Security Price Process Models: Do These Have the Correct Properties For Understanding Options Values?

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Standing on the Shoulders of Giants:

To my friends Michael Selby and Stewart Hodges

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Acknowledgements

From the day I began this research in 1988, my path has never been straight or easy. However, for this entire period one man has consistently been my guide and helped me move forward: Dr. Michael Selby. For this reason, I co-dedicate this research to him. Even though Michael ceased to be my PhD. advisor when he left the London School of Economics, he devoted his valuable time to my research. At any time, Michael would drop his own research and spend hours on the telephone or at his flat providing me guidance or cheering me up when my spirits sank. Michael also assisted me in providing the derivation in Appendix 1.1 that demonstrates the relationship between the log stochastic volatility model and the Hull and White stochastic volatility model. When I had problems modelling the three stochastic volatility models examined in this research, Michael helped me find the solution and write the appropriate computer code. The debt owed to him is incalculable.

Later, when I shifted my registration from the London School of Economics to the University of Warwick another individual took on the task of guiding my research. To him, I have also co-dedicated my research and without the help of Stewart Hodges, I have little doubt this work would never have been completed. Being a part-time PhD. candidate imposes special pressures both on the candidate and the supervisor. Since it was not always feasible to meet Stewart at Warwick, there were innumerable times that I was required to telephone him at home (and I apologise to his wife Barbara) or to meet at my offices in London. There was never an instance where Stewart did not have time for me and this constant level of support made it possible to complete this research in the remarkably short time of two years.

Along the way, a number of other individuals have either provided me assistance or provided insights. My friend Sheldon Natenberg provided me with the insight to compare the objective to the risk neutral dispersion processes using a Monte Carlo simulation. On one of his trips to London, he stopped at my office and explained that he had used a similar approach to determine the expected smile shapes and this was a crucial element in his trading activities. Also, his method of standardising strike prices when estimating the standardised implied volatility smiles led to the approach used in this research.

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In retrospect, it is clear that the effort and expense required to complete this research was much higher than I could have possibly imagined. The only way to complete this research (in a timely manner) was to allow all other projects to have a lower priority. Thus, over the last two years, my business has effectively been run down. This led most of my staff to find alternative employment, as it was not possible to supervise their activities. However, almost all my staff played a key role in obtaining and cleaning the data we eventually used for the research. My current employees, Paolo Fornasiero and Beth Ragheb, have been absolutely critical in the completion of the research. While I personally wrote all the computer code for all this research, Paolo made sure that it would work efficiently and more importantly, would work on our limited computer resources. While I typed the entire manuscript, Beth diligently proof-read the Chapters and generally kept the company afloat allowing me to devote my time exclusively to research.

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Declaration

The methodology for the standardisation of the implied volatility surfaces that is outlined in Chapter 7 has already appeared in Tompkins (1994). The results for the determination of the objective dispersion processes and the standardisation of the implied volatility surfaces (solely for the four stock index options) will appear in a Chapter titled ***Measuring Equity Volatilities*** in the book ***Equity Derivatives*** that is due to be published by RISK publications shortly. Portions of Chapters 1 and 7 will appear in my new book, ***EXOTIC OPTIONS***, which will be published by John Wiley & Sons in 1998. This book will be co-authored by Peter James and David DeRosa.

Finally, I had a paper recently published in the journal **NETEXPOSURE** (November 1997). The title of this paper is: ***Static versus Dynamic Hedging of Exotic Options: An Evaluation of Hedge Performance via Simulation.*** This paper relied heavily upon the results of Chapter 4 of this research. In this paper, I investigated the hedging costs for a variety of contingent claims assuming stochastic volatility (and transaction costs). The simulations used to estimate these hedging costs were drawn from the simulations in Chapter 4. In addition, the choice of the appropriate stochastic volatility model and the parameter values were based on the results of this Chapter.

Robert George Tompkins

Abstract

It is well known that the market price of options are inconsistent with option pricing models that assume the innovation of the underlying price follows Geometric Brownian Motion. What is not clear is why this occurs.

The testing of option pricing models requires a joint hypothesis to be tested that the options pricing models are correct and that the options markets are efficient. To test the option pricing models, we will examine the relationship between the objective and risk neutral dispersion processes for twelve financial futures markets. These markets have been selected so that we can investigate the dynamics of equity, fixed income and foreign exchange asset classes.

Our analysis of the objective dispersion processes allows us to reject the hypothesis that the prices of these twelve markets follow Geometric Brownian motion. For all twelve markets and for various sub-periods of analysis, we find that the optimal models for capturing the dynamics of the objective dispersion process include jump diffusion and stochastic volatility.

For the risk neutral dispersion process, we chose to examine the implied volatility surfaces from the closing prices of options (on these same futures markets). It appears that both within and between markets similar dynamics determine the shapes of the implied volatility surfaces. By employing an Analysis of Covariance approach, we found that important consistencies exist within asset classes and between markets. The first order strike price effect (skewness) differs widely among markets but is fairly consistent within the same asset class. The second order strike price effect (kurtosis) is consistent among all markets. The dependency of both strike price effects on the time to expiration is also similar across all markets and suggests that both jump diffusion and stochastic volatility play a role.

A comparison of option prices, which are consistent with the objective dispersion process, with actual options prices suggests that significant divergences exist. The actual smile patterns display greater variation in the amplitude compared to those from the objective function. The least degree of discrepancy exists for the foreign exchange markets. Both the stock index and fixed income options dispersion processes display behaviours that diverge considerably. This divergence is primarily due to the existence of negative skews that are not justified by the objective dispersion processes. This suggests that other mechanisms are at work for the risk neutral dispersion processes for these asset classes. The most likely explanations are the existence of risk premia associated with stochastic volatility and non-diversifiable jumps or that transaction costs are relevant.

INTRODUCTION

This research is a direct consequence of two events that occurred at almost the same moment in time and at the same place. In the spring of 1973 in Chicago, the Chicago Board Options Exchange opened an organised market in equity options. For the first time in the modern era, option prices (for what had to that point been an obscure and relatively unimportant security) were transparent and an empirical record of their price dynamics could be recorded with accuracy. Secondly, at the University of Chicago, the Journal of Political Economy published the seminal work on option pricing by Black and Scholes (1973).

This seeming coincidence spurred the development of a major branch of financial economics: contingent claims analysis. These two simultaneous events meant that the analysis of contingent claims could extend along two paths: one theoretical and one empirical. There is little doubt that this field experienced rapid advances in such a short period of time due to the interaction of both research approaches. This was due to the fact that the development of theoretical models could then be compared to actual market prices. With this feedback mechanism, both theoreticians and practitioners could benefit from iterative refinement in both areas.

Likewise, in this research, we must choose which path we will take in the investigation of option pricing. Our choice is the empirical route. It is clear that the actual option prices suggest that the original model by Black and Scholes (and subsequent models using similar assumptions) do not capture the dynamics of the options markets. One of the most serious violations of these models implies that the market prices of options appear to be inconsistent with the assumption that the underlying price process follows Geometric Brownian motion. This assumption lies at the heart of the Black Scholes model. The clearest indication of this is that the standard deviations implied by options prices vary for the same expiry depending upon the strike price and also vary across time. These effects have been referred to as the strike price effect or the term structure of volatility and have led many to question whether the Black Scholes model is correct. As with any feedback process, this had led theoreticians to modify the Black Scholes approach to explain the actual market dynamics. One reason why options prices were thought to diverge from Black Scholes values was that the assumed dispersion process for the underlying asset price was incorrect.

Given the mathematical elegance (and tractability) of the Black Scholes approach, much of the early research concentrated on the development of revised models which assuming other distributional forms for the underlying price innovation. As long as the contingent claim and the underlying asset were perfectly substitutable and investors preferred more to less, the pricing of the options would be preference-free. However, for many of these approaches, the models [such as the CEV model of Cox and Ross (1976)] still failed to explain the actual options prices. Again, one possible reason for this failure was that the alternative dispersion processes for the underlying asset remained incorrect.

Thus, our first objective for this research will be to examine the observed (or objective) dispersion process for twelve financial futures markets. This will allow us to examine how well alternative models describe these dynamics. With these models, we can now compare what option prices would be if they were consistent with our best fitting models. If we simply change the drift, we can determine some set of options prices which could be described by the objective processes. Since the effect of realistic levels of risk premia (for volatility smiles) is likely to be small, it is interesting to examine the structure of prices from this procedure.

Another possible explanation for why option prices may differ from these models is that the nature of risk premia are more complicated than we thought. Alternatively, transaction costs may have an impact or the markets may be inefficient. For example, the assumption of constant variance in the Black Scholes model may be incorrect and thus risk premia exists in the option prices. This could suggest that perfect substitutability may not exist for the option and the underlying asset and option pricing may not be preference-free. To assess if this is the case or not, we will examine the empirical record of options prices on these same twelve financial futures markets and compare the risk-neutral to the objective dispersion processes.

Our next objective is to understand the dynamics of the risk-neutral dispersion processes implied by options prices. This will entail standardisation of implied volatilities and comparison of the implied volatility surface both across time and across markets. We will then examine the implied volatility surfaces that are consistent with the objective dispersion processes and compare these to the actual implied volatility surfaces. The motivation for this to examine the degree to which options prices diverge from the assumptions of the Black Scholes model and ascertain as to why this may occur. Our focus is to compare the objective processes and the empirical option prices. Once we have determined what process best describes the movements of asset prices, we will examine if these processes describe options prices without the need for potentially complicated risk premia or the consideration of transaction costs. The answer is that these processes alone do capture most of the dynamics of options prices.

While it is clear that empirical investigations of options prices have appeared in the literature, the research provides a major contribution by extending this work. We will examine simultaneously a wider variety of asset classes at the same points in time. Secondly, much of the empirical work on options pricing has concentrated on the markets in the United States and primarily the stock option market. It is not clear if the conclusions of these papers apply equally across other asset classes or for options on non-U.S. markets. This research will rectify this by examining stock index, fixed income and foreign exchange options and futures markets, where half of the markets are not U.S. Dollar based. Given that many of these markets have appeared recently, such a cross-sectional examination of the empirical dynamics could only be completed at this time when a sufficiently long empirical record had been established. It is also important that we have created an unprecedented data resource both for this research and for future research. We have obtained and cleaned 33,239 futures prices and 1,263,317 options prices for twelve major financial markets. The research that follows was only possible by creating this data resource with sufficient observations to allow for long-term analysis of the dynamics of these processes.

The goals for the first five Chapters are threefold. Initially, this research will discern the nature of the objective dispersion processes which can be observed for return series for futures contracts on three categories of financial assets: fixed income, stock indices and foreign exchange. The primary goal is to understand those statistical properties of futures returns most relevant for option pricing.

Secondly, to capture those statistical properties of relevance we will apply an existing methodology in a new way. Burghardt and Lane (1990) identified the use of volatility cones. Their principal motivation was to assess if options prices were mis-priced. We will also use this method. Our motivation is to understand the variability of unconditional (historical) volatilities as a function of the time horizon of estimation. This of particular relevance to option pricing, as those who price options must make a forecast of the actual volatility that will occur over the life of the option and the volatility cone analysis can provide an indication of the degree of forecast error. One contribution of this research is to provide a new methodology to correct the biases in the volatility cone method introduced by the analysis of overlapping observations.

Thirdly, we will identify five key target conditions that can be used to describe the empirical dispersion processes. These target conditions include: 1) the coefficient of variation for the volatility at a 20 day time horizon, 2) the rate of decay of the volatility of volatility as the lag is increased, 3) the kurtosis of the time series, and 4) & 5) two measures of the average autocorrelation of absolute returns for short and intermediate lag periods. We will also examine whether these target conditions display similar dynamics over different time periods.

Finally, with these target conditions for each of the twelve markets, we will examine alternative models to understand the nature of these dispersion processes. The models will include four alternatives. Two models will assume that the variance remains constant but that the underlying price series either follow geometric Brownian motion or a Student-t distribution. The Student-t distribution will serve as a proxy for a jump diffusion model. The remaining models will assume that the variance is stochastic and that the underlying price series either follow geometric Brownian motion or a Student-t distribution. The findings will suggest that the objective dispersion process for markets are better described by Student-t distributions than by geometric Brownian motion. Further explanatory improvement (in almost all cases) can be obtained by the selection of an appropriate stochastic volatility model.

Finally, if we assume that the variance is stochastic and the underlying price series follow a Student-t distribution, almost all the markets under investigation can be adequately modelled. This suggests that both jump diffusion and stochastic volatility models are both required to understand the objective dispersion processes for markets.

The goal of the second part of the dissertation is twofold. First, this research will discern the nature of the risk neutral dispersion processes implied by options on these same twelve futures contracts. The primary goal is to understand how the prices of options contracts provide information regarding the expected dispersion processes of futures markets. This analysis will examine the volatilities implied from the prices of options on these futures. By examining the available universe of options prices available, we will examine the volatility smile structures. We will demonstrate a methodology for standardising the volatility smiles allowing for direct comparisons of the dynamics within and between markets.

As a result of this standardisation, we will show that the implied volatility surfaces display empirical regularities. Individual markets display regularity in their strike price effects (both first and second order). There is also consistency between markets in the same asset class and consistencies across all markets. To better quantify and analyse these consistencies, we will apply an Analysis of Covariance approach, which will allow us to carefully examine and compare these effects for all markets.

Second, we examine if the empirical smile structures are related to the objective processes of futures prices that underlie these options. It will be shown that consistencies do exist between the objective probabilities, associated with futures returns, and the risk neutral probabilities associated with options on these futures contracts. As with the first portion of this research, we will discern the rationale for the divergence from a lognormal dispersion process and examine the relative importance of jump processes and stochastic volatility models.

Even though we will demonstrate that a link exists between the objective and risk neutral processes, we observe divergences. The actual smile patterns display greater variation in the amplitude compared to those from the objective function. Another divergence is due to the existence of negative skews in the empirical smile patterns that are not justified by the objective dispersion processes. This suggests that other mechanisms are at work for the risk neutral dispersion processes for these asset classes.

CHAPTER ONE
THE ANALYSIS OF OBJECTIVE PROBABILITIES IN FUTURES MARKETS: LITERATURE REVIEW AND EMPIRICAL DESIGN

1.1 INTRODUCTION

The objective of the first five Chapters is threefold. Initially, this research will discern the nature of the objective dispersion processes which can be observed for return series for futures contracts on three categories of financial assets: fixed income, stock indices and foreign exchange. The primary goal is to understand the statistical properties of the volatility of futures returns.

Secondly, to capture those statistical properties of relevance we will apply an existing methodology in a new way. Burghardt and Lane (1990) identified the use of volatility cones. Their principal motivation was to assess if options prices were mis-priced. We will also use this method. Our motivation is to understand the variability of unconditional (historical) volatilities as a function of the time horizon of estimation. This of particular relevance to option pricing, as those who price options must make a forecast of the actual volatility that will occur over the life of the option and the volatility cone analysis can provide an indication of the degree of forecast error. One contribution of this research is to provide a new methodology to correct the biases in the volatility cone method introduced by the analysis of overlapping observations.

Thirdly, we will concentrate on those aspects of the processes which are most relevant to option prices. This requires the identification of five key attributes that can be used to describe the empirical volatility series. These attributes include: 1) the coefficient of variation for the volatility at a 20 day time horizon, 2) the rate of decay of the volatility of volatility as the lag is increased, 3) the kurtosis of the time series, and 4) & 5) two measures of the average autocorrelation of absolute returns for short and intermediate lag periods.

With these attributes for each of the twelve markets, we will examine alternative models to understand the nature of these processes. The models will include four alternatives. Two models will assume that the variance remains constant but that the underlying price series either follow geometric Brownian motion or a Student-t distribution. Two models will assume that the variance is stochastic and that the underlying price series either follow geometric Brownian motion or a Student-t distribution. The findings suggest that all markets are better described by Student-t distributions than by geometric Brownian motion. Further explanatory improvement (in almost all cases) can be obtained by the selection of an appropriate stochastic volatility model. Finally, if we assume that the variance is stochastic and the underlying price series follow a Student-t distribution, almost all the markets under investigation can be adequately modelled.

In this first Chapter, we will provide a literature review, present new analysis of the sampling properties of volatility cone estimation and discuss the alternative models selected for the later analysis of the objective processes.

1.2 REVIEW OF THE LITERATURE ON VOLATILITY PROCESSES

The variance of the rate of return for assets is one of the cornerstones of modern financial theory. Variance is a key concept for capital asset pricing and for portfolio analysis. If we are able to better able to measure financial risk, we can more accurately value risky assets. In the form of the standard deviation (or volatility), it plays a critical role in the pricing of contingent claims. Because of the importance of variance in financial theory and the fact that variance and volatilities exhibit stochastic behaviour over time, much research has been devoted to understanding the nature of this behaviour.

The aim of this research is to further our understanding of this behaviour for a wide variety of financial assets. According to Cox and Ross (1976), the option valuation problem is really equivalent to the problem of determining the distribution of the underlying variable, which is the return series for the underlying asset. They established that an option's price equals its expected payoff discounted at the risk-free rate where the expectation is taken over the 'risk-neutral' (rather than the true) distribution of the underlying asset. Much research has examined the characteristics of objective distributions, particularly in the equity markets. My research will extend this analysis to include equities, fixed income and currencies. I will compare these objective processes to those implied from options on these same assets. The goal is to assess the relationship between the conditional and risk-neutral distributions for a considerable historical period.

In this first part of this research, we will examine the unconditional (objective) volatility of financial futures markets. This will entail examining the historical record to assess the objective dynamics of the dispersion processes for twelve markets. Later, in the second part of this research, we will examine the volatilities implied from option prices on the same twelve financial futures markets.

Review of Research on Objective Dispersion Processes for Assets

To date, most of the research on this area has concentrated on the analysis of equity variances. Christie (1982) examined the stochastic behaviour of common stock variances and Schwert (1989, 1990) and Turner and Weigel (1992) have examined the stochastic behaviour of stock market indices. Our research extends these results by examining the stochastic behaviour of financial futures for four of the largest equity markets, fixed income markets and currency markets. By examining futures contracts, we remove many of the difficulties that have been observed when examining the underlying assets themselves.

For equities, problems with volatility estimation occur when discrete dividends are paid or when stock splits occur (Cox & Rubinstein 1985, page 260). Furthermore, there may be systematic patterns in the returns of equities due to corporate reorganisations, mergers and acquisitions or other changes in the nature of the firms that the equity represents. One solution would be to only analyse stock indices (as a more consistent and broadly based measure of the overall market). However, stock indices themselves present problems.

A major problem for the analysis of the variance of stock indices is the assumption that the returns for both the equities (which comprise the stock index) and the stock index returns are lognormally distributed. This introduces a paradox because such a condition can only exist if the index is a product of the underlying stock prices. Such an index is the Value Line Index (which is geometrically weighted stock index). Since in our research we are examining simple market capital weighted indices such as the S&P 500 index, it is not obvious that the sum of lognormal returns for the individual equities will result in a lognormal distribution. This is based upon the well-known result that the sum of lognormal distributions will not itself be lognormal. However, since we are using futures on the S&P 500, this asset may follow a lognormal distribution even if the underlying stock index does not. Samuelson (1965b) pointed out that even if systematic and non-lognormal patterns in spot prices (he examined commodities) exist, the futures price will fluctuate randomly and will follow a lognormal dispersion process. We will extend this finding directly and without loss of generality to all financial markets. This is our principal rationale for examining futures contract variances instead of spot market variances.

Thus, futures on stock indices represent a more consistent measure of the overall equity market that remains standardised through time, should follow a distributional form which lends itself to standard analysis and thus simplifies the time series analysis of volatilities. The use of stock index futures to model equity market variance has been used increasingly in other recent research. For example, Jackwerth and Rubinstein (1996) chose for estimation of volatility for stock indices, to use futures-based index levels throughout their research.[1]

In the case of bonds, the nature of the bond changes through time as the bond approaches maturity. This leads to problems in variance estimation for an asset whose stochastic behaviour is time dependent [see Schaefer & Schwartz (1987)]. In the case of currencies, the interest rates, inflation rates and purchasing power parity relationships all impact the variance of the spot exchange rate [Tucker and Pond (1988)].

An additional reason for using futures is the availability of transparent prices and sufficient data with homogeneous standardised instruments. The fact that the instruments do not change over time reduces the problems in examining the longer-term behaviour of a time series (where the underlying instrument may change through time). Another reason for using futures is that in the second part of this dissertation, I will examine the risk neutral distributions based upon option prices. Since most of

[1] The reason they state for the use of this data is related to the interest rate parameter that must be used to determine the implied volatilities of options. They concluded that the use of S&P 500 futures prices to estimate the implied interest rate was significantly less variable that either using T-Bill rates or interest rates implied from options on the cash S&P 500. (Page 1617).

these markets, the options are based upon the futures, this will allow conclusions from the first part of the research to be compared directly to those latter findings.

Random Walk Processes, Stationary Series and Objective Volatility

Before we can address the issues involved in the estimation of financial futures volatilities, we must define a few terms and review previous work in this field.

The theory of random walk and security price dynamics began at the start of the twentieth century with the work of Bachelier (1900). The fundamental problem with Bachelier's work was that he assumed prices to be described by a normal distribution. The same question was addressed later by Kruizenga (1956) and Samuelson (1965a): they showed that proportional price changes are the metric of interest and suggested that a lognormal distribution was more appropriate. Continuing along this line were, Kendall (1953), Osborne (1959), Roberts (1959), Alexander (1961) and Sprenkle (1961). All investigated the dynamics of asset price behaviour in relation to the theory that these prices follow geometric Brownian motion (and therefore the Random Walk Hypothesis). An excellent summary of the early work on the random walk hypothesis in financial markets appears in Cootner (1964).

Given that the focus of this research is on understanding options values, the papers by Bachelier (1900), Kruizenga (1956), Samuelson (1965a) and Sprenkle (1961) are of most interest. Each of these papers examined the relationship between security price dynamics and contingent claim prices. Unfortunately, these papers were unable to derive satisfactory option pricing models. This is because the resulting solutions were either not preference-free or option prices could violate the boundary conditions. Most of these approaches assumed that the security price process followed Geometric Brownian motion. Nevertheless, these papers established a framework within which subsequent research could derive satisfactory option pricing models.

Black and Scholes (1973) also assumed that the security price process followed Geometric Brownian motion. However, their major contribution was to use the principle of no arbitrage to create a risk-free portfolio so that risk-neutral pricing could be used. This led to a preference-free solution for the problem of pricing European call options that did not violate boundary conditions. Since that time, most models of asset price behaviour start with the assumption of a generalised Wiener process. Given that the drift and variance of asset returns can be both a function of the level of the return and time, an Ito Process is used. Finally, for most examples of the pricing of derivative securities, geometric Brownian motion has been used as the most plausible Markov stochastic process. Under this process, the proportional rate of return in any small interval of time is normally distributed and the returns in any two intervals are independent. These are the conditions associated with the Random Walk hypothesis.

A random walk formally states that there is no difference between the distribution of returns conditional on a given information set and the objective distributions of returns. Random walks are much stronger conditions than fair games or martingale processes because they require all the parameters of a distribution to be the same with or without an information set. In addition, successive returns drawn over time must be both independent and have to be taken from the same distribution. Thus, we can define the expected conditional variance as equal to the previously realised objective variances. Unless all the factors which influence the underlying probability distribution of returns remain unchanged (or stationary) over time, one would expect the properties of the probability distributions to change over time. This has been examined extensively in the literature and will be re-examined in this research.

A series is stationary if the mean value of the process is independent of time and the autocovariance is dependent only on the lag and is independent of the absolute value of time. The process is then said to be stationary. The implication that the probability distribution of returns may not be stationary is of critical importance to many areas of financial theory. For example, one of the key assumptions in options pricing theory is that the probability distribution is stationary. In the seminal Black and Scholes paper (1973), one of the ideal conditions for both the price of the underlying asset [stock] and for the options was that "The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus, the distributions of possible stock prices at the end of any finite interval is lognormal [and] The variance rate of the return on the stock is constant." (Page 640).

While most research on the dispersion of returns concentrate on the process driving returns to be normal. This has inherent problems. According to Copeland and Weston (1988) [p. 208], "Obviously, returns on assets cannot be normally distributed because the largest negative return possible, given limited liability of the investor, is minus 100%". They continue to state that while this assumption might lead to negative asset prices (to allow for infinite negative returns), the probability of observing returns as low as minus 100% is so low as to be irrelevant from a practical standpoint.

While this may be irrelevant in practice, a more serious problem exists if for all returns (and not just those approaching minus 100%), the distributional form does not follow Geometric Brownian motion and therefore (prices) fails to evolve in a lognormal manner. It is logical that during the early work on the random walk hypothesis many of the papers (published at the time) were for the most part empirical [see Mandelbrot (1963a, 1963b), Fama (1963,1965) and Cootner (1964) for a survey of the early empirical studies]. These papers cast serious doubt on whether a lognormal distribution was an accurate model for security price behaviour.

According to Merton (1982),

"It was the standard practice in early studies to assume that the logarithm of the ratio of successive prices had a Gaussian distribution with time-homogenous independent increments and stationary parameters. However, the sample characteristics of the time series were frequently inconsistent with these assumed population properties. One of the most important inconsistencies was the empirical distributions were often too 'peaked' to be consistent with the Gaussian distributions. That is, the frequency of extreme observations is too high to be consistent with samples from a normal distribution." (page 21).

The papers that Merton refers to are referenced above. Two approaches were attempted to resolve these discrepancies. The first was proposed by Mandelbrot (1963a, 1963b) and Fama (1963,1965). Finding (from an empirical investigation of daily returns on commons shares) that the empirical distribution displays leptokurtic behaviour and that the variance is not finite, these authors proposed a more general stable Paretian (Levy) distribution but retained the assumptions of independent increments and stationarity parameters. As was pointed out by Cootner (1964) the fact that the variance was infinite implies that not only will this render most of the standard statistical tools useless but also implies that the expected value of the arithmetic price change cannot exist. A key question remains: How can variance be modelled if the actual distribution of security prices suggests that such a variance is not finite?

Cootner (1964) suggested an alternative approach: finite-moment processes where the distributions are non-stationary. Merton (1982) points out that models requiring the underlying process to be a mixture of diffusion (Geometric Brownian Motion, for example) and Jump processes (Poisson-directed processes) can accommodate a wide range of market dynamics. Examples of these include the reflecting barrier model proposed by Cootner (1964) and the Rosenberg (1972) model which was a Gaussian model with a changing (but forecastable) variance rate. Rosenberg demonstrated that his model could explain the observed leptokurtic behaviour in stock market returns.

Other approaches (using finite-moment processes) to solve for the existence of non-normally distributed returns include Blattberg and Gonedes (1974) proposing a Student t distribution, Bookstaber and McDonald (1987) suggesting a Wiebull distribution and Press (1967) with a Poisson mixture of normals approach.

An alternative process to explain the high degree of kurtosis in the return data could be a GARCH (Generalised Auto-Regressive Conditional Heteroscedasticity) process. The empirical success of GARCH models is partly due to their ability to generate fat tail distributions, with finite or infinite objective variances. Much research has demonstrated that many time series display conditional heteroscedasticity. This means that clusters of extreme volatility occur within the time series. The first ARCH (Auto-Regressive Conditional Heteroscedasticity) model was introduced by Engle (1982) and was later generalised by Bollerslev (1986) with the GARCH model. The Bollerslev formulation has been found to be sufficiently 'general' to capture most types of conditionally heteroscedastic behaviour. Since this initial work, many variations on the basic GARCH model that have been introduced in the past 10 years. For excellent reviews on the broad literature on GARCH models in finance, see Bollerslev, Chou and Kroner (1992) and Bollerslev, Engle and Nelson (1994).

Of these alternative approaches, Merton (1982) states that taking the finite-moment process approach would be more promising (for modelling the price processes in continuous time). We will follow these lines in our research by examining the time series dynamics of asset returns and will compare our findings to assumptions of Geometric Brownian Motion, a Student-t distributional model and a range of models that examine finite-moment processes with stochastic volatility parameters.

Regardless of the approach chosen, assumptions must be made whether the variance is stationary or not. If stationary, then we should estimate the objective variance by using as much data as possible from the past to reduce the statistical sampling error. Furthermore, these observations should all be equally weighted distant observations being as important as recent ones. If the variance is non-stationary, this would imply that this may no longer be appropriate and the objective variance must place more emphasis on recent observations.

Fischer Black (1975) suggests that this might be ill advised to assume that variance is stationary when he indicates "But the volatility does change, so more weight should be given to recent months and less weight should be given to distant months." Therefore, the question remains how to estimate the volatility of asset markets in the presence of non-stationarity. Several possibilities for the estimation of asset return volatility exist. The historical standard deviation of returns is an obvious one, Black and Scholes (1972) used it. "Although the historical volatility is a reasonable estimator when volatility is stationary, it fails to capture instaneous changes, and thus a proxy based upon contemporaneous observations should be more appropriate for the study of volatility dynamics "[Merville and Piepta (1989) page 197]

When tests were done for stock market volatility by Schwert (1989), he found that the estimate of historical volatility (standard deviation of monthly stock returns) from the period from 1857 to 1987 varied from two to twenty percent over the period. Schwert concluded that "Tests for whether differences this large could be attributable to estimation error strongly reject the hypothesis of constant variance." [Schwert (1989) page 1115]. While this may suggest that volatility is not constant over time, it is not clear whether this result is due to sampling error or reflects statistically significant differences. Schwert argued for the latter.

There is further evidence that the historical volatility measure is not an unbiased estimator. A comparison of the linear (ARIMA) and the non-linear auto-regressive conditional heteroscedastic (ARCH) time-series behaviour of historical price variance (volatility) yields disturbing results. Barnaud and Dabouineau (1992) have found this for the variance and volatility of crude oil prices from 1987 to 1992. "Strange autocorrelation and partial autocorrelation phenomena can be found, strongly connected to the frequency of data used. However, these disturbances are not observed for volatilities extracted from simulated ARIMA data and cannot be reported as just resulting from the mathematical aggregation of data. The simple combination of actual data and such estimators may therefore be technically misleading rather than explanatory." [Barnaud and Dabouineau, page 109].

Schwert (1989) also examined whether the returns were independent over time. He compared daily returns for the Standard and Poor's (S&P) composite portfolio from January 1928 through to December 1987 and used daily estimates of the returns for the Dow Jones composite portfolio from February 1885 to December

1927. Schwert claimed that "Using non-overlapping samples of daily data to estimate the monthly variance creates estimation error that is uncorrelated through time." Schwert (1989) page 1117.[2] This result suggests that subsequent monthly variances are independent.

However, other researchers have found that time series behaviour of stock market returns is inconsistent with the hypothesis of stationarity. For example, Turner and Weigel (1992) examined daily return variability of the S&P 500 and Dow Jones indices over the period from 1928 to 1989. They used the close-to-close standard deviation of returns and compared this approach to alternative estimators including the daily high and low of the index and a robust estimator to measure the volatility of stock price returns. Their evidence rejected the hypothesis that stock market volatility is stationary some of their specific results are that: "Extreme-return days are preceded by significant losses and are intertemporally clustered. There is no evidence of short-term market reversals after either positive or negative jumps in stock index returns"(page 1586). Other authors have also found that there is evidence of positive serial correlation in equity returns. Poterba and Summers (1988) and Lo and MacKinley (1988), are only two of many.

A nagging problem is still whether these results could have been obtained simply by chance. The obvious solution to reduce error from small samples is to add more observations. However, a paradox (for the estimation of stock index volatility) is

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[2]. He goes further to identify the expected variance of the volatility estimator as: "If the data are normally distributed, the variance of the estimator $\hat{\sigma}_t$ is $\sigma_t^2 / 2N_t$, where σ_t^2 is the true variance (Kendall and Stuart (1969, p. 243). Thus, for monthly observations of $N_t = 22$, and $\sigma_t = 0.04$, the standard error of $\hat{\sigma}_t$ is 0.006, which is small relative to the level of σ_t." (page 1117). He goes further to state that " Since this is a classic errors-in-variables problem, the autocorrelations of the estimates $\hat{\sigma}_t$ will be smaller than, but will not decay at the same rate as, the autocorrelations of the true values, σ_t." (page 1117)

that adding more observations appears to make the estimate even worse. Jackwerth and Rubinstein (1996) examined the historical volatility over various sub-periods from 1980 to 1995. They found that at very small sample sizes (of 25 observations), the average kurtosis was almost 0 which is consistent with a lognormal distribution. To quote them "Unfortunately, the sample kurtosis systematically rises as a function of the sample size so that at sample size 720 [observations]... the average kurtosis... is quite different from lognormal"(Page 1612-1613). They concluded that "This kind of systematic increase in the average sample kurtosis is, of course, exactly what one would predict from a random volatility model, first postulated over two decades ago by Barr Rosenberg (1972)" (pp 1613). An additional problem was that "Historically measured volatility varies significantly over different time periods" Jackwerth and Rubinstein (1996) [Page1613].

On the other hand, Stein and Stein (1991) concluded that S&P 500 volatility was stationary over time. They drew upon other research by Bookstaber and Mc Donald (1987). They quoted their findings thus: "They [Bookstaber and McDonald] note that while one- and five- day stock returns have significantly fatter tails than lognormals of the same variance, returns over longer horizons (e.g. 250 days) are much better described by a lognormal distribution. This observation, taken together with our analytical results, would seem to provide indirect support for the hypothesis that volatility follows a stationary process. If volatility were nonstationary, then our results would lead one to expect long-horizon returns that look substantially fatter-tailed than lognormals." page 742.

Even if the nature of the objective volatility is fully understood and we are able to determine whether it is stationary or not, we will still not be able to ascertain if the instaneous volatility is stationary. This is simply because we can never directly observe the instaneous or true volatility. As Ball (1993) states, "The greatest problem is the unobservability of volatility. It must be filtered from the observable security prices." (page 20). Thus, this research must also be limited to the examination of the objective volatility.

This research differs (from the previous papers) in that we will use futures contracts as surrogates for the actual underlying assets and will assess the relative dynamics of the dispersion processes across the different markets. Our particular interest is the impact on the realised volatility upon the values of options on these markets. Later in this research, we will examine the relationship between the objective processes examined in this first portion of the research and compare these results with the risk-neutral processes implied by options prices.

Methods to Determine Objective Volatility

The measure of the futures price changes used is based upon the continuously compounded returns:

$$r_t = \ln(P_t / P_{t-1}) \tag{1.1}$$

where P_t and P_{t-1} refer to the level of the nearby futures contract at date t and t-1, respectively. ln refers to the natural logarithm and r_t refers to the return for the futures contract on date t.

The estimation of the standard deviation of the returns is determined using:

$$\sigma_N = \sqrt{\frac{\sum_{t=1}^{N}(r_t - \bar{r})^2}{N-1}} \tag{1.2}$$

Alexander (1996, page 235) shows that in the instance where the average return $\bar{r}$, is equal to zero, the estimation of the historical volatility can be expressed as:

$$\sigma_N = \sqrt{\frac{\sum_{t=1}^{N} (r_t)^2}{N}} \quad (1.3)$$

Neuberger (1994) points out that if the drift, $\bar{r}$, is close to zero, the sample second moment provides a better estimate of the population standard deviation than the sample standard deviation.

We will refer to the standard deviation (from equation 1.2) as the objective (or historical) volatility. Furthermore, it is a key input into the familiar stochastic differential equation:

$$d(\ln P) = \mu dt + \sigma dZ \quad (1.4)$$

where Z(t) is a standard Wiener Process and μ and σ are constants, thus return series r_t, has a normal distribution and

$$r_t = \mu + \sigma Z_t \text{ with } Z_t \sim N(0,1). \quad (1.5)$$

Furthermore, the Z_t's are independent and identically distributed (i.i.d.). If the r_t's are normally distributed, then the objective variance would follow chi-squared distribution with mean σ^2 and variance equal to $(\sigma^2)^2 / N$. In the instance where r_t's are i.i.d. but follow a fat tailed distribution, the objective variance has a variance $(K-1)\cdot(\sigma^2)^2 / N$, where K is the kurtosis μ_4 / σ^4. While these equations (1.4 and 1.5) assume that the price processes follow Geometric Brownian motion and given that much of the empirical evidence suggested above refutes this, it would appear to be inconsistent. However, we will use this as our starting point for analysis. The simple reason is that many of the option pricing formulae make this assumption. One key question in this research is assessing whether the objective processes are consistent with GBM. If not, we can then assume that options for these markets are also not well explained by a GBM process.

Equations 1.2 and 1.3, imply that the estimation of the variances will result in an average of the individual observations. Often such objective variance is measured for a fixed time horizon moving through time. The result is that the measure of (the sampled) objective variance always represents the same time period, N. This approach is commonly known as a moving average method, and causes a significant problem when estimating the objective variance, if the return series is not stationary. Suppose an extreme return event occurs. If the time period of estimation, N, is fixed and the individual returns, r_t, are equally weighted, then a 'ghost' feature can result. This means that the extreme event will impact the estimation of the objective variance with the same impact for as long as it remains in the estimation window (until t = N). The most common method for dealing with the problem of "ghost" features is the use of an exponentially weighted moving average (EWMA).

The EWMA approach reduces the problems of "ghosts" by placing more weight on recent observations. This provides the double benefit of reducing the impact of extreme events (as the date of the event lengthens from the current date) and retaining all the observations to maintain a sufficiently large sample of data to allow meaningful estimation of the sample variance. Most of the EWMA approaches, which appear most commonly in the literature, weight the previous observations in the form:

$$EV = \alpha \cdot r_1^2 + \alpha(1-\alpha) r_2^2 + \alpha(1-\alpha) r_3^2 + \ldots \tag{1.6}$$

where EV is the exponentially weighted variance, the α is the weight assigned to each observation and the r_t's correspond to the individual returns. The exponentially weighted variance will itself have a sampling variance equal to:

$$(K-1)\cdot(\sigma^{2)^{2}}\cdot\frac{\alpha}{2-\alpha} \tag{1.7}$$

For an equally weighted moving average approach of N equal weighed observations, it can be easily shown that both approaches will have the same sampling variance when $\alpha = 2/(N+1)$.

Another approach to exponential weighting is to have a moving average of past returns (t - N/2) and future observations (t + N/2) for a fixed sample size N. This is the approach which will be used in this research to gauge the general level of the objective variance series. This weighting scheme can be expressed as:

$$EV = \alpha^2(K-1)(\sigma^2)^2[1+(1-\alpha)^2+(1-\alpha)^4+...] \tag{1.8}$$

This formula combines equations (1.6) and (1.7). For an equally weighted moving average approach of N equal weighed observations, it can be easily shown that both approaches will have the same sampling variance when $\alpha = 2/2(N+1)$. This is approximately equal to 1/N. Thus, if α is equal to 0.10, then it is the same expected sampling variance as an equally weighted moving average of approximately 10 returns. If α is equal to 0.01, then it is the same expected sampling variance as an equally weighted moving average of approximately 100 returns.

These simple approaches are by no means the only alternatives for the estimation of historical volatility. Other methods discussed in the literature include estimators by Parkinson (1980) and Garman and Klass (1980). Both estimators incorporate high and low prices, and have been shown to be statistically more efficient that the commonly used close-to-close return standard deviation. These estimators are:

(Parkinson)

$$\sigma_N = \sqrt{\frac{\sum_{t=1}^{N}\left[(H_t - L_t)^2\right] / 4 \cdot \ln(2)}{N}} \tag{1.9}$$

(Garman and Klass)

$$\sigma_N = \sqrt{\frac{\sum_{t=1}^{N}\left[0.50 \cdot (H_t - L_t)^2 - 0.39 \cdot (C_t - C_{t-1})^2\right]}{N}} \tag{1.10}$$

where H_t and L_t are the natural logarithms of the high and low price of the futures contract during time period t. The terms C_t and C_{t-1} correspond to the natural logarithms of the closing prices at time periods t and t-1, respectively.

Marsh and Rosenfeld (1986) have shown these parameters to be 5.2 times more efficient (for the Parkinson estimator) and 7.4 times more efficient (for the Garman and Klass estimator). While other methods have been proposed in the literature, we have chosen to apply the close to close measure as not all the markets we had obtained data for had high and low prices available. The reason for estimating the variance with extreme prices rather than closing prices is that the extreme values contain more information than do the closing prices

These methods for estimation of historical volatility share the assumption that the volatility is itself normally distributed. Evidence in this research refutes this for both return series and for the objective volatility. To deal with departures from normality that may exist, another approach has been suggested which includes various proposed robust estimators that work close to "best" under a variety of distributions. One such measure has been proposed by Iglewicz (1983) and is known as the interquartile range:

$$IQ = (Q_{3t} - Q_{1t}) \tag{1.11}$$

where Q_{3t} is the 75th percentile of asset returns computed from close-to-close price changes and Q_{1t} is the 25th percentile of the asset returns. It can be shown that for a normal distribution, the interquartile range is approximately 4/3s times the standard deviation. This robust estimator measures dispersion around the sample median, a measure that is less sensitive to outliers than the sample mean. This approach is well known in non-parametric statistical tests.

Furthermore, recent evidence has suggested that using close to close data for the estimation of variances is as good a measure of volatility as using the extreme methods. Clewlow and Xu (1994) compared the estimation of historical volatilities using the alternative approaches that include all the available information with the simple close to close approach. While they found that the estimators using the full information available have the smallest variation, they are not significantly different from the classic estimator (which solely uses close to close data). Therefore, for this analysis, we will use daily closing prices for the twelve markets examined. The choice of daily observations has been discussed in the literature. According to Fischer Black (1976a), "In my view, if you want to study changes in volatility, you have got to use daily data" (page 177).

Problems in the Estimation of the Objective Volatility of Futures

A problem with futures prices is that when expiration is approached, the contract approaching expiration begins to become less actively traded and the next contract starts to have more trading volume. Some authors who have examined futures data [Merville and Pieptea (1989)] have chosen to switch the analysis from the nearby futures contract to the first deferred contract prior to the expiration. For Merville and Piepta, "ten trading days prior to expiration the nearby contract is replaced by the next to nearby contract to avoid data contamination related to any excess volatility induced by program trading and speculative positions between options and futures markets" (p195-196).

We chose to take the futures price series to the day prior to expiration and switch the series on that day to the next delivery period. With this method, we assured that each return series was calculated using the changes in the appropriate underlying assets.

Another problem with the estimation of the objective volatility of futures contracts is that (for many of the markets we will examine) the length of the historical record is relatively short. Levy and Yoder (1989) examined the problem of estimation of historical volatility for a small number of observations. Their rationale was to address the problem of using the Black/Scholes model after a sudden and permanent change in the volatility of the underlying stock resulting from some shock. This was motivated by the volatility of stocks pre and post the 1987 crash. They assumed that the conditional volatility was known and they tested the impacts of sampling errors on the estimates of this parameter by simulation. The simulation was based upon the well-known relationship between the sample and true variance:

$$\tilde{S}^2 \approx \left(\frac{\sigma^2}{N-1}\right)\cdot\chi^2 \qquad (1.12)$$

where $\tilde{S}^2$ is distributed as Chi-square with N-1 degrees of freedom. They estimated the sample variance for samples from 4 to 30 observations and with this determined call option prices using Black/Scholes. They found that errors in the prices of the options which were consistently below the price of the options using the conditional volatility. They demonstrate that this bias can be almost eliminated by modification of the estimated variance by the factor N -1.5 rather than N -1. This is only relevant for estimation periods of the volatility less than 60 days. "If sixty or more observations are used to calculate the sample variance, the difference between E($\tilde{C}$) and C is small [see Boyle and Ananthanarayanan (1977)]" (p 106).

Since for our purposes, we will be estimating volatility with at least 1200 observations, this result may not seem relevant to our situation. Nevertheless, we still face a problem when estimating longer-term volatility. For example, if we wish to estimate 20-day volatility from 1200 daily observations, we will only have 60 observations that are independent and non-overlapping. Three solutions exist for this problem. The first is the adjustment proposed by Levy and Yoder. However, it is not obvious that this will apply to the estimation of variance for observations with a significant lag occurring between their occurrence. Secondly, one can add more observations. This is not possible, as we have obtained all the observations available for our markets. The third solution is to overlap the estimation periods to obtain more observations. However, this introduces a potential bias since the samples are no longer independent. This problem is one of the major theoretical drawbacks to the volatility estimation technique we will now discuss that is commonly referred to as the volatility cone approach.

1.3 VOLATILITY CONES - COMPARING VOLATILITIES OVER TIME

Determination of Volatility Cones

The principal reason why volatility estimation is so critical is that the fair value of a wide range of contingent claims depends directly upon a correct estimation of the volatility. Those who sell options both price and hedge these products based upon the expected realised volatility for the asset that underlies the option. What Neuberger (1994) demonstrates is that variability in realised volatility has a dramatic impact on the costs of hedging options. He proposes that a solution to this problem would be the creation of a derivative contract based upon the natural logarithm of the price of the underlying asset. This is because sellers of options are concerned with the quadratic variation of the underlying asset price process, which is what we commonly refer to as the volatility. Another technique has the potential of providing us information about the volatility of volatility. Furthermore, this technique could provide an estimate of the likely probability distribution of forecast errors of volatility at different time horizons. This is the volatility cone technique.

Option prices are based upon some estimate of expected actual volatility. Tompkins (1997) has demonstrated that the replication error from dynamic hedging of options is directly related to the realised volatilities of the underlying asset over the life of the option. Therefore, the distribution of the potential hedging error is related to the distributions of realised volatility and that is why this approach is so critical to our research.

This approach is based on the assumption that the realised volatilities for assets should bear some relationship to previously realised volatilities assuming the volatility is a stationary series. If that is the case, it would be reasonable for this realised future volatility probably not to exceed the highest actual volatility having occurred over a comparable time period in the past or be below the lowest actual volatility having ever occurred. Burghardt and Lane (1990) first identified this approach.

To estimate the volatility cones, daily returns for the nearby underlying futures contract were determined using the formula:

$$\ln (P_t/P_{t-1}) * \sqrt{252} \qquad (1.13)$$

On the day the nearby futures expired, the return for the next day would reflect the difference between the final day of the preceding contract and the first day of the next nearby. To remove this bias, on this switch day the return was estimated using the change in the logarithms of the price of the next nearby contract from the previous day (for this same contract).

To estimate the empirical behaviour of the actual volatility series of the underlying markets, the following steps were taken:

(1) A variety of sample time horizons were selected over 20 day intervals from the expiration of the futures contract and extending out to 500 days until expiration. This led to twenty-five volatility series to be compared for each asset under consideration. The actual volatilities were estimated (a postiori) on a daily basis and a rolling basis for the periods outlined above. In this way, a running series of twenty-five historical volatilities were estimated and were updated as each business day passed.

(2) For the purpose of drawing the cones, the maximum, minimum, median and the 25th and 75th quartile volatility values were determined for the entire period of the analysis.

As was pointed out by Burghardt and Lane (1990), the narrowing between the maximum and minimum levels for the volatility cones as the time horizon is extended provides a clue to the nature of the quadratic variation in volatilities. Of particular interest in our research is the standard deviation of the volatility estimated at the various horizons. As was indicated in the previous section, volatility cones are biased due to the fact that overlapping data were used to estimate the volatilities. To understand the true nature of volatility correctly, we must correct for the bias in the estimated standard deviations of the volatilities. Once this is done, we will understand the true nature of quadratic variation that is so critical to those dealing in option contracts.

Problems with Overlapping and Non-Overlapping Data

Suppose one wished to estimate the volatility for a financial futures time series. The first step would be the determination of daily price relatives for the entire period of analysis. For ten years of data the number of daily price relatives will be approximately N=2500. These price relatives will be grouped into periods of analysis from one day (n=1) to a period which is one half of the number of the observations (n = N/2). With these groupings, standard deviations (or the volatility) will be estimated for the grouped periods. From this analysis, summary statistics will be produced for the volatility estimates. We are interested in the maximum, average and minimum observed values. These estimations (apart from when n=1) will entail overlapping estimation periods. For the observation period n = N/2, we will obtain N/2 observed standard deviations but only two observations will be from non-overlapping (and thus independent periods). The high degree of correlation between such overlapping observations will dampen the true variability of the volatility and makes the estimation of long-term volatility using this method unreliable. We will address this problem by estimation of the standard deviations for observed periods that are both overlapping and non-overlapping and compare the empirical bias that occurs from estimation with overlapping periods. In addition, a theoretical model has been derived which addresses the question of the theoretical relationship between the standard deviation of the volatility estimated in overlapping and non-overlapping periods. The first part of the research will compare this theoretical bias to the observed bias. It will be demonstrated that the theoretical model correctly explains the bias. This is a significant contribution to the problem of long term volatility estimation where the limited number of available independent observations is not sufficient to allow a statistically meaningful estimate of the volatility parameter to be determined. Thus, it will now be possible to estimate the long-term volatility using overlapping data and use the theoretical model to derive confidence intervals for the results that will quantify the bias. With the bias corrected for, we will now examine the unbiased dispersion that occurs for the volatility of these price series. This will allow us to correctly assess the true nature of the volatility dispersion and test for mean reversion effects that have been identified in the literature. This will identify if the process is independent and identically distributed or not.

Clearly, it would be best to have non-overlapping observations. However, this is a significant problem when there are not enough non-overlapping periods to determine statistically significant results. Secondly, by restricting the analysis solely to non-overlapping samples, we circumvent problems with serial correlation but have sacrificed observations in the process.

One method of addressing the problem of overlapping data in volatility estimation is the use of panel regression techniques. This is a common econometric technique that corrects for the technical problems that arise when using overlapping data in financial time series analysis. Dunis and Keller (1995) used this for the examination of currency option volatilities. They pointed out that when data were overlapping it led to dependence between the observations. This approach has the following economic consequences: "most notably, it results in an under-estimation of the calculated variances of the estimators occurs as a result of the strong autocorrelation of the residuals"(Dunis & Keller, p 349). They demonstrated that the use of the panel regression technique significantly improved the Durbin-Watson statistic that measured the degree of autocorrelation of the residuals. The panel regression approach is a much less burdensome alternative to the approach followed by Hansen and Hodrick (1980).

The Hansen and Hodrick approach requires the construction of an appropriate variance/covariance matrix. This is achieved by estimating the parameters of interest by sampling data at fixed intervals of the data set. This method is known as the Generalised Method of Moments estimation (GMM) and was also employed by Hansen (1982) and later by Newey and West (1987).

In the analysis of mean estimates for overlapping and non-overlapping periods, Müller (1993) found that for series with limited data points, one would increase the precision of the estimate by overlapping the data to the sampling scheme. Although he pointed out that "The exact answer of this approach depends on the statistical properties of the analysed variable." Müller (1993, p 1)

Müller used the simplest case of a Gaussian random walk process assuming a i.i.d. random variable with zero mean and a variance of one in his study. With this given distribution, he generated a series of data and then analysed the results by sampling data from the generated data series and estimating the sample mean and variance. He found that the reduction in the variance of the variance estimator could be expressed as:

$$\frac{E\{[E_{overlap}(y_i^2)-m]^2\}}{E\{[E_{no-overlap}(y_i^2)-m]^2\}} \approx \frac{2}{3}+\frac{1}{3\cdot m^2}, \text{for } N >> m, \qquad (1.14)$$

where y_i is the difference series of the standard variable x_i (which has a mean of zero and variance of 1.0) and m is the period of the difference between the estimation of the differences. While it is clear that this is a simplistic example, Müller found that the variance reduction from overlapping the data can be reduced by approximately

2/3s by overlapping the data compared to non-overlapping the data for large m. Thus, the use of overlapping data does reduce the error of estimation of the variance substantially.

Müller does indicate that this gain in reduction of error variation is only applicable to his ideal situation. He goes on to state: "The beneficial effect of overlapping is probably lower than in the 'ideal' example if there is already considerable serial dependence in the underlying x_i, e.g. if the time series x_i is a computed series that already, implicitly used some overlapping in its computation formula. If, on the other hand, the x_i are known to have negative autocorrelation... we expect to gain more by using overlapping data intervals than in the Gaussian case."

Another solution to the overlapping data problem which has been addressed in the literature is the jack-knife method. Yang and Robinson (1986) present an example of this. This method is based on strongly overlapping resamples of the original sample. The final method that has appeared in the literature to addressing the overlapping data problem is the use of the "bootstrapping" method.

One popular technique is the ".632" bootstrap method developed by Efron (1983). In this method, a bootstrap sample is generated from the original sample; that is, a sample of size n is drawn with replacement. This is compared to the analysis from the entire sample by repeated samples. The name ".632" is taken from the fact that in a sample of size n taken with replacement from n items, the probability of any given item appearing is $1-(1-1/n)^n \rightarrow 1 - e^{-1} \approx 0.632$ and $n \rightarrow \infty$.

For the panel data, bootstrapping, jack-knife and indeed the Hansen and Hodrick approaches, sampling approaches are employed which address the problem of serial correlations by changing how the estimates are drawn such that they are no longer linked directly through time. An alternative approach is to examine the actual biases of the estimated variances for overlapping and non-overlapping periods and adjust the error of the overlapping estimates by this factor. Müller (1993) has examined this for an idealised distribution and we will examine this for actual market distributions.

A New Method for Unbiasing Overlapping Samples

One of the results of the analysis on autocorrelations (to be presented in Chapter 2), is that the absolute difference series are positively correlated. Thus, this approach must be rejected for the actual estimation of empirical series of returns and differences. This has lead to a more realistic derivation of the impact of the reduction in the error variances that have been completed for this research.

If we have large samples, then the sampling variance of $\hat{\sigma}_\tau^2$ is given as:

$$\frac{(K-1)^\tau}{\tau}\cdot(\sigma^2)^2 \qquad \textbf{(1.15)}$$

where variance is estimated from τ daily returns and K is the measure of kurtosis. For smaller and overlapping samples this estimate is biased. However, since we know the weights involved, we can estimate new values (to replace 1/τ) to take this into account. This is the method used for this research and derived specifically for this research.

For this analysis, we wish to understand how we would expect volatility cones to behave (because of overlapping observations) if returns were i.i.d. We assume k observations to a volatility estimate, and n separate volatility estimates.

1 Non-overlapping case

$$n\left\{\begin{array}{lll}\dfrac{R_1^2}{k}\cdots\dfrac{Rk^2}{k} & & \\ & \dfrac{R_{k+1}^2}{k}\cdots\dfrac{R^2}{k} & \\ & & \dfrac{R^2}{k}\cdots\dfrac{R^2}{k}\end{array}\right.$$

$$\text{Variance} = \frac{(k-1)}{k}\sigma^4$$

where K is the kurtosis of the underlying

2 Overlapping case

Weights one: 1/k on each disturbance = *n/nk* less weight from mean: *e.g.* $\begin{array}{l}k=3\\ n=5\end{array}$

	x	x	x					
		x	x	x				
from			x	x	x			n
disturbances				x	x	x		
					x	x	x	
	$\leftarrow (k-1) \rightarrow$		$\leftarrow (n-k+1) \rightarrow$			$(k)\leftarrow$	$(k-1) \rightarrow$	

$$\begin{array}{lllllllll} k=\{ & 1 & 2 & \vert\; 3 & 3 & 3 & \vert\; 2 & 1 & \}/12 \\ \;\;=\{ & 1\ldots & (k-1) & \vert\; k & k & k & \vert\; (k-1) & \ldots 1 & \}/(nk) \end{array}$$

$$W_{i,j} : i = 1,n : j = 1,n+k-1,$$

$$\text{total days} = n+k-1$$

$$T = n+k-1$$

$$n = T-k+1$$

$$Variance = \sum W_{ij}^2 / n$$

$$\sum W_{ij}^2 = \underset{(k-1)\cdot terms}{(n-1)^2 + 2(n-2)^2 \ldots (k-1)(n-k+1)^2} \;\Big\vert\; \underset{(n-k+1)\cdot terms}{k(n-k)^2 \ldots k(n-k)^2} \;\Big\vert\; \underset{(k-1)terms \text{ as first weight}(nk)^2 n}{\ldots 2(n-2)^2 + (n-1)^2}$$

What we are interested in solving is:

$$\sum_{i=1}^{k-1} i \cdot (n-i)^2 \tag{1.16}$$

It can be shown algebraically that the solution to overlapping weights in this summation is:

$$\frac{k \cdot (k-1) \cdot (3k^2 - k \cdot (8n+3) + 2n \cdot (3n+2))}{12} \tag{1.17}$$

The summation of all the weights is equal to:

$$2 \cdot \left(\frac{k \cdot (k-1) \cdot (3k^2 - k \cdot (8n+3) + 2n \cdot (3n+2))}{12} \right) + k \cdot (n-k+1) \cdot (n-k)^2 \tag{1.18}$$

This result can be simplified to yield:

$$-\frac{k \cdot (3k^3 - 10k^2 n + 3k \cdot (4n^2 - 1) - 2n \cdot (3n^2 - 2))}{6} \tag{1.19}$$

By grouping together and simplifying, the final result is:

$$\sum W_{ij}^2 / n = \frac{1}{k} - \frac{4n + 3k(4n^2 - 1) - 10k^2 n + 3k^3}{6k \cdot n^3} \tag{1.20}$$

for the case $n \geq k+1$ i.e. $T \geq 2k$

for $n < k+1$: $(T < 2k)$

```
x ¦ x ¦ x   x ¦
  ¦ x ¦ x   x ¦ x
```

$$\Sigma W^2/n = \left\{1.(n-1)^2 + 2(n-2)^2 + \ldots (n-1).1^2\right\} \cdot \frac{2}{\left\{(nk)^2 n\right\}}$$

$$= \frac{(n^2-1)}{6.k^2.n} \tag{1.21}$$

Using these solutions, the estimated ratio of the non-overlapping variance to the overlapping variance is simply:

$$\left(\frac{(K-1)}{k} \sigma^4 \right) \div \left(\frac{(n^2-1)}{6.k^2.n} \right) \tag{1.22}$$

If K (the kurtosis) is assumed to be 3.00 then the numerator of the equation becomes:

$$\left(\frac{(2)}{k}\sigma^4\right) \quad (1.23)$$

Using this above assumption, we used the following formula to determine the adjustment factor to correct for the bias from overlapping.

k = period of estimation

n = number of data points

Y = adjustment Factor

$$Y = \frac{1}{k} - \frac{\left(4(n-k+1)+3k\left(4(n-k+1)^2-1\right)-10k^2(n-k+1)+3k^3\right)}{6k(n-k+1)^3} \quad (1.24)$$

In the next Chapter, we will test this adjustment factor by comparing the theoretical ratio of the variances of non-overlapping to overlapping observations to the actual ratio of these two approaches. This analysis will be examined for different periods of analysis to assess the consistency of the approach. It will be shown that this technique is extremely powerful in explaining the nature of the bias when estimating volatility from overlapping data.

1.4 AUTOCORRELATIONS OF ABSOLUTE RETURNS

It is well established that stock market returns contain little serial correlation [Fama (1970) and Taylor (1986)]. This is consistent with the weak form efficient markets hypothesis. However, this empirical fact does not necessarily imply that the returns are i.i.d.. Taylor (1986) found that the return process is characterised by substantially more correlation between absolute or squared returns than there is between the returns themselves.

A number of papers have found that the stock returns exhibited time-varying and predictable volatility [see Ho, Perraudin and Sørensen (1996) who examined a selection of US equities]. We have previously referred to GARCH models that have also indicated that volatilities are inter-related through time. Rather than choose one particular GARCH model, we have decided to examine autocorrelation dynamics directly. One such approach is the use of autocorrelograms previously employed by Taylor (1986) and Ding, Granger and Engle (1993).

The differences in these two approaches is that Ding, Granger and Engle (1993) extended the work of Taylor (1986) by examining the autocorrelograms for the return, absolute returns and squared returns for the S&P 500 for lags from 1 to 5 observations and 10, 20, 40, 70 and 100 observations after that. They considered the properties of the absolute return of the return series for the S&P 500 stock market index from January 3, 1928 to August 30, 1991. They chose this metric, as it appeared to exhibit less noise. For the same reason, we will examine the autocorrelations of absolute returns.[3]

The benefits of this approach is that if one is interested in assessing if the distribution of asset returns were i.i.d., one can use a simple 95% confidence interval test assuming the distribution was i.i.d. In their study, Ding, Granger and Engle found that S&P 500 was inconsistent with an i.i.d. assumption. They state: "about one quarter of the sample autocorrelations with lag 100 are outside the 95% confidence interval for a i.i.d. process... [the] stock market return series have a very small positive first order autocorrelation. The small positive first order autocorrelation suggests that the [returns] do have some memory although it is very short... The second lag autocorrelation... is significantly negative which supports the so-called 'mean-reversion' behaviour of stock market returns. This suggests that the S&P 500 stock market return series is not a realisation of an i.i.d. process." (page 86-87).

It is interesting in the Ding, Granger and Engle (1993) paper that if the return series is an i.i.d. process any transformation will also be i.i.d. They found that the sample autocorrelation of the absolute return series and the squared return series were all outside the 95% confidence interval, and that the autocorrelations were all positive even over long lags. From this evidence, they concluded: "It is clear that the S&P 500 stock market return process is not an i.i.d. process." (page 87).

Ding, Granger and Engle (1993) also found that the serial correlation of absolute returns was greater than the serial correlation of squared returns. Thus, it might make sense to estimate averages of absolute returns as an alternative to the volatility cone. Ding, Granger and Engle (1993) found in their autocorrelograms that the autocorrelations decreased fast in the first month and then very slowly. They found that the autocorrelograms displayed consistently positive autocorrelations for the S&P 500 stock market returns.

They needed to measure whether this difference was statistically significant or not. For this they used a simple test making no assumptions of normality in the underlying volatility series, only that the series was assumed to be i.i.d. and the sample autocorrelations were assumed to be distributed normally. The 95% confidence interval for estimated sample autocorrelations was $\pm 1.96/\sqrt{T}$. Where T was the total number of observations in the sample. Ding, et. al. indicated that what Bartlett demonstrated in 1946: that if the returns are described by an i.i.d. process then the sample autocorrelation ρ_τ is approximately $N(0,1/T)$.

Previous research by Poterba and Summers (1986) examined the S&P Composite Stock index for the period from 1928-1984 (note pre crash). They found

[3] Even if the returns are normally distributed, the squared returns would follow a Chi-squared distribution, which are extremely skewed. Given that we know that the actual distributions of returns have fat-tails, a squared series would have been even worse.

results consistent with the conclusions of Ding, Granger and Engle. They stated: "We find that shocks to volatility decay rapidly... The results suggest that while volatility is serially correlated, changes in current volatility have relatively small effects on volatility forecasts over even short horizons...the results again suggest that these shocks are only weakly serially correlated." (page 1142) In terms of magnitude, they found that "Estimates based on both actual and *ex ante* volatilities indicate that these volatility shocks have half-lives of less than six months, and in some cases as short as one month." (page 1150).

Given the power of this approach and its establishment as a methodology in the literature for testing autocorrelation effects, we will also employ this technique for our twelve markets.

1.5 MEAN REVERSION IN HISTORICAL VOLATILITY

It will be shown in the next Chapter that the unbiased standard deviations of the volatility are clearly decaying at a much slower pace than for an i.i.d. single distribution. One possible reason for this effect is mean reversion. To analyse this effect properly, we must digress and examine the literature on mean reversion effects.

There is solid empirical evidence for the application of a mean reversion model to volatility. Merville and Pieptea (1989) examined option prices on twenty-five (US) stocks and calls on the S&P 500 stock index futures over a ten year period from 1975 to 1985. They found that the ex ante (objective) market volatility follows a mixed mean reverting diffusion with noise process. They concluded that strong forces pull the volatility back to its long-term value. Likewise, Scott (1987) also found that for the S&P 500, the monthly estimates of the historical volatility were not serial independent and the volatilities had a strong tendency to return to an average level. Wiggins (1987) and Hull and White (1987b) have reported similar findings.

Haugen, Talmor and Torous (1991) also found evidence for mean reversion in a study of the Dow Jones Industrial Average from 1897 to 1988. They stated: "There appears to be a relationship between the level of volatility and volatility changes: volatility increases tend to occur when the level of volatility is relatively low, while volatility decreases tend to occur when the level of volatility is relatively high." (pages 991-992). Recently, Clewlow and Xu (1994) also concluded that the realised volatilities for S&P 500 futures are mean-reverting. They stated: "For most samples (contracts), only the first order autocorrelations and partial autocorrelations are significant and consequently an AR(1) model is adequate." (page 8).

The main focus of mean reversion in financial series has been associated with the modelling of the term structure of interest rates. Vasicek (1977) developed a simple model which was based upon zero coupon bonds (and spot rates) and assumed that all the uncertainty was being driven by a single source of variability, the so-called instantaneous 'spot rate'. The mean-reverting stochastic process he suggested for all interest rates was defined by:

$$dr = k(\theta - r)dt + \sigma \cdot dZ_1 \qquad (1.25)$$

where dr is the change in the interest rate of interest, r; θ is the long term average the interest rate is being drawn to , k is the positive rate at which the interest rate is being pulled back to the long term average, σ is the volatility of interest rates and Z_1 is the standard Ito process (assuming geometric Brownian motion) for modelling uncertainty. One problem with the Vasicek model is the assumption that interest rates are normally distributed (although mean reverting). As with Bachelier (1900), this implies that interest rates can be negative and thus introduce an arbitrage opportunity.

Cox, Ingersoll and Ross (1985) solved this problem when they modified the Vasicek model to remove the possibility of negative interest rates. This is a slightly different example of a mean-reverting stochastic process. This process can be summarised as:

$$dr = k(\theta - r)dt + \sigma\sqrt{r} \cdot dZ_1 \tag{1.26}$$

The difference is that the second term (the volatility function) has the square root of the current interest rate. When (and if) the interest rates approach zero, the square root of zero causes the volatility to go to zero and the rate will solely be influenced by the drift term (and pulled upwards). This assumed model means that interest rates follow a chi-square distribution.

The Cox, Ingersoll, Ross approach can be applied to model the behaviour of volatility [Leong (1991)]. The model would simply substitute volatility for the interest rates and the second term in the model would be considered the volatility of the volatility. This implies that the volatility itself would be assumed to follow a non-central chi-square distribution. This process is also known as a modified Bessel function.

While these models examined the nature of mean reversion for interest rates, similar approaches have been applied to the analysis of volatility. These will be discussed in the next section. In addition, we will discuss other models that have been proposed to explain the anomalies we observe in the objective processes.

1.6 MODELS TO EXPLAIN THE NATURE OF EMPIRICAL VOLATILITY.

Given the extensive evidence in the literature that indicates that return series are not normally distributed and volatility is not stationary, a number of theories have been proposed to explain these results. Broadly speaking there are three possible explanations for the anomalies that have been observed for all asset markets. These include a Constant Elasticity of Variance Model, Jump Processes or the existence of Stochastic Volatility. At this point, I will provide a brief review of these approaches and indicate why this research examined only two of the possible alternatives.

Constant Elasticity of Variance Model

Cox and Ross (1976) first proposed the constant elasticity of variance model. In this model the stock price has a volatility of $\sigma S^{-\alpha}$ for some α where $0 \leq \alpha \leq 1$. Thus, the level of volatility decreases as the stock price increases. This model would also address the divergence from a lognormal process that occurs for stock index futures and individual equities.

The rationale for this model is often referred to as the leverage effect. The logic is that all firms have fixed costs that have to be met regardless of the firm's operating performance. When the stock price declines, it is obvious that the equity value of the firm has fallen while the costs have remained fixed. Given that the variability of the equity price can be seen as a function of the variability of earnings and it is assumed that the variability of earning is constant, a lower equity value for the same earnings volatility will cause the volatility of the equity to rise.

Given that significant skewness effects are observed for the stock index futures markets, this model should be considered. Unfortunately, while this model may explain the skewness effect observed for the stock index futures we examined, it will not address the fat-tailed nature of these or the other markets. This severe limitation has led many market participants to ignore it.

Pricing Models which Incorporate Jumps in the Underlying Asset Price

While the CEV models assume that the underlying asset price can change continuously, another class of models has been developed with the underlying asset price following a jump process. Robert Merton proposed such a model in 1976. In his model the underlying asset price has jumps that are superimposed upon a geometric Brownian motion process. This approach is referred to as a jump diffusion process. An alternative jump model was first suggested by Cox and Ross and elaborated on in the seminal paper by Cox, Ross and Rubinstein (1979) outlining the binomial process for option pricing. For each small and discrete interval of time, Δt, the stock price has the probability $\lambda\Delta t$ of moving from S to Us and a probability of $1 - \lambda\Delta t$ of moving from S to $Se^{-\omega\Delta t}$. Most of the time, the stock price declines at rate ω. However, occasionally it exhibits jumps equal to $u - 1$ times the current asset price. In the limit as $\Delta t \rightarrow 0$, jumps occur according to a Poisson process at rate λ. The terminal stock price distribution is log-Poisson and the price of a call is determined in the same way as was discussed in the previous references.

An interesting conclusion of this approach is that the inclusion of jump processes into the pricing models causes the distributions to display fatter tails than for continuous lognormal processes. In Merton's model, jumps can be either positive or negative. An interesting result is that the longer the maturity of the option the more the impact of the jump process is neutralised. This is consistent with the empirical evidence from market data. In the equity markets, Jarrow and Rosenfeld (1984) and Ball and Torous (1985) found significant jump components in the return series. Bodurtha and Courtadon (1987) suggested that the excess kurtosis they observed in foreign exchange returns could be explained by the presence of jumps. This result has also suggested by Jorion (1988) and Vlaar and Palm (1993). Thus, jump processes may be a better approach for adjusting the lognormal dispersion process to yield theoretical results that are consistent with empirical objective returns.

Merton states in his 1976 paper that, "Indeed, since empirical studies of stock price series tend to show far too many outliers for a simple, constant-variance log-normal distribution, there is a 'prima facie' case for the existence of such jumps." (p 127). He offers an explanation for this in the footnote 4 of his 1976 paper, where he says: "There have been a variety of alternative explanations for these observations [too many outliers in stock price series]. Among them non-Stationarity in Cootner (1964); finite variance, subordinated processes in Clark (1973); non-local jump processes in Press (1967); non-stationary variance in Rosenberg (1972); stable Paretian, infinite variance processes in Mandelbrot (1963) and Fama (1965). The latter stable Paretian hypothesis is not, in my opinion, a reasonable description of security returns because it allow for negative prices as does the corresponding finite-variance, Gaussian hypothesis. Of course, limited liability can be imposed by specifying that the logarithmic returns are stable Paretian, and therefore, the distribution of stock prices would be log-stable Paretian (the analogue to log-normal for the Gaussian case). However, under this specification, the expected (arithmetic) return on such securities would be infinite, and it is not clear in this case that the equilibrium interest rate would be finite." (page 127).

Regarding our problem of fitting non-normal return series, the jump models do have promise. Beckers (1981) shows that the Merton (1976) Jump Diffusion model will yield a leptokurtic distribution and "therefore might better describe the actual price return behaviour than the pure lognormal model". (page 128). Recently, Scott (1994) stated that "On occasions, there are large rapid movements resembling jumps, and the volatility of stock returns changes randomly over time. Both of these features serve to explain the leptokurtosis". (page 1) Later papers by Jarrow and Rosenfeld (1984) and Ball and Torous (1985) have also modelled this feature by including leptokurtic jump components. Jorion (1988) found evidence that both ARCH effects and jumps exist in financial data and are required to describe the empirical behaviour of the return series.

The problem with the jump process model proposed by Merton is that it requires both a risk premium for jumps and the expected returns for options to solve. Merton himself states in his 1976 paper, "Moreover the power and beauty of the original Black-Scholes derivation stems from not having to know α [jump risk premium] or $g(S,\tau)$ [the expected return on the option] to compute the option's value, and both are required to solve [equation for the jump model] (12)." page 133.

Given that no securities yet exist to allow hedging of this risk premium component, it might seem that these models are of limited practical use. Nevertheless, Bates (1991) and Naik and Lee (1990) examined options on securities with nondiversifiable jump risk. Since our goal is to understand the nature of the objective processes, such limitations (risk premia or nondiversifiable risks) may not be as restrictive as feared for our purposes. We can still understand the dynamics of the markets without the markets necessarily being complete.

Models which Incorporate Stochastic Volatility

Given that the empirical evidence rejects the hypothesis that volatility is constant, another pricing approach allows the underlying asset price volatility to be stochastic as well. Stochastic volatility models bear an intimate relationship to the ARCH approaches to modelling the objective volatility. This is because a diffusion model with normally distributed innovations and an instantaneous volatility that depends upon a mean reverting Ornstein-Uhlenbeck process can potentially yield conditional heteroscedasticity. Nelson (1990) has shown that the continuous-time limit of standard ARCH and GARCH models is a stochastic volatility model with increments to volatility that are independent to those of the asset price. Thus, this approach provides links between two well established branches of the literature, contingent claims analysis and empirical modelling.

Hull and White (1987a) were the first to consider this approach. They demonstrate that when the volatility is uncorrelated with the stock price, the option's price would be the Black and Scholes price if the integration was done over the distribution of the average variance rate during the life of the option. Simulations of their approach show that a constant volatility (Black-Scholes) model overprices options that are at-the-money or close to the money, and underprices options that are deep in or deep out of the money. Thus, once again we can explain the existence of leptokurtosis in the markets.

In the case where the stock price and volatility are instantaneously correlated, Hull and White show how either Monte Carlo simulation or a series expansion can be used to obtain option prices. When the correlation is negative, the situation would predict an implied distribution that is consistent with equity markets. The rationale is that when the stock price decreases, volatility tends to increase. This means that very low stock prices are more likely than under geometric Brownian motion.

While the effects of stochastic volatility have been examined by a number of authors, [Bailey and Stulz (1989), Johnson and Shanno (1987), Scott (1987,1991) and Wiggins (1987), Chesney and Scott (1989) and more recently by Stein and Stein (1991) and Heston (1993)] Taylor (1994) probably provides the best survey of this work.

Taylor defines a stochastic volatility process as an extension of the standard stochastic differential equation presented earlier.

$$r_t = \mu + \sigma_t U_t \text{ with } Z_t \sim N(0,1). \qquad (1.27)$$

In this model, the constant volatility term σ is replaced with a positive random variable, σ_t. Quoting from Taylor (1994): "Whenever the returns process [r_t] can be represented by this equation. I will call σ_t the stochastic volatility for period t. A normal distribution for $(r_t - \mu)/\sigma_t$ is an essential component of the definition of stochastic volatility adopted here. The stochastic process [σ_t] will generate realised volatilities [σ^*_t], which are in general not observable. For any realisation σ^*_t,

$$r_t \mid \sigma_t = \sigma^*_t \;\; \sigma^*_t \sim N(\mu, \sigma^{*2}_t). \qquad (1.28)$$

A mixture of these conditional normal distributions defines the objective distribution of r_t, which has excess kurtosis wherever σ_t has positive variance and is independent of U_t." (page 185).

A number of options pricing models have been published which allow both the asset price and the volatility to each follow a diffusion process. At least four processes for the volatility have been proposed.

(1) The Hull & White approach (1987a) assumes that the variance follows a lognormal diffusion process and the square root of this process will provide the volatility parameter. This process can be written as:

$$d\sigma = k\sigma(\theta - \sigma)dt + \xi\sigma \cdot dZ_1 \qquad (1.29)$$

When Hull and White actually modelled the stochastic process for the volatility, they chose to transform it into a variance process.

$$d\sigma^2 = k\sigma^2(\theta - \sigma)dt + \xi\sigma^2 \cdot dZ_1 \qquad (1.30)$$

They state generate variances according to the formula (on page 290):

$$V_i = V_{i-1} \cdot e^{[k(\theta-\sigma)-\xi^2/2]dt+\xi \cdot dZ_1} \quad (1.31)$$

The volatility estimate, σ, is simply $\sqrt{V_i}$. For the estimation the stochastic volatility both are required but for testing purposes, we will only examine the $\sqrt{V_i}$, which is the estimated volatility, σ. Again, time is expressed as the percentage that one day represents in a trading year (of 252 days).

(2) Alternatively, the Stein and Stein (1991) approach assumes that the volatility (absolute level) follows the Ornstein-Uhlenbeck (O-U) process. This model can be seen as the equivalent for volatilities that the Vasicek model is for interest rates. This process can be written as:

$$d\sigma = -k(\sigma-\theta)dt + \xi \cdot dZ_1 \quad (1.32)$$

In our case, τ_1-τ_0 is equal to 1/252 (which is the percentage that one day represents in our assumed trading year). This time increment was chosen because it is the same time increment used for the empirical estimation of the volatility cones presented previously.

(3) Scott (1987,1991), Wiggins (1987) and Chesney and Scott (1989) suppose that the logarithm of the volatility follows the Ornstein-Uhlenbeck (O-U) process. This process can be written as:

$$d(\ln\sigma) = k'(\theta' - \ln\sigma)dt + \xi \cdot dZ_1 \quad (1.33)$$

This model can be expressed in terms of volatility as:

$$d\sigma = (k'\theta' + \frac{1}{2}\xi^2 - \kappa'\ln\sigma)\sigma \cdot dt + \xi\sigma \cdot dZ_1{}^4 \quad (1.34)$$

In this model, κ',θ' and ξ are unknown parameters which must be estimated. It should be noted that the two parameters with primes (κ' and θ') are so indicated because these will not be of the same scale as the parameters for the other models.

While this approach seems to be different from the previous models, this model is in fact a variation of the Hull & White (1987a) model. This makes sense given that Hull & White assumed a log normal dispersion process for volatilities as does this class of models.

For the same random series of draws from a normal distribution, Z_1, an almost exact series of volatilities will be produced if the following adjustments are made to the parameter values of the Hull & White model.

The long term volatility term, θ, in the Hull & White model becomes:

$$\theta' = \ln(\theta) - \frac{1}{2} \cdot \left(\frac{\xi^2}{\kappa'} \right) \tag{1.35}$$

and the rate of reversion for Hull & White, κ, becomes:

$$\kappa' = \kappa \cdot \theta \tag{1.36}$$

Due to the fact that the log model will produce almost identical results as the Hull & White approach, once the parameters have been adjusted, this class of models is redundant.

(4) Heston (1993) proposes another stochastic volatility model that states that the variance follows the following process:

$$d\sigma^2 = k(\theta - \sigma^2)dt + \xi\sigma \cdot dZ_1 \tag{1.37}$$

which Taylor (1994) described in terms of the volatility process (page 186) as:

$$d\sigma = \frac{1}{\sigma}(\theta - k\sigma^2)dt + \xi \cdot dZ_1 \tag{1.38}$$

Other authors who have tried this approach include: Bates and Pennacchi (1990), Gennotte and Marsh (1991) and Ball and Roma (1993). This model is the volatility equivalent of the Cox, Ingersoll and Ross (1985) model for interest rates.

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[4] The proof of this can be seen in the appendix to this Chapter.

The problem with simulating this model to generate a series of volatilities (in discrete steps) is that the process assumes a non-central Chi-square (or Bessel process) distribution. While this is more difficult to model than the lognormal dispersion process used by Hull & White, it can be estimated. To simulate volatilities, one can use the method Cox, Ingersoll and Roll (1985) used in their paper. On pages 391 and 392 of their paper, they indicate how to model this process. For this exposition, I will replicate their formula and use their symbols (which are almost identical to ours but I will clarify when differences exist). From CIR, we start with the following stochastic process for interest rates:

$$dr = k(\theta - r)dt + \sigma\sqrt{r} \cdot dZ_1 \qquad \textbf{(1.39)}$$

The Heston (1993) process for volatility can be expressed (using our notation) as:

$$d\sigma^2 = k(\theta - \sigma^2)dt + \xi\sigma \cdot dZ_1 \qquad \textbf{(1.40)}$$

In this formula, we are working with variances so the σ^2 replaces the 'r' in the CIR model. Instead of the $\sqrt{r}$ in the CIR model, we input the square root of the variance which is the volatility. Finally, instead of representing the stochastic nature of the process by σ (which for CIR is the volatility of interest rates), we use ξ which is the volatility of the variance. CIR indicate in equation (18) of their paper what the process is that describes their model:

$$f(r(s), s; r(t), t) = c \cdot e^{-u-v}\left(\frac{v}{u}\right)^{q/2} \cdot I_q(2(uv)^{1/2}) \qquad \textbf{(1.41)}$$

where:

$$c \equiv \frac{2\kappa}{\sigma^2(1 - e^{-\kappa(s-t)})}, \qquad \textbf{(1.41a)}$$

$$u \equiv c \cdot r(t) \cdot e^{-\kappa(s-t)}, \qquad \textbf{(1.41b)}$$

$$v \equiv c \cdot r(s), \qquad (1.41c)$$

$$q \equiv \frac{2\kappa\theta}{\sigma^2} - 1, \qquad (1.41d)$$

and $I_q(\cdot)$ is the modified Bessel function of the first kind of order q. The distribution function is the noncentral chi-square, χ^2[2cr(s); 2q+2, 2u], with 2q+2 degrees of freedom and parameter of noncentrality 2u proportional to the current spot rate. (pages 391-392).

For the modelling of volatility, (s-t) was equal to 1 (or one day); this was in turn expressed in terms of a trading year (divided by 252), r(t) was replaced by the variance from the previous day (σ^2_{t-1}). The terms κ and θ are the same as CIR. However, one must replace σ and σ^2 by the volatility (or variance) of the volatility, ξ or ξ^2.

While other models have been proposed by a number of authors [Johnson & Shanno (1987), Hull and White (1987b, 1988) and Melino and Turnbull (1990)], we will choose to ignore them given that they may not incorporate a mean reversion feature and the literature has shown that mean reversion does exist. Alternatively, it can be shown that these models are equivalent to those we have already selected.

Bailey and Stulz (1989) and Scott (1992) have examined the choice of volatility models and found that they have non-trivial differences and economic implications for the determination of contingent claims. Given the considerable interest in these models, it is interesting to note that there have been very few studies that examine how well these models fit the empirical behaviour of asset returns. The two most important papers, which examined the effectiveness of the various stochastic volatility models on fitting empirical return series are Longstaff (1989) and Ho, Perraudin and Sørensen (1996). Longstaff estimated a continuous-time capital asset-pricing model using a generalised method of moments (GMM) on time aggregated returns. The Ho, Perraudin and Sørensen paper examined a multifactor continuous-time arbitrage pricing model that included both stochastic volatility and jumps. Their research was restricted to the examination of equity returns in the United States. Perraudin and Sørensen (1997) later applied a similar approach to the foreign exchange market. They refined their approach by examining a model with included jumps, stochastic volatility and correlations between interest and exchange rates. All of these papers suggest that combinations of models may be required to understand the return series of assets. Ho, Perraudin and Sørensen (1996) conclude their paper with suggestions for further research thus:

"Another important area for further research is the development and adaptation to this type of model of estimation methods that may be more efficient that our present method, which is based on information from the objective moments of the [underlying asset] returns" (page 40).

For this research, we will extend this analysis by examining a wider range of financial assets (represented by financial futures) and incorporate the objective moments of the underlying assets returns to assess the efficacy of the various approaches to understanding the dispersion processes of financial assets. Therefore, an important element in this research is to examine how well these models explain the actual empirical behaviour of the standard deviations of the volatility estimates at different time horizons. Furthermore, we will determine the optimal parameters for these models which explains the dynamics of these markets. This will be examined in Chapters 4 and 5.

1.7 CONCLUSION

In this Chapter, we have provided a review of the literature that examined the stochastic process for assets. We will examine the nature of this process in the next four chapters. With this assumption of the process driving the asset prices, we will concentrate on understanding the nature of the volatility parameter into these processes. Also from this research review, we have gained insights into the key elements that must be understood to fully explain the nature of the objective dispersion process. The methodology will entail examining the time-series behaviour of asset returns and assessing how they are empirically distributed and the nature of autocorrelations of absolute returns. It is clear that features such as fat-tailed tendencies of the actual dispersion processes will impact options. Therefore, we must capture this effect. We must also capture the autocorrelations in the objective process. Finally, the volatility cones will provide us an indication of the possible forecast errors in the realised volatility, which is critical to those pricing options. Once, we have found statistics that will capture these key factors, we will use these to understand the dynamics of the volatility process. By capturing these dynamics, we can then test a variety of processes to assess which best describe the empirical record. With this achieved, we can then examine whether these asset processes have the correct properties for understanding option values.

Appendix 1.1 Conversion of the log volatility process to a simple volatility process

$$d(\ln v) = \kappa(\theta - \ln v)dt + \xi dZ \quad (1)$$

where v is the volatility, κ is the rate of mean reversion, θ is the long term log of the volatility and ξ is the volatility of the volatility.

Let $\sigma = \ln v \therefore v = e^{\sigma}$, with this substitution to the above equation, becomes

$$d\sigma = \kappa(\theta - \sigma)dt + \xi dZ \quad (2)$$

$$v_{\sigma} = e^{\sigma} = v, \text{ and } v_{\sigma\sigma} = e^{\sigma\sigma} = v \quad (3)$$

From Ito: $v \equiv v(\sigma)$, therefore:

$$dv = v_{\sigma} d\sigma + \frac{1}{2} v_{\sigma\sigma} (d\sigma)^2, \text{ or} \quad (4)$$

$$dv = v d\sigma + \frac{1}{2} v (d\sigma)^2, \text{ thus} \quad (5)$$

$$dv / v = d\sigma + \frac{1}{2} (d\sigma)^2 \quad (6)$$

from above (2), we can substitute, to yield:

$$dv / v = \kappa(\theta - \sigma)dt + \xi dZ + \frac{1}{2} \xi^2 dt \quad (7)$$

this can be simplified and written as:

$$dv / v = (\kappa\theta + \frac{1}{2} \xi^2 - \kappa\sigma)dt + \xi dZ \quad (8)$$

then substituting ln v for σ, we obtain:

$$dv / v = (\kappa\theta + \frac{1}{2} \xi^2 - \kappa \ln v)dt + \xi dZ \quad (9)$$

or expressed in terms of dv,

$$dv = (\kappa\theta + \frac{1}{2} \xi^2 - \kappa \ln v)v dt + \xi v dZ \quad (10)$$

CHAPTER TWO
DATA, DESCRIPTIVE STATISTICS AND DERIVED STATISTICS

2.1 INTRODUCTION

In the previous Chapter, we identified a clear path to follow. We will examine the historical return series for a series of twelve financial futures. Previous research has suggested that we must examine the statistical moments of these return series and has led us to expect that non-normality will be observed. We expect that the absolute returns will display autocorrelations over time. This will also be examined. Given the problems we face with relatively short time series of returns, we will extensively use the Volatility Cone approach discussed in the previous Chapter. This will allow us to gain a clearer insight into how the unconditional forecasts of volatility behave as a function of the time horizon of prediction. We will then integrate these various facets of the processes into five key summary statistics, that we will later use as attributes. These attributes capture the essence of the objective dynamics relevant to option prices. By doing this, we will determine a concrete metric for the comparison of various security price process models and to assess which would be best for pricing options. There is also some evidence that the dynamics of the objective process are not stationary. To assess this, we will split our return series into two sub-samples (of roughly equal size) and examine this by comparing the statistical properties in different time periods.

2.2 DATA SOURCES

The first portion of the results will examine the return series for a variety of underlying assets over various time horizons. These assets will include four fixed income futures contracts: US Treasury Bond Futures, UK Gilt Futures, German

Bundesanleihen Futures, and Italian Government Bond Futures (BTPs), four equity index futures: S&P 500 Futures, FTSE 100 Futures, DAX Index Futures, and Nikkei 225 Futures, and four currency futures: US Dollar/Deutsche Mark, US Dollar/British Pound, US Dollar/ Japanese Yen and US Dollar/Swiss Franc. For all of these assets, the analysis period extends back in time to either the start of these contracts or to include all the available data that was in the public domain. For most of the instruments data was available for more than 10 years, apart from the DAX Index Futures which had daily data only from January 1, 1992 and the BTP futures which only had data available from December 1991.

For the following underlying assets, the following time periods of analysis were completed with the associated number of daily prices. These can be seen in Table 2.1.

Underlying Asset	Time Period of Analysis	Number of Observations
S&P 500 Futures	3/1/1985 - 20/12/1996	3031
FTSE Futures	4/5/1984 - 20/12/1996	3198
Nikkei 225 Futures	16/9/1990 - 13/12/1996	1576
DAX Futures	3/1/1992 - 20/12/1996	1254
Bund Futures	30/9/1988 - 5/12/1996	2073
BTP Futures	9/9/1991 - 4/12/1996	1325
Gilt Futures	19/11/1982 - 27/12/1996	3568
US T-Bond Futures	23/8/1977 -31/12/1996	4885
Deutsche Mark /US Dollar	3/1/1985 - 16/12/1996	3027
British Pound / US Dollar	3/1/1985 - 16/12/1996	3028
Japanese Yen / US Dollar	3/1/1985 - 16/12/1996	3022
Swiss Franc / US Dollar	24/1/1984 - 16/12/1996	3265

Table 2.1, Markets Included in Research, Time Period of Data, Number of Observations

For the analysis of the futures and options contacts that trade on the London International Financial Futures Exchange (LIFFE) this includes: BTPs (Italian Government Bonds), Bunds (German Government Bonds), Gilts (British Government Bonds) and the Financial Times 100 Stock Index (FTSE), data was obtained directly from the LIFFE. This data includes closing prices of the futures contracts and other information including volume, open interest, opening, high and low prices.

For the futures and options contracts traded at the Chicago Board of Trade (US T-Bond Futures), the data was obtained directly from the CBT on floppy disks. This data included only the closing prices. For the futures and options contracts traded at the Chicago Mercantile Exchange and this includes the data on S&P 500, Nikkei 225, Deutsche Mark, British Pound, Swiss Franc and Japanese Yen (all versus US Dollar) Futures. The data was obtained directly from the CME. This data included only the closing prices of the futures. For the futures and options contracts traded at the Deutsche Terminbörse (i.e. the DAX futures and options) this data was obtained directly from the exchange. It included all tick by tick prices during the day.

Cleaning the Data Series

Given that this research is empirical, a major effort was made to assure the validity of the data used in the analysis and that the analytic methods employed were correct. This was achieved in a number of ways. Firstly, we compared the futures price series with the options price series for the same days to identify obvious errors in recording either price series. This comparison was achieved by comparing the put-call parity values of the options with the underlying futures prices for every single date in our database (and for all twelve markets). A screening procedure was imposed such that if futures or options prices diverged by more than the normal bid/offer spread (of one tick), the observations were flagged. Once this was done, each price was compared with the original daily price sheets to confirm if a 'keypunch' error had occurred. We discovered that only 1-2% of the data had such errors. Nevertheless, these errors were of a sufficient magnitude that they did influence the results and therefore required correction.

For the currency futures, we also constructed a time series of theoretical futures prices. This was done by obtaining a series of spot exchange rates for the four currency pairs examined from Knight-Ridder for the same period as the futures in our sample (see Table 2.1). Then we obtained time series of one week, one month and three-month Euro-currency interest rates for the entire period from The Bank of England statistics department for each of the countries represented in the currency pairs (five countries including the US Dollar). Given that we knew the exact data that each futures contract expired, we could use the spot rate and the two interest rates for each country (assuming the Interest Rate Parity Theorem) determine a theoretical forward price. The interest rates used in the estimation were linear interpolations of the interest rate time series obtained from the Bank of England. While differences do exist between futures and forward prices [see French (1983)], for the currency markets, Cornell (1977) demonstrated that the Interest Rate Parity Theorem will lead to an unbiased estimate of the future spot exchange rate. The purpose of this analysis was to assess if the futures prices obtained from the exchanges were significantly different from theoretical forward prices. If the divergence exceeded the arbitrage range suggested by French (1983), these observations were also flagged and we returned to original sources to verify the data points. This procedure only revealed four errors in the futures prices. However, these errors had a substantial impact on the analyses. For example, the British Pound futures prices had an recording error (one day a price of 131, the next data 113 and the final day 131) which caused the return series to display extreme excess kurtosis. This was spotted using the theoretical forward price and corrected. This also occurred for other currency futures from the CME and for the FTSE 100 futures.

The most arduous of the data cleaning process was the ongoing examination of the data as results of the analysis were obtained. This is another important reason why this research examined twelve markets simultaneously and four markets in each of the three asset classes. Our prior assumption was that similar dynamics might be found for futures in the same asset class. While we expected these to vary somewhat, if results were dramatically different, we re-examined the data series for this anomalous result and also verified the computer programmes we had written to complete the analysis. This feedback mechanism allowed us to spot potential errors (either in data or in programming). Throughout this research process, we were required to re-run the analysis as many as five times to assure all errors had been corrected. At each stage, we devised additional tests to confirm our results. Only after each result was verified by re-running all the analysis from first principles did we move on. This assured that the data series employed in this research was as accurate as is humanly possible.

In this section, we have discussed the steps involved in cleaning the futures price series. We also used a number of methods to clean the options data. How this data was cleaned will be discussed in Chapter 7 when we discuss the options data used in this research.

2.3 UNCONDITIONAL RETURN DISTRIBUTIONS FOR VARIOUS HORIZONS

For all the markets, we examined the return statistics for daily, weekly and monthly returns. These returns were calculated using non-overlapping data. For the weekly and monthly series, the return was determined by the logarithm of the price relatives at time T=i and T=i+5 for the weekly series and T=i and T=i+20 for the monthly series. For this first stage of analysis, all the available data was used. The results of these analyses were summarised and can be seen in Table 2.2.

Underlying Asset	Mean	Std Dev	Skew	Uncond. Kurtosis	Range	Observations
S&P 500 Futures						
Daily Returns	.000389	.012034	-7.1699	254.5062	.53262	3030
Weekly Returns	.001894	.021015	-1.6373	17.72106	.26380	606
Monthly Returns	.007348	.042618	-0.91634	7.92878	.35209	151
FTSE Futures						
Daily Returns	.000243	.010438	-1.66561	29.47038	.24814	3197
Weekly Returns	.001156	.023284	-1.51148	14.95273	.27467	639
Monthly Returns	.004526	.050165	-2.90344	25.47266	.52044	159
Nikkei 225 Futures						
Daily Returns	-.00025	.014895	0.148381	4.7400	0.12068	1575
Weekly Returns	-.00121	.030316	2.093259	5.0933	0.23367	315
Monthly Returns	-.00420	.067262	0.150368	4.3889	0.40259	78
DAX Futures						
Daily Returns	.000251	.009551	-0.28886	5.7004	0.10657	1253
Weekly Returns	.001227	.020923	-0.10233	4.3205	0.14927	250
Monthly Returns	.005578	.041813	-0.55121	3.0734	0.18659	62
Bund Futures						
Daily Returns	.000067	.003476	-0.13334	6.87162	0.042452	2072
Weekly Returns	.000368	.007816	-1.00293	7.65522	0.073231	414
Monthly Returns	.001358	.016227	-0.90301	6.66585	0.113173	103
BTP Futures						
Daily Returns	.000235	.005874	-0.32498	5.05899	0.055337	1324
Weekly Returns	.001119	.014089	-0.67822	4.95154	0.102704	264
Monthly Returns	.004333	.030907	-0.51087	3.36595	0.148770	66
Gilt Futures						
Daily Returns	.000080	.005638	0.018851	5.75804	0.063795	3567
Weekly Returns	.000390	.012163	0.028687	3.91758	0.091405	713
Monthly Returns	.001380	.026667	0.383716	3.66232	0.150335	178
US T-Bond Futures						
Daily Returns .	.000104	.007701	0.024003	5.36237	0.075696	4884
Weekly Returns	.000510	.017804	0.316367	5.55104	0.183041	975
Monthly Returns	.002074	.037440	0.186522	4.67991	0.282151	243
Deutsche Mark /US Dollar						
Daily Returns	.000210	.007443	0.123807	5.44599	0.081446.	3026
Weekly Returns	.001104	.016605	0.135496	4.21552	0.136368	605
Monthly Returns	.004444	.032521	0.05563	2.92998	0.161141	151
British Pound / US Dollar						
Daily Returns	.000232	.007481	0.096509	6.53195	0.090289	3026
Weekly Returns	.001217	.017207	-0.18558	7.46481	0.194957	605
Monthly Returns	.004789	.035838	-0.19297	4.99446	0.273316	151
Japanese Yen / US Dollar						
Daily Returns	.000176	.007063	0.386986	7.78965	0.095401	3021
Weekly Returns	.000907	.015405	0.292853	4.77537	0.125520	605
Monthly Returns	.003585	.032930	0.612899	3.49379	0.180435	151

Underlying Asset	Mean	Std Dev	Skew	Uncond. Kurtosis	Range	Observations
Swiss Franc / US Dollar						
Daily Returns	.000081	.00819	0.155202	5.045	0.089553	3264
Weekly Returns	.000487	.017942	-0.00905	3.85769	0.131532	652
Monthly Returns	.001934	.037694	0.077388	3.08828	0.209201	163

Table 2.2, Statistics of the Daily, Weekly and Monthly Returns for Twelve Financial Futures for the Whole Period of Available Observations

For almost all the series, the dispersion of returns is not well described by a normal distribution. For almost all series (and for almost all horizon periods), the kurtosis measure indicates significant leptokurtosis exists. The kurtosis measure has a critical level above 3.0 for rejection as mesokurtic. The most extreme violations from the assumption of normality are for the equity index futures, especially the S&P 500 and the FTSE 100. The DAX and Nikkei do not display the most extreme negative skewness or the high level of kurtosis. This is most probably due to the lack of data for these series that does not include the crash of 1987 or the mini-crash of 1989.

One of the problems we encountered in the estimation of historical volatility particularly for the stock index futures contracts was that the inclusion or exclusion of the return data surrounding the 1987 crash (and the 1989 mini crash) had significant impacts on the measurements of the objective processes.

While it is tempting to drop the 1987 crash from the analysis, it has been argued that this is a valid observation (although extreme). In a recent paper by Jackwerth and Rubinstein (1996), they state: "What is virtually certain is that the crash should not be omitted as an outlier." Jackwerth and Rubinstein continue, "Apart from the special problems created by the stock market crash, many other difficulties are encountered sampling from an inherently nonstationary time series such as stock market prices."

For the other asset classes, while the skewness is not significant, all the daily returns display fat-tailed behaviour. For most of the fixed income markets, the longer the estimation period of the returns is the lower the measure of kurtosis. However, for almost all the bond futures markets, significant kurtosis remains at the monthly estimation period. For the currency markets, while the daily kurtosis is significantly different than mesokurtic, as the observation periods are extended to monthly, most markets approach a normal distribution.

These findings are by no means unique. The consensus in the literature is that the distributions display excessive leptokurtosis and can be skewed depending on the asset class examined. One possible reason for these results is that the time period of analysis is atypical. It could be that the return series is not stationary and that analysis for different periods would yield different results. To examine this, we split the analysis horizon into two parts. This was achieved by simply dividing the overall observation period in half. Whenever possible, the number of data points in the first and second period were the same and no data overlapped in the two periods. The time periods of the analysis and the number of observations can be seen in Table 2.3.

Underlying Asset	1st Time Period Of Analysis	Number of Observations	2nd Time Period Of Analysis	Number of Observations
S&P 500 Futures	3/1/1985 - 27/12/1990	1515	28/12/1990 - 20/12/1996	1517
FTSE Futures	4/5/1984 - 29/8/1990	1599	30/8/1990 - 20/12/1996	1600
Nikkei 225 Futures	26/9/1990 - 3/11/1993	788	4/11/1993 - 13/12/1996	789
DAX Futures	3/1/1992 - 1/7/1994	628	4/7/1994 - 20/12/1996	628
Bund Futures	30/9/1988 - 3/11/1992	1037	4/11/1992 - 5/12/1996	1037
BTP Futures	9/9/1991 - 26/4/1994	663	27/4/1994 - 4/12/1996	663
Gilt Futures	19/11/1982 - 6/12/1989	1784	7/12/1989 - 27/12/1996	1785
US T-Bond Futures	23/8/1977 -28/4/1987	2444	29/4/1987 - 31/12/1996	2445
Deutsche Mark	3/1/1985 - 26/12/1990	1515	27/12/1990 - 16/12/1996	1514
British Pound	3/1/1985 - 26/12/1990	1515	27/12/1990 - 16/12/1996	1514
Japanese Yen	3/1/1985 - 28/12/1990	1511	31/12/1990 - 16/12/1996	1512
Swiss Franc	24/1/1984 - 6/7/1990	1632	9/7/1990 - 16/12/1996	1634

Table 2.3, Periods & Observations for Markets Under Analysis, Broken into Two Sub-Periods

For each portion, the return series summary statistics were determined. The results of these analyses for the first half and the second half of the available data were summarised and can be seen in Tables 2.4 and 2.5.

(First Period) Underlying Asset	Mean	Std Dev	Skew	Uncond. Kurtosis	Range	Observations
S&P 500 Futures						
Daily Returns	.00031	.01549	-6.7117	185.34	.5326	1514
Weekly Returns	.00147	.02561	-1.8674	15.62	.2638	302
Monthly Returns	.00582	.05428	-0.9195	5.88	.3521	75
FTSE Futures						
Daily Returns	.00020	.01168	-2.391	35.9850	.24814	1598
Weekly Returns	.00095	.02687	-2.1209	15.7083	.27168	319
Monthly Returns	.00486	.06155	-3.2018	23.2482	.52044	79
Nikkei 225 Futures						
Daily Returns	-.0004	.01634	0.1527	3.9912	0.1207	787
Weekly Returns	-.0023	.03291	0.6757	5.0661	0.2337	157
Monthly Returns	-.008	.07705	0.4655	4.4768	0.3842	39
DAX Futures						
Daily Returns	.00010	.01037	-0.3181	6.1734	0.1066	627
Weekly Returns	.00053	.02256	-0.1895	4.1130	0.1330	125
Monthly Returns	.00187	.04936	-0.5243	2.5569	0.1866	31
Bund Futures						
Daily Returns	-.00004	.00340	0.00865	9.5135	0.04245	1036
Weekly Returns	-.00016	.00763	-1.1493	11.393	0.07323	207
Monthly Returns	-.00073	.01854	-0.9872	6.9802	0.11317	51
BTP Futures						
Daily Returns	.00014	.00538	-0.2544	6.8939	0.05534	662
Weekly Returns	.00073	.01315	-1.0325	7.3166	0.09364	132
Monthly Returns	.00292	.02606	0.1666	2.3185	0.09825	33
Gilt Futures						
Daily Returns	.000013	.00587	-0.1026	5.3372	0.05534	1783
Weekly Returns	.00003	.01270	0.1254	4.0554	0.0914	356
Monthly Returns	.00016	.02827	0.5992	3.9202	0.1503	89
US T-Bond Futures						
Daily Returns	.000029	.00894	0.01534	4.5418	0.0757	2443
Weekly Returns	.000065	.02104	0.33825	4.8295	0.183	488
Monthly Returns	.000168	.04409	0.22475	4.1408	0.282	122
Deutsche Mark /US Dollar						
Daily Returns	.00035	.00759	0.2566	5.3573	0.08096	1513
Weekly Returns	.00195	.01714	0.2894	4.6190	0.13637	302
Monthly Returns	.00809	.03349	0.0079	2.6955	0.1428	75
British Pound / US Dollar						
Daily Returns	.00045	.00791	0.29876	6.18583	0.07451	1513
Weekly Returns	.00236	.01842	0.43812	6.22321	0.15732	302
Monthly Returns	.00970	.03769	0.39467	3.81009	0.21654	75
Japanese Yen / US Dollar						
Daily Returns	.00031	.00718	0.3368	7.1017	0.09516	1510
Weekly Returns	.00157	.01656	0.4540	4.2001	0.1121	302
Monthly Returns	.00644	.03615	0.5695	3.1028	0.174	75
Swiss Franc / US Dollar						
Daily Returns	.00012	.00828	0.22729	5.0305	0.08659	1631
Weekly Returns	.00073	.01806	0.17394	3.4223	0.12147	326
Monthly Returns	.00284	.03686	0.46234	2.7030	0.15468	81

Table 2.4, Statistics of the Daily, Weekly and Monthly Returns for Twelve Financial Futures for the First Half of the Observation Period

(Second Period) Underlying Asset	Mean	Std Dev	Skew	Uncond. Kurtosis	Range	Observations
S&P 500 Futures						
Daily Returns	.00047	.00705	-0.0502	5.8823	.08195	1516
Weekly Returns	.00229	.01503	0.00902	3.6838	.0973	327
Monthly Returns	.00941	.02633	0.88922	4.5881	.14134	82
FTSE Futures						
Daily Returns	.00028	.00903	0.0361	4.5168	.09532	1599
Weekly Returns	.00126	.01936	0.355	3.8616	.12566	319
Monthly Returns	.00523	.0346	0.0356	3.1165	.17052	79
Nikkei 225 Futures						
Daily Returns	-.000037	.01330	0.15845	5.8671	0.11801	788
Weekly Returns	-.000082	.02726	0.53047	4.9355	0.18699	157
Monthly Returns	-.000108	.05439	0.36817	3.7983	0.26146	39
DAX Futures						
Daily Returns	.000399	.00867	-0.2080	4.2104	0.07232	626
Weekly Returns	.001917	.01815	-0.601	3.1927	0.08586	125
Monthly Returns	.008113	.03029	-0.7951	3.3508	0.12507	31
Bund Futures						
Daily Returns	.000175	.00354	-0.2669	4.6621	0.0302	1036
Weekly Returns	.000826	.00778	-0.7939	4.0912	0.0449	207
Monthly Returns	.003084	.01347	-0.7993	3.2145	0.0579	51
BTP Futures						
Daily Returns	.00033	.00633	-0.3766	3.9391	0.04234	662
Weekly Returns	.00167	.01492	-0.4943	3.5725	0.08056	132
Monthly Returns	.00629	.03208	-0.7296	3.2166	0.12919	33
Gilt Futures						
Daily Returns	.000147	.00539	0.1821	6.2497	0.05846	1784
Weekly Returns	.000691	.01188	0.0032	4.6908	0.09894	356
Monthly Returns	.002652	.02394	-0.0116	3.1823	0.12192	89
US T-Bond Futures						
Daily Returns	.00018	.00621	0.00834	5.8301	0.06929	2444
Weekly Returns	.00091	.01406	0.68975	8.946	0.15615	488
Monthly Returns	.00372	.02926	0.46439	5.1053	0.19616	122
Deutsche Mark /US Dollar						
Daily Returns	.000067	.00729	-0.0301	5.5237	0.06914	1513
Weekly Returns	.000366	.01659	-0.1611	4.4694	0.12176	302
Monthly Returns	.00143	.03248	-0.2363	3.069	0.17051	75
British Pound / US Dollar						
Daily Returns	.000015	.007026	-0.2167	6.8068	0.07951	1512
Weekly Returns	.000117	.016241	-1.3800	9.7949	0.14067	302
Monthly Returns	.000507	.032503	-1.4532	7.5824	0.21059	75
Japanese Yen / US Dollar						
Daily Returns	.000043	.00695	0.43888	8.5897	0.08961	1511
Weekly Returns	.000211	.01381	0.37715	3.8694	0.09165	302
Monthly Returns	.000889	.02875	-0.0869	4.1441	0.17557	75
Swiss Franc / US Dollar						
Daily Returns	.000045	.00810	0.07762	5.0630	0.07915	1633
Weekly Returns	.000259	.01851	0.00663	3.9843	0.12344	326
Monthly Returns	.001196	.03615	-0.1304	2.783	0.18457	81

Table 2.5, Statistics of the Daily, Weekly and Monthly Returns for Twelve Financial Futures for the Second Half of the Observation Period

When comparing the observation period split into two parts, we find that the series display significantly different behaviours. For the stock index futures, the first period includes the 1987 and 1989 crashes for the S&P 500 and the FTSE 100[1]. For the DAX and Nikkei, the first period was a period of relative stability. We observe extremely high kurtosis for the S&P 500 and the FTSE 100 and significantly negative skewness. For the DAX and Nikkei, the kurtosis is moderately high and the skewness measures are insignificantly different than zero. In the second period, the S&P 500 and FTSE 100 now have much smaller levels of kurtosis and the skewness has been reduced to either a small negative level (or even positive for the weekly and monthly observations for the FTSE). For the DAX and Nikkei, the latter period was also different from the first period. In this instance, there was greater kurtosis in the later period for both markets. For the DAX there appears to be slightly negative skewness for daily returns, while the Nikkei has a slightly positive skew.

From this comparison, it is clear that the empirical reason for the existence of extreme kurtosis and skewness in equity markets is due to a few extreme events. Therefore, the statistical behaviour of the equity markets is not stationary over time depending on whether these extreme events are included or not. In the next Chapter, we will model the behaviour of the twelve markets using a variety of different theoretical approaches. All tests will be completed for the entire period of analysis and for the two sub periods. In addition, we will examine the impact solely of the 1987 crash on the modelling of the S&P 500, where the impact was the most significant.

For the fixed income markets, while some differences exist between the first and second period, it appears that the kurtosis and skewness behaviours are more

[1] These results are similar to those reported by Gemmill (1991) for the FTSE 100 (see Appendix 2,

stationary. The only important difference is that for the Bund and BTP, the kurtosis is higher in the second period relative to the first. The Gilt and US T-Bond markets display somewhat lower kurtosis in the first period relative to the second period. For none of the fixed income markets does skewness play a significant role. Depending on the period of analysis, the skewness is either slightly negative or positive and this could be due simply to sampling variability.

For the foreign exchange markets, most of the currencies display similar behaviour in the first and second period. The only exceptions are for the Japanese Yen and the British pound, which has a slightly higher kurtosis in the second period relative to the first period. This is due to turbulence in the markets during the early 1990s. As will be demonstrated later, for the British Pound the ERM crisis in 1992 is responsible for the greater kurtosis in the second period. Likewise, for the Japanese Yen, extreme turbulence occurred during 1994/1995 as the U.S. Dollar went to post-World War II lows.

The historical record demonstrates that the stationarity of the objective process for the twelve financial futures markets under investigation is not constant. For the foreign exchange and fixed income markets, it would appear that the dispersion processes tend to display more stationary behaviour. For the stock index futures markets, the dispersion processes appear to be less stable over different time periods. Key questions to be answered include: Why do these series all deviate from normality and what causes this deviation to change over time?

period 28/7/85 to 30/7/90). The slight differences that are observed are due to the fact that our first period of analysis includes approximately one more year of data.

Review of Other Work on the Determination of Objective Processes

As was discussed in the last Chapter, an alternative process to explain the high degree of kurtosis in the return data could be a GARCH process. However, even though the GARCH approach has won a loyal following among econometricians, it may be inadequate for our purposes of understanding the high degree of excess kurtosis we have observed in the twelve markets. If GARCH-normal processes could adequately account for all the observed leptokurtosis, then one would expect the evidence against normality to be eliminated (once the GARCH effects are taken into account). Nevertheless, Bidarkota and McCulloch (1996) tested this and found GARCH did not explain the high degree of excess kurtosis we observe empirically. They concluded: "There is strong evidence that, even after accounting for the seasonal scales and GARCH-like behaviour, real returns are still significantly non-normal." (p 11). Therefore, we must look elsewhere to understand the excess kurtosis we observe in financial markets.

One explanation for the degree of leptokurtosis we observe in the return series is that the conditional returns follow a distributional form, which is not normal. A variety of distributions have been proposed to account for the thick tails, primary among these being the stable distributions, Student-t distributions, mixtures or normals and the Weibull distribution. The general conclusion is that stationary stable distributions with infinite variance do not fit the data as well as mixtures of normals with different variances (i.e. subordinated processes). However, this is only one possible approach to explain these results. Alternative approaches will be examined in this research, when we test for the mixture of normals (stochastic volatility) and for alternative dispersion processes such as the Student - t distribution.

Another problem is that even if we can understand the high degree of excess kurtosis in a subset of the data series, it is by no means obvious that such behaviour will exist in other periods. This was demonstrated by examination of the statistical moments of the twelve markets in the two sub-periods. According to Bidarkota and McCulloch (1996), "If stock returns are truly i.i.d. stable stochastic processes, then one expects to find identical estimates of the characteristic exponent for the returns data sampled at daily, weekly and monthly frequencies. Failure to obtain such matching estimates has been cited as evidence against stability [Akgiray and Booth (1988)]" (p. 4). Therefore, an important element in our analysis must be the comparison of the statistics for the return series measured at different time horizons (to assess if any models we uncover are stable over time).

Similar research has been done for equity indices (including the S&P 500 that is included in our study). What we expect to find would be results consistent with these studies. For example, Ding, Granger and Engle (1993) considered the properties of the absolute return of the return series for the S&P 500 stock market index from January 3, 1928 to August 30, 1991. They found that the kurtosis measure was significantly higher than for a normal distribution. The application of a Jarque-Bera test for normality indicated that the returns were far from a normal distribution. From using the absolute returns they also found that these confirmed the findings of Mandelbrot (1963) and Fama (1965) that large absolute returns are more likely than small absolute returns to be followed by large absolute returns.

For the currency markets, Malz (1996) completed similar research. He concluded that "For currency exchange rates, most investigations find the log price relatives or nominal returns to be stationary and serially uncorrelated. The hypothesis that the log price relatives are nominal i.i.d. has been in doubt since the advent of floating exchange rates in the 1970s. The distribution of $S_t - S_{t-1}$ violates normality in three crucial respects. First, it is leptokurtic.... Second, the distribution appears to be skewed....Finally, the variance of the price relatives appears to be time varying." (page 131)

When Malz examined the degree of autocorrelation in the returns, he found that "the lack of correlation in $S_t - S_{t-1}$ indicates that its variance rather than its mean varies. Thus, nominal returns are not both i.i.d. and normally distributed, suggesting two approaches to characterising nominal exchange rate returns: nonnormal distributions and time-varying distribution parameters." (page 131).

Such nonnormal distributions could include jump diffusion processes or perhaps stochastic volatility models that are a mixture of normals. While time-varying parameter approaches could possibly be represented by autoregressive conditional heteroscedasticity models such as ARCH, it is possible that both of these models could account both for kurtosis as well as for the time variation of volatility. However, we will not follow this path, for the reason that Malz finds that "There is strong evidence that flexible exchange rate returns follow jump diffusions, that is, a sum of i.i.d. normal and Poisson-distributed jump components, which can account both for the kurtosis and the skew in nominal returns." (page 131). Given that this same evidence has been presented by Akgiray and Booth (1988), Tucker and Pond (1988) and Jorion (1988), we will restrict our further analysis to the tests of these models. Furthermore, we have demonstrated that the key factor, which causes the dispersion processes to diverge from normality, is the excess kurtosis and not the skewness. Therefore, our primary objective is to understand the leptokurtic behaviour that is found in all twelve markets. Thus, we will not concentrate on skewness directly. However, we will examine the impacts of jumps that would be associated with fat-tailed distributions and the occurrence of non-symmetrical jumps could lead to skewness in the objective process. This will be examined later when we examine the risk-neutral dispersion processes implied by options prices.

2.4 THE UNCONDITIONAL VOLATILITY SERIES FOR TWELVE MARKETS

Using the two-sided EWMA approach, which was described in the first Chapter, we determined the unconditional volatility for each of the markets under examination. For this analysis we used three separate weights. We chose $\alpha = 0.1$, $\alpha = 0.05$, and $\alpha = 0.02$. As was discussed in that section, this corresponds to a normal moving average with N=10, N=20 and N=50.

These weights were applied to the individual returns and from these time series a volatility estimate was made. The resulting time series of EWMA volatilities can be seen in Figure 2.1a for the four stock index futures, Figure 2.1b for the four fixed income futures and Figure 2.1c for the four currency futures.

For the stock index futures, one can clearly see the impact of the 1987 crash for both the S&P 500 and the FTSE 100. This is evident from the single spike that occurs in October 1987 in all three volatility time series. The Nikkei 225 and the DAX futures range between roughly 10% to 40% and it appears that the variability of the volatility series is similar across time.

For the fixed income markets, the Bund and BTP futures display similar volatility time series (for the period where they share data). For the Bund, there was extreme volatility at two points: in late 1989 and in 1994 (when the Federal Reserve raised short-term interest rates). The BTP experienced fairly low volatility in 1991 but had a shock in 1992 when an EMU crisis was precipitated by the Danish rejection of the Maastricht treaty. Thereafter, extreme events occurred also in 1994 (Federal

Reserve rate hike) and in 1995 when internal Italian politics led to instability. The odd market out is the US-T Bond futures. From 1977 to 1979, the volatility time series was at the lowest level in the history of this instrument. A dramatic change in Federal Reserve policy in Autumn of 1979 (shifting from targeting interest rates to managing money supply) led to a dramatic increase in volatility that remained until 1987. Thereafter, the volatility time series remained in range between 5% and 15% with what appears to be more stability.

The four currency markets, they appear to display similar volatility characteristics over the analysis period. The exception is the British Pound that has extreme events in 1985/1986 and in the third quarter of 1992 (ERM crisis). Thereafter, the Pound displayed similar behaviour to the other currency markets. The volatilities for the four markets ranged between 10% and 25% for the period. This result is consistent with the analysis of the returns presented previously.

2.5 AUTOCORRELATION TESTS FOR TWELVE MARKETS

As was outlined in the first Chapter of this dissertation, substantial evidence has been published indicating that absolute return series are autocorrelated. While a number of techniques can be used to identify this effect, we have chosen to use the autocorrelogram to understand this behaviour for the twelve markets under study.

When Merville and Pieptea (1989) completed an autocorrelation analysis for a set of US common shares and the S&P 500 Futures, they found that the autocorrelation coefficient is a decreasing function of the lag in all cases. Therefore, it is not possible to accept the hypothesis that the volatilities at different points are unrelated. Furthermore, they found that the shorter the time lag, the higher the correlation and thus the greater the dependence. They conclude that since the autocorrelation increases significantly as the lag becomes smaller which suggests that volatility paths have a continuous character.

When Merville and Pieptea examined the autocorrelation of changes in the volatility they found significantly lower autocorrelation coefficients. As the lag increased beyond one period (they examined weekly changes), the coefficients approached 0.0. They concluded that the changes in volatility are more independent than the volatility itself and that the relatively low autocorrelations of volatility changes supported the contention that volatility has a diffusion component.

Merville and Pieptea concluded: "The stock volatility is subject to a diffusion process (with a continuous path) on which noise is superimposed. For short periods the general level of the process changes little, with most variations form the average being due to noise. Over long periods, however, the general level of the stock volatility changes, the noisy variations from a moving average becomes smaller in relations to the deviations from the long-term average, which renders discontinuities less significant, and thus a higher autocorrelation is observed. Discontinuities are enlarged by analysis of a detailed portion of the sample path. Although shortening the sample period can decrease the diffusion-related variance, the noise-related variance remains constant. By shortening the period under analysis enough, we should see only discontinuities. Trend becomes less important and noise takes over. " (page 201)

All the autocorrelograms for the twelve markets can be seen in Figures 2.2a, 2.2b and 2.2c. For the four stock index futures, we observed similar patterns. For the S&P 500 Futures, we observe an initial positive correlation for out to lag 100. However, this pattern is not significantly different at the 95% level past 60 lags. The number of observations we had was 2771 and therefore would have yielded a 95 % confidence interval around zero of ± 0.0372. Thereafter, when we extended the analysis for absolute returns past 60 days, almost all the autocorrelations fell within the 95% confidence interval. Thus, in this range, it would appear that the autocorrelations of the absolute returns are not statistically different than what would be expected from a series following an i.i.d. distribution. The overall pattern for the S&P 500 futures resembles the pattern produced by Ding, Granger and Engle (1993) in their paper. However, they found statistically significant positive autocorrelations out to a lag of 2500. This could be due to the fact that they had significantly more observations in their study (N=17054) which would have reduced the 95% confidence interval to such a tight range that even small positive autocorrelations would be significant. Nevertheless, even if we imposed their tight confidence interval, we would still reject the hypotheses that the series was not i.i.d. after about 200 observations. To verify, we had completed the analysis correctly, we compared our results to a similar autocorrelogram produced by the Financial Options Research Centre at the University of Warwick (Figure 2.2d) and found that even though FORC used less data than was used in our analysis, the patterns were practically identical.

For the other stock index futures, we obtained slightly different results. For the FTSE 100 futures, a positive autocorrelation existed out to 140 lags. However, the autocorrelations fell within the 95% confidence interval from about 60 observations until the 500th lag. It is interesting to note that after about the 120th lag most of the time the autocorrelations were negative (although insignificant). The 95% confidence interval was determined using 3197 observations and yielded a confidence interval of ± 0.0347.

For the Nikkei Dow futures, the pattern displayed a similar pattern to the other stock index futures. Initially, a positive autocorrelation was recorded to lag 100. However, this failed to be significant after the 60th lag. From lags 100 to 290, the autocorrelation was negative and between lags 200 to 250, was significantly so. From 290 to 500, the autocorrelations became positive again and many were significant at a 95% confidence interval from the lags 310 to 480. The 95% confidence interval was determined using 1378 observations and yielded a confidence interval of ± 0.0528.

For the DAX futures, the autocorrelogram pattern was the most dissimilar to the other equity markets. The positive autocorrelation fell dramatically becoming insignificant after the 30th lag. Thereafter, the patterns display extreme variability, reaching both positive and negative values. However, most of the time, the autocorrelations are not significant. One possible reason for the divergence of the DAX futures from the other equity index futures is that the period of analysis for this instrument is only for 5 years. It is not possible to examine the longer-term autocorrelation behaviour of the DAX futures as data is not available. To gain a better view of the true longer-term dynamics of the autocorrelation process, we examined the DAX cash index. This was initially done for the cash index for the same time period as the futures. This can be seen in the graph titled Figure 2.2e. The cash series is identified as DAX2 and the futures series for the same time period is identified as DAX. As one can see, the pattern is similar to that of the futures contract. Thereafter, we examined the cash index from 1970 to 1996. This can be seen in the graph titled "DAXIND". For this series, we observe a pattern similar to that observed by Ding, Granger and Engle when they examined the long-term autocorrelation behaviour of the S&P 500 cash index.

Nevertheless, the focus of our research is on the autocorrelation behaviour of the equity futures markets. It may very well be that the nature of autocorrelations in the data differs for the periods of the 1980s and 1990s compared with the longer-term dynamics.

In the case of the Fixed Income futures, all the four markets display a generally similar pattern of initially being positively autocorrelated and then decaying to insignificant levels of autocorrelation, the patterns for each market however displays certain idiosyncratic behaviour.

The Bund futures has the most divergent of the autocorrelogram patterns. It displays a positive autocorrelation of the absolute returns out to 130 lags and then for a period from 130 to 280 lags is insignificantly different than an i.i.d. distribution. After the 280th lag, the autocorrelations are negative most of the time and most of these are statistically significant. The 95% confidence interval was determined using 2072 observations and yielded a confidence interval of ± 0.0430.

For the Italian Government Bond (BTP) futures, the autocorrelation is also significantly positive out to about 90 days. From 90 days to 360, the autocorrelations are insignificantly different than an i.i.d. distribution. After 360, the autocorrelations turn positive again and many of these observations are significant. The 95% confidence interval was determined using 1324 observations and yielded a confidence interval of ± 0.0539.

For the UK Gilt futures, the autocorrelation graph is significantly positive out to about 80 days and then hovers within the 95% confidence interval. Thereafter, the autocorrelations are generally within the confidence interval. It is interesting that the autocorrelations tend to be negative after 150 days and at a number of points are significantly negative. The 95% confidence interval was determined using 3567 observations and yielded a confidence interval of ± 0.0328.

For the US T-Bond futures, the autocorrelations remain positive for almost the entire analysis period. This is the first result that most resembles the results for the S&P 500 presented by Ding, Granger and Engle (1993) which indicated consistent positive autocorrelations of the absolute returns. What is different from the Ding, et al. results is that the pattern of autocorrelation decay appears to be almost linear, while Ding, Granger and Engle (1993) found that the rate of decay was a complicated polynomially decreasing function. However, for the period out to 500 observations, the pattern is similar apart from the lack of an extremely positive autocorrelation for very short-term lags. [see Figure 3.8 of Ding, Granger and Engle (1993)]. Only after the 460th lag does the autocorrelation become statistically insignificant. The 95% confidence interval was determined using 4690 observations and yielded a confidence interval of $\pm$ 0.0286.

This is most probably due to the unusual behaviour of the volatility series for the US T-Bond (see Figure 2.1b). For all the fixed income markets, no other market had such a shift in the levels of volatility and the variability of the volatility as the US T-Bonds (from 1977 to 1979 - very low and 1979-1987 - very high). We will later examine if this is the case by splitting the time series of the data into two periods and examining the dynamics separately. The first period will include the turbulent period from 1977 to 1987 which we will then compare with the autocorrelograms from the period from 1987 to 1996.

For the Currency futures analysed, the autocorrelograms for the four examined currency pairs displayed similar patterns to those observed for the stock index futures. Even so, it is clear that among the individual currencies slightly different patterns exist.

For $/DM, the autocorrelations were positive only until lag 60 and of these only the lags out to 30 days were statistically significant. After that the autocorrelations oscillated between positive and negative values out to the 500th lagged observation. However, for most of the observations beyond 30 days, the autocorrelations are within the 95% confidence interval and not significantly different than the assumption of an i.i.d. distribution for the absolute returns. The 95% confidence interval was determined using 2773 observations and yielded a confidence interval of ± 0.0372.

For $/£, the positive autocorrelations decay at the slowest rate of the four currencies. The autocorrelations are positive until the 90th lag. From that point onwards, the autocorrelations are not significantly different from an i.i.d. assumed distribution. The 95% confidence interval was determined using 2774 observations and yielded a confidence interval of ± 0.0372.

For the $/Yen, the positive autocorrelations also steeply drop off and are no longer significant after about 20 lags. Thereafter, the autocorrelations display much greater variability compared to the other currencies becoming at certain points significantly negative and positive. The 95% confidence interval was determined using 2768 observations and yielded a confidence interval of ± 0.0373.

For $/Swiss Franc, the autocorrelations were positive until lag 290. These positive autocorrelations, were significant only out to the 190th lag. After this point, the autocorrelations hovered within the 95% confidence interval until the 500th lag. Although, the autocorrelations become negative after 460 lags, these were not significant. The 95% confidence interval was determined using 3011 observations and yielded a confidence interval of ± 0.0357.

For all twelve markets, certain patterns are consistent. For all the markets, the autocorrelations are initially positive but the rate of decay differs between the markets. For many of the markets the rate of decay is much faster than the result found by Ding, Granger and Engle (1993) for the S&P 500 stock market index. In addition, for a number of the markets, the autocorrelations become significantly negative.

One possible reason for this result is that the relatively low number of observations allows the 95% confidence interval to be sufficiently large to exclude significance. For those markets with the greatest number of observations (US T Bond and $/SwF), the observations display a much slower rate of autocorrelation decay and a longer period of significantly different positive autocorrelations. Another possible explanation is that autocorrelations in markets are not stationary and have changed in the past 5 to 10 years compared with the period from 1928 to 1991. A third explanation is that the futures prices react more quickly than the stock index itself and shocks die out more quickly.

To examine the possibility that patterns of autocorrelations may be non stationary, we also split the time period of analysis in half and examined the patterns in each period. These can be seen in Figures 2.3a, 2.3b and 2.3c for the first half of the period and in Figures 2.4a, 2.4b and 2.4c for the second half of the period.

From the comparisons of the autocorrelograms for the whole period to the two subperiods, we find that the behaviours do diverge. For the equity index futures, the relatively high autocorrelations are a function of the first period rather than the second period. This is hardly surprising since the 1987 and 1989 crashes are causing this effect for the S&P 500 and the FTSE 100. Thus, the autocorrelograms for the entire period are most similar to the period with these extreme events. For the second period (which does not include the crashes), the autocorrelations between the stock index futures is now much lower. In fact, for the DAX, it appears that the autocorrelations are not significantly different than zero in the second period. One interesting result is that for the Nikkei (for all periods of analysis), the autocorrelations become significantly negative after a lag of approximately 150 to 200 days.

For the fixed income and foreign exchange markets, the autocorrelograms seem to be relatively stable when broken into the two subperiods. When these figures are compared to the entire period, there does not seem to be the same degree of divergence that we observed in the stock index futures markets. One exception is the US T-Bond market, where the autocorrelogram converges at a faster rate to an insignificant level in the second period compared with the first period (or for the overall period). Thus, it would appear that the unusual patterns of the autocorrelograms are due to the volatility shock that occurred in the autumn of 1979, which fundamentally changed the degree of uncertainty in this market.

Regardless of the reason for the differences between these results and those presented by Ding, Granger and Engle. Our findings suggest that the absolute returns are not i.i.d. in the short term. However, the impact of autocorrelations for most markets dies out after between 30 and 60 lags. The fact that many series subsequently have negative autocorrelations could indicate a strong tendency for mean reversion.

2.6 COMPOSITE MEASURES OF AUTOCORRELATION

For the purposes of this research, we wanted to determine some composite measures of the autocorrelations be determined for the markets. Given the relatively rapid decay of the autocorrelations, we chose to calculate the average autocorrelation from lag 1 to 20 and the average correlation from lag 51 to 70. These two measures then represent the short term autocorrelation and the longer term average provides an indication of how quickly the autocorrelations die out. For the twelve markets, these average autocorrelations for the two sets of lags appear in the following Table. In addition to the average autocorrelations, the standard error of the autocorrelations appears to the immediate right of all results and appear in italics. This was determined by estimating the variance of the autocorrelations, dividing this result by 20 and then taking the square root.

Underlying Asset	Average Autocorrelation From Lags 1 to 20		Average Autocorrelation From Lags 51 to 70	
S&P 500 Futures	0.1387	*(0.0167)*	0.0549	*(0.0069)*
FTSE Futures	0.1428	*(0.0118)*	0.0398	*(0.0041)*
Nikkei 225 Futures	0.1557	*(0.0077)*	0.0629	*(0.0053)*
DAX Futures	0.0782	*(0.0049)*	0.0207	*(0.0057)*
Bund Futures	0.1661	*(0.0063)*	0.1009	*(0.0041)*
BTP Futures	0.1638	*(0.0063)*	0.0826	*(0.0040)*
Gilt Futures	0.1116	*(0.0051)*	0.0663	*(0.0027)*
US T-Bond Futures	0.2080	*(0.0196)*	0.1643	*(0.0131)*
Deutsche Mark /US Dollar	0.0804	*(0.0047)*	0.0139	*(0.0047)*
British Pound / US Dollar	0.1140	*(0.0041)*	0.0655	*(0.0036)*
Japanese Yen / US Dollar	0.0577	*(0.0054)*	0.0087	*(0.0046)*
Swiss Franc / US Dollar	0.0660	*(0.0032)*	0.0158	*(0.0030)*

Table 2.6, Average Autocorrelations of Absolute Returns for the Lags 1-20 and 51-70 for Twelve Markets for the Entire Period of Analysis.

The results of the average correlations are consistent with autocorrelograms. The autocorrelations for most markets decay quite rapidly and approach zero by between 50 and 70 days. The main exception is the US T-Bond futures that experienced a slow rate of decay in the autocorrelations for the period. The standard errors of the variances also indicate that the choice of the averaging period (1-20 and 51-70) does not display an inordinate amount of variation among the autocorrelations being averaged. Thus the averaging process seems to provide a reasonable estimate of the autocorrelation behaviour of the twelve markets and captures the general patterns of decay in the autocorrelations.

To assess if these results are period specific, the same analysis was completed for the entire data series broken into two equally sized (non-overlapping) halves. The results of this analysis appears in the following two Tables, Table 2.7 And 2.8.

Underlying Asset (First Period)	Average Autocorrelation From Lags 1 to 20		Average Autocorrelation From Lags 51 to 70	
S&P 500 Futures	0.1247	*(0.0190)*	0.0304	*(0.0081)*
FTSE Futures	0.1615	*(0.0162)*	0.0321	*(0.0064)*
Nikkei 225 Futures	0.1458	*(0.0115)*	0.0456	*(0.0060)*
DAX Futures	0.0883	*(0.0081)*	0.0153	*(0.0077)*
Bund Futures	0.1890	*(0.0085)*	0.1103	*(0.0055)*
BTP Futures	0.1971	*(0.0076)*	0.0843	*(0.0069)*
Gilt Futures	0.1228	*(0.0066)*	0.0784	*(0.0046)*
US T-Bond Futures	0.2349	*(0.0053)*	0.1845	*(0.0043)*
Deutsche Mark /US Dollar	0.0711	*(0.0064)*	-0.0139	*(0.0057)*
British Pound / US Dollar	0.0917	*(0.0053)*	0.0335	*(0.0050)*
Japanese Yen / US Dollar	0.0495	*(0.0078)*	0.0005	*(0.0050)*
Swiss Franc / US Dollar	0.0665	*(0.0059)*	0.0024	*(0.0057)*

Table 2.7, Average Autocorrelations of Absolute Returns for the Lags 1-20 and 51-70 for Twelve Markets for the First Half Period of Analysis.

A comparison of Tables 2.6 and 2.7 suggests that (for the most part) the levels of the averaged autocorrelations are similar for the first half of the observations compared to the entire period. The average autocorrelation for the stock index futures for the short-term lag (1-20) are similar for both periods. For the longer-term average autocorrelation (lags 51-70), the average autocorrelation is somewhat less positive for the first period. For the fixed income futures, the short-term and the longer-term average autocorrelations are slightly higher in the first period compared to the entire period. For the currency futures, the average autocorrelations are both less positive in the first period compared with the overall period. It is of interest that the longer-term autocorrelation measures are insignificantly different from zero in both the first period and overall. To assess if these autocorrelation patterns remain consistent across time, we also examined the average autocorrelations for the short and long-term lags for the second half of the available observations. These results can be seen in Table 2.8.

Underlying Asset (Second Period)	Average Autocorrelation From Lags 1 to 20		Average Autocorrelation From Lags 51 to 70	
S&P 500 Futures	0.0596	*(0.0066)*	0.0224	*(0.0041)*
FTSE Futures	0.0888	*(0.0074)*	0.0430	*(0.0054)*
Nikkei 225 Futures	0.1383	*(0.0082)*	0.0704	*(0.0089)*
DAX Futures	0.0481	*(0.0109)*	0.0161	*(0.0085)*
Bund Futures	0.1329	*(0.0079)*	0.0805	*(0.0057)*
BTP Futures	0.1014	*(0.0090)*	0.0109	*(0.0081)*
Gilt Futures	0.0950	*(0.0063)*	0.0504	*(0.0043)*
US T-Bond Futures	0.0641	*(0.0054)*	0.0348	*(0.0039)*
Deutsche Mark /US Dollar	0.0884	*(0.0057)*	0.0379	*(0.0075)*
British Pound / US Dollar	0.1309	*(0.0055)*	0.0940	*(0.0058)*
Japanese Yen / US Dollar	0.0657	*(0.0057)*	0.0110	*(0.0060)*
Swiss Franc / US Dollar	0.0655	*(0.0068)*	0.0318	*(0.0073)*

Table 2.8, Average Autocorrelations of Absolute Returns for the Lags 1-20 and 51-70 for Twelve Markets for the Second Half Period of Analysis.

A comparison of Table 2.6, 2.7 and 2.8 now indicates a significant shift in the autocorrelation dynamics for the twelve markets has occurred. For example, three of the four stock index futures have a significantly lower average correlation for both the short and long-term measures in the second period. The exception is the Nikkei 225, which is lower in the second period (compared to the first) but the drop is not as significant. For all the fixed income futures, significant reductions have occurred for both the short and long-term measures. The most significant reduction occurred for the US T-Bond futures. This can be cross-checked by examination of the autocorrelation graphs, Figure 2.3b and 2.4b. Comparing the figures, it becomes clear that the period from 1987 to 1996 is fundamentally different from the period 1977 to 1987 for the US T-Bond market. The reason for this result has already been discussed. The average autocorrelations for the four currencies are essentially unchanged in the second period compared with the first period, except for the British Pound. Again the impact of the 1992 ERM crisis led to a much higher average autocorrelation measures for the British pound. This is most probably due to the fact that the disruptive price behaviour during this period took a fairly long time to dissipate.

2.7 DETERMINATION OF VOLATILITY CONES

In Figures 2.5a, 2.5b and 2.5c, all the cones are displayed for the twelve markets examined for the entire period of available data. These have not been corrected for the overlapping bias. This will be done in the following sections. For each of these figures, the minimum, median and maximum levels of the volatility for that time horizon are presented, as well as the 2nd and 3rd quartiles. All the markets analysed displayed the usual cone shape of the original Burghardt and Lane article in Journal of Portfolio Management (1990). The dispersion between the maximum and minimum levels becomes more compact the further one goes out in time. The widest margin for error in the historical volatility for the one-month sample horizon where the range has the greatest spread.

The most ambitious of the research on volatility cones has suggested that mean reversion plays a major role in the variability of volatility [see Leong (1991)]. However, one portion of this research will suggest that the narrowing of the cone as the time period of volatility estimation is lengthened is simply due to the Central Limit Theorem. It is well established result that a sample statistic (in this case volatility) approaches a normal distribution as the sample size increases and the standard error of the estimate falls as a function of the number of observations ($\sigma/\sqrt{2n}$). Even if the true distribution of volatilities is not normally distributed, the Central Limit Theorem would still explain this effect for nonparametric statistical tests.

An important question is whether the volatilities are normally distributed or not. In the tables preceeding the volatility cone graphs, one will find the summary statistics for all the maturity sectors of the volatility series. These series are based upon the volatility estimate for the relevant period and are annualised. The statistics for these twelve volatility cones appear in Tables 2.5a, 2.5b and 2.5c.

One can glance across the kurtosis and skewness rows to see that for most of the markets, the kurtosis measure is well above 3.0 (indicating that volatility is leptokurtic). Furthermore, most of the skewness measures are significantly positive. This effect is most pronounced for the shortest time horizon (which is 20 days). As the time horizon is extended to 500 days, the kurtosis and skewness measures tend to fall. However, in two instances (S&P 500, British Pound) they remain significantly different from the normal distribution hypothesis. Thus, it appears that for short time horizons, the distribution of volatility is non-normal. Yet, as the time horizon is lengthened, the volatility dynamics approach normality. Again, this could simply be due to the Central Limit Theorem. However important these results may be for understanding the dynamics of the volatility process, our objective is to discern some measure of the overall variability of the volatility.

In these tables, the statistics of particular interest are the average and standard deviation measures. While recognising that the distributional form of volatility may not be normal, we chose to concentrate on the shortest time horizon (that was 20 days) and to compare this to the standard deviation of the volatility at 20 days to the average level of volatility at this time horizon. The resulting coefficient of variation provides a relative comparison of how variable the volatility of the volatility is. The results solely for the 20 day time horizon are presented in Table 2.9.

The 20-day Coefficient of Variation as a Measure of the Variability of Volatility

Markets	20 Day Average	20 Day SD	Coefficient of Variation
S&P 500 Futures	14.3148%	12.9482%	0.9045
FTSE Futures	14.7577%	7.3546%	0.4984
DAX Futures	14.4124%	4.8873%	0.3391
Nikkei Dow Futures	21.8033%	8.7890%	0.4031
Bund Futures	5.0278%	2.2721%	0.4519
BTP Futures	8.5045%	3.6296%	0.4268
Gilt Futures	8.3592%	3.1009%	0.3710
US T-Bond Futures	11.0792%	5.1377%	0.4637
Deutsche Mark /US Dollar	11.1990%	3.8243%	0.3415
British Pound / US Dollar	11.0049%	4.4720%	0.4064
Japanese Yen / US Dollar	10.5328%	3.7818%	0.3590
Swiss Franc / US Dollar	12.4243%	3.8648%	0.3111

Table 2.9, Statistics of the 20 Day Volatility for Twelve Financial Futures

To determine the stability of the volatility cones, the analysis period was divided into two with the same number of observations. The cones and the accompanying tables of statistics for each period appear in Figures 2.6a, 2.6b and 2.6c for the first half of the time period and in Tables 2.6a, 2.6b and 2.6c. The analysis for the second time period appear in Figures 2.7a, 2.7b and 2.7c for the first half of the time period and in Tables 2.7a, 2.7b and 2.7c.

Given the importance of the 20 day volatility behaviour as a primary condition in explaining one element in the dynamics of the volatility, we have also provided the tables which describe these results for each of the sub-periods. The results for the 20 day time horizon using the first half of the available data are presented in Table 2.10.

Markets (First Period)	20 Day Average	20 Day SD	Coefficient of Variation
S&P 500 Futures	18.0299%	17.2744%	0.9581
FTSE Futures	15.7199%	9.3741%	0.5963
DAX Futures	15.4274%	5.5725%	0.3612
Nikkei 225 Futures	24.2710%	8.4771%	0.3493
Bund Futures	4.7991%	2.4834%	0.5175
BTP Futures	7.2859%	3.7839%	0.5193
Gilt Futures	8.6368%	3.3022%	0.3823
US T-Bond Futures	12.7198%	6.1861%	0.4863
Deutsche Mark /US Dollar	11.4364%	3.7839%	0.3309
British Pound / US Dollar	11.7722%	4.3800%	0.3721
Japanese Yen / US Dollar	10.7438%	3.6349%	0.3383
Swiss Franc / US Dollar	12.5805%	3.8775%	0.3082

Table 2.10, Statistics of the 20 Day Volatility for Twelve Financial Futures for the first half of the observation period.

These results for the 20 day time horizon for the latter half of the available data are presented in Table 2.11.

Markets (Second Period)	20 Day Average	20 Day SD	Coefficient of Variation
S&P 500 Futures	10.5877%	3.3799%	0.3192
FTSE Futures	13.7056%	4.2751%	0.3119
DAX Futures	13.2396%	3.7388%	0.2824
Nikkei 225 Futures	19.3100%	8.4599%	0.4381
Bund Futures	5.2894%	2.0193%	0.3818
BTP Futures	9.6321%	3.0596%	0.3176
Gilt Futures	8.1012%	2.8730%	0.3546
US T-Bond Futures	9.3757%	2.9374%	0.3113
Deutsche Mark /US Dollar	10.9019%	3.8319%	0.3515
British Pound / US Dollar	10.1976%	4.4399%	0.4354
Japanese Yen / US Dollar	10.2762%	3.8983%	0.3793
Swiss Franc / US Dollar	12.2809%	3.8659%	0.3148

Table 2.11, Statistics of the 20 Day Volatility for Twelve Financial Futures for the second half of the observation period.

For all the markets, the variability of the volatility (as measured by the COV) has been significantly reduced in the second period compared to the first period. As with the reduction in the autocorrelations that also occurred for the second period (compared with the first period), this could be due to the increased efficiency with which volatility shocks are absorbed into price behaviour. Furthermore, many of the extreme events which caused excessive volatility such as the 1987 and 1989 mini-crashes and turbulence in the currency markets in the 1980s, are represented in the first period of analysis and not the second. Regardless of the reasons, it appears that volatility is becoming a more benign time series through time.

2.8 BIAS CORRECTION OF THE VOLATILITY CONE ANALYSIS

It has been pointed out in Chapter one, that there is an issue of bias inherent in the use of overlapping data to estimate the volatility cone. As this bias is potentially serious, it must be corrected. Leong (1991) states: "There are, admittedly, some flaws in this research design [the volatility cone approach]. First, the sampling design is unbalanced - there are 24 data points for the one-month bucket but only 13 for the one-year bucket. Second, and more important, the samples are not totally independent of each other. For example, in the sampling of one-year volatilities, the adjacent samples are almost [totally] overlapping, except for one month. This means that they are sampling essentially from largely the same information pool." (pag 45). Leong goes on to state: "Both flaws then to inflate the uncertainty in short-term volatility relative to that of long-term volatility. If such biases are eliminated, there should still be a volatility term structure. However, the volatility cone should be narrower at the shorter time buckets." (pag 45).

Thus, we applied the new adjustment factor to assess the true behaviour of the standard deviations of the volatilities. This produces a series of unbiased standard deviations of the volatility for each estimation horizon period. To test the effectiveness of the bias correction factor, we examine the true relationship between the variances of the overlapping and non-overlapping estimation techniques. Then we assess how well the biasing factor predicts the actual biasing from overlapping.

This simple test involved comparing the variance of the overlapping volatilities from the cone to the variance of volatilities estimated solely with non-overlapping observations. Given the bias inherent in overlapping, we would expect the variance of the overlapping analysis to be lower. We estimated the ratio between the non-overlapping variances of the volatilities to the overlapping variances of the volatility and this was graphed against the predicted bias from the new formula (1.24). All these graphs can be seen in Figures 2.8a, 2.8b and 2.8c.

These graphs indicate that the predicted bias fits extremely well out to at least the 200th estimation horizon. Thereafter, significant divergences occur. However, this is most extreme for those series (DAX and Nikkei) which have the smallest number of observations. For the S&P 500 and the FTSE, the theoretical ratio somewhat overstates the increase in the ratio of the non-overlapping to overlapping variances (compared to the actual ratio). However, given that the general trend in the ratio is predicted well out to the 200th estimation horizon, we remain comfortable that we have correctly determined this relationship.

For the fixed income futures markets, the model seems to work even better than for the stock index futures. For the time horizons out to the 300th estimation horizon, the theoretical ratio seems to provide the best quadratic fit. For the fixed income market where we had the most data (and would then have the most accurate measure of the true ratio of the non-overlapping variances), the US T-Bond futures, the theoretical ratio provides an almost perfect fit even out to the 500th estimation horizon.

For the currency futures, there is much greater variability at the longer time horizons. However, for the shorter time horizons (out to the 200th estimation period), the results are good. Any deviations between the theoretical and actual variance ratios seem to be random and the theoretical ratio provides (what appears) to be an adequate quadratic fit. As with the stock index futures, for longer time periods, more discrepancies occur between the theoretical ratio and the actual ratio of variances. However (and apart from the British Pound), these deviations seem to be behaving randomly.

Thus, we conclude that beyond the 200-day estimation period, we do not have sufficient non-overlapping observations to draw reasonable conclusions for all our markets. Nevertheless, for those markets where we do have more observations, the fit is extremely good. Thus, we will conclude that out to the 200th day estimation period, the model effectively describes the effects of the biases introduced by overlapping data when estimating volatilities in the volatility cone. With the assumption that the bias correction model fits the empirical data well, the unbiased standard deviations of the volatility out to the 200th estimation period will be used for the remainder of the analysis.

