Global hydroelastic model for liquid cargo ships

Šime Malenica[1,*], Nikola Vladimir[2], Young Myung Choi[3], Ivo Senjanović[2], and Sun Hong Kwon[4]

[1] Bureau Veritas, Neuilly sur Seine, France
[2] University of Zagreb, Faculty of Mechanical Engineerng and Naval Architecture, Zagreb, Croatia
[3] Korea Research Institute of Ships and Ocean Engineering, Daejeon, South Korea
[4] Pusan National University, Busan, South Korea

Abstract. In this paper the problem of the global hydroelastic response of the ships carrying the liquid cargo (LNG ships, tankers, FPSO-s, dry cargo ships in ballast conditions...) is considered. The potential flow assumptions are adopted for the hydrodynamic part and the resulting Boudary Value Problems (BVP) are solved using the classical Boundary Integral Equation Method (BIEM) and that both for the external (sea-keeping) and the internal (sloshing) fluid flow. The dynamic equation is solved in a fully coupled sense using the generalizd modal approach. For the time being the linear frequency domain appraoch is considered only.

Key words: *hydroelasticity; sloshing; springing*

1. Introduction

The global hydroelastic response of ships can become an important part of the total ship structural response influencing both the extreme structural response as well as the fatigue life of some structural details. This is particularly true for large ships since their natural frequencies are much lower. In the case of the dry cargo ships, such as large container ships, the overall methodology for calculating the global hydroelastic response is fairly well established and an example of these types of models is presented in [9]. The method is based on the generalized modal approach where the modal basis was chosen to be the structural natural modes calculated using the classical finite element method (FEM) and the hydrodynamic problems are solved under the potential flow assumptions using the BIEM.

In the case of the ships carrying the large amount of the liquid cargo (LNG ships, tankers, FPSO-s, dry cargo ships in ballast conditions...) the global hydroelastic response can be influenced by the dynamics of the liquid motion. Indeed, the presence of the liquid cargo does not only change the structural

*Correspondence to: `sime.malenica@bureauveritas.com`

natural frequencies but also introduce the additional resonant conditions due to the natural sloshing modes. In order to properly take into account the hydroelastic interaction in between two resonant systems (the floating body and the tanks) a fully coupled hydroelastic model should be used. The method which we chose follows the generalized modal appraoch and closely follows the theory described in [9]. The main additional technical difficulty concerns the evaluation of the sloshing hydrodynamics for arbitrary boundary conditions at the tank boundaries.

2. Theory

The initial theoretical framework for hydroelastic model of the LNG ship was presented in [4] and we briefly recall the basic principles herebelow. For the sake of simplicity the problem with zero forward speed in water of infinite depth is considered only. For the case with forward speed and finite water depth, only the seakeeping part of the hydrodynamic solution is changing and the overall coupling procedure remains the same.

The basic configuration is shown in Figure 1. The main principles of the

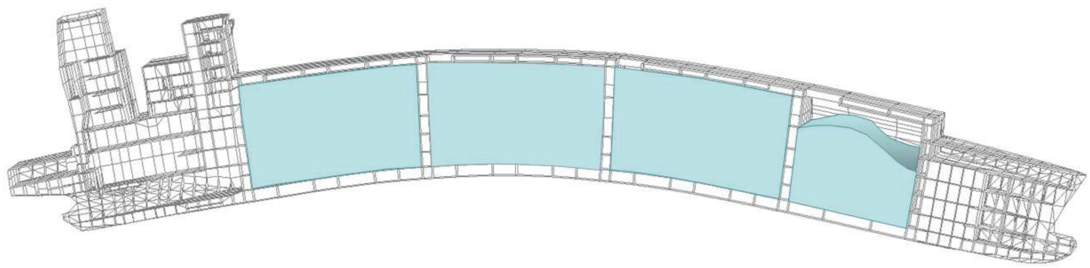

Figure 1. Basic configuration.

generalized modal approach [9] are closely followed and the original six degrees of freedom dynamic system of the floating rigid body is extended with a certain number of flexible structural modes. At the same time the linear frequency domain approach is adopted, and we formally write for the displacement of one point on the body:

$$\boldsymbol{H}(\boldsymbol{x},\omega) = \sum_{i=1}^{N} \xi_i(\omega)\boldsymbol{h}^i(\boldsymbol{x}) \tag{1}$$

where N is the total number of modes (rigid + flexible), $\boldsymbol{x} = (x, y, z)$ describes the position of one point in the structure and $\boldsymbol{h}^i(\boldsymbol{x})$ is i^{th} modal displacement vector:

