to ten years. Second, and more importantly, we ask whether there is any additional economic value to a utility-maximizing investor who uses exchange rate forecasts from a monetary-fundamentals model relative to an investor who uses forecasts from a naive random walk model. We quantify the economic value of predictability in a Bayesian framework that allows us to account for uncertainty surrounding parameter estimates in the forecasting model. Indeed, parameter uncertainty or 'estimation risk' is likely to be of importance, especially over long horizons.

Our work is related to and builds on earlier research by Kandel and Stambaugh (1996) and Barberis (2000), who use a Bayesian framework to study asset allocation between a riskless asset and risky equities. Our work differs from theirs in three important ways. First, since we consider the economic gains (losses) to an investor whose problem is allocating her wealth between two assets that are identical in all respects except the currency of denomination, our focus is on exchange rate prediction. Put differently, in our framework risk only enters the investor's problem through the nominal exchange rate. Second, we allow the investor to hold short positions in the assets, which is an important feature in real-world foreign exchange markets (e.g., Lyons, 2001). Third, while we analyze the impact of predictability and parameter uncertainty on optimal allocation decisions, our primary goal is to evaluate the out-of-sample economic value of exchange rate predictability.

The remainder of the thesis is set up as follows: in Chapter 2 it is proposed a term-structure forecasting model of exchange rates based on a regime-switching vector equilibrium correction model which is novel in this context. It is concluded that our model significantly outperforms both a random walk and, to a lesser extent, a linear term-structure vector equilibrium correction model for four major dollar rates across a range of horizons.. In Chapter 3, we propose a vector equilibrium correction model of stock returns that exploits the information in the futures market, while also allowing for regime-switching behavior and international spillovers across stock market indices. Using data for three major stock market indices since 1989, we find that: (i) in sample, our model outperforms several alternative models on the basis of standard statistical criteria; (ii) in out-of-sample forecasting, our model does not produce significant gains in terms of point forecasts relative to more parsimonious alternative specifications, but it does so both in terms of market timing ability and in density forecasting performance. In Chapter 4, we estimate, using a framework that allows for parameter uncertainty, the economic and utility gains to an investor who manages her portfolio based on exchange rate forecasts from a monetary fundamentals model. In contrast to much previous research, we find that the economic value of the exchange rate forecasts implied by monetary fundamentals can be substantially greater than the economic value of forecasts obtained using a random walk across a range of horizons.

A final chapter briefly summarises the key findings of the thesis and concludes.

Chapter 2

The role of forward rates in forecasting exchange rates: can we beat a random walk?

The Meese and Rogoff (1983a,b) studies marked a watershed in empirical exchange rate economics. In particular, their robust finding that standard empirical exchange rate models could not outperform a simple random walk forecast was at the time seen as devastating.[1] Even with the benefit of twenty years of hindsight, moreover, the random walk remains the standard comparator for exchange rate forecasting and models which consistently and significantly outperform a naive random walk are still elusive (e.g. see Mark, 1995). A parallel finding in the exchange rate literature, also dating from the early 1980s, was that the forward rate is not an optimal predictor of the future spot exchange rate (see e.g. Hansen and Hodrick, 1980; Frankel, 1980; Bilson,1981), or equivalently that the forward premium is not an optimal predictor of the rate of depreciation, as the efficient markets hypothesis, at least in its risk-neutral formulation, would suggest (see Frankel and Rose, 1995; Taylor, 1995). Attempts to locate the source of this failure of the risk-neutral efficient markets hypothesis either in the presence of stable, significant and plausible risk premia, or in some sense in the failure of rational expectations when applied to the foreign exchange market as a whole, have also met with mixed and very

[1] See also Meese and Rogoff (1988) and the discussion on the 'exchange rate disconnect puzzle' by Obstfeld and Rogoff (2000).

limited success (see Pope and Peel, 1991; Lewis, 1995; Taylor, 1995; Coakley and Fuertes, 2001b). Thus, from the early 1980s onward, exchange rate forecasting in general became increasingly to be seen as a hazardous occupation, and this remains largely the case.

A ray of hope in an otherwise murky environment was, however, provided by Clarida and Taylor (1997), who argued that the failure of the forward rate optimally to predict the future spot rate did not necessarily imply that forward rates did not contain valuable information for forecasting future spot exchange rates. Clarida and Taylor develop what they term an 'agnostic' framework for linking spot rate and forward rate movements without assuming anything at all specific about risk premia or expectations formation except that departures from the risk-neutral efficient markets hypothesis (RNEMH) drive at most a stationary wedge between forward and expected future spot rates. This is sufficient to establish the existence of a linear vector equilibrium correction model (VECM) for spot and forward exchange rates. Using this framework, Clarida and Taylor are able to extract sufficient information from the term structure of forward premia to outperform the random walk forecast - and a range of alternative forecasts - for several exchange rates in out-of-sample forecasting. Indeed, at the one-year forecasting horizon, their improvement over the naive random walk is of the order of 40 percent in terms of root mean square errors.

Alongside the work on exchange rate forecasting, another strand of the literature has developed in which increasingly strong evidence of nonlinearities of one sort or another in exchange rate movements has been reported[2]. One element of this, dating at least to Booth and Glassman (1987), has been the mounting evidence that the conditional distribution of nominal exchange rate changes is well described by a mixture of normal distributions and that, consequently, a Markov switching model may be a logical characterization of exchange rate behavior (e.g. see Engel and Hamilton, 1990; LeBaron,

[2] See Coakley and Fuertes (2001a) and the references therein.

1992; Engel, 1994; Engel and Hakkio, 1996; Engel and Kim, 1999). However, although Markov-switching models fit nominal exchange rate data very well, in general they do not produce superior forecasts to a random walk or the forward rate on the basis of conventional forecasting criteria (see e.g. Engel, 1994). An exception in this context is the study by Engel and Hamilton (1990), who apply the Markov-switching model developed by Hamilton (1988, 1989) to dollar exchange rate data and show that the model generates better forecasts than a random walk. In the light of the subsequent literature, however, these forecasting results appear to be somewhat fragile. Overall, in fact, the literature on nonlinear modelling of exchange rates has produced models that fit satisfactorily and forecast well in sample but that in general fail to beat simple random walk models or linear specifications in out-of-sample forecasting (e.g. see Diebold and Nason, 1990; Engel, 1994; Meese and Rose, 1990, 1991).

In the present chapter, we investigate whether allowing for nonlinearities in the underlying data-generating process for the term structure yields superior exchange rate forecasts. This is done through estimating a fairly general three-regime Markov-switching vector equilibrium correction model (MS-VECM) for spot rates and the term structure of forward rates which is essentially based on an extension of Markovian regime shifts to a nonstationary framework, for which the underlying econometric theory has recently been developed. Given the evidence of significant regime-switching behavior in exchange rate movements discussed above, this seems a natural way to extend the Clarida-Taylor analysis, even though this involves estimating and forecasting from a sophisticated multivariate nonlinear model. Indeed, to the best of our knowledge, the research reported in this chapter represents the first application of Markov-switching in a multivariate cointegrated framework to exchange rate modelling and forecasting.

Using weekly data since 1979 for four major dollar exchange rates, we are able to replicate the Clarida-Taylor forecasting results in a linear VECM framework. However, we also show that conventional linear VECMs reveal significant residual nonlinearity and are easily rejected when tested against the alternative of an MS-VECM. Finally, we show that allowing for nonlinearities, using an MS-VECM, results in forecasts which are superior to random walk forecasts and, to a lesser extent, to the linear VECM exchange rate forecasts. We thus confirm that the information contained in the term structure of forward rates is indeed valuable for forecasting spot exchange rates but that statistically significant improvements can be made over linear forecasting models by allowing for nonlinearities.

The remainder of the chapter is set out as follows. In Section 2.1 we discuss a general framework for linking spot and forward rates, as a motivation for our multivariate modelling. In Section 2.2 we briefly set out the econometrics of Markov-switching multivariate models as applied to nonstationary processes and cointegrated systems. In the following section we report our empirical testing, estimation and forecasting results. A final section concludes.

2.1 The information in the term structure of forward exchange premia

Let s_t and $f_t^{h(k)}$ be, respectively, the spot exchange rate and the $h(k)$-period forward exchange rate, each at time t.[3] It is now well documented that nominal exchange rates between the currencies of the major industrialized economies are well described by unit root processes. We can therefore write the spot exchange rate as the sum of two components:

$$s_t = m_t + q_t, \tag{2.1}$$

where m_t is a unit-root process evolving as a random walk with drift, and q_t is a stationary process having mean zero and a finite variance (Beveridge and Nelson, 1981; Stock and

[3] In our empirical work, we consider forward rates of 4, 13, 26 and 52 weeks maturity, so that in our notation, $h(1) = 4$, $h(2) = 13$, $h(3) = 26$, and $h(4) = 52$.

Watson, 1988). If agents are risk-neutral and the market is efficient in the sense that exchange rates fully reflect all information in a given information set $_t$ (so that, in effect, the market conforms to the rational expectations hypothesis) then the forward exchange rate $f_t^{h(k)}$ should predict the $h(k)$-period ahead future value of the spot exchange rate optimally given $_t$. This is the essence of the risk-neutral efficient market hypothesis (RNEMH). There now exists a large literature rejecting the RNEMH, although it is unclear whether rejection is due to a failure of the assumptions of risk neutrality or of rational expectations or of both (e.g. see Peel and Pope, 1995; Taylor, 1995; Sarno and Taylor, 2002 Chapter 2).

Following Clarida and Taylor (1997), we may in general define departures from the RNEMH, due either to the presence of risk premia or to a failure of rational expectations, or both, as follows:

$$\gamma_t \equiv f_t^{h(k)} - E\left(s_{t+h(k)} \middle| \ _t\right), \tag{2.2}$$

where $E(.| \ _t)$ denotes the mathematical expectation conditional on $_t$. From (2.1) and (2.2) we can obtain:

$$f_t^{h(k)} = \gamma_t + h(k)\theta + E_t\left(q_{t+h(k)} \middle| \ _t\right) + m_t, \tag{2.3}$$

where θ is the drift of the random walk process m_t. Subtracting (2.1) from (2.3), we achieve an expression for the forward premium at time t:

$$f_t^{h(k)} - s_t = \gamma_t + h(k)\theta + E_t\left(q_{t+h(k)} - q_t \middle| \ _t\right). \tag{2.4}$$

Equation (2.4) says that if the departure from the RNMEH γ_t is stationary, given $q_t \sim I(0)$, the forward premium $\left(f_t^{h(k)} - s_t\right)$ must also be stationary. This implies that forward and spot rates exhibit a common stochastic trend and are cointegrated with cointegrating vector $[1, -1]$. Moreover, since this is true for any $h(k)$, if we consider the vector of forward rates of tenor $h(1)$ to $h(m)$ periods, together with the current spot rate, $[s_t, f_t^{h(1)}, f_t^{h(2)}, f_t^{h(3)}, \dots f_t^{h(m)}]'$, then this must be cointegrated with m unique cointegrating vectors, each given by a row of the matrix $[-\iota, I_m]$, where I_m is an m-dimensional identity matrix and ι is an m-dimensional column vector of ones. Further, by the Granger Representation Theorem (Engle and Granger, 1987) the same set of forward and spot rates must possess a VECM representation in which the term structure of forward premia plays the part of the equilibrium errors. Clarida and Taylor (1997) exploit this framework and use exactly a linear VECM representation to demonstrate that a large amount of information may be extracted from the term structure in order to forecast the spot exchange rate, even though the forward rate is not an optimal spot rate predictor. Indeed, dynamic out-of-sample forecasts up to one year ahead indicate that the VECM is superior to a range of alternative forecasts, including a random walk and standard spot-forward regressions.

2.2 Markov-switching equilibrium correction

In this section we outline the econometric procedure employed in order to model regime shifts in the dynamic relationship between spot exchange rates and the term structure of forward rates. The procedure essentially extends Hamilton's (1988, 1989) Markov-switching regime framework to nonstationary systems, allowing us to apply it to cointegrated vector autoregressive (VAR) and VECM systems (see Krolzig, 1997, 1999).

Consider the following M-regime p-th order Markov-switching vector autoregression (MS(M)-VAR(p)) which allows for regime shifts in the intercept term:[4]

$$y_t = \nu(z_t) + \textstyle\sum_{i=1}^{p} \Pi_i y_{t-i} + \varepsilon_t, \tag{2.5}$$

where y_t is a K-dimensional observed time series vector, $y_t = [y_{1t}, y_{2t}, \dots, y_{Kt}]'$; $\nu(z_t) = [\nu_1(z_t), \nu_2(z_t), \dots, \nu_K(z_t)]'$ is a K-dimensional column vector of regime-dependent in-

[4] Although, for expositional simplicity, this section focuses on equation (2.5), clearly a more general formulation of (2.5) may be considered which allows for all parameters of the model to be conditioned on the state z_t.

tercept terms; the Π_i's are $K \times K$ matrices of parameters; $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{Kt}]'$ is a K-dimensional vector of Gaussian white noise processes with covariance matrix Σ, $\varepsilon_t \sim NID(\mathbf{0}, \Sigma)$. The regime-generating process is assumed to be an ergodic Markov chain with a finite number of states $z_t \in \{1, \ldots, M\}$ governed by the transition probabilities $p_{ij} = \Pr(z_{t+1} = j \mid z_t = i)$, and $\sum_{j=1}^{M} p_{ij} = 1 \; \forall i, j \in \{1, \ldots, M\}$.[5]

A standard case in economics and finance is that y_t is nonstationary but first-difference stationary, i.e. $y_t \sim I(1)$. Then, given $y_t \sim I(1)$, there may be up to $K - 1$ linearly independent cointegrating relationships, which represent the long-run equilibrium of the system, and the equilibrium error (the deviation from the long-run equilibrium) is measured by the stationary stochastic process $u_t = \alpha' y_t - \beta$ (Granger, 1986; Engle and Granger, 1987). If indeed there is cointegration, the cointegrated MS-VAR (2.5) implies a Markov-switching vector equilibrium correction model or MS-VECM of the form:

$$\Delta y_t = \nu(z_t) + \textstyle\sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \Pi y_{t-1} + \varepsilon_t, \tag{2.6}$$

where $\Gamma_i = -\sum_{j=i+1}^{p} \Pi_j$ are matrices of parameters, and $\Pi = \sum_{i=1}^{p} \Pi_i - \mathbf{I}$ is the long-run impact matrix whose rank r determines the number of cointegrating vectors (e.g. Johansen, 1995; Krolzig, 1999).[6]

Although, for expositional purposes, we have outlined the MS-VECM framework for the case of regime shifts in the intercept alone, shifts may be allowed for elsewhere. The present application focuses on a multivariate model comprising, for each of the four major dollar exchange rate analyzed, the spot exchange rate and the forward rates at one

[5] To be precise, z_t is assumed to follow an ergodic M-state Markov process with transition matrix

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1M} \\ p_{21} & p_{22} & \cdots & p_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ p_{M1} & p_{M2} & \cdots & p_{MM} \end{bmatrix},$$

where $p_{iM} = 1 - p_{i1} - \ldots - p_{i,M-1}$ for $i \in \{1, \ldots, M\}$.

[6] In this section it is assumed that $0 < r < K$, implying that y_t is neither purely difference-stationary and non-cointegrated (i.e. $r = 0$) nor is a stationary vector (i.e. $r = K$).

month (four weeks), three months (thirteen weeks), six months (twenty-six weeks) and twelve months (fifty-two weeks) to maturity (hence $y_t = [s_t, f_t^{h(1)}, f_t^{h(2)}, f_t^{h(3)}, f_t^{h(4)}]' = [s_t, f_t^4, f_t^{13}, f_t^{26}, f_t^{52}]'$), for which, following the reasoning of Section 2.1, four unique, independent cointegrating relationships should exist. As discussed in Section 2.3 below, in our empirical work, after considerable experimentation, we selected a specification of the MS-VECM which allows for regime shifts in the intercept as well as in the variance-covariance matrix. This model, the Markov-Switching-Intercept-Heteroskedastic-VECM or MSIH-VECM, may be written as follows:

$$\Delta y_t = v(z_t) + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \Pi y_{t-1} + u_t, \tag{2.7}$$

where $\Pi = \alpha\beta'$, $u_t \sim NIID(\mathbf{0}, \Sigma(z_t))$ and $z_t \in \{1, \ldots, M\}$.

An MS-VECM can be estimated using a two-stage maximum likelihood procedure. The first stage of this procedure essentially consists of the implementation of the Johansen (1988, 1991) maximum likelihood cointegration procedure in order to test for the number of cointegrating relationships in the system and to estimate the cointegration matrix. In fact, in the first stage use of the conventional Johansen procedure is legitimate without modelling the Markovian regime shifts explicitly (see Saikkonen, 1992; Saikkonen and Luukkonen, 1997). The second stage then consists of the implementation of an expectation-maximization (EM) algorithm for maximum likelihood estimation which yields estimates of the remaining parameters of the model (Dempster, Laird and Rubin, 1977; Hamilton, 1990; Kim and Nelson, 1999; Krolzig, 1999).

We now turn to a brief discussion of our data set and then to our empirical analysis.

2.3 Empirical results

2.3.1 Data, unit root tests and cointegration analysis

Our data set comprises weekly observations of spot and 4-, 13-, 26- and 52-week forward US dollar exchange rates among the G5 countries (dollar-franc, dollar-mark, dollar-yen and dollar-sterling) for the period January 7 1979 to December 31 1998, a total of 1,043 observations for each series.[7] In our empirical work, we carried out our estimations over the period January 1979-December 1995, reserving the last three years of data for out-of-sample forecasting tests.

As a preliminary exercise, we tested for unit root behavior of the spot rate and the four forward rate time series examined for each of the four dollar exchange rates under investigation by calculating standard augmented Dickey-Fuller (ADF) test statistics. In each case, the number of lags was chosen such that no residual autocorrelation was evident in the auxiliary regressions. In keeping with the very large number of studies of unit root behavior for these time series, we were in each case unable to reject the unit root null hypothesis at conventional nominal levels of significance. On the other hand, differencing the series did appear to induce stationarity in each case. Hence, the unit root tests clearly indicate that each of the time series examined is a realization from a stochastic process integrated of order one, which suggests that testing for cointegration between s_t, f_t^4, f_t^{13}, f_t^{26}, and f_t^{52}, is the logical next step.

We then employed the Johansen (1988, 1991) maximum likelihood procedure in a VAR for $y_t = [s_t, f_t^4, f_t^{13}, f_t^{26}, f_t^{52}]'$ and an unrestricted constant term.[8] On the basis of the Johansen likelihood ratio test statistics for cointegrating rank (based on the maximal eigenvalue and on the trace of the stochastic matrix), we could strongly reject the hypothesis of three independent cointegrating vectors against the alternative of four, but were not able to reject the hypothesis of exactly four cointegrating vectors for each exchange rate examined at conventional nominal test sizes.

In order to identify the cointegrating vectors uniquely, we then imposed and tested the over-identifying restrictions on the β' matrix of cointegrating coefficients suggested

[7] We are grateful to the Bank for International Settlements (BIS) for supplying the data.

[8] We allowed for a maximum lag length of five and chose, for each dollar exchange rate, the appropriate lag length on the basis of conventional information criteria.

by the framework discussed in Section 2.1:

$$\beta' y_t = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_t \\ f_t^4 \\ f_t^{13} \\ f_t^{26} \\ f_t^{52} \end{bmatrix}. \tag{2.8}$$

As the results reported in Table 2.1 clearly show, for each currency examined these restrictions were in fact rejected by the data. While these rejections are clearly statistically significant, we proceeded to examine whether the departures from the null hypothesis were large by imposing the following exactly-identifying restrictions:

$$\beta' y_t = \begin{bmatrix} -1 & \phi_4 & 0 & 0 & 0 \\ -1 & 0 & \phi_{13} & 0 & 0 \\ -1 & 0 & 0 & \phi_{26} & 0 \\ -1 & 0 & 0 & 0 & \phi_{52} \end{bmatrix} \begin{bmatrix} s_t \\ f_t^4 \\ f_t^{13} \\ f_t^{26} \\ f_t^{52} \end{bmatrix}, \tag{2.9}$$

where the ϕ_i parameters are unrestricted. This yielded the results reported in Table 2.2. The results suggest that, consistent with the recent studies by Naka and Whitney (1995) and Luintel and Paudyal (1998), the departure from the overidentifying restrictions, although statistically significant at conventional test sizes, is actually very small in magnitude. Indeed all of the estimated ϕ_i coefficients, except for the last two cointegrating relationships obtained on French and German data, are in the range between 0.98 and 1.04 and, therefore, very close indeed to the theoretical value of 1. Thus, rejection of the hypothesis $H_0 : \phi_i = 1 \; \forall i$ may be due to tiny departures from the null hypothesis (due, for example, to tiny data imperfections) which are not economically significant, but which appear as statistically significant given our very large sample size.[9] The framework we discussed in Section 2.1 provides strong economic priors in favor of the unity restrictions. Moreover, given that we know that covered interest parity holds strongly among Eurodeposit interest rates (Taylor, 1987, 1989), coefficients different from unity

[9] Leamer (1978, Chapter 4) points out that classical hypothesis testing will lead to rejection of any null hypothesis with a sufficiently large sample: 'Classical hypothesis testing at a fixed level of significance increasingly distorts the interpretation of the data against a null hypothesis as the sample size grows. The significance level should consequently be a decreasing function of sample size' (p. 114). See also Berkson (1938).

would imply a unit root in international interest rate differentials which seems highly implausible for the countries considered. Given this reasoning, we report below results obtained with the unity restrictions imposed.[10]

2.3.2 Linearity testing and MS-VECM estimation results

We next estimated a standard linear VECM using full-information maximum likelihood (FIML) methods:

$$\Delta y_t = \nu + \textstyle\sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \Pi y_{t-1} + u_t \tag{2.10}$$

where $y_t = [s_t, f_t^4, f_t^{13}, f_t^{26}, f_t^{52}]'$, assuming $p = 5$ as suggested by both the Akaike Information Criterion, the Schwartz Information Criterion and the Hannan-Quinn Criterion. Employing the conventional general-to-specific procedure, we obtained fairly parsimonious models for each exchange rate with no significant residual serial correlation. We then applied two fairly general linearity tests to the residuals from the estimated linear VECMs, namely Ramsey's (1969) RESET test and the Brock, Dechert and Sheinkman (BDS) (1991) test for testing the null hypothesis that the residuals from (10) are independent and identically distributed (iid) against an unspecified alternative.[11] Application of both of these tests provided very strong empirical evidence that the linear VECM fails to capture important nonlinearities in the data generating process, as linearity is

[10] We did, however, execute all of the empirical analysis discussed below *without* imposing the unity restrictions and using instead the estimates of the cointegrating parameters reported in Table 2.2. The results were quantitatively extremely similar (virtually identical) and qualitatively identical to those reported below.

[11] The BDS test for a series u_t is calculated in the following way. Let $\mathbf{u}_{t,v}$ be a set of consecutive terms from u_t : $\mathbf{u}_{t,v}$ $\{u_t, u_{t+1}, \dots, u_{t+v-1}\}$. The pair of vectors $\mathbf{u}_{t,v}$ and $\mathbf{u}_{s,v}$ are said to be no more than ς apart if $| u_{t+j} - u_{s+j} | \leq \varsigma$ for $j = 0, 1, \dots, v-1$. Thus, the correlation integral $C_v(\varsigma)$ is defined as the product of the limit of T^{-2} (T being the number of observations) times the number of ς-close pairs (s, t), essentially measuring the probability that the pairs of points (s, t) are within ς of each other. The BDS statistic is then constructed as $S(v, \varsigma) = \widehat{C}_v(\varsigma) - [\widehat{C}_1(\varsigma)]^v$ for some v and ς. Under the null hypothesis that u_t is iid, $\sqrt{T}[S(v, \varsigma)] \sim N(0, \xi)$, where the variance ξ is a function of v and ς. Rejection of the null implies that some form of nonlinearity is present in u_t, although the type of nonlinearity cannot be exactly determined under the BDS test. BDS (1991) suggest that the choice of v and, particularly, the choice of ς, are crucial for the power of the test, which is reasonably powerful only in large samples. BDS (1991) also suggest values of ς between 0.5 and 1.5 times the standard deviation of u_t, whereas the value of v should preferably be such that $(T/v) > 200$.

rejected with marginal significance levels (p-values) of virtually zero (see Table 2.3).[12]

We then proceeded to investigate the presence of nonlinearities further through the estimation of a fairly general Markov-switching model of the form:

$$\Delta y_t - \delta(z_t) = \alpha\left[\beta' y_{t-1} - \mu(z_t)\right] + \sum_{i=1}^{p-1} \Gamma_i \left[\Delta y_{t-i} - \delta(z_t)\right] + \omega_t, \tag{2.11}$$

where $y_t = [s_t, f_t^4, f_t^{13}, f_t^{26}, f_t^{52}]'$, $\delta(z_t)$ is the (5×1) regime-dependent vector of means of the short-run dynamics, and $\mu(z_t)$ is the (4×1) regime-dependent vector of means of the long-run equilibrium relationships.

Next we applied the conventional 'bottom-up' procedure designed to detect Markovian shifts in order to select the most adequate characterization of an M-regime p-th order MS-VECM for Δy_t.[13] The VARMA representations of the time series suggested in each case that the number of regimes was in the range between two and three. The linearity test indicates in each case the rejection of the linear VECM in favor of a nonlinear alternative model (see Table A.1 in the Appendix).

However, for any MS-VECM estimated the implicit assumption that the regime shifts affect only the intercept term of the VECM was found to be inappropriate. In fact, we checked the relevance of conditional homoskedasticity by estimating an MS-VECM where the Gaussian innovation is allowed to be regime-dependent, $\varepsilon_t \sim NID(\mathbf{0}, \Sigma(z_t))$. We then tested the hypothesis of no regime dependence in the variance-covariance matrix using a likelihood ratio (LR) test of the type suggested by Krolzig (1997, p. 135-6), in addition to constructing an LR test for the null hypothesis of no regime dependent intercept. The results (see Table A.1 in the Appendix) indicated very strong rejection of the null of no regime dependence, clearly suggesting that an MS-VECM that allows for shifts

[12] We also compare below the forecasting performance of the linear VECM to that of an MS-VECM.

[13] Essentially, the bottom-up procedure consists of starting with a simple but statistically reliable Markov-switching model by restricting the effects of regime shifts on a limited number of parameters and checking the model against alternatives. In such a procedure, most of the structure contained in the data is not attributed to regime shifts, but explained by observable variables, consistent with the general-to-specific approach to econometric modelling. For a technical discussion of the bottom-up procedure, see Krolzig (1997); for a more intuitive discussion see Sarno and Valente (2000).

in both the intercept *and* the variance-covariance matrix, namely an MSIH-VECM(p), is the most appropriate model within its class in the present application. Further in the same spirit of the test for regime-conditional homoskedasticity, we executed a test in order to select the most parsimonious VECM appropriately representing the dynamic relationship between spot and forward exchange rates. In particular, we tested the null of MSIH-VECM(1) against the alternative of MSIH-VECM(p) and, as may be seen in Table A.1 (Appendix), for all currencies examined, we were not able to reject this null hypothesis at standard significance levels.

Finally, in order to discriminate between models allowing for two regimes against models governed by three regimes we also constructed the upper bound LR test of Davies (1977, 1987). The results produced (see Table A.1, last column) very large statistics, suggesting that three regimes may be appropriate in all cases.[14] Therefore, in spite of parsimony considerations, we allowed for three regimes in our MS-VECM.

As stressed in some recent contributions[15], it is instructive to note that model (2.11), where the regime shifts occur in the drift of the VECM as well as the equilibrium mean of the cointegration relationships, can be equivalently represented by means of an MSI-VECM.[16] Hence, the final result of the procedure identifies in all countries an MS-VECM

[14] It is important to note here that the regularity conditions under which the Davies (1977, 1987) test is valid are violated, since the Markov model has both a problem of nuisance parameters and a problem of 'zero score' under the null hypothesis. Moreover, even if the Davies bound suggested by Krolzig is appropriate, it is possible that it will only be valid if the null model is a linear model with iid errors; in the present case, it is difficult to believe that this condition is met since exchange rate innovations are not homoskedastic, which would induce some distortion. Therefore, the distribution of the LR test is likely to differ from the adjusted χ^2 distribution proposed by Davies (1977, 1987), and this is why we do not report marginal significance levels for the LR tests. For extensive discussions of the problems related to LR testing in this context, see Hansen (1992, 1996) and Garcia (1998). See Garcia and Perron (1996) for an empirical application. We are thankful to Bruce Hansen for clarifying several econometric issues related to LR testing in the present chapter.

[15] See, for example, Krolzig (1997) and the references therein.

[16] In order to recognize the shifts in the drift of the VECM separately from the ones occurring in the long-run equilibrium mean, consistent with the standard theoretical literature on multiple cointegrated time series, it is possible to decompose the shifts in the intercept term $v(z_t)$ into changes in the drift of the system $\delta(z_t) = \beta_\perp (\alpha'_\perp \beta_\perp)^{-1} \alpha'_\perp v(z_t)$ ($\perp$ denoting the orthogonal complement) and the equilibrium mean $\mu(s_t) = -(\beta'\alpha)^{-1} [\beta' v(z_t)]$.

model governed by three different regimes that can be written as follows:

$$\Delta y_t = v(z_t) + \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \omega_t, \tag{2.12}$$

where $\Pi = \alpha\beta'$, $\omega_t \sim NIID(\mathbf{0}, \Sigma(z_t))$ and $z_t = 1, 2, 3$. We estimated the MSIH-VECM (2.12) using an EM algorithm for maximum likelihood (Dempster, Laird and Rubin, 1977), for each of France, Germany, Japan and the UK.[17] With few exceptions, the estimation yields fairly plausible estimates of the coefficients for the VECMs estimated, including the adjustment coefficients in α, which were generally found to be strongly statistically significantly different from zero.

For each country we find that three regimes are appropriate in describing the data, and that in each case the three regimes are driven mainly by the joint variability of spot and forward exchange rates. Shifts from one regime to another appeared to be due largely to shifts in the variance of the term structure equilibrium. On the other hand, shifts in the intercept terms were found to be relatively smaller in magnitude, albeit massively statistically significant. This appears to be in line with the extensive empirical literature investigating the time-varying nature of exchange rates risk premia. One tentative interpretation of our MSIH-VECM is, in fact, in terms of shifts in the mean and variance of foreign exchange returns consistent with deviations from the equilibrium levels implied by conventional macroeconomic fundamentals that may be caused, for example, by 'peso problems' or by other kinds of departures from the standard efficient markets hypothesis (see Engel and Hamilton, 1990).

2.3.3 Forecasting spot exchange rates out-of-sample with the MSIH-VECM

The procedure we have applied so far allowed us to achieve a reliable estimation of the dynamic relationship between spot exchange rates and the whole term structure of forward premia. The exercise conducted by Clarida and Taylor (1997) showed that

[17] These estimation results are reported in the Appendix A, Tables A.2-A.5.

the term structure of forward rates, embedding a larger information set than a single forward rate, is able to predict the dynamics of the spot exchange rate out-of-sample with a higher precision achieving average gains of 40% at long horizons with respect to the random walk benchmark model (in terms of root mean square errors).

In order to assess the usefulness of our nonlinear VECM characterization of the term structure, dynamic out-of-sample forecasts of the spot rate were constructed using the MSIH(3)-VECM(1) estimated and described in the previous section. In particular we performed forecasting exercises on the period January 1996-December 1998 with forecast horizons up to 52 weeks ahead.[18] The out-of-sample forecasts for a given horizon $j = 1, \ldots, 52$ are constructed according to the recursive procedure described in Clarida and Taylor (1993), namely conditional only upon information up to the data of the forecast and with successive re-estimation as the date on which forecasts are conditioned moves through the data set.

It is well known in the literature that forecasting with nonlinear models raises special problems.[19] We therefore adopt a very general forecasting procedure based on Monte Carlo integration which is capable of producing forecasts virtually identical to the analytical forecasts for a wide range of models. In particular, we forecast the path for s_{t+j} for $j = 1, \ldots, 52$ using Monte Carlo simulations calibrated on the estimated MSIH-VECMs. The simulation procedure is repeated with identical random numbers 10,000 times and the average of the 10,000 realizations of the sequence of forecasts is taken as the point forecast. Since we use a large number of simulations, by the Law of Large Numbers this procedure should produce results virtually identical to those which would result from calculating the exact forecast analytically (Gallant, Rossi and Tauchen, 1993; Brown and Mariano, 1984, 1989; Granger and Terasvirta, 1993, Chapter 8; Franses and

[18] For a description of the procedure of out-of-sample forecasting in a Markov-switching framework, see Hamilton (1993, 1994). See also Franses and van Dijk (2000).

[19] See Brown and Mariano (1984, 1989) and, for a general discussion, Granger and Terasvirta (1993, Chapter 8) and Franses and van Dijk (2000, Chapters 3-4).

van Dijk, 2000, Chapters 3-4).

Forecast accuracy is evaluated using absolute and square error criteria, namely the mean absolute error (MAE) and the root mean square error (RMSE). We compared the forecasts produced by the MSIH-VECM to the forecasts generated by a simple random walk benchmark as well as the forecasts generated by the linear term-structure VECM originally proposed by Clarida and Taylor (1997). Further, in order to assess the accuracy of forecasts derived from two different models we employ the Diebold and Mariano (1995) test:

$$DM = \frac{\overline{d}}{\sqrt{\frac{2\pi \widehat{f}(0)}{T}}} \tag{2.13}$$

where $\overline{d}$ is an average (over T observations) of a general loss differential function and $\widehat{f}(0)$ is a consistent estimate of the spectral density of the loss differential function at frequency zero. Diebold and Mariano show that the DM statistic is distributed as standard normal under the null hypothesis of equal forecast accuracy. Consistent with a large literature (see, *inter alia*, Mark, 1995; Kilian, 1999; Kilian and Taylor, 2003) the loss differential function we consider is the difference between the (absolute and square) forecast errors. A consistent estimate of the spectral density at frequency zero $\widehat{f}(0)$ is obtained using the method of Newey and West (1987) where the optimal truncation lag has been selected using the Andrews (1991) AR(1) rule.[20]

Several problems may arise using DM statistics in small sample as well as taking into account parameter uncertainty (see also West, 1996; West and McCracken, 1998; and McCracken, 2000). In our case, where we are dealing with nested competing forecasting models - one of which is nonlinear - and with multi-step-ahead forecasts, the asymptotic distribution of the DM statistic is non-standard and unknown. Therefore, the marginal

[20] The rule is implemented as follows: we estimated an AR(1) model to the quantity d_t obtaining the autocorrelation coefficient $\widehat{\rho}$ and the innovation variance from the AR(1) process $\widehat{\sigma}^2$. Then the optimal truncation lag A for the Parzen window in the Newey-West estimator is given by the Andrews rule $A = 2.6614 \left[\widehat{\zeta}(2)T\right]^{1/5}$ where $\widehat{\zeta}(2)$ is a function of $\widehat{\rho}$ and $\widehat{\sigma}^2$. The Parzen window minimizes the mean square error of the estimator (Gallant, 1987, p. 534).

significance levels reported below should be interpreted with caution.[21]

Tables 2.4-2.7 give detailed results of the accuracy of the forecasts for the dollar-franc, dollar-mark, dollar-yen and dollar-sterling systems respectively, using MAE and RMSE criteria for forecast accuracy. The results provide evidence in favor of the predictive superiority of the MSIH-VECM models against the naive random walk and, to a lesser extent, against linear VECM models. Comparing our results to those obtained using the pure random walk we can see that, across countries, the MSIH-VECM models give very much more accurate forecasts. At the 4-week horizon we achieve average improvements ranging between 28% and 38% across currencies using the MAE and between 27% and 31% using the RMSE. At the 52-week horizon we obtain average improvements ranging between 8% and 70% using the MAE and between 5% and 68% using the RMSE, with a maximum reduction of 70% in the case of the dollar-yen rate using the MAE. The statistical significance of these results is confirmed executing the DM test, although the marginal significance levels reported should be treated with caution in the light of our earlier discussion on the asymptotic properties of the DM statistic in the present context.

These results extend the findings of Clarida and Taylor (1997) who, using a linear VECM framework for the term structure of forward foreign exchange premia, were able to provide out-of-sample forecasts of spot exchange rates which were superior to alternative conventional forecasting methods. By explicitly incorporating nonlinearity into the modelling framework, we have in the present analysis been able to improve upon the Clarida-Taylor results as in almost all cases we are able to improve the forecasting performance of the linear VECM models. In particular, the gains we obtained relative

[21] Clark and McCracken (2001) derive the asymptotic distributions of two standard tests in this context for one-step-ahead forecasts from nested linear models. Their results are, unfortunately, not directly applicable to our case because we are dealing with multi-step-ahead forecasts from nested models, and because one of the competing models is a Markov-switching model. Therefore, our case is one for which the asymptotic theory of the DM statistic is unknown at the present time. A possible solution involves calculating the marginal significance levels by bootstrap methods using a variant of the bootstrap procedure proposed by Kilian (1999) and Kilian and Taylor (2003), although this procedure is computationally very demanding and it is unknown whether it is valid in the context of MSIH-VECMs.

to the linear VECMs range, on average across currencies, between 1% and 10% at the 4-week horizon and between 10% and 38% at the 52-week horizon using the MAE. Using the RMSE the gains range between 1% and 7% at the 4-week horizon and between 10% and 38% at the 52-week horizon.[22] Therefore, the gain from using an MSIH-VECM rather than a linear VECM is relatively small at short horizon, albeit generally statistically significant; however, this gain increases with the forecast horizon and becomes substantial at the 52-week horizon.

2.4 Conclusion

In this chapter we have reported what we believe to be the first analysis of spot and forward exchange rates in a multivariate Markov-switching framework, and in particular we have applied that framework to exchange rate forecasting. Our research was inspired by encouraging results previously reported in the literature on the presence of nonlinearities (and particularly by the success of Markov-switching models) in the context of exchange rate modelling, as well as by the relative forecasting success of the 'agnostic' linear VECM model of the term structure of forward premia.