To further test the effectiveness of the method to correct biases when using overlapping observations, we repeated the simple test (between the theoretical and actual ratio of the variances) for the two sub-periods of the available data. As before, the data was split evenly into two and we investigated the relationship in both periods. These results can be seen in Figures 2.9a, 2.9b and 2.9c. for the first half of the observation period and in Figures 2.10a, 2.10b and 2.10c for the second half of the observation period.

An obvious problem with extending this analysis to the two sub-periods is that we have reduced the number of observations in half. This means that our estimates of the actual ratios of the non-overlapping variance to the overlapping variance will now display even greater variability. Furthermore, for certain markets, we fail to have sufficient data to even estimate the variance on a non-overlapping basis for the longer time horizons. Nevertheless, the results for the two sub-periods suggest that the theoretical model does fit the actual ratio of the variances well for time horizons less that 200 days. It is interesting to notice that even for those markets which had the least amount of data (DAX, Nikkei and BTP), the model displays an extremely good fit out to the 200th horizon point.

Comparison of the Unbiased Standard Deviations of Volatility to the Standard Deviations of Volatility Consistent with an i.i.d. dispersion process.

Having justified our confidence in the unbiasing methodology, it is now possible to accurately measure how the standard deviations of the volatility are related to an increase in the time horizon of estimation. As was stated previously, one possible reason for the reduction in the standard deviation of the volatilities (and the narrowing of the volatility cone) as the time horizon was extended was the biasing effects of overlapping the data. This has been corrected for. Another reason for the reduction in the standard deviations of the volatility could be the increase in the sample size of the estimation horizon that would be expected from the Central Limit Theorem. To test this effect, we took the (square root) of the unbiasing factor for the variances to estimate unbiased standard deviations of the volatility at different estimation horizons. These series were then plotted against the time of the estimation horizon. These series were compared to the biased standard deviations (unadjusted for the overlapping problems) and to a hypothetical series that would be drawn from a single (i.i.d.) distribution one would expect this process to be a function of the number of observations (or more correctly $1/\sqrt{2n}$). This last series would produce the result we would expect if solely the Central Limit Theorem (and sampling theory) were at work. Specifically, since we are starting at the 20th observation, this process is estimated using:

$$\sigma_{20} \cdot \sqrt{\frac{20}{N}} \qquad \textbf{(2.1)}$$

where σ_{20} is the standard deviation of the volatility at the 20th observation and N is the number of observations in the time horizon.

For all the series, we started with an intercept at σ_{20} and examined the behaviour of the standard deviations out to the 200th time horizon. These results can be seen in Figures 2.11a, 2.11b and 2.11c.

For all these graphs, we find that both the biased and unbiased standard deviations of the volatilities diverge sharply from the assumption of a single (i.i.d.) dispersion process. The unbiased standard deviations decay through time at a much slower rate than either the biased standard deviations or the case of the single i.i.d. distribution process. However, the effect of the unbiasing adjustment is fairly slight (apart from the longer-term time horizons). These results suggest that the standard deviation of the volatility remains at a fairly high level not decaying as one would expect from sampling theory. Thus, the narrowing of the volatility cones would be more extreme if the dynamics of volatility followed a single i.i.d. distributional form. The effects of the unbiasing factor would have a minimal impact on widening the cone at longer-term estimation horizons.

For the stock index futures, this effect is most extreme for the S&P 500 and the FTSE. However, even for the DAX and Nikkei, the unbiased standard deviations of the volatility decay at a slower rate than for an i.i.d. dispersion process. The fixed income futures resemble the dynamics of the stock index futures, for the most part. Where a divergence occurs it is for the currency futures. For all the currencies except the British Pound, the unbiased standard deviations of the volatility decay in a pattern that is approaching an i.i.d. dispersion process. This is especially true for the longer-term time horizons.

It is clear from these figures that the dynamics of volatility are multi-faceted. It appears that one factor, the Central Limit Theorem, is relatively more important for some markets than others. For those markets, where the standard deviations of volatility fail to decay at a sufficiently rapid rate, another factor is causing the variability of volatility to remain high as the time horizon is extended. It is not clear what this mechanism is. The most likely possibility is that the volatility itself is stochastic. It remains to determine for each market the relative impact of these two factors in describing the volatility dynamics through time. This will be a central focus of this research and will be examined in the next three Chapters. Three classes of stochastic volatility models will be examined in this research with a wide variety of parameter inputs. This will compare the effectiveness of alternative models in capturing the dynamics of the unconditional standard deviation of the volatility and assess the impacts of mean reversion. But before we can model these factors, we must ask what the assumption of an i.i.d. dispersion process would imply for the behaviour of the underlying prices series.

One interpretation for the single distribution hypothesis is that this could be consistent with the possibility that jumps occur in the price of the underlying asset. Another possibility is that i.i.d. distribution follows geometric Brownian motion. Later in this research, we will test this directly and assess which distribution would be the best candidate for the single distributional form describing the markets if a single i.i.d. distribution were chosen. Evidence will be presented in the next Chapter that this form follows a Student-t distribution, which would be consistent with a jump process.

Finally, this research will combine both a stochastic volatility and jump process to assess which model best describes the observed probabilities from the volatility cones. As has been pointed out by Bates (1996), there may be both jumps and stochastic volatility intermixed in the actual dispersion process of volatility. However, the relative importance of each element in describing the actual empirical volatility processes is not clear. This approach will allow us to determine both the relative importance of the two elements and to calibrate the stochastic volatility model for simulations. The key result is that we must build a model including both jump diffusion processes and stochastic volatility in such a way that we are able to explain the empirical standard deviation patterns of the volatility.

In order to assess if this result is period specific, we examined the relationship between the decay in the standard deviation of the volatility (and compared these to the assumption of an i.i.d. distribution) for the first half and second half of the available observations. This can be seen in Figures 2.12a, 2.12b and 2.12c for the first half and in Figures 2.13a, 2.13b and 2.13c for the second half of the observation period.

In these figures, we find similar dynamics for the two sub-periods compared to the entire period. The only exceptions are for the Nikkei in the second period that has a time decay in the standard deviation which is somewhat unusual (likewise for the DAX and the BTP). The most probable explanation is that these series have the least amount of data for analysis and we are experiencing a problem with insufficient observations. For the currencies, the trend towards an i.i.d. dispersional form is even stronger than for the entire period (especially in the first period). Even with these slight variations, the patterns of the standard deviation decay seem fairly stable.

With this analysis, we now have a clearer understanding of the relationship between the dynamics of volatility and the time horizon of estimation. However, we are missing some composite measure of how the standard deviations of the volatility decay as a function of time. This measure will be determined in the next section by determining the functional form best fitting the empirical decay in the standard deviation of the volatility.

2.9 A SIMPLE MODEL FOR THE STANDARD DEVIATION OF VOLATILITY AS A FUNCTION OF THE TIME HORIZON

With the variance of the volatility series across time unbiased for the overlapping and non-overlapping observations, we can now examine how deviations in volatility decrease as a function of time. The approach was to assess the functional form of the decay of the empirical standard deviations of the unbiased volatility estimates from the cone analysis. We examined the standard deviations from the 20-day horizon to the 200-day horizon. From the previously presented results, the decay of the standard deviation was downward sloping. However, this decay was not as extreme as would be predicted from the functional form $se_{20}*\sqrt{20/N}$.

It is important to decide which sort of functional form would be appropriate for our purposes. It would appear that either exponential or power forms would be the only approaches for this problem yielding the same shape regardless of the time measure. Furthermore, the shape of the unbiased standard deviation of the volatility suggests that of the two approaches the power function may work better. Therefore, given that the standard deviation from the single distribution should display behaviour of the form $\sqrt{2T}$, we instead tried to find the power form of T^f that would best fit the observed data. Here T replaces what was previously referred to as N for the sake of the formula.

This was achieved by solving the following equation for each of the standard deviation curves:

$$\min \sum_{T=20}^{200} (se_T - se_{20} * T^f)^2 \tag{2.2}$$

For this equation, we are solving for *f*, which is the attribute of how the standard deviations decay through time. Since we are starting at the standard deviation estimated at a 20-day horizon, this is defined to be the starting point for the weighting attribute.

Another approach was used for the estimation of the decay attribute, *f*, which used the following linear regression equation:

$$\ln(\hat{\sigma}_T) = \alpha + \beta(\ln(T)) \tag{2.3}$$

From this regression equation, we then estimated the optimal decay attribute, *f*, and then estimated the actual standard deviations of the volatility using the formula:

$$e^{\alpha+\beta(\ln(T))} \tag{2.4}$$

It should be noted that for the single i.i.d. distribution, we would expect f to be 0.5. To test this, we ran a simulation of asset prices that assumed geometric Brownian motion with constant volatility. This is the assumption of the Black and Scholes (1973) model. The results of the analysis of the slope of the unbiased standard deviation of the volatility do indicate that (as expected) the rate of decay is approximately equal to the square root of time (-0.5000). For the naive fitted time decay coefficient, the value was -0.4714. For the regression of the logged volatilities and logged time, the slope coefficient was -0.4886. Given that all the empirical series decay at a much slower rate, the decay attributes, *f*, must be less than 0.5.

The results of these two analysis techniques are presented in Figures 2.14a, 2.14b and 2.14c and the attributes are presented in the following Table 2.12 for all twelve markets.

Markets	Time Factor	Regression Estimation	
		Alpha	Beta
S&P 500 Futures	-0.0498	2.6943	-0.0448
FTSE Futures	-0.0899	2.2961	-0.0956
DAX Futures	-0.1740	2.1967	-0.1905
Nikkei Dow Futures	-0.1681	2.7802	-0.1886
Bund Futures	-0.0814	1.0961	-0.0863
BTP Futures	-0.1665	2.0046	-0.2124
Gilt Futures	-0.1775	1.6889	-0.1820
US T-Bond Futures	-0.0572	1.8123	-0.0573
Deutsche Mark /US Dollar	-0.2432	2.1663	-0.2637
British Pound / US Dollar	-0.1613	2.0614	-0.1780
Japanese Yen / US Dollar	-0.2792	2.2765	-0.3029
Swiss Franc / US Dollar	-0.2458	2.1531	-0.2592

Table 2.12, Time Decay Factors for the Standard Deviation of Volatility for Twelve Financial Futures Markets.

It can be seen from the figures, that the fitting approach is extremely close to the empirical decay function. In most instances, the fit is almost perfect and the errors between the predicted function and the empirical function are less than 0.1% at all the time horizons.

As expected, the decay factors are significantly below what would be expected from a geometric Brownian motion model with constant variance. The slowest rate of time decay occurs for the S&P 500, FTSE, Bund and US-T Bond futures. The DAX, Nikkei, BTP and Gilt futures all share similar time decay parameters. For the currencies, most of the currencies (apart from the British Pound) share similar time decay parameters. It is interesting to note that (for the most part) those markets which have the highest kurtosis for the period also have the lowest time decay factors. An interesting comparison is between the decay factors determined in the two alternative methods. The first approach can be seen as a curvilinear regression method (on the absolute levels) and the second a linear regression method (on the logarithm of the levels versus the logarithm of time). Both approaches produce similar results, which suggests that either approach could be used. For the purposes of this research, the first method will be chosen due primarily to its simplicity and the fact that we are not required to perform an exponential transformation to produce our results. Finally, the second approach requires the inclusion of an alpha parameter, which can cause the estimated time decay series to diverge from the starting standard deviation of the volatility at the 20th day horizon.

To test the stability of these time decay factors, we reran the analysis splitting the time period into two halves. The results of these two analysis techniques are presented in Figures 2.15a, 2.15b and 2.15c for the first half of the observations and in Figures 2.16a, 2.16b and 2.16c for the first half of the observations. The time decay factors are also presented in the following Tables 2.13 and 2.14 for both periods.

Markets (First Period)	Time Factor	Regression Estimation Alpha	Beta
S&P 500 Futures	-0.0498	2.9548	-0.0352
FTSE Futures	-0.0569	2.4089	-0.0542
DAX Futures	-0.1740	2.4996	-0.2259
Nikkei Dow Futures	-0.1783	2.6543	-0.1686
Bund Futures	-0.0065	0.8906	0.0065
BTP Futures	-0.1500	1.9753	-0.1886
Gilt Futures	-0.1535	1.6139	-0.1419
US T-Bond Futures	-0.0399	1.9583	-0.0418
Deutsche Mark /US Dollar	-0.3000	2.3674	-0.3273
British Pound / US Dollar	-0.1900	2.2052	-0.2226
Japanese Yen / US Dollar	-0.4014	2.7312	-0.4537
Swiss Franc / US Dollar	-0.2748	2.2619	-0.2912

Table 2.13, Time Decay Factors for the Standard Deviation of Volatility for Twelve Financial Futures Markets For the First Half of the Available Observations

Markets (Second Period)	Time Factor	Regression Estimation Alpha	Beta
S&P 500 Futures	-0.2535	1.9811	-0.2513
FTSE Futures	-0.1856	1.9375	-0.1667
DAX Futures	-0.1274	1.6034	-0.0983
Nikkei Dow Futures	-0.1056	2.4835	-0.1069
Bund Futures	-0.1004	1.0291	-0.1018
BTP Futures	-0.2600	2.2730	-0.3340
Gilt Futures	-0.1602	1.6020	-0.1727
US T-Bond Futures	-0.1906	1.6864	-0.1974
Deutsche Mark /US Dollar	-0.1570	1.8445	-0.1610
British Pound / US Dollar	-0.0846	1.6886	-0.0693
Japanese Yen / US Dollar	-0.1876	1.9531	-0.1915
Swiss Franc / US Dollar	-0.1745	1.8562	-0.1675

Table 2.14, Time Decay Factors for the Standard Deviation of Volatility for Twelve Financial Futures Markets for the Second Half of the Available Observations.

For almost all the markets in both periods, the line fitting function provides an extremely close fit to the observed dynamics. The exception is the Bund futures in the first period, where at first glance a significant deviation seems to occur. However, one should realise that the scaling for this graph is extremely small. Thus, the maximum deviation between the (first methods) fitted line and the empirical decay dynamics is only off by a maximum of 0.06.

It is more interesting that the levels of the time decay factors vary considerably for many of the markets for the diverse periods. For the stock index and fixed income futures, the time decay factors are much higher in the second period. As with the case with the summary statistics, coefficients of variation (of the 20th day volatility) and the average autocorrelation measures for both periods, it appears that these series are displaying less abnormal behaviour in the second period compared to the first. The currencies have a lower time decay factor in the second period compared to the first period. This is especially pronounced for the British Pound for the second period. While this result may seem inconsistent with the previous conclusion that the currencies tend to be better described by a single i.i.d. dispersion process, it is important to remember that this time decay fitting is restricted to the

200th time horizon. For the earlier comparisons, we found that by the 500th time horizon point, the currencies were approaching the time decay expected from an i.i.d. dispersion process.

With this final analysis, we have identified the key factors that describe the dynamics of empirical volatility for financial futures markets. At this point, we will summarise what elements of the volatility process they describe and how these factors will be used to model the empirical dynamics.

2.10 ATTRIBUTES THAT CAPTURE EMPIRICAL VOLATILITY DYNAMICS

With this last analysis complete, we now have sufficient attributes to describe the multi-faceted nature of empirical volatility that has been discussed in the literature. At this point, one could argue that a similar analysis could be completed by a maximum likelihood approach or by a simple generalised methods of moments. It is true that we would be able to fit the objective processes using these approaches. We have chosen to examine each of the five attributes separately because it is important to understand which aspects the models are able to capture. Furthermore, since each of these conditions has an economic interpretation, we gain insights by separately examining how the models address each condition.

The first element is the general level of the volatility of the volatility. This factor will provide an insight to the stochastic nature of volatility itself and provide an overall measure for how variable volatility is. This will be measured by the coefficient of variation of the volatility estimated at the 20-day increment. This attribute can be taken from the volatility cone analysis and by dividing by the average level of the volatility at that point in time will serve to eliminate the problem of scaling. This coefficient of variation will be defined as:

$$COV_{20} = \sigma_{20} / \mu_{20} \tag{2.5}$$

Where COV_{20} is the coefficient of variation, σ_{20} is the standard deviation of the volatility measured at a 20 day time horizon and μ_{20} is the mean volatility measured at a 20 day time horizon.

The coefficient of variation value at 20 days should be the standard deviation of the volatility divided by the expected average volatility. In the case of an i.i.d normal. dispersion process, this should be equal to 0.1622[2]. This can be determined by simply dividing the formula for the standard deviation of the volatility by the formula for volatility. The standard deviation of the volatility is equal to the usual volatility formula divided by the square root of twice the number of observations. The formulae for this relationship appear below with the X's indicating the daily returns:

$$\left(\sqrt{\frac{\sum_{i=1}^{20}(X_i - \overline{X})^2}{19}} \div \sqrt{2 \cdot 19} \right) \div \left(\sqrt{\frac{\sum_{i=1}^{20}(X_i - \overline{X})^2}{19}} \right) \tag{2.6}$$

It should be noted that this relationship assumes that the price series follows a normal i.i.d. dispersion process because the expected kurtosis is equal to three (3). Suppose that the daily returns are i.i.d., then it is a simple matter to determine the average volatility for the time horizon. We are particularly interested in the sampling distribution of the volatility estimate. While the above does provide our expected value from an i.i.d. normal dispersion process, if the results are not equal to this, we are not able to tell whether this result is due to the process not being i.i.d. or non-

[2] This functional form is because this is a Chi-squared distribution.

normal. From Figures 2.11a,b & c, 2.12a,b & c and 2.13a,b & c (comparing the decay of the volatility relative to an i.i.d. process), we can see that it is clearly not i.i.d. From the initial examinations of the return series in Tables 2.1, 2.2 & 2.3, we can see that the returns are non-normal (displaying leptokurtic dynamics for daily returns).

It is clear that additional attributes must be included that address both of these elements. The kurtosis is related to the variance of the variance and must be considered. The evidence above suggests that even if we think we know what the true volatility is (the average), this may not help given that the existence of excess kurtosis will indicate the degree of quadratic variation. Therefore, the second criterion is the natural logarithm of the kurtosis measure for daily observations. The logarithm (rather than the absolute level) of kurtosis was used because of the extremely wide range of kurtosis measures for these twelve markets. Taking the natural logarithm of the kurtosis serves to reduce the scaling problem. For the sake of presentation, the actual kurtosis appears in the all the tables in this research instead of the logarithm of the kurtosis. However, all the analysis was done using the logarithm.

The third criterion for describing the empirical nature of the markets is the time decay function which best fits the empirical unbiased standard deviations of the volatility. We chose the simple line fit rather than the regression estimate. This was examined in the previous section. This will address one aspect of non-i.i.d. dynamics of the volatility series. In addition, this attribute will indicated how difficult it is to forecast volatility for different time horizons. As was suggested previously, this is one of the key issues for those pricing options.

Another possible reason for the non-i.i.d. nature of the price series is the existence of autocorrelations in the absolute returns. To include this element in describing the dynamics, the fourth and fifth attributes incorporated the autocorrelations for the twelve markets. To obtain a meaningful estimate of both shorter term and longer-term autocorrelations, we took the averages of the autocorrelations from the first lag to the 20th lag (0-20) and the averages of the autocorrelations from the 50th lag to the 70th lag. These values were estimated from the results previously presented in Tables 2.6, 2.7 and 2.8.

One important issue to address is whether or not the omission of a measure of the skewness of returns is a gross omission, which will limit the ability to capture all the dynamics of the dispersion processes for the twelve markets. A skewness factor would capture the fact that the volatility process is not independent of the underlying price innovations. There has been considerable research (presented in the last Chapter) that suggests that stock volatility is dependent upon the level of the stock price. However, this is not the case for other asset classes such as foreign exchange. Perraudin and Sørensen (1997) indicate that in the foreign exchange markets there is no evidence that price and volatility changes are negatively correlated.

Furthermore, in Tables 2.2, 2.4 and 2.5 where the summary statistics are presented for the returns of the twelve markets, only for two stock index futures (S&P 500 and FTSE 100) is the skewness statistically significantly negative. In addition, this is due to extreme events that occurred in the period from 1984(5) to 1990. It is obvious that this includes the 1987 stock market crash and the 1989 mini-crash. Gemmill (1991) also concluded that significantly negative skewness in the unconditional distribution of returns in the FTSE 100 is not observed in the normal course of events. When such skewness is observed it is generally associated with singular extreme events. In Appendix 2 of his paper, he reports skewness measures for the distributions of returns for the FTSE 100 for the periods from 28/7/85 to 30/7/90. In this analysis, he compared the statistics for the daily returns over various periods and examined the statistics for period before and (one month) after the 1987 crash. When the crash was excluded, he found that the skewness statistic was similar in magnitude and slightly negative. Thus, it makes little sense to include as a key factor in describing the objective process of all markets one that is based solely upon a single event.

Furthermore, for almost all the other markets, the skewness factor is insignificant and by no means consistently negative. Thus, we would argue that including skewness as a attribute is unnecessary. Inclusion of this factor may allow us to capture the dynamics of the 1987 crash for two of our markets but at the expense of being irrelevant for all other markets and for the time period where the 1987 crash is not represented.

The five attributes are summarised below for the empirical volatilities determined for the twelve markets and appear in Table 2.15.

Markets	Coefficient Of Variation	Time Factor	Unconditional Kurtosis	Autocorr (0-20)	Autocorr (50-70)
S&P 500 Futures	0.9045	-0.0498	254.50	0.1387	0.0549
FTSE Futures	0.4984	-0.0899	29.46	0.1428	0.0398
DAX Futures	0.3391	-0.1740	5.70	0.0782	0.0207
Nikkei 225 Futures	0.4031	-0.1681	4.74	0.1557	0.0629
Bund Futures	0.4519	-0.0814	6.87	0.1661	0.1009
BTP Futures	0.4268	-0.1665	5.06	0.1638	0.0826
Gilt Futures	0.3710	-0.1775	5.76	0.1116	0.0663
US T-Bond Futures	0.4637	-0.0572	5.37	0.2080	0.1643
D Mark /US $	0.3415	-0.2432	5.44	0.0804	0.0139
Pound / US $	0.4064	-0.1613	6.53	0.1140	0.0655
Yen / US $	0.3590	-0.2792	7.79	0.0577	0.0087
S-Franc / US $	0.3111	-0.2458	5.05	0.0660	0.0158
Average of Parameter Values	0.4397	-0.1578	28.52	0.1236	0.0580
Standard Deviation Of Attribute Values	0.1568	0.0756	71.49	0.0467	0.0445

Table 2.15, Attributes That Describe the Empirical Dynamics of Twelve Financial Futures Dispersion Processes

In these tables, we also present the standard deviations of the attribute values across the markets. As one can see, these values are fairly large reflecting the wide range of values observed for the different markets. These are most extreme for the standard deviations of the coefficient of variation and the kurtosis. Most markets seem to share similar dynamics for the average autocorrelations.

Finally, to examine the stability of these attributes, we examined the conditions for the first and second halves of the available data series. These can be seen in Tables 2.16 and 2.17.

These five attributes are summarised below for the empirical volatilities determined for the twelve markets using only the first half of the available observations and these appear in Table 2.16.

Markets (First Period)	Coefficient Of Variation	Time Factor	Unconditional Kurtosis	Autocorr (0-20)	Autocorr (50-70)
S&P 500 Futures	0.9581	-0.0438	185.34	0.1247	0.0304
FTSE Futures	0.5963	-0.0569	35.99	0.1615	0.0321
DAX Futures	0.3612	-0.1740	6.17	0.0883	0.0153
Nikkei 225 Futures	0.3493	-0.1783	3.99	0.1458	0.0456
Bund Futures	0.5175	-0.0065	9.51	0.1890	0.1103
BTP Futures	0.5193	-0.1500	6.89	0.1971	0.0833
Gilt Futures	0.3823	-0.1535	5.34	0.1228	0.0784
US T-Bond Futures	0.4863	-0.0399	4.54	0.2349	0.1845
D Mark /US $	0.3309	-0.3000	5.36	0.0711	-0.0114
Pound / US $	0.3721	-0.1900	6.18	0.0917	0.0335
Yen / US $	0.3383	-0.4014	7.10	0.0495	0.0005
S-Franc / US $	0.3082	-0.2748	5.03	0.0665	0.0024
Average of Parameter Values	0.4599	-0.1641	23.45	0.1286	0.0504
Standard Deviation Of Attribute Values	0.1820	0.1185	51.72	0.0584	0.0559

Table 2.16, Attributes That Describe the Empirical Dynamics of Twelve Financial Futures Dispersion Processes for the First Half of the Available Observations.

These five attributes are summarised below for the empirical volatilities determined for the twelve markets for the second half of the available observations and these appear in Table 2.17.

Markets (Second Period)	Coefficient Of Variation	Time Factor	Unconditional Kurtosis	Autocorr (0-20)	Autocorr (50-70)
S&P 500 Futures	0.3192	-0.2535	5.88	0.0596	0.0224
FTSE Futures	0.3119	-0.1856	4.52	0.0888	0.0430
DAX Futures	0.2824	-0.1274	4.21	0.0481	0.0161
Nikkei Dow Futures	0.4381	-0.1056	5.87	0.1383	0.0704
Bund Futures	0.3818	-0.1004	4.66	0.1329	0.0805
BTP Futures	0.3176	-0.2600	3.94	0.1014	0.0109
Gilt Futures	0.3546	-0.1602	6.25	0.0950	0.0504
US T-Bond Futures	0.3133	-0.1906	5.83	0.0641	0.0348
D Mark /US $	0.3515	-0.1570	5.52	0.0884	0.0379
Pound / US $	0.4354	-0.0846	6.81	0.1309	0.0940
Yen / US $	0.3794	-0.1876	8.59	0.0657	0.0110
S-Franc / US $	0.3148	-0.1745	5.06	0.0655	0.0318
Average of Parameter Values	0.3500	-0.1656	5.60	0.0899	0.0419
Standard Deviation Of Attribute Values	0.0500	0.05576	1.28	0.0310	0.0273

Table 2.17, Attributes That Describe the Empirical Dynamics of Twelve Financial Futures Dispersion Processes for the Second Half of the Available Observations.

An important result is that the dynamics of the twelve financial futures markets (as measured by the attributes) are dissimilar when the period of analysis is split into the two segments. For almost all the stock index futures and bond futures markets the degree of excess kurtosis drops significantly in the second half of the analysis period. Thus, it would appear that markets displayed a much greater tendency to being fat-tailed during the latter half of the 1980s when compared to the first half of the 1990s. This conclusion is somewhat difficult to generalise since a number of the markets under investigation (DAX, Nikkei, BTP and Bund) did not have observations in the 1980s. Nevertheless, it appears to be the general pattern that as we approach 1996, the levels of excess kurtosis have for the most part fallen. The exception to this result was the Nikkei 225 that both displayed more excess kurtosis in the latter period of analysis. In addition, all the stock index futures and bond futures markets had much lower autocorrelations in the absolute returns.

For the currencies, the volatility dynamics appear more stable between the period of the latter half of the 1980s and the first half of the 1990s. The levels of excess kurtosis, autocorrelation behaviours and the coefficients of variations are almost identical for the Deutsche Mark and the Swiss Franc for the two periods. The only divergence is with the slope of the time decay of unbiased standard deviation of the volatility. In both cases, the slope is much lower in the latter period. The British Pound and the Japanese Yen do display somewhat more variability in their dynamics for the two periods. For the British Pound, the degree of excess kurtosis has fallen dramatically, as has the coefficient of variation of 20-day volatility. The autocorrelations in absolute returns have increased and the line fit has decreased. For the Japanese Yen, the coefficient of variation is slightly higher in the second period and is only the second financial market under study that had a higher excess kurtosis for the latter period (along with the Nikkei 225). In addition, the autocorrelations are also more positive in the second period (although remain close to zero).

The fact that the volatility dynamics of the twelve markets change over time suggests that either the results are due to sampling variations for the true dynamics of the markets or that the behaviour of volatility is not stationary. One possible explanation is that the two markets which displayed more leptokurtic behaviour and higher autocorrelations in the second period were both associated with the Japanese economy. Tompkins (1983) first showed that volatility for financial assets is related to the business cycle. Tompkins demonstrated that for the last five business cycles in the United States, stock market volatility (as measured by the S&P 500) is significantly higher during periods of recession and lower in periods of expansion. While this research only examined the absolute levels of volatility, it is possible that in periods of recession, unusual patterns of volatility behaviour exist. Therefore, it is relevant to note that during the mid 1990s, the Japanese economy was in recession while the countries represented by the other financial markets in this study were expanding. Furthermore, in the first period of the analysis, the Japanese economy was expanding while other countries were experiencing contracting economies. This may suggest that both the absolute levels of volatility and the dynamics of the volatility behaviour vary over time and this could be related to the current state of the overall economy.

Another explanation for the divergent behaviour of the volatility dynamics in the latter period of analysis is that the latter period does not include the stock market crashes of 1987 and 1989. The elimination of these extreme events could solely explain the dynamics for the S&P and FTSE. However, this would fail to address why the Bond markets and British Pound display a much less extreme degree of leptokurtic behaviour in the second period.

Regardless of the reasons for these divergences, it is important to examine the overall dynamics of volatility for both long periods and for shorter periods. This will provide us with a clearer picture of the stability of volatility dynamics and allow us to model why volatility dynamics may differ over time. Although at first sight, this evolution displays a disturbing lack of stability, we will find that they still have sufficient power to discriminate between different models.

This research will use these attributes using a modified method of moments approach to test the ability of alternative models to describe the empirical dynamics of objective dispersion processes. It is critical that the choice of the attributes captures the relevant elements of empirical non-normality. These factors should be independent, lend themselves to economic interpretation and be measurable.

The first issue we examine are whether or not they are independent. If this were not the case, it could imply that one or more attributes are redundant. To examine this, we estimated the correlations between the attribute values for the 12 financial markets for all three periods of analysis. This can be seen below in Table 2.18.

	COV	KURTOSIS	CORR (0-20)	CORR (50-70)	LINE FIT
COV	1.0				
KURTOSIS	0.8811	1.0			
CORR (0-20)	0.4837	0.1189	1.0		
CORR (50-70)	0.2617	-0.0403	0.8615	1.0	
LINE FIT	0.6171	0.3639	0.7248	0.6883	1.0

Table 2.18, Correlation Matrix of Attributes for the 12 Financial Futures Markets for the Three Periods of Analysis

In this table, it appears that a number of the attributes are measuring similar dynamics. For example, the coefficient of variation (COV) is extremely highly correlated to the level of kurtosis. The COV also is relatively highly correlated to the Line Fit (of the decay of the standard deviation of volatility over time). Furthermore, there exists a positive correlation between the COV condition with the autocorrelations of absolute returns. However, if the COV and kurtosis of daily returns are redundant, one would expect a similar relationship between these conditions and the other conditions. For the autocorrelation conditions, the kurtosis condition is uncorrelated. Thus, the COV could be measuring the interaction between autocorrelations and the kurtosis that might be not be captured by the correlation measure. The Line fit condition is also highly related to the autocorrelation conditions. This is not surprising considering that the decay in the standard deviation of volatility should be related to the degree of autocorrelations of absolute returns. Nevertheless, the correlations are sufficiently less than one to suggest different dynamics are being measured. Therefore, it would appear that these conditions capture two primary factors that have been identified as causing the volatility behaviour to diverge from stationarity: excess kurtosis and positive autocorrelations in absolute returns. We argue that even though many of these attributes are highly correlated, they are not redundant. It is likely that they measure both the overall factors identified in the empirical literature and more complex interactions that can only be captured by inclusion of all the attributes.

2.11 CONCLUSION

In this Chapter, we have examined the dynamics of the objective processes for twelve financial futures markets. The statistics of the return series suggest the twelve markets deviate from the assumption of lognormality. Our attributes are providing insights into how. Examinations of the autocorrelations of the absolute returns suggest that significant relationships exist between the time series of returns. However, we find that these autocorrelations tend to decay fairly rapidly to insignificance after the 20th lag. From this Chapter, we have set the stage for what will follow. Our research aim is to understand the appropriate security price process that is consistent with the historical record. To compare the alternative approaches, we must have some metrics for testing. In this Chapter, we have identified five attributes that capture the non-normal dynamics of the objective process. With these, we will test a variety of models to assess which captures these dynamics. The next three Chapters will demonstrate that a considerable amount of information about the dynamics of the process can be discerned from the volatility cone approach (once the overlapping bias problem has been corrected).

CHAPTER THREE
THE ANALYSIS OF OBJECTIVE PROBABILITIES IN FUTURES MARKETS: EXPLAINING THE EMPIRICAL DYNAMICS WITH MODELS ASSUMING CONSTANT VARIANCE

3.1 INTRODUCTION

In the last Chapter, we identified five attributes that capture the key dynamics of the volatility process. With these conditions we can now compare the empirical dynamics of the twelve financial futures markets to simulated dynamics for a variety of models. Given that these attributes provide a broad and fairly complete picture of the multi-faceted nature of volatility dynamics, this approach promises to provide a powerful tool for the comparison of models. First of all, we will examine the simplest possible model, which is geometric Brownian motion with constant variance. Thereafter, we will examine progressively more complex models that include fat-tailed distributions (as a proxy for jump diffusion processes), stochastic volatility models, and finally combinations of models. In this Chapter, we will examine only those models that assume the variance is constant. Specifically, we will compare models that assume the underlying asset follows either geometric Brownian motion or a Student-*t* distribution.

3.2 TESTING A GEOMETRIC BROWNIAN MOTION MODEL

The first test will simulate a series of asset prices using a constant level of volatility and comparing the results to empirical attributes of the twelve markets. The volatility chosen was approximately equal to the long-term volatility for each market and the prices were determined from the standard asset pricing model of the following form:

$$dS = S\mu \cdot dt + S\hat{\sigma} \cdot dZ_1 \qquad (3.1)$$

In this formula, we assumed an interest rate of 0.0. Therefore, the μ term is solely equal to $\hat{\sigma}^2/2$. This is possible since we have used futures prices rather than spot prices. As was discussed by Bates (1991), the cost of carry (drift term) for the futures can be assumed to be zero.

This is the standard Geometric Brownian motion formula that generates asset price changes, thus, the series of asset prices were generated using the formula:

$$S_t = S_{t-1} \cdot e^{\mu \cdot dt + \hat{\sigma}_{t-1} \cdot dZ_1} \qquad (3.2)$$

This is the simple Euler approach. The term $\hat{\sigma}$, reflects the volatilities estimated from the various models tested and the previous day's volatility estimate is used to estimate today's new asset price. Given that the volatilities are assumed to be constant, this does not change.

With a series of 2000 prices determined, these prices were imported into the same programmes used to describe the empirical dynamics of the twelve financial futures markets. These programmes included the determination of the kurtosis of each series, the volatility cone analysis and the estimation of the autocorrelograms. The results were compared to each futures market using a method that minimises the sum of squared errors for each of the attributes compared to those conditions for each of the twelve markets. To aid the interpretation, the squared errors were divided by the standard deviation of the attributes across the markets. By standardising the test statistic, we are removing the impacts of scaling for the different attributes. This test statistic can be written as:

$$\min \sum \left(\frac{M_i - X_i}{\sigma_i} \right)^2 \qquad (3.3)$$

where M_i is the attribute for the financial futures, X_i is the attribute of the price series generated by the model and σ_i is the standard deviation of the attributes across all the financial futures for the relevant period of analysis. The division by the standard deviation means that all the statistics have been standardised and the summation of the deviations can be compared directly without the requirement for any weighting scheme. These standard deviations can be seen at the bottom of Table 2.15 for the entire period of analysis, Table 2.16 for the first half of the available observations and Table 2.17 for the second half of the available observations. (See the previous Chapter).

In spirit, this method bears some resemblance to the Generalised Method of Moments (GMM) technique where a single model is assumed and the determination of the summation of the squared errors are for the individual moments for that model. This is based upon modelling the sample properties of the moments within the model.[1] This approach is similar to the technique used by Longstaff (1989) when he examined stochastic volatility models and to Ho, Perraudin and Sørensen (1996) when they modelled stock returns with stochastic volatility and jumps. We will refer to this test as MSSE (for the minimised sum of the squared errors) in the following three Chapters.

This test will achieve two goals. It will allow us to examine which elements of geometric Brownian motion are consistent with the dynamics of the futures markets and will also provide an overall measure of the goodness of fit for the comparison of different models. Clearly, the dynamics of a simulated GBM process will depend upon the random numbers selected. It is possible that our results could be biased from an unrepresentative selection of random variables. Fortunately, we can determine what theoretical values should be obtained for a geometric Brownian motion process.

[1] In the approach used in this research, the test metric is across models which aims to obtain a single best fit. As we are not modelling from the standard errors of the moments, this approach is not a traditional GMM technique. As GMM is in reality an extension of OLS on sample moments.

In the previous Chapter, we showed that if the dispersion process of the price series is i.i.d., then the expected coefficient of variation would be $1/\sqrt{(2*19)}$ or approximately 0.1622. The kurtosis result should be equal to 3.00, while the line slope should be equal to -0.5000 and both the autocorrelation should be zero.

To test whether our simulations conform to these expected results, we generated 100 series of 2000 random prices that were consistent with GBM and constant variance (from equation 3.2). Using a Box-Muller technique we approximated each draw from the normal distribution. We also determined the standard deviation for each of the attributes for the 100 simulations. Finally, we tested the null hypothesis that the average results were significantly different than the theoretical values. These results can be seen in Table 3.1

	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)
Average Random Result	0.1618	-0.5056	2.996	-0.00107	-0.00024
Standard Deviation	0.0104	0.07351	1.0446	0.004905	0.005732
Theoretical Value	0.1622	-0.5000	3.000	0.000000	0.000000
t-test	15.59	13.13	261.15	4.58	23.88

Table 3.1, Estimated Values of the Attributes for a GBM price series with Constant Variance.

For all the attributes, we can reject at above a 99% level that the average results differ from the expected theoretical values.[2] These results confirm both our theoretical hypotheses about the expected values for these attributes and suggest that the process we are using for generating random numbers will be on average consistent with GBM. Another benefit is that because of relatively small standard deviations,

[2] Strictly speaking, we cannot really use a t-test given that this test is generally used for testing differences in sample moments from two samples. What we have is a sample result for a simulation

these attributes are relatively insensitive to the selection of the random numbers. This suggests that the simulations are being correctly estimated.

In the last Chapter, we examined whether the attributes from the twelve markets that measure different aspects of volatility dynamics might be redundant. One possible reason for the patterns observed in Table 2.18 is that the results are due simply to sampling error. To examine this, we estimated the correlations between the attribute values for the 100 simulations of a GBM process. This can be seen below in Table 3.2.

	COV	KURTOSIS	CORR (0-20)	CORR (50-70)	LINE FIT
COV	1.0				
KURTOSIS	0.361	1.0			
CORR (0-20)	0.601	0.003	1.0		
CORR (50-70)	-0.053	-0.055	-0.016	1.0	
LINE FIT	0.216	0.0715	0.375	0.323	1.0

Table 3.2, Correlation Matrix of Attributes for the 100 simulations of a GBM price series with Constant variance.

While some interesting points can be discerned by comparing the two tables, we must recognise that we are not comparing like with like. The previous table, (Table 2.18) is a cross sectional analysis, while this table is only examining the effects of sampling. In this table, most of the attributes appear to be measuring different aspects of volatility behaviour. The highest correlation is between the coefficient of variation and the average autocorrelation from 0 to 20 days. This is not surprising since a relatively high (or low) coefficient of variation would suggest a correspondingly high (or low) short-term autocorrelation. The line fit is somewhat positively correlated with the autocorrelation measures. Again this is not surprising

compared to the theoretical values we expect (and this is not a sample). However, the use of this test since the decay in the standard deviation of the volatilities should have some impact on the measures of autocorrelations. Nevertheless, these correlations are of a small enough magnitude to indicate they are measuring different elements. The remaining factors seem to be independent of one another.

In the previous table (Table 2.18), which examined the actual correlation behaviour of the twelve markets, significant divergences arise from the GBM case. For the twelve markets, the COV condition has a much higher correlation with the kurtosis. In fact most of the conditions are much more correlated than compared to the GBM case. The only exception is that the kurtosis remains relatively uncorrelated with the autocorrelation conditions. Thus, the results we have obtained from the twelve markets diverge significantly from the assumption that the twelve markets follow GBM. Nevertheless, it appears that the chosen conditions are capturing different elements in the volatility process both in an assumed GBM framework and from the empirical dynamics.

For all such methods of moments testing, key elements in the choice of the conditions are that they are independent, have an economic interpretation and can be measured accurately. The correlation matrices presented suggest that these five attributes meet these criteria. Therefore, our conclusion is that we are measuring different aspects of dynamics of volatility and which can serve as criteria for testing models of volatility behaviour.

While the average result provides some degree of comfort that the GBM simulation process is correct, it cannot be used for testing purposes. This is due to the fact that an average of the 100 draws would also not provide a single series of 2000 prices that would have the appropriate dynamics. Therefore, for testing purposes, we

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allows us insights into the natures of the simulated data.

choice to identify a representative draw of the 100 sample draws that was closest to the theoretical dynamics. This series was chosen using the same minimisation of the sum of squared errors (for the five attributes) that will be used extensively in the first part of this research (see equation 3.3). The dynamics of this representative series (and the comparison of this series to the average series and the expected theoretical results) appear below in Table 3.3.

	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)
Average Random Result	0.1618	-0.5056	2.996	-0.00107	-0.00024
Selected Result	0.1612	-0.5331	2.989	-0.00124	+0.0031
Theoretical Value	0.1622	-0.5000	3.000	0.000000	0.000000

Table 3.3, Comparison of the Attributes for the Average of 100 GBM price series with Constant Variance to a Selected GBM price series with Constant variance.

As would be expected, the closest series of simulated GBM prices diverges somewhat from both the average random result and the expected theoretical results. While the coefficient of variation is only slightly different, the rate of time decay (at -0.5331) is somewhat more extreme. However, these results are not statistically different than the expected results if we apply the same t-test used in Table 3.2. Even so, it is possible that this choice of random variables could impact the results. This point is taken. However, we counter this by interpreting the small size of the standard deviations of the attributes for the 100 simulations (in Table 3.2) as indicating the choice of the price series will not be that critical[3].

[3] This representative normal distribution was used for all subsequent analysis that relied upon a GBM innovation for the underlying price process. The only important impacts of the choice of a single (but representative) random series of prices is that we have sum of squared deviations that are not coloured by the choice of the random series but reflects how the empirical attributes differ from the attributes from the model. Secondly, the choice of a representative normal distribution allows for meaningful

Clearly, the choice of a 'representative' series of random numbers could be seen as problematic. It could be argued that our results may be due to the selection of the appropriate random number series and not to the theoretical models we aim to examine. The alternatives open to us included drawing more random numbers and using Monte Carlo methods to reduce sampling variation. This is the approach used by Hull & White (1987a, 1988) to test their models. However, we wished to analyse a series (of 2000 sample prices) that was close to the number of actual futures prices we were examining. Therefore, a Monte Carlo approach was not feasible. This was due to computer limitations and the fact that our analysis relied on the use of EXCEL (version 97). The other alternative was to draw a number of samples of random numbers and examine the sampling properties of these draws. Once we could confirm that the average of these results corresponded to the expected theoretical results, we could have confidence that the procedure for drawing random variables was consistent with GBM. However, it still remained necessary to select a single draw to run our simulations and this raises the issue that the results are due simply to our selection of random numbers.

To examine this potentiality, we drew one series of 2000 numbers at random. Due to the choice of the random variables from the Box-Muller method, we obtained a series that diverged somewhat from our expected results (although these results for the attributes were within the range of sample variation). The actual results for this sample were:

parameter values for the stochastic volatility models in the next Chapter. This will allow our optimal parameter values to be comparable to those presented in the literature (see Guo (1996)) that used alternative and more powerful methods to determine a GBM process. It is important that the relative performance of the models was not affected and our conclusions would have been identical. The difference is that our results are less effected by the sampling error from drawing from a single normal distribution that turned out to be less variable that is expected.

	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)
Sample Random Result	0.0824	-0.6100	1.380	-0.0009	-0.0010

With this selected series, we completed all the analysis for the first portion of the research. As the reader can see, this series has an extremely low kurtosis measure, a time factor for the decay of volatilities that is too high and a low coefficient of variation. In essence, we have drawn a somewhat platykurtic distribution.

With this series of random prices, we re-ran all the analysis for the next three chapters. Given that our sample of normal prices was platykurtic, this altered the selection of the optimal parameters for the stochastic volatility models in the next Chapter and lead to somewhat higher sums of the squared errors. However, it was comforting to find that the overall conclusions of this research were not altered (see conclusions to Chapter 5), however, it was no longer possible to compare our results to elsewhere in the literature that used alternative methods to determine a more representative normal distribution.[4]

GBM Test Results for Entire Period of Analysis

With this particular series of random prices, we examined the ability of geometric Brownian motion to explain the dynamics of the twelve financial futures markets. Specifically, once the series of 2000 prices was chosen, these prices were imported into the programmes used to describe the empirical dynamics of the twelve

[4] In the previous footnote, we comment on the fact that our results using the representative normal distribution were extremely similar to those found for the Heston model by Guo (1996) for the Deutsche Mark / US Dollar for a similar period of analysis. Thus, the choice of the representative normal distribution appears to provide results which are similar to those obtained using alternative approaches. By testing the results using another draw of the normal distribution, we are confident that

financial futures markets. These programmes included the determination of the kurtosis of each series, the volatility cone analysis and the estimation of the autocorrelograms. When all the conditions for the 'best' GBM price series were estimated, the sum of the squared errors were determined comparing this series with each of the twelve markets' conditions. The results of these tests appear in Table 3.4 and the analysis period includes all the available observations.

Markets (GBM Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	0.9045	-0.0498	254.50	0.1387	0.0549	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**70.9208**
FTSE	0.4984	-0.0899	29.461	0.1428	0.0398	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**60.0982**
DAX	0.3391	-0.1740	5.7004	0.0782	0.0207	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**22.6601**
Nikkei	0.4031	-0.1681	4.74	0.1557	0.0629	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**45.0403**
Bund	0.4519	-0.0814	6.8716	0.1661	0.1009	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**65.9331**
BTP	0.4268	-0.1665	5.0590	0.1638	0.0826	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**48.1248**
Gilt	0.3710	-0.1775	5.7580	0.1116	0.0663	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**37.0817**
US T-Bond	0.4637	-0.0572	5.3650	0.2080	0.1643	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**88.0371**
D-mark	0.3415	-0.2432	5.4460	0.0804	0.0139	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**22.3236**
Pound	0.4064	-0.1613	6.53	0.1140	0.0655	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**35.5084**
Yen	0.3590	-0.2792	7.7897	0.0577	0.0087	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**17.2707**
S-Franc	0.3111	-0.2458	5.0450	0.0660	0.0158	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**36.2483**

Table 3.4, Comparisons of the Empirical Dynamics of Twelve Financial Futures Dispersion Processes with the Dynamics of a GBM Price Series with Constant Variance for the Entire Period of Available Observations.

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the results obtained are due to correct testing of the models and not to the selection of the random variables for the simulations.

These results clearly demonstrate that all the financial futures markets fail the test of a lognormal dispersion process (GBM) with constant variance. The coefficient of variation is much too low, the rate of decay in the volatility of the volatility is much too high and the kurtosis is much smaller than what we observe in the financial futures markets. Finally, the GBM series displays autocorrelations in the absolute daily returns which are insignificantly different than zero. For all the twelve markets, most the average autocorrelations are significantly positive.

GBM Test Results for SubPeriods of Analysis

To examine whether these results are consistent over time and are not period specific, we reran the analysis for two sub-periods. The results for this analysis in the first half can be seen in Table 3.5.

Markets (GBM Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	0.9581	-0.0438	185.344	0.1247	0.0304	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**49.8419**
FTSE	0.5963	-0.0569	35.9850	0.1615	0.0321	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**32.7907**
DAX	0.3612	-0.1740	6.1734	0.0883	0.0153	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**12.8280**
Nikkei	0.3493	-0.1783	3.9912	0.1458	0.0456	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**16.4677**
Bund	0.5175	-0.0065	9.5135	0.1890	0.1103	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**37.5815**
BTP	0.5193	-0.1500	6.8939	0.1971	0.0833	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**27.2153**
Gilt	0.3823	-0.1535	5.3372	0.1228	0.0784	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**17.7907**
US T-Bond	0.4863	-0.0399	4.5418	0.2349	0.1845	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**45.9846**

Markets (GBM Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
D-mark	0.3309	-0.3000	5.3573	0.0711	-0.0114	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**6.3656**
Pound	0.3721	-0.1900	6.18	0.0917	0.0335	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**12.8628**
Yen	0.3383	-0.4014	7.1017	0.0495	0.0005	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**3.2769**
S-Franc	0.3082	-0.2748	5.0305	0.0665	0.0024	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**6.7588**

Table 3.5, Comparisons of the Empirical Dynamics of Twelve Financial Futures Dispersion Processes with the Dynamics of a GBM Price Series for the First Half of the Available Observations

Markets (GBM Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	0.3192	-0.2535	5.8823	0.0596	0.0224	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**49.0282**
FTSE	0.3119	-0.1856	4.5168	0.0888	0.0430	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**62.0476**
DAX	0.2824	-0.1274	4.2104	0.0481	0.0161	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**64.0125**
Nikkei	0.4381	-0.1056	5.8671	0.1383	0.0704	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**125.2491**
Bund	0.3818	-0.1004	4.6621	0.1329	0.0805	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**110.5935**
BTP	0.3176	-0.2600	3.9391	0.1014	0.0109	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**46.4193**
Gilt	0.3546	-0.1602	6.2497	0.0950	0.0504	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**83.6713**
US T-Bond	0.3133	-0.1906	5.8301	0.0641	0.0348	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**61.9444**
D-mark	0.3515	-0.1570	5.5237	0.0884	0.0379	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**77.8149**
Pound	0.4354	-0.0846	6.8103	0.1308	0.0937	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**138.0695**
Yen	0.3794	-0.1876	8.5897	0.0657	0.0110	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**85.3851**
S-Franc	0.3148	-0.1745	5.0631	0.0655	0.0318	
GBM	**0.1612**	**-0.5331**	**2.989**	**-0.00124**	**0.0031**	**62.3183**

Table 3.6, Comparisons of the Empirical Dynamics of Twelve Financial Futures Dispersion Processes with the Dynamics of a GBM Price Series for the Second Half of the Available Observations.

For both of the sub-periods, the results are similar to those obtained for the entire period. The sum of the squared errors is very large and suggests that none of the markets in any of the three periods is well explained by an assumption of geometric Brownian motion. It is interesting to note that the latter period (which roughly corresponds to the first half of the 1990s) is much worse than the earlier period (which roughly corresponds to the last half of the 1980s).

Given that we can reject the usual assumptions that all three categories of financial futures are lognormally distributed with constant variance, our task is now to understand what processes do describe the dynamics of these markets.

3.3 ALTERNATIVE MODELS TO EXPLAIN VOLATILITY DYNAMICS

At this point, it makes sense to state explicitly the objectives of this part of the research. The key objectives are: (1) to fit the observed empirical volatility cones, (2) to fit the observed distributions of returns and (3) to explain the autocorrelations of the absolute returns. The choice of the five attributes identified above will serve as the criteria for meeting these objectives.

At this point, we have found that the (unbiased) standard deviation of the volatility is much higher than we would expect as the time horizon is lengthened. Thus, we must distinguish between the three possible hypotheses for this result: (1) we could have a single distribution with a sampling problem as the only effect, (2) we could have diverse distributions due to the variability of volatility and, finally, (3) we have a mixture of diverse distributions of variable volatility.

Furthermore, the empirical examination of the twelve markets clearly indicates that for all the series significant excess leptokurtosis exists. In addition, many of the other attributes that describe the nature of volatility for these markets diverge significantly from geometric Brownian motion. Much research has concentrated on explaining the fat-tailed nature of financial asset returns. The key question remains; what is causing these divergences? Broadly speaking there are three possible explanations for the anomalies that have been observed for all asset markets. These include a Constant Elasticity of Variance Model, Jump Processes or the existence of Stochastic Volatility. In the first Chapter, we examined these three models. For this research, we are concerned primarily with the quadratic variation in volatility. Therefore, we have chosen to examine solely Jump processes and Stochastic Volatility models to explain this phenomenon.

3.4 TESTING A FAT-TAILED DISTRIBUTION MODEL

As was discussed in Chapter one, numerous papers have indicated that asset price series display excess kurtosis. One proposed explanation is that the dispersion process for the underlying asset prices is not lognormal. Another explanation is that perhaps other factors such as jump processes might be explaining the behaviour of the price series. To test both these possibilities, we will introduce a fat-tailed distribution for the generation of the price series and assume a constant volatility over time.

Our approach will follow the lines of Blattberg and Gonedes (1974) and use a Student-*t* distribution instead of a normal dispersion process. While it is possible to include other dispersion processes that also produce excess kurtosis, the Student-*t* distribution was chosen because of the simplicity of the approach and the ease with which that such a price series could be simulated. This approach has also been applied by Bollerslev (1987) who formulated an ARCH-type model with heavy-tailed innovations based on a Student-*t* distribution. For this simulation, all the previous steps remain the same with the one alteration that instead of drawing from a normal dispersion process in determining the price of the asset, we used a Student- t distribution.

This t distribution was approximated using the following approach. To obtain a fat-tailed distribution, we simulated a t-distribution with 5 degrees of freedom. This was achieved by taking 5 draws from a normal distribution that used the Box-Muller technique. The draws were squared and summed. This result was divided by five (5) and then the square root was taken. Finally, another standard draw from a normal distribution was taken and this was divided by this scaling factor. This final simulation resulted in a symmetrical distribution that had a mean insignificantly different from zero, somewhat higher than unit variance and the fat tails we are looking for. Theoretically, the expected kurtosis should be equal to nine (9). However, this will depend on the draw of random variables used to estimate the series. We drew eight series of random variables that yielded different kurtosis results. The first series had a kurtosis of 6.10 and will be referred to as T1. The second draw was approximately equal to the expected kurtosis of 9.16 and will be referred to as T2. The third draw yielded a kurtosis of 12.23 (which will be referred to as T3) and the final draw yielded a price series with a kurtosis of 15.26 (that will be referred to as T4).

One concern is that as with the selection of the normal distributions, the results may be subject to the choice of the random variables that are generating the Student-*t* distributions. Thus, we selected four additional Student-*t* distributions that all had similar levels of kurtosis. This are referred to as T5, T6, T7 and T8. For the purposes of comparison, all the eight Student-*t* distribution statistical moments are summarised in the following table, Table 3.7.

Moments	T1	T2	T3	T4	T5	T6	T7	T8
Mean	-0.04	-0.003	0.04	0.03	0.03	0.08	0.009	-0.02
Std. Dev.	1.280	1.297	1.317	1.303	1.264	1.223	1.296	1.224
Skewness	0.067	0.075	0.791	0.959	0.134	0.133	-0.079	-0.134
Kurtosis	6.10	9.15	12.23	15.26	4.79	4.54	4.89	4.58

Table 3.7, Sample Moments of Eight Student-t Distributions.

From the simulated distribution series, the means are all close to zero and the standard deviation is between 1.20 and 1.30 (as opposed to a standardised normal distribution with a standard deviation of 1.00). The skewness statistic is for the most part positive and increasingly so the greater the level of kurtosis. The kurtosis measure increases for the series T1 to T4 and is roughly the same for the series T5 to T8.

Student-t Test Results for Entire Period of Analysis

With these fat-tailed price series thus obtained, we now tested the hypothesis that financial futures are better described by a Student-*t* distribution with constant variance. As was indicated in Chapter one, this will also test the hypothesis that jump processes (which would be associated with such fat-tailed distributions) are crucial in explaining the dynamics of financial futures volatility. To test these joint hypotheses, we ran another series of simulations where the volatility was assumed to be constant but the price series follows a Student-*t* distribution. Thus, for each market and for each of the eight possible Student-*t* distributions, we reran the analysis and completed the minimised SSE comparisons. This can be seen in Tables 3.8a, 3.8b and 3.8c for the three categories of financial assets (and utilised the entire period of available observations for each market). For the sake of comparison, the GBM results for each market are also presented (*in Italics*).

Markets (T-dist.)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.9045**	**-0.0498**	**254.51**	**0.1387**	**0.0549**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*70.9208*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	54.8337
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	66.7552
T3	0.2891	0.0000	12.23	0.0085	-0.0026	29.7040
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	41.7646
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	58.2730
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	96.6956
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	62.6669
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	72.8226
FTSE	**0.4984**	**-0.0899**	**29.46**	**0.1428**	**0.0398**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*60.0982*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	29.0038
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	41.2433
T3	0.2891	0.0000	12.23	0.0085	-0.0026	12.5310
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	21.4832
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	30.9975
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	65.6446
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	34.7264
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	43.2353
DAX	**0.3391**	**-0.1740**	**5.70**	**0.0782**	**0.0207**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*22.6601*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	11.1503
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	20.8941
T3	0.2891	0.0000	12.23	0.0085	-0.0026	8.0903
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	7.8243
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	11.8606
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	39.4521
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	14.5965
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	20.7017
Nikkei	**0.4031**	**-0.1681**	**4.74**	**0.1557**	**0.0629**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*45.0403*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	21.8178
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	32.8385
T3	0.2891	0.0000	12.23	0.0085	-0.0026	17.8154
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	18.8386
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	23.2944
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	51.1647
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	26.1680
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	32.8182

Table 3.8a, Results for Student-t Distribution Models for Four Stock Index Futures Assuming Price Series the Variance is Constant.

Markets (T-dist.)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
Bund	**0.4519**	**-0.0814**	**6.87**	**0.1661**	**0.1009**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*65.9331*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	34.9203
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	48.9112
T3	0.2891	0.0000	12.23	0.0085	-0.0026	18.9436
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	29.7329
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	36.6975
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	71.5223
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	40.7052
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	49.0282
BTP	**0.4268**	**-0.1665**	**5.06**	**0.1638**	**0.0826**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*27.2153*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	24.7704
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	35.9979
T3	0.2891	0.0000	12.23	0.0085	-0.0026	20.4784
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	21.7445
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	26.4198
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	54.3309
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	29.3752
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	36.0666
Gilt	**0.3710**	**-0.1775**	**5.76**	**0.1116**	**0.0663**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*37.0817*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	15.7906
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	26.0301
T3	0.2891	0.0000	12.23	0.0085	-0.0026	13.1780
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	12.8546
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	16.9322
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	43.9928
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	19.7461
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	25.8520
US T-Bond	**0.4637**	**-0.0572**	**5.37**	**0.2080**	**0.1643**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*88.0371*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	53.0587
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	68.8317
T3	0.2891	0.0000	12.23	0.0085	-0.0026	34.1694
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	48.2561
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	55.2315
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	91.6486
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	59.6832
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	68.4477

Table 3.8b, Results for Student-t Distribution Models for Four Fixed Income Futures Assuming Price Series the Variance is Constant.

Markets (T-dist.)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
D Mark	**0.3415**	**-0.2432**	**5.45**	**0.0804**	**0.0139**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*22.3236*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	6.8676
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	14.3251
T3	0.2891	0.0000	12.23	0.0085	-0.0026	13.0748
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	5.0201
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	7.6299
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	29.5886
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	9.5371
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	14.3649
Pound	**0.4064**	**-0.1613**	**6.53**	**0.1140**	**0.0655**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*35.5084*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	17.7874
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	28.7720
T3	0.2891	0.0000	12.23	0.0085	-0.0026	13.2803
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	14.8546
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	18.8594
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	47.2573
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	21.8852
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	28.3522
Yen	**0.3590**	**-0.2792**	**7.79**	**0.0577**	**0.0087**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*17.2707*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	4.0627
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	9.8614
T3	0.2891	0.0000	12.23	0.0085	-0.0026	14.7800
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	2.3600
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	4.8110
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	23.8944
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	6.3119
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	10.3854
S-Franc	**0.3111**	**-0.2458**	**5.05**	**0.0660**	**0.0158**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*36.2483*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	5.6883
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	13.0135
T3	0.2891	0.0000	12.23	0.0085	-0.0026	12.4847
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	4.1261
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	6.2605
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	27.9575
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	8.1502
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	12.7547

Table 3.8c, Results for Student-t Distribution Models for Four Foreign Exchange Futures Assuming Price Series the Variance is Constant.

The general conclusion is that the inclusion of the fat-tailed distributions has led to an improvement relative to the previous GBM model (except t-distribution T6). Table 3.9 compares the GBM case to the best of the t-distributions.

Markets (Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.9045**	**-0.0498**	**254.50**	**0.1387**	**0.0549**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	70.9208
T3	0.2891	0.0000	12.23	0.0085	-0.0026	29.7040
FTSE	**0.4984**	**-0.0899**	**29.461**	**0.1428**	**0.0398**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	60.0982
T3	0.2891	0.0000	12.23	0.0085	-0.0026	12.5310
DAX	**0.3391**	**-0.1740**	**5.7004**	**0.0782**	**0.0207**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	22.6601
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	7.8243
Nikkei	**0.4031**	**-0.1681**	**4.74**	**0.1557**	**0.0629**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	45.0403
T3	0.2891	0.0000	12.23	0.0085	-0.0026	17.8154
Bund	**0.4519**	**-0.0814**	**6.8716**	**0.1661**	**0.1009**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	65.9331
T3	0.2891	0.0000	12.23	0.0085	-0.0026	18.9436
BTP	**0.4268**	**-0.1665**	**5.0590**	**0.1638**	**0.0826**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	48.1248
T3	0.2891	0.0000	12.23	0.0085	-0.0026	20.4784
Gilt	**0.3710**	**-0.1775**	**5.7580**	**0.1116**	**0.0663**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	37.0817
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	12.8546
US T-Bond	**0.4637**	**-0.0572**	**5.3650**	**0.2080**	**0.1643**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	88.0371
T3	0.2891	0.0000	12.23	0.0085	-0.0026	34.1694
D-mark	**0.3415**	**-0.2432**	**5.4460**	**0.0804**	**0.0139**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	22.3236
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	5.0201
Pound	**0.4064**	**-0.1613**	**6.53**	**0.1140**	**0.0655**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	35.5084
T3	0.2891	0.0000	12.23	0.0085	-0.0026	13.2803
Yen	**0.3590**	**-0.2792**	**7.7897**	**0.0577**	**0.0087**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	17.2707
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	2.3600
S-Franc	**0.3111**	**-0.2458**	**5.0450**	**0.0660**	**0.0158**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	36.2483
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	4.1261

Table 3.9, Comparisons of the Empirical Dynamics of Twelve Financial Futures Dispersion Processes with the Dynamics of a GBM Price Series with Constant Variance and the Best Student-t distribution with Constant Variance.

For all twelve markets, either Student-*t* distributions T3 or T4 were the optimal distributions. It is interesting to note that these two distributions have the highest kurtosis statistics of any of the series.

These results are hardly surprising. Bates (1991) and Naik and Lee (1990) indicate that the 1987 stock market crash was clearly an indication of a jump and both papers go on to develop models for pricing options on securities with nondiversifiable jump risks. In the currency markets, Bates (1996) could not reject the absence of jumps in data for the currency markets during the 1980s and Jorion (1988) found jumps in the US Dollar versus Deutsche Mark exchange rate in the 1970s.

Student-t Test Results for SubPeriods of Analysis

An important issue is whether these results are period specific. Therefore, to test this hypothesis, the analysis was rerun for the observation period split into two portions. The analysis of the first half of the available observations can be seen in Tables 3.10a, 3.10b and 3.10b.

Markets (T-dist.)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.9581**	**-0.0438**	**185.34**	**0.1247**	**0.0304**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*49.8419*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	34.3921
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	38.2731
T3	0.2891	0.0000	12.23	0.0085	-0.0026	21.1225
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	25.6531
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	37.0813
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	53.6804
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	38.9655
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	44.0655
FTSE	**0.5963**	**-0.0569**	**35.99**	**0.1615**	**0.0321**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*32.7907*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	20.5465
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	25.7418
T3	0.2891	0.0000	12.23	0.0085	-0.0026	10.3718
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	15.4610
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	22.3723
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	38.1241
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	24.0825
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	28.5711
DAX	**0.3612**	**-0.1740**	**6.17**	**0.0883**	**0.0153**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*12.8280*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	5.7743
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	10.0271
T3	0.2891	0.0000	12.23	0.0085	-0.0026	4.4459
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	4.5614
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	6.3753
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	17.9018
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	7.5079
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	10.3277
Nikkei	**0.3493**	**-0.1783**	**3.99**	**0.1458**	**0.0456**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*16.4677*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	9.7863
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	14.7138
T3	0.2891	0.0000	12.23	0.0085	-0.0026	9.0880
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	9.4645
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	10.5498
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	21.7733
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	11.6829
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	14.5607

Table 3.10a, Results for Student-t Distribution Models for Four Stock Index Futures Assuming Price Series the Variance is Constant for the First Half of the Available Observations.

Markets (T-dist.)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
Bund	**0.5175**	**-0.0065**	**9.5135**	**0.1890**	**0.1103**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*37.5815*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	24.8480
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	33.0390
T3	0.2891	0.0000	12.23	0.0085	-0.0026	14.4586
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	22.6986
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	27.3637
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	44.3793
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	29.4599
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	34.0004
BTP	**0.5193**	**-0.1500**	**6.89**	**0.1971**	**0.0833**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*27.2153*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	18.5938
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	23.9204
T3	0.2891	0.0000	12.23	0.0085	-0.0026	15.3758
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	16.9749
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	20.0642
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	32.2760
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	21.4138
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	24.9040
Gilt	**0.3823**	**-0.1535**	**5.34**	**0.1228**	**0.0784**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*17.7907*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	10.2935
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	15.3748
T3	0.2891	0.0000	12.23	0.0085	-0.0026	8.0175
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	9.3950
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	11.1236
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	23.1141
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	12.4660
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	15.4176
US T-Bond	**0.4863**	**-0.0399**	**4.54**	**0.2349**	**0.1845**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*45.9846*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	35.6233
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	43.3963
T3	0.2891	0.0000	12.23	0.0085	-0.0026	26.9706
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	34.2772
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	37.1946
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	52.7146
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	39.2502
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	43.0655

Table 3.10b, Results for Student-t Distribution Models for Four Fixed Income Futures Assuming Price Series the Variance is Constant for the First Half of the Available Observations.

Markets (T-dist.)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
D Mark	**0.3309**	**-0.3000**	**5.36**	**0.0711**	**-0.0114**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*6.3656*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	2.1262
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	4.5387
T3	0.2891	0.0000	12.23	0.0085	-0.0026	7.9757
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	2.1549
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	2.5647
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	9.8710
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	3.0408
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	4.8131
Pound	**0.3721**	**-0.1900**	**6.18**	**0.0917**	**0.0335**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*12.8628*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	6.0568
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	10.5300
T3	0.2891	0.0000	12.23	0.0085	-0.0026	6.2353
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	5.8328
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	6.5149
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	17.3894
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	7.6246
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	10.2354
Yen	**0.3383**	**-0.4014**	**7.10**	**0.0495**	**0.0005**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*3.2769*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	0.8417
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	1.6302
T3	0.2891	0.0000	12.23	0.0085	-0.0026	12.1470
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	1.4303
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	1.3284
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	5.1646
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	1.3514
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	2.2251
S-Franc	**0.3082**	**-0.2748**	**5.03**	**0.0665**	**0.0024**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*6.7588*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	2.1932
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	5.0413
T3	0.2891	0.0000	12.23	0.0085	-0.0026	6.8104
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	2.2300
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	2.5668
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	10.6567
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	3.1903
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	5.0640

Table 3.10c, Results for Student-t Distribution Models for Four Foreign Exchange Futures Assuming Price Series the Variance is Constant for the First Half of the Available Observations.

For the most part, the inclusion of the fat-tailed distributions has improved the fit of the model to the five attributes relative to the case of geometric Brownian motion. Table 3.11 compares the GBM case to the best of the t-distributions.

Markets (Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.9581**	**-0.0438**	**185.34**	**0.1247**	**0.0304**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	49.8419
T3	0.2891	0.0000	12.23	0.0085	-0.0026	21.1225
FTSE	**0.5963**	**-0.0569**	**35.99**	**0.1615**	**0.0321**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	32.7907
T3	0.2891	0.0000	12.23	0.0085	-0.0026	10.3718
DAX	**0.3612**	**-0.1740**	**6.1734**	**0.0883**	**0.0153**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	12.8280
T3	0.2891	0.0000	12.23	0.0085	-0.0026	4.4459
Nikkei	**0.3493**	**-0.1783**	**3.9912**	**0.1458**	**0.0456**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	16.4677
T3	0.2891	0.0000	12.23	0.0085	-0.0026	9.0880
Bund	**0.5175**	**-0.0065**	**9.5135**	**0.1890**	**0.1103**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	37.5815
T3	0.2891	0.0000	12.23	0.0085	-0.0026	14.4586
BTP	**0.5193**	**-0.1500**	**6.8939**	**0.1971**	**0.0833**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	27.2153
T3	0.2891	0.0000	12.23	0.0085	-0.0026	15.3758
Gilt	**0.3823**	**-0.1535**	**5.3372**	**0.1228**	**0.0784**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	17.7907
T3	0.2891	0.0000	12.23	0.0085	-0.0026	8.0175
US T-Bond	**0.4863**	**-0.0399**	**4.5418**	**0.2349**	**0.1845**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	45.9846
T3	0.2891	0.0000	12.23	0.0085	-0.0026	26.9706
D-mark	**0.3309**	**-0.3000**	**5.3573**	**0.0711**	**-0.0114**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	6.3656
T1	0.2400	-0.3925	6.10	0.0045	0.0023	2.1262
Pound	**0.3721**	**-0.1900**	**6.18**	**0.0917**	**0.0335**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	12.8628
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	5.8328
Yen	**0.3383**	**-0.4014**	**7.1017**	**0.0495**	**0.0005**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	3.2769
T1	0.2400	-0.3925	6.10	0.0045	0.0023	0.8417
S-Franc	**0.3082**	**-0.2748**	**5.0305**	**0.0665**	**0.0024**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	6.7588
T1	0.2400	-0.3925	6.10	0.0045	0.0023	2.1932

Table 3.11, Comparisons of the Empirical Dynamics of Twelve Financial Futures Dispersion Processes with the Dynamics of a GBM Price Series with Constant Variance and the Best t-distribution with Constant Variance, for the First Half of the Available Observations.

For the first period, the best Student-*t* distribution for most of the markets remained the T3 distribution (kurtosis of 12.23). For the currencies, three of the four markets were best fit by the T1 distribution (kurtosis of 6.10). The analysis for the second half of the observations can be seen in Tables 3.12a, 3.12b and 3.12c.

Markets (T-dist.)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.3192**	**-0.2535**	**5.88**	**0.0596**	**0.0224**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*49.0282*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	12.4521
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	28.8889
T3	0.2891	0.0000	12.23	0.0085	-0.0026	35.7603
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	25.5149
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	16.2858
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	57.3784
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	20.0671
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	31.2111
FTSE	**0.3119**	**-0.1856**	**4.52**	**0.0888**	**0.0430**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*62.0476*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	27.3332
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	53.6580
T3	0.2891	0.0000	12.23	0.0085	-0.0026	41.4783
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	48.8945
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	28.8973
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	79.4517
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	34.4323
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	47.3464
DAX	**0.2824**	**-0.1274**	**4.21**	**0.0481**	**0.0161**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*64.0125*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	28.4112
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	58.5486
T3	0.2891	0.0000	12.23	0.0085	-0.0026	31.0492
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	51.1586
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	27.6476
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	87.1313
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	34.2595
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	48.1721
Nikkei	**0.4381**	**-0.1056**	**5.87**	**0.1383**	**0.0704**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*125.2491*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	67.0493
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	93.5484
T3	0.2891	0.0000	12.23	0.0085	-0.0026	48.3928
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	68.6718
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	74.8994
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	139.3796
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	82.7041
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	102.2892

Table 3.12a, Results for Student-t Distribution Models for Four Stock Index Futures Assuming Price Series the Variance is Constant for the Second Half of the Available Observations.

Markets (T-dist.)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
Bund	**0.3818**	**-0.1004**	**4.66**	**0.1329**	**0.0805**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*110.5935*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	62.3857
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	94.0839
T3	0.2891	0.0000	12.23	0.0085	-0.0026	51.4817
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	76.2243
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	66.6627
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	130.6208
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	74.5850
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	92.3330
BTP	**0.3176**	**-0.2600**	**3.94**	**0.1014**	**0.0109**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*46.4193*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	21.9242
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	45.5553
T3	0.2891	0.0000	12.23	0.0085	-0.0026	58.0725
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	50.7561
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	22.4005
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	61.7353
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	26.1278
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	36.7198
Gilt	**0.3546**	**-0.1602**	**6.25**	**0.0950**	**0.0504**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*83.6713*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	34.2612
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	56.2471
T3	0.2891	0.0000	12.23	0.0085	-0.0026	30.9329
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	39.9800
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	40.1223
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	95.7030
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	46.2131
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	61.8273
US T-Bond	**0.3133**	**-0.1906**	**5.8301**	**0.0641**	**0.0348**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*61.9444*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	20.4269
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	41.4303
T3	0.2891	0.0000	12.23	0.0085	-0.0026	28.6058
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	32.1379
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	24.0482
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	74.5533
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	29.2832
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	42.4454

Table 3.12b, Results for Student-t Distribution Models for Four Fixed Income Futures Assuming Price Series the Variance is Constant for the Second Half of the Available Observations.

Markets (T-dist.)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
D Mark	**0.3515**	**-0.1570**	**5.5237**	**0.0884**	**0.0379**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*77.8149*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	32.0372
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	55.9770
T3	0.2891	0.0000	12.23	0.0085	-0.0026	31.5012
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	42.2019
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	36.3953
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	92.3542
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	42.5531
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	57.9384
Pound	**0.4354**	**-0.0846**	**6.8103**	**0.1308**	**0.0937**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*138.0695*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	73.8484
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	99.4308
T3	0.2891	0.0000	12.23	0.0085	-0.0026	46.0396
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	69.5594
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	83.1021
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	150.7592
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	91.4571
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	111.7285
Yen	**0.3794**	**-0.1876**	**8.5897**	**0.0657**	**0.0110**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*85.3851*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	27.7165
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	40.7714
T3	0.2891	0.0000	12.23	0.0085	-0.0026	20.8299
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	19.7352
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	36.5883
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	89.5831
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	41.6962
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	57.7572
S-Franc	**0.3148**	**-0.1745**	**5.0631**	**0.0655**	**0.0318**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*62.3183*
T1	0.2400	-0.3925	6.10	0.0045	0.0023	23.2820
T2	0.2601	-0.4889	9.16	-0.0067	-0.0047	47.5528
T3	0.2891	0.0000	12.23	0.0085	-0.0026	31.2467
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	39.4081
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	25.5508
T6	0.1986	-0.6300	4.54	-0.0041	0.0062	78.2949
T7	0.2064	-0.4241	4.89	-0.0026	-0.0031	31.2269
T8	0.1801	-0.4802	4.58	-0.0102	0.0018	44.6914

Table 3.12c, Results for Student-t Distribution Models for Four Foreign Exchange Futures Assuming Price Series the Variance is Constant for the Second Half of the Available Observations.

Again, the inclusion of the fat-tailed distributions has improved the fit of the model to the five attributes relative to the case of geometric Brownian motion. Table 3.13 compares the GBM case to the best of the t-distributions for the latter period.

Markets (Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.3192**	**-0.2535**	**5.8823**	**0.0596**	**0.0224**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	49.0282
T1	0.2400	-0.3925	6.10	0.0045	0.0023	12.4521
FTSE	**0.3119**	**-0.1856**	**4.5168**	**0.0888**	**0.0430**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	62.0476
T1	0.2400	-0.3925	6.10	0.0045	0.0023	27.3332
DAX	**0.2824**	**-0.1274**	**4.2104**	**0.0481**	**0.0161**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	64.0125
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	27.6476
Nikkei	**0.4381**	**-0.1056**	**5.8671**	**0.1383**	**0.0704**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	125.2491
T3	0.2891	0.0000	12.23	0.0085	-0.0026	48.3928
Bund	**0.3818**	**-0.1004**	**4.6621**	**0.1329**	**0.0805**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	110.5935
T3	0.2891	0.0000	12.23	0.0085	-0.0026	51.4817
BTP	**0.3176**	**-0.2600**	**3.9391**	**0.1014**	**0.0109**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	46.4193
T1	0.2400	-0.3925	6.10	0.0045	0.0023	21.9242
Gilt	**0.3546**	**-0.1602**	**6.2497**	**0.0950**	**0.0504**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	83.6713
T3	0.2891	0.0000	12.23	0.0085	-0.0026	30.9329
US T-Bond	**0.3133**	**-0.1906**	**5.8301**	**0.0641**	**0.0348**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	61.9444
T1	0.2400	-0.3925	6.10	0.0045	0.0023	20.4269
D-mark	**0.3515**	**-0.1570**	**5.5237**	**0.0884**	**0.0379**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	77.8149
T3	0.2891	0.0000	12.23	0.0085	-0.0026	31.5012
Pound	**0.4354**	**-0.0846**	**6.8103**	**0.1308**	**0.0937**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	138.0695
T3	0.2891	0.0000	12.23	0.0085	-0.0026	46.0396
Yen	**0.3794**	**-0.1876**	**8.5897**	**0.0657**	**0.0110**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	85.3851
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	19.7352
S-Franc	**0.3148**	**-0.1745**	**5.0631**	**0.0655**	**0.0318**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	62.3183
T1	0.2400	-0.3925	6.10	0.0045	0.0023	23.2820

Table 3.13, Comparisons of the Empirical Dynamics of Twelve Financial Futures Dispersion Processes with the Dynamics of a GBM Price Series with Constant Variance and the Best t-distribution with Constant Variance for the Second Half of the Available Observations.

In the second period, the optimal Student-*t* distribution now varies somewhat from either the first period or the overall period. Once again the T1 and T3 distributions are the most common distributions which best explain the twelve markets. Two stock index futures are now better fit by the T1 distribution, as is the

BTP, US T-Bond and Swiss Franc. The T3 distribution is the best for another five financial futures. Thus, it would appear that since the same asset class will require different Student-*t* distributions at different times, we can conclude that the kurtosis of markets is consistently leptokurtic but the degree of excess kurtosis varies over time.

In the latter period and for each of the twelve markets, the inclusion of an appropriate fat-tailed distribution will reduce the sum of the squared errors substantially relative to the case of the GBM model. In fact, of the 36 comparisons being made (the entire period, first and second sub periods for the 12 markets), in every case, the inclusion of a fat-tailed distribution improves the fit. The principal reasons for the improvement in the fit is due to the increased coefficient of variation (COV) condition and the increased kurtosis. A review of Tables 3.9, 3.11 and 3.13 indicates that the fat-tailed model does not address the autocorrelation conditions or the time (decay) factor of the unbiased standard deviations of the volatility.

3.5 CONCLUSION

In this Chapter, we have examined whether the time series of returns for twelve financial futures markets can be understood by security price process models that assume constant variance. Rather than accepting or rejecting these models based upon a single factor (such as excess kurtosis or autocorrelations in the absolute returns), we have examined a number of factors that address most of the divergences from normality pointed out in the literature. As a result of this analysis, we can identify exactly what aspects of non-normality exist for the empirical return series and which of these factors the models may address.

Based on our analysis, we reject the hypothesis that the twelve financial futures markets return series follow Geometric Brownian motion. For each of the attributes, the actual return series show behaviours that do not conform to the dynamics of a simulated time series of returns that conforms to Geometric Brownian motion. This has important implications for the understanding of options values.

Given that many of the popular option pricing models [such as Black and Scholes (1973)] assume the asset return series conforms to GBM, this would suggest that these models do not conform to the empirical record. This result has profound implications for those who price options. Consider option market makers that assumed that the conditional dispersion process for the underlying asset price followed GBM for this period by using the Black Scholes formula. Then, over the life of the option, the actual dispersion processes of these markets did not conform to this assumption. This would imply that the prices for these options would have been ex ante incorrect. Therefore, options prices (for these twelve markets) would not be well understood by a Geometric Brownian Motion model (for this period of analysis).

An alternative model for security price processes assumes that returns follow a Student-t distribution. This approach would be consistent with the jump-process model proposed by Merton (1976). When we compared the dynamics of the objective process for the twelve markets to eight series of simulated prices that were generated with such a distributional form, we observed that this model also failed to address many of the dynamics of the unconditional return series. It was shown that these models could explain two of the attributes (coefficient of variation of 20-day return volatility and the excess kurtosis) but failed to address the other three attributes (autocorrelations of absolute returns and the time decay factor of the volatility of volatility). Thus, it would appear that the jump-process model for option pricing also fail to conform to the empirical return series. Simply said, if this model cannot capture the dynamics of the securities returns, we cannot expect it to capture the dynamics of contingent claims based upon these same securities.

While it is clear that progress has been made in addressing the kurtosis and COV conditions, it is clear that we must evaluate models that can also address the other attributes that capture the dynamics of the objective process. We must examine models that explain the existence of positive autocorrelations in the absolute returns and allow us to capture the slow rate of decay of the volatility of volatility that is observed empirically. Such a family of models (which should address these conditions) are the stochastic volatility models outlined in Chapter one. These will be examined in the next two Chapters.

CHAPTER FOUR
THE ANALYSIS OF OBJECTIVE PROBABILITIES IN FUTURES MARKETS: EXPLAINING THE EMPIRICAL DYNAMICS WITH MODELS ASSUMING STOCHASTIC VARIANCE

4.1 INTRODUCTION

In the previous Chapter, we have developed a methodology to test how well models with constant volatility explain the dynamics of empirical volatility for twelve financial futures markets. This process required the selection of attributes that were measurable, meaningful and independent gauges of volatility dynamics. Then, we minimised the sum of the standardised squared errors for the attributes generated by the model compared to the empirical attributes.

The conclusions are that none of the twelve financial futures markets is well described by a geometric Brownian motion model. A better fit could be obtained by the inclusion of a Student-t distribution. However, the fat-tailed distribution failed to explain three of the attributes; the time decay of the standard deviation of the volatility and the average autocorrelations of the absolute returns.

In this Chapter, we will examine an alternative family of security price process models that could potentially explain all the five attributes (and thus the dynamics of empirical volatility). These are the stochastic volatility models. We will continue the analysis completed for the previous Chapter. This will entail generating simulated price series that are consistent with the potential stochastic volatility models we will examine. In this Chapter, a significant difference is that we will employ an optimisation technique for the selection of the optimal parameters for each of the selected stochastic volatility models.

4.2 STOCHASTIC VOLATILITY MODELS USED IN THE ANALYSIS

As was discussed in Chapter One, we have restricted our analysis to three models. These include:

(1) The Hull & White approach (1987a) assumes that the variance follows a lognormal diffusion process and the square root of this process will provide the volatility parameter. This process can be written as:

$$d\sigma = k(\theta - \sigma)dt + \xi\sigma \cdot dZ_1 \quad (4.1)$$

It should be noted that all the variables are defined in Chapter One and remain the same in this Chapter. While it is possible to model the stochastic volatility using the variance transformation suggested by Hull & White (1987a):

$$V_i = V_{i-1} \cdot e^{[k(\theta-\sigma)-\xi^2/2]dt+\xi \cdot dZ_1} \quad (4.2)$$

Then, to determine the volatility estimate, σ, one could simply estimate $\sqrt{V_i}$. To confirm the accuracy of our computer code, we compared the results to a simple Euler approach with the same random variables and input parameters.

$$\sigma_{t+1} = \sigma_t + d\sigma \quad (4.3)$$

There, the volatility tomorrow is equal to the discrete change in the volatility from today with the addition of the volatility change projected by equation (4.1).

In this simulation, time is expressed as the percentage that one day represents in a trading year (of 252 days). The results of a comparison of two series of volatilities generated using formulae 4.2 and 4.3 were almost identical (out to the fourth decimal place and using the same series of draws from the normal distribution function). This test confirmed the accuracy of the computer code for both simulations and given the ease and accuracy of the Euler approach, this approach was used for the analysis. To save space, all simulations of this model will be referred to as "H&W".

(2) Alternatively, the Stein and Stein (1991) approach assumes that the volatility (absolute level) follows the Ornstein-Uhlenbeck (O-U) process. This model has the same functional form (seen as an equivalent) for volatilities as the Vasicek (1977) model has for interest rates. This process can be written as:

$$d\sigma = -k(\sigma - \theta)dt + \xi \cdot dZ_1 \qquad \textbf{(4.4)}$$

A problem exists when simulating the O-U process. At first glance, the O-U process should be straightforward to simulate given that at any given point in time it is similar to a normal distribution. The difference for the O-U process is that time is measured as an exponential factor. For both a unit normal distribution [N(0,1)] and an O-U process, if we assume geometric Brownian motion, the initial estimation of the instaneous change will be identical. However, in the next unit of time, the O-U process no longer is distributed as a unit normal distribution. Therefore, to simulate the process correctly, we must substitute the expected mean and variance of the O-U process. The expected mean can be expressed as:

$$E_{T_0}[\tilde{\sigma}_T] = \mu_T = \sigma_{T_0} \cdot e^{-\kappa(T_1 - T_0)} + \theta[1 - e^{(-\kappa(T_1 - T_0))}] \qquad \textbf{(4.5)}$$

and the expected variance can be expressed as:

$$Var_{T_0}[\tilde{\sigma}_T] = \xi^2 / 2\kappa \cdot [1 - e^{-(2\kappa(T_1 - T_0))}] \qquad \textbf{(4.6)}$$

In our case, T_1-T_0 is equal to 1/252 (which is the percentage that one day represents in our assumed trading year). This time increment was chosen because it is the same time increment used for the empirical estimation of the volatility cones presented previously. With the modified mean and variance, we have all the elements required to simulate the O-U process. The mean of the process as well as the standard deviation can be estimated from the above formula. To generate the simulated volatility series, we simply apply the following formula:

$$\hat{\sigma}_T = \mu_T + [\sqrt{\tilde{\sigma}_{T-1}}^2 \cdot N(0,1)] \tag{4.7}$$

Once we have estimated the mean using the previous simulated volatility estimate, we then determine the standard deviation of the volatility process and multiply this by a standard unit normal distribution function that used a Box-Muller approach.

As with the Vasicek model, where a possibility exists of negative interest rates, there is a possibility of negative volatilities. To correct for this problem, we used the principle of reflection about the origin to assure if any resulting volatility was negative, the absolute value was taken. Another approach to correct for possible negative results is to take the square of the Ornstein-Uhlenbeck process and hence is reflected at zero. Perraudin and Sørensen (1997) point out that for some parameter values, this approach is exactly the same as the Heston (1993) approach in which the instantaneous variance is a square-root process.[1] Thus, to provide some differentiation between the two models, we chose to reflect the estimated volatility at the origin rather than square the volatility and thus be redundant with the Heston model.

To make sure the model was correctly coded we then estimated the volatility series for the same series of random variables and the same parameters using a simple Euler approach. The results were almost identical (out to four decimal places) which confirms the accuracy of the computer coding. As with the Hull & White (1987a) model, for ease of estimation, we chose to use the Euler approach rather than use the O-U approach. As before, this approach used:

$$\sigma_{t+1} = \sigma_t + d\sigma \tag{4.8}$$

[1] If one applies Ito's lemma, it can be shown for some parameter values that squaring an Ornstein-Uhlenbeck process yields a square-root process.

There, the volatility tomorrow is equal to the discrete change in the volatility from today with the addition of the volatility change projected by equation (4.4). Tests of this model will be referred to as the "S&S" model.

(3) Heston (1993) proposes another stochastic volatility model that states that the variance follows the following process:

$$d\sigma^2 = k(\theta - \sigma^2)dt + \xi\sigma \cdot dZ_1 \tag{4.9}$$

Taylor (1994) described this in terms of the volatility process (page 186) as:

$$d\sigma = \frac{1}{\sigma}(\theta - k\sigma^2)dt + \xi \cdot dZ_1 \tag{4.10}$$

When these simulations were run, we then applied a simple Euler method to estimate the volatility series. We found that the results were identical out to the third decimal place compared to the Bessel approach suggested by Cox, Ingersoll and Ross (1985). Thus, for all of our simulations, we applied the Euler approach to estimate the volatility series generated by this model. Again, this model applied the following formula:

$$\sigma_{t+1} = \sigma_t + d\sigma \tag{4.11}$$

There, the volatility tomorrow is equal to the discrete change in the volatility from today with the addition of the volatility change projected by equation (4.10). Tests of this model will be referred to at the "HES" model.

For all of these models, κ indicates the degree of mean reversion, σ is the observed unconditional volatility, θ is the long-term conditional (instaneous) volatility and ξ is the volatility of the volatility. These variables are the same as the ones used in Chapter One.

As previously discussed, we have identified what we consider to be the key attributes that describe the empirical distribution of the financial futures markets.

With these observed attributes, we can now test the three alternative stochastic volatility models and find those models (and appropriate parameters) that minimise the difference for all these attributes between the models and the markets. For these models we have three unknown parameters: κ, the degree of mean reversion, θ, the long-term conditional volatility and ξ, the volatility of the volatility. The unconditional volatility, σ, was set equal to the long-term volatility at the initial of the simulation.

As with the test performed for normality, we will continue to use the method of minimising the sum of squared differences between the attributes generated by the models and the attributes associated with the markets. In addition, these tests will provide useful feedback as to which attributes the models are able to explain and exactly why they fail.

This approach is similar in spirit to research on models of the short-term interest rate completed by Chan, Karolyi, Longstaff and Sanders (1992). In their research, they examined all eight alternative models that had been proposed for modelling short-term interest rates. By using a similar Generalised Method of Moments estimation technique, they were able to find the models that most successfully capture the dynamics of the short-term interest rate. This also is consistent with the approach used by Ho, Perraudin and Sørensen (1996) where they formulated and tested a pricing model for equities that included both random jumps and stochastic volatility. This research will include both elements of these papers. We will also test for models that include stochastic volatility alone and later incorporate jumps in a similar spirit to the latter paper. We differ from that paper, in that we will also assess which stochastic volatility models are more consistent with the dynamics of volatility (both alone and combined with jumps). Thus, using all the relevant categories of stochastic volatility models, we will simulate volatility processes. Then we will compare these simulated processes with the empirical results. This will also allow us to demonstrate which models and which parameters are best in describing the dynamics of the volatility process.

4.3 CALIBRATION OF STOCHASTIC VOLATILITY MODELS

A key problem in the use of the stochastic volatility model is the calibration of the input parameters into the model. Specifically, we are interested in determining both the rate of mean reversion for the volatility process and the variance of the volatility itself. Scott (1994) estimates the latter by choosing parameters such that the selected moments from the model are close to the sample moments computed from empirical asset price changes. We will employ a similar approach. The difference is that we will run simulations of the models and then optimise the simulations to fit the observed empirical moments.

To test the degree that these models fit the empirical cone data and the unbiased standard deviations of the volatility we were able to determine, the three models were simulated to generate a series of 2000 random daily volatilities. These volatilities were then used as input into a standard asset pricing model of the usual form:

$$dS = S\mu \cdot dt + S\hat{\sigma} \cdot dZ_1 \qquad \textbf{(4.12)}$$

As with the previous test for assessing if the financial futures prices follow a lognormal dispersion process with constant variance, we will assume the interest rates are zero. Again, this is reasonable since we are assuming the price series are futures prices and can adjust the usual drift terms.

The same formula (as equation (3.2)) was used for the generation of the price series when testing whether the twelve financial futures markets followed the assumption of GBM with constant variance. For these simulations, two alternations were made. The first is that the term $\hat{\sigma}$, reflects the volatilities estimated from the various models tested and the previous day's volatility estimate is used to estimate today's new asset price. The second modification was for all of these simulations, one series of random normal distributions were used. This series was the one selected previously as being the closest to the average results (and theoretical expectations) for 100 simulations of random prices. This was to allow for direct comparison of the models without having the water muddied due to the random nature of the normal distribution generation function.[2]

With the series of 2000 prices determined these prices were imported into the programmes used to describe the empirical dynamics of the twelve financial futures markets. These programmes included the determination of the kurtosis of each series, the volatility cone analysis and the estimation of the autocorrelograms. For each of the three stochastic volatility models, 200 simulations were run with a variety of different parameter inputs. These results were run through the MSSE technique to select the best performing models. Thereafter, an optimisation technique was utilised to find the mix of parameters that minimised the sum of the standardised squared differences.

Optimisation was achieved by comparing the best performing model to models that varied each of the three parameters. Given that we allowed the parameters to vary up and down by small increments, there were eight possible candidates. We found that

[2] This may introduce biases. To test for this a trial run was completed for one of the simulations with another set of randomly chosen normal distributions and the results were within the range of variation that would be expected from the sampling error of the 100 simulations run previously.

the key factors were the rate of reversion, κ, and the volatility of the volatility, ξ. The long-term volatility, θ, had almost no effect. Thus, our analysis only varied κ and ξ. This meant that we were searching for the better fitting model from a neighbouring square. For each simulation, we varied the two parameters and if one of the corners of the neighbouring square (four alternative models) was a better fit, this would replace the initial best fitting model. The search routine continued looking at the neighbouring square (a new set of four alternative models) until none of the new four models yielded a better result. The search first used fairly high increments in the search (for example 1.0 for κ and 0.1 for ξ) and then we progressively reduced the increments to find the optimal solution (for example, as low as 0.01 for κ and 0.0001 for ξ).

For the Hull & White (1987a) model, we had to input four parameters, namely the current volatility, σ, the long-term volatility, θ, the rate of reversion, κ and the volatility of the volatility, ξ. For the H&W simulations, the initial seed parameter values were taken from Hull & White (1987a). Using their values, it became clear that their choice of parameter values could lead to difficulties. For a number of simulations, it became impossible to arrive at a reasonable result. The reason was that the simulated volatility series would fall to zero and remain at this level for the remainder of the simulation. This was due to the volatility of the volatility parameter being too large. Hull and White (1987a) indicate this problem with their model (when they were pricing options) and they indicate that if the volatility of the volatility parameter is too large, the series will not converge properly. We found that an upper limit of the volatility of volatility parameter exists, beyond which the model ceases to work properly. From the Hull & White formula, we can determine the critical level by setting the first term of the equation equal to the second term. When this is done, it is trivial to show that:

$$\xi \leq \sqrt{2\kappa\theta} \quad (4.13)$$

This states that the maximum allowable volatility of volatility must be equal to the square root of two times the mean reversion factor times the long-term volatility. With this result, we were able to restrict our selection of parameters to provide meaningful results. After 200 simulations were completed, we selected the best 8 models using the MSSE criteria. These input parameters are presented in Tables 4.1.

Simulation Run	σ	θ	κ	ξ
H&W1	0.20	0.20	16	0.63
H&W2	0.20	0.20	16	0.32
H&W3	0.20	0.20	8	0.45
H&W4	0.15	0.15	16	0.55
H&W5	0.15	0.15	16	0.27
H&W6	0.15	0.15	8	0.39
H&W7	0.15	0.15	2	0.39
H&W8	0.10	0.10	16	0.45

Table 4.1, Initial Parameter Values for Hull & White Simulations

In the spirit of the Stein and Stein (1991) paper [pages 736-737] various seed values were input which would characterise different asset classes. While Stein and Stein ran simulations for stock indices and individual stocks, we only drew upon the parameters for the stock index futures we were examining. Once again 200 simulations were run and the eight best are presented in Table 4.2.

Simulation Run	σ	θ	κ	ξ
S&S1	0.20	0.20	4	0.15
S&S2	0.20	0.20	8	0.15
S&S3	0.20	0.20	16	0.30
S&S4	0.15	0.15	4	0.1150
S&S5	0.15	0.15	4	0.1125
S&S6	0.05	0.05	4	0.025
S&S7	0.05	0.05	4	0.0375
S&S8	0.20	0.20	2	0.15

Table 4.2, Initial Parameter Values for Stein & Stein Simulations

The final approach to modelling the stochastic volatility was the use of the Heston model. The parameter values were similar to those used by Hull & White

(1988) and Heston (1993). As with the previous approaches 200 simulations were run and the best eight models are presented in Table 4.3. These parameters were:

Simulation Run	σ	θ	κ	ξ
HES1	0.20	0.20	4	0.30
HES2	0.20	0.20	8	0.30
HES3	0.20	0.20	8	0.60
HES4	0.15	0.15	4	0.225
HES5	0.15	0.15	8	0.45
HES6	0.15	0.15	4	0.1125
HES7	0.10	0.10	4	0.15
HES8	0.05	0.05	2	0.0375

Table 4.3, Initial Parameter Values for Heston Simulations

4.4 ANALYSIS OF STOCHASTIC VOLATILITY MODELS (ENTIRE PERIOD)

Optimal Parameters for the Three Stochastic Volatility Models (Entire Period)

This yielded 24 potential candidates to explain the empirical dynamics for the twelve financial futures markets. After applying the optimisation search method outlined above, we selected the best models for each of the twelve financial futures. In the following Table, we present the input parameters for the best fitting models.

Market / Model	σ	θ	κ	ξ
S&P 500				
H&W	0.20	0.20	18.97	1.530
S&S	0.15	0.15	5.01	0.853
HES	0.20	0.20	3.00	0.56
FTSE				
H&W	0.15	0.15	16.7	1.04
S&S	0.20	0.20	2.96	0.25
HES	0.20	0.20	2.40	0.482
DAX				
H&W	0.20	0.20	19.6	0.865
S&S	0.15	0.15	5.72	0.1805
HES	0.15	0.15	7.55	0.422
Nikkei 225				
H&W	0.20	0.20	15.82	1.124
S&S	0.15	0.15	3.81	0.2155
HES	0.20	0.20	3.41	0.461

Market / Model	σ	θ	κ	ξ
Bund				
H&W	0.10	0.10	15.02	0.949
S&S	0.20	0.20	1.62	0.220
HES	0.10	0.10	2.79	0.311
BTP				
H&W	0.20	0.20	15.28	1.161
S&S	0.05	0.05	3.63	0.0745
HES	0.20	0.20	2.43	0.463
Gilt				
H&W	0.20	0.20	15.54	0.937
S&S	0.15	0.15	3.55	0.1675
HES	0.05	0.05	2.33	0.0805
US T-Bond				
H&W	0.15	0.15	1.48	0.878
S&S	0.20	0.20	0.84	0.236
HES	0.20	0.20	1.01	0.381
D Mark				
H&W	0.20	0.20	19.58	0.736
S&S	0.05	0.05	4.15	0.041
HES	0.05	0.05	7.43	0.125
Pound				
H&W	0.20	0.20	16	0.980
S&S	0.20	0.20	2.93	0.211
HES	0.20	0.20	2.86	0.374
Yen				
H&W	0.20	0.20	16.29	0.484
S&S	0.20	0.20	8.17	0.249
HES	0.20	0.20	10.6	0.594
S-Franc				
H&W	0.20	0.20	19.53	0.670
S&S	0.15	0.15	4.41	0.1175
HES	0.15	0.15	5.72	0.277

Table 4.4, Parameter values for the Best of the Three Stochastic Volatility Models for the Twelve Financial Futures Markets.

For all the models, most of the parameter values are similar across the different markets. The exception is for the US T-Bond that has an extremely low mean reversion parameter. Furthermore, the S&S and HES parameters seem to be related. One will observe that the volatility of volatility parameter of the HES model approximately twice the S&S parameter for the same market. This is to be expected [see Perraudin and Sørensen (1997) referred to previously].

Results for the Three Stochastic Volatility Models (Entire Period)

The results of the optimisation appear in Tables 4.5a, 4.5b and 4.5c. For the sake of comparisons, the GBM model appears in Italics.

Markets (Best Model)	Coefficient Of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.9045**	**-0.0498**	**254.50**	**0.1387**	**0.0549**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*70.9208*
H&W	0.4399	-0.1198	4.91	0.2242	0.0692	21.5903
S&S	0.4711	-0.2143	5.05	0.2203	0.0293	20.3151
HES	0.4606	-0.1316	4.62	0.2579	0.1128	22.9015
FTSE	**0.4984**	**-0.0899**	**29.461**	**0.1428**	**0.0398**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*60.0982*
H&W	0.3477	-0.1097	3.935	0.1611	0.0692	3.9898
S&S	0.3370	-0.1380	3.63	0.1562	0.0704	4.6166
HES	0.3911	-0.1250	4.03	0.2044	0.0875	5.8586
DAX	**0.3391**	**-0.1740**	**5.7004**	**0.0782**	**0.0207**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*22.6601*
H&W	0.2532	-0.1865	3.13	0.0907	0.0342	0.6909
S&S	0.2849	-0.2203	3.25	0.1138	0.0303	1.1510
HES	0.2858	-0.2579	3.27	0.1130	0.0184	2.0412
Nikkei	**0.4031**	**-0.1681**	**4.74**	**0.1557**	**0.0629**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*45.0403*
H&W	0.3427	-0.1279	3.86	0.1580	0.0591	0.4476
S&S	0.3448	-0.1608	3.66	0.1619	0.0599	0.2007
HES	0.3962	-0.1189	4.15	0.1985	0.0897	0.2358

Table 4.5a, Best Fitting Models for Four Stock Index Futures Assuming Price Series are Lognormally Distributed and the Variance is Stochastic.

Markets (Best Model)	Coefficient Of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
Bund	**0.4519**	**-0.0814**	**6.8716**	**0.1661**	**0.1009**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*65.9331*
H&W	0.3800	-0.0734	4.41	0.1792	0.0958	0.4187
S&S	0.3621	-0.0879	3.88	0.1729	0.1049	0.5426
HES	0.4397	-0.1289	4.48	0.2376	0.0924	2.8489
BTP	**0.4268**	**-0.1665**	**5.0590**	**0.1638**	**0.0826**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*48.1248*
H&W	0.3586	-0.1208	4.01	0.1696	0.0641	0.7532
S&S	0.3577	-0.1553	3.74	0.1734	0.0658	0.4432
HES	0.3907	-0.1140	4.12	0.1944	0.0877	0.9210
Gilt	**0.3710**	**-0.1775**	**5.7580**	**0.1116**	**0.0663**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*37.0817*
H&W	0.2922	-0.1450	3.43	0.1201	0.0484	0.7741
S&S	0.2938	-0.1654	3.33	0.1225	0.0509	0.6061
HES	0.2918	-0.1308	3.39	0.1176	0.0620	0.7973
US T-Bond	**0.4637**	**-0.0572**	**5.3650**	**0.2080**	**0.1643**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*88.0371*
H&W	0.4589	-0.0449	5.84	0.2164	0.1494	0.1753
S&S	0.4233	-0.0499	4.37	0.2173	0.1591	0.1489
HES	0.5308	-0.0937	5.84	0.2612	0.1288	0.3342

Table 4.5b, Best Fitting Models for Four Fixed Income Futures Assuming Price Series are Lognormally Distributed and the Variance is Stochastic.

Markets (Best Model)	Coefficient Of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
D Mark	**0.3415**	**-0.2432**	**5.4460**	**0.0804**	**0.0139**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*22.3236*
H&W	0.2262	-0.2102	2.96	0.0707	0.0297	1.0850
S&S	0.2290	-0.2169	2.95	0.0730	0.0316	1.0752
HES	0.2678	-0.2609	3.17	0.0988	0.0191	0.6015
Pound	**0.4064**	**-0.1613**	**6.53**	**0.1140**	**0.0655**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*35.5084*
H&W	0.3033	-0.1425	4.256	0.1286	0.0500	0.6766
S&S	0.2992	-0.1461	4.219	0.1264	0.0595	0.5611
HES	0.3078	-0.1427	4.246	0.1315	0.0606	0.5749
Yen	**0.3590**	**-0.2792**	**7.7897**	**0.0577**	**0.0087**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*17.2707*
H&W	0.1888	-0.2574	2.76	0.0436	0.0251	2.0723
S&S	0.2490	-0.2836	3.01	0.0804	0.0166	1.3337
HES	0.2606	-0.3162	3.16	0.0887	0.0098	1.5363
S-Franc	**0.3111**	**-0.2458**	**5.0450**	**0.0660**	**0.0158**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*36.2483*
H&W	0.2140	-0.2245	2.89	0.0618	0.0276	0.7027
S&S	0.2442	-0.2062	3.03	0.0842	0.0343	0.6185
HES	0.2512	-0.2142	3.08	0.0889	0.0316	0.4771

Table 4.5c, Best Fitting Models for Four Foreign Exchange Futures Assuming Price Series are Lognormally Distributed and the Variance is Stochastic.

At this stage, we will compare the three models discussed so far: the GBM, the best of the Student-t distributions and the best of the stochastic volatility models that assume the underlying asset follows GBM. This can be seen in Table 4.6.

Markets (Model)	Coefficient Of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.9045**	**-0.0498**	**254.50**	**0.1387**	**0.0549**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	70.9208
T3	0.2891	0.0000	12.23	0.0085	-0.0026	29.7040
S&S	0.4711	-0.2143	5.05	0.2203	0.0293	20.3151
FTSE	**0.4984**	**-0.0899**	**29.461**	**0.1428**	**0.0398**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	60.0982
T3	0.2891	0.0000	12.23	0.0085	-0.0026	12.5310
H&W	0.3477	-0.1097	3.935	0.1611	0.0692	3.9898
DAX	**0.3391**	**-0.1740**	**5.7004**	**0.0782**	**0.0207**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	22.6601
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	7.8243
H&W	0.2532	-0.1865	3.13	0.0907	0.0342	0.6909
Nikkei	**0.4031**	**-0.1681**	**4.74**	**0.1557**	**0.0629**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	45.0403
T3	0.2891	0.0000	12.23	0.0085	-0.0026	17.8154
S&S	0.3448	-0.1608	3.66	0.1619	0.0599	0.2007

Markets (Model)	Coefficient Of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
Bund	**0.4519**	**-0.0814**	**6.8716**	**0.1661**	**0.1009**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	65.9331
T3	0.2891	0.0000	12.23	0.0085	-0.0026	18.9436
H&W	0.3800	-0.0734	4.41	0.1792	0.0958	0.4187
BTP	**0.4268**	**-0.1665**	**5.0590**	**0.1638**	**0.0826**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	48.1248
T3	0.2891	0.0000	12.23	0.0085	-0.0026	20.4784
S&S	0.3577	-0.1553	3.74	0.1734	0.0658	0.4432
Gilt	**0.3710**	**-0.1775**	**5.7580**	**0.1116**	**0.0663**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	37.0817
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	12.8546
S&S	0.2938	-0.1654	3.33	0.1225	0.0509	0.6061
US T-Bond	**0.4637**	**-0.0572**	**5.3650**	**0.2080**	**0.1643**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	88.0371
T3	0.2891	0.0000	12.23	0.0085	-0.0026	34.1694
S&S	0.4233	-0.0499	4.37	0.2173	0.1591	0.1489
D-Mark	**0.3415**	**-0.2432**	**5.4460**	**0.0804**	**0.0139**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	22.3236
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	5.0201
HES	0.2678	-0.2609	3.17	0.0988	0.0191	0.6015
Pound	**0.4064**	**-0.1613**	**6.53**	**0.1140**	**0.0655**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	35.5084
T3	0.2891	0.0000	12.23	0.0085	-0.0026	13.2803
S&S	0.2992	-0.1461	4.219	0.1264	0.0595	0.5611
Yen	**0.3590**	**-0.2792**	**7.7897**	**0.0577**	**0.0087**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	17.2707
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	2.3600
S&S	0.2490	-0.2836	3.01	0.0804	0.0166	1.3337
S-Franc	**0.3111**	**-0.2458**	**5.0450**	**0.0660**	**0.0158**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	36.2483
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	4.1261
HES	0.2512	-0.2142	3.08	0.0889	0.0316	0.4771

Table 4.6, Comparisons of the Empirical Dynamics of Twelve Financial Futures Dispersion Processes with the Dynamics of a GBM Price Series with Constant Variance versus the Best t-distribution with Constant Variance and the Best Stochastic Volatility Model that assumes the Underlying Price Series is Lognormal.

From this table, we can see that the best of the stochastic volatility models have much greater success (compared to the previous models with constant variance) in explaining the dynamics of the twelve financial futures markets. All the models seem to explain well the time attribute and the autocorrelation attributes but fail to address the COV conditions and do not explain the high kurtosis we observe in the markets. Of the twelve markets, the S&P 500 and the FTSE 100 have the worst fits, while the Nikkei and the US T-Bond have the closest fits. Even so, most of the other markets have SSEs that are below 1.0.

Regarding which stochastic volatility model is best, it appears that that all the models are equally effective in explaining the dynamics of the twelve markets. On the margin, it does appear that the Stein & Stein model is slightly better than the Hull & White model, which is in turn slightly better than the Heston model. Therefore, we conclude that the choice of the stochastic volatility model does not appear to be critical. Nevertheless, all the models still fail to address the relatively high kurtosis (and COV) conditions that we observe empirically.

4.5 ANALYSIS OF STOCHASTIC VOLATILITY MODELS (FIRST PERIOD)

Optimal Parameters for the Three Stochastic Volatility Models (First Period)

To test whether these results are period specific the observation period was split into halves and the analysis rerun for each sub-period. The optimised parameter values appear for the first period in Table 4.7.

Market / Model	σ	θ	κ	ξ
S&P 500				
H&W	0.20	0.20	20.7	1.7
S&S	0.15	0.15	3.2	0.2325
HES	0.20	0.20	3.1	0.41
FTSE				
H&W	0.15	0.15	27.5	1.51
S&S	0.20	0.20	3.00	0.27
HES	0.20	0.20	3.00	0.50
DAX				
H&W	0.20	0.20	21.9	0.99
S&S	0.15	0.15	4	0.1525
HES	0.15	0.15	4.8	0.305
Nikkei 225				
H&W	0.20	0.20	24.2	1.31
S&S	0.15	0.15	4.4	0.2125
HES	0.20	0.20	4.4	0.5

Market / Model	σ	θ	κ	ξ
Bund				
H&W	0.10	0.10	12.2	0.83
S&S	0.20	0.20	1.1	0.220
HES	0.10	0.10	2.9	0.28
BTP				
H&W	0.20	0.20	15.3	1.36
S&S	0.05	0.05	2.90	0.0875
HES	0.20	0.20	2.4	0.46
Gilt				
H&W	0.20	0.20	12.4	0.93
S&S	0.15	0.15	3.20	0.1725
HES	0.05	0.05	1.8	0.0775
US T-Bond				
H&W	0.15	0.15	2.4	0.97
S&S	0.20	0.20	0.70	0.28
HES	0.20	0.20	0.5	0.37
D Mark				
H&W	0.20	0.20	39.3	1.13
S&S	0.05	0.05	4.00	0.025
HES	0.05	0.05	4.1	0.06
Pound				
H&W	0.15	0.15	22	0.85
S&S	0.20	0.20	3.1	0.18
HES	0.20	0.20	4.2	0.39
Yen				
H&W	0.20	0.20	19.2	0.33
S&S	0.20	0.20	9.00	0.15
HES	0.20	0.20	17.7	0.71
S-Franc				
H&W	0.20	0.20	40.6	1.16
S&S	0.15	0.15	4.3	0.1025
HES	0.15	0.15	7.7	0.335

Table 4.7, Parameter values for the Best of the Three Stochastic Volatility Models for the Twelve Financial Futures Markets for the First Half of the Available Observations.

For the most part, the optimised parameter values for the first half of the available observations are similar to those of the entire period. For example, for the fixed income markets, the parameters are almost unchanged. The only important differences are for the Gilt, where the Hull & White κ increased slightly and the κ parameters also changed for the US T-Bond, falling for the Hull & White model and rising for the other two models. The biggest change in the parameter values occurred for the stock index futures and currency futures.

For the S&P 500, the volatility of the volatility parameter (ξ), rose for the S&S and HES models. However, the rate of mean reversion remained relatively stable. For the H&W models, both factors fell. For the FTSE, the S&S model is essentially unchanged, while the κ parameter fell for HES and rose for H&W. The DAX was almost the same for the H&W model but both ξ and κ rose for the other two models. For the Nikkei, the S&S and HES are similar in both periods, although the κ is lower for the HES model and the H&W model.

For the currency futures, significant differences in the parameter values are observed. The British Pound had an increase in the κ, but a reduction in the ξ parameters (except for the Heston model). The D-Mark and the S-Franc both display different changes relative to the British Pound but similar to each other. For the H&W models, the κ and the ξ parameters drop significantly. The S&S models are basically the same and the HES models see an increase in both parameters for the Deutsche Mark[3] and a reduction in the parameters for the Swiss Franc. For the Japanese Yen, all the model parameters are significantly different for the first period relative to the overall period. The κ parameters have fallen for all three models and the ξ parameters have risen for the H&W and S&S models, while falling for the HES model.

Considering the stability of the parameter values is important from a theoretical as well as practical point of view. First of all, the parameter κ, indicates the rate of mean reversion and thus indicates how quickly volatility shocks are dissipated.

[3] While our approach uses the objective return series to obtain the optimal parameters, other approaches have appeared in the literature. Guo (1996) also examined the US Dollar/Deutsche Mark for a period similar to our first period. The first difference is that he looked at the currency options that are based on the spot currency and are traded at the Philadelphia Stock Exchange. The second difference is that he calibrated his Heston model using a non-linear least square estimator which was applied to the risk-neutral stochastic variance process (that is option prices). He decided to follow the Whaley (1982) and Bates (1996) equally-weighted non-linear least square approaches they suggested. The parameters were chosen to minimise the distance between the observed option prices and the predicted option prices from the Heston model. The parameter values he obtained were almost exactly twice those we obtained. From subsequent simulations, we were able to determine that the parameters for these stochastic volatility models have a homogeneity property, where if all the parameter values are multiplied by a constant the results are not substantially changed. Thus, when we doubled the parameter values for the Heston model reported in Table 4.7 and reran the simulation, the results were only slightly different. In Table 4.8c, we obtained a sum of the squared errors of 2.1342 for our parameters and a sum of squared errors of 3.5212 for the parameter values of Guo. While these are not exactly the same, it must be remembered that we are looking at different markets and for not exactly

The higher the value, the faster this dissipation occurs. Secondly, the volatility of volatility parameter, ξ, indicates how unstable the volatility is. Clearly, by examining how these parameters change overtime, we can gain theoretical insights into changes in the behaviour of market volatility. From a practical standpoint, it is relevant if the parameter values differ widely over different time horizons, which would suggest that the fitting of these models is period specific. This would imply that out of sample applications of these models would be greatly reduced. While variability does exist in the parameters, they are for the most part similar in both periods, especially for the fixed income futures markets.

Given that the rate of mean reversion factor, κ, is lower in the first period (overall) relative to the entire period, this might suggest that volatility shocks were slower to dissipate in the early period. This could lead to the conclusion that overall, the dissipation is occurring at a faster rate. To examine this properly, we will examine the parameter values of the best fitting stochastic volatility models for the second half of the observations to see if the rate of mean reversion has continued to increase.

Results for the Three Stochastic Volatility Models (First Period)

With these parameters and assuming the underlying price series follows GBM, the attributes were estimated. These were then compared to the attributes of the twelve markets and the sum of the squared deviations were determined. This can be seen in Tables 4.8a, 4.8b and 4.8c for the three asset classes.

the same time period. Nevertheless, it is comforting that our results are similar to those presented elsewhere.

Markets (Best Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.9581**	**-0.0438**	**185.34**	**0.1247**	**0.0304**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*49.8419*
H&W	0.4903	-0.1139	5.59	0.2543	0.0756	19.0367
S&S	0.3826	-0.1397	3.94	0.1926	0.0797	26.5192
HES	0.3245	-0.1457	3.58	0.1449	0.0622	27.2188
FTSE	**0.5963**	**-0.0569**	**35.99**	**0.1615**	**0.0321**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*32.7907*
H&W	0.4237	-0.1269	4.72	0.2134	0.0634	4.8381
S&S	0.3551	-0.1364	3.75	0.1703	0.0751	7.4042
HES	0.3797	-0.1388	3.92	0.1966	0.0759	6.8844
DAX	**0.3612**	**-0.1740**	**6.17**	**0.0883**	**0.0153**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*12.8280*
H&W	0.2714	-0.1843	3.25	0.1042	0.0343	0.6725
S&S	0.2512	-0.2386	3.06	0.0885	0.0252	1.0775
HES	0.2832	-0.2004	3.27	0.1130	0.0360	1.0318
Nikkei	**0.3493**	**-0.1783**	**3.99**	**0.1458**	**0.0456**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*16.4677*
H&W	0.3337	-0.1682	3.77	0.1491	0.0388	0.0336
S&S	0.3304	-0.1734	3.55	0.1504	0.0517	0.0356
HES	0.3382	-0.1803	3.63	0.1570	0.0469	0.0344

Table 4.8a, Best Fitting Models for Four Stock Index Futures Assuming Price Series are Lognormally Distributed & the Variance is Stochastic (First Half of Available Observations).

Markets (Best Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
Bund	**0.5175**	**-0.0065**	**9.51**	**0.1890**	**0.1103**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*37.5815*
H&W	0.3572	-0.0690	4.21	0.1602	0.0940	1.4893
S&S	0.4148	-0.0614	4.31	0.2110	0.1441	1.8641
HES	0.4181	-0.1268	4.42	0.2121	0.0799	3.9158
BTP	**0.5193**	**-0.1500**	**6.89**	**0.1971**	**0.0833**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*27.2153*
H&W	0.4257	-0.1053	4.75	0.2163	0.0795	0.5697
S&S	0.4371	-0.1431	4.42	0.2320	0.0830	0.7728
HES	0.3959	-0.1185	4.15	0.1982	0.0899	0.9253
Gilt	**0.3823**	**-0.1535**	**5.34**	**0.1228**	**0.0784**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*17.7907*
H&W	0.3142	-0.1197	3.63	0.1358	0.0606	0.4499
S&S	0.3098	-0.1510	3.44	0.1350	0.0592	0.5707
HES	0.3088	-0.1081	3.54	0.1281	0.0746	0.6801
US T-Bond	**0.4863**	**-0.0399**	**4.54**	**0.2349**	**0.1845**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*45.9846*
H&W	0.5052	-0.0451	6.61	0.2441	0.1636	0.2528
S&S	0.4644	-0.0341	4.55	0.2521	0.2024	0.0606
HES	0.5008	-0.0489	5.41	0.2420	0.1674	0.2113

Table 4.8b, Best Fitting Models for Four Fixed Income Futures Assuming Price Series are Lognormally Distributed & the Variance is Stochastic (First Half of Available Observations).

Markets (Best Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
D Mark	**0.3309**	**-0.3000**	**5.36**	**0.0711**	**-0.0114**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*6.3656*
H&W	0.2441	-0.2662	3.07	0.0810	0.0152	0.7117
S&S	0.1818	-0.2905	2.72	0.0392	0.0230	2.1950
HES	0.2023	-0.2456	2.82	0.0531	0.0268	2.1342
Pound	**0.3721**	**-0.1900**	**6.18**	**0.0917**	**0.0335**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*12.8628*
H&W	0.2660	-0.1620	4.17	0.1004	0.0415	0.3928
S&S	0.2638	-0.1655	4.15	0.0989	0.0473	0.4352
HES	0.2739	-0.1901	4.17	0.1062	0.0392	0.3290
Yen	**0.3383**	**-0.4014**	**7.10**	**0.0495**	**0.0005**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*3.2769*
H&W	0.1612	-0.3782	2.63	0.0256	0.0186	1.7893
S&S	0.1825	-0.3677	2.70	0.0395	0.0148	1.7989
HES	0.2404	-0.4009	3.13	0.0673	0.0042	0.9272
S-Franc	**0.3082**	**-0.2748**	**5.03**	**0.0665**	**0.0024**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*6.7588*
H&W	0.2436	-0.2694	3.07	0.0804	0.0146	0.3706
S&S	0.2077	-0.2404	2.84	0.0573	0.0277	1.0692
HES	0.2428	-0.2763	3.02	0.0808	0.0180	0.5255

Table 4.8c, Best Fitting Models for Four Foreign Exchange Futures Assuming Price Series Lognormally Distributed & the Variance is Stochastic (First Half of Available Observations).

As before, we will summarise the relative goodness of fit for the three models discussed so far just for the period that has the first half of the available data.

Markets (Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.9581**	**-0.0438**	**185.34**	**0.1247**	**0.0304**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	49.8419
T3	0.2891	0.0000	12.23	0.0085	-0.0026	21.1225
H&W	0.4903	-0.1139	5.59	0.2543	0.0756	19.0367
FTSE	**0.5963**	**-0.0569**	**35.99**	**0.1615**	**0.0321**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	32.7907
T3	0.2891	0.0000	12.23	0.0085	-0.0026	10.3718
H&W	0.4237	-0.1269	4.72	0.2134	0.0634	4.8381
DAX	**0.3612**	**-0.1740**	**6.1734**	**0.0883**	**0.0153**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	12.8280
T3	0.2891	0.0000	12.23	0.0085	-0.0026	4.4459
H&W	0.2714	-0.1843	3.25	0.1042	0.0343	0.6725
Nikkei	**0.3493**	**-0.1783**	**3.9912**	**0.1458**	**0.0456**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	16.4677
T3	0.2891	0.0000	12.23	0.0085	-0.0026	9.0880
H&W	0.3337	-0.1682	3.77	0.1491	0.0388	0.0336

Markets (Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
Bund	**0.5175**	**-0.0065**	**9.5135**	**0.1890**	**0.1103**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	37.5815
T3	0.2891	0.0000	12.23	0.0085	-0.0026	14.4586
H&W	0.3572	-0.0690	4.21	0.1602	0.0940	1.4893
BTP	**0.5193**	**-0.1500**	**6.8939**	**0.1971**	**0.0833**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	27.2153
T3	0.2891	0.0000	12.23	0.0085	-0.0026	15.3758
H&W	0.4257	-0.1053	4.75	0.2163	0.0795	0.5697
Gilt	**0.3823**	**-0.1535**	**5.3372**	**0.1228**	**0.0784**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	17.7907
T3	0.2891	0.0000	12.23	0.0085	-0.0026	8.0175
H&W	0.3142	-0.1197	3.63	0.1358	0.0606	0.4499
US T-Bond	**0.4863**	**-0.0399**	**4.5418**	**0.2349**	**0.1845**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	45.9846
T3	0.2891	0.0000	12.23	0.0085	-0.0026	26.9706
S&S	0.4644	-0.0341	4.55	0.2521	0.2024	0.0606
D-Mark	**0.3309**	**-0.3000**	**5.3573**	**0.0711**	**-0.0114**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	6.3656
T1	0.2400	-0.3925	6.10	0.0045	0.0023	2.1262
H&W	0.2441	-0.2662	3.07	0.0810	0.0152	0.7117
Pound	**0.3721**	**-0.1900**	**6.18**	**0.0917**	**0.0335**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	12.8628
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	5.8328
HES	0.2739	-0.1901	4.17	0.1062	0.0392	0.3290
Yen	**0.3383**	**-0.4014**	**7.1017**	**0.0495**	**0.0005**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	3.2769
T1	0.2400	-0.3925	6.10	0.0045	0.0023	0.8417
HES	0.2404	-0.4009	3.13	0.0673	0.0042	0.9272
S-Franc	**0.3082**	**-0.2748**	**5.0305**	**0.0665**	**0.0024**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	6.7588
T1	0.2400	-0.3925	6.10	0.0045	0.0023	2.1932
H&W	0.2436	-0.2694	3.07	0.0804	0.0146	0.3706

Table 4.9, Comparisons of the Empirical Dynamics of Twelve Financial Futures Dispersion Processes with the Dynamics of a GBM Price Series with Constant Variance versus the Best t-distribution with Constant Variance and the Best Stochastic Volatility Model that assumes the Underlying Price Series is Lognormal (Analysis Period included the First Half of the Available Observations).

As with the entire period, the inclusion of the stochastic volatility model significantly improves the explanatory power compared to the assumption that volatility is constant and the underlying price series follows GBM. However, for one market, the Japanese Yen, the best of the t-distributions (assuming constant volatility) is slightly better at explaining this market's dynamics than the best stochastic volatility model. The stochastic volatility models almost perfectly explain the behaviour of the

US T-Bond and Nikkei 225 futures for this period. While the stochastic volatility models for the other markets are better than the models assuming constant volatility, there is still a substantial error for the S&P 500 and FTSE 100. The remaining error is primarily due to the inability of the models to explain the high excess kurtosis in these markets and address the relatively high COV conditions.

Nevertheless, the results are similar to those observed for the entire period. The conclusion is that stochastic volatility models are superior to models which assume constant volatility for the entire period from the mid 1980s to the mid 1990s and in the period of the late 1980s. Now, we will examine how well these models perform in the first half of the 1990s.

4.6 ANALYSIS OF STOCHASTIC VOLATILITY MODELS (SECOND PERIOD)

Optimal Parameters for the Three Stochastic Volatility Models (Second Period)

As before, the previous best fitting models for the entire period were examined and the optimisation was run to achieve the best fit for each of the twelve markets. The accompanying parameter values of these best fitting models appear below in Table 4.10.

Market / Model	σ	θ	κ	ξ
S&P 500				
H&W	0.20	0.20	36.1	1.34
S&S	0.15	0.15	5.475	0.1825
HES	0.20	0.20	7.32	0.544
FTSE				
H&W	0.15	0.15	20.1	1.03
S&S	0.20	0.20	3.95	0.225
HES	0.20	0.20	4.02	0.404
DAX				
H&W	0.20	0.20	19.6	0.86
S&S	0.15	0.15	3.1	0.12
HES	0.15	0.15	1.56	0.157
Nikkei 225				
H&W	0.20	0.20	17.0	1.31
S&S	0.15	0.15	2.925	0.2225
HES	0.20	0.20	2.99	0.499

Market / Model	σ	θ	κ	ξ
Bund				
H&W	0.10	0.10	12.6	0.83
S&S	0.20	0.20	2.15	0.225
HES	0.10	0.10	2.4	0.204
BTP				
H&W	0.20	0.20	23.4	1.06
S&S	0.05	0.05	5.9	0.07
HES	0.20	0.20	6.6	0.526
Gilt				
H&W	0.20	0.20	14.1	1.00
S&S	0.15	0.15	3.9	0.195
HES	0.05	0.05	2.04	0.0815
US T-Bond				
H&W	0.15	0.15	21.8	1.04
S&S	0.20	0.20	4.575	0.2325
HES	0.20	0.20	4.27	0.401
D Mark				
H&W	0.20	0.20	15.8	1.02
S&S	0.05	0.05	6.325	0.0825
HES	0.05	0.05	3.68	0.104
Pound				
H&W	0.15	0.15	8.4	0.91
S&S	0.20	0.20	1.525	0.2225
HES	0.20	0.20	1.31	0.359
Yen				
H&W	0.20	0.20	35.5	1.37
S&S	0.20	0.20	7.375	0.4025
HES	0.20	0.20	5.78	0.2947
S-Franc				
H&W	0.20	0.20	20.7	0.999
S&S	0.15	0.15	4.3	0.165
HES	0.15	0.15	3.73	0.274

Table 4.10, Parameter values for the Best of the Three Stochastic Volatility Models for the Twelve Financial Futures Markets for the Second Half of the Available Observations.

As with the comparison of the first half of the observations to the entire period, the optimised parameter values for the second half of the available observations are similar to those of the other period. For example, for the fixed income markets, the ξ parameters are almost unchanged. However, the rate of mean reversion has risen substantially for all the fixed income futures markets. The most dramatic increase in the κ parameter is for the US T-Bond. This result is consistent with the different pattern of the autocorrelogram observed for US T-Bond in the second period. This was presented in the second Chapter. For the other asset classes, we find that the volatility of volatility parameter has remained essentially the same and the only significant differences were in the mean reversion parameter. For the

S&P 500, FTSE (S&S & HES), Deutsche Mark (S&S) and Japanese Yen (H&W), the κ parameter has risen. For the DAX, Nikkei, British Pound and Swiss Franc, the κ parameter has fallen.

Again, it is important to consider the stability of the parameter values. It appears that the volatility of volatility parameter is fairly stable over time. What does differ significantly over time is the rate of the mean reversion factor, κ. It appears that in the first half of the 1990s, this parameter value is higher suggesting that most of the markets under investigation are absorbing volatility shocks at a faster rate. This implies that mean reversion is becoming relatively more important in the dynamics of financial futures volatility.

<u>Results for the Three Stochastic Volatility Models (Second Period)</u>

With these optimised parameter values, we reran the analysis for the second period. This analysis appears in Tables 4.11a, 4.11b and 4.11c.

Markets (Best Model)	Coefficient Of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.3192**	**-0.2535**	**5.88**	**0.0596**	**0.0224**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*49.0282*
H&W	0.2905	-0.2276	3.41	0.1137	0.0187	9.7687
S&S	0.2718	-0.2189	3.18	0.1043	0.0308	11.3546
HES	0.2849	-0.2514	3.29	0.1107	0.0205	10.2973
FTSE	**0.3119**	**-0.1856**	**4.52**	**0.0888**	**0.0430**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*62.0476*
H&W	0.2897	-0.1669	3.39	0.1181	0.0400	2.9198
S&S	0.2868	-0.1771	3.28	0.1167	0.0457	3.2152
HES	0.2877	-0.1797	3.32	0.1167	0.0436	3.0586
DAX	**0.2824**	**-0.1274**	**4.21**	**0.0481**	**0.0161**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*64.0125*
H&W	0.2520	-0.1874	3.12	0.0898	0.0340	5.6524
S&S	0.2509	-0.1731	3.09	0.0890	0.0437	5.7853
HES	0.2380	-0.1373	3.10	0.0731	0.0489	4.9376
Nikkei	**0.4381**	**-0.1056**	**5.87**	**0.1383**	**0.0704**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*125.2491*
H&W	0.3885	-0.1218	4.32	0.1907	0.0655	5.9381
S&S	0.3800	-0.1331	3.92	0.1905	0.0833	8.0306
HES	0.3845	-0.1324	4.06	0.1910	0.0740	7.1501

Table 4.11a, Best Fitting Models for Four Stock Index Futures Assuming Price Series are Lognormally Distributed & the Variance is Stochastic (Second Half of Available Observations).

Markets (Best Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
Bund	**0.3818**	**-0.1004**	**4.66**	**0.1329**	**0.0805**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*110.5935*
H&W	0.3523	-0.0720	4.14	0.1572	0.0910	1.6543
S&S	0.3476	-0.1098	3.74	0.1634	0.0879	2.5147
HES	0.3508	-0.1239	3.78	0.1675	0.0798	2.7445
BTP	**0.3176**	**-0.2600**	**3.94**	**0.1014**	**0.0109**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*46.4193*
H&W	0.2846	-0.1837	3.35	0.1136	0.0340	3.6506
S&S	0.2920	-0.2207	3.29	0.1192	0.0301	2.1956
HES	0.2902	-0.2357	3.32	0.1158	0.0246	1.5724
Gilt	**0.3546**	**-0.1602**	**6.25**	**0.0950**	**0.0504**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*83.6713*
H&W	0.3238	-0.1231	3.70	0.1436	0.0600	9.0557
S&S	0.3239	-0.1669	3.52	0.1457	0.0540	9.9379
HES	0.3102	-0.1153	3.53	0.1305	0.0721	10.1372
US T-Bond	**0.3133**	**-0.1906**	**5.83**	**0.0641**	**0.0348**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*61.9444*
H&W	0.2832	-0.1775	3.41	0.1137	0.0187	9.2693
S&S	0.2797	-0.1952	3.23	0.1110	0.0387	9.9182
HES	0.2790	-0.1891	3.25	0.1100	0.0397	9.6783

Table 4.11b, Best Fitting Models for Four Fixed Income Futures Assuming Price Series are Lognormally Distributed & the Variance is Stochastic (Second Half of Available Observations).

Markets (Best Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
D Mark	**0.3515**	**-0.1570**	**5.52**	**0.0884**	**0.0379**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*77.8149*
H&W	0.3206	-0.1800	3.66	0.1415	0.0566	7.2055
S&S	0.3271	-0.2252	3.54	0.1440	0.0274	9.0777
HES	0.3063	-0.1652	3.46	0.1314	0.0510	7.6615
Pound	**0.4354**	**-0.0846**	**6.81**	**0.1308**	**0.0937**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*138.0695*
H&W	0.3927	-0.0637	4.62	0.1859	0.1056	7.3074
S&S	0.3703	-0.0830	3.95	0.1789	0.1108	10.7054
HES	0.3820	-0.0831	4.10	0.1826	0.1105	9.6288
Yen	**0.3794**	**-0.1876**	**8.59**	**0.0657**	**0.0110**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*85.3851*
H&W	0.3360	-0.1758	3.79	0.1498	0.0351	22.7850
S&S	0.3363	-0.2477	3.62	0.1462	0.0175	24.2428
HES	0.2947	-0.2222	3.32	0.1239	0.0308	25.2071
S-Franc	**0.3148**	**-0.1745**	**5.06**	**0.0655**	**0.0318**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*62.3183*
H&W	0.2787	-0.1755	3.31	0.1097	0.0370	6.3877
S&S	0.2737	-0.1922	3.20	0.1066	0.0394	7.0211
HES	0.2740	-0.1778	3.22	0.1062	0.0432	6.7714

Table 4.11c, Best Fitting Models for Four Foreign Exchange Futures Assuming Price Series Lognormally Distributed & the Variance is Stochastic (Second Half of Available Observations).

While for each of the markets some improvement in the fit is achieved with the addition of the optimal stochastic volatility models (and parameters), it is interesting to note that the sum of the squared errors is much higher than in the two other periods. This is primarily due to the fact that the stochastic volatility models fail to address the excess kurtosis in the markets.

As with the two previous analyses, we will compare the GBM model, the best of the Student-t distributions and the best of the stochastic volatility models. This can be seen in Table 4.12.

Markets (Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.3192**	**-0.2535**	**5.8823**	**0.0596**	**0.0224**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	49.0282
T1	0.2400	-0.3925	6.10	0.0045	0.0023	12.4521
H&W	0.2905	-0.2276	3.41	0.1137	0.0187	9.7687
FTSE	**0.3119**	**-0.1856**	**4.5168**	**0.0888**	**0.0430**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	62.0476
T1	0.2400	-0.3925	6.10	0.0045	0.0023	27.3332
H&W	0.2897	-0.1669	3.39	0.1181	0.0400	2.9198
DAX	**0.2824**	**-0.1274**	**4.2104**	**0.0481**	**0.0161**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	64.0125
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	27.6476
HES	0.2380	-0.1373	3.10	0.0731	0.0489	4.9376
Nikkei	**0.4381**	**-0.1056**	**5.8671**	**0.1383**	**0.0704**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	125.2491
T3	0.2891	0.0000	12.23	0.0085	-0.0026	48.3928
H&W	0.3885	-0.1218	4.32	0.1907	0.0655	5.9381
Bund	**0.3818**	**-0.1004**	**4.6621**	**0.1329**	**0.0805**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	110.5935
T3	0.2891	0.0000	12.23	0.0085	-0.0026	51.4817
H&W	0.3523	-0.0720	4.14	0.1572	0.0910	1.6543
BTP	**0.3176**	**-0.2600**	**3.9391**	**0.1014**	**0.0109**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	46.4193
T1	0.2400	-0.3925	6.10	0.0045	0.0023	21.9242
HES	0.2930	-0.2318	3.30	0.1217	0.0275	1.6896
Gilt	**0.3546**	**-0.1602**	**6.2497**	**0.0950**	**0.0504**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	83.6713
T3	0.2891	0.0000	12.23	0.0085	-0.0026	30.9329
H&W	0.3238	-0.1231	3.70	0.1436	0.0600	9.0557
US T-Bond	**0.3133**	**-0.1906**	**5.8301**	**0.0641**	**0.0348**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	61.9444
T1	0.2400	-0.3925	6.10	0.0045	0.0023	20.4269
H&W	0.2832	-0.1775	3.41	0.1137	0.0187	9.2693

Markets (Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
D-Mark	**0.3515**	**-0.1570**	**5.5237**	**0.0884**	**0.0379**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	77.8149
T3	0.2891	0.0000	12.23	0.0085	-0.0026	31.5012
H&W	0.3206	-0.1800	3.66	0.1415	0.0566	7.2055
Pound	**0.4354**	**-0.0846**	**6.8103**	**0.1308**	**0.0937**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	138.0695
T3	0.2891	0.0000	12.23	0.0085	-0.0026	46.0396
H&W	0.3927	-0.0637	4.62	0.1859	0.1056	7.3074
Yen	**0.3794**	**-0.1876**	**8.5897**	**0.0657**	**0.0110**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	85.3851
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	19.7352
H&W	0.3360	-0.1758	3.79	0.1498	0.0351	22.7850
S-Franc	**0.3148**	**-0.1745**	**5.0631**	**0.0655**	**0.0318**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	62.3183
T1	0.2400	-0.3925	6.10	0.0045	0.0023	23.2820
H&W	0.2787	-0.1755	3.31	0.1097	0.0370	6.3877

Table 4.12, Comparisons of the Empirical Dynamics of Twelve Financial Futures Dispersion Processes with the Dynamics of a GBM Price Series with Constant Variance versus the Best t-distribution with Constant Variance and the Best Stochastic Volatility Model that assumes the Underlying Price Series is Lognormal (Analysis Period included the Second Half of the Available Observations).

As with the two previous analyses, the inclusion of the stochastic volatility model significantly improves the explanatory power compared to the assumption that volatility is constant and the underlying price series follows GBM. However, it is interesting to note that for one market, the Japanese Yen, the best of the t-distributions (assuming constant volatility) is better at explaining these market's dynamics than the best stochastic volatility model. When compared to the same analysis for the second period, the stochastic volatility models are much less effective in explaining the behaviour of the twelve markets. Not a single market has a SSE less than 1.0.

4.7 CONCLUSION

In this Chapter, we have examined whether the dynamics of the objective processes for twelve financial futures markets can be understood by security price process models that assume returns following GBM but allow the volatility to be stochastic. As with the previous Chapter, we have examined a number of factors that address most of the divergences from normality pointed out in the literature and tested which of these factors the three stochastic volatility models may address.

Having previously rejected Geometric Brownian motion as a solution for capturing the dynamics of the objective process, we have found that the inclusion of a stochastic volatility to the GBM assumption does capture significantly more of the dynamics of the returns series for the twelve financial futures markets. We conclude that stochastic volatility models are superior to models that assume constant volatility for all three periods. However, the models are less effective in the mid-1990s than either for the late 1980s or for the overall period. Furthermore, we found that there was not a significant difference in the three stochastic volatility models tested once the optimal parameters for each model had been selected.

We found that the (best) stochastic volatility model was able to explain the three attributes that were not addressed by the Student-t distribution model examined in the previous Chapter. Specifically, this class of models was able to capture the dynamics of the time (decay) factor of the volatility of the volatility and the two autocorrelation factors. Unfortunately, these models were not able to capture the coefficient of variation (of the unconditional volatility at 20 days) or the excess kurtosis of the return series. Thus, it would appear that a security price process model that assumes GBM and stochastic volatility would not be adequate to understand the objective processes for the twelve financial futures under investigation, either. As before, if these models fail to capture all the dynamics of the underlying asset dispersion process, we cannot expect to understand the dynamics of contingent claims on these same assets.

At this point, we have demonstrated that in all cases, the inclusion of a fat-tailed distribution provides a better fit for observed financial futures markets volatilities than an assumption of GBM. Furthermore, in 33 of the 36 cases, we were able to determine that some (optimised) stochastic volatility model would fit the observed empirical behaviours better than a fat-tailed distribution with constant volatility. However, these two approaches were able to capture different aspects (attributes) of the unconditional process. Given this, it is not unreasonable to expect that by combining both approaches, the resulting models might be able to capture all the dynamics of the objective processes. Bates (1996) and Ho, Perraudin and Sørensen (1996) have previously suggested this in the literature. Both papers suggested that both jump process and stochastic volatility are required together to explain the dynamics of objective processes. Therefore, we will turn our attention in the next Chapter to this class of models that combine both approaches.

CHAPTER FIVE
THE ANALYSIS OF OBJECTIVE PROBABILITIES IN FUTURES MARKETS: EXPLAINING THE EMPIRICAL DYNAMICS WITH MODELS ASSUMING STOCHASTIC VARIANCE AND THE UNDERLYING ASSET PRICES FOLLOW A STUDENT-T DISTRIBUTION

5.1 INTRODUCTION

For this final Chapter of the first part of the dissertation, we will combine both approaches discussed in the previous two Chapters combining fat-tailed distributions and stochastic volatility models. As before, we will optimise the results to achieve the best fitting model. Unfortunately, we must now optimise over four parameters. As before, we must examine three parameters for the stochastic volatility models: κ, the degree of mean reversion, θ, the long term conditional volatility and ξ, the volatility of the volatility. For this simulation, we must also simulate across different possible Student-t distributions that have different levels of excess kurtosis. We will demonstrate that this approach lead to significant improvement in understanding the dynamics of the objective processes.

5.2 ANALYSIS OF COMBINATION MODELS FOR THE ENTIRE PERIOD

The procedures followed these steps. First, we selected the same eight simulated t-distributions (that were used in Chapter 3) and the simulated volatility series generated by the three stochastic volatility models (examined in the last Chapter). Then we simulated new price series that assumed the price innovation followed (each of) the Student-t distributions and the volatility dynamics were captured by (each of) the stochastic volatility models. Thereafter, the best model (and t-distribution) was used as the seed for an optimisation routine to minimise the SSE.

The best fitting parameter values for each stochastic volatility model and the optimal Student-t distribution are summarised in Tables 5.1a, 5.1b and 5.1c.

Parameter Estimation for Combination Models for the Entire Period

Markets (Best Model)	Initial Volatility	Long Term Volatility	Mean Reversion	Volatility of Volatility	t- dist
S&P 500					
H&W	0.20	0.20	21.7	2.111	T2 (k=9.16)
S&S	0.20	0.20	1.52	1.11	T2 (k=9.16)
HES	0.20	0.20	1.61	0.701	T2 (k=9.16)
FTSE					
H&W	0.20	0.20	12.1	1.375	T2 (k=9.16)
S&S	0.20	0.20	1.79	0.371	T2 (k=9.16)
HES	0.20	0.20	1.91	0.589	T2 (k=9.16)
DAX					
H&W	0.15	0.15	3.82	0.652	T4 (k=15.26)
S&S	0.20	0.20	1.095	0.1625	T1 (k=6.10)
HES	0.20	0.20	0.9	0.25	T1 (k=6.10)
Nikkei					
H&W	0.10	0.10	3.78	0.856	T4 (k=15.26)
S&S	0.20	0.20	1.69	0.298	T3 (k=12.23)
HES	0.20	0.20	1.3	0.44	T3 (k=12.23)

Table 5.1a, Parameters for the Best Fitting Models for Four Stock Index Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic.

Markets (Best Model)	Initial Volatility	Long Term Volatility	Mean Reversion	Volatility of Volatility	t- dist
Bund					
H&W	0.10	0.10	3.37	0.853	T3 (k=12.23)
S&S	0.20	0.20	0.68	0.252	T3 (k=12.23)
HES	0.20	0.20	1.7	0.62	T3 (k=12.23)
BTP					
H&W	0.10	0.10	3.71	0.901	T4 (k=15.26)
S&S	0.20	0.20	1.37	0.338	T3 (k=12.23)
HES	0.20	0.20	1.91	0.525	T3 (k=12.23)
Gilt					
H&W	0.15	0.15	1.87	0.735	T4 (k=15.26)
S&S	0.20	0.20	1.57	0.23	T3 (k=12.23)
HES	0.05	0.05	2.1	0.12	T1 (k=6.10)
US T-Bond					
H&W	0.10	0.10	3.75	1.023	T3 (k=12.23)
S&S	0.20	0.20	0.08	0.322	T3 (k=12.23)
HES	0.15	0.15	0.19	0.67	T3 (k=12.23)

Table 5.1b, Parameters for the Best Fitting Models for Four Fixed Income Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic.

Markets (Best Model)	Initial Volatility	Long Term Volatility	Mean Reversion	Volatility of Volatility	t- dist
D-Mark					
H&W	0.15	0.15	9.35	0.725	T4 (k=15.26)
S&S	0.20	0.20	2.46	0.212	T1 (k=6.10)
HES	0.20	0.20	2.44	0.36	T1 (k=6.10)
Pound					
H&W	0.20	0.20	2.2	0.770	T4 (k=15.26)
S&S	0.20	0.20	1.21	0.216	T4 (k=15.26)
HES	0.20	0.20	1.25	0.435	T3 (k=12.23)
Yen					
H&W	0.15	0.15	7.77	0.599	T4 (k=15.26)
S&S	0.20	0.20	2.71	0.174	T4 (k=15.26)
HES	0.20	0.20	3.11	0.355	T1 (k=6.10)
S-Franc					
H&W	0.15	0.15	9.4	0.685	T4 (k=15.26)
S&S	0.20	0.20	2.53	0.195	T3 (k=12.23)
HES	0.20	0.20	1.82	0.372	T1 (k=6.10)

Table 5.1c, Parameters for the Best Fitting Models for Four Foreign Exchange Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic.

It is interesting to note that the parameter values for the mean reversion and the volatility of volatility have changed dramatically compared with the parameters for the stochastic volatility models with the GBM assumption. It is clear why this is the case. The parameter values are now interacting with the inclusion of the fat-tailed distribution. Given that different fat-tailed distributions are appropriate to different markets, it is not possible to interpret the changes in the parameter values in any meaningful manner. It will suffice to say that the stochastic volatility parameters have fallen for the most part due to the fact that the inclusion of the fat-tailed distribution is addressing some of the elements these models had previously addressed. Given this difficulty in the interpretation of changes in parameter values for the stochastic volatility models, we will not attempt to lend economic rationale to changing parameter values from this point forward.

Results for Combination Models for the Entire Period

Nevertheless, the inclusion of the fat-tailed distribution to the stochastic volatility models has a dramatic effect in explaining the dynamics of empirical volatility. This can be seen in Tables 5.2a, 5.2b and 5.2c.