$$\boldsymbol{h}^i(\boldsymbol{x}) = h^i_x(\boldsymbol{x})\boldsymbol{i} + h^i_y(\boldsymbol{x})\boldsymbol{j} + h^i_z(\boldsymbol{x})\boldsymbol{k} \tag{2}$$

where $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are the unit vectors defining the global coordinate system.
It should be kept in mind that the modal definition formally includes also the

local rotation angles (ϕ, θ, ψ) and we can write:

$$\boldsymbol{h}_r^i(\boldsymbol{x}) = h_\phi^i(\boldsymbol{x})\boldsymbol{i} + h_\theta^i(\boldsymbol{x})\boldsymbol{j} + h_\psi^i(\boldsymbol{x})\boldsymbol{k} \tag{3}$$

2.1. Structural dynamics

In order to calculate the flexible natural modes of the dry structure the finite element method (FEM) is usually employed. In practice, two types of models are usually employed: the nonuniform beam model and the 3D finite element model. Indeed, the fact that we are interested in the global body behavior only, and we consider only the limited number of modes, the nonuniform beam approximation shows to be sufficiently accurate for many ship types. The main and only advantage of using the beam model lies in its simplicity. For some ship types (container ships, some passenger ships, multihull ships ...) the simplified beam model become rather innacurate and the important non-trivial modifications need to be performed in order to improve the accuracy [8]. The alternative solution is to use the 3D FE model of the ship which is, at least theoretically, representative for any type of ships. The obvious disadvantage of this method is related to the practical difficulties of producing the 3D FE model of the complex ship structure. Whatever the method used for evaluating the dynamic characteristics of the dry structure, the general dynamic equation for the structural response under arbitrary external loading can be written in the following form:

$$\left\{-\omega^2[\,\mathbf{m}\,] - i\omega[\,\mathbf{b}\,] + [\,\mathbf{k}\,]\right\}\{\boldsymbol{\xi}\} = \{\boldsymbol{F}\} \tag{4}$$

where:

$[\,\mathbf{m}\,]$	-	modal mass
$[\,\mathbf{k}\,]$	-	modal structural stiffness
$[\,\mathbf{b}\,]$	-	modal structural damping
$\{\xi\}$	-	modal amplitudes
$\{\boldsymbol{F}\}$	-	external forces (modal)

The dimension of the system is equal to the number of flexible modes N_{flex}.

2.2. Hydrodynamics

2.2.1. Seakeeping

Within the potential flow theory the body boundary condition for the total potentail φ states that the total normal velocity is equal to zero:

$$\frac{\partial \varphi}{\partial n} = 0 \tag{5}$$

where φ stands for the total velocity potential which can be decomposed into the incident part φ_I and the perturbation part φ_P:

$$\varphi = \varphi_I + \varphi_P \tag{6}$$

Thanks to the assumptions of linearity, the perturbation part can be further decomposed into the diffracted part φ_D which is independent of the body motions/deformations and the radiation parts φ_{Rj} which accounts for the body motions:

$$\varphi_P = \varphi_D - i\omega \sum_{j=1}^{N} \xi_j \varphi_{Rj} \tag{7}$$

This leads to the following body boundary conditions for each potential:

$$\frac{\partial \varphi_D}{\partial n} = -\frac{\partial \varphi_I}{\partial n} \quad , \quad \frac{\partial \varphi_{Rj}}{\partial n} = \boldsymbol{h}^j \boldsymbol{n} \tag{8}$$

In addition to the body boundary condition, the velocity potential should also satisfy the Laplace equation in the fluid, adequate free surface condition and the radiation condition at infinity. The BVP for each of the potentials has the same form and only the body boundary condition changes. We can write for the generic potential φ the following BVP:

$$\left.\begin{aligned} &\Delta\varphi = 0 && \text{in the fluid} \\ &-\nu\varphi + \frac{\partial \varphi}{\partial z} = 0 && z = 0 \\ &\frac{\partial \varphi}{\partial n} = V_n && \text{on } S_B \\ &\lim\Big[\sqrt{\nu r}\Big(\frac{\partial \varphi}{\partial r} - i\nu\varphi\Big)\Big] = 0 && r \to \infty \end{aligned}\right\} \tag{9}$$

where ν is the infinite depth wave number $\nu = \omega^2/g$, r is the radial distance, S_B is the wetted body surface and the normal velocity V_n is given by (8).