Using weekly data on spot and forward dollar exchange rates for the G5 countries over the period January 1979 through December 1995, we found strong evidence of the presence of nonlinearities in the term structure, which appeared to be modelled well by a multivariate three-regime Markov-switching VECM that allows for shifts in both the intercept and in the covariance structure. We then used this model to forecast dynamically out of sample over the period January 1996 through to December 1998. The forecasting results were impressive. The MS-VECM forecasts were found to be strongly superior to the random walk forecasts at a range of forecasting horizons up to

[22] Note, however, that there is heterogeneity across countries. For example, the gain for the French data is fairly small, whereas the gain for the German data is substantial. The MSIH-VECM for dollar-sterling is arguably the model which performs worse in forecasting and is beaten by a random walk at the 52-week horizon on the basis of the RMSE.

52 weeks ahead, using standard forecasting accuracy criteria and on the basis of standard tests of significance. Moreover, the nonlinear VECM also outperformed, in general, a linear VECM for spot and forward rates in out-of-sample forecasting of the spot rate, although the magnitude of the gain from using a Markov-switching VECM relative to a linear VECM is rather small in magnitude at short horizon.

In this research, we have been primarily concerned with providing sound models of exchange rate forecasting and have therefore adopted an 'agnostic' approach both with respect to the sources of the underlying departures from the risk-neutral efficient markets hypothesis and in the sources of the underlying nonlinearities. Future research might, therefore, usefully analyze the sources of these nonlinearities further and attempt to improve on the parametric nonlinear formulation proposed in this chapter. Possible extensions include the allowance for different equilibrium correction terms (speeds of adjustment towards equilibrium) in different regimes, and the endogeneization of the probability of switching from one regime to another, which might, for example, be made a function of macroeconomic fundamentals.

With regard to the evaluation of forecasting models, although the relevant literature has traditionally focused on accuracy evaluations based on point forecasts, several authors have recently emphasized the importance of evaluating the forecast accuracy of economic models on the basis of density - as opposed to point - forecasting performance (see, *inter alia*, Diebold, Gunther and Tay, 1998; Granger and Pesaran, 1999; Tay and Wallis, 2000; Timmerman, 2000; Pesaran and Skouras, 2002; Sarno and Valente, 2003). Especially when evaluating nonlinear models, which are capable of producing highly non-normal forecast densities, it would seem appropriate to consider a model's density forecasting performance. This is an immediate avenue for future research.

Given the difficulty in beating random walk forecasts using fundamentals-based models - first highlighted by Meese and Rogoff (1983a,b) - as well as the well-known failure of the forward rate optimally to predict the future spot rate, the evidence provided by our results that the term structure of forward rates is powerful in forecasting spot exchange rates is rather striking. In particular, it seems that, notwithstanding the failure of the simple (risk-neutral) efficient markets hypothesis in this context, forward rates may contain more useful information to forecast spot exchange rates than do conventional fundamentals. It seems plausible that important microstructural effects may be responsible for this phenomenon, as argued, for example, by Lyons (2001). Sarno and Taylor (2001) and Evans and Lyons (2002). Understanding the exact nature of this incremental information remains an important challenge in the research agenda.

Table 2.1: Tests of the null hypothesis that independent forward premia comprise a basis for the cointegration space

	$\chi^2(g)$ {p-value}
France	21.6 $\{5.6\times10^{-3}\}$
Germany	51.7 $\{1.8\times10^{-8}\}$
Japan	41.9 $\{1.4\times10^{-6}\}$
UK	53.6 $\{8.1\times10^{-9}\}$

Notes: The test is a χ^2 version of the test of the overidentifying restrictions on the β' matrix described in equation (2.8); g is the number of restrictions imposed.

Table 2.2: Long-run equilibrium parameters

$h(k)$	France	Germany	Japan	UK
4 weeks	0.995 (0.002)	1.006 (0.002)	1.002 (0.001)	0.999 (0.002)
13 weeks	0.987 (0.007)	1.020 (0.006)	1.005 (0.004)	1.001 (0.007)
26 weeks	0.976 (0.013)	1.042 (0.011)	1.010 (0.007)	1.004 (0.012)
52 weeks	0.964 (0.022)	1.084 (0.022)	1.018 (0.013)	1.015 (0.022)

Notes: The table gives the estimated long-run slope parameter for the forward rate at each maturity. Figures in parentheses denote asymptotic standard errors.

Table 2.3: Linearity tests on the residuals from the linear VECM (2.10)

Panel a): RESET tests: p-values

	France	Germany	Japan	UK
Δs_t	4.1×10^{-60}	1.2×10^{-49}	3.6×10^{-37}	3.1×10^{-18}
Δf_t^4	3.7×10^{-48}	4.1×10^{-58}	1.2×10^{-34}	7.3×10^{-19}
Δf_t^{13}	5.2×10^{-56}	3.1×10^{-58}	2.1×10^{-52}	2.3×10^{-4}
Δf_t^{26}	4.4×10^{-22}	2.6×10^{-19}	1.4×10^{-46}	7.2×10^{-57}
Δf_t^{52}	1.0×10^{-26}	2.6×10^{-27}	2.9×10^{-16}	1.5×10^{-54}

Panel b): BDS tests: p-values

	$v=2$			$v=3$		
	$\varsigma=0.5\sigma_u$	$\varsigma=1.0\sigma_u$	$\varsigma=1.5\sigma_u$	$\varsigma=0.5\sigma_u$	$\varsigma=1.0\sigma_u$	$\varsigma=1.5\sigma_u$
France						
Δs_t	4.9×10^{-2}	3.0×10^{-2}	7.2×10^{-3}	5.4×10^{-5}	2.0×10^{-4}	3.4×10^{-5}
Δf_t^4	1.0×10^{-2}	1.4×10^{-2}	4.2×10^{-3}	5.3×10^{-5}	1.1×10^{-4}	1.8×10^{-5}
Δf_t^{13}	9.3×10^{-3}	9.9×10^{-3}	4.2×10^{-3}	7.0×10^{-5}	1.6×10^{-4}	4.0×10^{-5}
Δf_t^{26}	2.1×10^{-3}	3.4×10^{-3}	6.0×10^{-4}	3.5×10^{-5}	1.1×10^{-4}	1.0×10^{-5}
Δf_t^{52}	2.0×10^{-4}	1.2×10^{-4}	2.1×10^{-5}	1.7×10^{-5}	1.7×10^{-5}	2.4×10^{-6}
Germany						
Δs_t	5.7×10^{-2}	1.2×10^{-2}	4.4×10^{-3}	2.0×10^{-5}	2.5×10^{-5}	5.7×10^{-6}
Δf_t^4	3.9×10^{-2}	1.2×10^{-2}	2.5×10^{-3}	8.4×10^{-6}	1.9×10^{-5}	2.2×10^{-6}
Δf_t^{13}	7.7×10^{-2}	8.8×10^{-3}	1.9×10^{-3}	3.9×10^{-5}	8.2×10^{-6}	2.3×10^{-6}
Δf_t^{26}	3.1×10^{-2}	6.8×10^{-3}	1.7×10^{-3}	5.2×10^{-6}	4.8×10^{-6}	2.2×10^{-6}
Δf_t^{52}	3.4×10^{-2}	5.8×10^{-3}	1.5×10^{-3}	2.0×10^{-5}	1.5×10^{-6}	3.1×10^{-6}
Japan						
Δs_t	9.7×10^{-5}	8.4×10^{-5}	1.0×10^{-4}	3.4×10^{-6}	5.4×10^{-6}	2.0×10^{-5}
Δf_t^4	4.5×10^{-5}	5.9×10^{-5}	7.5×10^{-5}	7.2×10^{-7}	2.7×10^{-6}	1.3×10^{-5}
Δf_t^{13}	3.4×10^{-4}	1.7×10^{-4}	1.1×10^{-4}	6.3×10^{-7}	5.3×10^{-6}	1.8×10^{-5}
Δf_t^{26}	8.6×10^{-5}	9.4×10^{-5}	2.9×10^{-4}	2.7×10^{-8}	1.1×10^{-6}	5.2×10^{-5}
Δf_t^{52}	1.1×10^{-10}	2.9×10^{-10}	6.7×10^{-8}	2.0×10^{-15}	3.2×10^{-13}	3.2×10^{-9}
UK						
Δs_t	2.1×10^{-2}	1.7×10^{-2}	7.7×10^{-8}	3.6×10^{-3}	2.1×10^{-4}	1.6×10^{-4}
Δf_t^4	1.0×10^{-2}	1.3×10^{-2}	6.0×10^{-3}	1.4×10^{-4}	1.5×10^{-4}	1.1×10^{-4}
Δf_t^{13}	1.9×10^{-2}	1.0×10^{-2}	7.1×10^{-3}	2.9×10^{-4}	1.2×10^{-4}	1.2×10^{-4}
Δf_t^{26}	5.8×10^{-3}	4.9×10^{-3}	5.1×10^{-3}	2.4×10^{-5}	3.4×10^{-5}	6.8×10^{-5}
Δf_t^{52}	4.5×10^{-3}	1.2×10^{-3}	2.2×10^{-3}	1.1×10^{-5}	2.0×10^{-6}	2.2×10^{-5}

Notes: The RESET test statistics were computed considering an alternative model with a quadratic and a cubic term; they are distributed as $F(2, T-m-3)$ under the null hypothesis of linearity (no misspecification), where T is the number of observations and m is the number of regressors (inclusive of the intercept). The BDS test statistic tests the null hypothesis that a series is iid against the alternative of a realization from an unspecified nonlinear process, as described in the text. The critical values, from the normal distribution, are 1.960 and 2.576 at the five percent and one percent nominal levels of significance respectively. Given that the choices of v and ς are crucial for the power of the test, we report the results for different plausible values of v and ς, as suggested by BDS (1991) and explained in the text; σ_u is the standard deviation of the residuals from the linear VECM (2.10). For both RESET tests and BDS tests, only p-values are reported.

Table 2.4: Forecasting exercise: France

	Mean Absolute Errors				
k	MSVECM	RW	VECM	DM1(A)	DM2(A)
4	0.0037	0.0057	0.0037	1.3×10^{-1}	1.7×10^{-1}
13	0.0037	0.0066	0.0038	6.9×10^{-3}	3.0×10^{-2}
26	0.0047	0.0827	0.0049	1.2×10^{-2}	1.7×10^{-6}
52	0.0066	0.0115	0.0073	3.9×10^{-3}	2.2×10^{-16}

	Root Mean Square Errors				
k	MSVECM	RW	VECM	DM1(A)	DM2(A)
4	0.0043	0.0062	0.0043	1.0×10^{-1}	3.0×10^{-1}
13	0.0047	0.0074	0.0048	4.9×10^{-3}	2.2×10^{-2}
26	0.0055	0.0092	0.0058	1.0×10^{-2}	3.7×10^{-8}
52	0.0095	0.0147	0.0106	2.1×10^{-3}	1.4×10^{-13}

Notes: MSVECM, RW and VECM are the average level of the (absolute or square) forecast error obtained by the MSIH-VECM, random walk and linear VECM respectively. The forecast errors are obtained by recursive estimation of out-of-sample dynamic forecasts up to $k = 4, 13, 26, 52$ steps ahead over the period 1996:1-1998:52. DM1(A) is the Diebold-Mariano statistic comparing the forecast errors of the MSIH-VECM model with the ones obtained by a driftless random walk. DM2(A) is the Diebold-Mariano statistic comparing the forecast errors of the MSIH-VECM model with the ones obtained by the linear VECM. A is the optimal truncation lag constructed according to the AR(1) Andrews (1991) rule. For the Diebold and Mariano's statistics only $p-$values are reported. 0 indicates $p-$values below 10^{-100}, which are considered as virtually zero.

Table 2.5: Forecasting exercise: Germany

	Mean Absolute Errors				
k	MSVECM	RW	VECM	DM1(A)	DM2(A)
4	0.0040	0.0064	0.0042	5.4×10^{-2}	2.7×10^{-1}
13	0.0043	0.0082	0.0056	1.3×10^{-3}	1.1×10^{-10}
26	0.0055	0.0121	0.0075	2.1×10^{-7}	7.8×10^{-4}
52	0.0062	0.0160	0.0100	1.3×10^{-15}	0

	Root Mean Square Errors				
k	MSVECM	RW	VECM	DM1(A)	DM2(A)
4	0.0046	0.0066	0.0047	7.9×10^{-2}	3.7×10^{-1}
13	0.0054	0.0094	0.0068	1.0×10^{-3}	1.8×10^{-17}
26	0.0066	0.0138	0.0088	7.0×10^{-8}	3.3×10^{-4}
52	0.0080	0.0190	0.0128	7.0×10^{-12}	0

Notes: See Notes to Table 2.4

Table 2.6: Forecasting exercise: Japan

Mean Absolute Errors					
k	MSVECM	RW	VECM	DM1(A)	DM2(A)
4	0.0020	0.0032	0.0022	1.2×10^{-2}	5.9×10^{-1}
13	0.0032	0.0042	0.0032	1.9×10^{-2}	8.9×10^{-1}
26	0.0040	0.0077	0.0046	6.1×10^{-6}	2.8×10^{-2}
52	0.0053	0.0179	0.0082	3.9×10^{-14}	0
Root Mean Square Errors					
k	MSVECM	RW	VECM	DM1(A)	DM2(A)
4	0.0026	0.0036	0.0028	4.4×10^{-2}	5.4×10^{-1}
13	0.0042	0.0052	0.0042	4.6×10^{-2}	9.6×10^{-1}
26	0.0050	0.0094	0.0058	2.9×10^{-7}	5.6×10^{-4}
52	0.0074	0.0230	0.0112	0	0

Notes: See Notes to Table 2.4

Table 2.7: Forecasting exercise: UK

Mean Absolute Errors					
k	MSVECM	RW	VECM	DM1(A)	DM2(A)
4	0.0044	0.0061	0.0046	1.8×10^{-2}	3.4×10^{-1}
13	0.0025	0.0047	0.0028	1.3×10^{-3}	1.4×10^{-1}
26	0.0042	0.0059	0.0053	5.1×10^{-3}	6.9×10^{-7}
52	0.0135	0.0147	0.0167	5.1×10^{-1}	0
Root Mean Square Errors					
k	MSVECM	RW	VECM	DM1(A)	DM2(A)
4	0.0049	0.0067	0.0052	2.1×10^{-2}	2.9×10^{-1}
13	0.0034	0.0054	0.0036	2.1×10^{-3}	3.5×10^{-1}
26	0.0054	0.0069	0.0069	1.2×10^{-2}	7.5×10^{-13}
52	0.0186	0.0069	0.0225	4.4×10^{-1}	0

Notes: See Notes to Table 2.4

Appendix A

Markov-Switching-VECM results

Table A.1: 'Bottom-up' identification procedure

	$LR1$	$LR2$	Davies
France	5007.8 {0}	0.4 {0.999}	1305.1
Germany	3158.6 {0}	21.8 {0.648}	440.5
Japan	3320.9 {0}	15.5 {0.928}	487.3
United Kingdom	1668.5 {0}	35.0 {0.128}	477.7

Notes: $LR1$ is a test statistic of the null hypothesis of no regime dependent variance-covariance matrix (ie. MSI(M)-VECM(p) versus MSIH(M)-VECM(p)). $LR2$ tests the null hypothesis that the model having autoregressive component of order one is equivalent to another with a higher autoregressive order (i.e. MS(M)-VECM(1) versus MS(M)-VECM(p)). Both tests are constructed as $2(\ln L^* - \ln L)$, where L^* and L represent the unconstrained and the constrained maximum likelihood respectively. Those tests are distributed as $\chi^2(g)$ where g is the number of restrictions imposed. *Davies* is the upper bound of the likelihood ratio test where the model is not identified under the null hypothesis due to the existance of nuisance parameters. In this case it tests the null hypothesis that the model with two regimes is equivalent to the model with three regimes. Figures in braces denote $p-$values, and {0} indicates $p-$values below 10^{-100}, which are considered as virtually zero.

Table A.2: MSIH(3)-VECM(1) estimation results: France

Panel A:

$$\widetilde{\Gamma}_1 = \begin{bmatrix} 0.089 & -0.844 & 1.094 & -0.291 & -0.048 \\ (0.23) & (0.39) & (0.19) & (0.42) & (0.26) \\ 0.090 & -0.865 & 1.148 & -0.322 & -0.049 \\ (0.23) & (0.38) & (0.17) & (0.41) & (0.26) \\ 0.072 & -0.867 & 1.178 & -0.319 & -0.063 \\ (0.23) & (0.38) & (0.15) & (0.40) & (0.26) \\ 0.039 & -0.875 & 1.263 & -0.344 & -0.080 \\ (0.23) & (0.37) & (0.17) & (0.40) & (0.26) \\ 0.088 & -0.915 & 0.941 & 0.150 & -0.265 \\ (0.22) & (0.38) & (0.26) & (0.41) & (0.26) \end{bmatrix};$$

$$\widetilde{v}(z_1) = \begin{bmatrix} -1.15\times 10^{-3} \\ (1.12\times 10^{-3}) \\ -1.44\times 10^{-3} \\ (1.14\times 10^{-3}) \\ -1.46\times 10^{-3} \\ (1.17\times 10^{-3}) \\ -1.41\times 10^{-3} \\ (1.29\times 10^{-3}) \\ -1.59\times 10^{-3} \\ (1.50\times 10^{-3}) \end{bmatrix}; \widetilde{v}(z_2) = \begin{bmatrix} 1.12\times 10^{-4} \\ (1.49\times 10^{-5}) \\ 1.25\times 10^{-4} \\ (8.81\times 10^{-6}) \\ 1.25\times 10^{-4} \\ (2.28\times 10^{-5}) \\ 1.20\times 10^{-4} \\ (4.48\times 10^{-5}) \\ 1.25\times 10^{-4} \\ (7.91\times 10^{-5}) \end{bmatrix}; \widetilde{v}(z_3) = \begin{bmatrix} -6.49\times 10^{-5} \\ (5.92\times 10^{-4}) \\ -1.06\times 10^{-5} \\ (5.87\times 10^{-4}) \\ -1.34\times 10^{-5} \\ (5.64\times 10^{-4}) \\ -2.07\times 10^{-5} \\ (5.35\times 10^{-4}) \\ 1.27\times 10^{-5} \\ (5.09\times 10^{-4}) \end{bmatrix};$$

$$\widetilde{\alpha} = \begin{bmatrix} -0.708 & 0.989 & -0.422 & 0.013 \\ (0.50) & (0.20) & (0.20) & (0.11) \\ -0.487 & 0.931 & -0.440 & 0.018 \\ (0.50) & (0.19) & (0.20) & (0.11) \\ -0.918 & 1.245 & -0.500 & 0.004 \\ (0.49) & (0.16) & (0.19) & (0.11) \\ -0.850 & 0.906 & -0.191 & 0.075 \\ (0.50) & (0.16) & (0.20) & (0.11) \\ -0.796 & 0.951 & -0.263 & 0.050 \\ (0.53) & (0.22) & (0.23) & (0.12) \end{bmatrix};$$

(continued)

Panel B:

$$\widetilde{\sum}(z_1) = \begin{bmatrix} 8.05\times10^{-5} & & & & \\ 7.60\times10^{-5} & 8.59\times10^{-5} & & & \\ 7.24\times10^{-5} & 8.48\times10^{-5} & 9.14\times10^{-5} & & \\ 7.19\times10^{-5} & 8.66\times10^{-5} & 9.95\times10^{-5} & 1.15\times10^{-4} & \\ 7.11\times10^{-5} & 8.91\times10^{-5} & 1.08\times10^{-4} & 1.32\times10^{-4} & 1.63\times10^{-4} \end{bmatrix};$$

$$\widetilde{\sum}(z_2) = \begin{bmatrix} 4.03\times10^{-5} & & & & \\ 4.02\times10^{-5} & 4.01\times10^{-5} & & & \\ 4.00\times10^{-5} & 3.99\times10^{-5} & 3.97\times10^{-5} & & \\ 3.96\times10^{-5} & 3.96\times10^{-5} & 3.94\times10^{-5} & 3.92\times10^{-5} & \\ 3.89\times10^{-5} & 3.89\times10^{-5} & 3.88\times10^{-5} & 3.86\times10^{-5} & 3.82\times10^{-5} \end{bmatrix};$$

$$\widetilde{\sum}(z_3) = \begin{bmatrix} 4.55\times10^{-5} & & & & \\ 4.49\times10^{-5} & 4.44\times10^{-5} & & & \\ 4.34\times10^{-5} & 4.31\times10^{-5} & 4.22\times10^{-5} & & \\ 4.15\times10^{-5} & 4.13\times10^{-5} & 4.07\times10^{-5} & 3.97\times10^{-5} & \\ 3.96\times10^{-5} & 3.96\times10^{-5} & 3.93\times10^{-5} & 3.88\times10^{-5} & 3.88\times10^{-5} \end{bmatrix};$$

$$\widetilde{\mathbf{P}} = \begin{bmatrix} 0.68 & 0.01 & 0.06 \\ 0.07 & 0.93 & 0.09 \\ 0.25 & 0.06 & 0.85 \end{bmatrix}; \widetilde{\xi} = \begin{bmatrix} 0.09 \\ 0.55 \\ 0.36 \end{bmatrix};$$

$\rho(A) = 0.1515$

LR linearity test: 2430.01 {0}

Notes: Tildes denotes estimated values obtained using the EM algorithm for maximum likelihood (Dempster, Laird and Rubin, 1977). Figures in parentheses are asymptotic standard errors. Symbols are defined as in equation (2.12). P and ξ denote the $M \times M$ transition matrix and the M-dimensional ergodic probabilities vector respectively. $\rho(A)$ is the spectral radius of the matrix A calculated as in Karlsen (1990). It can be tought as a measure of stationarity of the MS-VECM. The LR linearity test is a Davies (1987)-type test checking the hypothesis that the true model is a linear VECM against the alternative of a MSIH-VECM. Its p−value is calculated as in Davies (1987).

Table A.3: MSIH(3)-VECM(1) estimation results: Germany

Panel A:

$$\widetilde{\Gamma}_1 = \begin{bmatrix} -2.972 & 3.440 & 1.112 & -1.452 & -0.145 \\ (0.14) & (0.16) & (0.12) & (0.09) & (0.11) \\ -2.939 & 3.362 & 1.159 & -1.438 & -0.160 \\ (0.12) & (0.13) & (0.09) & (0.08) & (0.11) \\ -2.922 & 3.341 & 1.142 & -1.401 & -0.175 \\ (0.10) & (0.08) & (0.05) & (0.05) & (0.11) \\ -2.911 & 3.193 & 1.358 & -1.466 & -0.187 \\ (0.10) & (0.10) & (0.06) & (0.06) & (0.11) \\ -2.549 & 2.724 & 1.245 & -1.170 & -0.266 \\ (0.20) & (0.27) & (0.20) & (0.15) & (0.13) \end{bmatrix};$$

$$\widetilde{v}(z_1) = \begin{bmatrix} -2.95\times10^{-3} \\ (1.00\times10^{-3}) \\ -2.90\times10^{-3} \\ (9.98\times10^{-4}) \\ -2.90\times10^{-3} \\ (9.86\times10^{-4}) \\ -2.89\times10^{-3} \\ (1.01\times10^{-3}) \\ -3.00\times10^{-3} \\ (1.15\times10^{-3}) \end{bmatrix}; \widetilde{v}(z_2) = \begin{bmatrix} -1.01\times10^{-3} \\ (1.66\times10^{-4}) \\ -9.79\times10^{-4} \\ (1.70\times10^{-4}) \\ -9.96\times10^{-4} \\ (1.61\times10^{-4}) \\ -9.43\times10^{-4} \\ (1.54\times10^{-4}) \\ -9.27\times10^{-4} \\ (1.58\times10^{-4}) \end{bmatrix}; \widetilde{v}(z_3) = \begin{bmatrix} -5.47\times10^{-4} \\ (1.34\times10^{-5}) \\ -5.27\times10^{-4} \\ (6.47\times10^{-6}) \\ -5.12\times10^{-4} \\ (9.70\times10^{-6}) \\ -5.09\times10^{-4} \\ (1.91\times10^{-5}) \\ -5.11\times10^{-4} \\ (4.21\times10^{-5}) \end{bmatrix};$$

$$\widetilde{\alpha} = \begin{bmatrix} 7.641 & -4.107 & -0.235 & 0.570 \\ (0.15) & (0.06) & (0.06) & (0.02) \\ 7.973 & -4.248 & -0.223 & 0.571 \\ (0.11) & (0.05) & (0.06) & (0.02) \\ 7.445 & -3.874 & -0.292 & 0.554 \\ (0.05) & (0.06) & (0.06) & (0.02) \\ 7.525 & -4.100 & -0.092 & 0.502 \\ (0.09) & (0.14) & (0.09) & (0.02) \\ 7.584 & -3.903 & -0.276 & 0.541 \\ (0.27) & (0.27) & (0.16) & (0.04) \end{bmatrix};$$

(continued)

Panel B:

$$\widetilde{\sum}(z_1) = \begin{bmatrix} 9.20\times10^{-5} & & & & \\ 9.06\times10^{-5} & 8.98\times10^{-5} & & & \\ 8.86\times10^{-5} & 8.89\times10^{-5} & 8.98\times10^{-5} & & \\ 8.58\times10^{-5} & 8.73\times10^{-5} & 9.05\times10^{-5} & 9.45\times10^{-5} & \\ 8.29\times10^{-5} & 8.63\times10^{-5} & 9.29\times10^{-5} & 1.01\times10^{-4} & 1.17\times10^{-4} \end{bmatrix};$$

$$\widetilde{\sum}(z_2) = \begin{bmatrix} 3.85\times10^{-5} & & & & \\ 3.85\times10^{-5} & 3.84\times10^{-5} & & & \\ 3.81\times10^{-5} & 3.81\times10^{-5} & 3.78\times10^{-5} & & \\ 3.75\times10^{-5} & 3.75\times10^{-5} & 3.73\times10^{-5} & 3.69\times10^{-5} & \\ 3.73\times10^{-5} & 3.73\times10^{-5} & 3.72\times10^{-5} & 3.70\times10^{-5} & 3.74\times10^{-5} \end{bmatrix};$$

$$\widetilde{\sum}(z_3) = \begin{bmatrix} 4.36\times10^{-5} & & & & \\ 4.36\times10^{-5} & 4.36\times10^{-5} & & & \\ 4.36\times10^{-5} & 4.37\times10^{-5} & 4.37\times10^{-5} & & \\ 4.36\times10^{-5} & 4.36\times10^{-5} & 4.37\times10^{-5} & 4.37\times10^{-5} & \\ 4.35\times10^{-5} & 4.36\times10^{-5} & 4.36\times10^{-5} & 4.37\times10^{-5} & 4.39\times10^{-5} \end{bmatrix};$$

$$\widetilde{\mathbf{P}} = \begin{bmatrix} 0.57 & 0.11 & 0.01 \\ 0.36 & 0.79 & 0.04 \\ 0.07 & 0.10 & 0.95 \end{bmatrix}; \widetilde{\xi} = \begin{bmatrix} 0.08 \\ 0.28 \\ 0.64 \end{bmatrix};$$

$\rho(A) = 0.1243$

LR linearity test: 3586.70 {0}

Notes: See Notes to Table A.2

Table A.4: MSIH(3)-VECM(1) estimation results: Japan

Panel A:

$$\widetilde{\Gamma}_1 = \begin{bmatrix} -1.627 & 1.947 & 0.499 & -0.788 & 0.021 \\ (0.11) & (0.13) & (0.10) & (0.09) & (0.05) \\ -1.574 & 1.939 & 0.447 & -0.796 & 0.037 \\ (0.09) & (0.11) & (0.08) & (0.07) & (0.05) \\ -1.652 & 2.143 & 0.303 & -0.791 & 0.050 \\ (0.06) & (0.06) & (0.05) & (0.05) & (0.05) \\ -1.617 & 2.110 & 0.351 & -0.865 & 0.072 \\ (0.06) & (0.07) & (0.05) & (0.04) & (0.05) \\ -1.551 & 2.154 & -0.021 & -0.504 & -0.028 \\ (0.16) & (0.23) & (0.18) & (0.17) & (0.07) \end{bmatrix};$$

$$\widetilde{v}(z_1) = \begin{bmatrix} -1.25\times10^{-4} \\ (1.93\times10^{-4}) \\ -1.46\times10^{-4} \\ (1.60\times10^{-4}) \\ -2.28\times10^{-4} \\ (1.69\times10^{-4}) \\ -3.35\times10^{-4} \\ (3.23\times10^{-4}) \\ -5.58\times10^{-4} \\ (6.87\times10^{-4}) \end{bmatrix}; \widetilde{v}(z_2) = \begin{bmatrix} 3.08\times10^{-4} \\ (1.62\times10^{-5}) \\ 3.09\times10^{-4} \\ (1.45\times10^{-5}) \\ 3.01\times10^{-4} \\ (1.74\times10^{-5}) \\ 3.52\times10^{-4} \\ (4.40\times10^{-5}) \\ 3.95\times10^{-4} \\ (8.67\times10^{-4}) \end{bmatrix}; \widetilde{v}(z_3) = \begin{bmatrix} 2.61\times10^{-4} \\ (4.49\times10^{-4}) \\ 2.75\times10^{-4} \\ (4.47\times10^{-4}) \\ 2.28\times10^{-4} \\ (4.36\times10^{-4}) \\ 2.93\times10^{-4} \\ (4.36\times10^{-4}) \\ 2.58\times10^{-4} \\ (4.68\times10^{-4}) \end{bmatrix};$$

$$\widetilde{\alpha} = \begin{bmatrix} 1.309 & -1.133 & 0.476 & 0.015 \\ (0.12) & (0.12) & (0.13) & (0.10) \\ 1.848 & -1.360 & 0.462 & 0.034 \\ (0.09) & (0.11) & (0.11) & (0.10) \\ 1.579 & -1.087 & 0.340 & 0.050 \\ (0.07) & (0.11) & (0.10) & (0.09) \\ 1.913 & -1.546 & 0.580 & 0.017 \\ (0.09) & (0.12) & (0.10) & (0.09) \\ 2.470 & -1.681 & 0.384 & 0.109 \\ (0.18) & (0.16) & (0.16) & (0.10) \end{bmatrix};$$

(continued)

Panel B:

$$\widetilde{\sum}(z_1) = \begin{bmatrix} 8.02\times10^{-5} & & & & \\ 7.99\times10^{-5} & 8.06\times10^{-5} & & & \\ 7.93\times10^{-5} & 8.21\times10^{-5} & 8.77\times10^{-5} & & \\ 7.75\times10^{-5} & 8.34\times10^{-5} & 9.52\times10^{-5} & 1.12\times10^{-4} & \\ 7.44\times10^{-5} & 8.63\times10^{-5} & 1.10\times10^{-4} & 1.45\times10^{-4} & 2.17\times10^{-4} \end{bmatrix};$$

$$\widetilde{\sum}(z_2) = \begin{bmatrix} 3.50\times10^{-5} & & & & \\ 3.50\times10^{-5} & 3.49\times10^{-5} & & & \\ 3.49\times10^{-5} & 3.49\times10^{-5} & 3.48\times10^{-5} & & \\ 3.50\times10^{-5} & 3.50\times10^{-5} & 3.49\times10^{-5} & 3.49\times10^{-5} & \\ 3.49\times10^{-5} & 3.50\times10^{-5} & 3.50\times10^{-5} & 3.50\times10^{-5} & 3.54\times10^{-5} \end{bmatrix};$$

$$\widetilde{\sum}(z_3) = \begin{bmatrix} 3.55\times10^{-5} & & & & \\ 3.47\times10^{-5} & 3.43\times10^{-5} & & & \\ 3.33\times10^{-5} & 3.33\times10^{-5} & 3.30\times10^{-5} & & \\ 3.19\times10^{-5} & 3.23\times10^{-5} & 3.27\times10^{-5} & 3.34\times10^{-5} & \\ 3.13\times10^{-5} & 3.23\times10^{-5} & 3.34\times10^{-5} & 3.50\times10^{-5} & 3.86\times10^{-5} \end{bmatrix};$$

$$\widetilde{\mathbf{P}} = \begin{bmatrix} 0.47 & 0.08 & 0.05 \\ 0.49 & 0.90 & 0.04 \\ 0.04 & 0.02 & 0.91 \end{bmatrix}; \widetilde{\xi} = \begin{bmatrix} 0.12 \\ 0.68 \\ 0.20 \end{bmatrix};$$

$\rho(A) = 0.1594$
LR linearity test: 4110.68 $\{0\}$

Notes: See Notes to Table A.2

Table A.5: MSIH(3)-VECM(1) estimation results: UK

Panel A:

$$\widetilde{\Gamma}_1 = \begin{bmatrix} -0.472 & 0.839 & 0.031 & 0.042 & -0.438 \\ (0.34) & (0.17) & (0.31) & (0.30) & (0.28) \\ -0.422 & 0.704 & 0.165 & -0.002 & -0.443 \\ (0.34) & (0.14) & (0.30) & (0.30) & (0.28) \\ -0.471 & 0.722 & 0.245 & -0.061 & -0.434 \\ (0.33) & (0.09) & (0.28) & (0.29) & (0.28) \\ -0.498 & 0.562 & 0.650 & -0.323 & -0.389 \\ (0.33) & (0.11) & (0.28) & (0.29) & (0.28) \\ -0.435 & 0.350 & 0.763 & -0.286 & -0.392 \\ (0.36) & (0.28) & (0.35) & (0.33) & (0.28) \end{bmatrix};$$

$$\widetilde{v}(z_1) = \begin{bmatrix} -6.09\times10^{-4} \\ (1.31\times10^{-3}) \\ -6.18\times10^{-4} \\ (1.31\times10^{-3}) \\ -6.11\times10^{-4} \\ (1.27\times10^{-3}) \\ -5.30\times10^{-4} \\ (1.20\times10^{-3}) \\ -4.18\times10^{-4} \\ (1.12\times10^{-3}) \end{bmatrix}; \widetilde{v}(z_2) = \begin{bmatrix} 2.30\times10^{-4} \\ (1.10\times10^{-5}) \\ 2.38\times10^{-4} \\ (8.37\times10^{-6}) \\ 2.55\times10^{-4} \\ (8.57\times10^{-6}) \\ 2.58\times10^{-4} \\ (1.58\times10^{-5}) \\ 2.45\times10^{-4} \\ (2.52\times10^{-5}) \end{bmatrix}; \widetilde{v}(z_3) = \begin{bmatrix} -3.71\times10^{-5} \\ (2.85\times10^{-5}) \\ -3.83\times10^{-5} \\ (3.01\times10^{-5}) \\ -4.80\times10^{-5} \\ (2.83\times10^{-5}) \\ -7.45\times10^{-5} \\ (4.37\times10^{-5}) \\ -9.39\times10^{-5} \\ (7.30\times10^{-5}) \end{bmatrix};$$

$$\widetilde{\alpha} = \begin{bmatrix} -1.379 & 1.713 & -0.861 & 0.178 \\ (0.40) & (0.31) & (0.32) & (0.06) \\ -1.060 & 1.623 & -0.889 & 0.188 \\ (0.39) & (0.30) & (0.31) & (0.05) \\ -1.593 & 1.980 & -0.974 & 0.185 \\ (0.37) & (0.29) & (0.31) & (0.05) \\ -1.612 & 1.793 & -0.789 & 0.138 \\ (0.37) & (0.29) & (0.31) & (0.05) \\ -1.556 & 1.962 & -1.024 & 0.215 \\ (0.43) & (0.35) & (0.33) & (0.06) \end{bmatrix};$$

(continued)

Panel B:

$$\widetilde{\sum}(z_1) = \begin{bmatrix} 5.62\times10^{-5} & & & & \\ 5.52\times10^{-5} & 5.44\times10^{-5} & & & \\ 5.30\times10^{-5} & 5.25\times10^{-5} & 5.12\times10^{-5} & & \\ 4.99\times10^{-5} & 4.97\times10^{-5} & 4.87\times10^{-5} & 4.71\times10^{-5} & \\ 4.60\times10^{-5} & 4.60\times10^{-5} & 4.55\times10^{-5} & 4.45\times10^{-5} & 4.29\times10^{-5} \end{bmatrix};$$

$$\widetilde{\sum}(z_2) = \begin{bmatrix} 2.53\times10^{-5} & & & & \\ 2.53\times10^{-5} & 2.53\times10^{-5} & & & \\ 2.52\times10^{-5} & 2.52\times10^{-5} & 2.50\times10^{-5} & & \\ 2.49\times10^{-5} & 2.49\times10^{-5} & 2.48\times10^{-5} & 2.46\times10^{-5} & \\ 2.44\times10^{-5} & 2.44\times10^{-5} & 2.43\times10^{-5} & 2.42\times10^{-5} & 2.38\times10^{-5} \end{bmatrix};$$

$$\widetilde{\sum}(z_3) = \begin{bmatrix} 5.72\times10^{-5} & & & & \\ 5.73\times10^{-5} & 5.74\times10^{-5} & & & \\ 5.77\times10^{-5} & 5.78\times10^{-5} & 5.83\times10^{-5} & & \\ 5.84\times10^{-5} & 5.86\times10^{-5} & 5.92\times10^{-5} & 6.02\times10^{-5} & \\ 5.93\times10^{-5} & 5.95\times10^{-5} & 6.03\times10^{-5} & 6.14\times10^{-5} & 6.31\times10^{-5} \end{bmatrix};$$

$$\widetilde{\mathbf{P}} = \begin{bmatrix} 0.81 & 0.01 & 0.05 \\ 0.02 & 0.88 & 0.07 \\ 0.17 & 0.10 & 0.88 \end{bmatrix};\widetilde{\xi} = \begin{bmatrix} 0.15 \\ 0.33 \\ 0.51 \end{bmatrix};$$

$\rho(A) = 0.1876$
LR linearity test: 2090.66 {0}

Notes: See Notes to Table A.2.

Chapter 3

Modelling and forecasting stock returns: Exploiting the futures market, regime shifts and international spillovers

A large body of research on modeling and forecasting stock returns has investigated the relationship between spot and futures prices in stock index futures markets. In particular, a number of empirical studies have focused on the persistence of deviations from the cost of carry and have investigated the relationship between spot and futures prices in the context of vector autoregressions using cointegration or equilibrium correction models (see Dwyer, Locke and Yu, 1996; Neely and Weller, 2000, and the references therein). The rationale underlying this line of research is that the cost of carry model and variants of it predict that spot and futures prices cointegrate and their long-run relationship is characterized by a long-run equilibrium defined by the futures basis, implying both mean reversion in the basis and the existence of a vector equilibrium correction model (VECM) for spot and futures prices. This literature, discussed in greater detail in the next section, has generally reported evidence that the futures market contains valuable information for modeling and/or forecasting stock returns.