Markets (Best Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.9045**	**-0.0498**	**254.50**	**0.1387**	**0.0549**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*70.9208*
H&W	0.7508	-0.1285	39.04	0.1809	0.0509	4.3262
S&S	0.6668	-0.1334	35.05	0.1602	0.0756	6.1777
HES	0.6194	-0.1488	36.00	0.1359	0.0463	5.6107
FTSE	**0.4984**	**-0.0899**	**29.46**	**0.1428**	**0.0398**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*60.0982*
H&W	0.5929	0.0000	27.90	0.1302	0.0572	0.8267
S&S	0.5693	-0.1452	26.16	0.1243	0.0638	1.1632
HES	0.5857	-0.1366	27.73	0.1283	0.0600	0.9613
DAX	**0.3391**	**-0.1740**	**5.70**	**0.0782**	**0.0207**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*22.6601*
H&W	0.3783	-0.2062	8.08	0.0790	0.0396	0.4914
S&S	0.3752	-0.1923	7.79	0.0669	0.0399	0.4109
HES	0.3873	-0.1791	7.98	0.0753	0.0451	0.3895
Nikkei	**0.4031**	**-0.1681**	**4.74**	**0.1557**	**0.0629**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*45.0403*
H&W	0.4707	-0.1515	9.38	0.1362	0.0720	0.7212
S&S	0.4576	-0.1649	8.74	0.1348	0.0697	0.5657
HES	0.4989	-0.1446	9.67	0.1569	0.0858	0.4800

Table 5.2a, Results for the Best Fitting Models for Four Stock Index Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic.

Markets (Best Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
Bund	**0.4519**	**-0.0814**	**6.87**	**0.1661**	**0.1009**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*65.9331*
H&W	0.4920	-0.0793	10.69	0.1605	0.1086	0.2100
S&S	0.4836	-0.0837	7.87	0.1590	0.1256	0.2575
HES	0.5462	-0.1355	11.40	0.1775	0.0874	1.1437
BTP	**0.4268**	**-0.1665**	**5.06**	**0.1638**	**0.0826**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*48.1248*
H&W	0.5046	-0.1459	10.03	0.1549	0.0809	0.6114
S&S	0.4781	-0.1518	9.04	0.1472	0.0811	0.4695
HES	0.4958	-0.1614	10.08	0.1524	0.0692	0.6276
Gilt	**0.3710**	**-0.1775**	**5.76**	**0.1116**	**0.0663**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*37.0817*
H&W	0.4196	-0.1721	8.61	0.1060	0.0573	0.2493
S&S	0.4108	-0.1701	7.89	0.1076	0.0587	0.1670
HES	0.4965	-0.2054	9.39	0.1250	0.0406	1.0665
US T-Bond	**0.4637**	**-0.0572**	**5.37**	**0.2080**	**0.1643**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*88.0371*
H&W	0.5705	-0.0673	13.10	0.2054	0.1349	1.3832
S&S	0.5225	-0.0457	7.43	0.1865	0.1691	0.4379
HES	0.5867	-0.0757	9.23	0.2164	0.1418	1.0986

Table 5.2b, Results for the Best Fitting Models for Four Fixed Income Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic.

Markets (Best Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
D-Mark	**0.3415**	**-0.2432**	**5.44**	**0.0804**	**0.0139**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*22.3236*
H&W	0.3725	-0.2379	8.21	0.0695	0.0246	0.2515
S&S	0.3866	-0.2508	7.85	0.0696	0.0227	0.2602
HES	0.3862	-0.2504	7.89	0.0700	0.0218	0.2500
Pound	**0.4064**	**-0.1613**	**6.53**	**0.1140**	**0.0655**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*35.5084*
H&W	0.4291	-0.1654	5.166	0.1117	0.0591	0.5518
S&S	0.4215	-0.1605	5.122	0.1060	0.0568	0.5336
HES	0.4533	-0.1556	5.103	0.1365	0.0689	0.7597
Yen	**0.3590**	**-0.2792**	**7.79**	**0.0577**	**0.0087**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*17.2707*
H&W	0.3445	-0.2727	7.92	0.0524	0.0187	0.0728
S&S	0.3473	-0.2792	7.58	0.0546	0.0114	0.0141
HES	0.3679	-0.2757	7.61	0.0602	0.0149	0.0152
S-Franc	**0.3111**	**-0.2458**	**5.05**	**0.0660**	**0.0158**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*36.2483*
H&W	0.3657	-0.2477	8.14	0.0650	0.0221	0.2640
S&S	0.3668	-0.2465	8.71	0.0744	0.0265	0.3165
HES	0.4217	-0.2100	8.38	0.0896	0.0388	0.8836

Table 5.2c, Results for the Best Fitting Models for Four Foreign Exchange Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic.

The inclusion of both the optimised stochastic volatility model and Student-t distribution had led to a dramatic improvement in the explanatory power of the model. For almost all markets, at least one model exists which has a sum of the squared errors less than one. The only exception is the S&P 500. Even so, the SSE for the S&P 500 has dropped significantly. Furthermore, all the stochastic volatility models appear to perform relatively well in describing the dynamics of the empirical volatility once the fat-tailed distributions have been included.

At this point, we will compare all the proposed models and assess which best explains the dynamics of the twelve financial futures markets for the entire period of the analysis period. This can be seen in the following Table 5.3.

Markets (Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.9045**	**-0.0498**	**254.50**	**0.1387**	**0.0549**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	70.9208
T3	0.2891	0.0000	12.23	0.0085	-0.0026	29.7040
S&S	0.4711	-0.2143	5.05	0.2203	0.0293	20.3151
H&W (t)	0.7508	-0.1285	39.04	0.1809	0.0509	4.3262
FTSE	**0.4984**	**-0.0899**	**29.46**	**0.1428**	**0.0398**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	60.0982
T3	0.2891	0.0000	12.23	0.0085	-0.0026	12.5310
H&W	0.3477	-0.1097	3.935	0.1611	0.0692	3.9898
H&W (t)	0.5929	0.0000	27.90	0.1302	0.0572	0.8267
DAX	**0.3391**	**-0.1740**	**5.70**	**0.0782**	**0.0207**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	22.6601
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	7.8243
H&W	0.2532	-0.1865	3.13	0.0907	0.0342	0.6909
HES (t)	0.3873	-0.1791	7.98	0.0753	0.0451	0.3895
Nikkei	**0.4031**	**-0.1681**	**4.74**	**0.1557**	**0.0629**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	45.0403
T3	0.2891	0.0000	12.23	0.0085	-0.0026	17.8154
S&S	0.3448	-0.1608	3.66	0.1619	0.0599	0.2007
HES (t)	0.4989	-0.1446	9.67	0.1569	0.0858	0.4800
Bund	**0.4519**	**-0.0814**	**6.87**	**0.1661**	**0.1009**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	65.9331
T3	0.2891	0.0000	12.23	0.0085	-0.0026	18.9436
H&W	0.3800	-0.0734	4.41	0.1792	0.0958	0.4187
H&W (t)	0.4920	-0.0793	10.69	0.1605	0.1086	0.2100
BTP	**0.4268**	**-0.1665**	**5.06**	**0.1638**	**0.0826**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	48.1248
T3	0.2891	0.0000	12.23	0.0085	-0.0026	20.4784
S&S	0.3577	-0.1553	3.74	0.1734	0.0658	0.4432
S&S (t)	0.4781	-0.1518	9.04	0.1472	0.0811	0.4695
Gilt	**0.3710**	**-0.1775**	**5.76**	**0.1116**	**0.0663**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	37.0817
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	12.8546
S&S	0.2938	-0.1654	3.33	0.1225	0.0509	0.6061
S&S (t)	0.4108	-0.1701	7.89	0.1076	0.0587	0.1670
US T-Bond	**0.4637**	**-0.0572**	**5.37**	**0.2080**	**0.1643**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	88.0371
T3	0.2891	0.0000	12.23	0.0085	-0.0026	34.1694
S&S	0.4233	-0.0499	4.37	0.2173	0.1591	0.1489
S&S (t)	0.5225	-0.0457	7.43	0.1865	0.1691	0.4379

Markets (Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
D-Mark	**0.3415**	**-0.2432**	**5.45**	**0.0804**	**0.0139**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	22.3236
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	5.0201
HES	0.2678	-0.2609	3.17	0.0988	0.0191	0.6015
HES (t)	0.3862	-0.2504	7.89	0.0700	0.0218	0.2500
Pound	**0.4064**	**-0.1613**	**6.53**	**0.1140**	**0.0655**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	35.5084
T3	0.2891	0.0000	12.23	0.0085	-0.0026	12.9998
S&S	0.2992	-0.1461	4.219	0.1264	0.0595	0.5611
S&S (t)	0.4215	-0.1605	5.122	0.1060	0.0568	0.5336
Yen	**0.3590**	**-0.2792**	**7.79**	**0.0577**	**0.0087**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	17.2707
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	2.3600
S&S	0.2490	-0.2836	3.01	0.0804	0.0166	1.3337
S&S (t)	0.3473	-0.2792	7.58	0.0546	0.0114	0.0141
S-Franc	**0.3111**	**-0.2458**	**5.05**	**0.0660**	**0.0158**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	36.2483
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	4.1261
HES	0.2512	-0.2142	3.08	0.0889	0.0316	0.4771
H&W (t)	0.3657	-0.2477	8.14	0.0650	0.0221	0.2640

Table 5.3, Comparisons of the Empirical Dynamics of Twelve Financial Futures Dispersion Processes with the Dynamics of a GBM Price Series with Constant Variance versus the Best t-distribution with Constant Variance, the Best Stochastic Volatility Model that assume the Underlying Price Series is Lognormal and the Best Stochastic Volatility Model that assumes the Underlying Price Series follows a Student-t-distribution.

As one can see from these tables, the inclusion of both fat-tailed distributions for the price series and the stochastic volatility models dramatically improve the fit of the models. In nine of the twelve markets, the addition of the fat-tailed distribution has led to improvements in the stochastic volatility models. The exceptions are the Nikkei and the BTP, where the addition of the fat-tailed distributions causes a small, marginal increase in the sum of squared errors. The US T-Bond is also somewhat worse by inclusion of the fat-tailed distribution. The optimal stochastic volatility model, which assumed GBM, had a sum of the squared errors of only 0.1489. The best fat-tailed stochastic volatility model has a sum of squared errors of 0.4379. However, these results should be put in perspective. If we compare the original errors from the GBM with constant volatility models (or even the fat-tailed distribution with constant volatility), it is clear that both versions of the stochastic volatility models are a vast improvement. However, the greatest absolute improvement to explaining the dynamics of financial futures volatility is due to the choice of the appropriate stochastic volatility model. The choice of the dispersion process of the underlying asset price is secondary. However, it is clear that both elements contribute to explaining the dynamics of empirical volatility. The stochastic volatility models cover the autocorrelation and line fit conditions, while the fat-tailed distributions address the COV and kurtosis conditions.

For the stock index futures, the DAX has the best fit, while both the FTSE and the Nikkei have SSE measures below one. The S&P 500 is the only market where the SSE measure still remains unacceptably high. Nevertheless, it has been reduced five-fold by the inclusion of the fat-tailed distribution relative to the simulation assuming normality (S&S). The failure to model the S&P 500 perfectly is due to the extremely high kurtosis and the high coefficient of variation condition values that are anomalous. This is due to the impact of the 1987 stock market crash that affected the S&P 500 more than any other of the assets under examination. To assess if this hypothesis is correct, we reran the analysis for the S&P 500 excluding the observations from the 15th to the 25th of October, 1987. Specifically, these ten days were assigned returns of zero and the analysis proceeded as before.

<u>Results of the Combination Models - S&P 500 (including/excluding the 1987 Crash)</u>

The exclusion of these ten days from the analysis has a dramatic impact on the dynamics of the S&P 500 return series. In Table 5.4, we compare the key attributes for the S&P 500 with and without the crash included.

S&P 500 (Crash or Not)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)
With Crash	0.9045	-0.0498	254.50	0.1387	0.0549
Without Crash	0.4404	-0.1076	11.53	0.1069	0.0706

Table 5.4, Comparison of the S&P 500 Futures Dynamics including/excluding 1987 crash.

Clearly, the exclusion of the 10 days surrounding the 1987 crash has a dramatic impact on the dynamics of the time series. The kurtosis drops by a factor of twenty. In addition, the coefficient of variation attribute has been reduced by one half. The time factor has now doubled when the crash is omitted. It is interesting that the autocorrelation attributes do not change as dramatically.

With the crash excluded, we reran the simulations with the stochastic volatility models and the range of Student-t distributions to assess if it would now be possible to fit the S&P 500. The results of this analysis can be seen in Table 5.5.

S&P 500 (Best Model)	Initial Volatility	Long Term Volatility	Mean Reversion	Volatility of Volatility	t- dist
Excluding 1987 Crash					
S&P 500 (Parameter Values)					
H&W	0.20	0.20	3.8	0.84	T2 (k=9.16)
S&S	0.20	0.20	0.8	0.22	T4 (k=15.26)
HES	0.20	0.20	0.91	0.32	T3 (k=12.23)

Markets (Best Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.4404**	**-0.1076**	**11.53**	**0.1069**	**0.0706**	
H&W	0.4845	-0.1250	15.60	0.1015	0.0697	0.4696
S&S	0.4459	-0.1283	8.53	0.1246	0.0793	0.3576
HES	0.4239	-0.1440	7.77	0.1183	0.0728	0.4694

Table 5.5, Best Fitting Models for S&P 500 Futures excluding the 1987 crash.

When the 1987 crash is excluded, each model performs well given that the sum of squared deviations from the SSE technique is close to zero. It is an important result that almost all the simulations require the Student-t distribution assumption (for the price series) to adequately fit the data. Although, a stochastic volatility model performs fairly well (with the assumption of GBM for the underlying price series) and the a Student-t model with constant variance also performs fairly well.

It is interesting to note that when we compare the parameter values across the twelve markets, they differ widely. This means both among the best fitting models for a particular asset and among different assets. In addition, no one model is the best for all the markets but there is a fair amount of consistency in the performance of all the models. This is not surprising as they bear an intimate theoretical relationship. Overall, it appears that all the models can fit the empirical results equally well if the appropriate parameter values and the fat-tailed distribution are carefully chosen. Given the divergences in parameter values for the models among the markets, it appears that the stochastic nature of volatility is not consistent across all financial futures markets.

5.3 ANALYSIS OF COMBINATION MODELS FOR THE FIRST PERIOD

Finally, we examined the fitting ability in the two subperiods of the observations to test the robustness of the combined model. As before, the data was split evenly in two and the optimisation was rerun.

Parameter Estimation for Combination Models for the First Period

The optimised parameter values can be found in Tables 5.6a, 5.6b and 5.6c for the first half of the available observations and the results of these optimised models appear in Tables 5.7a, 5.7b and 5.7c.

Markets (Best Model)	Initial Volatility	Long Term Volatility	Mean Reversion	Volatility of Volatility	t- dist
S&P 500					
H&W	0.20	0.20	31.1	2.61	T2 (k=9.16)
S&S	0.15	0.15	1.65	1.155	T2 (k=9.16)
HES	0.15	0.15	3.1	0.51	T2 (k=9.16)
FTSE					
H&W	0.20	0.20	16	1.68	T2 (k=9.16)
S&S	0.05	0.05	1.99	0.31	T2 (k=9.16)
HES	0.10	0.10	2.9	0.32	T3 (k=12.23)
DAX					
H&W	0.15	0.15	4.2	0.67	T4 (k=15.26)
S&S	0.15	0.15	3.2	0.20	T1 (k=6.10)
HES	0.20	0.20	3.1	0.5	T1 (k=6.10)
Nikkei					
H&W	0.20	0.20	15.5	1.37	T5 (k=4.79)
S&S	0.15	0.15	2.65	0.275	T3 (k=12.23)
HES	0.20	0.20	3.4	0.60	T3 (k=12.23)

Table 5.6a, Parameters for the Best Fitting Models for Four Stock Index Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic for the First Half of the Available Observations.

Markets (Best Model)	Initial Volatility	Long Term Volatility	Mean Reversion	Volatility of Volatility	t- dist
Bund					
H&W	0.10	0.10	4.1	0.96	T3 (k=12.23)
S&S	0.20	0.20	0.3	0.32	T3 (k=12.23)
HES	0.20	0.20	0.67	0.371	T3 (k=12.23)
BTP					
H&W	0.10	0.10	4.7	1.02	T4 (k=15.26)
S&S	0.05	0.05	1.3	0.315	T3 (k=12.23)
HES	0.10	0.10	2.6	0.33	T6 (k=4.54)
Gilt					
H&W	0.15	0.15	2.7	0.80	T4 (k=15.26)
S&S	0.15	0.15	1.41	0.175	T3 (k=12.23)
HES	0.05	0.05	1.5	0.1075	T1 (k=6.10)
US T-Bond					
H&W	0.10	0.10	6.1	1.11	T6 (k=4.54)
S&S	0.20	0.20	0.01	0.41	T3 (k=12.23)
HES	0.05	0.05	0.16	0.406	T8 (k=4.58)

Table 5.6b, Parameters for the Best Fitting Models for Four Fixed Income Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic for the First Half of the Available Observations.

Markets (Best Model)	Initial Volatility	Long Term Volatility	Mean Reversion	Volatility of Volatility	t- dist
D-Mark					
H&W	0.15	0.15	5.3	0.52	T4 (k=15.26)
S&S	0.20	0.20	4.3	0.24	T3 (k=12.23)
HES	0.05	0.05	3.7	0.10	T1 (k=6.10)
Pound					
H&W	0.20	0.20	15.9	1.17	T5 (k=4.79)
S&S	0.20	0.20	3.05	0.26	T6 (k=4.54)
HES	0.20	0.20	3.8	0.51	T8 (k=4.58)
Yen					
H&W	0.15	0.15	4.8	0.37	T4 (k=15.26)
S&S	0.20	0.20	2.9	0.105	T4 (k=15.26)
HES	0.20	0.20	17.5	0.73	T1 (k=6.10)
S-Franc					
H&W	0.15	0.15	3.8	0.51	T4 (k=15.26)
S&S	0.15	0.15	2.9	0.15	T3 (k=12.23)
HES	0.15	0.15	6.2	0.385	T6 (k=4.54)

Table 5.6c, Parameters for the Best Fitting Models for Four Foreign Exchange Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic for the First Half of the Available Observations.

The only point of note in these tables is that different fat-tailed distributions are required to explain the market dynamics. While this may suggest that the results may be due solely to the choice of the Student-t distribution, cross-checking the results with alternative Student-t distributions also lead to improvements in the SSE. The results presented are only for the best of the Student-t distributions.

Results for Combination Models for the First Period

Now we will examine the effectiveness of the combined model in explaining the dynamics of the twelve markets volatility for the first period. This can be seen in Tables 5.7a, 5.7b and 5.7c.

Markets (Best Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.9581**	**-0.0438**	**185.34**	**0.1247**	**0.0304**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*49.8419*
H&W	0.8359	-0.1352	44.75	0.1995	0.0387	3.3567
S&S	0.6734	-0.1299	35.70	0.1626	0.0776	7.6953
HES	0.5735	-0.1783	29.08	0.1167	0.0293	8.1910
FTSE	**0.5963**	**-0.0569**	**35.99**	**0.1615**	**0.0321**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*32.7907*
H&W	0.6564	-0.1283	32.61	0.1522	0.0560	0.6578
S&S	0.6568	-0.1514	33.88	0.1542	0.0574	2.0109
HES	0.4934	-0.1271	12.35	0.1405	0.0427	1.9140
DAX	**0.3612**	**-0.1740**	**6.17**	**0.0883**	**0.0153**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*12.8280*
H&W	0.3848	-0.1995	8.15	0.0832	0.0420	0.3303
S&S	0.3780	-0.2153	6.24	0.0990	0.0471	0.7815
HES	0.4364	-0.2244	11.25	0.1101	0.0288	0.9815
Nikkei	**0.3493**	**-0.1783**	**3.99**	**0.1458**	**0.0456**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*16.4677*
H&W	0.4642	-0.1520	7.90	0.1227	0.0454	0.8435
S&S	0.4369	-0.1909	7.04	0.1349	0.0641	0.7063
HES	0.4675	-0.2020	11.72	0.1279	0.0402	1.4995

Table 5.7a, Results for the Best Fitting Models for Four Stock Index Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic (First Half)

Markets (Best Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
Bund	**0.5175**	**-0.0065**	**9.51**	**0.1890**	**0.1103**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*37.5815*
H&W	0.5356	-0.0708	12.04	0.1857	0.1238	0.3857
S&S	0.5161	-0.0489	7.39	0.1826	0.1643	1.4466
HES	0.5281	-0.0967	9.45	0.1773	0.1227	1.5160
BTP	**0.5193**	**-0.1500**	**6.89**	**0.1971**	**0.0833**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*27.2153*
H&W	0.5836	-0.1381	11.92	0.1958	0.0994	0.2919
S&S	0.5741	-0.1373	12.54	0.1884	0.0947	0.4623
HES	0.4810	-0.1470	7.90	0.1605	0.0799	0.6778
Gilt	**0.3823**	**-0.1535**	**5.34**	**0.1228**	**0.0784**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*17.7907*
H&W	0.4618	-0.1529	9.24	0.1311	0.0693	0.3431
S&S	0.4439	-0.1511	8.28	0.1280	0.0741	0.2345
HES	0.4661	-0.1887	8.95	0.1123	0.0521	1.0414
US T-Bond	**0.4863**	**-0.0399**	**4.54**	**0.2349**	**0.1845**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*45.9846*
H&W	0.5918	-0.0505	13.27	0.2352	0.1699	1.0560
S&S	0.5615	-0.0342	7.86	0.2090	0.1946	0.7597
HES	0.5629	-0.0768	7.39	0.2119	0.1534	0.9066

Table 5.7b, Results for the Best Fitting Models for Four Fixed Income Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic (First Half).

Markets (Best Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
D-Mark	**0.3309**	**-0.3000**	**5.36**	**0.0711**	**-0.0114**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*6.3656*
H&W	0.3333	-0.2811	7.81	0.0474	0.0198	0.5628
S&S	0.3896	-0.3100	7.68	0.0698	0.0091	0.3847
HES	0.3789	-0.2900	7.67	0.0652	0.0127	0.4377
Pound	**0.3721**	**-0.1900**	**6.18**	**0.0917**	**0.0335**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*12.8628*
H&W	0.3895	-0.1797	4.907	0.0826	0.0304	0.4040
S&S	0.3576	-0.2181	4.798	0.0866	0.0437	0.3663
HES	0.3630	-0.1944	4.712	0.0905	0.0418	0.2236
Yen	**0.3383**	**-0.4014**	**7.10**	**0.0495**	**0.0005**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*3.2769*
H&W	0.3033	-0.3687	8.09	0.0248	0.0074	0.2951
S&S	0.3037	-0.3904	7.83	0.0220	-0.0015	0.4676
HES	0.3386	-0.4287	6.93	0.0397	-0.0041	0.1815
S-Franc	**0.3082**	**-0.2748**	**5.03**	**0.0665**	**0.0024**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*6.7588*
H&W	0.3357	-0.2659	7.77	0.0510	0.0244	0.3533
S&S	0.3725	-0.2476	8.83	0.0777	0.0264	0.4064
HES	0.3130	-0.3215	5.63	0.0525	0.0163	0.5676

Table 5.7c, Results for the Best Fitting Models for Four Foreign Exchange Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic (First Half).

As with the entire period, the inclusion of both the optimal stochastic volatility model and the appropriate fat-tailed distribution leads to a significant improvement in the explanatory power of the models. Of the twelve markets, eleven have a SSE that is less than 1.0. The only exception is the S&P 500. This is most probably due to the 1987 crash. The excess kurtosis associated with this event is so anomalous that even the inclusions of extremely leptokurtic student-t distributions are not sufficient to capture this event.

As before, we will summarise the relative goodness of fit for the three models discussed so far just for the period that has the first half of the available data. This will allow us to compare directly the effectiveness of the four models and demonstrate how far we have come in explaining the dynamics of volatility behaviour. This can be seen in Table 5.8.

Markets (Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.9581**	**-0.0438**	**185.34**	**0.1247**	**0.0304**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	49.8419
T3	0.2891	0.0000	12.23	0.0085	-0.0026	21.1225
H&W	0.4903	-0.1139	5.59	0.2543	0.0756	19.0367
H&W (t)	0.8359	-0.1352	44.75	0.1995	0.0387	3.3567
FTSE	**0.5963**	**-0.0569**	**35.99**	**0.1615**	**0.0321**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	32.7907
T3	0.2891	0.0000	12.23	0.0085	-0.0026	10.3718
H&W	0.4237	-0.1269	4.72	0.2134	0.0634	4.8381
H&W (t)	0.6564	-0.1283	32.61	0.1522	0.0560	0.6578
DAX	**0.3612**	**-0.1740**	**6.17**	**0.0883**	**0.0153**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	12.8280
T3	0.2891	0.0000	12.23	0.0085	-0.0026	4.4459
H&W	0.2714	-0.1843	3.25	0.1042	0.0343	0.6725
H&W (t)	0.3848	-0.1995	8.15	0.0832	0.0420	0.3303
Nikkei	**0.3493**	**-0.1783**	**3.99**	**0.1458**	**0.0456**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	16.4677
T3	0.2891	0.0000	12.23	0.0085	-0.0026	9.0880
H&W	0.3337	-0.1682	3.77	0.1491	0.0388	0.0336
S&S (t)	0.4369	-0.1909	7.04	0.1349	0.0641	0.7063
Bund	**0.5175**	**-0.0065**	**9.51**	**0.1890**	**0.1103**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	37.5815
T3	0.2891	0.0000	12.23	0.0085	-0.0026	14.4586
H&W	0.3572	-0.0690	4.21	0.1602	0.0940	1.4893
H&W (t)	0.5356	-0.0708	12.04	0.1857	0.1238	0.3857
BTP	**0.5193**	**-0.1500**	**6.89**	**0.1971**	**0.0833**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	27.2153
T3	0.2891	0.0000	12.23	0.0085	-0.0026	15.3758
H&W	0.4257	-0.1053	4.75	0.2163	0.0795	0.5697
H&W (t)	0.5836	-0.1381	11.92	0.1958	0.0994	0.2919
Gilt	**0.3823**	**-0.1535**	**5.34**	**0.1228**	**0.0784**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	17.7907
T3	0.2891	0.0000	12.23	0.0085	-0.0026	8.0175
H&W	0.3142	-0.1197	3.63	0.1358	0.0606	0.4499
S&S (t)	0.4439	-0.1511	8.28	0.1280	0.0741	0.2345
US T-Bond	**0.4863**	**-0.0399**	**4.54**	**0.2349**	**0.1845**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	45.9846
T3	0.2891	0.0000	12.23	0.0085	-0.0026	26.9706
S&S	0.4644	-0.0341	4.55	0.2521	0.2024	0.0606
S&S (t)	0.5615	-0.0342	7.86	0.2090	0.1946	0.7597

Markets (Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
D-Mark	**0.3309**	**-0.3000**	**5.36**	**0.0711**	**-0.0114**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	6.3656
T1	0.2400	-0.3925	6.10	0.0045	0.0023	2.1262
H&W	0.2441	-0.2662	3.07	0.0810	0.0152	0.7117
S&S (t)	0.3896	-0.3100	7.68	0.0698	0.0091	0.3847
Pound	**0.3721**	**-0.1900**	**6.18**	**0.0917**	**0.0335**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	12.8628
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	5.8328
HES	0.2739	-0.1901	4.17	0.1062	0.0392	0.3290
HES (t)	0.3630	-0.1944	4.712	0.0905	0.0418	0.2236
Yen	**0.3383**	**-0.4014**	**7.10**	**0.0495**	**0.0005**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	3.2769
T1	0.2400	-0.3925	6.10	0.0045	0.0023	0.8417
HES	0.2404	-0.4009	3.13	0.0673	0.0042	0.9272
HES (t)	0.3386	-0.4287	6.93	0.0397	-0.0041	0.1815
S-Franc	**0.3082**	**-0.2748**	**5.03**	**0.0665**	**0.0024**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	6.7588
T1	0.2400	-0.3925	6.10	0.0045	0.0023	2.1932
H&W	0.2436	-0.2694	3.07	0.0804	0.0146	0.3706
H&W (t)	0.3357	-0.2659	7.77	0.0510	0.0244	0.3533

Table 5.8, Comparisons of the Empirical Dynamics of Twelve Financial Futures Dispersion Processes with the Dynamics of a GBM Price Series with Constant Variance versus the Best t-distribution with Constant Variance, the Best Stochastic Volatility Models that assumes the Underlying Price Series is Lognormal or follows a Student-t distribution (First Half).

As with the analysis of the entire period, the addition of the fat-tailed distribution leads to a significant improvement in the explanatory performance of the stochastic volatility models. This is most pronounced for those markets that had the highest level of kurtosis in the period (S&P 500 and FTSE 100). For two markets, the addition of the fat-tailed distribution led to worse results. As before (for the entire period), this included the Nikkei 225 and the US T-Bond. In both cases, they were almost perfectly described by the optimal stochastic volatility model that assumed GBM. Nevertheless, the fat-tailed stochastic volatility models for both markets did produce SSEs that were well below 1.0. Overall, ten of the twelve markets were better explained by the inclusion of the fat-tailed distribution to the stochastic volatility model in the first period. So far, of twenty-four models that included stochastic volatility, nineteen were improved by including a fat-tailed distribution for the evolution of the underlying asset.

Previously, when we compared the stochastic volatility models in the first period, we found that for one market the best fat-tailed distribution had a lower SSE. This was for the Japanese Yen. With the inclusion of the optimal fat-tailed distribution in the stochastic volatility model, we find that this is a substantial improvement over the fat-tailed distribution model with constant variance. Thus, the combined model is superior to the fat-tailed model (constant variance) in all twenty-four cases examined so far.

5.4 ANALYSIS OF COMBINATION MODELS FOR THE SECOND PERIOD

The final analysis was to evaluate the combined model for the second half of the available observations. The optimised parameter values can be found in Tables 5.9a, 5.9b and 5.9c for the second half of the available observations.

Parameter Estimation for Combination Models for the Second Period

Markets (Best Model)	Initial Volatility	Long Term Volatility	Mean Reversion	Volatility of Volatility	t- dist
S&P 500					
H&W	0.20	0.20	20.28	1.104	T6 (k=4.54)
S&S	0.15	0.15	3.5	0.1625	T6 (k=4.54)
HES	0.20	0.20	3.7	0.41	T6 (k=4.54)
FTSE					
H&W	0.15	0.15	14.8	0.866	T6 (k=4.54)
S&S	0.20	0.20	3.4	0.25	T8 (k=4.58)
HES	0.20	0.20	2.95	0.41	T8 (k=4.58)
DAX					
H&W	0.20	0.20	8.1	0.64	T8 (k=4.58)
S&S	0.15	0.15	1.41	0.0965	T8 (k=4.58)
HES	0.20	0.20	2.1	0.27	T8 (k=4.58)
Nikkei					
H&W	0.20	0.20	8.33	1.063	T4 (k=15.26)
S&S	0.15	0.15	2.3	0.2825	T8 (k=4.58)
HES	0.20	0.20	1.79	0.449	T8 (k=4.58)

Table 5.9a, Parameters for the Best Fitting Models for Four Stock Index Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic (Second Half).

Markets (Best Model)	Initial Volatility	Long Term Volatility	Mean Reversion	Volatility of Volatility	t- dist
Bund					
H&W	0.10	0.10	11.87	0.919	T8 (k=4.58)
S&S	0.15	0.15	1.77	0.1935	T8 (k=4.58)
HES	0.20	0.20	1.9	0.44	T8 (k=4.58)
BTP					
H&W	0.20	0.20	18.6	1.00	T7 (k=4.89)
S&S	0.20	0.20	5.96	0.325	T8 (k=4.58)
HES	0.20	0.20	5.89	0.581	T8 (k=4.58)
Gilt					
H&W	0.20	0.20	12.38	1.062	T6 (k=4.54)
S&S	0.20	0.20	1.5	0.1425	T6 (k=4.54)
HES	0.05	0.05	26	0.43	T8 (k=4.58)
US T-Bond					
H&W	0.15	0.15	13.29	0.793	T6 (k=4.54)
S&S	0.20	0.20	2.2	0.19	T6 (k=4.54)
HES	0.15	0.15	1.9	0.30	T6 (k=4.54)

Table 5.9b, Parameters for the Best Fitting Models for Four Fixed Income Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic (Second Half).

Markets (Best Model)	Initial Volatility	Long Term Volatility	Mean Reversion	Volatility of Volatility	t- dist
D-Mark					
H&W	0.20	0.20	14.09	1.089	T6 (k=4.54)
S&S	0.05	0.05	3.9	0.075	T8 (k=4.58)
HES	0.20	0.20	3.0	0.44	T8 (k=4.58)
Pound					
H&W	0.15	0.15	7.74	1.014	T8 (k=4.58)
S&S	0.20	0.20	1.1	0.26	T8 (k=4.58)
HES	0.20	0.20	1.74	0.458	T8 (k=4.58)
Yen					
H&W	0.15	0.15	16.79	0.921	T4 (k=15.26)
S&S	0.15	0.15	2.42	0.158	T4 (k=15.26)
HES	0.20	0.20	1.79	0.449	T1 (k=6.10)
S-Franc					
H&W	0.20	0.20	11.7	0.86	T6 (k=4.54)
S&S	0.15	0.15	3.61	0.1725	T8 (k=4.58)
HES	0.15	0.15	2.6	0.26	T8 (k=4.58)

Table 5.9c, Parameters for the Best Fitting Models for Four Foreign Exchange Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic (Second Half).

It is interesting to note that the choice of Student-t distributions is completely different from the first half of the analysis (or indeed for the entire period). The optimal Student-t distributions had a fairly low level of excess kurtosis. The exception was for the Nikkei and the Japanese Yen, which both had relatively high kurtosis in the second period.

Results for Combination Models for the Second Period

Our final evaluation is for the combined model in the second period. As with the two previous analyses, the inclusion of the fat-tailed distributions has led to a significant improvement (overall) in explaining the market dynamics. This can be seen in Tables 5.10a, 5.10b and 5.10c.

As with the two previous periods, the inclusion of both the optimal stochastic volatility model and the appropriate fat-tailed distribution has led to improvement in the explanatory power of the models. Compared with the previous analysis, the improvement is not as consistent. In this period only seven of the twelve markets have SSE measures that are less than 1.0. Nevertheless, the models are all better than the best stochastic volatility model that assumed GBM. This can be seen in Table 5.11, where all the models for the second period are summarised.

Markets (Best Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.3192**	**-0.2535**	**5.88**	**0.0596**	**0.0224**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*49.0282*
H&W	0.3188	-0.2548	5.66	0.0617	0.0259	0.0523
S&S	0.3533	-0.2360	5.97	0.0826	0.0380	0.1843
HES	0.3264	-0.2453	5.72	0.0672	0.0330	0.1278
FTSE	**0.3119**	**-0.1856**	**4.52**	**0.0888**	**0.0430**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*62.0476*
H&W	0.3208	-0.1894	5.89	0.0669	0.0418	2.0115
S&S	0.3458	-0.1685	5.50	0.0799	0.0483	1.1389
HES	0.3355	-0.1730	5.39	0.0762	0.0500	1.0938
DAX	**0.2824**	**-0.1274**	**4.21**	**0.0481**	**0.0161**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*64.0125*
H&W	0.2829	-0.1513	5.16	0.0442	0.0370	1.5989
S&S	0.2746	-0.1530	5.16	0.0380	0.0368	1.7733
HES	0.2909	-0.1627	5.20	0.0486	0.0384	1.9218
Nikkei	**0.4381**	**-0.1056**	**5.87**	**0.1383**	**0.0704**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*125.2491*
H&W	0.4418	-0.1070	7.36	0.1313	0.0696	1.0508
S&S	0.4560	-0.1491	6.37	0.1460	0.0798	0.7172
HES	0.4183	-0.1199	6.11	0.1278	0.0808	0.4387

Table 5.10a, Results for the Best Fitting Models for Four Stock Index Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic (Second Half).

Markets (Best Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
Bund	**0.3818**	**-0.1004**	**4.66**	**0.1329**	**0.0805**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*110.5935*
H&W	0.3912	-0.0917	6.27	0.1092	0.0769	2.4900
S&S	0.4010	-0.1169	6.02	0.1146	0.0803	1.9355
HES	0.4008	-0.1319	5.88	0.1178	0.0726	1.9164
BTP	**0.3176**	**-0.2600**	**3.94**	**0.1014**	**0.0109**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*46.4193*
H&W	0.3204	-0.2251	5.67	0.0566	0.0174	5.2982
S&S	0.3346	-0.2531	5.46	0.0674	0.0234	3.7670
HES	0.3343	-0.2530	5.40	0.0687	0.0226	3.4997
Gilt	**0.3546**	**-0.1602**	**6.25**	**0.0950**	**0.0504**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*83.6713*
H&W	0.3615	-0.1686	6.42	0.0919	0.0515	0.0682
S&S	0.3484	-0.1542	6.14	0.0849	0.0616	0.3116
HES	0.3768	-0.1507	5.72	0.1020	0.0601	0.3093
US T-Bond	**0.3133**	**-0.1906**	**5.83**	**0.0641**	**0.0348**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*61.9444*
H&W	0.3114	-0.1894	5.81	0.0614	0.0410	0.0595
S&S	0.3354	-0.1773	5.95	0.0759	0.0521	0.1848
HES	0.3305	-0.1831	5.91	0.0730	0.0493	0.1198

Table 5.10b, Results for the Best Fitting Models for Four Fixed Income Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic (Second Half).

Markets (Best Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
D-Mark	**0.3515**	**-0.1570**	**5.52**	**0.0884**	**0.0379**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*77.8149*
H&W	0.3532	-0.1865	6.22	0.0859	0.0450	0.6503
S&S	0.3717	-0.1813	5.66	0.0945	0.0490	0.5283
HES	0.3617	-0.1641	5.58	0.0918	0.0522	0.2143
Pound	**0.4354**	**-0.0846**	**6.81**	**0.1308**	**0.0937**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*138.0695*
H&W	0.4329	-0.0822	6.87	0.1346	0.0911	0.0293
S&S	0.4546	-0.0988	6.63	0.1481	0.0988	0.2243
HES	0.4454	-0.1105	6.35	0.1445	0.0924	0.5197
Yen	**0.3794**	**-0.1876**	**8.59**	**0.0657**	**0.0110**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*85.3851*
H&W	0.3944	-0.2403	8.77	0.0784	0.0177	1.2193
S&S	0.3872	-0.2294	8.06	0.0782	0.0239	1.0456
HES	0.3916	-0.2143	8.02	0.0745	0.0332	1.0691
S-Franc	**0.3148**	**-0.1745**	**5.06**	**0.0655**	**0.0318**	
GBM	*0.1612*	*-0.5331*	*2.989*	*-0.00124*	*0.0031*	*62.3183*
H&W	0.3191	-0.1959	5.85	0.0656	0.0400	0.6566
S&S	0.3106	-0.1973	5.23	0.0583	0.0347	0.2610
HES	0.3202	-0.1571	5.37	0.0663	0.0457	0.2193

Table 5.10c, Results for the Best Fitting Models for Four Foreign Exchange Futures Assuming Price Series follow a Student-t Distribution and the Variance is Stochastic (Second Half).

Markets (Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
S&P 500	**0.3192**	**-0.2535**	**5.88**	**0.0596**	**0.0224**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	49.0282
T1	0.2400	-0.3925	6.10	0.0045	0.0023	12.4521
H&W	0.2905	-0.2276	3.41	0.1137	0.0187	9.7687
H&W (t)	0.3188	-0.2548	5.66	0.0617	0.0259	0.0523
FTSE	**0.3119**	**-0.1856**	**4.52**	**0.0888**	**0.0430**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	62.0476
T1	0.2400	-0.3925	6.10	0.0045	0.0023	27.3332
H&W	0.2897	-0.1669	3.39	0.1181	0.0400	2.9198
HES (t)	0.3355	-0.1730	5.39	0.0762	0.0500	1.0938
DAX	**0.2824**	**-0.1274**	**4.21**	**0.0481**	**0.0161**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	64.0125
T5	0.2128	-0.3900	4.79	-0.0041	0.0002	27.6476
HES	0.2380	-0.1373	3.10	0.0731	0.0489	4.9376
H&W (t)	0.2829	-0.1513	5.16	0.0442	0.0370	1.5989
Nikkei	**0.4381**	**-0.1056**	**5.87**	**0.1383**	**0.0704**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	125.2491
T3	0.2891	0.0000	12.23	0.0085	-0.0026	48.3928
H&W	0.3885	-0.1218	4.32	0.1907	0.0655	5.9381
HES (t)	0.4183	-0.1199	6.11	0.1278	0.0808	0.4387
Bund	**0.3818**	**-0.1004**	**4.66**	**0.1329**	**0.0805**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	110.5935
T3	0.2891	0.0000	12.23	0.0085	-0.0026	51.4817
H&W	0.3523	-0.0720	4.14	0.1572	0.0910	1.6543
HES (t)	0.4008	-0.1319	5.88	0.1178	0.0726	1.9164
BTP	**0.3176**	**-0.2600**	**3.94**	**0.1014**	**0.0109**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	46.4193
T1	0.2400	-0.3925	6.10	0.0045	0.0023	21.9242
HES	0.2902	-0.2357	3.32	0.1158	0.0246	1.5724
HES (t)	0.3343	-0.2530	5.40	0.0687	0.0226	3.4997
Gilt	**0.3546**	**-0.1602**	**6.25**	**0.0950**	**0.0504**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	83.6713
T3	0.2891	0.0000	12.23	0.0085	-0.0026	30.9329
H&W	0.3238	-0.1231	3.70	0.1436	0.0600	9.0557
H&W (t)	0.3615	-0.1686	6.42	0.0919	0.0515	0.0682
US T-Bond	**0.3133**	**-0.1906**	**5.83**	**0.0641**	**0.0348**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	61.9444
T1	0.2400	-0.3925	6.10	0.0045	0.0023	20.4269
H&W	0.2832	-0.1775	3.41	0.1137	0.0187	9.2693
H&W (t)	0.3114	-0.1894	5.81	0.0614	0.0410	0.0595

Markets (Model)	Coefficient of Variation	Time Factor	Uncond. Kurtosis	Autocorr (0-20)	Autocorr (50-70)	Sum of Squared Deviations
D-Mark	**0.3515**	**-0.1570**	**5.52**	**0.0884**	**0.0379**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	77.8149
T3	0.2891	0.0000	12.23	0.0085	-0.0026	31.5012
H&W	0.3206	-0.1800	3.66	0.1415	0.0566	7.2055
HES (t)	0.3617	-0.1641	5.58	0.0918	0.0522	0.2143
Pound	**0.4354**	**-0.0846**	**6.81**	**0.1308**	**0.0937**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	138.0695
T3	0.2891	0.0000	12.23	0.0085	-0.0026	46.0396
H&W	0.3927	-0.0637	4.62	0.1859	0.1056	7.3074
H&W (t)	0.4329	-0.0822	6.87	0.1346	0.0911	0.0293
Yen	**0.3794**	**-0.1876**	**8.59**	**0.0657**	**0.0110**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	85.3851
T4	0.3165	-0.3294	15.26	0.0006	-0.0062	19.7352
H&W	0.3360	-0.1758	3.79	0.1498	0.0351	22.7850
S&S (t)	0.3872	-0.2294	8.06	0.0782	0.0239	1.0456
S-Franc	**0.3148**	**-0.1745**	**5.06**	**0.0655**	**0.0318**	
GBM	0.1612	-0.5331	2.989	-0.00124	0.0031	62.3183
T1	0.2400	-0.3925	6.10	0.0045	0.0023	23.2820
H&W	0.2787	-0.1755	3.31	0.1097	0.0370	6.3877
HES (t)	0.3202	-0.1571	5.37	0.0663	0.0457	0.2193

Table 5.11, Comparisons of the Empirical Dynamics of Twelve Financial Futures Dispersion Processes with the Dynamics of a GBM Price Series with Constant Variance versus the Best t-distribution with Constant Variance, the Best Stochastic Volatility Models that assume the Underlying Price Series is Lognormal or follows a t-distribution (Second Half).

As with the analysis of the entire period, the addition of the fat-tailed distribution leads to a significant improvement in the explanatory performance of the stochastic volatility models. This is most pronounced for those markets that had the highest level of kurtosis in the period (Gilt, Japanese Yen and the British Pound). For two markets, the addition of the fat-tailed distribution led to worse results. In this case, this included the BTP and the Bund futures. In both cases, the SSEs were well above 1.0 and the GBM assumption led to the better fit. Nevertheless, the fat-tailed stochastic volatility models for both markets did produce SSEs that were well below those models that assumed constant volatility.

Overall, ten of the twelve markets were better explained by the inclusion of the fat-tailed distribution to the stochastic volatility model. This compares with nineteen markets of the twenty-four examined that showed improvement (for the first and the entire periods). Overall, of thirty-six comparisons made, twenty-nine were improved by including a fat-tailed distribution to the stochastic volatility models.

An interesting pattern emerges when we examine which markets are better explained by stochastic volatility models that assume GBM. In five of the seven cases, the asset class that performed better was a fixed income futures contract. The BTP and US T-Bond were both better explained with the GBM-stochastic volatility model in two of the three analysis periods. However, the Bund was the only fixed income futures better explained in the latter period. For the stock index futures, the Nikkei was the only market that was better explained by the GBM-stochastic volatility model. In all cases and for all periods, the best model for currencies was the combination model. Furthermore, when we compared the combination model to the fat-tailed model with constant variance, we found that in all cases, the combination models were superior.

5.5 CONCLUSION

We examined the dynamics of twelve financial futures markets from a period extending over a period from the middle of the 1980s until the end of 1996. Initially, we examined the statistical moments of the return series and found they diverged significantly from geometric Brownian motion. Our analysis then concentrated on the evaluation of the volatility of these return series. As a result of this analysis, we identified five key attributes that addressed the dynamics of empirical volatility and explained the divergences from an i.i.d. normal dispersion process. With these attributes, we were able to test a number of modelling approaches to explain these dynamics. To further the rigour of the tests, we examined the dynamics for the entire period and for two sub-periods where the observations were evenly split.

We rejected the assumption that the twelve financial futures prices follow geometric Brownian motion with constant variance for any of the time periods. We were able to show that a class of models that assumes the underlying prices follows a Student-t distribution with constant variance was a significant improvement in explaining the dynamics of these twelve futures markets. For each market and for every period, this Student-t distribution model was superior to the GBM model.

When stochastic volatility models were examined (which assumed GBM for the underlying price movement), thirty-four of the thirty-six cases showed improvement over the fat-tailed model (with constant variance).

Finally, when we combined the stochastic volatility models with fat-tailed distributions, we found that in all cases this combination model was superior to the fat-tailed model with constant variance. In addition, the combination model was superior to the GBM-stochastic volatility models in twenty-nine of the thirty-six cases. For currencies, the combination model was always the best model. For the stock index futures, only the Nikkei 225 futures was better explained by the GBM-stochastic volatility model. In almost all the instances where the GBM-stochastic volatility model was superior, the market analysed was a Fixed Income futures. This may suggest that their prices are more likely to follow geometric Brownian motion but that their volatility is stochastic.

It should not be surprising that different models, parameters and fat-tailed distributions are required to understand the dynamics of different markets. Given that each market has different volatilities and indeed different dispersion characteristics, it is not surprising that it will require different models to understand how these dispersion processes vary. Nevertheless, it is a significant contribution to demonstrate that across almost all financial assets examined in this research a combination of stochastic volatility models with fat-tailed distributions can serve to explain their behaviour almost perfectly.

We surmise that by uncovering those models that best capture the dynamics of the objective processes for these twelve markets, we would also be able to understand the values of contingent claims on these same assets better. However, to assess if this is the case or not, we will now shift the emphasis of this research to examine options prices directly. In the next four Chapters, we will model the dispersion processes implied in options prices. Our objective is to capture these dynamics as well as compare them to the dynamics we have uncovered for the objective processes for these twelve markets.

Security Price Process Models: Do These Have the Correct Properties For Understanding Options Values?

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6.1 INTRODUCTION

The objective of the second part of the dissertation is threefold. Firstly, this research will discern the nature of the risk neutral dispersion processes which can be observed for options on futures contracts on three categories of financial assets: fixed income, stock indices and foreign exchange. The primary goal is to understand how the prices of options contracts provide clues as to the expected dispersion processes of futures markets. This analysis will examine the volatilities implied from the prices of options on these futures. By examining the available universe of options prices available, we will examine the volatility smile structures. We will demonstrate a methodology for standardising the volatility smiles allowing for direct comparisons of the dynamics within and between markets.

This investigation will allow us to compare the expectations of market participants to the findings in the first portion of this research, that the volatility of futures returns is neither stationary nor lognormal. The implied dispersion processes drawn from option prices will also indicate significant degrees of skewness and kurtosis. We will demonstrate that the most extreme divergences from an expected GBM dispersion process occurs as the time period until the expiration of the option becomes shorter. Furthermore, we will demonstrate that certain characteristics of smile structures are consistent through time after they have been standardised relative to the price of the underlying asset and the time to expiration of the option.

Secondly, we will compare the implied levels of skewness and kurtosis to those measures observed for the time series of futures prices that underlie these options. It will be shown that consistencies exist between the objective probabilities, associated with futures returns, and the risk neutral probabilities associated with options on these futures contracts. As with the first portion of this research, we will discern the rationale for the divergence from a lognormal dispersion process and examine the relative importance of jump processes and stochastic volatility models.

Thirdly, recent research has pointed to the use of implied dispersion processes from options to price and hedge options. We will extend this analysis via Monte Carlo hedging simulations to examine the impact of hedging both simple and complex contingent claims. This will allow us to examine the true values of European call options and a wide variety of European exotic options given the objective probability dispersion process (estimated in the first portion of this research) is known. We will compare the costs of hedging assuming the risk manager follows the models that assume GBM with constant variance, but the actual dispersion process has stochastic variance. In addition, we will examine the relative impacts of transactions costs in the true cost of hedging this contingent claims.

6.2 REVIEW OF THE LITERATURE ON RISK NEUTRAL EVALUATION

With the introduction of option contracts on stock index futures, probability density functions can be determined directly from the prices of the options. Given that the prices of the options could be determined theoretically in a variety of ways, a comparison of market prices of options with their theoretical prices would provide insights into how market participants evaluate the dispersion processes of the underlying markets. This dispersion process is known as the Risk Neutral Density Function.

The logic for this approach is drawn from the most general result in modern option pricing theory. Under certain conditions, a contingent claim that depends on the terminal stock price (a European option for example) can be priced by describing the contract as a bundle of state-contingent claims, which are in turn multiplied in each state by the corresponding "Arrow-Debreu" state price and summed across states.

Thus, given N different states, the time t price of a contingent claim expiring at time T would be calculated by the equation:

$$C(t) = \sum_{s=1}^{N} V(s)\, p(s) \qquad (6.1)$$

where V(s) describes the payout at time T and p(s) is the Arrow-Debreu price of state s. If we assume that the risk-free rate, r, is constant, then the sum of all the state prices must be multiplied by a continuous discounted factor, $e^{-r(T-t)}$. The previous equation (6.1) can now be rewritten as:

$$C(t) = \sum_{s=1}^{N} e^{-r(T-t)} V(s) \frac{p(s)}{e^{-r(T-t)}} \qquad (6.2)$$

$$\equiv \sum_{s=1}^{N} e^{-r(T-t)} V(s)\, \pi(s) \qquad (6.3)$$

In the second formula (6.3), the π(s) sum to 1 and the set of these terms has the essential properties of a probability density function.

In the instance that the market participants were risk neutral, then for each state, these π(s) terms would be the same as the objective probability of that state. Said in another way, the set of state-contingent prices divided by the discount factor would be the underlying probability density function.

When the state space is continuous, the price of any contingent claim is derived by simply integrating the payoff over the risk-neutral density of the underlying asset and then discounted at the risk-free interest rate. This can be expressed as:

$$C(t) = e^{-r(T-t)} \int_0^{\infty} V(s) f(s) ds \tag{6.4}$$

where *f(s)* is the risk-neutral density. For the simplest of contingent claims, European call and put options, *V(s)* represents the terminal payoff function. For the B-S-M approach, a constant-variance assumption on the underlying asset price is imposed and the resulting density function *f(s)* is used to price the options.

The Use of Risk Neutrality in Pricing Contingent Claims

Cox and Ross (1976) established that an option's price equals its expected payoff discounted at the risk-free rate where the expectation is taken over the 'risk-neutral', rather than the true, distribution of the underlying asset. As Grundy (1991) points out: "Linking the risk-neutral distribution implicit in option prices to the true distribution remains a comparative mystery. A necessary condition for the risk-neutral pricing methodology to be applicable is that the true and the risk-neutral distribution share a common support. The only information about the true distribution that can be obtained from observed option prices *alone* is information about that support." (page 1045).

According to Bates (1991), "Fundamental to the pricing of European and American options is the derivation of the actual distribution of the asset price of an equivalent "risk neutral" distribution that summarises the prices of relevant Arrow-Debreu state-contingent claims. Options are then priced at the discounted expected value of their future payoffs, using this risk-neutral distribution. For processes such as geometric Brownian motion and constant elasticity of variance for which options are redundant assets that can be replicated by a dynamic trading strategy in the underlying asset and a riskless bond, the equivalent risk-neutral distribution can be derived via no-arbitrage conditions. For more complicated processes such as stochastic volatility and jump-diffusion processes, such replication is not feasible. Deriving the appropriate risk-neutral probability measure in those cases requires pricing volatility risk or jump risk, which typically requires additional restrictions on distributions and/or preferences. The two standard approaches are: 1) assume the additional risk is non-systematic and therefore has price zero; or 2) assume the representative investor has time-separable power utility, and preferably log utility, so that Cox, Ingersoll and Ross (1985) separability results can be invoked to price the additional risk."

Examples of the assumption that the additional risk is non-systematic include: Hull and White (1987a), Johnson and Shanno (1987), and Scott (1987) for pricing options under stochastic volatility and Merton (1976) for pricing options under jump-diffusions. Examples of the use of the time-separable power utility include Wiggins (1987), Melino and Turnbull (1990) for stochastic volatility and Bates (1988), Naik and Lee (1990) and Bates (1991).

Merton (1976) summarises the complication arising when the dispersion of the underlying asset prices depends on both a normal change in prices (that is driven by a geometric Brownian motion) and an abnormal change (that occurs when sudden important new information for an underlying asset market arrives). Such an abnormal change is modelled as a jump process.

6.3 REVIEW OF IMPLIED VOLATILITY

Of the theoretical option models available, the Black-Scholes-Merton (1973) option pricing model has proven to be the most popular tool for the pricing of options and for the management of hedged positions in derivative markets. A major reason for the success of the model is its simplicity, which in turn relies on simplistic assumptions regarding the dispersion of the underlying asset price movements. Option pricing formulae, such as the Black-Scholes-Merton model (1973), relate the price of options to five parameters: the underlying price, the strike price, maturity of the option, the risk free interest rate and the volatility of the underlying asset.

All the parameters are known apart from the volatility that must be estimated. In the first portion of this research, we examined the volatility estimate from the historical dispersion process for the underlying assets. This follows in the spirit of Black-Scholes (1972). We were able to demonstrate that the historical dispersion processes may not be stationary and certainly do not conform to the assumption of an i.i.d. dispersion process.

An alternative approach to determining the volatility input is to take the market prices of options and invert the option pricing formula to determine the volatility implied in the price of the option. Under the strict assumptions of the B-S-M models, the implied volatility is interpreted as the market's estimate of the constant volatility parameter. Even if the underlying asset's volatility is allowed to vary deterministically over time, the implied volatility can be interpreted as the market's assessment of the average volatility over the remaining life of the option. Hull and White (1987a) have shown that even when the volatility is allowed to vary according to some stochastic process, the average volatility for the term of the option's life will provide the correct price of the option.

Probably the greatest advantage of implied volatility is that it is forward looking, while the historical (objective) volatility estimates only examine past dynamics. Even so, some systematic and economically significant divergences have been reported between the options prices predicted by the Black-Scholes-Merton formula and the actual options prices that are observed in the market. The B-S-M formula tends to overprice at-the-money options and underprice out-of-the money options. By inverting the B-S-M formula to solve for the only unknown parameter (the volatility), this means that the volatilities differ across the strike prices. This effect is referred to as the volatility smile.

Therefore, we face a theoretical dilemma: the fact that a multiplicity of volatilities exist is inherently contradictory with the assumptions of most pricing models, which assume volatility is constant and that the dispersion of the underlying asset follows a geometric Brownian motion leading to a lognormal distribution for asset prices. Furthermore, under the assumption of constant volatility that underlies diffusion option pricing, this is theoretically impossible. Nevertheless, there is an abundance of evidence that demonstrates that the objective volatility of assets does not remain constant. Not only was this demonstrated in the first portion of this research, but has also be presented by a number of studies [see Taylor (1994)]. It is of relevance to this portion of the research that there is evidence in the time series behaviour of the implied volatilities that they are not constant over time either [see Shastri and Wethyavivorin (1987)].

Initially, much of the research on implied volatilities concentrated on the ability of implied volatility to predict the future realised volatility (for the time horizon of the option's life). The initial research interest examined common stock options [Latané and Rendleman (1976), Trippi (1977), Chiras and Manaster (1978) and Beckers (1981a)]. Later, more emphasis was placed on stock index options and options on futures. Examples of such research include Fung and Hsieh (1991), Canina and Figlewski (1993) and Fleming (1993). Day and Lewis (1988,1990), Lamoureux and LaStrapes (1991) and Harvey and Whaley (1992) have examined the behaviour of implied volatilities as predictors of future volatility on the S&P 100 stock index. Feinstein (1989) and Park and Sears (1985) examined options on the S&P 500 index futures. They presented evidence that implied volatilities from stock index futures options contain a significant amount of information about futures volatility. However, their interest was in the predictive ability of implied volatility and not to examine how the divergences of implied volatilities across strike prices (and time) provided insights into the risk-neutral dispersion process.

For the currency futures markets, Jorion (1995) examined the predictive ability of implied volatilities on the currency options traded at the Chicago Mercantile Exchange for predicting the actual volatility of the currency futures. Jorion examined three of the currencies that were also examined in this research (Japanese Yen, Deutsche Mark and Swiss Franc). Scott and Tucker (1989) examined the implied volatilities for the options on the spot currencies traded at the Philadelphia Stock Exchange (PHLX). For options on fixed income futures, there have been a number of papers that have examined the empirical behaviours of US T-Bond futures options. The earliest of these papers were by Belongia and Gregory (1984), Latane and Rendelman (1976) and Merville and Overdahl (1986).

Another area of research on implied volatilities popular in the literature is the examination of the implied volatility estimates from multiple options on the same underlying asset. Examples of such research on stock options include Brenner and

Galai (1984), Emanuel and MacBeth (1982), Rubinstein (1985) and Whaley (1982). This is the area of the literature that this research aims to enhance.

It is of interest to us (in this research) to compare the analysis of implied volatilities across a wider range of financial assets. In the first portion of this research, where we chose to use futures contracts on stock indices, fixed income markets and currencies as surrogates for these asset classes. In this portion of the research, we will examine options on these same futures contracts. This analysis follows in the steps of similar research on implied volatilities for these same asset classes completed by Fung and Hsieh (1991). They restricted their analysis to the option on the S&P 500 futures, the option on the US T-Bond futures and the option on the US Dollar/Deutsche Mark futures only. It should be noted that these are also contained within our analysis. The time period of analysis for their research was for option prices for the S&P 500 and US T-Bond futures from March 1, 1983 to July 31, 1989 and for the options on the Deutsche Mark futures from February 26, 1985 to July 31, 1989.

Our analysis differs significantly from Fung and Hsieh (1991) in that they concentrated on understanding the empirical behaviour of at-the-money implied volatilities. Their goal was to assess how effective ATM implied volatility is as a predictor of the actual conditional volatility for the underlying assets. While they acknowledged the existence of the volatility smile (they referred to it as the strike price bias), they chose to ignore out-of-the-money options due to the fact that the inclusion of these would have introduced further errors into their analysis. This makes sense considering that their objective was to understand the informational content of implied volatilities (for prediction of the conditional volatility). Here, we are interested in the informational content of the volatility smile and will not examine the ability of implied volatilities to forecast the actual volatilities.

Determination of Implied Volatility

Most option pricing formulas cannot be inverted analytically, so implied volatility must be calculated numerically. For a good review of the procedures, see Manaster and Koehler (1982). In general, this calculation is accomplished by minimising the absolute difference of the fair value to the market price, $|C(\sigma) - C_M|$, where $C(\sigma)$ is the volatility input into the pricing formula and C_M is the market price of the option. This is minimised using some root-finding program. The most common approaches are the Newton-Raphson algorithm or the method of bisection.[1] The Newton-Raphson algorithm is highly efficient and accurate in the context of estimating implied volatilities of European call options. The method of bisection is somewhat simpler and guaranteed to work for all classes of derivative securities. In our research, we used both techniques. For all European style options, we used the Newton-Raphson algorithm and for American options, we used the method of bisection to obtain the implied volatilities. Applying the little known fact that the time value of an option is an approximate linear function of the volatility can enhance this process.

Brenner and Subrahmanyam (1994) and Feinstein (1988) used this fact to demonstrate that it is relatively simple to estimate the implied volatility directly. For most users of the Black-Scholes model, the implied volatility is computed by a cumbersome numerical procedure. They show that when the price of the underlying of an option is equal to the present value of the strike price, the implied volatility is approximately equal to:

[1] From Financial Options, page 97-99, Figlewski, et al.

$$IV = \hat{\sigma} \approx \frac{2.5}{\sqrt{t}} \cdot (C/S) = \frac{2.5}{\sqrt{t}} \cdot (P/S) \tag{6.5}$$

where: $S = K \cdot e^{-rt}$, which implies that $F = S \cdot e^{rt} = K$. This means that the forward price is equal to the strike price. The time term, t, indicates the time to expiration of the option as a percentage of one year and C (P) is the price of the call (put). While in this paper they restricted the use of the technique to the at-the-money option, Brenner and Subrahmanyam (1988) demonstrated that the formula works quite well even for options that may be as much as 10% on either side of the at-the-money forward. Chance (1993) also examined this approach and made suggestions that enhanced the estimation of the implied volatilities for further out-of-the-money options. Using these approaches, we were able to determine a good starting value for estimating the implied volatility numerically and converge to the correct value rapidly.

Regardless of the method used for the determination of the implied volatility, problems exist with the interpretation of the results. Since in the B-S-M model, the volatility is the only free parameter, in theory, the result should provide us the market's estimate of the average instaneous variance for the remaining period of the options life. However, this is only true if all the other (perfect market) assumptions of the model hold. In fact the procedure inherently incorporates into the volatility estimate all sources of mispricing, including data errors, effects of the bid-ask spread and temporary imbalances in supply and demand. Additional factors could include the assumption of constant interest rates and the existence of dealing costs (apart from the bid-ask spread).

The problem of data errors is simply that the price data for the underlying markets and the options may not be simultaneous. Many studies have attempted to solve this problem by using only closing prices for both securities. This is the approach we used in this research. Even so, it has been pointed out that that problems remain even when the estimation of implied volatilities uses closing prices. Galai (1977) considers this problem. He examined the price behaviour of stock options at the Chicago Board Options Exchange (CBOE) during the first 7 months of its existence. He tested three hypotheses jointly: the mathematical structure of the Black-Scholes formula, the methodology for measurement of its inputs (particularly volatility), and the efficiency of the options market in pricing options.

Regarding the simultaneity of option and underlying stock prices, Cox and Rubinstein (1985) state: "Galai's principal caveat – the questionable accuracy of daily closing prices – is critical. In contrast to stock, where empirical research often makes do with monthly – or, at best weekly – prices, use of even daily prices for options poses several significant difficulties. First, since an option's value depends on the contemporaneous price of its underlying stock, it is very important to know the stock price at the time of the options close. However, as we have already emphasised, since the stock and option often close at different times, the stock close, in many cases, is an inadequate approximation of the stock price at the time of the option close. Second, knowing only the option close, we cannot tell whether the close is at the bid, at the ask, or in between, so we cannot actually know at what price options could have been bought or sold. There remains a band of uncertainty around the close equal to twice the spread, and the magnitude of the spread- though usually one-fourth – is also uncertain. In contrast to stock, this is particularly important for options, since the spread is apt to be a relatively large percentage of the option price. Third, with only closing prices, we have no information about the depth of the market at that price – how many contracts could have been bought or sold at that price. Fourth, for certain other reasons relating to Market Maker behaviour, closing prices may not be representative of the actual trades that could have been made near the end of the day. Fifth, because the underlying conditions that determine an option's value change over time, efficient statistical procedures necessitate use of a large sample of data over small periods of time. For many purposes, daily data do not occur with sufficient frequency." (p 341).

Galai realised that these problems could exist with daily data. To address these problems, Galai used a small sample of CBOE transactions to test for the occurrences of riskless arbitrage opportunities. He did find some instances during the trading day when boundary conditions were breached and appeared to be of sufficient magnitude to provide opportunities for riskless arbitrage. However, it is not clear whether these occurred due to the immaturity of the CBOE market at that time or because it was unclear if transactions could have happened at the prices recorded for the stocks and options could have actually been transacted.

Later research pointed to the same problem. Mayhew (1995) points out that "Even if market participants were to price options according to Black-Scholes, price discreteness, transactions costs, and nonsynchronous trading would cause implied volatilities to differ across options." (page 9). Day and Lewis (1988) also suggest that differences in implied volatilities for different strike prices are due to structural inefficiencies that they refer to as noise. They state: "Two significant sources of noise are the inability to determine whether [closing] option and stock prices reflect bid or ask levels and the failure to observe the option price and the price of the underlying security simultaneously." (page 104).

In the analysis completed for this research, the underlying assets were futures prices that closed at the same point in time as the options contracts. In addition, the only option maturities considered corresponded exactly to the same expiration date for the underlying futures contract. This assures that a simultaneity bias was eliminated. To further address the potential problem of non-simultaneity in data, we restricted our analysis of implied volatilities solely to out-of-the-money options. Thus, if the strike price were below the underlying futures price, we examined put options and if the strike price were above the underlying futures price, we examined call options. If prices are not simultaneous, this will have a direct impact on the intrinsic value of options but a reduced impact on the time value of the options. For in-the-money options, Bates (1991) and Gemmill (1991) have shown that much greater deviations occur in the implied volatilities relative to the out-of-the-money options. They claim that the most possible reason for this is the fact that these options are hardly traded and therefore the prices of the options and the underlying may not be simultaneous.

Another problem in the estimation of implied volatilities is the choice of the appropriate time measure. Belongia and Gregory (1984) estimated the time to expiration on the option contract in terms of trading days instead of calendar days. According to Merville and Overdahl (1986) this is incorrect. This is in error because the time input for the B-S-M model assumes calendar time and not trading time. While French (1984) puts forth an argument that trading time should be used (instead of calendar time), Messerschmidt (1984) demonstrated the error in (the Belongia and Gregory approach of) determining implied volatilities by using trading days. For our analysis, we will use calendar days for the estimation of the empirical implied volatilities. Later, when we test the smile behaviour that would be consistent with the models that best describe the unconditional (objective) dispersion processes for the underlying futures markets (see Chapters 3, 4 and 5), we will use trading time. This is done to be consistent with the manner with which these models were estimated. However, we will present the results in calendar time. In addition, the impact of the time parameter in the B-S-M models is related to the percentage in one year that the time remaining represents. We will simulate the trading day objective models with exactly the same percentage time factor as corresponds to the estimation of the volatility smiles from option markets.

Weighting Implied Volatilities

A problem with the estimation of the implied volatility as a true estimate of the instaneous variance is that the Black-Scholes model prices some options more accurately than others. Brown (1990) points out that since that value of certain options is more sensitive to the choice of the correct measure of volatility than others, it makes sense to weight those options more heavily than others. Schmalensee and Trippi (1978) and Trippi (1977) dealt with is potential problem by simply throwing out the implied volatilities for options that were near to expiration or that had strike prices which were far from the current price of the underlying asset. This assumed that the Black-Scholes model is more effective in pricing at-the-money options with neither too long or too short a period to expiration.

This approach has its limitations as the number of observations is restricted and this leads to a potential sampling issue. Another approach is to include all the available implied volatilities, but provide some weighting scheme to arrive at the estimate. Essentially, we are attempting to determine a point estimate of the instaneous volatility. The idea behind this approach is straightforward: if the model is correct, then deviations from the predicted prices represent noise, and noise can be minimised by using more observations. Brown (1990) suggests a number of weighting schemes. The simplest approach is to use equal weights. This approach was suggested by Trippi (1977) and by Schmalensee and Trippi (1978).

Another approach was to compute the sensitivity of the options price to the implied volatility (commonly referred to as vega) and use this to weight each observed implied volatility. This approach was suggested by Latané and Rendleman (1976). This approach has the advantage of weighting the volatilities according to their sensitivities to the correct level of volatility. The logic is that those options that are most sensitive to a mis-specification in the volatility parameter will lead traders to more carefully price these securities and assure the best estimate of the volatility. The problem with this approach is that the weighting scheme does not sum to 1 and thus the weighted average must be divided by the sum of the weights. Another approach was suggested by Chiras and Manaster (1978) that suggested weighting not by the vegas but by the volatility elasticities.

Alternatively, a method suggested by Brown minimises the sum of squares of the differences between all observed market prices for options with theoretical option prices estimated with a single volatility input. This approach was suggested both by Beckers (1981a) and Whaley (1982).

A key question is: Which of these approaches is best? The early literature addressed this question by examining which approach provided the most accurate estimate of the actual future volatility that occurred over the life of the options. Beckers (1981a) addressed this question empirically using daily prices of equity options from 1975 to 1976. Of the three approaches (equal weighting, vega weighting and minimised least square errors), he found that the squared-error-minimising technique led to better forecasts than the simple equal weighting scheme. He also found that simply using the implied volatility of the option with the highest vega, performed better than any of the other methods.

Similar research by Whaley (1982) found that the squared-error-minimising technique also appeared to work better than the other approaches for US stock options from 1975 to 1978. Gemmill (1986) examined equity options on the London Traded Option Market and found similar results to Beckers. Scott and Tucker (1989), Fung, Lie and Moreno (1990), Edey and Eliot (1992) examined various weighting schemes for currency option's implied volatilities. They found that all the weighting schemes provided approximately the same predictive power.

The most promising use of weighted-average implied volatilities has been for the construction of volatility indices, which have been suggested as a means to hedge changes in volatility. Gastineau (1977) and Whaley (1993) have suggested such indices.

The key premise underlying these weighting approaches is that the existence of different implied volatilities at different strike prices (for the same underlying asset) is a sampling or noise problem. One would expect such problems would become less pronounced as the options markets became more mature. The empirical findings were exactly the reverse. Not only did such patterns continue to exist in certain markets, but became even more extreme for certain markets. Thus, in the literature, a new tact was taken for the analysis of implied volatilities. Rather than denigrate the divergences in implied volatilities across strike prices (as sampling errors or noise), these divergences were examined in detail as providing clues to expected market dynamics. This approach is commonly referred to as the volatility smile and is the thrust of this portion of the research.

6.4 DIVERGENCES OF IMPLIED VOLATILITIES ACROSS STRIKE PRICES: THE VOLATILITY SMILE

The starting point for the estimation of implied volatility is the assumption that the Black-Scholes-Merton model accurately describes conditions in actual options markets. The major assumptions are that the prices of underlying assets evolve through time lognormally with a constant volatility σ at any time and market level. The fact that the implied volatilities differ across strike prices for the same maturity and across diverse expiration periods has led many to question the efficacy of the traditional Black-Scholes-Merton methodology and all those pricing models based upon similar assumptions.

If market participants accept the assumptions of the B-S-M model, then the volatilities should be identical across the strike prices. The fact that the volatilities differ systematically across the strike prices has been well documented and has been referred to in the academic literature as the strike price bias and among practitioners as the volatility smile, smirk or skew. This result and subsequent examination of these patterns leads to a paradox. It is fundamentally inconsistent with the theory used to derive the models (that we in turn use to estimate the implied volatilities) to even consider these patterns. The implied volatilities are estimated from a model that assumes a constant volatility. This assumption is critical to the derivation of the model in the first place. The existence of different implied volatilities means that the Black-Scholes - Merton model must be rejected. Thus, the implied volatilities from the B-S-M model are in a real sense meaningless and can no longer be interpreted as the market's assessment of the underlying market's volatility.

For our purposes, the volatility smile will represent a U shaped function that is symmetrical (and horizontally tangent) about the at-the-money implied volatility. The smirk or skew is a shape (that may also be convex) that is higher on one side of the strike price ranges (away from the at-the-money implied volatility) compared to the other (tangent to some non-horizontal line). For many of the markets examined (stock index futures and to a lesser extent fixed income futures), the implied volatilities increase monotonically as the option exercise price falls relative to the underlying price level. Dumas, Fleming and Whaley (1996) have noted this effect and coined this phenomenon the 'smirk'. They also demonstrated that the smirk shape is time dependent; becoming more extreme for options as the time to expiration comes closer. We will also examine this time effect and will report similar results.

For the purpose of this research, we shall refer to all patterns of implied volatilities relative to their strike prices as the ***smile***. Other authors have referred to these patterns in a number of other ways. As was discussed above, some researchers have referred to this pattern as the smirk. Other authors have referred to it as the volatility skew. In some ways, the inconsistent manner in which the divergences of implied volatilities across strike prices are referred to in the literature is unfortunate. When these patterns are referred to as 'skews' that suggests a formal statistical concept of skewness. However, these patterns also display curvature, which is why they are sometimes referred to as smiles or smirks. These patterns suggest that the curvature is convex relative to some linear function.

In this research, we will choose our terms carefully, because we will show that when the patterns of implied volatilities are examined, both skewness and curvature exist. It is critical is to separate both elements in the patterns and understand the statistical implications of these results. Specifically, these risk neutral dispersion processes may help us to understand how market participants expect the actual dispersion process to vary from GBM. Given that from a statistical perspective we require additional moments to model a non-normal distribution completely, the separation of the implied volatility patterns will allow a proxy for measuring the pure skewness and kurtosis imbedded in the risk neutral distributions implied by option prices. Therefore, we will only refer to the patterns of implied volatilities as the ***smile***. Later, when we investigate the multifaceted dynamics of these patterns, we will distinguish between the skew and the curvature at the appropriate points.

Furthermore, for this research, we will refine our definition of the volatility smile. In this research, the volatility smile is defined as the graph of the (relative) implied volatilities determined by the appropriate model, plotted against a standardised strike price of the option. Given that the overall levels of the implied volatilities differ widely both for single markets and across markets, scaling adjustments are required for both cross-sectional and time series comparisons.

While a number of approaches have been proposed to standardise the implied volatilities, the simplest approach is to create an index where the implied volatilities at each strike price are expressed as the ratio to the implied volatility of the option closest to the at-the-money level. Fung and Hsieh (1991), Tompkins (1994) and Natenberg (1994) have all used this approach. All these approaches take the simple ratio between the implied volatility at each strike price divided by the ATM implied volatility and multiply the result by 100 (or express the result in percentages). This transformation is required because the levels of volatility are not constant. The logic behind this approach is that the relative relationships between the volatilities and not the absolute levels are of interest.

The strike prices must also be standardised to allow comparisons to be drawn. A simple approach suggested by Tompkins (1994) was to take the ratio of the strike price to the underlying price. Jackwerth and Rubinstein (1996) used the same approach. A similar approach was used by Fung and Hsieh (1991), the difference being that they inverted this ratio. While this has practical advantages for market participants (namely it is simple to reverse the equation to obtain actual strike prices), the approach is inconsistent with the assumptions of the B-S-M option pricing models. Natenberg (1994) has proposed a more consistent approach. Specifically, strike price is expressed as the ratio of the strike price X of the option relative to the underlying futures price F, standardised for both the time remaining until the expiration of the option and the level of the strike prices and underlying prices. This will be expressed as:

$$\frac{\ln(X_\tau / F_\tau)}{\sigma\sqrt{\tau / 365}} \qquad (6.6)$$

where X is the strike price of the option, F is the underlying futures price and the square root of time factor reflects the percentage in a year of the remaining time until the expiration of the option. The sigma (σ) is the at-the-money volatility. For this analysis, we will assume that the relevant time is calendar days and will express time as the percentage of calendar days remaining in the options life to the total trading time in a year (which we assume is 365 days).

This transformation is consistent with the assumptions of the B-S-M model, where the movement in the underlying asset is measured on a logarithmic scale. From the first term in the Black-Scholes model [N(d1)], the relationship between the exercise price of an option and the current underlying price is expressed as the logarithm of the exercise price divided by the underlying price. At the same time, GBM assumes that movement over time is governed by a square root relationship, so that in the Black-Scholes model, the relative amount of movement (for the underlying asset) to reach a strike price is fully expressed by the above formula 6.6. Finally, the inclusion of the at-the-money volatility will allow us to express the strike prices in standard deviation terms. This will allow us to compare the smile relationships between and within markets more directly.

By adjusting the strike prices of the options using this formula, we are removing the impacts of the B-S-M models, which would obscure the true relationships between the implied volatilities. We have removed the impacts of non-constant levels of volatility and the impacts of time on the patterns of volatilities across strikes.

Review of the Literature on Volatility Smiles

When implied volatility patterns across strike prices have been examined in the literature, authors have identified both the existence of skewed and curved shapes. Many have offered explanations for their findings.

Fischer Black was the first person to comment on the existence of implied volatility patterns. He proposed that a negative correlation between stock price changes and volatility changes should result in a negatively skewed relationship. He came to this conclusion from his examination of the objective processes for equities. He pointed out in Black (1975): "A stock that drops sharply in price is likely to show a higher volatility in the future (in percentage terms) than a stock that rises sharply in price" (page 7). It is interesting that when he examined the actual implied volatility patterns for option on equities, the empirical results were exactly opposite to his hypothesis. He found that the skew was positive. That is, the lower strike price options (in-the-money calls) had lower volatilities and higher strike price options (out-of-the-money calls) had higher volatilities. While this is an interesting result, it could be due to the inefficiency of the options markets in the early to mid 1970s. Most of subsequent research on equity volatilities displays the opposite pattern.

MacBeth and Merville (1979) next re-examined the biases of the B-S-M implied volatilities across strike prices. They reported that the B-S-M model systematically undervalues in-the-money calls (strike prices lower than the current market price) and overvalues out-of-the-money calls (strike prices higher than the current market price). In terms of implied volatilities, this means that the implied volatilities of lower strike price calls are higher than the at-the-money implied volatility and the implied volatilities of higher strike price calls are lower than that at-the-money implied volatility. This effect is the "volatility skew" we discussed previously. Subsequent research found the contrary result and in certain instances the skew was reversed. These results are summarised in Galai (1983,1987).

Black (1975) also identified for the first time the existence of smiles. He states "Options that are way out of the money tend to be overpriced, and options that are way into the money tend to be underpriced". Black suggested that this result could be due to the assumption of constant variance in his model. Black (1976a) states: "... if the volatility of a stock changes over time, the option formulae that assume a constant volatility are wrong." (page 177).

Probably the most complete of the early studies which examined implied volatility discrepancies was that of Rubinstein (1985). He completed nonparametric tests of implied volatilities of options and compared them to the assumption of a constant volatility. Rubinstein found that the implied volatilities of call options with higher strike prices (out-of-the-money) are systematically higher for options with shorter times to expiration. His other results were contradictory and led to the conclusion that while systematic deviations from the B-S-M model appear to exist, the pattern of the divergences varies over time. A similar study was completed by Culumovic and Welch (1994) for more recent data and also confirmed that the patterns diverge over time.

Many other authors have found evidence for volatility smiles (in a wide variety of markets). These include Shastri and Tandon (1986), Kemna (1989), Xu and Taylor (1993) and Heynen, Kemna and Vorst (1994). One general result from all of these is that ever since the stock market crash of 1987, the market evidently does not price all options according to the Black-Scholes formula. The general consensus is that the model works well for at-the-money options with between 30 and 60 days to expiration. However, for options with strike prices far from the current level of the underlying market price and with either a long or short time to expiry for the option, consistent and systematic biases occur. Since the Black-Scholes model describes certain options well and others poorly, it makes sense for different options to have different B-S-M implied volatilities.

Rubinstein (1994) examined options on the S&P 500 index and found that prior to the 1987 stock market crash the implied volatilities failed to display any skewed relationship. Dumas, Fleming and Whaley (1996) confirm this finding. They report that prior to the crash, the implied volatilities formed a symmetrical smile pattern. However, after the 1987 crash, Rubinstein observed that the implied volatilities of options with strike prices below the current underlying price were higher than options with strike prices that were above the current underlying price. He states: "One is tempted to hypothesise that the stock market crash of October 1987 changed the way market participants viewed index options. Out-of-the-money puts (and hence in-the-money calls perforce by put-call parity) became valued much more highly". Dumas, Fleming and Whaley (1996) also report this result.

Other authors who have examined the behaviour of implied volatility patterns for stock index futures pre- and post- stock market crash include Clewlow and Xu (1994). They report from their analysis of options on the S&P 500 futures (for the years 1987 to 1989) that: "The implied volatilities exhibit substantial skews in addition to the smiles. This indicates that changes in volatility are negatively correlated with the returns as we demonstrate with the properties of the Hull & White (1988) model." (page 14). They go on to note that "The most striking facet is the persistent negative skew (that is the slope is negative with respect to increasing strike price) in the 1989 data which is not present to the same degree in 1987." (page 9).

A later paper by Jackwerth and Rubinstein (1996) confirms these same results. They state: "We find that the implied probability distributions in the pre-crash period are some-what left-skewed and platykurtic.... After the crash in the fourth quarter of 1987, we find a period of adjustment where the distributions become more left-skewed and change from platykurtic to leptokurtic. This adjustment is completed by mid-1988. Thereafter, we observe very consistent levels for both skewness and kurtosis." (page 1629).

As was mentioned earlier, Dumas, Fleming and Whaley (1996) indicated that the natures of the divergences in the implied volatility patterns are time dependent. It is well established that the closer the option is to its expiration, the more extreme the divergences are. According to Barnaud and Dabouineau (1992), "Volatility smile curves are intended to overcome the presence of excessive skewness and/or excess kurtosis. Different strike prices would imply different volatilities for the same maturity, in spite of the Black-Scholes constant volatility hypothesis. Moreover, the basic U-shape around the money is not stable and its depth seems to be technically correlated to time-to-maturity and liquidity, as measured by trading volume and open interest. The smile curve effects flatten out when considering options on the same asset with different maturities. The term structure of implied volatility incorporates smoothing properties in longer-term price return stochastic processes." (Page 110).

Thus, there is ample evidence in the literature that implied volatility varies across time as well as the strike price. Generally speaking, the pattern of implied volatilities, which are observed across time, is referred to as the "term structure of volatility". When market participants combine the term structure and the volatility smiles, they determine a "volatility matrix" which displays both patterns across strike prices and time. This will be discussed in some detail later in this Chapter. The volatility matrix is the most common (and practical) approach for dealing with discrepancies in the implied volatilities from the B-S-M assumptions. In the next Chapter, we will describe exactly how such a volatility matrix is estimated using historical volatility smiles.

While there is little doubt that such patterns exist, most of the literature has concentrated either on explaining why implied volatilities diverge from the assumption of constant variance or methods to correct for the biases in the B-S-M models.

Theoretical Reasons for the Existence of Volatility Smiles

The possible reasons for the existence of the volatility smiles and term structure of volatility is that either: 1) market imperfections exist that systematically prevent option prices from taking their true B-S-M values or 2) the underlying asset price process differs from the lognormal diffusion process assumed by the B-S-M model.

Considering market imperfections that could explain the existence of the smile, researchers have chosen to examine problems with the liquidity for out-of-the-money options. An additional impact of discrepancies in the supply and demand for leverage that these out-of-the-money options offer to speculators could be combined with this factor. On the other hand, the sellers of options would be devastated when the out-of-the-money options became in-the-money and would only sell them if they received a minimum amount for their trouble. There is an old adage in the options market never to sell an option for less than 2 ticks. If this is the case, then the actual options prices and their associated implied volatilities might be much higher than the fair price and "fair" volatility of the option. Cochrane and Saá-Requejo (1996) suggest that the reason for the existence of smiles is that "Out-of-the-money options are harder to hedge with the underlying asset, suggesting that the Black-Scholes formula is more sensitive to infrequent trading for such options" (page 23).

What this line of argument really suggests is that transaction costs may cause the volatility smiles. Figlewski (1989) examined the effects of transaction costs on the boundary values for options by simulating a large number of price paths and found that the arbitrage boundaries were 'disturbingly wide'. These costs can be only one of many factors that is impounded in the implied volatility. As Figlewski points out: "[the] implied volatility serves as a free parameter. It impounds expected volatility and *everything else* that affects option supply and demand but is not in the model." (page 13). Clearly, the Black-Scholes model does not include transaction costs and they could be a major element in the divergences of volatilities across strike prices. Nevertheless, Constantinides (1996) who also examined the impact of transaction costs on the behaviour of implied volatilities across strike prices arrived at a different conclusion. He examined the boundaries of option prices derived in the presence of proportional transaction costs. He found that the bounds are sufficiently tight to reject the hypothesis that transaction costs can account for the volatility smile in an otherwise Black-Scholes-Merton market environment. Thus, we must look elsewhere for the reasons.

Another explanation is that out-of-the-money options are quoted in discrete prices and are often subject to some minimum level. This could imply that simply bridging the bid/offer spread could result in a disproportionally large impact on the estimation of the implied volatility. Consider a deep out of the money option that had a theoretical value of 0.02 and the bid price was 0.01 offered at 0.03. If the evaluation of the implied volatility was made at the offered price of 0.03, this would lead to a much higher implied volatility for that option than at its fair price. Because the theoretical options pricing models cannot incorporate transaction costs or the bid/offer spread this exaggerates the impact of the implied volatility result from inverting the options pricing model. Cox and Rubinstein (1985) state their belief that one true volatility does exist but differences "will be due primarily to [the lack of simultaneity in quoted stock and option prices and] the inherent coarseness of prices that are quoted in units of 12.5 cents (or 6.7 cents) rather than one cent" (page 278).

Another factor that could easily impact the volatility smiles is that option prices (in the traded markets we are examining) are subject to some minimum level. For example, in the FTSE 100 options, the price can never fall below 0.5 (which represents £5). In this case, when a range of strike prices are far enough from the underlying market price, they approach a theoretical value of 0. However, if this is not allowed, then as option strike prices become further and further out-of-the-money and the option price remains locked at the minimum level, the Black-Scholes model will assign higher implied volatilities for the further out-of-the-money options. Merton

(1973) demonstrated that the further away from the underlying price a strike price is, the lower the option price must become. The establishment of a minimum option price would by definition suggest that an (butterfly) arbitrage exists and would cause the extremely out-of-the-money options to have progressively higher implied volatilities. Most empirical research that examines the strike price biases associated with option prices have chosen to ignore these options [see Jackwerth and Rubinstein (1996)], as will we. Nevertheless, this suggests that discrete nature of the option prices may have a profound influence on the determination of implied volatilities that assume continuous prices.

According to Cox and Rubinstein (1985), another possible reason for different volatilities in the smile is the early exercise potential of American options. They also explain the existence of different volatilities at different maturities as a direct result of stochastic volatility. They state: “In principle, different options on the same underlying stock may have different implied volatilities. If the volatility is changing over time, then options with different expiration dates would not be expected to have the same implicit volatility. Even for options with the same expiration date, some difference should arise because the possibility of early exercise means that the actual lifetime of the options may not be the same.” (page 278). Geske and Roll (1984) also suggested that the empirical biases were a result of mispricing due to the phenomenological formula for the pricing of American options proposed by Black (1975).

Dumas, Fleming and Whaley (1996) suggest that the behaviour of market participants may be the reason. They state: "... the "smile" [skew] problem may not be a deficiency of the Black/Scholes model. After the October 1987 crash, portfolio insurers began buying exchange-traded index options to replace dynamic portfolio insurance schemes. Buying out-of-the-money index puts surely drives Black/Scholes implied volatilities higher if no one is actively arbitraging according to the Black/Scholes model. With institutional buying pressures for out-of-the-money puts and no naturally offsetting selling pressure, index put prices rise to a level where market makers are eventually willing to step in and accept the bet that the index level will not fall below the exercise price before the option's expiration (i.e. they sell naked puts). So, even if the Black/Scholes model is correct, trading costs combined with option series clienteles may induce patterns in implied volatilities, with these patterns implying little in terms of the distributional properties of the underlying index." (page 21).

Figlewski (1989a) also suggests that the reason for the existence of volatility smiles is due to the demands of option users. He suggests that the higher prices (and resulting higher implied volatility) associated with out-of-the-money options exists because people simply like the combination of a large potential payoff and limited risk. He likens out-of-the-money options to lottery tickets with prices such that they embody an expected loss. Nevertheless, this does not dissuade some from purchasing them.

Another possible reason for the existence of smiles is that the volatility of the underlying asset could be correlated to the level of interest rates. Scott (1994) tested this using data for the S&P 500 and 3 month T-bills. He found that the correlations between monthly changes in stock return volatility and changes in T-bill rates is –0.08 for the period 1970-1990. If the stock market crash of 1987 is omitted, this correlation is only +0.06. Thus, we can conclude that there is practically no relationship between changes in interest rates and changes in stock market volatility.

For equity markets, some have argued that the existence of the skew is due to a simple economic mechanism; the fact that there is an inverse relationship between the levels of stock prices and the variance of return. One of the earliest explanations is that as the firm's stock price falls, the market value of its equity tends to fall more rapidly than the market value of its debt, causing the debt-to-equity ratio to rise. Thus, the riskiness of the stock increases. A similar effect can be observed even if a firm has almost no debt, since every firm faces fixed costs, which have to be met regardless of income. If a decrease in income occurs, the value of the firm will decrease and at the same time the riskiness will increase. These operating and financial leverage effects have been one possible reason for the skewed relationship in volatilities. They were pointed out by Black (1975) and Schmalensee and Trippi (1978).

Apart from these possible reasons for the existence of smiles, most research suggests as the most plausible explanation that the market does not believe the dispersion of the underlying asset to follow a lognormal distribution. There are a wide variety of possible distributions that could be explain the existence of smiles. The choice of these distributions depends on whether one is attempting to explain the skew or the kurtosis effects we observed.

Most of this research follows a parallel path to the models that attempt to capture the dynamics of the objective processes. As with the literature review in Chapter one, the same models have been suggested as candidates for underlying the dynamics of the risk-neutral distributions.

Generally speaking two dispersion processes have been proposed to explain the existence of excess kurtosis implied from the volatility smiles: stochastic volatility models and jump processes. Johnson and Shanno (1987) state: "It is possible that exercise price biases can be explained in a variety of ways, e.g. with a jump process, with the displaced diffusion model of Rubinstein (1983), or with stochastic interest rates; however, only a model with stochastic changing variance appears capable of explaining both the stock return data and the options data." (page 144).

Their conclusion comes from the extreme rich literature on the derivation and empirical testing of stochastic volatility models. Apart from Johnson and Shanno (1987), the effects of stochastic volatility upon options prices have been examined by Hull & White (1987b), Scott (1987), Wiggins (1987), Stein & Stein (1991) and Heston (1993). All these papers show that predicted European option prices from these models tend to produce options prices that are lower than the Black-Scholes option prices for at the money options and greater for out of the money options. For deep out of the money options, the results are sensitive to the parameters of the stochastic process describing changes in volatility and the correlation between changes in volatility and the price of the underlying asset.

Hull and White (1987b) discuss what they term the time-to-maturity effect This effect was assessed by running simulations of options prices using their model and comparing the implied volatilities of these prices using the Black-Scholes formula. When this is done, it appears that the longer the maturity of the option, the lower the implied volatility of the at-the-money option and the higher the implied volatility of out-of-the-money options. Thus, the Hull and White stochastic volatility model would predict smiles to become more extreme in shape the longer the maturity of options.

Stein & Stein (1991) also examined the impacts of stochastic volatility on option prices by running simulations with a variety of parameter values for the initial volatility level, σ_0, the long term volatility θ, the rate of mean reversion, δ and the volatility of the volatility, κ. Apart from determining options prices for a variety of maturities, they also determined the implied volatility of these prices from a Black-Scholes formula. They concluded: "Several observations emerge from the table. First, stochastic volatility exerts an upward influence on all options prices. Whenever $\sigma_0 = \theta$, the new price exceeds the Black-Scholes price for the same θ. Second, stochastic volatility is "more important" for away-from-the-money options than for at-the-money options, in the sense that the implied volatilities corresponding to the new prices exhibit a U-shape as the strike price is varied. Implied volatility is lowest at-the-money and rises as the strike price moves in either direction." (page 738).

Subsequent research by Sheikh (1991) and Heynen (1993) used the Rubinstein nonparametric tests to examine implied volatility patterns in index options. Sheikh found smile effects for the US index options market (S&P 100) and concluded that the existence of the smile constitutes evidence against the B-S-M model and in favour of a stochastic volatility model. On the other hand, Heynen examining European index option implied volatilities also found systematic smile effects. However, when he reviewed the predictions of various stochastic volatility models, he found the observed smile pattern to be inconsistent with them. He suggested that market imperfections were the most likely explanation for the existence of volatility smiles.

Ball (1993) notes that there is evidence that stochastic volatility models do explain the smile effect in option pricing. Hull and White (1987b) make the most significant contributions in this analysis. According to Ball, "Using their expectation paradigm, and noting that the Black-Scholes pricing formula is everywhere concave in variance only for at the money options, we may invoke Jensen's inequality. Consequently, we have that the stochastic volatility option price is lower than its constant volatility counterpart for at the money options. This result breaks down for out of the money options and simulations by H&W indicate the extent of the smile effect under these conditions." (page 16).

Hilliard and Schwartz (1996) also addressed the question: "Why does the smile exist?" by postulating a stochastic volatility model. Similar to research presented in the first portion of this research, they found that most sensitive parameter for the determination of the degree of leptokurtosis was the volatility of the volatility input. They were able to create simulated smiles that appear to be similar to shapes observed in the market by increasing the volatility of the volatility parameter to an extremely high level. They fail to report any of the other statistics of the underlying price series dynamics that would have been associated with this approach. Given other research on simulating stochastic volatility models, it would be expected that the longer the term to expiration, the greater the impact of the model on the prices of the options. Therefore, we would expect that the smiles would become more extreme the further out in time one goes.

Thus, we observe in all the tests of stochastic volatility models that the longer the time period to expiration for the option, the greater the impact of the stochastic volatility on the prices of options. However, the predicted results of these models are inconsistent with the empirical evidence. Bookstaber and McDonald (1987) found that longer term options are better described by a lognormal distribution. They state, "If volatility were nonstationary, then our results would lead one to expect long-horizon returns that look substantially fatter-tailed than lognormals." (page 742). This would cause the volatility smiles to flatten out as the time to expiration is extended. This is exactly the opposite of what would be predicted by this class of models.

Apart from the fact that the stochastic volatility models fail to produce smile behaviour that is consistent with actual smiles, there is a more fundamental problem with these models. Neuberger and Hodges (1996) examined the impacts on options prices when the market is incomplete due to the lack of a traded asset that is volatility sensitive. When they introduced such a volatility-sensitive asset into their economy, they report the gains from introducing such an asset is relatively minor. Scott (1992) examined the impact of options prices in a market in which volatility and interest rates change randomly and options are priced correctly. He concludes that: "The empirical results suggest that there is an economically significant negative risk premium associated with volatility" (page 3). The lack of a volatility traded asset and the problem of estimating risk premiums for heterogeneous market participants is a serious limitation for this model.

A new tact in the literature is to examine stochastic volatility models that are consistent with the implied volatility smile and use this information to provide insights into the assumed dispersion process. Dupire (1992) also examined the possibility that the existence of smile structures is caused by stochastic volatility. The presence of stochastic volatility implies that, since volatility itself is not a traded asset, it is not possible to create a riskless hedge. To price options, equilibrium arguments must be used and a risk premium for volatility must be specified. Dupire's contribution is to treat the prices of standard options as exogenous inputs and use these to derive the nature of the stochastic process driving both the underlying asset price and the volatility which allows no arbitrage opportunities. This approach is similar in spirit to the modelling of interest rates by Ho and Lee (1986) and Heath, Jarrow and Morton (1989).

Dupire's aim is not to explain the standard option prices observed in the market but rather to assume that since they are traded assets they are fairly priced otherwise arbitrage would exist. Assuming such equilibrium exists, we can use the prices of these assets to assess the appropriate dispersion process, which is consistent with these prices. This approach will be examined in detail later in this Chapter.

Another problem with the stochastic volatility models is that they fail to explain the high degree of skewness that is observed in the volatility smiles of many markets. Most of the literature has concentrated on the analysis of skews for equities and stock index options. However, Merville and Overdahl (1986) identified that a negative skew also exists for options on US T-Bond futures. Significantly, they suggest that the skew had remained essentially unchanged over the period from 1982 to 1985. In later research (on financial futures), Fung and Hsieh (1991) pointed out that the observed patterns of volatility smiles for options on financial futures could be a combination of the effects of stochastic volatility and a possible correlation existing between volatility and price levels. This would cause the standard option pricing models to imply volatilities of the same underlying asset to vary across strike prices and in the patterns observed in the market.

With this approach in mind, Heston (1993) examined a stochastic volatility model that incorporated correlations between the levels of the asset price and volatility. He states: "An important insight from the analysis is the distinction between the effects of stochastic volatility per se and the effects of correlation of volatility with the spot return. If the volatility is uncorrelated with the spot return, then increasing the volatility of volatility (σ) increases the kurtosis of spot returns, not the skewness. In this case, random volatility is associated with increases in the prices of far-from-the-money options relative to near-the-money options. In contrast, the correlation of volatility with the spot return produces skewness. And positive skewness is associated with increases in the prices of out-of-the-money options relative to in-the-money options. Therefore, it is essential to choose properly the correlation of volatility with spot returns as well as the volatility of volatility." (page 339).

However, significant problems arise from the use of stochastic volatility models to price options. Under constant volatility, the absence of arbitrage implies that the option prices must satisfy a fundamental partial differential equation in security price and time. The terminal payoffs of the option determine the current option's price as a unique solution to this partial differential equation. As Ball (1993) points out: "With stochastic volatility, a second factor is introduced requiring the option to satisfy a bivariate fundamental p.d.e. Furthermore, since the second factor, volatility, is not spanned by assets in the economy, we can no longer price options by no arbitrage techniques and the explicit exogenous market price of volatility risk must be introduced." (page 3). The solutions to this bivariate equation can be complex and a number of solutions have been proposed. Furthermore, researchers have tackled the problem of volatility not being a traded asset by examining either a risk premium for volatility or proposing new assets that allow the direct trading of the volatility. Such solutions to the bivariate p.d.e. include numerical approaches [see Wiggins(1987)] or the analytic approaches examined earlier in this research and summarised by Taylor (1994).

Furthermore, Hilliard and Schwartz (1996) point out that stochastic volatility models cannot even be used when there is non-zero correlation between stochastic volatility and price. Hull & White (1987a) acknowledge this problem and provide some sensitivity analysis to measure the effects of correlation using a Monte Carlo simulation approach to solving the fundamental pde.

Even if one were able to model such a correlation sensitive stochastic volatility model, problems will occur if the correlation between volatility and price levels vary. Recent research has pointed out that the existence of skewed distributions for equity indices could be a function of instabilities in the correlations between the equities which comprise the index. Kelly (1994) in a RISK magazine article, pointed out that the steep volatility skew that is found for equity index options is not as extreme for options on the individual stocks which comprise the index. He proposes that the relatively extreme skew pattern for equity indices is due to an expected increase in the correlations between stock when the underlying equity markets fall.[2]

One of the earliest models to address the skewness observed in the implied volatility smiles was the Constant Elasticity of Variance Option pricing model proposed by Cox and Ross (1976). Cox (1996) states that one result of this model would be to explain the skewness observed in the volatility smiles for stock indices. This is achieved by making the volatility proportional to a power of the stock price. According to Cox (1996), "The market's current smile is of course a complex phenomenon, but most would now agree that the negative correlation between stock price changes and volatility changes is a primary ingredient. Indeed, this inverse relationship is the foundation for a number of recent articles on the volatility smile, including Derman and Kani (1994), Dupire (1994) and Rubinstein (1994)." (page 16) These studies are similar in spirit to the constant elasticity of variance model, but they determine the precise relationship between stock price and volatility endogenously from market data rather than specifying it in advance. These will all be discussed in the later section on the determination of implied distributions from the volatility smiles.

2 Kelly, Michael. "Stock Answer," Risk Magazine, August 1994 Volume 7, Number 8, pp. 40-43.

Another possible approach which would explain why the distributions of equity indices might be skewed is the compound option model of Geske (1979). However, this approach requires modelling the options on equities as compound options on the assets of a firm. While this may make inherent sense for individual equity options, it may not be as valid for stock indices, which are a composite measure of all equities.

As with the first portion of this research on the objective dispersion processes, another hypothesis that could explain the existence of smiles are that jumps exist. The Merton (1976) model, which assumes that the underlying asset prices are driven by a jump process, will generate certain smile behaviours. Pappalardo (1996) summarises the impacts of such a jump model on the smile dynamics thus: "an option on a stock whose price is driven by a process with jumps is more valuable than a stock whose price is driven by a geometric Brownian motion. Moreover, the formula (for pricing options with a jump process), which assumes that the jumps have a lognormal distribution, explains (at least qualitatively) many discrepancies between the Black and Scholes formula and the option prices observed in the market, such as the fact that either deep in the money and deep out of the money options are underestimated by the Black and Scholes model." (page 13). Pappalardo (1997) later examined the local volatility function that would be associated with a lognormal jump process. He was able to demonstrate the existence of volatility smiles and that the smiles become more extreme as the time period to expiration is approached. Furthermore, for options with maturities of greater than six months, there appears to be almost no strike price effect. This appears to be closer to what we observe in the empirical implied volatility smiles.

Beckers (1981b) suggests that jumps are responsible for the existence of smiles. He states: "it can be proven that this distribution (jump diffusion model) is leptokurtic and therefore might better describe the actual stock price return behaviour than the pure lognormal model." (page 128). Cox and Rubinstein (1985) examined the impact of a Diffusion-Jump Process for the pricing of options. They found that as the jump component becomes more significant, this will increase the value of out-of-the-money options and decrease the value of at-the-money options. They also found that the increase of the jump component leads to the greatest impact for options that are close to expiration. (page 371).

Recent research by Masson and Perrakis (1996) finds that the smile behaviour of the S&P 500 stock index options are not explained by models based on systematically-varying volatility of the underlying asset, or by transactions costs. But they state: "it is fully consistent with the existence of jumps in the price of the underlying asset." (page 2). Other research by Shastri and Tandon (1986) and Bodurtha and Courtadon (1987) suggest that currencies do also follow jump processes. Significantly, Taylor and Xu (1993) predicted that the smile shapes will become more pronounced as the options contracts approach maturity if this approach is used. As was stated earlier, that is exactly what we observe.

Another approach to solving for the biases observed in the implied volatility patterns is to derive an analytic option pricing model which assumes the underlying price series follows a different evolutionary process than Geometric Brownian motion. One such attempt to derive an approximate analytic model was done by Jarrow and Rudd (1982). They were able to derive an option price as the sum of a Black-Scholes price plus adjustment terms which depended on the second and higher moments of the underlying security stochastic process. This allowed the option price to also depend upon the skewness and kurtosis of the underlying asset's distribution. They used a generalised Edgeworth series expansion to determine the higher moments. Recently, Corrado and Su (1996, 1997) have extended the Black-Scholes model to include skewness and kurtosis in the options-implied distributions. From this methodology, they can estimate the options-implied coefficients of skewness and kurtosis. Their analysis covered both stock options [Corrado and Su (1997)] and options on stock indices [Corrado and Su (1996)].

A final possible explanation for the existence of the kurtosis dynamics is that both jumps and stochastic volatility are responsible in some complex interaction. Scott (1994) suggests that the extreme kurtosis (which describes the existence of smiles) could be caused either by jumps or stochastic volatility. He states: "On occasions, there are large, rapid price movements resembling jumps, and the volatility of stock returns changes randomly over time. Both of these features serve to explain the kurtosis." (page 1).

As with the first portion of this research, the aim of this portion will be to examine these alternative models and assess which best describes the empirical dynamics of the risk-neutral dispersion processes. A potentially important comparison is between the models that have been determined which explain the objective dispersion processes for our twelve financial futures markets and how well these models explain the dynamics of the risk-neutral dispersion process implied by options on these same twelve financial futures.

6.5 APPROACHES FOR DEALING WITH VOLATILITY SMILES

Wisse (1995) points out that there are essentially three approaches for adjusting option pricing methods to correct for the existence of volatility smiles. The first (and simplest) approach is to take the implied volatilities from existing option prices and apply Black-Scholes pricing using these volatilities. While this certainly addresses some of the problems with the constant volatility assumption, it is not clear whether these implied volatilities will retain the same volatility patterns at future estimation dates. One solution that was discussed previously is to examine the historical record of smile patterns, standardise them and assess if these patterns are consistent across time. This is the basis for the construction of volatility matrices and will be discussed extensively in the next Chapter, where such comparisons will be examined for the twelve markets under investigation.

An important area of research is to examine how effective the Black-Scholes model performs in market pricing terms if such volatility matrices are used to price options at some future date. Assuming a consistency exists in the relative patterns of smiles at given points in time, this could provide a computationally inexpensive and effective method of pricing options for risk management purposes. This is precisely the aim of the next Chapter.

The second approach that Wisse (1995) suggests is to assume that the volatility parameter follows some stochastic process. Specifically, this entails modelling stochastic volatility, which we addressed in the first portion of this research for the objective dispersion processes of the twelve futures markets. Wisse points out that this approach has been shown to be successful for pricing currency options and does partly address the smile effect. However, he goes on to point out that "A major difficulty lies in estimating the parameters of the underlying random walk [process]" (page 3). We have addressed this issue using the minimised sum of squared methodology used in Chapter 4. From this analysis, we will examine what would be the sorts of volatility smiles that would be generated from the optimised stochastic volatility models and will compare these to the actual smile patterns for each of our twelve markets.

The third approach suggested by Wisse (1995) is to consider the market prices of options as an "exact reflection of the option's value, assuming markets are efficient." (ibid.). Under this assumption, it is possible to find the stochastic process followed by the price of the underlying asset that is consistent with all the available option prices. This will be examined later in this Chapter when we examine the dispersion processed implied by option prices.

Determination of Risk Neutral Probabilities Empirically

As was stated above, a major new area of the literature has been to examine probability functions for volatilities implied from options prices. Since the determination of these probabilities depends upon the utilisation of risk neutral assumptions (by using option pricing models to assess the volatilities implied in these options prices), the probabilities will themselves be risk neutral estimates. This will depend upon the observed biases of implied volatility across different strike prices for options that share the same time to expiration.

One of the first papers that examined the distributions implied by option prices was Bates (1991). His goal was to answer the question whether the 1987 stock market crash was predicted by the prices of options on the S&P 500. He states (page 1010) that "the instantaneous set of call and put option prices across all exercise prices gives a very direct indication of market participants' aggregate subjective distributions." While Bates assumed a model with jumps and determined a means to parameterise his model using a Least Squares Method, his major contribution was to start others along a similar path. Gemmill (1991) examined whether prices of options on the FTSE 100 predicted the 1987 crash and found similar results to Bates.

The new approach suggested in recent research by Rubinstein (1994) among others [Derman and Kani (1994) and Dupire (1994)] is to start with the market prices of options and find the density function that is consistent with those prices. The variance of this market-implied distribution may then be used as a measure of future volatility over the remaining life of the option. It is important to remember that this market-implied distribution is not the variance of the true probability distribution but rather of the distribution in a risk-neutral world. The theoretical relationship between the two depends on the equilibrium price process in the economy, which in turn depends on investor preferences.

Potentially, this implied risk-neutral distribution contains even richer information about the market's expectations for future movements in the underlying asset price. This distribution can then be used to calibrate a binomial or trinomial tree that is consistent with the observed prices of all options. Methods for this implementation have been suggested by Rubinstein (1994).

These methods find a process such that the model is consistent with the implied volatility of the market and thus the model is complete. Given that the price of a series of European options, $C_i(K_i, T_j)$ can be made a function of the strike price i and maturity j, Dupire (1992) suggests a way to find a process for the stock returns which follows this form of diffusion:

$$dS_t = r(t)S_t dt + \sigma(S_t, t)S_t dZ \quad \textbf{(6.7)}$$

where $r(t)$ is the interest rate for the period, S_t is the asset price at the current time, the volatility, σ is a complex function of the level of the asset price and time and dZ is a standard Brownian motion. Pappalardo (1996) states that the determination of the smile surface associated with $C_i(K_i, T_j)$ can lead to ambiguities. He states "In general, the diffusion contains more information than the conditional law it generates at a fixed time, i.e. different diffusions can give the same conditional law... This ambiguity can be removed if we restrict ourselves to observe the process under the risk neutral probability measure." This is exactly what is done by Dupire (1992).

Differentiating the above equation twice with respect to the strike prices, K_I, will provide the relationship between the risk neutral probability density and the smile surface:

$$\varphi_T(K) = \frac{\partial^2 C(K,T)}{\partial K^2} \qquad \textbf{(6.8)}$$

All that is required given the conditional probability density function, φ_T, is to search for a risk neutral diffusion process that generates it.

Rubinstein (1994) discusses three methods for estimating the risk-neutral density function implied by option prices. Jackwerth and Rubinstein (1996) point out that given that observed option prices are only available at discretely spaced striking price levels, it can be problematic to determine a continuous risk-neutral density function that requires a continuous range of strike prices. The first method (suggested by Rubinstein) is the Longstaff (1993) approach which simply derives a step-function approximation to the risk-neutral density function, where the step function is a coarse as the interval between successive strike prices of traded options. However, Rubinstein shows that for some parameter values, this approach does not work.

The second method, which is attributed to Shimko (1991, 1993), relies on the fact that the risk-neutral density function is equal to the second derivative of the option price relative to the strike price. Breeden and Litzenberger (1978) were the first to demonstrated this. This approach does rely upon a continuum of observable option prices. Since these do not exist, Shimko suggests interpolating the prices of market-traded options and then deriving the risk-neutral density function from these interpolated values. While many methods exist for such an interpolation, Shimko (1993) suggests the use of a least squares method to fit a quadratic function to the volatility smile. This approach can be expressed as:

$$\hat{\sigma}(X,\tau)=\alpha(\tau)+\beta_1(\tau)X+\beta_2(\tau)X^2+\varepsilon \qquad (6.9)$$

where X is the (unadjusted) strike price, τ is point in time at which the implied volatility smile is estimated, and the coefficients of the regression measure the intercept of the regression, the first and second order effects of the strike price biases. This formula will be used extensively in this research both to produce a continuous implied volatility function and to capture the two elements of the volatility smile that capture the higher moments of the distributional function (β_1 for the skewness and β_2 for the kurtosis).

Rubinstein suggests a third method, which is to choose the distribution that is closest (in the least squares sense) to some "prior" distribution. Given the estimation of the objective density functions completed in the first portion of this Chapter, this would serve as a good starting point or "prior" distribution to calibrate the implied density function.

Jackwerth and Rubinstein (1996) also derive the underlying asset risk-neutral probability distributions implied by European option prices on the S&P 500 index. They used non-parametric methods to choose probabilities that minimise some objective function subject to requiring that the probabilities are consistent with observed option and underlying prices. Their contribution is that they demonstrate a new and faster optimisation technique for determining the implied probability distributions by maximising the smoothness of the resulting distribution.

6.6 RATIONALE FOR EXAMINATION OF IMPLIED VOLATILITY SMILES RATHER THAN IMPLIED DISTRIBUTIONS

Given that the current emphasis in the literature is on the examination of implied distributions rather than the smile surfaces, we could choose to examine the risk neutral dispersion approach by modelling implied distributions. We have chosen not to do this but to concentrate on the implied volatility surfaces. There are a number of reasons for this. Firstly, it is not clear which of the available approaches suggested in the previous section are most appropriate. As Jackwerth and Rubinstein (1996) demonstrate significant problems exist from both fitting a continuous diffusion process with discretely observed options and adjustments must be made to the tails of the implied distribution to assure the sum of the probabilities equal 1.0. They are required to interpolate implied volatilities between discrete observations and employ a 'clamping' technique to assure that the probabilities equal 1.0 (see page 1627).

We did attempt to determine implied distributions using the simple approach of Shimko (1991,1993). However, we found significant difficulties in arriving at reasonable results. Firstly, the interpolation process for implied volatilities between strike prices was found to introduce significant errors[3]. Therefore, it appeared that almost any approach to smooth the jagged implied volatility patterns would introduce errors. The nature of these errors is somewhat difficult to understand and we felt that this would introduce further noise into our analysis. Finally, we had to make assumptions about the nature of the distribution beyond observable option prices (that

[3] In the next Chapter, we found that estimation of at-the-money implied volatility using a quadratic approach led to considerable errors The best estimate was a simple linear interpolation of the implied volatilities of the strike price closest to and below the underlying futures price (using OTM puts) and the strike price closest to and above the underlying futures price (using OTM calls).

is in the tails). As was indicated above Jackwerth and Rubinstein (1996) were able to 'clamp' the distribution to achieve non-negative probabilities and a sum of probabilities equal to 1.0. However, to arrive at this result certain assumptions were made regarding a "prior" distribution [see Rubinstein (1994)]. We were not sufficiently comfortable in the choice of this prior distribution.

Thus, the estimation of implied distributions required us to interpolate options implied volatilities, where we often had too few observations (or fairly wide strike price intervals) to have any confidence in the final result. We also found that a quadratic approach led to considerable errors in the estimation of the at-the-money implied volatility. Finally, we must extrapolate the distribution beyond the range of available options prices. We simply do not know what the true prices of options would be at these extreme strike prices. This would require us to fill in options values and we prefer to work with the options available rather than estimate options. We were uncomfortable estimating these deep out-of-the-money options. Given that the implied distributions are estimated in a number of steps removed from the actual option prices (and require unverifiable assumptions), we chose instead to examine the actual implied volatility surfaces. It was felt that minimal assumptions were made in the determination of these surfaces that would introduce errors. A potential problem is that this approach could limit the ability to compare risk neutral and objective dispersion processes. One benefit of the implied distribution process is that we can then identify the summary statistics of this process and compare these directly to the summary statistics of the objective processes.

In our research, we will compare the risk neutral and objective dispersion processes by comparing options prices for both. It is clear that for the risk neutral dispersion processes, we have options prices and implied volatilities and minimal assumptions must be made to examine these. For the objective dispersion process, we will determine option prices at discrete strike price intervals using a Monte Carlo technique that assumes the distributional form of the underlying asset follows the objective dispersion processes identified previously. This will be done in Chapter 9. In this way, we assure that comparisons between the risk neutral and objective processes are done using a metric (the implied volatility) surface that has less potential estimation problems compared with the implied distribution approach.

6.7 CONCLUSION

Options prices provide a rich resource for determination of the risk-neutral dispersion processes of assets returns. Through the estimation of the free parameter in the B-S-M option pricing models, one can determine the volatility implied in option prices. We have discussed how this measure is somewhat difficult to interpret and explained why there is need for standardisation for hypothesis testing. We also established the goal of this portion of the research. Our objective is to understand the nature of the risk-neutral dispersion processes that can be determined by volatility smiles. We seek to understand the time series nature of volatility smiles for a wide variety of financial assets, compare the smile behaviours both for single underlying assets and across asset classes and determine the relationship with the objective dispersion processes for these markets uncovered in the first portion of the research.

CHAPTER SEVEN
THE ANALYSIS OF RISK NEUTRAL PROBABILITIES IN OPTIONS ON FUTURES: STANDARDISATION OF IMPLIED VOLATILITY SMILES

7.1 INTRODUCTION

In the last Chapter, we indicated that the empirical analysis of implied volatilities has examined a wide range of theoretical issues. Such research has investigated the methods of estimating the implied standard deviation, the time series behaviour of the at-the-money implied volatilities and weighting implied volatilities at different strike prices and time to maturity to arrive at a composite estimate. Much of the work on the implied volatilities was to examine the informational content of this parameter and the motivation was to assess if the implied volatility is an unbiased estimation of the realised volatility.

A recent trend in the empirical investigation of implied volatilities has been to concentrate on understanding the behaviour of implied volatilities across strike prices and time to expiration. This line of research assumes implicitly that these divergences are not solely due to sampling problems or errors in measurement, but provide information about the dynamics of the options markets. Specifically, a number of recent papers have suggested that the divergences of implied volatilities across strike prices may be providing information about the expected dispersion process for underlying asset prices.

In these papers, the emphasis has been on examining the current implied volatility surface given current options prices and using this information to determine the implied dispersion process. One line of research that has not yet been examined is whether these implied volatility surfaces display any consistencies either within the same market or if consistencies exist across markets. This research will deal with time (in some cases more than 10 years) to conduct meaningful analysis.[1] What is critical for our research is that most of the option data series exist for a period similar to the period for the underlying futures contracts. This will allow comparison between the risk-neutral dispersion processes (from the options markets) and the unconditional objective dispersion processes (from the futures markets).

For the following underlying assets, the following time periods of analysis were examined:

Underlying Asset	Time Period of Analysis
Stock Index Options	
S&P 500 Futures	25/03/1986 - 24/12/1996
FTSE Futures	02/01/1985 - 20/12/1996
Nikkei Dow Futures	25/09/1990 - 16/12/1996
DAX Futures	02/01/1992 - 20/12/1996
Fixed Income Options	
Bund Futures	20/04/1989 - 21/11/1996
BTP Futures	11/10/1991 - 21/11/1996
Gilt Futures	13/03/1986 - 22/11/1996
US T-Bond Futures	02/01/1985 - 15/11/1996
Currency Options	
Deutsche Mark /US Dollar	03/01/1985 - 09/12/1996
British Pound / US Dollar	25/02/1985 - 09/12/1996
Japanese Yen / US Dollar	05/03/1986 - 09/12/1996
Swiss Franc / US Dollar	25/02/1985 - 09/12/1996

For the analysis of the options contracts that trade on the London International Financial Futures Exchange (LIFFE) [this includes the BTPs, Bunds, Gilts and the FTSE 100], data was obtained directly from the LIFFE. This data includes closing prices of the options contracts, the implied volatility using the BLACK 1976 model and other information including volume, open interest, opening, high and low prices of the futures and options. For the FTSE 100, options data was only available from

[1] In total, the number of option prices examined for all twelve markets was 1,263.317. Given that we also had the underlying futures prices for the same dates (and at the same time) as the options, we were

1992 from the LIFFE and only for the European options. To extend the analysis, we obtained additional data from Professor Gordon Gemmill, of City University Business School, who had compiled data for American options on the FTSE 100 (from the financial press) from 1985 to 1992. For our analysis we merged both data series. One problem was that only five option strike prices are reported in the financial press (Financial Times) and the limited number of option strike prices made it somewhat difficult to estimate the entire range of smile behaviours. For the data from the LIFFE, all the available options are reported, thus, we were able to gain more data points for estimation. Nevertheless, the addition of the seven years of options data will allow examination of smile behaviour before and after the 1987 stock market crash. For the estimation of implied volatilities, we required the closing futures prices for the same dates. These were drawn from the same data set as was used in the first portion of this research (see Chapter 2 for the description of this data).

For the futures and options contracts traded at the Chicago Board of Trade (US T-Bond Futures and Options), the data was obtained directly from the CBOT on floppy disks. This data included closing prices of the futures and options contracts.

For the futures and options contracts traded at the Chicago Mercantile Exchange [S&P 500, Nikkei Dow, Deutsche Mark, British Pound, Swiss Franc and Japanese Yen (all versus US Dollar) Futures and Options], the data was obtained directly from the CME on floppy disks. This data included closing prices of the futures and options contracts. From the original data files obtained, there were significant gaps in the price series for the S&P 500, the British Pound and Japanese Yen. Data was missing for entire years during the late 1980s. To fill in the missing

able to assure that both time series were consistent to each other. From this analysis, we were able to clean both series and assure our analysis was minimally impacted by errors in data.

data, we obtained copies of the daily price sheets from the CME and we entered by hand all the futures and options prices that were missing.

For the futures and options contracts traded at the Deutsche Terminbörse (i.e. the DAX futures and options) this data was obtained directly from the exchange. It included all the tick by tick prices of the futures and options contracts during each trading day. Given that the rest of the analysis was done on closing prices, we sorted the data by time of trade and chose the options and futures prices within one hour of the close. For this, we would only select options for analysis if a futures trade occurred within 3 minutes of the option trade, otherwise, we ignored the option. In most instances, the options selected were within the last 30 minutes of the close and the accompanying underlying futures contract traded within 1 minute of the option.

Eight of the options under examination in this research were American style options on Futures. These included the FTSE 100 (prior to 1992), S&P 500 and Nikkei 225, the US T-Bond, and all the currency options. These options are paid for up-front and are American style. To estimate the implied volatilities correctly, we chose to use the Barone-Adesi and Whaley (1987) model. According to Clewlow and Xu (1994), this model has been shown to be very accurate for option maturities shorter than twelve months. For this model, an interest rate parameter must be included in order to estimate the implied volatilities. While Clewlow and Xu (1994) used London euro-currency interest rates to approximate the riskless rate in their option pricing models, for our analysis, we chose to use for all US Dollar based options, the US Treasury Bill interest rate for that day whose maturity fell most closely to the actual expiration date of the options. This data was obtained directly from the Federal Reserve Bank in Washington D.C. These contracts that required the

US Dollar interest rates included all the currencies, the US T-Bond, the S&P 500 and the Nikkei 225 contracts.

It should be noted that some researchers have determined the implied volatilities of American options on futures using a European pricing model [Black (1976a)]. Jorion (1995) determined the implied volatilities of currency options on futures traded at the Chicago Mercantile Exchange using the Black (1976a) model. He justified this by demonstrating that the use of the European model introduced a upward bias in the estimated volatility that was small enough to be within the normal bid/offer spread for at the money options. However, it was shown by Whaley (1986) that this bias will depend not only on the level of the interest rate but also the absolute level of the volatility, the time to expiration and the degree to which the option strike price is away from the ATM level. For short dated options, there is little difference for ATM options, however, for out-of-the-money options, the biases become more severe. Given these potential problems and our objective to model the behaviours of out-of-the-money options we decided to use the Barone-Adesi Whaley model for all American options on futures under investigation (except for the LIFFE fixed income options).

The FTSE 100 option (after 1992) was a European style option which also had stock type settlement (this means the premium was paid up-front). The options expire on the same date as the FTSE 100 futures contract and LIFFE determined the implied volatilities using the Black (1976a) model. At first, we were tempted to use these volatilities directly. However, when we obtained the additional (and earlier) data from Professor Gemmill, we were obliged to re-estimate the implied volatilities. Given that some of the options were American style and some were European style, we used the Barone-Adesi Whaley model for all the American options (pre 1992) and the Black

(1976b) model for all the European options (post 1992). To determine the interest rate parameter, we obtained weekly average Sterling interest rates from the Bank of England. When we compared our results to the implied volatilities produced by LIFFE, we found significant differences. After investigation with exchange staff and comparing exactly the same options, we learned that the implied volatilities provided are not accurate. LIFFE staff informed us that these are solely for indication purposes and are subjectively altered from the Black (1976b) implied volatilities before being released. This being the case, we estimated all the implied volatilities again using the appropriate models, made no adjustments to these results and used these numbers in the analysis.

Most of the remaining options under examination in this research were American style options on Futures. All these options are traded at the LIFFE. These include the Bund, BTP and Gilt options. For these options the mechanics of margining of both the underlying futures and options removed the possibility of early exercise. For these options, it is possible to estimate the implied volatilities using the Black (1976b) model and interest rates can be ignored.

These approaches addressed all the markets under examination except the DAX index options. For this contract, the option was based upon the cash index. This was selected because the option on the DAX futures is extremely illiquid and the date of settlement is exactly the same as the expiration of the futures. Thus, this contract is de facto an option on the futures. This option contract is European style and has stock type settlement. This means that the appropriate model is the Black and Scholes (1973) model with an interest rate input. No dividend yield is required because the DAX index is a total return index where the dividends are assumed to be automatically reinvested in the index. The interest rate input was the average weekly 3-month LIBOR for Deutsche Marks obtained from The Bank of England.

Finally, given that this research is empirical in nature it was of utmost importance that the data we examined was carefully screened to remove errors. This was achieved in a number of ways. Firstly, we compared the futures price series with the options price series for the same days to identify obvious errors in recording either price series. This comparison was achieved by comparing the put-call parity values of the options with the underlying futures prices for every single date in our database (and for all twelve markets). A screening procedure was imposed such that if futures or options prices diverged by more than the normal bid/offer spread (of one tick), the observations were flagged. Once this was done, each price was compared with the original daily price sheets to confirm if a 'keypunch' error had occurred. We discovered that only 1-2% of the data had such errors. Nevertheless, these errors were of a sufficient magnitude that they did influence the results and therefore required correction.

One of the most important methods of data cleaning was comparing the results among markets in the same asset classes. At each stage of the analysis, we found anomalous results. For example, after the indexing the at-the-money implied volatility to 100, we expected the levels of the at-the-money implied volatility to be equal to 100. We found unusual results. For the US T-Bond, the values were much lower. This was due to the fact that the initial approach we used for estimating the at-the-money implied volatility was biased [a quadratic approach suggested by Shimko (1991,1993)]. In addition, for a number of the other markets (S&P 500 and FTSE 100), the final day of options prices provided by the exchanges did not correspond to the expiration date. This meant that our estimate of the implied volatility was using the wrong period of time. Once this was corrected, the results corresponded to our expectations. Thus, by comparing the expected results to the actual results, it was possible to check for errors in a number of alternative ways. Finally, it should be mentioned that we had 1,263,317 options prices and all the analysis was standardised, this allowed us to check for any unusual results. From this we were able to screen all results for anomalies and in most cases this was due to errors in the data. By combining these varied approaches (both pre and post analysis), we assured that the options prices examined were as free of data as is humanly possible.

7.3 HISTORICAL RECORD OF IMPLIED VOLATILTIES

In the last Chapter, it was noted that a substantial literature exists examining the time series behaviour of at-the-money implied volatilities. In this research, we have chosen not to cover the same ground.

Our objective is to examine the behaviours of implied volatilities, given that we have identified the objective processes that drive the assets underlying these options markets. Previous research has linked the objective and risk-neutral dispersion processes by testing option market efficiency (of the risk neutral dispersion processes relative to the objective processes). Guo (1996) examining this and most of the research on the information content of the risk-neutral processes addressed similar issues. We will not take this approach in this research. Instead, we are interested in answering a more general question: can the dynamics of the risk neutral process be better understood in light of the processes explaining the dynamics of the objective processes?

An important starting point for this investigation is to state what behaviours of implied volatilities we are interested in understanding. Given that we will not examine the informational content of the implied volatilities, we have a number of areas we could investigate. One such area of investigation could be the time series behaviour of the implied volatilities. As was discussed in the last Chapter, this area of research has primarily examined the dynamics of either the at-the-money implied volatility or some weighted average of all implied volatilities.

As was stated in the last Chapter, the latter approach assumes that the only substantive difference in the volatilities between strike prices is due to sampling errors in their estimation. As was pointed out at that time, the process of weighting eliminates these differences and if this effect is not due to sampling but is some systematic behaviour, these patterns will be lost. Subsequently, research [Jackwerth and Rubinstein (1996), for example] has suggested that these differences in the implied volatilities are indeed systematic and provide information about the risk neutral dispersion processes implied by market participants. Therefore, we will choose to examine the time series dynamics of implied volatilities by concentrating on the implied volatilities of those options that are at-the-money.

Once again, we must decide which time series dynamics we are interested in understanding. The area of time series analysis is indeed so rich that it offers fertile ground for potentially important research. Even so, many time series models have previously been examined in the literature and will not be the focus of this research. This research will concentrate on understanding the implied volatility surface.

To set the stage for the analysis that follows, our first task will be to examine the dynamics of the implied volatility processes for the twelve markets under investigation. For this analysis, we will restrict ourselves to the behaviour of the at-the-money volatility for the twelve markets in our study. Figure 7.1a displays the at-the-money volatility for the four stock index options under investigation. Figures 7.1b and 7.1c display similar charts for the four fixed income and four foreign exchange options.

A general conclusion is that the levels of the implied volatilities vary considerably over time. For the stock index options (Figure 7.1a) one can see the extreme spike that occurred at the time of the 1987 stock market crash. However, for the other asset classes of fixed income and foreign exchange options similar spikes occur from time to time. To understand the process we are dealing with better, summary statistics have been estimated for all the at-the-money implied volatilities for a number of time horizons. We have examined the implied volatility on a daily basis and compared the implied volatilities in 5-day increments from 5 days to 90 days to expiration The summary statistics for the four stock index options ATM implied volatilities can be found in Table 7.1a. Likewise, the summary statistics for the ATM implied volatilities for the four fixed income options appear in Table 7.1b and for the four foreign exchange options in Table 7.1c.

Examining the figures as well as the tables, it becomes clear that the at-the-money implied volatilities data series are highly non-normal. Considering the stock index option (daily) implied volatilities, all four markets display significantly positive skewness. For the two markets (S&P and FTSE) that include the 1987 crashes, there is also significant excess kurtosis in the implied volatility series. This is due to the fact that the implied volatilities experienced jumps of 160% for the S&P and 92.93% for the FTSE as of the 19th of October 1987. Of particular interest to our research is the relationship between the implied volatilities statistical moments as a function of the time to expiration. By including the implied volatilities measured at fixed time periods to expiration, we can assess how the distributional dynamics vary as the time to expiration varies. It is clear that the dynamics are not constant. Consider the variability of the volatility. The standard deviation of the implied volatilities at different times to expiration does display variability. While the variability may be a purely a sampling problem (we can only have as many of these observations as we have contracts), it would appear that the closer we are to expiration, the more variable the volatility is (as measured by the standard deviation).

For the fixed income options markets, similar patterns exist. Three of the four fixed income options experience significant positive skewness with the BTP market the exception. Again, three of the four markets also have significant excess kurtosis (statistics above 3.00) for the series of daily implied volatilities. Again, it appears that when the implied volatilities are measured at fixed points to expiration, the closer we are to expiration, the greater the observed standard deviations of the implied volatilities. In all cases, the standard deviations of the implied volatilities at five (calendar) days to expiration are significantly greater than for the standard deviations of the daily implied volatility series.

Finally, for the four foreign exchange options markets, we again observe similar dynamics. All four markets display positive skewness for the daily implied volatility series and three of the four have significant excess kurtosis. As with the fixed income options, the closer the time to expiration for the option, the greater the standard deviation of the implied volatilities measured at that point. Once again, these standard deviations are significantly greater than for the daily at-the-money implied volatility series.

Any interpretation of this result is difficult to draw since each of these implied volatilities measured at fixed points until expiration represents observations which are approximately three months apart. Nevertheless, the appropriate comparison would be between estimates all with three months lags but with progressively less time to expiration. Thus, on a relative basis, it would appear that the variance of the implied volatility is inversely related to the time to expiration of the option.

While this impact is somewhat puzzling and certainly warrants further investigation, we have chosen to limit our research to understanding the second order impact of the implied volatility smile.

7.4 RESULTS OF THE IMPLIED VOLATILITY ANALYSIS

To gain an overview of the smile estimation process and the implications for this research, we examined the implied volatility patterns as of the 2nd of November 1996 for each of the twelve markets under investigation.

The general viewpoint regarding volatility estimation assumes that the Black-Scholes-Merton model accurately describes conditions in actual options markets. The major assumptions are that the prices of underlying assets evolve through time log-normally with a constant volatility σ at any time and market level. If this were correct, then the volatilities implied from the actual option prices in the market would be the same regardless of the strike price of the option or its maturity. To examine whether this is true or not, we examined the options for our twelve markets as of May 7, 1996. The implied volatilities were determined using the appropriate option pricing model, interest rate parameter and the closing price of the option and the futures. This was done for all the reported options and the results for the twelve markets are listed in Appendix 7.1. For one of these markets, the FTSE 100, the options prices and the implied volatilities (as of this date) appear in Table 7.2.

One obvious result is that more options are available for the contract that is nearest to maturity (June 1996) compared with the deferred maturities (September 1996 and December 1996). This is most probably due to the fact that most trading activity in exchange traded options is concentrated in the nearest maturity options. For this reason, we will restrict our analysis to only those exchange traded options that are the closest to maturity. Secondly, for the June 1996 FTSE options, many of the series fail to have implied volatilities. In all instances these missing values occur for deep in-the-money call or put options (that are of critical interest to our research). The failure to determine an implied volatility is due to the fact that these options are trading at their intrinsic value alone. Given that no time (or extrinsic) value exists, it is not possible to invert the option pricing model to determine an implied volatility. This problem is mirrored for those options with the same strike price but deep out-of-the-money. For example, the deep out-of-the-money calls and puts, are all quoted at the minimum allowable price of 1.0. The prices for the put options from a strike price of 3125 to 2625, are all equal to one. According to option pricing theory, the options prices should be monotonically decreasing as they become further out-of-the-money. Given that this is not possible due to the minimum price imposed by the LIFFE, the resulting implied volatilities compensate for this artificial minimum price level by increasing monotonically as the strike prices decrease. This effect would cause the convex curvature we observe (and is referred to as the smile). Whenever, such discrete price increments exist with minimum price levels, a volatility smile would result. This effect can also be observed for the out-of-the-money calls.

Clearly, this effect is structural and cannot provide us with reliable information about the risk-neutral dispersion processes. The obvious solution is to remove these options from the analysis. Jackwerth and Rubinstein (1996) removed these options in their analysis of the implied volatilities of the S&P 500 by examining the butterfly arbitrage condition. This states that it is not possible to sell two options at one strike price and use the proceeds to purchase two options at adjacent strike prices (one above and one below) for no cost (or a premium inflow). This is due the curved relationship between option prices and the underlying which omits such arbitrage [see Merton (1973) for a proof of this]. We have also chosen to impose this condition that will eliminate the consideration of these deep out-of-the-money options in our analysis.

One will further notice that this problem does not appear to exist for those options with longer time periods to analysis. While it would be tempting only to consider these options in our analysis, these options often have extremely poor liquidity and one must accept a trade-off between actively traded options and options with prices that are sufficiently small to approach the minimum levels. We chose to select those actively traded options and eliminate options that allowed arbitrage.

Nevertheless, we are interested in the behaviours of implied volatilities as a function of time. While this requires a time series analysis of these options, it is also possible to examine the effects of the time to maturity for the implied volatilities of options by examining on a single date the implied volatilities for different maturities. For this analysis, one simply examines all the available maturities for the options (and their associated futures contracts). Finally, one can plot the raw (and unadjusted) implied volatilities versus the strike prices of the options. These graphs can be seen in Figure 7.2a (for the four stock index options), Figure 7.2b (for the four fixed income options) and Figure 7.2c (for the four currency options).

For the same reason discussed previously regarding the potential problem of nonsynchronous prices for the options and underlying futures, only those implied volatilities from the available out-of-the-money option contracts (not admitting arbitrage) were examined. These were then plotted versus each option's strike price and options were grouped by their expiration date. Finally, to assess if any consistent patterns exist, a line is drawn connecting the implied volatilities for each maturity. For these graphs, the implied volatility smiles were estimated using a simple linear interpolation between the adjacent point estimates.

It is clear that the implied volatilities differ across strike prices for the same maturity and across diverse expiration periods for these twelve instruments. This is by no means an unusual result. This has been identified across underlying asset classes: the implied volatilities deviate from the constant and uniform level predicted by a lognormal dispersion assumption. However, the pattern of divergence does differ between different asset classes.

For the options on the stock index futures (in Figure 7.2a), the implied volatility patterns generally display a convex shape and are skewed to the left. This implies that the market prices of options with lower strike prices are higher than the theoretical prices obtained from the option-pricing model using the at-the-money volatility. In addition, the market prices of options with higher strike prices are lower than what would be predicted based on the assumptions of the option-pricing model. This result has been identified extensively in the literature [see references in Chapter 6].

While for the S&P 500, FTSE 100 and DAX options the smile relationships are skewed to the lower strike prices, the Nikkei displays a more symmetrical shape for the September 1996 maturity. Thus, all the stock index options display the skew behaviour previously identified in the literature. Regarding the curvature of the implied volatility patterns, it appears that the closer we come to expiration (and excluding the minimum price effect), a more extreme curvature is observed. If one could restrict the evaluation of the patterns solely to the curvature effect, it is now clear why this pattern has been referred to as a 'smile'.

For the options on the fixed income futures (in Figure 7.2b), we also observe both skewness and curvature effects. Unfortunately, we did not have multiple option maturities to examine the effects of time on the implied volatilities. The only market that allows us some comparison is the Gilt with two available maturities. This is due to the fact that there is an even greater concentration of trading activity on the options that are closest to maturity. Nevertheless, it appears from the Gilt options that a similar pattern exists for those observed for the stock index options. The curvature of the implied volatility patterns becomes more extreme the closer we are to expiration of the option.

For the options on the foreign exchange futures (in Figure 7.2c) we fortunately have more option maturities to examine. This will allow us to gain some insights in the effects of time on the implied volatility patterns. For all four markets, there does not appear to be a systematic skewed relationship between strike prices and implied volatilities. However, one can clearly see that as the options are closer to maturity, there is much greater curvature. Furthermore, the levels of the implied volatilities differ significantly across maturities.

While it is clear that these patterns are not consistent with the assumptions of the option pricing models, many questions remain. The first is whether the divergences are due to the mis-specification of the option pricing models or are due to inefficiencies in the markets. Regardless of the reason for the divergences, the question is whether these patterns are random or systematic. If they are a function of time, or either the level of the underlying futures or the level of the volatility, it may be possible to discern some deterministic function that governs the behaviour of these volatility smiles. Dumas, Fleming and Whaley (1996) have already attempted to test a deterministic volatility function (DVF) option valuation model that is based solely upon the strike price and time [also postulated by Rubinstein (1994), Dupire (1994) and Derman and Kani (1994)]. We will extend this analysis by enriching their model with additional factors. Our primary objective is to understand the dynamics of the implied volatility surface. Through this research we will examine how well the deterministic approaches suggested by existing research capture the dynamics of implied volatilities both cross-sectionally and across time. This will be done in the next Chapter. However, before we can complete this analysis, we must carefully prepare the data for analysis. Since our objective is to compare implied volatility functions between markets and within markets, we will require some standardisation to allow this to occur. To achieve this, we will examine such standardisation techniques for individual markets, as are used for the construction of historical implied volatility relationships.

Therefore, our objective for the rest of this Chapter will be to examine the patterns of volatility smiles through time and across strike prices for individual markets. To do this we will first introduce Volatility Matrices and demonstrate how they can be constructed for a single equity index option market: the FTSE 100. Then, we will concentrate on the patterns of implied volatilities for the same maturity that has been called the "Smile". In this section, we will demonstrate how practitioners standardise these patterns to allow comparison both across time and among different markets. Finally, we will demonstrate how to split out the effects of skewness and kurtosis that will allow these to examined separately. When the overall methodology has been presented, the final steps in the analysis will be done for all twelve markets under investigation.[2]

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[2] To consider the nature of the strike price biases, we have chosen to examine only one market, the FTSE 100. This is done to address the general issues involved in the volatility smile without what would become repetition of repeating the step by step analysis for each market. However, once the

7.5 DETERMINATION OF VOLATILITY MATRICES

Given that the implied volatilities display variability across strike prices and maturities, there may exist complex interactions between price and time. Dumas, Fleming and Whaley (1996) identified this in the development of their DVF. One method of achieving a similar result is the construction of a volatility matrix that displays the implied volatilities in two dimensions: across strike prices and across time. Wisse (1995) indicates that this is the simplest way to correct for the existence of volatility smiles (and the term structure) and this approach has become one of the most important tools in the repertoire of option traders and analysts. Essentially, historical patterns of implied volatilities are examined both across strike prices and time and some assumption is made that these patterns are stationary. This historical analysis is then projected at future points in time to estimate what the implied volatilities should be at that point. Such a volatility matrix is presented in Table 7.3 for options on the Financial Times 100 Stock Index Futures (FTSE) and for Over the Counter (OTC) options on the FTSE with expirations out to five years in duration.

	-30%	-20%	-10%	-5%	ATM	5%	10%	20%	30%
1 Month	25.75	22.00	19.00	17.75	16.50	16.00	15.50	14.75	14.00
3 Months	24.75	21.25	18.75	17.75	16.75	16.25	15.75	15.00	14.25
1 Year	22.50	20.25	18.50	17.75	17.25	16.75	16.25	15.50	14.75
2 Years	21.75	19.75	18.25	17.75	17.25	17.00	16.75	16.25	15.75
3 Years	21.5	20	19	18.5	17.75	17.5	17.25	17	16.75
4 Years	21	20.25	19.5	19	18.5	18	17.75	17.5	17
5 Years	21	20.75	20	19.25	18.75	18.25	18	17.75	17.25

Table 7.3, Volatility Matrix for Options on the FTSE Index.

methodology has been detailed for the FTSE 100 options, the final steps will be presented for all markets. This is done both to save paper and not try the patience of those reading this research.

The data for this matrix was compiled on 9 August 1994 and used by BZW, London[3] to make prices for options on the FTSE Index. Specifically, the bank used the standard option pricing model for European options on Futures [Black (1976b)] but instead of assuming a constant volatility parameter, would override the model by inputting different volatilities depending on the strike price and maturity of the option required. In this table, it is clear to see that different implied volatilities are utilised across the range of standardised strike price range and maturities. The implied volatility for the option series that has a strike price closest to the current forward price level was deemed to be the at-the-money (or ATM) option. This can be seen in the middle of the top line of the matrix. Thereafter, option strike prices are represented in 10% increments. These are analysed up to 30% above and below the current level of the underlying FTSE forward price. To aid our analysis, the results of the volatility matrix are graphed to show the patterns that exist both across strike

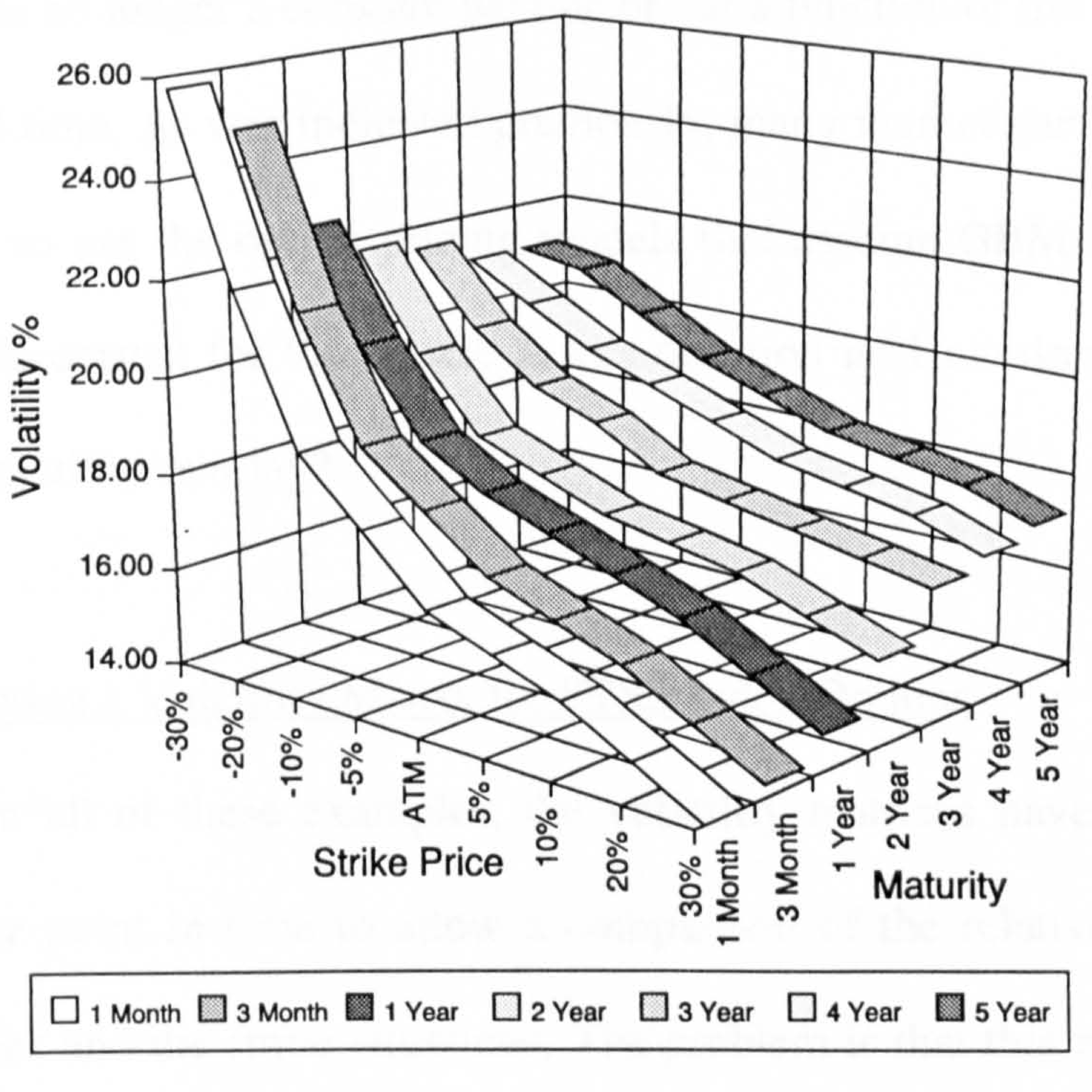

Figure 7.3, Volatility Smiles of Options on the FTSE Index.

[3]Many thanks to Niran de Silva and Leigh Baxendale of BZW, London for kindly providing this information.

prices and time. This has been produced in Figure 7.3. As a result, almost all the implied volatilities are different. For the same maturity, the further away from the ATM level the more divergent the implied volatilities.