The above BVP's are solved using the Boundary Integral Equation (BIE) method based on source formulation. Within the source formulation, the potential at any point in the fluid is expressed in the following form:

$$\varphi = \int_{S_B} \sigma G dS \tag{10}$$

where G stands for the Green function, and σ is the unknown source strength which is found after solving the following integral equation:

$$\frac{1}{2}\sigma + \int_{S_B} \sigma \frac{\partial G}{\partial n} dS = V_n \quad , \quad on \quad S_B \tag{11}$$

This equation is solved numerically, after discretizing the wetted part of the body into a number of flat panels over which the constant source distribution is assumed. Pressure is calculated using the linearized Bernoulli equation according to which, the total hydrodynamic pressure is composed of the part associated with the variation of the hydrostatic pressure and the part associated with the time derivative of the velocity potential:

$$p = -\varrho g \Delta z + i\omega\varrho\varphi_T \tag{12}$$

where Δz denotes the vertical displacement of one point on the body and φ_T is the total velocity potential (6).

After projecting the pressure on different motion/deformation modes, and integrating it over the body surface, each part of the pressure will give the corresponding hydrodynamic forces. It is a common practice to decompose the resulting total hydrodynamic force $\{\boldsymbol{F}\}$ in the following form:

$$\{\boldsymbol{F}\} = \{\boldsymbol{F}^{DI}\} + \left\{\omega^2[\,\mathbf{A}\,] + i\omega[\,\mathbf{B}\,] - [\,\mathbf{C}\,]\right\}\{\boldsymbol{\xi}\} \tag{13}$$

where:

$\{\boldsymbol{F}^{DI}\}$	-	excitation force vector
$[\,\mathbf{A}\,]$	-	hydrodynamic added mass matrix
$[\,\mathbf{B}\,]$	-	hydrodynamic damping matrix
$[\,\mathbf{C}\,]$	-	hydrostatic restoring matrix
$\{\boldsymbol{\xi}\}$	-	modal amplitudes

The hydrodynamic coefficients associated with the velocity potential are defined as follows:

$$F_i^{DI} = i\omega\varrho \int_{S_B} (\varphi_I + \varphi_D)\boldsymbol{h}^i \boldsymbol{n} dS \tag{14}$$

$$\omega^2 A_{ij} + i\omega B_{ij} = \varrho\omega^2 \int_{S_B} \varphi_{Rj} \boldsymbol{h}^i \boldsymbol{n} dS \tag{15}$$

Concerning the hydrostatic restoring matrix, the situation is more complicated as discussed in [5]. The total restoring matrix C_{ij} is decomposed into the pure hydrostatic part C_{ij}^H and the gravity part C_{ij}^g. The hydrostatic part is obtained after accounting for the change of the hydrostatic part of the pressure, when integrating it over the instantaneous wetted body surface which also changes:

$$\tilde{\boldsymbol{F}}_i^{Hs} = -\iint_{\tilde{S}_B} z\tilde{\boldsymbol{h}}^i \tilde{\boldsymbol{n}} d\tilde{S} = -\iint_{S_B+\delta S_B} (Z+\delta Z)(\boldsymbol{h}^i + \delta\boldsymbol{h}^i)[\boldsymbol{n}dS + \delta(\boldsymbol{n}dS)] \tag{16}$$

where the sign " ˜ " is used to denote the instantaneous values and δ denotes the change of the corresponding quantity due to the body motion/distortion. The final expression for the restoring matrix is obtained after introducing the notion of the deformation gradient $\underline{\underline{\nabla_X \boldsymbol{h}^i}}$ (see [5] for details):

$$C_{ij}^H = \iint_{S_B} \left\{Z[\nabla_X \boldsymbol{h}^j \boldsymbol{h}^i \cdot \boldsymbol{n} + (\underline{\underline{\nabla_X \boldsymbol{h}^i}} \cdot \boldsymbol{h}^j - \underline{\underline{\nabla_X \boldsymbol{h}^j}} \cdot \boldsymbol{h}^i)\cdot \boldsymbol{n}] + h_Z^j \boldsymbol{h}^i \boldsymbol{n}\right\} dS \tag{17}$$

At the same time, the gravity part of the stiffness C_{ij}^g is obtained as:

$$C_{ij}^g = g \iiint_{V_B} (\boldsymbol{h}^j \nabla_X) h_Z^i dm \tag{18}$$

2.3. Hydroelastic coupling without internal liquids

The coupled hydroelastic equation is easily obtained after combining the hydrodynamic loading (16) and the structural dynamic equation (4). The following final expression is obtained:

$$\left\{-\omega^2([\,\mathbf{m}\,]+[\,\mathbf{A}\,]) - i\omega([\,\mathbf{b}\,]+[\,\mathbf{B}\,]) + ([\,\mathbf{k}\,]+[\,\mathbf{C}\,])\right\}\{\boldsymbol{\xi}\} = \{\boldsymbol{F}^{DI}\} \qquad (19)$$