A related line of research emphasizes that trading activity does not take place for one index per unit of time (e.g. see Eun and Shin, 1989; Engle and Susmel, 1994;

Koutmos and Booth, 1995; Lee, 1995; Karoly and Stulz, 1996). Indeed, it is more likely that traders place orders and take positions simultaneously using different indices given that stock and futures markets for different indices are closely linked by both hedging activities and cross-market arbitrage. This may generate comovements across stock market indices and, in turn, the cross-correlation between different indices may be potentially very useful in improving empirical models of stock returns. In particular, it seems possible that, in the unknown dynamic model governing the relationship between futures and stock prices, stock returns for a particular index respond not only to the disequilibrium in the relevant stock index market but also to disequilibria in stock index markets that are linked to the relevant stock index by hedging activities and cross-market arbitrage (e.g. Martens and Poon, 2001; Ang and Bekaert, 2001; Goetzmann, Li and Rouwenhorst, 2001).[1]

Alongside the work on modeling and forecasting stock prices and returns, another strand of the literature has developed where increasingly strong evidence of nonlinearities in stock price movements has been documented[2]. One element of this has been the mounting evidence that the conditional distribution of stock returns is well described by a mixture of normal distributions (e.g. see Rydén, Teräsvirta and Åsbrink, 1998, and the references therein) and that, consequently, a Markov switching model may be a logical characterization of stock returns behavior (e.g. see, *inter alia*, LeBaron, 1992; Hamilton and Susmel, 1994; Hamilton and Lin, 1996; Ramchand and Susmel, 1998a,b; Rydén, Teräsvirta and Åsbrink, 1998; Susmel, 1999; Perez-Quiros and Timmermann, 2001). Also, not only Markov-switching models fit stock returns data well, but they have often been proved to produce superior forecasts to several alternative conventional

[1] For example, Ang and Bekaert (2001) find that cross-country predictability is stronger than predictability using local instruments. Goetzmann, Li and Rouwenhorst (2001) also document the correlation structure of several major equity returns over 150 years.

[2] See Abhyankar, Copeland and Wong (1995, 1997) and Lane, Peel and Reaburn (1995) and the references therein.

models of stock returns (e.g. see Hamilton and Susmel, 1994; Hamilton and Lin, 1996).[3]

In this chapter, we tie together these somewhat different, albeit related, strands of research. In particular, we investigate whether allowing for nonlinearities and international spillovers in the underlying data-generating process for a VECM that links spot and futures prices yields an improvement, in terms of both in-sample fit and out-of-sample forecasting, over models of stock returns that do not allow for nonlinearities and/or international spillovers. This is done through estimating a fairly general Markov-switching VECM (MS-VECM) for stock and futures prices that is based on an extension of Markovian regime shifts to a nonstationary framework, for which the underlying econometric theory has recently been developed. Given the evidence of significant regime-switching behavior in stock returns and the evidence on international cross-correlations of stock returns discussed above, this seems a natural way to extend current econometric procedures applied to stock returns modeling and forecasting, even though this involves estimating and forecasting from a sophisticated multivariate nonlinear model.

Using weekly data since 1989 for three major stock market indices - the S&P 500, the NIKKEI 225 and the FTSE 100 indices - we confirm that the futures market does contain some valuable information to explain stock returns in a linear VECM framework. However, we show that conventional linear VECMs, even when allowing for international spillovers in the equilibrium correction equations, display significant residual nonlinearity and are rejected when tested against the alternative of an MS-VECM. Thus, we show that allowing for nonlinearities and for international spillovers in an MS-VECM results in a superior empirical model which explains a large proportion of the stock returns

[3] Other studies in this literature have provided ample empirical evidence that the dynamic relationship linking stock and futures prices may be characterized by significant nonlinearities that can be well characterized using threshold models of various sort. These nonlinearities are rationalized on the basis of factors such as non-zero transactions costs or infrequent trading or simply the existence of regime shifts in the dynamic adjustment of stock and futures price changes towards their long-run equilibrium values (e.g. see, *inter alia*, Yadav, Pope and Paudyal, 1994; Dwyer, Locke and Yu, 1996; Martens, Kofman and Vorst, 1998; Gao and Wang, 1999; Aslanidis, Osborn and Sensier, 2002). See also Andreou, Osborn and Sensier (2000), Perez-Quiros and Timmermann (2001) and Timmermann (2001).

examined over our sample. We then compare the performance of our proposed model to several alternative linear and nonlinear models in an out-of-sample forecasting exercise. The evaluation of the relative performance is based on conventional statistical criteria for point forecasting performance as well as on the ability of the models to forecast the true predictive density of stock returns out of sample.[4] In fact, we argue and provide evidence that density forecast accuracy is more appropriate for evaluating our competing models since stock returns are non-normally distributed and we are considering nonlinear models consistent with non-normal densities (see, *inter alia*, Diebold, Gunther and Tay, 1998; Granger and Pesaran, 1999; Tay and Wallis, 2000; Timmermann, 2000).

To anticipate our forecasting results, we find that the MS-VECM that allows for international spillovers does not outperform the competing models examined in terms of point forecasting performance. However, our model significantly outperforms all of the competing models in terms of density forecasting performance in that it generates predictive densities that are much closer to the true predictive density of the data. Overall, these results suggest that, while the statistical performance of the linear and nonlinear models examined in this chapter differs little in terms of conditional mean, inspection of the predictive densities implied by the various models shows greater ability to discriminate between models, indicating that both multiple regimes and the allowance for international spillovers are important ingredients for a model to produce satisfactory predictive densities.[5]

We illustrate the practical importance of our results on density forecasting with an application to financial risk management. In recent years, trading accounts at large financial institutions have shown a dramatic growth and become increasingly more com-

[4] By true predictive density of the data we mean the density of the data over the chosen forecast period. Therefore, no forecast is in fact carried out in this case, and the term 'predictive' simply refers to the fact that the density in question does not refer to the full sample but only to the forecast period. Also note that we use the terms 'predictive density' and 'forecast density' interchangeably below.

[5] This finding is consistent with the results of Clements and Smith (2000) and Perez-Quiros and Timmermann (2001) in a related context.

plex. Partly in response to this trend, major trading institutions have developed risk measurement models designed to manage risk. These models generally employ the Value-at-Risk (VaR) methodology, where VaR is defined as the expected maximum loss over a target horizon within a given confidence interval (Jorion, 1997; Basak and Shapiro, 2001).[6] Users of the VaR methodology generally assume that expected returns are normally or t-distributed. However, this assumption contrasts with the large amount of empirical evidence suggesting that the distribution of financial asset returns is not normal, confirmed in the present chapter. Point forecast analysis and testing procedures based upon it do not take into account these features, so that VaR analysis often relies on dubious parametric distributional assumptions. In our simple application we analyze the out-of-sample forecasting performance of our proposed empirical models of stock returns and we investigate the implications of these forecasts for a risk manager who has to quantify the risk associated with holding the stock indices in question over a one-week horizon.

The remainder of the chapter is set out as follows. In Section 3.1 we describe our empirical framework for modeling stock and futures prices allowing for international spillovers and nonlinear dynamics. We also briefly set out the econometrics of Markov-switching multivariate models as applied to nonstationary processes and cointegrated systems. In Section 3.2 we describe the data and report the results relating to the in-sample empirical analysis. In Section 3.4 we report our forecasting results, including evidence of point forecast accuracy, market timing ability, density forecast accuracy and an illustrative application to risk management aimed at investigating the importance of density forecasting in the context of stock returns. A final section concludes.

[6] More formally, VaR is an interval forecast, typically a one-sided 95 or 99 percent interval of the distribution of expected wealth or returns.

3.1 Modeling stock returns: an empirical framework

In this section we outline our empirical framework for modeling stock returns, which we apply to our data in the subsequent sections. First, we use a conventional cost of carry model to show that futures and stock prices must be cointegrated and, therefore, linked by a VECM that can be used both to explain and forecast stock returns. Second, we generalize the VECM linking stock and futures prices to take into account potentially important regime switches of the kind reported by a large empirical literature. Third, we further generalize our empirical framework by also taking into account the observed cross-correlations between major stock market indices, which leads us to consider a panel of VECMs which explicitly allows for both regime shifts and international spillovers across major stock market indices.

3.1.1 The information in the futures market

A useful starting point for building an empirical framework to model stock returns is the relationship between stock prices and stock futures prices, as described by a conventional cost of carry model with no transaction costs:

$$F(t,T) = S(t)\exp\left\{\sum_{k=1}^{T-t} c(t+k)\right\} \tag{3.1}$$

where $S(t)$ is the stock index price, $F(t,T)$ is the futures price at time t for delivery of the stock at time $T \geq t$ and $c(t+k)$ represents the expected net cost of carry for period $t+k$. Taking logs, equation (3.1) can be rewritten as

$$\log F(t,T) - \log S(t) = \sum_{k=1}^{T-t} c(t+k) \tag{3.2}$$

where $\log F(t,T) - \log S(t)$ is the log-basis. Following Low, Muthuswamy, Sakar and Terry (2002), suppose that market expectations about the cost of carry for each period are drawn from independent and identical normal distributions, each with mean $\overline{c}$ and variance σ_c^2. Then the log-basis will be normally distributed with mean $\overline{c}(T-t)$ and variance $\sigma_c^2 (T-t)$. This implies that both the first and second moments of the log-basis will be functions of the time to maturity $(T-t)$ (see Low, Muthuswamy, Sakar and Terry, 2002).[7] If the expected cost of carry for each period has a stationary distribution, then equation (3.2) implies cointegration between futures and spot prices with the cointegrating relationship given by

$$Z_t = \log F(t,T) - \log S(t) - \overline{c}(T-t). \tag{3.3}$$

Equation (3.3) implies that the futures and the underlying spot prices cointegrate with a cointegrating vector which differs from the usual cointegrating vectors investigated in the empirical literature on the cost of carry model (see, *inter alia*, Lien and Lou, 1993; 1994; Kroner and Sultan, 1993, Gagnon and Lypny, 1995) as a result of the presence of a time to maturity term $\overline{c}(T-t)$.[8] Given equation (3.3), Z_t may be seen as the stationary deviation from the cost-of-carry model. In turn, the Granger Representation Theorem (Engle and Granger, 1987) implies that the futures and stock prices must possess a VECM representation where the log-basis adjusted for the time to maturity term (Z_t) plays the part of the equilibrium error.[9] We exploit this framework and use exactly a VECM representation to demonstrate that valuable information may be extracted from

[7] In the case of futures, c is explained by time-varying interest rates and dividend yields. In principle, if one had data at weekly frequency on dividend yields, it would be possible to calculate the basis using interest rates and dividend yields to match the remaining time to maturity. However, typically weekly data on dividend yields are difficult to obtain for some of the indices used in this paper and need to be interpolated (under an assumed process for dividends), potentially reducing the accuracy of the basis calculations. While many studies use the 'de-meaning' method (not using interest rates and dividends) per day (using a large number of intraday observations for each day) and assume that the time to maturity is approximately constant (e.g. Dwyer, Locke and Yu, 1996), this approach cannot be applied to an entire data set of weekly data. Hence, given these difficulties, we follow the method of Low, Muthuswamy, Sakar and Terry (2002) in our calculation of the log-basis, where we assume a distribution for the cost of carry and also adjust for the time to maturity $T-t$.

[8] Note that the framework described in this section differs from the conventional cost of carry model which, strictly speaking, applies to forwards, not futures. For futures, interest rates, dividends and the time-to-maturity are not constant. As a result, the variance of the basis is not constant, which requires the refinement to the conventional cost of carry model discussed above to yield an adequate calculation of the basis - e.g. see Dwyer, Locke and Yu (1996) and Low, Muthuswamy, Sakar and Terry, (2002) for further details.

[9] The precise definition of cointegration requires that the cointegrating vector to be covariance stationary. Because equation (3.3) implies that the variance of the cointegrating vector will be a function of the time to maturity, the futures and underlying spot price cannot be cointegrated in a strict sense. However, Hansen (1992a) shows that much of the statistical theory developed under strict definition of cointegration still holds when heteroskedasticity is permitted in the cointegrating vector. See Low,

the futures market in order to forecast stock returns (Low, Muthuswamy, Sakar and Terry, 2002).

3.1.2 Regime-switching equilibrium correction in stock index futures markets

A large literature has documented evidence of nonlinearities in stock returns. One element of this has been the mounting evidence that the conditional distribution of stock returns is well described by a mixture of normal distributions (e.g. see Rydén, Teräsvirta and Åsbrink, 1998, and the references therein) and that, consequently, a Markov switching model may be a logical characterization of stock returns behavior (e.g. see, *inter alia*, LeBaron, 1992; Hamilton and Susmel, 1994; Hamilton and Lin, 1996; Ramchand and Susmel, 1998a,b; Rydén, Teräsvirta and Åsbrink, 1998; Susmel, 1999). In fact, the relevant literature suggests that not only Markov-switching models fit stock returns data well, but they often perform satisfactorily in forecasting (e.g. see Hamilton and Susmel, 1994; Hamilton and Lin, 1996).

In the present chapter, we investigate whether allowing for regime-switching in the VECM implied by the framework described in the previous subsection yields superior stock returns forecasts relative to several alternative specifications. This is done through estimating a fairly general MS-VECM for stock and futures prices which is based on an extension of Markovian regime shifts to a nonstationary framework. In the rest of this subsection we outline the econometric procedure employed in order to model regime shifts in the dynamic relationship between stock and futures prices. The procedure essentially extends Hamilton's (1988, 1989) Markov-switching regime framework to nonstationary systems, allowing us to apply it to cointegrated vector autoregressive (VAR) and VECM systems (see Krolzig, 1997, 1999, 2000).

Consider the following M-regime p-th order Markov-switching vector autoregression

Muthuswamy, Sakar and Terry (2002) for a detailed discussion of the cointegrating properties of the cost of carry model in this context.

(MS(M)-VAR(p)) which allows for regime shifts in the intercept term:

$$y_t = \nu(\omega_t) + \sum_{i=1}^{p} \Pi_i y_{t-i} + \varepsilon_t, \tag{3.4}$$

where y_t is a K-dimensional observed time series vector, $y_t = [y_{1t}, y_{2t}, \dots, y_{Kt}]'$; $\nu(\omega_t) = [\nu_1(\omega_t), \nu_2(\omega_t), \dots, \nu_K(\omega_t)]'$ is a K-dimensional column vector of regime-dependent intercept terms; the Π_i's are $K \times K$ matrices of parameters; $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}, \dots, \varepsilon_{Kt}]'$ is a K-dimensional vector of Gaussian white noise processes with covariance matrix Σ, $\varepsilon_t \sim NID(\mathbf{0}, \Sigma)$. The regime-generating process is assumed to be an ergodic Markov chain with a finite number of states $\omega_t \in \{1, \dots, M\}$, governed by the transition probabilities $p_{ij} = \Pr(\omega_{t+1} = j \mid \omega_t = i)$, and $\sum_{j=1}^{M} p_{ij} = 1 \; \forall i, j \in \{1, \dots, M\}$.

A standard case in economics and finance is that y_t is nonstationary but first-difference stationary, i.e. $y_t \sim I(1)$. Then, given $y_t \sim I(1)$, there may be up to $K - 1$ linearly independent cointegrating relationships, which represent the long-run equilibrium of the system, and the equilibrium error (the deviation from long-run equilibrium) is measured by the stationary stochastic process $u_t = \alpha' y_t - \beta$ (Granger, 1986; Engle and Granger, 1987). If indeed there is cointegration, the cointegrated MS-VAR (3.4) implies an MS-VECM of the form:

$$\Delta y_t = \nu(\omega_t) + \sum_{i=1}^{p-1} \Lambda_i \Delta y_{t-i} + \Pi y_{t-1} + \varepsilon_t, \tag{3.5}$$

where $\Lambda_i = -\sum_{j=i+1}^{p} \Pi_j$ are matrices of parameters, and $\Pi = \sum_{i=1}^{p} \Pi_i - \mathbf{I}$ is the long-run impact matrix whose rank r determines the number of cointegrating vectors (e.g. Johansen, 1995; Krolzig, 1999).

Although, for expositional purposes, we have outlined the MS-VECM framework for the case of regime shifts in the intercept alone, shifts may be allowed for elsewhere. The present application focuses on a multivariate model comprising, for each of the three major stock index futures markets analyzed, the futures price and the stock price (hence $y_t = [f_t, s_t]'$) where f_t and s_t denote the logarithmic futures and stock prices respectively. As discussed in Section 3.2 below, in our empirical work, after considerable experimentation, we selected a specification of the MS-VECM which allows for regime shifts in the intercept, the autoregressive structure and in the variance-covariance matrix. This model, the Markov-Switching-Intercept-Autoregressive-Heteroskedastic-VECM or MSIAH-VECM, may be written as follows:

$$\Delta y_t = \nu(\omega_t) + \sum_{i=1}^{p-1} \Lambda_i(\omega_t)\Delta y_{t-i} + \Pi(\omega_t)y_{t-1} + u_t, \tag{3.6}$$

where $\Pi(\omega_t) = \alpha(\omega_t)\beta'$, $u_t \sim NIID[\mathbf{0}, \Sigma(\omega_t)]$ and $\omega_t \in \{1, \dots, M\}$. Intuitively, the shifts in the variance-covariance matrix allow us to capture the well-documented heteroskedasticity of stock returns over the sample examined. On the other hand, the need for shifts in the intercept and the autoregressive structure is consistent with the well-known evidence that analyses of forecasting that implicitly rule out structural breaks and regime shifts in the economy ignore an aspect of the real world responsible for some episodes of predictive failure (Clements and Hendry, 1996). These corrections therefore may offer greater protection against unforeseen regime shifts, enhancing the forecasting performance of the model.

An MS-VECM can be estimated using a two-stage maximum likelihood procedure. The first stage essentially consists of the implementation of the Johansen (1988, 1991) maximum likelihood cointegration procedure in order to test for the number of cointegrating relationships in the system and to estimate the cointegration matrix. In fact, in the first stage use of the conventional Johansen procedure is valid without modeling the Markovian regime shifts explicitly (see Saikkonen, 1992; Saikkonen and Luukkonen, 1997). The second stage then consists of the implementation of an expectation-maximization (EM) algorithm for maximum likelihood estimation which yields estimates of the remaining parameters of the model (Dempster, Laird and Rubin, 1977; Hamilton, 1990; Kim and Nelson, 1999; Krolzig, 1999).

3.1.3 Separation and cointegration in modeling stock returns

Although conventional time series models employed to explain or forecast stock returns treat a particular asset or index in isolation, a vast literature in finance has pointed out that trading activity does not take place for one index per unit of time (see, *inter alia,* Eun and Shin, 1989; Engle and Susmel, 1994; Koutmos and Booth, 1995; Lee, 1995; Karoly and Stulz, 1996). This literature generally emphasizes that hedging activities and cross-market arbitrage may generate comovements across different stock market indices (Martens and Poon, 2001; Ang and Bekaert, 2001; Goetzmann, Li and Rouwenhorst, 2001) and, in turn, the correlation between different indices may be potentially very useful in improving empirical models of stock returns. In particular, it is possible that, in a VECM for futures and stock prices, stock price changes respond not only to the disequilibrium in the relevant stock index market but also to disequilibria in stock index markets that are linked to the relevant stock index.

Figure 3.1, which documents the time-varying contemporaneous correlation for the three different futures bases examined in this study (the S&P 500, the FTSE 100, and the NIKKEI 225) for different time windows (namely estimating the correlation over rolling windows of 1, 3, and 6 months respectively), provides clear visual evidence that the three futures bases of these indices display substantial and statistically significant cross-correlation, especially for the 1-month window. Althought the graphs in Figure 3.1 indicate that the correlation appears to vary substantially over the sample, it is clear that the correlation reported indicates clear interdependencies between these three major stock market indices, suggesting that the allowance for spillovers in our spot-futures VECM may yield substantial improvements relative to individual VECM estimation due to the incremental information yielded by the cross-correlation of the indices examined.

This line of reasoning suggests the possibility of enriching our MS-VECM framework by allowing for spillovers through the equilibrium correction terms, that is the possibility that equilibrium correction terms from one cointegrating relationship for a particular stock market index may have explanatory power in the equilibrium correction equation driving the returns of another stock market index. This approach is consistent with the notion of separation and cointegration - popularized by Konishi and Granger (1993), Konishi (1993), Granger and Swanson (1996) and Granger and Haldrup (1997) - which therefore provides a useful way of describing formally the above ideas.

Consider, for example, the MS-VECM (3.5) and define an n-dimensional cointegrated vector $Y_t = [y_t^1, y_t^2, y_t^3]'$, where $y_t^j = [f_t^j, s_t^j]'$ for $j = 1, 2, 3$ is of dimension of n_j (i.e. $n = n_1 + n_2 + n_3$) and y_t^1, y_t^2 and y_t^3 have no variable in common. We can then generalize equation (3.5) to a VECM that exploits the information in the futures market while also allowing for both regime shifts and international spillovers. This VECM may be written as follows:

$$\Delta Y_t = \boldsymbol{\nu}(\omega_t) + \sum_{i=1}^{p-1} \Lambda_i \Delta Y_{t-i} + \boldsymbol{\alpha}\boldsymbol{\beta}' Y_{t-1} + \varepsilon_t, \tag{3.7}$$

where Λ_i is an $n \times n$ matrix of autoregressive parameters, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}'$ denote the $n \times r$ loading matrix and the $r \times n$ cointegration matrix (or matrix of cointegrating vectors) respectively, and r is the cointegration rank. The cointegration matrix $\boldsymbol{\beta}'$ can be factorized as

$$\boldsymbol{\beta}' = \begin{bmatrix} \beta'_{11} & 0 & 0 \\ 0 & \beta'_{22} & 0 \\ 0 & 0 & \beta'_{33} \end{bmatrix} \tag{3.8}$$

where β'_{jj} is $r^j \times n_j$, for $j = 1, 2, 3$. The system is said to have *separate cointegration* with cointegration ranks for each subsystem given by n_1, n_2 and n_3 respectively. If we then factorize the loading matrix as follows

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{22} & 0 \\ 0 & 0 & \alpha_{33} \end{bmatrix}, \tag{3.9}$$

where α_{jj} is $n_j \times r^j$ for $j = 1, 2, 3$, we have *type B-separation* or separation in the equilibrium correction. Finally, if we factorize the matrix Λ_i as

$$\Lambda_i = \begin{bmatrix} \Lambda^j_{11} & 0 & 0 \\ 0 & \Lambda^j_{22} & 0 \\ 0 & 0 & \Lambda^j_{33} \end{bmatrix} \tag{3.10}$$

we have *type A-separation* or separation in the dynamic adjustment towards the long-run equilibrium defined for each y_t^j for $j = 1, 2, 3$ (e.g. Granger and Haldrup, 1997). If all of the conditions (3.8)-(3.10) hold there is *complete separation*, while if condition (3.8) is associated with either (3.9) or (3.10) we have *partial separation*.

Our earlier discussion on spillovers in the dynamics of stock returns is consistent with a situation where, although two or more different stock indices are 'separated in the long-run' (i.e. condition (3.8) holds), there may be important short-run relationships between them and, therefore, the deviation from the equilibrium relationship (defined by the futures basis) from one index may enter the equilibrium correction equation of another index (i.e. condition (3.9) does *not* hold).

This 'amalgamation' is applied to the case of cointegration analysis across different stock indices in the world economy, which seems intuitively appealing given the high degree of integration of global capital markets during the last fifteen years or so. In particular, our framework is consistent with a situation where, for any stock index k, a long-run equilibrium relationship is established in a static cointegrating equation involving stock and futures prices for index k, as predicted by the standard cost of carry model. Hence, stock and futures prices for any other index $j \neq k$ do not enter the long-run cointegrating equation defining the equilibrium value of the stock price of index k. Despite long-run separation (that is the equilibrium value of the stock price of any index k is *fully* determined by the equilibrium relationship between stock and futures prices of the index k itself), however, the individual short-run relationships may be characterized by the equilibrium error from one equation entering another equilibrium correction equation of the system. This is the approach followed below, where we start by estimating cointegrating relationships and, therefore, equilibrium correction terms, which imply plausible parameters and are consistent with the definition of the adjusted log-basis. Thus, we estimate a nonlinear MS-VECM where, for each stock index examined, the lagged deviation from equilibrium (equilibrium correction term) in other stock indices is allowed to enter the equilibrium correction equation in addition to the own-index lagged deviation from equilibrium (equilibrium correction term) in order to exploit the information content of international spillovers.

3.2 Empirical analysis I: modeling

3.2.1 Data and preliminary statistics

The data set comprises weekly time series on prices of futures contracts written on the S&P 500, the NIKKEI 225 and the FTSE 100 indices, as well as price levels of the corresponding underlying cash indices. The data set is obtained from *Datastream*. Specifically, we use price levels of each stock index and corresponding futures contracts at the close of trade of every day. The data is collected to coincide with the lenght of the available futures contract. The futures data are constructed according to standard conventions (e.g. Low, Muthuswamy, Sakar and Terry, 2002). In particular, a single time series of future prices is spliced together from individual futures contract prices. For liquidity, the nearest contract's prices are used until the first day of the expiration month, then the next nearest is used (see, *inter alia*, Ahn, Boudoukh, Richardson and Whitelaw, 2002, p. 667-8). The adjusted log-basis has been constructed as in Low, Muthuswamy, Sakar and Terry (2002). We used equation (3.3) to calculate the log-basis adjusted for the time-to-maturity of each futures contract. In practice, for each contract and stock index, we regressed the difference between log-futures price and log-spot price on the time to maturity. The residual is then the log-basis for the specific contract and stock index. All of the series considered have initially been constructed from daily data, from which we then obtained the weekly series from the daily series by using Wednesday prices, or Thursday prices when Wednesday prices were unavailable, in order to avoid potential weekend price effects (French, 1980; Gibbons and Hess, 1981; Low, Muthuswamy, Sakar and Terry, 2002).

The sample period examined spans from January 1989 to December 2002. We choose this sample period for two reasons. First, the NIKKEI 225 stock index futures was first traded on September 1988 in the Osaka Stock Exchange (OSE).[10] Second, given the focus of the present chapter on investigating the importance of allowing for nonlinearity and regime switching in modeling stock returns, using data after the 1987 crash should reduce the risk that the nonlinearity detected and modeled in the empirical analysis could be determined by or attributed to a unique and perhaps exceptional event occurred over the sample. In our empirical work, we carried out estimations over the period January 1989-December 1998, reserving the last four years of data for out-of-sample forecasting tests.

A number of related studies motivated by microstructure considerations or focusing on modeling intraday or short-lived arbitrage have used intraday data at various intervals or daily data.[11] In order to reduce the noise element in the data, we choose to employ data at weekly frequency. However, we carried out a fraction of the estimation work reported below also using daily data. These estimation results were qualitatively identical, suggesting that aggregation from daily to weekly may not have particularly important effects on the regime-switching properties of our stock returns data.[12]

It is worth noting that in estimating our (linear or nonlinear) VECMs for stock and futures prices (recorded at the close of trade for each Wednesday), there is no overlap of

[10] More precisely, NIKKEI 225 futures contracts were first traded in 1986 in the Singapore International Monetary Exchange (SIMEX). Since NIKKEI 225 futures contracts are more actively traded in the OSE than the SIMEX we prefer to use the OSE data (see Pan and Hsueh, 1998, for further discussion of the institutional details of trading the NIKKEI 225 stock index futures contracts).

[11] For example, Miller, Muthuswamy and Whaley (1994) and Dwyer, Locke and Yu (1996) use 15- and 5-minute intervals respectively.

[12] Nevertheless, given the high computational burden of executing the simulations discussed below in the forecasting exercise, using weekly (rather than daily) data allowed us to be more ambitious in terms of the amount of overall empirical work carried out.

the returns time series across the markets analyzed. The first market to open among the three considered here is the NIKKEI 225. At the opening of the NIKKEI 225 market on any Wednesday (trading day) t, the closing prices of both the S&P 500 and the FTSE 100 on the previous Wednesday (and in fact also on the previous trading day $t-1$) are known. Hence, for example, the US equilibrium correction term at time $t-1$ (which can be defined as trading day or trading week) is available when estimating the NIKKEI VECM equation at time t.[13]

Table 3.1 provides summary statistics of the logarithm of the futures price, f_t and the logarithm of the spot price, s_t. As one would expect, for each stock index, the first moment of the futures price is larger than the first moment of the spot price (although it is not the case that $f_t > s_t$ at each point in time), while the second moment of the spot price is larger than the second moment of the futures price, suggesting that the futures price is larger on average and less volatile than the spot price. The partial autocorrelation functions, reported in Table 3.1 up to order 12, suggest that each spot and futures price examined displays very strong first-order serial correlation, while none of these series appears to be significantly serially correlated at higher lags. This is confirmed by the visual evidence provided in Figure 3.2, which plots the time series to be predicted in our VECMs, namely Δf_t and Δs_t, over the full sample period.

3.2.2 Unit root and cointegration tests

As a preliminary exercise, we tested for unit root behavior of each of the (log) futures price and spot price time series by calculating standard augmented Dickey-Fuller test statistics. In each case, the number of lags was chosen such that no residual autocorrelation was evident in the auxiliary regressions. In keeping with the very large number

[13] Also, we were careful in avoiding the problems caused by nonsynchronous market closure. For example, given that in the UK the futures market ceases trading at 16.10 and the underlying index closes at 16.30, we use FTSE 100 index levels at 16.10. Similarly, given that for the S&P 500 the futures market ceases trading at 16.15 EST and the underlying index closes at 16.00 EST, we use S&P 500 futures index levels at 16.00 EST.

of studies of unit root behavior for these time series and conventional finance theory, we were in each case unable to reject the unit root null hypothesis at conventional nominal levels of significance. On the other hand, differencing the series did appear to induce stationarity in each case. Overall, the unit root tests clearly indicate that both f_t and s_t are realizations from stochastic processes integrated of order one, which suggests that testing for cointegration between f_t and s_t is the logical next step.

We employed the Johansen procedure in a VAR for f_t and s_t allowing for an unrestricted constant term (see Table 3.2)[14]. Both Johansen likelihood ratio (LR) test statistics (based on the maximum eigenvalue and on the trace of the stochastic matrix respectively) clearly suggested that a cointegrating relationship existed. Also, the hypothesis that the cointegrating parameter associated with s_t equals unity could not be rejected at conventional nominal levels of significance for each of the estimated VARs.[15] However, although these cointegration results prove that futures and spot prices cointegrate with a unity parameter, they do not provide us with the most appropriate equilibrium correction terms for estimating a VECM for Δf_t and Δs_t since the cointegrating relationship tested does not allow for the time-varying nature of dividend yields and interest rates and for the time-to-maturity effect discussed in Section 3.1.1. Since the equilibrium correction term we wish to use in the VECM estimation is, in fact, the log-basis, we calculated the log-basis following Low, Muthuswamy, Sakar and Terry (2002) and as discussed in Sections 3.1.1 and 3.2.1. The VECM results reported below are obtained using this measure of the futures basis as the equilibrium correction term.

[14] Using a constant term restricted to the cointegration space or adding a deterministic trend in the contegration space produced qualitatively identical results to the ones reported in Table 3.2.

[15] LR tests of the hypothesis that the coefficient associated with s_t equals unity could not be rejected with p-values equal to 0.610, 0.572 and 0.607 for the S&P 500, the NIKKEI 225 and the FTSE 100 respectively.

3.2.3 Linear dynamic modeling

As a further preliminary to considering an MS-VECM, we estimated a standard linear bivariate VECM for Δf_t and Δs_t, which is implied by the finding of cointegration between f_t and s_t reported in the previous sub-section (Granger, 1986; Engle and Granger, 1987). Thus, using full-information maximum likelihood (FIML) methods, we estimated for each stock index a bivariate VECM of the form

$$\Delta y_t = \nu + \sum_{i=1}^{p-1} \Lambda_i \Delta y_{t-i} + \Pi y_{t-1} + \varepsilon_t, \tag{3.11}$$

where $y_t = [f_t, s_t]'$. We allowed for a maximum lag length of five, which was the longest lag lenght suggested by the Akaike information criterion (AIC) and the Schwartz information criterion (SIC). Employing the conventional general-to-specific procedure, we then obtained, for each stock index examined, fairly parsimonious models for Δf_t and Δs_t which display no residual serial correlation.

Further, in order to test for cointegration and separation of the type discussed in Section 3.1.3, we estimated the following model:

$$\Delta Y_t = \boldsymbol{\nu} + \sum_{i=1}^{p-1} \Lambda_i \Delta Y_{t-i} + \Pi Y_{t-1} + \boldsymbol{\varepsilon}_t, \tag{3.12}$$

where $Y_t = [f_t^{SP500}, s_t^{SP500}, f_t^{NK225}, s_t^{NK225}, f_t^{FTSE100}, s_t^{FTSE100}]'$. We tested for *type B-separation* (separation in the equilibrium correction) by estimating model (3.12) and testing the zero restrictions in (3.10) using a standard likelihood ratio (LR) test. The results allow us to reject the zero restrictions under the null hypothesis (3.10), implying that there is no separation in the equilibrium correction, or put differently, that the disequilibrium (deviation of the basis from its equilibrium level) in one index influences the dynamics of stock returns of other indices.

As a check of adequateness of the models as well as an additional motivation for the need of employing a nonlinear model to characterize the dynamic relationship between stock and futures prices, however, we employed two fairly general tests for linearity of the residuals from the VECMs (3.11) and (3.12), namely Ramsey's (1969) RESET test and the Brock, Dechert and Sheinkman (BDS) (1991) test for testing the null hypothesis that the residuals from (3.11) and (3.12) are independent and identically distributed (iid) against an unspecified alternative. Application of both of these tests provided strong evidence that the linear VECM fails to capture important nonlinearities in the data generating process, as linearity is rejected with marginal significance levels (p-values) of virtually zero - see Table A.1 in Appendix A.[16]

3.2.4 MS-VECM estimation results

Next, we applied the conventional 'bottom-up' procedure designed to detect Markovian shifts in order to select the most adequate characterization of an M-regime p-th order MS-VECM for Δy_t of the form discussed in Section 1. However, for each MS-VECM estimated the implicit assumption that the regime shifts affect only the intercept term of the VECM was found to be inappropriate. In fact, we checked in turn the relevance of regime-conditional heteroskedasticity and regime-conditional autoregressive structure. We then tested the hypothesis of no regime dependence using an LR test of the type suggested by Krolzig (1997, p. 136). The results suggest very strong rejection of the null of no regime dependence, clearly indicating that an MS-VECM that allows for shifts in the intercept v, the autoregressive structure Λ_i, the cointegrating matrix Π and the variance-covariance matrix Σ, that is an MSIAH(M)-VECM(p), is the most appropriate model within its class in the present application. We also tested for the significance of the autoregressive structure and found that $p = 1$ is the lag length which better characterizes the dynamics of the series. For simplicity, we assume, as done in much recent literature on Markov-switching models (see, *inter alia*, Cecchetti, Pok-Sang and Mark, 1990, 2000; Hamilton and Lin, 1996; Richmond and Susmel, 1998a,b; Perez-Quiros and Timmermann, 2001), the presence of two regimes for each stock index.

[16] However, we also used the linear VECMs to forecast the future stock price and compared these forecasts to the forecasts obtained from an MS-VECM, as discussed below.

Thus, we selected and estimated a bivariate MSIAH(2)-VECM(1) for Δy_t of the form

$$\begin{aligned}\Delta y_t &= \nu(\omega_t) + \sum_{i=1}^{p-1} \Lambda_i(\omega_t)\,\Delta y_{t-i} + \Pi(\omega_t)\,y_{t-1} + \varepsilon_t \\ \varepsilon_t &\sim NID[\mathbf{0}, \Sigma(\omega_t)] \qquad \omega_t = 1, 2 \end{aligned} \tag{3.13}$$

using the EM algorithm for maximum likelihood estimation discussed in Section 1. In order to test for cointegration and type B-separation we also estimated the following model

$$\begin{aligned}\Delta Y_t &= \boldsymbol{\nu}(\omega_t) + \sum_{i=1}^{p-1} \Lambda_i(\omega_t)\,\Delta Y_{t-i} + \Pi(\omega_t)\,Y_{t-1} + \varepsilon_t \\ \varepsilon_t &\sim NID[\mathbf{0}, \Sigma(\omega_t)] \qquad \omega_t = 1, \ldots, 2^3. \end{aligned} \tag{3.14}$$

Making no assumption on the relationship between the regime shifts occurring in the stock indices examined (see Krolzig, 1997; Hamilton and Lin, 1996) implies that the number of regimes incorporated in model (3.14), and consequently the dimension of the transition matrix, is $2^3 = 8$ (for further technical details see Appendices B and C).

The empirical results are very encouraging on a number of fronts. The estimation yields plausible results for each VECM estimated. The impact effect of regime shifts also appears to be substantial on the variance-covariance matrix and the autoregressive structure. Furthermore, we computed an LR test statistic for linearity (LR1), which essentially tests the hypothesis that the true model is a linear VECM against the alternative of the MSIAH-VECM, reported in Table 3.3. The test was carried out using lag lenght of five in each of the linear VECM and the MSIAH-VECM. Even by invoking the upper bound of Davies (1977, 1987), the linearity hypothesis is rejected very strongly, with a p-value of virtually zero, providing convincing evidence of the need of employing a regime-switching model. Moreover, even in the context of Markov-switching models, type B-separation is rejected by the data. In fact the likelihood ratio test (LR2) reported in the second column of Table 3.3 strongly rejects the null of separation in the equilibrium correction terms.