The graph shows that the shape of the pattern is curved upwards the lower the strike price. When the strike price is higher then the ATM level the implied volatility falls; reaching the lowest level with the strike price 30% above the current market (forward) price. These results are similar to the volatility smiles reproduced in Figure 7.2a for the FTSE 100 as of May 7, 1996. Given that these shapes bear some resemblance, one could hypothesise that perhaps some consistency occurs over time. More precisely, some deterministic function may exist that will allow us to model these relationships and test whether they are consistent across markets and over time or not.

These findings suggest that the volatility input necessary for the pricing formula is no longer a constant parameter but a function of the price of the underlying asset and time. As was indicated previously, many market participants have chosen to continue to use the option pricing models that assume GBM but include a volatility overlay to correct for the biases. A key question is: How do practitioners determine such a volatility overlay?

Constructing a Volatility Matrix for FTSE Index Options

In all of these examples, the volatility matrices have been constructed at a particular point in time to allow a comparison of the relative levels of the implied volatilities and the smile structures. The problem is that this matrix can only be used on that one day, for the next day the entire structure may change. An example can be found in Figure 7.4 that displays a time varying three-dimensional graph. This graph displays the smile structure for the nearby options on the FTSE 100 during 1996 determined at five-day intervals. One thing that can be discerned from the graph is that the smile structures do vary through time. However, some sort of consistent cyclical dynamics seem to be occurring. This is because the level of the underlying FTSE futures as well as the levels of the implied volatilities was fairly stable over this period. Tompkins (1994) demonstrated that these patterns will appear to be much more random when the levels of the underlying and implied volatilities vary considerably over the time period of analysis.[4]

Apart from this obvious conclusion, these kinds of charts can be extremely difficult to interpret. This is because the scale at the front lists a fixed range of strike prices. It is not clear where the underlying futures contract was on that day and which strike price was at-the-money. Secondly, the overall level of the implied volatilities could be rising or falling and that could cause the curvature of the smile to appear more or less pronounced than the figure demonstrates. Finally, and this is a major concern, the graph provides little information about the stability of smile patterns.

Thus, the goal of estimating the smile structure must be to construct a consistent and predictable method for both pricing the options and identifying whether enough consistency in the patterns of implied volatility exists to be used to project the implied volatility surface at future points in time. This can only be achieved if some consistency exists both over the strike price ranges and time and if this relationship can be understood.

To accomplish this, we will employ the standardisation techniques for the volatility smiles outlined in the last chapter. Specifically, we will convert the levels of the implied volatilities into index form. The denominator (in the ratio) is the at-the-

[4] Tompkins (1994) examined the BTP options for the period of 1993-1994 and observed extreme

money volatility. Given that it is unlikely that the underlying futures will be exactly equal to one of the discrete strike prices available, we must estimate the level of the at-the-money volatility. We used two methods. The first was to determine the quadratic functional form that fits the volatility smile. This used formula 6.9 from the last Chapter [see Shimko (1991,1993)].[5] This equation will allow us (in theory) to estimate the implied volatility that would exist for a strike price that would be exactly equal to the underlying futures price. In essence, this is equal to the α of equation 6.9. We found two major problems with this approach. The first is that for many days, we had barely enough degrees of freedom (options prices) to determine the quadratic form. This led to significant errors in the estimation of the at-the-money volatility. We observed implied volatilities that were significantly different from the volatilities of adjacent strike prices. Secondly, it became apparent that there were problems with this quadratic functional form. Many of our markets (the US T-Bond market in particular) were not well described by a quadratic function. It appears that higher moments are required and given that we have a limited number of degrees of freedom, this approach was not tenable. The second approach was to take a simple linear interpolation for the two implied volatilities of the strike prices that bracketed the underlying asset price (one below and one above). By reducing the grid to these two closest observations, many of the problems encountered with the quadratic approach disappeared.

Finally, with an estimated at-the-money volatility, we could standardise each observed implied volatility by indexing each to this level. As was discussed in the last Chapter, this standardisation is necessary due to the non-constant levels of the implied

variability of the implied volatility smiles for this period. The patterns failed to display any cyclical behaviour due to wide swings occurring in the underlying market during that period.

[5] There was a slight modification to formula 6.9. The strike price was expressed as the natural logarithm of the ratio of the strike price to the underlying futures price.

that are observed over time. This variability in the implied volatility makes it difficult to compare the relative levels of implied volatilities to the level of the ATM volatility. Furthermore, this standardisation will allow us to compare implied volatility patterns cross-sectionally (as the effects of scaling are removed).

The second standardisation was to index the strike prices to the level of the underlying futures price. This standardisation was achieved using formula (6.6) from the last Chapter. As a review, this is expressed as:

$$\frac{\ln(X_\tau / F_\tau)}{\sigma\sqrt{\tau/365}} \tag{7.1}$$

Where X_τ is the strike price of the option at time to expiration τ, F_τ is the underlying futures price at time to expiration τ, and the square root of time factor reflects the percentage in a (calendar) year of the remaining time until the expiration of the option. Finally, the sigma term (σ) represents the at-money-volatility.

Again this adjustment is required because the underlying asset price was constantly changing throughout the period. If one were to draw each smile relative to the same fixed strike prices, this is misleading. This is because the strike associated with the ATM option will be a function of the price of the underlying asset price. Since our objective is to assess a deterministic volatility function, such standardisation will allow for us not only to estimate such a function but also test the stability of this function. It is acknowledged that whenever some method of standardisation is employed, a loss of information (detail) results. However, given our objective is to compare smile behaviours both cross-sectionally and across time, we believe the loss of information by standardising is more that made up by the ability to compare smile dynamics within and between markets more directly.

A key question for our research was the selection of the term to maturity of the options in this analysis. Essentially, how far back in time should we estimate our standardised smiles? We observed from the collection of the option and futures prices that the liquidity for both assets is not uniform over time. While options and futures could be offered with up to nine months to expiration, these contracts rarely traded until they became the nearest contracts to expiration. Furthermore, for the options markets, we must have a wide range of strike prices to be able to determine a meaningful quadratic function to ascertain the shapes. The problems suggested in the previous chapter of illiquidity and nonsynchronous were significant until the futures and options contracts became the nearest contracts to expiration. Therefore, for our analysis, we restricted our smile evaluations only to those options and futures that were the nearest contracts to expiration. Furthermore, we restricted our analysis to the quarterly expiration schedule of March, June, September and December maturities.

This restriction might seem to reduce the information about the smile patterns that could be obtained. For a number of the options markets (such as the currency options offered at the CME and the options offered at the LIFFE) serial delivery contracts were introduced in the mid-1990s. These options were based upon the quarterly futures delivery cycle but had monthly expirations. As an example, the options on the Bunds traded at the LIFFE would be based on the June 1996 Bund futures but would expire in March, April and May. We ignored the March and April expirations and only examined the options that expired as close to the expiration of the futures as possible. Our rationale for this exclusion was twofold. First, since these serial options were not available for the entire period of our data analysis, we are unable to examine whether some consistency existed over time, due either to the nature of these serial options or whether some sort of month of the year effect exists. Secondly, these contracts have tended to be illiquid. The most actively traded options have remained the options that are based on the same quarterly trading cycle as the underlying futures. Thus, to minimise errors in our analysis that could be introduced by inclusion of options with poor liquidity, we chose to exclude these serial options.

It is clear that at some point these effects should be examined for serial options. However, we have chosen to leave to later research to ascertain, when and if these contracts become more mature and a longer time series of data is available for analysis.

By restricting our analysis solely to the nearby options and only on the same quarterly trading cycle as the underlying futures, the options we examined tended to have a maximum maturity of approximately 90 days to expiration. As an example, we would track the options as soon as the quarterly cycle for the previous option had expired. For example, on March 15th, 1996, the March option on the FTSE expired (and this coincided with the expiration of the futures). From that point, we would examine the June 1996 FTSE options until they expired on the 21st of June 1996. Thereafter, we would then examine the September 1996 FTSE options until they expired and so forth and so on.

Thus, for each of the twelve markets, we determined the expiration date of each option and then worked backwards in time to the previous expiration date. From this, we determined a series of nearby contracts that were distinct and independent. For each of these contracts, we examined the implied volatility patterns separately using the appropriate underlying futures contracts that the options were based upon.

This meant that our analysis would have standardised implied volatility smiles for maturities from (approximately) 90 days until the expiration day. These would be estimated 4 times per year and if we had 12 years of options data, we would have 48 separate contracts that would have been analysed. These 48 contracts could be seen as distinct observations of the relationship between implied volatilities, standardised strike prices and time. Our objective was to compare the smile behaviours at different times to expiration for these 48 contracts to assess if patterns could be discerned both within the contracts and between the contracts. To reduce the data required for this analysis, we restricted our analysis of the smiles to eighteen time points from (the date nearest to) 90 calendar days to expiration to (the date nearest) 5 calendar days to expiration in 5-day increments.

Thus, for every contract, we had standardised strike prices, the standardised implied volatility index, the level of the underlying futures, the interest rate level, and the level of the absolute ATM volatility for eighteen observations.

In the determination of the implied volatilities, we placed certain restrictions on the observations. Along the lines of Jackwerth and Rubinstein (1996), we tested for the absence of arbitrage opportunities. Firstly, we examined the options prices at adjacent strike prices and determined whether a butterfly arbitrage was possible. If this was the case, these options were excluded. Examples of such exclusions can be seen in Table 7.2. This was discussed previously in some detail. Furthermore, we eliminated any options that were quoted at a price of zero. Finally, given the standardisation of the strike prices implied that they were expressed in percentage terms, we had to choose a reasonable range including the available options. From casual observation, we found that the options we examined would not be excluded from the analysis over a range that was 4.5 standard deviations away from the underlying asset price. Given that our standardisation of the strike price was expressed in standard deviation terms, we had eliminated the problem of scaling that would occur from a simple ratio of the strike price to the underlying price [Tompkins (1994) or from the approach suggested by Natenberg (1994)].

The steps in the construction of this VSI are as follows. For every day that the smile structure is estimated, the actual strike prices are replaced by a relative strike price index. As a review, this is constructed by taking the ratio of the strike price to the current price of the underlying asset and dividing this result by the square root of time and the at-the-money volatility (see equation 7.1). This technique standardises all strike prices in the smiles so that they can be compared to the same relative distance from the current market price. The at-the-money (ATM) was determined by a linear interpolation of the implied volatilities for the two strike prices that bracketed the underlying asset price. Then all the implied volatilities were indexed to this level and stored in two arrays, which contain the standardised volatilities, and their accompanying standardised strike prices.

For the purposes of presentation, we have restricted the reported range of the standardised strike prices to ± 4.5 standard deviations away from the current underlying price. These results are presented in Figure 7.5. This figure can be compared to that presented in Figure 7.4, for the unadjusted raw implied volatilities and strike prices for the FTSE 100.

Again, it should be noted that these figures represent four separate expiration cycles. Even when these have been standardised, it is obvious that they change through time. Moreover, a systematic pattern seems to be emerging for the smiles. Each of the option cycles represents a different underlying asset. The patterns differ because at each point of estimation the options have different remaining time to expiration. However, if the standardised smile structures are compared with the same time to expiration a consistent and regular pattern seems to emerge.

In Figure 7.6 the smile structures are compared across different underlying FTSE 100 futures contracts but in this instance with the same time to expiration. It now appears that there is a certain degree of consistency if the time to expiration is held constant across the four underlying contracts. While it is clear that the patterns display some degree of random behaviour, for all the option cycles, the smile starts out with 90 days to expiration as a relatively linear function (although skewed for certain markets). As the option approaches expiration, the skew remains but the curvature becomes more extreme as expiration approaches. Since this curvature represents the kurtosis of the smile, we can hypothesise that both skewness and kurtosis factors determine the shape. However, the impacts of these two are not consistent. The skewness factor does seem to be relatively stable but the kurtosis is a function of the time to expiration. However, it appears that the evolution of the dynamics of the smile structure suggests that some deterministic volatility function may exist which is a function of time remaining to expiration and the levels of the strike prices.

Splitting the First Order and Second Order Strike Price Effects

While this analysis seems to suggest that consistencies exist for the shapes of the smiles when standardisation has occurred, it would be useful to understand the dynamics of the pattern better. As we outlined above, smiles contain both measures of skewness and excess kurtosis compared to a GBM process. To split this effect, we determined the skewness of the standardised smiles by taking the first term in the quadratic regression of the following form. This followed the form:

$$VSI = \alpha + \beta_1 \cdot \frac{\ln(X_\tau / F_\tau)}{\sigma\sqrt{\tau/365}} + \beta_2 \cdot \left[\frac{\ln(X_\tau / F_\tau)}{\sigma\sqrt{\tau/365}}\right]^2 + \varepsilon \tag{7.2}$$

It is noticeable that this equation bears some resemblance to equation (6.9), which is the approach suggested by Shimko (1991,1993). The sole differences are that in this analysis the strike price is standardised in formula (7.2) and is unstandardised in formula (6.9). In this regression equation, X_τ is the strike price of the option at time to expiration τ and F_τ is the underlying futures price (at time to expiration τ) that underlies the option. The procedure used the ordinary least squares (OLS) regression approach. From this regression, we obtained a slope coefficient, β_1. At this point, we must be careful of how we interpret this result. We could interpret this in two ways, firstly as an indication of a degree of 'moneyness' where the implied volatility is minimised and secondly as a surrogate for skewness. Since the coefficient of this independent variable measures the slope of the implied volatility curve at the money, it is clear that this must provide some information about the skewness of the implied distribution. Therefore, we will choose to interpret this as the latter.

The other slope coefficient, β_2, can be interpreted loosely as the measure of kurtosis. It is clear that such a quadratic curve will yield a bowl shape, which is similar to the curvature we observe in implied volatility patterns. For the period of 1996, the results of the regression for the FTSE 100 options can be found in Table 7.4 for each maturity from 90 days to 5 days to expiration.

There are a number of interesting results in this analysis. First of all, the R squared of the quadratic regression decreases as we approach the expiration of the options. Nevertheless, they are sufficiently high to suggest that we have explained most of the variance in the implied volatility patterns. We also find that the intercept is approximately equal to the expected value of 100. However, there is a tendency for the intercept to be below 100 (although not statistically significant). The first Beta intercept (measuring the first order strike price effect) suggests that the degree of negative skewness decreases as we approach expiration. The second Beta intercept (measuring the second order strike price effect) suggests that as the time to expiration is approached, the degree of curvature is becoming more extreme. Later in this research, we will examine these effects for all twelve markets and for different periods of analysis. It will be demonstrated that certain general strike price effects are indeed consistent over time. Before this, we will plot the standardised implied volatility patterns as a function of time to expiration.

Maturity	Intercept α	Standardised Strike β_1	Standardised Strike² β_2	R Squared
90 days	100.67	-10.452	2.22198	0.96591
85 days	98.634	-9.2783	2.35433	0.95754
80 days	100.27	-9.2852	2.42936	0.97052
75 days	99.344	-9.3741	2.62722	0.96767
70 days	100.96	-8.8053	2.76079	0.95430
65 days	101.00	-8.2035	2.75210	0.94295
60 days	99.898	-7.7579	2.91665	0.94879
55 days	98.760	-10.033	2.43167	0.96638
50 days	100.76	-8.8040	2.44500	0.95441
45 days	100.79	-7.7993	2.78914	0.92242
40 days	99.599	-6.3950	3.47758	0.95191
35 days	99.126	-7.0287	3.52888	0.94002
30 days	98.760	-5.9621	3.95725	0.92618
25 days	99.125	-4.6260	4.56055	0.91257
20 days	98.783	-3.2520	5.10602	0.88571
15 days	98.502	-2.9058	5.20245	0.91936
10 days	99.305	-3.0363	5.51127	0.91898
5 days	99.628	-2.4739	5.41991	0.92142

Table 7.4, Quadratic Regression Results for Standardised Implied Volatility (VSI) as a function of the Standardised Strike Price and Standardised Strike Price².

Using the results from this quadratic analysis, we can represent the results of this regression graphically. The first graph (Figure 7.7) displays the results of the overall quadratic regression. This can be compared to the earlier Figure 7.6, with the lines smoothed as a result of the regression technique. It is clear from this figure that we have both first order and second order strike price effects. To examine these separately, we determined the fitted line for the first slope coefficient and then determined the fitted line using only the second slope coefficient.

To identify the pure impact of the first order effect (for the regression), we estimated the standardised implied volatility (VSI) using the following equation:

$$VSI_{skew} = \alpha + \beta_1 \cdot \frac{\ln(X_\tau / F_\tau)}{\sigma\sqrt{\tau/365}} \tag{7.3}$$

From this equation, we will estimate the linear function that can be interpreted as the skew effect of the strike price bias. The resulting VSI estimates were determined for the time increments from 90 days to 5 days to expiration and a plot of these values can be seen in Figure 7.8a for the FTSE 100 option. Again, all the time periods to expiration are grouped together so that we can compare the estimates for each period to expiration.

From this graph, one can see that the skewness does change through time; appearing to be related to how close each option is (in its cycle) to expiration. For all the periods, the skewness is negatively sloped. The slope is remarkably similar for all the option cycles from 90 days to 30 days to expiration. Thereafter, the skewness flattens consistently as expiration is approached, becoming flat as we are between 5 and 10 days to expiration. To see this effect better, we have constructed a simple two-dimensional graph that displays the relationship between the first Beta of the quadratic regression (which captures the first order strike price effect) relative to the time to expiration. This can be seen in Figure 7.8b. As with the previous graph, it is clear that the closer to the expiration of the option, the less the degree of the negative skew.

To capture the kurtosis, we interpret the second coefficient of the regression as a surrogate measure. From this regression, we then plotted the results of the line fit using solely the second slope coefficient. This provides us with a clearer picture of pure effect of the excess kurtosis of the series, using the following equation:

$$VSI_{kurtosis} = \alpha + \beta_2 \cdot \left[\frac{\ln(X_\tau / F_\tau)}{\sigma\sqrt{\tau / 365}} \right]^2 \quad (7.4)$$

Again, the resulting VSI estimates were determined for the time increments from 90 days to 5 days to expiration and a plot of these values are presented for the FTSE 100 in Figure 7.9a.

The shapes of the patterns indicate a clear pattern of leptokurtic dynamics and this seems to be displaying a cyclical behaviour. The closer we come to expiration of each option, the more extreme the kurtosis becomes. This behaviour is consistent across all the contract cycles. At 90 days to expiration, the curvature is the least extreme and becomes progressively more extreme as expiration is approached. Given that we have corrected for the time to expiration (by dividing these results by the square root of time), the results clearly indicate a divergence from GBM. To see this relationship between the second order strike price effect and time better, we have produced a simple two-dimensional graph of the second Beta coefficient of the quadratic regression relative to time. This can be seen in Figure 7.9b. This graph clearly demonstrates that the curvature of the implied volatility pattern becomes more extreme as we approach expiration.

7.6 IMPLICATIONS OF NONSTATIC IMPLIED VOLATILITY SURFACES FOR DETERMINISTIC IMPLIED VOLATILITY MODELS

Another implication of this analysis is that the shape of the implied volatility smile appears not to be static. Not only does the pattern evolve in a similar manner over time, but it also appears that the implied volatility pattern remains centred at the current underlying forward price. Suppose we consider two classes of alternative pricing models to Black and Scholes. If the volatility evolution is captured by a stochastic volatility model (perhaps with a correlation between the evolution of the volatility and the underlying) or follows a jump diffusion process, we expect the implied volatility pattern to be centred at the current forward price. However, the research presented by Dupire (1992,1994), Derman and Kani (1994) and Rubinstein (1994) and discussed in the last Chapter, suggests that the dynamics of implied volatility patterns follow a deterministic form. This form can be determined by assuming the current implied volatility pattern associated with European options does not allow arbitrage. Therefore, as for Ho and Lee (1986) with interest rates, the implied volatility pattern is assumed to be an exogenous input and the solution for the implied dispersion processes is derived assuming no arbitrage. This would suggest that the implied volatility surface is fixed at the current point in time and will provide an expectation for the evolution of implied volatilities at future points in time and for different levels of the underlying asset.

The implications that the implied volatility patterns may be centred at the current forward price would suggest that these approaches may be mis-specified. To examine this issue, we return to equation 7.2. An equivalent form of this equation is:

$$VSI = \beta_2 \cdot \left(\frac{\ln(X_\tau / F_\tau)}{\sigma\sqrt{\tau/365}} + \frac{1}{2} \cdot \frac{\beta_1}{\beta_2} \right)^2 + \alpha - \frac{1}{4} \cdot \frac{\beta_1^2}{\beta_2} + \varepsilon \qquad \textbf{(7.2a)}$$

If we determine the minimum of the VSI, this is achieved where:

$$\frac{\ln(X_\tau / F_\tau)}{\sigma\sqrt{\tau/365}} = -\frac{1}{2} \cdot \frac{\beta_1}{\beta_2} \qquad \textbf{(7.2b)}$$

By applying algebra and solving for ln (X) and subsequently for X, we obtain:

$$\ln(X_\tau) = -\frac{1}{2}\sigma\sqrt{\tau/365} \cdot {}^{\beta_1}\!/_{\beta_s} + \ln(F_\tau), \text{ and} \qquad \textbf{(7.2c)}$$

$$X_\tau = F_\tau \cdot e^{\left(-\frac{1}{2}\sigma\sqrt{\tau/365} \cdot {}^{\beta_1}\!/_{\beta_s}\right)} \qquad \textbf{(7.2d)}$$

With the minimum of X_τ thus determined, we conducted a number of tests to understand the relationship between the point of the minimum of the implied volatility pattern and the underlying futures price, F_τ. The first test was to construct a scatterplot of the relationship between the strike price where the VSI was minimised relative to the underlying futures price. This was done using all the implied volatility patterns for the FTSE for 1996. This plot can be seen in Figure 7.10a. In this representation, all the point estimates are connected with a line to indicate their evolution over time. According to our theory, if the shape of the implied volatility surface remains centred at the futures price, the evolution of the scatterplot would be at a 45 degree angle. This plot does shows that there is a positive relationship between the minimum of the VSI and the underlying futures price. However, it is not clear that this relationship is one to one. The shapes for the individual contracts suggest some complex time dynamic may be occurring which is to be expected given that we have already demonstrated the FTSE implied volatility smile during 1996 had a consistent negatively skewed slope. Given that this skew exists, the minimum strike price (from equation 7.4b) should be higher than the current futures price. If this skewed relationship remained constant, it might very well be that the minimum strike price would have a more consistent relationship to the underlying futures. However, in Table 7.4, we demonstrated that the skewness effect was not constant across time. In that table, it was shown that if we used a quadratic regression to capture the first two moments of the strike price effect, the first order strike price effect (skewness) was reduced as the time to expiration was approached.

To examine whether the shapes of the implied volatility smiles followed the underlying futures price, we had to control for this fact that the skewness was decreasing as we approached expiration. This test entailed comparison of the difference between the underlying futures price and the minimum strike price at the same points in time to expiration. Given that at each point, we had different futures prices and minimum strike prices, we could gain a clearer insight into whether the shapes were consistent over time. This plot appears in Table 7.10b. If the implied volatility shape remained fixed relative to the underlying futures price, we would expect the percentage difference between the futures price and the minimum strike price to be the same at all points in time to expiration. In the plot we observe that for the period from 40 days to 5 days to expiration, this was indeed the case. Thus, it would appear that during this period the shapes of the implied volatility smile remained fixed relative to the level of the underlying futures. From 90 days to 40 days, three of the four contracts (all except December) had a similar relationship. This could be due to the fact that the levels of the underlying futures seemed to remain in a range between 3650 and 3900 (see Figure 7.10a). In December, the level of the futures was much higher (between 3900 and 4100). This led to a greater percentage difference between the futures price level and the achieved minimum strike price. Again this was probably due to an increase in the skew relationship in the volatility smile during this period. Nevertheless, even for the December contract the impact of the higher futures price level did not cause the percentage difference to be divergent relative to the other three contracts from 40 days to 5 days to expiration. Thus, it would appear from this preliminary analysis that the implied volatility smiles seem to move in a consistent manner remaining tied to the level of the underlying futures for the period from 40 days to 5 days to expiration. From 90 days to 40 days, three of the four contracts displayed a similar and consistent relationship.

While this analysis does suggest that the Dupire (1992,1994), Derman and Kani (1994) and Rubinstein (1994) models may be mis-specified (particularly as the options approach expiration), it remains for further research to identify a better approach to test this hypothesis. As we have indicated, the existence of a first order strike price effect related to the time to expiration requires careful thought as to how to remove this effect.

Even so, we have demonstrated that for the period from 40 days to 5 days to expiration, these results are somewhat anomalous to the work of Dupire (1992,1994), Derman and Kani (1994) and Rubinstein (1994) as they suggest there may exist arbitrage opportunities. Given that all three approaches depend upon a no-arbitrage constraint to assess the risk-neutral dispersion process. The implications of his findings are that the current implied volatility smile pattern will be an unbiased estimate of the actual implied volatility observed if the underlying price moved such that it were equal to the strike prices of the option. These results suggest that the implied volatility patterns retain the same shape and generally follow the prevailing underlying price. Thus, because smile patterns appear to remain centred at the underlying asset price something similar to arbitrage appears to be possible.

For example, assuming the Dupire model with a symmetrical smile shape, and the underlying market price at 100, the 110 call option would have a higher implied volatility than the 100 call option. If the underlying market price were to rise to 110, the Dupire model would expect the shape of the implied volatilities to be static. Thus, a higher implied volatility would be realised for the 110 call and a lower implied volatility would exist for the 100 call option. However, if the smile follows the underlying price, the 110 call would have a lower implied volatility than the 100 call option. To benefit from this effect one could sell the 110 call and buy the 100 call. The trader would immunise the exposure to the underlying asset price by dynamic (delta) hedging in the standard manner and unwind the transaction for a profit when the price of the underlying rose to 110.

While this appears to be an arbitrage, in practice it is not. The exposures of each option to implied volatility ($\partial C / \partial \sigma$) are not identical and would require different number of options to remain neutral to changes in implied volatility. In addition, these exposures would be constantly changing requiring extensive revisions both to the dynamic hedge of the underlying and to the levels of the implied volatility. Finally, while the overall position might be continually rebalanced to remain neutral to the changes in implied volatility, in practice this would require some hedge for the overall level of implied volatility. While Neuberger (1994) and Whaley (1993) have suggested instruments to achieve this goal, they are not currently available. Furthermore, the trade would involve extensive transaction costs and it is not clear that such a trade would provide a net profit.

Finally, it may be that the volatility smile dynamics are determined by additional factors apart from time and strike price. Suppose a fixed smile exists that remains tied at the level of the underlying asset price. This may still omit an arbitrage because we have more variables than states, so that the volatility pattern must evolve through time to disallow arbitrage. However, this is probably a second order effect. For this empirical research, our objective is to understand the regularities of market development. Therefore, the algebraic interpretations of our model should not be taken literally. We will provide subsequent evidence that the magnitude of the residuals would remove any existence of practicable arbitrage.

7.7 STANDARDISED IMPLIED VOLATILITY SMILES FOR ALL TWELVE MARKETS

Given the relative stability of the volatility smile graphs both for the overall levels, the skewness measures and the kurtosis measures (when compared at the same point in time), an index of the volatility smile patterns can now be constructed for all the markets under investigation. Fitting the above regression (equation 7.2) achieves this goal and will allow us to compare the patterns across markets. This analysis has been done for all the twelve options markets and appears for the overall VSI levels in Figures 7.11a for the four stock index options, 7.11b for the four fixed income options and 7.11c for the four foreign exchange options. This analysis was completed using all options data from 1996.

In Figure 7.11a, one can see that some consistent relationships seem to exist among the four stock index options markets. The S&P 500 and the DAX options display almost identical patterns with similar dynamics over time. Both markets start with 90 days to expiration with a relatively linear left skewed shape. This shape flattens somewhat as expiration is approached and the patterns display progressively greater curvature. One can see that the FTSE and the Nikkei also display similar dynamics. The implied volatility patterns are more convex at 90 days compared to the other stock index options and this pattern becomes more convex as expiration is approached.

In Figure 7.11b, there are also consistencies among the four fixed income options markets. The Bund, BTP and Gilt options display similar patterns over time. In some ways, these shapes are reminiscent of those observed for the FTSE and Nikkei smiles. All markets start with 90 days to expiration with a relatively linear left skewed shape with a convex shape. As expiration is approached, the skew flattens and the convexity increases. The odd market out is the US T-Bond, where there is little evidence of a skew but the convexity can clearly be seen as increasing as expiration approaches.

In Figure 7.11c, there appears to be even greater consistency among the four foreign exchange options markets. All four markets display almost identical patterns with similar dynamics over time. In many ways, there appears to be a constant shape across time for all the four markets. The only time dependent variation that seems to occur is that the convexity is becoming more extreme as the time to expiration is reduced.

From all these graphs, it is difficult to assess the time varying characteristics of the skewness and convexity. This is due to the smile patterns containing both elements. Therefore, to assess the pure effect of the skewness we applied the regression method to the indexed volatility smile patterns to determine the first slope coefficient. In this instance, the estimated VSI results are based upon results from the first term of the quadratic regression for each time period to expiration. These results can be seen in two sets of graphs. In Figures 7.12a (for the four stock index options), 7.12b (for the four fixed income options) and 7.12c (for the four foreign exchange options), three-dimensional graphs are displayed that demonstrated the time varying behaviour of the first order strike price effect. For the sake of clearer presentation, we have also produced the simpler two-dimensional graph of the first order strike price effect relative to time. These can be seen in Figures 7.13a (for the four stock index options), 7.13b (for the four fixed income options) and 7.13c (for the four foreign exchange options). In these graphs, the solid line represents the actual slope coefficient from the simple quadratic regression. In these figures a dotted line also appears. This series is based upon a more complete model that will be developed in the next Chapter. At that time, we will return to these figures for the comparison of the two approaches.

In Figures 7.12a and 7.13a, we can confirm the conclusions suggested from Figure 7.11a regarding the time dynamics of the skew. For all four markets, the degree of the negative skew is most extreme as the time to maturity is greatest. As expiration is approached, the skewness flattens considerably. Nevertheless, the degree to which this flattening occurs is idiosyncratic to each market. For the S&P 500, the skewness remains fairly consistent for the option cycle until the last 5 days of the options life. The FTSE 100 options display a more consistent flattening from approximately 30 days until expiration. The Nikkei-225 displays a slightly different behaviour. For this market, the least amount of skew is observed. Nevertheless, it also flattens as expiration is approached. Finally, the DAX option bears a remarkable resemblance to the one observed for the S&P 500 (although the S&P displays more skewness, note the different scales on the graphs).

In Figures 7.12b and 7.13b, once again we can observe that the three fixed income markets on the LIFFE (Gilt, BTP and Bund) display similar time dependent dynamics. As with the stock index options, the degree of the skew is directly related to the time to expiration of the option. As expiration is approached (especially in the last 5 days), the skew flattens considerably. As before (Figure 7.10b), the US T-Bond displays a different pattern. Essentially, there is no skewed relationship and the implied volatilities (from the pure first order strike price effect) are falling as expiration is approached. Thus, there must be a relatively greater second order effect for this market.

In Figures 7.12c and 7.13c, for all four foreign exchange markets, there is essentially no consistent skewed relationship in the smiles. This confirms the conclusions drawn from Figure 7.11c. Thus, the entire strike dependent effects for these markets must be related to the second order effect.

By examining the second slope coefficient of the regression, we can now obtain a measure of the pure kurtosis of the volatility smile patterns for the twelve markets. As with the previous graphs, these plots consider the results from the quadratic regression for the same time period to expiration. The three-dimensional graphs are presented in Figure 7.14a for the four stock index options, Figure 7.14b for the four fixed income options and Figure 7.14c for the four foreign exchange options. As with the first order strike price effects, we have also chosen to present the two-dimensional graphs that describe the second order strike price relationship relative to time. This can be seen in Figure 7.15a for the four stock index options, Figure 7.15b for the four fixed income options and Figure 7.15c for the four foreign exchange options. As with the previous figures, the solid line represents the second beta coefficient from the simple quadratic regression. The dotted line represents an alternative way to estimate this effect. The method of estimation will be discussed in the next Chapter.

It is interesting in the comparison of these three figures how remarkably similar the shapes are for all twelve markets. In Figures 7.14a and 7.15a, the evolution of the kurtosis for the DAX and the FTSE 100 display almost the same patterns. For both markets, an excess kurtosis is implied but becomes more extreme the closer we are to the expiration of the option. While the S&P 500 options display a similar pattern, the major difference is that it is more extreme. We observe almost a flat line when the option is at 90 days to expiration and the change in the curvature is thus much more extreme as expiration is approached. For the Nikkei, we seem to observe more curvature for the longer maturity options. Some flattening occurs in the maturities from 60 to 20 days and increases again as expiration is approached.

In Figures 7.14b and 7.15b (for the four fixed income option markets), we observe consistency in the kurtosis effects for the Gilt, BTP and Bund markets. As was suggested in Figure 7.10b, the US T-Bond displays a much more consistent degree of curvature throughout the option cycle (although the curvature does become slightly more extreme as expiration is approached). Finally in Figures 7.14c and 7.15c, the four foreign exchange options display almost an identical time varying pattern to the kurtosis. As was indicated in the analysis of Figure 7.11c, there appears to be an almost consistent implied kurtosis through the life cycle of the options. If any divergence occurs it is clear that the convexity increases slightly as expiration is approached. However, across all twelve markets, if appears that the kurtosis effect is a function of the time to expiration and the closer we are to expiration, the more extreme the kurtosis becomes.

These graphical presentations suggest that smile patterns both for individual markets, within asset classes and indeed among all markets display some similar behaviours. If this is indeed the case, it has profound implications on the modelling of implied volatility patterns.

Consistency in smile structures though time and across markets could suggest that some systematic dynamics might be driving all markets. This could mean that it might be possible to discern some consistent (and possibly deterministic) volatility functional form that will allow us to understand the dynamics of the volatility smiles better.

Examination of Standardised Smile Structures for the Twelve Markets in Sub-Periods

Finally, to allow later comparisons to be drawn, we estimated the quadratic regressions for all the options examined (and for all twelve markets) for three separate time periods. The first period covered the entire period for each market (where data was available). The other two periods correspond (roughly) to the same split for the underlying futures date that occurred in Chapter 2 (see Table 2.3). The exact breakdown of the option prices (in terms of the time periods) appears in Table 7.5.

Underlying Asset	1st Time Period Of Analysis	2nd Time Period Of Analysis
S&P 500 Options	25/03/1986 - 27/12/1990	28/12/1990 - 20/12/1996
FTSE Options	02/01/1985 - 29/08/1990	30/08/1990 - 20/12/1996
Nikkei 225 Options	25/09/1990 - 03/11/1993	04/11/1993 - 13/12/1996
DAX Options	03/01/1992 - 01/07/1994	04/07/1994 - 20/12/1996
Bund Options	20/04/1989 - 03/11/1992	04/11/1992 - 05/12/1996
BTP Options	11/10/1991 - 26/04/1994	27/04/1994 - 04/12/1996
Gilt Option	13/03/1986 - 06/12/1989	07/12/1989 - 27/12/1996
US T-Bond Options	02/01/1985 - 28/04/1987	29/04/1987 - 31/12/1996
D Mark Options	03/01/1985 - 26/12/1990	27/12/1990 - 16/12/1996
Pound Options	25/02/1985 - 26/12/1990	27/12/1990 - 16/12/1996
Yen Options	05/03/1986 - 28/12/1990	31/12/1990 - 16/12/1996
S-Franc Options	25/02/1985 - 06/07/1990	09/07/1990 - 16/12/1996

Table 7.5, Periods & Observations for Markets Under Analysis, Broken into Two Sub-Periods

Thus, the average VSI values and the accompanying smile shapes for the entire period can be seen in Figures 7.16a, 7.16b and 7.16c. For the earlier portion of the analysis period, these results appear in Figures 7.17a, 7.17b and 7.17c. Finally for the latter portion of the analysis period, these results appear in Figures 7.18a, 7.18b and 7.18c.

7.8 CONCLUSION

From a comparison of all the figures, it appears that a great deal of consistency exists for the same markets over all three periods of analysis. For example, the stock index options all display a similar first order and second order strike price effect. While these patterns seem relatively stable, it appears that for some markets (S&P 500, for example) a greater first-order negative skew effect is apparent in the second period. For all the fixed income options, the shapes are almost identical across all periods. This also appears to be the case for the currency options. If such consistency exists, this has profound implications for understanding the dynamics of the implied volatility patterns.

While this type of graphical presentation does provide insights into the behaviour of smiles and suggests that some consistencies may exist in the nature of the implied volatility surface, it is not possible to draw definitive conclusions simply by comparing graphs over different periods. While there is little doubt that such graphs may provide clues to understand the general consistency of patterns, (in any case) rigorous statistical testing is required to assess if the dynamics of these implied volatility surfaces are similar or not. Thus, the primary benefit from the graphs developed in this Chapter is to provide insights into what the form that the statistical analysis and what questions should be examined.

In the next chapter, we will examine whether some functional form for the implied volatility surface can be determined statistically. From this analysis, we will be able to answer a multitude of questions, which have been raised in the literature. Namely, we will assess whether the smiles are stationary for the same underlying. Secondly, we will ascertain if the smiles at the same time to expiration are similar for the same markets (but for different contracts). We will also examine the stability of the moments of the smiles estimated by the quadratic regression. This will allow us to draw conclusions about the stability of the skewness and kurtosis relationships. Given the evidence that suggests that the markets were not skewed prior to the 1987 stock market crash, we will examine whether this is the case or not and how the patterns behave over time. Thirdly, we will compare smiles at the same time to expiration across different instruments. This will allow us to assess if consistencies exist in smile dynamics across all markets. In addition, we will extend this analysis within the three asset classes. This will allow us to determine whether smiles within an asset class display consistent (or related) behaviours or not.

CHAPTER EIGHT
THE ANALYSIS OF RISK NEUTRAL PROBABILITIES IN OPTIONS ON FUTURES. COMPARISON OF IMPLIED VOLATILITY SMILES WITHIN AND BETWEEN MARKETS: ANALYSIS USING MULTIPLE REGRESSION /ANALYSIS OF VARIANCE

8.1 INTRODUCTION

The objective of this Chapter is to assess the extent to which the implied volatility structures for the twelve markets under investigation share characteristics and to compare differences. From the last Chapter, it appears that some sort of consistency may exist for the shapes of implied volatility patterns across strike prices. It is somewhat surprising that apart from the implied distributions proposed by Rubinstein (1994), Derman and Kani (1994), Dupire (1994) and Jackwerth and Rubinstein (1996), there are been few attempts to ascertain if consistencies in the implied volatility functions exist over time.

Recently, Corrado and Su (1996, 1997) have extended the Black-Scholes model to include skewness and kurtosis in the options-implied distributions. Based on this methodology, they estimated the options-implied coefficients of skewness and kurtosis. Their analysis covered both stock options [Corrado and Su (1997)] and options on stock-indices [Corrado and Su (1996)]. While this approach does seem to capture significant elements in the strike price effect, it remains to be seen whether these effects are consistent over time and whether the inclusion of other factors may improve our understanding of the implied volatility surface.

In this Chapter, we will fill a gap in the literature. We will examine the consistency of implied volatility patterns across time and between markets. This will allow us to assess the dynamics that drive the implied volatility surface. When similar research had been done the emphasis was on assessing a deterministic implied volatility function. This is not our objective. We will instead examine a more general functional form of the implied volatility process. However, we will build upon the work completed on the estimation of deterministic implied volatility models.

8.2 DETERMINISTIC IMPLIED VOLATILITY MODELS

Our approach is related to the work by Dumas, Fleming and Whaley (1996), who attempted to assess a deterministic volatility function using implied volatilities. We will not follow the same ground for the simple reason that we have already rejected the existence of a deterministic implied volatility function when we found that the implied volatility pattern remained centred at the level of the underlying asset price. Therefore, we are asking a different question: What factors are required to capture the dynamics of the implied volatility surface. It will be demonstrated that certain elements of the Dumas, Fleming and Whaley work will be necessary for our task. So, even though we will not aim to assess a deterministic implied volatility function, it will be our starting point.

Dumas, Fleming and Whaley (1996) indicated that they had no prior assumptions of what the function of $\sigma(X,T)$ might be, so they tested a number of arbitrary models. Using their notation, the four models they examined were:

Model 0: $\sigma = a_0$ **(8.1a)**

Model 1: $\sigma = a_0 + a_1X + a_2X^2$ **(8.1b)**

Model 2: $\sigma = a_0 + a_1X + a_2X^2 + a_3T + a_5XT$ **(8.1c)**

Model 3: $\sigma = a_0 + a_1X + a_2X^2 + a_3T + a_4T^2 + a_5XT$ **(8.1d)**

For their models, the implied volatility (σ) is the 'raw' implied volatility determined by the Black-Scholes model. The strike price is also unadjusted and is referred to as X. Finally, the term T is simply the time to expiration. One can see immediately, that they are testing with model 0 the assumption of constant volatility. Model 1 is the first approach used by this research [and Shimko (1991,1993)] to determine a quadratic fit for the implied volatilities at a single point in time. Models 2 and 3 are determined by means of a Taylor series expansion to degree two. Model 2 only included the first expansion relative to time and the interaction term between time and the strike price, while Model 3 includes both second order terms.

In a sense, the selection of these models [as Dumas, Fleming and Whaley (1996) point out] is arbitrary. This is somewhat unfortunate since there is extensive evidence of other factors that may influence the behaviour of implied volatilities in the literature.

8.3 MODELLING THE IMPLIED VOLATILITY SURFACE

In this research, we will extend the analysis of Dumas, Fleming and Whaley (1996) to include additional factors, which may influence the behaviours of volatility smiles. These include the evidence suggested by Rubinstein (1994) that the behaviour of volatility smiles changed after the 1987 stock market crash. If this is the case for stock index futures, it may very well be that volatility smile behaviour might have changed after shocks that are specific to other individual markets. We have already shown that by using raw implied volatilities, some of the smile effect could be due to the manner with which the Black-Scholes model assumes price movement over time. Thus, by correcting for this impact, we will gain a clearer picture of the true behaviour of the implied volatilities that is not due to the nature of the Black-Scholes model. Finally, as we demonstrated in the previous Chapter, there seems to be consistency in the behaviour of the smiles for the same underlying market, for the same standardised strike prices and at the same time to expiration. What we require is a quantitative approach to test whether these casual observations suggested from the figures in the last Chapter are justified.

To achieve our objective of understanding the implied volatility processes more generally, we decided to analyse the standardised implied volatilities (VSI) by running an analysis of covariance (ANCOVA). [See Scheffé (1959) and Lindman (1974)]. The research design employed follows the lead of Davies (1954) for such experiments. Essentially, our analysis involved establishing a multiple regression with dummy variables (this is the definition of ANCOVA). For this analysis, we will examine all the relevant interactions between terms by careful review of the residuals. The computer programme used for the analysis was STATISTICA for Windows (version 5.0). [See STATISTICA (1995)].

Thus, our analysis will include a number of other variables and extend the analysis of Dumas, Fleming and Whaley (1996) to include higher order terms for both the strike price and the time factors. In addition, we will include all the interaction terms for these factors. This will lead to a fairly complex model with a substantial number of variables. Dumas, Fleming and Whaley (1996) suggested that the more complicated the model to capture implied volatility dynamics, the more the model could be accused of overfitting in sample (and be potentially exposed to data mining). Given that our models at first glance appear to be fairly complex, we will diverge briefly to describe our model and the rationale for the inclusion of our variables to address the issue of overfitting.

Variables Used in the Analysis

Our objective is to understand the general dynamics of the implied volatility process. In the last Chapter we have identified that the first and second order strike price effects depend on the time to expiration. Therefore, we constructed variables that would allow us to examine these effects. We will start with Model 3 of Dumas, Fleming and Whaley (1996) and found in this Chapter as equation 8.1d. As was mentioned previously, they took a Taylor's series expansion to degree two. Consider a Taylor's series expansion to degree three.

Expanding the function $\sigma = f(x,t)$ with Taylor's expansion series

$$\sigma = \sum_{i,j=0}^{\infty} \frac{1}{(i+j)!} \frac{\partial^{(i+j)}\sigma}{\partial x^i \partial t^j} x^i t^j \tag{8.1e}$$

and stopping the expansion at the third degree ($i + j \leq 3$), we obtain

$$\begin{aligned} \sigma \approx & \frac{\partial \sigma}{\partial x} x + \frac{\partial \sigma}{\partial t} t + \frac{1}{2!} \frac{\partial^2 \sigma}{\partial x^2} x^2 + \frac{1}{2!} \frac{\partial^2 \sigma}{\partial t^2} t^2 + \frac{1}{2!} \frac{\partial^2 \sigma}{\partial x \partial t} xt + \\ & + \frac{1}{3!} \frac{\partial^3 \sigma}{\partial x^3} x^3 + \frac{1}{3!} \frac{\partial^3 \sigma}{\partial t^3} t^3 + \frac{1}{3!} \frac{\partial^3 \sigma}{\partial x^2 \partial t} x^2 t + \frac{1}{3!} \frac{\partial^3 \sigma}{\partial x \partial t^2} xt^2 \end{aligned} \tag{8.1f}$$

Given that we have nine derivatives in the expansion, we have constructed nine variables to capture these effects.

However, we are aware of findings in the literature that the strike price effects have not remained the same over time. This was discussed extensively by Rubinstein (1994), where he claimed that the skewness effect was a result of the 1987 stock market crash. Thus, our model must examine this by inclusion of a variable that allows us to compare strike price effects prior and post 1987 crash. The most reasonable variables to capture this effect would be a dummy variable, which assumes a value of 0 prior to the crash and 1 thereafter. To assess the impacts on the strike price effect, this dummy variable will be multiplied by the first and second order strike price variables from equation 8.1f. It is not unreasonable to surmise that if the shock associated with the 1987 stock market crash affected the strike price effect for the S&P 500 futures, other shocks may change the strike price effect for other markets. Specifically, idiosyncratic shocks may affect the strike price effects for individual markets. Thus, another two series of dummy variables were constructed that were similar to the dummy variable estimated for the 1987 stock market crash. The difference is that these shock variables related to each of the twelve markets. Once again composite strike price variables were estimated (similar to the composite strike price variables relative to the 1987 crash) to examine the effects of shocks on the strike price effects.

Finally, market practitioners have claimed that the strike price bias is a function of the level of the implied volatility. While this has not been investigated extensively in the literature, we will rectify this here. This will be done by including the level of the at-the-money implied volatility in the model. Once again, since our objective is to understand the strike price effects, combination variables will be estimated which are the products of the first and second order strike price effects in equation 8.1f with the level of the at-the-money implied volatility.

This will lead to a fairly complex model with a large number of variables. While some may claim that this model may amount to data-mining, it is important to realise that the choice of this model has been done on an a priori basis and the rationale for the choice of all the variables in the model come either from a Taylor's series expansion or have been identified in the literature as being relevant.

An additional consideration is that normally when evaluates an equation with so many independent variables, there are too many parameters to identify

Column	Factor
1	VSI (dependent variable)
2-4	CLASS (3 Dummy Variables)
5-16	MARKET (12 Dummy Variables)
17-64	CONTRACT (48 Dummy Variables)
65	TIME (% of a Calendar Year)
66	$TIME^2$ (% of a Calendar Year)
67	$TIME^3$ (% of a Calendar Year)
68	STRIKE PRICE (Standardised)
69	STRIKE $PRICE^2$ (Standardised)
70	STRIKE $PRICE^3$ (Standardised)
71	INTERACTION1 (STRIKE PRICE * TIME)
72	INTERACTION2 (STRIKE PRICE * $TIME^2$)
73	INTERACTION3 (STRIKE $PRICE^2$ * TIME)
73	INTERACTION4 (STRIKE $PRICE^2$ * $TIME^2$)
74	87CRASH (Dummy Variable for 1987 Crash)
75	CRASH&S (87CRASH*STRIKE PRICE)
76	CRASH&S2 (87CRASH * STRIKE $PRICE^2$)
77	SHOCK1 (1st Major Shock to Unconditional Returns)
78	SHOCK1&S (SHOCK1*STRIKE PRICE)
79	SHOCK1&S2 (SHOCK1 * STRIKE $PRICE^2$)
80	SHOCK2 (2nd Major Shock to Unconditional Returns)
81	SHOCK2&S (SHOCK1*STRIKE PRICE)
82	SHOCK2&S2 (SHOCK1 * STRIKE $PRICE^2$)
83	ATMVOL (Unadjusted ATM Implied Volatility)
84	ATMVOL&S (ATMVOL*STRIKE PRICE)
85	ATMVOL&S2 (ATMVOL * STRIKE $PRICE^2$)

The STRIKE price variables will capture the pure effects of the first order relationship between the strike price and implied volatility (STRIKE1), the second order effect (STRIKE2), and STRIKE3 was added to assess if further and higher moments are required. From these variables, we will determine the statistical relationships for the skewness and kurtosis (plus another higher moment) which was suggested by the graphical presentations in the last Chapter.

The TIME variables will examine the relationship between implied volatilities and time. Given that these effects may be complex, a number of combined variables were examined to assess the interactions between time and other variables. The four interaction terms capture two elements of the strike price effect. As with the analysis of Dumas, Fleming and Whaley (1996), we wish to understand the time varying dynamics of the volatility skew and implied kurtosis. INTERACTION1 and INTERACTION2 examine the first order and second order impact of time on the skew of the implied volatility pattern. In a similar manner, INTERACTION3 and INTERACTION4 examine the first order and second order impact of time on the kurtosis of the implied volatility pattern.

Inclusion of Variables to Capture Shocks

We will clarify some of the new factors. First, the factors, SHOCK1 and SHOCK2 require clarification. While it is clear that the 87CRASH variable reflects a dummy variable with 0 value prior to October 19th, 1987, these variables achieve a similar goal for the other markets. To determine when such a shock had occurred, we returned to Figures 2.1a, 2.1b and 2.1c. In these figures, we displayed the (forward and backward) exponentially weighted unconditional volatility time series for the twelve markets of interest. From these figures, we can determine extreme spikes in the unconditional volatility that occurred over the period of analysis.

Underlying Asset	First Shock	Second Shock
Stock Index Options		
S&P 500 Futures	19/10/1987	13/10/1989
FTSE Futures	19/10/1987	16/10/1989
Nikkei Dow Futures	21/08/1992	07/07/1995
DAX Futures	05/10/1992	02/03/1994
Fixed Income Options		
Bund Futures	21/02/1990	13/06/1994
BTP Futures	05/10/1992	16/06/1994
Gilt Futures	30/09/1986	02/06/1994
US T-Bond Futures	09/06/1986	28/04/1994
Currency Options		
Deutsche Mark /US Dollar	23/09/1985	21/08/1991
British Pound / US Dollar	23/09/1985	16/09/1992
Japanese Yen / US Dollar	23/09/1985	05/01/1988
Swiss Franc / US Dollar	23/09/1985	05/01/1988

Table 8.1, Dates on Which Two Major Shocks in Variance Occurred for the Twelve Markets Under Examination.

Specifically, we determined the day at which the first and second major spike occurred (that fell within the period of which we had implied volatility estimates). These are summarised in Table 8.1

On all of these dates, major economic events caused extreme levels of unconditional volatility. For the S&P 500 and FTSE 100 futures, these two events are the 1987 stock market crash and the 1989 mini-stock market crash. For the DAX and Nikkei (where we only had options data post these events), the shocks to the unconditional returns were country specific.

For the DAX, the October 1992 shock was due to the aftermath of the EMS crisis, when a number of major German trading partners (Britain and Italy) were ejected from the exchange rate mechanism. This was considered to hurt the competitiveness of the German economy and thus impact future corporate profits. At the same time, the Bundesbank attempted to ameliorate the situation by lowering short-term interest rates. This seemed to have the opposite effect; indicating the gravity of the lower expected corporate profits. The March 1994 shock was also associated with a Bundesbank change in interest rate policy. In response to a burgeoning M3 money supply surge, the Bundesbank decided not to reduce interest rates further. According to the Wall Street Journal on March 3, 1994 [Roth and Whitney (1994)], the failure to reduce interest rates in line with market expectations of such a decrease caused "waves of selling". This caused the DAX to drop by 2.3% in one day and increased the uncertainty of future interest rate policy.

The 1992 shock to the Nikkei, was due to increased uncertainty regarding corporate profits due to dismal government economic statistics reporting in July 1992 that the economy had slowed down to a then record low of 0.2% (which was down 0.7 percentage points from the previous month). This led to the Nikkei index to fall to under 15000. The August 1995 shock was again due to a collapse of the Nikkei index to below the 15,000 level for the first time in 34 months. This was associated with the all-time strengthening of the Yen relative to the US Dollar (to below ¥80) which was seen to have serious implications for the export oriented Japanese economy.

For the fixed income markets, the first shocks tended to be country specific. For example the first shock in the Bund market reflected uncertainty after the re-unification of West and East Germany and the expectation that the issuance of massive quantities of the special Government bonds [Treuhand Anleihen] would be required for financing the re-unification.

The first shock for the BTP occurred in the Summer of 1992, when the Danish people rejected the Maastricht treaty in a referendum. This cast the entire monetary union issue into question, which was particularly relevant to the Italian market, with a relatively weak economy and an overvalued currency (in the fall of that year the ERM crisis resulted in the expulsion of the Lire from the exchange rate mechanism).

For the Gilt, the first major shock to the unconditional returns occurred in September 1986. This was due to the delay in an expected interest rate cut by the Bank of England. However, the Bank had concerns about monetary and credit growth and together with the weakening of the exchange rate (largely oil-related), caused sentiments to reverse. At the September meeting of the Bank of England no change in interest rate policy was announced. Subsequently, on the 14th of October of that same year, interest rates were raised by 1%. This change in policy lead to a substantial sell-off in all UK Government stock.

For the US T-Bond market, it is clear from Figure 2.1b that significant shocks came in the period from the late 1970s to the late 1980s. Unfortunately, we only have options data for the US T-Bond from 1985. Over this period, the first major shock occurred in June 1986. This shock seems to an aftermath of the Economic Summit meeting in Tokyo of the G-7 members during early May. Due to a swelling trade deficit between the US and Japan, the US contingent had pressed the Japanese to cut interest rates to ease pressure on a depreciated US Dollar. The Japanese, having refused this suggestion, forced the US to reconsider their own interest rate policy. To that point, the bond markets had been expected a co-ordinated cut in interest rates. As the US Dollar continued to weaken in early June, it became apparent that the anticipated boost to US exports and growth was not being sustained and expectations of another downward adjustment in US interest rates were revived. Finally, after an increase in US unemployment was reported (confirming other Government statistics that the economy was weakening), the markets began to believe that a further easing of U.S. Monetary policy was imminent. This lead to further downward pressure on the US Dollar, as foreign holders of US Government Bonds began dumping their holdings.

It is interesting that for all the fixed income markets, the second shock is shared. This shock occurred in spring 1994 as the US Federal Reserve surprised the bond markets by raising the Discount Rate to stem what they felt was inflationary pressures. This lead to a world-wide rise of interest rates, fall in bond prices and an increase in bond market volatilities. However, the impact for the other world-wide bond markets was delayed somewhat. The full impact was felt approximately one month after the shock hit the US markets.

For the currency markets, the first shock was shared for all the four currencies examined. This shock occurred in 1985 as the US Dollar has risen to an extremely over-valued level (versus all major currencies). The shock occurred when a weekend meeting of the Group of Seven (G7) resulted in a concerted effort by all the central banks to sell Dollars to return the economic systems to what was perceived as a sustainable equilibrium. The second shock was shared by two of the four currencies, the Swiss Franc and Japanese Yen. This occurred in January 1988. For the Swiss Franc, as a consequence of the 1987 Stock Market Crash, Swiss investors had been reducing their holdings of US investments. The Swiss Franc had by that time become one of the strongest currencies versus the US Dollar and there was concern as the Swiss National Bank indicated that the strengthening of the currency could have adverse long-term impacts on the economy. For the Japanese Yen, a similar reason exists for the shock. The spot rate of the Yen had risen to the highest level since the Second World War (¥120) but thereafter dropped sharply. This occurred as confidence in international economic policy co-ordination began to take hold after a G-7 meeting, which occurred in January. The first major shock for the British Pound also occurred in 1985, when the over-valued US Dollar collapsed after the G-7 meeting. The second shock came on September 16th, 1992 when the British Pound was ejected from the European Monetary System by speculative pressures. Even though this crisis was related primarily between the British Pound and the Deutsche Mark, the volatility spilled over to the US Dollar/British Pound Exchange rate. The second shock for the Deutsche Mark was deemed to have occurred in August, 1991. Again this was due to market uncertainty regarding the success of the re-unification of West and East Germany. The specific event which caused the shock was the Bundesbank raising the discount interest rate from 6.5% to 7.5% (on the 16th of August) in response to the overheating of the economy.

A number of points should be made about the inclusion of these shock dummy variables. Firstly, for the S&P and the FTSE, the 1987 stock market crash was the first major shock. Thus, it would be redundant to include both 87CRASH and SHOCK1 in the analysis. Therefore, we ignored SHOCK1 for these markets. Secondly, the inclusion of the 1987 stock market crash may seem irrelevant for the non-equity asset classes. However, the economic rationale is that the occurrence of the 1987 stock market crash may have changed the way that market participants addressed the risk management of options more generally. Finally, the economic rationale for the inclusion of the other shocks is to extend to each market what Rubinstein (1994) and others have claimed occurred for the S&P 500. Specifically, the question is: do market participants change the way they evaluate options after the occurrence of extreme events?

For each of these shock variables, we have included interactions with the STRIKE and STRIKE2 variables. These will allow us to examine whether the behaviours of the skew (STRIKE) and the implied kurtosis (STRIKE2) changed after these events. This will allow a direct test of the Rubinstein hypothesis. In addition, since we are including the CRASH interaction for all markets, we can assess the global effect of the 1987 stock market crash on the risk characteristics of contingent claims in non-equity related markets.

Inclusion of Additional Variables

Given that almost all the other factors are self-evident as to their meaning, we would like to clarify the factor ATMVOL. This is determined by taking a linear interpolation of the implied volatilities of the two (out-of-the-money) strike prices that bracketed the underlying futures price. This variable is included to assess the errors we may have introduced into our analysis by our way of estimating the at-the-money implied volatility.

The inclusions of the interaction variables that include ATMVOL have another purpose. The variable ATMVOL&S will indicate whether the degree to which the skew is manifested in our twelve markets is related to the level of the at-the-money volatility. In a similar manner, ATMVOL&S2 will indicate whether the curvature (implied excess kurtosis) from the implied volatility patterns is also related to the level of the at-the-money implied volatility. If these factors do turn out to be significant, we will have a clearer understanding of how absolute levels of expected variance affects market participants implied dispersion processes.

Final clarification must be given for the CONTRACT dummy variables. Our data set included the earliest observation, in January 1985. These observations were for the March 1985 contract. Since we are looking at quarterly contracts until the end of 1996, this means we have 48 contracts in total. It should be noted that not all markets have observations for all these contracts. However, since all the markets examined are examined for the same quarterly cycle, we will be able to test for contemporaneous volatility effects that may have occurred during the period of analysis.

Finally (and on a technical note) we sorted the data to understand the time series behaviour better. This is because this analysis bears some resemblance to a panel analysis. This sorting was done first by CONTRACT so that all the volatilities in our analysis were grouped from 1 to 48. Then, these were sorted by the actual strike price. This allowed the same option contract to be examined. Finally, we sorted by time to expiration. In this way, we obtained a series of options with the same strike price and contract expiration date and could analyse these through time. Unfortunately when an individual option would expire, another option would immediately follow in the data array (but with the next highest strike price) with the longest time period to expiration. Of course, this will lead to unusual time series characteristics in the residuals. However, this approach will allow us (in a limited manner) to examine the residuals for effects of serial correlation (as we would expect would exist when one option expires and the next one replaces it in the analysis, these panel sets would be uncorrelated at that point).

8.4 INTERPRETATION OF THE MODEL

Before we present the results of our analysis, we should pause to examine the overall research design and how this will address our research objectives. To begin, we are interested in understanding the nature of the implied volatility processes associated with smiles. Therefore, our first objective is to understand the higher order moments of the distribution, namely the skewness and kurtosis. Given that the skewness is captured by the STRIKE variable and the kurtosis is captured by the STRIKE2 variable, we can group all these together to yield the following equation:

$$\begin{aligned} VSI &= \alpha + STRIKE \cdot (\beta_1 + \beta_2 \cdot TIME + \beta_3 \cdot TIME^2 + \beta_4 \cdot CRASH + \beta_5 \cdot SHOCK1 + \beta_6 \cdot SHOCK2 + \beta_7 \cdot ATMVOL) \\ &+ STRIKE^2 \cdot (\beta_8 + \beta_9 \cdot TIME + \beta_{10} \cdot TIME^2 + \beta_{11} \cdot CRASH + \beta_{12} \cdot SHOCK1 + \beta_{13} \cdot SHOCK2 + \beta_{14} \cdot ATMVOL) \\ &+ \beta_{15} \cdot STRIKE^3 + \beta_{16} \cdot CRASH + \beta_{17} \cdot SHOCK1 + \beta_{18} \cdot SHOCK2 + \beta_{19} \cdot ATMVOL + \beta_{20} \cdot TIME + \beta_{21} \cdot TIME^2 \\ &+ \beta_{22} \cdot TIME^3 + \varepsilon \end{aligned} \tag{8.2}$$

From this equation, we can examine the multi-faceted nature of the implied dispersion processes. For example, we can examine whether the implied volatilities are constant (equation 8.1a) or not. Simply said, this assumption of constant volatility can be rejected if any of the independent variables are found to be significant. Our second objective is to understand the first order strike price effects (skewness). This can be understood better by examining the Beta coefficients from β_1 to β_7. If a skew effect does exist, we would expect that if it were not time varying, only the first Beta would be significant. If it is time varying, then β_2 and β_3 should be significant. If any of the shocks caused a change in the nature of the skew, then β_4, β_5 and β_6 will capture these effects. This should allow us to assess differences in the differences in the first order strike price effect before and after shocks. Finally, β_7 will allow us to examine the interaction between the expected level of future variance and the first order strike price effect.

For the examination of the second order effect, we should gain insights by examining the Beta coefficients from β_8 to β_{14}. Similar to the analysis of the first order strike price effects, if kurtosis effects do exist and they are not time varying, only β_8 would be significant. If these effects are time varying, then β_9 and β_{10} would indicate this. If any of the shocks caused a change in the nature of the implied kurtosis, then this will be captured by β_{11}, β_{12} and β_{13}. This should allow us to examine whether differences in the levels of implied kurtosis occur before and after shocks. Finally, β_{14} will allow us to examine the interaction between the expected level of future variance and the expected kurtosis.

The remaining variables exist for two primary purposes. The first is that the previous variables will provide insights into the average slope relationships that exist for the strike price effects. These variables will allow us to understand the dynamics of the process. The second reason is that for a number of these variables we have a prior expectation that they should not be significant in the regression. These would include the dummy variables that identify the occurrence of the 1987 stock market crash and the individual market shocks. We would also expect that the level of the at-the-money implied volatility would not be significant. These expected results are due to the fact that we have standardised our implied volatilities (indexing them to 100). Therefore, we should not observe significant results in these variables. If these variables are significant, this may serve as a diagnostic for potential errors in variables.

It may very well be that further moments are required to understand the implied dispersion process beyond skewness and kurtosis. For this reason, β_{15} will show us if this is the case or not ($STRIKE^3$). The inclusion of the CRASH, SHOCK1 and SHOCK2 variables will allow to examine if the manner in which we have estimated the standardised implied volatilities (or the other independent variables) is inconsistent over time. This would be measured again by statistically significant coefficients for β_{16}, β_{17} and β_{18}. The ATMVOL variable is included to measure whether the method we have used to standardise the smile patterns may be biased. We would expect that some errors might exist from the way we have estimated the at-the-money volatility. What we would hope is that this does not lead to systematic biases across all the analysis. Finally, the time variables will examine whether the implied volatilities are systematically varying over time. Significant results for β_{20}, β_{21} and β_{22} would suggest that the overall levels of implied volatilities rise or fall as time to expiration is approached. However, this would not be expected to be significant given that we have indexed the implied volatilities to the at-the-money implied volatility.

8.5 ORDINARY LEAST SQUARES REGRESSION RESULTS

With the dependent variable, the standardised implied volatility (VSI), and all the factors determined for each option in the observation period, the OLS regression with dummy variables (ANCOVA) was run. The first approach was to run an ordinary least squares regression with all the variables indicated above except: CLASS, MARKET and CONTRACT. This was done for all twelve markets for the entire period of the available data. The results of these statistical procedures can be seen in

Table 8.2a, 8.2b and 8.2c for the three asset classes, stock indices, fixed income instruments and foreign exchange.

In these tables, the coefficients of the regression are presented along with the standard error of the estimates and the t-statistic. The t-statistics for all the independent variables indicate whether the coefficient is statistically significantly different than zero. For the intercept, the t-statistic indicates whether the coefficient (alpha) is statistically significantly different than 100. This is the appropriate test level given that our standardisation technique has indexed all the volatilities to this level. Therefore, we would expect that the intercept of this regression would equal this level. For all variables that have a significant t-statistic (at a 95% level), the results are presented in **bold.** For all results that are in normal text, these were not significantly different from zero for the independent variables or 100 for the intercept. Finally, the ordinary least squares regression technique used employed a forward stepwise approach. This was done because the covariance matrix was in some instances non-singular. This is to be expected as a number of the variables are highly correlated by design. For example, all the TIME, STRIKE and INTERACTION variables are products of other included variables and will thereby be highly correlated. However, the forward stepwise approach will select in descending order those variables that provide the highest explanatory power given all previous variables that have been included in the regression at that point. As will be discussed later, this technique allowed us to ameliorate the effects of multi-collinearity that may be associated with our research design. In the instance that the variable was not selected in the forward stepwise regression, this is represented by "-.--".

We have also included the number of observations included in the analysis, the adjusted R-squared statistic and the Durbin-Watson statistic to measure possible problems with serial correlations in the residuals.

In Table 8.2a, we find that almost all of the included independent variables are statistically significant for the four stock index options. Exceptions include the variables that include SHOCK1 for the S&P and FTSE and variables that include CRASH for the DAX and Nikkei. This is hardly surprising since these variables were not appropriate for these markets. The reader may recall that for the S&P and FTSE, the CRASH and SHOCK1 were identical. Thus, we did not include SHOCK1 for these markets. In addition, for the DAX and Nikkei, the available observations occurred after the CRASH and thus, it made no sense to include a variable with no variance in the analysis. In addition, for most of the markets, the dummy variables SHOCK1 and SHOCK2 were not statistically significant. For almost all the other variables, they provided significant explanatory power to the model.

The extremely high explanatory power of each of the models is somewhat surprising. The adjusted R-squared statistic is between 0.9084 (for the Nikkei) to 0.9573 (for the S&P). These results (at first glance) seem to indicate that we are explaining almost all the variance in the implied volatility processes. Nevertheless, we must be careful with interpreting these results since the regressions might be biased and therefore the statistics could be misleading. For example, the Durbin-Watson statistics suggest that (for three of the four markets) some of the variance explained by the regressions is due to serial correlations in the residuals. Given that this indicates the OLS assumption of IID residuals is violated, this could also mean that the coefficients of the regression are biased and that we are reporting standard errors that are much too low. At this point, we cannot distinguish between the true variance our model is explaining and the extent these results may be due to errors in our research design. Correcting these errors could potentially change our results dramatically as once the violations in the regression assumptions are corrected for, these variables may no longer be significant and the amount of true variation we are explaining may be reduced.

Given that this methodology is at the heart of this portion of the research and that our understanding of the implied volatility process depends entirely upon the interpretation of unbiased results, we will examine each possible violation of the OLS assumptions. This will be achieved by modifying the manner of the regression analysis to address every violation of the OLS assumptions. It is of prime interest to assess how divergence the coefficients, standard errors and R-squared are once the factors introducing biases are corrected for. We will present results that suggest the regressions are remarkably robust and that even after revisions have been made to the regression models (to correct each of the potential violations of the OLS regression assumptions), there is not a significant difference in our results. Given that this is the case, we shall interpret the regression results in Tables 8.2a, 8.2b and 8.2c knowing that later approaches correcting the regression models will not substantially alter our conclusions.

Modelling the First Order Strike Price Effect (SKEWNESS)

From a casual review of these three tables, we observe that the coefficients vary significantly from each other for the same independent variables. However, we find that certain consistencies exist within the asset classes. For example, for many of the markets (in the same asset class) the coefficients are of the same sign (for the most part) and this suggests that similar dynamics may be affecting each asset class. To see

In Figure 7.13a, we estimated the first order strike price effect using the above regression methodology.[1] This was done so that we could gain a better insight of the overall skewness effect captured by the above model. For this estimate, we only included the strike and time related variables. The dotted line in this Figure represents this estimated relationship. As one would expect, the initial approach to capturing the skewness effect relied simply on a quadratic regression and was plotting as the first Beta of this regression (see equation 7.2). The dotted line is the best fitting quadratic function for this skewness effect. This relies on a richer model, which includes the time effects to degree two. In some ways, this approach is a quadratic model that minimises the squared error for the skewness effect. Therefore, it is hardly surprising that the dotted lines are a smoothed function that best fits the observed skewness parameters for 1996. From this graph, we can see that the overall impacts of the skewness estimated by the OLS model. These figures demonstrate that for all four markets (during 1996), a negative skew existed and this degree of skew is reduced as the expiration date of the option is approached.

The third coefficient (measuring STRIKE and TIME^2), β_3, is also consistent and positive for all four markets. This variable is somewhat difficult to interpret. This is due to two reasons. Firstly, the variable considers a higher power of time. Secondly, the overall impact of time is affecting both this variable and the previous variable (that considers the first order time impact). Thus, this variable appears to be acting with the previous variable and it would appear to be reducing the overall impact of the first variable. This overall effect is captured by the dotted line in Figure 7.13a.

[1] The results of the regression in Table 8.3a were not used for this analysis. This is due to the fact that this analysis was for the entire period of analysis. In Figure 7.12a, this represents the first order strike price effect solely for 1996. Therefore, we had to re-run the model only using data for 1996 to allow a comparison to be drawn. The results of these regression are available from the Author upon request. However, there was not a substantial difference between the results presented above for the entire period with the results for 1996.

The fourth, fifth and sixth coefficients allows us examine whether the nature of the volatility skew changed after shocks, including the 1987 stock market crash. We find that for the S&P and the FTSE, this result is significantly negative. Thus, it would confirm the Rubinstein (1994) contention that the nature of the skew differed after the 1987 crash. For the other markets, that only had observations past the crash, we find that the skew was slightly (but significantly) positive. However, after the second shock for three of the four markets, this increased the negative degree of the skew. Only for the Nikkei was the skew more positive after the second shock. Thus, for the DAX and Nikkei the first coefficient, β_1, appears to already include the negative bias in the skew that resulted from the 1987 stock market crash.

The final interaction variable provides an interesting result. It appears that the level of the expected volatility has a significant impact on the degree of the skew. For the S&P and Nikkei, the coefficient of β_7 is significantly negative. This suggests that the higher the expected variance, the greater the degree of the skew. For the DAX and the FTSE, the effect is the opposite. The greater the implied volatility, the flatter the skew. This effect could reflect the expectation imbedded in the implied volatility of jumps occurring. The negative coefficients can be interpreted as incorporating a negative jump, while the positive coefficients could reflect the expectation of positive jumps.

Now, let us examine the results for the independent variables describing the nature of the first order strike price effect for the four fixed income options under investigation. These results appear in Table 8.3b for each of the four fixed income option markets.

Variables Coefficient	STRIKE β_1	TIME β_2	TIME^2 β_3	CRASH β_4	SHOCK1 β_5	SHOCK2 β_6	ATMVOL β_7
Bund	5.534	-26.088	85.086	-.--	-1.679	-2.496	-96.526
BTP	-0.044	-32.213	113.758	-.--	-.--	-2.994	-44.109
Gilt	2.895	-9.662	25.352	-1.585	-0.657	-4.156	-22.411
US T-Bond	-0.969	-4.085	0.947	0.781	-2.490	0.452	-7.141

Table 8.3b, Regression Results for the First Order Effect of the Strike Price for Four Fixed Income Options.

For the first regression coefficient, β_1, we find that the pure effect of the strike price is positive for three of the four markets. This indicates that in the absence of the interaction terms, the normal skew effect is positive. For all the four markets, the second coefficient (measuring the interaction of STRIKE and TIME), β_2, is of the same negative sign. It is interesting to note that the Bund and BTP (both continental European bond markets) have coefficients that are almost identical. This suggests that as time falls, the degree of the skewness decreases. This is consistent with Figures 7.12b and 7.13b, which suggested that the degree of skewness flattens as we approach the expiration of the option. The dotted line in these graphs represents the skewness effect from the above regression (solely for 1996).

The third coefficient (measuring STRIKE and TIME^2), β_3, is also consistent and positive for all four markets. As with the first order strike price effect for the stock index options, this variable has similar problems in interpretation. All that can be said is that both time variables are working together to describe the time sensitive behaviour of the first order strike price effect. The overall effect can be seen in Figure 7.13b. This best fitting line suggests that three of the four markets display a curvilinear shape indicating a reduction in the skewness effect as the expiration of the option is approached. The odd market out is the US T-Bond that seems to display a more consistent skewness effect over time.

Now, we will examine the dynamics of the first order strike price effects for the third asset class: foreign exchange. The results for those independent variables associated with the STRIKE appear in Table 8.3c for each of the four foreign exchange option markets.

Variables Coefficient	STRIKE β_1	TIME β_2	TIME^2 β_3	CRASH β_4	SHOCK1 β_5	SHOCK2 β_6	ATMVOL β_7
D-Mark	3.933	1.045	-.--	-0.595	-1.929	0.522	-15.234
Pound	-1.002	1.929	-.--	-0.741	-.--	0.333	4.471
Yen	1.247	-3.428	10.701	-1.052	1.809	1.830	-6.647
S-Franc	2.850	2.177	-4.231	-.--	-2.126	-.--	-7.234

Table 8.3c, Regression Results for the First Order Effect of the Strike Price for Four Foreign Currency Options.

For the first regression coefficient, β_1, we find that the pure effect of the strike price is significant for all four markets. It is interesting that three of the four are significantly positive (D-Mark, Yen and S-Franc). Only for the British Pound, the relationship is a negative one. For three of the four markets, the second coefficient (measuring the interaction of STRIKE and TIME), β_2, is of the same positive sign. This suggests that as time falls, the skew becomes slightly more positive. This is not consistent with Figures 7.12c and 7.13c, which suggested that the degree of skewness was independent of the time to expiration of the option. However, this Figure only represents the smile behaviour for 1996. The analysis presented in Table 8.3c, includes all the observations from 1985 until 1996. As was pointed out earlier (see footnote 1), the plotted line that indicates the skew effect from the above regression approach was also determined using only the data from 1996. In Figure 7.13c, we see that either the dotted lines are flat (for the D-Mark) or display some slight curvilinear pattern. However, these patterns are not consistently positive or negative and for the Swiss Franc, vary between positive and negative values. Thus, for this period, it would appear that any skewness effect is negligible or fairly insensitive to the time to expiration of the option.

The third coefficient (measuring STRIKE and $TIME^2$), β_3, is statistically insignificant for the four markets.

The fourth, fifth and sixth coefficients allow us to assess whether the nature of the volatility skew for foreign exchange options changed after shocks, including the 1987 stock market crash. Here we find that no systematic patterns emerge. It is clear that shocks occur related to specific dynamics of the particular market, being either positive or negative. As these occur, the strike price effects adjust to reflect new market expectations. For the 1987 stock market crash, there was a negative impact on the strike price effect for the D-Mark and Yen, however, the impact was positive for the Swiss-Franc.

The final interaction variable provides consistency with the results obtained both for stock and bond options. It again appears that the level of the expected volatility generally has negative impact on the degree of the skew. For three of the four foreign exchange options markets, the higher the expected variance, the greater the degree of the skew. The interpretation of this result is that when market participants are concerned about jumps, they tend to fear negative ones. This is then incorporated into a higher level of expected variance to compensate for the risk premium required for such events.

From this initial analysis we can draw some preliminary conclusions. Both stock index and fixed income options display statistically significantly negative first order strike price effects. These skew effects are time dependent in a similar way and are more extreme after shocks occur. For the foreign exchange options, the first order strike price effect seems to be either flat or slightly positive. This positively skewed relationship is also time-dependent in a similar manner to the stock index and fixed income option markets. In all markets, the skewed relationship becomes more positive (or less negative) as the time to expiration decreases. The impact of shocks for foreign exchange options can yield either more positive or more negative skewed relationships. However, the effect is not consistent. For all three asset classes, there appears to be a significant impact on the first order strike price effect from the level of the expected future variance. In most cases (8 of 12 markets), as the expected variance increases, the degree of the skew becomes more negative. The only notable exception is for the Nikkei and this could be due to the fact that during most of the time period of analysis for this market, the index was in a downward trend and a significant rebound in the market would have been unusual.

Modelling the Second Order Strike Price Effect (KURTOSIS)

At this point, we will now examine the second order strike price effect for three asset classes. As before, we can separate these effects from equation 8.2 as follows:

$$STRIKE^2 \cdot (\beta_8 + \beta_9 \cdot TIME + \beta_{10} \cdot TIME^2 + \beta_{11} \cdot CRASH + \beta_{12} \cdot SHOCK1 + \beta_{13} \cdot SHOCK2 + \beta_{14} \cdot ATMVOL) \tag{8.4}$$

From Table 8.2a, we have selected only the slope coefficients outlined in equation 8.4 and these appear in Table 8.4a for each of the four stock index options markets.

For the first regression coefficient, β_8, we find that the pure effect of the strike price squared is positive for all four markets and is similar in size. This indicates the pure kurtosis effect. For all the four markets, the second coefficient (measuring the interaction of STRIKE^2 and TIME), β_9, is of the same negative sign and again of the same degree. This suggests that as time falls, the degree of the kurtosis s. This is consistent with Figures 7.14a and 7.15a, which suggested that the e of the implied volatility pattern becomes more extreme as we approach the n of the option.

STRIKE² β_8	TIME β_9	TIME² β_{10}	CRASH β_{11}	SHOCK1 β_{12}	SHOCK2 β_{13}	ATMVOL β_{14}
5.899	**-12.295**	**15.557**	**0.681**	**-.--**	**-0.717**	**-4.200**
8.963	**-6.491**	**-.--**	**-1.359**	**-.--**	**-1.317**	**-9.147**
7.222	**-13.868**	**17.144**	**-.--**	**-.--**	**-1.310**	**-11.392**
4.031	**-12.205**	**29.648**	**-.--**	**1.170**	**-.--**	**-4.172**

la, Regression Results for the Second Order Effect of the Strike Price for Four Stock tions.

n Figure 7.15a, the dotted line represents the kurtosis effect as measured is OLS approach for 1996. One can see that for three of the four stock index the kurtosis effect is monotonically increasing as we approach expiration. ly divergence from this pattern is for the Nikkei that has relatively high with 90 days to expiration and then drops at 50 days and increases again as roach expiration. One possible reason for this could be that this option is not as traded (with longer times to expiration) as the other options in the analysis true kurtosis behaviour is only captured as these contracts become more liquid me to expiration is approached.

The third coefficient (measuring STRIKE² and TIME²), β_{10}, is also positive e of the four markets (although this is not statistically significant for the As was stated earlier for the second order time impact for the first order strike ect, it is somewhat difficult to provide an economic interpretation for. This aid is that both time factors interact in a complex manner impacting the rder strike price effect. It would appear that the second order time factor is in the level of all implied volatilities. Secondly, as the implied volatility surface is indexed to the at-the-money implied volatility, if the at-the-money implied volatility rises then this alone would serve to dampen the degree of the implied curvature.

Now, we will examine the second order strike price effects for the fixed income options markets. From Table 8.2b, we have selected only the slope coefficients outlined in equation 8.4 and these appear in Table 8.4b for each of the four fixed income option markets.

Variables Coefficient	STRIKE^2 β_8	TIME β_9	TIME^2 β_{10}	CRASH β_{11}	SHOCK1 β_{12}	SHOCK2 β_{13}	ATMVOL β_{14}
Bund	4.547	-20.470	46.894	-.--	1.538	-1.325	-.--
BTP	5.082	1.847	-52.302	-.--	0.870	0.167	-17.698
Gilt	4.411	-17.584	32.372	1.499	1.226	-0.757	-22.729
US T-Bond	5.652	-8.763	15.837	0.350	-0.505	-0.318	-5.578

Table 8.4b, Regression Results for the Second Order Effect of the Strike Price for Four Fixed Income Options.

For the first regression coefficient, β_8, we find that the pure effect of the strike price squared is positive for all four markets and is similar in size. This again indicates the pure kurtosis effect and is similar to the results presented for the four stock index options. For three of the four markets, the second coefficient (measuring the interaction of STRIKE^2 and TIME), β_9, is of the same negative sign and again of roughly the same degree. This suggests that as time falls, the degree of the kurtosis increases. This is again consistent with Figures 7.14b and 7.15b, which suggested that the curvature of the implied volatility pattern becomes more extreme as we approach the expiration of the option. The odd market out is the BTP market that has a positive (but insignificant) coefficient. This would suggest that the kurtosis becomes more extreme the further the time to expiration. However, in Figure 7.14b it does appear that the curvature for the BTP is relatively more stable over the option cycle. Nevertheless, we would expect some increased curvature to exist for this market and this can be found in the third variable (measuring $STRIKE^2$ and $TIME^2$) and it's coefficient, β_{10}. For the BTP, this coefficient is large and negative. Overall, it would appear that the combination of these two factors would lead to an overall increase in curvature as we approach expiration. For the other three markets, the coefficient is also consistently positive and roughly of the same magnitude. These three markets bear very similar results to those observed for the four stock index options.

In Figure 7.15b, we have displayed the estimated kurtosis effect for these four markets over the period 1996 that come from the OLS model. We can now see that for all four markets the level of the kurtosis rises as we approach expiration. Thus, for the BTP, the combined effect of these two time dependent variables does cause an overall increase in the kurtosis as was suggested above.

The fourth, fifth and sixth coefficients allow us to examines if the curvature of the implied volatility patterns changed after shocks, including the 1987 stock market crash. We find that both for the 1987 stock market crash and the first shock to each bond market, the expected excess kurtosis increases in almost all instances (the exception is the US T-Bond after the first shock, but the magnitude of the negative coefficient is small). The second shock has the universal effect of lowering the expected kurtosis for three of the four markets. Perhaps, market participants had anticipated the extreme movements associated with the second shock already in the expected excess kurtosis prior to the event. After this event was realised, the expected excess kurtosis fell as a result.

The final interaction variable also provides a result that is consistent with the findings for the four stock index options. It appears once again that the level of the expected volatility has a significantly negative impact on the degree of the expected kurtosis. As with the stock index options, this suggests that the higher the expected variance, the lower the degree of the expected kurtosis.

Finally, we extended the same analysis for the third asset class, foreign exchange, by selecting from Table 8.2c only those slope coefficients (outlined in equation 8.4) that relate to the second order strike price effect. These appear in Table 8.4c for each of the four foreign exchange options markets.

Variables Coefficient	$STRIKE^2$ β_8	TIME β_9	$TIME^2$ β_{10}	CRASH β_{11}	SHOCK1 β_{12}	SHOCK2 β_{13}	ATMVOL β_{14}
D-Mark	**7.838**	**-16.273**	**36.311**	**0.156**	**-1.087**	**-0.127**	**-17.166**
Pound	**9.154**	**-14.431**	**31.282**	**-1.254**	**-1.176**	**0.391**	**-20.620**
Yen	**6.340**	**-12.943**	**24.408**	**-0.750**	**0.576**	**0.266**	**-12.820**
S-Franc	**8.367**	**-19.269**	**53.228**	0.159	**-1.876**	**0.342**	**-19.014**

Table 8.4c, Regression Results for the Second Order Effect of the Strike Price for Four Foreign Exchange Options.

For the first regression coefficient, β_8, we find that the pure effect of the strike price squared is positive for all four markets and almost identical in magnitude. This indicates the pure kurtosis effect. For all the four markets, the second coefficient (measuring the interaction of $STRIKE^2$ and TIME), β_9, is of the same negative sign and again of roughly the same degree (apart from the Japanese Yen). This suggests that as the expiration of the option is approached the degree of the kurtosis increases. This is consistent with Figures 7.14c and 7.15c, which suggested that the curvature of the implied volatility pattern becomes more extreme as we approach the expiration of the option. From examination of the dotted line in Figure 7.15c, we observe the overall kurtosis effect predicted from the OLS regression (for 1996). These graphs do indicate that the overall kurtosis is increasing as the expiration is approached. However, the degree to which this occurs is much less extreme than we observed for the other asset classes. Thus, there appears to be a more consistent level of kurtosis overtime for foreign exchange options relative to stock index or fixed income options. Nevertheless, the kurtosis effect does increase as the time to expiration falls.

The third coefficient (measuring $STRIKE^2$ and $TIME^2$), β_{10}, is also consistently positive for three of the four markets. For the Japanese Yen, this variable had a significantly negative impact. This is due to the previous slope coefficient (β_9) being somewhat smaller in magnitude relative to the other currencies.

The fourth, fifth and sixth coefficients allow us to examine the impact of shocks on the nature of the curvature of the implied volatility patterns for foreign exchange options. We find that the effect is not consistent (including the 1987 stock market crash). For many of the markets, there is no effect from such shocks and when such an impact is significant, the magnitude of the effect is small. Thus, it would appear that the curvature we observe in foreign exchange options is endemic and not that sensitive to the occurrence of shocks.

The final interaction variable is remarkably consistent, both among the four foreign exchange markets and with the two previous analyses for the stock and bond markets. It once again appears that the level of the expected volatility has a significantly negative impact on the degree of the expected kurtosis. Once again, we can conclude that the greater the implied volatility, the flatter the shape of the implied volatility pattern. It is truly remarkable that the magnitudes of the coefficients are extremely close and in some instances not statistically significantly different (for example, S-Franc and Pound and D-Mark and Yen).

From this initial analysis, we will also draw some preliminary conclusions. For almost all the markets we have examined, consistencies exist in the sign (and in many instances the magnitude) of the independent variables. This could suggest that similar dynamics are at work across all markets. For example, all twelve markets have similar slope coefficients to the pure kurtosis effect (β_8). Furthermore, for 11 of the 12 markets, the interaction between time and the squared strike price is negative. Of these 11 markets, 10 display a similar magnitude in the coefficients. Of the two markets which are divergent, BTP and Yen, the interaction is taken by the second order interaction term ($STRIKE^2$ and $TIME^2$). Thus, it would appear that a similar dynamic process is explaining the time dependent behaviour of the second order impact of the strike price. The fact that the inclusion of shocks has a minimal impact on the change in the second order strike price effect suggests that these behaviours existed throughout the time period for all twelve markets and are not event sensitive. Thus, we can confirm the Rubinstein (1994) conjecture that the curvature of the smile existed before and after the 1987 stock market crash and what changed was the skewness (see analysis above). For all three asset classes, there appears to be a significant impact on the second order strike price effect from the level of the expected future variance. In all cases, as the expected variance increases, the degree of the expected kurtosis lessens. For most markets, the magnitude of the impact is extremely similar. The only notable exception is for the DAX, where the coefficient is much greater than for any other market.

Now that we have examined the first and second order impacts of the strike prices on the dynamics of the implied volatility patterns, we will now examine the other variables included in the analysis.

The Impacts of Other Variables on Implied Volatility Surfaces

From equation 8.2, we can separate the remaining variables to assess their impacts on explaining the dynamics of the implied volatility process. These remaining variables have been split out and appear in equation 8.5. As was discussed previously, the earlier analysis examined the average slopes of the strike price effects and these variables provide both an indication of the dynamics of the strike price effects and act as a diagnostic for errors in variables.

$$VSI = \alpha + \beta_{15} \cdot STRIKE^3 + \beta_{16} \cdot CRASH + \beta_{17} \cdot SHOCK1 + \beta_{18} \cdot SHOCK2 + \beta_{19} \cdot ATMVOL + \beta_{20} \cdot TIME + \beta_{21} \cdot TIME^2 + \beta_{22} \cdot TIME^3 + \varepsilon \tag{8.5}$$

The results for these additional independent variables appear in Table 8.5a for each of the four stock index options markets.

Variables Coefficient	Intercept α	$STRIKE^3$ β_{15}	CRASH β_{16}	SHOCK1 β_{17}	SHOCK2 β_{18}	ATMVOL β_{19}	TIME β_{20}	$TIME^2$ β_{21}	$TIME^3$ β_{22}
S&P	97.163	0.801	1.465	-.--	0.441	-1.835	54.822	-554.37	1538.4
FTSE	98.027	0.388	2.921	-.--	-0.886	-.--	-.--	-291.48	1306.9
DAX	98.280	0.329	-.--	-.--	-.--	4.093	1.755	-.--	-.--
Nikkei	98.115	0.316	-.--	-.--	-.--	-.--	48.428	-275.02	436.43

Table 8.5a, Regression Results for the Remaining Independent Variables for Four Stock Index Options.

We would expect from the procedure of implied volatility standardisation that the intercept, α, would be approximately equal to 100. We observe that it is somewhat lower (and this difference is statistically significant), which could be due to a number of factors. One possible explanation is that errors may have been introduced from the method of estimating the at-the-money volatility. We chose to use a simple linear interpolation of adjacent strike price implied volatilities. This was used because when using other methods to estimate the at-the-money implied volatilities (such as a quadratic regression approach), the coefficients were substantially (and significantly) different than 100. When we re-estimated the at-the-money volatility using the simpler technique, the intercepts increased and were all in the range between 97 and 99. Thus, while we recognise that a coefficient lower than 100 suggests that some other errors in the standardisation of the implied volatilities may have been introduced, the impact appears to be consistent across the stock index options. However, for the sake of comparison, the effects of the independent variables should still be comparable

The fact that the higher order strike variable ($STRIKE^3$) appears as a significant factor in the regression suggests that for all four stock index options, the strike price effect extends beyond a second order impact. For all four stock index options, β_{15} is positive and of similar magnitude. While the implications of the inclusion of this variable are somewhat difficult to interpret (due to the fact that this implied statistical moments above kurtosis), we can understand why this variable is significant if we return to Figure 7.11a. For all the stock index options, the smile shapes resemble an inverted 'J' shape. Thus, some sort of interaction is occurring between the skewness and kurtosis, which is being captured by the $STRIKE^3$ variable. It remains for further research to assess whether further higher order effects exist (beyond the cubic) exist and what possible economic rationale would explain it.

The impacts of the pure shock dummy variables are mixed. We would expect that these would not be significant. Indeed for most of the markets, the coefficients for CRASH and SHOCK1 and SHOCK2 are not significant. When the effect is significant, the impact is slight and appears to bring the overall intercept level back to the 100 level we expected.

We would expect that all the impacts of at-the-money volatility would have been captured by previously included independent variables. If this were not the case, the coefficients for the ATMVOL variable would be statistically significant. As was mentioned previously, this variable provides some indication of whether problems may exist in how we are estimating the at-the-money volatility. When using the quadratic regression approach, three of the four markets registered significant results.

However, using the simpler linear extrapolation method, none of the four markets had a significant result. Therefore, the reason why our intercepts are less than 100 has to be caused by the other dummy variables or by some impact of time.

Time is most likely the culprit (explaining our intercept term less than 100). For the S&P and FTSE, which had the lowest intercepts, most of the TIME variables had a statistically significant contribution to the regression equation. However, the TIME impacts were not as significant for the DAX or the Nikkei. When the coefficients for the first order TIME variable were significant, the coefficients were all positive. Likewise for the second order time effect, $TIME^2$, all significant results were negative and for the third order effect, $TIME^3$, were positive again. There are at least two explanations for this result. The first is that we have not correctly measured the time to expiration of the options. We have measured time as the calendar time to expiration and have used full days to measure time. It may be possible that as expiration is approached and the options expire during the final trading day, this difference in the exact time to expiration (i.e. hours) may be significant.

The second explanation may be that we are capturing a more subtle effect of time decay on the implied volatilities of options. The relative magnitudes of the coefficients suggest that we are capturing what may be a negative exponential of time function. For market practitioners, this effect is known as the premium erosion effect. The inclusion of the TIME variables to degree three should capture these effects. Suppose that the functional form of time is of the form: e^{-T}. This can be approximated by an expansion of the form:

$$T \cdot [(1-X) + \frac{X^2}{2!} + \frac{X^3}{3!} + higher \cdot order \cdot terms] \qquad \textbf{(8.6)}$$

T is fairly small (as it is in our case). Thus, we would expect to capture most of the dynamics of the negative exponential form by the time we had expanded to the third moment. It would (therefore) be consistent for the regression results to be in line with what would be expected from such a dynamic. Essentially, these results suggest that as time decreases, the implied volatility also tends to decrease. The inclusion of the two higher order time variables suggests that this has a negative exponential form. Even so, (a quick glance at) Table 8.2a indicates that these variables have t-statistics that are (for the most part) not very significant. Nevertheless, it appears that the overall level of implied volatilities decays as the expiration of an option approaches.

The results for the additional independent variable for the four fixed income option markets appear in Table 8.5b.

Variables Coefficient	Intercept α	STRIKE³ β_{15}	CRASH β_{16}	SHOCK1 β_{17}	SHOCK2 β_{18}	ATMVOL β_{19}	TIME β_{20}	TIME² β_{21}	TIME³ β_{22}
Bund	**98.144**	**0.134**	-.--	-0.401	-.--	**9.895**	**16.737**	-.--	**-185.08**
BTP	**98.471**	**0.342**	-.--	**-0.772**	**-0.703**	**16.471**	-.--	-.--	**70.187**
Gilt	**97.224**	**0.151**	**0.819**	-.--	-0.278	**12.233**	-11.485	**240.89**	**-700.84**
T-Bond	**93.853**	**0.214**	-.--	**2.134**	**-2.552**	**10.087**	**46.468**	**-321.26**	**789.73**

Table 8.5b, Regression Results for the Remaining Independent Variables for Four Fixed Income Options.

As with the stock index options, there is concern is that the intercept, α, is statistically significantly lower that the result of 100 that we expected. In this case, the most likely explanation is our method for determining the at-the-money volatility. For all four markets, the ATMVOL variable has a significantly positive beta coefficient. This would suggest that the simple linear interpolation method is underestimating the at-the-money volatility. However, when we applied the Shimko quadratic approach, the results were even worse. This suggests that further research may be warranted to determine a better estimate of at-the-money volatility when the observed strike prices are discrete. Another possible reason for this result could be that critical variables have been omitted, causing the intercept (especially for the US T-Bond) to differ from

100. Later, we will examine this issue and demonstrate that this is the likely explanation for the US T-Bond option market. Nevertheless, the error we observe for the intercepts seems to be similar across the four markets and given that we want to understand the relative contributions of the other independent variables in the regression, we will demonstrate latter that this problem is of little significance.

As was observed for the four stock index options, the β_{15} is positive and of similar magnitude for all four bond options. As with the stock index options, this variable suggests that the smile shapes resemble an inverted 'J' shape (see Figure 7.11b). Therefore, some sort of interaction is also occurring between the skewness and kurtosis, which is being captured by the $STRIKE^3$ variable. Again, this may suggest that further higher order strike price effects may be occurring.

The impacts of the pure shock dummy variables are mixed. There appears to be little effect of the 1987 crash on the volatility of fixed income options. The only significant impact was for the GILT options. The impacts of the market specific shocks are not consistent. Overall, they tend to reduce the overall level of the standardised implied volatilities. Once again, this could be due to the realisation of an expected shock reducing future expectations of variance. Nevertheless, this impact is not consistent.

As was stated above, we find that for all four markets the ATMVOL variable is positive and statistically significant. As we discussed this does cause some concern that this variable has been mis-estimated. Even so, when other methods were used to estimate the at-the-money volatility, even more significant results were obtained. Therefore, we must accept that some errors in estimation exist. However, it would appear that the effects are similar for all four markets. As before, since our aim is to understand the relative effects across the variables, we maintain that our that for the Japanese Yen the intercept has the highest statistically significantly divergence from 100. Once again, there are a number of possible explanations for this problem. However, we now have a clearer idea as to why the intercepts are statistically different from 100. If one examines the coefficient for the ATMVOL variable, this coefficient is negative for the first time and of a substantial magnitude. As was discussed previously, this may suggest that our method of estimating the level of the at-the-money volatility is introducing errors. In this instance, it appears that have over-estimated the implied volatility and this has led to both a negative coefficient for β_{19} and for the first time to intercepts above 100. In all the previous results, the relationship was reversed. Therefore, it appears that the problem in our analysis may be caused by our method of estimating the at-the-money volatility.

Variables Coefficient	Intercept α	$STRIKE^3$ β_{15}	CRASH β_{16}	SHOCK1 β_{17}	SHOCK2 β_{18}	ATMVOL β_{19}	TIME β_{20}	$TIME^2$ β_{21}	$TIME^3$ β_{22}
D-Mark	**101.67**	**-0.089**	**0.522**	0.532	**-0.714**	**-13.729**	-.--	-.--	-.--
Pound	100.21	**0.032**	**0.517**	-.--	**-0.283**	**-10.127**	**19.445**	**-223.22**	**651.22**
Yen	**103.29**	**-0.169**	**3.956**	**-4.215**	**1.664**	**-19.256**	**-3.350**	-.--	-.--
S-Franc	**102.40**	**-0.068**	**1.878**	-0.599	**-2.566**	**-25.721**	**48.242**	**-285.33**	-.--

Table 8.5c, Regression Results for the Remaining Independent Variables for Four Foreign Exchange Options.

Nevertheless, this will only be a problem if it affects the overall results (apart from the intercept and the β_{19} coefficient for the ATMVOL variable). We will demonstrate that this error does not substantially impact the coefficients for the other variables in the analysis or the explanatory power of the model. Therefore it is sufficient to acknowledge problems exist with our method of estimating the at-the-money volatility and continue our line of investigation. In the next three sections we show how robust our regression models are once the possible errors have been corrected for.

Regarding the higher order strike price variable, $STRIKE^3$, the regression results diverge compare to those obtained from either the stock index or bond markets. The regression coefficient for this variable, β_{15}, is positive for only one market, British Pound and of a minuscule magnitude. For the other three foreign exchange options, the coefficient is negative and also of a relatively small magnitude. Therefore, the interaction that is occurring between the skewness and kurtosis takes an inverse form to what we observed for the other asset classes.

The impacts of the pure shock dummy variables are mixed. There appears to be some consistent effect from the 1987 crash on the volatility of foreign exchange options. The positive coefficients are somewhat difficult to interpret as we are modelling with a standardised volatility index and the strike price effects have been captured with previously included variables. We would expect these variables to be insignificant in our model unless these shocks had a more subtle impact on implied volatilities beyond the strike price effects. This possibly served to increase the overall levels of implied volatilities. As with the fixed income options, the impacts of the market specific shocks add little to the explanatory power of the model. Only two of the first period shocks (SHOCK1) has an impact on volatility: they reduce it. Once again, this could be due to the realisation of an expected shock reducing future expectations of variance. Nevertheless, this impact is not consistent for all markets or for both shocks, nor would one expect the impacts to be of the same sign or magnitude as each shock was market specific.

Finally, the analysis of the coefficients for the ATMVOL variable was discussed above. For the foreign exchange options, this effect has a significantly negative impact for all four markets. As we suggested this is the most probable reason for the intercept having a level statistically different from 100. We will examine the impacts of this potential error in the next three sections.

Finally, the impacts of the three TIME variables are substantially different compared to the analysis for the two previous asset classes. The first TIME variable is only significant for three of the four markets and positive only for the British Pound and the Swiss Franc. Again, this might suggest that some premium erosion effect could be occurring or that some errors in the expiration date are occurring. However, this is unlikely since all four markets have exactly the same expiration date. If some negative exponential of time effect is occurring, it is not clear why this result is not found for all four markets. For the stock index options some negative exponential of time was clearly suggested, while for the foreign exchange options such a pattern is only observed for the British Pound and the Swiss Franc. In addition, only these two markets have the second order time variable, $TIME^2$, that is statistically significant and negative. Finally, only one of the foreign exchange options (British Pound) has a significant result for the third time variable, $TIME^3$. Again, this is of the correct sign (positive) and magnitude which is consistent with the findings for the four stock index options and the US T-Bond option.

From this initial analysis, we can draw some preliminary conclusions. The fact that the intercepts are statistically significantly different from 100 and that the β coefficient of the ATMVOL is non-zero may indicate we have problems with our regressions. Furthermore, from Tables 8.2a, 8.2b and 8.2c, the Durbin-Watson statistics indicated that problems with serial correlations in the residuals exist.

8.6 TESTING THE ROBUSTNESS OF THE REGRESSION MODELS

The results presented seem to achieve our primary objective of understanding the dynamics of the implied volatility process for all twelve markets. We still have to examine however, how robust these models are.

Classical regression theory states that, a number of conditions can bias a regression model:

1) Omission of Critical Variables

2) The use of a prediction from a confidence interval based on the assumption that the conditional distributions of the independent variables are normal and have equal variances. This is known as a homoscedasticity condition.

3) The residuals of the regression are IID with an N(0,1) distribution. and the residuals are not correlated across time.

4) Independent variables are highly correlated and therefore redundant to the model. If this is the case, the true effect of each variable on the regression model will not be clear. This is the classic problem of Multi-collinearity.

Each of these conditions could lead to biased regression results. This would imply that the coefficients for our models could be incorrect and that the standard errors much higher than is reported. Both results could mean that our analysis can not be used unless we can correct for these problems and in doing so find that our results are not substantially altered.

Testing for An Exclusion of Critical Variables Problem

Of these problems, probably the most difficult to deal with is the exclusion of critical variables. It is extremely difficult to know what variables have been excluded. One obvious variable for inclusion could be the level of the underlying futures price. In our research, we did include the futures price as an independent variable. When this was done, we found that it did fit into our models but with a barely significant slope coefficient. Furthermore, the actual coefficients were not consistent. They tended to be negative for the stock index options but varied for the other markets. By introducing this variable, we also experienced problems in modelling the time varying nature of the residuals.

This is due to the fact that we are (essentially) running a panel regression through time. The panel is the individual (standardised) implied volatilities across strike prices at single points in time. Due to the nature of the data, we do not either have the same number of observations for each date nor will the same options be available in the subsequent period. Furthermore, by sorting the option data (described previously) in order to examine the time series behaviour of a particular option contract (with the same strike price), we are splitting the panel into individual option contracts. Each one will have the same series of futures prices as the following option contract (with the next highest strike price). This means that the same series of futures prices will appear for each option contract. The time series analysis of the residuals including this repeating futures price series will greatly complicate our analysis of the serial correlations. For this reason, we excluded the futures price series.

Likewise, we examined whether the inclusion of interest rates was relevant for modelling the implied volatility dynamics. We found that the contribution was marginal and only relevant for the fixed income markets and foreign exchange. For the stock index option markets, we found no significant contribution. This is consistent with the findings of Scott (1994) that there is no significant correlation between interest rates and the unconditional volatility of the S&P 500 futures. Therefore, for the same reasons are valid as for the exclusion of the futures prices, and we chose to omit interest rates from our model.

However, it is not obvious which other variables could be included. It is apparent that shocks do play a role in the dynamics of implied volatility processes. However, it is not clear that we have chosen only the relevant events. Perhaps, we missed period specific events. We have chosen to address these events (and perhaps other omitted variables) by rerunning our ordinary least squares regression including all the individual contracts from 1985 to 1996 as dummy variables. If any period specific effect exists (from whatever source), this would now appear in our regression.

These regressions were run for all three asset classes (and all twelve markets) and appear in Table 8.6a for the stock index options, Table 8.6b for the fixed income options and in Table 8.6c for the foreign exchange options.

In these tables, we have only included the coefficients for the variables examined previously (and excluded the coefficients for each of the 48 dummy variables). The rationale for this is three-fold: firstly including the coefficients for all the included (contract) variables would dramatically increase the space required to present the results. Secondly, we found that hardly any of the contract dummy variables added significant explanatory power to the models. Finally, our principle interest is to assess the impacts from the possible exclusion of critical variables on the previously included independent variables. If the inclusion of these variables does not substantively alter our previous results, we can conclude that even while biases may exist in our initial regression, they will not alter our general conclusions.

Comparisons between Tables 8.6a, 8.6b and 8.6c to Tables 8.2a, 8.2b and 8.2c show that there has been no appreciable difference in the initial results. In all cases this had led to an improvement in the adjusted R-squared statistics for the models, however, this improvement is minuscule and given the penalty imposed by the inclusion of so many new variables, this addition has questionable merit. Other changes in the regression results include different intercept values and changes in the size and sign of the ATMVOL coefficient and of the impact of the market shock dummy variables. As was stated previously, the intercept and ATMVOL coefficient were most probably due to the method in which we estimated the at-the-money implied volatility. It is interesting to note that for the stock index options (where we were most concerned), the sign of the ATMVOL coefficient has now turned significantly negative (where before it was insignificant). Furthermore, the t-statistics of this variable has for the most part become much smaller in magnitude. Also the CRASH coefficient has now become much larger for the S&P and the FTSE. For the FTSE, the intercept is now not statistically different from 100 and the Nikkei has an intercept that is above 100. The inclusion of the additional contract variables has reduced the intercept results for the S&P and DAX.

Apart from these changes, almost none of the coefficients for the other variables in the model have changed appreciably. All the strike price dependent variables that we are examining in this research retain essentially the same coefficient values and standard errors.

A similar result is found for the fixed income options markets. In Table 8.6b, the only significant changes in the regression results are for the intercept value and the coefficients for the non-strike price dependent variables. It is interesting to note that the relatively low intercept value of the US T-Bond has now risen to 99.076 from the previous 93.853 and is no longer statistically different from 100. Furthermore, the ATMVOL coefficient has now become extremely negative. Thus, it would appear that if some exclusion of critical variables has occurred, the sole impact is on variables which are not of primary interest to understanding the dynamics of the strike price effect. As with the stock index options, the coefficient values for all the strike price dependent variables are essentially unchanged. Furthermore, the addition of all these new variables has once again had a minimal effect on improving the adjusted R-squared of the model. While we have not reported the Durbin-Watson statistics for these regressions, they were almost identical to those found in the initial OLS analysis.

In Table 8.6c, we find similar results for the foreign exchange options. The only significant changes that have occurred by including the CONTRACT dummy variables are a general reduction in the intercept of the regression and changes in those variables that are not strike price dependent. The only exceptions are for the Deutsche Mark and the British Pound, which now have higher intercept coefficients. The CRASH variable is now relatively more significant for three of the four currencies and some of the market specific shock variables now have a different (although small) impact. However, as with the previous asset classes, this inclusion of variables has failed to change the coefficients or standard errors of the strike related variables in a substantial manner.

Given that the inclusion of the CONTRACT variables has not appreciably affected the coefficients of the strike-related variables in our analysis, we are confident that our model is fairly robust to the exclusion of critical variables problem. Furthermore, we are less concerned about the potential bias of our method of estimating the at-the-money volatility. The inclusion of the new variables only served to modify the intercept and ATMVOL coefficients (and the other non-strike dependent variables). Since the coefficient of the ATMVOL experienced dramatic changes (often assuming an opposite sign), while leaving the coefficients for all the strike dependent variables essentially unchanged, we conclude that the errors in this variable may indeed be random and are in any case, inconsequential to our conclusions.

Correction for Heteroscedasticity in the Regressions

Still, we may face problems if the residuals for the regression display heteroscedasticity. To examine this, we plotted the residuals of the simple OLS regression as a function of the standardised strike price. We observed a systematic 'dumb-bell' shape to the errors. Thus, the further we went away from the at-the-money level, the more extreme the variation of the errors became. To address this problem, we applied a weighted least squares regression approach to all twelve markets. This approach followed the lines of Neter, Wasserman & Kutner (1985) and Kvålseth (1985).

The steps taken in this analysis were as follows. First we saved all the residuals from the original OLS regressions. Then, we merged the original data file with the absolute value of these residuals. A second OLS regression was run of the form:

$$
\begin{aligned}
AR &= \alpha + STRIKE \cdot (\beta_1 + \beta_2 \cdot TIME + \beta_3 \cdot TIME^2 + \beta_4 \cdot CRASH + \beta_5 \cdot SHOCK1 + \beta_6 \cdot SHOCK2 + \beta_7 \cdot ATMVOL) \\
&+ STRIKE^2 \cdot (\beta_8 + \beta_9 \cdot TIME + \beta_{10} \cdot TIME^2 + \beta_{11} \cdot CRASH + \beta_{12} \cdot SHOCK1 + \beta_{13} \cdot SHOCK2 + \beta_{14} \cdot ATMVOL) \\
&+ \beta_{15} \cdot STRIKE^3 + \beta_{16} \cdot CRASH + \beta_{17} \cdot SHOCK1 + \beta_{18} \cdot SHOCK2 + \beta_{19} \cdot ATMVOL + \beta_{20} \cdot TIME + \beta_{21} \cdot TIME^2 \\
&+ \beta_{22} \cdot TIME^3 + \varepsilon
\end{aligned}
\tag{8.7}
$$

This is essentially the same regression model as appeared in equation 8.2. The only difference is that the standardised volatility (VSI) has been replaced by the absolute value of the residual from that first regression (AR). When this regression was run, the predicted value of the regression was saved. A weight was constructed using the following form:

$$WEIGHT = 1/MAX(1, PREDICTED) \qquad \textbf{(8.8)}$$

This standard weighting scheme was chosen to reduce the problem of the heteroscedasticity observed in the residuals. The rationale using the maximum of 1 and the predicted value was to avoid weights that would assume enormous values when the predicted value was close to zero and to avoid any potential problems with negative weights. This approach is common in the literature on weighted least squares regression techniques (see previous references).

With the weights determined for each of the twelve markets, the weighted least squares analysis was run using STATISTICA for Windows (version 5.0). [See STATISTICA (1995)]. The results for the twelve markets are again grouped by asset class and can be seen in Table 8.7a for the four stock index options markets, Table 8.7b for the four fixed income options markets and Table 8.7c for the four foreign exchange options markets.

As expected, the weighted least squares results did diverge from those obtained from the OLS approach. One would expect that the weighted least squares to experience a significant decrease in the explanatory power of the model. However, we find that the weighted least squares approach has only marginally reduced the adjusted R-squared of the models. If we compare Table 8.7a with Table 8.2a, we find that the weighted least squares regression has led to an increase in the magnitude of the regression coefficients for almost all the strike related variables of interest. Furthermore, in many cases, the results are even more statistically significant (greater absolute t-statistics). At the very least, we can say that the effects we have identified remain in our model even after corrections have been made for heteroscedasticity. In fact, they become even more pronounced and significant when such a correction has been made.

Similar conclusions can be drawn for the fixed income options markets (by comparisons of Table 8.7b to Table 8.2b) and for the foreign exchange options markets (by comparisons of Table 8.7c to Table 8.7c). While for the fixed income options markets we do not observe the increase in the significance of the coefficients for the independent variables in the regression, we can see that in most cases there is no significant difference in the obtained coefficient values (significant in the sense of a t-test for the differences in the coefficient values). This result can also be found for the four foreign exchange options.

Thus, we conclude that the original OLS regression does experience problems of heteroscedasticity. However, once this problem has been corrected, we find no appreciable difference in the nature of our regression model. In fact, the model has been enhanced for many markets.

Corrections for Serial Correlations in the Residuals of the Regressions

There is a third major problem which could bias our regression results and lead to inappropriate conclusions. The Durbin-Watson statistics for the original OLS regressions indicate that we have significant levels of serial correlations in the residuals of most of our models (the DAX model did not experience these problems).

To address this problem, we selected one market from each of the asset classes, which displayed the most extreme problem with serial correlations in the residuals. These selected markets were Nikkei for the stock index options, Bund for the fixed income options, and the Japanese Yen for the foreign exchange options. The approach chosen to solve the problem of serial correlations in the residuals is the Generalised Differences approach to Generalised Least Squares (GLS).

To run this analysis, we determined new dependent and independent variables by applying a transformation of the following form:

$$VSI_{GLS} = VSI_t - \rho \cdot VSI_{t-1} \qquad \textbf{(8.9a)}$$

(for the dependent variable and)

$$X_{GLS} = X_{j,t} - \rho \cdot X_{j,t-1} \qquad \textbf{(8.9b)}$$

(for N dependent variables)

In this equation, the term ρ refers to the autocorrelation coefficient of the previous OLS regression residual terms. For the Nikkei, the serial correlation in the residuals was 0.617015. For the Bund, the serial correlation in the residuals was 0.555563 and for the Japanese Yen, the serial correlation in the residuals was 0.331315. With these inputs, we applied the above equations 8.9a and 8.9b to transform all the variables in the analysis. Once this was completed, we reran the OLS regression. The results for these three markets can be seen in Table 8.8.

It is not surprising that the intercept has significantly dropped as we are now dealing with generalised differences. This implies that nothing can be gained by comparisons of these coefficients to those obtained in Tables 8.2a, 8.2b and 8.2c for the relevant markets. It is of interest to us how much we have lost in explanatory power for our models and whether the coefficients (and standard errors) of the independent variables in our models have substantively changed. For the Nikkei, the adjusted R-squared has certainly been reduced. However, the drop is only from 0.9319 in the OLS result to 0.8490 for the GLS result. Furthermore, the reported Durbin-Watson statistic in Table 8.8 for the GLS regression indicates that the problem of serial correlations has been corrected (at 1.80624). Finally, it is apparent that some changes in the coefficients of the independent variables have occurred. Nevertheless, for most of the strike-related variables this is not a statistically significant difference (from a t-test for differences in the coefficient results). For the differences that are significant, we find that for the first order strike price effect, the impacts of the first shock on the strike (STRIKE*SHOCK1) has a reduced impact. In addition, the interaction between the skewness and the at-the-money implied volatility has been reduced (STRIKE*ATMVOL). For the second order strike price effects, there were more significant changes. Notably, the pure second order strike price effect ($STRIKE^2$) has increased and is more significant. The time interactions (both first and second order) are now more pronounced. As with the first order strike price effect, the first shock now has a somewhat lessened impact. Finally, there appears to be a more important impact of the at-the-money volatility level on the degree of curvature in the smile ($STRIKE^2$*ATMVOL). Even though these changes are significant, what we are interested in is to capture the general dynamics of the model. For almost all the strike related variables in our model, the signs and relative magnitudes of the impacts are similar once the potential biases from the serial correlations in the residuals have been corrected for. This is a satisfactory result.

For the Bund GLS regression, we obtain similar results. The adjusted R-squared statistic has fallen from the OLS result of 0.8098 to 0.7509. However, the Durbin-Watson statistic indicates the problem of serial correlation has been corrected (with a value of 1.844168). As with the Nikkei, there are some changes in the coefficients of the strike-related variables and in some instances these changes are significant. However, the signs and the relative magnitudes of the coefficients remain at similar levels. The most notable change in the strike-related variable coefficients can be observed for the $STRIKE^2$*ATMVOL and the STRIKE*ATMVOL variables, which are now more statistically significant (negative). It is interesting to note that the second order strike price effect (relative to the at-the-money volatility) was not even significant for the OLS model. Otherwise, the two models yield similar results. The only substantial changes have occurred for the non-strike price related variables, which (a priori) we would have expected to be insignificant.

Finally, for the Japanese Yen, the GLS regression also suggests that our model is extremely robust. The adjusted R-squared statistic has only dropped from 0.8634 for the OLS model to 0.8263 for the GLS model (with a now healthy Durbin-Watson statistic of 1.950257). As with the two previous examples, while some changes have occurred in the coefficients (and standard errors) of the independent variables, the sign and magnitude of the results are even more similar for both approaches. Some notable changes can be observed for the $STRIKE^2$*TIME and $STRIKE^2$*$TIME^2$ variables which have reduced effects. For all the second order strike price interactions with shocks, the impacts are more pronounced. The impact of the at-the-money implied volatility has a reduced strike price effect and is now more significant on its own. As with the two previous markets, once we have corrected for the serial correlation problem in the residuals, we find some differences between the GLS and OLS models. Even so, the critical independent variables have a similar sign and magnitude of impact on the estimation of the standardised implied volatilities.

<u>Assessing Multi-Collinearity Problems in the Regressions</u>

Earlier, we indicated that four conditions exist that can bias a regression model. At this point, we have dealt with the first three (omission of critical variables, heteroscedasticity and serial correlation in the residuals). Now we will examine potential problems of multi-collinearity. It is clear from our research design, that many of the variables in the regression are highly correlated. According to Wonnacott and Wonnacott (1977) this can be a tricky problem. They state:

"To keep multicollinearity to a minimum, we should design an experiment or collect data that has as little relation as possible among the regressors. In other words, we try to get our regressors as unrelated as possible; but having done this, we then must live with them. Multicollinearity should not make us simply omit a regressor that we believe to be important, since this omission would introduce bias." (page 382).

This potential problem of multicollinearity was partially addressed by the choice of a step-wise selection of variables in our OLS to minimise this problem. This approach would select variables in the order of their explanatory contribution to the model. If variables are highly correlated, then this approach will minimise the problem of multicollinearity by only including new variables in the model if they can explain variance not explained by previously included variables. Given the significance of many of the included variables and the fact that they allow us to draw useful (and consistent) economic interpretations, we feel that the danger of risking multicollinearity is more than outweighed by the danger of losing information by omitting important variables.

One potential approach that could be taken at this point would be to combine all the approaches to correct simultaneously for the omission of variables problem (including contract dummy variables), heteroscedasticity, serial correlations and multicollinearity. This was not possible due to the constraints imposed by the size of the data sets we were working with. Simply said, our computers did not have sufficient capacity to run such an analysis. However, given that for each and every test of the simple analysis no substantive changes were found for the independent variables of interest, we surmise that even if such an all encompassing model were run our results would not be significantly different.

At this point and after rigorous testing of the regression models under investigation, we can conclude that the results we are obtaining will remain substantively the same even when biases in the regression models are corrected for. We conclude that we have demonstrated that these models are extremely robust in measuring the true relationships between the implied volatility process and the independent variables in our model. Given the models are so robust, it will make little difference which approach we use to present our further results. Therefore, to keep things simple (and given constraints imposed by the size of our database and the capacity of our statistical programme), we will use the results from the OLS regression for the next two sections of this Chapter. We recognise that the coefficients and standard errors of the estimates will be biased, however, the general tendencies will not change as these biases are corrected for.

Testing the Regression Models for Different Time Periods

One reason for running the regressions with all contracts included as dummy variables (results in Tables 8.6a, 8.6b and 8.6c) was to assess if the regression results would hold across time. If the strike price effects were significantly different over time, this effect should be indicated by significant coefficients for the contract dummy variables and important changes in the coefficients of the key variables in our regressions. What we observed was that few of the contract dummy variables were statistically significant and there appeared to be minor impacts on changes in the coefficients of the key variables of interest to this research. Nevertheless, it is important to assess if the overall conclusions we have drawn from the regression results would hold if the regressions were run for sub-periods of the data set.

To test the stability of the regression equations, the observations for each of the twelve markets was split into two sub-periods. These periods correspond to the data divisions used in Chapter Seven for the analysis of the implied volatility surfaces and can be found in Table 7.5.

With these divided sets of observations, we re-ran the OLS regression using equation 8.2. The only difference in these sets of regressions is that the inclusion of the Crash and Shock related variables would depend on whether this event occurred within the period of the data sample. If the data sample included this event, it was included in the regression and was ignored if the event occurred either before or after the period of analysis.

The results for the regressions estimated solely using the first portion of the available observations can be seen in Table 8.9a for the four Stock Index options, Table 8.9b for the four Fixed Income options and finally, Table 8.9c for the four Foreign Exchange options. The results for the second period can be seen in Tables 8.10a, 8.10b and 8.10c.

All of these tables should be compared to one another (for the same underlying markets) and to Tables 8.2a, 8.2b and 8.2c. First, let us examine the three regression results for the Stock Index options. In the regressions for the split periods, we find that the problem of the intercept being significantly different from 100 has now been corrected for more than half of the regression equations. This implies that the errors in variables (primarily the manner the at-the-money implied volatility was estimated) is no longer relevant. This is particularly evident in the second period for the S&P 500. One will also observe that a number of the shock and crash variables are no longer significant in Tables 8.9a and 8.10a. This is because these events were not contained within both of the periods and were therefore excluded from the analysis.

The importance of this analysis is to assess if one had run the regression solely for the first period of analysis how comparable would the regression results be relative to the entire period of analysis. Consider first the S&P 500. In the first period regression (Table 8.9a), the coefficients are remarkably similar to those observed for the entire period (Table 8.2a). For most of the first and second order strike price variables, the signs and magnitudes of the regression coefficients are similar. In most cases, these are not statistically different (using a T-test). There are some differences in the relationship between the first order strike price effect and the level of the at-the-money implied volatility, and the second order impact of time on the kurtosis ($STRIKE^2*TIME^2$), both of which are more extreme in the first period compared to the overall period. It also appears that errors in the estimation of the at-the-money implied volatilities are important for the first period but not overall. While it is clear that we have lost some explanatory power in the first period, we still have an acceptable R squared statistic of 0.8942.

While it is comforting that the regression would be similar for both the first period and overall (for the S&P 500), the true acid test is to compare the regression results for the first period of analysis to the second period (outside of sample). As was indicated throughout this Chapter, we are interested in understanding the strike price effects. Therefore, of relevance, is a comparison of the first and second strike price effects from the regression model from the first period with the actual coefficients for the strike price effects in the second period. At first glance, there appears to be significant differences between the coefficient for the first order strike price effect in the first period and in the second period. In the first period, the coefficient for the first order strike price effect (STRIKE) was +4.444 and for the second period, the coefficient was -10.2432 (from Tables 8.9a and 8.10a). However, this is an incorrect interpretation. As was indicated previously, the overall first-order strike price effect includes a number of variables. A truer comparison is if one includes both the pure strike price effects and the coefficients for the dummy variables for shocks (and the 1987 stock market crash). If the reader reviews equation 8.3 and includes the pure strike effect and the effect from the 1987 crash (for the first period), the estimate for the first order strike price effect is -8.8030. This is much closer to the actual slope coefficient of -10.2432 observed in the second period. Likewise, the other first-order strike price effects relative to time are fairly similar. While there may be some differences in the coefficients observed, the relative signs and magnitudes are similar. The comparisons of the overall first order strike price effect for the first period with the second period for all twelve markets appears in Table 8.11.

Underlying Asset	OLS Regression First Period First Order Strike Price Effect (Overall Effect: $\beta_1+\beta_4+\beta_5+\beta_6$)	OLS Regression Second Period First Order Strike Price Effect (Actual Effect β_1)
S&P 500 Futures	-8.8030	-10.2432
FTSE Futures	-6.8200	-7.7698
Nikkei 225 Futures	-2.7584	-4.7400
DAX Futures	+1.1748	-10.4075
Bund Futures	+3.9193	+4.8400
BTP Futures	-3.1144	-2.3450
Gilt Futures	-1.2300	+0.5950
US T-Bond Futures	-2.4910	-2.6050
Deutsche Mark /US Dollar	+1.7420	+0.5693
British Pound / US Dollar	-1.9280	-1.4330
Japanese Yen / US Dollar	+0.8820	+2.5850
Swiss Franc / US Dollar	+0.4590	+0.5448

Table 8.11, Comparisons of the First Order Strike Price Effect for the First Period and the Actual First Order Strike Price Effect for the Second Period.

We observe that in most cases, the predicted first order strike price effect in the second period is of the same sign and relative magnitude as the results obtained for the first period. Significant differences exist for the DAX and Gilt options. This seeming aberration is explained by a change in the relationship between the at-the-money implied volatility and the first order strike price (STRIKE*ATMVOL). For both these markets, a change in the level of expected variance had a significant effect on the first order strike price effect in the second period. Nevertheless, the absolute impact on the Gilt is slight. For the DAX, this change can be considered more important. However, the first period of analysis only had 928 observations. It is probable that with a longer period of analysis, a better result would be obtained. Nevertheless, in ten of the twelve markets, the first order strike price effect estimated in the first period would have provided a reasonable estimate of the first order strike price effect in the second period.

For the second order strike price effect (for the S&P 500), at first glance it appears that less stability exists in the two periods. However, it appears that most of the coefficients of the variables have the same sign and relatively similar magnitudes. Two exceptions are for the pure second order strike variable ($STRIKE^2$) and the interaction for this variable with the at-the-money implied volatility. In the first period there is both a reduced second order strike price effect and ATM implied volatility interaction effect. The larger coefficients in the second period (but of the same sign) could be offsetting each other to some degree. Again, we will compare the second order strike price effect estimated using the first portion of the observations to the actual second order strike price effect observed in the latter period. This can be seen in Table 8.12. Again, the estimated second order strike price effect from the first period uses the pure independent variable ($STRIKE^2$) and all the dummy variables with the second order strike price effect relative to shocks. The latter period second order strike price effect is simply the pure independent variable.

Underlying Asset	OLS Regression First Period Second Order Strike Price Effect (Overall Effect: $\beta_8+\beta_{11}+\beta_{12}+\beta_{13}$)	OLS Regression Second Period Second Order Strike Price Effect (Actual Effect β_8)
S&P 500 Futures	+4.6300	+7.6897
FTSE Futures	+10.4200	+6.2181
Nikkei 225 Futures	+4.3874	+4.5970
DAX Futures	+6.9221	+5.9285
Bund Futures	+5.2010	+7.5620
BTP Futures	+5.7600	+5.9090
Gilt Futures	+8.8670	+6.6030
US T-Bond Futures	+5.9690	+5.1710
Deutsche Mark /US Dollar	+6.7790	+7.0862
British Pound / US Dollar	+6.8850	+6.4710
Japanese Yen / US Dollar	+6.2270	+6.0860
Swiss Franc / US Dollar	+5.8710	+7.6436

Table 8.12, Comparisons of the Second Order Strike Price Effect for the First Period and the Actual Second Order Strike Price Effect for the Second Period.

In this instance, we find much greater stability between the estimated second order strike price effect from the first period and the realised second order strike price effect observed in the latter period. In many instances, the predicted and realised results are not statistically difference (using a T-test for coefficients). Nevertheless for all twelve markets, we find that the sign and relative order of magnitude is stable over both periods. Where differences in magnitude are found (for the S&P, the Swiss Franc and Bund), this divergence appears to be related to a change in the relationship between the second order strike price effect and the at-the-money implied volatility ($STRIKE^2$*ATMVOL). In all instances, a more negative relationship was observed in the second period. Nevertheless, for nine of the twelve markets, the prediction in the predict strike price effects well in a latter period. From this analysis, we conclude that the regression results display stability over time. By comparison of OLS results for three time periods and finding similar coefficients for the variables in the regressions, we conclude the factors influencing the implied volatility surfaces appear to be stable since the 1987 stock market crash. Furthermore, given that the regression results appear to be similar in the first and second period, it is not unreasonable to expect that using the regression equation obtained in the first period to predict the implied volatility surface in the second period would be effective. This is an important result and provides the clearest evidence that the results indicate regularities in the dynamics of the implied volatility surfaces and are not simply over-fitting our data set within sample.

Now that we have successfully modelled each of the twelve individual markets under analysis, tested the results for biases in the regressions and demonstrated that the models display consistent regularities over time, we now turn our attention to whether consistencies exist in the dynamics of the implied volatility process for the same asset class. For example, we will assess if the implied volatility patterns for each stock index option market have similar dynamics compared with all stock index options markets (and for fixed income and foreign exchange options markets as well). Finally, we will examine whether consistencies exist across all options markets.

8.7 COMPARISONS OF IMPLIED VOLATILTY MODELS WITHIN ASSET CLASSES

At this point we have gained important insights into the dynamics of the implied volatility process for twelve individual markets. For this analysis, (for each asset class) we merged the four individual options markets data files into one. This yielded three new files we will refer to as ALLSTOCKS, ALLBONDS and ALLFX. With these new files, we ran another regression of the form:

$$VSI = \alpha + STRIKE \cdot (\beta_1 + \beta_2 \cdot TIME + \beta_3 \cdot TIME^2 + \beta_4 \cdot CRASH + \beta_5 \cdot SHOCK1 + \beta_6 \cdot SHOCK2 + \beta_7 \cdot ATMVOL)$$
$$+ STRIKE^2 \cdot (\beta_8 + \beta_9 \cdot TIME + \beta_{10} \cdot TIME^2 + \beta_{11} \cdot CRASH + \beta_{12} \cdot SHOCK1 + \beta_{13} \cdot SHOCK2 + \beta_{14} \cdot ATMVOL)$$
$$+ \beta_{15} \cdot STRIKE^3 + \beta_{16} \cdot CRASH + \beta_{17} \cdot SHOCK1 + \beta_{18} \cdot SHOCK2 + \beta_{19} \cdot ATMVOL + \beta_{20} \cdot TIME + \beta_{21} \cdot TIME^2$$
$$+ \beta_{22} \cdot TIME^3 + \beta_{23} \cdot MARKET1 + \beta_{24} \cdot MARKET1 * STRIKE + \beta_{25} \cdot MARKET1 * STRIKE^2 + \beta_{26} \cdot MARKET2 +$$
$$\beta_{27} \cdot MARKET2 * STRIKE + \beta_{28} \cdot MARKET2 * STRIKE^2 + \beta_{29} \cdot MARKET3 + \beta_{30} \cdot MARKET3 * STRIKE +$$
$$\beta_{31} \cdot MARKET3 * STRIKE^2 + \beta_{32} \cdot MARKET4 + \beta_{33} \cdot MARKET4 * STRIKE + \beta_{34} \cdot MARKET4 * STRIKE^2 + \varepsilon$$

(8.10)

This is essentially the same regression model that appeared in equation 8.2 for each of the individual markets. The difference for this model is that we have included dummy variables for each of the four individual markets contained within this asset class. These dummy variables (MARKET1, MARKET2, MARKET3 and MARKET4) correspond to S&P, FTSE, Nikkei and DAX for the stock index options, US T-Bond, Gilt, BTP and Bund for the fixed income options and British Pound, Japanese Yen, Swiss Franc and Deutsche Mark for the foreign exchange options. For each of these market specific variables, we have also constructed first and second order strike price variables to examine how the strike price effects differ across the markets.

With these additional variables, the OLS regression was rerun for all three asset classes and the results can be seen in Table 8.9a for the stock index options, Table 8.9b for the fixed income options and in Table 8.9c for the foreign exchange options. For the sake of comparisons, the OLS regression results (from Tables 8.2a, 8.2b and 8.2c) have been included in these tables.

Comparisons of Models For Stock Index Options

In the case of the stock index options, we are interested in the general tendencies that are consistent across all stock index options markets. In Table 8.9a, it is clear that the coefficients for the overall stock index options market data file (ALLSTOCKS) and each individual stock index option market do vary. However, given that we are interested in general relationships, we will concentrate on those results where the sign of the coefficients (and the relative magnitude) for each independent variable are either similar or dissimilar.

For example, for the first order strike price effects, we find that most of the STRIKE-related variables are consistently of the same sign and relative magnitude. However, the pure strike price effect varies between the four markets. For the S&P and all stocks, the coefficient is *positive.* This suggests that controlling for all the other variables, the skew for these markets is positive. However, it is clear that the negative sign and magnitude of the combination of the first order strike price effect with the crash is causing an overall negative skew for both these markets. For the FTSE, DAX and Nikkei, the pure first order strike price effect is negative. For the DAX and Nikkei, this is hardly surprising given that both of these markets only had observations after the 1987 and 1989 stock market crashes. This suggests that the crashes had impacts for all stock index options markets. For the options markets observed both before and after these events the effect is found in the STRIKE*CRASH variable and in the STRIKE variable for those markets only observed after the crash.

Caution has to be exercised because as with the previous analysis for the single markets, the overall strike price effects are the aggregate of what is now a number of variables. To gauge the overall first order strike price effect, one must compare the STRIKE related variables for all markets and include the STRIKE related variables associated with the dummy variables for each market.

Apart from this difference, the interaction variables that combine STRIKE and TIME are similar both between and within all stock index options (both first order and second order effect). The interpretation of these variables is that as the expiration of the option is approached the negative skew begins to flatten. In some ways, it is difficult to interpret the coefficients of these two variables between the markets. This is because they are a combined effect. Nevertheless, the signs and magnitudes of the effects are of a similar dimension.

Regarding the second order strike price effect, again many of the coefficients for the stock index options (and the aggregated file) share the same sign and similar magnitudes. For all the markets, the pure kurtosis effect ($STRIKE^2$) is positive and of a similar magnitude. The first order impact of $STRIKE^2$ with TIME is negative for all the models. Again, this suggests that the curvature of the smiles becomes more extreme as expiration is approached. When considering the second order time effect ont he curvature, the impacts were not consistent across all the markets. Even so, the effect tends to be a positive one (for those markets where the impact was significant).

One interesting result is the relationship between the expected kurtosis implied in the implied volatility patterns and the expected level of variance. We find that a significantly negative coefficient for this variable ($STRIKE^2$*ATMVOL) exists. This suggests that the higher the level of the expected variance, the flatter the curve of the implied volatility pattern.

Another significant result is that for all four stock index options and for the overall aggregate of all stock index options, there are significant third order strike price effects. The $STRIKE^3$ variable is positive for all five models and roughly of the same order of magnitude. Thus, it would appear that this variable can be seen as the interaction of the first and second order strike price effects (by definition, $STRIKE^3$

=STRIKE* STRIKE^2). We must now consider higher order terms to model the dynamics of stock index options implied volatility patterns correctly. Furthermore, even though the magnitude of these coefficients is small, the t-statistics are among the highest in the models (and can thus be interpreted as being among the most significant effects). This result suggests that we must be careful when applying a quadratic form to fit the implied volatility surface. As was mentioned previously, the quadratic approach that was suggested by Shimko (1991,1993) did not allow us to correctly assess the level of the at-the-money volatility. We found that a simple linear interpolation between the implied volatilities at adjacent strike prices was better. Furthermore, the implied volatility surfaces in the last Chapter suggested that a more complex dynamic existed. Given these results, it is not surprising a higher order strike price effect was uncovered.

Apart from modelling the strike price effects, we find that a number of the other variables are important in understanding the nature of the implied volatility processes for stock index options. As was discussed previously many of these variables would not be expected to be significant in the regression model. The CRASH variable has now been eliminated from our model for all stock index options. While the two shock variables do appear, the interpretation of their coefficients is not obvious. Clearly, they are acting to correct for the fact that the intercept of the regression is not equal to 100. Roughly speaking one might state that the first shock tended to increase implied volatility levels and the second to reduce implied volatility levels. However, since all the volatilities have been standardised, this conclusion is somewhat difficult to justify. The ATMVOL variable is also not significant for all stock index options markets. Thus, it would appear that no systematic effect is appearing to suggest that we have mis-estimated this variable for these markets.

One interesting result is that the time effects appear to be fairly consistent across all the stock index option markets. While the first order effect is not significant for all stock index options markets, the second order effect is significantly negative and the third order effect significantly positive. As was suggested earlier, this could be capturing the negative exponential of time observed for two of the individual markets. Indeed this is fortunate result, because an argument can be made now that for a non-linear regression approach as the more appropriate approach to modelling implied volatility, given that we know that the time effect to be exponential in nature.

Finally, in Table 8.9a, we are interested in assessing if the individual markets have strike price effects that are not captured by the model for all stock index options. The coefficients for the market dummy variables should be insignificant. However, for the S&P and Nikkei, they have a significantly positive result. Again, these results are difficult to interpret as our dependent variable has been indexed. Nevertheless, they could be indicating that the overall levels of the indexed implied volatilities are higher for these markets compared to the other stock index options. Furthermore, for three of the four stock index options, there are at least one significant strike price effect. The first order strike price effect for the S&P is significantly negative suggesting that this market displays more of a negative skew than the other markets. The FTSE, on the other hand, has a positive first order strike price effect: it is less negatively skewed compared to other stock index options. For the second order strike price effects, the S&P displays less curvature, and the FTSE and the DAX slightly more.

Implications of the Findings for Stock Index Options Smile Surfaces

Thus, it would appear that when modelling individual stock index option markets, we observe different coefficients for the models that describe the implied volatility dynamics. The significance of the market related strike price variables suggests that somewhat different dynamics are occurring for the implied volatility patterns of the same asset class. Nevertheless, it is interesting to note that many of the variables in these models do influence all stock index options markets in similar ways. All stock index options display a negatively skewed first order strike price effect. This effect differs before and after the 1987 stock market crash. Thus, we conclude that the 1987 stock market crash changed this effect for all stock index options. It is further interesting to note that the degree of the skewness lessens as the expiration of the option is approached for each of the four stock index options and across all the stock index options markets.

The pure second and third order strike price effects are consistently of the same sign and of similar magnitude for each of the stock index options markets. Apart from the consistency in how the second shock effected the degree of the second order strike price effect, the major consistency between all the markets is the relationship of the second order strike price effect to the at-the-money implied volatility. For all the markets, the higher the at-the-money implied volatility, the lower the curvature of the second order effect.

One conclusion from these findings is that consistency exists between the dynamics of the implied volatility patterns observed for these markets with the dynamics of the objective dispersion processes we examined previously in this research. In the first part of this research, we observed that both stochastic volatility models and jump processes are required to understand the dynamics of the objective dispersion processes. Given the relationships between the time and expected variances with the first order and second order strike price effects, we may be seeing a similar effect.

For example, if market participants are concerned about negative jumps, this would clearly lead to a negative (skewed) first order strike price effect. However, if the expected variance already incorporates this event and was accordingly higher to include the risk premium of such a jump, this effect would be amplified in a more extreme negative strike price effect. The negative relationship of the first order strike price effect with time could point to the inclusion of some stochastic volatility model. It has been pointed out in the literature [Hull & White (1987a) and Stein & Stein (1991)] that the longer the maturity of an option, the greater the impact of uncertainty in the nature of the dispersion process on the deviations of the option price from the Black-Scholes assumptions. Therefore, it may be that the longer the term of the option, the greater the probability of a negative jump occurring.

Clearly, the implied volatilities are stochastic and would therefore require a stochastic volatility model to understand their behaviours. The consistent nature of the pure second and third order strike price effects could suggest that either stochastic volatility models or jump processes are the possible reasons. However, the fact that the second order strike price effect becomes more extreme as the expiration of the option approaches may suggest that jumps are the more likely reason. Nevertheless, it is more probable that both elements are responsible for the risk-neutral dispersion processes (as with the objective dispersion processes). We will examine this question in more detail in the next Chapter, where we will show that both factors play their part in describing the implied volatility dynamics we observe for stock index options markets.

Comparisons of Models For Fixed Income Options

For the fixed income options markets, there is a similar degree of consistency in the importance of the independent variables to each model (compared to the stock index options). In Table 8.9b, there are five first order STRIKE-related independent variables that are fairly consistent for all bond option markets. These include both the two TIME interaction variables. As with the stock index options, these indicate that the skewness has a negative relationship with TIME and a positive relationship with $TIME^2$. Again, the interpretation of this coefficient is that if a negative skew exists, it will flatten as the expiration date of the option is approached. Another STRIKE-related variable that is consistent across all fixed income option markets exists for the level of the at-the-money volatility (ATMVOL). The consistent negative coefficient indicates that the higher the level of the expected variance, the more the volatility pattern is skewed. This is similar to the result observed for the stock index options markets. Finally, it appears that market specific shocks have an important impact on the nature of the first order strike price effect. For almost all shocks (apart from the second shock for the US T-Bond), the impact was to increase the negative skew of the implied volatility pattern.

As with the stock index options, there is a consistently positive kurtosis effect measured by the coefficient for $STRIKE^2$. There is also consistency in the first order impact of TIME. This result suggests that the curvature of the implied volatility patterns becomes more extreme as the options expiration date is approached. The only exception exists for the BTP. However (as was pointed out earlier), the increased curvature effect comes from the second order time factor, $TIME^2$. Finally, the only other consistent independent variable (for the second order strike price effect) is the relationship between the curvature and the level of the at-the-money implied volatility. As with the stock options markets, the significantly negative coefficient for this variable ($STRIKE^2$*ATMVOL) suggests that the higher the level of the expected variance, the flatter the curve of the implied volatility pattern.

As with the stock index options, we observe that higher strike price effect moments are significant in modelling the implied volatility dynamics. The third order strike price variable ($STRIKE^3$) is positive and of a similar magnitude for all the fixed income options examined.

Most of the other independent variables do not display the same degree of consistency either in terms of the sign or the magnitude. The only important exception is for the ATMVOL variable, which is significantly positive for all the four fixed income option markets (and across all markets). As was stated previously, this probably suggests errors in how we have measured this variable. Nevertheless, it is interesting to compare the results for the stock and fixed income options. We find that for the stock index options, this variable was not significant and for the fixed income options it was. However, the other strike-price related independent variables are fairly consistent between both asset classes. Thus, while the at-the-money implied volatilities may be introducing errors for the fixed income options markets, they probably have little or no effect on the other variables that are the focus of this research.

Finally, the individual dummy variables for each market do suggest that significant divergences do exist among the fixed income options markets. Of the pure dummy variables, only the US T-Bond and the BTP are significant. Given that these variables are difficult to interpret, we will instead concentrate on the market specific strike price effects. Regarding the first order strike price effect, the BTP and Bund options markets display more negative skewness. For the second order effect, the US T-Bond has more curvature, while the Gilt has somewhat less. Apart from these differences, it appears that the overall model captures most of the first and second order strike price dynamics.

At this point, it would appear that both stock index and fixed income options share a number of similar factors capturing the nature of the implied volatility process for these markets. It is interesting to reflect that the adjusted R-squared statistic for ALL BONDS is 0.8620 for the simple OLS model and for ALL STOCKS the adjusted R-squared statistic is 0.9082. While it is acknowledged that biases in the regressions make these numbers suspect, we have demonstrated that when corrections are made to correct for these biases, we would expect the resulting models to approach this degree of explanatory power. In any event, it does appear that we are explaining a substantial amount of the variance in the implied volatility process for both asset classes.

Comparisons of Models For Foreign Exchange Options

For the foreign exchange options markets, we observe less consistency among the four markets we examined, and there is also less consistency with the results found for the previous two asset classes. In Table 8.9c, we find that that overall first order strike price effect is not consistent (or even significant for the four markets). It would appear that skewness in not normally endemic for these assets. While the pure first order effect (skew) does tend to be positive (in three of four markets), the effect for all foreign exchange options is insignificant. There also does not seem to be any consistency in the relationship between the first order strike price effect with either of the two TIME interaction variables or for the level of the at-the-money volatility. The only first order strike price effect somewhat consistent across the foreign exchange markets is that the CRASH tended to cause a negative skew to appear in the implied volatility patterns. However, this effect is slight. An opposite effect exists for the occurrence of the second shock for each market. This tended to cause the first order effect to be slightly more positive. One could claim that our interpretations of these coefficients are misleading given that we have chosen the foreign currency as our numeraire. All these options (and the underlying futures) are expressed as the number of US Dollars per unit of foreign currency. If these were expressed as the inverse, then the coefficients would reverse sign. However, given that the coefficients are of different signs for the four markets and in many instances insignificant, even with this transformation of the price series, the first order strike price effect is not as important for foreign exchange as it is for the fixed income and stock index markets.

On the other hand, there is a great deal of consistency in the second order strike price effects between the foreign exchange options and the two previous asset classes. As with both the two previous asset classes, there is a consistently positive kurtosis effect measured by the coefficient for $STRIKE^2$. There is also consistency in the first and second order impacts of TIME. Again, this result suggests that the curvature of the implied volatility patterns becomes more extreme as the options expiration date is approached. Finally, the only other consistent independent variable (for the second order strike price effect) is the relationship between the curvature and the level of the at-the-money implied volatility. As with the other two asset classes, the significantly negative coefficient for this variable ($STRIKE^2$*ATMVOL) suggests that the higher the level of the expected variance, the flatter the curve of the implied volatility pattern.

There also appears to be a higher order strike price effect required for understanding the dynamics of foreign exchange implied volatilities. The third order strike price variable, ($STRIKE^3$), is also significant for all the four markets (and overall). However, while for the previous two asset classes this relationship was positive, this relationship is a negative one for all markets apart from the British Pound (which was barely significant). Thus, we could conclude that some other higher order dynamics influence this market and it is left to further research to uncover the nature of these dynamics.

Of the other independent variables, the only variable that is consistent across all the four markets and overall is the ATMVOL variable. This is significantly negative for all the models. As was stated previously, this is most probably due to errors in our method of estimating the at-the-money implied volatility. Yet, while we acknowledge that suggests potential problems in our models, we are unsure as how to estimate this variable better. Furthermore, we have demonstrated that when alternative modelling approaches were examined, this variable did not provide the same degree of contribution to the regression. Thus, we conclude that the biasing effect introduced by this variable is not critical to our interpretation of the other independent variables in the model.

Finally, the individual dummy variables for each market do suggest that some substantive divergences exist among the foreign exchange options markets. The only one of the simple market dummy variables that was statistically significant was the British Pound. All the other market specific effects came from strike price effects. Each of the other three foreign exchange options had significant first and second order strike price effects. For the Deutsche Mark, Swiss Franc and Japanese Yen, the first order strike price effect was positive. Both the Deutsche Mark and Japanese Yen also had a higher second order strike price effect (although small). Only the Swiss Franc displayed a slightly flattened second order curvature in the implied volatility patterns.

Overall, it would appear that foreign exchange options are not as homogeneous in the nature of the implied volatility process as stock index or fixed income options. Nevertheless, the adjusted R-squared statistic has only been reduced to 0.8484 for the model of the aggregated foreign exchange options markets. While we must interpret this result with care, it is clear that this model approaches the explanatory power of the models for the previous two asset classes. Thus, it could be said that while fewer variables are required to understand the nature of the foreign exchange options implied volatility dynamics, the significant variables provide relatively more explanatory power.