It is important to mention that the dimension of the system is $N = 6 + N_{flex}$ contrary to (4) where the dimension is N_{flex}. This is because the rigid body modes, which can not be obtained from the structural modal analysis because of the zero stiffness, were added to the system of equations. That is why this approach is called generalized modal approach and therefore can also be used for rigid body case with the following definition of the mode shapes:

$$\boldsymbol{h}^1 = \boldsymbol{i} \quad , \quad \boldsymbol{h}^2 = \boldsymbol{j} \quad , \quad \boldsymbol{h}^3 = \boldsymbol{k} \qquad (20)$$

$$\boldsymbol{h}^4 = \boldsymbol{i} \wedge (\boldsymbol{R} \wedge \boldsymbol{R}_G) \quad , \quad \boldsymbol{h}^5 = \boldsymbol{j} \wedge (\boldsymbol{R} \wedge \boldsymbol{R}_G) \quad , \quad \boldsymbol{h}^6 = \boldsymbol{k} \wedge (\boldsymbol{R} \wedge \boldsymbol{R}_G) \qquad (21)$$

where $\boldsymbol{R}$ is the position vector and $\boldsymbol{R}_G$ defines the center of gravity.
The above equation (19) represents the final fully coupled hydroelastic equation of the body. The solution of this equation gives the modal amplitudes from which all the relevant quantities can be obtained by simple summation of the different modal contributions.

2.4. Sloshing

The procedure for the inclusion of the sloshing dynamics is very similar to seakeeping. Indeed, the additional "flexible" sloshing BVP's are defined by projecting the mode shapes onto the internal wet structure of the tank:

$$\frac{\partial \varphi^{T}_{Rj}}{\partial n} = \boldsymbol{h}^j \boldsymbol{n} \qquad (22)$$

Special care should be given to the free surface condition inside the tank. Here we follow the method proposed in [3] and we write:

$$-\nu\varphi + \frac{\partial \varphi}{\partial z} = -i\omega\zeta_v^A \qquad (23)$$

Due to the arbitrary tank deformation which should be assumed, the vertical displacement of the waterplane ζ_v^A can not be expressed simply as in [3] because the volume of the tank can also change. According to Figure 2. we can formally write the following expression for the conservation of the liquid volume:

$$\Delta V - S_W \zeta_v^A = 0 \qquad (24)$$

where ΔV is the change of the tank position and the tank volume due to the motions and deformations, and S_W is the initial waterplane area.

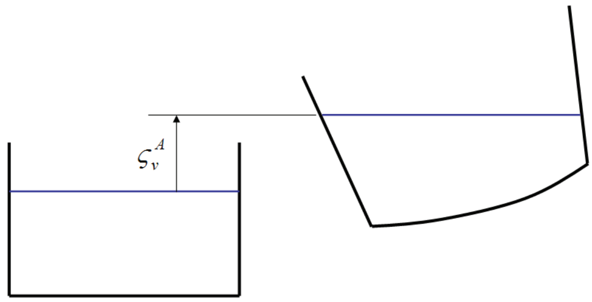

Figure 2. General tank motion/deformation.

The waterplane area can be easily calculated, and for the change of the tank position and volume we can write:

$$\Delta V = \sum_{i=1}^{N} \xi_i \iint_{S_T} \boldsymbol{h}^i \boldsymbol{n} dS \tag{25}$$

where S_T denotes the wetted surface of the tank.
In order to simplify the notations let us write the above expressions in a more compact form:

$$\zeta_v^A = \sum_{i=1}^{N} \xi_i \zeta_v^{Ai} \quad , \quad \zeta_v^{Ai} = \frac{\iint_{S_T} \boldsymbol{h}^i \boldsymbol{n} dS}{S_W} \tag{26}$$

With this in mind the final BVP for the sloshing potential becomes:

$$\left.\begin{aligned} &\Delta\varphi_{Rj}^{T} = 0 && \text{in the fluid} \\ &-\nu\varphi_{Rj}^{T} + \frac{\partial\varphi_{Rj}^{T}}{\partial z} = \zeta_v^{Aj} && z = 0 \\ &\frac{\partial\varphi_{Rj}^{T}}{\partial n} = \boldsymbol{h}^j \boldsymbol{n} && \text{on } S_T \end{aligned}\right\} \tag{27}$$