We also compute coefficients of determination $\overline{R}^2$, which were adjusted both for the bias towards preferring a larger model relative to a smaller one as well as for the fact that the model allows for regime-dependent heteroskedasticity, and conventional information criteria (namely AIC and BIC). The results are reported in Table 3.4. Under these measures of goodness of fit, two facts arise. First, the role of non-separation in the equilibrium correction terms is important to explain the variability of futures and spot returns: columns 2 and 4 highlight the improvement in the in-sample predictive performance of the models when the futures bases from different stock markets are incorporated as explanatory variables in the returns equations. Second, the role of nonlinearities appears to be very important to better explain stock returns. Columns 3 and 4 show how nonlinearities of the type specified in Section 3.1 help to capture the general features exhibited by the time series under investigation. Thus, examining the last column of Table 4, where international spillovers and nonlinearities are both explicitly taken into account, suggests that the in-sample performance of the model is very satisfactory. Even correcting for the larger number of parameters of the MSIAH(8)-VECM(1) model, the coefficient of determination is at least four times larger than the coefficient of determination obtained for the bivariate MS-VECM models and more than 10 times larger than the coefficient of determination of the standard linear VECM models.[17]

3.3 Empirical analysis II: forecasting

3.3.1 Point forecasting performance and market timing tests

One of our results, corroborating some previous findings in the relevant literature, is that futures prices contain valuable information that can be exploited to explain a sizable proportion of stock prices and returns, at least in sample. In order to better evaluate the gain from using a sophisticated nonlinear empirical model, dynamic out-of-sample

[17] Note that all of our estimated MS-VECMs are stationary, as confirmed by calculating the value of the spectral radius as in Karlsen (1990).

forecasts of stock returns were constructed using the MSIAH-VECM estimated and discussed in the previous section. In particular, we calculated one-step-ahead forecasts over the period January 1999-December 2002.[18] The out-of-sample forecasts for a given horizon are constructed according to a recursive procedure that is conditional only upon information available up to the date of the forecasts and with successive re-estimation as the date on which forecasts are conditioned moves through the data set.

It is well known in the literature that forecasting with nonlinear models is in general much more difficult than forecasting with linear models because of the need to condition on the distribution of future exogenous shocks whose conditional expectation may be zero in a linear framework but not in a nonlinear framework. However, given that we compute one-step-ahead forecasts, the procedure often suggested in the literature that involves implementing numerical integration using Monte Carlo methods is not required as the one-step-ahead forecasts can be calculated analytically for our models (see, *inter alia*, Brown and Mariano, 1984, 1989; Granger and Teräsvirta, 1993, chapter 8; Franses and van Dijk, 2000, chapters 3-4; Krolzig, 2000).

Forecast accuracy is evaluated using several criteria. Panel a) of Table 3.5 shows the mean absolute errors (MAE) and the root mean square errors (RMSE) for each of the estimated models. The MSIAH-VECM (3.14) (i.e. the nonlinear VECM which allows for international spillovers) exhibits the best out-of-sample performance: the MAEs and RMSEs are always lower than the ones obtained from each of the alternative models suggesting that *both* nonlinearities *and* spillovers are important to explain, even out-of-sample, the dynamics of stock returns. However, the results of the Diebold and Mariano (1995) test (or DM test), reported in parentheses in Panel a) of Table 3.5, indicate that we are not able to reject the null of equal predictive accuracy in each case. Hence the differences in terms of MAEs and RMSEs reported in Table 3.5 are not statistically

[18] For a description of the econometric issues related to out-of-sample forecasting in a Markov-switching framework, see Hamilton (1993).

significant and do not enable us to discriminate across the models examined.[19] [20]

Alternative formal comparisons of the predicted and actual stock index returns can be obtained in a variety of ways. Hence, we consider the 'hit' ratio, calculated as the proportion of correctly predicted signs over the whole forecast period. Further, we consider a set of tests for market timing ability of the competing models. In particular, we carried out the tests proposed by Henriksson and Merton (1981), by Cumby and Modest (1987), and by Bossaerts and Hillion (1999) - HM, CM and BH tests from now onwards. The idea behind the HM test is that there is evidence of market timing if the sum of the estimated conditional probabilities of correct forecasts (that is the probability of correct forecast sign either when the market is bullish and bearish) exceed one. The HM test statistics is given by:

$$HM = \frac{n_{11} - \frac{n_{01}n_{10}}{n}}{\sqrt{\frac{n_{01}n_{10}n_{20}n_{02}}{n^2(n-1)}}} \sim N(0,1) \tag{3.15}$$

where n_{11} is the number of correct bear market forecast; n_{01}, n_{10} are the number of bear markets and bear market forecasts respectively, while n_{02} and n_{20} denote the number of bull market and bull market forecasts respectively. The total number of evaluation periods is denoted by n. The CM test extends the HM test to take into account not only the sign of the realized returns, but also their magnitude. This involves the estimation of the auxiliary regression:

$$\Delta s^i_{t+1} = \phi_0 + \phi_1 I_{\left\{\widetilde{\Delta s^i_{t+1}}>0\right\}} + error\ term \tag{3.16}$$

where Δs^i_{t+1} is the time series of the realized returns for the stock index i and $I_{\left\{\widetilde{\Delta s^i_{t+1}}>0\right\}}$ is the indicator function equal to unity when the forecast returns for the index i $\widetilde{\Delta s^i_{t+1}} > 0$ and equal to zero otherwise. Finally, the BH test involves estimating the following

[19] See footnote 20 in Chapter 2.

[20] Note that the non-rejection of the null of equal point forecast accuracy may be due to the well documented low power of the Diebold-Mariano test statistic in finite sample (see Kilian and Taylor, 2003, and the references therein).

auxiliary regression:

$$\Delta s_{t+1}^{i} = \zeta_0 + \zeta_1 \widetilde{\Delta s_{t+1}^{i}} + error\ term \tag{3.17}$$

where $\widetilde{\Delta s_{t+1}^{i}}$ is the time series of the forecast returns for the stock index i. For both CM and BH tests, the null hypothesis of no market timing ability is that the slope coefficients ϕ_1 and ζ_1 are equal to zero against the one-side alternative that they are positive. The results from executing these tests are reported in Panel b) of Table 3.5. Under these measures of market timing ability, we find a very different picture from the one suggested by the Diebold-Mariano tests for equal point forecast accuracy, but a very similar picture to the one portraied by the in-sample analysis. The role of non-separation in the equilibrium correction terms is important to explain out-of-sample futures and spot returns: columns 2 and 4 highlight the improvement in the predictive performance of the models when the futures bases from different stock markets are incorporated as explanatory variables in the returns equations. Further, the role of nonlinearities appears to be very important to better explain stock returns. Columns 3 and 4 show how nonlinearities of the type specified in Section 1 help to capture the general features exhibited by the time series under investigation. Thus, examining the last column of Table 3.5, where international spillovers and nonlinearities are both explicitly taken into account, suggests that the market timing performance of the model is highly satisfactory.

3.3.2 Density forecasting performance: main results

The findings in the previous subsection deserve further discussion. The estimated linear and nonlinear models produced a series of dynamic out-of-sample forecasts. Using different criteria to evaluate their predictive accuracy we obtained somewhat conflicting results. How can one reconcile, for example, the finding that the MSIAH-VECM with international spillovers displays satisfactory market timing ability to the various alternative models with its inability to beat the alternative models on the basis of MAEs and

RMSEs?

One explanation is that by focusing only on the first two moments of the stock returns distributions, which is effectively what one does when using point forecast accuracy tests, we do not exploit the whole information provided by the MS-VECMs out-of-sample predictions. In particular, the MSIAH-VECM (3.14) may exhibit the best performance across the models considered in terms of 'closeness' of the predicted moments to the true moments of stock returns data over the forecast period, although this might not be clear if one considers only the first two moments of the distribution of stock returns.

A logical next step then involves testing formally the hypothesis that the forecast density implied by the MSIAH-VECM (3.14) is equal to the true predictive density of the data. A large body of literature in financial econometrics has recently focused on evaluating the forecast accuracy of empirical models on the basis of density, as opposed to point, forecasting performance (see, *inter alia*, Diebold, Gunther and Tay, 1998; Diebold, Hahn and Tay, 1999; Granger and Pesaran, 1999; Tay and Wallis, 2000; Timmermann, 2000; Pesaran and Skouras, 2002; Sarno and Valente, 2003). Several researchers have proposed methods for evaluating density forecasts. For example, Diebold, Gunther and Tay (1998) extend previous work on the probability integral transform and show how it is possible to evaluate a model-based predictive density and to test formally the hypothesis that the predictive density implied by a particular model corresponds to the true predictive density. In general, they propose the calculation the probability integral transforms of the actual realizations of the variables (i.e. stock returns for the different stock indices under investigation) over the forecast period, $\left\{\Delta s_{t+1}^{i}\right\}_{t=1}^{n}$ with respect to the models' forecast densities, denoted by $\left\{p_t\left(\Delta s_{t+1}^{i}\right)\right\}_{t=1}^{n}$:

$$z_t = \int_{-\infty}^{\Delta s_{t+1}^{i}} p_t\left(u\right) du \qquad t = 1, \ldots, n. \tag{3.18}$$

When the model forecast density corresponds to the true predictive density, denoted by $f_t\left(\Delta s_{t+1}^{i}\right)$, then the sequence of $\left\{z_t\right\}_{t=1}^{n}$ is iid $U\left[0,1\right]$ distributed. The idea is therefore to evaluate whether the realizations of the data over the forecast period do come from the selected forecast density by testing whether the $\{z_t\}$ series depart from the iid uniformity assumption. Following Clements and Smith (2000), we assess uniformity by plotting the empirical distribution function against the 45^0 line.[21] Berkowitz (2001) suggests that rather than working with the $\{z_t\}$ series it may be fruitful to take the inverse normal cumulative distribution function (CDF) transform of the series $\{z_t\}$, denoted by $\{x_t\}$. Under the null hypothesis of equality of the model density and the true predictive density, $\{x_t\}$ is distributed as standard normal, and Berkowitz proposes an LR test for zero mean, unit variance and independence.

While, under general conditions, the linear VECMs forecast densities are easy to calculate analytically (they are in fact multivariate normal distributions with means and variances given by simple functions of the estimated parameters), the implied MSIAH-VECM forecast densities can, in general, be obtained analytically only for one-step ahead forecasts. The MSIAH-VECM forecast densities are mixture of multivariate normal distributions with weights given by the predicted regime probabilities. In general the MSIAH-VECM forecast densities are nonnormal, neither symmetric, homoskedastic nor regime invariant. Following Krolzig (2000), the one-step ahead MSIAH-VECM forecast density is given by:

$$p_{t+1}\left(\Delta y_{t+1}\right) = \sum_{j=1}^{M}\left\{\sum_{i=1}^{M} p_{ij}\mathbf{P}\right\} p_{t+1}\left(\Delta y_{t+1} \mid \omega_{t+1} = j, \quad {}_t\right) \tag{3.19}$$

where $p_{ij} = \Pr(\omega_{t+1} = j \mid \omega_t = i)$ are the transition probabilities, $\mathbf{P}$ is the transition matrix conditional on the information set at time t, ${}_t$ and $p_{t+1}\left(\Delta y_{t+1} \mid \omega_{t+1} = j, \quad {}_t\right)$ is the regime-conditional forecast gaussian density with mean $\left[\sum_{i=1}^{p-1} \Lambda_i\left(\omega_t\right) \Delta y_{t-i} + \Pi\left(\omega_t\right) y_{t-1}\right]$ and variance $\sum\left(\omega_t\right)$.

We now turn to the evaluation of the probability integral transforms. The null of iid uniformity is a joint hypothesis and according to the suggestions of Diebold, Gunther

[21] It is important to notice that the confidence intervals reported are only valid under the assumption of independence.

and Tay (1998) we consider each part of the hypothesis in turn. The iid assumption is tested by executing the Ljung-Box (1978) test for serial correlation up to the fourth-order. The results are reported in Panel a) of Table 3.6. In order to take into account the dependence occurring in the higher moments, we also consider $(z-\overline{z})^j$ for j up to three. The results tell us that in most of the cases we are not able to reject the null hypothesis of no serial correlation. This finding applies particularly for the most general model, the MSIAH-VECM in equation (3.14), while sporadic rejections occur in the second moment in the case of linear VECMs (3.11) and (3.12) estimated for FTSE 100 and S&P 500 and MSIAH-VECM (3.13) only for FTSE 100. We assess the uniformity aspect by plotting the actual CDFs of the $\{z_t\}$ series against the theoretical CDF (i.e. 45^0 line). The results are plotted in Figure 3.3. The confidence intervals reported have been calculated by Monte Carlo simulation with 50,000 replications. Figure 3 clearly indicates that it is possible to distinguish among the different competing models. In fact, the models which consider in turn nonlinearity and international spillovers generally reject the null hypothesis of uniformity and in all cases we can see that the empirical CDFs exhibit an S-shape around the 45^0 line. This could occur because the point forecast is a biased predictor of the mean of the true forecast density or it could be due to any of the higher moments failing to match. A different picture can be seen by looking at the last column in Figure 3.3. The most general model incorporating both nonlinearities and international spillovers does not exhibit the same S-shape pattern and, most importantly, we are not able to reject the null hypothesis of uniformity. The same results can be found in Panel b) of Table 3.6, where we report the LR tests of zero mean, unit variance and independence proposed by Berkowitz (2001). In fact the only model which fail to reject the null hypothesis is again the MSIAH-VECM (3.14).

Summing up, the forecasting results in this section suggest that, in terms of density forecasting perfomance, the general MSIAH-VECM that allows for international spillovers performs significantly better than any other linear and nonlinear model considered in this chapter in terms of explaining the out-of-sample behavior of stock returns. Taken together, the results in Section 3.3.1 and 3.3.2 suggest that, while the forecasting performance of the general MSIAH-VECM is not statistically different from the performance of the alternative models in terms of point forecasting, the MSIAH-VECM is superior when one evaluates out-of-sample performance on the basis of the ability of the model to match the full out-of-sample predictive density of stock returns. Clearly, this finding is due to the allowance for both international spillovers and multiple regimes in our model. In particular, shifts in the variance-covariance matrix allow us to capture satisfactorily the heteroskedasticity of stock returns, while the intercept shifts offer the model greater protection against unforeseen regime shifts, overall enhancing the forecasting performance of the model.

3.3.3 The economic relevance of density forecasts: a simple example of Value-at-Risk analysis

Under the 1997 Amendment to the Basle Accord, banks are able to seek approval for the adoption of their own in-house risk models in order to calculate the minimum regulatory required capital to cover their market risk. Given that banks are permitted to develop different risk models, it is necessary to assess the relative performance of the alternative models. Therefore it is interesting to further investigate the practical implications of the density forecasting results reported in the previous sub-section in the context of a simple risk management problem. Given the predictions of the four competing models examined here, assume that a risk manager wishes to quantify the one-week-ahead risk associated with one stock index.[22] The different competing models provide the one-week-ahead density forecasts of Δs_{t+1}^{i} and on the basis of these densities the risk manager calculates the Value-at-Risk (VaR) of the stock index as a one-sided confidence interval on losses

[22] Of course, a more complicated example would involve considering the joint density for all of the stock indices considered. We limit ourselves to the simplest case, given the illustrative nature of the application in the present section.

such that:

$$\Pr\left(\Delta s^{i}_{t+1} < VaR_{it+1}\right) = 1 - c, \tag{3.20}$$

where Δs^{i}_{t+1} is the realized end-of-week return and c denotes the given confidence level. In our example the VaR is a 99 percent confidence level for losses (i.e. $c = 0.99$), for all models. Equation (3.20) simply states that the probability that the change in the value of the portfolio is less than the Value-at-Risk is equal to the significance level $1 - c$. Summary statistics are reported in Panel a) of Table 3.7. For all competing models we record the average VaR and the standard deviation of the estimated VaR over the forecast period and the realized violations, that are the number of times that $\Delta s^{i}_{t+1} < VaR_{it+1}$. The results in Table 3.7 suggest that, for all stock indices, the MSIAH-VECM (3.14) exhibits the highest average VaR (in absolute value), the highest standard deviation for the estimated VaR, and the lowest number of violations (i.e. zero). Although the latter result may suggest a conservative behavior in predicting future risk, the high variability and the positive and significant correlation between the estimated VaR_{it+1} and the realized series of returns Δs^{i}_{t+1} are instead supportive of a very satisfactory performance of the MSIAH-VECM (3.14) in terms of accuracy and efficiency.[23]

In the recent literature, there is no definitive measure of VaR model performance, thus in order to evaluate the performance of the competing models, we present a variety of different metrics. To assess the relative size and relative variability of the VaR estimates produced by the competing models we use the mean relative bias statistics (MRB) and root mean squared relative bias statistics (RMSRB), introduced by Hendricks (1996). The MRB statistics is calculated as:

$$MRB_i = \frac{1}{n}\sum_{j=1}^{n}\frac{VaR_{it+j} - \overline{VaR_{t+j}}}{VaR_{it+j}} \tag{3.21}$$

where VaR_{it} is the estimated Value-at-Risk for model i at time t and $\overline{VaR_t}$ is the cross-sectional average (over the competing models) Value-at-Risk at time t. This statistics

[23] This measure has been introduced by Hendricks (1996, p. 161).

indicates a measure of size for each estimated VaR, independent of the scale of the simulated portfolio, that is relative to the average of all the competing models. The RMSRB statistics is calculated as:

$$RSMRB_i = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(\frac{VaR_{it+j} - \overline{VaR_{t+j}}}{VaR_{it+j}}\right)^2} \tag{3.22}$$

This measure provide us with information about the extent to which the estimated VaR tend to vary around the average VaR for a given date t. Another statistics, introduced by Christoffersen and Diebold (2000) is given by the autocorrelation coefficient of the binary variable V_t which is equal to 1 if a violation occurs and 0 otherwise. A significant autocorrelation coefficient denotes a persistent series of violations which in turn implies a nonsatisfactory performance of a model in estimating the VaR. Finally the last measure considered is the LR test for the statistical significance of the violation rate (i.e. the ratio between the number of violations over the observations comprised in the forecasting period) If the realized violation rate is larger than the theoretical violation rate (i.e. $1-c = 0.01$) the model failed in estimating the VaR (Kupiec, 1995; Christoffersen, 1998).

The results, reported in Panel b) of Table 3.7, confirm the findings in Panel a). In fact the MRB and RSMRB statistics show that the MSIAH-VECM(3.14) produces higher VaRs (compared to the average VaR produced by all competing models) and it also produces more volatile VaRs (around the average VaR produced by all competing models). Since the MSIAH-VECMs (14) have not been violated over the forecasting period, we are not able to obtain the further two statistics. For the remaining competing models we can see that the MSIAH-VECM (3.13) always reject the null hypothesis that the realized violation rate is higher than the theoretical one and the MSIAH-VECM (3.13) and the linear VECM (3.12) estimated for the FTSE 100 produced VaRs which experience persistent violations.

In this simple application we have shown that the forecasting performance of alternative models can be very different when analyzed under different metrics. Conventional measures of predictive accuracy based on MAEs and RMSEs fail to recognize differences in higher moments of the predictive distributions, which may be very relevant, for example, when assessing risk. In our example, although all the competing models were indistinguishable from the MSIAH-VECM (3.14) in terms of point forecasting accuracy, they have produced forecasts that did not capture satisfactorily the higher moments of the predictive distribution of stock returns, generating VaRs that underestimate the probability of large losses. Differently, the most general model incorporating both nonlinearities and international spillovers, did better than all of the linear and nonlinear competing models at matching the higher moments of the predictive distribution of stock returns and produced VaRs that are generally in line with the target violation rate of 1 per cent.

3.4 Conclusion

This article has re-examined the dynamic relationship between spot and futures prices in stock index futures markets using data since 1989 at weekly frequency for three major stock market indices - the S&P 500, the NIKKEI 225 and the FTSE 100 indices. In particular, we propose a nonlinear, Markov-switching vector equilibrium correction model that explicitly takes into account the mounting evidence that the conditional distribution of stock returns is well characterized by a mixture of normal distributions. Also, we use the recently developed notion of 'separation and cointegration' to provide a richer characterization of the dynamics of stock returns that explicitly allows for international spillovers across these stock index and stock index futures markets.

The empirical results provide evidence in favor of the existence of international spillovers across these major stock markets and a well-defined long-run equilibrium relationship between spot and futures prices which is consistent with mean reversion in the futures basis. Linear vector equilibrium correction models were rejected when tested against a Markov-switching vector equilibrium correction model which allows for shifts in the intercept, the autoregressive structure and the variance-covariance matrix. Our preferred nonlinear specification explains a significant fraction of the stock returns examined, with the $\overline{R}^2$ ranging from 0.08 for the NIKKEI 225 index returns to 0.12 for the FTSE 100 index returns.

Using the estimated models in an out-of-sample forecasting exercise we found that both nonlinearity and international spillovers are important in forecasting future stock returns. However, their importance is not apparent when the forecasting ability of our proposed nonlinear VECM is evaluated on the basis of conventional point forecasting criteria. In fact, these criteria neglect the fact that stock returns may be non-normally distributed and that the nonlinear models employed in this chapter imply non-normal predictive densities. In order to measure more adequately the forecasting ability of our nonlinear model and discriminate among competing models we calculated hit ratios, employed tests for market timing ability and also evaluated the density forecasting performance of both linear and nonlinear models.

Overall, the evidence reported in this chapter suggests that the statistical performance of the linear (single-regime) and nonlinear (multiple-regime) models examined differs little in terms of conditional mean, regardless of whether allowance is made for international spillovers across the stock indices examined. However, the hit ratios and the tests of market timing ability as well as inspection of the predictive densities, which fully consider the higher order conditional moments implied by the various models, shows greater ability to discriminate between competing models. In particular, exploration of the model-based forecast densities indicates the rejection of single-regime models as well as multiple-regime models with no international spillovers against a multiple-regime model with international spillovers, leading us to the conclusion that both multiple regimes and the allowance for international spillovers are important ingredients for a model to produce satisfactory out-of-sample forecasting performance. The implication of our findings are further investigated in the context of a simple application to Value-at-Risk which highlights how better density forecasts of stock returns, of the type recorded in this chapter, can potentially lead to substantial improvements in risk management and more precisely, to better estimates of downside risk.

While these results aid the profession's understanding of the behavior of stock returns, we view our model as a tentatively adequate characterization of the data which appears to be superior to linear equilibrium correction modeling in a number of respects, but which nevertheless may be capable of improvement. In particular, while we focused on the information provided by the futures market for forecasting future stock returns, it would be interesting to investigate the presence of regime-switching behavior in the context of conventional models involving dividend yields or other fundamentals. Also, while the model used here is fairly general and flexible, the evidence we document suggests that global stock index and stock index futures markets are characterized by very complex dynamic interactions. Much more work needs to be done to understand these relationships.

Table 3.1: Preliminary data statistics

	S&P 500		NIKKEI 225		FTSE 100	
	f_t	s_t	f_t	s_t	f_t	s_t
Minimum	7.333	7.318	10.589	10.569	8.837	8.829
Maximum	5.645	5.632	9.025	9.029	7.506	7.491
Mean	6.489	6.483	9.833	9.829	8.227	8.220
Std Dev	0.521	0.520	0.321	0.317	0.375	0.378
PACF:						
lag 1	0.997	0.997	0.990	0.990	0.995	0.996
lag 2	0.029	0.030	-0.024	-0.015	0.066	0.052
lag 3	0.010	0.003	0.002	-0.006	0.016	0.008
lag 4	-0.013	-0.010	0.000	-0.006	-0.002	-0.012
lag 5	0.013	0.019	0.003	0.007	0.056	0.069
lag 6	0.018	0.016	-0.044	-0.044	0.035	0.028
lag 7	-0.017	-0.016	0.013	0.015	-0.023	-0.024
lag 8	0.011	0.012	0.012	0.013	0.011	0.010
lag 9	0.024	0.012	-0.032	-0.029	0.026	0.009
lag 10	-0.011	-0.009	0.015	0.012	0.008	0.016
lag 11	0.001	0.002	-0.005	0.001	0.026	0.014
lag 12	-0.006	-0.006	-0.011	-0.018	-0.033	-0.040

Notes: f_t and s_t denote the log-level of the futures price and the log-level of the spot price respectively. PACF is the partial autocorrelation function, and its standard deviation can be approximated by the square root of the reciprocal of the number of observations, $T = 730$.

Table 3.2: Johansen maximum likelihood cointegration procedure

Panel a) LR tests based on the maximum eigenvalue of the stochastic matrix

H_0	H_1	$LR_{S\&P}$	LR_{NIKKEI}	LR_{FTSE}	$95\%CV$
$r=0$	$r=1$	50.42	66.93	54.98	14.06
$r\leq 1$	$r=2$	0.76	1.58	0.11	3.84

Panel b) LR tests based on the trace of the stochastic matrix

H_0	H_1	$LR_{S\&P}$	LR_{NIKKEI}	LR_{FTSE}	$95\%CV$
$r=0$	$r\geq 1$	51.19	68.52	55.08	15.41
$r\leq 1$	$r=2$	0.76	1.58	0.11	3.84

Notes: The cointegration tests refer to a cointegrating VAR model for f_t and s_t. H_0 and H_1 denote the null hypothesis and the alternative hypothesis respectively; r denotes the number of cointegrating vectors. The test statistics are calculated including an unrestricted constant term $\alpha\left(\beta' y_{t-1}+\rho_0\right)+\alpha\perp\gamma_0$ (case 3; Johansen, 1995).

Table 3.3: Likelihood ratio (LR) tests

	LR1	LR2
Bivariate VECM for S&P 500	359.44	—
	$\{8.00\times 10^{-69}\}$	—
Bivariate VECM for NIKKEI 225	270.10	—
	$\{4.22\times 10^{-50}\}$	—
Bivariate VECM for FTSE 100	340.60	—
	$\{7.35\times 10^{-65}\}$	—
Multivariate VECM (all indices)	1999.56	1698.30
	$\{7.09\times 10^{-144}\}$	$\{1.72\times 10^{-98}\}$

Notes: $LR1$ tests the null hypothesis of a linear VECM against the alternative hypothesis of an MSIAH-VECM with $M=2$ or 2^3 regimes. $LR2$ is the likelihood ratio test calculated to test the restrictions in (3.9) for the estimated MSIAH-VECMs. $LR2$ is distributed as $\chi^2\left(g\right)$ where g is the number of restrictions imposed. Figures in braces denote p-values.

Table 3.4: In-sample performance

	VECM (11)	VECM (12)	MSIAH-VECM (13)	MSIAH-VECM (14)
		$\overline{R}^2$		
S&P 500 s_t	0.0058	0.0233	0.0262	0.1019
NIKKEI 225 s_t	0.0015	0.0071	0.0058	0.0812
FTSE 100 s_t	0.0277	0.0292	0.0278	0.1188
		Information Criteria		
AIC	0.938	0.964	0.979	–
BIC	0.878	0.897	0.917	–

Notes: $\overline{R}^2$ is the adjusted coefficient of determination calculated as Krolzig (1997, p. 133-4). AIC and BIC are the ratios of the AIC and the BIC, respectively, from each index preferred MSIAH-VECM(3.14) to the corresponding goodness-of-fit measures for the alternative competing models. AIC and BIC criteria reported are calculated for the whole (linear and nonlinear) VECM systems.

Table 3.5: Out-of-sample performance: point forecasting

Panel a) Mean absolute errors, root mean square errors and Diebold-Mariano tests

	VECM (11)	VECM (12)	MSIAH-VECM (13)	MSIAH-VECM (14)
		S&P 500		
MAE	0.022	0.022	0.021	0.015
	(0.923)	(0.927)	(0.925)	–
RMSE	0.028	0.028	0.027	0.019
	(0.995)	(0.996)	(0.996)	–
		NIKKEI 225		
MAE	0.026	0.025	0.025	0.020
	(0.933)	(0.939)	(0.938)	–
RMSE	0.033	0.032	0.032	0.024
	(0.995)	(0.995)	(0.995)	–
		FTSE 100		
MAE	0.021	0.021	0.021	0.012
	(0.922)	(0.923)	(0.921)	–
RMSE	0.027	0.027	0.027	0.015
	(0.996)	(0.996)	(0.996)	–

Panel b) Market Timing Test

	VECM (11)	VECM (12)	MSIAH-VECM (13)	MSIAH-VECM (14)
		S&P 500		
HR	0.510	0.553	0.563	0.745
HM	3.54×10^{-1}	4.02×10^{-2}	1.19×10^{-3}	1.05×10^{-13}
CM	1.41×10^{-1}	2.46×10^{-2}	1.24×10^{-7}	8.88×10^{-20}
BH	3.84×10^{-2}	1.09×10^{-5}	7.61×10^{-11}	2.89×10^{-42}
		NIKKEI 225		
HR	0.534	0.601	0.519	0.697
HM	9.38×10^{-1}	6.85×10^{-3}	9.60×10^{-1}	6.65×10^{-8}
CM	2.51×10^{-1}	1.44×10^{-1}	1.68×10^{-1}	9.49×10^{-11}
BH	4.62×10^{-1}	7.89×10^{-2}	4.79×10^{-3}	5.01×10^{-29}
		FTSE 100		
HR	0.529	0.587	0.529	0.760
HM	2.30×10^{-1}	4.23×10^{-3}	2.30×10^{-1}	5.14×10^{-14}
CM	3.39×10^{-2}	5.43×10^{-4}	9.47×10^{-3}	1.39×10^{-21}
BH	5.54×10^{-7}	7.73×10^{-8}	4.59×10^{-9}	3.13×10^{-64}

Notes: *Panel a):* MAE and $RMSE$denote the mean absolute error and the root mean square error respectively. Figures in parentheses are p-values from executing Diebold-Mariano (1995) test statistics for the null hypothesis the model i=VECM(3.11), VECM(3.12), MSIAH-VECM(3.13) have equal point forecast accuracy of MSIAH-VECM(3.14). The spectral density of the loss differential function at frequency zero $\hat{f}(0)$ is estimated using the optimal truncation lag according to the AR(1) Andrews's (1991) rule. The p-values were calculated by bootstrap methods using a variant of the procedure suggested by Kilian (1999). *Panel b):* HR is the Hit-ratio calculated as the proportion of correctly predicted signs. HM is the Henriksson and Merton (1981) test for market timing. CM is the Cumby and Modest (1987) test for the significance of the t-statistics of the slope coefficient in the regression $\Delta s_t^i = \phi_0 + \phi_1 I_{\{\widehat{\Delta s_t^i} > 0\}} + \epsilon$ where Δs_t^i are the realized returns for the index i = $S\&P500, NIKKEI225, FTSE100$ and I is the indicator function equal to 1 when the forecasted returns for the index i $\widehat{\Delta s_t^i} > 0$ and equal to zero otherwise. BH is the Bossaerts and Hillion (1999) test for the significance of the t-statistics of the slope coefficient in the regression $\Delta s_t^i = \zeta_0 + \zeta_1 \widehat{\Delta s_t^i} + \epsilon$ where $\widehat{\Delta s_t^i}$ are the forecasted returns for the index i = $S\&P500, NIKKEI225, FTSE100$. For the HM, CM, and BH test statistics only p-values are reported.

Table 3.6: Out-of-sample performance: density forecasting

Panel a) Test for iid based upon probability integral transforms

	VECM (11)	VECM (12)	MSIAH-VECM (13)	MSIAH-VECM (14)
			S&P 500	
z	0.457	0.330	0.201	0.423
$(z-\overline{z})^2$	0.052	0.031	0.350	0.934
$(z-\overline{z})^3$	0.441	0.307	0.337	0.270
			NIKKEI 225	
z	0.501	0.505	0.417	0.968
$(z-\overline{z})^2$	0.957	0.934	0.489	0.175
$(z-\overline{z})^3$	0.507	0.436	0.333	0.477
			FTSE 100	
z	0.221	0.278	0.278	0.403
$(z-\overline{z})^2$	0.016	0.007	0.017	0.413
$(z-\overline{z})^3$	0.093	0.121	0.167	0.275

Panel b) Berkowitz (1999) LR test

VECM (11)	VECM (12)	MSIAH-VECM (13)	MSIAH-VECM (14)
		S&P 500	
2.73×10^{-19}	1.04×10^{-19}	7.34×10^{-17}	1.18×10^{-1}
		NIKKEI 225	
3.86×10^{-3}	5.11×10^{-3}	2.78×10^{-9}	4.77×10^{-2}
		FTSE 100	
7.08×10^{-11}	6.12×10^{-13}	1.80×10^{-12}	7.59×10^{-2}

Notes: *Panel a)* Figures denote p-values for the Ljung and Box (1978) χ^2 test of serial correlation up to fourth-order. *Panel b)* Figures denote the p-values for the LR test of Berkowitz (2001). The tests is calculated including considering an alternative model with a quadratic and a cubic term lagged up to order 4. The test statistics is distributed under the null as a $\chi^2\,(q)$ where q is the number of restrictions imposed

Table 3.7: Value-at-Risk exercise

Panel a) Summary statistics

	VECM (11)	VECM (12)	MSIAH-VECM (13)	MSIAH-VECM (14)
		S&P 500		
Mean VaR	-0.053	-0.052	-0.051	-0.058
S.D. VaR	8.39×10^{-6}	1.54×10^{-5}	5.58×10^{-5}	4.16×10^{-4}
n. violations	5	5	8	0
$corr(\Delta s_t, VaR)$	0.117	0.303**	0.198**	0.731**
		NIKKEI 225		
Mean Var	-0.078	-0.078	-0.070	-0.085
S.D. VaR	5.38×10^{-6}	1.86×10^{-5}	5.19×10^{-5}	5.92×10^{-4}
n. violation	5	4	7	0
$corr(\Delta s_t, VaR)$	-0.046	0.125	0.048	0.621**
		FTSE 100		
Mean VaR	-0.054	-0.054	-0.051	-0.061
S.D. VaR	1.17×10^{-5}	1.35×10^{-5}	2.85×10^{-5}	5.75×10^{-4}
n. violation	5	5	8	0
$corr(\Delta s_t, VaR)$	0.290**	0.294**	0.226**	0.814**

Panel b) VaR backtests

	VECM (11)	VECM (12)	MSIAH-VECM (13)	MSIAH-VECM (14)
		S&P 500		
MRB	-0.001	-0.011	-0.044	0.056
RMSRB	0.106	0.096	0.124	0.265
CD	-0.025	-0.025	-0.040	–
K	0.084	0.084	0.002	–
		NIKKEI 225		
MRB	0.010	0.013	-0.093	0.074
RMSRB	0.083	0.083	0.138	0.229
CD	-0.024	-0.020	-0.035	–
K	0.085	0.235	0.007	–
		FTSE 100		
MRB	-0.002	-0.007	-0.067	0.076
RMSRB	0.109	0.106	0.128	0.308
CD	-0.024	0.180**	0.219**	–
K	0.085	0.085	0.002	–

Notes: Panel a) Mean Var and S.D. VaR denote the mean and standard deviation of the calculated VaRs from model (3.11)-(3.14) respectively. N. of violations denotes the number of times where the realized returns exceeded the estimated VaR. $corr(\Delta s_t, VaR)$ is the correlation coefficient between the estimated VaR and the realized data as in Hendricks (1996). Panel b) MRB and RMSRB are mean relative bias and square-root mean relative bias respectively calculated as in Hendricks (1996). CD is the Christoffersen and Diebold (2000) test for the sample autocorrelation of the variable V_t which is equal to 1 if violation occurs and zero otherwise. K is the Kupiec (1995) LR test for the null hypothesis that the violation rate is equal to the theoretical violation rate (i.e. 1%). The test statistic is distributed under the null as $\chi^2(1)$ For the K test only p-values are reported. *, ** denote significant at 5% and 1% respectively.