From this analysis, we can compare the factors that describe the implied volatility process across asset classes. It is clear that individual option markets have idiosyncratic features that cause them to differ even within their own asset class. Nevertheless, there does appear to be consistency within an asset class for those independent variables that explain the implied volatility dynamics. This consistency seems to be similar across the three asset classes we have examined. Even so, we must reject the hypothesis that within the same asset class, the dynamics of the implied volatility process are identical.

Finally, we will address the question, which factors explaining the implied volatility processes are consistent across all markets.

8.8 COMPARISONS OF IMPLIED VOLATILTY MODELS FOR ALL MARKETS

For this final analysis, we created an aggregated data file of all the three asset classes. Thus, we obtained one file that we referred to as ALLMARKETS. This was a combination of ALLSTOCKS, ALLBONDS, and ALLFX. The regression model for this data set took the following form:

$$
\begin{aligned}
VSI &= \alpha + STRIKE\cdot(\beta_1+\beta_2\cdot TIME+\beta_3\cdot TIME^2+\beta_4\cdot CRASH+\beta_5\cdot SHOCK1+\beta_6\cdot SHOCK2+\beta_7\cdot ATMVOL)\\
&+STRIKE^2\cdot(\beta_8+\beta_9\cdot TIME+\beta_{10}\cdot TIME^2+\beta_{11}\cdot CRASH+\beta_{12}\cdot SHOCK1+\beta_{13}\cdot SHOCK2+\beta_{14}\cdot ATMVOL)\\
&+\beta_{15}\cdot STRIKE^3+\beta_{16}\cdot CRASH+\beta_{17}\cdot SHOCK1+\beta_{18}\cdot SHOCK2+\beta_{19}\cdot ATMVOL+\beta_{20}\cdot TIME+\beta_{21}\cdot TIME^2\\
&+\beta_{22}\cdot TIME^3+\beta_{23}\cdot BOND+\beta_{24}\cdot BOND*STRIKE+\beta_{25}\cdot BOND*STRIKE^2+\beta_{26}\cdot STOCK\\
&+\beta_{27}\cdot STOCK*STRIKE+\beta_{28}\cdot STOCK*STRIKE^2+\beta_{29}\cdot FX+\beta_{30}\cdot FX*STRIKE+\beta_{31}\cdot FX*STRIKE^2\\
&+\beta_{32}\cdot USTB+\beta_{33}\cdot USTB*STRIKE+\beta_{34}\cdot USTB*STRIKE^2+\beta_{35}\cdot GILT+\beta_{36}\cdot GILT*STRIKE\\
&+\beta_{37}\cdot GILT*STRIKE^2+\beta_{38}\cdot BTP+\beta_{39}\cdot BTP*STRIKE+\beta_{40}\cdot BTP*STRIKE^2+\beta_{41}\cdot BUND\\
&+\beta_{42}\cdot BUND*STRIKE+\beta_{43}\cdot BUND*STRIKE^2+\beta_{44}\cdot S\&P+\beta_{45}\cdot S\&P*STRIKE+\beta_{46}\cdot S\&P*STRIKE^2\\
&+\beta_{47}\cdot FTSE+\beta_{48}\cdot FTSE*STRIKE+\beta_{49}\cdot FTSE*STRIKE^2+\beta_{50}\cdot Nikkei+\beta_{51}\cdot Nikkei*STRIKE\\
&+\beta_{52}\cdot Nikkei*STRIKE^2+\beta_{53}\cdot DAX+\beta_{54}\cdot DAX*STRIKE+\beta_{55}\cdot DAX*STRIKE^2+\beta_{56}\cdot BP\\
&+\beta_{57}\cdot BP*STRIKE+\beta_{58}\cdot BP*STRIKE^2+\beta_{59}\cdot JY+\beta_{60}\cdot JY*STRIKE+\beta_{61}\cdot JY*STRIKE^2+\beta_{62}\cdot SF\\
&+\beta_{63}\cdot SF*STRIKE+\beta_{64}\cdot SF*STRIKE^2+\beta_{65}\cdot DM+\beta_{66}\cdot DM*STRIKE+\beta_{67}\cdot DM*STRIKE^2+\varepsilon
\end{aligned}
\tag{8.11}
$$

This regression includes all the variables used in the OLS regression in equation 8.2 for each of the individual markets. The difference for this model is that we have included three dummy variables for each asset class (STOCK, BOND, FX) and dummy variables for each individual market. For each of these dummy variables, we determined two more strike price dependent variables. Each of these variables was determined by simply multiplying the strike (or strike squared) by the dummy variable. With these additional variables, the OLS regression was rerun for all the options markets in this research and the results can be seen in Table 8.14. For the sake of comparisons, we have included the OLS results for the three asset classes that appeared in Tables 8.9a, 8.9b and 8.9c

We observe that the coefficients of the regression model differ substantively among the asset classes and for the aggregate of all markets. Therefore (and as expected), we can see that implied volatility dynamics are not the same across all asset classes. Nevertheless, as with the previous analyses, we can see that a number of general relationships are shared among asset classes and all markets. Therefore, we will once again concentrate on those results where the sign of the coefficients (and the relative magnitude) for the independent variables are similar.

For the four models presented in Table 8.14, we do observe a number of general tendencies that are shared across markets. For the first order strike price effects, the first STRIKE-related variable that is consistently of the same sign and relative magnitude is the interaction between the STRIKE and the CRASH. It appears that for almost all markets, the occurrence of the crash led to a significantly negative skew after that event. Secondly, for all markets, there is a significant impact from the level of the expected variance and the degree of the volatility skew. The coefficient of the STRIKE*ATMVOL variable is extremely significantly negative. Thus for all markets, the higher the level of the expected variance the more negative the volatility skew is. It is of interest that this effect is most pronounced not for the stock index options markets but for the foreign exchange and fixed income options markets.

Regarding the second order strike price effect ($STRIKE^2$), many more of the coefficients for all markets share the same sign and similar magnitudes. For all the models, the pure kurtosis effect is positive and of a similar magnitude. The first and second order impacts of $STRIKE^2$ with TIME are also similar for all the models. Again, this suggests that the curvature of the smiles becomes more extreme as expiration is approached for all markets. Finally, the only other consistent independent variable (for the second order strike price effect) is the relationship between the curvature and the level of the at-the-money implied volatility. For all markets, the significantly negative coefficient for this variable ($STRIKE^2*ATMVOL$) suggests that the higher the level of the expected variance, the flatter the curve of the implied volatility pattern.

The higher order strike price effect ($STRIKE^3$) is relevant for all the asset classes and indeed across all markets. However, given that the impact for the foreign exchange options markets is of the opposite sign of the impact for the stock index and fixed income options, this suggests different dynamics may be driving different asset classes.[2] Nevertheless, this suggests that further work is needed to understand these higher order impacts.

For the remaining variables, there is little consistency across asset classes or the overall aggregate of all options markets. One interesting result is that relationship of the ATMVOL variable to the model for ALLMARKETS has now assumed a negative sign. If one examines the coefficients of this variable across all asset classes, one will observe both positive and negative signs. The results of our previous tests of the regression models had us to assume that this effect is most probably due to random error in the estimation of the at-the-money implied volatility and inconsequential to our general conclusions.

It is also of interest that strong TIME impacts are observed overall for all markets. However, this effect may either indicate errors in time estimation or may be indicating some negative exponential of time impact. It is interesting to note that this seems to be offsetting the coefficient of the ATMVOL variable. Thus, this could be some complex correction for the error we have identified there.

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[2] However, it may be a question of the choice of the numeraire for the four foreign exchange options. If we chose the numeriare as the US Dollar, this effect would be consistent across all markets.

Finally, in Table 8.14, a number of the market specific dummy variables have significant coefficients. For the pure dummy variables, BOND, STOCK, US T-Bond, S&P, Nikkei and British Pound are significant. As before, the interpretation of these coefficients is somewhat difficult.

Of more interest are the strike-related interactions with these market dummy variables. We observe that for all stock index options there is a significant negative first order strike price effect. The S&P and the DAX have a further significant negative first order effect, which is consistent for the analysis presented in Table 8.9a. The FTSE has a positive first order strike price effect that suggests it is less negatively skewed than the other options markets examined.

The BOND and FX asset classes all show a positive first order strike price effect. However, for individual markets within these asset classes, the first order effect is also significant. For the fixed income options markets, the GILT is more positively skewed, while the BTP and BUND are negatively skewed. For the foreign exchange options, only the British Pound (negative) and Swiss Franc (positive) have first order strike price effects.

For the second order strike price effects, the only significant asset class is the foreign exchange class. Here the coefficient is slightly positive suggesting that more curvature exists for this asset class. There are, however, a number of individual markets that have significant second order strike price effects. The US T-Bond, Bund, Japanese Yen (barely) and FTSE all have slightly positive coefficients, while the S&P, Nikkei and Swiss Franc have negative coefficients. While these do suggest divergences, the levels of these market specific impacts are relatively small.

8.9 CONCLUSION

In this Chapter, we have been able to develop a robust modelling approach that relies on the use of a simple OLS regression technique with dummy variables. The ANCOVA approach has allowed us to explain the majority of the variance in the implied volatility surfaces for the twelve option markets under investigation.

From this analysis, we can draw a number of conclusions. The implied volatility dynamics are clearly different for different markets. Nevertheless, there are a number of variables that have a similar impact on the modelling of the surfaces both in terms of the sign of the impact and the relative magnitude. It appears that all markets share a similar degree of absolute implied kurtosis (measured by the $STRIKE^2$ variable). All markets experience more implied kurtosis as the expiration of the option is approached. In addition, the strike price effects (both first and second order) for all markets are inversely related to the level of the expected variance. An increase in the at-the-money volatility serves to increase a negative skew in the implied volatility pattern and decreases the curvature of the implied kurtosis in the pattern.

As was stated previously, these results suggest that consistencies may exist between the dynamics of the implied volatility patterns observed for these markets with the dynamics of the objective processes we examined previously in this research. We demonstrated in Chapter 5 that both stochastic volatility models and jump processes are required to understand the dynamics of the objective processes. One could interpret the first order and second order strike price effects we have identified in this Chapter as reflecting both these factors.

The existence of a consistent negative (skewed) first-order strike price effect might suggest that market participants are concerned about negative jumps. However, we did not observe a significant negative skew effect for the unconditional return series. The only two markets that displayed a significantly negative skew in the returns were the S&P 500 and FTSE 100 and this was entirely due to the 1987 stock market crash. Yet, for all the stock index and fixed income options a negative (skewed) first-order strike price effect was found. From our analysis, we established that this is due to the occurrence of the crash. Thus, it would appear that even though such a negative skew is not necessary warranted by the objective process, market participants consistently price options as if it were[3]. There are a number of possible explanations for this. The most obvious is that the 1987 stock market crash led participants to include a risk premium of such a negative jump. Thus, it would be appropriate to expect jumps in order to understand this effect. However, the negative relationship of the first order strike price effect with time could point to the inclusion of some stochastic volatility model. It has been mentioned previously that the longer the maturity of an option, the greater the impacts of uncertainty about the nature of the dispersion process (and the effect on the deviations of the option price from the Black-Scholes results). Therefore, it may be that the longer the term of the option, the greater the probability of a negative jump occurring and that is one conclusion that can be drawn from the results of our models.

Furthermore, in the last Chapter we demonstrated that the implied volatilities are indeed stochastic (see Figures 7.1a, 7.1b and 7.1c). Thus, it is consistent that an understanding of the dynamics of implied volatilities requires a stochastic volatility model. As with the analysis of Chapter 5, we demonstrated that both jumps and stochastic volatility models are necessary to capture the dynamics of volatility. In this Chapter, (our results suggest again that) the consistent nature of the pure second and

[3] Jackwerth and Rubinstein (1996) completed a detailed analysis of the skewness effect. They

third order strike price effects suggest that both stochastic volatility models and jump processes act in combination. However, the fact that the second-order strike price effect becomes more extreme as the expiration of the option approaches may allow a conclusion that jumps are the more likely explanation.

One interpretation of these results is that similar models are required to understand both the objective and risk-neutral processes. To test this hypothesis (that a combination model best explains both dispersion processes), we will now examine the relationships between the actual implied volatility dynamics and the implied volatility processes that are consistent with these combination models.

suggested that this is a particularly difficult paradox to unravel. We have, therefore, concentrated on somewhat different issues

CHAPTER NINE
THE ANALYSIS OF RISK NEUTRAL PROBABILITIES IN OPTIONS ON FUTURES COMPARISON OF IMPLIED VOLATILITY SMILES WITH SIMULATED SMILES DERIVED FROM OBJECTIVE DISPERSION PROCESS MODELS

9.1 INTRODUCTION

In the first portion of this dissertation, we examined a variety of models to explain the objective processes associated with twelve financial futures markets. With these models we have a means to generate simulated price series, which were demonstrated to be effective in explaining the dynamics of the objective processes for these markets.

With this price generation methodology, we can now assess the dynamics of the prices of options on these futures markets that would be consistent with these dispersion processes. To achieve this we employed a Monte Carlo simulation to estimate call options with a variety of strike prices. This is similar to the approach by Johnson and Shanno (1987). As with this research, they were interested in understanding the exercise price bias, which was pointed out by MacBeth and Merville (1979) and Rubinstein (1985). However, these exercise price biases were only identified for stock options and we will extend this analysis for fixed income and foreign exchange options.

9.2 SIMULATING IMPLIED VOLATILITY DYNAMICS CONSISTENT WITH THE OBJECTIVE PROCESS MODELS

The first step required the generation of price series of 62 (trading) days, which were determined by either a Student-t model with constant variance, the optimal stochastic volatility model (assuming price innovation followed GBM) or the optimal model which incorporated stochastic volatility and a Student-t distribution.

These 62 trading days are comparable to the 90 calendar days used in the previous two chapters. The following table compares the number of trading days (in a year of 252 trading days) compared to the calendar days (in a year of 365 days).

Calendar Days	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90
Business Days	3	7	10	14	17	21	24	28	31	35	38	41	45	48	52	55	59	62

The reason why we must work with trading days is that the best fitting models in the first portion of this research assumed that time was measured as trading days rather than calendar days. In Chapter 6, we discussed that previous research had pointed out that to correctly estimate implied volatility one should use calendar time. Thus, we will estimate the price generation series using trading time and then for comparison's sake, we will convert this result using the above schedule to compare this with the implied volatilities estimated using calendar time.

For the Monte Carlo simulation we used an anti-thetic approach suggested by Boyle (1977). Using a Box-Muller approximation, we determined 5000 draws of 62 observations from a N(0,1) distribution. Then to apply the anti-thetic approach, we multiplied each of these Z's by minus one (-1). This generated the series of 62 draws from a normal distribution (10000 times) that had an average of 0.0 and a variance of approximately 1.0 (0.999998). These random numbers were then stored and used for all subsequent Monte Carlo simulations. This was done so that all results could be compared without introducing variation from the selection of the random numbers.

The three models we examined included: 1) the model that assumed the underlying price series followed a Student-t distribution with constant variance, 2) the model that assumed the underlying price series followed Geometric Brownian motion but the volatility evolution followed some stochastic process, and 3) the model that assumed the underlying price series followed a Student-t distribution and the volatility followed some stochastic process. The choice of the models and the parameter values were drawn from Chapters 3, 4 and 5, where we determined the optimum models using the minimised sum of squared errors approach.

An apparent inconsistency in this research approach is that the use of Monte Carlo simulations to price options assumes that a risk-neutrality condition exists. As was discussed in Chapter 6, this suggests that the state space is continuous and spanned across that space by existing securities. From the examination of these models, we are introducing both jumps and stochastic volatility into the state space. As we indicated earlier there are not securities in existence, which allow us to span a state space where the volatility displays such dynamics. Therefore, these models do not allow us to invoke a unique risk neutral measure and therefore will not allow us (in the strictest sense) to price the options unambiguously. This is the apparent theoretical inconsistency. Our objective is to compare the simulated options prices (and their implied volatilities) to those we observe empirically. The prices are determined by expectations under the usual form of risk neutral adjustment. While we cannot justify these as no-arbitrage prices, we can justify them as possible prices. Given the incompleteness of our markets, we must assume that a risk premium is likely to exist due to the inability to span the volatility state space. Such a comparison between these simulated options prices and the actual options prices will provide us an insight into the nature of this risk premium.

9.3 SIMULATING IMPLIED VOLATILITY DYNAMICS CONSISTENT WITH A STUDENT-t DISTRIBUTION MODEL

For the first model (the Student-t distributed underlying price evolution with constant variance), we drew 62 random numbers from a normal distribution and adjusted these using the Student-t estimation technique described in Chapter 3. This t-distribution was approximated using the following approach. To obtain a fat-tailed distribution, we simulated a t-distribution with 5 degrees of freedom. This was achieved by taking 5 draws from a normal distribution that used the Box-Muller technique. The draws were squared and summed. This result was divided by five (5) and then the square root was taken. To sample from the Student-t distribution, a random normal variate is divided by this factor. This is repeated for each of the 62 days to expiration and the entire simulation was repeated 10000 times. This final simulation resulted in a symmetrical distribution that had a mean that was exactly equal to zero, somewhat higher than unit variance and the fat tails we are looking for. Theoretically, the expected kurtosis should be equal to nine (9).

The summary statistics of the 62 Student-t draws done 10000 times were as following:

Moments	62 Draws (10000 times)
Mean	0.0000
Std. Dev	1.2910
Skewness	0.0000
Kurtosis	8.969

From this series, we have obtained somewhat less than the kurtosis we were expecting, but the difference is slight. As with the draws from the normal distribution, these draws from the Student-t distribution were saved and used for all subsequent simulations that used the Student-t distribution.

With these draws from the Student-t distribution, we generated a series of 62 (trading) days prices using the following formula:

$$S_t = S_{t-1} \cdot e^{\mu \cdot dt + \hat{\sigma}_{t-1} \cdot dT_1} \tag{9.1}$$

This is the same Euler approach used in Chapter 3. The term $\hat{\sigma}$, reflects the volatilities estimated from the various models tested and the previous day's volatility estimate is used to estimate today's new asset price. Given that the volatilities are assumed to be constant for the first model, this does not change. It was set for the simulations at 20% per annum. One should note that the price generation process does not use dZ_1 but rather dT_1. This notation reflects that we are using the Student-t distribution rather than assuming Geometric Brownian motion. The function dT_1 represents the volatility input, σ, time the square root of time, √1/252, times the draw from the Student-t distribution. For these simulations we assumed the interest rate was 0% (zero), so that the drift term is expressed as: $-\frac{1}{2}\sigma^2 * 1/252$[1].

With these 10000 simulations of 62 trading days, we then estimated the Monte Carlo prices for call options with strike prices from 80 to 120 in 2 point increments. The initial price of the underlying asset was set to 100. Using the above schedule for trading days that corresponded to the same number of calendar days (as was examined in the last two chapters for the implied volatilities), we determined the average price for the call options. For example, for the call options, which correspond to 5 calendar days to expiration, the Monte Carlo price of the call options were determined with 3 trading days ahead of the initial starting value of 100. From these call option prices, we determined a matrix of call option prices that varied across the range of strike prices and had different times to expiration. These can be seen in Table 9.1. For the sake of comparison, time is represented both in trading days and calendar days.

With these call option prices determined, we then input these prices into the Black 1976 option pricing formula to estimate the implied volatilities for each of the strike prices at each time to expiration. For this estimation, the initial underlying price was 100, the interest rates were set to zero (0%) and time is measured as calendar time. It should be noted that since the time parameter in this model is the percentage of a year, the time input actually used corresponds to the appropriate percentage that the trading days (of each option) represents in a 252 trading day year. From this we were able to obtain a series of implied volatilities across strike prices and across time. These results can be seen in Table 9.2.

Finally, using the standardisation technique that was discussed in Chapter 7, the implied volatilities were indexed. The final transformed volatility matrix can be seen in Table 9.3. To aid evaluation, the results of this matrix have been drawn to produce a series of volatility smiles at different times to expiration. This can be seen in Figure 9.1 (where the strike price was transformed by the formula: $[\ln(X/F)/\sigma\sqrt{t/252}]$).

One interesting result is that this model displays many of the features we observe in the empirical volatility smiles (see Figures 7.11a, 7.11b and 7.11c., Figures 7.16a, 7.16b and 7.16c., Figures 7.17a, 7.17b and 7.17c and Figures 7.18a, 7.18b and 7.18c for the empirical smile patterns). To make these comparisons easier, we have produced a series of graphs that contain the actual implied volatility surface for each of the twelve markets (and for each period) and included the simulated smiles from the three alternative models. These figures can be seen as Figures 9.2a, 9.2b, 9.2c, 9.2d for the stock index options. For the fixed income options, these results can be

[1] Since in this simulation the process is non-Gaussian, this is not strictly correct. However, given that the time increment (dt) is a small number this is a good approximation.

seen in Figures 9.3a, 9.3b, 9.3c and 9.3d. Finally for the Currency options, these results can be seen in Figures 9.4a, 9.4b, 9.4c and 9.4d.

The smiles generated from the Student-t model are relatively flat with 62 trading days (90 calendar days) to expiration. As the expiration is approached, the smiles become more curved. This result is consistent with the finding in the last Chapter that the second order strike price becomes more extreme as expiration is approached. Nevertheless, we must reject this model on two grounds. The first is that it is clear that from a comparison of the smile patterns the Student-t model simulated smile displays more of a time effect on the curvature of the implied volatility pattern than we observe empirically. Secondly, it is clear from the summary statistics (presented in Tables 7.1a, 7.1b and 7.1c) for the at-the-money implied volatilities that the implied volatilities do follow a stochastic process and we must reject (from a theoretical sense) any model which assumes constant variance.

Now that we have simulated what smile behaviour would be consistent with a Student-t distribution model with constant variance, we will now compare the structure of the smile surface from this model to the actual smile behaviour of the twelve option markets. While graphs can be helpful in comparing results, we will instead use a standard minimisation of errors method to test statistically how well this model behaves.

9.4 TESTING THE EXPLANATORY POWER OF THEORICAL PRICING MODELS FOR THE IMPLIED VOLATILITY SMILE PATTERNS

To achieve this, we first needed to fit a two-dimensional surface using a polynomial function. This approach was to fit the data from Table 9.3, using all the strike prices from 80 to 120 and all the times to expiry[2].The surface we need to fit is the matrix of VSI values from the Student-t distribution model with constant variance. We chose to fit this surface with the following equation:

$$VSI = \alpha_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3 + \beta_4 XT + \beta_5 X^2 T + \beta_6 XT^2 \tag{9.2}$$

Where, X is the standardised strike price and T is the time to expiration of the option expressed in the percentage in a (trading) year. While this equation bears resemblance to equation used by Dumas, Fleming and Whaley (1996) to determine their Deterministic Volatility Function (see equation 8.1d), this model is based upon the findings from the previous Chapter that indicated that additional higher order terms were required to understand the complex relationship between strike prices and time. For each VSI observation, we have a standardised strike price (X) and a specific time to expiration (T).

The results from the above polynomial equation for the Student-t distribution model appear in Table 9.4.

Factor		Coefficient	Standard Error	T-Statistic
Intercept	α	99.9132	0.1272	-0.04039
Strike	β_1	0.0525	0.2237	0.23457
Strike^2	β_2	2.5721	0.0245	104.9837
Strike^3	β_3	0.0065	0.0085	0.7647
Strike*Time	β_4	6.0387	3.7425	1.6136
Strike^2*Time	β_5	-11.4791	0.5458	-21.0317
Strike* Time^2	β_6	-35.0199	14.4992	-2.4153
R-Squared	**0.977402**			

Table 9.4 Results from Fitting the Student-t Model Theoretical Implied Volatility Surface with a Polynomial Function.

Of most importance in this table, is the fact that the R-squared confirms that this approach is a good approximation to the points estimated numerically. Of less

[2] This was necessary because the actual options (we are trying to explain) have discrete strike prices that will not necessarily correspond exactly to the strike prices of our simulation. Rather than re-running each Monte Carlo simulation for each observed option, we wished to fit the theoretical surface and then use this to predict the observed options.

interest is whether the independent variables are statistically significant (this is indicated by a bold typeface). As we expected, all the independent variables that include the second order strike price effect are significant. The first order strike price effect is not captured apart from the interaction of Strike and $Time^2$ (and this is barely significant). Furthermore, this effect is also of the opposite sign to what we observed for the actual markets (see Table 8.14 for example). In addition, this model fails to include the higher order strike price variable, $STRIKE^3$, as a significant contribution to the regression. For all the actual option markets, this effect was extremely significant.

From this preliminary analysis, we would conclude that substantive differences exist between the Student-t model with constant variance and what we observe in the actual options markets. It is hardly surprising that we have failed to capture the first and third order strike price effects as this model was only intended to capture the second order strike price effect.

With this equation, we generated a predicted series of VSI values for each option available for the twelve markets. With these predicted results, the test of whether this model is effective or not was to run a simple linear regression of the form:

$$VSI_{actual} = \alpha + \beta \cdot VSI_{predicted} + \varepsilon \qquad (9.3)$$

From this regression, we are interested in assessing the intercept, the slope coefficient, and most importantly the R^2 of the regression. What we will focus on is the adjusted R^2 of the regression as a measure of the ability of the model to explain the actual behaviour of the implied volatilities and the slope coefficient to indicate if the models display the appropriate dynamics. The intercepts and the slope coefficients will provide us a method of assessing what may be occurring in the regression. We expect that the intercept should have a value of 0.0 and the slope coefficient should have a value of 1.0. To assess if the slope coefficient is statistically different than 1.0, we used a t-test for each slope coefficient with the critical value equal to 1.0. If the regression results are significantly different than what we expect, this will provide an indication that problems could exist with the models.

9.5 TESTING THE EXPLANATORY POWER OF A STUDENT-t DISTRIBUTION MODEL FOR THE IMPLIED VOLATILITY PATTERNS

Using this approach, the Student-t distribution model was applied to all twelve markets and for both time periods. These results for the twelve markets, for the entire period and the first and second portions of the data appear in Tables 9.5a, 9.5b and 9.5c.

Markets	Intercept	β Coefficient	Standard Error	T-Statistic[3]	Adjusted R-squared
S&P 500	-125.87	2.3473	0.04496	29.9667	0.1803
FTSE	-115.12	2.2015	0.03038	39.5491	0.4292
DAX	-114.47	2.1827	0.05562	21.2639	0.3574
Nikkei	-92.37	1.9468	0.05648	16.7635	0.2521
Bund	-76.40	1.7972	0.03374	23.6277	0.2559
BTP	-91.79	1.9318	0.03311	28.1426	0.2839
Gilt	-62.48	1.6573	0.02664	24.6734	0.2993
US T-Bond	-85.43	1.9324	0.01957	47.6444	0.5057
D-mark	-97.47	2.0090	0.01553	64.9710	0.6018
Pound	-96.79	1.9981	0.01645	60.6748	0.6165
Yen	-121.08	2.2465	0.01300	95.8846	0.6969
S-Franc	-78.96	1.8211	0.01549	53.0084	0.5388

Table 9.5a, Regression Results for the Predicted Smile Behaviour from a Constant Volatility Student-t Distribution model against the Actual Smile Behaviour for Twelve Option Markets for the Entire Period of the Analysis.

[3] The t-test is relative to an expected Beta coefficient of 1.0.

Markets	Intercept	β Coefficient	Standard Error	T-Statistic	Adjusted R-squared
S&P 500	-103.94	2.1006	0.05963	18.4572	0.2065
FTSE	-194.00	2.9467	0.06884	28.2786	0.5045
DAX	-165.45	2.6736	0.14730	11.3619	0.2616
Nikkei	-60.65	1.6293	0.08449	7.4482	0.2521
Bund	-91.14	1.9514	0.04930	19.2982	0.3044
BTP	-94.89	1.9637	0.04085	23.5912	0.3902
Gilt	-80.11	1.8185	0.04139	19.7753	0.4542
US T-Bond	-74.67	1.8313	0.05401	15.3916	0.4165
D-mark	-99.97	2.0486	0.02741	38.2561	0.5891
Pound	-88.73	1.9185	0.02812	32.6636	0.5482
Yen	-111.13	2.1553	0.02502	46.1751	0.6463
S-Franc	-71.62	1.7628	0.02714	28.1061	0.5077

Table 9.5b, Regression Results for the Predicted Smile Behaviour from a Constant Volatility Student-t Distribution model against the Actual Smile Behaviour for Twelve Option Markets for the First Half of the Available Observations.

Markets	Intercept	β Coefficient	Standard Error	T-Statistic	Adjusted R-squared
S&P 500	-140.44	2.5204	0.06259	24.2914	0.1754
FTSE	-102.22	2.0887	0.03499	31.1146	0.4074
DAX	-106.80	2.1148	0.06575	16.9551	0.3598
Nikkei	-114.95	2.1766	0.07561	15.5614	0.3509
Bund	-64.51	1.6733	0.04578	14.7073	0.2224
BTP	-90.24	1.9156	0.04777	19.1668	0.2443
Gilt	-56.17	1.5994	0.03285	18.2466	0.2601
US T-Bond	-86.98	1.9466	0.02117	44.7142	0.5165
D-mark	-95.82	1.9847	0.01821	54.0747	0.6231
Pound	-99.97	2.0294	0.02069	49.7535	0.6427
Yen	-125.62	2.2881	0.01504	85.6450	0.7214
S-Franc	-83.27	1.8561	0.01841	46.0190	0.5678

Table 9.5c, Regression Results for the Predicted Smile Behaviour from a Constant Volatility Student-t Distribution model against the Actual Smile Behaviour for Twelve Option Markets for the Second Half of the Available Observations.

From this analysis, we find that all the slope coefficients are higher than 1.0. This indicates that the actual curvature of the implied volatility smiles for the options is more extreme than for the smiles predicted by the Student-t models. Given that these coefficients are higher, the intercept is correspondingly negative. This is because the intercept adjusts the mean to correspond to the general level of 100. The intercept is of little interest to our analysis as it simply is a scaling adjustment.

We are also interested in the R-squared. This statistic indicates which model has the best shape. One potential source of concern for this analysis is we are comparing simulations always based upon five degrees of freedom (for the determination of the Student-t distribution). Previously in Chapter 5, we examined eight draws from a Student-t distribution with five degrees of freedom. Given the selection of the random numbers, the realised draws would imply different degrees of freedom. However, subsequent analysis indicated that the results from Chapter 5 were not substantively altered for a Student-t result consistent with five degrees of freedom

Overall, the results are somewhat disappointing. While we are explaining a significant portion of the variance in the actual implied volatility patterns, in most cases we are explaining less than 50% of the variance. The markets that are best explained (by a R-squared criterion) tend to be the foreign exchange options. If our criterion for the best model is a slope coefficient closest to 1.0, then the models best explain the fixed income markets. The Beta for most of the markets indicates that the amplitude of the observed smiles is roughly double what the models predict.

One possible reason for the R-squared result is that the models for the objective processes do not capture the first order strike price effect. As was discussed previously, we chose not to examine the skewness in the objective processes. This was justified because negative skewness was only important for two of the twelve markets (S&P 500 and FTSE 100) and appeared to be solely a result of the 1987 crash. Nevertheless, the implied volatility patterns for all stock index options and fixed income options have a significant first order strike price effect (even though the objective processes do not seem to justify this). Thus, it is not possible for our model to capture this effect with our Student-t model.

Another reason is that this model is failing to correctly capture the second order strike price effect. As was indicated above, the regression results suggest that the amplitude of the actual implied volatility surface is approximately twice what the models predict. This is not surprising given that in the comparison of the three models for the objective processes (see Chapter 5, Tables 5.3, 5.8 and 5.11), the Student-t distribution was never the best model. Therefore, our prior would be to assume that if the Student-t model was not the best model for the objective process, it would also fail to best explain the risk-neutral dispersion process.

9.6 SIMULATING IMPLIED VOLATILITY DYNAMICS CONSISTENT WITH GBM STOCHASTIC VOLATILITY MODELS

To test the second model (stochastic volatility models with the assumption of GBM for the underlying asset price), we needed to generate another series of 62 days prices that were consistent with the appropriate model. For this we needed to generate a series of 62 volatilities 10000 times (again using the anti-thetic method). This was done by determined a new series of random N(0,1) draws using the Box-Muller approach. These were determined in the same manner as the random draws for the N(0,1) used for the price generation (and the estimation of the Student-t draws). However, these new random draws were independent of the initial draws. These were saved and used for all subsequent estimations of the stochastic volatility models.

Then we chose the appropriate stochastic volatility model that was the best fitting model (that was determined in Chapter 4) for each of the twelve markets and for the whole period, the first half and the second half of the available observations. For these models, we generated a series of random volatilities. With these, we then estimated a series of 62 days prices using the following formula:

$$S_t = S_{t-1} \cdot e^{\mu \cdot dt + \hat{\sigma}_{t-1} \cdot dZ_1} \qquad (9.4)$$

Again this is the simple Euler approach used previously in Chapter 4. The term $\hat{\sigma}$, reflects the volatilities estimated from the various models tested and the previous day's volatility estimate is used to estimate today's new asset price. For this simulation, the dZ_1 represents a standard Geometric Brownian motion process and is the draw from the random N(0,1)'s which we initially estimated and were saved for all subsequent simulations. This generated a price series from a starting value of 100 and we captured each daily price until 62 days forward in time. As before, we assumed the interest rate was 0% (zero), so that the drift term (μ) is expressed as: $-\frac{1}{2}\sigma^2 * 1/252$.

For each of the simulations, we examined the best stochastic volatility models for each of the twelve markets and for each of the three estimation periods. The best fitting models and the optimal parameter values can be seen in Tables 4.4,4.6 (for the entire period),4.7,4.9 (for the first period), and 4.10 and 4.12 (for the second period).

Then we repeated the same procedures used previously to determine standardised implied volatilities across strike prices and time. As was done previously, the results of this matrix have been drawn to produce a series of volatility smiles at different times to expiration. These graphs can be seen as Figures 9.2a, 9.2b, 9.2c, 9.2d for the stock index options. For the fixed income options, these results can be seen in Figures 9.3a, 9.3b, 9.3c and 9.3d. Finally for the Currency options, these results can be seen in Figures 9.4a, 9.4b, 9.4c and 9.4d. For each of these plots, we have included the standardised (and averaged) VSI graphs for the actual implied volatility for the entire period of analysis, the VSI graph associated with the Student-t distribution, the VSI graph associated with the best fitting stochastic volatility model and the VSI graph associated with the best combination model (Student-t distribution and stochastic volatility model). After these figures, there also appear similar graphs for the earlier portion of the data and for the latter period of the data.

These figures confirm the results suggested by the earlier research by Hull & White (1988), Stein & Stein (1991) and Heston (1993). These stochastic volatility models will increase the second order strike effect the longer the term to maturity of the option. This effect was rejected in the analysis of the last Chapter. There we demonstrated that the second order strike price effect is inversely related to the time remaining to expiration, which is exactly the opposite of what is generated by these stochastic volatility models. Thus, we would expect that the explanatory power of these models would be low when explaining the risk-neutral dynamics associates with actual options implied volatilities.

Now that we have simulated what smile behaviour would be consistent with the best stochastic volatility models for each market (assuming the underlying asset price follows GBM), we will now compare the dynamics of this model to the actual smile behaviour of the twelve option markets. To assess the effectiveness of the stochastic volatility models in explaining the empirical smile behaviour, the same analysis was completed as was done previously for the Student-t distribution model with constant variance.

9.7 TESTING THE EXPLANATORY POWER OF STOCHASTIC VOLATILITY MODELS FOR THE IMPLIED VOLATILITY PATTERNS

The regression results for the best fitting stochastic volatility models for all three periods can be found in Tables 9.6a, 9.6b and 9.6c for all twelve markets. For the sake of comparisons, the adjusted R-squared statistics from the Student-t distribution model also appear.

Markets	Intercept	β Coefficient	Standard Error	T-Statistic	Adjusted R-squared	*Student-t R-squared*
S&P 500	-383.43	4.8299	0.03968	96.520	0.5446	*0.1803*
FTSE	-253.55	3.5385	0.03691	68.775	0.5684	*0.4292*
DAX	-729.38	8.2599	0.10591	68.548	0.6873	*0.3574*
Nikkei	-202.55	2.9957	0.05277	37.819	0.4778	*0.2521*
Bund	-141.86	2.4229	0.03168	44.915	0.4149	*0.2559*
BTP	-124.50	2.2144	0.02450	49.567	0.4876	*0.2839*
Gilt	-136.75	2.3674	0.02853	47.929	0.4319	*0.2993*
T-Bond	-90.347	1.9304	0.01755	53.014	0.5595	*0.5057*
D-mark	-136.35	2.3768	0.01887	72.962	0.5887	*0.6018*
Pound	-157.03	2.5755	0.02963	53.172	0.4514	*0.6165*
Yen	-266.94	3.6728	0.02440	109.541	0.6355	*0.6969*
S-Franc	-248.91	3.4961	0.02689	92.826	0.5883	*0.5388*

Table 9.6a, Regression Results for the Predicted Smile Behaviour from the Optimal Stochastic Volatility Model assuming that Underlying asset follows GBM versus the Actual Smile Behaviour for Twelve Option Markets for the Entire Period of the Analysis.

Markets	Intercept	β Coefficient	Standard Error	T-Statistic	Adjusted R-squared	*Student-t R-squared*
S&P 500	-124.40	2.2317	0.03997	30.816	0.3955	*0.2065*
FTSE	-367.07	4.6336	0.14346	25.328	0.3668	*0.5045*
DAX	-704.57	8.0168	0.23279	30.142	0.5611	*0.2616*
Nikkei	-185.88	2.8354	0.07936	23.129	0.3906	*0.1571*
Bund	-185.29	2.8690	0.05244	35.641	0.4555	*0.3044*
BTP	-97.45	1.9622	0.03875	24.831	0.4151	*0.3902*
Gilt	-100.06	2.0095	0.06616	15.258	0.2844	*0.4542*
T-Bond	-35.149	1.3846	0.03382	11.372	0.5101	*0.4165*
D-mark	-241.59	3.4233	0.04106	59.019	0.6408	*0.5891*
Pound	-189.05	2.8917	0.04642	40.752	0.5030	*0.5482*
Yen	-269.81	3.6940	0.03447	78.155	0.7388	*0.6463*
S-Franc	-194.74	2.9563	0.03900	50.162	0.5841	*0.5077*

Table 9.6b, Regression Results for the Predicted Smile Behaviour from the Optimal Stochastic Volatility Model assuming that Underlying asset follows GBM versus the Actual Smile Behaviour for Twelve Option Markets for the First Half of the Available Observations.

Markets	Intercept	β Coefficient	Standard Error	T-Statistic	Adjusted R-squared	*Student-t R-squared*
S&P 500	-401.10	5.0080	0.04633	86.510	0.6052	*0.1754*
FTSE	-289.47	3.8916	0.04248	68.070	0.6183	*0.4074*
DAX	-1306.77	14.0226	0.19064	68.310	0.7463	*0.3598*
Nikkei	-247.76	3.4506	0.07856	31.194	0.5574	*0.3509*
Bund	-187.72	2.8751	0.05184	36.171	0.3972	*0.2224*
BTP	-246.18	3.4176	0.05222	46.296	0.4627	*0.2443*
Gilt	-150.43	2.5075	0.03426	44.002	0.4429	*0.2601*
T-Bond	-184.16	2.8529	0.02390	77.527	0.6429	*0.5165*
D-mark	-131.10	2.3280	0.02968	44.744	0.4614	*0.6231*
Pound	-152.24	2.5357	0.04045	37.965	0.4235	*0.6427*
Yen	-157.79	3.5868	0.02179	118.715	0.6121	*0.7214*
S-Franc	-165.41	2.6633	0.02885	57.653	0.5239	*0.5678*

Table 9.6c, Regression Results for the Predicted Smile Behaviour from the Optimal Stochastic Volatility Model assuming that Underlying asset follows GBM versus the Actual Smile Behaviour for Twelve Option Markets for the Second Half of the Available Observations.

Somewhat surprisingly, the stochastic volatility models have performed much better when explaining the variance of the actual implied volatility patterns compared to the Student-t model. By examining the R-squared statistics, it appears these models are better. Unfortunately, if we examine the slope coefficients, these are also higher. So this suggests that while these models are better at capturing the overall shape of the implied volatility surface, the amplitude is even more incorrect.

It is somewhat surprising that these models have a higher R-squared because the smile surfaces in the above mentioned figures for the simulated stochastic volatility models displayed more curvature the further the time to expiration and that is opposite to what we observe. Nevertheless, this result might be expected since these models were also better at explaining the objective process (see Tables 4.6, 4.9 and 4.12). If these classes of models better explain the objective processes, one might expect to also better capture the dynamics of the risk-neutral processes. Only for the Foreign exchange options (in the majority of cases) does the Student-t model explain more of the variance in the actual implied volatility patterns.

These puzzling results may be better understood by reviewing how the smile surfaces for stock index, fixed income and foreign exchange markets vary. For the actual smile surfaces, the stock index and fixed income options display both a significant negative first order strike price effect (skewness) and second order (kurtosis) strike price effect. For the foreign exchange options, the only significant strike price effect is second order (kurtosis). Given that these model perform poorly for foreign exchange options, it is reasonable that this is because the stochastic volatility models do not display sufficient excess kurtosis to capture the second order strike price effect. Indeed, this appears to be the case from the review of the figures and is consistent with previously referenced work (see Chapter 6) that suggests that the stochastic volatility models will not produce sufficiently high enough kurtosis (to be consistent with observed implied volatility surfaces). Thus, the foreign exchange options are better explained by a Student-t distribution that displays more kurtosis. Nevertheless, these models are better at capturing the overall dynamics of the volatility surface for stock index and fixed income options. The most likely reason is that they must be addressing the first order strike price effect.

From the last Chapter, we demonstrated that the time dependency of the first order strike price effect for both stock index and fixed income options was negative (see Tables 8.13a and 8.13b). This implies that the longer the time period to expiration of the option, the more the smile surface was negatively skewed. From an examination of the figures (9.2 to 9.10), we also find that the stochastic volatility models display more curvature as the time to expiration is lengthened. In this regression analysis, we are comparing the standardised implied volatilities of individual options to predicted implied volatilities from the model. If we tend to have all strike price options represented equally, then the increased curvature of the stochastic volatility models would better explain the first order dynamics of lower strike price options and would poorly explain the first order dynamics of higher strike price options. Overall, we would expect the ability to capture this effect would be poor. However, we found that lower strike price options were much more likely to be represented than higher strike price options in the observed data sets.

This being the case, it would appear that the stochastic volatility models fit the stock index and fixed income options smile surfaces better because they capture the important time dependent dynamics of this effect. Given our data set of options prices tends to have more lower strike price options compared with higher strike price options, the fact that the skewness becomes more negative with time is consistent with the dynamics of these models.

Even though these models fit the stock index and fixed income options implied volatility patterns better than the Student-t model, they still barely explain half of the variance. One reason could be that these models fail to capture the first order strike price effect for options with higher strike prices (see previous two paragraphs). Secondly, it is clear (from the figures 9.2a to 9.10d) that the general tendency of these models is to increase the curvature of the implied volatility smile the further the time to expiration (which is the opposite of what we observe empirically). Even given these factors, it does confirm what we have observed previously that implied volatilities do follow some stochastic process and these models are capturing some of these dynamics.

It is interesting to note that this analysis of the risk neutral dispersion processes is following a similar sequence to our previous research on the objective processes. At similar stages in both lines of research, we concluded that the stochastic volatility models (for the most part) were better at explaining the process than a simple Student-t distribution with constant variance. In some ways this is comforting given that we know that implied volatilities follow some stochastic process and we must (therefore) reject the Student-t model. Even so, we have an entire asset class (foreign exchange options) where the Student-t model is superior. This presents a dilemma: how can a model, which assumes constant volatility, provide a better fit than models which allow the volatility to evolve stochastically. Fortunately (as with our previous investigation into the objective processes), we have a solution. As was demonstrated in Chapter 5, a combination model of both stochastic volatility with a Student-t evolution of the underlying asset return was in the vast majority of cases the best approach. It is not unreasonable to surmise that similar dynamics might also work for the risk-neutral processes. As was suggested previously, it may be that the stochastic volatility models are required to capture the dynamics of the first order strike price effect and jumps are required to capture the second order strike price effects.

9.8 SIMULATING IMPLIED VOLATILITY DYNAMICS CONSISTENT WITH STOCHASTIC VOLATILITY & STUDENT-t DISTRIBUTION MODELS

To test this class of models, we followed the same steps as were previously taken. As before, we generated new series of 62 daily prices that were consistent with the appropriate optimised model and repeated this process 10000 times using the anti-thetic approach. To allow comparisons to be drawn, the same random numbers were used. As with the previous analysis, we examined the best fitting model (that was determined in Chapter 5) for each of the twelve markets and for the whole period, the first half and the second half of the available observations. These prices were estimated using the following formula:

$$S_t = S_{t-1} \cdot e^{\mu \cdot dt + \hat{\sigma}_{t-1} \cdot dT_1} \qquad (9.5)$$

This is the simple Euler approach used previously in Chapter 5. The term $\hat{\sigma}$, reflects the volatilities estimated from the various models tested and the previous day's volatility estimate is used to estimate today's new asset price. For this simulation, the dT_1 represents the Student-t distributions that were used for the previous simulations. This generated a price series from a starting value of 100 and we captured each daily price until 62 days forward in time. As before, we assumed the interest rate was 0% (zero), so that the drift term (μ) is expressed as: $-\frac{1}{2}\sigma^2 * 1/252$. With 10000 of these price series estimated, we then evaluated the values of the options at a range of strike prices and finally determined a series of implied volatilities from the Black 1976 model.

For each of the simulations, we examined the best combined stochastic volatility and Student-t models for each of the twelve markets and for each of the three estimation periods. One difference between these simulations and the analysis done in Chapter 5 is that previously we could only use a single Student-t distribution. In that analysis, optimisation was run both over different Student-t distributions and different parameter values for the stochastic volatility models. Thus, it could be possible that the results are due to the selection of the random numbers for each of the eight Student-t distributions. To test this, we re-ran all the previous optimisations using a Student-t distribution which matched the statistical moments of the average Student-t distributions in the Monte Carlo simulation (see the summary statistics of the Student-t moments presented earlier in this Chapter). When this was completed, we found that the results were only slightly different. Thus, we confirmed that the selection of the Student-t distribution has a minor effect on the model and these results are insensitive to the choice of the random numbers generating the Student-t distribution.

The best fitting models and the optimal parameter values can be found in Chapter 5. For the entire period, these appear in Table 5.1a, 5.1b and 5.1c. For the first period, these appear in Tables 5.6a, 5.6b and 5.6c. Finally, for the second period, these appear in Tables 5.9a, 5.9b and 5.9c.

Again, we then estimated Monte Carlo prices for call options with strike prices from 80 to 120 in 2 point increments. From these call option prices, we determined thirty-six matrices of call option prices that varied across the range of strike prices and had different times to expiration. Finally, implied volatilities were determined.

As before, to better compare the smile behaviour that is associated with these best fitting models, graphs were produced for each of the twelve markets and for all three periods. These graphs can be seen as Figures 9.2a, 9.2b, 9.2c, 9.2d for the stock index options. For the fixed income options, these results can be seen in Figures 9.3a, 9.3b, 9.3c and 9.3d. Finally for the Currency options, these results can be seen in Figures 9.4a, 9.4b, 9.4c and 9.4d. As was discussed previously, these graphs compare the smile behaviour of the actual options market to the three models we determined for the objective processed for the underlying futures markets. As before these graphs are for the entire period of analysis. The graphs for the first and second periods of the analysis period appear in subsequent figures and are clearly labelled.

From an examination of the graphs, it would appear that the combination model does appear to fit the (average) empirical smiles better. The combination of the stochastic volatility models with a Student-t evolution for the underlying asset price yields implied volatility smiles that are the closest to what we observe for the actual implied volatility smiles. We appear to be capturing the second order strike price effect well. For the actual implied volatility patterns, a curved pattern that exists for the entire period from 90 days until 5 days until expiration. As was indicated in the last two Chapters, this pattern becomes more extreme as the expiration of the option is approached. Also clear is that our combination model fails to capture the first order strike price effect. This is hardly surprising given that we did not attempt to capture this effect for the objective processes in the first portion of this research. Even though, the patterns appear to fit better, the acid test is whether the errors from these models are less than the two previous models tested.

Now that we have simulated what smile behaviour would be consistent with the best stochastic volatility models for each market (assuming the underlying asset price follows a Student-t distribution), we will now compare the dynamics of this model to the actual smile behaviour of the twelve option markets. To assess the effectiveness of the stochastic volatility models in explaining the empirical smile behaviour, the same analysis was completed as was done previously for the two earlier models.

9.9 TESTING THE EXPLANATORY POWER OF STOCHASTIC VOLATILITY & STUDENT-t MODELS FOR THE IMPLIED VOLATILITY PATTERNS

The regression results for the best fitting combination model for all three periods can be found in Tables 9.7a, 9.7b and 9.7c for the three periods of the analysis. For the sake of comparisons, the adjusted R-squared results from the previous two models also appear.

Markets	Intercept	β Coefficient	Standard Error	T-Statistic	Adjusted R-squared	*Sto-Vol R-squared*	*Student-t R-squared*
S&P 500	-59.97	1.5950	0.01483	40.121	**0.4828**	***0.5446***	***0.1803***
FTSE	-88.76	1.8755	0.01591	55.028	**0.6657**	***0.5684***	***0.4292***
DAX	-127.36	2.2979	0.05237	43.878	**0.4102**	***0.6873***	***0.3574***
Nikkei	-80.25	1.7886	0.04104	43.819	**0.3503**	***0.4778***	***0.2521***
Bund	-81.72	1.8144	0.02387	34.118	**0.4210**	***0.4149***	***0.2559***
BTP	-26.61	1.2378	0.01698	14.005	**0.3824**	***0.4876***	***0.2839***
Gilt	-47.84	1.4745	0.01729	27.443	**0.4454**	***0.4319***	***0.2993***
T-Bond	-21.58	1.2042	0.00729	28.011	**0.7414**	***0.5595***	***0.5057***
D-mark	-85.92	1.8660	0.00983	88.098	**0.7649**	***0.5887***	***0.6018***
Pound	-84.95	1.8429	0.01115	75.596	**0.7485**	***0.4514***	***0.6165***
Yen	-105.93	2.0696	0.00875	122.240	**0.8113**	***0.6355***	***0.6969***
S-Franc	-81.14	1.8213	0.01022	80.364	**0.7286**	***0.5883***	***0.5388***

Table 9.7a, Regression Results for the Predicted Smile Behaviour from the Optimal Stochastic Volatility Model assuming that Underlying asset follows a Student-t Distribution against the Actual Smile Behaviour for Twelve Option Markets for the Entire Period of the Analysis.

Markets	Intercept	β Coefficient	Standard Error	T-Statistic	Adjusted R-squared	*Sto-Vol R-squared*	*Student-t R-squared*
S&P 500	-25.12	1.2350	0.02310	10.173	**0.3749**	***0.3955***	***0.2065***
FTSE	-164.99	2.6182	0.06360	25.443	**0.4849**	***0.3668***	***0.5045***
DAX	-178.53	2.7942	0.13669	13.126	**0.3102**	***0.5611***	***0.2616***
Nikkei	-26.00	1.2384	0.04118	5.789	**0.3123**	***0.3906***	***0.1571***
Bund	-81.32	1.8107	0.02917	27.792	**0.5185**	***0.4554***	***0.3044***
BTP	-74.93	1.7315	0.02777	26.341	**0.5184**	***0.4151***	***0.3902***
Gilt	-39.10	1.3783	0.02705	13.985	**0.5282**	***0.2844***	***0.4542***
T-Bond	5.57	0.9446	0.01489	-3.721	**0.7144**	***0.5101***	***0.4165***
D-mark	-88.04	1.8833	0.01367	64.616	**0.8296**	***0.6408***	***0.5891***
Pound	-73.99	1.7310	0.01631	44.819	**0.7461**	***0.5030***	***0.5482***
Yen	-97.98	1.9837	0.01528	64.378	**0.8058**	***0.7388***	***0.6463***
S-Franc	-74.01	1.7753	0.02323	33.375	**0.5881**	***0.5841***	***0.5077***

Table 9.7b, Regression Results for the Predicted Smile Behaviour from the Optimal Stochastic Volatility Model assuming that Underlying asset follows a Student-t Distribution against the Actual Smile Behaviour for Twelve Option Markets for the First Half of the Available Observations.

Markets	Intercept	β Coefficient	Standard Error	T-Statistic	Adjusted R-squared	*Sto-Vol R-squared*	*Student-t R-squared*
S&P 500	-233.12	3.4670	0.04219	58.474	0.4698	*0.6052*	*0.1754*
FTSE	-135.56	2.3633	0.02472	55.150	0.6384	*0.6183*	*0.4074*
DAX	-133.63	2.3724	0.06702	20.477	0.4051	*0.7463*	*0.3598*
Nikkei	-126.59	2.2481	0.05632	22.161	0.5098	*0.5574*	*0.3509*
Bund	-52.82	1.5248	0.03305	15.879	0.3131	*0.3972*	*0.2224*
BTP	-94.19	1.9057	0.03036	29.832	0.4420	*0.4627*	*0.2443*
Gilt	-72.21	1.7243	0.02345	30.887	0.4452	*0.4429*	*0.2601*
T-Bond	-109.67	2.1214	0.01570	71.427	0.6974	*0.6429*	*0.5165*
D-mark	-78.39	1.7840	0.01162	67.470	0.7665	*0.4614*	*0.6231*
Pound	-90.14	1.8898	0.01433	62.094	0.7650	*0.4235*	*0.6427*
Yen	-104.19	2.0436	0.00971	107.477	0.8322	*0.6121*	*0.7214*
S-Franc	-91.92	1.9219	0.01303	70.752	0.7374	*0.5239*	*0.5678*

Table 9.7c, Regression Results for the Predicted Smile Behaviour from the Optimal Stochastic Volatility Model assuming that Underlying asset follows a Student-t Distribution against the Actual Smile Behaviour for Twelve Option Markets for the Second Half of the Available Observations.

Generally, we find that the combination model is much better at explaining the variance of the actual implied volatility patterns. If our criterion is R-squared, in most cases, this model is best. In twenty-three of the thirty-six cases, the combination model has the highest adjusted R squared (and thus the minimised error). All cases where this model is not the best, the pure stochastic volatility model was slightly superior. Regarding our other criterion, the slope coefficients are now much closer to one. Thus, we conclude that these models are better than the stochastic volatility models at capturing the amplitude of the empirical implied volatility surfaces. Of the three asset classes, the model best explains the behaviour of the foreign currency options. In many cases, the explained variance is above 75%. For the other asset classes, the explanatory power is probably being reduced given that the implied volatility patterns for these markets have significant first order strike price effects we are not capturing. Thus, it would appear that the pure stochastic volatility model is better at capturing the skewness effect found in the stock index and fixed income options. Nevertheless, it appears some class of stochastic volatility models best explains both the objective and risk-neutral processes for these twelve markets.

9.10 COMPARISON OF THE THREE MODELS FOR EXPLAINING THE DYNAMICS OF THE IMPLIED VOLATILITY SMILES

Comparison by Minimised Least Squares

Now that we have completed the least squares analysis for each market and for all three time periods, we can summarise our results. From Tables 9.7a, 9.7b and 9.7c, we have all the adjusted R-squared statistics for the three candidate models. These results are summarised in Table 9.8a for the four stock index options, in Table 9.8b for the four fixed income options and in Table 9.8c for the four foreign exchange options. We have also included a column that indicates which model is best.

Markets (PERIOD)	*Combo R-squared*	*Sto-Vol R-squared*	*Student-t R-squared*	*BEST MODEL*
S&P 500				
Whole Period	0.4828	**0.5446**	0.1803	***Stochastic Volatility***
First Period	0.3749	**0.3955**	0.2065	***Stochastic Volatility***
Second Period	0.4698	**0.6052**	0.1754	***Stochastic Volatility***
FTSE				
Whole Period	**0.6657**	0.5684	0.4292	***Combination***
First Period	0.4849	0.3668	**0.5045**	***Student-t***
Second Period	**0.6384**	0.6183	0.4074	***Combination***
DAX				
Whole Period	0.4102	**0.6873**	0.3574	***Stochastic Volatility***
First Period	0.3102	**0.5611**	0.2616	***Stochastic Volatility***
Second Period	0.4051	**0.7463**	0.3598	***Stochastic Volatility***
Nikkei				
Whole Period	0.3503	**0.4778**	0.2521	***Stochastic Volatility***
First Period	0.3123	**0.3906**	0.1571	***Stochastic Volatility***
Second Period	0.5098	**0.5574**	0.3509	***Stochastic Volatility***

Table 9.8a, Comparisons of the Adjusted R-Squares Statistics for the Three Possible Models to explain the Dynamics of the Implied Volatility Smiles for Four Stock Index Options.

Combo Markets (PERIOD)	Sto-Vol R-squared	Student-t R-squared	BEST R-squared	MODEL
Bund				
Whole Period	**0.4210**	0.4149	0.2559	*Combination*
First Period	**0.5185**	0.4554	0.3044	*Combination*
Second Period	0.3131	**0.3972**	0.2224	*Stochastic Volatility*
BTP				
Whole Period	0.3824	**0.4876**	0.2839	*Stochastic Volatility*
First Period	**0.5184**	0.4151	0.3902	*Combination*
Second Period	0.4420	**0.4627**	0.2443	*Stochastic Volatility*
Gilt				
Whole Period	**0.4454**	0.4319	0.2993	*Combination*
First Period	**0.5282**	0.2844	0.4542	*Combination*
Second Period	**0.4452**	0.4429	0.2601	*Combination*
T-Bond				
Whole Period	**0.7414**	0.5595	0.5057	*Combination*
First Period	**0.7144**	0.5101	0.4165	*Combination*
Second Period	**0.6974**	0.6429	0.5165	*Combination*

Table 9.8b, Comparisons of the Adjusted R-Squares Statistics for the Three Possible Models to explain the Dynamics of the Implied Volatility Smiles for Four Fixed Income Options.

Markets (PERIOD)	Combo R-squared	Sto-Vol R-squared	Student-t R-squared	BEST MODEL
D-mark				
Whole Period	**0.7649**	0.5887	0.6018	*Combination*
First Period	**0.8296**	0.6408	0.5891	*Combination*
Second Period	**0.7665**	0.4614	0.6231	*Combination*
Pound				
Whole Period	**0.7485**	0.4514	0.6165	*Combination*
First Period	**0.7461**	0.5030	0.5482	*Combination*
Second Period	**0.7650**	0.4235	0.6427	*Combination*
Yen				
Whole Period	**0.8113**	0.6355	0.6969	*Combination*
First Period	**0.8058**	0.7388	0.6463	*Combination*
Second Period	**0.8322**	0.6121	0.7214	*Combination*
S-Franc				
Whole Period	**0.7286**	0.5883	0.5388	*Combination*
First Period	**0.5881**	0.5841	0.5077	*Combination*
Second Period	**0.7374**	0.5239	0.5678	*Combination*

Table 9.8c, Comparisons of the Adjusted R-Squares Statistics for the Three Possible Models to explain the Dynamics of the Implied Volatility Smiles for Four Foreign Exchange Options.

From these results, we can confirm that in almost all cases a stochastic volatility model is required to best understand the implied volatility surface. For the stock index options, a simple stochastic volatility model tends to explain the highest variance. However, for the FTSE-100, the combination is better for the overall period and for the second period and for the first period the Student-t model is the (only instance) best model for the FTSE-100. This is not surprising when we consider that the FTSE-100 has the highest second order strike price effect for the period. This can be seen in Table 8.2a where the coefficient (β_8) for the STRIKE^2 was highest for the FTSE-100 and in Table 8.14 where the coefficient (β_{49}) for the FTSE* STRIKE^2 also indicated a higher level of this effect. Thus, the Student-t distribution may required to address this effect.

For the four fixed income options, the combination model also tends to be the best performing model (in terms of variance explained). The only exception is for the BTP (and the Bund in the second period). This could be due to the fact that the BTP has the highest degree of negative skewness relative to the other fixed income options. This effect can be seen in Table 8.13b, where the coefficient (β_{30}) for the BTP* STRIKE variable demonstrates this fact. What is interesting is that it appears that the simple stochastic volatility may perform better when a significant negative skew exists. However, the combination model is best when the second order strike price effect (kurtosis) is dominant.

Finally, for the foreign exchange options, the combination model is in all cases the best model. Fortunately, this means that we have a model that is consistent with the fact that we have a stochastic volatility process. Nevertheless, as was shown for the objective processes, the simple stochastic volatility models did not produce enough excess kurtosis relative to what we observe in the markets. A comparison of Figures 7.14a, 7.14b and 7.14c demonstrates that the foreign exchange options tend to have the highest levels of implied kurtosis. Thus, it is hardly surprising that the combination model captures this effect well when the kurtosis effect is consistently dominating the skewness effect.

Comparison by Averaged Differences in Predicted and Actual Results

While the previous comparisons may be valid if our sole concern is minimised sum of squared errors, we may be drawing the wrong conclusions. Clearly, this test is relevant as we wish to minimise the errors from the models we predict. However, it is important to recognise that the R-squared of the regression measures the fit after adjusting the intercept and rescaling the effect of the predicted variable. Ideally, we would wish to obtain results with an intercept centred at the origin and a Beta coefficient of 1.0. For almost all the regression results, the intercepts are significantly different than zero (and negative) and the slope coefficients are significantly different than 1.0 (and greater).

This suggests that although our models display a similar implied volatility surface, the scaling is wrong. Thus, we would be able to capture the shape of the implied volatility surface but would be unable (a priori) to accurately predict the actual observed implied volatilities. Only after the estimation of the regressions would we know that these predictions would require rescaling to be used.

To better assess how well these models perform, another test was done that examined how different the raw predictions are compared to the observed standardised implied volatilities. This test took the averaged (absolute) differences between the predicted and observed observations. Specifically, we estimated this test using the following formula:

$$\frac{1}{N} \cdot \sum_{i=1}^{N} \sqrt{\left(VSI_{pred} - VSI_{act}\right)^2} \qquad (9.5)$$

With this test, the resulting statistic will indicate the degree of divergence between the observed and predicted standardised volatilities and for our purposes, the best fitting model has the lowest average deviation. The results of this test for the twelve markets and for the three periods of analysis appear in Tables 9.9a (for the stock index options), 9.9b (for the fixed income options) and 9.9c (for the four foreign exchange options). In these tables the column heading for the test statistic appears as RMSE.

Markets (PERIOD)	*Combo RMSE*	*Sto-Vol RMSE*	*Student-t RMSE*	*BEST MODEL*
S&P 500				
Whole Period	**22.053**	27.797	29.234	***Combination***
First Period	**19.210**	21.263	23.893	***Combination***
Second Period	**29.486**	30.078	32.125	***Combination***
FTSE				
Whole Period	**16.495**	21.216	20.953	***Combination***
First Period	**12.125**	13.580	12.555	***Combination***
Second Period	**20.488**	23.429	23.169	***Combination***
DAX				
Whole Period	**16.964**	18.629	17.411	***Combination***
First Period	**11.001**	11.437	11.241	***Combination***
Second Period	**19.639**	22.192	19.807	***Combination***
Nikkei				
Whole Period	**15.677**	16.179	17.127	***Combination***
First Period	**14.252**	15.354	16.329	***Combination***
Second Period	**16.358**	17.670	18.111	***Combination***

Table 9.9a, Comparisons of An Average Difference Test for the Three Possible Models to explain the Dynamics of the Implied Volatility Smiles for Four Stock Index Options.

Markets (PERIOD)	*Combo RMSE*	*Sto-Vol RMSE*	*Student-t RMSE*	*BEST MODEL*
Bund				
Whole Period	**10.658**	11.532	12.208	***Combination***
First Period	**10.210**	12.319	12.550	***Combination***
Second Period	**10.689**	11.635	11.939	***Combination***
BTP				
Whole Period	**13.106**	13.490	14.998	***Combination***
First Period	**10.109**	11.159	11.669	***Combination***
Second Period	**14.988**	16.472	16.746	***Combination***
Gilt				
Whole Period	**8.811**	10.205	10.700	***Combination***
First Period	**7.244**	9.598	8.854	***Combination***
Second Period	**9.617**	10.681	11.266	***Combination***
T-Bond				
Whole Period	**12.533**	20.285	22.326	***Combination***
First Period	**10.671**	15.876	19.920	***Combination***
Second Period	**20.223**	22.801	22.784	***Combination***

Table 9.9b, Comparisons of An Average Difference Test for the Three Possible Models to explain the Dynamics of the Implied Volatility Smiles for Four Fixed Income Options.

Markets (PERIOD)	Combo RMSE	Sto-Vol RMSE	Student-t RMSE	BEST MODEL
D-mark				
Whole Period	8.128	10.121	10.055	*Combination*
First Period	8.557	12.214	11.663	*Combination*
Second Period	7.174	9.921	9.064	*Combination*
Pound				
Whole Period	8.912	12.132	11.030	*Combination*
First Period	7.228	10.368	9.723	*Combination*
Second Period	9.662	13.474	11.879	*Combination*
Yen				
Whole Period	10.806	14.175	12.617	*Combination*
First Period	10.630	14.069	13.209	*Combination*
Second Period	10.242	12.932	12.339	*Combination*
S-Franc				
Whole Period	7.695	10.270	9.256	*Combination*
First Period	9.875	10.929	10.621	*Combination*
Second Period	7.227	9.227	8.446	*Combination*

Table 9.9c, Comparisons of An Average Difference Test for the Three Possible Models to explain the Dynamics of the Implied Volatility Smiles for Four Foreign Exchange Options.

The interpretation of the results is somewhat simplified given that the VSI methodology has indexed volatilities to 100. One could roughly interpret the deviations as the average percentage error between the predicted and actual standardised volatilities. Clearly, this error is highest for the stock index options. This varies between 11% to 30%. The most probable reason is these models cannot capture the negatively skewed first order strike price effect. The errors for the fixed income options are in the range of 7% to 30%. Again, this is most probably due to a similar reason. For the foreign exchange options, the errors are between 7% and 11%. As with the previous test, it appears that for markets where the second order strike price is dominant, our models perform best.

Overall, this testing approach suggests that in all cases the combination model is the best alternative for capturing the dynamics of the actual implied volatility dynamics. While it is clear that significant prediction errors exist, it appears that the best models for explaining the objective processes are also best in capturing the dynamics in the implied volatility surface.

Implications of the Results

At this point, it is useful to consider the implications of these results. We have already demonstrated that the implied volatility process is stochastic and we would therefore expect a stochastic volatility model to capture some of this effect. Unfortunately, even our best fitting stochastic volatility and combination models fail to explain all the variance in the implied volatility surface. The most likely reason for this is that the implied volatilities represent a *risk-neutral* surface and our optimal combination models represent an *objective* surface. Only under certain situations would we expect these to be identical.

The existence of skews in the risk-neutral processes that are not justified by the historical processes suggests that other factors are at work. As was suggested previously, this could be due to a risk premium associated with the occurrence of negative jumps or even transaction costs. Nevertheless, there are also important consistencies between the two approaches to modelling the processes.

In Chapter 5, we showed that the optimal combination model was superior to the optimal stochastic volatility in twenty-nine of the thirty-six comparisons. Furthermore, in all cases, some stochastic volatility model was superior in describing the objective process compared with the Student-t model. Similar results were obtained in this Chapter for the risk-neutral process. Of the thirty-six cases, only one was better described by the Student-t model. In the remaining thirty-five cases, the combination model was superior in twenty-three cases (if our criterion is R-squared). In the case that our criterion is based upon the average deviation in the predicted and actual standardised implied volatilities, in all cases the combination model was superior.

Thus, it would appear that both processes could best be understood with some form of a combination model including stochastic volatility and jumps. These results imply that the implied binomial tree methodology suggested by Rubinstein (1992), Derman and Kani (1994) and Dupire (1992,1994) is mis-specified. These approaches assume that the underlying price process follows some diffusion process and this is inconsistent with the existence of stochastic volatility and jumps that we have indicated exist in the implied volatility surfaces.

Even so, it must be recognised that the implied volatility surfaces display more skewness and extreme curvature than is consistent with the objective processes. Thus, only a portion of the features of the implied volatility surface are explained by knowledge of the objective processes.

Nevertheless, this result is consistent with previous research on the risk-neutral processes associated with options prices. Bates (1996) found similar results when he derived a model for pricing American options on stochastic volatility/jump-diffusion processes under systematic jump and volatility risk. He estimated the parameters implicit in Deutsche mark (DM) options for his model and tested various other models for the period 1984 to 1991 using a non-linear generalised least squares approach. Finally, these models were tested for consistency with $/DM futures prices and the implicit volatility sample path. He also found that a GBM stochastic volatility model could not explain the 'volatility smile' evidence of implicit excess kurtosis, except under parameters implausible given the time series properties of implicit volatilities. He concluded that Jump fears were necessary to explain the smile.

Other research has also pointed to a similar conclusion. Jones (1984) has also argued that option pricing models should be derived that allow for both large and small asset-price changes, given the empirical evidence of such jumps.

Unfortunately, our findings present a challenge for further research. Firstly, it can be extremely difficult to incorporate jumps (which will no longer indicate that options can be priced using the preference free assumptions) into option pricing. Derman (1996) indicated that Fischer Black had suggested this problem. Black indicated that any model which described volatility smiles must include jumps. Derman states "...when Iraj Kani and I were working on models of the volatility smile, Fischer [Black] always insisted that anything we did would be unsatisfactory as long as we ignored the effect of the expectation of jumps on the implied distribution of the index, no matter how difficult that might be. No excuse about the difficulty of doing so, or the fact that jump models were usually not preference-free, placated him, because jumps were there." (page 22)

Thus, this research points ominously to the fact that many of the standard tools required for the determination of contingent claims may not be adequate in the presence of jumps. Given most of the best fitting models for both the objective and risk-neutral processes include Student-t distributions (which are a surrogate for such jumps), it is clear that as Black would no doubt say, these must be included to properly understand the nature of the dispersion processes of markets.

9.11 CONCLUSIONS

In this Chapter, we examined implied volatility surfaces that are consistent with models that explain the behaviours of the objective processes. By employing a Monte Carlo simulation technique, we estimated call options with a variety of strike prices. The simulations assumed that the underlying price process either followed a Student-t distribution, a GBM process with stochastic volatility, or a combination of the two.

Our findings were broadly consistent with those obtained for the objective processes. All markets display stochastic volatility and therefore require stochastic volatility models to understand their dynamics. Furthermore, jump processes are equally important and must be incorporated to fully understand the behaviour of the volatility surfaces. Depending upon the criteria used for the determination of the best models, the combination of stochastic volatility and jump process models is optimal.

These models were especially effective for explaining the implied volatility surface for foreign exchange options. Nevertheless, they were less effective for stock index and fixed income options. These models fail to explain the negative skewness effect for these asset classes. The reason is simple: such extreme negative skewness is not significant in the objective processes so our models do not capture this.

These findings suggest that significant divergences exist between the objective process and the risk neutral process implied by options prices. While the expected dispersion process for the underlying asset is a major determinant of option prices, market imperfections and perhaps risk premia seem be present. An alternative explanation is that we have assumed the simplest possible structure for risk premia and it is possible that this actual process may be more complex. Potential market imperfections could include transaction costs, the discrete nature of underlying and options prices or the lack of continuous trading in markets.

It is left for future research to understand the nature of these differences in the objective and risk neutral processes. However, this research does point the way forward. This research has developed a clear methodology for the determination of the key conditions in the objective process that impact options values. This allows us to determine smile surfaces consistent with these processes. One could surmise (if the structure of the risk premium is as simple as we have assumed) the difference between the objective surface and the empirical implied volatility surface could then be examined to understand the reasons.

Of all these effects, the two major effects that must be examined are the negative skewness (implied in the actual implied volatility surface) and the difference in amplitude in the curvature. Research must examine the nature of these effects. It could very well be that the skewness effect is due to concerns about negative shocks or jumps. Given that such events are unhedgable, then the implied volatilities would include a risk premium for this event. The extreme amplitude in the risk-neutral volatility surface could be due to a number of complicated factors. These could include transaction costs, the fact that option sellers will not offer options at a price below some absolute minimum or other structural reasons. Nevertheless, because this research has been able to strip out the volatility surface explained by the objective processes, this future research will be considerably easier.

CHAPTER TEN
CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

Our initial objective was to understand the dynamics of the objective processes for twelve financial futures markets. Our first conclusion is that the objective processes for all twelve markets do not conform to the assumption of lognormality. Although we reject that the objective processes for these markets are stationary, we conclude that the objective processes display similar dynamics over different time periods.

The second objective of this research was to determine which models best explain the dynamics of the objective process. We tested three classes of models: a jump diffusion model (simulated by a Student-t distribution) with constant variance, three stochastic volatility models (assuming the underlying price innovation followed Geometric Brownian motion) and a model combining jump diffusion with stochastic volatility. We found that for each market and for every period, a jump diffusion model was superior to a Geometric Brownian motion model. For the stochastic volatility models (assuming GBM price innovation), in thirty-four of the thirty-six cases, these models were superior to the jump diffusion model. Finally, for the combination model, we found that in all cases this combination model was superior to a jump diffusion model with constant variance. In addition, the combination model was superior to the (GBM) stochastic volatility models in twenty-nine of the thirty-six cases.

In conclusion, we found that for all markets, to understand the objective process, one must allow volatility to be stochastic. Secondly, the evidence suggests that a jump diffusion process also describes the objective process. For some markets, solely knowing the stochastic volatility process is sufficient to capture the dynamics of the objective dispersion process. However, for the vast majority of the markets (and time periods) under investigation, both a jump diffusion process and stochastic volatility interact in defining the behaviour of the objective process.

This research then investigated the risk-neutral dispersion processes implied by options prices on these same twelve markets. The breadth and scope of this research goes beyond what has previously appeared in the literature. This was done by examination of the implied volatilities across strike prices and time (generating an implied volatility surface). To allow comparison of these processes within and between markets, a method for standardisation was determined. This standardisation allowed implied volatility surfaces to be drawn and directly compared. We conclude that a significant degree of consistency exists for the implied volatility surfaces for the same markets over all periods of analysis. Consistency is defined as displaying a similar shape and similar dynamics as a function of time. By assuming that the dynamics of the implied volatility surface at a single point in time could be captured as a quadratic function of the strike price, we were able to split the strike price biases into first order and second order effects. We also observed consistency in both of these effects for the same markets.

An important result is that we also observed similarities in the implied volatility surfaces for markets in the same asset class. The stock index, fixed income and foreign exchange options all display similar first order and second order strike price effects. Given consistency exists, this suggests that similar mechanisms may be causing the strike price biases within asset classes. We then estimated functional forms for the implied volatility surface using an analysis of covariance approach. We conclude that while the implied volatility dynamics are clearly different for different markets, there are a number of factors that affect all options markets in a similar way. We observe similar effects both within each asset class and for all markets.

Specifically, it appears that all markets share a similar degree of absolute kurtosis and that the time dependency of this expected kurtosis is similar. This is somewhat surprising given the observed kurtosis found in the objective return series differed across the markets. The expected kurtosis rises as the expiration of the option is approached. For all markets, strike price effects (both first and second order) are inversely related to the level of the expected variance. An increase in the at-the-money volatility serves to increase a negative skew in the implied volatility pattern and decreases the curvature of the implied kurtosis in the pattern.

These results suggest that the dynamics impacting the objective dispersion processes may also be affecting the risk-neutral dispersion processes. The existence of a consistent negative (skewed) first-order strike price effect might suggest that market participants are concerned about negative jumps. However, we did not observe a significant negative skew effect for the unconditional return series. Only two markets (and for a single period) of our twelve markets had significant negative skewness statistics in the unconditional return series. Yet, for all the stock indexes and fixed income options a negative (skewed) first-order strike price effect was found. Thus, it would appear that even though such a negative skew is not necessary warranted by the objective dispersion process, market participants consistently price options as if it were. We conclude that this effect is consistent with expectations of negative jumps. Therefore, it is consistent with the conclusions from modelling the objective dispersion processes: a jump-diffusion process is required. Furthermore, the time-dependency of the negative skew suggested that if such a jump diffusion process existed, the volatility for this process was stochastic. Finally, we established that the implied volatility process is itself stochastic. Thus, for both the objective and risk neutral dispersion processes both jump diffusion and stochastic volatility models are required to capture the dynamics. This leads directly into the final Chapter where we assessed the relative contribution each of these models to understanding the dynamics of the risk-neutral processes.

The final objective of this research was to examine the relative contributions of jump diffusion and stochastic volatility models to the dynamics of the risk neutral processes and to assess the relationship between the objective and risk neutral processes. This was achieved using a Monte Carlo simulation to determine implied volatility surfaces that were consistent with models that capture the dynamics of the objective processes. Our findings were broadly consistent with those obtained for the objective processes. All markets display stochastic volatility and therefore require stochastic volatility models to understand their dynamics. Furthermore, jump processes are equally important and must be incorporated to fully understand the behaviour of the volatility processes. Depending upon the criteria used for the determination of the best models, the combination of stochastic volatility and jump process models are optimal.

These models were especially effective for capturing the dynamics of the implied volatility surface for foreign exchange options. Even so, it became clear that important discrepancies existed. By employing minimised least squared errors (OLS regression) as one of our criterion, we were able to understand some of the divergences between the objective and risk neutral processes. We observed that the slope coefficients for the regression between the objective and empirical implied volatilities were consistently above one. This suggests that while the optimal objective models will produce (qualitatively) the right shape, the empirical volatility surfaces display greater amplitude.

Furthermore, we observed that the optimal objective models were somewhat less effective for stock index and fixed income options (displaying further discrepancies). These models fail to explain the negative skewness effect for these asset classes. One reason for this is that negative skewness was not observed for the unconditional return series of most of the markets under investigation. This suggests that significant divergences exist between the objective dispersion processes and the risk neutral dispersion processes implied by options prices. While the expected dispersion process for the underlying asset is a major determinant of option prices, market imperfections and perhaps risk premia seem be present. An alternative explanation is that we have assumed the simplest possible structure for risk premia and it is possible that this actual process may be more complex. What is clear is that understanding the objective process does explain a majority of the dynamics of the implied volatility surface. What remains for further research is to understand the nature of the residual dynamics in the implied volatility surface. This may come from examination of a more complex risk premia functional form or from market imperfections. Such imperfections could include transaction costs, the discrete nature of underlying and options prices or the lack of continuous trading in markets. Furthermore, since negative jumps and stochastic volatility are known to exist and these factors are not currently spanned by instruments in the capital markets, risk premia may exist which effect options prices.

This research points in a number of directions. The most obvious path to take is the understand the nature of the risk premium implied by the difference between the objective and risk neutral volatility surfaces. One approach would be to test whether the close relationship we have identified would allow profitable trading opportunities. Of particular interest would be an examination of the risk reward trade-offs from such a strategy. It would also be interesting to assess the impacts of transaction costs

Another line of research would be to examine the time series consistencies of the objective and risk neutral processes. Specifically, it would interesting to assess the time series behaviour of the at-the-money implied volatilities and compare these to the dynamics of implied volatilities consistent with the objective processes. This research would need to filter the actual implied volatilities to correct for noise and should estimate the true variability of the time series.

Clearly one implication that both the objective and risk neutral processes include both jump processes and stochastic volatility is that the hedging of contingent claims must be re-examined. Tompkins (1997) examined the impact on hedging costs for both simple European calls and a range of Exotic options assuming volatility is stochastic. What should be done is to examine the costs of dynamic hedging contingent claims when both stochastic volatility and jumps are included. This research will provide future researchers an insight into what models would be consistent for such analysis.

Finally, a number of papers have appeared in the literature that suggest that the diffusion process can be captured by knowledge of the implied volatility surface. We have shown that this approach is mis-specified. The first evidence is that the smile surface is not static but tends to remain centred at the forward price of the underlying asset. Secondly, these models assume a diffusion process. Since we have demonstrated that both the objective and risk neutral processes include stochastic volatility and jumps, this is clearly incorrect. What would be valuable would be to rework these models to be consistent with the processes suggested from this research.

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Security Price Process Models: Do These Have the Correct Properties For Understanding Options Values?

ROBERT GEORGE TOMPKINS

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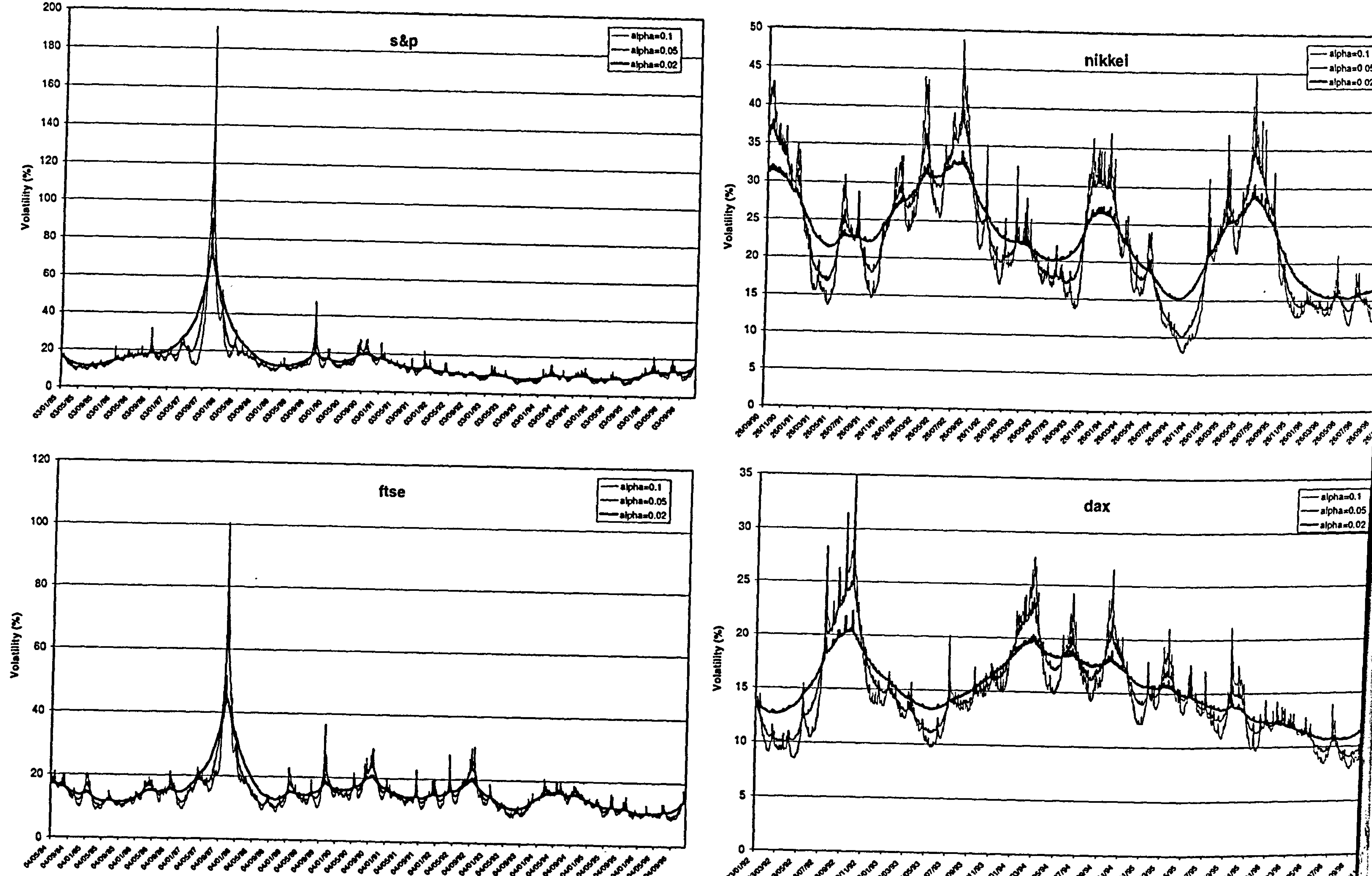


Figure 2.1a Exponentially Weighted Unconditional Volatility for Four Stock Index Futures

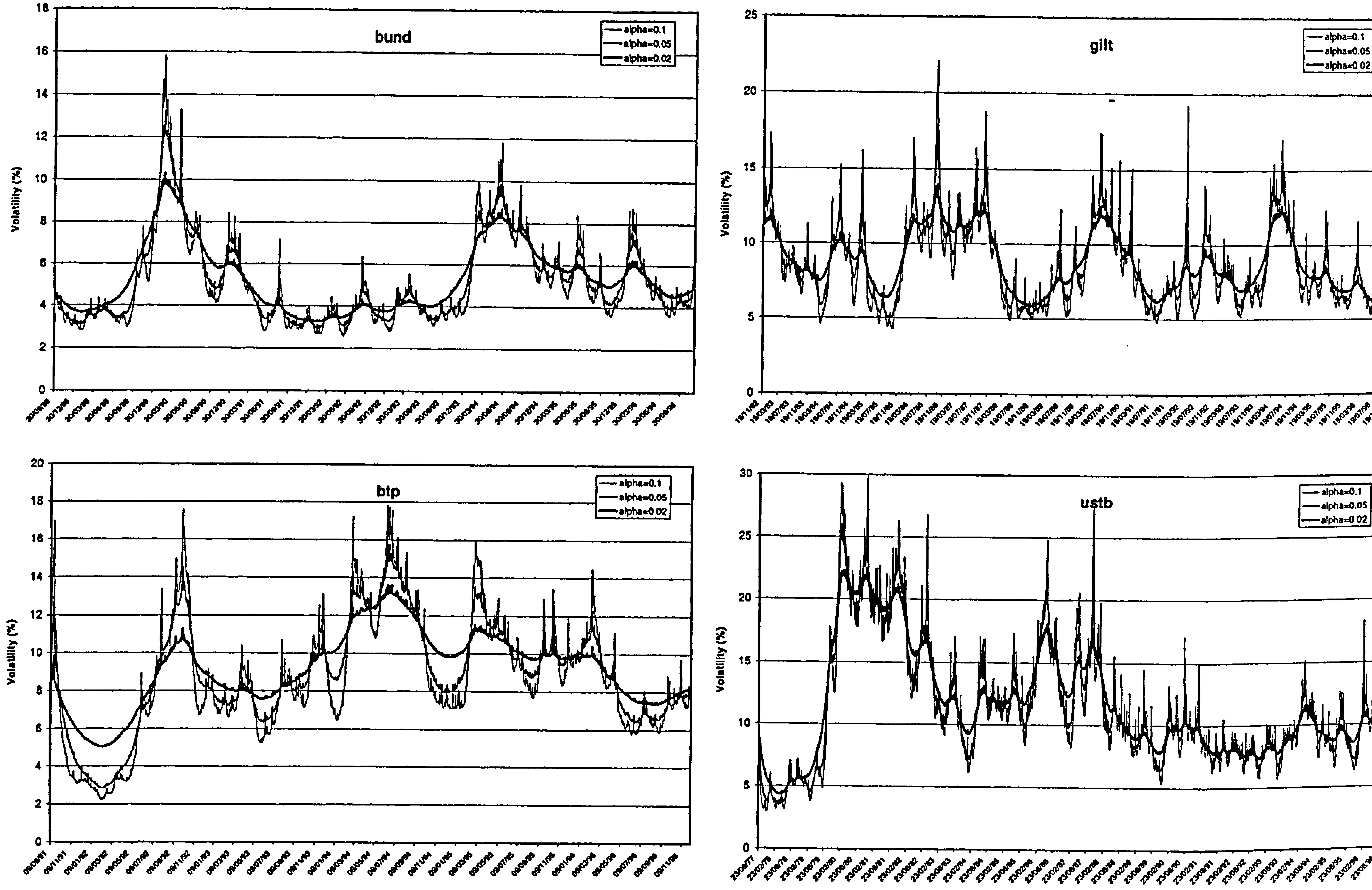


Figure 2.1b Exponentially Weighted Unconditional Volatility for Four Fixed Income Futures

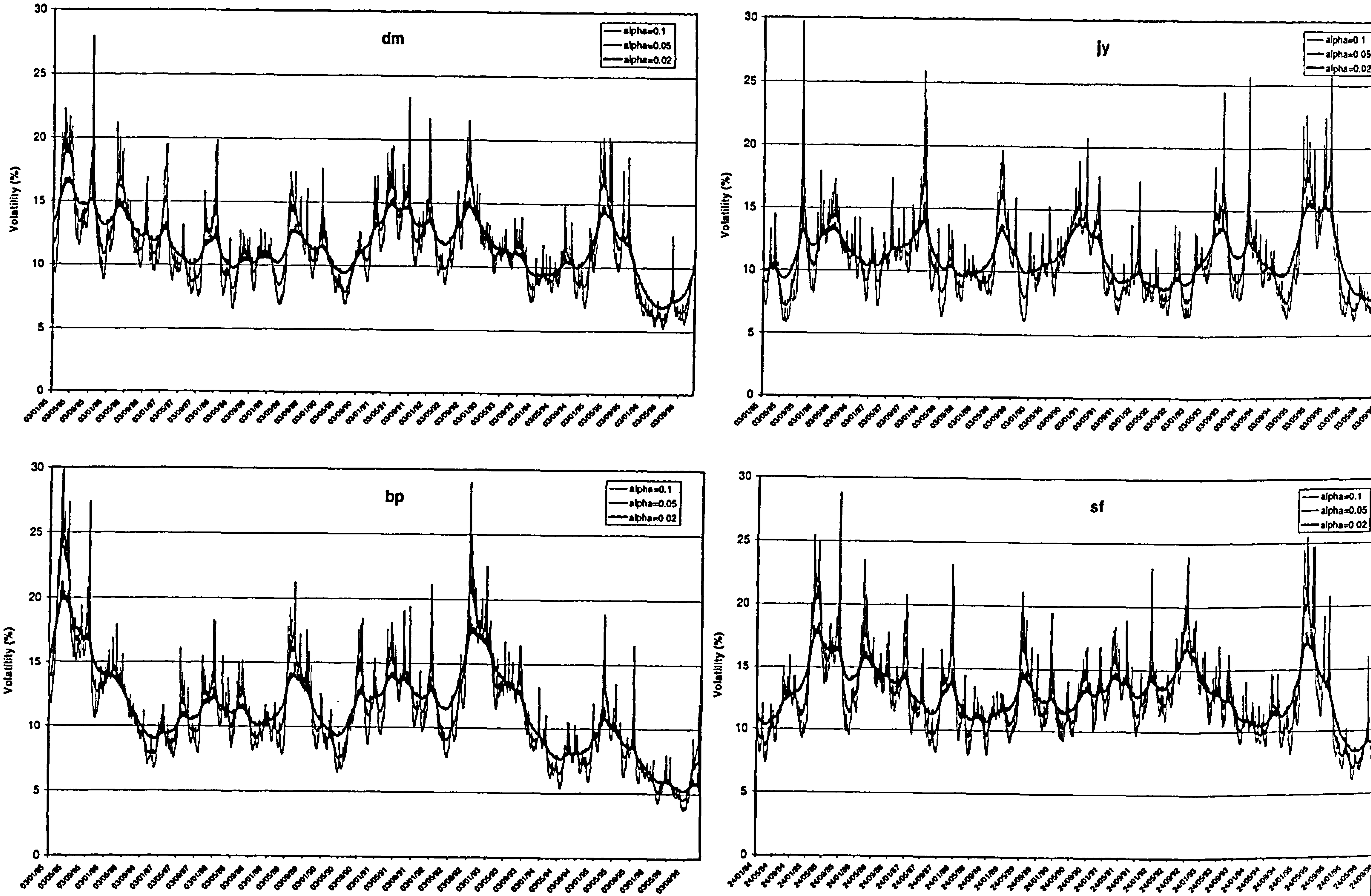
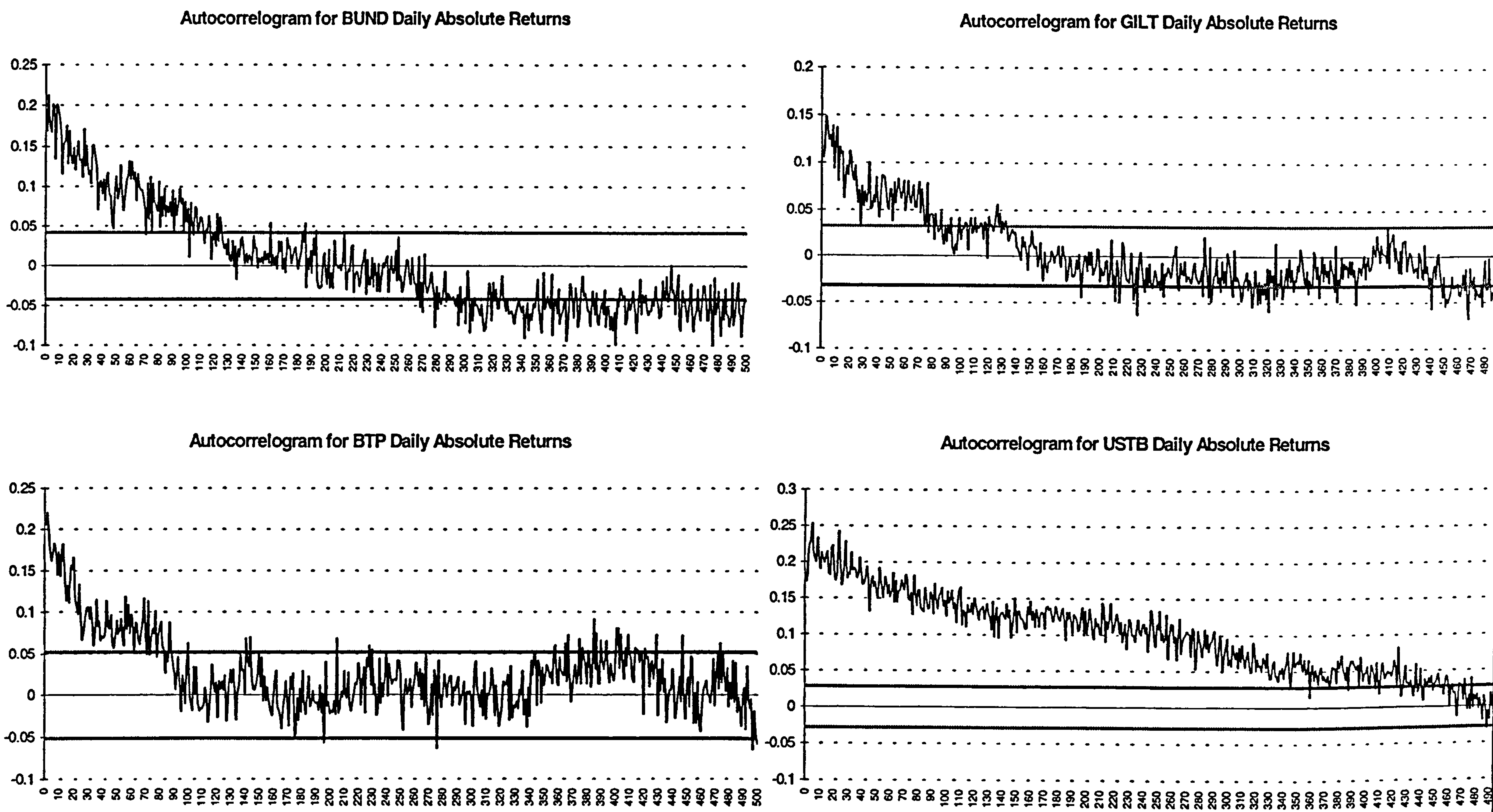


Figure 2.1c Exponentially Weighted Unconditional Volatility for Four Foreign Exchange Futures

Figure 2.2a Autocorrelogram for four Stock Index Futures absolute daily returns.



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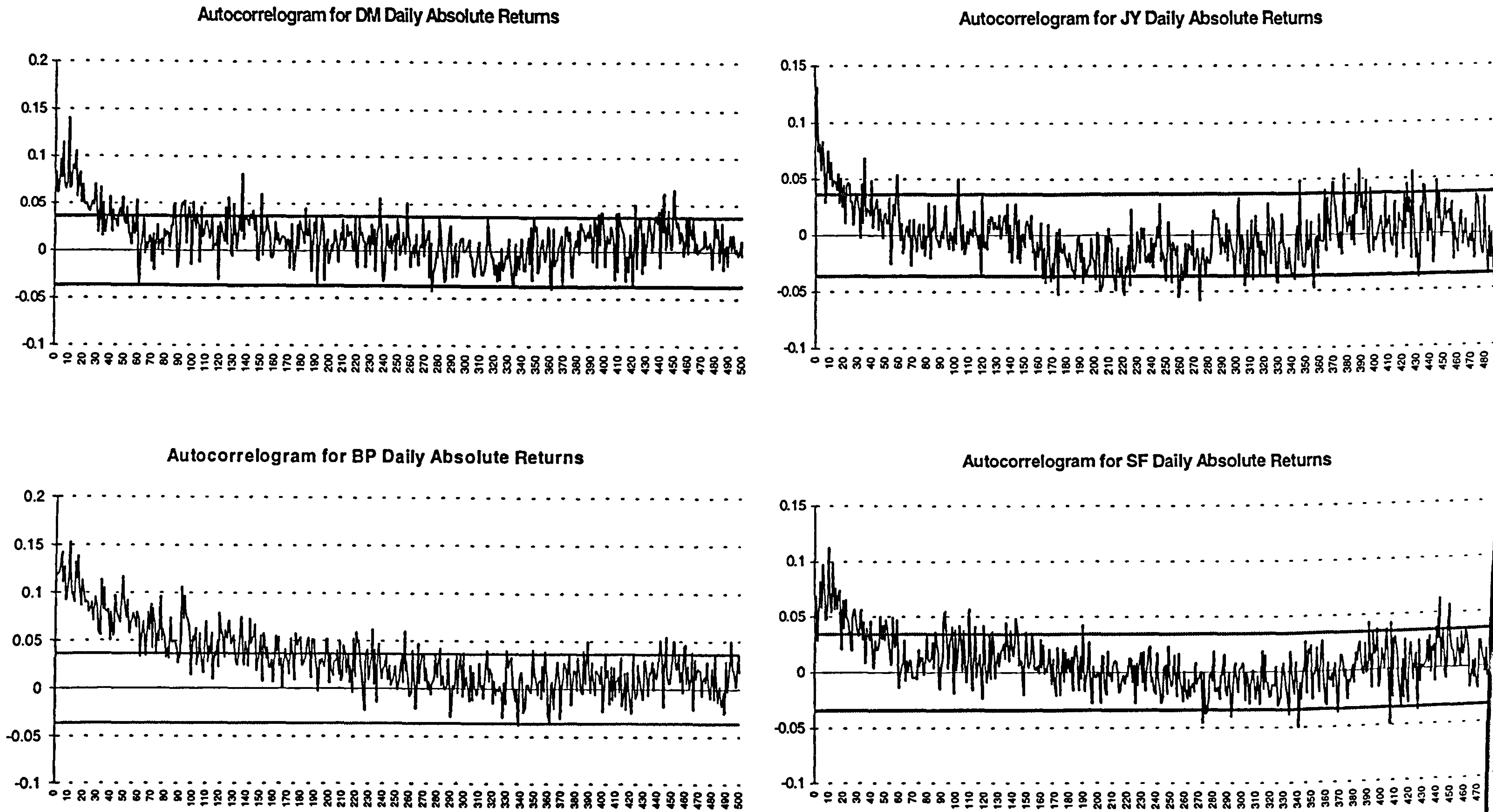

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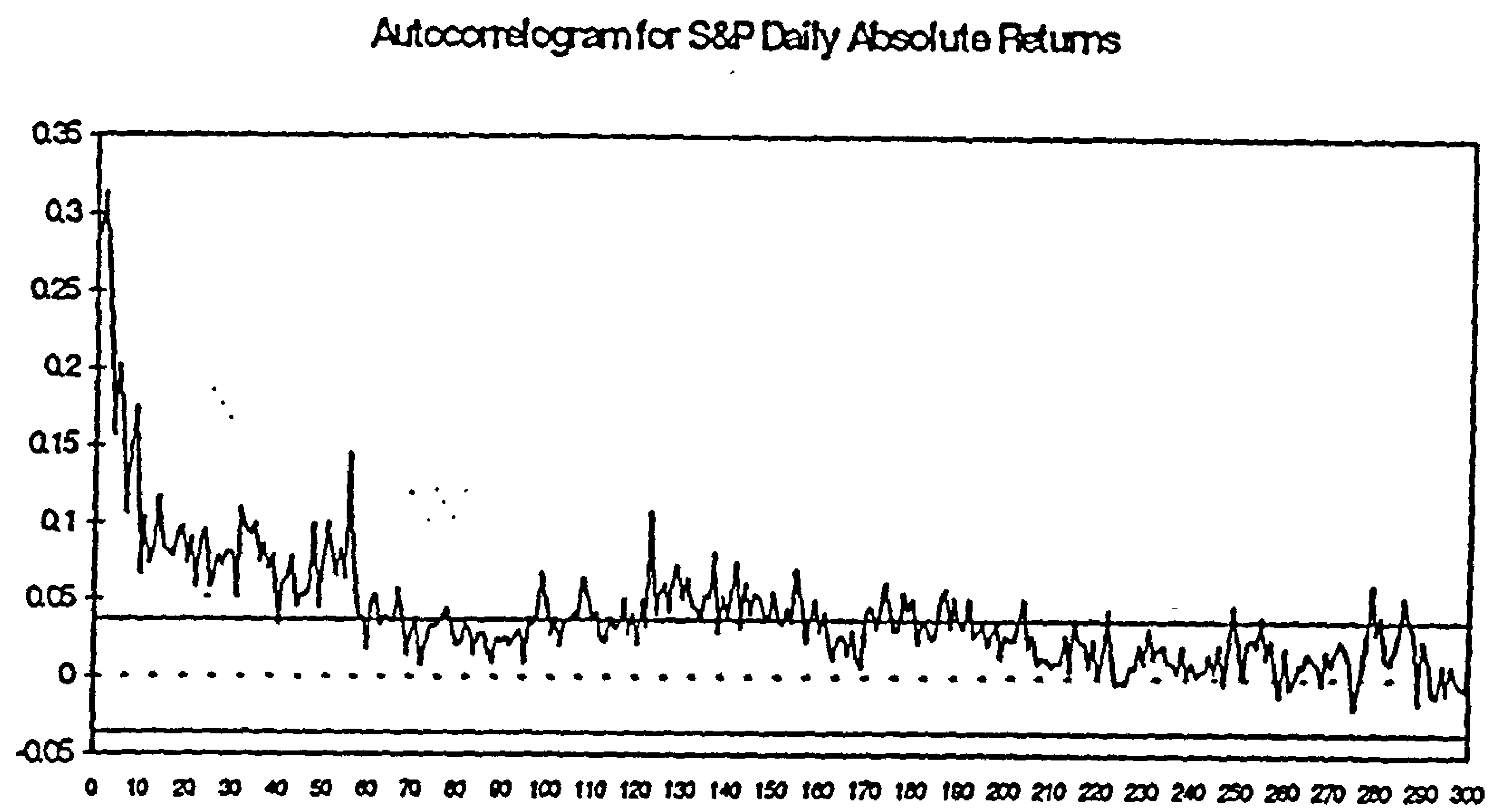


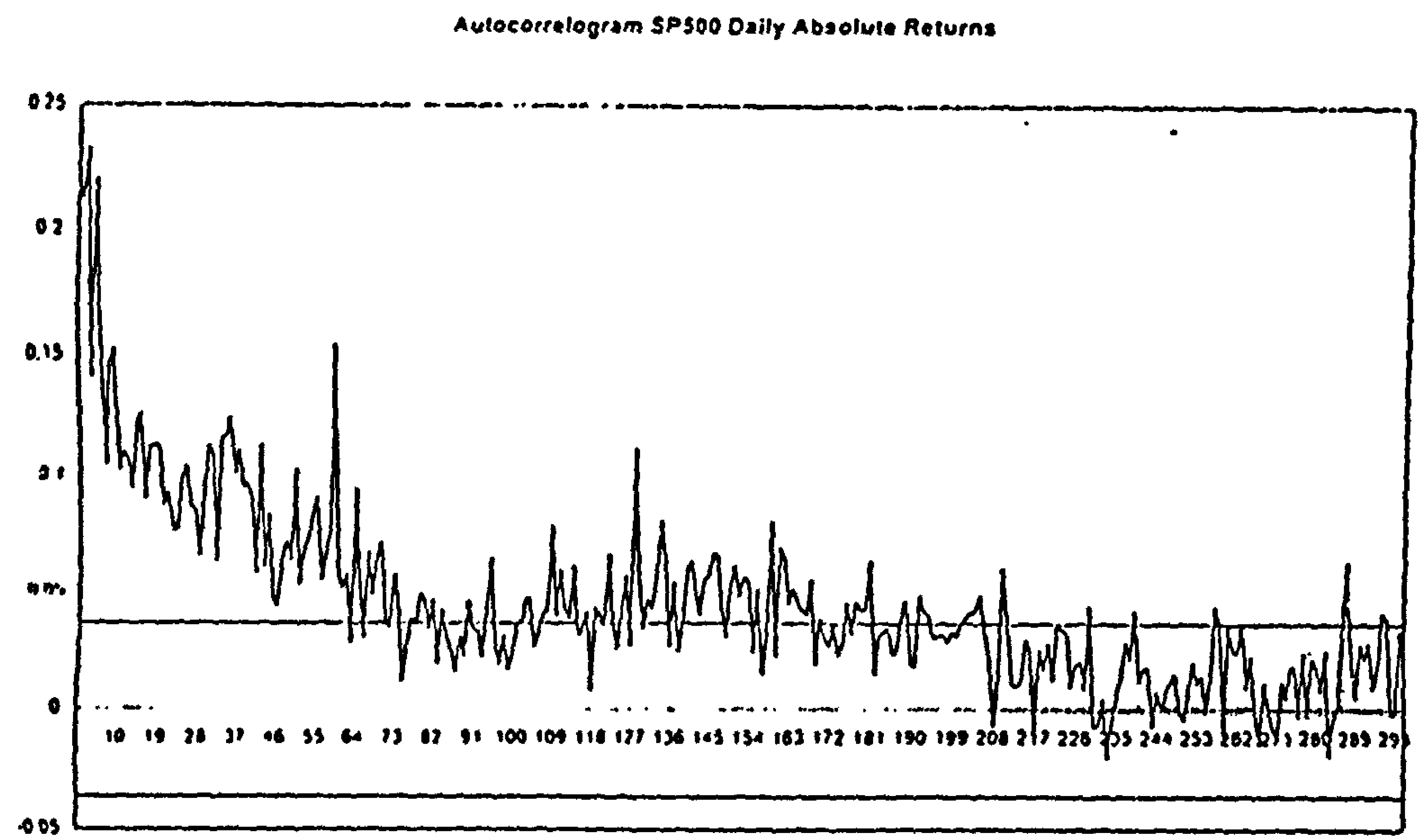
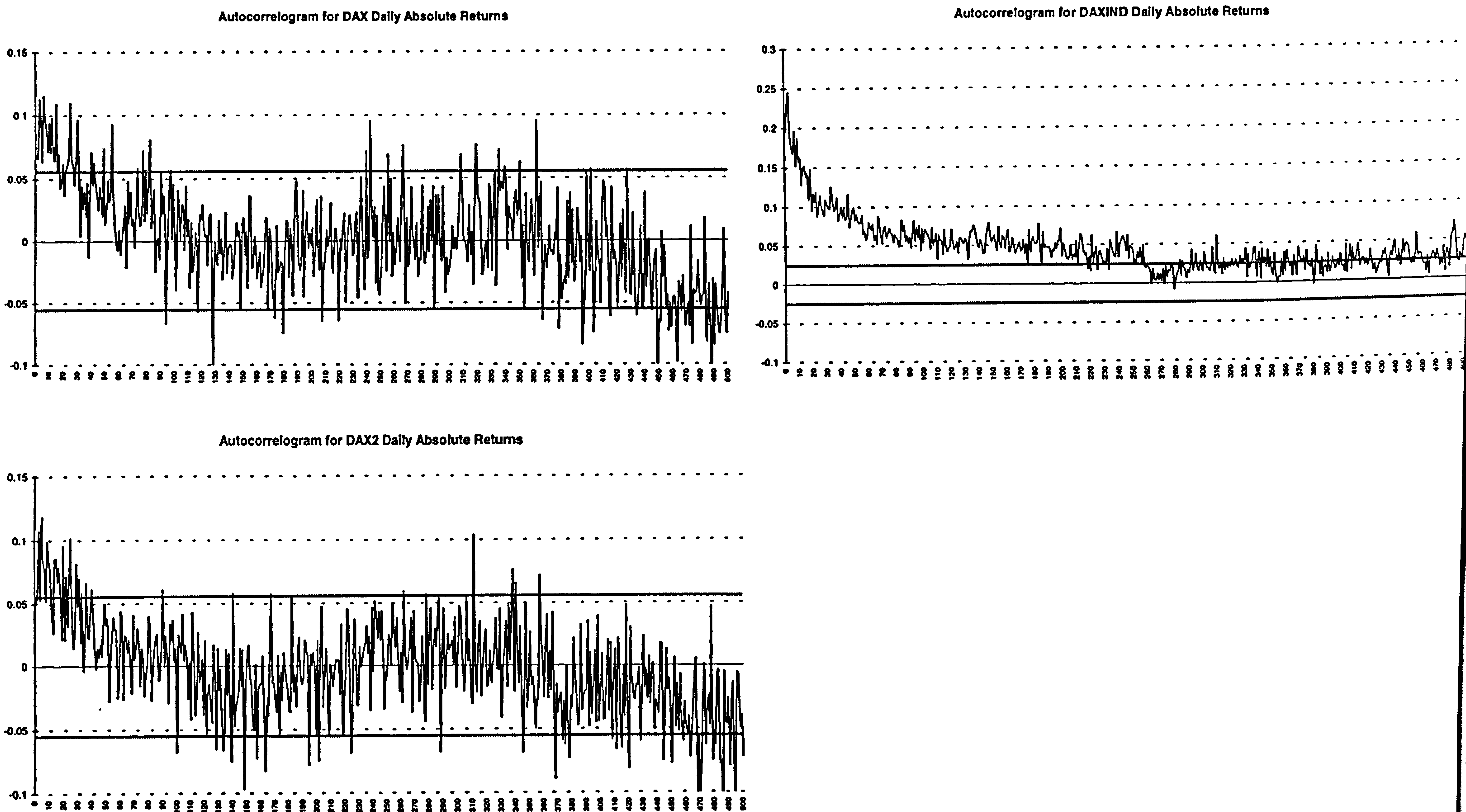


Figure 2.2d Comparison of autocorrelograms on S&P-500 Futures



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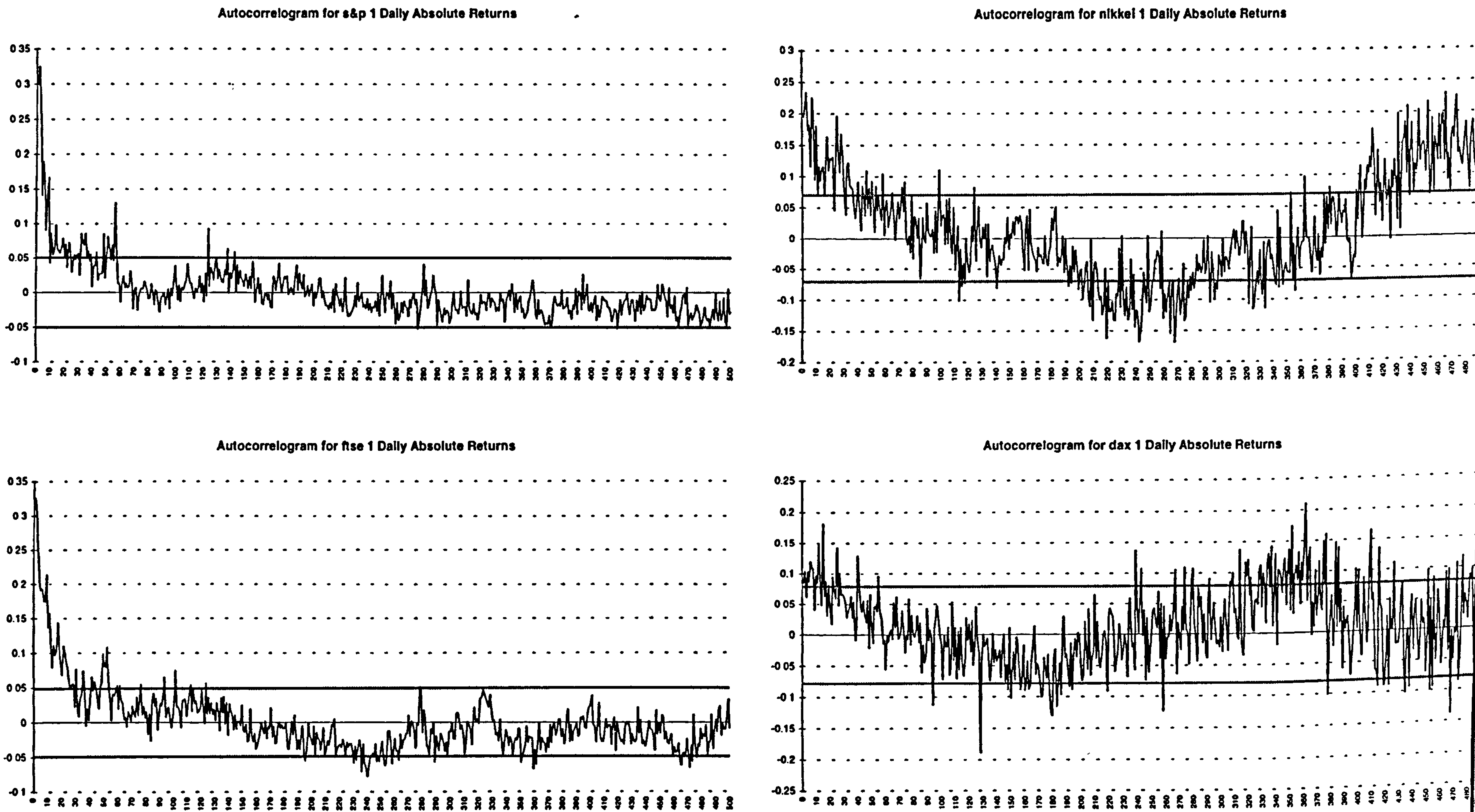

Figure 2.3a First period autocorrelogram for four Stock Index Futures absolute daily returns.

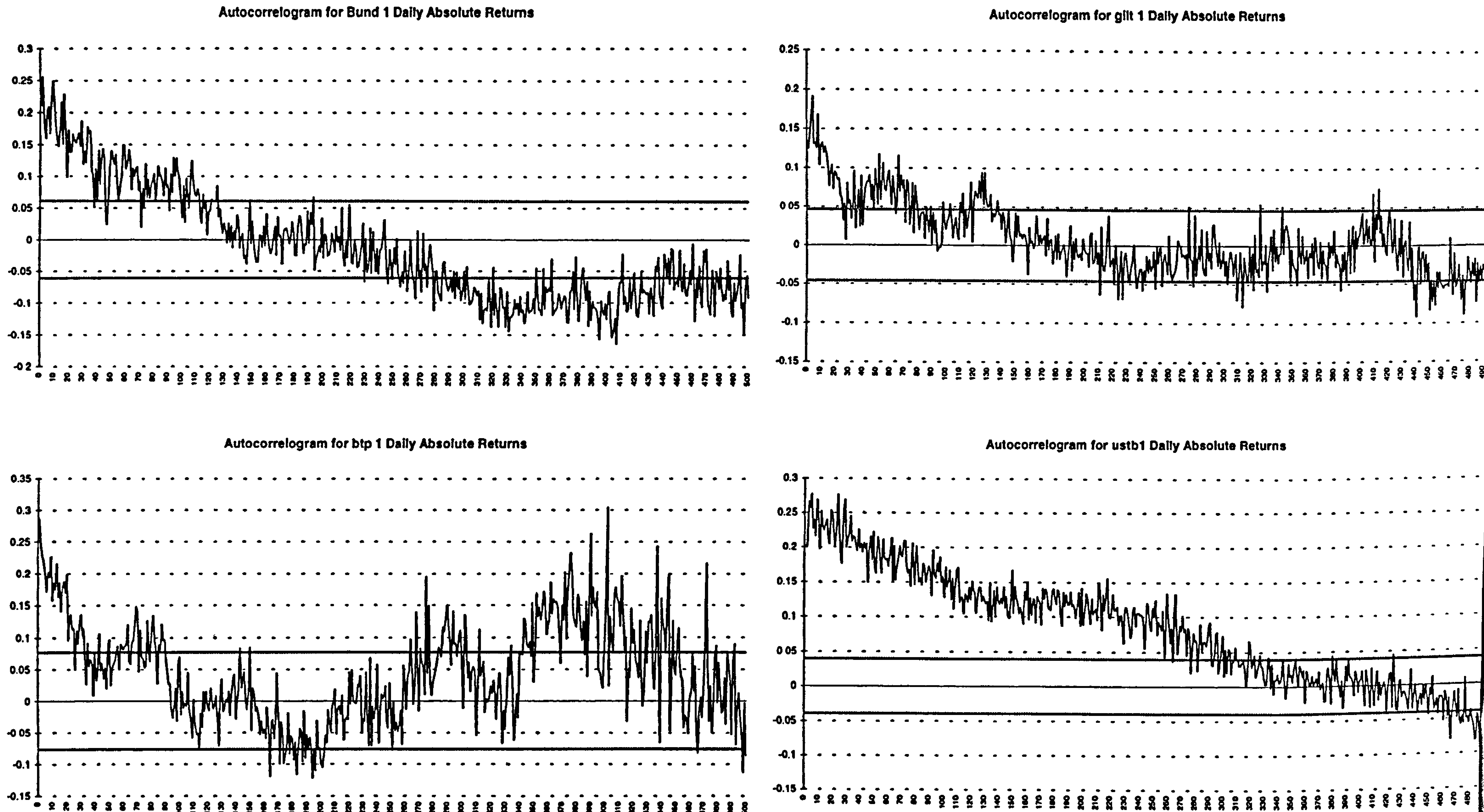

Figure 2.3b First period autocorrelogram for four Fixed income Futures absolute daily returns.

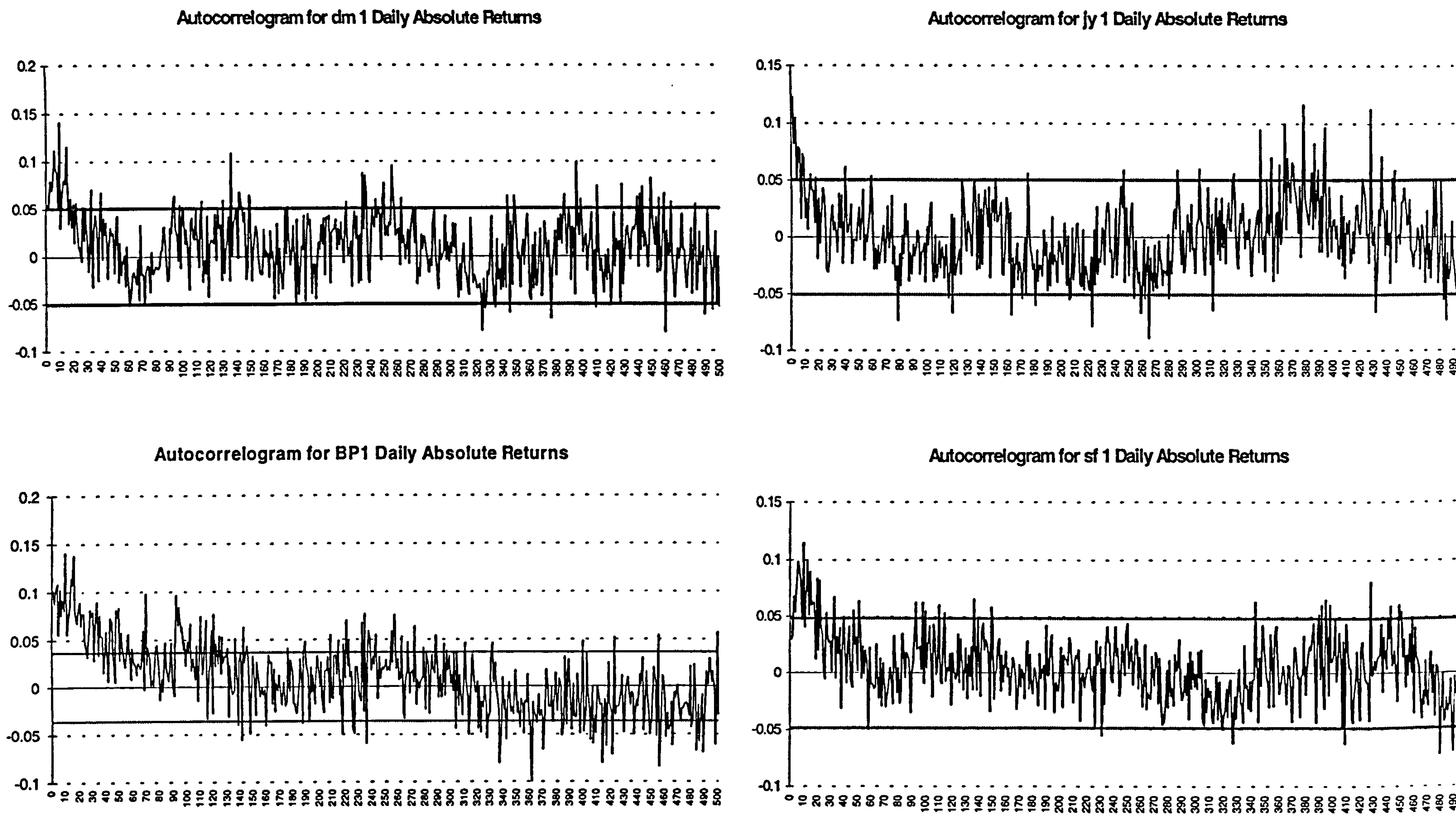

Figure 2.3c First period autocorrelogram for four Foreign exchange Futures absolute daily returns.

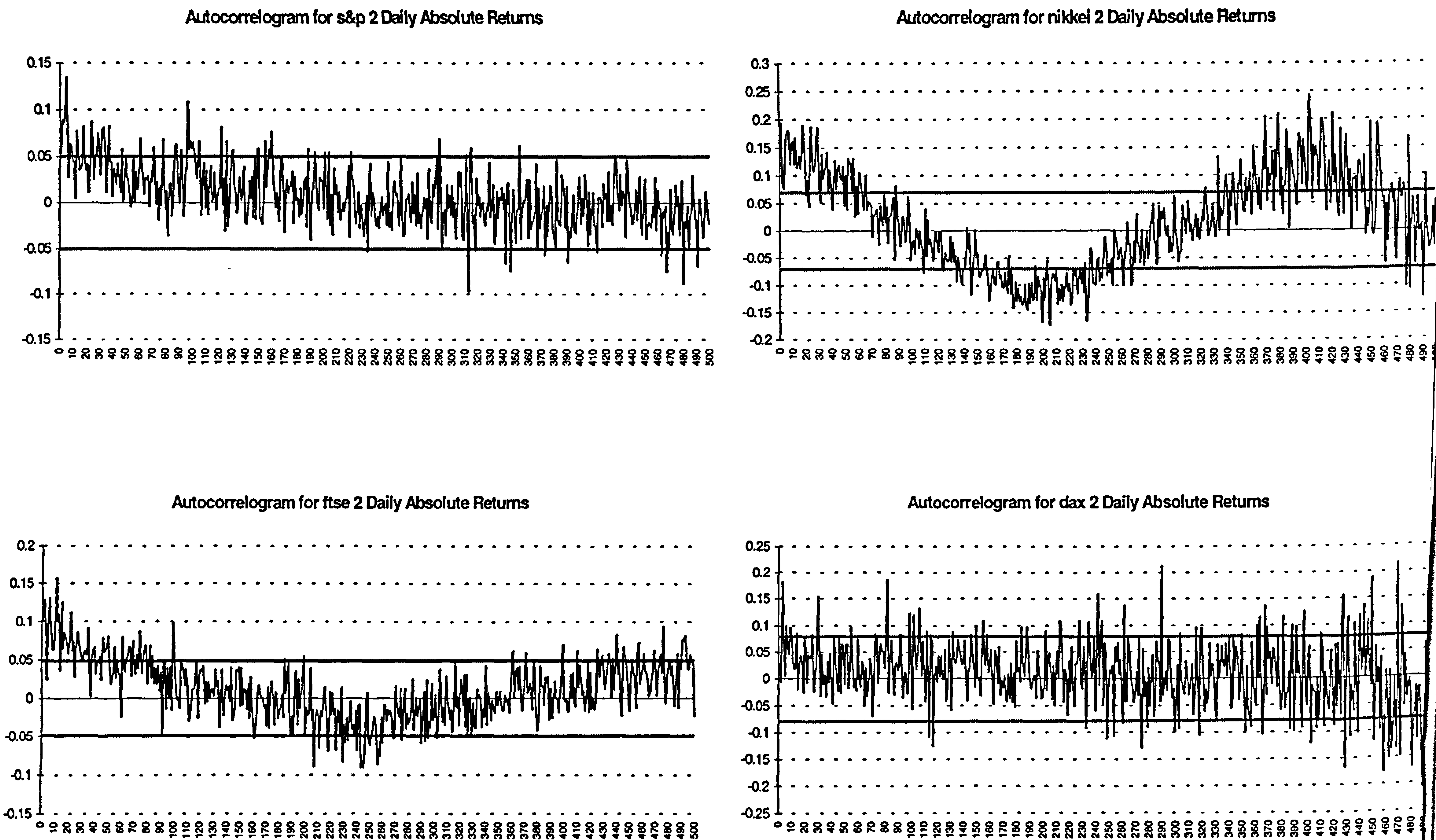

Figure 2.4a Second period autocorrelogram for four Stock Index Futures absolute daily returns.

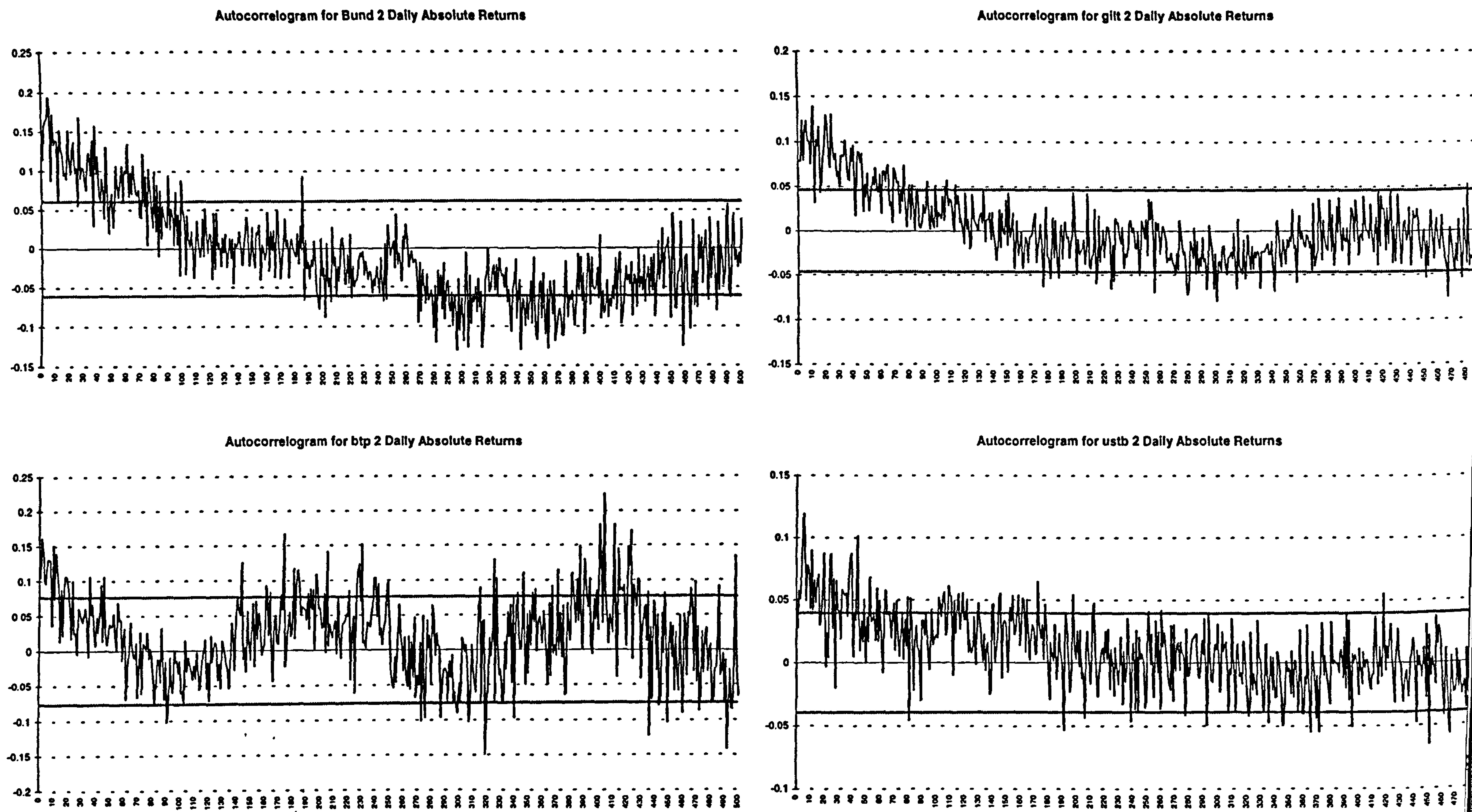

Figure 2.4b Second period autocorrelogram for four Fixed Income Futures absolute daily returns.

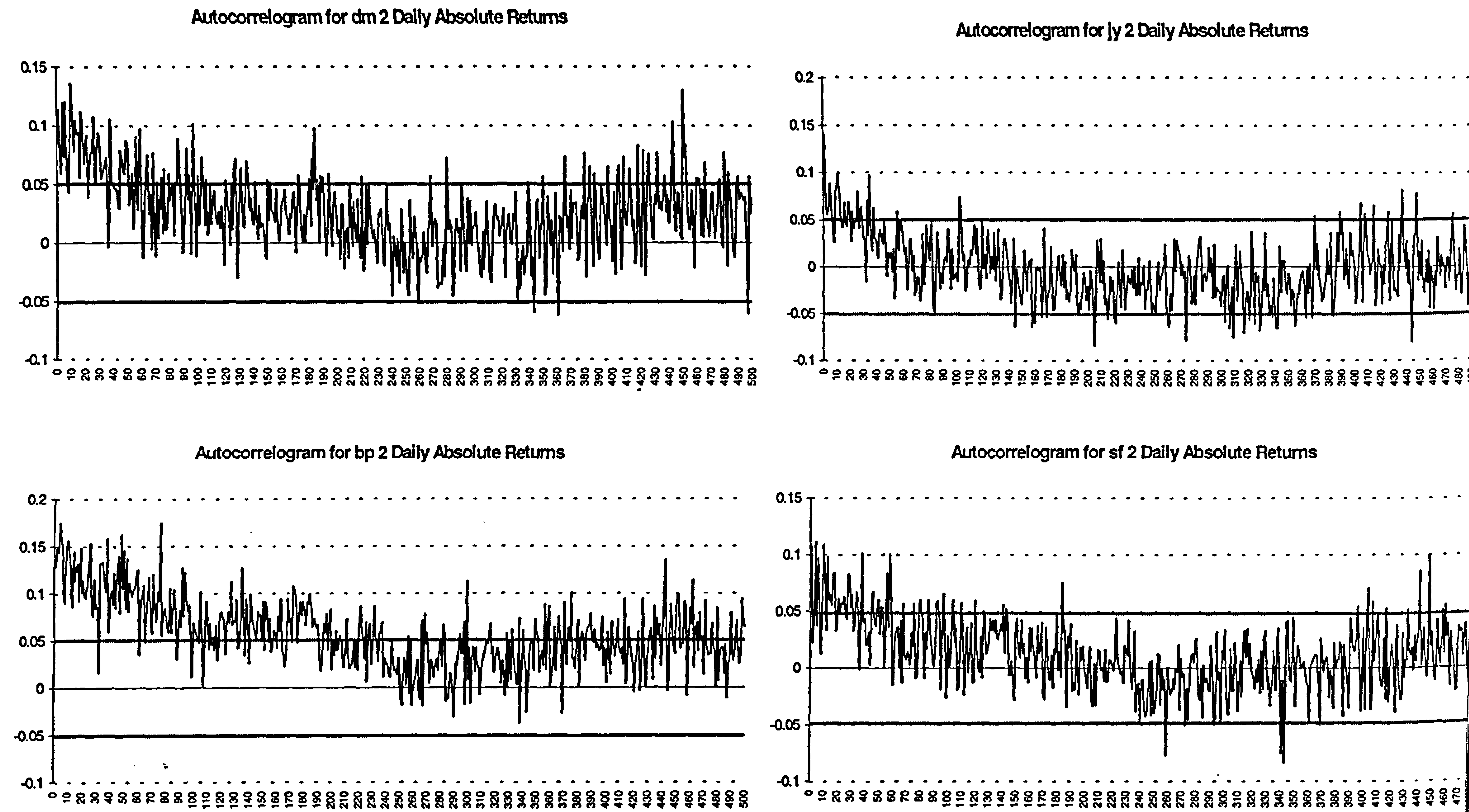

Figure 2.4c Second period autocorrelogram for four Foreign Exchange Futures absolute daily returns.