In principle, the same BIE method as that for seakeeping can be used to solve this BVP. However, due to the fact that we are using the idealised potential flow assumptions in closed domain some precautions are necessary. Indeed, the potential flow approach in the closed fluid domain leads to the infinite solution for the potential around the natural frequencies of sloshing. It is clear that the infinite solution is due to the fact that we used the idealized potential flow assumptions and in reality the physical dissipation (wave breaking, viscous effects...) will make the response finite around the resonance. The detailed analysis of the real sloshing dynamics is beyond the scope of the present work, and can not be done consistently using the potential flow theory, so that more sophisticated numerical approaches based on the solution of the Navier Stokes equations are necessary. Since our goal is not to model in details the sloshing phenomena, but just its global effects, what we propose here is an approximate solution based on the introduction of the artificial damping in the free surface condition. For that purpose the free surface condition is modified as follows:

$$\frac{\partial \varphi}{\partial z} = \nu \varphi + i\epsilon_{FS} \varphi \tag{28}$$

where ϵ_{FS} is the artificial damping coefficient to be calibrated by comparisons with model tests or with the dedicated CFD simulations.
This modification of the free surface condition leads to the modified Boundary Integral Equations and requires the meshing of the internal free surface too thus increasing the total number of unknowns. Within this approach the potential representation remains the same, as for the case without the artificial damping (10), but the BIE changes to:

$$\frac{1}{2}\sigma + \int_{S_T+S_{FS}} \sigma \frac{\partial G}{\partial n} dS = V_n \quad on \quad S_B \tag{29}$$

$$\sigma + i\epsilon_{FS} \int_{S_T+S_{FS}} \sigma G dS = V_n \quad on \quad S_{FS} \tag{30}$$

where S_T denotes the wetted tank surface and S_{FS} the waterplane surface.
Once the above BVP solved, the corresponding pressure can easily be calculated and integrated over the wet surface of the tank. This results in the additional added mass and damping effects defined by:

$$\omega^2 A_{ij}^T + i\omega B_{ij}^T = \varrho \omega^2 \int_{S_T} \varphi_{Rj}^T \boldsymbol{h}^i \boldsymbol{n} dS \tag{31}$$

Now we turn to the evaluation of the hydrostatic restoring part due to liquid cargo.First of all, it is important to mention that the calculation of the hydrostatic restoring, for the seakeeping part, does not change when the tanks are included, and we can directly use (17,18). However, when calculating C_{ij}^g, the volume integral should exclude the mass of the liquid in the tanks!
Concerning the hydrostatic restoring related to the liquid cargo, we proceed in a similar way and we write for the instantaneous hydrostatic loading:

$$\begin{aligned} \tilde{\boldsymbol{F}}_{ij}^{Hs} &= -\varrho g \iint_{\tilde{S}_T} (z - \zeta_v^{Aj} - Z_0^T) \tilde{\boldsymbol{h}}^i \tilde{\boldsymbol{n}} d\tilde{S} \\ &= -\varrho g \int\int_{S_T+\delta S_T} (Z - \zeta_v^{Aj} - Z_0^T + \delta Z)(\boldsymbol{h}^i + \delta \boldsymbol{h}^i)[\boldsymbol{n} dS + \delta(\boldsymbol{n} dS)] \end{aligned} \tag{32}$$

After few manipulations, similar to those used for seakeeping part, the expression for the hydrostatic restoring of the tank becomes:

$$\begin{aligned} C_{ij}^T &= \varrho g \int\int_{S_T} \Big\{ (Z - Z_0^T)[\nabla_X \boldsymbol{h}^j \boldsymbol{h}^i \cdot \boldsymbol{n} + (\underline{\underline{\nabla_X \boldsymbol{h}^i}} \cdot \boldsymbol{h}^j - \underline{\underline{\nabla_X \boldsymbol{h}^j}} \cdot \boldsymbol{h}^i) \cdot \boldsymbol{n}] \\ &\quad + (h_Z^j - \zeta_v^{Aj}) \boldsymbol{h}^i \boldsymbol{n} \Big\} dS \end{aligned} \tag{33}$$

where Z_0^T denotes the initial vertical position of the free surface in the tank.

We can now write the total modal force vector $\{\boldsymbol{F}\}^T$ due to sloshing:

$$\{\boldsymbol{F}\}^T = \left\{\omega^2[\ \mathbf{A}\ ]^T + i\omega[\ \mathbf{B}\ ]^T - [\ \mathbf{C}\ ]^T\right\}\{\boldsymbol{\xi}\} \tag{34}$$

where:

$[\ \mathbf{A}\ ]^T$	-	tank added mass matrix
$[\ \mathbf{B}\ ]^T$	-	tank damping matrix
$[\ \mathbf{C}\ ]^T$	-	tank restoring stiffness matrix
$\{\boldsymbol{\xi}\}$	-	body modal amplitudes [same as in (19)]

This force can be simply added to the dynamic motion equation (19) and there is no need for any special coordinate transformations, as it was the case in [3].