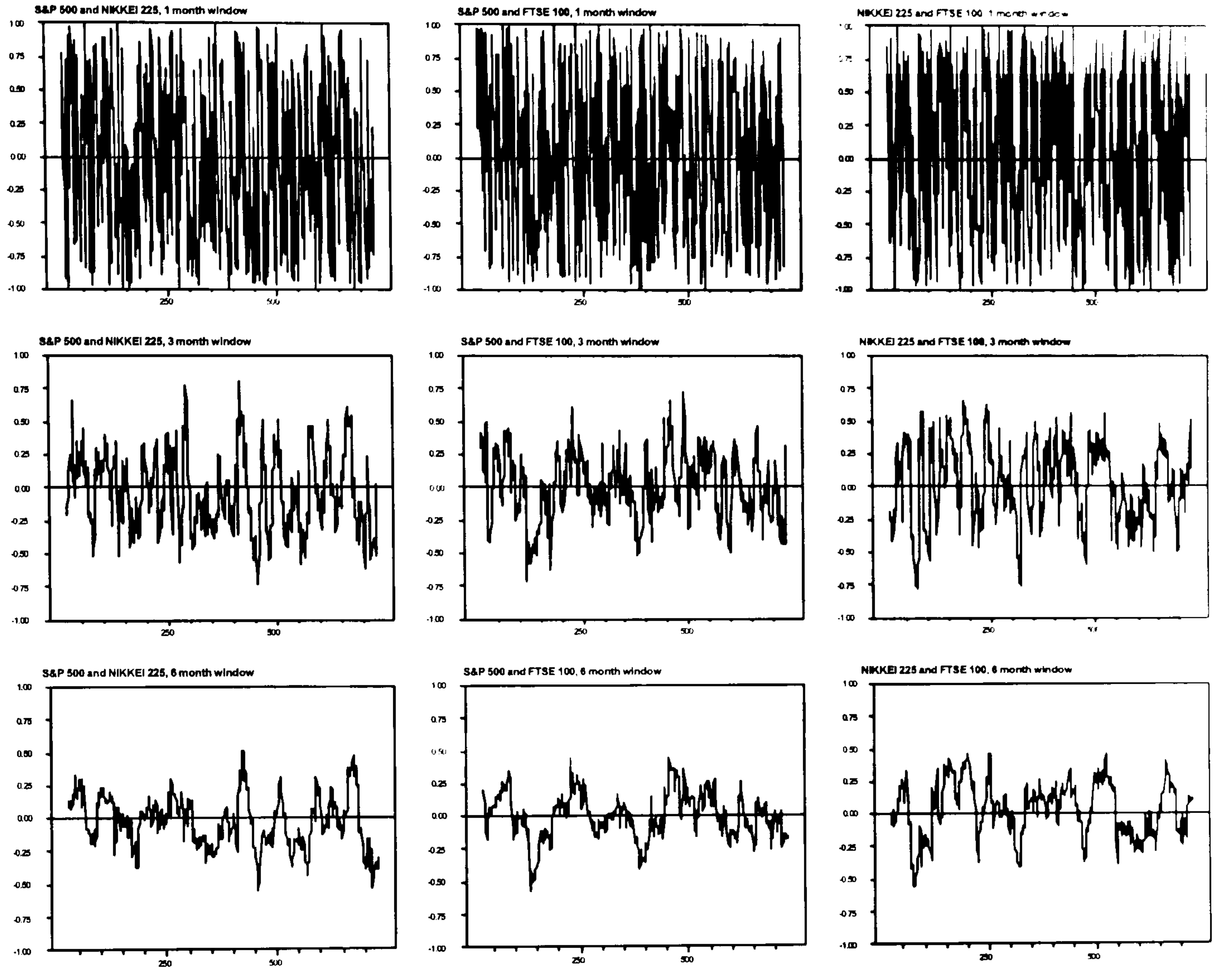

Figure 3.1: **Futures bases correlation**

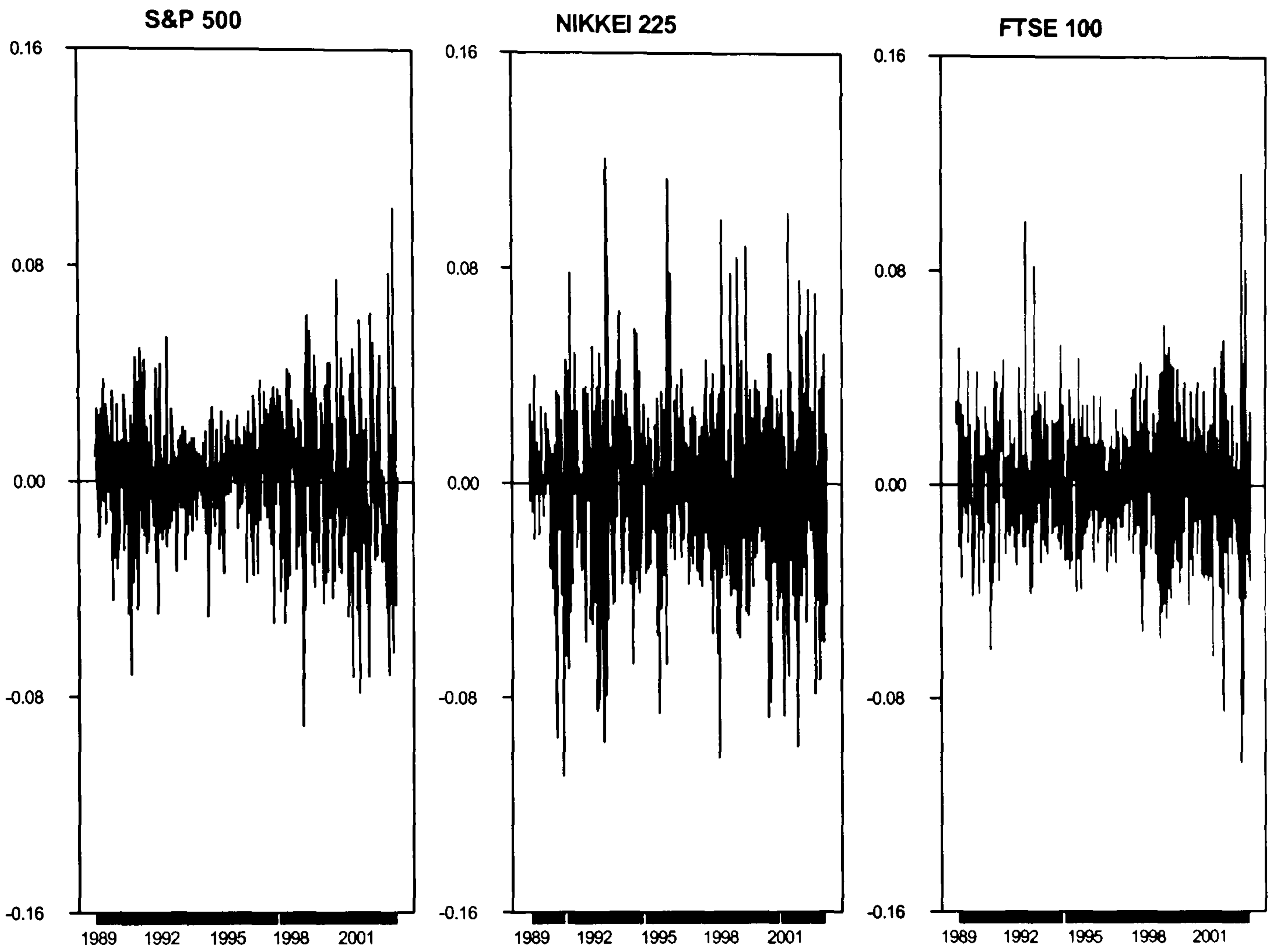

Figure 3.2: **Weekly log-differences of stock prices**

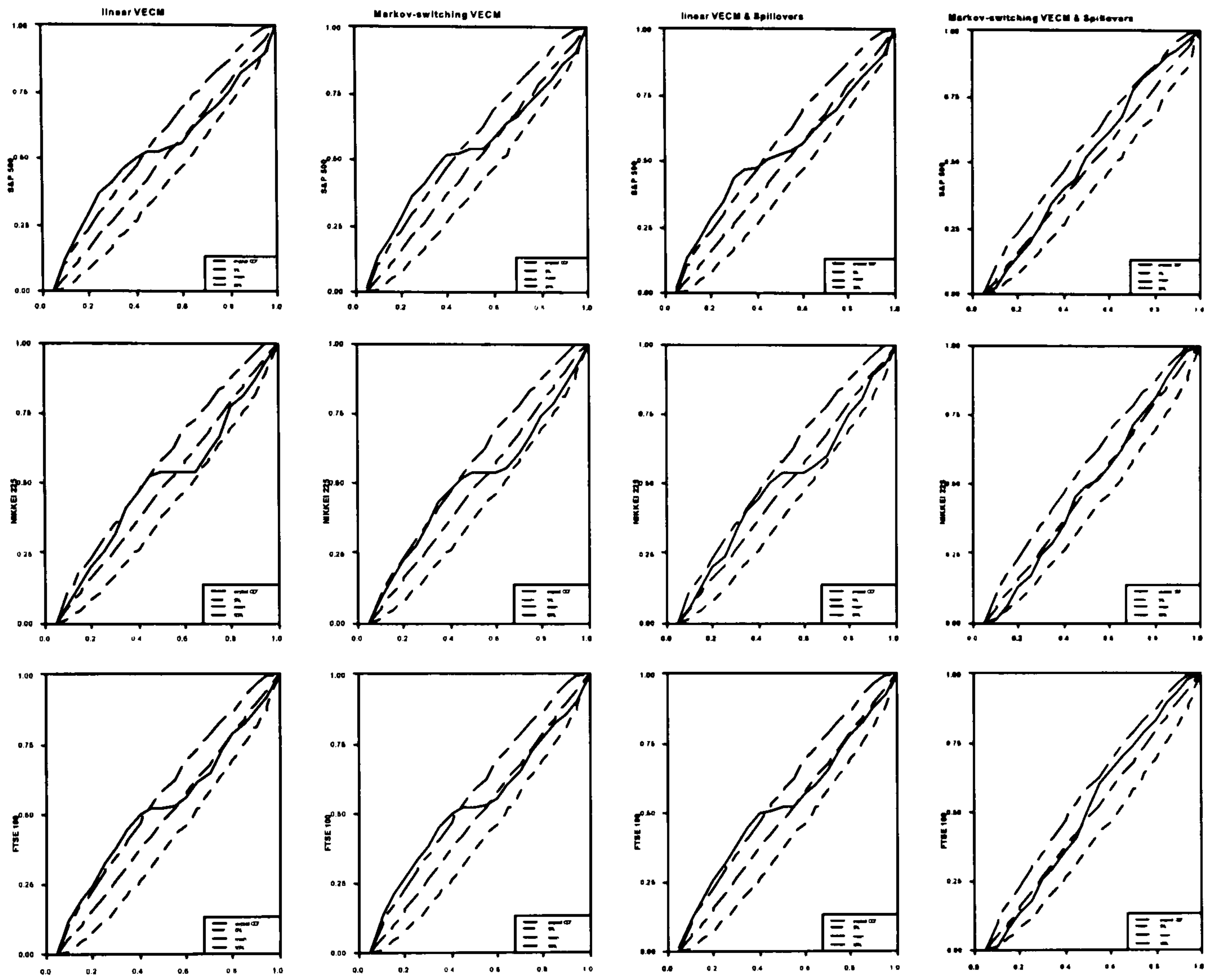

Figure 3.3: **CDFs of z-values**

Appendix A

Linearity tests

Under the RESET test statistic, the alternative model involves a higher-order polynomial to represent a different functional form; under the null hypothesis, the statistic is distributed as $\chi^2(q)$ with q equal to the number of higher-order terms in the alternative model.

The BDS test for a series u_t is calculated in the following way. Let $\mathbf{u}_{t,v}$ be a set of consecutive terms from u_t : $\mathbf{u}_{t,v}$ $\{u_t, u_{t+1}, \ldots, u_{t+v-1}\}$. The pair of vectors $\mathbf{u}_{t,v}$ and $\mathbf{u}_{s,v}$ are said to be no more than ς apart if $| u_{t+j} - u_{s+j} | \leq \varsigma$ for $j = 0, 1, \ldots, v-1$. Thus, the correlation integral $C_v(\varsigma)$ is defined as the product of the limit of T^{-2} (T being the number of observations) times the number of ς-close pairs (s,t), essentially measuring the probability that the pairs of points (s,t) are within ς of each other. The BDS statistic is then constructed as $S(v,\varsigma) = \widehat{C}_v(\varsigma) - [\widehat{C}_1(\varsigma)]^v$ for some v and ς. Under the null hypothesis that u_t is iid, $\sqrt{T}[S(v,\varsigma)] \sim N(0,\xi)$, where the variance ξ is a function of v and ς. Rejection of the null implies that some form of nonlinearity is present in u_t, although the type of nonlinearity cannot be exactly determined under the BDS test. BDS (1991) suggest that the choice of v and, particularly, the choice of ς, are crucial for the test power. BDS (1991) also suggest values of ς between 0.5 and 1.5 times the standard deviation of u_t, whereas the value of v should preferably be such that $(T/v) > 200$.

Table A.1: Linearity tests on the residuals from linear VECMs

Panel a) Linear VECM (3.11) (complete separation)

RESET tests	S&P 500	NIKKEI 225	FTSE 100
futures equation	0	0	5.82×10^{-2}
spot equation	1.47×10^{-2}	1.25×10^{-3}	7.47×10^{-2}

BDS tests	$v=2$			$v=3$		
	$\varsigma=0.5$	$\varsigma=1.0$	$\varsigma=1.5$	$\varsigma=0.5$	$\varsigma=1.0$	$\varsigma=1.5$
			S&P 500			
futures equation	3.12×10^{-6}	8.82×10^{-7}	4.11×10^{-7}	2.49×10^{-7}	1.10×10^{-7}	3.52×10^{-8}
spot equation	1.19×10^{-5}	6.20×10^{-7}	2.26×10^{-6}	2.02×10^{-7}	4.28×10^{-8}	5.77×10^{-8}
			NIKKEI 225			
futures equation	3.15×10^{-8}	2.13×10^{-7}	1.10×10^{-6}	2.78×10^{-14}	4.74×10^{-10}	1.39×10^{-8}
spot equation	2.40×10^{-9}	4.36×10^{-8}	3.83×10^{-7}	2.41×10^{-15}	6.36×10^{-11}	8.40×10^{-9}
			FTSE 100			
futures equation	2.55×10^{-1}	2.72×10^{-1}	2.93×10^{-1}	5.79×10^{-2}	4.86×10^{-2}	3.33×10^{-2}
spot equation	4.32×10^{-1}	3.25×10^{-1}	3.20×10^{-1}	7.14×10^{-2}	4.47×10^{-2}	1.45×10^{-2}

Panel b) Linear VECM (3.12) (type-B separation)

RESET tests	S&P 500	NIKKEI 225	FTSE 100
futures equation	7.72×10^{-4}	1.06×10^{-3}	0
spot equation	1.04×10^{-5}	9.58×10^{-3}	0

BDS tests	$v=2$			$v=3$		
	$\varsigma=0.5$	$\varsigma=1.0$	$\varsigma=1.5$	$\varsigma=0.5$	$\varsigma=1.0$	$\varsigma=1.5$
			S&P 500			
futures equation	5.51×10^{-7}	7.54×10^{-7}	1.32×10^{-6}	7.39×10^{-9}	4.30×10^{-8}	9.11×10^{-8}
spot equation	2.11×10^{-6}	2.58×10^{-7}	4.28×10^{-6}	1.55×10^{-8}	5.14×10^{-9}	8.06×10^{-8}
			NIKKEI 225			
futures equation	1.31×10^{-5}	1.67×10^{-5}	3.12×10^{-5}	1.92×10^{-9}	1.71×10^{-7}	1.01×10^{-6}
spot equation	5.36×10^{-7}	6.18×10^{-6}	2.47×10^{-5}	2.98×10^{-10}	5.36×10^{-8}	1.43×10^{-6}
			FTSE 100			
futures equation	2.34×10^{-1}	3.64×10^{-1}	2.77×10^{-1}	3.79×10^{-2}	3.53×10^{-2}	1.24×10^{-2}
spot equation	3.13×10^{-1}	3.12×10^{-1}	4.34×10^{-1}	2.52×10^{-2}	1.57×10^{-2}	9.43×10^{-3}

Notes: *Panel a):* Under the RESET test statistic, the alternative model involves a higher-order polynomial to represent a different functional form; in the present context we computed the RESET test considering an alternative model with a quadratic and a cubic term under the null of linearity. The RESET test statistic is distributed as $\chi^2(q)$ with q equal to the number of higher-order terms in the alternative model. *Panel b):* The BDS test statistic tests the null hypothesis that a series is iid against the alternative of a realization from an unspecified nonlinear process. The critical values, from the normal distribution, are 1.960 and 2.576 at the five percent and one percent nominal levels of significance respectively. For both RESET tests and BDS tests, we report p-values; p-values lower than equal to zero at the 8th decimal point are reported as 0.

Appendix B

The transition matrix of the MSIAH-VECM

In Section 3.1.2 we mentioned that the underlying regime-generating process is assumed to be an ergodic Markov chain with a finite number of states $\omega_t \in \{1, \ldots, M\}$ governed by the transition probabilities $p_{ij} = \Pr(\omega_t = j \mid \omega_{t-1} = i)$, and $\sum_{j=1}^{M} p_{ij} = 1 \ \forall i, j \in \{1, \ldots, M\}$. If we move from the perspective of a single system of variables (i.e. futures and spot returns in a single stock market) towards a model where several systems of variables are jointly considered (i.e. non-separation is explicitly considered, MSIAH-VECM (3.14)), we need to specify the joint process governing the transitional dynamics of the whole system. Define ω_t^{SP}, ω_t^{NK} and ω_t^{FT} the unobserved variable governing the transitional dynamics of the S&P 500, NIKKEI 255 and FTSE 100 indices respectively, and assume $M = 2$.

In order to achieve greater flexibility, at the cost of a high computational burden, we make no assumption about the relationship between the shifts occurring in the three markets examined, so that ω_t^{υ} would be an outcome of a Markov chain with transition probabilities p_{ij}^{υ} where ω_t^{υ} is independent of ω_t^{ψ} with $\psi \neq \upsilon$ for any t. In order to analyze the whole dynamics of the MSIAH-VECM (3.14) we construct the following latent variable

$$
\begin{aligned}
\xi_t &= 1 \quad \text{if } \omega_t^{SP} = 1,\ \omega_t^{NK} = 1 \text{ and } \omega_t^{FT} = 1 \\
\xi_t &= 2 \quad \text{if } \omega_t^{SP} = 2,\ \omega_t^{NK} = 1 \text{ and } \omega_t^{FT} = 1 \\
\xi_t &= 3 \quad \text{if } \omega_t^{SP} = 1,\ \omega_t^{NK} = 2 \text{ and } \omega_t^{FT} = 1 \\
\xi_t &= 4 \quad \text{if } \omega_t^{SP} = 2,\ \omega_t^{NK} = 2 \text{ and } \omega_t^{FT} = 1 \\
\xi_t &= 5 \quad \text{if } \omega_t^{SP} = 1,\ \omega_t^{NK} = 1 \text{ and } \omega_t^{FT} = 2 \\
\xi_t &= 6 \quad \text{if } \omega_t^{SP} = 2,\ \omega_t^{NK} = 1 \text{ and } \omega_t^{FT} = 2 \\
\xi_t &= 7 \quad \text{if } \omega_t^{SP} = 1,\ \omega_t^{NK} = 2 \text{ and } \omega_t^{FT} = 2 \\
\xi_t &= 8 \quad \text{if } \omega_t^{SP} = 2,\ \omega_t^{NK} = 2 \text{ and } \omega_t^{FT} = 2. \qquad \text{(B.1)}
\end{aligned}
$$

Under this formalization the latent variable ξ_t governing the transitional dynamics of the whole system MSIH-VECM (3.14) follows an 8-state Markov chain whose transition probabilities can be easily calculated from the probabilities of the chain governing ω_t^{SP}, ω_t^{NK} and ω_t^{FT}. For example:

$$
\begin{aligned}
\Pr\left(\xi_t = 1 | \xi_{t-1} = 1\right) &= \Pr\left(\omega_t^{SP} = 1 | \omega_{t-1}^{SP} = 1\right) \cdot \Pr\left(\omega_t^{NK} = 1 | \omega_{t-1}^{NK} = 1\right) \cdot \\
&\quad \Pr\left(\omega_t^{FT} = 1 | \omega_{t-1}^{FT} = 1\right) \\
&= p_{11}^{SP} p_{11}^{NK} p_{11}^{FT}. \qquad \text{(B.2)}
\end{aligned}
$$

Appendix C

Code (Ox 3.20) for estimating MSIAH-VECM (3.14)

C.1 Main program

```
#include <oxstd.h>
#import <msvar130>
#import <database>
#include ``OtherRout.ox''
main()
{
    decl time=timer();
    decl msvar = new MSVAR();

    msvar->IsOxPack(FALSE);
    msvar->Load(``Data01w.in7'');
    msvar->SetOptions(FALSE,FALSE,FALSE);
    msvar->SetPrint(FALSE,FALSE);
    msvar->SetEmOptions(1e-6, 100, 4);

    decl M = 8; // Number of regimes
    decl p = 1; // Autoregressive lag lenght
    decl obse = 522; // Observations
    decl fModel=MSIAH; // Model used

    msvar->Select(Y_VAR, { ``DLUSF'', 0, p, ``DLUSS'', 0, p, ``DLJAPF'', 0, p,
``DLJAPS'', 0, p, ``DLUKF'', 0, p, ``DLUKS'', 0, p });
    msvar->Select(X_VAR, { ``BUS'', 1, 1, ``BJAP'', 1, 1, ``BUK'', 1, 1});
    msvar->SetSample(1,1,obse,1);
    msvar->SetModel(fModel, M);
    msvar->Estimate();

    // Next section is devoted to loading database of the time series used in
the estimation

    decl dbase, var, X;
    dbase = new Database();
```

```
    dbase->LoadIn7(``Data01w.in7'');
    dbase->Select(0, { ``DLUSF'', 0, 0, ``DLUSS'', 0, 0, ``DLJAPF'', 0, 0,
``DLJAPS'', 0, 0, ``DLUKF'', 0, 0, ``DLUKS'', 0, 0, ``BUS'', 0, 0, ``BJAP'',
0, 0, ``BUK'', 0, 0});
    dbase->SetSample(p+2,1,obse,1);
    var = dbase->GetGroup(0);
    X = ones(rows(var),1)~var;

    // Next section is devoted to gathering estimation results

    decl v, B, U, S, P, AIC, BIC, HQ, Info;
    v = msvar->GetMu();
    B = msvar->GetB();
    U = msvar->GetU()';
    S = Errors(U, obse, p, M);
    P = msvar->GetTrans();
    AIC = msvar->GetAIC();
    BIC = msvar->GetSC();
    HQ = msvar->GetHQ();
    Info = AIC~BIC~HQ;

    // Print results using routine OtherRout.ox

    PrintResults(P, Info, X, v, B, S, M, 10);
    print(``\n\n****\ttime passed:  '', timespan(time), ``\t****\n'');
}
```

C.2 Routine OtherRout.ox

```
Errors(const U, const obse, const p, const M)
{
    decl err, i;

    err = zeros(columns(U), 1);

        for(i=0; i<M; ++i)
        {
            err = err~variance(U[i*(obse-p-1):(i+1)*(obse-p-1)-1][]);
        }
        return err[][1:];
}

PrintResults(const P, const Info, const X, const v, const B, const S, const M,
const npar)
{
    decl i, coeff, se;

    print(``\n\nEstimated Transition Matrix'');
    print(P);
    print(``\n\nInformation Criteria'');
    print(``%c'', {``AIC'', ``BIC'', ``HQ''}, Info);
```

```
    for(i=0; i<M; ++i)
    {
        coeff = v[][i]~B[][i*(M+1):((i+1)*(M+1))-1];
        se = reshape(sqrt(diagonal(S[][i*rows(B):((i+1)*rows(B))-1]
**invert(X'*X))), rows(B),npar);

        print(''\n\n*** Regime n.'', i+1, '' ***'');
        print(''\n\nRegime-conditional Estimated Parameters'');
        print(''%c'', {''v'', ''USf'', ''USs'', ''JAPf'', ''JAPs'', ''UKf'',
''UKs'', ''USb'', ''JAPb'', ''UKb''},
                ''%r'', {''USf'', ''USs'', ''JAPf'', ''JAPs'', ''UKf'',
''UKs''}, coeff);

        print(''\nEstimated Standard Errors'');
        print(''%c'', {''v'', ''USf'', ''USs'', ''JAPf'', ''JAPs'', ''UKf'',
''UKs'', ''USb'', ''JAPb'', ''UKb''},
                ''%r'', {''USf'', ''USs'', ''JAPf'', ''JAPs'', ''UKf'',
''UKs''}, se);

        print(''\nT-statistics'');
        print(''%c'', {''v'', ''USf'', ''USs'', ''JAPf'', ''JAPs'', ''UKf'',
''UKs'', ''USb'', ''JAPb'', ''UKb''},
                ''%r'', {''USf'', ''USs'', ''JAPf'', ''JAPs'', ''UKf'',
''UKs''}, coeff./se);

        print(''\nEstimated Regime-conditional Variance-Covariance Matrix'');
        print(''%c'', {''USf'', ''USs'', ''JAPf'', ''JAPs'', ''UKf'', ''UKs''},
                ''%r'', {''USf'', ''USs'', ''JAPf'', ''JAPs'', ''UKf'',
''UKs''}, S[][i*rows(B):((i+1)*rows(B))-1]);

    }
}
```

Chapter 4

Re-examining the predictive ability of the monetary model of exchange rate determination: statistical testing or economic value?

In an influential series of papers, Meese and Rogoff (1983a,b, 1988) noted that the out-of-sample forecasts of exchange rates produced by structural models based on fundamentals are no better than those obtained using a naive random walk or no-change model of the nominal exchange rate. These results, seen as devastating at the time, spurred a large literature that has re-examined the conclusions of the Meese-Rogoff studies. Some recent research, using techniques that account for several cumbersome econometric problems, including small sample bias and near-integrated regressors in the predictive regressions, suggests that models based on monetary fundamentals can explain a small amount of the variation in exchange rates (e.g., Mark, 1995; Mark and Sul, 2001). However, others remain skeptical (e.g., Berkowitz and Giorgianni, 2001; Faust, Rogers and Wright, 2003). Thus, even with the benefit of almost twenty years of hindsight, the Meese-Rogoff results have not been convincingly overturned: evidence that exchange rate forecasts obtained using fundamentals models are better than forecasts from a naive random walk is still elusive (e.g., Cheung, Chinn and Pascual, 2002; Neely and Sarno, 2002).

Prior research on the ability of monetary-fundamentals models to forecast exchange rates relies on *statistical* measures of forecast accuracy, like mean squared errors. Surprisingly little attention has been directed, however, to assessing whether there is any *economic* value to exchange rate predictability (i.e., to using a model where the exchange rate is forecast using economic fundamentals)[1]. The study fills this gap. We investigate the ability of a monetary-fundamentals model to predict exchange rates by measuring the economic or utility-based value to an investor who relies on this model to allocate her wealth between two assets that are identical in all respects except the currency of denomination. We focus on two key questions. First, as a preliminary to the forecasting exercise, we ask how exchange rate predictability and parameter uncertainty affect optimal portfolio choice for investors with a range of horizons up to ten years. Second, and more importantly, we ask whether there is any additional economic value to a utility-maximizing investor who uses exchange rate forecasts from a monetary-fundamentals model relative to an investor who uses forecasts from a naive random walk model. We quantify the economic value of predictability in a Bayesian framework that allows us to account for uncertainty surrounding parameter estimates in the forecasting model. Indeed, parameter uncertainty or 'estimation risk' is likely to be of importance, especially over long horizons.

Our results with regard to the two questions addressed in this chapter, obtained using three major US dollar exchange rates during the recent float and considering forecast horizons from 1 to 10 years, are as follows. First, we find that each of exchange rate predictability and parameter uncertainty substantially affect, both quantitatively and qualitatively, the choice between domestic and foreign assets for all currencies and across different levels of risk aversion. Specifically, exchange rate predictability can

[1] An exception is West, Edison and Cho (1993), who compare the utility gains from competing models for forecasting the volatility of exchange rates.

generate optimal weights to the foreign asset that are substantially different (in magnitude and, sometimes, in sign) from the optimal weights generated under a random walk model. Further, we find that taking into account parameter uncertainty causes the allocation to the foreign asset to fall (in absolute value) relative to the case when parameter uncertainty is not taken into account, effectively making the foreign asset look more risky. Second, our main result is that we find evidence of economic value to exchange rate predictability across all exchange rates examined and for a wide range of plausible levels of risk aversion. In particular, the realized end-of-period wealth, utility and certainty equivalent return achieved by a US investor over a ten-year horizon using a monetary fundamentals-exchange rate model for forecasting the exchange rate are higher than the corresponding end-of-period wealth, utility and certainty equivalent return obtained by an investor who acts as if the exchange rate were a random walk. Our results show that the economic value of predictability can be substantial also over relatively short horizons and across different levels of risk aversion, regardless of whether the investment strategy is static or dynamic and whether parameter uncertainty is taken into account. We view our findings as suggesting that the case against the predictive power of monetary fundamentals may be overstated.

Our work is related to and builds on earlier research by Kandel and Stambaugh (1996) and Barberis (2000), who use a Bayesian framework to study asset allocation[2] between a riskless asset and risky equities. Our work differs from theirs in three important ways. First, since we consider the economic gains (losses) to an investor whose problem is allocating her wealth between two assets that are identical in all respects except the currency of denomination, our focus is on exchange rate prediction. Put differently, in our framework risk only enters the investor's problem through the nominal exchange rate[3]. Second, we allow the investor to hold short positions in the assets, which is an

[2] This decision-theoretic approach has also been used recently by Avramov (2001), Bauer (2000), Cremers (2002), Shanken and Tamayo (2001) and Tamayo (2002).

[3] See Karolyi and Stulz (2002) for an elegant survey of asset allocation in an international context.

important feature in real-world foreign exchange markets (e.g., Lyons, 2001). Third, while we analyze the impact of predictability and parameter uncertainty on optimal allocation decisions, our primary goal is to evaluate the out-of-sample economic value of exchange rate predictability. We do this by comparing the end-of-period wealth, end-of-period utility and certainty equivalent return obtained using a standard monetary fundamentals model of the exchange rate with the corresponding measures of economic value obtained using a naive random walk, which remains the standard benchmark in the exchange rate forecasting literature.

Another related paper is Campbell, Viceira and White (2003), who study long-horizon currency allocation using a vector autoregressive framework where the predictive variables are the real interest rate and the real exchange rate. Our study differs from theirs in at least two ways. First, our basic forecasting instrument is the conventional set of monetary fundamentals proposed by exchange rate determination theory and used in the exchange rate forecasting literature since the Meese-Rogoff studies. Second, our framework allows for parameter uncertainty, which may be relevant over long investment horizons.

The rest of the chapter is organized as follows. Section 4.1 provides a brief outline of the theoretical background, while in Section 4.2 we describe the framework used to analyze the economic value of exchange rate predictability both with and without parameter uncertainty. Next, in Section 4.3, we discuss our empirical results relating to the asset allocation choice of our investor over various horizons. In Section 4.4 we report the results from an out-of-sample forecasting exercise, where we compare the realized end-of-period wealth, utility gains and certainty equivalent return for an investor who relies on the monetary fundamentals model and one who uses a random walk model. Section 6 concludes. Details of the estimation procedure and the numerical methods used are provided in a Technical Appendix.

4.1 Exchange Rates and Monetary Fundamentals

A large literature in international finance has investigated the relationship between the nominal exchange rate and monetary fundamentals. This research focuses on the deviation, say u, of the nominal exchange rate from its fundamental value:

$$u_t = s_t - f_t, \tag{4.1}$$

where s denotes the log-level of the nominal bilateral exchange rate (the domestic price of the foreign currency); f is the long-run equilibrium of the nominal exchange rate determined by the monetary fundamentals; and t is a time subscript.

The fundamentals term is, most commonly, given by

$$f_t = (m_t - m_t^*) - \phi(y_t - y_t^*), \tag{4.2}$$

where m and y denote the log-levels of the money supply and income respectively; ϕ is a constant; and asterisks denote foreign variables. Here f may be thought of 'as a generic representation of the long-run equilibrium exchange rate implied by modern theories of exchange rate determination' (Mark and Sul, 2001, p. 32). For example, equation (4.2) is implied by the monetary approach to exchange rate determination (Frenkel, 1976; Mussa, 1976, 1979; Frenkel and Johnson, 1978) as well as by Lucas' (1982) equilibrium model and by several 'new open economy macroeconomic' models (Obstfeld and Rogoff, 1995, 2000; Lane, 2001). Hence, the link between monetary fundamentals and the nominal exchange rate is consistent with both traditional models of exchange rate determination based on aggregate functions as well as with more recent microfounded open economy models.

While it has been difficult to establish the empirical significance of the link between monetary fundamentals and the exchange rate due to a number of cumbersome econometric problems[4], some recent research suggests that the monetary fundamentals

[4] E.g., see Mark (1995), Berben and van Dijk (1998), Kilian (1999), Berkowitz and Giorgianni (2001).

described by equation (4.2) co-move in the long run with the nominal exchange rate and therefore determine its equilibrium level (Groen, 2000; Mark and Sul, 2001; Rapach and Wohar, 2002). This result implies that current deviations of the exchange rate from the equilibrium level determined by the monetary fundamentals induce future changes in the nominal exchange rate which tend to correct the deviations from long-run equilibrium, so that estimation of a regression of the form

$$\Delta_k s_{t+k} = \alpha + \beta u_t + \varepsilon_{t+k} \tag{4.3}$$

(where Δ_k denotes the k-difference operator) often produces statistically significant estimates of β (e.g., Mark, 1995; Mark and Sul, 2001). Indeed, equation (4.3) is the equation analyzed by a vast literature investigating the ability of monetary fundamentals to forecast the nominal exchange rate out of sample at least since Mark (1995)[5]. In this chapter, we follow this literature and use equation (4.3) in our empirical analysis, imposing the conventional restriction that $\phi = 1$ in the definition of f_t given by equation (4.2) (e.g., Mark, 1995; Taylor and Peel, 2000; Mark and Sul, 2001).

4.2 International Asset Allocation, Predictability and Parameter Uncertainty: Methodology

In this section we describe our framework for measuring the economic value of predictability of exchange rates, both with and without parameter uncertainty. Our work is related to and builds on the empirical finance literature that analyzes asset allocation in a Bayesian framework, including the work of Kandel and Stambaugh (1996) and

[5] See Mark (2001, Ch. 4) for a recent review of the relevant studies. Also, note that equation (4.3) implicitly assumes that deviations from long-run equilibrium are restored via movements in the exchange rate; however, it seems possible that they may be restored also via movements in the fundamentals. Notably, Engel and West (2002) show analytically that, in a stylized rational expectations present value model, the exchange rate follows a near random walk if fundamentals are nonstationary and the discount factor is close to unity. Under these conditions, therefore, the exchange rate is exogenous but an exchange rate-monetary fundamentals relationship may still exist where fundamentals bear the burden of adjustment towards long-run equilibrium.

Barberis (2000)[6]. We consider a utility-maximizing US investor who faces the problem of choosing how to invest in two assets that are identical in all respects except the currency of denomination. As a result we can focus on evaluating the economic and utility gains to an investor who relies on the monetary-fundamentals model to forecast exchange rates. Our benchmark is an investor who does not believe in predictability or, in other words, believes that the exchange rate follows a random walk - the benchmark used in the exchange rate literature since Meese and Rogoff (1983a,b). In our framework, the investor uses the forecasts from the model (either the fundamentals model or the random walk model) to construct strategies designed to decide how much of her wealth to invest in the domestic and foreign assets respectively.

We consider the following two cases. First, we study the problem of an investor who has to decide at time T how much of her wealth to invest in a nominally safe (or riskless) domestic bond and a foreign bond which is nominally safe in local currency over a time period $\widetilde{T}$ using a simple buy-and-hold strategy. Second, we allow our investor to optimally re-balance her portfolio at the end of every year over her investment horizon. Finally, for each of these two cases - buy-and-hold and dynamic rebalancing strategies - we consider both cases with and without parameter uncertainty in estimating the monetary-fundamentals model.

4.2.1 Buy-and-Hold Strategy

Consider first the problem of an investor who has to decide at time T how much of her wealth to invest in nominally safe domestic and foreign bonds respectively. These two bonds yield the continuously compounded returns r and r^* respectively, each expressed in local currency. The investor wishes to hold the portfolio for $\widetilde{T}$ periods.

The exchange rate may be modelled using a vector autoregression (VAR) of the following form (Campbell, 1991; Bekaert and Hodrick, 1992; Hodrick, 1992; Barberis,

[6] Lewis (1989) is an example of an early application of Bayesian techniques to the foreign exchange market. See also Lewis (1995).

2000; Campbell, Viceira and White, 2003):

$$z_t = a + Bx_{t-1} + \eta_t, \tag{4.4}$$

where $z_t' = (\Delta s_t, x_t')$, $x_t = (x_{1,t}, x_{2,t}, \ldots, x_{n,t})'$, and $\eta_t \sim iid(0, \Sigma)$.[7] The first component of z_t, namely Δs_t, is the change in the nominal exchange rate between period t and $t-1$. The remaining components of z_t consist of variables useful for predicting the change in the exchange rate, such as the deviation from the long-run equilibrium level of the exchange rate as measured by the monetary fundamentals (u_t as defined by equations (4.1)-(4.2)). Thus, the VAR (4.4) comprises a first equation which specifies the exchange rate change as a function of the predictor variables, while the other equations govern the stochastic evolution of the predictor or state variables.

In our empirical work, we implement the VAR (4.4) assuming a monetary fundamentals equation of the form (4.3) as the predictive regression and a first-order autoregressive process for the deviations from the fundamentals, u_t. This amounts to estimating a bivariate VAR with $z_t' = (\Delta s_t, u_t)$; a is a 2×1 vector of intercept terms; B is a 2×1 vector of parameters; the predictor variables vector comprises only one variable, namely the deviation from the fundamental exchange rate equilibrium level, i.e., $x_t = u_t$; and $\eta_t' = (\eta_{1t}, \eta_{2t})$ where η_{jt} is the error term of the jth equation in the VAR, for $j = 1, 2$. In the case of no predictability of the exchange rate, Δs_t equals a drift term plus a random error term.

Given initial wealth $W_T = 1$ and defining ω the allocation to the foreign bond, the end-of-horizon or end-of-period wealth is

$$W_{T+\widetilde{T}} = (1-\omega)\exp\left(r\widetilde{T}\right) + \omega\exp\left(r^*\widetilde{T} + \Delta_{\widetilde{T}} s_{T+\widetilde{T}}\right). \tag{4.5}$$

The investor's preferences over end-of-period wealth are governed by a constant relative

[7] We term the model in equation (4.4) a VAR to adhere to the standard terminology used in this literature (e.g., Kandel and Stambaugh, 1996; Barberis, 2000).

risk-aversion (CRRA) power utility function of the form

$$v(W) = \frac{W^{1-A}}{1-A}, \tag{4.6}$$

where A is the coefficient of risk aversion.

The investor's problem may then be written as follows:

$$\max_{\omega} E_T \left\{ \frac{\left[(1-\omega)\exp\left(r\widetilde{T}\right) + \omega \exp\left(r^*\widetilde{T} + \Delta_{\widetilde{T}} s_{T+\widetilde{T}}\right)\right]^{1-A}}{1-A} \right\}, \tag{4.7}$$

where the expectation operator $E_T(\cdot)$ reflects the fact that the investor calculates the expectation conditional on her information set at time T. A key issue in solving this problem relates to the distribution the investor uses in calculating this expectation, which depends both upon whether the exchange rate is predictable and on whether parameter uncertainty is taken into account.