S&P-500	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	4.57	5.23	6.26	6.37	6.88	6.70	6.59	7.02	7.32	7.68	7.88	8.06	8.14	8.43	8.50	8.50	8.67	8.68	8.72	8.80	8.91	8.92	9.02	9.01	9.05
1st Quart	9.53	10.01	10.14	10.29	10.28	10.30	10.13	10.18	10.08	10.04	9.98	10.07	10.20	10.19	10.11	10.01	10.05	10.05	10.03	9.96	10.10	10.06	10.10	10.15	10.17
Average	14.31	14.77	15.05	15.27	15.47	15.64	15.78	15.92	16.05	16.17	16.29	16.40	16.51	16.62	16.72	16.82	16.92	17.01	17.11	17.20	17.30	17.38	17.47	17.56	17.65
Median	12.14	12.37	12.26	12.51	12.77	12.66	12.68	12.66	12.86	12.96	12.95	13.00	13.18	13.57	13.80	14.01	14.34	14.71	14.76	14.82	14.82	14.82	14.96	15.02	15.25
3rd Quart	15.65	16.11	16.14	16.53	17.03	17.40	17.62	17.65	17.53	17.38	17.17	17.17	17.23	17.56	17.49	17.37	17.27	17.35	17.33	17.30	17.20	17.12	17.34	17.56	17.49
Max	157.98	112.65	95.16	83.16	74.85	68.65	64.10	60.35	57.19	54.60	52.30	50.28	48.59	47.16	45.80	44.56	43.46	42.38	41.41	40.57	39.79	39.06	38.41	37.73	37.13
Stdev	12.95	12.35	12.01	11.78	11.58	11.42	11.28	11.16	11.04	10.94	10.84	10.74	10.65	10.56	10.48	10.39	10.32	10.24	10.16	10.09	10.02	9.95	9.88	9.82	9.75
Kurt	88.89	48.44	33.51	25.71	20.63	17.06	14.52	12.59	11.06	9.83	8.81	7.95	7.21	6.57	6.02	5.53	5.10	4.71	4.37	4.06	3.78	3.53	3.30	3.09	2.90
Skew	8.43	6.30	5.26	4.61	4.11	3.72	3.42	3.17	2.95	2.76	2.60	2.45	2.31	2.18	2.07	1.96	1.86	1.76	1.67	1.59	1.51	1.43	1.36	1.29	1.22
CF	1.0067	1.0135	1.0205	1.0276	1.0349	1.0424	1.0500	1.0578	1.0659	1.0741	1.0825	1.0911	1.0999	1.1089	1.1182	1.1277	1.1375	1.1475	1.1577	1.1683	1.1791	1.1902	1.2016	1.2133	1.2253
COV	0.9045																								

FTSE-100	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	7.26	7.97	8.32	8.93	9.07	9.30	9.74	9.72	9.69	9.86	9.86	9.79	9.78	9.83	10.16	10.22	10.23	10.20	10.46	10.44	10.46	10.66	10.70	10.85	10.89
1st Quart	10.66	11.42	11.74	11.73	11.92	12.11	12.30	12.52	12.69	12.70	12.96	13.14	13.17	13.25	13.25	13.29	13.60	13.63	13.68	13.74	13.86	14.08	14.17	14.18	14.14
Average	14.76	15.01	15.19	15.30	15.40	15.49	15.57	15.64	15.72	15.79	15.86	15.93	16.00	16.07	16.13	16.20	16.26	16.32	16.38	16.44	16.49	16.55	16.61	16.66	16.71
Median	13.58	13.75	14.12	14.52	14.39	14.53	14.55	14.62	14.51	14.54	14.80	14.79	14.80	14.80	15.04	15.12	15.13	15.07	15.05	15.01	15.00	15.01	15.13	15.22	15.22
3rd Quart	16.62	16.93	16.54	16.39	16.30	16.40	16.57	16.51	16.50	16.51	16.58	16.60	16.61	16.95	16.84	16.87	16.77	16.68	16.70	16.65	16.65	16.69	16.91	17.09	17.05
Max	90.32	68.44	58.33	51.41	46.82	43.38	40.64	38.42	36.69	35.37	34.10	33.06	32.17	31.33	30.51	29.78	29.06	28.40	27.76	27.35	26.90	26.44	26.03	25.61	25.31
Stdev	7.35	6.91	6.63	6.40	6.22	6.06	5.93	5.80	5.67	5.55	5.43	5.31	5.19	5.07	4.96	4.85	4.75	4.65	4.55	4.46	4.38	4.29	4.21	4.12	4.04
Kurt	49.24	33.62	25.52	20.56	17.02	14.47	12.56	11.10	9.93	8.96	8.13	7.43	6.83	6.30	5.84	5.42	5.05	4.74	4.45	4.19	3.95	3.72	3.52	3.33	3.14
Skew	5.60	4.79	4.23	3.82	3.47	3.19	2.96	2.77	2.60	2.45	2.32	2.20	2.09	1.99	1.90	1.81	1.73	1.65	1.59	1.52	1.46	1.40	1.35	1.29	1.23
CF	1.0063	1.0128	1.0194	1.0261	1.0330	1.0400	1.0472	1.0546	1.0621	1.0698	1.0776	1.0857	1.0939	1.1023	1.1109	1.1198	1.1288	1.1381	1.1475	1.1573	1.1672	1.1774	1.1879	1.1986	1.2096
COV	0.4984																								

Nikkei-225	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	6.85	8.41	8.38	8.71	9.39	10.01	10.80	13.03	13.43	13.82	14.11	13.95	14.31	14.53	14.77	15.94	16.86	18.05	19.79	19.84	19.70	19.77	19.96	20.07	20.07
1st Quart	15.02	15.58	15.77	16.26	16.66	17.60	18.11	18.92	19.27	19.42	19.89	20.56	21.23	21.67	21.50	21.53	21.51	21.93	21.84	21.70	21.47	21.29	21.08	20.89	20.78
Average	21.80	22.02	22.13	22.22	22.31	22.39	22.51	22.62	22.74	22.86	22.98	23.10	23.22	23.34	23.46	23.58	23.66	23.71	23.74	23.74	23.73	23.71	23.68	23.64	23.59
Median	19.97	20.98	21.15	21.29	21.71	21.64	22.20	22.57	23.18	23.49	23.54	23.25	23.10	22.99	23.22	23.06	22.92	22.67	22.43	22.36	22.65	22.83	23.05	23.22	23.39
3rd Quart	27.63	27.89	27.52	27.26	27.91	27.72	27.19	26.53	26.52	26.30	26.20	25.92	25.31	25.00	25.08	25.19	25.34	25.30	25.72	26.31	26.33	26.63	26.69	26.45	26.34
Max	48.40	43.49	39.98	37.34	36.50	36.47	35.13	33.95	33.54	32.99	32.30	31.87	31.31	30.59	29.80	29.43	29.15	28.96	28.56	28.13	27.78	27.59	27.55	28.05	28.79
Stdev	8.79	7.86	7.28	6.86	6.44	6.09	5.78	5.49	5.23	4.95	4.66	4.35	4.03	3.71	3.40	3.11	2.89	2.73	2.66	2.63	2.62	2.64	2.66	2.68	2.67
Kurt	2.59	2.35	2.19	2.12	2.09	2.12	2.12	2.13	2.21	2.30	2.41	2.55	2.72	2.85	2.87	2.72	2.44	2.10	1.82	1.60	1.43	1.32	1.27	1.29	1.33
Skew	0.60	0.45	0.34	0.27	0.21	0.17	0.12	0.06	0.02	-0.02	-0.05	-0.06	-0.06	0.00	0.11	0.29	0.46	0.54	0.48	0.37	0.26	0.19	0.14	0.11	0.08
CF	1.0130	1.0265	1.0406	1.0554	1.0709	1.0871	1.1041	1.1218	1.1405	1.1601	1.1808	1.2025	1.2253	1.2494	1.2747	1.3015	1.3297	1.3596	1.3911	1.4244	1.4596	1.4968	1.5361	1.5777	1.6215
COV	0.4031																								

DAX	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	4.89	7.48	7.67	8.63	9.02	9.20	9.44	9.87	10.09	10.41	10.58	10.44	10.48	10.68	11.08	11.47	11.46	11.59	11.66	11.87	11.90	12.23	12.47	12.61	12.57
1st Quart	10.94	11.06	12.03	12.15	12.44	12.68	12.91	12.99	13.06	13.24	13.36	13.60	13.77	13.90	13.96	14.34	14.96	15.03	15.01	14.95	14.97	15.26	15.30	15.34	15.30
Average	14.41	14.63	14.78	14.92	15.06	15.18	15.30	15.39	15.45	15.48	15.50	15.51	15.54	15.57	15.60	15.63	15.67	15.71	15.75	15.80	15.85	15.90	15.94	15.99	16.03
Median	13.69	13.90	14.29	14.22	14.46	14.71	14.76	15.03	15.38	16.08	16.27	16.14	16.13	16.14	16.16	16.01	16.04	16.12	16.14	16.19	16.17	16.29	16.57	16.60	16.58
3rd Quart	16.48	16.46	17.25	17.81	17.70	17.66	18.34	18.23	18.00	17.71	17.48	17.35	17.14	16.98	16.91	16.89	16.77	16.73	17.05	17.18	17.10	17.09	17.16	17.08	16.97
Max	33.45	28.04	26.38	24.63	22.97	21.66	21.02	20.13	19.60	19.67	19.65	19.34	19.07	18.86	18.66	18.36	18.22	18.14	17.97	17.83	17.89	17.71	17.57	17.66	17.82
Stdev	4.89	4.30	3.98	3.72	3.50	3.30	3.09	2.92	2.77	2.63	2.49	2.36	2.24	2.13	2.01	1.91	1.84	1.77	1.70	1.64	1.58	1.51	1.46	1.40	1.33
Kurt	4.06	3.41	2.92	2.52	2.19	1.99	1.83	1.76	1.75	1.82	1.94	2.09	2.21	2.30	2.37	2.54	2.71	2.79	2.81	2.80	2.82	2.79	2.91	3.10	3.34
Skew	1.01	0.88	0.68	0.49	0.34	0.20	0.07	-0.08	-0.19	-0.26	-0.33	-0.41	-0.48	-0.53	-0.57	-0.64	-0.73	-0.80	-0.86	-0.89	-0.93	-0.96	-1.02	-1.10	-1.19
CF	1.0164	1.0337	1.0520	1.0713	1.0918	1.1136	1.1367	1.1612	1.1874	1.2153	1.2450	1.2769	1.3109	1.3473	1.3863	1.4281	1.4730	1.5210	1.5725	1.6276	1.6864	1.7489	1.8152	1.8850	1.9580
COV	0.3391																								

Table 2.5a Summary statistics of volatility cones for four Stock Index Futures

Bund	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	2.00	2.51	2.79	2.84	3.01	2.97	2.93	3.02	3.12	3.13	3.09	3.13	3.16	3.36	3.33	3.37	3.38	3.41	3.40	3.48	3.51	3.50	3.52	3.56	3.60
1st Quart	3.43	3.51	3.60	3.65	3.69	3.73	3.81	3.84	3.85	3.85	3.87	3.90	3.93	3.94	3.93	3.93	3.93	3.97	4.09	4.22	4.22	4.21	4.23	4.24	4.25
Average	5.03	5.11	5.16	5.20	5.24	5.27	5.31	5.34	5.37	5.40	5.42	5.44	5.46	5.48	5.50	5.52	5.54	5.56	5.56	5.55	5.55	5.54	5.53	5.53	5.52
Median	4.23	4.44	4.47	4.40	4.53	4.75	4.99	5.19	5.21	5.30	5.25	5.25	5.26	5.33	5.47	5.48	5.57	5.64	5.69	5.76	5.75	5.81	5.89	5.91	5.91
3rd Quart	6.16	6.13	6.37	6.28	6.09	6.02	6.04	6.16	6.30	6.50	6.65	6.72	6.82	6.93	7.03	6.99	6.92	6.86	6.85	6.79	6.72	6.70	6.64	6.62	6.59
Max	15.96	14.15	12.47	11.79	10.99	10.49	10.09	9.76	9.35	9.08	8.81	8.53	8.35	8.26	8.11	8.00	7.88	7.74	7.61	7.48	7.34	7.20	7.08	7.02	6.91
Stdev	2.27	2.14	2.05	1.96	1.92	1.87	1.82	1.77	1.73	1.69	1.65	1.61	1.57	1.53	1.50	1.46	1.42	1.39	1.36	1.33	1.30	1.27	1.24	1.20	1.17
Kurt	5.84	4.96	4.30	3.91	3.53	3.23	2.96	2.71	2.46	2.24	2.06	1.92	1.81	1.73	1.68	1.64	1.62	1.60	1.57	1.56	1.55	1.56	1.57	1.59	1.62
Skew	1.55	1.40	1.28	1.20	1.11	1.02	0.92	0.81	0.70	0.59	0.50	0.41	0.33	0.25	0.17	0.08	0.00	-0.08	-0.15	-0.20	-0.25	-0.30	-0.36	-0.41	-0.45
CF	1.0098	1.0199	1.0304	1.0412	1.0524	1.0640	1.0760	1.0884	1.1013	1.1147	1.1285	1.1429	1.1578	1.1733	1.1894	1.2061	1.2235	1.2416	1.2605	1.2801	1.3005	1.3218	1.3440	1.3672	1.3913
COV	0.4519																								

BTP	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	2.02	2.18	2.51	2.64	2.70	2.75	2.80	2.85	3.57	3.90	5.00	5.18	6.08	7.48	7.38	7.42	7.41	7.39	7.33	7.45	7.49	7.40	7.33	7.41	7.42
1st Quart	6.02	6.71	6.91	7.11	7.12	7.44	7.50	8.06	8.18	8.37	8.44	8.58	8.64	8.79	8.75	8.76	8.76	8.77	8.90	8.98	9.20	9.53	9.46	9.40	9.35
Average	8.50	8.70	8.83	8.94	9.04	9.15	9.26	9.36	9.47	9.56	9.63	9.69	9.74	9.77	9.79	9.80	9.82	9.85	9.87	9.90	9.92	9.94	9.96	9.98	10.00
Median	7.93	8.53	8.63	9.28	9.52	9.63	9.71	9.88	9.85	9.73	9.63	9.50	9.45	9.50	9.64	9.87	9.99	10.09	9.99	9.90	10.02	10.09	10.20	10.24	10.18
3rd Quart	10.86	11.13	10.83	10.82	10.95	10.85	10.58	10.53	10.46	10.65	10.74	10.74	10.96	11.01	11.10	11.18	11.12	11.01	10.88	10.79	10.97	10.99	11.01	10.99	10.97
Max	18.55	16.31	15.31	14.80	14.61	14.39	14.22	13.85	13.31	12.88	12.66	12.45	12.16	12.28	12.17	12.15	12.03	11.92	11.75	11.65	11.63	11.56	11.45	11.37	11.40
Stdev	3.63	3.28	3.05	2.88	2.73	2.56	2.37	2.19	1.99	1.82	1.65	1.53	1.43	1.37	1.34	1.31	1.28	1.24	1.21	1.18	1.16	1.14	1.12	1.10	1.08
Kurt	2.67	2.52	2.55	2.73	2.95	3.20	3.46	3.66	3.57	3.49	2.99	2.54	1.96	1.77	1.74	1.73	1.75	1.81	1.91	2.05	2.25	2.44	2.54	2.60	2.72
Skew	0.39	0.09	-0.13	-0.28	-0.40	-0.51	-0.61	-0.65	-0.59	-0.50	-0.28	-0.09	0.13	0.19	0.12	0.03	-0.05	-0.13	-0.24	-0.36	-0.48	-0.60	-0.69	-0.77	-0.84
CF	1.0155	1.0318	1.0490	1.0671	1.0862	1.1064	1.1278	1.1505	1.1746	1.2001	1.2273	1.2562	1.2869	1.3197	1.3547	1.3921	1.4320	1.4746	1.5201	1.5687	1.6205	1.6756	1.7341	1.7959	1.8610
COV	0.4268																								

Gilt	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	3.01	3.86	4.24	4.62	4.88	5.11	5.20	5.31	5.36	5.50	5.67	5.68	5.72	5.77	5.86	5.98	6.23	6.23	6.24	6.25	6.37	6.53	6.51	6.59	6.72
1st Quart	5.94	6.24	6.37	6.51	6.68	6.90	7.08	7.09	7.09	7.19	7.25	7.23	7.29	7.40	7.44	7.52	7.65	7.76	7.83	7.89	7.99	7.98	7.94	7.95	8.04
Average	8.36	8.47	8.52	8.56	8.59	8.61	8.64	8.66	8.69	8.71	8.73	8.75	8.76	8.78	8.80	8.82	8.84	8.85	8.87	8.89	8.90	8.91	8.92	8.93	8.94
Median	7.68	7.83	8.11	8.16	8.12	8.13	8.14	8.15	8.22	8.39	8.43	8.49	8.51	8.54	8.60	8.59	8.64	8.68	8.68	8.77	8.81	8.85	8.90	8.92	9.02
3rd Quart	10.24	10.38	10.24	10.26	10.21	10.30	10.37	10.34	10.30	10.29	10.25	10.24	10.25	10.23	10.16	10.16	10.07	9.98	9.87	9.85	9.80	9.70	9.60	9.52	9.47
Max	22.59	17.63	15.24	14.48	13.85	13.54	13.38	13.38	13.07	12.85	12.56	12.38	12.27	12.46	12.58	12.42	12.24	12.16	12.11	11.99	12.23	12.18	12.11	12.04	11.93
Stdev	3.10	2.74	2.53	2.40	2.29	2.20	2.12	2.06	1.99	1.92	1.85	1.79	1.73	1.67	1.61	1.56	1.50	1.44	1.39	1.34	1.29	1.25	1.21	1.17	1.14
Kurt	3.96	2.86	2.44	2.35	2.26	2.13	2.06	2.05	2.03	2.00	1.97	1.95	1.96	2.00	2.08	2.16	2.26	2.39	2.52	2.64	2.77	2.89	3.00	3.10	3.17
Skew	0.98	0.71	0.58	0.56	0.52	0.46	0.42	0.42	0.40	0.39	0.37	0.36	0.33	0.32	0.31	0.32	0.33	0.35	0.36	0.37	0.39	0.40	0.42	0.44	0.47
CF	1.0057	1.0114	1.0173	1.0233	1.0294	1.0356	1.0420	1.0484	1.0550	1.0618	1.0687	1.0757	1.0828	1.0901	1.0976	1.1052	1.1130	1.1210	1.1291	1.1374	1.1458	1.1545	1.1634	1.1724	1.1817
COV	0.3710																								

USTB	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	2.17	2.94	2.87	3.04	3.44	3.42	3.65	3.60	3.63	3.58	3.60	3.86	4.14	4.18	4.23	4.44	4.51	4.54	4.57	4.55	4.53	4.56	4.72	4.76	4.78
1st Quart	7.61	7.92	8.07	8.10	8.15	8.18	8.35	8.48	8.61	8.61	8.58	8.67	8.72	8.75	8.82	8.91	8.96	9.06	9.09	9.14	9.17	9.20	9.18	9.19	9.19
Average	11.08	11.20	11.28	11.35	11.41	11.45	11.50	11.54	11.57	11.61	11.64	11.67	11.70	11.73	11.76	11.80	11.83	11.86	11.89	11.93	11.96	11.99	12.02	12.06	12.09
Median	9.85	10.00	10.14	10.23	10.30	10.34	10.50	10.47	10.39	10.31	10.34	10.38	10.33	10.25	10.21	10.36	10.60	10.78	10.96	11.11	11.22	11.29	11.35	11.38	11.39
3rd Quart	13.51	13.48	13.57	13.42	13.42	13.68	13.62	13.83	13.97	14.44	14.37	14.44	14.50	14.50	14.60	14.50	14.48	14.39	14.31	14.36	14.34	14.26	14.15	14.08	14.20
Max	31.41	28.58	26.62	25.44	24.29	23.74	23.00	22.54	22.29	22.48	22.93	22.63	22.37	22.09	22.06	21.88	21.71	21.44	21.31	21.13	21.18	21.17	21.35	21.31	21.28
Stdev	5.14	4.91	4.79	4.70	4.61	4.54	4.48	4.43	4.37	4.33	4.28	4.24	4.20	4.17	4.13	4.09	4.05	4.01	3.97	3.93	3.89	3.85	3.81	3.77	3.72
Kurt	3.88	3.41	3.23	3.10	3.01	2.94	2.88	2.83	2.80	2.78	2.77	2.76	2.75	2.75	2.75	2.74	2.74	2.73	2.71	2.70	2.69	2.67	2.66	2.65	2.63
Skew	1.03	0.92	0.88	0.84	0.82	0.79	0.77	0.75	0.73	0.72	0.71	0.71	0.70	0.70	0.70	0.69	0.69	0.68	0.68	0.67	0.67	0.68	0.68	0.69	0.69
CF	1.0041	1.0083	1.0125	1.0168	1.0212	1.0256	1.0301	1.0346	1.0392	1.0439	1.0486	1.0534	1.0583	1.0633	1.0683	1.0734	1.0786	1.0838	1.0892	1.0946	1.1001	1.1057	1.1113	1.1171	1.1230
COV	0.4637																								

Table 2.5b Summary statistics of volatility cones for four Fixed Income Futures

D-Mark	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	3.69	5.40	5.43	5.59	5.70	5.87	5.91	6.09	6.22	6.36	6.34	6.42	6.50	6.67	6.92	7.71	7.79	8.23	8.25	8.78	9.13	9.70	9.86	9.89	9.86
1st Quart	8.70	9.05	9.38	9.66	9.85	9.98	10.03	10.07	10.06	10.21	10.26	10.38	10.46	10.56	10.62	10.67	10.70	10.78	10.80	10.86	10.87	10.85	10.87	10.93	10.93
Average	11.20	11.39	11.46	11.50	11.53	11.55	11.57	11.58	11.60	11.61	11.63	11.64	11.66	11.68	11.69	11.71	11.71	11.72	11.72	11.73	11.73	11.73	11.72	11.72	11.71
Median	10.53	10.99	11.15	11.29	11.35	11.39	11.61	11.56	11.56	11.54	11.55	11.58	11.59	11.58	11.54	11.55	11.53	11.44	11.41	11.42	11.45	11.44	11.45	11.41	11.35
3rd Quart	12.96	13.24	13.41	13.43	13.30	13.33	13.37	13.42	13.24	13.22	13.19	13.04	12.92	12.88	12.85	12.89	12.89	12.75	12.64	12.54	12.47	12.57	12.56	12.54	12.51
Max	24.05	21.83	21.17	19.13	17.82	17.19	17.22	17.51	16.85	16.30	15.72	15.47	15.28	15.56	15.59	15.52	15.41	15.15	14.94	14.86	14.77	14.58	14.41	14.31	14.56
Stdev	3.82	3.28	2.98	2.73	2.52	2.37	2.26	2.16	2.07	1.97	1.87	1.78	1.70	1.62	1.54	1.46	1.39	1.32	1.25	1.19	1.14	1.11	1.08	1.07	1.05
Kurt	3.38	2.93	2.76	2.66	2.58	2.66	2.80	2.98	3.06	3.06	3.06	3.07	3.05	3.01	2.91	2.79	2.74	2.70	2.60	2.47	2.44	2.39	2.38	2.40	2.45
Skew	0.84	0.58	0.37	0.19	0.01	-0.10	-0.16	-0.19	-0.22	-0.27	-0.32	-0.33	-0.29	-0.20	-0.08	0.03	0.11	0.18	0.28	0.42	0.50	0.56	0.57	0.58	0.60
CF	1.0067	1.0135	1.0205	1.0276	1.0350	1.0424	1.0501	1.0579	1.0660	1.0742	1.0826	1.0912	1.1000	1.1091	1.1184	1.1279	1.1377	1.1477	1.1580	1.1685	1.1794	1.1905	1.2019	1.2136	1.2257
COV	0.3415																								

B-Pound	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	3.13	3.43	3.87	4.15	4.28	4.78	5.11	5.05	5.02	5.22	5.35	5.45	5.44	5.45	5.66	6.28	6.35	6.36	6.59	6.96	7.03	7.30	7.82	7.82	7.83
1st Quart	7.72	8.39	8.64	8.65	8.70	8.78	9.07	9.31	9.50	9.63	9.58	9.58	9.59	9.68	9.79	9.87	10.08	10.16	10.31	10.32	10.37	10.44	10.55	10.65	10.66
Average	11.00	11.18	11.25	11.27	11.29	11.30	11.31	11.32	11.34	11.34	11.35	11.37	11.38	11.40	11.41	11.43	11.44	11.45	11.46	11.47	11.48	11.49	11.50	11.51	11.52
Median	10.18	10.34	10.67	10.90	11.16	11.16	11.18	11.24	11.27	11.21	11.28	11.26	11.31	11.38	11.37	11.38	11.38	11.33	11.32	11.34	11.41	11.58	11.62	11.59	11.55
3rd Quart	13.65	13.62	13.64	13.44	13.28	13.18	13.21	13.12	13.16	13.10	12.98	12.91	12.94	12.87	12.96	12.88	12.86	12.76	12.66	12.56	12.63	12.71	12.67	12.57	12.49
Max	28.39	27.91	25.61	23.13	21.69	21.04	20.80	20.68	19.85	19.10	18.53	18.24	17.97	17.82	17.49	17.28	16.98	16.75	16.46	16.24	15.96	15.76	15.47	15.23	14.97
Stdev	4.47	4.03	3.79	3.58	3.39	3.23	3.11	3.00	2.90	2.79	2.68	2.59	2.50	2.41	2.33	2.25	2.17	2.10	2.03	1.97	1.91	1.86	1.82	1.79	1.76
Kurt	4.26	4.12	3.85	3.59	3.36	3.27	3.27	3.25	3.20	3.07	3.00	2.99	2.99	2.98	2.95	2.90	2.84	2.77	2.67	2.57	2.50	2.43	2.41	2.44	2.49
Skew	0.97	0.88	0.77	0.67	0.56	0.49	0.43	0.38	0.33	0.25	0.21	0.19	0.18	0.18	0.17	0.16	0.13	0.10	0.09	0.06	0.03	-0.01	-0.07	-0.14	-0.21
CF	1.0067	1.0135	1.0205	1.0276	1.0349	1.0424	1.0501	1.0579	1.0659	1.0741	1.0826	1.0912	1.1000	1.1091	1.1183	1.1279	1.1376	1.1476	1.1579	1.1685	1.1793	1.1904	1.2018	1.2135	1.2256
COV	0.4064																								

J-Yen	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	2.87	4.99	6.04	6.11	6.38	6.75	7.14	7.18	7.07	7.09	7.14	7.08	7.28	7.63	7.75	8.81	8.74	8.87	8.82	8.86	8.82	8.82	8.94	9.05	9.07
1st Quart	7.81	8.47	8.71	8.85	9.19	9.34	9.48	9.60	9.65	9.80	10.11	10.26	10.50	10.61	10.83	10.86	10.91	10.92	10.89	10.89	10.90	10.93	10.97	10.99	11.02
Average	10.53	10.76	10.87	10.94	11.00	11.06	11.11	11.16	11.20	11.23	11.26	11.28	11.31	11.34	11.36	11.37	11.38	11.38	11.38	11.38	11.38	11.37	11.36	11.34	11.33
Median	9.85	10.21	10.29	10.38	10.57	10.79	11.07	11.10	11.28	11.34	11.36	11.45	11.41	11.37	11.43	11.46	11.49	11.56	11.61	11.57	11.55	11.53	11.48	11.42	11.36
3rd Quart	12.27	12.27	12.90	13.01	12.90	12.87	12.73	12.65	12.45	12.40	12.28	12.30	12.28	12.23	12.15	12.10	12.05	11.96	11.91	11.85	11.79	11.77	11.80	11.83	11.77
Max	24.23	20.42	19.45	17.54	16.96	17.42	17.89	17.41	16.84	16.16	15.67	15.21	14.72	14.32	13.97	13.73	13.57	13.43	13.29	13.18	13.52	13.41	13.34	13.15	12.97
Stdev	3.78	3.15	2.82	2.59	2.39	2.22	2.09	1.97	1.86	1.76	1.65	1.55	1.44	1.33	1.23	1.14	1.06	0.99	0.93	0.89	0.85	0.83	0.81	0.79	0.77
Kurt	4.09	2.97	2.52	2.28	2.25	2.38	2.61	2.84	2.99	3.06	3.09	3.13	3.13	3.04	2.99	2.96	3.11	3.27	3.53	3.83	4.14	4.25	4.19	4.02	3.82
Skew	1.06	0.77	0.60	0.46	0.37	0.32	0.29	0.25	0.20	0.13	0.04	-0.06	-0.15	-0.20	-0.26	-0.32	-0.44	-0.54	-0.67	-0.78	-0.85	-0.84	-0.78	-0.68	-0.57
CF	1.0067	1.0135	1.0205	1.0277	1.0350	1.0425	1.0502	1.0580	1.0661	1.0743	1.0827	1.0914	1.1002	1.1093	1.1186	1.1282	1.1380	1.1480	1.1583	1.1689	1.1797	1.1909	1.2023	1.2141	1.2262
COV	0.3590																								

S-Franc	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	4.76	6.42	6.41	6.78	6.99	7.20	7.26	7.43	7.46	7.80	7.95	8.04	8.01	8.13	8.35	9.18	9.29	9.70	9.77	10.46	10.77	10.79	11.05	11.14	11.08
1st Quart	9.73	10.16	10.49	10.77	10.92	11.06	11.19	11.46	11.43	11.62	11.84	11.93	12.06	12.08	12.12	12.23	12.23	12.26	12.33	12.38	12.39	12.43	12.43	12.40	12.41
Average	12.42	12.60	12.71	12.78	12.85	12.90	12.94	12.98	13.01	13.04	13.07	13.10	13.13	13.17	13.20	13.22	13.23	13.23	13.24	13.25	13.25	13.24	13.23	13.22	13.21
Median	11.73	12.05	12.41	12.58	12.81	12.87	13.08	13.08	13.07	13.04	13.07	13.16	13.16	13.14	13.15	13.22	13.36	13.30	13.20	13.11	13.07	13.10	13.04	13.00	12.99
3rd Quart	14.43	14.52	14.64	14.42	14.34	14.26	14.27	14.23	14.34	14.43	14.72	14.70	14.65	14.54	14.40	14.40	14.34	14.25	14.16	14.09	14.08	13.97	14.09	14.06	14.03
Max	27.27	23.83	22.86	20.74	19.12	18.77	19.00	18.98	18.36	17.65	17.08	16.81	16.50	16.65	16.76	16.65	16.57	16.42	16.25	16.29	16.18	15.99	15.87	15.87	15.99
Stdev	3.86	3.28	2.98	2.73	2.54	2.40	2.28	2.19	2.10	2.02	1.93	1.84	1.75	1.65	1.56	1.47	1.40	1.33	1.27	1.21	1.17	1.13	1.10	1.08	1.06
Kurt	3.68	3.29	3.17	2.99	2.78	2.78	2.87	2.92	2.87	2.78	2.74	2.77	2.80	2.84	2.78	2.68	2.65	2.62	2.56	2.47	2.49	2.52	2.54	2.55	2.56
Skew	0.91	0.73	0.57	0.41	0.25	0.13	0.04	-0.04	-0.12	-0.19	-0.26	-0.31	-0.33	-0.31	-0.24	-0.16	-0.09	-0.01	0.10	0.22	0.29	0.36	0.41	0.45	0.49
CF	1.0062	1.0125	1.0190	1.0255	1.0323	1.0391	1.0462	1.0533	1.0607	1.0682	1.0758	1.0837	1.0917	1.0999	1.1083	1.1168	1.1256	1.1346	1.1438	1.1532	1.1629	1.1728	1.1829	1.1933	1.2040
COV	0.3111																								

Table 2.5c Summary statistics of volatility cones for four Foreign Exchange Futures

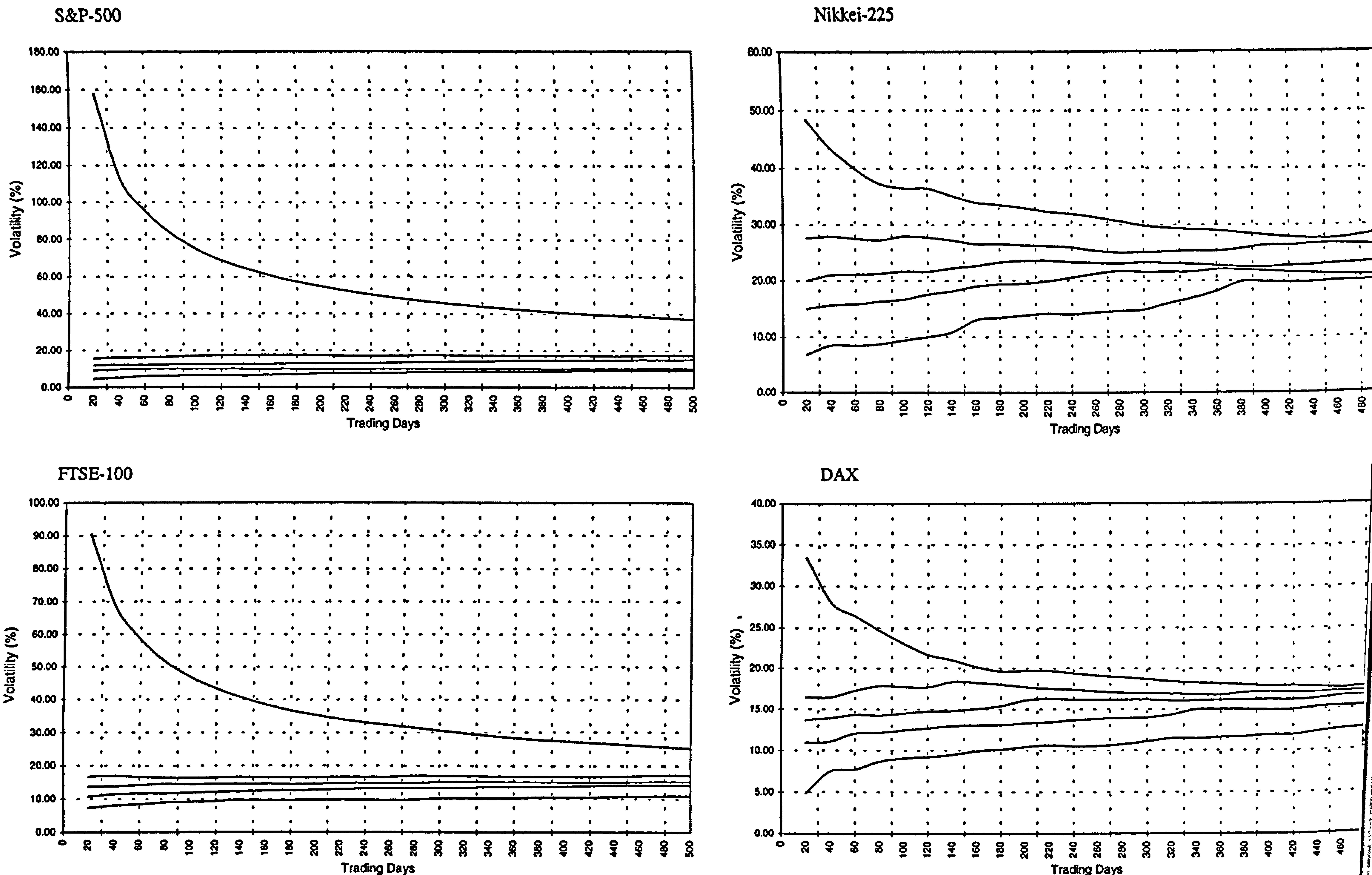

Figure 2.5a Volatility cones for four Stock Index Futures.

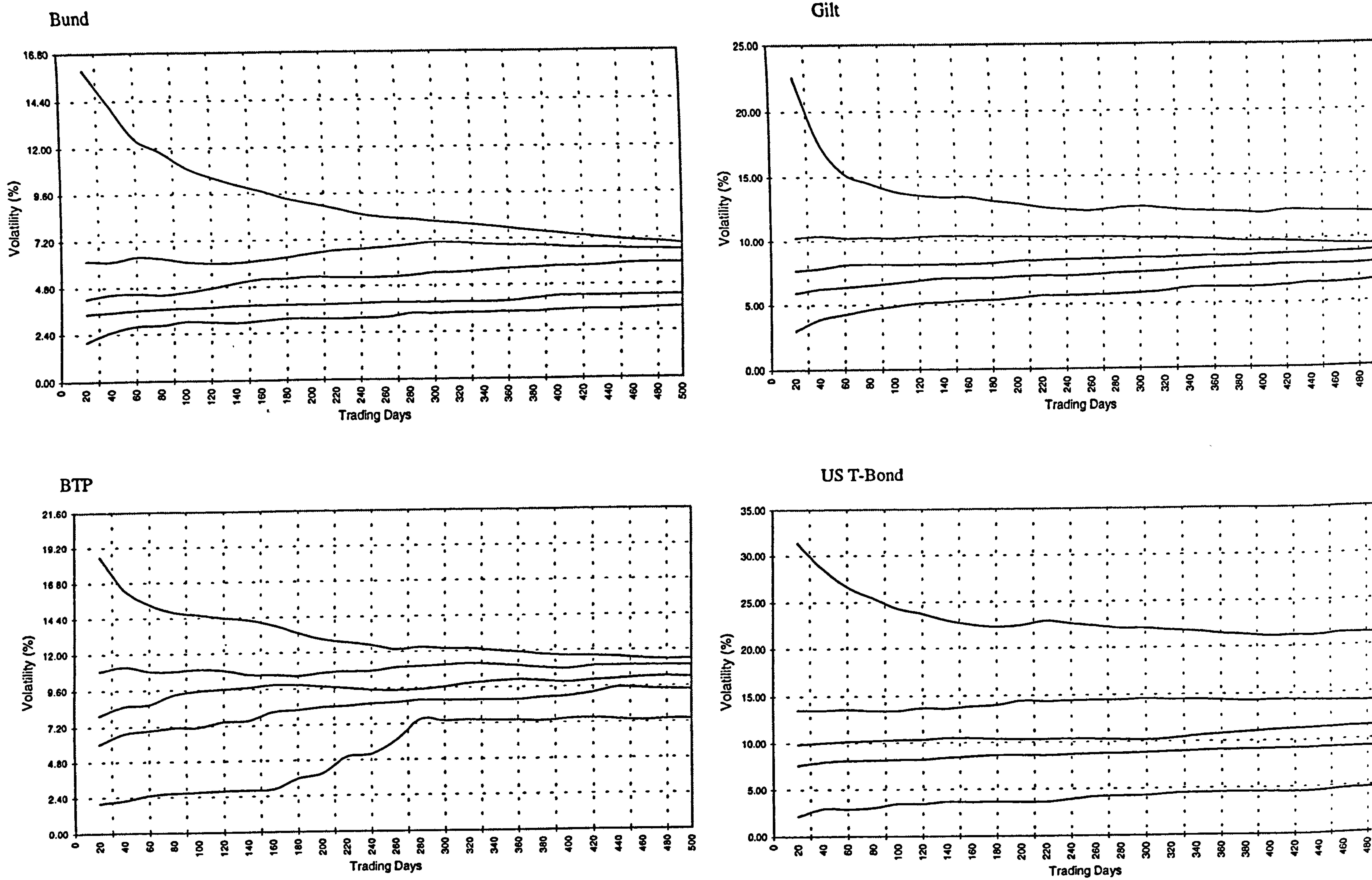

Figure 2.5b Volatility cones for four Fixed Income Futures.

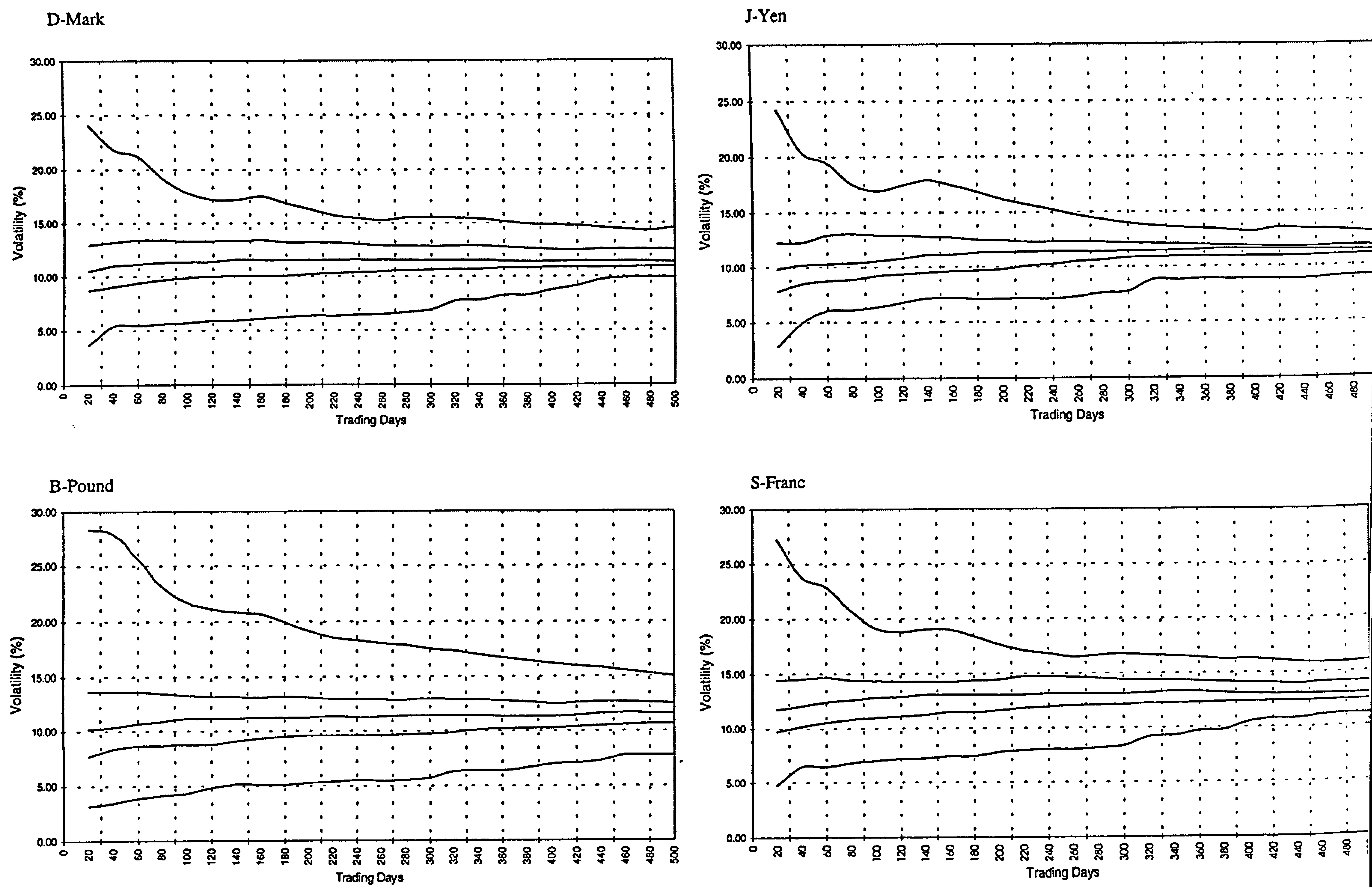

Figure 2.5c Volatility cones for four Foreign Exchange Futures.

S&P-500	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	6.78	8.76	9.20	9.02	9.68	9.51	9.88	10.16	10.34	10.41	10.55	10.71	11.57	11.53	11.83	12.33	12.69	12.88	13.11	13.36	13.46	14.17	14.22	14.30	14.34
1st Quart	11.71	12.08	12.38	12.30	12.53	12.88	13.03	13.73	14.04	14.48	14.83	15.08	15.16	15.10	15.16	15.21	15.15	15.06	15.08	15.12	15.30	15.49	15.64	15.88	16.00
Average	18.03	18.82	19.36	19.75	20.12	20.46	20.78	21.07	21.37	21.68	21.98	22.29	22.59	22.89	23.18	23.46	23.73	24.00	24.28	24.56	24.86	25.15	25.45	25.76	26.08
Median	14.25	15.49	15.71	15.94	16.51	16.96	17.49	17.29	17.32	17.25	17.03	16.86	16.80	16.76	16.85	17.09	17.32	17.52	17.38	17.34	17.19	17.04	17.13	18.03	20.18
3rd Quart	18.60	18.60	19.64	19.86	19.53	19.11	18.84	18.41	18.11	17.93	18.54	18.56	19.43	20.45	24.43	42.64	42.18	41.49	40.64	39.72	38.89	38.18	37.84	37.19	36.66
Max	157.98	112.65	95.16	83.16	74.85	68.65	64.10	60.35	57.19	54.60	52.30	50.28	48.59	47.16	45.80	44.56	43.46	42.38	41.41	40.57	39.79	39.06	38.41	37.73	37.13
Stdev	17.27	16.36	15.82	15.44	15.09	14.78	14.51	14.28	14.05	13.82	13.60	13.37	13.14	12.91	12.69	12.47	12.25	12.03	11.81	11.58	11.34	11.08	10.82	10.55	10.25
Kurt	50.71	26.78	18.05	13.53	10.61	8.59	7.14	6.03	5.15	4.45	3.87	3.39	2.98	2.63	2.33	2.07	1.85	1.66	1.49	1.36	1.24	1.15	1.09	1.04	1.03
Skew	6.53	4.77	3.91	3.37	2.96	2.63	2.37	2.15	1.95	1.78	1.63	1.48	1.35	1.22	1.10	0.99	0.88	0.77	0.67	0.56	0.46	0.35	0.24	0.13	0.01
CF	1.0135	1.0276	1.0424	1.0579	1.0741	1.0911	1.1090	1.1278	1.1475	1.1683	1.1902	1.2133	1.2377	1.2635	1.2907	1.3195	1.3500	1.3822	1.4164	1.4526	1.4909	1.5315	1.5745	1.6200	1.6679
COV	0.9581																								

FTSE-100	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	7.26	8.35	8.99	9.76	10.11	10.53	10.65	10.60	10.58	10.72	10.65	11.02	11.03	11.35	11.83	11.97	11.94	12.08	12.10	12.21	12.55	12.80	12.84	12.90	12.97
1st Quart	11.17	11.90	12.11	12.09	12.66	13.14	13.29	13.24	13.35	13.30	13.25	13.30	13.35	13.28	13.27	13.35	13.67	13.63	13.62	13.57	13.58	13.69	14.00	14.13	14.25
Average	15.72	16.10	16.37	16.54	16.68	16.81	16.94	17.06	17.19	17.31	17.42	17.51	17.61	17.73	17.83	17.94	18.06	18.18	18.30	18.42	18.54	18.66	18.79	18.92	19.06
Median	13.98	14.25	14.85	14.88	14.89	14.75	14.70	14.74	14.68	14.87	14.93	14.92	14.88	15.18	15.32	15.32	15.34	15.28	15.23	15.14	15.16	15.21	15.22	15.27	15.59
3rd Quart	17.38	17.34	17.02	16.69	16.45	16.74	16.67	16.54	16.49	16.34	16.47	16.40	16.33	16.38	16.73	22.82	27.39	27.13	26.75	26.32	26.00	25.66	25.38	25.10	24.74
Max	90.32	68.44	58.33	51.41	46.82	43.38	40.64	38.42	36.69	35.37	34.10	33.06	32.17	31.33	30.51	29.78	29.06	28.40	27.76	27.35	26.90	26.44	26.03	25.61	25.31
Stdev	9.37	8.89	8.56	8.29	8.05	7.86	7.70	7.55	7.41	7.27	7.13	6.98	6.84	6.69	6.56	6.43	6.30	6.18	6.05	5.92	5.79	5.66	5.53	5.39	5.24
Kurt	35.28	22.69	16.55	12.95	10.47	8.67	7.30	6.24	5.39	4.69	4.10	3.63	3.22	2.86	2.55	2.27	2.04	1.84	1.67	1.52	1.39	1.28	1.20	1.14	1.10
Skew	5.08	4.18	3.60	3.19	2.87	2.59	2.36	2.16	1.98	1.81	1.66	1.53	1.40	1.27	1.16	1.04	0.93	0.83	0.73	0.63	0.53	0.44	0.34	0.24	0.13
CF	1.0128	1.0261	1.0400	1.0546	1.0696	1.0857	1.1023	1.1196	1.1381	1.1573	1.1774	1.1986	1.2209	1.2444	1.2691	1.2952	1.3227	1.3517	1.3823	1.4146	1.4488	1.4848	1.5229	1.5631	1.6055
COV	0.5963																								

Nikkei-225	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	9.17	12.74	13.78	15.20	15.93	16.51	16.91	18.00	18.93	19.01	18.84	18.70	19.55	20.03	21.22	22.80	23.19	24.16	24.54	25.04	25.51	25.76	25.96	26.13	26.04
1st Quart	17.56	18.27	19.15	20.25	21.06	21.01	20.74	20.93	21.32	21.85	22.10	22.31	22.70	23.81	24.17	24.50	24.86	25.19	25.65	26.41	26.57	26.76	26.70	26.45	26.14
Average	24.27	24.53	24.68	24.78	24.86	24.98	25.13	25.27	25.43	25.60	25.77	25.97	26.16	26.38	26.61	26.75	26.84	26.90	26.97	27.01	26.96	26.93	26.88	26.79	26.63
Median	22.96	23.35	23.04	22.96	22.33	22.67	23.28	24.21	24.65	25.93	26.02	26.31	26.38	26.65	26.57	26.63	27.68	27.53	27.48	27.38	27.31	27.06	26.81	26.69	26.44
3rd Quart	30.07	29.68	30.13	30.22	30.29	29.86	29.60	29.30	29.25	28.87	28.51	29.75	29.56	29.31	29.07	28.98	28.59	28.35	27.93	27.78	27.47	27.19	27.13	27.05	26.89
Max	48.40	43.49	39.98	37.34	36.50	36.47	35.13	33.95	33.54	32.99	32.30	31.87	31.31	30.59	29.80	29.43	29.15	28.96	28.56	28.13	27.78	27.59	27.55	28.05	28.79
Stdev	8.48	7.35	6.51	5.99	5.48	5.12	4.83	4.62	4.47	4.29	4.05	3.73	3.39	2.98	2.54	2.20	1.91	1.61	1.27	0.92	0.65	0.41	0.30	0.46	0.58
Kurt	2.62	2.34	2.21	2.06	1.94	2.00	1.96	1.86	1.83	1.80	1.76	1.71	1.76	1.78	1.56	1.45	1.44	1.49	1.82	2.21	2.32	3.82	3.10	2.99	4.72
Skew	0.60	0.49	0.45	0.47	0.52	0.57	0.53	0.41	0.28	0.15	0.04	-0.05	-0.17	-0.23	-0.21	-0.24	-0.32	-0.42	-0.57	-0.72	-0.85	-1.23	-0.12	0.96	1.38
CF	1.0265	1.0554	1.0871	1.1218	1.1601	1.2025	1.2494	1.3015	1.3596	1.4244	1.4968	1.5777	1.6676	1.7671	1.8758	1.9926	2.1151	2.2397	2.3633	2.4872	2.6258	2.8283	3.2617	5.0969	
COV	0.3493																								

DAX	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	4.89	8.31	9.12	8.63	9.65	9.37	12.20	12.42	12.83	12.89	13.06	13.20	13.44	13.56	13.76	14.29	14.81	15.38	15.41	15.30	15.20	15.22	15.21	15.25	15.26
1st Quart	11.63	12.54	12.41	12.72	12.97	12.97	13.08	13.47	13.49	13.64	14.24	14.83	15.04	15.31	15.90	15.83	15.74	15.77	15.70	15.73	15.81	15.70	15.63	15.80	15.97
Average	15.43	15.65	15.84	16.03	16.15	16.23	16.32	16.36	16.35	16.28	16.17	16.09	16.03	15.99	15.98	15.98	16.01	16.07	16.15	16.24	16.36	16.49	16.56	16.59	16.61
Median	14.36	14.54	14.85	14.79	14.89	15.29	16.40	16.81	17.42	17.37	17.04	16.77	16.55	16.41	16.28	16.02	15.97	16.05	16.10	16.14	16.21	16.69	16.79	17.05	16.91
3rd Quart	17.82	18.01	18.90	19.77	20.16	19.64	19.14	18.77	18.34	17.88	17.61	17.34	17.06	16.73	16.42	16.38	16.32	16.34	16.58	16.80	16.94	17.15	17.30	17.26	17.09
Max	33.45	28.04	26.38	24.63	22.97	21.66	21.02	20.13	19.60	18.92	18.44	18.07	17.85	17.39	17.17	16.98	16.98	16.79	17.25	17.44	17.89	17.71	17.57	17.66	17.75
Stdev	5.57	4.88	4.52	4.20	3.82	3.42	2.97	2.65	2.35	2.08	1.82	1.56	1.32	1.07	0.85	0.61	0.41	0.36	0.49	0.63	0.73	0.80	0.84	0.82	0.72
Kurt	3.31	2.71	2.28	1.93	1.70	1.58	1.33	1.34	1.44	1.56	1.72	1.92	2.26	2.86	3.65	3.69	3.51	2.08	1.84	1.91	1.92	1.57	1.40	1.58	2.02
Skew	0.85	0.74	0.52	0.30	0.17	0.06	0.01	-0.16	-0.34	-0.46	-0.58	-0.71	-0.84	-1.04	-1.24	-1.00	-0.12	0.11	0.29	0.39	0.25	-0.12	-0.31	-0.43	-0.50
CF	1.0336	1.0712	1.1134	1.1609	1.2148	1.2762	1.3465	1.4270	1.5196	1.6258	1.7467	1.8823	2.0304	2.1857	2.3413	2.4969	2.6816	3.0308	4.4820						
COV	0.3612																								

Table 2.6a First period summary statistics of volatility cones for four Stock Index Futures

Bund	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	2.00	2.51	2.79	2.84	3.01	2.97	2.93	3.02	3.12	3.13	3.09	3.13	3.16	3.36	3.33	3.37	3.38	3.44	3.53	3.64	3.71	3.82	4.01	4.14	4.16
1st Quart	3.14	3.21	3.29	3.36	3.45	3.51	3.51	3.51	3.49	3.53	3.58	3.61	3.67	3.89	4.09	4.35	4.44	4.41	4.40	4.39	4.59	4.74	4.78	4.83	5.15
Average	4.80	4.89	4.95	5.00	5.06	5.12	5.18	5.24	5.30	5.36	5.43	5.50	5.57	5.65	5.72	5.79	5.86	5.91	5.94	5.96	5.98	6.00	6.01	6.02	6.02
Median	3.93	4.00	4.05	3.90	3.90	4.09	4.27	4.34	4.64	4.96	4.96	4.94	5.00	5.35	5.48	5.87	6.03	6.25	6.51	6.54	6.65	6.69	6.62	6.55	6.58
3rd Quart	5.82	5.81	5.96	6.02	5.99	5.83	6.04	6.24	6.71	6.91	7.29	7.32	7.60	7.54	7.56	7.55	7.40	7.27	7.16	7.18	7.13	7.07	6.99	6.92	6.83
Max	15.96	14.15	12.47	11.79	10.99	10.49	10.09	9.76	9.35	9.08	8.81	8.53	8.35	8.26	8.11	8.00	7.88	7.74	7.61	7.48	7.34	7.20	7.08	7.02	6.91
Stdev	2.48	2.39	2.32	2.27	2.22	2.18	2.14	2.10	2.05	2.00	1.95	1.90	1.83	1.77	1.70	1.62	1.55	1.48	1.41	1.35	1.28	1.20	1.13	1.07	1.01
Kurt	7.33	5.99	4.99	4.34	3.72	3.24	2.86	2.51	2.19	1.93	1.73	1.60	1.51	1.45	1.43	1.45	1.51	1.55	1.56	1.57	1.58	1.60	1.67	1.79	1.96
Skew	1.97	1.78	1.62	1.49	1.34	1.20	1.06	0.92	0.77	0.63	0.50	0.38	0.28	0.17	0.06	-0.06	-0.19	-0.31	-0.39	-0.45	-0.51	-0.56	-0.64	-0.72	-0.83
CF	1.0199	1.0412	1.0640	1.0884	1.1146	1.1428	1.1732	1.2060	1.2415	1.2799	1.3216	1.3669	1.4161	1.4697	1.5280	1.5913	1.6599	1.7340	1.8136	1.8983	1.9875	2.0801	2.1745	2.2691	2.3629
COV	0.5175																								

BTP	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	2.02	2.18	2.51	2.64	2.70	2.75	2.80	2.85	3.57	3.90	5.00	5.18	6.08	7.48	7.38	7.42	7.41	7.39	7.33	7.45	7.49	7.40	7.33	7.41	7.42
1st Quart	4.09	4.98	5.53	6.36	6.78	6.83	7.05	7.18	7.29	7.42	7.62	7.95	7.90	7.98	7.93	7.88	8.03	8.01	7.88	7.78	7.84	7.80	7.72	7.69	7.75
Average	7.29	7.43	7.49	7.55	7.64	7.72	7.82	7.92	8.04	8.14	8.25	8.33	8.40	8.43	8.41	8.41	8.41	8.42	8.43	8.44	8.39	8.34	8.29	8.25	8.20
Median	6.99	7.39	7.73	7.58	7.53	7.70	7.77	8.19	8.16	8.35	8.41	8.51	8.48	8.39	8.62	8.51	8.39	8.55	8.50	8.34	8.28	8.25	8.25	8.33	8.22
3rd Quart	9.33	9.02	9.46	9.48	9.60	9.87	9.94	9.75	9.65	9.36	9.17	8.98	9.01	8.94	8.81	8.86	8.84	8.81	8.93	8.97	8.89	8.77	8.60	8.49	8.43
Max	17.91	15.48	13.76	12.76	11.70	11.53	10.98	10.68	10.47	10.27	9.92	9.59	9.48	9.49	9.29	9.19	9.14	9.26	9.33	9.55	9.69	9.69	9.63	9.53	9.41
Stdev	3.78	3.35	3.01	2.82	2.68	2.49	2.26	2.03	1.76	1.49	1.18	0.92	0.68	0.53	0.50	0.50	0.50	0.50	0.56	0.62	0.65	0.67	0.64	0.60	0.55
Kurt	2.91	2.58	2.32	2.26	2.26	2.42	2.66	2.95	3.09	3.62	3.82	4.43	2.94	1.57	1.79	1.77	1.99	2.13	1.68	1.62	1.77	1.97	2.06	2.21	2.24
Skew	0.71	0.38	0.00	-0.21	-0.34	-0.51	-0.69	-0.83	-0.90	-1.05	-1.09	-1.14	-0.56	-0.07	-0.34	-0.39	-0.34	-0.39	-0.19	0.10	0.21	0.31	0.37	0.49	0.48
CF	1.0318	1.0670	1.1063	1.1504	1.1999	1.2559	1.3194	1.3916	1.4740	1.5680	1.6747	1.7948	1.9276	2.0705	2.2183	2.3653	2.5135	2.6931	3.0291	4.2875					
COV	0.5193																								

Gilt	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	3.01	4.08	4.24	4.89	4.88	5.11	5.20	5.31	5.36	5.50	5.67	5.68	5.72	5.77	5.86	5.98	6.23	6.23	6.24	6.25	6.37	6.53	6.61	6.79	6.91
1st Quart	5.93	6.11	6.20	6.39	6.74	7.11	7.13	7.09	7.09	7.25	7.39	7.33	7.48	7.66	7.91	8.13	8.29	8.41	8.54	8.49	8.50	8.55	8.58	8.56	8.63
Average	8.64	8.77	8.81	8.84	8.87	8.91	8.93	8.95	8.97	9.00	9.03	9.06	9.09	9.12	9.15	9.18	9.22	9.26	9.30	9.34	9.38	9.41	9.44	9.47	9.50
Median	8.36	8.71	8.52	8.36	8.46	8.73	8.60	8.79	8.85	8.96	8.94	8.95	8.94	8.94	8.93	8.89	8.87	8.85	8.92	8.96	8.98	8.97	8.98	9.11	9.18
3rd Quart	10.60	10.51	10.78	10.81	10.95	10.90	10.67	10.68	10.79	10.77	10.87	10.88	11.07	11.25	11.04	10.81	10.64	10.61	10.55	10.43	10.31	10.23	10.30	10.37	10.36
Max	22.59	17.63	15.24	14.48	13.78	13.05	13.38	13.38	13.07	12.85	12.56	12.38	12.27	12.46	12.58	12.42	12.24	12.16	12.11	11.99	12.23	12.18	12.11	12.04	11.93
Stdev	3.30	2.91	2.67	2.55	2.44	2.34	2.28	2.23	2.17	2.11	2.05	1.99	1.93	1.86	1.80	1.74	1.68	1.62	1.56	1.50	1.44	1.39	1.34	1.28	1.23
Kurt	4.27	2.87	2.24	2.04	1.92	1.82	1.80	1.84	1.84	1.82	1.80	1.80	1.81	1.86	1.96	2.04	2.12	2.22	2.32	2.40	2.46	2.47	2.45	2.40	2.32
Skew	0.97	0.62	0.40	0.32	0.24	0.14	0.10	0.11	0.11	0.10	0.10	0.09	0.07	0.06	0.07	0.08	0.08	0.10	0.11	0.13	0.16	0.19	0.25	0.31	0.35
CF	1.0114	1.0233	1.0356	1.0484	1.0618	1.0757	1.0901	1.1052	1.1210	1.1374	1.1545	1.1724	1.1911	1.2107	1.2312	1.2527	1.2752	1.2988	1.3235	1.3495	1.3767	1.4054	1.4354	1.4670	1.5001
COV	0.3823																								

US T-Bond	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	2.17	2.94	2.87	3.04	3.44	3.42	3.65	3.60	3.63	3.58	3.60	3.86	4.14	4.18	4.23	4.44	4.51	4.54	4.57	4.55	4.53	4.56	4.72	4.76	4.78
1st Quart	7.76	8.34	8.89	9.34	10.06	10.46	10.76	10.99	10.97	10.82	10.74	10.87	11.01	11.12	11.15	11.25	11.27	11.30	11.39	11.45	11.50	11.52	11.52	11.54	11.65
Average	12.72	12.86	12.98	13.09	13.19	13.28	13.37	13.45	13.52	13.59	13.66	13.72	13.76	13.81	13.86	13.91	13.96	14.02	14.08	14.14	14.21	14.27	14.34	14.42	14.49
Median	12.12	12.35	12.56	12.74	12.61	12.45	12.30	12.32	12.47	12.58	13.36	13.75	13.96	14.12	14.11	14.04	14.03	13.99	14.01	13.95	13.89	13.85	13.90	13.92	13.87
3rd Quart	17.22	17.63	18.15	18.26	18.37	18.27	18.05	17.91	17.88	18.00	18.10	18.18	18.24	18.25	18.18	18.21	18.17	18.20	18.17	18.07	18.20	18.07	18.01	17.97	17.83
Max	31.41	28.58	26.62	25.44	24.29	23.74	23.00	22.54	22.29	22.48	22.93	22.63	22.37	22.09	22.06	21.88	21.71	21.44	21.31	21.13	21.18	21.17	21.35	21.31	21.28
Stdev	6.19	5.96	5.84	5.73	5.63	5.53	5.43	5.34	5.25	5.16	5.08	5.00	4.93	4.86	4.78	4.71	4.63	4.54	4.46	4.36	4.27	4.16	4.06	3.95	3.84
Kurt	2.49	2.23	2.12	2.06	2.02	2.02	2.03	2.05	2.08	2.11	2.15	2.18	2.22	2.26	2.31	2.35	2.40	2.45	2.50	2.55	2.59	2.63	2.65	2.66	2.65
Skew	0.34	0.20	0.13	0.07	0.01	-0.04	-0.09	-0.13	-0.17	-0.19	-0.21	-0.22	-0.23	-0.24	-0.24	-0.25	-0.26	-0.26	-0.27	-0.27	-0.27	-0.26	-0.24	-0.22	-0.18
CF	1.0083	1.0168	1.0256	1.0346	1.0439	1.0534	1.0633	1.0734	1.0838	1.0946	1.1057	1.1171	1.1289	1.1411	1.1536	1.1666	1.1800	1.1938	1.2081	1.2229	1.2382	1.2540	1.2704	1.2873	1.3048
COV	0.4863																								

Table 2.6b First period summary statistics of volatility cones for four Fixed Income Futures

D-Mark	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	3.69	5.74	6.74	7.57	7.89	8.35	8.71	8.85	9.11	9.21	9.30	9.55	9.58	9.58	9.53	9.40	9.49	10.10	10.24	10.35	10.22	10.26	10.22	10.20	10.24
1st Quart	8.86	9.28	9.78	10.08	9.98	10.18	10.26	10.29	10.36	10.44	10.60	10.66	10.72	10.73	10.70	10.76	10.80	10.86	10.79	10.83	10.84	10.84	10.85	10.86	10.89
Average	11.44	11.66	11.72	11.74	11.73	11.72	11.72	11.71	11.72	11.71	11.70	11.70	11.69	11.68	11.68	11.67	11.66	11.64	11.63	11.61	11.59	11.57	11.55	11.54	11.53
Median	10.74	11.05	11.13	11.38	11.37	11.47	11.69	11.56	11.45	11.37	11.34	11.35	11.34	11.33	11.34	11.26	11.27	11.22	11.21	11.18	11.13	11.12	11.13	11.11	11.06
3rd Quart	12.91	13.35	13.25	13.25	13.12	13.04	12.84	12.64	12.74	12.66	12.67	12.61	12.61	12.50	12.41	12.39	12.60	12.66	12.49	12.34	12.20	12.19	12.13	12.10	12.07
Max	24.05	21.83	21.17	19.13	17.82	17.19	17.22	17.51	16.85	16.30	15.72	15.47	15.28	15.56	15.59	15.52	15.41	15.15	14.94	14.86	14.77	14.58	14.41	14.31	14.56
Stdev	3.78	3.18	2.81	2.48	2.19	2.00	1.87	1.78	1.71	1.60	1.51	1.44	1.41	1.39	1.37	1.33	1.27	1.22	1.17	1.13	1.10	1.08	1.06	1.04	1.02
Kurt	3.80	3.40	3.35	3.22	2.90	2.83	3.04	3.48	3.57	3.30	2.98	2.83	2.88	3.16	3.45	3.65	3.72	3.72	3.70	3.71	3.70	3.60	3.48	3.48	3.65
Skew	1.01	0.90	0.81	0.73	0.62	0.59	0.68	0.81	0.90	0.88	0.84	0.85	0.88	0.96	1.05	1.11	1.18	1.24	1.28	1.30	1.28	1.25	1.21	1.21	1.25
CF	1.0135	1.0276	1.0424	1.0579	1.0741	1.0912	1.1091	1.1279	1.1476	1.1685	1.1904	1.2135	1.2380	1.2637	1.2910	1.3198	1.3503	1.3826	1.4168	1.4531	1.4915	1.5322	1.5752	1.6207	1.6688
COV	0.3309																								

B-Pound	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	4.98	5.77	6.27	7.02	7.51	7.72	8.05	7.98	8.33	8.35	8.70	8.78	9.15	9.24	9.17	9.27	9.38	9.60	9.78	9.93	10.05	10.25	10.25	10.25	10.30
1st Quart	8.58	9.24	9.26	9.47	9.70	9.83	9.90	10.07	10.05	10.18	10.35	10.34	10.39	10.53	10.75	10.82	10.82	10.89	10.96	10.94	10.90	10.88	10.85	10.84	10.83
Average	11.77	11.95	12.01	11.97	11.90	11.85	11.81	11.78	11.76	11.73	11.71	11.70	11.69	11.68	11.67	11.66	11.64	11.62	11.60	11.57	11.54	11.52	11.50	11.48	11.47
Median	10.86	10.80	11.30	11.37	11.32	11.28	11.29	11.26	11.33	11.30	11.38	11.31	11.37	11.39	11.37	11.36	11.33	11.29	11.28	11.21	11.17	11.14	11.11	11.06	11.14
3rd Quart	13.95	13.84	13.84	13.62	13.32	13.29	13.22	13.02	12.85	12.76	12.56	12.42	12.28	12.03	11.97	11.89	11.78	11.73	11.82	11.84	11.79	11.74	11.71	11.72	11.77
Max	28.39	27.91	25.61	23.13	21.69	21.04	20.80	20.68	19.85	19.10	18.53	18.24	17.97	17.82	17.49	17.28	16.98	16.75	16.46	16.24	15.96	15.76	15.47	15.23	14.97
Stdev	4.38	3.93	3.64	3.35	3.07	2.87	2.72	2.59	2.46	2.29	2.15	2.03	1.92	1.83	1.73	1.64	1.55	1.47	1.39	1.31	1.24	1.18	1.12	1.07	1.02
Kurt	5.20	5.19	4.93	4.70	4.47	4.48	4.73	4.95	5.02	4.91	4.96	5.21	5.54	5.95	6.39	6.76	7.05	7.26	7.44	7.58	7.69	7.64	7.42	7.21	7.06
Skew	1.33	1.34	1.29	1.26	1.22	1.23	1.27	1.32	1.36	1.36	1.41	1.50	1.59	1.69	1.80	1.89	1.96	2.02	2.07	2.11	2.14	2.14	2.10	2.06	2.02
CF	1.0135	1.0276	1.0424	1.0579	1.0741	1.0911	1.1090	1.1278	1.1475	1.1683	1.1902	1.2133	1.2377	1.2635	1.2907	1.3195	1.3500	1.3822	1.4164	1.4526	1.4909	1.5315	1.5745	1.6200	1.6679
COV	0.3721																								

J-Yen	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	2.87	4.99	6.13	6.11	6.38	6.75	7.79	8.23	8.09	8.87	9.24	9.22	9.12	9.23	9.25	9.16	9.29	9.93	10.46	10.59	10.48	10.48	10.45	10.35	10.42
1st Quart	8.17	8.99	9.42	9.63	9.70	9.77	10.00	10.09	10.28	10.46	10.67	10.77	10.91	11.00	11.04	11.03	11.01	11.01	10.98	10.95	10.96	10.96	11.00	11.10	11.10
Average	10.74	10.98	11.07	11.11	11.15	11.20	11.24	11.28	11.32	11.33	11.33	11.34	11.35	11.37	11.38	11.40	11.40	11.41	11.41	11.40	11.38	11.37	11.36	11.35	11.35
Median	10.40	10.63	10.71	10.64	10.83	11.05	11.19	11.22	11.37	11.43	11.43	11.46	11.36	11.33	11.37	11.37	11.41	11.43	11.47	11.53	11.51	11.48	11.45	11.42	11.37
3rd Quart	12.58	12.18	12.24	12.70	12.62	12.52	12.52	12.46	12.32	12.12	12.05	12.00	12.02	11.91	11.87	11.81	11.80	11.78	11.76	11.71	11.68	11.67	11.63	11.62	11.60
Max	24.23	19.15	17.50	15.98	14.85	14.53	14.16	14.09	14.20	13.84	13.65	13.45	13.31	13.27	13.04	12.75	12.70	12.59	12.32	12.28	12.23	12.33	12.44	12.39	12.52
Stdev	3.63	2.84	2.40	2.10	1.86	1.63	1.46	1.34	1.22	1.13	1.05	0.96	0.89	0.80	0.72	0.64	0.56	0.49	0.45	0.43	0.42	0.43	0.45	0.46	0.47
Kurt	4.77	3.47	2.72	2.44	2.30	2.08	1.92	1.94	2.13	2.22	2.31	2.50	2.95	3.49	4.15	4.72	4.14	2.56	1.91	1.91	2.02	2.26	2.53	2.66	2.70
Skew	0.98	0.73	0.51	0.28	0.15	0.09	0.00	-0.10	-0.07	-0.05	-0.08	-0.21	-0.36	-0.49	-0.60	-0.68	-0.49	-0.05	-0.04	-0.18	-0.39	-0.30	-0.22	-0.16	-0.04
CF	1.0135	1.0277	1.0425	1.0580	1.0743	1.0914	1.1093	1.1282	1.1480	1.1689	1.1909	1.2141	1.2386	1.2645	1.2918	1.3208	1.3514	1.3838	1.4182	1.4546	1.4931	1.5340	1.5772	1.6229	1.6712
COV	0.3383																								

S-Franc	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	5.10	6.71	7.79	8.61	8.82	9.16	9.85	10.01	10.02	10.26	10.35	10.42	10.34	10.40	10.49	10.40	10.50	11.31	11.41	11.53	11.47	11.46	11.36	11.32	11.39
1st Quart	9.88	10.20	10.61	10.86	11.12	11.25	11.62	11.61	11.67	12.05	12.14	12.28	12.23	12.28	12.28	12.28	12.16	12.14	12.10	12.12	12.29	12.24	12.20	12.18	12.17
Average	12.58	12.81	12.93	13.04	13.12	13.20	13.25	13.28	13.31	13.34	13.37	13.40	13.43	13.46	13.47	13.46	13.45	13.44	13.44	13.43	13.42	13.41	13.40	13.39	13.39
Median	11.74	12.10	12.35	12.56	12.83	13.11	13.28	13.19	13.17	13.06	13.05	13.07	13.12	13.04	12.99	12.93	12.89	12.89	13.01	13.09	13.15	13.23	13.22	13.21	13.23
3rd Quart	14.50	14.79	14.91	14.75	14.71	14.52	14.41	14.50	14.54	14.78	14.90	14.88	14.92	14.95	14.86	14.71	14.69	14.87	14.83	14.79	14.66	14.58	14.59	14.56	14.53
Max	24.60	23.83	22.25	20.22	19.06	18.77	19.00	18.98	18.36	17.65	17.08	16.81	16.50	16.65	16.76	16.65	16.57	16.42	16.25	16.29	16.18	15.99	15.87	15.87	15.99
Stdev	3.88	3.26	2.89	2.58	2.33	2.14	2.02	1.95	1.88	1.81	1.74	1.68	1.62	1.57	1.54	1.51	1.49	1.46	1.44	1.42	1.41	1.39	1.37	1.36	1.34
Kurt	3.67	3.29	3.07	2.74	2.52	2.58	2.74	2.82	2.66	2.42	2.22	2.11	2.05	2.04	2.04	2.00	1.90	1.79	1.74	1.73	1.72	1.72	1.71	1.69	1.67
Skew	0.98	0.86	0.74	0.62	0.51	0.49	0.52	0.50	0.45	0.39	0.31	0.25	0.22	0.25	0.28	0.30	0.34	0.37	0.37	0.35	0.32	0.31	0.31	0.31	0.32
CF	1.0125	1.0256	1.0392	1.0534	1.0682	1.0837	1.0999	1.1169	1.1347	1.1533	1.1729	1.1934	1.2150	1.2376	1.2615	1.2866	1.3131	1.3410	1.3703	1.4013	1.4340	1.4685	1.5049	1.5433	1.5838
COV	0.3082																								

Table 2.6c First period summary statistics of volatility cones for four Foreign Exchange Futures

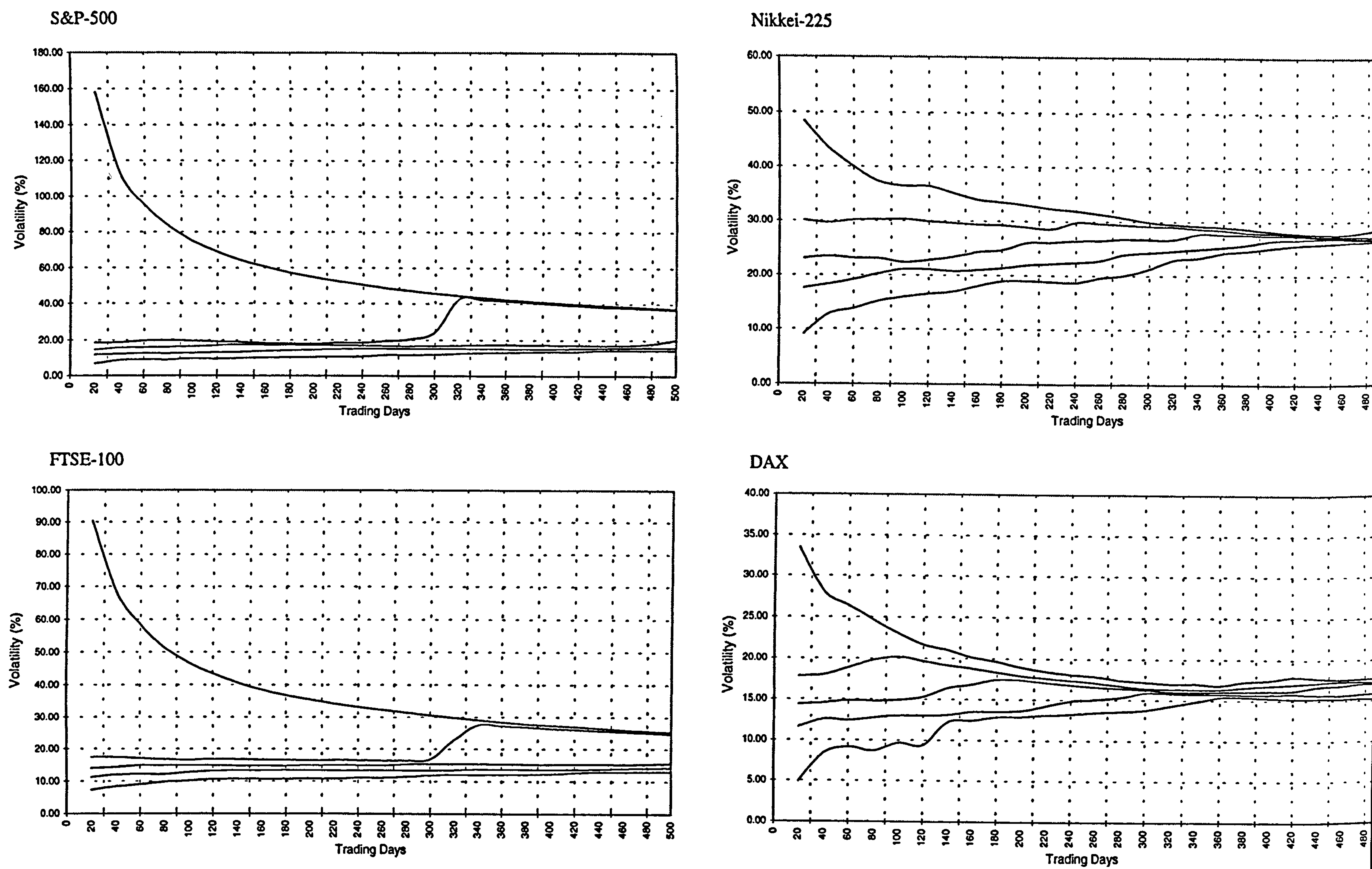

Figure 2.6a First period volatility cones for four Stock Index Futures.

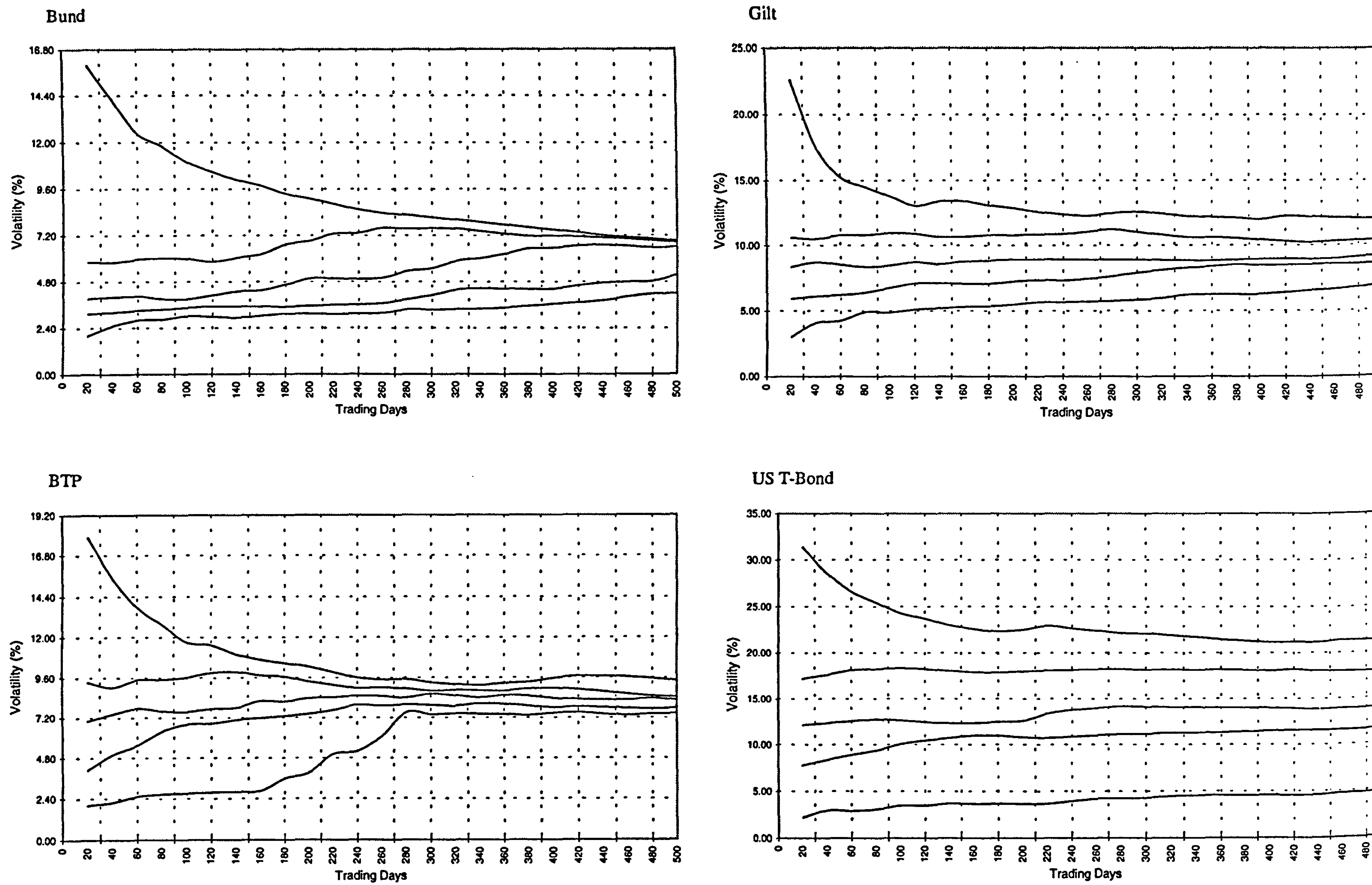

Figure 2.6b First period volatility cones for four Fixed Income Futures.

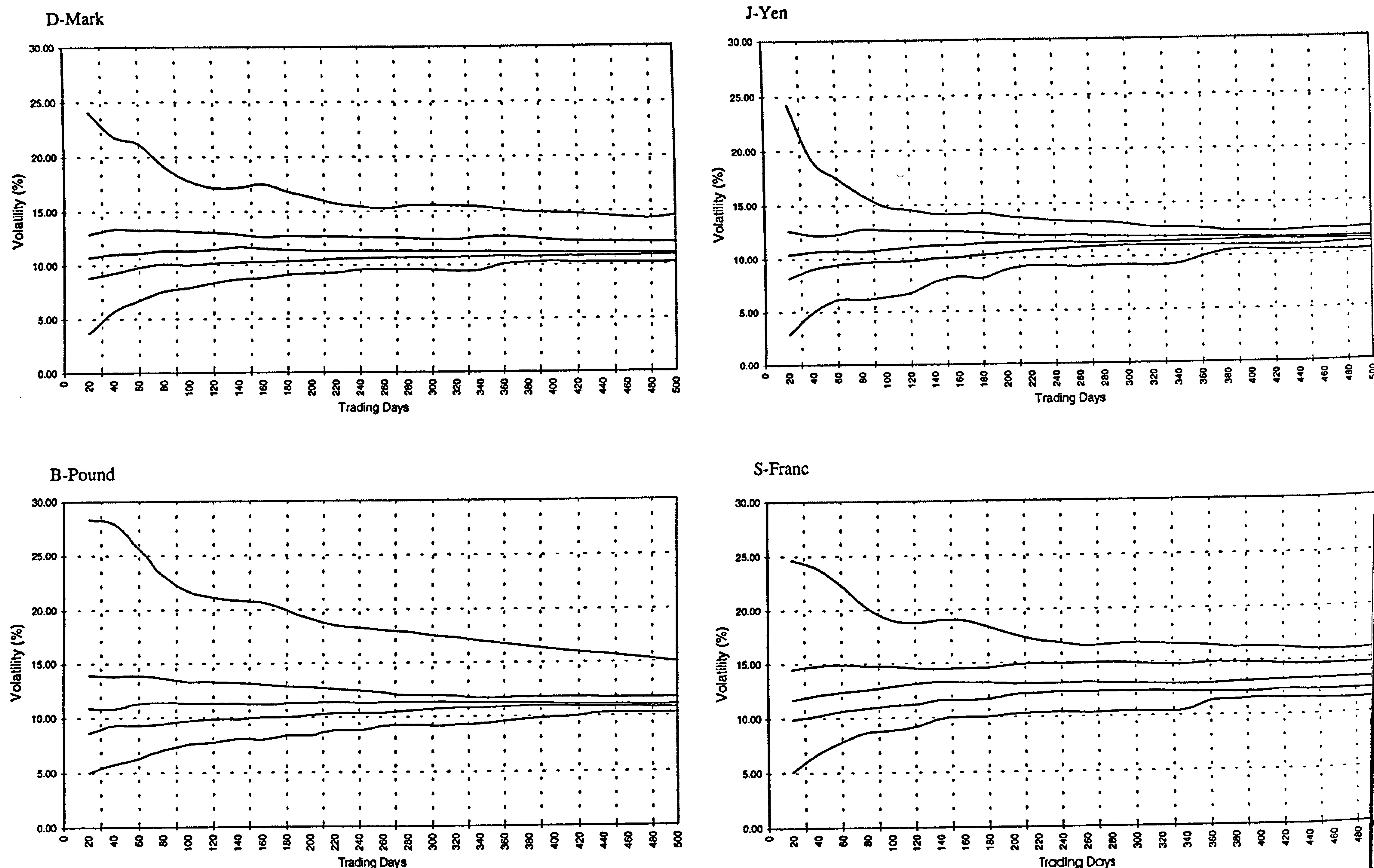

Figure 2.6c First period volatility cones for four Foreign Exchange Futures.

S&P-500	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	4.57	5.23	6.26	6.37	6.88	6.70	6.59	7.02	7.32	7.68	7.88	8.06	8.14	8.43	8.50	8.50	8.67	8.68	8.72	8.80	8.91	8.92	9.02	9.01	9.05
1st Quart	7.99	8.56	8.92	9.01	9.04	9.00	9.20	9.40	9.39	9.26	9.18	9.24	9.16	9.15	9.19	9.25	9.31	9.31	9.31	9.38	9.38	9.42	9.42	9.40	9.38
Average	10.59	10.65	10.65	10.64	10.62	10.59	10.54	10.51	10.47	10.44	10.41	10.37	10.32	10.28	10.25	10.22	10.19	10.16	10.14	10.12	10.10	10.09	10.07	10.06	10.05
Median	10.09	10.24	10.33	10.47	10.46	10.32	10.10	10.10	9.93	9.96	9.91	9.89	9.94	10.01	9.98	9.93	9.82	9.88	9.77	9.69	9.67	9.62	9.62	9.70	9.73
3rd Quart	12.49	12.42	11.99	12.06	12.65	12.42	12.32	12.21	11.97	11.84	11.69	11.56	11.46	11.35	11.24	11.06	10.96	11.05	10.99	10.92	10.87	10.81	10.77	10.68	10.68
Max	21.32	19.80	18.52	17.75	17.18	16.56	16.05	15.86	15.32	14.86	14.92	14.80	14.73	14.47	14.23	14.13	14.01	13.76	13.61	13.49	13.27	13.16	13.06	12.92	12.74
Stdev	3.38	2.79	2.48	2.28	2.11	1.99	1.88	1.79	1.72	1.66	1.59	1.51	1.42	1.33	1.26	1.20	1.14	1.08	1.04	0.99	0.95	0.91	0.88	0.84	0.82
Kurt	3.08	2.89	2.75	2.64	2.57	2.58	2.51	2.52	2.40	2.37	2.54	2.71	2.85	2.95	3.07	3.37	3.46	3.46	3.53	3.55	3.56	3.68	3.64	3.46	3.27
Skew	0.71	0.52	0.45	0.38	0.33	0.39	0.41	0.47	0.49	0.55	0.66	0.75	0.82	0.87	0.91	0.99	1.02	1.03	1.06	1.07	1.08	1.11	1.10	1.06	1.02
CF	1.0135	1.0276	1.0423	1.0578	1.0740	1.0910	1.1088	1.1276	1.1473	1.1680	1.1899	1.2130	1.2373	1.2630	1.2902	1.3189	1.3493	1.3815	1.4155	1.4516	1.4899	1.5303	1.5732	1.6185	1.6663
COV	0.3192																								

FTSE-100	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	7.29	7.97	8.32	8.93	9.07	9.30	9.74	9.72	9.69	9.86	9.86	9.79	9.78	9.83	10.16	10.22	10.23	10.20	10.46	10.44	10.46	10.66	10.70	10.85	10.89
1st Quart	10.23	10.92	11.02	11.22	11.07	11.13	11.49	11.43	11.33	11.41	11.52	11.65	11.82	12.18	12.56	12.84	13.03	13.07	13.34	13.57	13.67	13.74	13.80	13.99	14.00
Average	13.71	13.73	13.74	13.76	13.78	13.80	13.81	13.84	13.87	13.90	13.93	13.96	14.00	14.04	14.08	14.12	14.15	14.19	14.22	14.25	14.28	14.31	14.34	14.37	14.39
Median	13.15	13.20	13.60	13.68	13.81	13.71	13.80	14.04	14.06	14.04	14.09	14.18	14.30	14.41	14.43	14.43	14.50	14.59	14.64	14.66	14.68	14.59	14.57	14.45	14.39
3rd Quart	15.76	15.79	15.90	15.82	15.87	15.82	15.61	15.78	16.03	15.89	15.78	15.79	15.82	15.81	15.73	15.53	15.43	15.32	15.23	15.27	15.14	15.18	15.28	15.31	15.23
Max	31.61	26.36	24.23	22.78	21.41	20.42	20.37	19.67	19.05	18.57	18.42	18.09	17.77	17.56	17.39	17.15	16.95	16.75	16.72	16.56	16.35	16.29	16.31	16.25	16.43
Stdev	4.28	3.64	3.29	3.09	2.98	2.88	2.79	2.69	2.60	2.50	2.41	2.32	2.22	2.12	2.02	1.93	1.84	1.75	1.67	1.60	1.53	1.46	1.40	1.34	1.28
Kurt	5.18	4.41	3.66	3.09	2.61	2.29	2.16	2.05	1.94	1.87	1.87	1.90	1.94	1.99	2.07	2.19	2.34	2.53	2.72	2.93	3.12	3.26	3.35	3.37	3.32
Skew	1.26	1.03	0.81	0.62	0.48	0.37	0.30	0.23	0.15	0.07	-0.01	-0.07	-0.15	-0.24	-0.32	-0.40	-0.50	-0.61	-0.71	-0.82	-0.89	-0.95	-0.97	-0.95	-0.87
CF	1.0128	1.0261	1.0400	1.0545	1.0697	1.0856	1.1022	1.1197	1.1380	1.1571	1.1773	1.1985	1.2208	1.2442	1.2689	1.2949	1.3224	1.3514	1.3819	1.4142	1.4483	1.4843	1.5223	1.5625	1.6048
COV	0.3119																								

Nikkei-225	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	6.85	8.41	8.38	8.71	9.39	10.01	10.80	13.03	13.43	13.82	14.11	13.95	14.31	14.53	14.77	15.94	16.86	18.05	19.79	20.20	20.11	20.83	20.59	20.42	20.19
1st Quart	12.54	13.67	14.35	14.37	14.58	14.59	14.53	14.73	14.88	15.12	15.96	16.52	17.92	18.38	18.93	20.06	20.81	20.96	21.75	21.55	21.30	21.11	20.97	20.76	20.46
Average	19.31	19.40	19.40	19.37	19.40	19.44	19.54	19.66	19.78	19.94	20.12	20.32	20.55	20.81	21.09	21.35	21.57	21.73	21.78	21.79	21.77	21.68	21.51	21.33	21.12
Median	16.86	16.64	17.09	16.60	16.38	17.44	17.66	18.31	18.26	18.96	19.17	20.04	21.16	21.41	21.58	21.74	22.20	22.13	21.95	21.90	21.60	21.36	21.15	20.88	20.69
3rd Quart	25.32	24.30	25.67	25.06	24.50	23.99	24.79	25.23	24.83	24.54	23.91	24.01	24.13	23.93	23.47	23.07	22.77	22.44	22.24	22.15	22.08	21.80	21.53	21.30	21.14
Max	42.96	38.26	35.26	33.07	32.23	31.21	30.43	29.47	29.07	28.10	27.10	26.18	25.49	24.88	24.28	23.72	23.43	22.97	22.59	22.87	23.37	23.93	24.09	24.25	23.99
Stdev	8.46	7.58	7.15	6.73	6.36	6.00	5.68	5.38	5.05	4.70	4.34	3.96	3.55	3.10	2.61	2.07	1.52	1.00	0.65	0.57	0.70	0.83	0.94	0.99	1.00
Kurt	2.59	2.43	2.27	2.13	2.03	1.91	1.80	1.71	1.64	1.58	1.55	1.58	1.72	1.99	2.43	2.95	3.81	4.54	4.26	4.46	3.11	3.69	4.03	4.33	4.16
Skew	0.75	0.67	0.63	0.60	0.58	0.56	0.54	0.49	0.41	0.30	0.16	0.01	-0.19	-0.42	-0.67	-0.90	-1.16	-1.29	-1.45	-1.12	0.59	1.39	1.52	1.58	1.52
CF	1.0265	1.0553	1.0870	1.1217	1.1599	1.2021	1.2489	1.3009	1.3589	1.4235	1.4958	1.5764	1.6661	1.7652	1.8737	1.9902	2.1124	2.2368	2.3603	2.4839	2.6217	2.8208	3.2416	4.9545	
COV	0.4381																								

DAX	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	6.97	7.48	7.67	8.64	9.02	9.20	9.44	9.87	10.09	10.41	10.58	10.44	10.48	10.68	11.08	11.47	11.46	11.59	11.66	11.87	11.90	12.23	12.47	12.61	12.57
1st Quart	10.72	10.79	11.11	11.25	11.47	11.49	11.42	11.39	11.25	11.78	12.15	12.14	12.05	11.98	12.07	12.04	12.24	12.18	12.49	12.74	12.68	12.79	12.72	12.71	12.77
Average	13.24	13.34	13.39	13.40	13.38	13.35	13.35	13.37	13.39	13.39	13.39	13.39	13.39	13.40	13.43	13.45	13.43	13.43	13.45	13.47	13.48	13.51	13.51	13.50	13.51
Median	12.72	13.15	13.01	13.47	13.32	13.32	13.23	13.11	13.27	13.20	13.24	13.19	13.54	13.71	13.68	13.60	13.49	13.47	13.34	13.23	13.21	13.22	13.35	13.30	13.76
3rd Quart	15.47	15.19	15.10	15.43	15.31	15.15	15.03	14.86	14.88	14.73	14.62	14.69	14.55	14.41	14.42	14.33	14.30	14.66	14.80	14.73	14.63	14.46	14.29	14.24	14.16
Max	25.69	22.17	20.03	18.86	18.15	17.44	17.59	17.26	16.99	16.69	16.53	16.33	16.16	15.96	15.88	15.76	15.72	15.40	15.23	15.05	14.98	14.86	14.70	14.59	14.37
Stdev	3.74	3.21	2.96	2.71	2.49	2.32	2.22	2.12	2.04	1.93	1.83	1.74	1.65	1.56	1.45	1.39	1.32	1.22	1.14	1.05	0.97	0.86	0.80	0.73	0.65
Kurt	3.78	3.15	2.53	2.23	2.06	1.95	1.98	1.95	1.94	1.93	1.94	1.92	1.90	1.87	1.79	1.80	1.79	1.74	1.73	1.69	1.64	1.44	1.33	1.26	1.29
Skew	0.76	0.62	0.42	0.26	0.13	0.01	0.02	0.06	0.13	0.13	0.09	0.02	0.01	0.08	0.17	0.17	0.11	0.06	0.08	0.13	0.19	0.29	0.21	0.06	-0.15
CF	1.0337	1.0713	1.1136	1.1612	1.2153	1.2769	1.3473	1.4281	1.5210	1.6276	1.7490	1.8850	2.0336	2.1892	2.3450	2.5010	2.6878	3.0471	4.6035						
COV	0.2824																								

Table 2.7a Second period summary statistics of volatility cones for four Stock Index Futures

Bund	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	2.28	2.58	3.00	3.17	3.30	3.46	3.48	3.50	3.61	3.70	3.78	3.76	3.78	3.76	3.75	3.80	4.37	4.55	4.84	5.14	5.12	5.18	5.19	5.20	5.22
1st Quart	3.80	3.86	3.88	4.13	4.21	4.22	4.31	4.85	5.08	5.17	5.23	5.18	5.17	5.21	5.29	5.43	5.49	5.53	5.57	5.61	5.73	5.81	5.92	5.99	6.03
Average	5.29	5.38	5.44	5.50	5.56	5.61	5.66	5.71	5.75	5.79	5.83	5.86	5.90	5.94	5.99	6.05	6.10	6.14	6.17	6.19	6.21	6.22	6.23	6.23	6.24
Median	4.54	4.86	5.07	5.27	5.54	5.61	5.60	5.61	5.52	5.50	5.53	5.59	5.65	5.66	5.69	5.91	5.99	6.05	6.08	6.25	6.34	6.35	6.35	6.35	6.31
3rd Quart	6.62	6.96	6.60	6.53	6.34	6.21	6.37	6.54	6.55	6.69	6.77	6.82	6.90	6.99	7.05	6.98	6.86	6.84	6.78	6.76	6.66	6.58	6.57	6.62	6.59
Max	12.07	10.23	9.61	9.38	9.27	9.02	8.77	8.64	8.58	8.37	8.14	7.98	7.80	7.71	7.55	7.39	7.43	7.38	7.25	7.13	7.08	6.95	6.85	6.75	6.74
Stdev	2.02	1.85	1.74	1.65	1.58	1.51	1.45	1.38	1.33	1.27	1.22	1.17	1.12	1.06	0.99	0.90	0.81	0.73	0.67	0.61	0.56	0.52	0.49	0.45	0.42
Kurt	3.11	2.51	2.33	2.37	2.45	2.47	2.43	2.42	2.39	2.31	2.22	2.18	2.16	2.20	2.23	2.11	1.79	1.77	1.69	1.72	1.90	2.08	2.36	2.68	3.00
Skew	0.94	0.74	0.64	0.62	0.61	0.58	0.52	0.46	0.38	0.30	0.21	0.11	0.01	-0.08	-0.14	-0.12	-0.02	-0.03	-0.04	-0.12	-0.28	-0.46	-0.67	-0.86	-0.98
CF	1.0199	1.0412	1.0640	1.0884	1.1146	1.1428	1.1732	1.2060	1.2415	1.2799	1.3216	1.3669	1.4161	1.4697	1.5280	1.5913	1.6599	1.7340	1.8136	1.8983	1.9875	2.0801	2.1745	2.2691	2.3629
COV	0.3818																								

BTP	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	4.09	5.45	5.76	6.14	6.03	6.26	6.58	6.78	6.90	7.23	8.01	8.13	8.23	8.18	8.42	8.65	8.59	8.65	8.69	8.77	8.91	9.18	9.43	9.36	9.29
1st Quart	7.31	7.37	7.30	7.96	8.89	9.11	9.38	9.41	9.40	9.61	9.41	9.23	9.20	9.18	9.31	9.38	9.35	9.47	9.90	9.80	9.68	9.59	9.49	9.40	9.34
Average	9.63	9.74	9.74	9.75	9.76	9.77	9.80	9.85	9.90	9.95	9.99	9.99	9.99	9.99	9.99	9.98	9.98	9.99	10.01	10.01	10.02	10.02	9.99	9.94	9.91
Median	9.20	9.44	9.90	9.91	9.81	9.87	9.99	10.15	10.07	10.09	10.07	10.12	10.09	10.09	10.06	10.10	10.19	10.10	10.00	9.89	9.75	9.71	9.65	9.74	9.70
3rd Quart	11.64	11.53	11.10	10.83	10.68	10.69	10.58	10.57	10.50	10.59	10.56	10.54	10.51	10.52	10.55	10.50	10.32	10.23	10.29	10.45	10.48	10.44	10.52	10.36	10.39
Max	18.55	16.31	15.31	14.80	14.18	13.32	12.63	12.12	11.69	11.96	12.02	11.92	11.87	11.73	11.63	11.42	11.35	11.31	11.23	11.14	11.05	11.11	11.14	11.00	10.95
Stdev	3.06	2.71	2.42	2.16	1.89	1.64	1.42	1.21	1.07	0.98	0.97	0.94	0.92	0.88	0.83	0.79	0.75	0.71	0.65	0.60	0.57	0.57	0.60	0.60	0.60
Kurt	2.79	2.38	2.35	2.60	2.84	3.03	3.37	3.57	3.47	2.80	2.59	2.54	2.50	2.37	2.23	2.15	2.29	2.55	2.66	2.53	2.25	2.07	2.04	1.91	1.82
Skew	0.65	0.49	0.36	0.24	0.08	-0.22	-0.60	-0.83	-0.89	-0.47	-0.15	-0.06	0.06	0.10	0.02	-0.15	-0.26	-0.19	-0.08	0.08	0.35	0.75	0.78	0.69	0.58
CF	1.0318	1.0670	1.1063	1.1504	1.1999	1.2559	1.3194	1.3916	1.4740	1.5680	1.6747	1.7948	1.9276	2.0705	2.2183	2.3653	2.5135	2.6931	3.0291	4.2875					
COV	0.3176																								

Gilt	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	3.22	3.86	4.32	4.62	5.00	5.36	5.58	5.78	5.89	5.93	5.97	6.11	6.23	6.18	6.28	6.44	6.51	6.52	6.60	6.83	6.90	6.91	6.93	6.93	6.95
1st Quart	5.94	6.32	6.45	6.53	6.63	6.70	6.90	6.97	7.04	7.07	7.14	7.14	7.14	7.24	7.29	7.29	7.42	7.48	7.59	7.63	7.66	7.69	7.74	7.81	7.77
Average	8.10	8.20	8.25	8.29	8.31	8.32	8.33	8.34	8.34	8.35	8.35	8.35	8.35	8.35	8.36	8.36	8.37	8.38	8.39	8.40	8.41	8.42	8.43	8.45	8.46
Median	7.34	7.40	7.46	7.77	7.83	7.84	7.83	7.82	7.75	7.72	7.76	8.07	8.10	8.10	8.09	8.08	8.10	8.12	8.16	8.12	8.07	8.04	8.22	8.29	8.46
3rd Quart	9.68	10.18	9.70	9.36	9.54	9.42	9.29	9.25	9.27	9.21	9.12	9.15	9.40	9.42	9.57	9.51	9.54	9.51	9.50	9.52	9.47	9.39	9.34	9.31	9.27
Max	17.56	14.85	14.22	14.16	13.85	13.54	13.16	12.75	12.36	11.91	11.69	11.63	11.48	11.19	10.95	10.80	10.58	10.38	10.19	10.03	9.89	9.77	9.68	9.57	9.54
Stdev	2.87	2.55	2.38	2.26	2.15	2.05	1.96	1.87	1.78	1.69	1.61	1.53	1.46	1.39	1.32	1.24	1.17	1.10	1.03	0.98	0.93	0.89	0.85	0.82	0.79
Kurt	3.00	2.56	2.61	2.77	2.77	2.75	2.66	2.58	2.49	2.40	2.31	2.24	2.16	2.09	1.99	1.92	1.87	1.82	1.75	1.68	1.62	1.58	1.56	1.56	1.57
Skew	0.89	0.74	0.73	0.80	0.82	0.82	0.81	0.80	0.78	0.76	0.71	0.67	0.61	0.56	0.50	0.46	0.42	0.39	0.35	0.29	0.20	0.10	0.01	-0.07	-0.15
CF	1.0114	1.0233	1.0356	1.0484	1.0617	1.0756	1.0901	1.1052	1.1209	1.1373	1.1544	1.1723	1.1910	1.2106	1.2310	1.2525	1.2750	1.2985	1.3233	1.3492	1.3764	1.4050	1.4350	1.4665	1.4996
COV	0.3546																								

US T-Bond	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	3.79	5.28	5.67	5.95	6.10	6.79	6.95	7.20	7.13	7.24	7.29	7.39	7.47	7.52	7.50	7.45	7.44	7.50	7.53	7.58	7.57	7.55	7.60	7.61	7.66
1st Quart	7.55	7.83	7.97	7.97	7.97	7.98	7.99	8.07	8.13	8.19	8.24	8.26	8.26	8.31	8.37	8.41	8.44	8.46	8.47	8.50	8.52	8.58	8.74	8.77	8.72
Average	9.38	9.43	9.46	9.48	9.49	9.49	9.47	9.44	9.42	9.39	9.36	9.33	9.31	9.29	9.27	9.26	9.25	9.23	9.22	9.21	9.20	9.18	9.18	9.17	9.16
Median	8.88	8.84	8.96	9.02	9.11	9.22	9.17	9.10	9.07	9.07	9.10	9.16	9.21	9.25	9.21	9.20	9.21	9.23	9.25	9.26	9.25	9.24	9.21	9.20	9.19
3rd Quart	10.52	10.52	10.46	10.33	10.22	10.19	10.19	10.29	10.22	10.14	10.05	9.92	9.84	9.80	9.86	9.85	9.78	9.72	9.64	9.59	9.54	9.50	9.48	9.47	9.46
Max	27.10	21.85	19.51	18.65	17.99	17.29	16.66	16.33	16.57	16.19	15.68	15.28	14.84	14.56	14.55	14.27	14.08	13.87	13.68	13.49	13.37	13.16	12.99	12.85	12.69
Stdev	2.94	2.53	2.34	2.23	2.13	2.05	1.94	1.85	1.76	1.67	1.59	1.52	1.46	1.41	1.35	1.30	1.26	1.21	1.17	1.13	1.09	1.05	1.02	1.00	0.97
Kurt	10.63	8.01	7.56	7.32	7.13	7.22	7.07	6.97	6.95	6.70	6.50	6.40	6.28	6.30	6.27	6.14	6.04	5.92	5.82	5.75	5.68	5.60	5.53	5.50	5.45
Skew	2.04	1.79	1.77	1.80	1.83	1.91	1.89	1.86	1.83	1.76	1.70	1.66	1.62	1.60	1.56	1.50	1.45	1.39	1.34	1.30	1.25	1.21	1.17	1.15	1.12
CF	1.0083	1.0168	1.0256	1.0346	1.0439	1.0534	1.0633	1.0734	1.0838	1.0945	1.1056	1.1170	1.1288	1.1410	1.1535	1.1665	1.1799	1.1937	1.2080	1.2228	1.2380	1.2538	1.2702	1.2871	1.3046
COV	0.3133																								

Table 2.7b Second period summary statistics of volatility cones for four Fixed Income Futures

D-Mark	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	4.09	5.40	5.43	5.59	5.70	5.87	5.91	6.09	6.22	6.36	6.34	6.42	6.50	6.67	6.92	7.71	7.79	8.23	8.25	8.78	9.13	9.70	9.86	9.89	9.86
1st Quart	8.42	8.57	8.90	8.99	9.16	9.47	9.78	9.76	9.73	9.69	9.65	9.76	9.94	10.01	10.10	10.16	10.26	10.34	10.41	10.65	10.67	10.73	10.70	10.70	10.77
Average	10.90	11.05	11.14	11.20	11.23	11.26	11.29	11.32	11.33	11.35	11.38	11.41	11.44	11.47	11.50	11.52	11.54	11.56	11.58	11.59	11.60	11.60	11.58	11.56	11.53
Median	10.11	10.79	11.07	11.05	11.18	11.16	11.18	11.29	11.59	11.67	11.89	11.90	12.19	12.18	12.03	11.90	11.73	11.58	11.56	11.57	11.52	11.45	11.38	11.33	11.30
3rd Quart	12.87	12.89	13.51	13.68	13.57	13.66	13.75	13.58	13.51	13.42	13.29	13.09	12.94	12.88	12.84	12.77	12.78	12.66	12.53	12.41	12.34	12.28	12.31	12.44	12.30
Max	22.37	19.01	18.74	17.07	16.53	16.51	15.76	15.56	15.28	14.90	15.09	14.89	14.61	14.53	14.36	14.08	13.84	13.80	13.84	14.07	14.12	14.14	14.02	13.96	13.94
Stdev	3.83	3.37	3.16	2.98	2.82	2.70	2.60	2.49	2.37	2.26	2.15	2.05	1.94	1.81	1.68	1.56	1.47	1.39	1.31	1.24	1.18	1.14	1.11	1.08	1.05
Kurt	3.03	2.47	2.16	2.07	2.08	2.15	2.21	2.27	2.34	2.40	2.50	2.58	2.62	2.59	2.44	2.26	2.24	2.28	2.24	2.15	2.21	2.31	2.44	2.52	2.57
Skew	0.73	0.40	0.16	0.00	-0.12	-0.20	-0.29	-0.38	-0.47	-0.54	-0.58	-0.61	-0.62	-0.62	-0.58	-0.50	-0.44	-0.35	-0.18	0.08	0.30	0.48	0.58	0.64	0.67
CF	1.0135	1.0276	1.0424	1.0579	1.0741	1.0912	1.1091	1.1279	1.1476	1.1685	1.1904	1.2135	1.2380	1.2637	1.2910	1.3198	1.3503	1.3826	1.4168	1.4531	1.4915	1.5322	1.5752	1.6207	1.6688
COV	0.3515																								

B-Pound	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	3.13	3.43	3.87	4.15	4.28	4.78	5.11	5.05	5.02	5.22	5.35	5.45	5.44	5.45	5.66	6.28	6.35	6.36	6.59	6.96	7.03	7.30	7.82	7.82	7.83
1st Quart	6.92	7.07	7.65	7.68	7.78	7.73	7.70	7.81	7.94	8.02	8.04	8.20	8.43	8.72	8.77	8.76	8.81	8.77	8.78	8.82	8.80	8.72	8.69	8.69	8.73
Average	10.20	10.39	10.48	10.55	10.59	10.63	10.67	10.71	10.73	10.76	10.80	10.83	10.88	10.91	10.95	10.98	11.02	11.06	11.10	11.13	11.17	11.20	11.21	11.21	11.20
Median	9.37	9.61	9.56	9.80	10.23	10.32	10.50	10.43	10.28	10.11	10.06	9.90	9.83	9.97	10.05	10.26	10.16	10.24	10.27	10.27	10.18	10.22	10.44	10.43	10.70
3rd Quart	12.93	13.42	13.44	13.35	13.17	13.16	13.29	13.36	13.33	13.42	13.38	13.45	13.57	13.40	13.48	13.63	13.82	14.05	14.24	14.26	14.20	14.07	13.92	13.78	13.88
Max	25.84	21.99	21.13	20.89	20.05	19.12	18.34	17.87	17.43	17.08	16.77	16.57	16.39	16.04	15.90	15.57	15.29	15.04	14.82	14.67	14.73	14.60	14.50	14.60	14.51
Stdev	4.44	4.02	3.86	3.75	3.64	3.55	3.46	3.39	3.32	3.25	3.19	3.12	3.05	2.98	2.91	2.84	2.79	2.74	2.69	2.64	2.61	2.57	2.54	2.51	2.48
Kurt	3.20	2.74	2.50	2.43	2.40	2.33	2.23	2.16	2.09	2.02	1.96	1.90	1.85	1.79	1.72	1.64	1.57	1.49	1.40	1.33	1.27	1.23	1.22	1.24	1.26
Skew	0.76	0.59	0.48	0.44	0.41	0.37	0.32	0.27	0.23	0.21	0.20	0.18	0.18	0.18	0.18	0.18	0.16	0.15	0.14	0.14	0.12	0.10	0.08	0.05	0.02
CF	1.0135	1.0276	1.0424	1.0579	1.0741	1.0912	1.1091	1.1279	1.1476	1.1685	1.1904	1.2135	1.2380	1.2637	1.2910	1.3198	1.3503	1.3826	1.4168	1.4531	1.4915	1.5322	1.5752	1.6207	1.6688
COV	0.4354																								

J-Yen	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	4.53	5.62	6.04	6.56	6.58	6.83	7.14	7.18	7.07	7.09	7.14	7.08	7.28	7.63	7.75	8.81	8.74	8.87	8.82	8.86	8.82	8.82	8.94	9.05	9.07
1st Quart	7.45	7.88	7.97	8.25	8.40	8.59	8.78	9.01	9.13	9.24	9.21	9.26	9.31	9.46	9.74	9.95	9.98	9.99	10.26	10.41	10.57	10.51	10.46	10.59	10.58
Average	10.28	10.42	10.51	10.56	10.62	10.67	10.72	10.77	10.82	10.87	10.93	10.98	11.04	11.10	11.14	11.19	11.21	11.23	11.25	11.27	11.28	11.29	11.28	11.27	11.27
Median	9.12	9.23	9.58	9.75	9.77	9.86	9.95	10.17	10.36	10.54	10.92	10.86	10.96	11.15	11.18	11.23	11.16	11.28	11.59	11.54	11.52	11.54	11.46	11.35	11.29
3rd Quart	11.83	12.13	13.01	12.52	12.44	12.28	12.41	12.68	12.41	12.37	12.25	12.32	12.31	12.40	12.35	12.28	12.20	12.36	12.42	12.48	12.32	12.20	12.06	12.09	12.09
Max	22.67	20.42	19.45	17.54	16.96	17.42	17.89	17.41	16.84	16.16	15.67	15.21	14.72	14.32	13.97	13.73	13.57	13.43	13.29	13.18	13.52	13.41	13.34	13.15	12.97
Stdev	3.90	3.37	3.11	2.91	2.72	2.58	2.47	2.37	2.27	2.17	2.07	1.96	1.85	1.73	1.62	1.52	1.45	1.38	1.32	1.26	1.21	1.17	1.13	1.09	1.05
Kurt	3.73	2.92	2.62	2.37	2.35	2.46	2.64	2.80	2.81	2.74	2.62	2.50	2.35	2.16	1.98	1.83	1.81	1.80	1.87	2.02	2.21	2.35	2.40	2.39	2.35
Skew	1.17	0.94	0.83	0.74	0.69	0.65	0.65	0.64	0.60	0.51	0.42	0.31	0.23	0.19	0.13	0.07	-0.05	-0.16	-0.27	-0.38	-0.46	-0.50	-0.49	-0.45	-0.41
CF	1.0135	1.0277	1.0425	1.0580	1.0743	1.0913	1.1092	1.1281	1.1479	1.1687	1.1907	1.2139	1.2384	1.2642	1.2916	1.3205	1.3510	1.3834	1.4177	1.4541	1.4926	1.5334	1.5765	1.6222	1.6704
COV	0.3793																								

S-Franc	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
Min	4.76	6.42	6.41	6.78	6.99	7.20	7.26	7.43	7.46	7.80	7.95	8.04	8.01	8.13	8.35	9.18	9.29	9.70	9.77	10.46	10.77	10.79	11.05	11.14	11.08
1st Quart	9.52	10.08	10.29	10.56	10.68	10.74	10.77	10.84	10.80	10.85	10.87	11.06	11.32	11.59	11.52	11.72	11.87	12.23	12.32	12.55	12.66	12.61	12.55	12.50	12.57
Average	12.28	12.43	12.50	12.56	12.62	12.67	12.71	12.75	12.80	12.84	12.88	12.92	12.96	13.00	13.04	13.08	13.11	13.14	13.17	13.19	13.20	13.21	13.20	13.19	13.18
Median	11.73	12.03	12.46	12.66	12.81	12.85	13.03	13.15	13.32	13.48	13.39	13.37	13.38	13.45	13.65	13.64	13.58	13.55	13.50	13.47	13.40	13.32	13.24	13.14	13.12
3rd Quart	14.38	14.37	14.50	14.27	14.15	14.13	14.21	14.15	14.23	14.63	14.74	14.77	14.75	14.54	14.37	14.26	14.20	14.10	14.14	14.09	14.00	13.94	13.92	14.06	14.09
Max	27.27	23.05	22.86	20.74	19.12	18.67	18.29	17.63	16.93	16.50	15.96	15.84	15.63	15.52	15.42	15.17	15.06	15.05	14.90	14.72	14.68	14.84	14.76	14.73	14.71
Stdev	3.87	3.32	3.09	2.91	2.77	2.66	2.56	2.46	2.37	2.27	2.16	2.06	1.94	1.81	1.67	1.54	1.43	1.33	1.23	1.12	1.04	0.98	0.94	0.92	0.89
Kurt	3.62	3.17	3.10	2.95	2.65	2.51	2.45	2.41	2.37	2.33	2.36	2.43	2.53	2.59	2.54	2.46	2.54	2.65	2.62	2.40	2.42	2.42	2.39	2.31	2.24
Skew	0.83	0.59	0.43	0.30	0.15	0.00	-0.14	-0.26	-0.38	-0.47	-0.55	-0.63	-0.70	-0.75	-0.76	-0.77	-0.81	-0.84	-0.80	-0.67	-0.57	-0.46	-0.36	-0.28	-0.21
CF	1.0125	1.0255	1.0391	1.0533	1.0681	1.0836	1.0996	1.1167	1.1345	1.1531	1.1726	1.1931	1.2146	1.2372	1.2611	1.2861	1.3125	1.3403	1.3696	1.4006	1.4332	1.4676	1.5039	1.5422	1.5825
COV	0.3148																								

Table 2.7c Second period summary statistics of volatility cones for four Foreign Exchange Futures

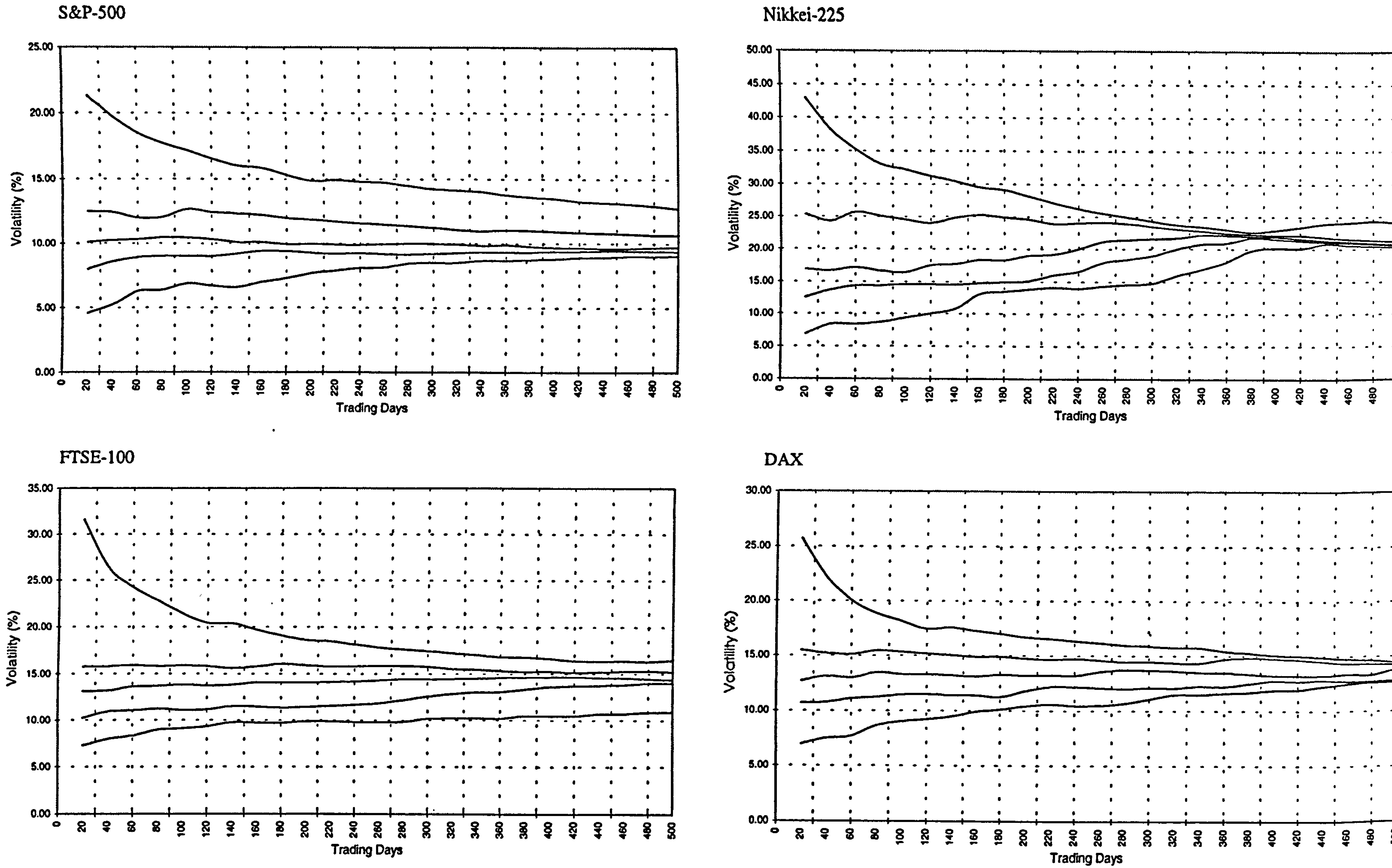

Figure 2.7a Second period volatility cones for four Stock Index Futures.

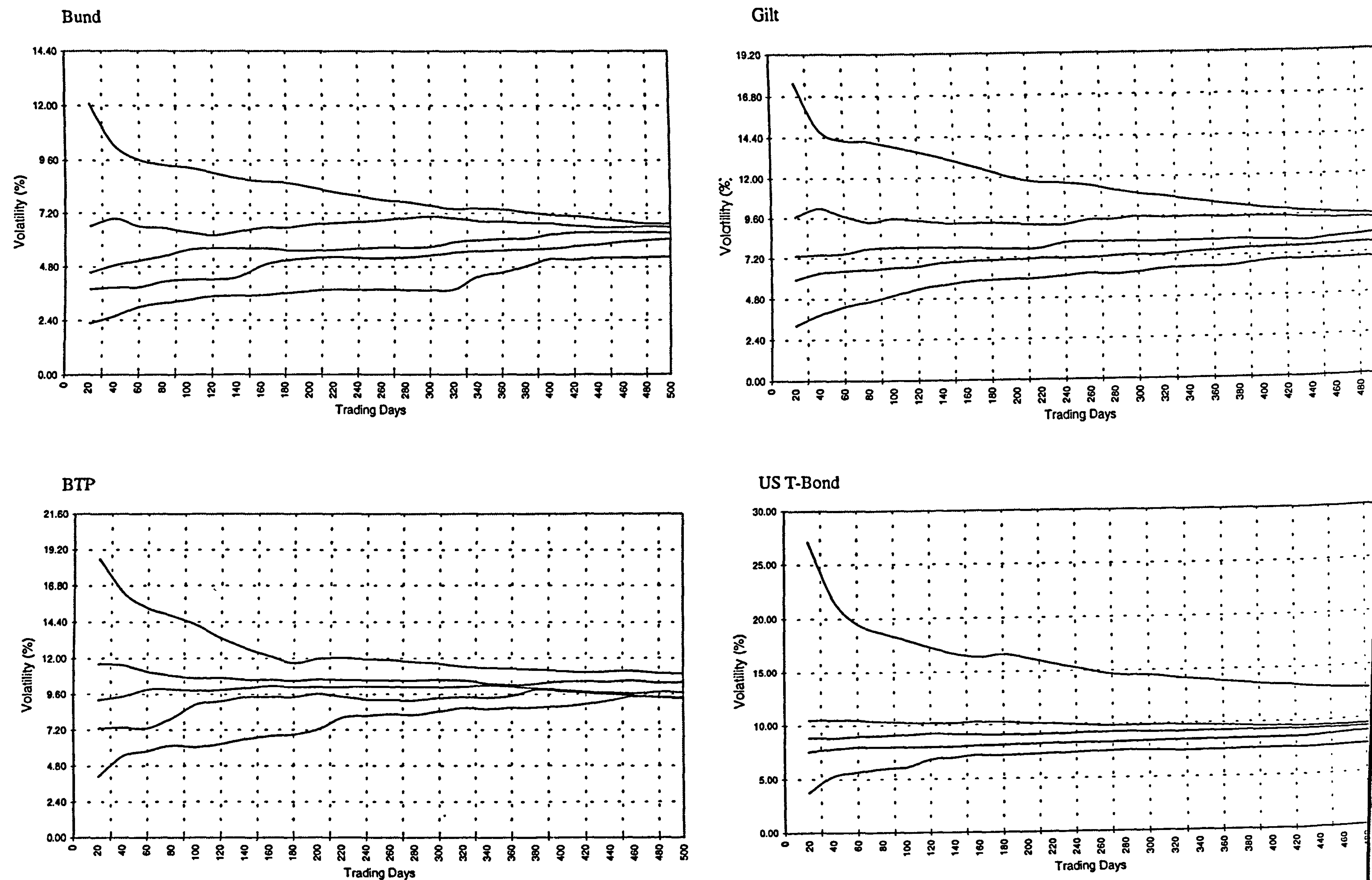

Figure 2.7b Second period volatility cones for four Fixed Income Futures.

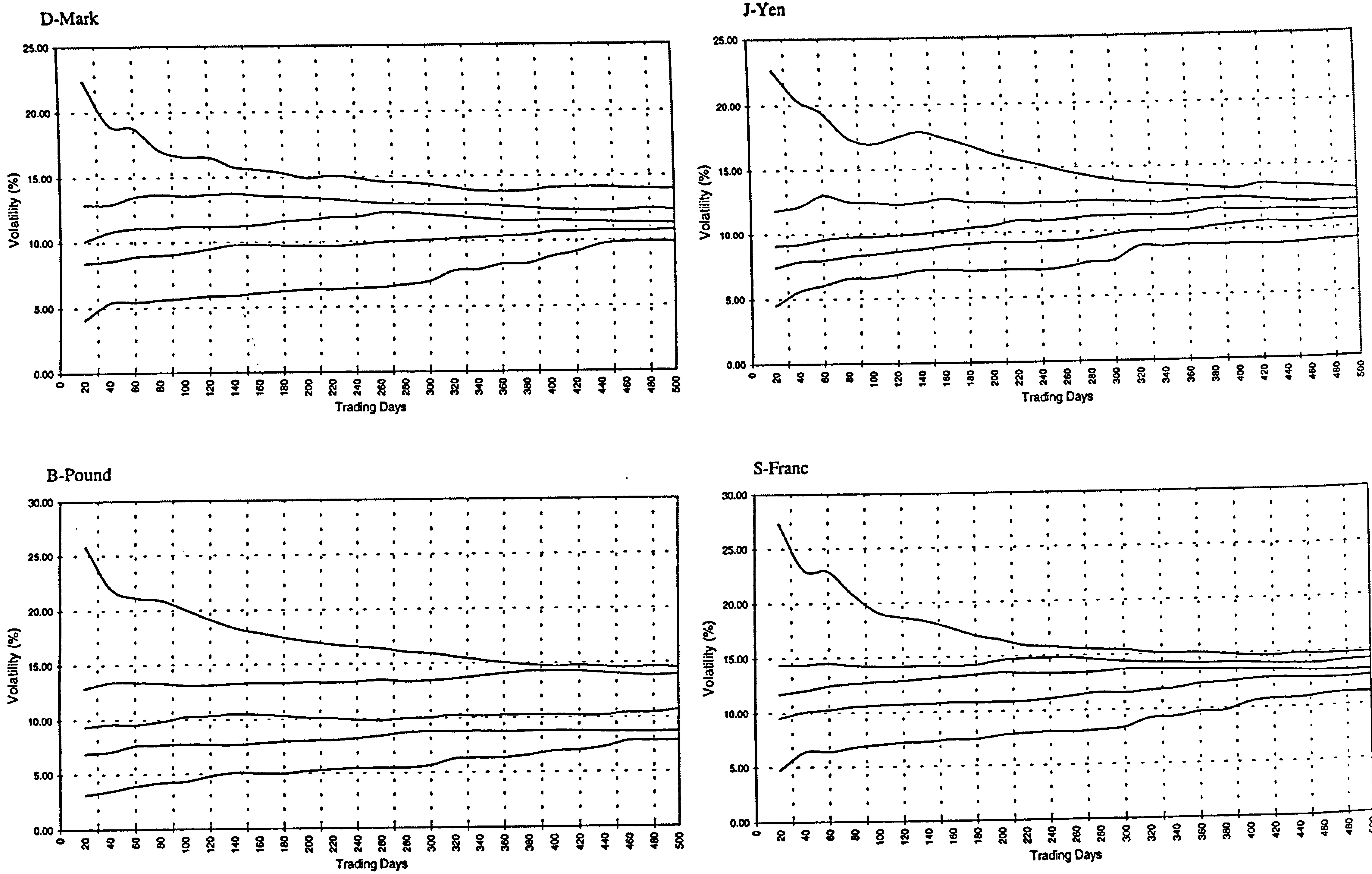

Figure 2.7c Second period volatility cones for four Foreign Exchange Futures.

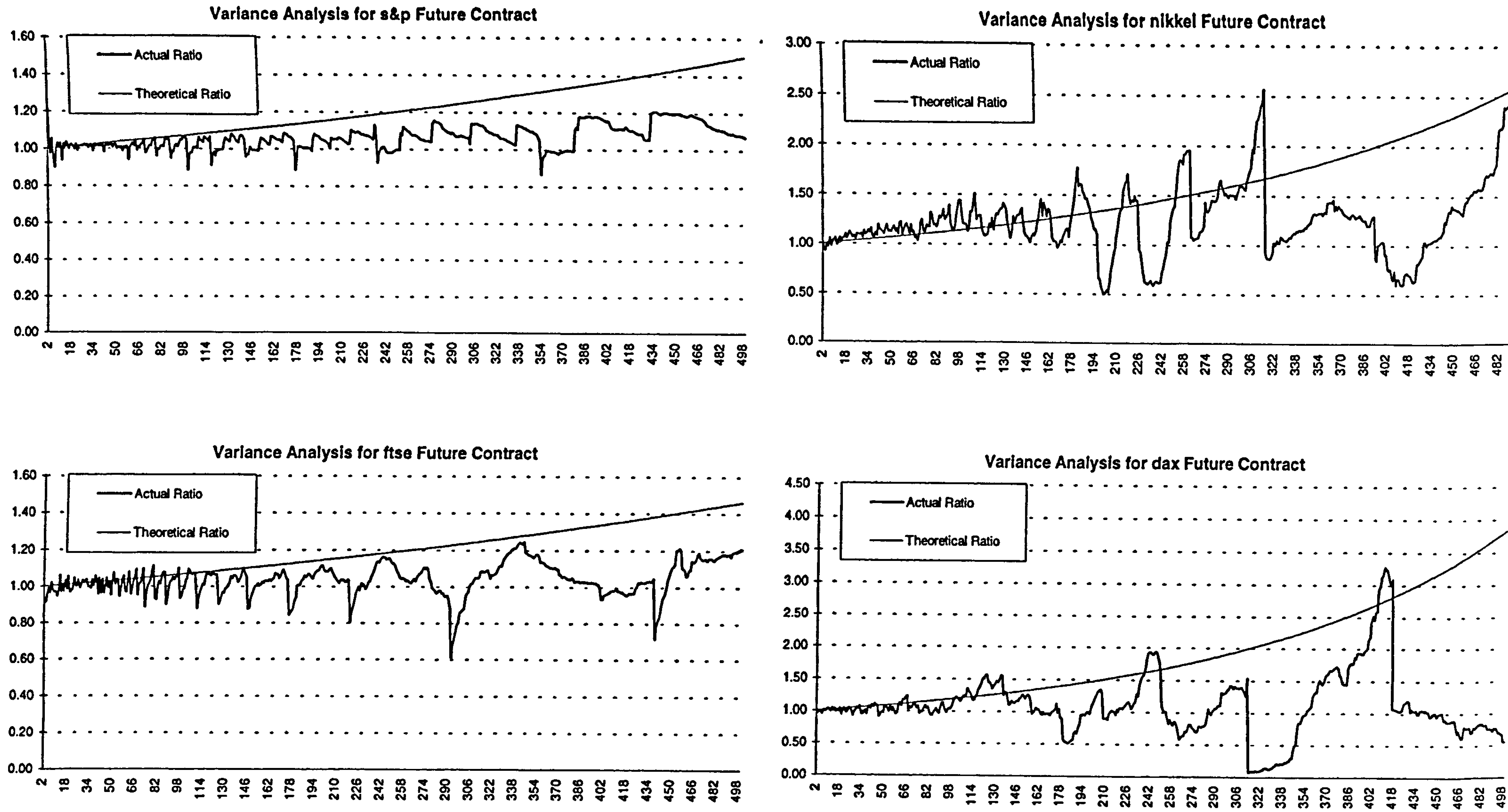

Figure 2.8a Comparison of volatility estimated using overlapping and non overlapping observations for four Stock Index Futures.

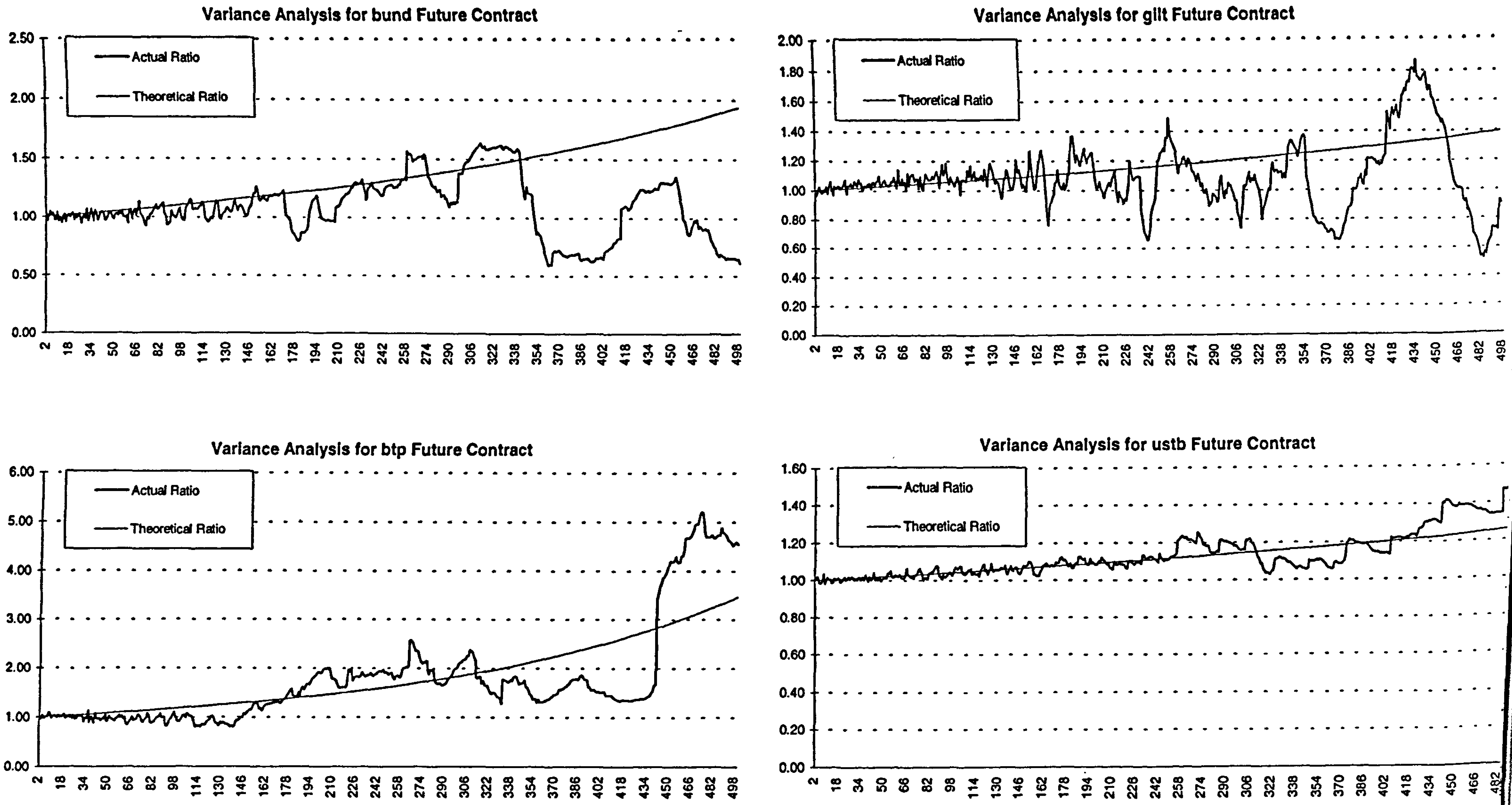

Figure 2.8b Comparison of volatility estimated using overlapping and non overlapping observations for four Fixed Income Futures.

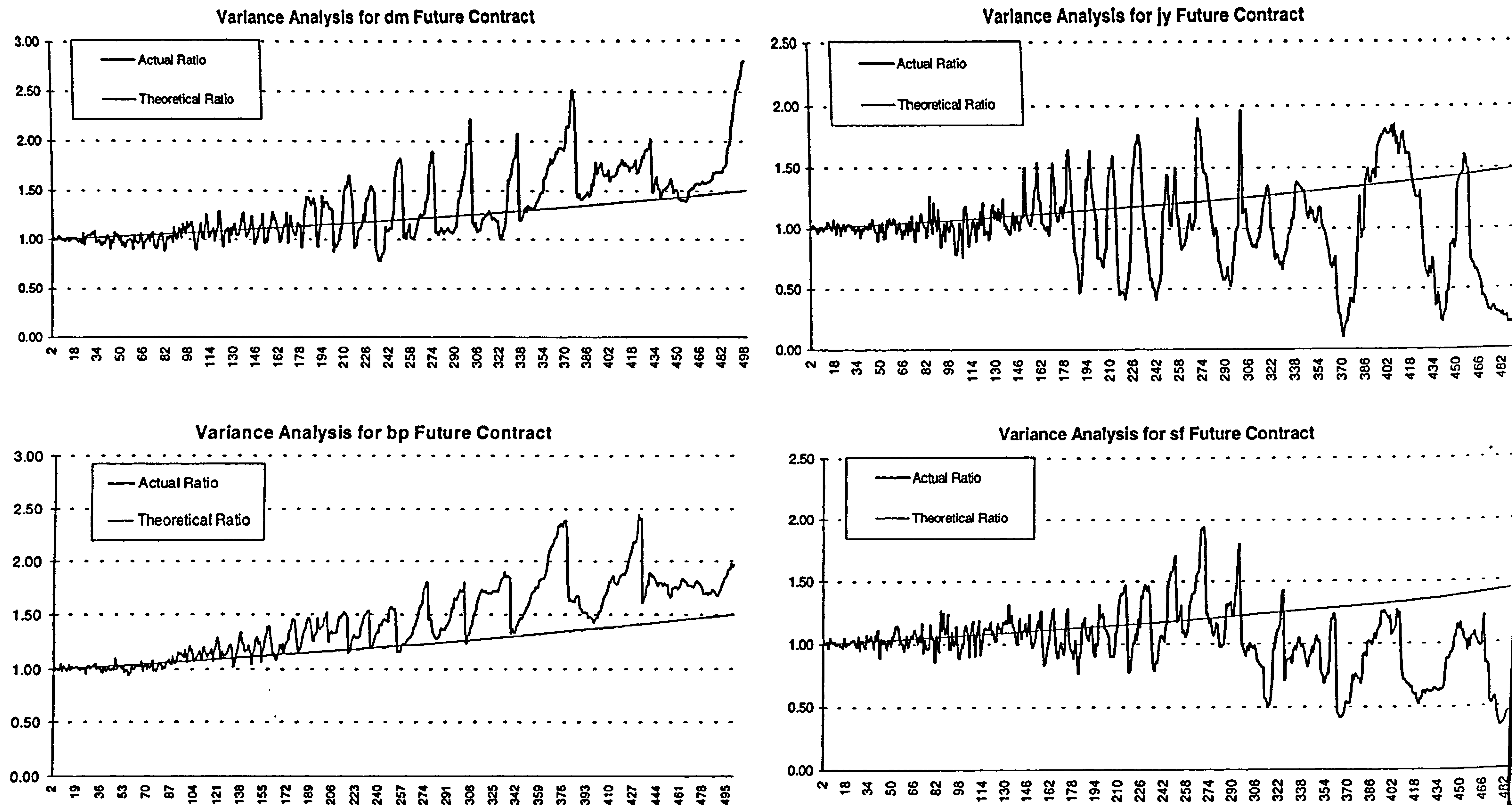

Figure 2.8c Comparison of volatility estimated using overlapping and non overlapping observations for four Foreign Exchange Futures.

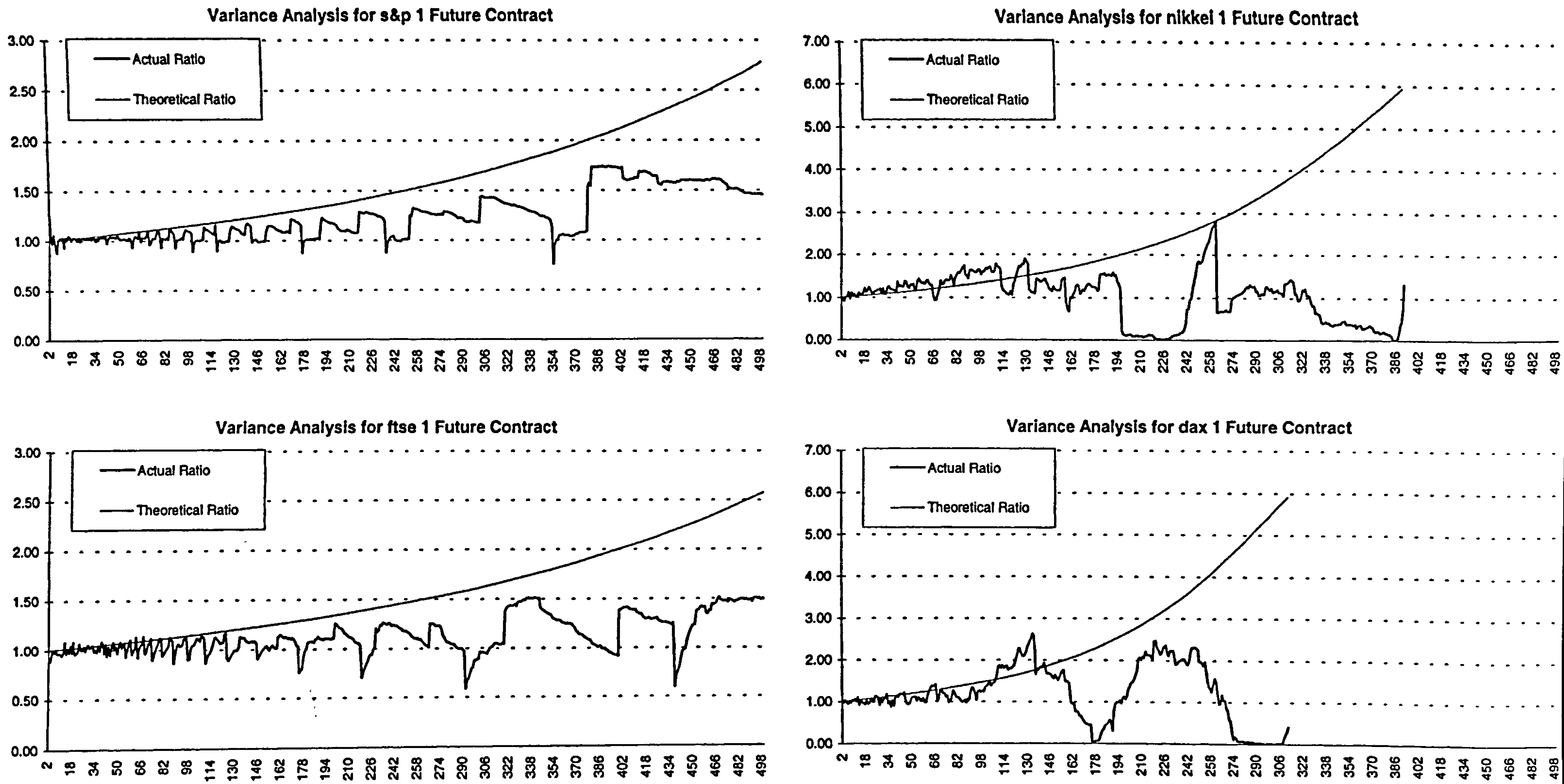

Figure 2.9a First period comparison of volatility estimated using overlapping and non overlapping observations for four Stock Index Futures.

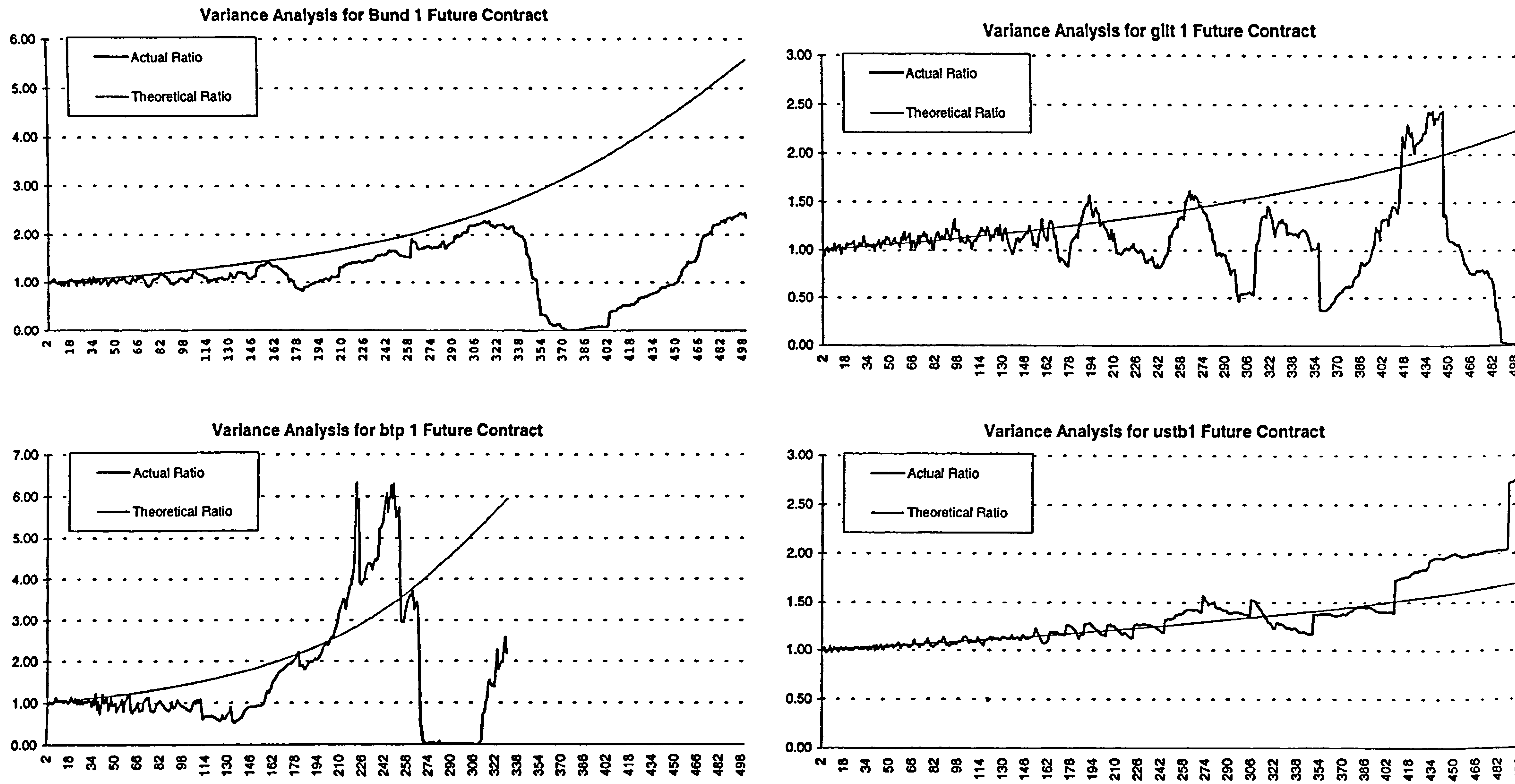

Figure 2.9b First period comparison of volatility estimated using overlapping and non overlapping observations for four Fixed Income Futures.

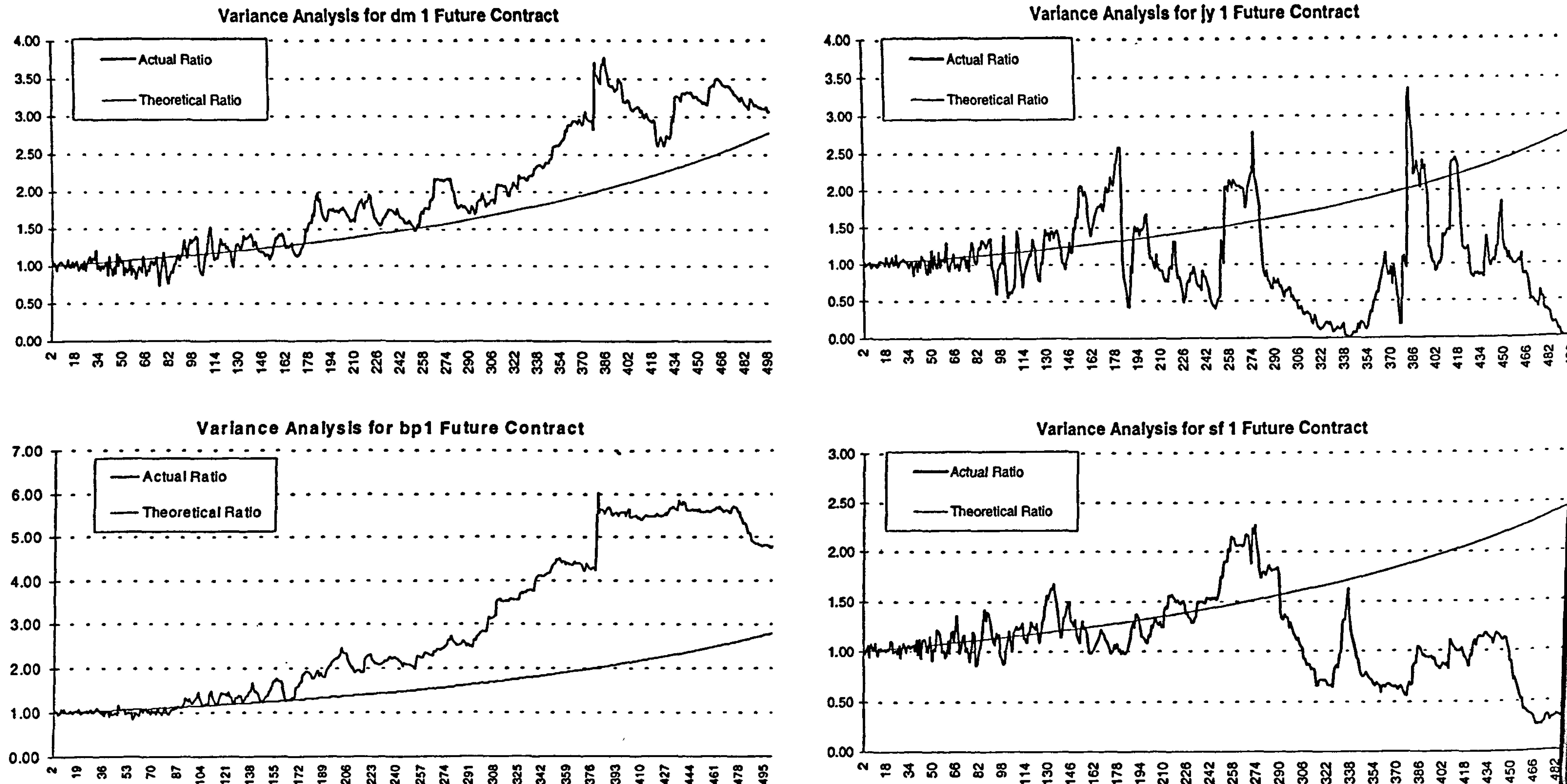

Figure 2.9c First period comparison of volatility estimated using overlapping and non overlapping observations for four Foreign Exchange Futures.

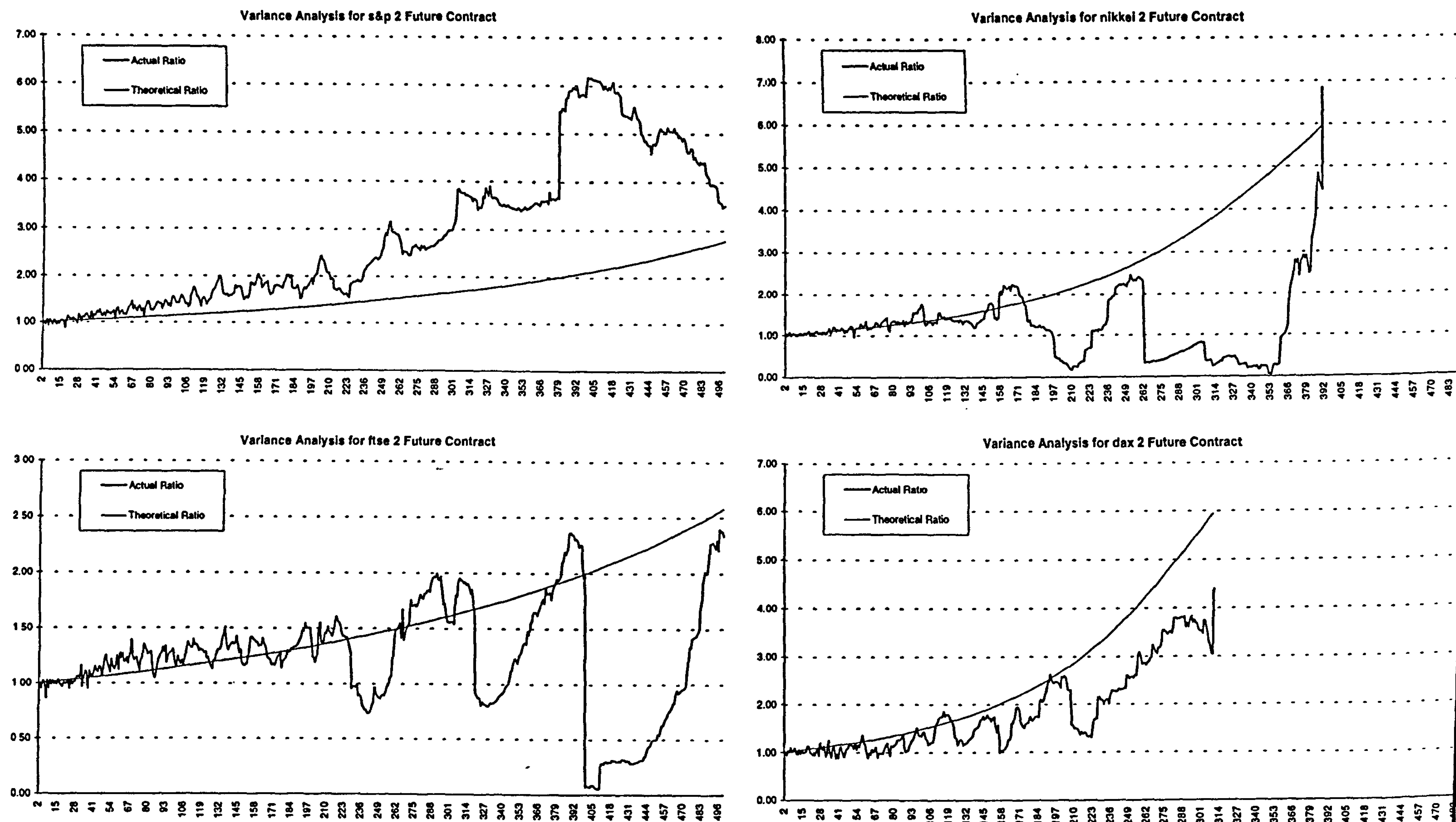

Figure 2.10a Second period comparison of volatility estimated using overlapping and non overlapping observations for four Stock Index Futures.