3. Simplified coupling model

We discuss here below a simplified model which can be used to describe the sloshing part of the problem. The main advantage of the simplified model is that we can use the existing codes and there is no need for further developments! However, for the moment it is not completely clear how accurate is this model and this should be carefully investigated.

Anyhow, the idea is to take into account only the rigid body motion of the tanks and disregard the local tank deformations. In that respect we write for the motions of the representative point of the tank and for the corresponding simplified modes:

$$\boldsymbol{H}(\boldsymbol{x}_T,\omega) = \sum_{j=1}^{N} \xi_j(\omega)\boldsymbol{h}^j(\boldsymbol{x}_T) \quad , \quad \boldsymbol{h}^j(\boldsymbol{x}) = \sum_{i=1}^{6} h_i^j(\boldsymbol{x}_T)\boldsymbol{h}^{Ri}(\boldsymbol{x}) \tag{35}$$

where it should be noted that the above expression formally includes both the translations (2) and the rotations (3) i.e. $h_i^j = (h_x^j, h_y^j, h_z^j)$ for $i = 1,3$ and $h_i^j = (h_\phi^j, h_\theta^j, h_\psi^j)$ for $i = 4,6$. The local rigid body modes $\boldsymbol{h}^{Ri}(\boldsymbol{x})$ define the motion of the tank with respect to $\boldsymbol{x}_T$.

Concerning the hydrodynamic part of the problem we can write for the sloshing potential the following expressions:

$$\varphi_{Rj}^{T} = \sum_{i=1}^{6} h_i^j(\boldsymbol{x}_T)\varphi_{Ri}^{R} \tag{36}$$

where φ_{Ri}^{R} represent the rigid body sloshing potential defined by (46) in [3].

With this in mind, the expression for the added masses becomes:

$$\begin{aligned} A_{ij}^{T} &= \varrho \iint_{S_T} \varphi_{Rj}^{T}\boldsymbol{h}^i\boldsymbol{n}dS = \varrho\sum_{k=1}^{6}\sum_{l=1}^{6} h_k^i(\boldsymbol{x}_T)h_l^j(\boldsymbol{x}_T)\iint_{S_T}\varphi_{Rl}^{R}\boldsymbol{h}^{Rk}\boldsymbol{n} \\ &= \varrho\sum_{k=1}^{6}\sum_{l=1}^{6} h_k^i(\boldsymbol{x}_T)h_l^j(\boldsymbol{x}_T)A_{kl}^{R} \end{aligned} \tag{37}$$

where A^{R}_{kl} is the rigid body added mass of the tank defined by (49) in [3]. Finally, concerning the hydrostatic restoring matrix, the expression (33) remains valid provided that the above defined mode shapes (35) are used.

4. Few results

4.1. Vertical column with the sloshing tank at the top

First we consider the simplified case of the vertical circular cylinder with the liquid filled tank at the top as shown in Figure 3. We assume that the tank is

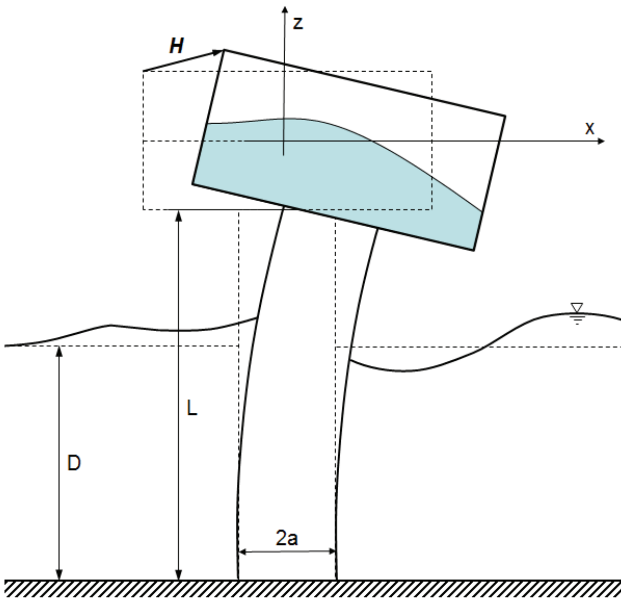

Figure 3. Vertical cylinder with liquid filled tank at the top.