To shed light on the impact of the predictability of exchange rates on portfolio decisions, we compare the allocation of an investor who ignores predictability to the allocation of an investor who takes it into account. This can easily be done by estimating the VAR model (4.4), with and without the deviations from fundamentals u_t, to obtain estimates of the parameters vector, say θ.[8] The model can be iterated forward with the parameters fixed at their estimated values. This gives a distribution of future exchange rates conditional on the estimated parameters vector, $p\left(\Delta_{\widetilde{T}} s_{T+\widetilde{T}} \mid \widehat{\theta}, z\right)$, where $z_t = (z_1, z_2, \ldots, z_T)'$ is the observed data up to the date when the investment begins. Thus, the investor's problem is

$$\max_{\omega} \int v\left(W_{T+\widetilde{T}}\right) p\left(\Delta_{\widetilde{T}} s_{T+\widetilde{T}} \mid \widehat{\theta}, z\right) d\Delta_{\widetilde{T}} s_{T+\widetilde{T}}. \tag{4.8}$$

In order to take into account parameter uncertainty, however, one can use the posterior distribution $p(\theta \mid z)$, which summarizes the uncertainty about the parameters given the data observed so far. Integrating over the posterior distribution, we obtain the predictive

[8] θ comprises a, B and the variance-covariance matrix of the error terms, say Σ.

distribution of exchange rate movements conditioned only on the data observed, not on the estimated parameters vector, $\widehat{\theta}$. Then the predictive distribution is

$$p\left(\Delta_{\widetilde{T}} s_{T+\widetilde{T}} \mid z\right) = \int p\left(\Delta_{\widetilde{T}} s_{T+\widetilde{T}} \mid \theta, z\right) p\left(\theta \mid z\right) d\theta, \tag{4.9}$$

which implies that the investor's problem under parameter uncertainty is

$$\begin{aligned} & \max_{\omega} \int v\left(W_{T+\widetilde{T}}\right) p\left(\Delta_{\widetilde{T}} s_{T+\widetilde{T}} \mid z\right) d\Delta_{\widetilde{T}} s_{T+\widetilde{T}} && (4.10) \\ = & \max_{\omega} \int v\left(W_{T+\widetilde{T}}\right) p\left(\Delta_{\widetilde{T}} s_{T+\widetilde{T}}, \theta \mid z\right) d\Delta_{\widetilde{T}} s_{T+\widetilde{T}} d\theta \\ = & \max_{\omega} \int v\left(W_{T+\widetilde{T}}\right) p\left(\Delta_{\widetilde{T}} s_{T+\widetilde{T}} \mid \theta, z\right) p\left(\theta \mid z\right) d\Delta_{\widetilde{T}} s_{T+\widetilde{T}} d\theta. && (4.11) \end{aligned}$$

Finally, given the optimal weights derived by the maximization problems (4.8) and (4.10), we can calculate the realized end-of-period wealth using the wealth function (4.5) for an investor who ignores parameter uncertainty - equation (4.8) - and an investor who recognizes it and takes it into account - equation (4.10). Given end-of-period wealth, we can then calculate also end-of-period utility of wealth using equation (6) and the certainty equivalent return[9] to measure the economic value of predictability.[10]

The maximization problems (4.8) and (4.10) are solved by calculating the integrals in these equations for values of $\omega = -100, -99, \ldots, 199, 200$ (in percentage terms), which essentially allows for short selling.[11] In our empirical analysis below, we report the value of ω that maximizes expected utility. The integrals are calculated by numerical methods, using 1,000,000 simulations in each experiment. In our case, the conditional distribution $p\left(\Delta_{\widetilde{T}} s_{T+\widetilde{T}} \mid \widehat{\theta}, z\right)$ is normal, so that the integral in (4.8) is approximated

[9] The certainty equivalent return (CER) can be defined as the return that, if earned with certainty, would provide the investor with the utility equal to the end-of-period utility calculated for a given allocation, $\overline{v}_{T+\widetilde{T}}$. In general, the CER can be obtained by solving the equation:

$$v\left[W_T\left(1 + CER\right)\right] = \overline{v}_{T+\widetilde{T}}$$

where W_T denotes wealth at time T and $v\left[\cdot\right]$ is the utility function in (4.6).

[10] See Section 4.5 for more details on these measures of economic value of predictability.

[11] Obviously no allowance for short selling would involve a weight ω between 0 and 100. Given the wide use of short selling in the foreign exchange market (e.g., Lyons, 2001) we allow ω to be defined between -100 and 200, which essentially allows for full proceeds of short sales and assumes no transactions costs.

by generating 1,000,000 independent draws from this normal distribution and averaging $v\left(W_{T+\widetilde{T}}\right)$ over all draws. For the maximization problem under parameter uncertainty, it is convenient to evaluate it in its reparameterized form (4.11) by sampling from the joint distribution $p\left(\Delta_{\widetilde{T}}s_{T+\widetilde{T}},\theta \mid z\right)$ - i.e., by first sampling from the posterior $p(\theta \mid z)$ and then from the conditional distribution $p\left(\Delta_{\widetilde{T}}s_{T+\widetilde{T}} \mid \theta, z\right)$ - and averaging $v\left(W_{T+\widetilde{T}}\right)$ over all draws[12].

4.2.2 Dynamic Rebalancing Strategy

We next consider an investor who optimally re-balances her portfolio at the end of every period using exchange rate forecasts based on the monetary-fundamentals model. We again analyze the optimal allocation both with and without parameter uncertainty. In this multi-period asset allocation problem, the optimal weights are now the solution to a dynamic programming problem that can be solved by discretizing the state space and using backward induction. We divide the investor's horizon starting at T and ending at $\widetilde{T}$ into K subperiods denoted by $[t_0, t_1], \ldots [t_{K-1}, t_K]$, where $t_0 = T$ and $t_K = T + \widetilde{T}$. Thus the investor now adjusts her portfolio K times over the investment horizon by changing ω, the allocation to the foreign bond, at the end of each sub-period. To simplify the notation we denote by W_k the quantity W_{t_k}, the investor's wealth at time t_k. The investor's problem now is

$$\max_{t_0} E_{t_0}\left(\frac{W_K^{1-A}}{1-A}\right), \tag{4.12}$$

where the investor maximizes over all remaining decisions from t_0 onwards. The law of motion of her wealth is given by

$$W_{k+1} = W_k\left\{(1-\omega_k)\exp\left(r\frac{\widetilde{T}}{K}\right) + \omega_k \exp\left(r^*\frac{\widetilde{T}}{K} + \Delta_{k+1}s_{k+1}\right)\right\}. \tag{4.13}$$

[12] For further details on the estimation procedure and the numerical methods used see the Technical Appendix A.

We can then define the indirect utility of wealth as

$$J\left(W_k, x_k, t_k\right) = \max_{t_k} E_{t_k}\left(\frac{W_K^{1-A}}{1-A}\right), \tag{4.14}$$

where the maximization is over all remaining decisions from t_k on. This can be written, using an induction argument, as

$$J\left(W_k, x_k, t_k\right) = \frac{W_k^{1-A}}{1-A} Q\left(x_k, t_k\right) \tag{4.15}$$

when $A \neq 1$. Accordingly, the Bellman equation is

$$Q\left(x_k, t_k\right) = \max_{\omega_k} E_{t_k}\left\{\left[\left(1-\omega_k\right)\exp\left(r\frac{\widetilde{T}}{K}\right) + \omega_k \exp\left(r^*\frac{\widetilde{T}}{K} + \Delta_{k+1}s_{k+1}\right)\right]^{1-A} \times Q\left(x_{k+1}, t_{k+1}\right)\right\}. \tag{4.16}$$

We first consider the case without parameter uncertainty. Here the expectation in equation (4.16) is evaluated conditional on fixed parameter values based on the posterior mean. When we allow for parameter uncertainty there are two main differences compared to the case with no parameter uncertainty. The first is that the expectation in the value function is now taken over the predictive distribution which incorporates parameter uncertainty. The second is that, in this multi-period case, parameter uncertainty may change over time and the investor updates her posterior distribution for the parameters. Thus, in addition to the hedging demand arising from the stochastic investment opportunity set (see Merton, 1973; Karolyi and Stulz, 2002), there may be an additional source of hedging demand arising from changes in the investor's beliefs about the model parameters over time.

Evaluating the joint dynamics of the state variables as well as the parameters in the model is a non-trivial dynamic programming problem. It is useful therefore to make some reasonable simplifying assumptions so that this task is numerically tractable. The dimensionality of the problem is reduced by assuming that the investor's beliefs about the parameters of the model do not change from what they are at the beginning of the investment horizon (e.g., Barberis, 2000). In other words, these beliefs are summarized by the posterior distribution calculated conditional only on the data observed at the beginning of the investment horizon. We can thus still use equation (4.16) to calculate the value function, but the expectation is now evaluated over $p(\Delta_{k+1}s_{k+1}, x_{k+1} \mid x_k)$ rather than over $p(\Delta_{k+1}s_{k+1}, x_{k+1} \mid \theta, x_k)$. The investor constructs a sample from the predictive distribution by taking a large number of draws from the posterior $p(\theta \mid z_1, \ldots, z_T)$ - conditional only on data until the horizon start date - and then, for each set of parameters values drawn, makes a draw from $p(\Delta_{k+1}s_{k+1}, x_{k+1} \mid \theta, x_k)$.

We now turn to a description of our data set, to which we apply the procedure outlined above.

4.3 Data

Our data set comprises monthly observations on money supply and income (industrial production) for the US, Canada, Japan and the UK, and spot exchange rates for the Canadian dollar, Japanese yen and UK sterling *vis-à-vis* the US dollar. The sample period covers most of the recent floating exchange rate regime, from 1977M01 to 2000M12, and the start date of the sample was dictated by data availability. The data are taken from the International Monetary Fund's *International Financial Statistics* data base. We use the monthly industrial production index (line code 66) as a proxy for national income since gross domestic product (GDP) is available only at the quarterly frequency.[13] Our measure of money is defined as the sum of money (line code 34) and quasi-money (line code 35) for the US, Canada, Japan, while for the UK we use M0. We deseasonalize the money and industrial production indices, following Mark and Sul (2001). The exchange rate is the end-of-month nominal bilateral exchange rate (line code AE). Our choice of countries reflects our intention to examine exchange rate data for major industrialized economies belonging to the G7 that have been governed by a pure float

[13] Note that a preliminary analysis of the statistical properties of the (quarterly) industrial production indices and GDP time series over the sample period and across the countries examined in this paper produced a coefficient of correlation higher than 0.95.

over the sample[14]. As a proxy for the nominally safe (riskfree) domestic and foreign bonds, we use end-of-month Euro-market bid rates with one month maturity for each of the US, Canada, Japan and the UK, provided by the Bank for International Settlements (BIS).

The data were transformed in natural logarithms prior to beginning the empirical analysis to yield time series for s_t, m_t, m_t^*, y_t and y_t^*. The monetary fundamentals series, f_t, was constructed with these data in logarithmic form according to equation (2) with $\phi = 1$; and s_t is taken as the logarithm of the domestic price of the foreign currency, with the US denoting the domestic country. In our empirical work, we use the data over the period January 1977-December 1990 for estimation, and reserve the remaining data for the out-of-sample forecasting exercise.[15] In addition, the domestic and foreign interest rates are treated as constant and set equal to their historical mean.

4.4 International Asset Allocation, Predictability and Parameter Uncertainty: Empirical Results

We now report our empirical results based on solving the maximization problems (4.8) and (4.10), which allow us to study the implications for portfolio weights when the exchange rate is either a random walk or predictable respectively. In each case our investor uses two different investment strategies. The first is a simple static buy-and-hold strategy, where the investor chooses the optimal weight to the foreign asset and does not change it until the end of the investment (forecast) horizon. The second is

[14]Note that, while Canada and Japan have experienced a free float since the collapse of the Bretton Woods system in the early 1970s, the UK was in the Exchange Rate Mechanism (ERM) of the European Monetary System (EMS) for about two years in the early 1990s. However, given the short length of this period, we consider sterling as a freely floating exchange rate in this paper. The remaining three G7 countries not investigated here, namely Germany, France and Italy, have all been part of the ERM for most of the sample period under investigation and in fact joined the European Monetary Union on 1 January 1999, when the euro replaced the national currencies of these three countries.

[15]It should be noted that the original Meese-Rogoff studies considered forecast horizons of up to 12 quarters ahead, while Mark (1995), for example, uses a maximum horizon of 16 quarters. In general, most studies in this literature have focused on horizons of up to 4 years ahead and therefore the forecast horizon considered in this paper is - to the best of our knowledge - the longest horizon considered in the relevant exchange rate literature to date.

a dynamic strategy where the investor optimally rebalances her portfolio at the end of each rebalancing period. We report results for four cases: random walk exchange rate and predictable exchange rate, and in each case both with and without parameter uncertainty. We begin by describing the case of a buy-and-hold investor in the next sub-section.[16]

4.4.1 Buy-and-Hold Strategy

As described in Section 4.2.1, a buy-and-hold investor with an horizon $\widetilde{T} = 1, ..., 10$ solves the problem in equation (4.8). Using a recursive Monte Carlo sampling procedure, we obtain an accurate representation of the posterior distributions of the estimated vector of parameters θ. Using data till December 1990, we estimate the posterior distribution of the parameters for all countries by drawing samples of size $1,000,000$. From these estimated distributions, we obtain out-of-sample forecasts for the investment horizon $\widetilde{T} = 1, ..., 10$ years when the investor takes into account parameter uncertainty and when she ignores it.

Figures 4.1-4.3 (which refer to the Canadian dollar, the Japanese yen and the UK sterling respectively) show the optimal weight ω (in percentage terms) allocated by a US investor to the foreign asset on the vertical axis, and the investment horizon (in years) on the horizontal axis. For each exchange rate, we show optimal weights for four different values of the coefficient of risk aversion, A, ranging from 2 to 20. The dotted and solid lines correspond to the case where the investor relies on the fundamentals model (predictability) with and without parameter uncertainty respectively. The dot-dash and dash lines refer to the cases where the investor uses a random walk model (no

[16] Preliminary estimation of the VAR model in equation (4.4) produced results consistent with a vast literature in this context (see Mark, 1995). Specifically, we find significant estimates of all parameters, with the parameter associated with the deviations from the fundamentals u_t being negative and very small in magnitude, suggesting slow adjustment of the exchange rate towards its equilibrium level. Also, the estimated AR(1) parameter on u_t is positive and quite large in magnitude, albeit clearly lower than unity, suggesting that u_t is stationary but persistent.

predictability) with and without parameter uncertainty respectively.[17]

It is important to note one point about the variability that would be attached to the estimate of ω obtained using this procedure. Barberis (2000) provides a detailed discussion of this issue and shows that, given the sample size used in the simulated draws (1, 000, 000), there is no significant variation in the estimate of ω. In other words, for this number of draws, the law of large numbers holds, resulting in a vanishing small variance of ω. As a result, we assume that we have converged to the optimal portfolio weight ω that would have been obtained if we could perform the integrations exactly (see Barberis, 2000, Appendix, for further details). Hence, in our empirical results, we do not report confidence intervals for ω given that its variability is 'virtually' zero for our number of draws.

The graphs show several interesting features that are common to all three exchange rates examined. We begin with an analysis of the case where the investor uses a random walk model (dash and dot-dash lines in Figures 4.1-4.3), which suggests the following results. First, if the investor does not account for parameter uncertainty (dash line in each of Figures 4.1 to 4.3), the optimal asset allocation does not vary with the investment horizon. This is consistent with studies on stock market data (Barberis, 2000) and may be seen as simply validating Samuelson's (1969) result that, under power utility, if asset prices follow a random walk then the optimal investment in the risky asset is constant regardless of the investment horizon.[18] Second, regardless of whether parameter uncertainty is accounted for, the optimal weight to the foreign bond, ω is lower (in absolute value) for higher levels of risk aversion, A (dash and dot-dash lines in Figures 4.1-4.3). Third, if the investor takes into account parameter uncertainty, we find that for low values of the coefficient of risk aversion (say $A = 2$), the optimal weight is virtually identical to the optimal weight obtained when parameter uncertainty is not accounted for

[17]Note that, within each figure, the graphs use different scales for clarity.

[18]Note, however, that Samuelson's result was obtained for an investor applying a rebalancing strategy, rather than a buy-and-hold strategy.

(i.e., dot-dash and dash lines are virtually identical). This suggests that, for low levels of risk aversion, parameter uncertainty does not influence asset allocation for our data and sample period. Fourth, for moderate to high values of the coefficient of risk aversion (say $A = 5, 10, 20$), if the investor takes into account parameter uncertainty (dot-dash line), we find a different optimal allocation across horizons: specifically, the absolute value of the initial optimal allocation to the foreign asset generally decreases with the length of the investment horizon. These results suggest that, under no predictability, parameter uncertainty matters more for optimal asset allocation the higher the coefficient of risk aversion and the longer the investment horizon.

We now turn to the case where the investor relies on the monetary-fundamentals model (solid and dotted lines in Figures 4.1-4.3), where we present our results on the impact of parameter uncertainty in an order similar to that in the preceding paragraph for the case of no predictability. First, in the case without parameter uncertainty (solid line in each of Figures 4.1 to 4.3), the absolute value of the initial optimal allocation to the foreign asset increases with the investment horizon. This result suggests that, if the investor believes in predictability of the exchange rate, she will be more prone to invest in the foreign asset the longer the investment horizon. This result contrasts with the invariance of the optimal weight over the investment horizon under no predictability and may be explained as follows. Under no predictability, the mean and the variance of the exchange rate increase linearly over time and, as shown by Samuelson (1969) for stock prices, this implies identical optimal weights for all investment horizons. However, as noted by Barberis (2000, p. 243-5), under predictability the variance of the exchange rate *may* grow less than linearly over time, making the foreign asset look less risky at longer investment horizons, leading to a higher optimal weight at longer horizons[19]. Second, regardless of whether parameter uncertainty is accounted for, the optimal allocation to

[19] However, note that this result may not hold if learning is taken into account (Xia, 2001).

the foreign bond, ω is lower (in absolute value) for higher levels of risk aversion, A (solid and dotted lines in Figures 4.1-4.3), essentially replicating the result discussed above for the case of no predictability. Third, if the investor takes into account parameter uncertainty, we find that, for low values of the coefficient of risk aversion (say $A = 2$), the optimal allocation line across horizons is virtually identical to the optimal allocation line obtained when parameter uncertainty is not accounted for (i.e., solid and dotted lines are identical). Again, this is similar to the case of no predictability and suggests that, for low levels of risk aversion, parameter uncertainty does not matter for asset allocation, for the exchange rates and sample period examined. Fourth, for moderate to high values of the coefficient of risk aversion (say $A = 5, 10, 20$), if the investor takes into account parameter uncertainty (dotted line), this implies a different optimal allocation across horizons where the absolute value of the initial optimal allocation to the foreign asset is generally non-decreasing with the length of the investment horizon. This result replicates the finding under no parameter uncertainty in a qualitative, but not quantitative, way.

In addition, with regard to the effects of predictability versus no predictability in determining the optimal weights to the foreign asset, our results clearly indicate that the optimal weights may differ significantly in these two cases. Indeed, the difference can be so large as to imply optimal weights with different signs, as reported, for example, in the cases of Canada and Japan (Figures 4.1-4.2). For the UK, however, the sign of the optimal weight is the same under predictability and no predictability, but the difference in the two corresponding weights is still sizable for higher levels of risk aversion (Figure 4.3). In addition, it is instructive to note that this result holds, in a qualitative sense, regardless of whether parameter uncertainty is taken into account.

A final observation, based on these results, is that the absolute value of the initial optimal allocation to the foreign asset for short investment horizons (say one or two years) is very similar for all cases examined here as the coefficient of risk aversion increases -

regardless of whether the investor recognizes predictability and/or takes into account parameter uncertainty. Intuitively this suggests that for very high levels of risk aversion neither predictability nor parameter uncertainty matter particularly for asset allocation at short investment horizons.

Overall, our results show that both predictability and parameter uncertainty play an important role in the investor's choices for all countries and for different values of the coefficient of risk aversion. Specifically, predictability implies different optimal weights to the foreign asset compared to no predictability. The difference can be as large as to generate weights with a different sign - effectively meaning that when a fundamentals model implies a long (short) position in the foreign asset the random walk model may imply a short (long) position in the foreign asset. Parameter uncertainty induces the foreign asset allocation to fall (rise) as the horizon increases when the models predict positive (negative) weights assigned to the foreign asset.[20] Intuitively, this means that parameter uncertainty makes the allocation to the foreign asset look more risky than without parameter uncertainty. Across different countries (on average), parameter uncertainty changes the optimal weight to the foreign asset, relative to the case without parameter uncertainty, by 33% in the case of no predictability (14% in the case of predictability) for a coefficient of risk aversion $A = 5$, and 44% in the case of no predictability (41% in the case of predictability) for a coefficient of risk aversion $A = 20$.

4.4.2 Dynamic Rebalancing Strategy

We now examine the case where the investor optimally rebalances over her investment horizon, assuming a rebalancing period of one year. Again, we analyze the cases with and without parameter uncertainty. The problem faced by the investor is as detailed

[20]Put differently, when the models would suggest buying the foreign asset, parameter uncertainty (by increasing the variance associated with the out-of-sample prediction) reduces the percentage of wealth invested in the foreign asset. This reduction is generally larger the longer is the investment horizon. If the models predict that the foreign asset be short sold, parameter uncertainty works in the opposite direction, by reducing the percentage of foreign asset to be sold short.

in Section 4.2.2. To solve the Bellman equation (4.16), we discretize the state space by taking intervals ranging from three standard deviations below to three standard deviations above the historical mean of the deviation from the monetary fundamentals, u and dividing it into 25 equally spaced grid points. We draw a sample of size 1,000,000 from the distributions of exchange rate changes as in the static buy-and-hold strategy. The number of grid points selected and the large number of replications used should guarantee satisfactory accuracy of the results.

We depict graphically, in Figures 4.4-4.6, changes over different horizons and for varying coefficients of risk aversion in the patterns of holding of a US investor who optimizes her portfolio annually. Our results, reported in the left-hand panels of Figures 4.4-4.6, show optimal allocations for the investor when parameter uncertainty is ignored. The graphs in the right-hand panels show the optimal allocation when parameter uncertainty is taken into account. Each graph refers to a different level of risk aversion and, in each graph, the lines plotted correspond to a different initial value of the predictor variable. In particular, each graph reports asset allocations relative to an initial value equal to three and one standard deviations below the historical mean, three and one standard deviations above the historical mean, and the historical mean itself.

Our results show that, even if different initial values of the predictor variable (i.e., the deviation from the fundamental exchange rate equilibrium value) influence the magnitude of the allocation to the foreign asset, the optimal allocation under dynamic rebalancing is qualitatively similar to the allocation implied by the static buy-and-hold strategy. The differences, for different initial values, in the foreign asset allocation under dynamic rebalancing are more pronounced for lower levels of risk aversion. Further, as in the static buy-and-hold case, parameter uncertainty affects asset allocation in the same way; that is, it causes the foreign asset allocation to fall (rise) as the horizon increases when the models predict positive (negative) weights assigned to the foreign asset.[21]

[21] However, although the results are qualitatively similar, the effect driving them is not the same in

It is interesting to note that the higher the initial value of the predictor variable, the lower (higher) is the proportion of wealth invested in the foreign asset when the underlying model predicts a positive (negative) weight to the foreign asset. Intuitively, for example, a high initial value of the predictor variable means that there is a large positive departure of the nominal exchange rate from its fundamental value. This in turn implies that, in order to restore equilibrium, the nominal exchange rate will decrease in the future - in other words, it will appreciate. A future appreciation of the nominal exchange rate will of course induce the US investor to invest less in the foreign asset and more in the domestic asset.

We now turn to the core of our empirical work, a quantitative analysis of the economic value of exchange rate predictability.

4.5 The Out-of-Sample Economic Value of Predictability

This section reports estimates of the economic value of predictability. We begin by calculating end-of-period wealth, as defined in equation (4.5) and normalizing its initial value $W_T = 1$. In these calculations ω is obtained from the utility maximization problems (4.10) and (4.16) for the static and dynamic rebalancing cases respectively. In our context, the random walk model and the fundamentals model may be seen as reflecting two polar approaches to exchange rate forecasting. Specifically, an investor who assumes predictability (believes in the fundamentals model) considers the fundamentals approach as a perfect description of reality. An investor who believes in the random walk approach assumes, on the other hand, that there is no variable able to predict the exchange rate. The wealth calculations on the basis of which we compare the two models are obtained using realized or *ex post* data in equation (4.5).[22] We also calculate the realized end-

that the increase in allocation across horizons in the case of a rebalancing strategy is due to hedging demand effects, as first described by Merton (1973) and reported, for example, by Barberis (2000). See also Karolyi and Stulz (2002).

[22] Thus, given equation (4.5), the forecasts produced by each of the two models considered affect the end-of-period wealth only through the choice of the optimal weight ω.

of-period utilities, using equation (4.6), and the realized certainty equivalent returns in order to compare the out-of-sample performance of the two competing models on the basis of various measures of economic and utility gains.

A related question involves the *ex ante* performance of each of the random walk model and the fundamentals model. In this case, the evaluation of the performance of the models would be based on an *ex ante* or expected end-of-period wealth calculation, where the change in the exchange rate $\Delta s_{T+\widetilde{T}}$ is the forecast of the exchange rate implied by the model being considered rather than its realized value. This calculation would provide information on the returns and on the economic value that the investor would expect given the data and investment horizon and given her belief in a particular model. Clearly, while this exercise can be implemented out-of-sample, it implicitly assumes that the model which provides the forecasts is the true data generating process - that is, no *ex post* realized data are used. However, this is helpful as it provides an estimate of expected returns or economic value, which the investor may use in deciding whether, given her belief in the model, the investment in foreign exchange is worthwhile *ex ante*. It should be clear, on the other hand, that such *ex ante* calculation does not address the key question in this chapter, which is about the out-of-sample forecasting ability of the fundamentals model relative to a random walk model. A pure out-of-sample comparison designed to evaluate the ability of a model to match the realized data can only be done by comparing the outcome from the model-based forecasts to the *ex post* data, which is the approach we follow in this chapter, in line with the literature on exchange rate forecasting.

We now turn to the core of the results in this section, which relates to the calculation of the *ex post* end-of-period wealth in each of our four cases (predictability and no predictability under each of parameter uncertainty and no parameter uncertainty) for both buy-and-hold and dynamic rebalancing strategies. We define the following measures of economic gain (loss): (i) the wealth ratio as the ratio of the end-of-period wealth from using the fundamentals model to the end-of-period wealth from using a random walk; (ii) the utility ratio as the ratio of the end-of-period utility from the fundamentals model to the end-of-period utility from using a random walk; (iii) the differences in certainty equivalent returns (CERs) as the annualized differences between the CER calculated from the utility from the fundamentals model and the CER corresponding to the utility using a random walk. It is important to emphasize that none of these measures of economic value has a standard error since they are based on a pure ex post out-of-sample evaluation which relies on the calculation of the end-of-period wealth given in equation (4.5) at time $\widetilde{T}$.[23]

Note that the end-of-period wealth is calculated on the basis of interest rates which are known (r and r^*), a realized value of the change in the exchange rate at time $\widetilde{T}$, and the value of ω implied by a particular investment strategy, risk aversion parameter and model. Hence, given that ω has a variance that may be regarded as 'virtually' zero for our number of draws, the end-of-period wealth obtained using equation (4.5) does not have an associated variance. As a result, our empirical results allow us to compare the ex post economic value across different models and investment strategies without having to test for statistical significance of the difference between different end-of-period wealths. Put differently, this means that in our framework if the results suggest that

[23] Although, as explained above, this is not directly relevant to the question addressed in this paper, as a preliminary exercise we also carry out the analysis on an *ex ante* basis. In particular, for each of static and dynamic strategies, we calculate the *ex ante* end-of-period wealth to verify that it is consistent with an *ex ante* economic value which would validate the belief of the investor (either in the random walk or the fundamentals model). In each case, the *ex ante* calculations indicate sizable increases in the end-of-period wealth up to ten years ahead. Indeed, the *ex ante* returns and measures of economic value are larger than their *ex post* corresponding measures we report later in the paper, especially for longer investment horizons. One advantage of the *ex ante* calculations is that it is possible to obtain a measure of the uncertainty surrounding the expected end-of-period wealth because the calculation is based on forecasts for exchange rates, obtained by drawing 1,000,000 times from the predictive distribution of exchange rates. This allows us to recover the distribution of the end-of-period wealth and hence to assign confidence intervals. Our general result is that, for each of the random walk model and the fundamentals model and for each of the two strategies employed here, the expected end-of-period wealth is not only large but also strongly statistically significant, which implies that any investor believing in either the random walk model or the fundamentals model would carry out the investment.

one strategy/model yields higher ex post returns than an alternative strategy/model, this implies the first strategy/model has greater economic value than the competing one, given the investment (forecast) horizon and sample period utilized.

In our discussion of the empirical results in this section, we focus mainly on end-of-period wealth and wealth ratios, since, as briefly reviewed below, the results from using the other measures of economic value of predictability (utility ratios and differences in certainty equivalent returns) are qualitatively identical. In Tables 4.1-4.6 we report our results from calculating the measures of economic gain (loss) defined above.

4.5.1 Buy-and-Hold Strategy

We first analyze the case of a buy-and-hold US investor and compute the end-of-period wealth for our investor over the period January 1991-December 2000 for each of the Canadian dollar, Japanese yen and UK sterling. The results for this case, reported in Tables 4.1-4.3, show the economic values and gains for different investment horizons $\widetilde{T} = 1, \ldots, 10$ and for different coefficients of risk aversion ($A = 2, 5, 10, 20$). For a given coefficient of risk aversion, Tables 4.1-4.3 report the end-of-period wealth both without and with parameter uncertainty (p.u.). The figures in parentheses, brackets and braces denote the wealth ratios, utility ratios and differences in CERs respectively, as defined above. Our results show that predictability using monetary fundamentals is, in general, of incremental economic value above that for a random walk specification. For example, for a less risk averse investor ($A = 2$), in the case of Canada, the wealth ratio is greater than unity at all horizons longer than one year, indicating that at all horizons longer than one year the end-of-period wealth achieved from using the fundamentals model is higher than the end-of-period wealth attained from using a random walk. Even allowing for parameter uncertainty, this still remains the case. For $A = 5, 10, 20$, the fundamentals model outperforms the random walk for all horizons except for 1 year. In the case of Japan, for $A = 2$ the end-of-period wealth under predictability is much higher than that for a naive no-change investor: the wealth ratio ranges from a low of 1.08 at the one-year horizon to a high of 1.60 at the ten-year horizon. The effects of predictability are dramatically reduced for a very risk averse investor ($A = 20$), with a wealth ratio ranging from 1.01 at the one-year horizon to a high of 1.05 at the ten-year horizon. For the UK, however, the use of predictability does not seem to be economically important for $A = 2$, although for more risk averse investors there is some gain from using the monetary fundamentals model compared with using a naive random walk model at medium to long horizons.

It is interesting to note that, in general, our results are not very sensitive to the length of the investment horizon for a low level of risk aversion. The results in Tables 4.1-4.3 also show that it is mainly at horizons longer than one year that monetary fundamentals predict future nominal exchange rates better than a naive random walk. However, we find that the wealth ratio is often greater than unity even for relatively short horizons such as $\widetilde{T} = 2$ and occasionally even for $\widetilde{T} = 1$. This is in sharp contrast with the conventional wisdom that monetary fundamentals can forecast the exchange rate only at horizons as long as 4 or 5 years ahead.[24] In the case of investors with greater risk aversion ($A = 20$), the results are qualitatively similar. We also find that allowing for parameter uncertainty at higher levels of risk aversion results in a lower relative wealth ratio. This indicates that the effect of parameter uncertainty at higher levels of risk aversion (in terms of reducing the absolute value of the optimal weight relative to the case without parameter uncertainty) is generally greater for the case of predictability than for the case of the random walk model.

However, note that, while wealth increases monotonically with the investment horizon both under predictability and no predictability, the wealth ratio measuring the gain

[24] As pointed out by Lyons (2002) : "The [...] puzzle [that macro variables cannot account for exchange rates empirically] does indeed remain unresolved. (Read 'exchange rates' as referring to major floating rates against the U.S. dollar and 'account for' as referring to horizons less than two years)." Note that the sentence in parentheses is in the original text.

from using the fundamentals model does not increase monotonically over the investment horizon. For example, for each of Canada and Japan, the wealth ratio drops at $\widetilde{T}$ equal to 5 or 6, while increasing again afterwards. Hence, while the wealth ratio always increases in period 10 as compared with period 1, its increase over the investment horizon is not monotonic. Nevertheless, it is notable that the return at the end of the 10-year investment horizon from employing a fundamentals model is relatively large, at least 120, 102 and 137 percent for Canada, Japan and the UK respectively.

Overall, these results provide evidence of economic value to exchange rate predictability across countries and for a range of values of the coefficient of risk aversion. This is clear from the fact that the end-of-period wealth achieved by the investor who assumes that the exchange rate is predictable is higher than that obtained by the investor who assumes that the exchange rate follows a random walk. The order of magnitude varies across countries and with the coefficient of risk aversion. In particular, we find that the difference between end-of-period wealth under predictable and unpredictable exchange rate changes is lower for higher levels of risk aversion. However, taken together, the results that the wealth ratio increases non-monotonically and that the return from employing a fundamentals model is large imply that the return from a random walk is also large in terms of economic value. This confirms the stylized fact that a random walk model is a very difficult benchmark to beat, even when the assessment of its predictive power is based on economic criteria.[25]

[25] Indeed, an extreme case is the UK for $A = 2$ (Table 4.3), where we report a wealth ratio of unity over the whole investment horizon. This is of course due to the fact that the optimal weights are the same under each of predictability and no predictability in this case (see top-left graph in Figure 4.3). Generally, although for the UK we record high returns in absolute terms from assuming predictability, these returns are not much larger than the returns obtained using a random walk specification. This result seems consistent with the difficulty to forecast the UK sterling during the 1990s often recorded in the literature even in studies where time-series models are found to beat a random walk (see Chapter 2).

4.5.2 Dynamic Rebalancing Strategy

We now turn to the forecasting results for an investor who uses a dynamic rebalancing strategy. Tables 4.4-4.6 report the end-of-period wealth (and the relevant wealth ratio, the utility ratio and the difference in CERs) for a US investor who dynamically rebalances her portfolio annually over an investment horizon of ten years. These results are obtained from solving the Bellman equation (4.16) by discretizing the state space and using backward induction. We take intervals ranging from three standard deviations ($\pm 3\sigma_u$) above and below the historical mean ($\mu(u)$) of the predictor variable, the deviation from the monetary fundamentals u. Intuitively, larger intervals for u imply the possibility of larger misalignments of the nominal exchange rate from its fundamental value. We report the expected end-of-period wealth calculated for five initial values of the predictor variable ranging from $-3\sigma_u$ to $+3\sigma_u$ at the end of the 10th year for different values of A. In the last column of Tables 4.4-4.6, we report for comparison the end-of-period wealth obtained under a static buy-and-hold strategy as well as the relevant wealth ratio.

The results in Tables 4.4-4.6 confirm, in general, the predictive ability of the monetary fundamentals model, as measured in terms of economic value. Except for Japan, where the random walk outperforms the monetary fundamentals framework for large negative initial values of u, the wealth and utility ratios recorded are almost always larger than unity, which is corroborated by the generally positive differences in the CERs, suggesting a higher CER for the fundamentals model. The results for the UK display virtually no change in end-of-period wealth and relative ratios for all values of A other than 20. This is not surprising given that the optimal weights from which these wealth calculations are derived do not show much variability over investment horizons and across lower levels of risk aversion (see Figure 4.6). The results in the last column of Table 4.4-4.6 clearly show that a static buy-and-hold strategy that recognizes predictability leads to the largest end-of-period wealth relative to all other strategies considered here for a forecast horizon equal to 10 years. Also, in general, a dynamic rebalancing strategy leads to worse outcomes relative to a static buy-and-hold strategy for a forecast horizon of 10 years.

At first glance, one might argue that this result is puzzling since it is always possible for the dynamic strategy to mimic the static strategy. In essence, the two strategies have the same weight at the end of the investment horizon $T + \widetilde{T}$. However, while the static strategy results in the same weight throughout the investment horizon, the dynamic strategy chooses weights by backward induction from time $T + \widetilde{T}$ to time $T + 1$; the weight is adjusted depending on the predicted path of the exchange rate between time T and $T + \widetilde{T}$ according to the Bellman equation (4.16). Therefore, in the dynamic strategy, maximization of expected utility occurs on the basis of the period-by-period predictive distributions of the exchange rate, whereas the static strategy maximizes expected utility on the basis of the $\widetilde{T}$-period predictive distribution of the exchange rate. This implies that, *ex ante*, when one knows or assumes the true data generating process of the exchange rate (and hence its distribution is known), the investor would always prefer the dynamic strategy to the static one[26]. However, this is not necessarily true *ex post* in finite sample. In our *ex post* evaluation over the sample period and exchange rates examined, the dynamic strategy performs worse than the static one. This suggests that, while the exchange rate forecasts at long horizons are accurate, as indicated by the evidence that the fundamentals model beats a random walk model *for both dynamic and static strategies*, the predicted dynamic adjustment path of the exchange rate towards its forecast at the end of the horizon $T + \widetilde{T}$ may be poor. This is not surprising since the model used for forecasting exchange rates with fundamentals is a classic long-horizon regression which does not attempt to model the short run dynamics. Clearly, a richer specification of the short-run exchange rate dynamics in our empirical model might well

[26] As a special case, note that dynamic and static strategies will imply identical weights ω only if the investor assumes a random walk for the exchange rate *and* does not take into account parameter uncertainty. In this case the weights do not change with the investment horizon (Samuelson, 1969).

yield the result that the dynamic strategy makes the investor better off relative to a static strategy. To sum up, what we take from the result that *ex post* the dynamic strategy performs worse than the static strategy on our data set is that if one uses a long-horizon regression out of sample the gain from using a dynamic strategy rather than a static one is not obvious.[27]

It is important to note that the results discussed above for end-of-period wealth and wealth ratios do not change qualitatively when looking at utility ratios and differences in CERs. In general, the utility ratios, reported in brackets in Tables 4.1-4.6, confirm that the investor using the fundamentals model enjoys higher utility than the investor using a random walk model. The gains increase, albeit non-monotonically, with the investment horizon, with a pattern that resembles the pattern of the wealth ratios. Finally, the differences in the CERs, reported in braces in Tables 4.1-4.6, indicate the certain return that would equate the end-of-period utility of the two investors. Our results show that the differences in CERs are almost always positive, suggesting that the end-of-period utility of the investor using a fundamentals model is generally higher than the end-of-period utility for the random-walk investor. Indeed, the positive differences in CERs can also be quite large in magnitude, suggesting that the difference in the utilities obtained under no predictability and predictability can be quite substantial.