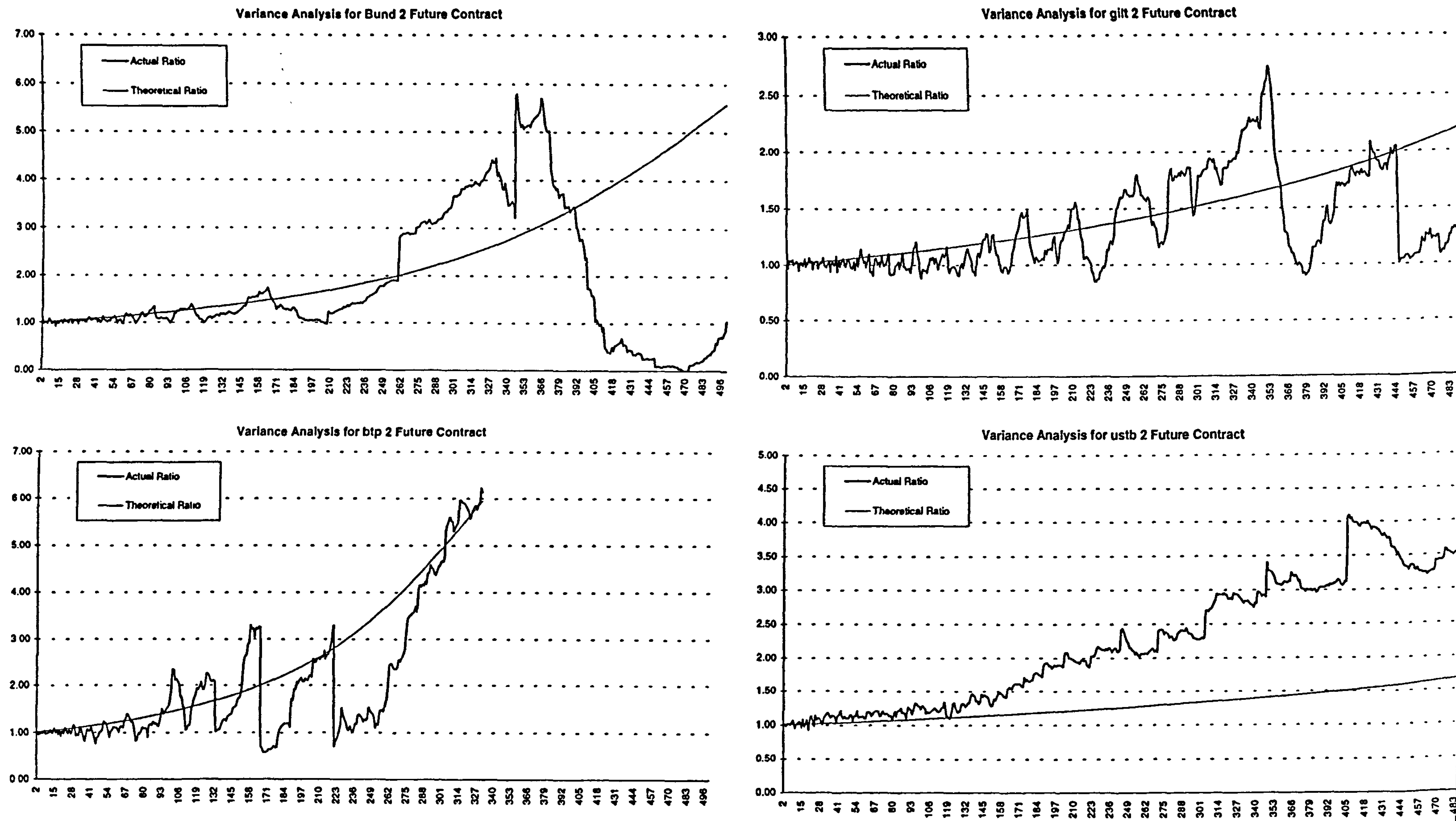

Figure 2.10b Second period comparison of volatility estimated using overlapping and non overlapping observations for four Fixed Income Futures.

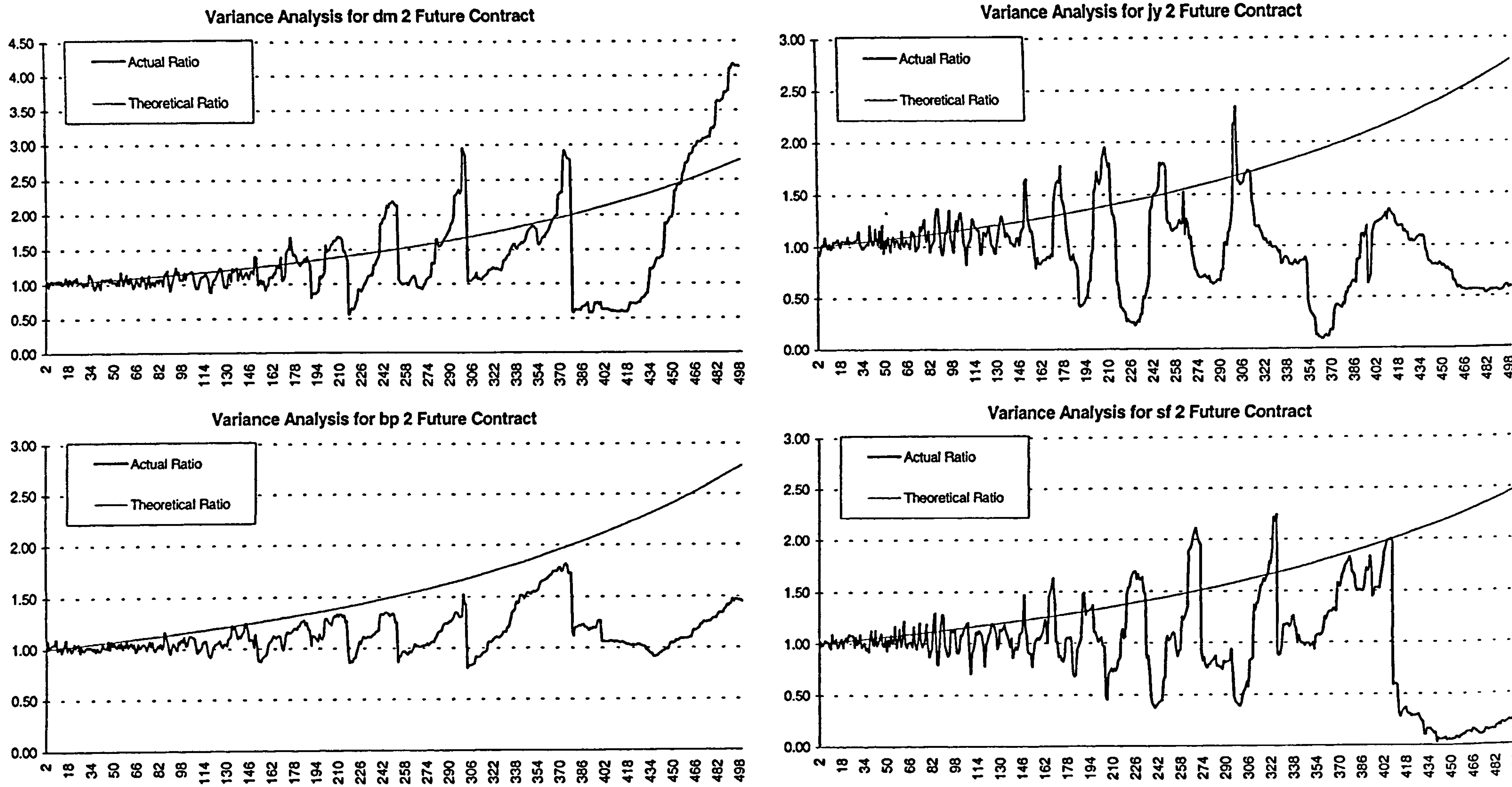

Figure 2.10c Second period comparison of volatility estimated using overlapping and non overlapping observations for four Foreign Exchange Futures.

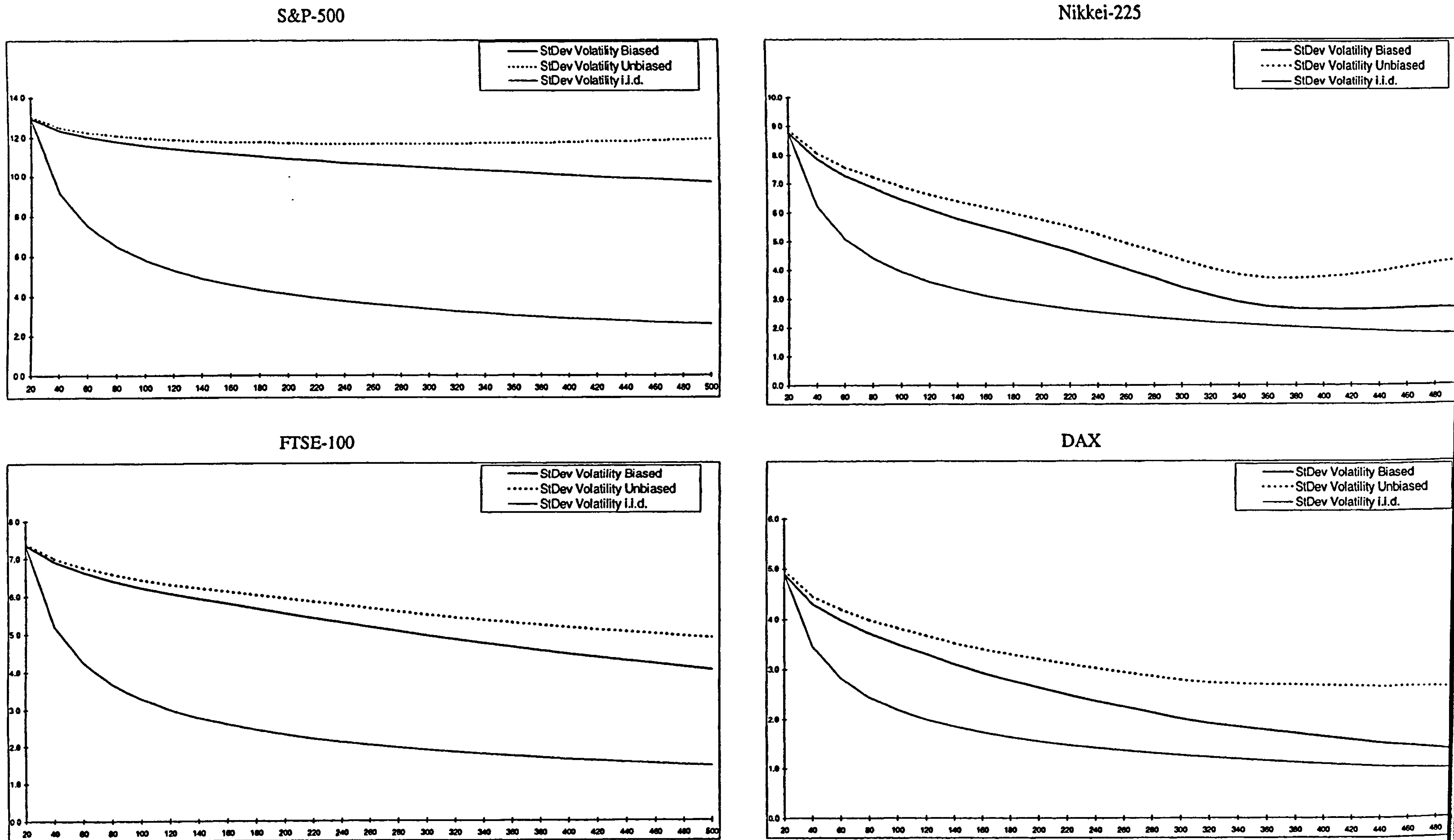

Figure 2.11a Standard deviation of the volatility cone, Biased vs. Unbiased vs. I.I.D. for four Stock Index Futures.

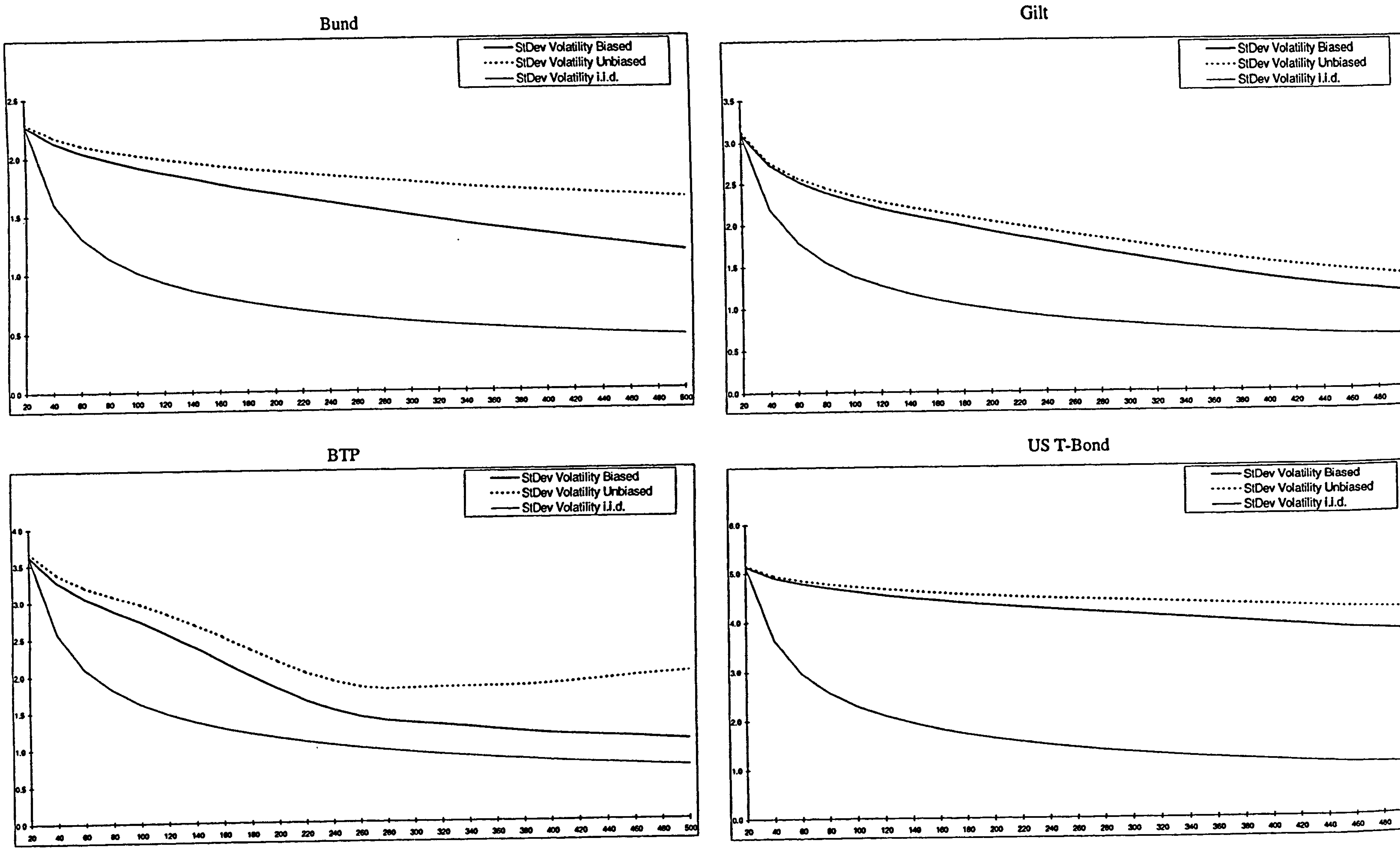

Figure 2.11b Standard deviation of the volatility cone, Biased vs. Unbiased vs. I.I.D. for four Fixed Income Futures.

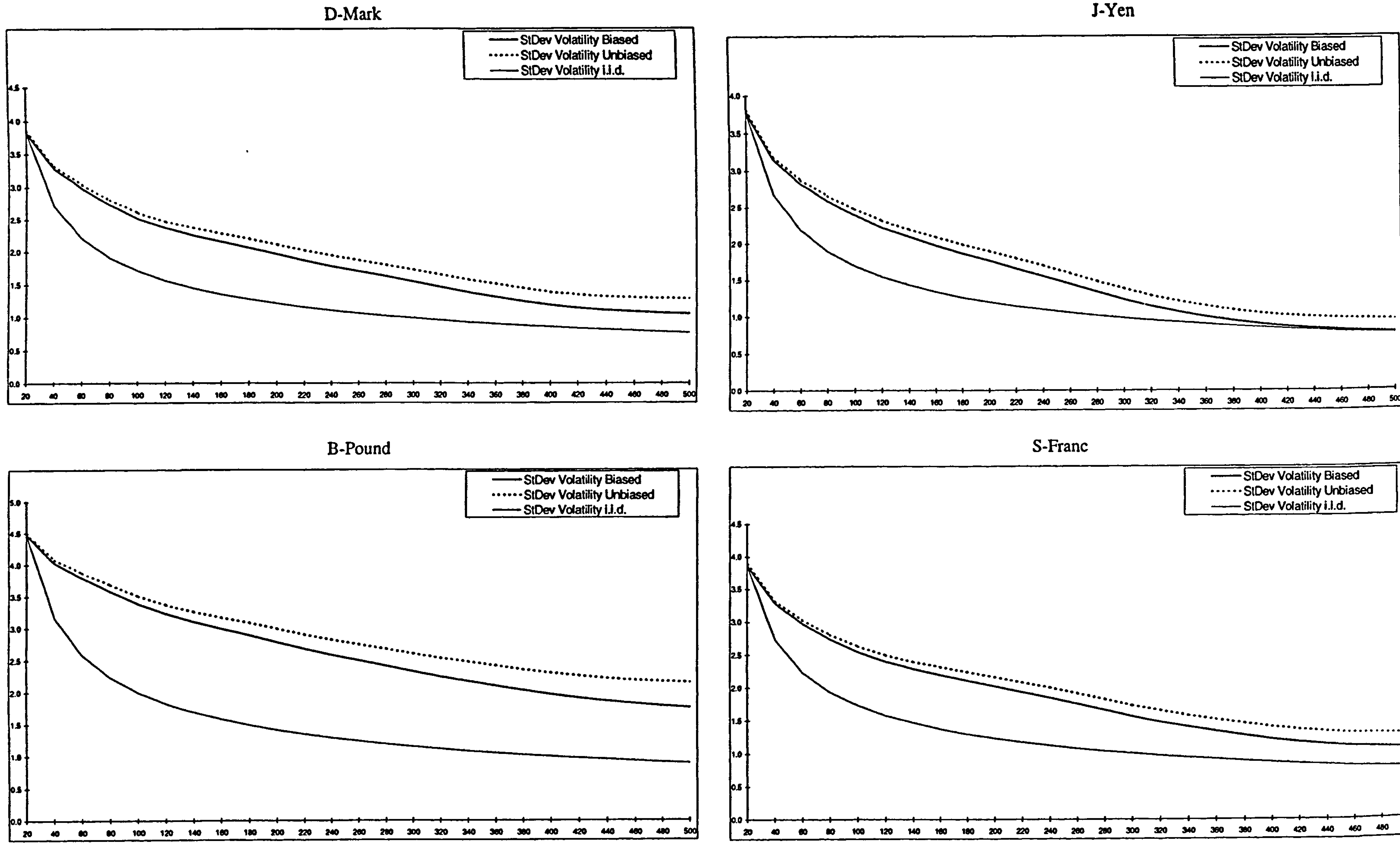

Figure 2.11c Standard deviation of the volatility cone, Biased vs. Unbiased vs. I.I.D. for four Foreign Exchange Futures.

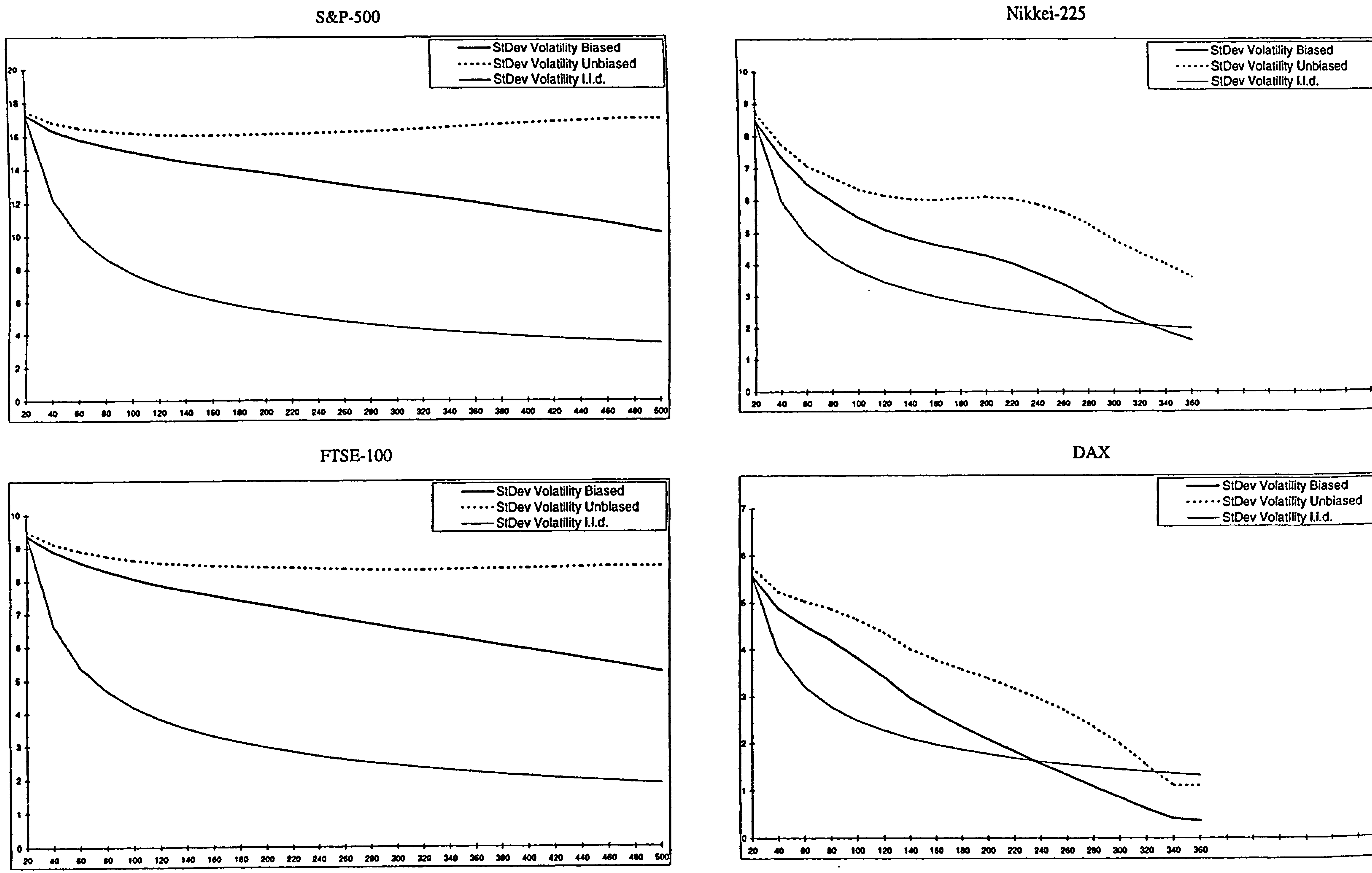

Figure 2.12a First period Standard deviation of the volatility cone, Biased vs. Unbiased vs. I.I.D. for four Stock Index Futures.

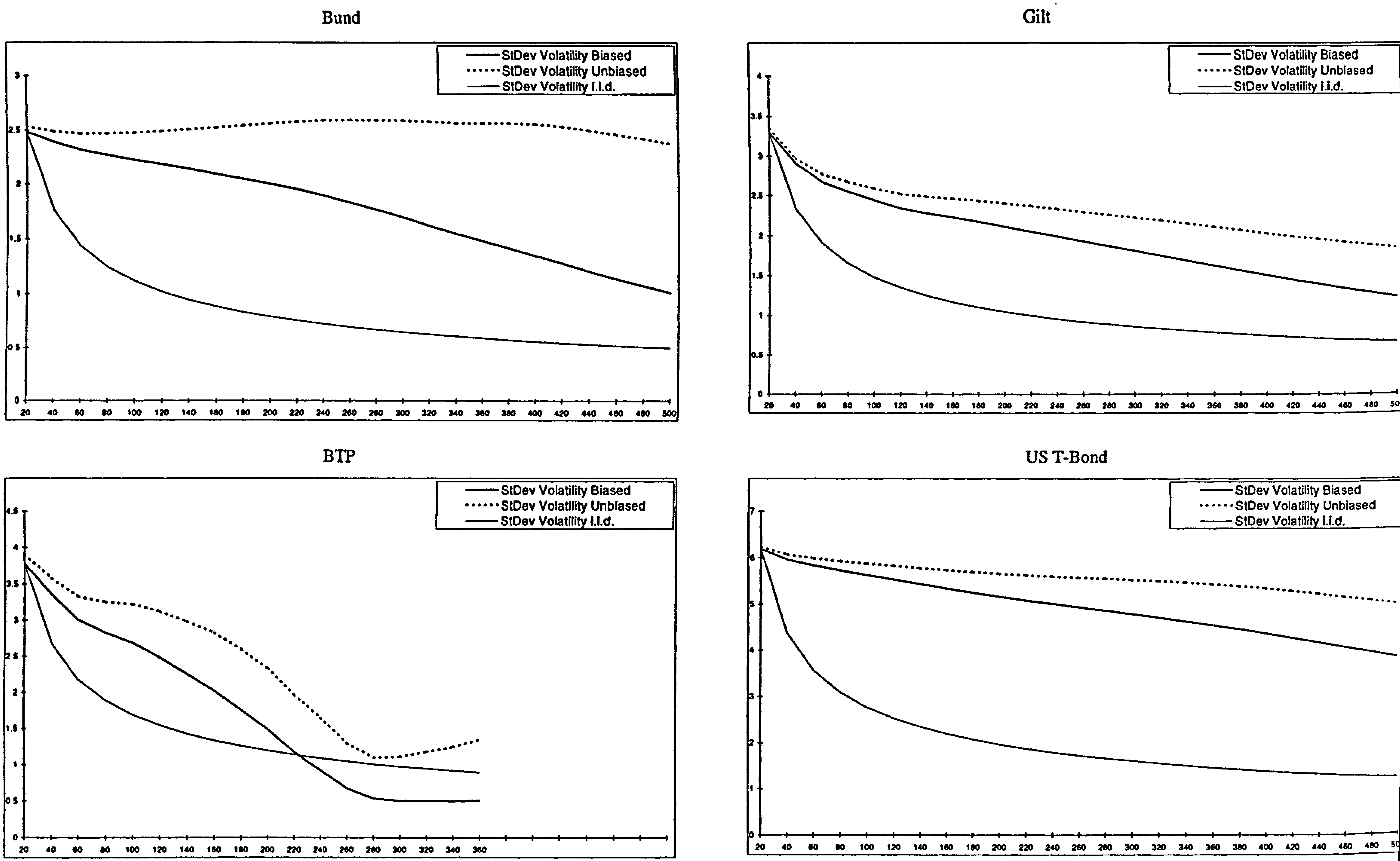

Figure 2.12b First period Standard deviation of the volatility cone, Biased vs. Unbiased vs. I.I.D. for four Fixed Income Futures.

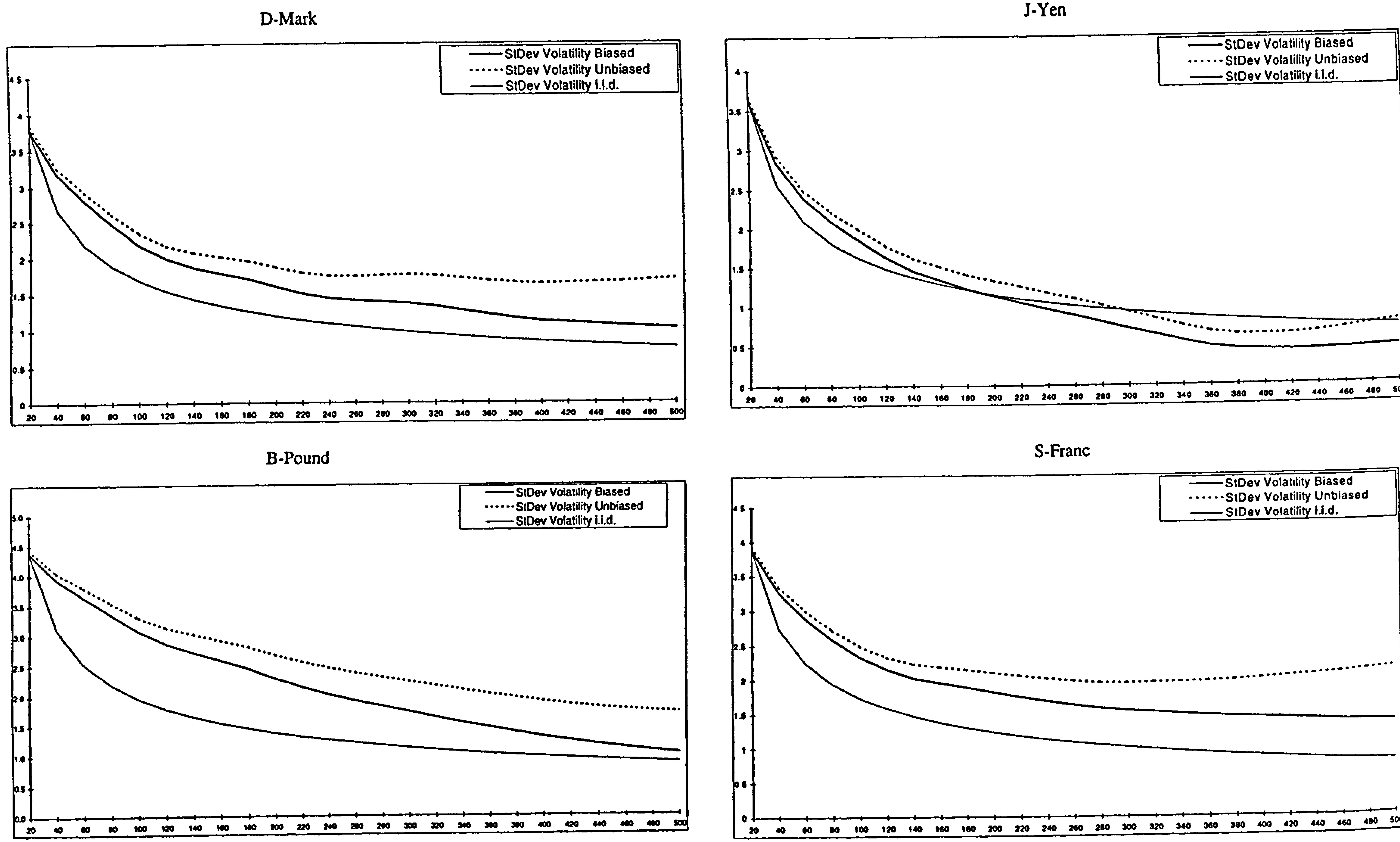

Figure 2.12c First period Standard deviation of the volatility cone, Biased vs. Unbiased vs. I.I.D. for four Foreign Exchange Futures.

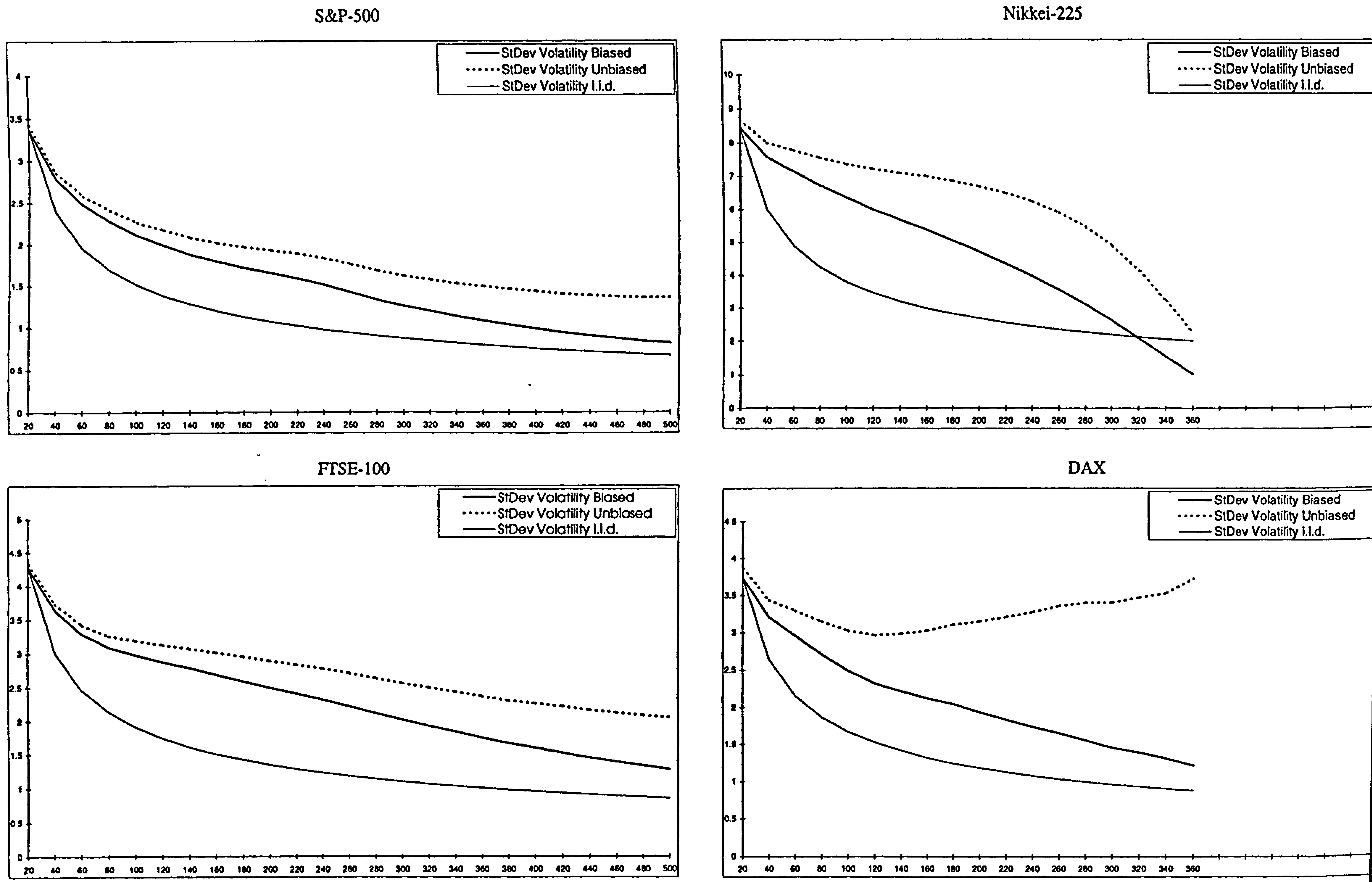

Figure 2.13a Second period Standard deviation of the volatility cone, Biased vs. Unbiased vs. I.I.D. for four Stock Index Futures.

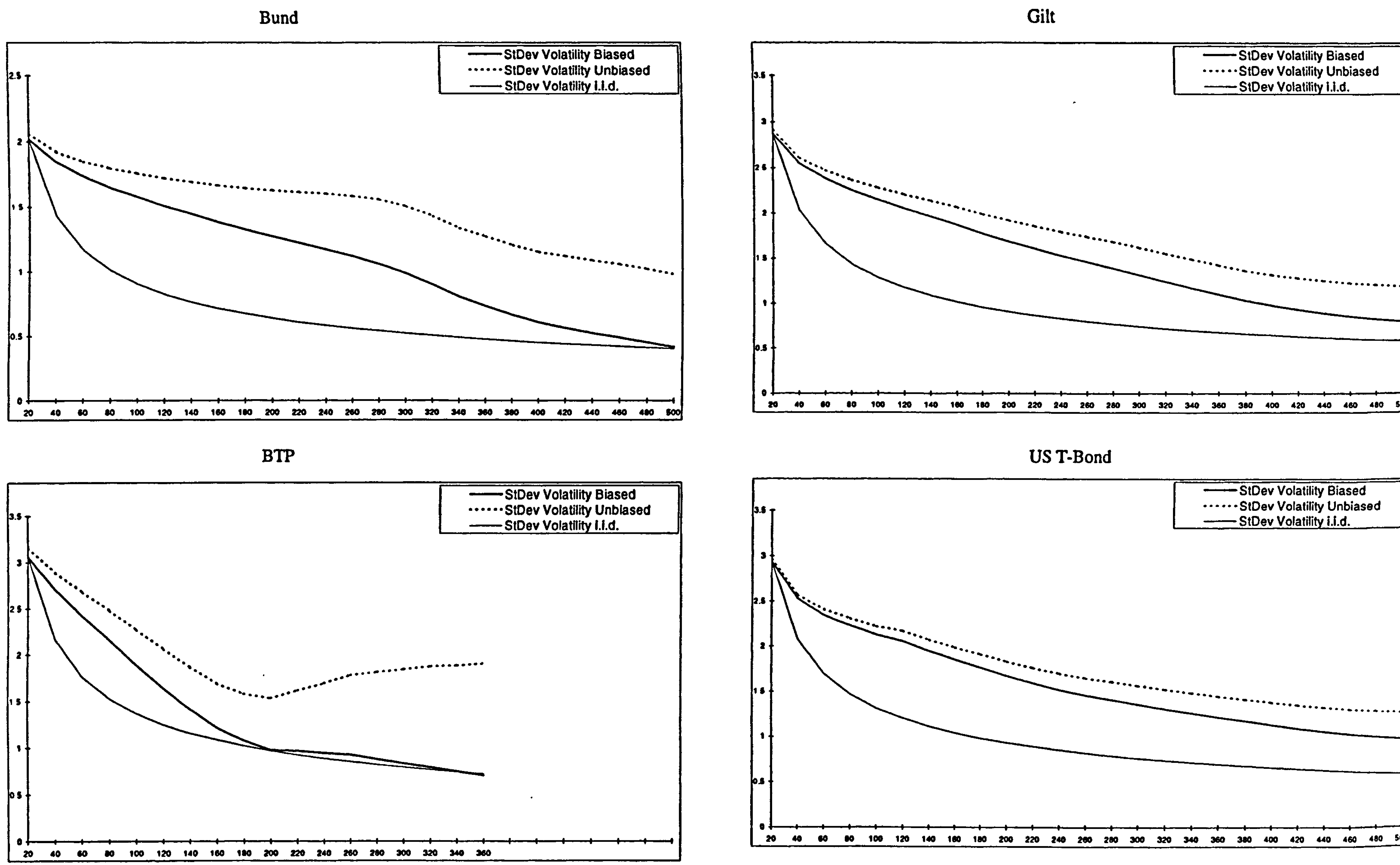

Figure 2.13b Second period Standard deviation of the volatility cone, Biased vs. Unbiased vs. I.I.D. for four Fixed Income Futures.

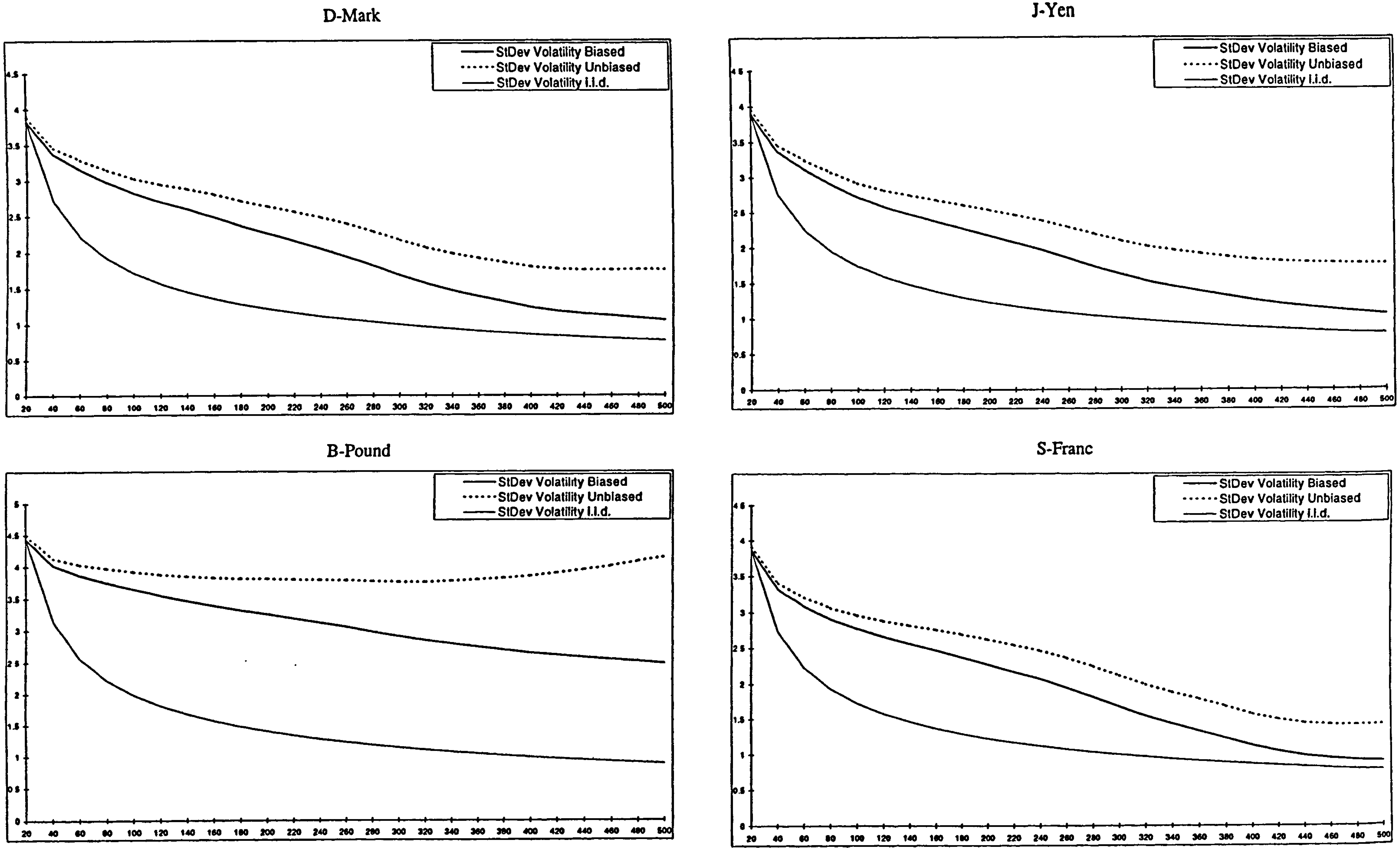

Figure 2.13c Second period Standard deviation of the volatility cone, Biased vs. Unbiased vs. I.I.D. for four Foreign Exchange Futures.

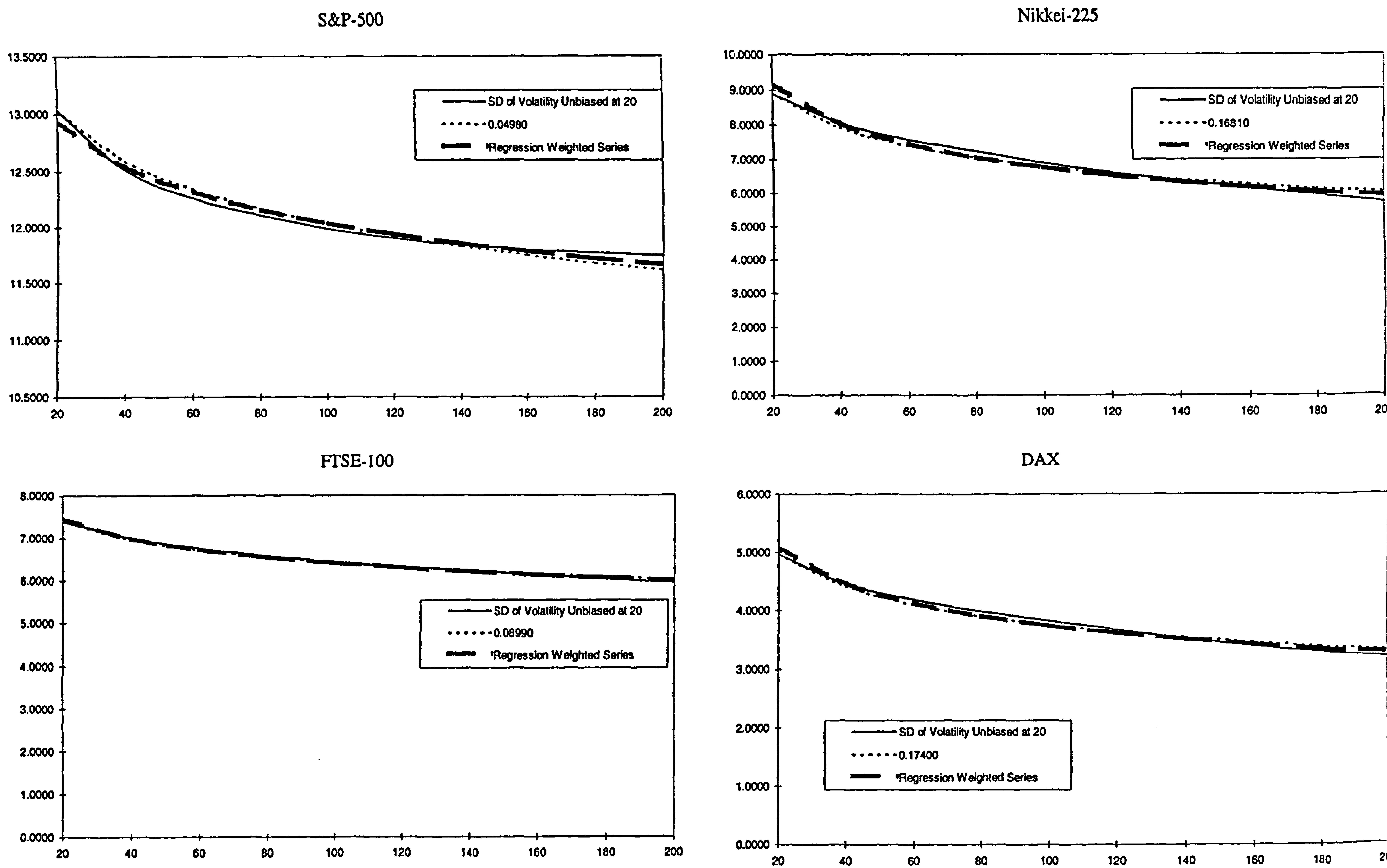

Figure 2.14a Time decay factors for the volatility of volatility for four Stock Index Futures.

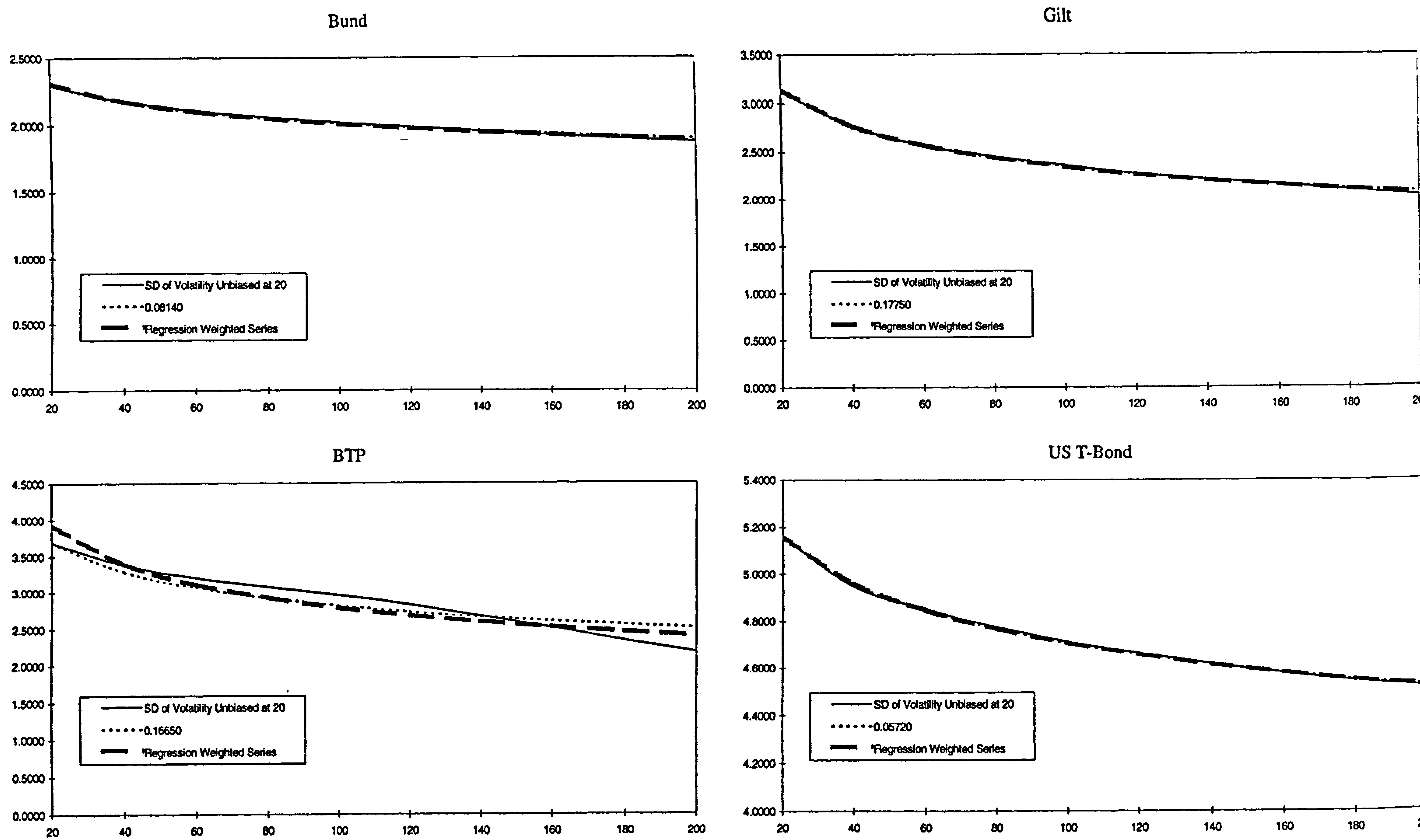

Figure 2.14b Time decay factors for the volatility of volatility for four Fixed Income Futures.

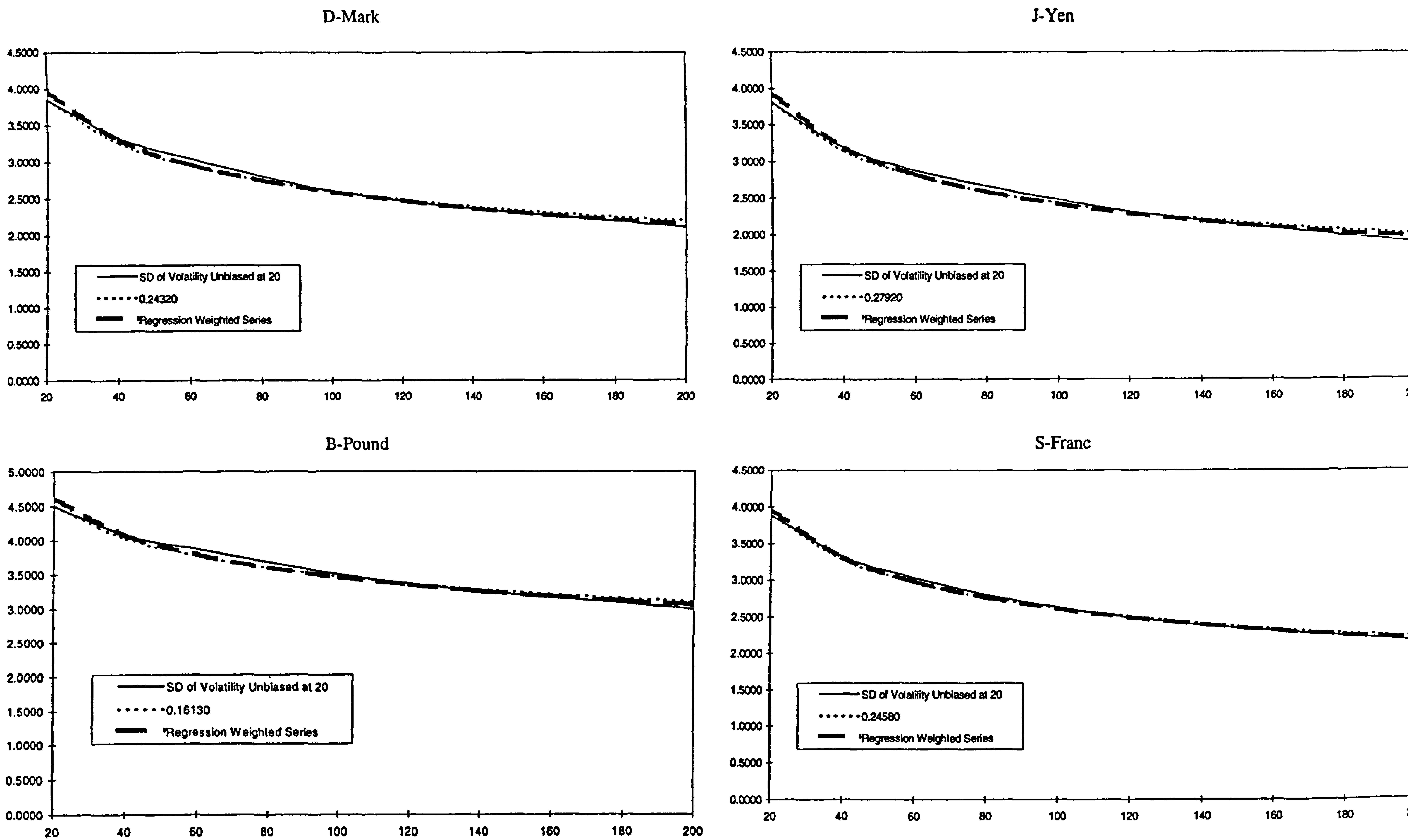

Figure 2.14c Time decay factors for the volatility of volatility for four Foreign Exchange Futures.

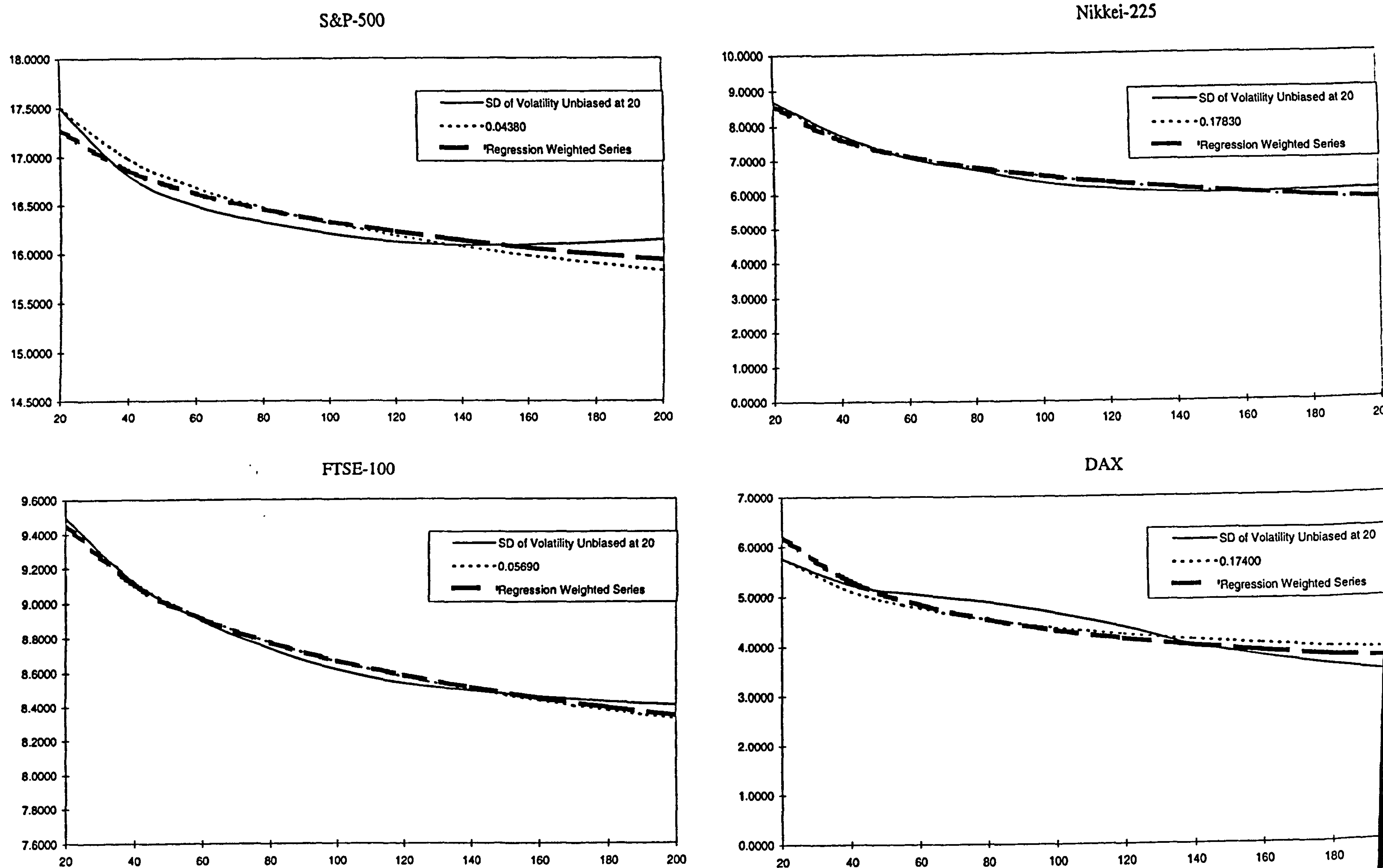

Figure 2.15a First period time decay factors for the volatility of volatility for four Stock Index Futures.

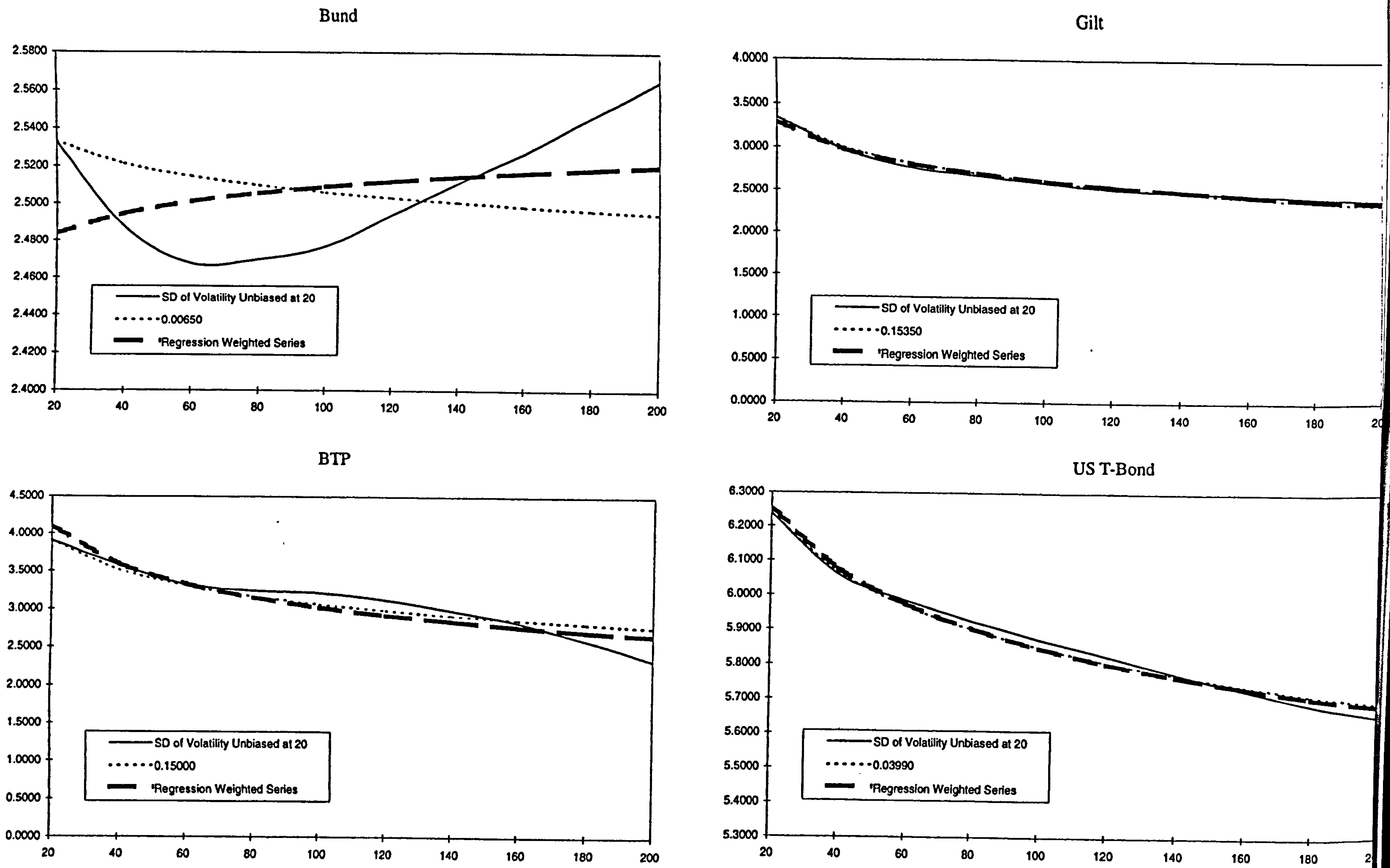

Figure 2.15b First period time decay factors for the volatility of volatility for four Fixed Income Futures.

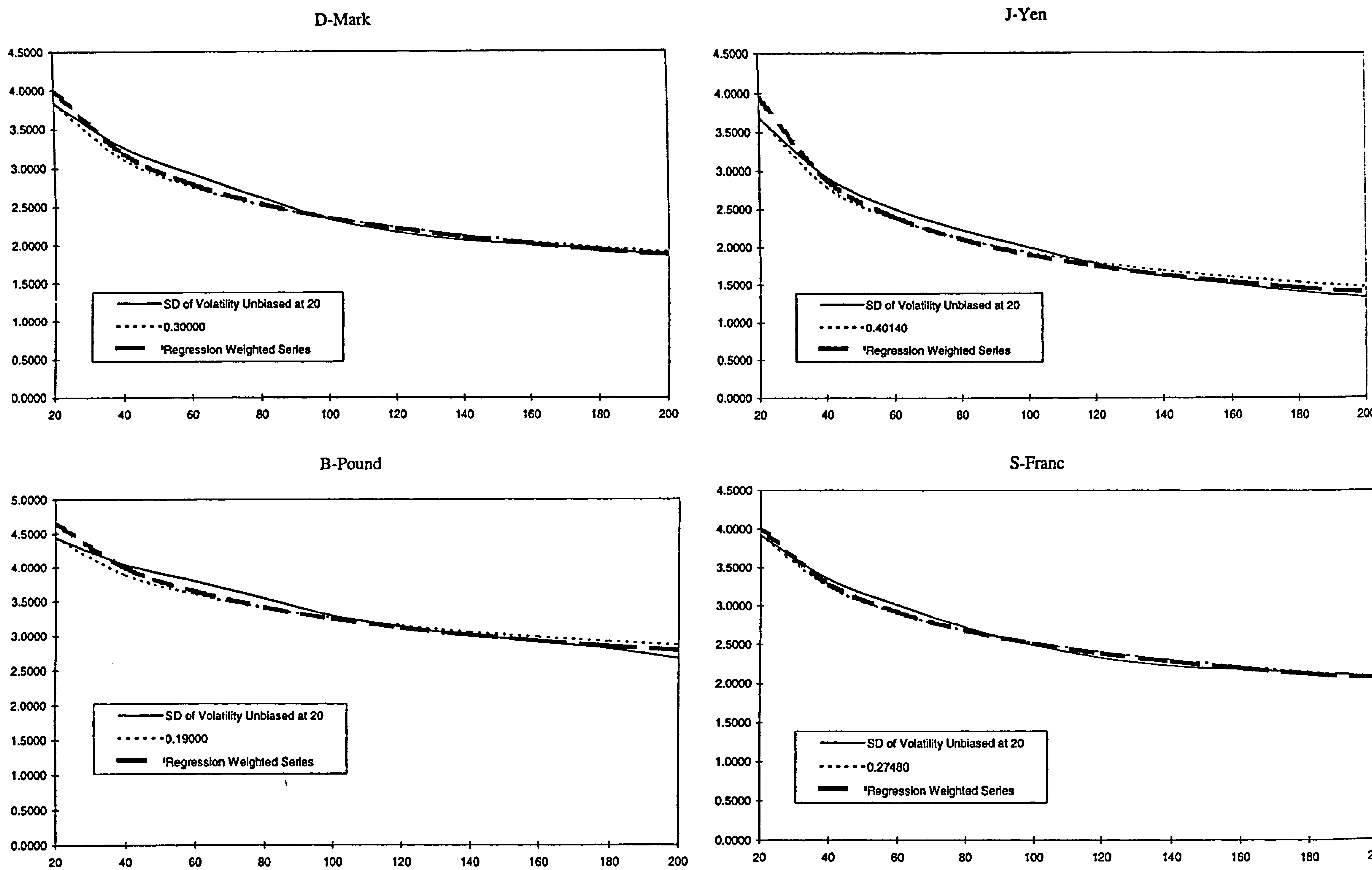

Figure 2.15c First period time decay factors for the volatility of volatility for four Foreign Exchange Futures.

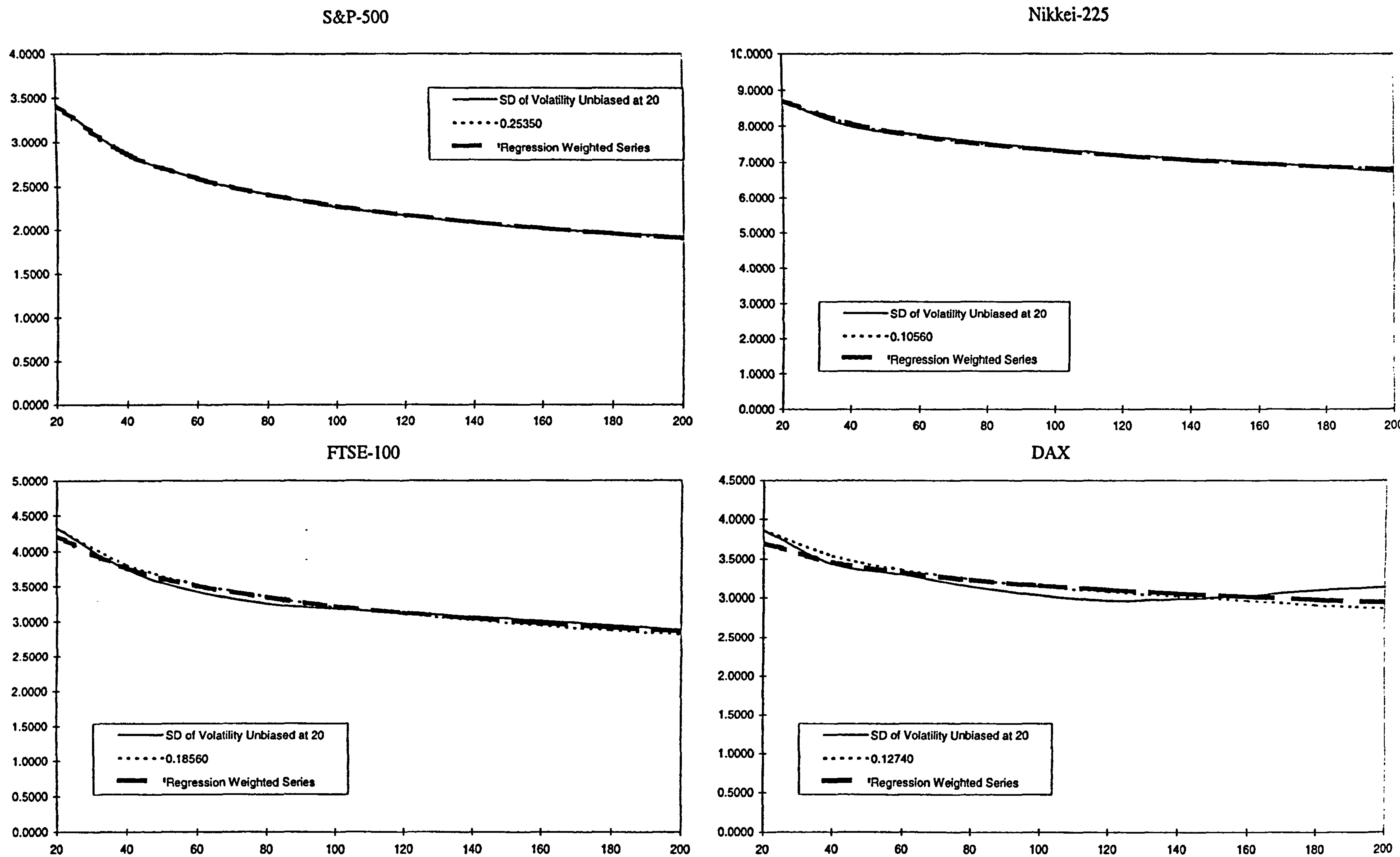

Figure 2.16a Second period time decay factors for the volatility of volatility for four Stock Index Futures.

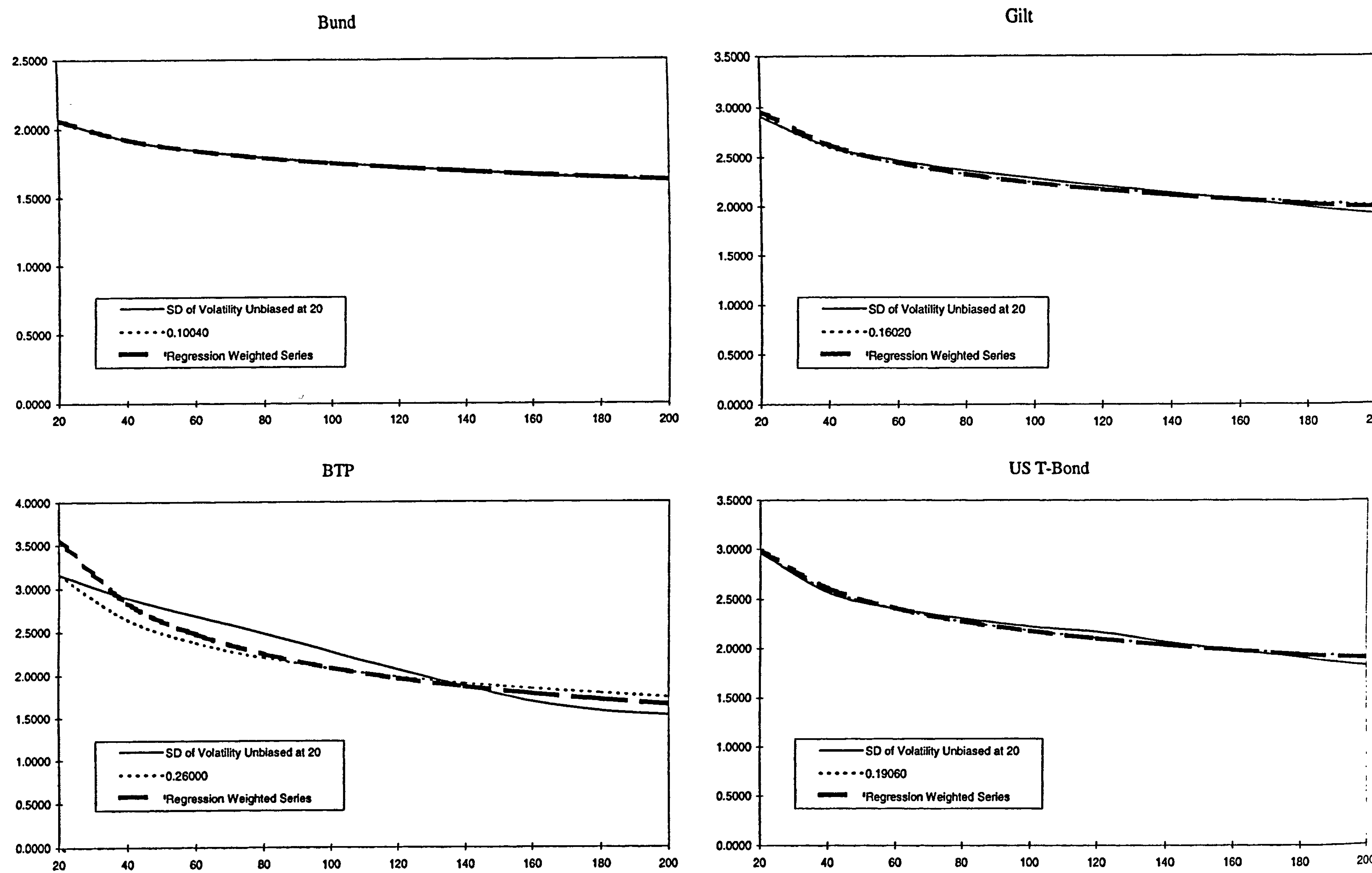

Figure 2.16b Second period time decay factors for the volatility of volatility for four Fixed Income Futures.

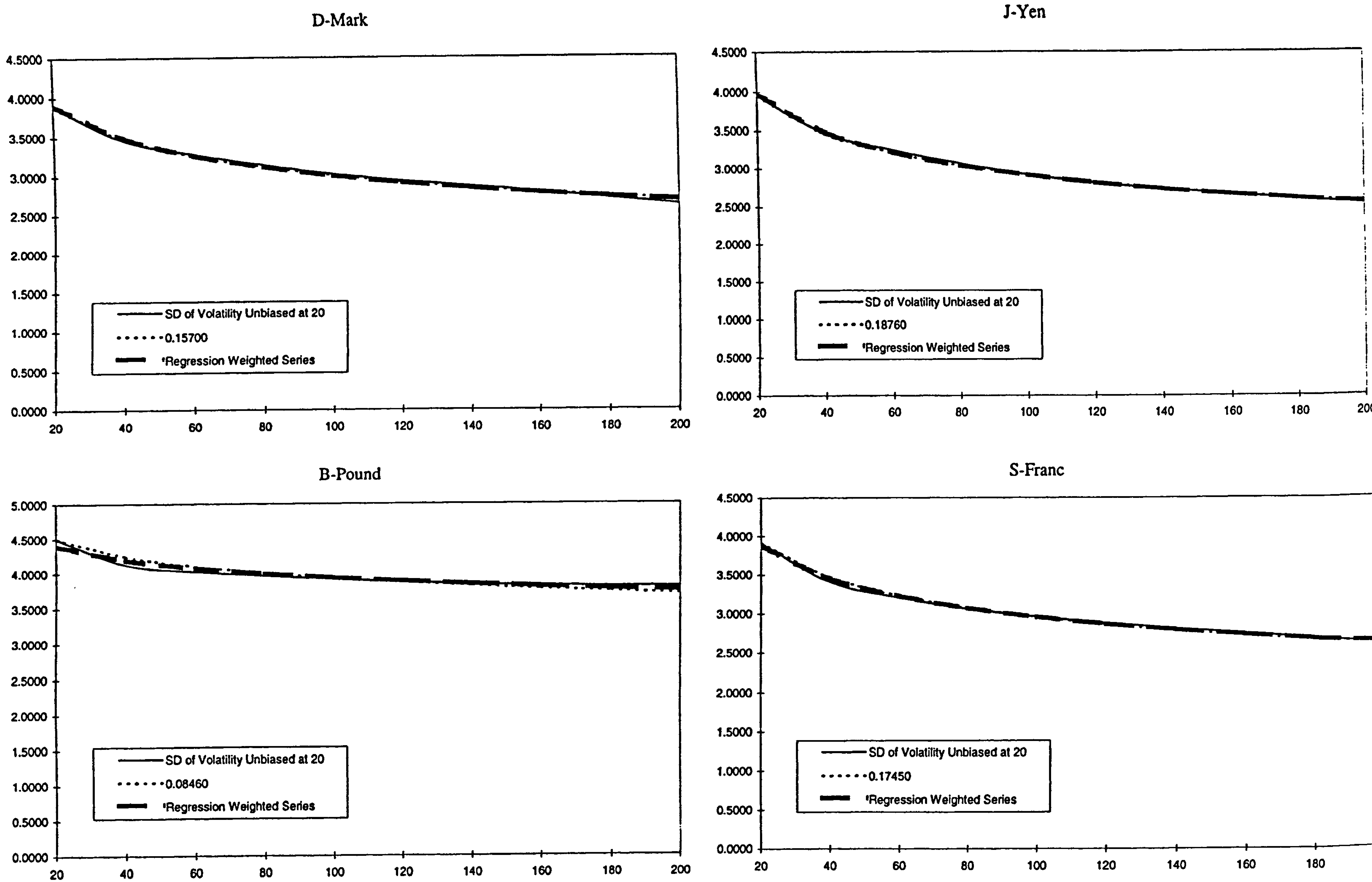

Figure 2.16c Second period time decay factors for the volatility of volatility for four Foreign Exchange Futures.

S&P-500	All	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5
Min	8.37%	8.09%	7.25%	7.46%	8.08%	7.80%	7.91%	8.53%	8.18%	7.75%	7.51%	8.67%	7.95%	7.94%	9.05%	8.92%	8.27%	7.96%	0.07589
1st quartile	12.17%	10.10%	10.15%	10.14%	10.56%	10.49%	10.08%	11.11%	10.56%	10.86%	10.20%	10.57%	10.65%	11.01%	11.20%	11.21%	11.70%	12.62%	0.108583
Average	15.94%	13.66%	13.57%	13.91%	13.55%	13.96%	13.88%	15.83%	14.55%	14.67%	14.53%	14.36%	14.50%	15.03%	15.31%	15.04%	15.18%	15.70%	0.153709
Median	14.78%	12.64%	12.81%	13.13%	13.00%	13.65%	13.21%	13.09%	12.54%	12.68%	13.27%	12.72%	13.06%	13.72%	13.52%	13.67%	14.37%	14.33%	0.145294
3rd quartile	18.03%	15.66%	15.80%	15.91%	15.47%	15.77%	16.06%	18.15%	15.72%	16.53%	17.43%	15.74%	16.16%	17.22%	17.06%	16.04%	17.16%	17.66%	0.166606
Max	160.00%	33.23%	29.58%	33.32%	20.96%	28.48%	31.25%	80.59%	51.27%	50.50%	39.04%	41.81%	39.34%	42.26%	40.46%	41.62%	39.41%	43.04%	0.423041
StDev	0.06659	0.04760	0.04528	0.04800	0.03536	0.04257	0.04688	0.12080	0.07513	0.07511	0.06110	0.06140	0.05936	0.06247	0.06417	0.05730	0.05477	0.05926	0.068938
Kurtosis	105.17225	8.61770	5.25974	7.99411	2.39585	4.77325	6.39299	29.77670	16.74609	15.53765	9.26459	12.74986	11.02799	10.88245	9.87851	13.54250	11.71889	13.20908	9.703666
Skewness	6.71069	1.88652	1.27088	1.73206	0.55138	1.12439	1.50902	4.92325	3.27867	3.12993	2.11781	2.73802	2.47944	2.33382	2.27085	2.71212	2.39046	2.56678	2.28097
Number of obs	2685	42	43	43	37	43	43	34	43	43	43	43	43	43	32	43	43	43	43

FTSE-100	All	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5
Min	7.47%	9.28%	9.67%	9.07%	8.79%	8.54%	8.68%	8.96%	9.00%	10.50%	10.89%	9.93%	9.63%	10.48%	9.82%	10.25%	10.71%	11.54%	0.111049
1st quartile	13.63%	13.59%	13.81%	14.21%	14.20%	14.24%	14.09%	13.63%	13.72%	13.13%	13.52%	14.39%	14.18%	13.84%	13.52%	13.62%	13.99%	14.79%	0.174457
Average	17.11%	16.88%	17.30%	16.93%	17.56%	18.35%	17.60%	17.43%	17.18%	17.46%	17.28%	17.79%	17.26%	17.58%	17.56%	17.78%	18.38%	19.15%	0.228056
Median	16.26%	16.46%	16.21%	16.34%	16.66%	16.60%	16.26%	16.24%	16.41%	16.34%	16.23%	16.42%	16.46%	16.39%	16.03%	15.78%	17.70%	18.18%	0.21128
3rd quartile	19.03%	18.34%	18.90%	18.40%	19.50%	19.87%	19.44%	19.02%	19.32%	19.40%	20.02%	20.21%	19.88%	20.89%	19.89%	19.09%	19.83%	21.22%	0.271736
Max	92.93%	38.00%	39.60%	38.65%	32.42%	76.46%	58.67%	52.37%	47.51%	50.74%	51.68%	52.58%	33.38%	34.95%	32.85%	35.90%	36.85%	36.56%	0.463356
StDev	0.05952	0.04862	0.05998	0.05028	0.04982	0.09750	0.07358	0.06556	0.05960	0.06421	0.06216	0.06534	0.04819	0.05257	0.05757	0.06565	0.05913	0.05790	0.071966
Kurtosis	31.43250	9.83518	9.65265	9.61908	4.02148	30.49408	23.79326	20.17069	16.63869	17.66581	22.74532	20.56769	4.59129	4.24088	3.42817	4.63929	5.12822	4.10662	4.7544
Skewness	3.84076	1.92616	2.29953	1.96385	0.92888	4.69687	3.92804	3.40751	2.91375	3.07297	3.69807	3.47204	0.98950	1.01651	1.04781	1.48459	1.45544	1.14626	1.09251
Number of obs	3012	46	48	48	42	48	48	47	48	48	48	46	48	48	48	48	48	47	42

Nikkei-225	All	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5
Min	9.72%	10.17%	10.62%	9.72%	8.78%	9.30%	9.42%	10.60%	10.31%	10.12%	13.00%	13.25%	12.36%	12.16%	14.34%	12.45%	15.34%	14.58%	0.122563
1st quartile	17.84%	18.02%	17.05%	17.68%	17.91%	18.02%	18.00%	17.94%	16.10%	16.06%	16.63%	16.74%	16.28%	15.73%	17.85%	17.99%	17.94%	19.90%	0.209342
Average	22.83%	21.31%	20.82%	21.26%	21.61%	21.86%	22.02%	21.63%	21.48%	21.92%	21.75%	20.71%	20.25%	21.08%	21.56%	21.98%	23.06%	25.28%	0.243183
Median	21.47%	20.48%	19.57%	21.26%	20.91%	21.68%	21.88%	20.32%	19.89%	19.30%	19.35%	19.11%	19.22%	20.63%	19.77%	22.64%	21.32%	23.83%	0.235114
3rd quartile	26.84%	22.75%	22.96%	24.10%	24.24%	25.37%	24.90%	24.27%	24.22%	25.77%	26.51%	23.68%	22.29%	24.34%	23.80%	26.12%	28.52%	30.08%	0.279965
Max	46.28%	35.11%	35.57%	35.20%	35.78%	38.55%	35.02%	39.70%	44.72%	47.41%	47.22%	38.43%	36.78%	36.59%	33.57%	30.40%	34.44%	39.46%	0.377316
StDev	0.06874	0.05826	0.05681	0.05685	0.06454	0.06703	0.06126	0.06377	0.07346	0.07927	0.07522	0.05638	0.05410	0.06008	0.05436	0.05333	0.05821	0.06793	0.062289
Kurtosis	3.15387	4.08767	4.51643	4.48059	3.31277	3.74163	2.83179	4.47572	5.88528	6.32139	7.26391	5.74843	5.33205	3.48326	3.14846	1.92016	2.01173	2.48491	3.12876
Skewness	0.77226	0.90417	1.02046	0.74564	0.42143	0.48706	0.18876	0.82872	1.29065	1.49682	1.81016	1.41627	1.24497	0.84348	0.90603	-0.18194	0.60215	0.54900	0.349997
Number of obs	1563	23	24	24	22	25	25	25	25	25	25	25	25	25	23	25	25	25	25

DAX	All	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5
Min	9.73%	9.02%	9.12%	9.41%	8.98%	9.26%	9.14%	8.89%	9.32%	10.02%	10.10%	9.11%	9.19%	8.86%	8.43%	8.29%	8.80%	9.58%	0.101214
1st quartile	13.41%	11.33%	12.14%	11.49%	13.19%	11.73%	11.32%	12.56%	12.79%	12.70%	12.37%	12.11%	12.47%	11.38%	11.49%	11.26%	11.76%	12.16%	0.133637
Average	15.62%	14.11%	14.70%	14.06%	15.00%	14.71%	14.23%	14.15%	14.58%	14.25%	13.89%	13.76%	14.21%	14.03%	14.25%	14.07%	14.71%	14.77%	0.156786
Median	15.02%	14.30%	14.09%	13.49%	16.83%	14.26%	13.43%	14.05%	14.18%	13.96%	13.76%	13.69%	13.39%	13.99%	14.15%	13.60%	14.42%	15.35%	0.151808
3rd quartile	17.81%	16.91%	18.02%	17.29%	17.32%	17.26%	17.42%	15.67%	16.37%	15.79%	14.93%	15.53%	16.34%	15.74%	16.97%	15.25%	16.32%	15.90%	0.171935
Max	25.10%	19.65%	19.86%	18.74%	18.38%	20.91%	21.69%	19.67%	20.47%	19.74%	18.98%	20.35%	21.34%	21.58%	22.22%	23.29%	29.57%	20.82%	0.241176
StDev	0.02985	0.03095	0.03372	0.03223	0.03587	0.03689	0.03927	0.03016	0.02939	0.02518	0.02650	0.02929	0.03261	0.03221	0.03735	0.04026	0.04536	0.02912	0.036117
Kurtosis	2.45980	1.83698	1.67731	1.59454	2.46913	1.86804	2.19887	2.56566	2.64660	3.01896	2.71865	3.18036	2.62373	3.20775	2.66472	3.13333	8.87830	2.81596	3.12076
Skewness	0.48220	0.07221	0.07072	0.22177	-1.07430	0.32529	0.49699	0.11095	0.29206	0.41266	0.46110	0.40479	0.31802	0.42180	0.31477	0.79820	1.93305	0.03179	0.659838
Number of obs	1246	17	17	13	8	16	14	14	19	17	17	18	20	19	19	20	19	17	20

Table 7.1a Summary Statistics for the At-the-money Implied Volatilities for Four Sock Index Options, Estimated on a Daily Basis and at 5-days increments until expiration.

Bund	All	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5
Min	2.91%	3.94%	4.00%	3.58%	3.77%	3.71%	3.68%	4.01%	4.19%	4.05%	3.87%	3.72%	3.69%	3.50%	3.51%	3.47%	3.11%	2.74%	0.034287
1st quartile	4.01%	4.56%	4.56%	4.55%	4.55%	4.67%	4.63%	4.65%	4.74%	4.88%	4.71%	4.84%	4.59%	4.94%	4.87%	4.77%	4.63%	4.64%	0.045514
Average	5.34%	5.69%	5.71%	5.67%	5.65%	5.62%	5.92%	5.98%	6.02%	5.90%	5.84%	5.70%	5.85%	5.81%	5.77%	6.00%	6.01%	6.33%	0.062418
Median	4.92%	5.08%	5.35%	5.27%	5.19%	5.09%	5.25%	5.20%	5.61%	5.36%	5.52%	5.32%	5.51%	5.43%	5.46%	5.76%	5.57%	5.35%	0.055757
3rd quartile	6.08%	5.98%	6.43%	6.23%	6.48%	6.09%	6.50%	6.68%	6.50%	6.32%	6.67%	6.51%	6.93%	6.38%	6.52%	6.76%	6.86%	6.80%	0.078701
Max	15.41%	14.15%	11.44%	12.80%	10.66%	10.95%	14.12%	12.74%	11.24%	11.08%	9.70%	8.71%	10.28%	9.89%	8.97%	11.46%	12.31%	18.27%	0.102977
StDev	0.01764	0.02033	0.01609	0.01823	0.01554	0.01532	0.02073	0.01952	0.01748	0.01613	0.01502	0.01270	0.01551	0.01362	0.01332	0.01682	0.02048	0.02938	0.021468
Kurtosis	4.62795	12.92420	7.89792	10.24063	5.51409	7.04505	10.74754	6.88436	5.18666	5.69353	3.17374	2.73856	3.75433	4.46571	3.27160	5.42338	4.81532	11.36142	2.140267
Skewness	1.31878	2.79329	1.88947	2.27824	1.45627	1.77778	2.38717	1.79281	1.51742	1.52432	0.91100	0.71217	1.01351	1.05693	0.63109	1.23511	1.17920	2.46396	0.647288
Number of obs	1917	30	28	30	30	30	30	29	29	28	30	29	30	31	31	31	30	31	31

BTP	All	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5
Min	2.34%	3.51%	3.71%	3.96%	3.85%	3.77%	3.94%	3.90%	4.43%	4.39%	4.33%	0.40%	4.14%	3.91%	4.81%	4.03%	4.02%	3.37%	0.033335
1st quartile	7.33%	7.58%	7.24%	7.12%	7.42%	7.11%	7.20%	6.91%	7.15%	7.54%	7.78%	6.98%	8.19%	7.96%	7.57%	7.92%	7.59%	7.42%	0.074558
Average	8.85%	8.08%	8.27%	8.42%	8.51%	8.75%	8.82%	8.87%	8.84%	9.35%	9.75%	8.66%	9.31%	9.33%	9.05%	8.79%	9.01%	9.13%	0.098055
Median	9.09%	8.39%	8.81%	8.67%	8.85%	8.72%	8.98%	8.72%	9.20%	9.12%	9.59%	9.29%	8.97%	8.88%	8.33%	9.03%	8.79%	8.30%	0.094881
3rd quartile	10.35%	9.39%	9.81%	9.51%	9.76%	10.27%	10.17%	9.98%	9.54%	10.70%	11.79%	10.83%	11.03%	10.38%	9.96%	9.80%	10.23%	10.01%	0.106444
Max	18.23%	11.66%	12.39%	12.87%	13.33%	15.90%	15.04%	15.84%	16.89%	15.03%	15.81%	12.65%	15.97%	15.85%	16.13%	14.49%	15.26%	17.49%	0.187519
StDev	0.02720	0.02229	0.02507	0.02410	0.02284	0.02783	0.02774	0.02855	0.02725	0.02784	0.03012	0.03066	0.02916	0.02999	0.03014	0.02502	0.02959	0.03472	0.042642
Kurtosis	3.26127	3.30873	2.70423	2.95480	3.51913	4.23331	3.43185	3.68992	6.11237	2.83254	2.58483	4.30125	3.52884	3.29020	3.32564	3.42928	3.06278	4.04014	3.229012
Skewness	-0.02003	-0.81555	-0.56062	-0.35272	-0.35628	0.43854	0.27699	0.59165	1.08359	0.37371	0.25826	-1.07418	0.37354	0.45788	0.78016	0.04622	0.38309	0.93772	0.507947
Number of obs	1283	20	18	20	20	20	20	20	20	19	19	20	20	20	21	21	20	21	21

Gilt	All	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5
Min	4.07%	5.40%	5.45%	5.68%	5.98%	6.55%	6.40%	5.84%	5.60%	6.80%	6.55%	6.19%	6.56%	6.61%	5.90%	6.16%	6.02%	5.71%	0.050314
1st quartile	7.01%	7.40%	7.44%	7.45%	7.59%	7.61%	7.71%	7.73%	7.93%	7.85%	8.00%	7.93%	7.83%	7.89%	7.74%	7.96%	7.75%	8.32%	0.074896
Average	8.74%	8.95%	8.80%	8.92%	9.14%	9.31%	9.52%	9.46%	9.72%	9.80%	9.83%	9.58%	9.46%	9.64%	9.58%	9.61%	9.65%	10.24%	0.099043
Median	8.14%	8.28%	8.19%	8.34%	8.73%	8.73%	8.70%	9.15%	9.45%	9.42%	9.43%	9.00%	8.56%	9.15%	8.67%	9.30%	9.33%	9.56%	0.097742
3rd quartile	10.38%	11.05%	10.34%	10.19%	10.24%	10.89%	11.06%	11.20%	11.28%	11.19%	11.19%	11.11%	11.58%	11.47%	11.61%	11.17%	11.17%	11.60%	0.11285
Max	17.24%	14.06%	12.56%	13.05%	13.65%	14.36%	15.19%	16.56%	15.45%	15.20%	16.13%	14.68%	13.91%	14.48%	15.89%	14.11%	16.70%	20.49%	0.172076
StDev	0.02220	0.02087	0.01862	0.01884	0.01889	0.02098	0.02300	0.02433	0.02521	0.02313	0.02356	0.02119	0.02160	0.02125	0.02231	0.02081	0.02499	0.02843	0.030337
Kurtosis	2.94996	2.32553	2.04077	2.42879	2.49789	2.61453	2.87645	3.42017	2.72119	2.65921	2.99037	2.53881	1.96903	2.02799	2.75350	2.21432	3.17628	5.95289	3.196191
Skewness	0.72952	0.48586	0.41689	0.59442	0.60072	0.65801	0.81018	0.82456	0.60866	0.71901	0.74689	0.62521	0.54436	0.44139	0.64282	0.39371	0.78016	1.36375	0.627913
Number of obs	2695	42	39	42	42	43	43	42	41	40	43	42	43	43	43	43	42	43	43

US T-Bond	All	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5
Min	5.85%	7.59%	7.82%	8.16%	8.16%	7.92%	7.76%	7.30%	7.75%	7.50%	7.57%	7.38%	6.54%	7.01%	7.85%	7.31%	7.34%	7.05%	0.086595
1st quartile	8.92%	9.64%	9.39%	9.70%	9.64%	9.35%	9.63%	10.08%	9.86%	9.86%	10.02%	10.45%	9.89%	10.91%	10.56%	11.17%	11.70%	12.30%	0.137959
Average	10.75%	11.62%	11.75%	12.08%	12.13%	12.36%	12.56%	13.01%	12.92%	13.50%	13.79%	13.99%	14.00%	15.04%	15.57%	15.23%	16.23%	16.99%	0.18648
Median	10.03%	10.83%	10.83%	10.82%	11.57%	11.18%	11.51%	12.14%	11.89%	12.45%	12.17%	12.64%	13.18%	14.62%	14.28%	14.07%	16.05%	16.48%	0.17538
3rd quartile	11.80%	13.35%	13.25%	14.11%	13.77%	14.62%	15.34%	15.80%	15.77%	16.54%	17.12%	17.35%	17.97%	18.57%	19.63%	18.93%	20.45%	19.76%	0.220834
Max	25.70%	19.56%	18.98%	20.21%	21.19%	22.93%	23.63%	23.89%	25.05%	27.16%	25.05%	24.04%	31.34%	31.97%	33.50%	31.35%	31.82%	36.20%	0.362288
StDev	0.02648	0.02732	0.02971	0.03270	0.03218	0.03755	0.03686	0.04026	0.04043	0.04423	0.04475	0.04706	0.05055	0.05326	0.05590	0.05333	0.05562	0.06030	0.065907
Kurtosis	6.33059	3.66599	2.77800	3.13579	4.34337	3.43385	3.27798	2.65280	3.39296	3.42855	2.31445	2.40252	4.24199	3.56620	3.50540	3.55446	2.68206	5.01062	3.125843
Skewness	1.60253	1.09083	0.90036	1.05117	1.20144	1.00518	0.85836	0.62502	0.82849	0.83436	0.54455	0.58440	0.90515	0.68940	0.79286	0.74009	0.49554	1.07731	0.81491
Number of obs	2902	44	44	44	34	45	45	46	43	47	47	41	48	48	48	48	48	48	40

Table 7.1b Summary Statistics for the At-the-money Implied Volatilities for Four Fixed Income Options, Estimated on a Daily Basis and at 5-days increments until expiration.

D-Mark	All	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5
Min	4.97%	9.50%	9.13%	9.49%	9.69%	8.77%	8.27%	8.45%	7.53%	8.36%	9.31%	9.89%	9.65%	9.80%	9.22%	8.08%	8.30%	6.74%	0.062172
1st quartile	10.08%	11.69%	11.78%	11.65%	11.80%	11.71%	11.42%	11.45%	11.89%	11.85%	11.72%	11.73%	11.64%	11.92%	11.43%	11.02%	11.83%	11.56%	0.119105
Average	11.71%	13.08%	13.23%	13.15%	13.26%	13.22%	13.19%	13.39%	13.57%	13.47%	13.44%	13.66%	13.48%	13.66%	13.67%	13.05%	13.66%	13.98%	0.151035
Median	11.62%	12.97%	12.91%	12.98%	13.15%	13.07%	13.04%	13.38%	13.48%	13.61%	13.27%	13.33%	13.00%	13.18%	13.35%	12.91%	13.78%	13.89%	0.144912
3rd quartile	12.98%	13.95%	14.56%	14.44%	14.63%	14.64%	14.49%	14.88%	14.66%	14.86%	14.33%	15.13%	15.26%	15.21%	15.58%	14.68%	15.32%	16.21%	0.17852
Max	24.95%	18.27%	18.26%	19.24%	18.89%	18.33%	18.90%	19.11%	21.56%	18.78%	20.53%	21.20%	19.66%	19.67%	19.68%	19.75%	19.64%	26.54%	0.430112
StDev	0.02517	0.02000	0.01973	0.02116	0.02196	0.02263	0.02374	0.02475	0.02495	0.02378	0.02368	0.02546	0.02395	0.02528	0.02726	0.02489	0.02585	0.03814	0.05985
Kurtosis	3.65757	3.31829	3.26312	3.07267	3.07708	2.80929	2.80828	2.76911	4.49681	2.65127	4.24792	3.92578	2.73114	2.62979	2.12092	2.84387	3.29496	4.28100	14.03185
Skewness	0.37533	0.66650	0.57313	0.55923	0.47696	0.44562	0.34303	0.24758	0.53790	0.07159	0.91264	0.91146	0.51540	0.72731	0.31684	0.50387	0.03095	0.51141	2.517838
Number of obs	2950	47	47	47	46	47	46	48	46	46	48	48	48	47	48	48	44	48	41

B-Pound	All	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5
Min	3.45%	5.87%	5.75%	5.62%	5.66%	5.48%	5.13%	4.97%	5.14%	5.07%	5.42%	7.44%	7.14%	7.05%	5.86%	5.73%	5.51%	5.34%	0.064678
1st quartile	8.94%	10.32%	10.09%	10.26%	10.01%	10.17%	10.23%	10.42%	10.74%	10.37%	10.70%	10.28%	10.54%	10.77%	10.53%	9.83%	9.87%	10.16%	0.103876
Average	11.18%	12.32%	12.38%	12.45%	12.63%	12.49%	12.59%	12.52%	12.61%	12.59%	12.59%	12.68%	12.77%	13.01%	12.97%	12.39%	12.90%	12.98%	0.137188
Median	11.22%	12.32%	12.31%	12.22%	12.35%	12.16%	12.61%	12.16%	12.20%	12.30%	12.23%	11.85%	11.98%	12.10%	12.44%	12.12%	12.48%	12.81%	0.134907
3rd quartile	13.02%	13.79%	14.26%	14.01%	14.47%	14.52%	14.45%	14.32%	15.01%	14.89%	14.20%	15.04%	15.39%	15.23%	15.04%	14.67%	15.53%	14.94%	0.17378
Max	23.45%	21.72%	21.09%	20.74%	20.75%	21.66%	22.83%	22.80%	22.08%	21.13%	20.56%	23.65%	22.31%	23.60%	22.30%	21.52%	23.51%	26.86%	0.217288
StDev	0.03202	0.03012	0.03015	0.03132	0.03339	0.03340	0.03459	0.03487	0.03317	0.03238	0.03154	0.03278	0.03276	0.03699	0.03553	0.03541	0.03912	0.04074	0.041733
Kurtosis	3.88983	4.35671	3.71405	3.43346	3.25987	3.46629	3.82681	3.79989	3.85171	3.20328	3.44031	4.50161	3.56590	3.91135	3.18146	3.30431	3.69844	4.85298	2.065258
Skewness	0.47402	0.62770	0.46387	0.50540	0.42459	0.45990	0.54076	0.55887	0.51580	0.23664	0.45463	1.08282	0.82905	0.91308	0.51556	0.71446	0.71991	0.73138	0.144242
Number of obs	2819	45	46	46	45	46	45	47	45	45	47	47	47	46	47	47	43	47	40

J-Yen	All	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5
Min	4.81%	8.36%	8.07%	8.64%	9.04%	9.19%	9.48%	9.76%	9.18%	8.90%	8.74%	10.10%	9.85%	9.08%	8.85%	9.15%	7.83%	9.22%	0.068827
1st quartile	8.82%	10.22%	10.36%	10.56%	10.58%	11.20%	10.65%	11.06%	11.19%	11.17%	11.19%	11.74%	11.68%	11.92%	11.83%	11.26%	11.03%	11.13%	0.115483
Average	10.47%	11.90%	12.05%	12.02%	12.25%	12.33%	12.43%	12.58%	12.67%	13.00%	12.85%	13.32%	13.35%	13.21%	13.36%	12.98%	13.01%	13.47%	0.143317
Median	10.10%	11.51%	11.85%	11.81%	11.97%	12.11%	12.16%	12.33%	11.91%	12.24%	12.52%	12.99%	13.22%	12.86%	12.74%	12.81%	12.55%	13.20%	0.138557
3rd quartile	11.93%	12.87%	12.88%	13.30%	13.41%	13.31%	13.52%	13.32%	13.82%	14.34%	14.14%	14.27%	14.65%	14.53%	14.75%	14.51%	14.78%	15.50%	0.17146
Max	19.55%	19.28%	19.80%	19.18%	20.42%	18.94%	19.61%	18.93%	19.87%	20.02%	19.73%	20.66%	19.62%	20.92%	19.35%	19.47%	22.96%	20.60%	0.304995
StDev	0.02242	0.02419	0.02406	0.02259	0.02318	0.02310	0.02276	0.02097	0.02240	0.02495	0.02217	0.02226	0.02095	0.02304	0.02461	0.02453	0.02962	0.02924	0.047075
Kurtosis	3.16241	4.05324	4.59815	4.28852	6.14640	4.28976	5.80541	4.38776	4.54373	3.54181	3.96314	4.70195	3.76969	4.99035	3.23940	2.92017	4.96287	2.78056	5.673583
Skewness	0.53629	1.07359	1.05951	0.94467	1.44535	0.99158	1.39235	1.14120	1.09683	0.90007	0.67153	1.12288	0.66710	1.04165	0.70666	0.66221	0.88150	0.51749	1.085401
Number of obs	2668	43	43	43	42	43	42	43	41	41	43	43	43	42	43	43	39	43	36

S-Franc	All	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5
Min	5.92%	9.56%	10.13%	10.38%	9.98%	9.97%	10.37%	10.20%	10.13%	10.21%	9.69%	9.66%	9.11%	7.39%	9.67%	9.67%	10.16%	7.53%	0.086795
1st quartile	10.48%	12.13%	12.26%	11.80%	11.83%	11.92%	11.72%	11.91%	11.89%	12.18%	12.02%	12.11%	11.73%	11.83%	11.84%	11.18%	11.82%	11.45%	0.114753
Average	12.19%	13.64%	13.61%	13.50%	13.71%	13.68%	13.62%	13.62%	13.86%	13.94%	13.73%	13.88%	13.86%	13.89%	13.95%	13.54%	13.91%	14.22%	0.151875
Median	12.08%	13.48%	13.51%	13.45%	13.90%	13.48%	13.59%	13.74%	13.54%	14.12%	13.65%	13.63%	14.02%	13.99%	13.40%	13.23%	13.66%	13.95%	0.153447
3rd quartile	13.69%	15.16%	14.85%	14.93%	15.23%	14.98%	14.71%	14.95%	15.31%	15.54%	15.31%	15.51%	15.46%	15.52%	16.05%	15.75%	15.93%	15.67%	0.172585
Max	20.23%	17.91%	18.66%	17.27%	18.53%	18.53%	18.15%	18.53%	18.23%	19.60%	18.75%	17.65%	21.22%	21.37%	20.67%	20.22%	20.05%	22.82%	0.277307
StDev	0.02336	0.01880	0.01830	0.01901	0.02003	0.02195	0.02008	0.02089	0.02137	0.02305	0.02185	0.02219	0.02675	0.02591	0.02752	0.02433	0.02400	0.03640	0.042273
Kurtosis	2.81892	2.68875	2.98327	2.04980	2.43029	2.52019	2.55061	2.32904	2.14473	2.46368	2.28740	1.91590	2.87596	3.70367	2.44182	2.57281	2.50156	3.32422	3.874419
Skewness	0.26430	0.28245	0.52004	0.31028	0.25758	0.52085	0.44546	0.19315	0.23301	0.18864	0.16998	0.09696	0.44286	0.32146	0.46594	0.41300	0.34646	0.78291	0.72313
Number of obs	2863	47	47	47	46	47	46	47	45	45	47	47	47	46	46	46	42	47	39

Table 7.1c Summary Statistics for the At-the-money Implied Volatilities for Four Foreign Exchange Options, Estimated on a Daily Basis and at 5-days increments until expiration.

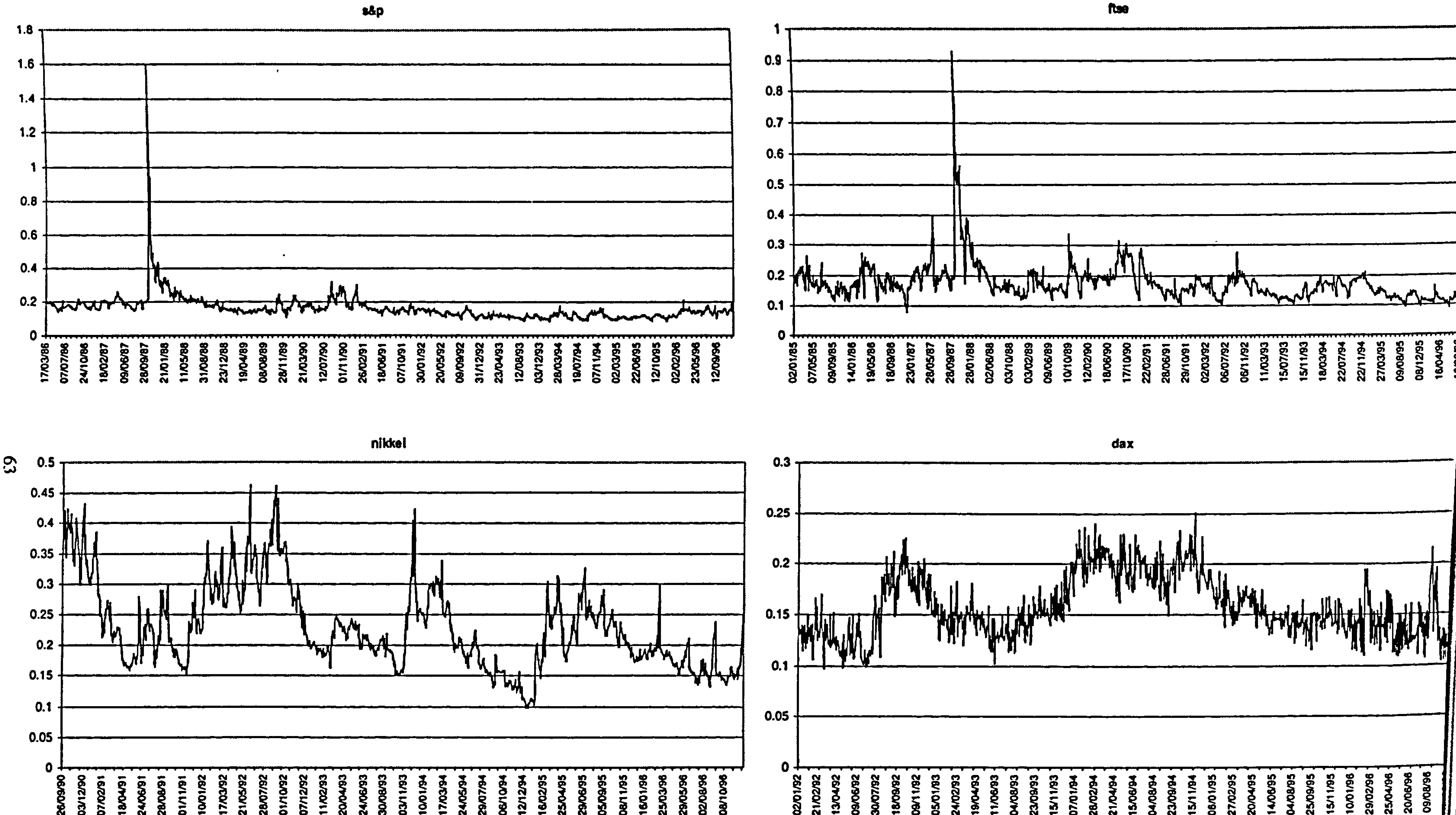

Figure 7.1a Time Series Plots of At-the-money Implied Volatilities for Four Stock Index Options

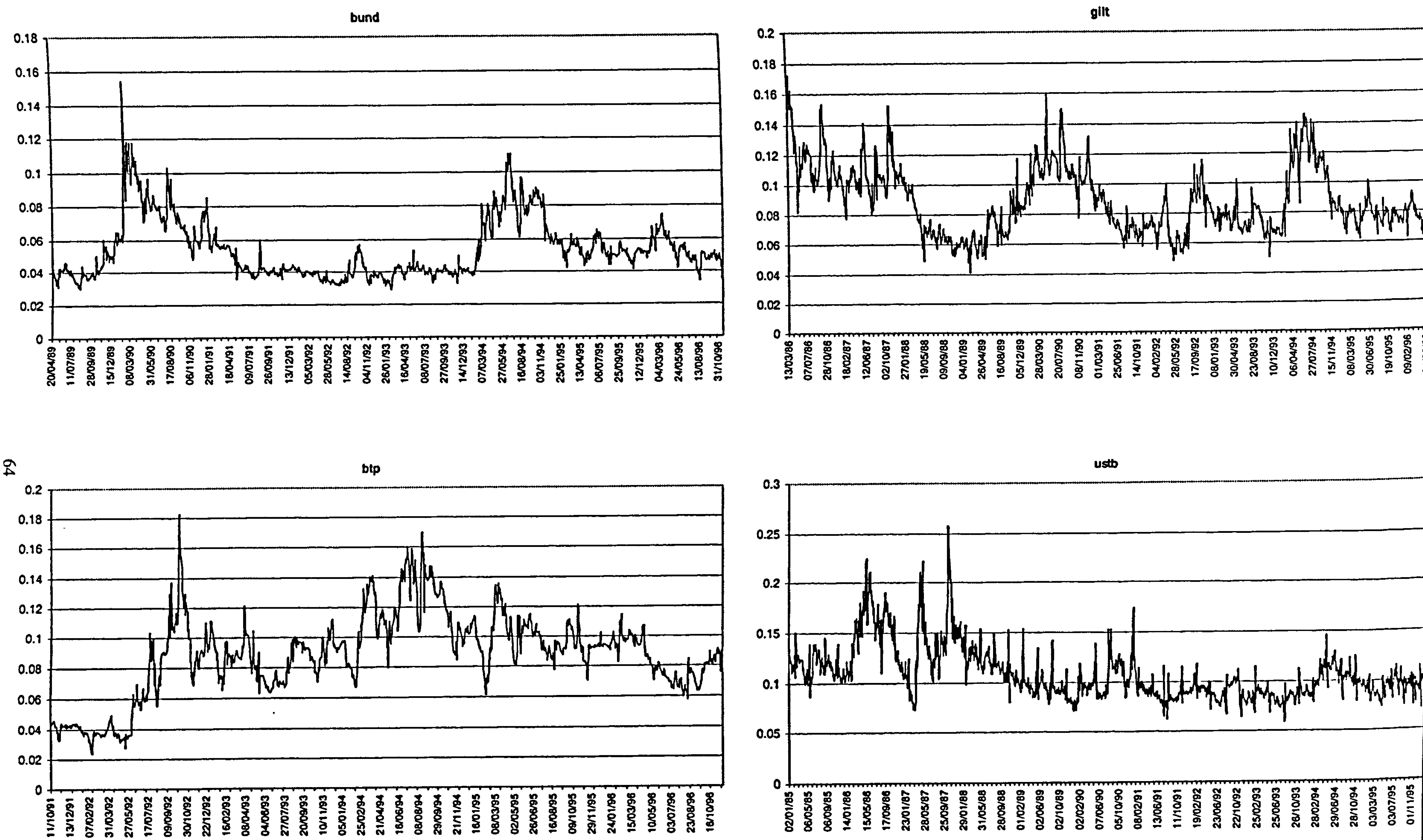

Figure 7.1b Time Series Plots of At-the-money Implied Volatilities for Four Fixed Income Options

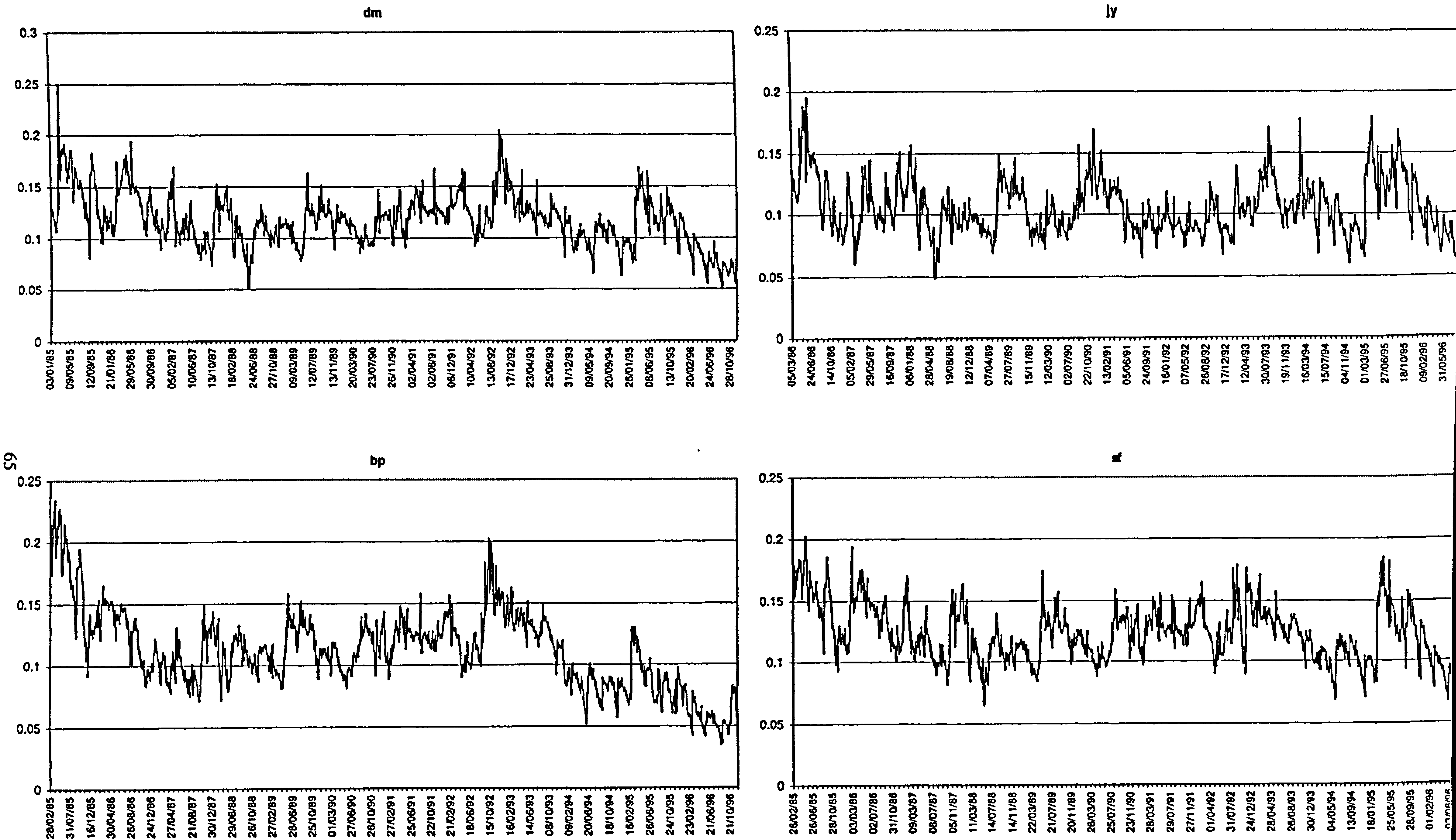

Figure 7.1c Time Series Plots of At-the-money Implied Volatilities for Four Foreign Exchange Options

Market: FTSE
Contract: June 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
2625	1092	#N/A	1	0.400817283
2725	992.5	#N/A	1	0.36199124
2825	893.5	#N/A	1	0.324289894
2875	843.5	#N/A	1	0.305821098
2925	794	#N/A	1	0.287585817
2975	744.5	#N/A	1	0.269568103
3025	695	#N/A	1	0.251751654
3075	645.5	#N/A	1	0.234119565
3125	595.5	#N/A	1	0.216654015
3175	546.5	#N/A	1.5	0.210819832
3225	497	#N/A	1.5	0.192841519
3275	448	#N/A	1.5	0.174960178
3325	399	#N/A	2	0.164465522
3375	351	0.162671451	3.5	0.161467282
3425	305	0.166067357	7.5	0.167370101
3475	258	0.155895921	10	0.156398421
3525	213	0.148750306	14.5	0.148725406
3575	169.5	0.140019668	21	0.141031439
3625	129.5	0.133184584	30.5	0.133747171
3675	92	0.123377088	42.5	0.1236255
3725	61	0.116513259	61	0.116513259
3775	38	0.112989004	88	0.113744441
3825	22	0.110769592	121.5	0.111368884
3875	12	0.109962382	161	0.110369231
3925	6	0.109072286	204.5	0.109106298
3975	2.5	0.106289642	251	0.109290654
4025	1.5	0.112134934	299	#N/A
4075	1	0.119285738	347.5	#N/A
4125	1	0.132922452	397	#N/A
4175	1	0.146202968	446.5	#N/A
4225	1	0.159161212	496.5	#N/A

Market: FTSE
Contract: September 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
3025	703.5	#N/A	5	0.188988847
3125	609.5	#N/A	8	0.180059308
3225	517	0.173841159	14	0.175384202
3325	425	0.1607915	19.5	0.16133554
3425	337.5	0.150968349	29.5	0.150894644
3525	257	0.143788947	46.5	0.143347446
3625	185.5	0.137642165	73.5	0.138165713
3725	125.5	0.132366142	110.5	0.131989493
3825	78.5	0.127625359	161.5	0.1275096
3925	48	0.127636106	229	0.127814124
4025	27.5	0.127304218	306	0.127084699
4125	15.5	0.128595854	392	0.128836961
4225	8	0.128465735	482	#N/A

Market: FTSE
Contract: December 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
3125	639.5	#N/A	18	0.174028237
3225	548	0.161959178	23	0.161719603
3325	462.5	0.156273404	34	0.156385276
3425	382.5	0.152273071	50	0.151974324
3525	309.5	0.149511042	73.5	0.149463335
3625	245	0.147867273	105.5	0.148005968
3725	187.5	0.145008806	144	0.144866048
3825	135.5	0.139338646	188.5	0.139365345
3925	93	0.133977403	242.5	0.134182217
4025	63	0.131914942	308.5	0.131826299
4125	42	0.131318153	384	0.131446942
4225	28	0.132121488	466	0.131817727
4325	18	0.132350426	553	#N/A

Table 7.2 Option prices and implied volatilities for FTSE-100 as of May 7th, 1996

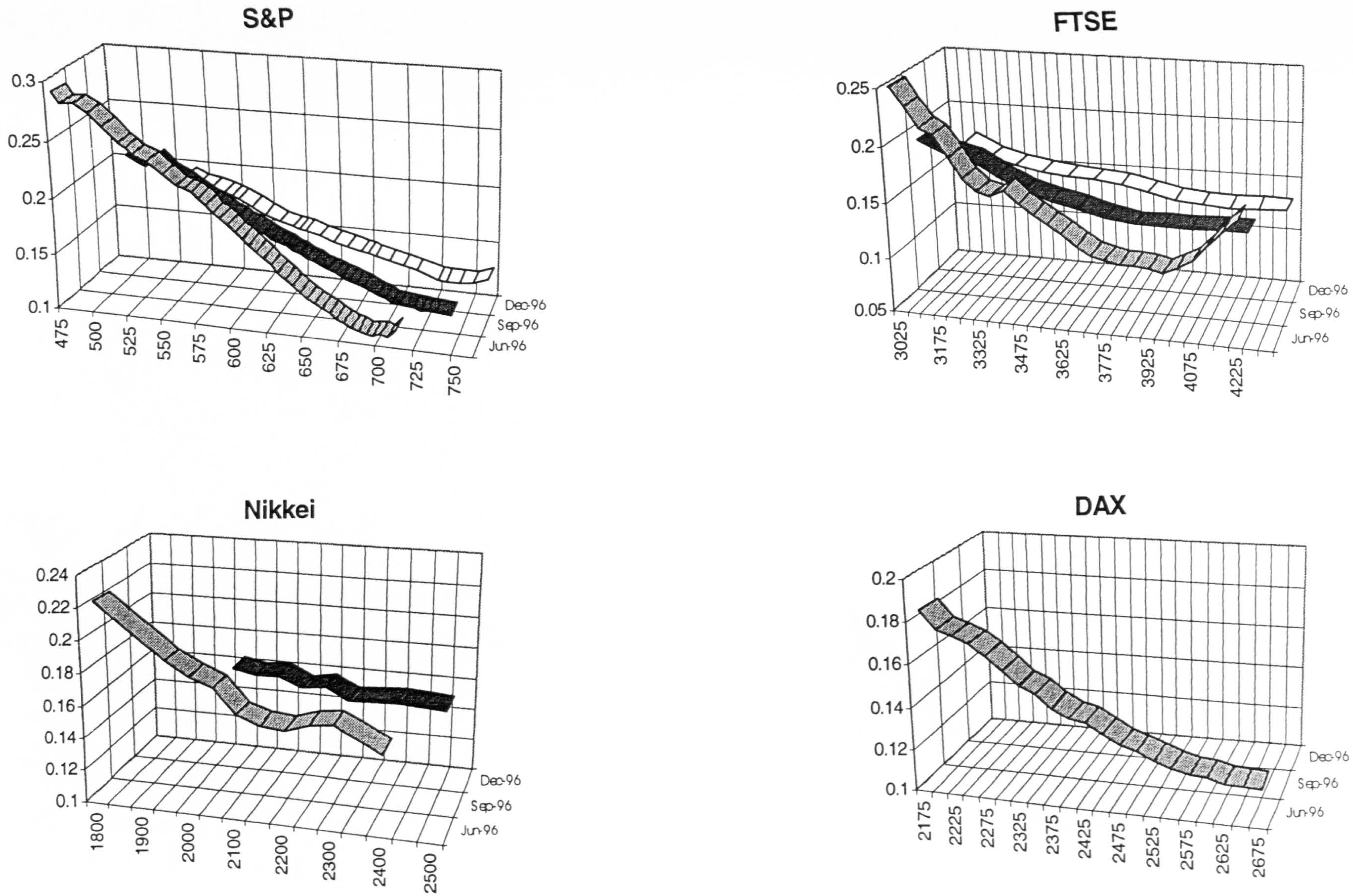

Figure 7.2a Implied Volatility Smiles for Four Stock Index Options as of May 7, 1996

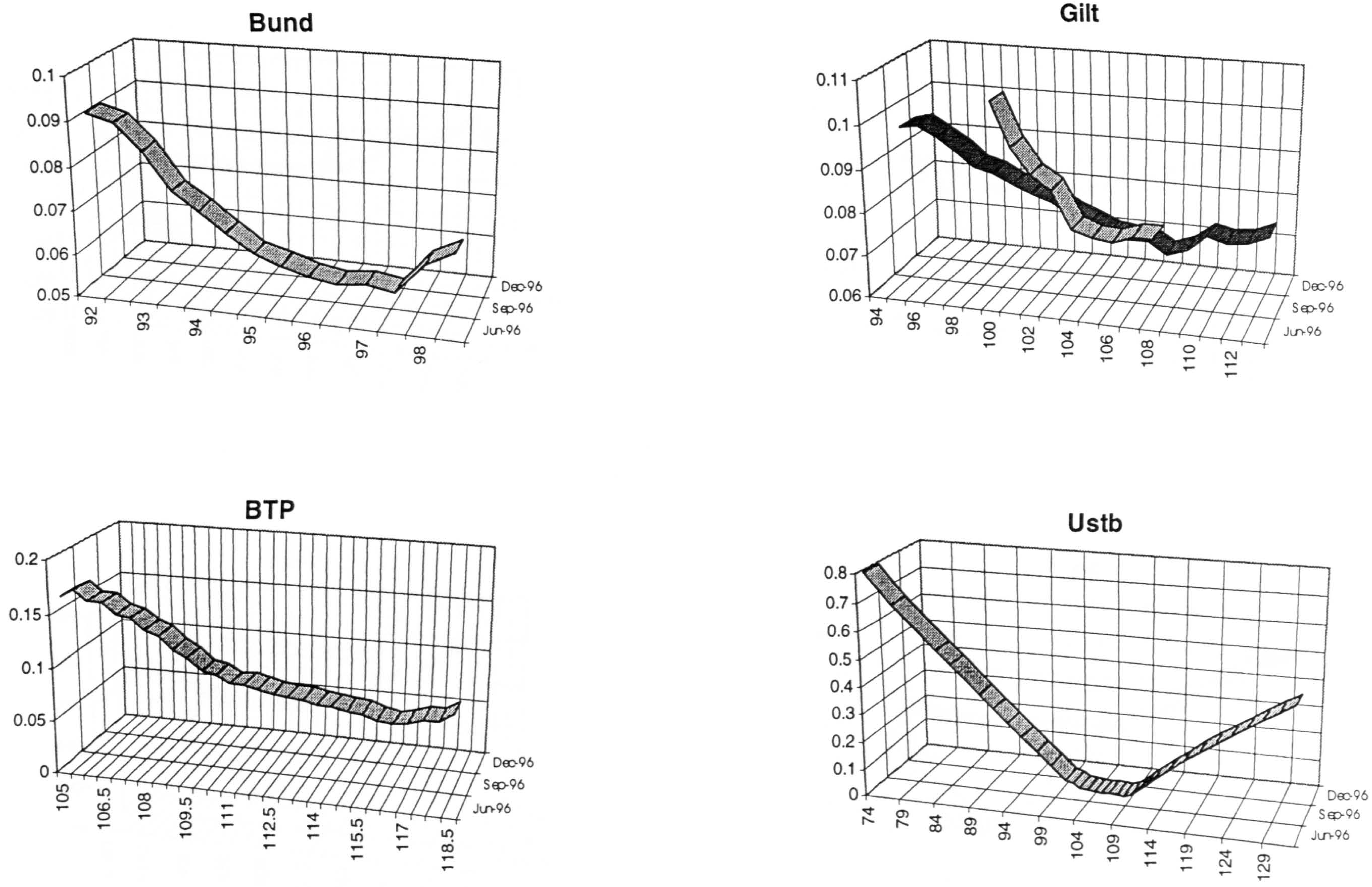

Figure 7.2b Implied Volatility Smiles for Four Fixed Income Options as of May 7, 1996

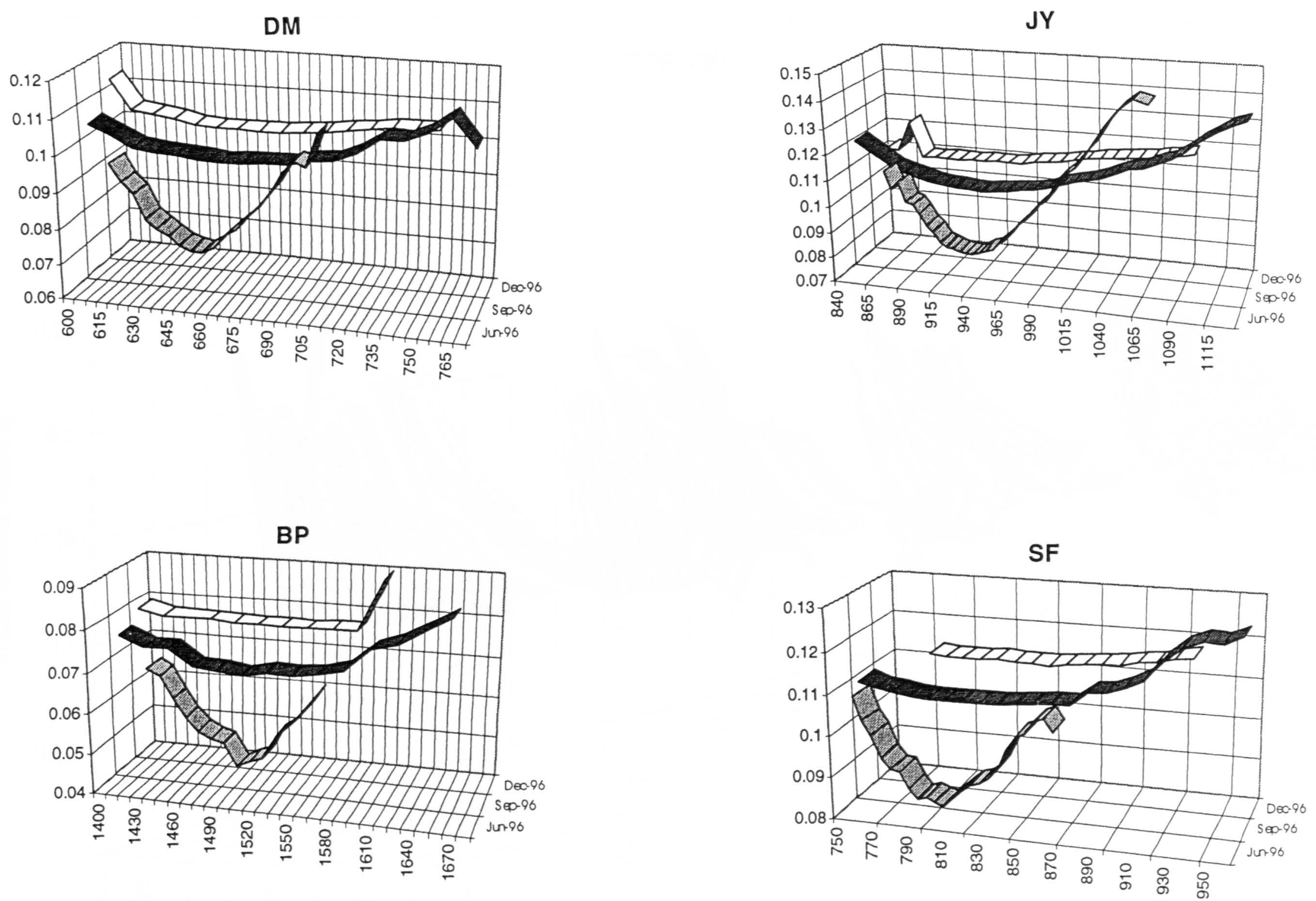

Figure 7.2c Implied Volatility Smiles for Four Foreign Exchange Options as of May 7, 1996

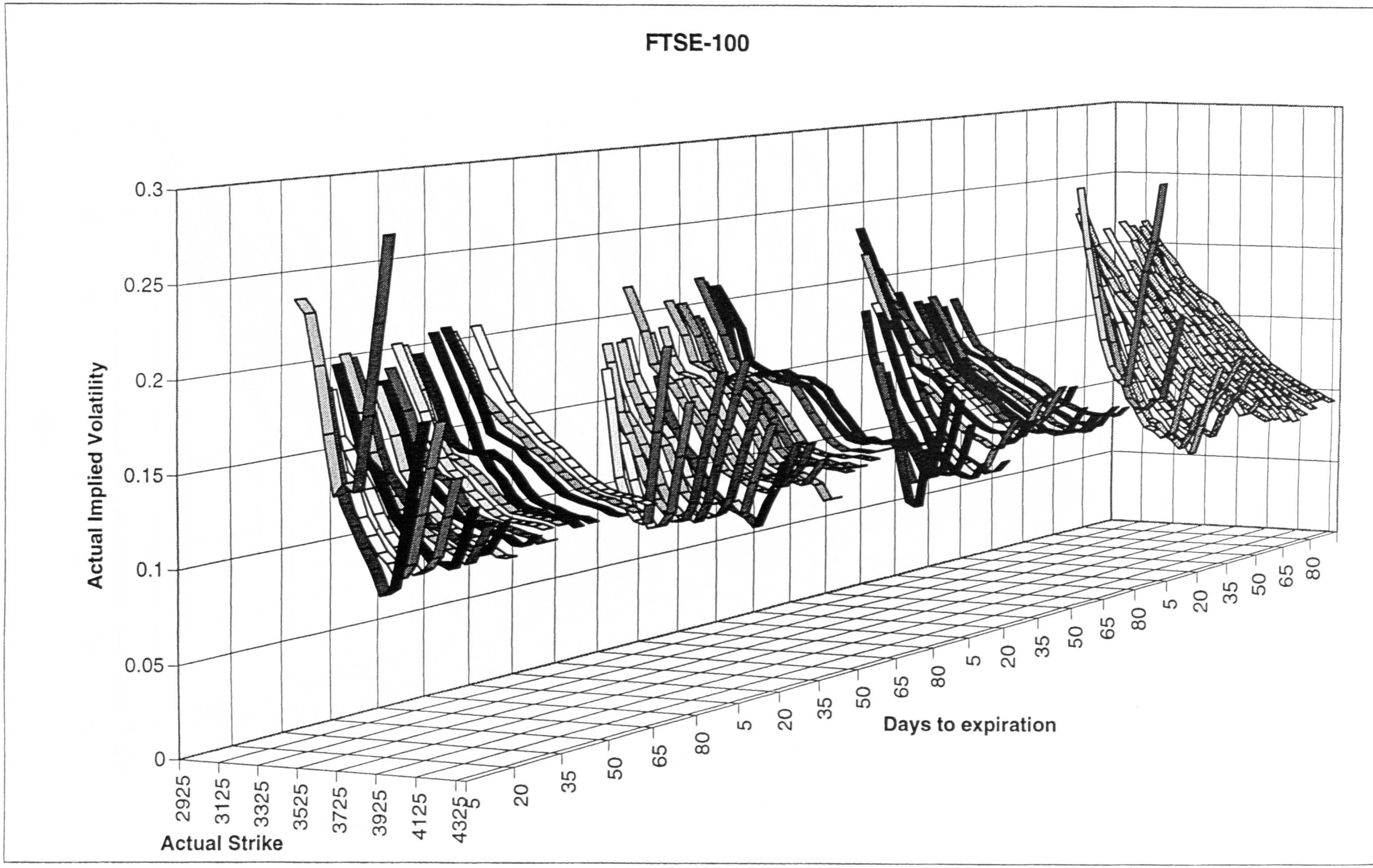

Figure 7.4 Unstandardized volatility smiles for 1996 FTSE-100 contracts

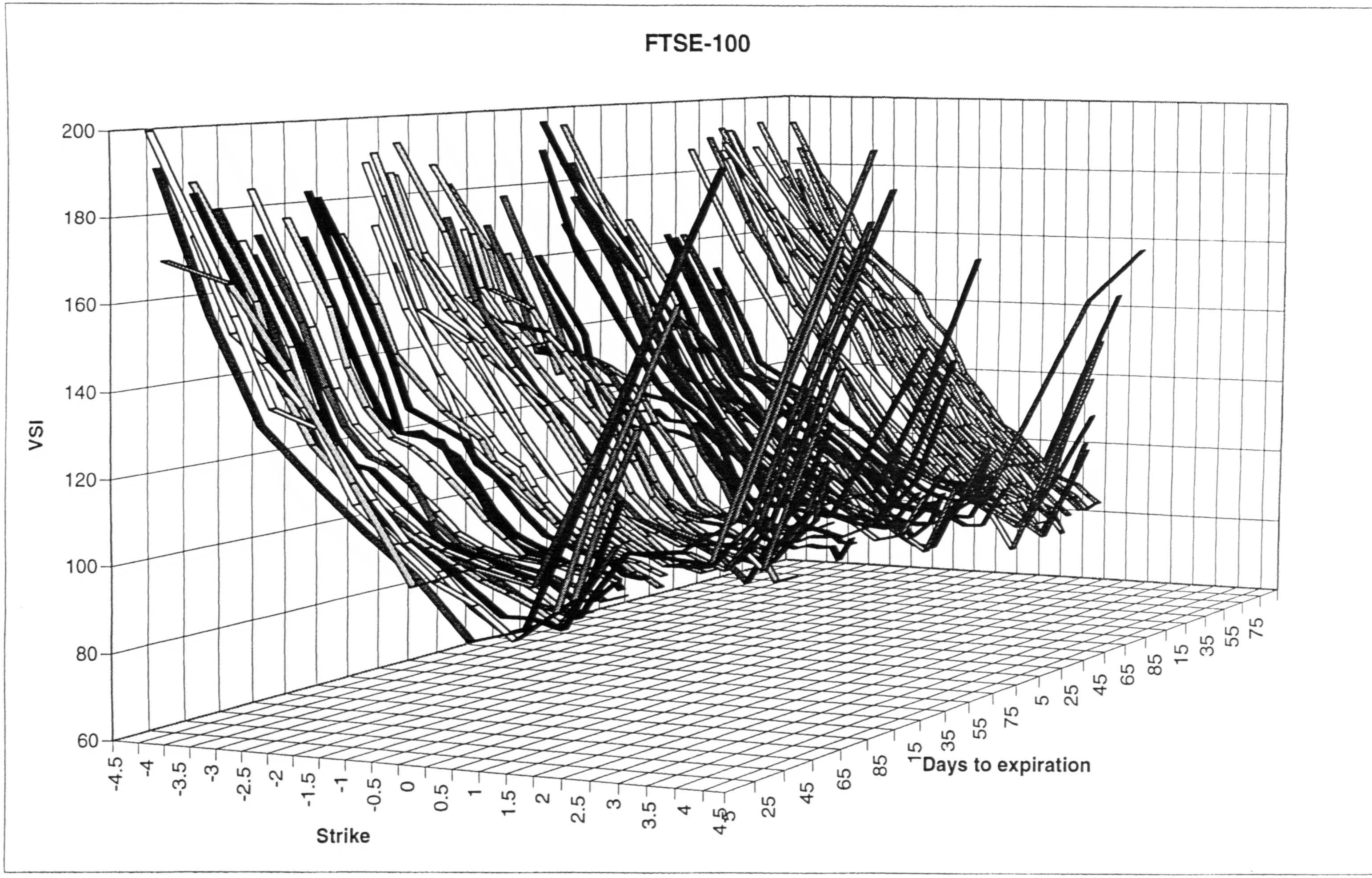

Figure 7.5 Standardized volatility smiles for 1996 FTSE-100 contracts grouped by contracts

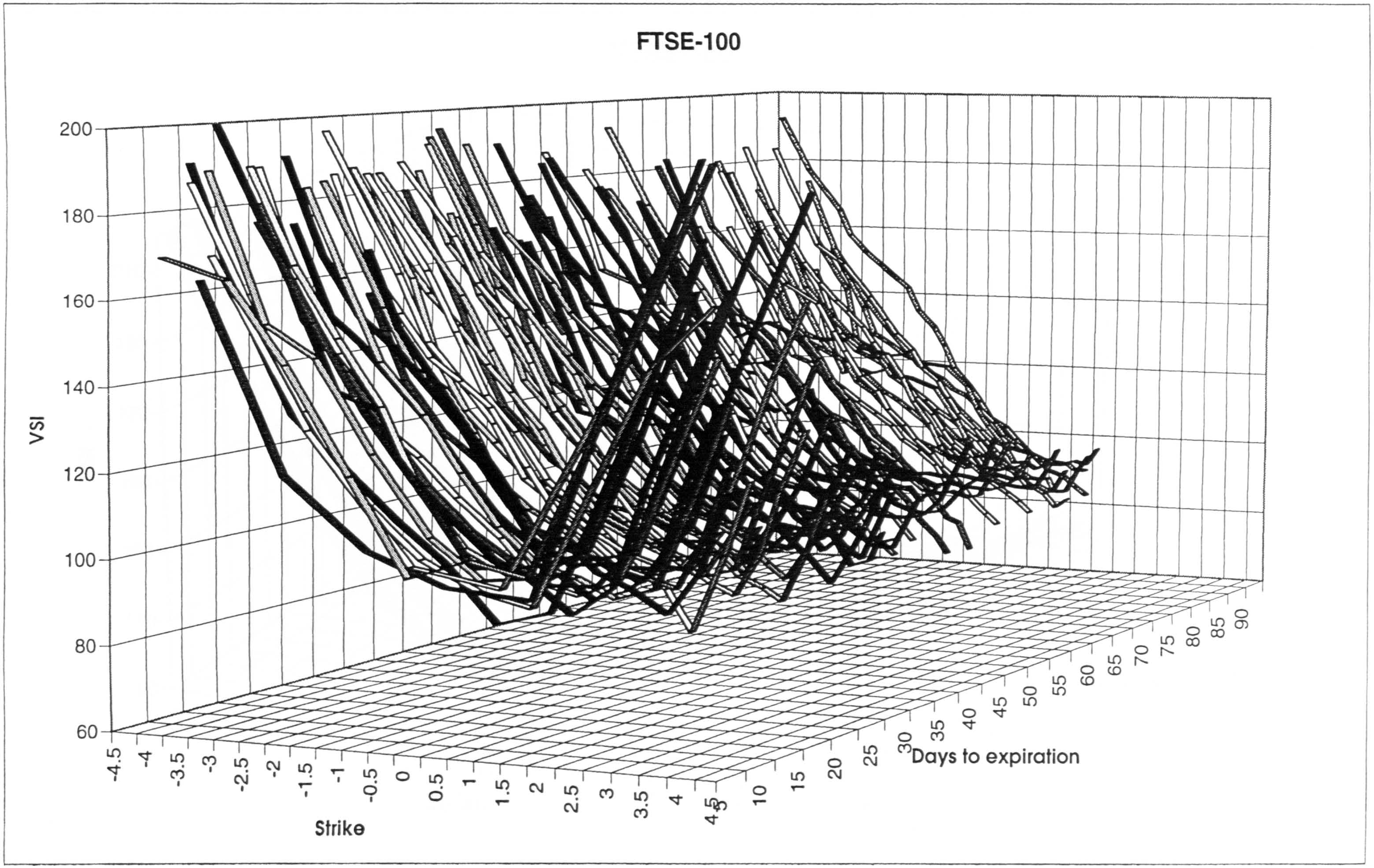

Figure 7.5 Standardized volatility smiles for 1996 FTSE-100 contracts grouped by days to expiration

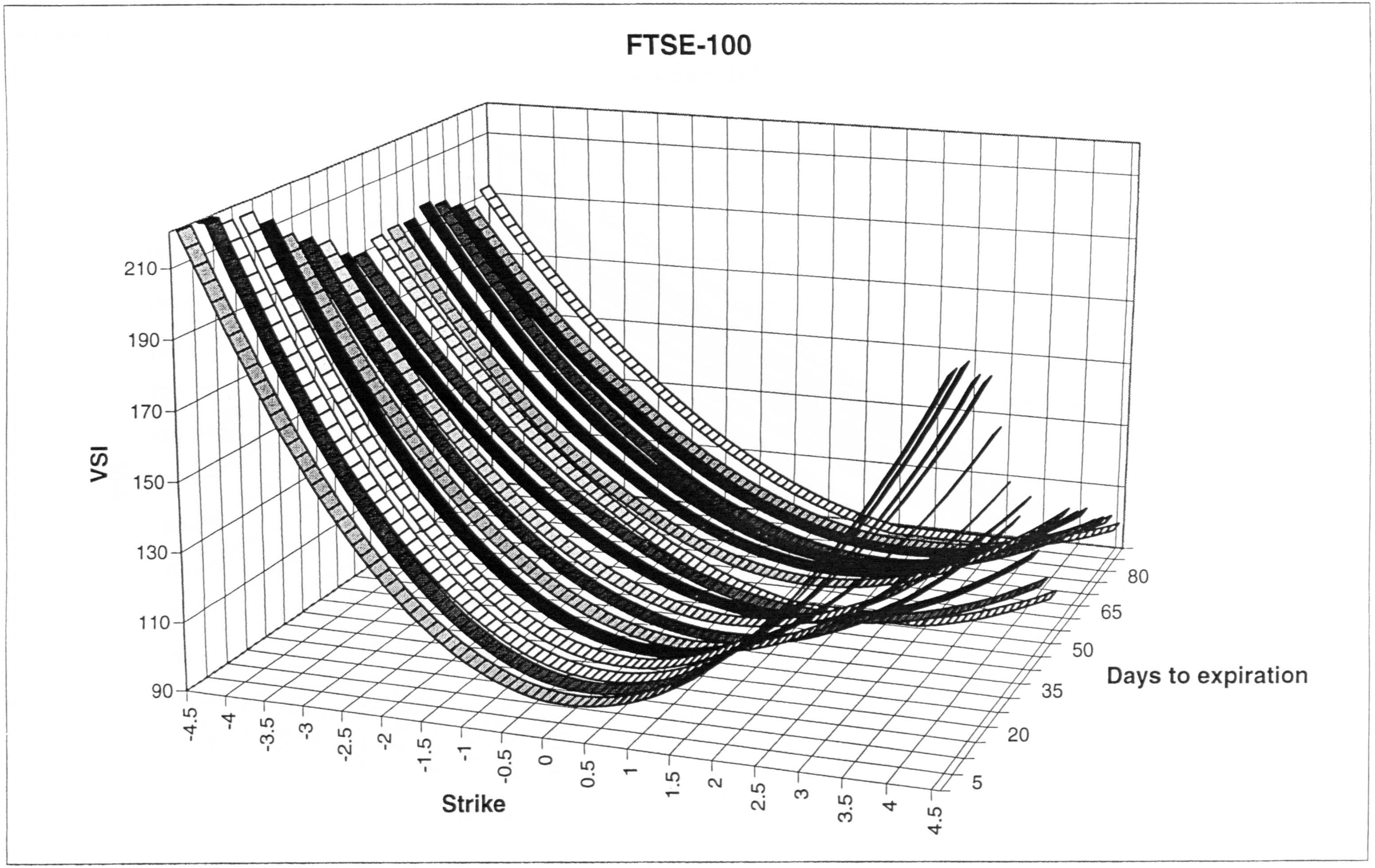

Figure 7.7 Interpolated volatility smiles for 1996 FTSE-100 contracts

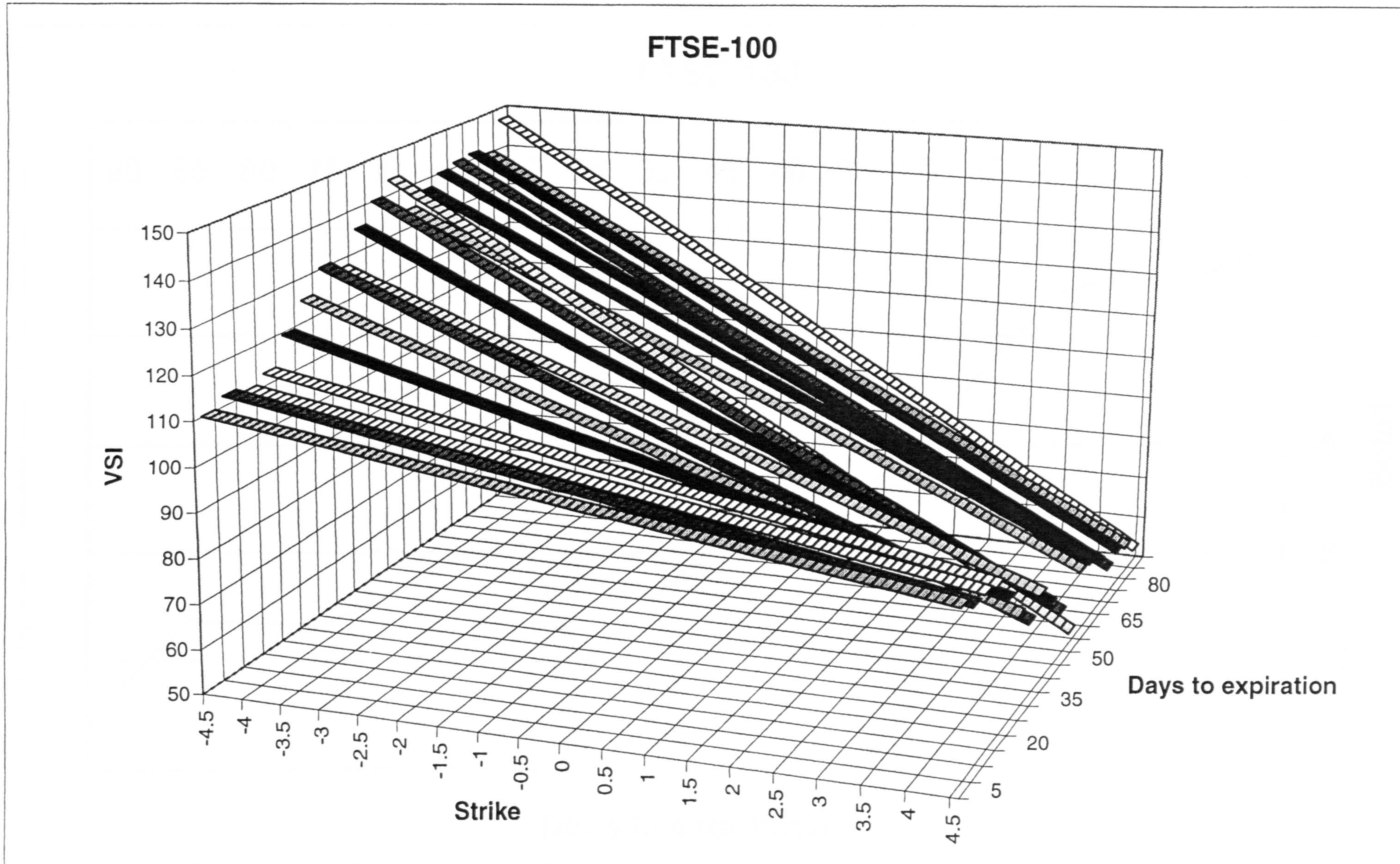

Figure 7.8a Skewness for 1996 FTSE-100 contracts

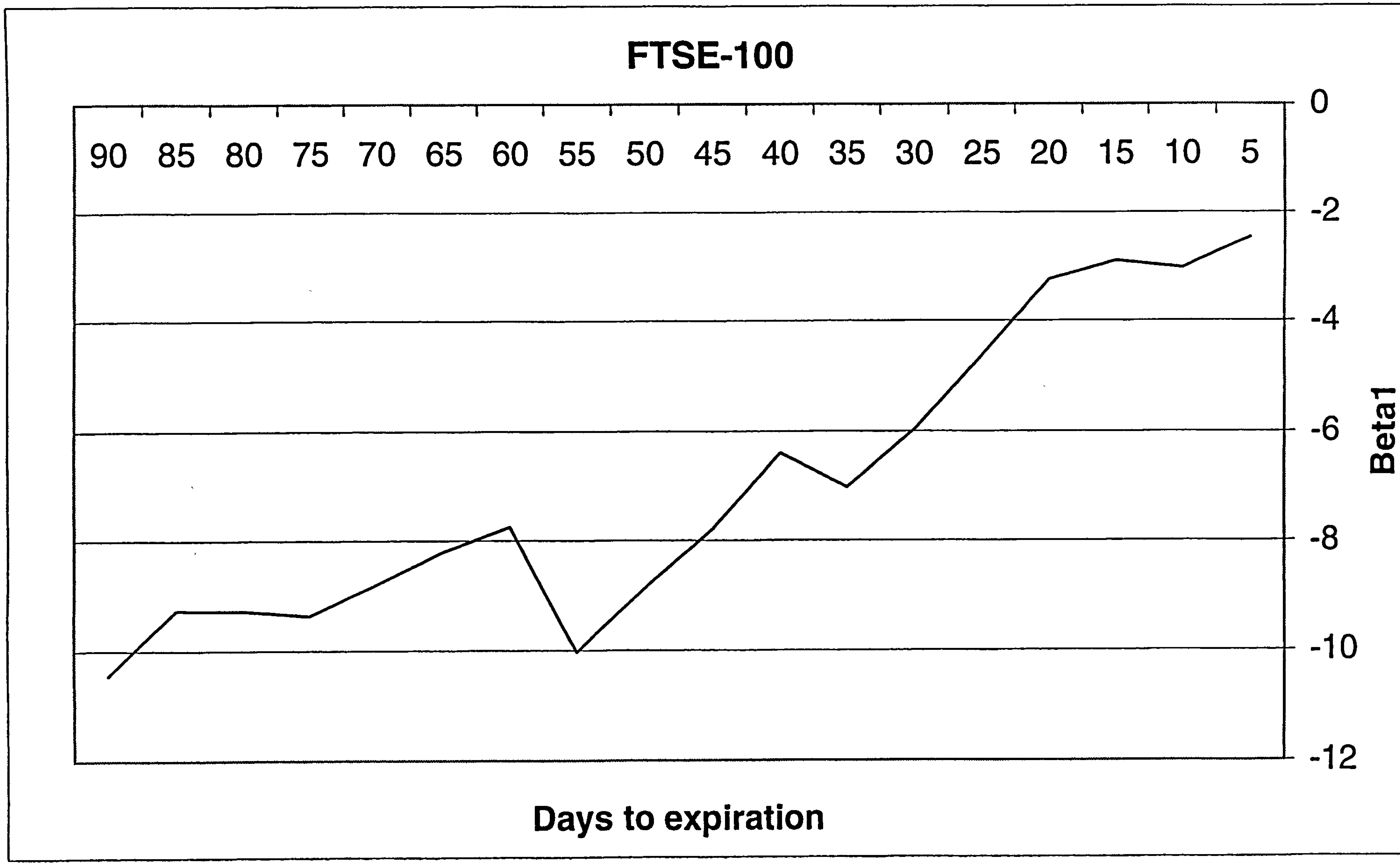

Figure 7.8b Skewness for 1996 FTSE-100 contracts

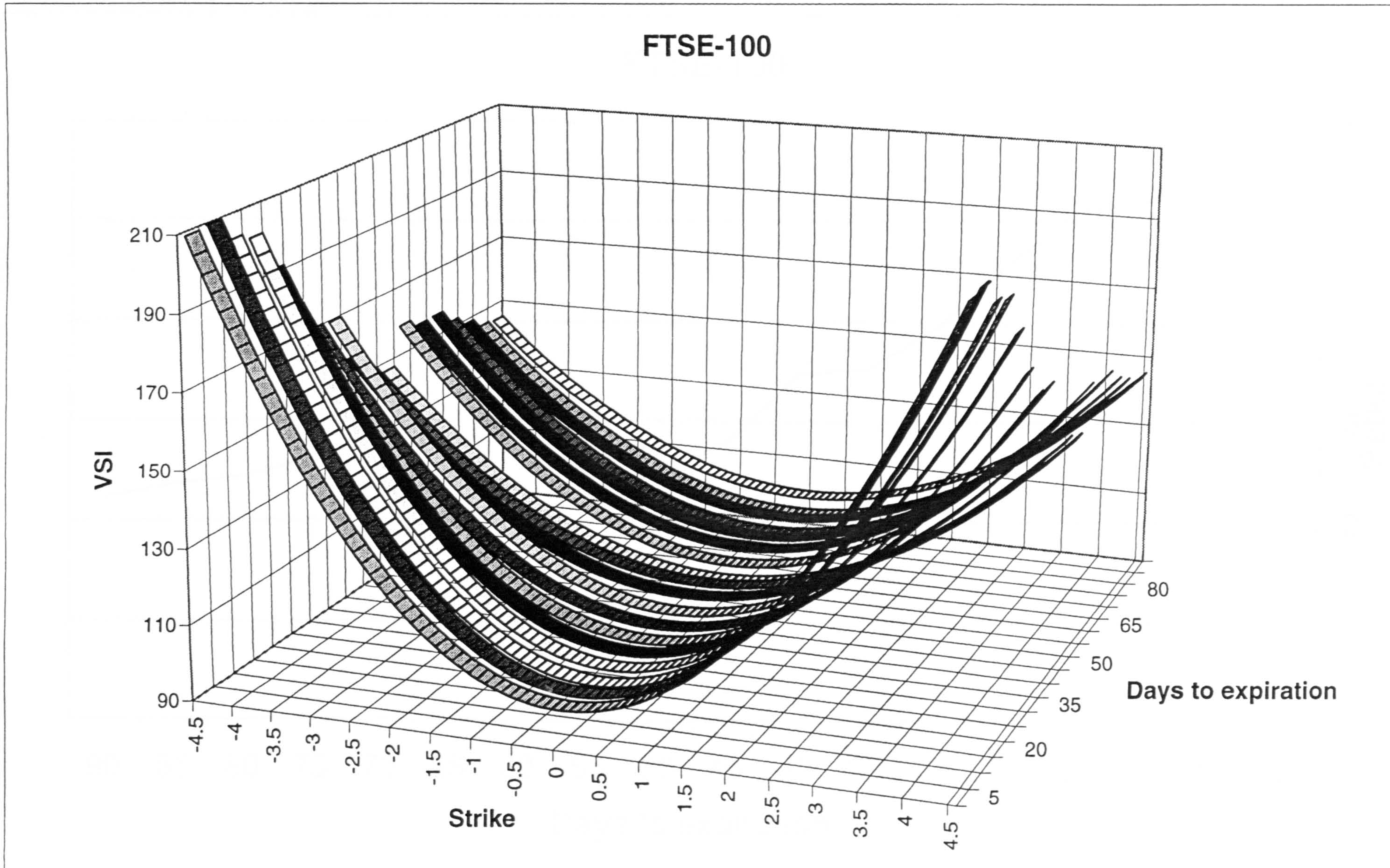

Figure 7.9a Kurtosis for 1996 FTSE-100 contracts

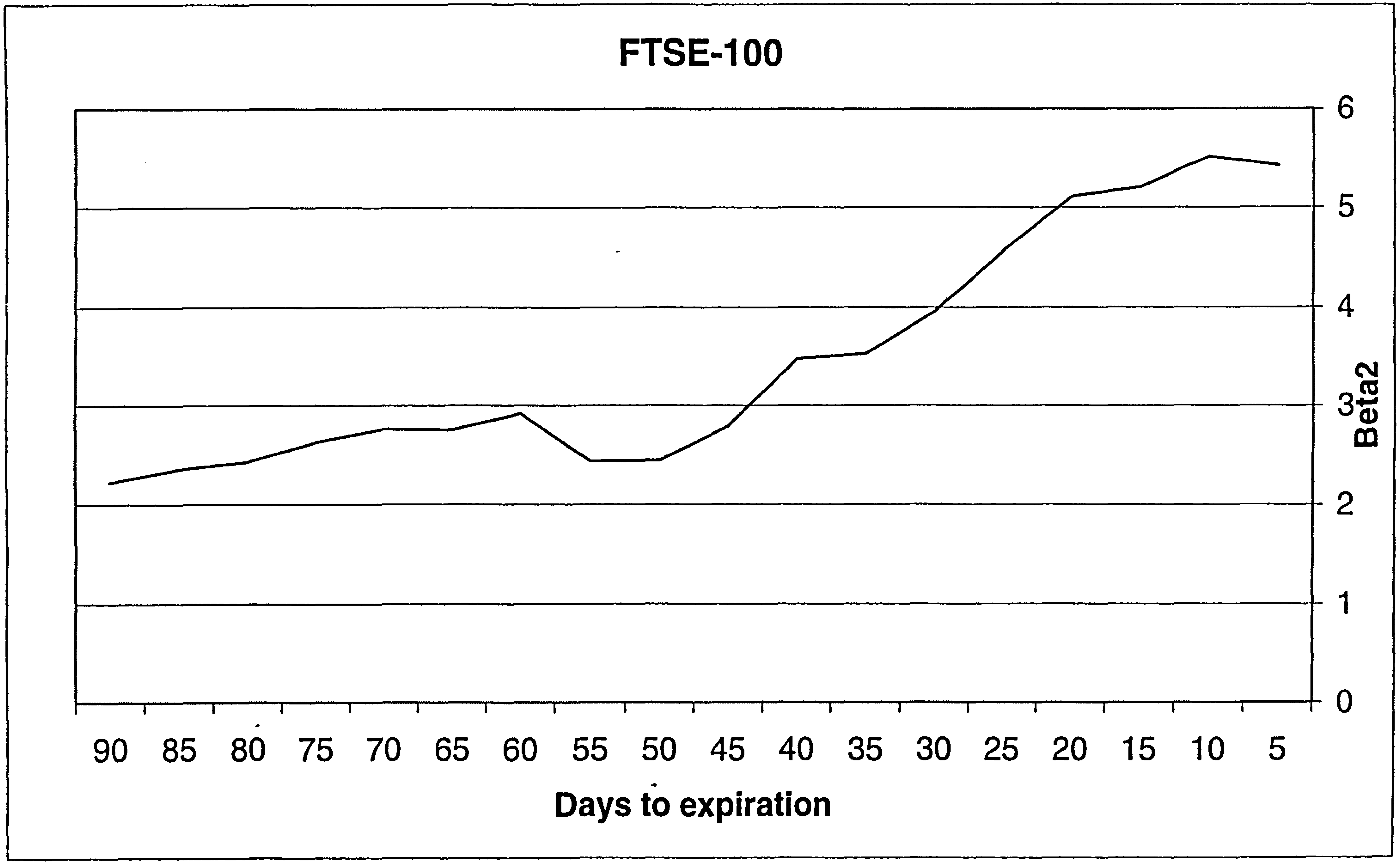

Figure 7.9b Kurtosis for 1996 FTSE-100 contracts

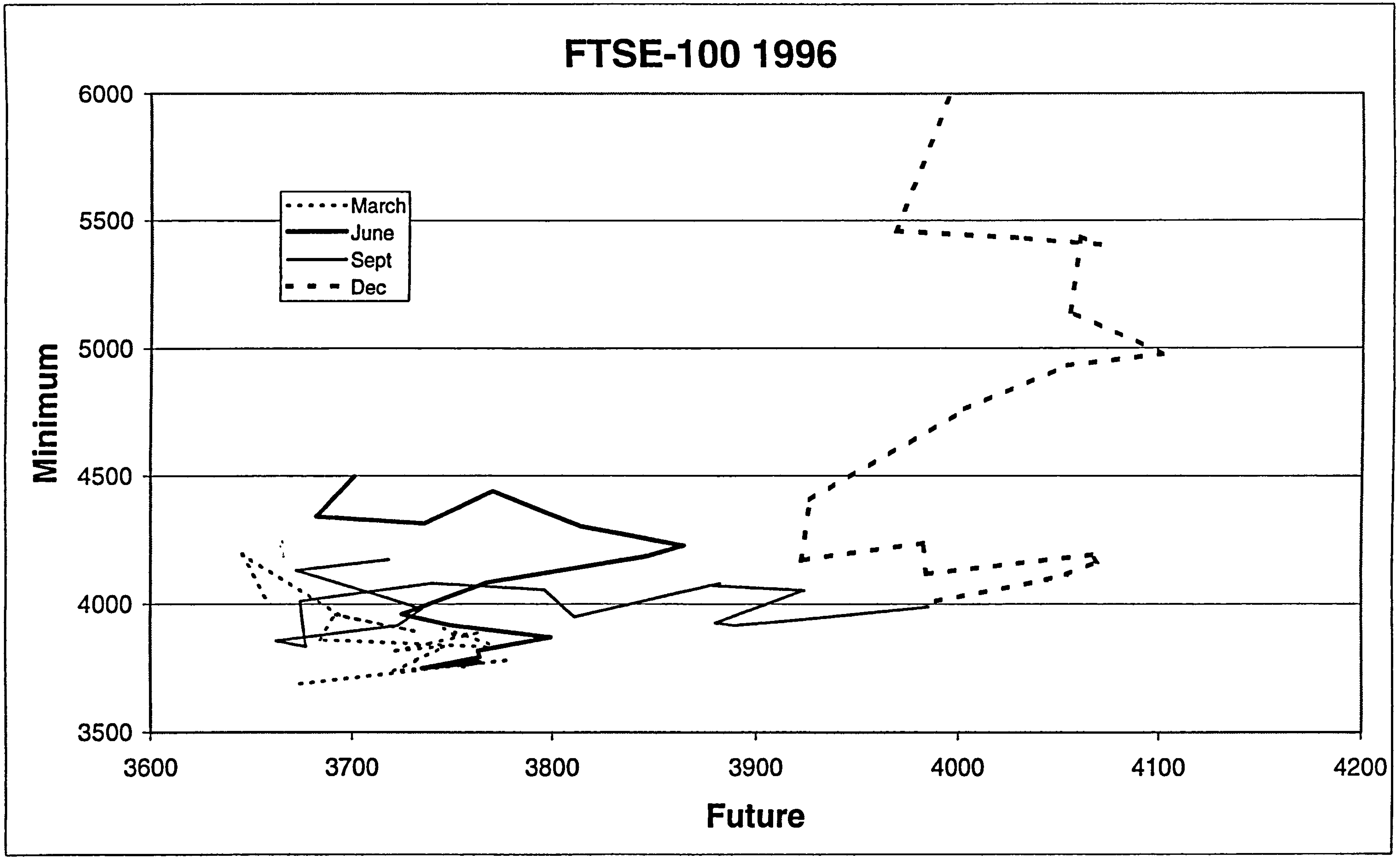

Figure 7.10a Scatterplots of minimum VSI strike price relative to underlying future price for the FTSE-100 during 1996.