rigid and we use simplified cupling model from Section 3. As already indicated, in this case we need to know only the rigid body added mass matrix of the tank with respect to the reference point. For the rectangular tank, the added mass can be calculated analytically using the methodology proposed either in [2] or [7]. The difference in between the two approaches is related to the decomposition of the total velocity potential.
In [2] authors decompose the total potential as follows:

$$\varphi^{R}_{Rj} = \Omega_j + \phi_j \tag{38}$$

where Ω_j represents the so called Stoke Joukowski potential which satisfies the rigid body boundary conditions at all fluid boundaries i.e. including the free surface while the potential ϕ_j satisfies the homogeneous Neumann boundary condition at the rigid boundaries and the following non homogeneous free surface boundary condition:

$$-\nu\varphi_j + \frac{\partial \varphi_j}{\partial z} = \zeta_v^{Aj} - h_z^j + \nu\Omega_j \tag{39}$$

It is easy to show that the sum of the two potentials satisfies the correct boundary conditions at all boundaries.
In [7] the decomposition of the total potential was made differently:

$$\varphi^{R}_{Rj} = \tilde{\phi}_j + \psi_j \tag{40}$$

where $\tilde{\phi}_j$ represents the infinite frequency radiation potential satisfying the rigid body boundary conditions at the tank boundaries and the infinite frequency free surface condition i.e.:

$$\tilde{\phi}_j = 0 \quad , \quad \text{at} \quad z = 0 \tag{41}$$

The potential ψ_j complement the solution by satisfying the homogeneous Neumann boundary condition at the rigid boundaries and the following non homogeneous free surface boundary condition:

$$-\nu\psi_j + \frac{\partial \psi_j}{\partial z} = \zeta_v^{Aj} - \frac{\partial \tilde{\phi}_j}{\partial z} \tag{42}$$

Once again, it is easy to show that the sum of the two potentials satisfies the correct boundary conditions at all boundaries.
It is clear that the two methods of solutions are completely equivalent but the second method has some advantages when formulating the problem in time domain.

Numerical example

The characteristics of the vertical cylinder are: cylinder radius is $a = 10m$, uniformly distributed mass along the length of the cylinder is the half of the displaced mass, a concentrated mass m_0 at the top of the cylinder (free surface level) is equal to the total displaced mass (liquid mass of the tank included), the stiffness of the cylinder is chosen such that the ratio EI/L^3 is equal to $0.41 m_0 s^{-2}$. Two sets of tank dimensions are chosen : (length$\times$ breadth $\times$ water level) = $(30\times30\times10)$ and $(23\times23\times10)$. In Figure 4. we present the hydroelastic response of the cylinder (motion and slop of the top of the cylinder) with and without the presence of the tank. As expected we can observe the important influence of sloshing on the global system dynamics, especially close to the resonant tank modes. Indeed, when the wave frequency is equal to the first sloshing frequency of the tank the total response becomes zero. In addition, and as expected, the presence of the tank change the natural frequencies of the system and, in this particular cases, we can observe two separate peaks in the response contrary to the case of the "frozen liquid". Finally we can observe good agreement in between the semi-analytical and numerical results which confirms the correct implementation of the method in *Homer* software.

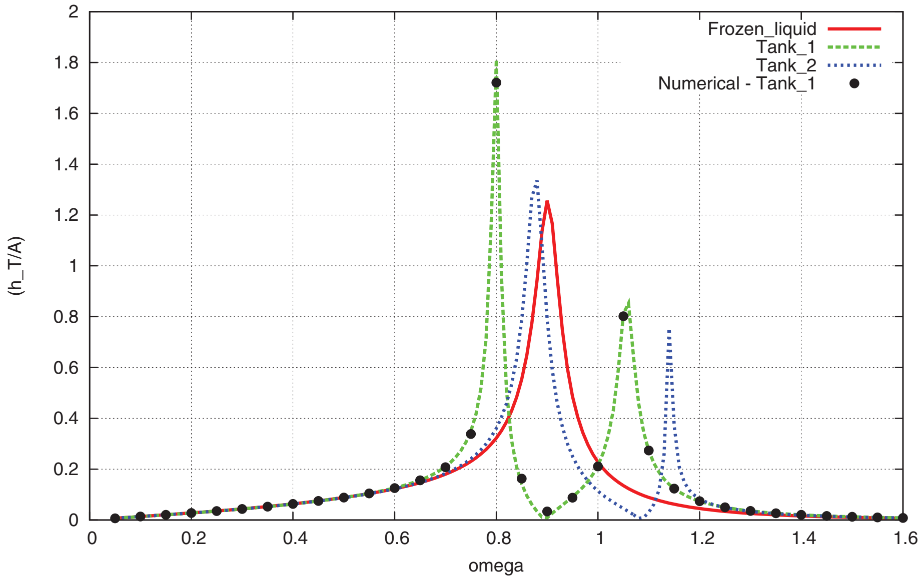

Figure 4. Deformation modes of the column without tank and linear RAO of the motion of the column top.