[27] Also, our result might be due to our choice of the rebalancing period, which is assumed to be one year. This may be suboptimal in light of the evidence that fundamentals are *most* powerful at predicting the exchange rate in the medium to long run, say 3 or 4 years (e.g., Mark, 1995). In principle, one would expect that the optimal rebalancing period is a function of the speed at which the exchange rate change adjusts to restore deviations of the exchange rate from its fundamental value in a way that the rebalancing is carried out over the horizon where the predictive power of the deviations from fundamentals is at its peak. Given the large amount of evidence in the literature (e.g., Mark, 1995; Mark and Sul, 2001) and in this paper that the predictive power of monetary fundamentals is higher at medium to longer horizons (albeit still being potentially substantial at shorter horizons) one would expect the optimal dynamic rebalancing period to be somewhat longer than one year. Rules of selection of the optimal rebalancing period are not investigated in this paper, but we consider this issue as an immediate avenue for future research.

4.5.3 Summing up the Forecasting Results

In general, our results provide evidence that there is economic value to predictability at various forecast horizons - which also include relatively short horizons - for a range of coefficients of risk aversion, regardless of whether the investment strategy is static or dynamic and whether parameter uncertainty is taken into account. However, the gain from assuming predictability appears to vary somewhat across currencies and increases non-monotonically over the 10-year investment horizon considered here. We find that the gain from using a fundamentals model is positively related to the investment horizon, negatively related to the level of risk aversion, and negatively related to parameter uncertainty. Of course, the results are based on a particular sample period for estimation and for out-of-sample prediction, so that our claims are subject to the caveat that they are sample specific. Nevertheless, for the sample period investigated, the evidence we present suggests that an investor using a fundamentals model in 1990 to take positions in domestic and foreign bonds would have been better off than an investor using a random walk model. Overall, these results may be viewed as suggesting that the case against the predictive power of monetary-fundamentals models may be overstated.

4.6 Conclusion

Meese and Rogoff (1983a,b, 1988) first noted that standard structural exchange rate models are unable to outperform a naive random walk model in out-of-sample exchange rate forecasting, even with the aid of *ex post* data. Despite the increasing sophistication of econometric techniques employed and quality of the data sets utilized, the original results highlighted by Meese and Rogoff continue to present a challenge and constitute a component of what Obstfeld and Rogoff (2000) have recently termed as the 'exchange rate disconnect puzzle'.

Prior research in this area has largely relied on statistical measures of forecast accuracy. Our study departs from this in that we focus instead on the metric of economic value to an investor in order to assess the performance of fundamentals models. This is particularly important given the several cumbersome econometric issues that plague statistical inference in this literature. Our study provides the first evidence on the economic value of the exchange rate forecasts provided by an exchange rate-monetary fundamentals framework. Specifically, we compare the economic value, to a utility maximizing investor, of out-of-sample exchange rate forecasts using a monetary-fundamentals model with the economic value under a naive random walk model. We assume that our investor faces the problem of choosing how much she will invest in two assets that are identical in all respects except the currency of denomination. This problem is studied in a Bayesian framework that explicitly allows for parameter uncertainty.

Our main findings are as follows. First, each of predictability and parameter uncertainty substantially affect, both quantitatively and qualitatively, the choice between domestic and foreign assets for all currencies and across different levels of risk aversion. Specifically, exchange rate predictability (characterized using the monetary-fundamentals model) can yield optimal weights to the foreign asset that may be very different (in magnitude and, sometimes, in sign) from the optimal weights obtained under a random walk model. Parameter uncertainty causes the foreign asset allocation to fall (rise) as the horizon increases when the models predict positive (negative) weights assigned to the foreign asset, effectively making the foreign asset look more risky. Second, and more importantly, our results lend some support for the predictive ability of the exchange rate-monetary fundamentals model. This finding holds for the three major exchange rates examined in this chapter using data for the modern floating exchange rate regime. The gain from using the information in fundamentals in order to predict the exchange rate out of sample (as opposed to assuming that the exchange rate follows a random walk) is often substantial, although it varies somewhat across countries. We find that the gain from using a fundamentals model is, in general, positively related to the investment horizon, negatively related to the level of risk aversion, and negatively related to parameter uncertainty. In turn, these findings suggest that the case against the predictive power of monetary-fundamentals models may be overstated.

There are a number of ways in which this study could be extended. First, one obvious concern is that our results may be sample specific. Our choice of exchange rates and sample period reflects our intention to focus on freely floating exchange rates over the post-Bretton Woods period and follows much previous research in the literature on exchange rate forecasting. Testing the robustness of our findings using other exchange rate data and/or sample periods is a logical extension. Second, we consider here a simple case where the investor allocates wealth between two assets; a more realistic scenario would be to allow for multiple assets. However, while this will require more complex estimation techniques, it would also take us away from the main point of this chapter, which is to draw attention to the economic value of forecasting fundamentals models rather than only on the use of statistical metrics for forecast comparison. Third, we use a simple power utility set up to illustrate our main point. However, in the context of an international investor, the use of other utility functions, such as those that allow for ambiguity aversion or habit formation, may also be of great interest.

Table 4.1: The economic value of predictability. Static buy-and-hold strategy: Canada

$\tilde{T}=$	1	2	3	4	5	6	7	8	9	10
					$A=2$					
without p.u.	1.0844	1.2167	1.3241	1.4497	1.5320	1.6441	1.8010	1.9798	2.0628	2.2501
	(0.96)	(1.04)	(1.05)	(1.09)	(1.02)	(1.01)	(1.06)	(1.14)	(1.05)	(1.10)
	[0.96]	[1.03]	[1.04]	[1.08]	[1.01]	[1.00]	[1.05]	[1.12]	[1.04]	[1.09]
	{-0.047}	{0.022}	{0.021}	{0.031}	{0.005}	{0.001}	{0.015}	{0.031}	{0.011}	{0.020}
with p.u.	1.0844	1.2167	1.3241	1.4497	1.5320	1.6441	1.8010	1.9798	2.0628	2.2501
	(0.96)	(1.04)	(1.05)	(1.09)	(1.02)	(1.01)	(1.06)	(1.14)	(1.05)	(1.10)
	[0.95]	[1.03]	[1.04]	[1.08]	[1.01]	[1.00]	[1.05]	[1.12]	[1.04]	[1.09]
	{-0.047}	{0.022}	{0.021}	{0.031}	{0.005}	{0.001}	{0.015}	{0.031}	{0.011}	{0.020}
					$A=5$					
without p.u.	1.0875	1.2167	1.3241	1.4497	1.5320	1.6441	1.8010	1.9798	2.0628	2.2501
	(0.96)	(1.04)	(1.05)	(1.09)	(1.02)	(1.01)	(1.06)	(1.14)	(1.05)	(1.10)
	[0.82]	[1.13]	[1.18]	[1.30]	[1.07]	[1.02]	[1.21]	[1.42]	[1.18]	[1.31]
	{-0.044}	{0.022}	{0.021}	{0.031}	{0.005}	{0.001}	{0.015}	{0.031}	{0.011}	{0.020}
with p.u.	1.0973	1.2092	1.3163	1.4377	1.5307	1.6437	1.7981	1.9738	2.0598	2.2432
	(0.97)	(1.03)	(1.04)	(1.08)	(1.02)	(1.01)	(1.06)	(1.14)	(1.05)	(1.10)
	[0.86]	[1.11]	[1.16]	[1.27]	[1.06]	[1.02]	[1.21]	[1.41]	[1.17]	[1.31]
	{-0.035}	{0.018}	{0.018}	{0.028}	{0.005}	{0.001}	{0.014}	{0.030}	{0.010}	{0.019}
					$A=10$					
without p.u.	1.0941	1.2145	1.3241	1.4497	1.5320	1.6441	1.8010	1.9798	2.0628	2.2501
	(0.97)	(1.04)	(1.05)	(1.09)	(1.02)	(1.01)	(1.06)	(1.14)	(1.05)	(1.10)
	[0.63]	[1.27]	[1.36]	[1.55]	[1.15]	[1.05]	[1.42]	[1.70]	[1.36]	[1.57]
	{-0.038}	{0.021}	{0.021}	{0.031}	{0.005}	{0.001}	{0.015}	{0.031}	{0.011}	{0.020}
with p.u.	1.0961	1.2065	1.3116	1.4260	1.5270	1.6423	1.7834	1.9416	2.0488	2.2225
	(0.97)	(1.03)	(1.04)	(1.08)	(1.02)	(1.01)	(1.05)	(1.12)	(1.04)	(1.09)
	[0.66]	[1.23]	[1.30]	[1.48]	[1.12]	[1.04]	[1.37]	[1.65]	[1.31]	[1.52]
	{-0.036}	{0.017}	{0.017}	{0.025}	{0.004}	{0.001}	{0.012}	{0.026}	{0.009}	{0.017}
					$A=20$					
without p.u.	1.0965	1.2091	1.3183	1.4481	1.5320	1.6441	1.8010	1.9798	2.0628	2.2501
	(0.98)	(1.02)	(1.03)	(1.07)	(1.01)	(1.00)	(1.05)	(1.11)	(1.04)	(1.07)
	[0.50]	[1.33]	[1.45]	[1.70]	[1.22]	[1.08]	[1.57]	[1.85]	[1.50]	[1.73]
	{-0.023}	{0.012}	{0.013}	{0.022}	{0.004}	{0.001}	{0.011}	{0.023}	{0.008}	{0.015}
with p.u.	1.0981	1.2035	1.3058	1.4148	1.5247	1.6415	1.7744	1.9187	2.0391	2.2018
	(0.98)	(1.02)	(1.02)	(1.03)	(1.01)	(1.00)	(1.02)	(1.03)	(1.01)	(1.02)
	[0.59]	[1.24]	[1.30]	[1.45]	[1.10]	[1.03]	[1.24]	[1.42]	[1.16]	[1.24]
	{-0.019}	{0.008}	{0.008}	{0.011}	{0.001}	{0.001}	{0.003}	{0.006}	{0.002}	{0.003}

Notes: These figures refer to the end-of-period (equal to 10 years) economic value, as measured by wealth levels, wealth ratios, utility ratios and certainty equivalent returns for the case of an investor acting on the basis of the static buy-and-hold strategy. Initial wealth is assumed to be equal to unity. A is the coefficient of risk aversion in the CRRA utility function defined by equation (4.6). $\tilde{T}$ is the investment horizon in years. 'With p.u.' and 'without p.u.' denote the case where the investor takes into account parameter uncertainty (p.u.) and the case where she ignores it respectively. Under each of these cases, the first row reports the end-of-period wealth calculated using the definition given by equation (4.5). Values in parentheses in the second row, for each of the two cases with and without p.u., are ratios of the end-of-period wealth levels obtained in the case of predictability to the end-of-period wealth levels obtained under a random walk exchange rate. Values in brackets in the third row are ratios of the end-of-period utility levels obtained in the case of predictability (with and without p.u.) to the end-of-period utility levels obtained under a random walk exchange rate model (with and without p.u.). Values in braces in the fourth row are differences of the end-of-period certainty equivalent return (CER) obtained in the case of predictability (with and without p.u.) and the end-of-period CER obtained under a random walk exchange rate model (with and without p.u.). The differences in CERs are annualized.

Table 4.2: The economic value of predictability. Static buy-and-hold strategy: Japan

$T=$	1	2	3	4	5	6	7	8	9	10
					$A=2$					
without p.u.	1.1079	1.2094	1.3846	1.6306	1.7437	1.7784	1.8217	2.1529	2.5325	2.5762
	(1.08)	(1.12)	(1.28)	(1.52)	(1.52)	(1.36)	(1.24)	(1.48)	(1.79)	(1.60)
	[1.07]	[1.10]	[1.21]	[1.34]	[1.34]	[1.26]	[1.19]	[1.32]	[1.44]	[1.37]
	{0.081}	{0.062}	{0.100}	{0.140}	{0.118}	{0.078}	{0.050}	{0.087}	{0.124}	{0.096}
with p.u.	1.1043	1.1949	1.3425	1.4922	1.6266	1.6755	1.7465	1.9814	2.2261	2.3038
	(1.08)	(1.10)	(1.24)	(1.39)	(1.41)	(1.28)	(1.19)	(1.36)	(1.58)	(1.43)
	[1.07]	[1.09]	[1.19]	[1.28]	[1.29]	[1.22]	[1.15]	[1.26]	[1.36]	[1.30]
	{0.078}	{0.055}	{0.086}	{0.105}	{0.095}	{0.061}	{0.039}	{0.066}	{0.090}	{0.069}
					$A=5$					
without p.u.	1.0857	1.1690	1.2859	1.4181	1.5407	1.6180	1.7014	1.8999	2.1150	2.2168
	(1.06)	(1.08)	(1.19)	(1.32)	(1.34)	(1.24)	(1.16)	(1.31)	(1.50)	(1.38)
	[1.20]	[1.25]	[1.49]	[1.67]	[1.68]	[1.57]	[1.44]	[1.65]	[1.80]	[1.72]
	{0.059}	{0.042}	{0.067}	{0.086}	{0.078}	{0.052}	{0.033}	{0.055}	{0.077}	{0.060}
with p.u.	1.0844	1.1639	1.2742	1.3860	1.4808	1.5645	1.6608	1.8155	1.9774	2.0919
	(1.05)	(1.06)	(1.14)	(1.22)	(1.22)	(1.16)	(1.10)	(1.17)	(1.27)	(1.19)
	[1.17]	[1.22]	[1.41]	[1.54]	[1.55]	[1.44]	[1.32]	[1.47]	[1.61]	[1.50]
	{0.052}	{0.035}	{0.053}	{0.062}	{0.053}	{0.035}	{0.022}	{0.033}	{0.046}	{0.033}
					$A=10$					
without p.u.	1.0807	1.1620	1.2641	1.3786	1.4782	1.5645	1.6593	1.8099	1.9818	2.1032
	(1.03)	(1.04)	(1.10)	(1.16)	(1.16)	(1.11)	(1.08)	(1.14)	(1.22)	(1.17)
	[1.24]	[1.31]	[1.56]	[1.73]	[1.73]	[1.61]	[1.48]	[1.69]	[1.83]	[1.75]
	{0.032}	{0.024}	{0.036}	{0.047}	{0.040}	{0.026}	{0.016}	{0.028}	{0.039}	{0.030}
with p.u.	1.0762	1.1551	1.2481	1.3514	1.4470	1.5398	1.6397	1.7733	1.9241	2.0503
	(1.03)	(1.03)	(1.07)	(1.11)	(1.10)	(1.07)	(1.04)	(1.08)	(1.12)	(1.09)
	[1.20]	[1.24]	[1.43]	[1.59]	[1.56]	[1.44]	[1.32]	[1.49]	[1.64]	[1.55]
	{0.026}	{0.017}	{0.025}	{0.032}	{0.025}	{0.016}	{0.009}	{0.016}	{0.023}	{0.017}
					$A=20$					
without p.u.	1.0758	1.1545	1.2467	1.3464	1.4444	1.5378	1.6397	1.7761	1.9285	2.0541
	(1.01)	(1.02)	(1.05)	(1.07)	(1.08)	(1.05)	(1.04)	(1.07)	(1.11)	(1.08)
	[1.23]	[1.32]	[1.56]	[1.73]	[1.74]	[1.62]	[1.49]	[1.71]	[1.85]	[1.78]
	{0.015}	{0.011}	{0.017}	{0.022}	{0.020}	{0.013}	{0.008}	{0.014}	{0.020}	{0.015}
with p.u.	1.0735	1.1513	1.2380	1.3341	1.4288	1.5254	1.6292	1.7536	1.8885	2.0200
	(1.01)	(1.01)	(1.03)	(1.05)	(1.05)	(1.03)	(1.02)	(1.04)	(1.06)	(1.05)
	[1.20]	[1.24]	[1.43]	[1.59]	[1.58]	[1.47]	[1.34]	[1.52]	[1.66]	[1.59]
	{0.013}	{0.008}	{0.012}	{0.015}	{0.013}	{0.008}	{0.005}	{0.008}	{0.011}	{0.009}

Notes: See Notes to Table 4.1

Table 4.3: The economic value of predictability. Static buy-and-hold strategy: UK

$T =$	1	2	3	4	5	6	7	8	9	10
					$A = 2$					
without p.u.	1.1241	1.0773	1.2109	1.4343	1.6020	1.9264	2.1020	2.3551	2.5288	2.6014
	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)
	[1.00]	[1.00]	[1.00]	[1.00]	[1.00]	[1.00]	[1.00]	[1.00]	[1.00]	[1.00]
	-	-	-	-	-	-	-	-	-	-
with p.u.	1.1241	1.0773	1.2109	1.4343	1.6020	1.9264	2.1020	2.3551	2.5288	2.6014
	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)	(1.00)
	[1.00]	[1.00]	[1.00]	[1.00]	[1.00]	[1.00]	[1.00]	[1.00]	[1.00]	[1.00]
	-	-	-	-	-	-	-	-	-	-
					$A = 5$					
without p.u.	1.1241	1.0773	1.2109	1.4343	1.6020	1.9264	2.1020	2.3551	2.5288	2.6014
	(1.01)	(0.96)	(0.97)	(1.01)	(1.02)	(1.06)	(1.08)	(1.09)	(1.09)	(1.07)
	[1.03]	[0.79]	[0.87]	[1.02]	[1.07]	[1.21]	[1.22]	[1.27]	[1.28]	[1.23]
	{0.009}	{-0.025}	{-0.011}	{0.002}	{0.006}	{0.018}	{0.018}	{0.023}	{0.022}	{0.016}
with p.u.	1.1241	1.0773	1.2109	1.4343	1.6020	1.9264	2.1020	2.3551	2.5288	2.6014
	(1.01)	(0.94)	(0.96)	(1.01)	(1.03)	(1.11)	(1.12)	(1.15)	(1.16)	(1.13)
	[1.03]	[0.73]	[0.81]	[1.04]	[1.11]	[1.33]	[1.36]	[1.43]	[1.44]	[1.38]
	{0.009}	{-0.033}	{-0.017}	{0.003}	{0.009}	{0.030}	{0.032}	{0.038}	{0.037}	{0.029}
					$A = 10$					
without p.u.	1.1199	1.0854	1.2114	1.4343	1.6020	1.9264	2.1020	2.3551	2.5288	2.6014
	(1.01)	(0.95)	(0.96)	(1.01)	(1.03)	(1.09)	(1.09)	(1.11)	(1.11)	(1.09)
	[1.06]	[0.36]	[0.57]	[1.08]	[1.20]	[1.52]	[1.54]	[1.61]	[1.60]	[1.54]
	{0.008}	{-0.030}	{-0.016}	{0.003}	{0.008}	{0.025}	{0.024}	{0.029}	{0.027}	{0.021}
with p.u.	1.1179	1.1004	1.2271	1.4301	1.5896	1.8804	2.0529	2.2950	2.4610	2.5465
	(1.01)	(0.95)	(0.97)	(1.01)	(1.03)	(1.08)	(1.09)	(1.13)	(1.13)	(1.10)
	[1.05]	[0.47]	[0.65]	[1.07]	[1.20]	[1.51]	[1.55]	[1.65]	[1.67]	[1.59]
	{0.007}	{-0.026}	{-0.013}	{0.003}	{0.007}	{0.024}	{0.025}	{0.032}	{0.031}	{0.024}
					$A = 20$					
without p.u.	1.1102	1.1429	1.2556	1.4226	1.5693	1.8129	1.9767	2.1909	2.3657	2.4657
	(1.00)	(0.97)	(0.98)	(1.01)	(1.02)	(1.06)	(1.07)	(1.09)	(1.10)	(1.08)
	[1.07]	[0.34]	[0.51]	[1.10]	[1.29]	[1.68]	[1.73]	[1.82]	[1.84]	[1.78]
	{0.004}	{-0.015}	{-0.008}	{0.002}	{0.005}	{0.017}	{0.018}	{0.023}	{0.024}	{0.019}
with p.u.	1.1089	1.1529	1.2657	1.4187	1.5554	1.7655	1.9141	2.1007	2.2628	2.3775
	(1.00)	(0.98)	(0.99)	(1.00)	(1.01)	(1.05)	(1.05)	(1.07)	(1.08)	(1.06)
	[1.05]	[0.52]	[0.65]	[1.07]	[1.20]	[1.58]	[1.61]	[1.72]	[1.76]	[1.69]
	{0.003}	{-0.011}	{-0.006}	{0.001}	{0.003}	{0.013}	{0.013}	{0.017}	{0.018}	{0.011}

Notes: See Notes to Table 4.1

Table 4.4: The economic value of predictability. Dynamic rebalancing strategy: Canada

	$-3\sigma_u$	$-1\sigma_u$	$\mu(u)$	$+1\sigma_u$	$+3\sigma_u$	Static
			$A=2$			
without p.u.	2.2269	2.2269	2.2269	2.2269	2.2269	2.2501
	(1.09)	(1.09)	(1.09)	(1.09)	(1.09)	(1.10)
	[1.08]	[1.08]	[1.08]	[1.08]	[1.08]	[1.09]
	{0.018}	{0.018}	{0.018}	{0.018}	{0.018}	{0.020}
with p.u.	2.2269	2.2269	2.2269	2.2269	2.2269	2.2501
	(1.09)	(1.09)	(1.09)	(1.09)	(1.09)	(1.10)
	[1.08]	[1.08]	[1.08]	[1.08]	[1.08]	[1.09]
	{0.018}	{0.018}	{0.018}	{0.018}	{0.018}	{0.020}
			$A=5$			
without p.u.	2.2269	2.2269	2.2269	2.2269	2.2269	2.2501
	(1.09)	(1.09)	(1.09)	(1.09)	(1.09)	(1.10)
	[1.29]	[1.29]	[1.29]	[1.29]	[1.29]	[1.31]
	(0.018)	(0.018)	(0.018)	(0.018)	(0.018)	(0.020)
with p.u.	2.1739	2.2102	2.2211	2.2269	2.2269	2.2432
	(1.06)	(1.08)	(1.09)	(1.09)	(1.09)	(1.10)
	[1.21]	[1.26]	[1.28]	[1.29]	[1.29]	[1.31]
	{0.012}	{0.016}	{0.017}	{0.018}	{0.018}	{0.019}
			$A=10$			
without p.u.	2.2269	2.2269	2.2269	2.2269	2.2269	2.2501
	(1.09)	(1.09)	(1.09)	(1.09)	(1.09)	(1.10)
	[1.53]	[1.53]	[1.53]	[1.53]	[1.53]	[1.57]
	{0.018}	{0.018}	{0.018}	{0.018}	{0.018}	{0.020}
with p.u.	2.1900	2.2084	2.2136	2.2165	2.2165	2.2225
	(1.07)	(1.08)	(1.08)	(1.08)	(1.08)	(1.09)
	[1.46]	[1.50]	[1.51]	[1.51]	[1.51]	[1.52]
	(0.014)	(0.016)	(0.016)	(0.017)	(0.017)	(0.017)
			$A=20$			
without p.u.	2.2269	2.2269	2.2269	2.2269	2.2269	2.2501
	(1.06)	(1.06)	(1.06)	(1.06)	(1.06)	(1.07)
	[1.67]	[1.67]	[1.67]	[1.67]	[1.67]	[1.73]
	{0.012}	{0.012}	{0.012}	{0.012}	{0.012}	{0.015}
with p.u.	2.1854	2.1912	2.1917	2.1935	2.1969	2.2018
	(1.01)	(1.01)	(1.01)	(1.01)	(1.01)	(1.02)
	[1.13]	[1.17]	[1.17]	[1.18]	[1.21]	[1.24]
	{0.001}	{0.002}	{0.002}	{0.002}	{0.002}	{0.003}

Notes: These figures refer to the end-of-period (equal to 10 years) economic value, as measured by wealth levels, wealth ratios, utility ratios and certainty equivalent returns for the case of an investor acting on the basis of the dynamic buy-and-hold strategy with a rebalancing period of 1 year. Initial wealth is assumed to be equal to unity. A is the coefficient of risk aversion in the CRRA utility function defined by equation (4.6). $\mu(u)$ denotes the historical mean of the predictor variable, u_t, calculated over the sample period September 1977 - December 1990. $\pm 3\sigma_u$ and $\pm 1\sigma_u$ denote three and one standard deviations above (below) the historical sample mean of the predictor variable. "Static" denotes the 10-year wealth obtained with a static buy-and-hold strategy under predictable exchange rates (as reported in Table 4.1). 'With p.u.' and 'without p.u.' denote the case where the investor takes into account parameter uncertainty (p.u.) and the case where she ignores it respectively. Under each of these cases, the first row reports the end-of-period wealth calculated using the definition given by equation (4.5). Values in parentheses in the second row, for each of the two cases with and without p.u., are ratios of the end-of-period wealth levels obtained in the case of predictability to the end-of-period wealth levels obtained under a random walk exchange rate. Values in brackets in the third row are ratios of the end-of-period utility levels obtained in the case of predictability (with and without p.u.) to the end-of-period utility levels obtained under a random walk exchange rate (with and without p.u.). Values in braces in the fourth row are differences of the end-of-period certainty equivalent return (CER) obtained in the case of predictability (with and without p.u.) and the end-of-period CER obtained under a random walk exchange rate (with and without p.u.). The differences in CERs are annualized.

Table 4.5: The economic value of predictability. Dynamic rebalancing strategy: Japan

	$-3\sigma_u$	$-1\sigma_u$	$\mu(u)$	$+1\sigma_u$	$+3\sigma_u$	Static
			$A=2$			
without p.u.	1.1399	1.1450	1.3715	1.5516	1.7627	2.5762
	(0.71)	(0.71)	(0.85)	(0.96)	(1.09)	(1.60)
	[0.58]	[0.59]	[0.82]	[0.96]	[1.08]	[1.37]
	{-0.047}	{-0.046}	{-0.023}	{-0.005}	{0.015}	{0.096}
with p.u.	1.3355	1.5413	1.7421	1.8038	1.9377	2.3038
	(0.83)	(0.96)	(1.08)	(1.12)	(1.20)	(1.43)
	[0.79]	[0.95]	[1.07]	[1.10]	[1.16]	[1.30]
	{-0.027}	{-0.007}	{0.013}	{0.019}	{0.032}	{0.069}
			$A=5$			
without p.u.	1.5928	1.6546	1.8605	2.0200	2.1693	2.2168
	(0.99)	(1.03)	(1.15)	(1.25)	(1.35)	(1.38)
	[0.95]	[1.10]	[1.43]	[1.59]	[1.69]	[1.72]
	{-0.001}	{0.004}	{0.024}	{0.040}	{0.055}	{0.060}
with p.u.	1.6185	1.7833	1.9891	2.0921	2.1641	2.0919
	(0.92)	(1.01)	(1.13)	(1.19)	(1.23)	(1.19)
	[0.60]	[1.05]	[1.38]	[1.50]	[1.56]	[1.50]
	{-0.014}	{0.002}	{0.023}	{0.033}	{0.040}	{0.033}
			$A=10$			
without p.u.	1.8656	1.9531	2.0149	2.0612	2.0509	2.1032
	(1.03)	(1.09)	(1.12)	(1.15)	(1.14)	(1.17)
	[1.28]	[1.52]	[1.64]	[1.70]	[1.69]	[1.75]
	{0.006}	{0.015}	{0.021}	{0.026}	{0.025}	{0.030}
with p.u.	1.9788	2.0406	2.0869	2.1230	2.1693	2.0503
	(1.05)	(1.09)	(1.11)	(1.13)	(1.16)	(1.09)
	[1.38]	[1.53]	[1.61]	[1.67]	[1.72]	[1.55]
	{0.010}	{0.016}	{0.021}	{0.024}	{0.029}	{0.017}
			$A=20$			
without p.u.	2.0200	2.0406	2.0818	2.1024	2.1127	2.0541
	(1.07)	(1.08)	(1.10)	(1.11)	(1.11)	(1.08)
	[1.70]	[1.75]	[1.83]	[1.86]	[1.87]	[1.78]
	{0.012}	{0.014}	{0.018}	{0.020}	{0.021}	{0.015}
with p.u.	2.0766	2.0921	2.1281	2.1744	2.2104	2.0200
	(1.08)	(1.09)	(1.11)	(1.13)	(1.15)	(1.05)
	[1.76]	[1.79]	[1.85]	[1.90]	[1.92]	[1.59]
	{0.015}	{0.016}	{0.020}	{0.024}	{0.028}	{0.009}

Notes: See Notes to Table 4.4.

Table 4.6: The economic value of predictability. Dynamic rebalancing strategy: UK

	$-3\sigma_u$	$-1\sigma_u$	$\mu(u)$	$+1\sigma_u$	$+3\sigma_u$	Static
			$A=2$			
without p.u.	2.6163	2.6163	2.6163	2.6163	2.6163	2.6014
	(1.01)	(1.01)	(1.01)	(1.01)	(1.01)	(1.00)
	[1.01]	[1.01]	[1.01]	[1.01]	[1.01]	[1.00]
	{0.001}	{0.001}	{0.001}	{0.001}	{0.001}	-
with p.u.	2.6163	2.6163	2.6163	2.6163	2.6163	2.6014
	(1.01)	(1.01)	(1.01)	(1.01)	(1.01)	(1.00)
	[1.01]	[1.01]	[1.01]	[1.01]	[1.01]	[1.00]
	{0.001}	{0.001}	{0.001}	{0.001}	{0.001}	-
			$A=5$			
without p.u.	2.6163	2.6163	2.6163	2.6163	2.6163	2.6014
	(1.07)	(1.07)	(1.07)	(1.07)	(1.07)	(1.07)
	[1.25]	[1.25]	[1.25]	[1.25]	[1.25]	[1.23]
	{0.018}	{0.018}	{0.018}	{0.018}	{0.018}	{0.016}
with p.u.	2.6163	2.6163	2.6163	2.6163	2.6163	2.6014
	(1.13)	(1.13)	(1.13)	(1.13)	(1.13)	(1.13)
	[1.39]	[1.39]	[1.39]	[1.39]	[1.39]	[1.38]
	{0.031}	{0.031}	{0.031}	{0.031}	{0.031}	{0.029}
			$A=10$			
without p.u.	2.6163	2.6163	2.6163	2.6163	2.4613	2.6014
	(1.10)	(1.10)	(1.10)	(1.10)	(1.03)	(1.09)
	[1.56]	[1.56]	[1.56]	[1.56]	[1.25]	[1.54]
	{0.023}	{0.023}	{0.023}	{0.023}	{0.007}	{0.021}
with p.u.	2.6163	2.6163	2.5624	2.5017	2.4186	2.5465
	(1.14)	(1.14)	(1.11)	(1.09)	(1.05)	(1.10)
	[1.68]	[1.68]	[1.61]	[1.52]	[1.36]	[1.59]
	{0.031}	{0.031}	{0.026}	{0.020}	{0.011}	{0.024}
			$A=20$			
without p.u.	2.5736	2.5736	2.5557	2.5332	2.5197	2.4657
	(1.13)	(1.13)	(1.12)	(1.11)	(1.11)	(1.08)
	[1.90]	[1.90]	[1.88]	[1.86]	[1.85]	[1.78]
	{0.029}	{0.029}	{0.027}	{0.025}	{0.024}	{0.019}
with p.u.	2.4546	2.4231	2.3580	2.2726	2.1760	2.3775
	(1.10)	(1.08)	(1.06)	(1.02)	(0.97)	(1.06)
	[1.83]	[1.78]	[1.64]	[1.27]	[0.35]	[1.69]
	{0.022}	{0.018}	{0.012}	{0.003}	{-0.005}	{0.014}

Notes: See Notes to Table 4.4.

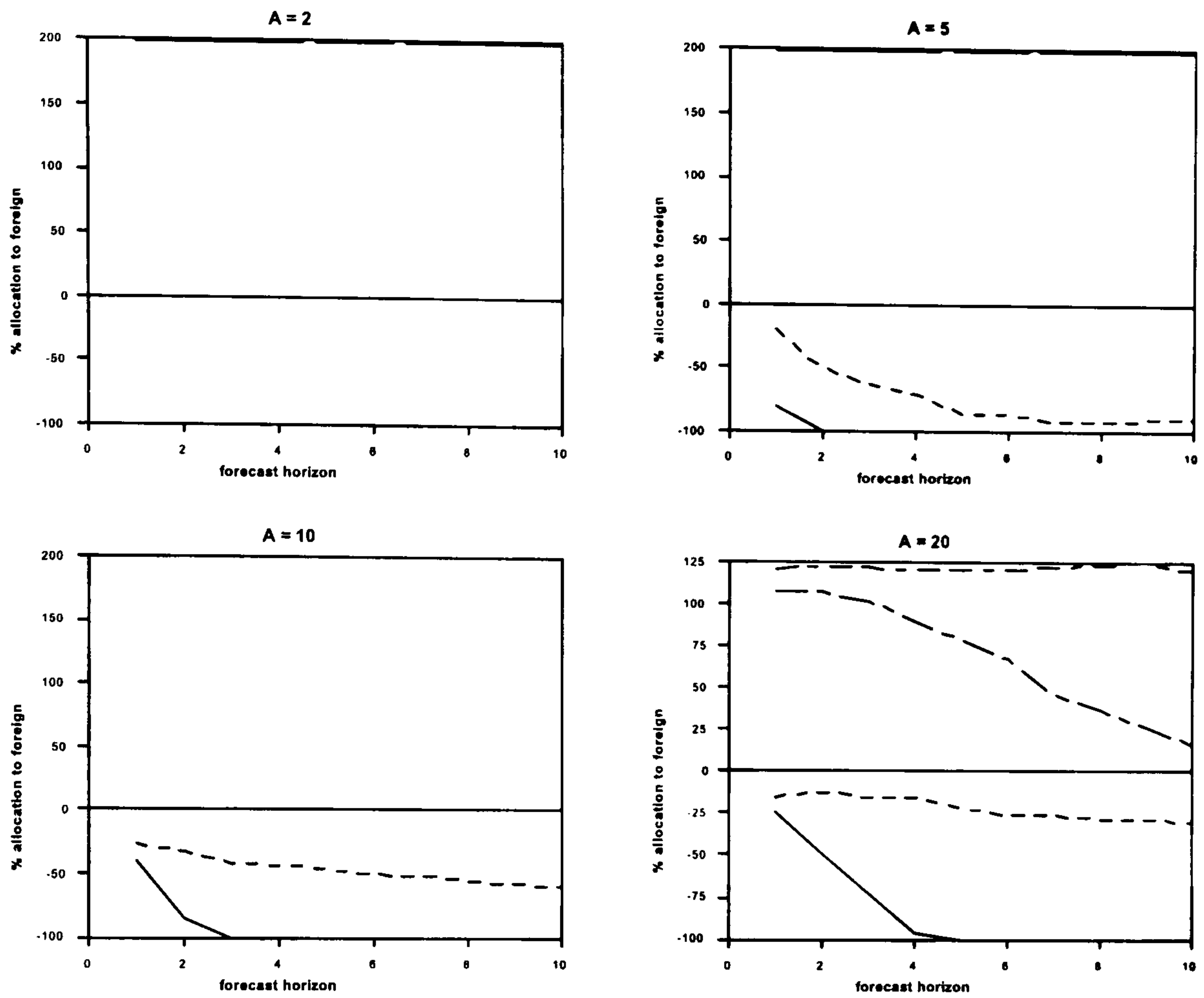

Figure 4.1: **US/Canada, static buy-and-hold strategy.** The figure shows the optimal weight ω to the foreign asset plotted against the investment horizon in years. The dotted and solid lines correspond to the cases where the investor assumes predictability with and without parameter uncertainty respectively. The dot-dash and dash lines correspond to the cases where the investor assumes that the exchange rate follows a random walk with and without parameter uncertainty respectively.

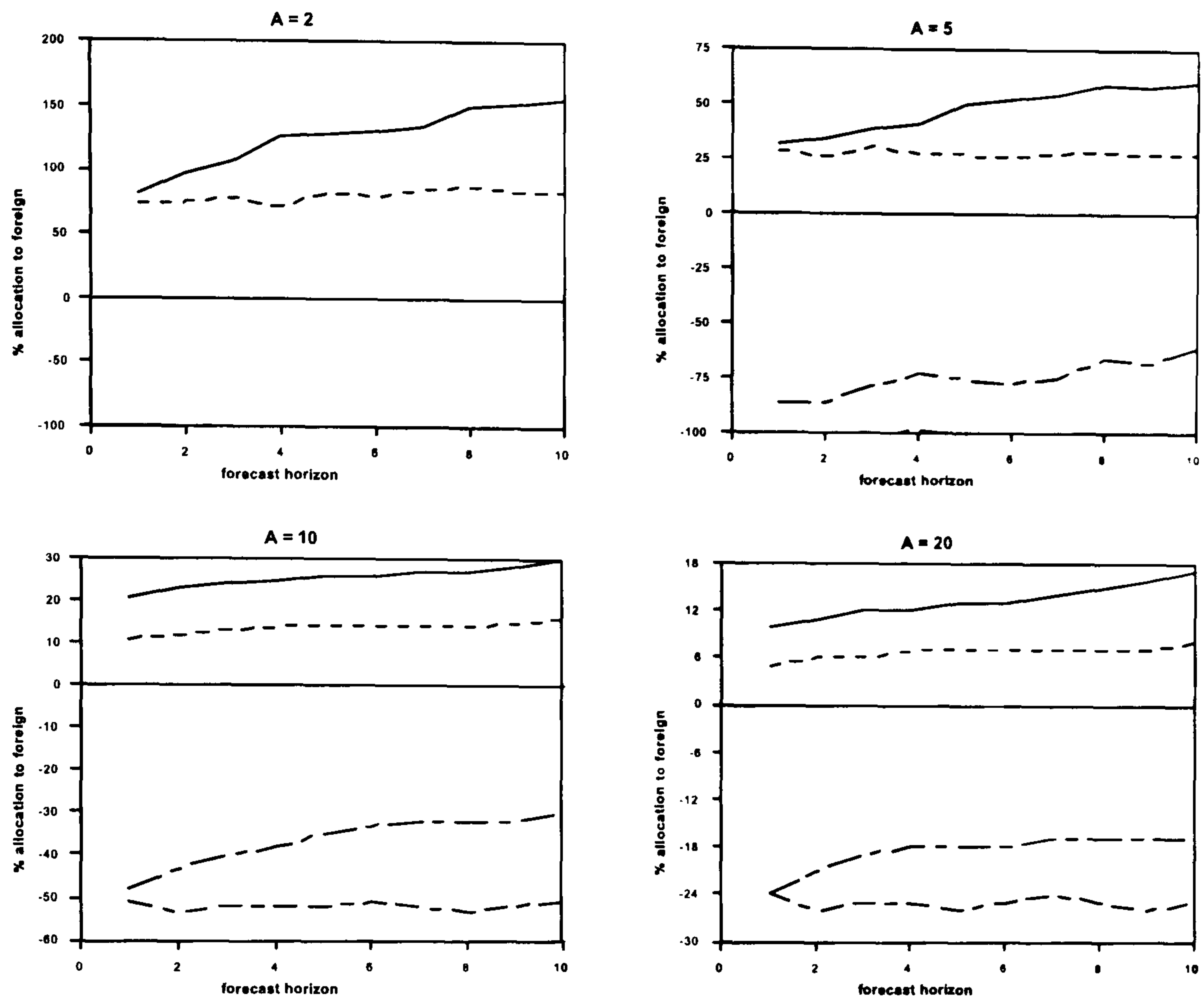

Figure 4.2: **US/Japan, static buy-and-hold strategy.** The figure shows the optimal weight ω to the foreign asset plotted against the investment horizon in years. The dotted and solid lines correspond to the cases where the investor assumes predictability with and without parameter uncertainty respectively. The dot-dash and dash lines correspond to the cases where the investor assumes that the exchange rate follows a random walk with and without parameter uncertainty respectively.