5. Conclusions and further work

We have presented here a method to take into account the influence of the sloshing on the global structural dynamics of the floating body. Both semi-analytical solution and the numerical one were built and they showed good agreement. The further work will consist in applying the method for general parctical cases such as the LNG ships (Figure 5.).

Acknowledgments

The authors acknowledge the support of the National Research Foundation of Korea (NRF) grant funded by the Korea Government (MSIP) through GCRC-SOP.

References

[1] Choi Y.M. & Malenica S. 2013. : "Semi analytical solution for second order hydroelastic response of the vertical circular cylinder in monochromatic water waves.", 28th IWWWFB, L'isle sur la Sorgue, France

[2] Faltinsen O. M. & Timokha A. 2009. : "*Sloshing*" Cambridge University Press

[3] Malenica S., Zalar M. & Chen X.B. 2003. : "Dynamic coupling of seakeeping and sloshing.", In Proceedings of ISOPE Conf., Hawaii, USA

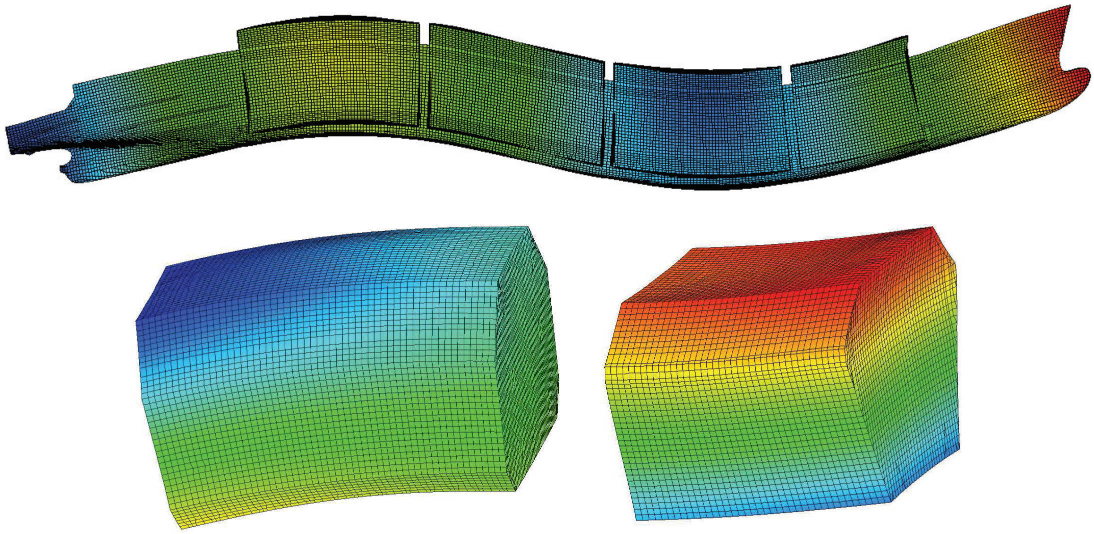

Figure 5. Modal deformations of the 175KLNG ship and the local deformation of the tank No 1 and the tank No 4.

[4] Malenica S, Bigot F., Chen X.B. & Bralic S. 2011. : "Global hydroelastic model for LNG ships.", 26th IWWWFB Athens, Greece.

[5] Malenica S, Molin B., Tuitman J.T., Bigot F. & Senjanovic I. 2009. : "Some aspects of hydrostatic restoring for elastic bodies.", 24th IWWWFB Saint Petersburg, Russia.

[6] Malenica S., Choi Y.M., Vladimir N., Kwon S.H. & Chen X.B. 2014. : "Wave induced hydroelastic behavior of the vertical circular cylinder with liquid filled tank at the top.", 29th IWWWFB, Osaka, Japan

[7] Molin B., Remy F., Rigaud S. & De Jouette Ch. 2002. : "LNG FPSO's: Frequency domain, coupled analysis of support and liquid cargo motions.", IMAM2002 Conference, Crete, Greece

[8] Senjanovic I., Vladimir N., Tomic M., Hadzic N. & Malenica S. 2014.: Global hydroelastic analysis of ULCS by improved beam structural model, International Journal of Naval Architecture and Ocean Engineering, Vol. 6, No. 4

[9] Tuitman J.T. & Malenica S. 2009. : "'Fully coupled seakeeping, slamming and whipping calculations."', Journal of Engineering for Maritime Environment. Vol 223, No M3