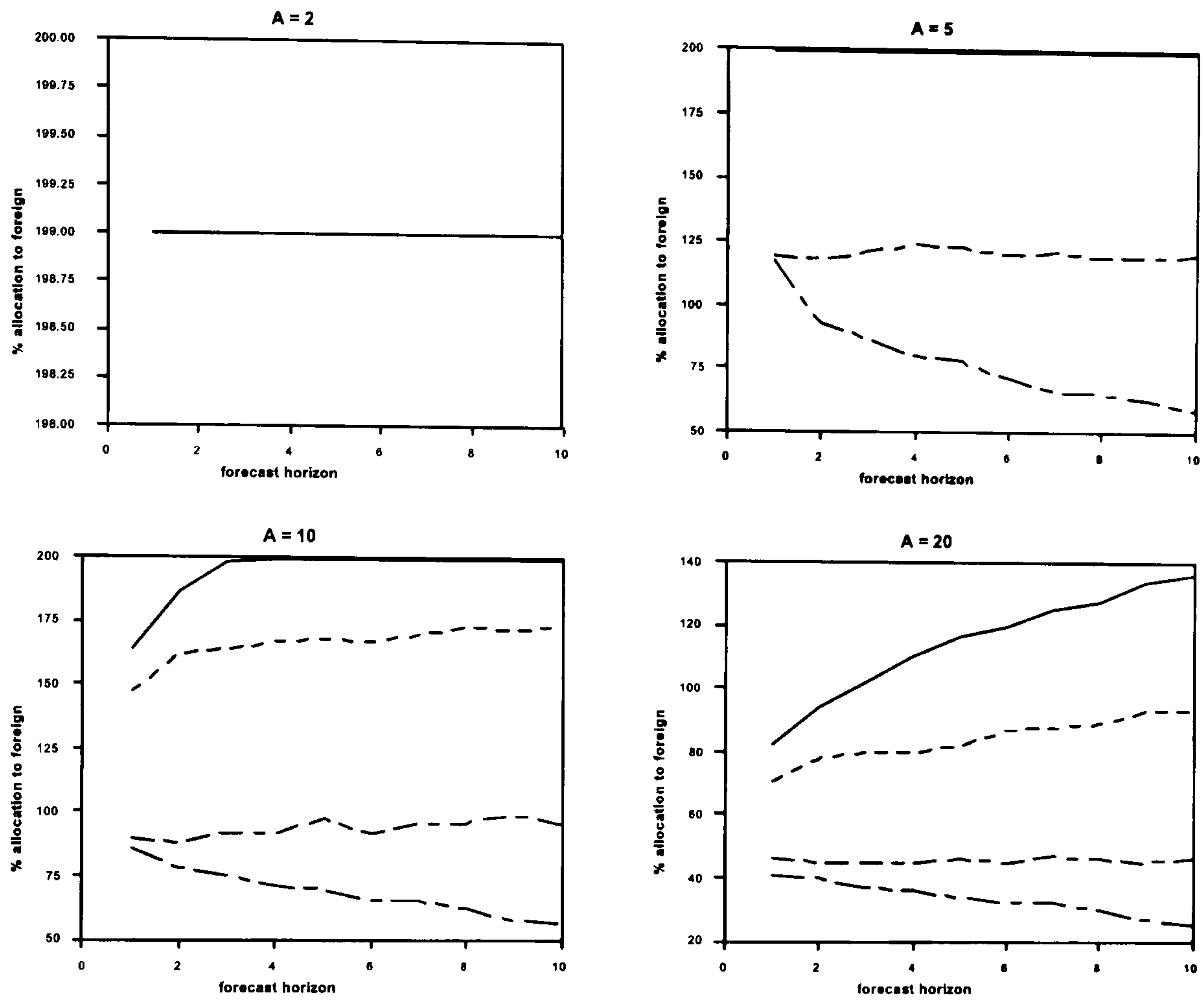

Figure 4.3: **US/UK, static buy-and-hold strategy.** The figure shows the optimal weight ω to the foreign asset plotted against the investment horizon in years. The dotted and solid lines correspond to the cases where the investor assumes predictability with and without parameter uncertainty respectively. The dot-dash and dash lines correspond to the cases where the investor assumes that the exchange rate follows a random walk with and without parameter uncertainty respectively.

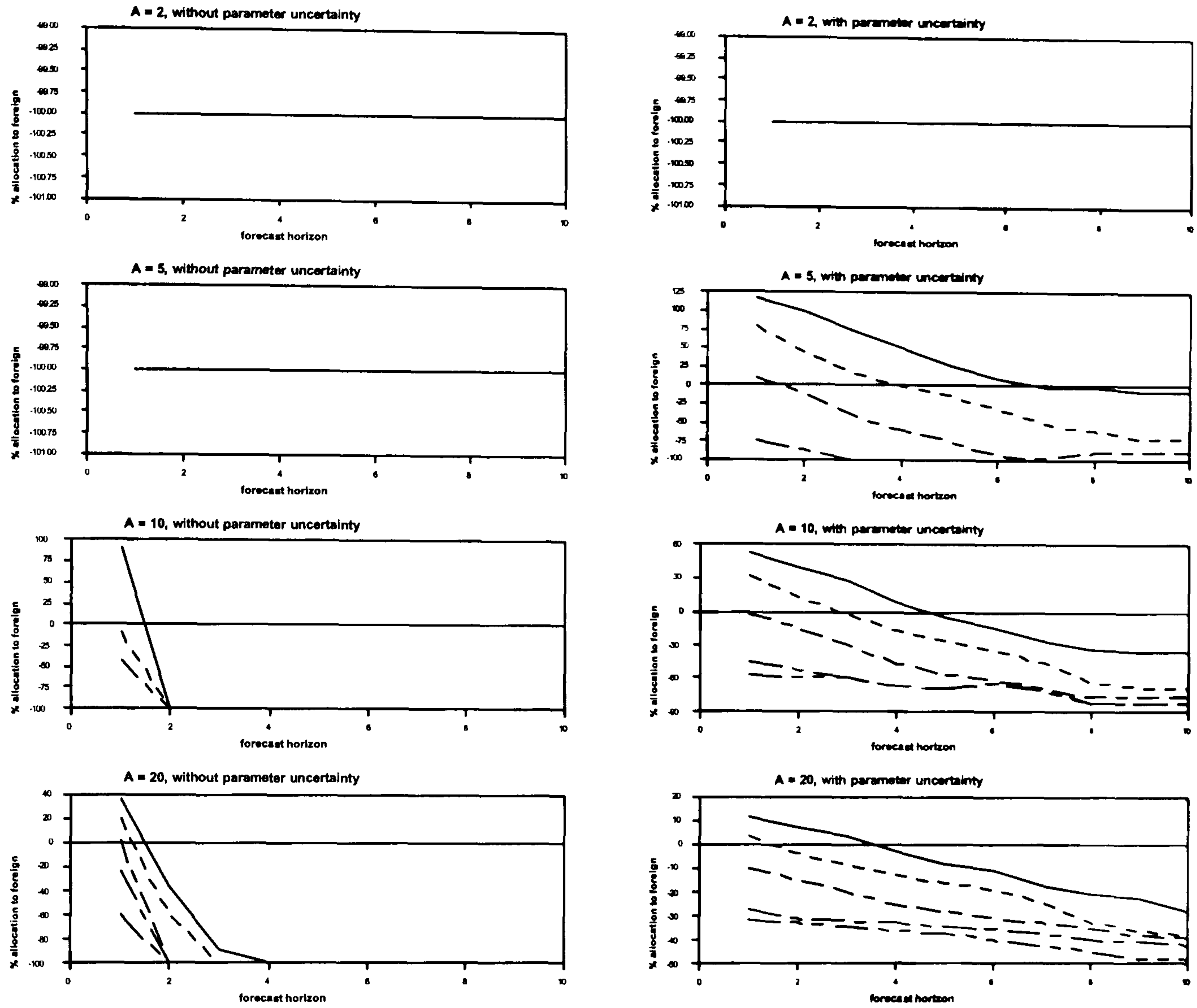

Figure 4.4: **US/Canada, optimal dynamic rebalancing strategy.** The figure shows the optimal weight ω to the foreign asset plotted against the investment horizon in years. The four graphs on the left refer to the case without parameter uncertainty, those on the right refer to the case with parameter uncertainty. The five lines within each graph correspond to different intial values of the predictor variable: $+3\sigma_u$ (solid), $+1\sigma_u$ (dotted), $\mu(u)$ (dash), $-1\sigma_u$ (dot/dash single), $-3\sigma_u$ (dot/dash double).

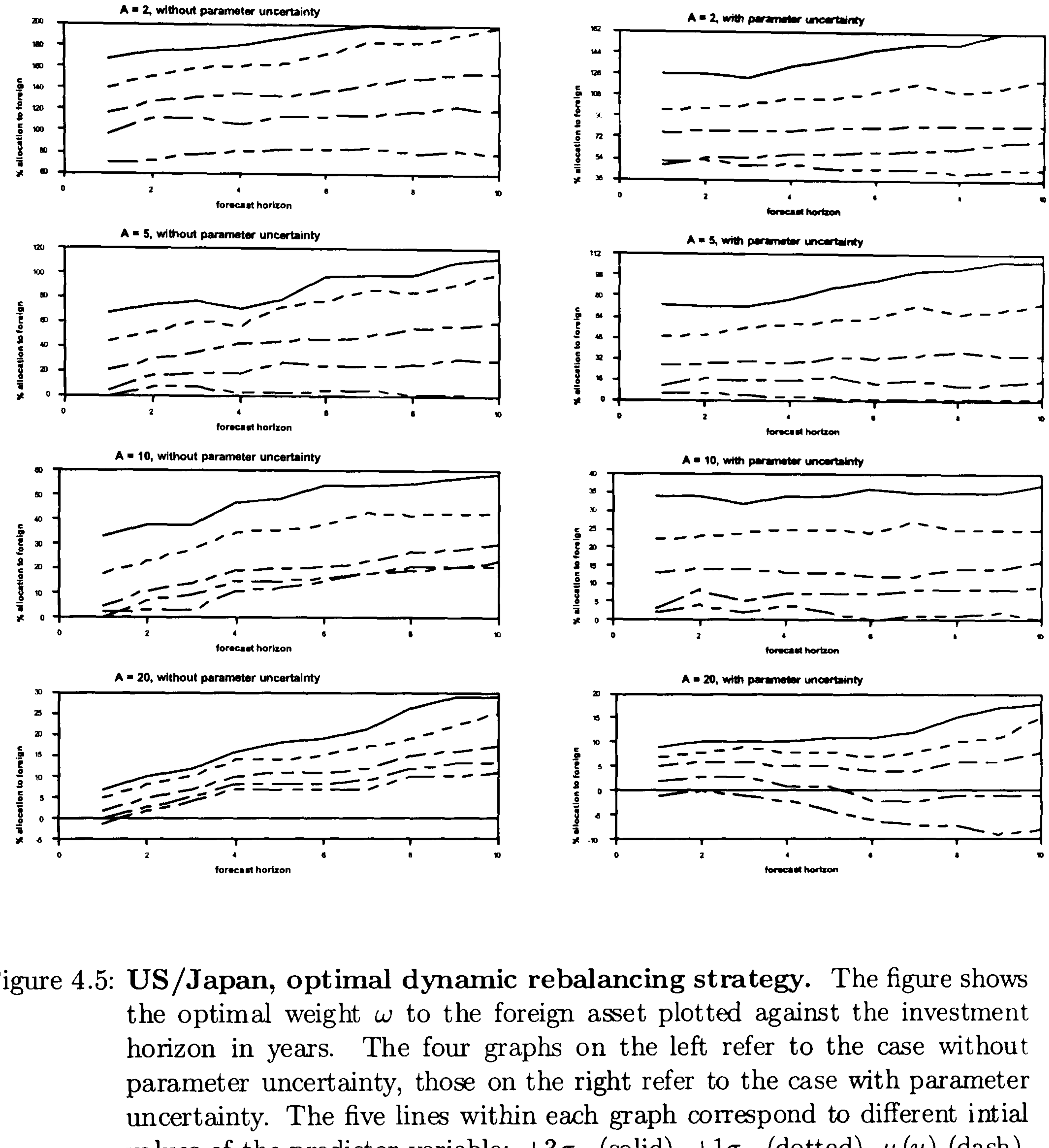

Figure 4.5: **US/Japan, optimal dynamic rebalancing strategy.** The figure shows the optimal weight ω to the foreign asset plotted against the investment horizon in years. The four graphs on the left refer to the case without parameter uncertainty, those on the right refer to the case with parameter uncertainty. The five lines within each graph correspond to different intial values of the predictor variable: $+3\sigma_u$ (solid), $+1\sigma_u$ (dotted), $\mu(u)$ (dash), $-1\sigma_u$ (dot/dash single), $-3\sigma_u$ (dot/dash double).

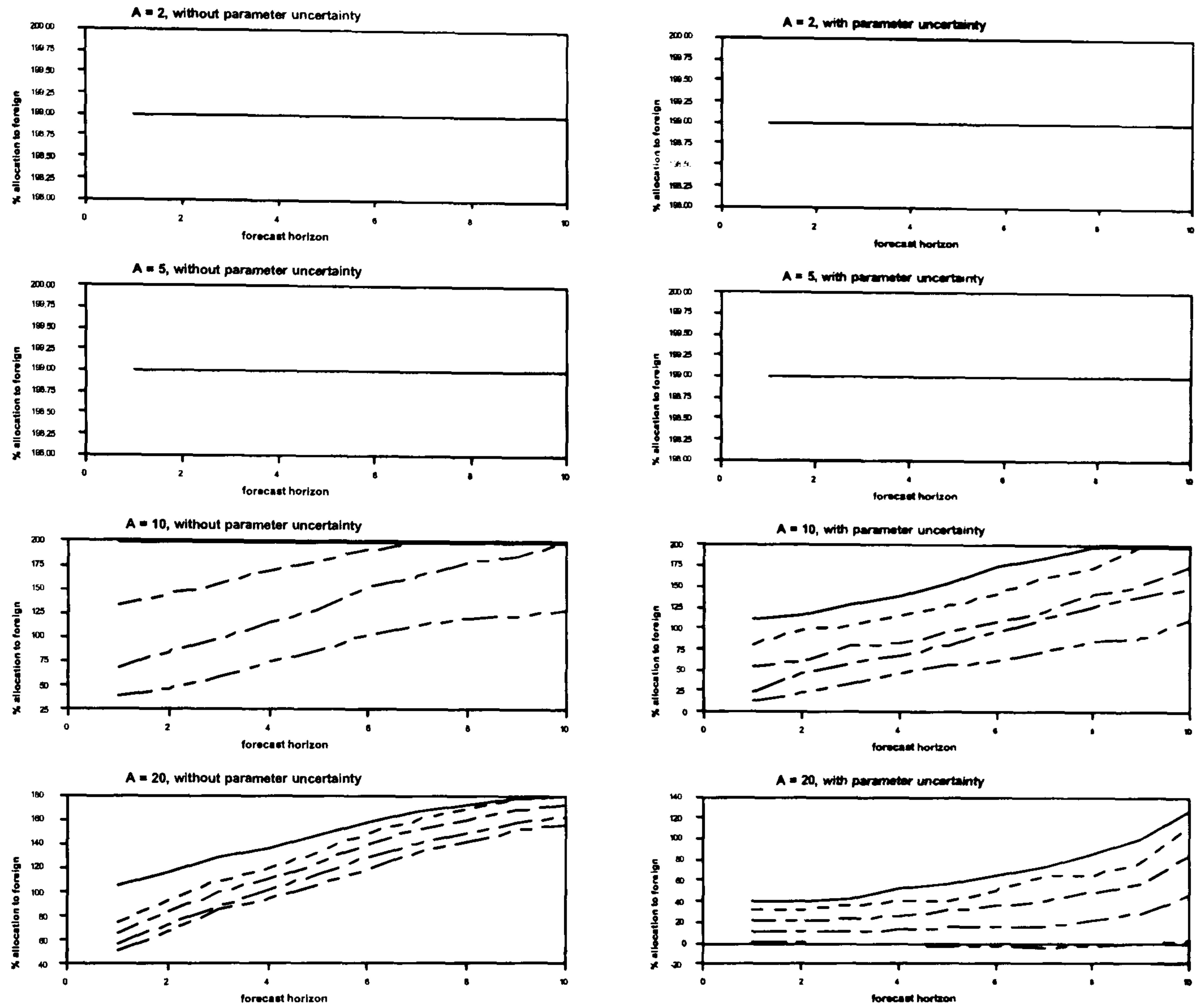

Figure 4.6: **US/UK, optimal dynamic rebalancing strategy.** The figure shows the optimal weight ω to the foreign asset plotted against the investment horizon in years. The four graphs on the left refer to the case without parameter uncertainty, those on the right refer to the case with parameter uncertainty. The five lines within each graph correspond to different intial values of the predictor variable: $+3\sigma_u$ (solid), $+1\sigma_u$ (dotted), $\mu(u)$ (dash), $-1\sigma_u$ (dot/dash single), $-3\sigma_u$ (dot/dash double).

Appendix A

Bayesian estimation techniques

This appendix provides details of the Bayesian econometric approach used in Chapter 4. We begin by describing the computations used in the static buy-and-hold case described in Section 4.2.1.

First, we assume that the exchange rate is a random walk with drift: $\Delta s_t = \mu + \varepsilon_t$, where Δs_t is the log-difference of the end-of-period nominal exchange rate, and Δ is the first-difference operator; and $\varepsilon_t \sim iidN(0, \sigma^2)$. We incorporate parameter uncertainty by using the predictive distribution of the nominal exchange rate, $p(\Delta_{\widetilde{T}} s_{T+\widetilde{T}} | \Delta s)$, where Δs is the vector of observed nominal exchange rate changes over the sample period. In the case without parameter uncertainty, on the other hand, we compute the expected value over the distribution of the future nominal exchange rate conditional on fixed parameters values, $p(\Delta_{\widetilde{T}} s_{T+\widetilde{T}} | \Delta s, \widehat{\mu}, \widehat{\sigma}^2)$. In both of these cases, the conditional distribution of the nominal exchange rate is a normal distribution. Under no parameter uncertainty, $p(\Delta_{\widetilde{T}} s_{T+\widetilde{T}} | \Delta s, \widehat{\mu}, \widehat{\sigma}^2)$ is a normal distribution, $N\left(\widetilde{T}\widehat{\mu}, \widetilde{T}\widehat{\sigma}^2\right)$, where $\widehat{\mu}$ and $\widehat{\sigma}^2$ denote the estimates of the mean and variance calculated over the sample period. When parameter uncertainty is accounted for, $p(\Delta_{\widetilde{T}} s_{T+\widetilde{T}}|\Delta s)$ is obtained using the value of the parameters μ and σ^2 obtained by iterative sampling from the marginal posterior distributions under a noninformative prior (that is, $p\left(\mu,\sigma^2\right) \propto \frac{1}{\sigma^2}$).[1] In other words, in order to get a sample $\left\{\Delta_{\widetilde{T}}^{(i)} s_{T+\widetilde{T}}\right\}_{i=1}^{M}$ from the two possible distributions, we draw M times from the normal distribution $N\left(\widetilde{T}\widehat{\mu}, \widetilde{T}\widehat{\sigma}^2\right)$ in the case of no parameter uncertainty; in the case of parameter uncertainty we draw M times from the normal distribution $N\left(\widetilde{T}\widehat{\mu}^{(i)}, \widetilde{T}\widehat{\sigma}^{2\,(i)}\right)$, where $\widehat{\mu}^{(i)}, \widehat{\sigma}^{2\,(i)}$ are values from the ith draw from $p\left(\sigma^2|\Delta s\right)$ and $p\left(\mu|\sigma^2, \Delta s\right)$.

Second, we consider the case when the exchange rate is predictable, that is $z_t = a + Bx_{t-1} + \eta_t$, where $z_t' = (\Delta s_t, x_t')$, $x_t = (x_{1,t}, x_{2,t}, \ldots, x_{n,t})'$, and $\eta_t \sim iidN(0, \Sigma)$. The vector of explanatory variables x_t are used for predicting the exchange rate; these include the deviation from the long-run equilibrium level of the exchange rate as measured by the monetary fundamentals. Here too we consider the effects of accounting for parameter uncertainty. In particular, under no parameter uncertainty, $p(z_{T+\widetilde{T}}|z, \widehat{a}, \widehat{B}, \widehat{\Sigma})$ is a bivariate normal distribution, $N_2\left(\widehat{\widehat{\mu}}, \widehat{\widehat{\Sigma}}\right)$, where

$$\begin{aligned}
\widehat{\widehat{\mu}} &= \widetilde{T}\widehat{a} + \left(\widetilde{T}-1\right)\widehat{B}_0\widehat{a} + \left(\widetilde{T}-2\right)\widehat{B}_0^2\widehat{a} + \ldots + \widehat{B}_0^{\widetilde{T}-1}\widehat{a} + \left(\widehat{B}_0 + \ldots + \widehat{B}_0^{\widetilde{T}}\right) z_T \\
\widehat{\widehat{\Sigma}} &= \widehat{\Sigma} + \left(I + \widehat{B}_0\right)\widehat{\Sigma}\left(I + \widehat{B}_0\right)' + \ldots + \\
&\quad \left(I + \widehat{B}_0 + \ldots + \widehat{B}_0^{\widetilde{T}-1}\right)\widehat{\Sigma}\left(I + \widehat{B}_0 + \ldots + \widehat{B}_0^{\widetilde{T}-1}\right)' \qquad \text{(A.1)}
\end{aligned}$$

and $\widehat{a}$, $\widehat{B}$, $\widehat{\Sigma}$ are estimates of the parameters in the VAR $z_t = a + Bx_{t-1} + \eta_t$, obtained over the sample period used; $\widehat{B}_0$ is a matrix obtained by adding an initial vector of zeros to $\widehat{B}$; and I is the identity matrix. If parameter uncertainty is taken into account, $p(z_{T+\widetilde{T}}|z)$ is computed using the value of the estimated parameters $\widehat{a}$, $\widehat{B}$, $\widehat{\Sigma}$ obtained by iterative sampling from the marginal posterior distributions under a noninformative

[1] The posterior distribution of the parameters conditional upon the data $p(\mu, \sigma^2|\Delta s)$ can be obtained by first sampling from the marginal distribution, $p(\sigma^2|\Delta s)$, an inverse gamma distribution, and then, given the draw for the variance, from the conditional distribution $p(\mu|\sigma^2, \Delta s)$, which is a normal distribution. See Zellner (1971).

prior (that is, $p(a,B,\Sigma) \propto |\Sigma|^{-(n+2)/2}$):[2]

$$\begin{aligned}
\widehat{\widehat{\mu}} &= \widetilde{T}\widehat{a}^{(i)} + \left(\widetilde{T}-1\right)\widehat{B}_0^{(i)}\widehat{a}^{(i)} + \left(\widetilde{T}-2\right)\widehat{B}_0^{2(i)}\widehat{a}^{(i)} + \ldots + \widehat{B}_0^{\widetilde{T}-1(i)}\widehat{a}^{(i)} + \left(\widehat{B}_0^{(i)} + \ldots + \widehat{B}_0^{\widetilde{T}(i)}\right) z_T \\
\widehat{\widehat{\Sigma}} &= \widehat{\Sigma}^{(i)} + \left(I+\widehat{B}_0^{(i)}\right)\widehat{\Sigma}^{(i)}\left(I+\widehat{B}_0^{(i)}\right)' + \ldots + \\
&\quad + \left(I+\widehat{B}_0^{(i)} + \ldots + \widehat{B}_0^{\widetilde{T}-1(i)}\right)\widehat{\Sigma}^{(i)}\left(I+\widehat{B}_0^{(i)} + \ldots + \widehat{B}_0^{\widetilde{T}-1(i)}\right)'
\end{aligned} \quad (A.2)$$

for $i = 1,\ldots,M$, where $\widehat{a}^{(i)}, \widehat{B}_0^{(i)}, \widehat{\Sigma}^{(i)}$ are values from the ith draw from $p(\Sigma^{-1}|z)$ and $p(vec(a,B)\,|\Sigma,\Delta s)$. By computing $p(z_{T+\widetilde{T}}|z,a,B,\Sigma)$ and $p(z_{T+\widetilde{T}}|z)$, we are able to extract a sample $\left\{\Delta_{\widetilde{T}}^{(i)} s_{T+\widetilde{T}}\right\}_{i=1}^{M}$ which represents the future expected nominal exchange rate for the horizon $\widetilde{T}$ under predictability, without and with parameter uncertainty respectively.

Finally, we approximate the integrals for expected utility (4.8) and (4.11) by using the sample $\left\{\Delta_{\widetilde{T}}^{(i)} s_{T+\widetilde{T}}\right\}_{i=1}^{M}$ from the two cases of no predictability and predictability and then computing

$$\frac{1}{M}\sum_{i=1}^{M}\frac{\left[(1-\omega)\exp\left(r\widetilde{T}\right)+\omega\exp\left(r^{*}\widetilde{T}+\Delta_{\widetilde{T}}^{(i)}s_{T+\widetilde{T}}\right)\right]^{1-A}}{1-A}. \quad (A.3)$$

To obtain an accurate representation of the posterior distributions, the data have been used to generate different sample sizes M. The results reported in Chapter 4 refer to a sample size of $M = 1{,}000{,}000$ and were produced using an initial value of the predictor variables vector (in our case simply u_t as defined in equations (4.1)-(4.2)) equal to its historical mean.

Next, we provide details of the computations related to the dynamic allocation problem described in Section 4.2.2. We solve this by discretizing the state space and then using backward induction to solve the Bellman equation. In particular, in the case of predictability, we take an interval ranging from three standard deviations below the historical mean of the predictor variables in x_t (simply u_t as defined in equations (4.1)-

[2] The posterior distribution of the parameters conditional upon the data is obtained in this case by first sampling from the marginal distribution $p(\Sigma^{-1}|z)$, a Wishart distribution, and then, given the draws for the variance-covariance matrix, from the conditional distribution $p(vec(a,B)\,|\Sigma,\Delta s)$, which is a multivariate normal distribution. See Zellner (1971).

(4.2)), to three standard deviations above and discretize this range using j equally spaced grid points. The maximization problem (4.16) in the main text can be solved as follows:

$$Q\left(x_k^j, t_k\right) = \max_{\omega} \frac{1}{M} \sum_{i=1}^{M} \left\{ \left[(1-\omega_k) \exp\left(r\frac{\widetilde{T}}{K} \right) + \omega_k \exp\left(r^* \frac{\widetilde{T}}{K} + \Delta_{k+1} s_{k+1}^{(i)} \right) \right]^{1-A} \times Q\left(x_{k+1}^j, t_{k+1}\right) \right\}, \tag{A.4}$$

where $Q\left(x_k^j, t_k\right)$ is the value function calculated for x_k^j for all j.[3] In order to carry out the backward induction we assume that $Q\left(x_{T+\widetilde{T}}, t_{T+\widetilde{T}}\right) = 1$ for all $x_{T+\widetilde{T}}$ and we use equation (A4) to approximate $Q\left(x_k^j, t_k\right)$. $\Delta_{k+1} s_{k+1}^{(i)}$ can then be computed as explained above in this appendix in the case of predictable exchange rates under the cases of both parameter uncertainty and no parameter uncertainty and for different values of the explanatory variable x^j. This calculation gives $Q\left(x_k^j, t_k\right)$ for all j. Solving through all of the rebalancing points allows us to obtain $Q\left(x_0^j, t_0\right)$ and hence the optimal allocation at time T. As in the static optimization problem the sample size used is $M = 1,000,000$. We performed additional robustness checks to investigate the effect of the number of grid points selected. We produced our results for $j = 15, 25, 35$ grid points and we selected $j = 25$ since the accuracy of our results was better than in the case of $j = 15$ but not qualitatively different from the case where $j = 35$.

[3] An alternative procedure would involve allowing for learning over the investment horizon. We did not explore the implications of learning for our results in this paper, although this is a logical exercise for future research (e.g., see Lewis, 1995; Xia, 2001).

Chapter 5

Concluding remarks

In this thesis we have investigated three different topics in financial and economic forecasting which are currently under debate, receiving widespread attention by researchers. In this final chapter, we briefly summarize our key findings and suggest potential avenues for future research.

In Chapter 2 we have reported what we believe to be the first analysis of spot and forward exchange rates in a multivariate Markov-switching framework, and in particular we have applied that framework to exchange rate forecasting. Our research was inspired by encouraging results previously reported in the literature on the presence of nonlinearities (and particularly by the success of Markov-switching models) in the context of exchange rate modelling, as well as by the relative forecasting success of the 'agnostic' linear VECM model of the term structure of forward premia.

Using weekly data on spot and forward dollar exchange rates for the G5 countries over the period January 1979 through December 1995, we found strong evidence of the presence of nonlinearities in the term structure, which appeared to be modelled well by a multivariate three-regime Markov-switching VECM that allows for shifts in both the intercept and in the covariance structure. We then used this model to forecast dynamically out of sample over the period January 1996 through to December 1998. The MS-VECM forecasts were found to be strongly superior to the random walk forecasts at a range of forecasting horizons up to 52 weeks ahead, using standard forecasting accuracy criteria and on the basis of standard tests of significance. Moreover, the nonlinear VECM also outperformed, in general, a linear VECM for spot and forward rates in out-of-sample forecasting of the spot rate, although the magnitude of the gain from using a Markov-switching VECM relative to a linear VECM is rather small in magnitude at short horizon.

In this research, we have been primarily concerned with providing sound models of exchange rate forecasting and have therefore adopted an 'agnostic' approach both with respect to the sources of the underlying departures from the risk-neutral efficient markets hypothesis and in the sources of the underlying nonlinearities. Future research might, therefore, usefully analyze the sources of these nonlinearities further and attempt to improve on the parametric nonlinear formulation proposed in this chapter. Possible extensions include the allowance for different equilibrium correction terms (speeds of adjustment towards equilibrium) in different regimes, and the endogenization of the probability of switching from one regime to another, which might, for example, be made a function of macroeconomic fundamentals.

With regard to the evaluation of forecasting models, although the relevant literature has traditionally focused on accuracy evaluations based on point forecasts, several authors have recently emphasized the importance of evaluating the forecast accuracy of economic models on the basis of density - as opposed to point - forecasting performance (see, *inter alia*, Diebold, Gunther and Tay, 1998; Granger and Pesaran, 1999; Tay and Wallis, 2000; Timmerman, 2000; Pesaran and Skouras, 2002; Sarno and Valente, 2003). Especially when evaluating nonlinear models, which are capable of producing highly non-normal forecast densities, it would seem appropriate to consider a model's density forecasting performance. This is an immediate avenue for future research.

Given the difficulty in beating random walk forecasts using fundamentals-based models - first highlighted by Meese and Rogoff (1983a,b) - as well as the well-known failure of the forward rate optimally to predict the future spot rate, the evidence provided by our results that the term structure of forward rates is powerful in forecasting spot exchange rates is rather striking. In particular, it seems that, notwithstanding the failure of the simple (risk-neutral) efficient markets hypothesis in this context, forward rates may contain more useful information to forecast spot exchange rates than do conventional fundamentals. It seems plausible that important microstructural effects may be responsible for this phenomenon, as argued, for example, by Lyons (2001), Sarno and Taylor (2001) and Evans and Lyons (2002). Understanding the exact nature of this incremental information remains an important challenge in the research agenda.

In Chapter 3 we have re-examined the dynamic relationship between spot and futures prices in stock index futures markets using data since 1989 at weekly frequency for three major stock market indices - the S&P 500, the NIKKEI 225 and the FTSE 100 indices. In particular, we propose a nonlinear, Markov-switching vector equilibrium correction model that explicitly takes into account the mounting evidence that the conditional distribution of stock returns is well characterized by a mixture of normal distributions. Also, we use the recently developed notion of 'separation and cointegration' to provide a richer characterization of the dynamics of stock returns that explicitly allows for international spillovers across these stock index and stock index futures markets.

The empirical results provide evidence in favor of the existence of international spillovers across these major stock markets and a well-defined long-run equilibrium relationship between spot and futures prices which is consistent with mean reversion in the futures basis. Linear vector equilibrium correction models were rejected when tested against a Markov-switching vector equilibrium correction model which allows for shifts in the intercept, the autoregressive structure and the variance-covariance matrix. Our preferred nonlinear specification explains a significant fraction of the stock returns examined, with the $\overline{R}^2$ ranging from 0.08 for the NIKKEI 225 index returns to 0.12 for the FTSE 100 index returns.

Using the estimated models in an out-of-sample forecasting exercise we found that both nonlinearity and international spillovers are important in forecasting future stock returns. However, their importance is not apparent when the forecasting ability of our proposed nonlinear VECM is evaluated on the basis of conventional point forecasting criteria. In fact, these criteria neglect the fact that stock returns may be non-normally distributed and that the nonlinear models employed in this paper imply non-normal predictive densities. In order to measure more adequately the forecasting ability of our nonlinear model and discriminate among competing models we calculated hit ratios, employed tests for market timing ability and also evaluated the density forecasting performance of both linear and nonlinear models.

Overall, the evidence reported in this chapter suggests that the statistical performance of the linear (single-regime) and nonlinear (multiple-regime) models examined differs little in terms of conditional mean, regardless of whether allowance is made for international spillovers across the stock indices examined. However, the hit ratios and the tests of market timing ability as well as inspection of the predictive densities, which fully consider the higher order conditional moments implied by the various models, shows greater ability to discriminate between competing models. In particular, exploration of the model-based forecast densities indicates the rejection of single-regime models as well as multiple-regime models with no international spillovers against a multiple-regime model with international spillovers, leading us to the conclusion that both multiple regimes and the allowance for international spillovers are important ingredients for a model to produce satisfactory out-of-sample forecasting performance. The implication of our findings are further investigated in the context of a simple application to Value-at-Risk which highlights how better density forecasts of stock returns, of the type recorded in this paper, can potentially lead to substantial improvements in risk management and more precisely, to better estimates of downside risk.

In Chapter 4 we have investigated what Obstfeld and Rogoff (2000) have recently termed as the 'exchange rate disconnect puzzle' under a new perspective. Prior research in this area has largely relied on statistical measures of forecast accuracy. Our study departs from this in that we focus instead on the metric of economic value to an investor in order to assess the performance of fundamentals models. This is particularly important given the several cumbersome econometric issues that plague statistical inference in this literature. Our analysis provides the first evidence on the economic value of the exchange rate forecasts provided by an exchange rate-monetary fundamentals framework. Specifically, we compare the economic value, to a utility maximizing investor, of out-of-sample exchange rate forecasts using a monetary-fundamentals model with the economic value under a naive random walk model. We assume that our investor faces the problem of choosing how much she will invest in two assets that are identical in all respects except the currency of denomination. This problem is studied in a Bayesian framework that explicitly allows for parameter uncertainty.

Our main findings are as follows. First, each of predictability and parameter uncertainty substantially affect, both quantitatively and qualitatively, the choice between domestic and foreign assets for all currencies and across different levels of risk aversion. Specifically, exchange rate predictability (characterized using the monetary-fundamentals model) can yield optimal weights to the foreign asset that may be very different (in magnitude and, sometimes, in sign) from the optimal weights obtained under a random walk model. Parameter uncertainty causes the foreign asset allocation to fall (rise) as the horizon increases when the models predict positive (negative) weights assigned to the foreign asset, effectively making the foreign asset look more risky. Second, and more importantly, our results lend some support for the predictive ability of the exchange rate-monetary fundamentals model. This finding holds for the three major exchange rates examined in this paper using data for the modern floating exchange rate regime. The gain from using the information in fundamentals in order to predict the exchange rate out of sample (as opposed to assuming that the exchange rate follows a random walk) is often substantial, although it varies somewhat across countries. We find that the gain from using a fundamentals model is, in general, positively related to the investment horizon, negatively related to the level of risk aversion, and negatively related to parameter uncertainty. In turn, these findings suggest that the case against the predictive power of monetary-fundamentals models may be overstated.

There are a number of ways in which this study could be extended. First, one obvious concern is that our results may be sample specific. Our choice of exchange rates and sample period reflects our intention to focus on freely floating exchange rates over the post-Bretton Woods period and follows much previous research in the literature on exchange rate forecasting. Testing the robustness of our findings using other exchange rate data and/or sample periods is a logical extension. Second, we consider here a simple case where the investor allocates wealth between two assets; a more realistic scenario would be to allow for multiple assets. However, while this will require more complex estimation techniques, it would also take us away from the main point of this paper, which is to draw attention to the economic value of forecasting fundamentals models rather than only on the use of statistical metrics for forecast comparison. Third, we use a simple power utility set up to illustrate our main point. However, in the context of an international investor, the use of other utility functions, such as those that allow for ambiguity aversion or habit formation, may also be of great interest.

Although a number of questions and unresolved puzzles remain after the writing of this thesis, our work enjoys some success in shedding light on the ability of sophisticated time series models to forecast exchange rates and stock prices.

Chapter 6

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