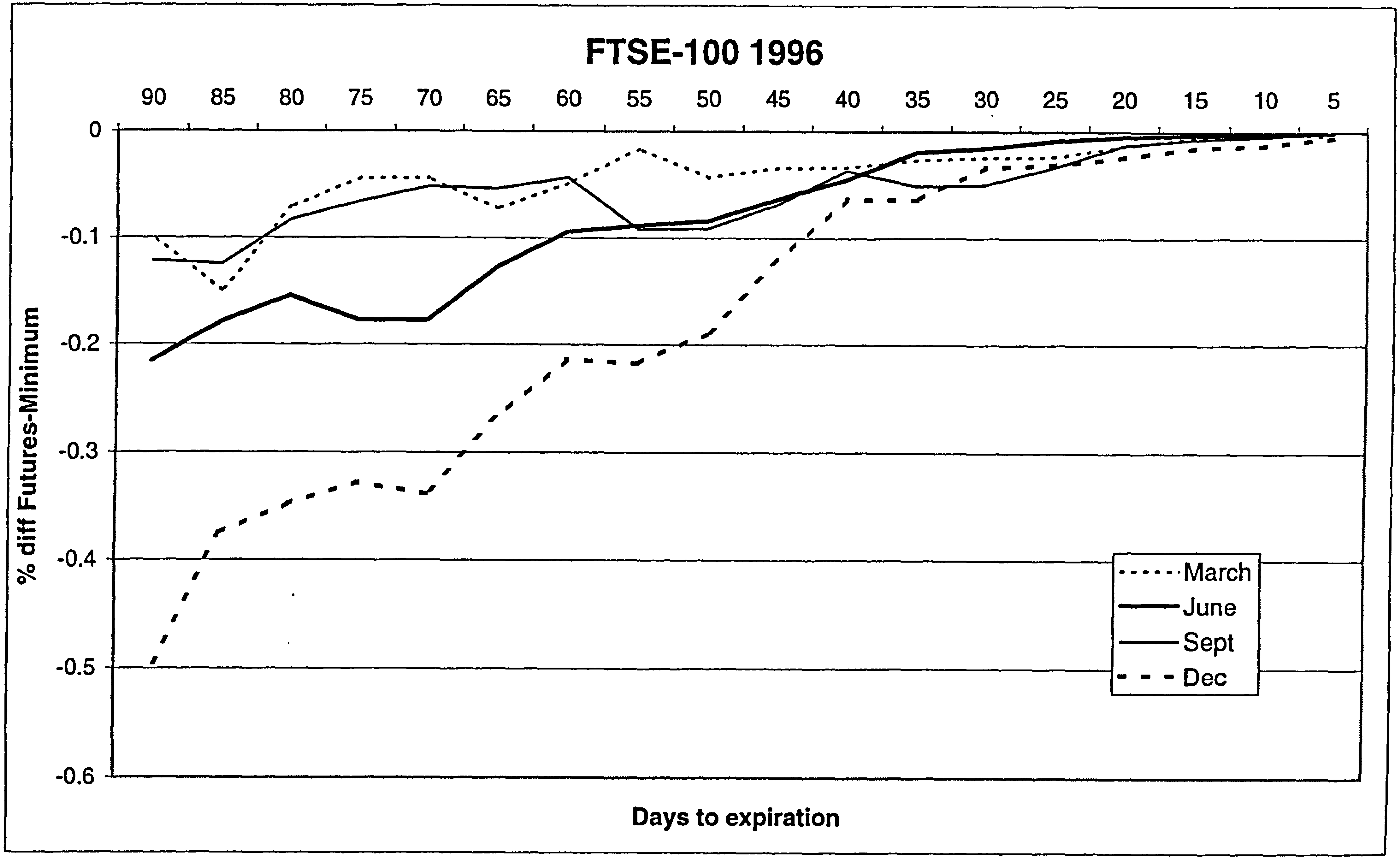

Figure 7.10b Plot of the percentage difference between the Underlying Futures Price to the minimum Strike Price for Four Options Cycles for the FTSE-100 during 1996.

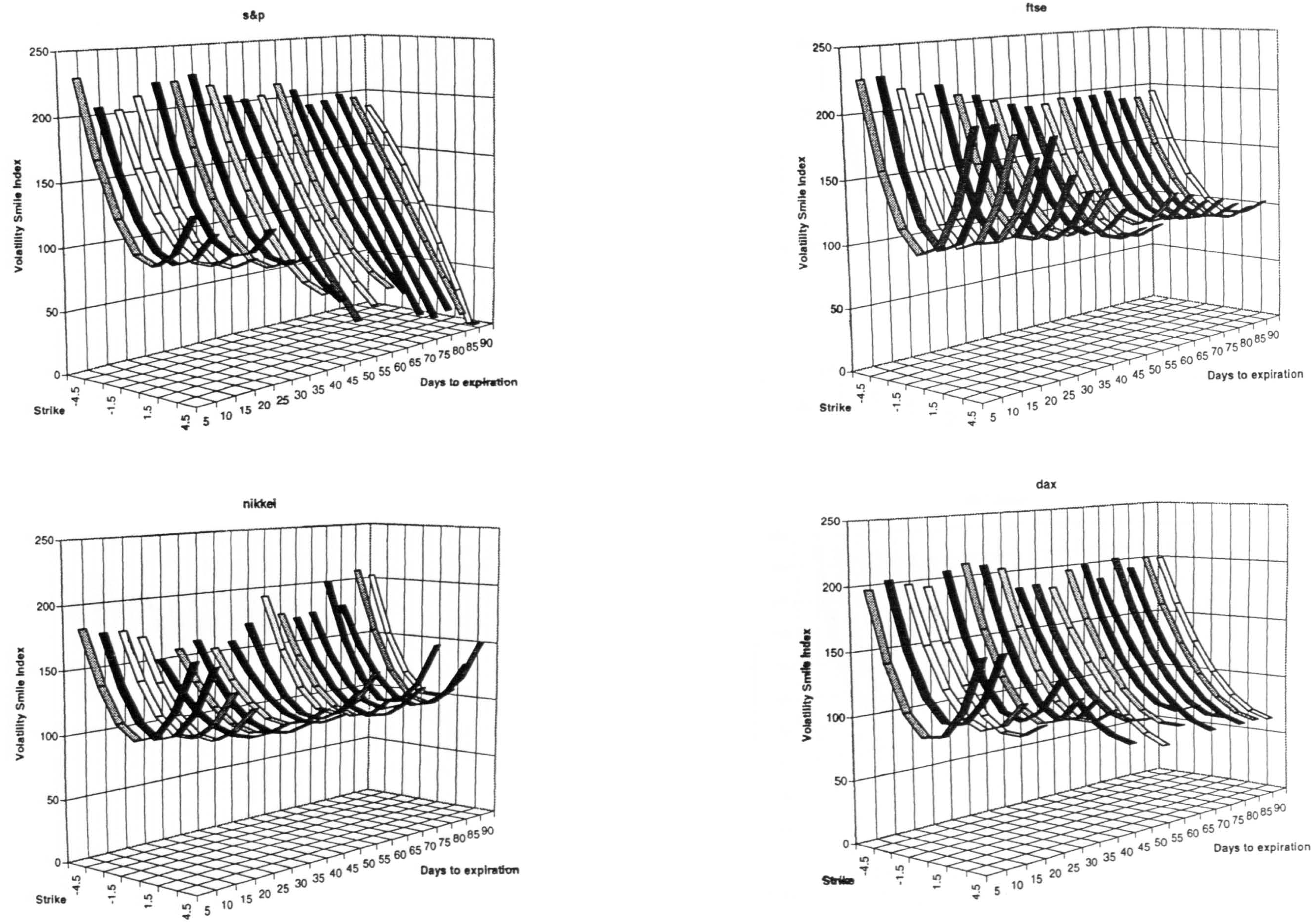

Figure 7.11a 1996 Volatility smiles for Four Stock Index Options

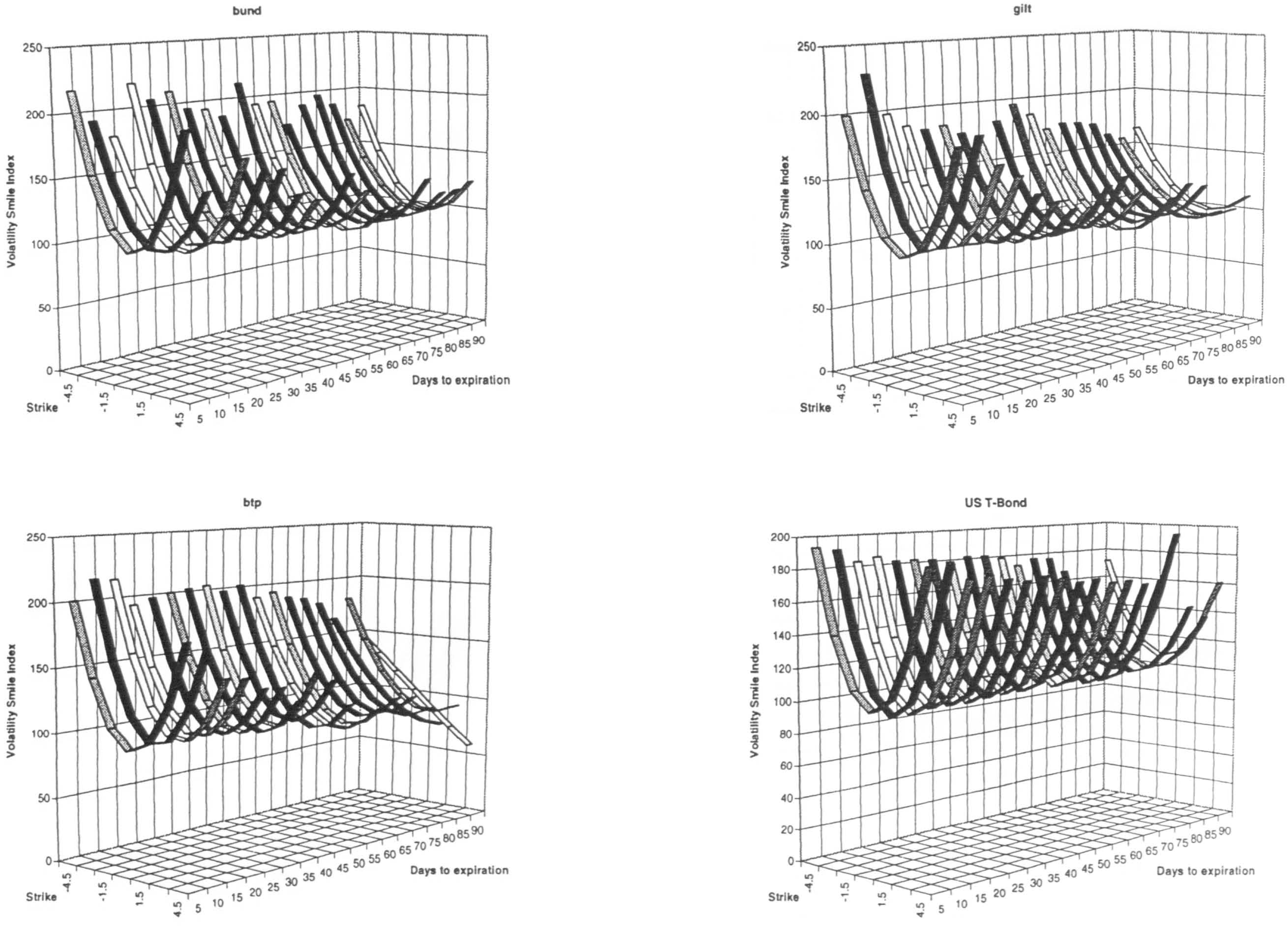

Figure 7.11b 1996 Volatility smiles for Four Fixed Income Options

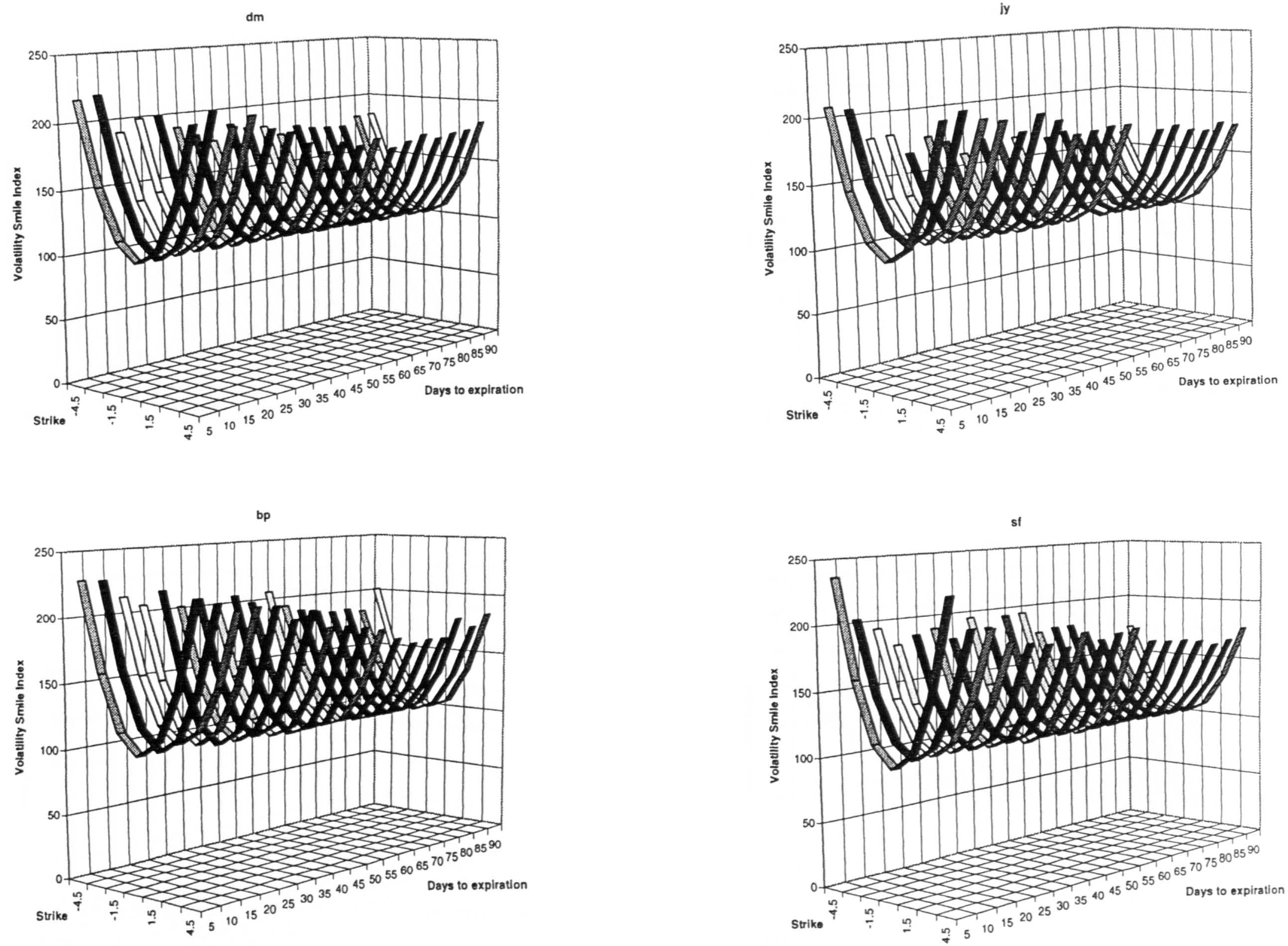

Figure 7.11c 1996 Volatility smiles for Four Foreign Exchange Options

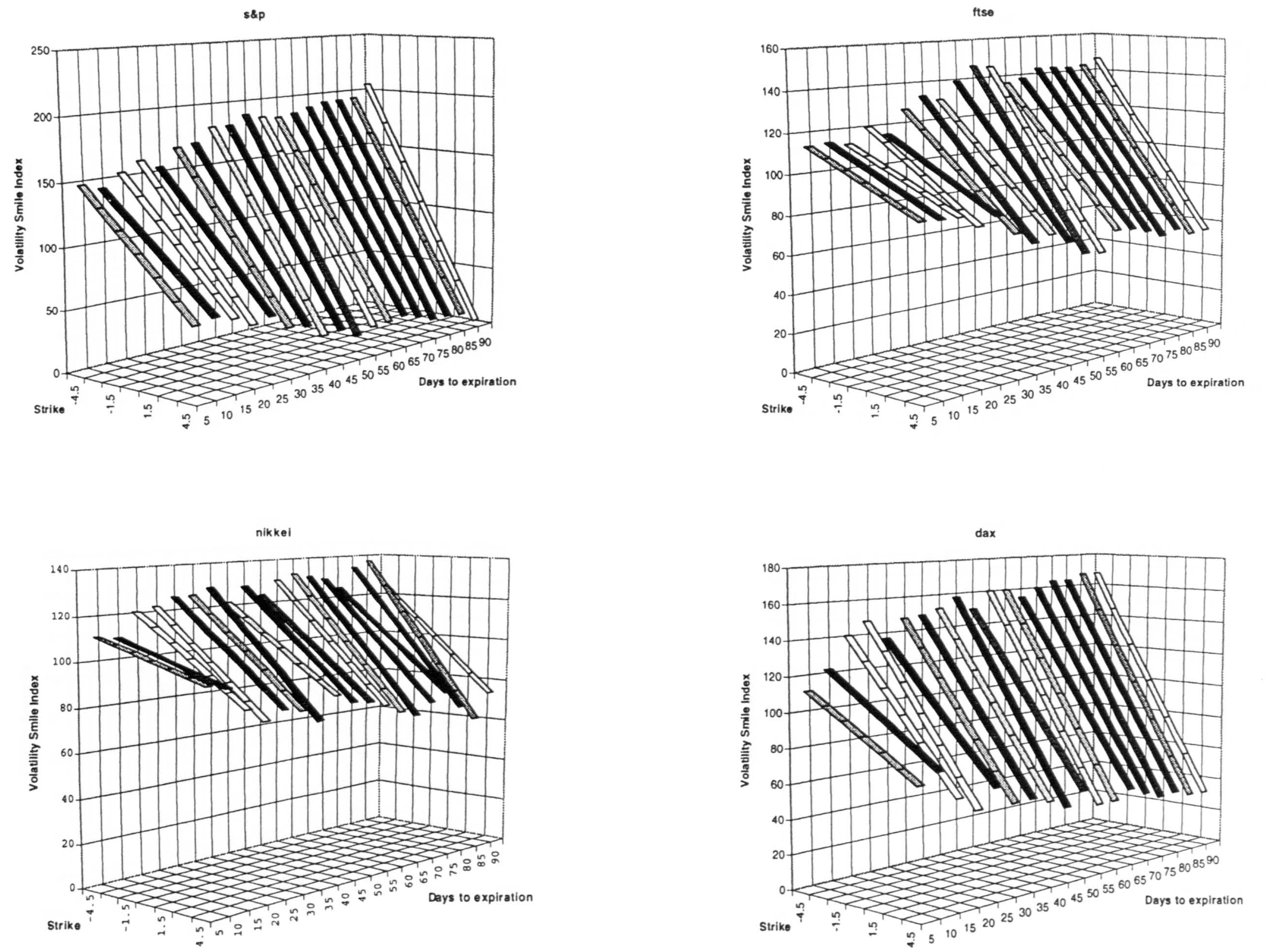

Figure 7.12a 1996 Skewness for Four Stock Index Options

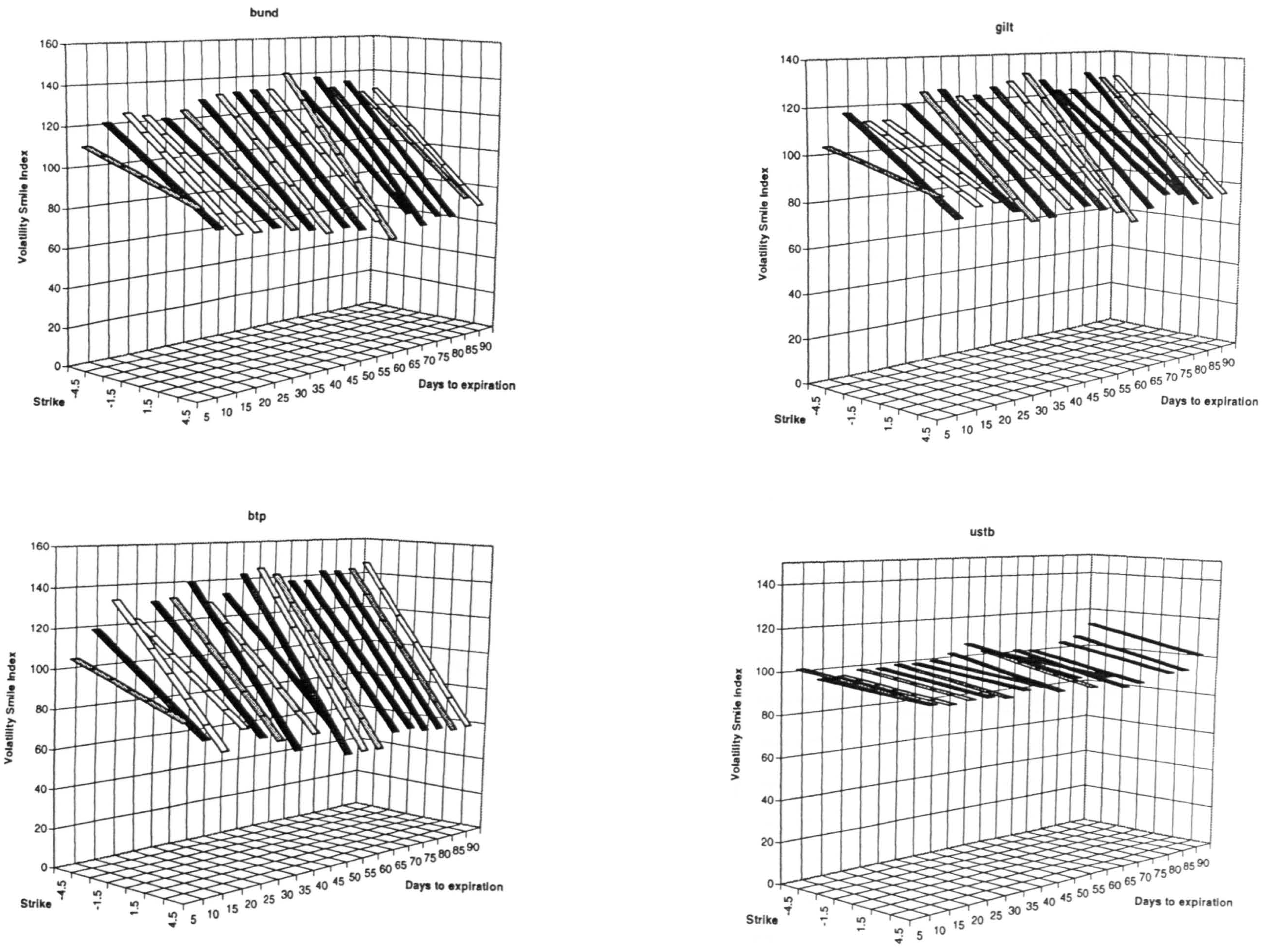

Figure 7.12b 1996 Skewness for Four Fixed Income Options

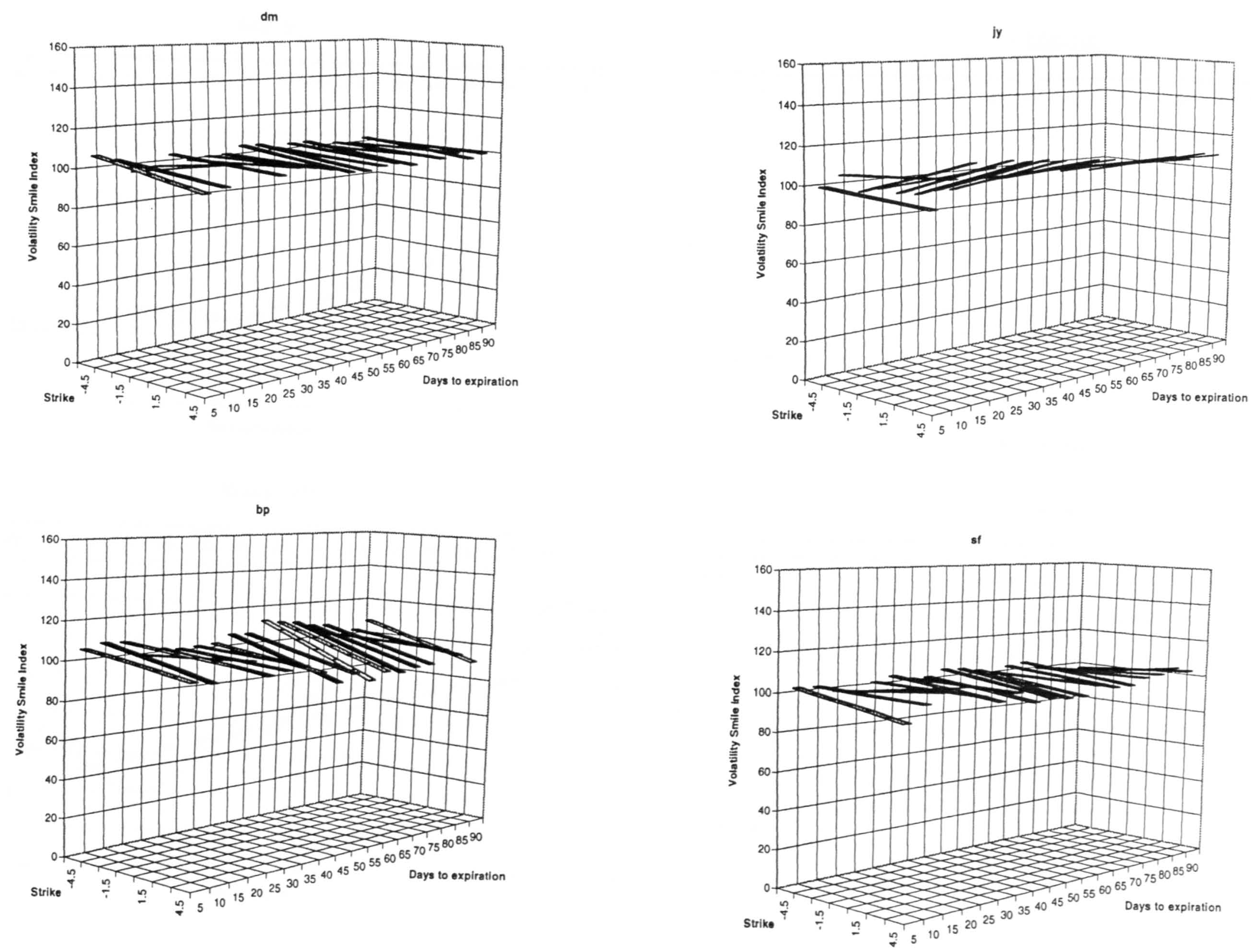

Figure 7.12c 1996 Skewness for Four Foreign Exchange Options

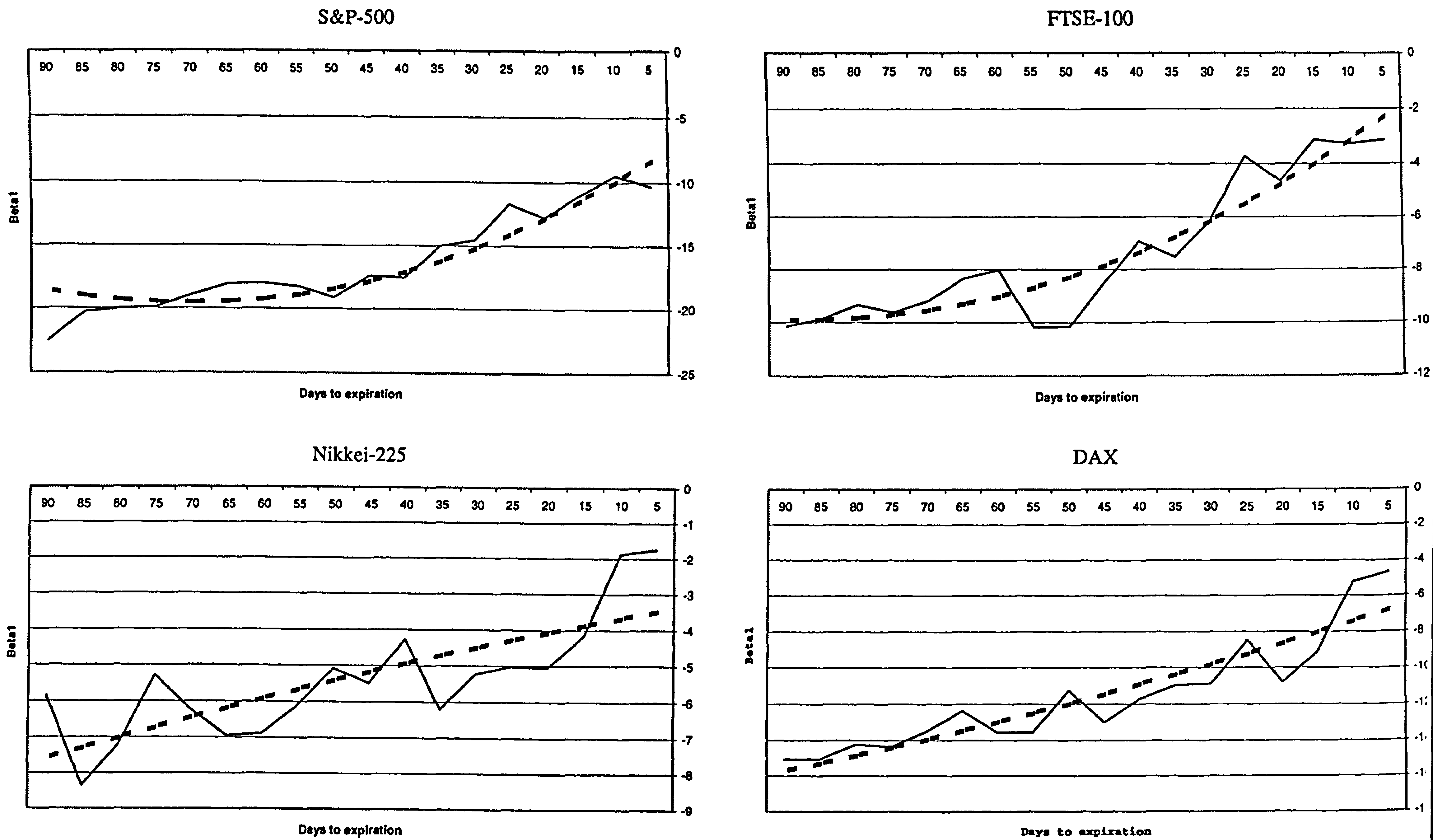

Figure 7.13a 1996 Skewness for Four Stock Index Options

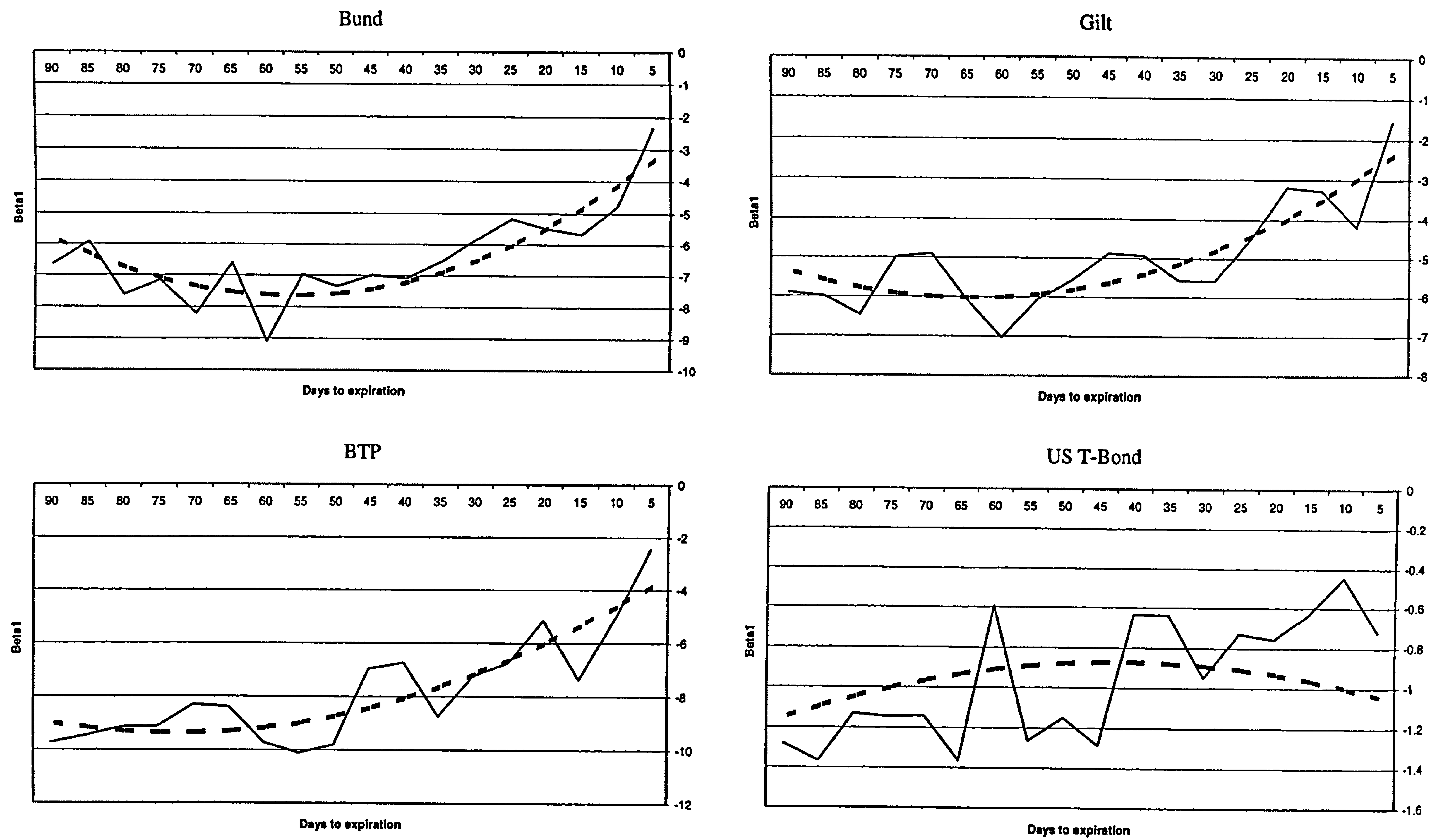

Figure 7.13b 1996 Skewness for Four Fixed Income Options

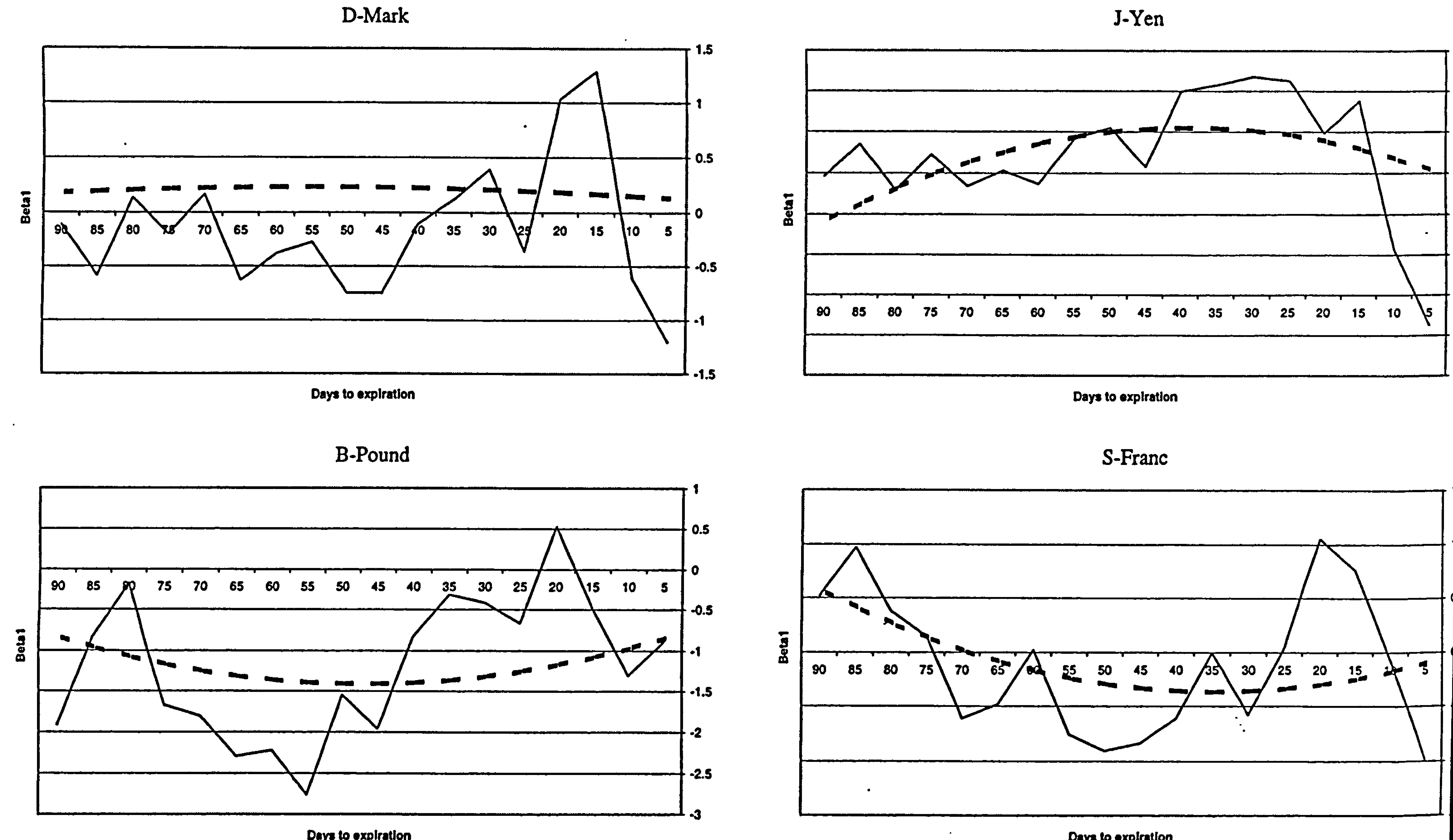

Figure 7.13c 1996 Skewness for Four Foreign Exchange Options

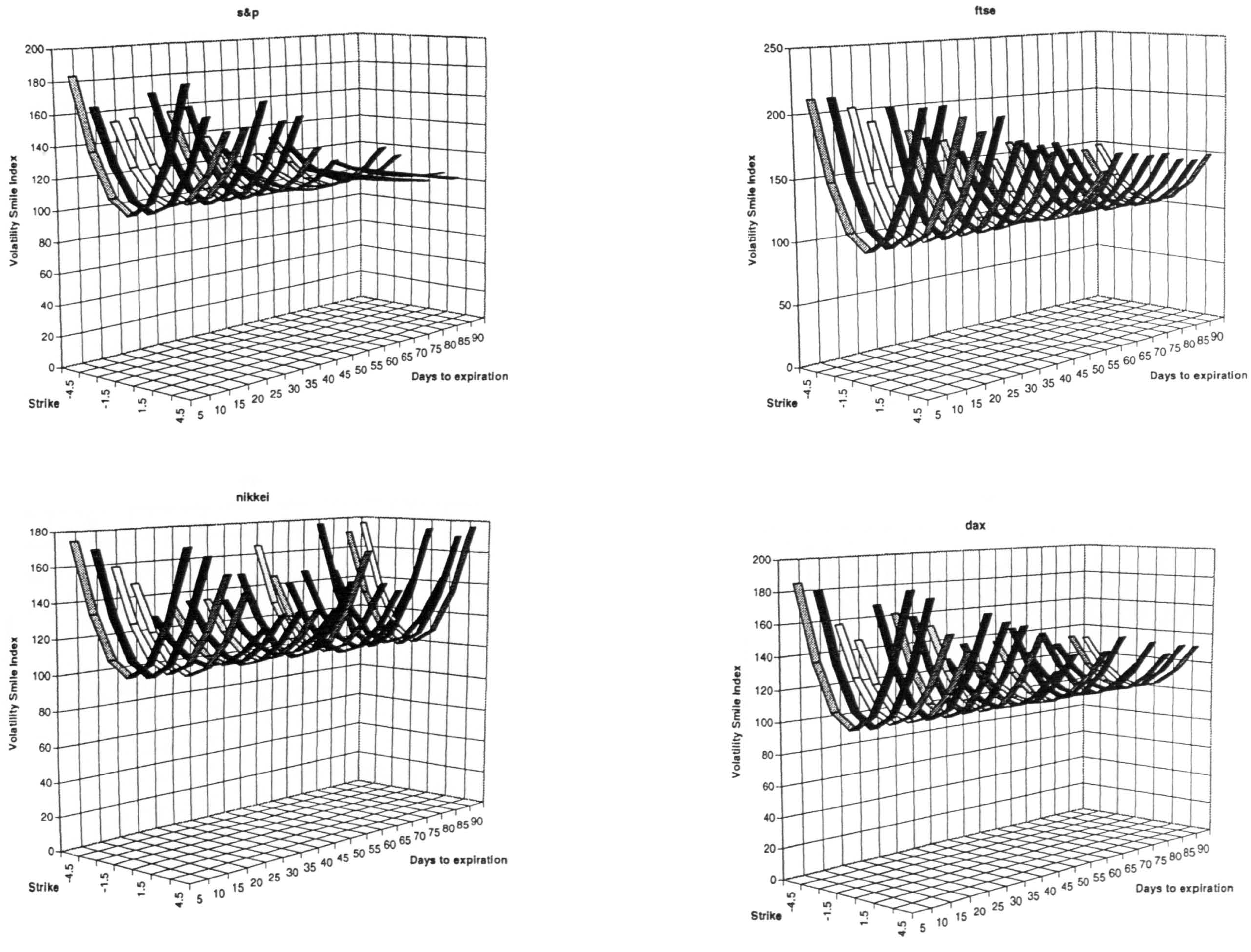

Figure 7.14a 1996 Kurtosis for Four Stock Index Options

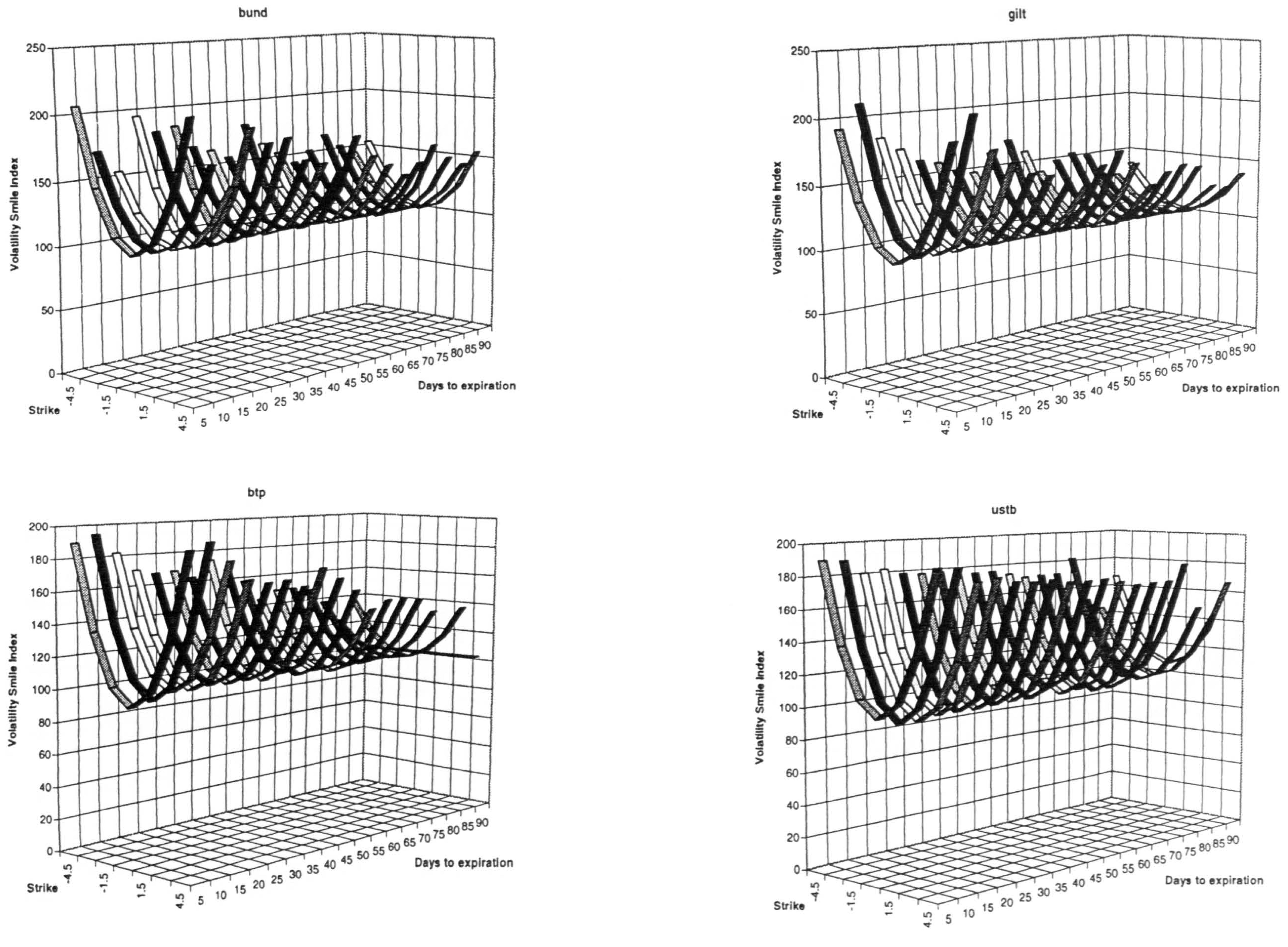

Figure 7.14b 1996 Kurtosis for Four Fixed Income Options

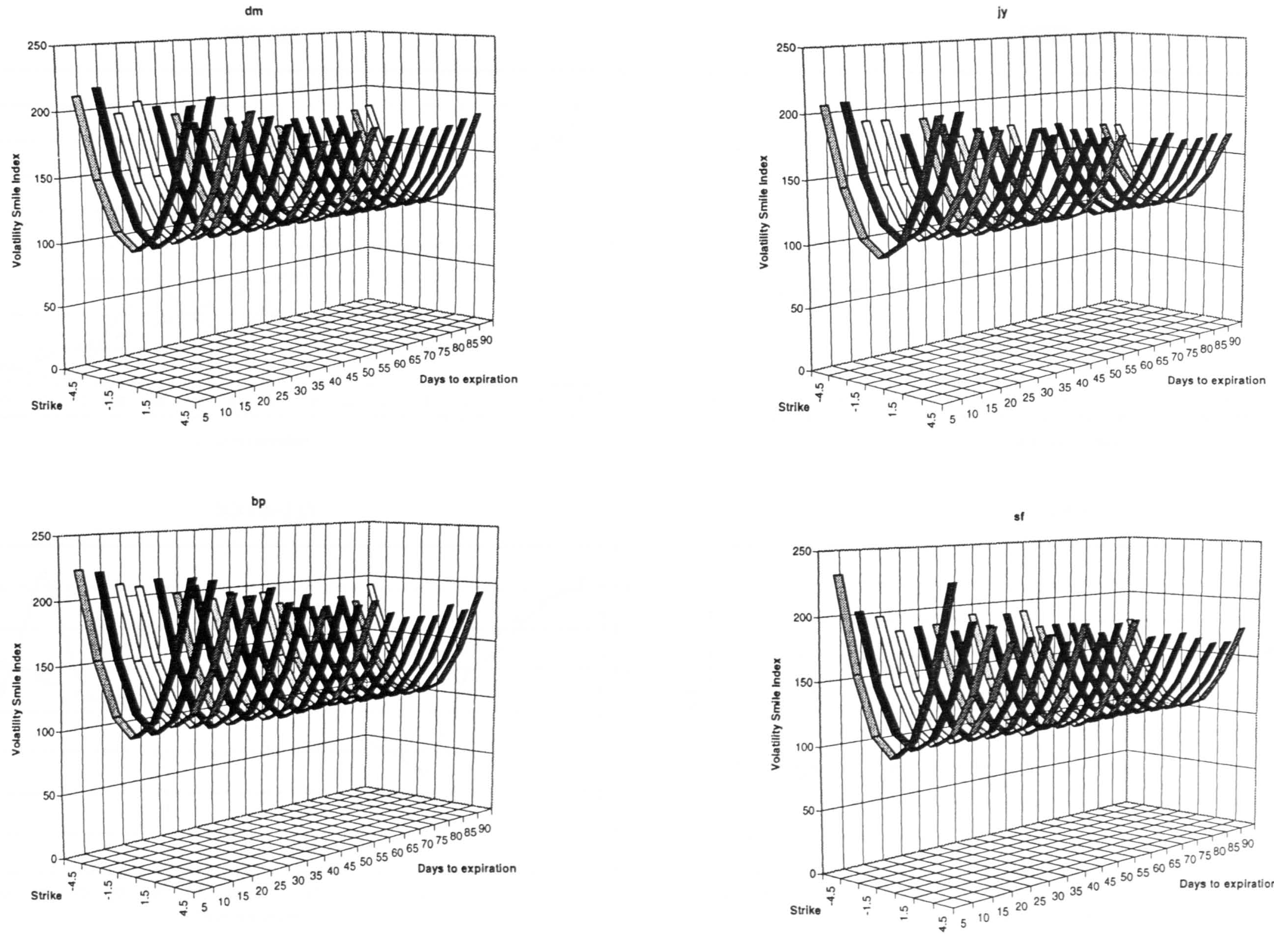

Figure 7.14c 1996 Kurtosis for Four Foreign Exchange Options

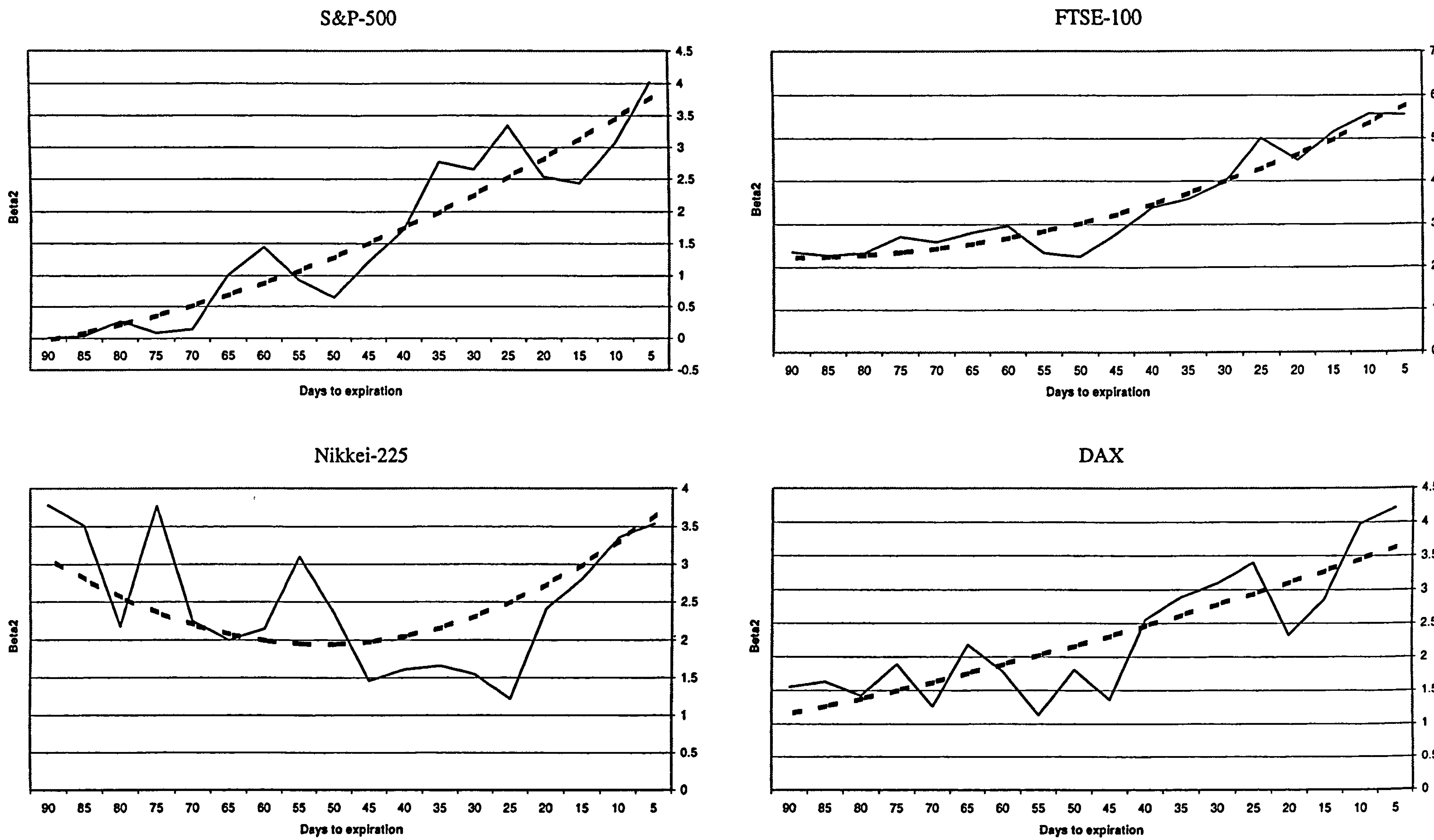

Figure 7.15a 1996 Kurtosis for Four Stock Index Options

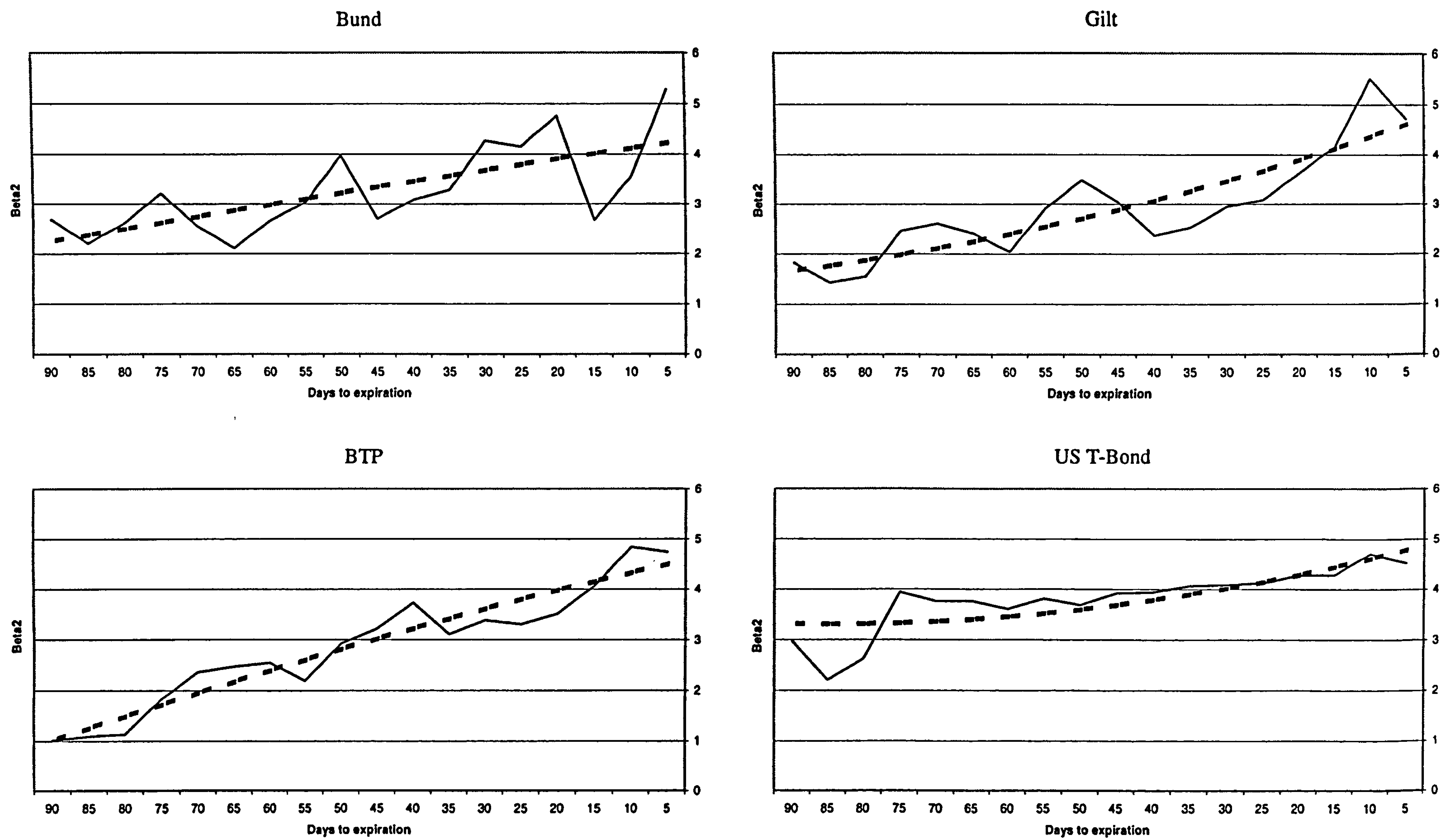

Figure 7.15b 1996 Kurtosis for Four Fixed Income Options

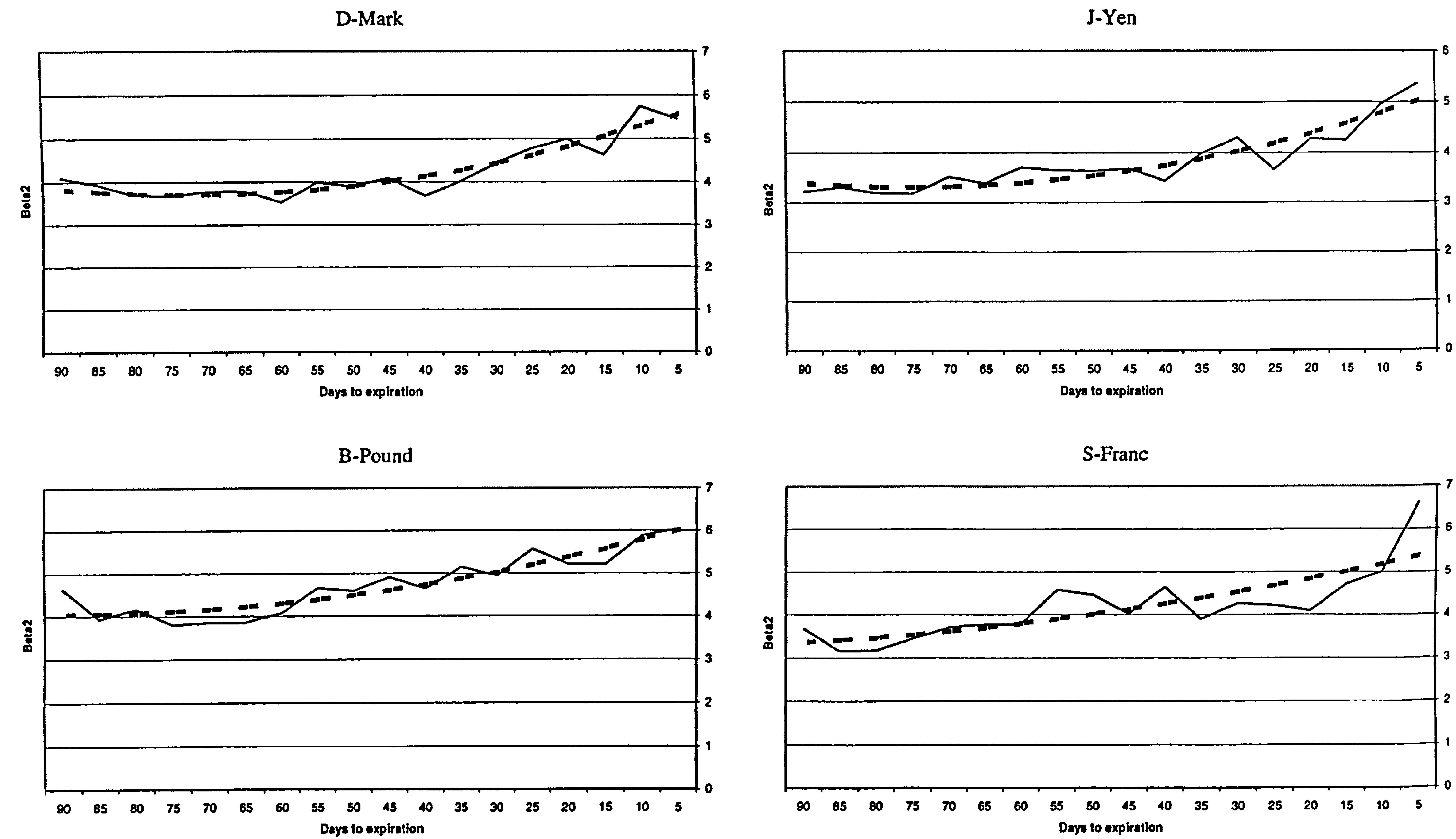

Figure 7.15c 1996 Kurtosis for Four Foreign Exchange Options

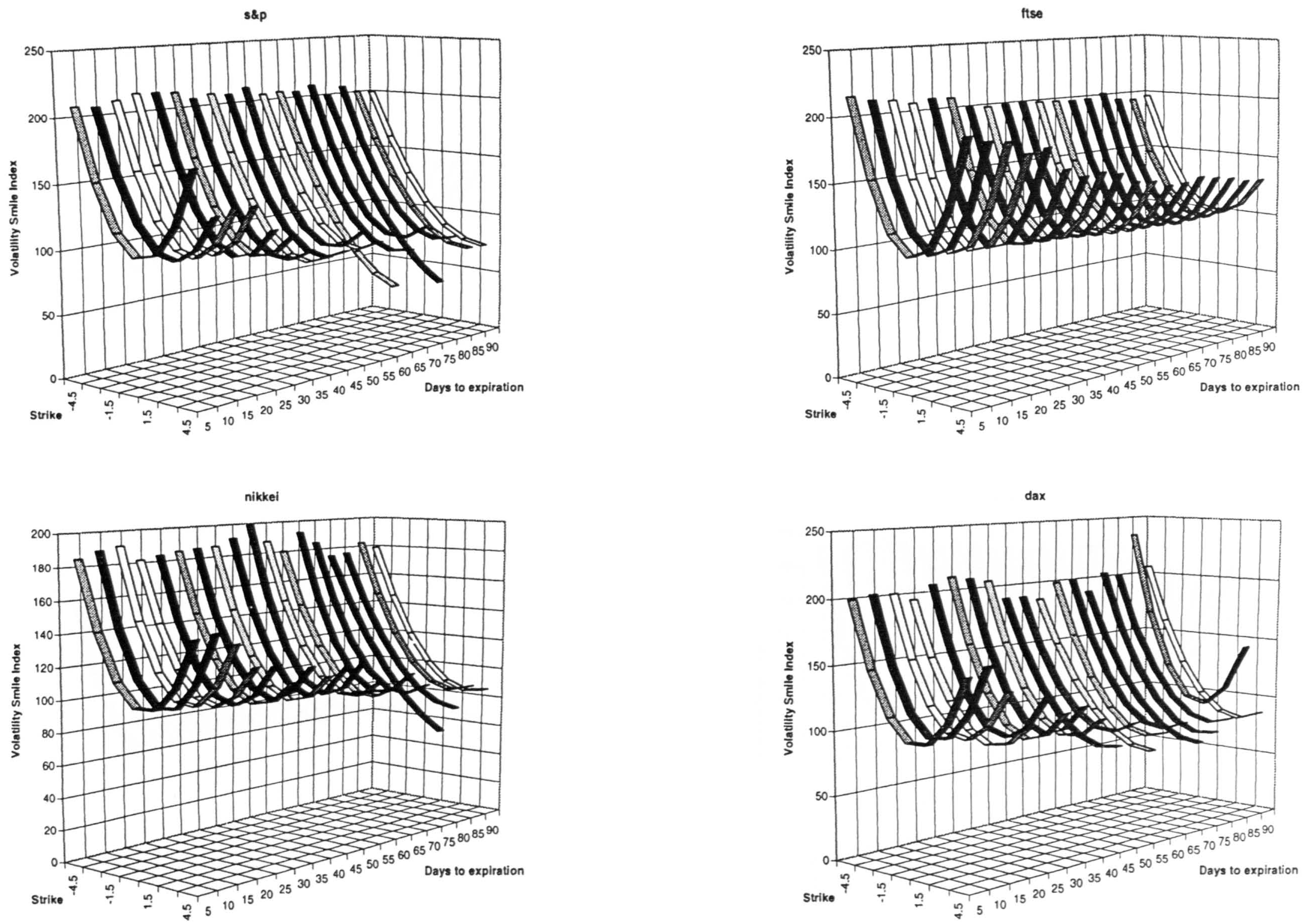

Figure 7.16a Standardized Volatility Smiles for Four Stock Index Options for the entire period of analysis.

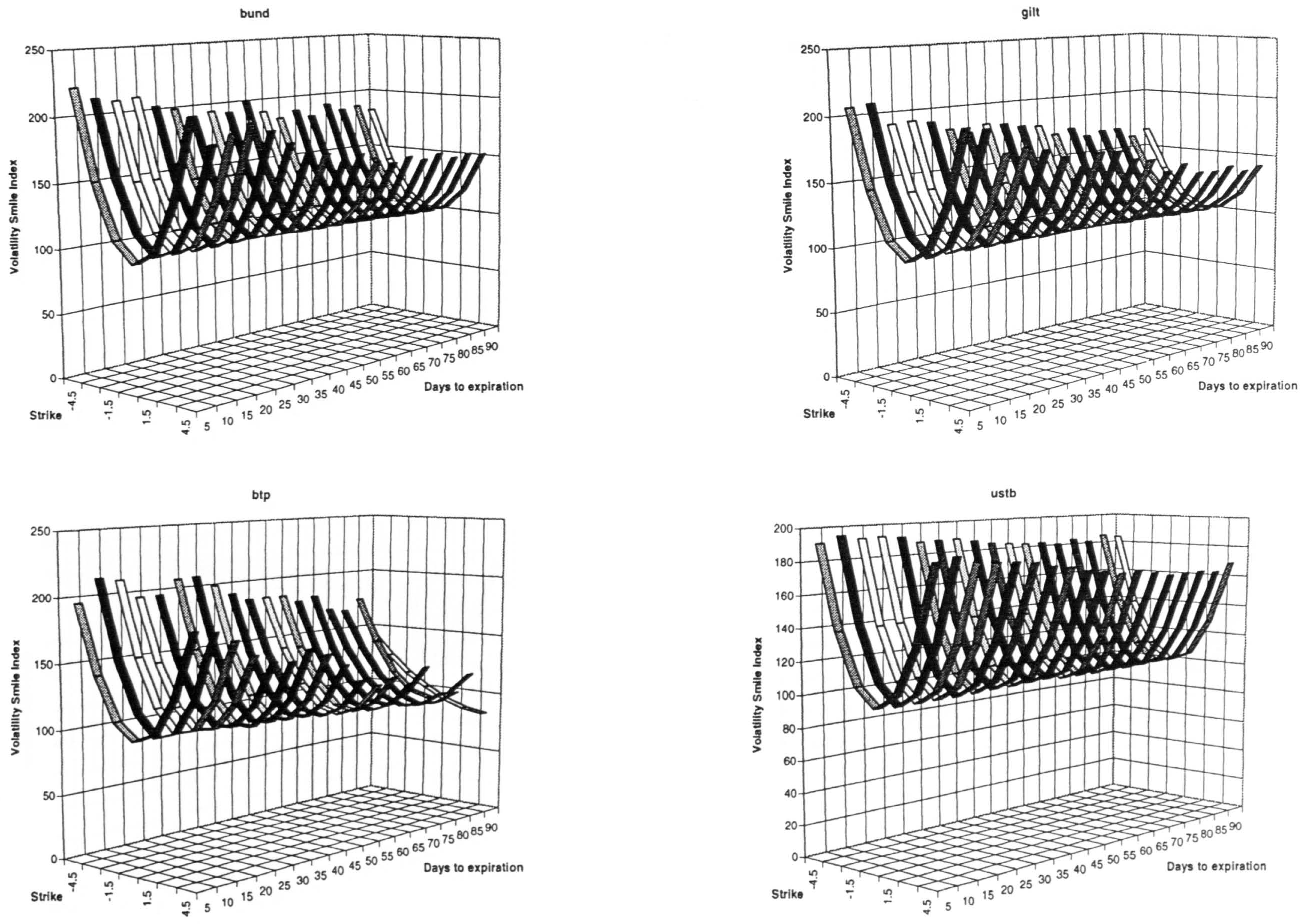

Figure 7.16b Standardized Volatility Smiles for Four Fixed Income Options for the entire period of analysis.

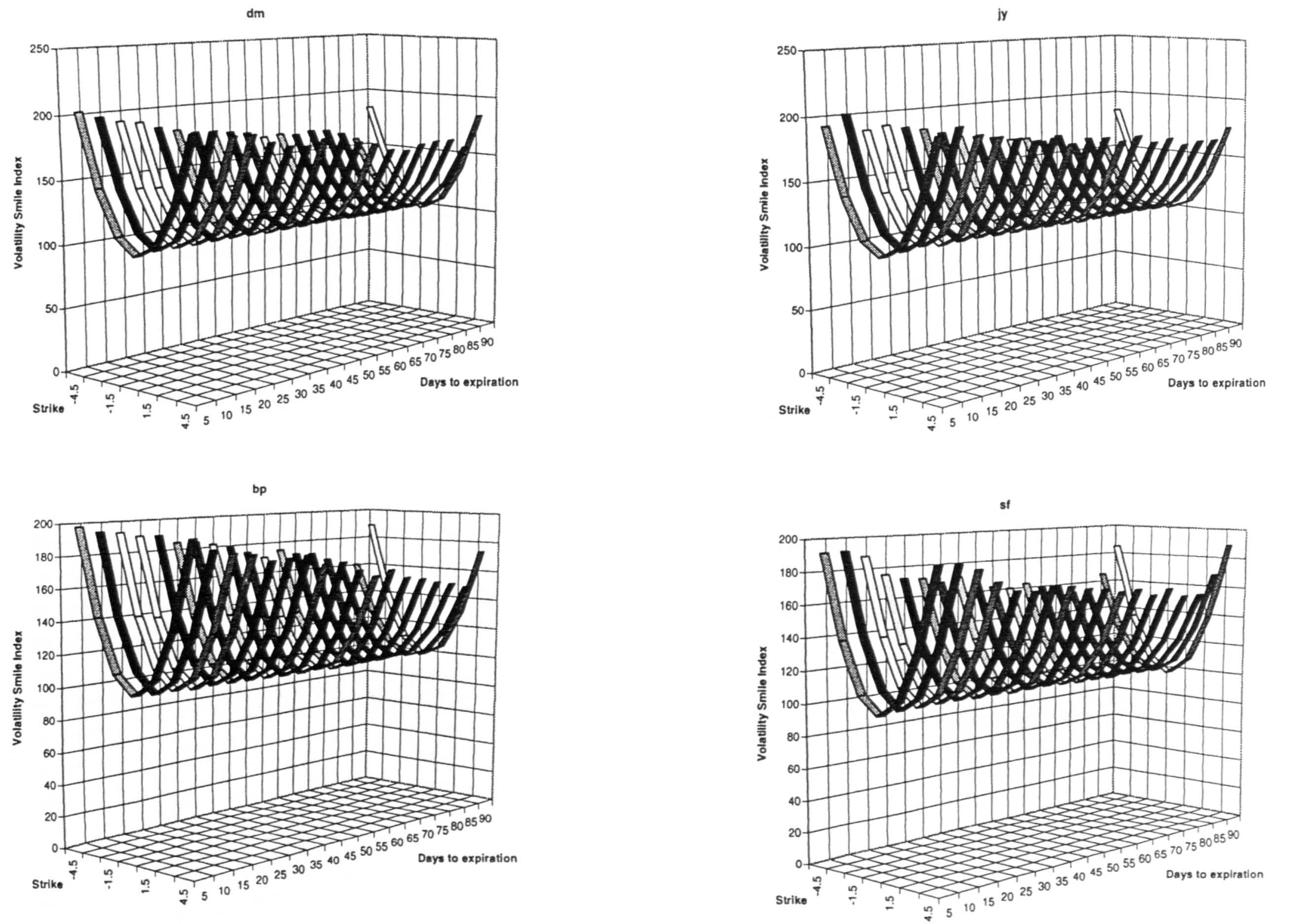

Figure 7.16c Standardized Volatility Smiles for Four Foreign Exchange Options for the entire period of analysis.

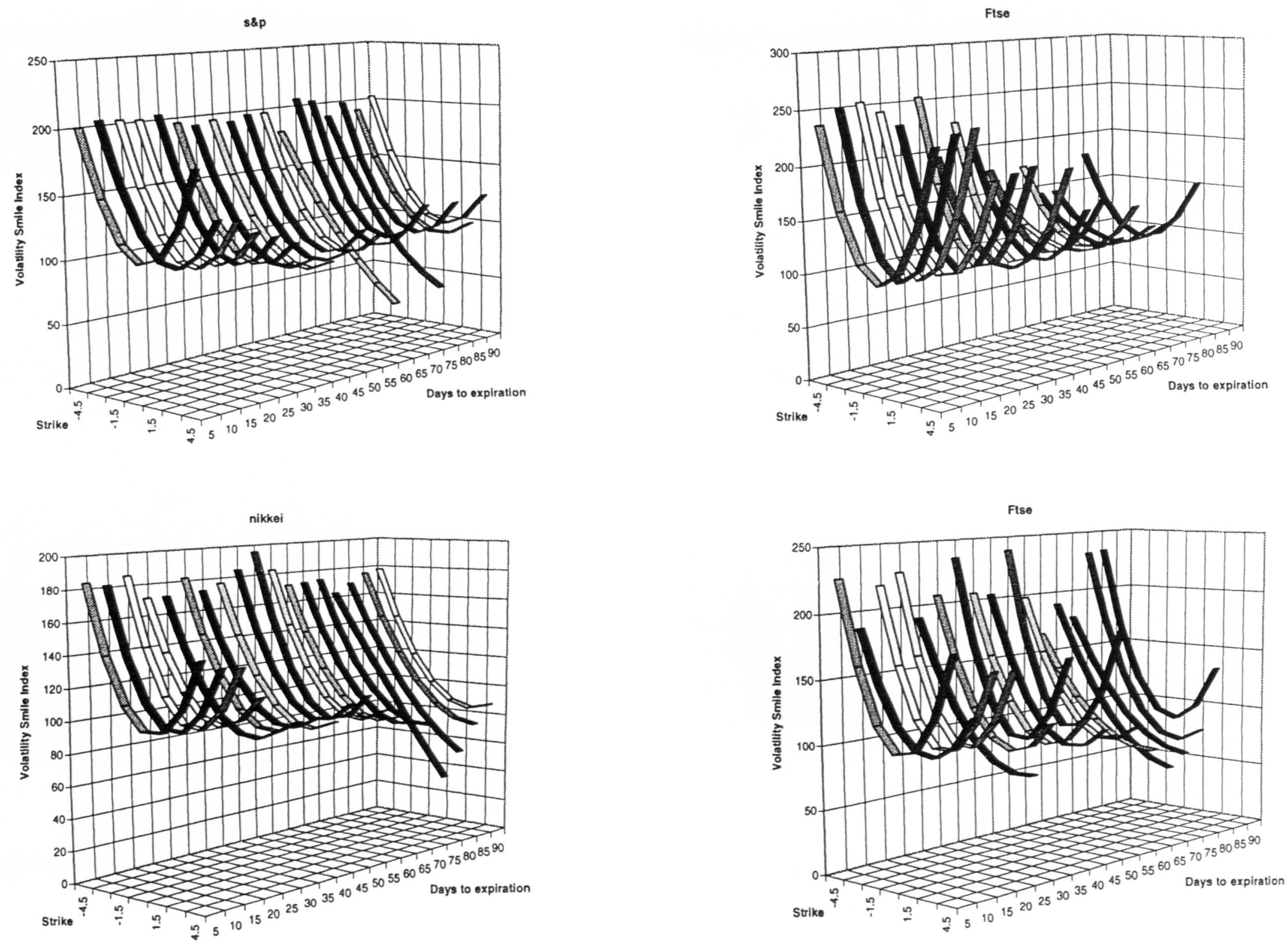

Figure 7.17a Standardized Volatility Smiles for Four Stock Index Options for the first portion of the available observations.

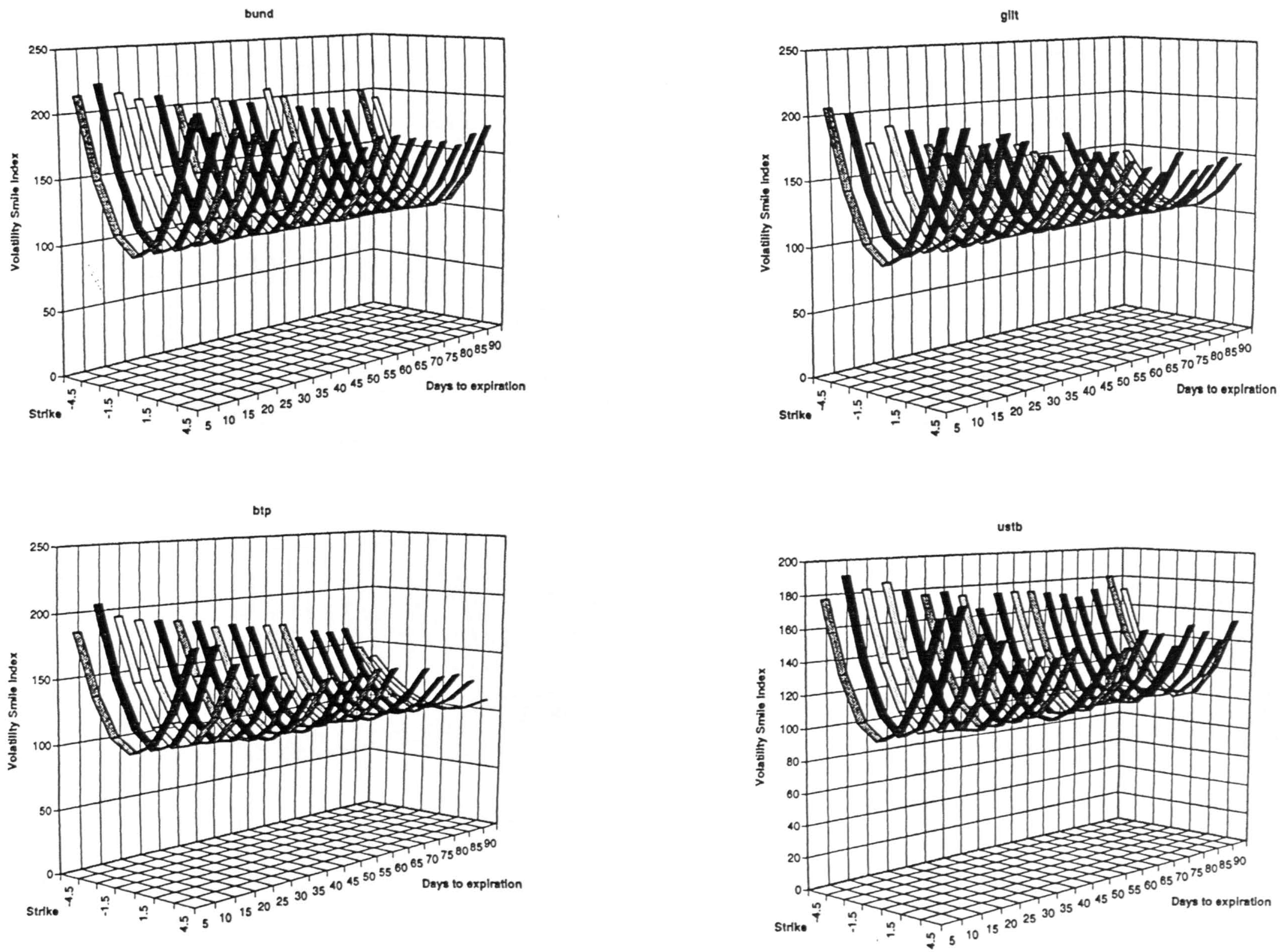

Figure 7.17b Standardized Volatility Smiles for Four Fixed Income Options for the first portion of the available observations.

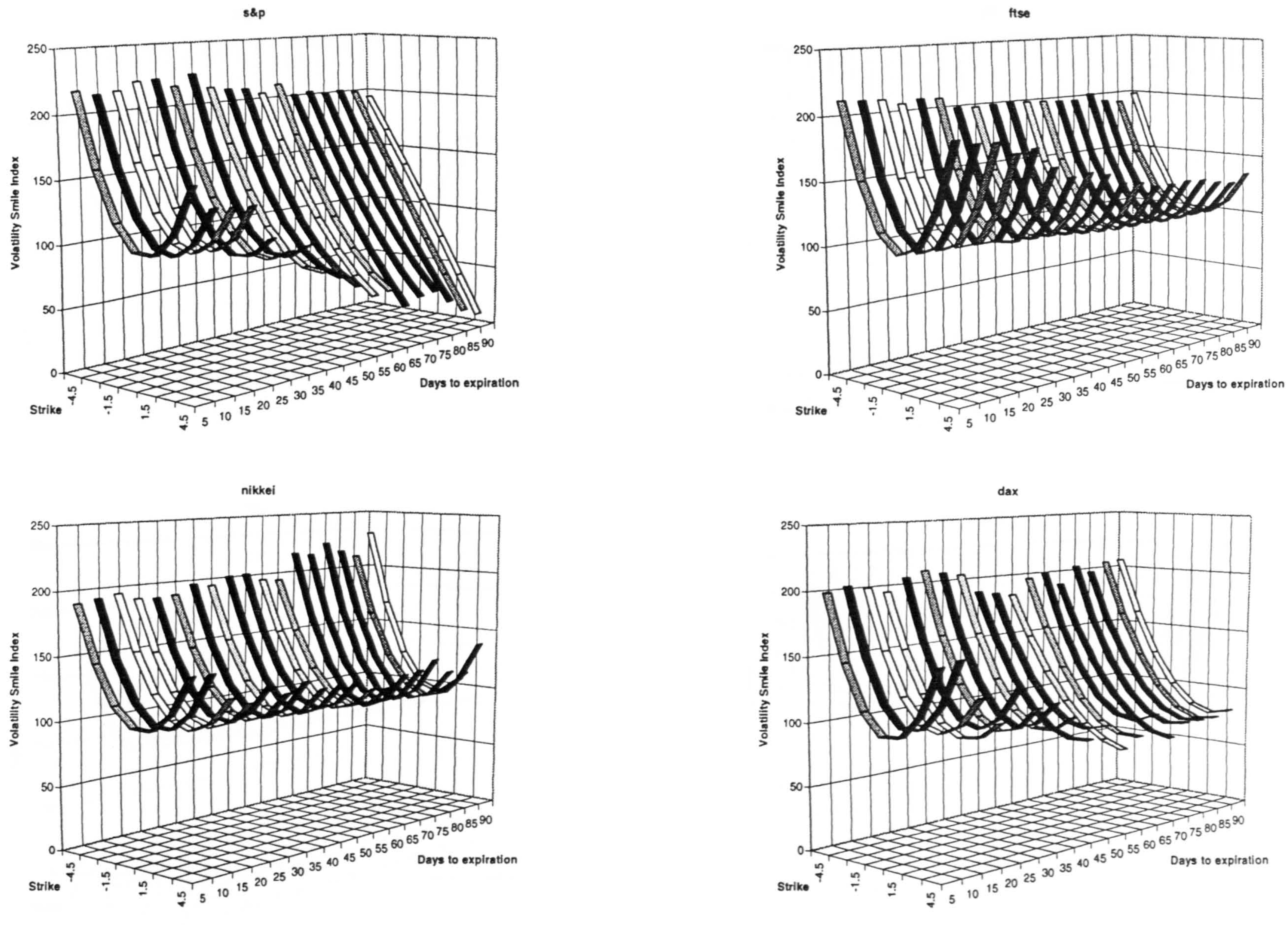

Figure 7.18a Standardized Volatility Smiles for Four Stock Index Options for the second portion of the available observations.

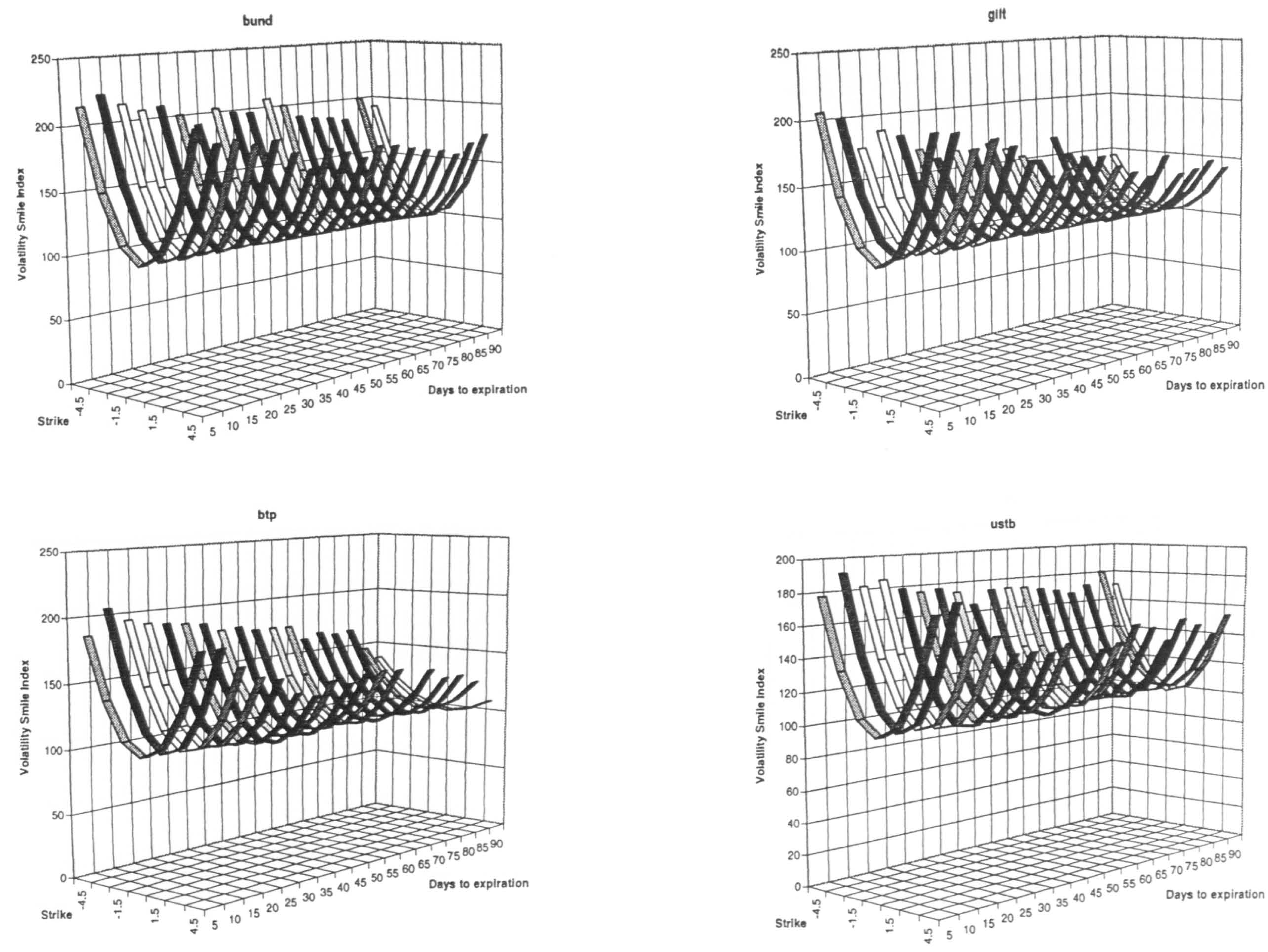

Figure 7.17b Standardized Volatility Smiles for Four Fixed Income Options for the first portion of the available observations.

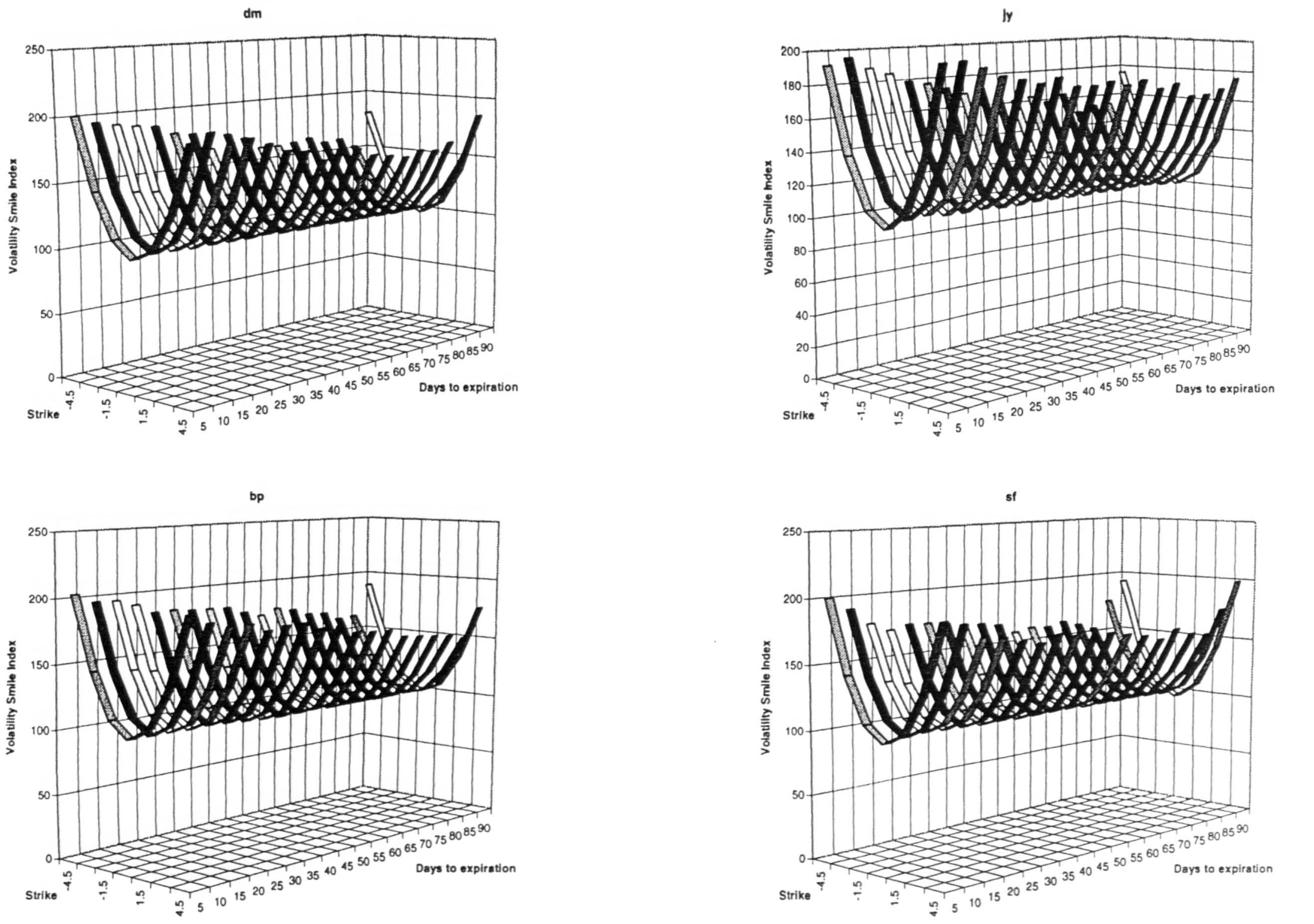

Figure 7.18c Standardized Volatility Smiles for Four Foreign Exchange Options for the second portion of the available observations.

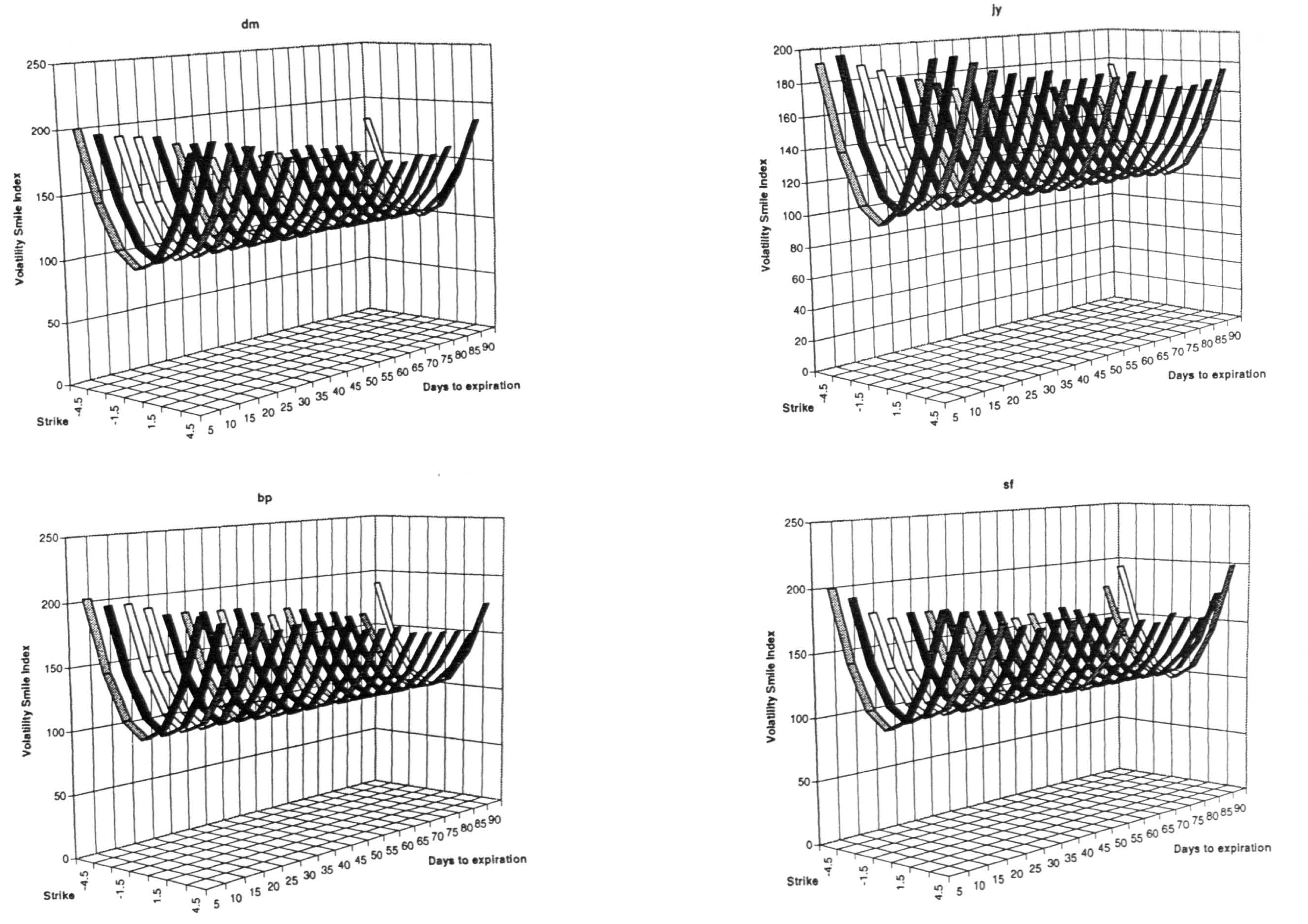

Figure 7.18c Standardized Volatility Smiles for Four Foreign Exchange Options for the second portion of the available observations.

APPENDIX 7.1

Market: S&P
Contract: June 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
475	16405	#N/A	5	0.289612079
480	15905	#N/A	5	0.280230418
490	#N/A	#N/A	10	0.281453716
500	13905	#N/A	15	0.27470694
510	12905	#N/A	20	0.264173584
520	11905	#N/A	25	0.251329516
525	11405	#N/A	30	0.247298346
530	10905	#N/A	35	0.242277389
540	9920	0.196614869	55	0.237698319
550	8940	0.200348154	75	0.227796743
560	7970	0.199553859	105	0.218943316
565	#N/A	#N/A	130	0.216659894
570	7025	0.200705056	160	0.214316285
575	6560	0.199998484	190	0.210296517
580	6090	0.195762098	220	0.204923778
585	#N/A	#N/A	250	0.198401112
590	5170	0.187688845	295	0.193963978
595	#N/A	#N/A	345	0.188982122
600	4280	0.179128729	400	0.183384083
605	#N/A	#N/A	470	0.178640195
610	3435	0.17076599	550	0.173599359
615	3035	0.16676626	645	0.168728486
620	2640	0.161284973	750	0.163118311
625	2270	0.15661553	875	0.157768811
630	1920	0.151788543	1020	0.152339863
635	1590	0.146525042	1190	0.14706047
640	1290	0.141599476	1385	0.141599476
645	1025	0.137244621	1615	0.136708119
650	785	0.132138075	1870	0.131020989
655	590	0.128343925	2175	0.127144368
660	425	0.124167606	2505	0.1221578
665	295	0.12034736	2875	0.118005222
670	195	0.116541584	3270	0.11268644
675	125	0.113539371	3700	0.10853133
680	75	0.110271575	4150	0.103196655
685	45	0.108388863	4620	0.097643907
690	25	0.106059118	#N/A	#N/A
695	15	0.105838027	#N/A	#N/A
700	10	0.107440344	6095	#N/A
705	5	0.105481208	#N/A	#N/A
710	5	0.112146201	7095	#N/A

Market: S&P
Contract: September 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
510	134.6	0.256015545	1.55	0.226263803
520	#N/A	#N/A	1.95	0.221050596
525	#N/A	#N/A	2.15	0.21768322
530	#N/A	#N/A	2.9	0.224851425
540	#N/A	#N/A	3.25	0.214093239
550	96.95	0.213539465	3.95	0.207781296
560	#N/A	#N/A	4.65	0.199592391
570	#N/A	#N/A	5.6	0.192738241
575	74.4	0.191215409	6.15	0.189363642
580	70.05	0.187240176	6.75	0.185953025
585	#N/A	#N/A	7.4	0.182462188
590	#N/A	#N/A	8.15	0.179311785
595	#N/A	#N/A	8.95	0.175960676
600	53.5	0.173586327	9.9	0.173212834
610	45.75	0.167135827	12	0.167099923
615	#N/A	#N/A	13.2	0.164037764
620	38.5	0.161349007	14.55	0.161247566
625	35.05	0.158435827	16	0.15830436
630	31.75	0.155657203	17.6	0.155496772
635	28.5	0.152294838	19.3	0.152432679
640	25.45	0.149285397	21.15	0.149390228
645	22.65	0.146879226	23.2	0.146632056
650	19.8	0.143101198	25.3	0.143143417
655	17.4	0.141110368	27.8	0.141121685
660	15.1	0.138631066	30.4	0.138610754
665	12.95	0.135948745	#N/A	#N/A
670	11	0.13337405	36.1	0.133284449
675	9.25	0.130910829	39.3	0.131149766
680	7.7	0.128584758	#N/A	#N/A
690	5.05	0.123234402	#N/A	#N/A
695	3.95	0.120183699	#N/A	#N/A
700	3	0.116873271	#N/A	#N/A
705	2.45	0.116712747	#N/A	#N/A
710	1.95	0.115983848	#N/A	#N/A
715	1.5	0.114611193	#N/A	#N/A
720	1.15	0.113527195	75.9	0.124601244
725	0.9	0.113204317	#N/A	#N/A
730	0.7	0.112924273	#N/A	#N/A
740	0.4	0.111783154	#N/A	#N/A

Market: S&P
Contract: December 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
540	#N/A	#N/A	5.2	0.196649233
550	103.75	0.197234999	6.1	0.19147353
560	#N/A	#N/A	7.45	0.189005126
570	#N/A	#N/A	8.9	0.185474435
575	#N/A	#N/A	9.65	0.183247521
580	#N/A	#N/A	10.4	0.180613104
590	#N/A	#N/A	12	0.174886152
600	62.45	0.169053842	13.75	0.16859696
610	54.9	0.163453214	15.95	0.163395121
620	48.05	0.159952065	18.8	0.159971015
625	44.8	0.158352145	20.4	0.158406605
630	41.7	0.156989854	22.15	0.157078385
640	35.65	0.153333039	25.8	0.153487088
650	30.05	0.149637031	29.85	0.149605678
660	25.1	0.146767277	34.6	0.146803098
670	20.6	0.14363376	39.75	0.143483807
675	18.5	0.141851209	42.5	0.141733529
680	16.6	0.140438626	45.45	0.140353938
690	13	0.136606189	51.6	0.136873272
700	10	0.133245729	58.35	0.133924825
710	7.55	0.130274766	#N/A	#N/A
725	4.55	0.124478029	#N/A	#N/A
730	3.9	0.123777667	#N/A	#N/A
740	2.95	0.12381632	#N/A	#N/A
750	2.25	0.124394694	#N/A	#N/A
760	1.95	0.128687322	109.85	0.148096422

Market: NIKKEI
Contract: June 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
1450	#N/A	#N/A	0	#N/A
1750	#N/A	#N/A	0	#N/A
1800	#N/A	#N/A	0.5	0.222627662
1900	245	#N/A	#N/A	#N/A
1950	199	0.178449683	4	0.190138656
2000	153	0.174385878	8	0.181441191
2050	111.5	0.173836527	16	0.176208522
2100	74	0.165353675	27	0.161595176
2150	43.5	0.156448563	47.5	0.156448563
2200	23.5	0.15472575	77	0.152820323
2250	12.5	0.159491193	116	0.157010332
2300	6	0.161762364	159	0.154077603
2400	0.5	0.147464172	#N/A	#N/A

Market: NIKKEI
Contract: September 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
2050	150	0.176298955	#N/A	#N/A
2100	118	0.173286192	#N/A	#N/A
2150	92	0.173915465	#N/A	#N/A
2200	67	0.167948611	#N/A	#N/A
2250	50	0.16945863	#N/A	#N/A
2300	33.5	0.163515782	#N/A	#N/A
2400	16	0.164578286	#N/A	#N/A
2500	6.5	0.162593747	#N/A	#N/A

Market: DAX
Contract: June 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
2175	300.6	0.183900326	1.4	0.185138239
2200	276	0.175733329	1.7	0.176827798
2225	251.9	0.173541081	2.5	0.174404504
2250	228	0.16999292	3.5	0.170700396
2275	204.3	0.165044752	4.7	0.165642269
2300	180.8	0.158694007	6.1	0.159210683
2325	157.8	0.152668136	8	0.153116186
2350	136.1	0.149954768	11.2	0.150338755
2375	114.6	0.143831218	14.7	0.144580011
2400	94.8	0.140082925	19.7	0.140393056
2425	77.3	0.138981518	26.9	0.138637779
2450	60.4	0.134190974	35.2	0.134764625
2475	45.4	0.129341336	45	0.129621933
2500	33.5	0.127244395	58.1	0.127832002
2525	23.3	0.123610517	72.7	0.123939141
2550	15.5	0.120609481	89.8	0.120996759
2575	9.9	0.11839222	109.2	0.119310697
2600	6.2	0.117476978	130.4	0.118668219
2625	3.6	0.115730211	152.6	0.116633166
2650	2	0.114380754	176	0.116812814
2675	1.1	0.113966116	199.9	0.116051774

Market: BUND
Contract: June 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
92	3.74	0.091499756	0.01	0.091499756
92.5	3.25	0.090058942	0.02	0.090058942
93	2.76	0.084303569	0.03	0.084303569
93.5	2.27	0.076045273	0.04	0.076045273
94	1.8	0.07196578	0.07	0.07196578
94.5	1.35	0.067446927	0.12	0.067446927
95	0.94	0.063675098	0.21	0.063675098
95.5	0.6	0.061580905	0.37	0.061580905
96	0.34	0.059656634	0.61	0.059656634
96.5	0.17	0.058546419	0.94	0.058546419
97	0.08	0.059283974	1.35	0.059283974
97.5	0.03	0.05837707	1.8	0.05837707
98	0.02	0.065477866	2.29	0.065477866
98.5	0.01	0.068614707	2.78	0.068614707

Market: BTP
Contract: June 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
105	8.82	0.165638024	0.01	0.165638024
105.5	8.33	0.171917199	0.02	0.171917199
106	7.83	0.162609821	0.02	0.162609821
106.5	7.34	0.162876153	0.03	0.162876153
107	6.84	0.153080261	0.03	0.153080261
107.5	6.35	0.150270953	0.04	0.150270953
108	5.85	0.140035103	0.04	0.140035103
108.5	5.36	0.135161204	0.05	0.135161204
109	4.86	0.124477716	0.05	0.124477716
109.5	4.37	0.117975703	0.06	0.117975703
110	3.87	0.106790199	0.06	0.106790199
110.5	3.4	0.104959715	0.09	0.104959715
111	2.92	0.097759941	0.11	0.097759941
111.5	2.49	0.098873805	0.18	0.098873805
112	2.06	0.095502884	0.25	0.095502884
112.5	1.66	0.09278145	0.35	0.09278145
113	1.3	0.090837516	0.49	0.090837516
113.5	0.99	0.089838244	0.68	0.089838244
114	0.71	0.086998827	0.9	0.086998827
114.5	0.5	0.086436754	1.19	0.086436754
115	0.32	0.08363016	1.51	0.08363016
115.5	0.2	0.082620349	1.89	0.082620349
116	0.1	0.077820367	2.29	0.077820367
116.5	0.05	0.07596849	2.74	0.07596849
117	0.03	0.078179581	3.22	0.078179581
117.5	0.02	0.081866384	3.71	0.081866384
118	0.01	0.081882192	4.2	0.081882192
118.5	0.01	0.090047796	4.7	0.090047796

Market: GILT
Contract: June 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
100	4.60938	0.107070582	0.01563	0.107070582
101	3.625	0.09744759	0.03125	0.09744759
102	2.67188	0.091621949	0.07813	0.091621949
103	1.79688	0.088584931	0.20313	0.088584931
104	1.01563	0.080354555	0.42188	0.080354555
105	0.48438	0.078809965	0.89063	0.078809965
106	0.1875	0.078440149	1.59375	0.078440149
107	0.0625	0.079979352	2.46875	0.079979352
108	0.01563	0.080020444	3.42188	0.080020444

Market: GILT
Contract: September 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
94	9.5625	0.095584492	0.0625	0.095584492
95	8.60938	0.096487601	0.10938	0.096487601
96	7.65625	0.093999503	0.15625	0.093999503
97	6.71875	0.091194011	0.21875	0.091194011
98	5.79688	0.087692216	0.29688	0.087692216
99	4.9375	0.086702785	0.4375	0.086702785
100	4.10938	0.084559131	0.60938	0.084559131
101	3.34375	0.082789799	0.84375	0.082789799
102	2.65625	0.081476518	1.15625	0.081476518
103	2.01563	0.078666884	1.51563	0.078666884
104	1.48438	0.076883727	1.98438	0.076883727
105	1.04688	0.075152795	2.54688	0.075152795
106	0.71875	0.07431702	3.21875	0.07431702
107	0.48438	0.074241768	3.98438	0.074241768
108	0.28125	0.071506792	4.78125	0.071506792
109	0.1875	0.073042361	5.6875	0.073042361
110	0.14063	0.076776501	6.64063	0.076776501
111	0.07813	0.075538565	7.57813	0.075538565
112	0.04688	0.076043558	8.54688	0.076043558
113	0.03125	0.077995721	9.53125	0.077995721

Market: USTB
Contract: June 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
74	#N/A	#N/A	0.01563	0.801788669
78	#N/A	#N/A	0.01563	0.696492029
84	#N/A	#N/A	0.01563	0.546468283
86	#N/A	#N/A	0.01563	0.498245953
90	#N/A	#N/A	0.01563	0.403985177
92	#N/A	#N/A	0.01563	0.357764893
94	#N/A	#N/A	0.01563	0.312010479
96	#N/A	#N/A	0.01563	0.266590188
98	8.95313	0.241293522	0.01563	0.221334582
100	6.95313	0.189522176	0.01563	0.176005098
102	4.95313	0.13820943	0.01563	0.130217996
103	#N/A	#N/A	0.03125	0.120112939
104	3	0.110855354	0.0625	0.109111524
105	2.09375	0.104865647	0.15625	0.104209543
106	1.3125	0.102382078	0.375	0.102166985
107	0.71875	0.101328493	0.78125	0.101341121
108	0.32813	0.099691422	1.39063	0.099942725
109	0.125	0.09918443	2.1875	0.099944457
110	0.03125	0.094733925	3.09375	0.097294131
111	0.01563	0.105777668	4.07813	0.111464016
112	0.01563	0.126946237	5.07813	0.134868675
113	0.01563	0.14745282	6.07813	0.157824823
114	0.01563	0.167404211	7.07813	0.180408802
116	0.01563	0.205916799	9.07813	0.224643577
118	0.01563	0.242876791	11.0781	0.267820262
120	0.01563	0.278526273	13.0781	0.310078081
122	0.01563	0.313031775	15.0781	0.351506706
124	0.01563	0.346516496	17.0781	0.392169731
126	0.01563	0.37907617	19.0781	0.432115614
128	0.01563	0.410787769	#N/A	#N/A
130	0.01563	0.441714746	23.0781	0.510009353
132	0.01563	0.471910473	25.0781	0.548011748

Market: DM
Contract: June 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
620	376	0.104177396	2	0.098454883
625	#N/A	#N/A	3	0.093610352
630	279	0.091818832	5	0.090609289
635	#N/A	#N/A	7	0.083966189
640	186	0.081379729	12	0.081756007
645	#N/A	#N/A	20	0.079711005
650	107	0.078058535	32	0.077562619
655	75	0.076549316	50	0.076400375
660	50	0.076110416	75	0.076258792
665	33	0.078058803	108	0.078547164
670	22	0.081485832	146	0.08082553
675	14	0.083795325	188	0.083448821
680	9	0.086765037	233	0.086944882
685	6	0.090580438	280	0.091641132
690	4	0.09421494	328	0.096695439
695	3	0.099757953	#N/A	#N/A
700	2	0.102888163	426	0.110265894
705	1	0.101708807	#N/A	#N/A
710	1	0.110509723	525	#N/A

Market: DM
Contract: September 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
600	#N/A	#N/A	0.9	0.104105804
610	#N/A	#N/A	1.5	0.10134975
620	43.1	0.099496431	2.4	0.098137421
630	34.8	0.097648197	4	0.097295115
640	27.3	0.09669105	6.4	0.096965076
650	20.8	0.096533568	9.7	0.096576914
660	15.3	0.096283721	14	0.096137122
670	10.9	0.096418551	19.4	0.096077977
680	7.5	0.096612222	25.9	0.096762933
690	5	0.096991042	33.2	0.096923915
700	3.3	0.098179459	41.4	0.098836833
710	2.1	0.09898082	50.1	0.100762168
720	1.4	0.101508477	59.4	0.106585774
730	1	0.105582916	69	0.115134272
740	0.6	0.105913315	78.7	#N/A
750	0.4	0.108457643	88.7	#N/A
760	0.3	0.112833644	98.7	#N/A
770	0.1	0.105501993	108.7	#N/A

Market: DM
Contract: December 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
600	66.6	0.112553449	#N/A	#N/A
610	57.7	0.107094071	3.5	0.1040288
620	49.5	0.105223698	5.1	0.10341486
630	41.8	0.103621575	7.2	0.10269081
640	34.6	0.101699005	9.8	0.101417892
650	#N/A	#N/A	13.2	0.100955316
660	22.7	0.100676283	17.4	0.100872591
670	18	0.100883414	22.4	0.101044933
680	14	0.100912802	28.1	0.101044926
690	10.8	0.101525302	34.6	0.101629755
700	8.2	0.102051801	41.8	0.102729591
710	6.2	0.10299961	#N/A	#N/A
720	4.6	0.10363685	#N/A	#N/A
730	3.4	0.104530151	#N/A	#N/A
740	2.4	0.104399494	75.4	0.111775979
750	1.7	0.104781858	84.8	0.118263122

Market BP
Contract: June 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
1440	#N/A	#N/A	2	0.071732541
1450	#N/A	#N/A	4	0.070027959
1460	#N/A	#N/A	6	0.064806042
1470	#N/A	#N/A	10	0.060615642
1480	#N/A	#N/A	18	0.05749464
1490	228	0.056525639	34	0.055867509
1500	156	0.055406118	62	0.055148419
1510	90	0.050078823	96	0.050094166
1520	52	0.051352316	156	0.050465188
1530	28	0.052624704	234	0.053360583
1540	18	0.057936901	322	0.057386962
1550	10	0.06038825	414	0.06028091
1560	6	0.063910007	510	0.064746299
1570	4	0.068419821	#N/A	#N/A
1580	2	0.069492282	#N/A	#N/A

Market BP
Contract: September 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
1400	#N/A	#N/A	1.2	0.075238243
1420	#N/A	#N/A	2.2	0.07270351
1440	#N/A	#N/A	4.4	0.07277132
1460	#N/A	#N/A	7	0.068629486
1480	40	0.06856317	12.2	0.068292189
1500	27.6	0.06738903	19.6	0.067562123
1520	18.6	0.068676266	30.2	0.068683056
1540	11.6	0.068553638	42.8	0.068375194
1560	7	0.069285545	58	0.069636977
1580	4.2	0.070915611	75	0.072114909
1600	3	0.076313018	#N/A	#N/A
1620	1.8	0.078045079	#N/A	#N/A
1660	0.8	0.084752894	#N/A	#N/A
1680	0.6	0.089327028	#N/A	#N/A

Market BP
Contract: December 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
1400	#N/A	#N/A	4.6	0.078385147
1420	#N/A	#N/A	7	0.07699717
1460	61.8	0.077380081	16	0.076913391
1480	#N/A	#N/A	22.6	0.076514772
1500	37.8	0.076408484	31	0.076396359
1520	28.6	0.076451732	41.2	0.076410341
1540	21	0.076271399	53	0.076197076
1560	15.2	0.076645539	#N/A	#N/A
1580	10.6	0.076557923	#N/A	#N/A
1600	7.2	0.076525147	#N/A	#N/A

Market: JY
Contract: June 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
880	#N/A	#N/A	1	0.114721622
885	#N/A	#N/A	1	0.107867966
890	#N/A	#N/A	2	0.111049842
895	#N/A	#N/A	2	0.103645958
900	570	0.106282449	3	0.102583836
905	#N/A	#N/A	4	0.099618608
910	472	0.096212335	5	0.095387512
915	#N/A	#N/A	7	0.093206228
920	376	0.08866243	9	0.089260757
925	331	0.088837753	13	0.087880818
930	287	0.087669904	19	0.087255123
935	245	0.086522965	27	0.08647408
940	206	0.085969147	38	0.086181607
945	170	0.085362697	52	0.085779026
950	140	0.087018969	71	0.086729366
955	113	0.087865662	94	0.087788747
960	90	0.088853773	121	0.088979469
965	72	0.091018611	153	0.091358765
970	57	0.092974621	187	0.092617274
975	46	0.096138013	#N/A	#N/A
980	37	0.099106759	267	0.099215872
985	29	0.101050419	309	0.101477988
990	24	0.104962596	353	0.104340153
995	19	0.107223081	398	0.106902516
1000	15	0.109401863	444	0.109489048
1005	13	0.114144973	#N/A	#N/A
1010	11	0.117968071	540	0.119235508
1020	7	0.122158014	636	0.125435786
1025	6	0.125857256	#N/A	#N/A
1030	5	0.128757284	734	0.13487542
1040	4	0.137299282	833	0.146805084
1050	3	0.143845717	932	0.157952342
1060	2	0.147566532	1031	#N/A
1070	1	0.146040875	1131	#N/A

Market: JY
Contract: September 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
840	#N/A	#N/A	0.5	0.119630674
860	#N/A	#N/A	0.9	0.113409058
870	#N/A	#N/A	1.2	0.11020301
880	88.9	0.115410042	1.7	0.108489501
890	79.6	0.110811949	2.4	0.107020698
900	70.6	0.107667422	3.4	0.106117761
910	62	0.105775796	4.7	0.105028185
920	53.9	0.104866033	6.5	0.104694405
930	46.4	0.104744117	8.8	0.104432275
940	39.3	0.103814001	11.6	0.103898359
950	33.1	0.104387748	15.2	0.104330464
960	27.6	0.105144375	19.5	0.104959755
970	22.7	0.105607656	24.5	0.105746023
980	18.5	0.106304245	30.1	0.106314803
990	15	0.107399652	36.4	0.107275887
1000	12.2	0.109203312	43.5	0.109429962
1010	9.8	0.110583905	50.9	0.110674553
1020	7.8	0.11180431	58.8	0.11233139
1030	6.2	0.113216621	67	0.113615722
1040	5	0.115331169	75.7	0.11631711
1050	4	0.117165843	84.7	0.119736212
1060	3.1	0.118022071	93.7	0.121690146
1070	2.5	0.120181496	#N/A	#N/A
1080	2	0.122065731	112.5	0.129863111
1090	1.7	0.125481521	122.2	0.13640219
1100	1.5	0.12966757	132	0.143929995
1110	1.3	0.133191983	141.9	0.15261782
1120	1.1	0.135961937	151.8	#N/A
1130	0.9	0.137826694	#N/A	#N/A

Market: JY
Contract: December 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
850	#N/A	#N/A	1.6	0.111829658
860	#N/A	#N/A	2.2	0.111713339
870	110.5	0.123043363	#N/A	#N/A
880	#N/A	#N/A	3.7	0.109438236
890	92.3	0.113613103	4.9	0.109452887
900	#N/A	#N/A	6.4	0.10955796
910	#N/A	#N/A	8.2	0.109525859
920	#N/A	#N/A	10.4	0.109698453
930	#N/A	#N/A	13	0.109882477
940	54.3	0.110189816	16	0.109968927
950	48	0.110128732	19.4	0.109896004
960	42.2	0.110229897	23.3	0.109983507
970	37	0.110790581	27.9	0.110874154
980	32.3	0.111447503	32.9	0.111507754
990	28.1	0.112241531	38.4	0.112279512
1000	24.3	0.112892253	#N/A	#N/A
1010	21	0.113833877	#N/A	#N/A
1020	18.1	0.114827533	#N/A	#N/A
1030	15.3	0.114826775	64.5	0.115159874
1040	13	0.115445235	#N/A	#N/A
1050	11	0.11604927	79.8	0.117261725
1060	9.2	0.116297996	#N/A	#N/A
1070	7.8	0.117308419	96.2	0.119709489
1080	6.5	0.117727073	104.8	0.121457846
1090	5.5	0.118817774	113.7	0.124089687

Market: SF
Contract: June 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
760	#N/A	#N/A	4	0.109417205
765	#N/A	#N/A	5	0.103961662
770	#N/A	#N/A	7	0.100723124
775	#N/A	#N/A	9	0.095424338
780	#N/A	#N/A	13	0.092839197
785	#N/A	#N/A	20	0.092549746
790	198	0.087163265	27	0.088550159
795	#N/A	#N/A	38	0.086408826
800	127	0.086894989	55	0.086894989
805	97	0.085910874	75	0.085910874
810	72	0.085364184	100	0.085364184
815	54	0.087353759	132	0.087353759
820	40	0.089419282	167	0.088247421
825	29	0.091096389	206	0.089752717
830	20	0.091576319	247	0.089960165
835	14	0.093125173	291	0.091130671
840	10	0.095436469	337	0.092938296
845	8	0.100349046	384	0.094124384
850	6	0.103510257	432	0.095541725
855	4	0.104230096	#N/A	#N/A
860	3	0.107267777	529	0.091019099
865	2	0.108212203	#N/A	#N/A
870	1	0.105202595	628	#N/A

Market: SF
Contract: September 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
750	#N/A	#N/A	2.3	0.109056061
760	#N/A	#N/A	3.4	0.10776552
770	#N/A	#N/A	5	0.107245608
780	#N/A	#N/A	7.1	0.106491788
790	#N/A	#N/A	9.9	0.106268647
800	27.7	0.106284192	13.4	0.106011546
810	22.1	0.105883818	17.7	0.106006435
820	17.4	0.106174244	22.8	0.106143797
830	13.4	0.106158751	28.6	0.105969457
840	10.2	0.1066477	35.3	0.106868237
850	7.6	0.10691139	42.5	0.106968482
860	5.6	0.107473052	50.4	0.108074219
870	4	0.107479286	58.7	0.108818968
880	3.1	0.110669064	67.7	0.112957724
890	2.2	0.111169284	76.7	0.114833251
900	1.6	0.112626061	#N/A	#N/A
910	1.2	0.114894888	#N/A	#N/A
920	1	0.119436799	#N/A	#N/A
930	0.8	0.12280265	#N/A	#N/A
940	0.6	0.124658548	#N/A	#N/A
950	0.4	0.124290224	#N/A	#N/A
960	0.3	0.126190776	#N/A	#N/A

Market: SF
Contract: December 1996

Strike	Call	Impl. vol. call	Put	Impl. vol. put
780	#N/A	#N/A	11.4	0.113224126
790	#N/A	#N/A	14.4	0.113009567
800	39.4	0.112967285	17.9	0.112756494
810	33.8	0.11302492	22	0.112795295
820	28.7	0.112901575	26.7	0.11305832
830	24.1	0.112612161	31.8	0.112742146
840	20.1	0.112607097	37.5	0.112712339
850	16.6	0.112566145	43.7	0.112647151
860	13.7	0.113075646	50.6	0.113586732
870	11.2	0.113452509	#N/A	#N/A
880	9.1	0.113890583	#N/A	#N/A
890	7.3	0.114075586	#N/A	#N/A
900	5.9	0.114857489	#N/A	#N/A
910	4.7	0.115289705	#N/A	#N/A
920	3.8	0.116389725	#N/A	#N/A
930	3	0.11684675	#N/A	#N/A

FACTOR		S&P COEFFICIENT	S&P Standard Error	S&P T-Statistic	FTSE COEFFICIENT	FTSE Standard Error	FTSE T-Statistic	DAX COEFFICIENT	DAX Standard Error	DAX T-Statistic	NIKKEI COEFFICIENT	NIKKEI Standard Error	NIKKEI T-Statistic
INTERCEPT	α	97.163	0.4823	-5.8822	98.027	0.3849	-5.1260	98.2797	0.4978	-3.4562	98.115	0.6739	-2.7972
Strike	$\beta1$	4.725	0.2804	16.8509	-2.062	0.4670	-4.4154	-8.1672	0.6239	-13.0912	-6.731	0.4883	-13.7846
Strike*Time	$\beta2$	-66.526	2.9015	-22.9281	-22.797	3.4891	-6.5338	-50.35	4.9292	-10.2146	-47.974	4.8015	-9.9915
Strike*Time^2	$\beta3$	159.673	11.2096	14.2443	56.272	13.4858	4.1727	89.7201	19.7946	4.5325	125.628	18.9691	6.6228
Strike*Crash	$\beta4$	-9.925	0.2373	-41.8247	-5.547	0.4840	-11.4607	-.--	-.--	-.--	-.--	-.--	-.--
Strike*Shock1	$\beta5$	-.--	-.--	-.--	-.--	-.--	-.--	0.8529	0.4175	2.0431	3.252	0.1776	18.3108
Strike*Shock2	$\beta6$	-6.083	0.1553	-39.1693	-0.518	0.3890	-1.3316	-2.0974	0.2883	-7.2761	1.213	0.1929	6.2882
Strike*ATMVol	$\beta7$	-17.17	0.8358	-20.5432	9.58	1.9842	4.8281	14.406	2.9011	4.9657	-9.103	1.2607	-7.2206
Strike^2	$\beta8$	5.899	0.1296	45.5170	8.963	0.1926	46.5369	7.2216	0.2958	24.4146	4.031	0.2067	19.5017
Strike^2*Time	$\beta9$	-12.295	1.2359	-9.9482	-6.491	0.4485	-14.4727	-13.8675	1.9932	-6.9576	-12.205	2.3352	-5.2265
Strike^2*Time^2	$\beta10$	15.557	4.8132	3.2322	-.--	-.--	-.--	17.1435	9.1866	1.8661	29.648	9.5672	3.0989
Strike^2*Crash	$\beta11$	0.681	0.1064	6.4004	-1.359	0.2802	-4.8501	-.--	-.--	-.--	-.--	-.--	-.--
Strike^2*Shock1	$\beta12$	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	1.17	0.0775	15.0968
Strike^2*Shock2	$\beta13$	-0.717	0.0624	-11.4904	-1.317	0.2396	-5.4967	-1.3096	0.1474	-8.8847	-.--	-.--	-.--
Strike^2*ATMVol	$\beta14$	-4.200	0.3970	-10.5793	-9.147	0.7184	-12.7325	-11.3916	1.4880	-7.6556	-4.172	0.5318	-7.8451
Strike^3	$\beta15$	0.801	0.0111	72.1622	0.388	0.0125	31.0400	0.3286	0.0211	15.5882	0.316	0.0207	15.2657
Crash	$\beta16$	1.465	0.2700	5.4259	2.921	0.4515	6.4695	-.--	-.--	-.--	-.--	-.--	-.--
Shock1	$\beta17$	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
Shock2	$\beta18$	0.441	0.1984	2.2228	-0.886	0.3528	-2.5113	-.--	-.--	-.--	-.--	-.--	-.--
ATMVol	$\beta19$	-1.835	1.0970	-1.6727	-.--	-.--	-.--	4.0932	3.0059	1.3617	-.--	-.--	-.--
Time	$\beta20$	54.822	10.7763	5.0873	-.--	-.--	-.--	1.7553	1.4661	1.1973	48.428	19.0513	2.5420
Time^2	$\beta21$	-554.37	86.3968	-6.4166	-291.477	33.2399	-8.7689	-.--	-.--	-.--	-275.02	156.3578	-1.7589
Time^3	$\beta22$	1538.426	207.5037	7.4140	1306.872	132.2727	9.8801	-.--	-.--	-.--	436.426	381.8629	1.1429
			(Observations) (12387)			(Observations) (6980)			(Observations) (2768)			(Observations) (3525)	
		R-Squared	0.9573		R-Squared	0.8875		R-Squared	0.9472		R-Squared	0.9084	
		Durbin-Watson	0.8765		Durbin-Watson	1.119042		Durbin-Watson	1.6069		Durbin-Watson	0.765779	

Table 8.2a Ordinary Least Squares Regression Results for Four Stock Index Options

FACTOR		BUND			BTP			GILT			USTB		
		COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic
INTERCEPT	α	98.144	0.4259	-4.3580	98.471	0.3713	-4.1179	97.224	0.5885	-4.7171	93.853	0.8332	-7.3776
Strike	β1	5.534	0.2575	21.4938	-0.044	0.2465	-0.1793	2.895	0.3527	8.2081	-0.969	0.3532	-2.7435
Strike*Time	β2	-26.088	2.6754	-9.7511	-32.213	3.0246	-10.6502	-9.662	2.2113	-4.3694	-4.085	2.1397	-1.9091
Strike*Time^2	β3	85.086	9.6950	8.7763	113.758	11.3120	10.0563	25.352	8.1773	3.1003	0.947	8.0677	0.1174
Strike*Crash	β4	-.--	-.--	-.--	-.--	-.--	-.--	-1.585	0.1442	-10.9917	0.781	0.1262	6.1886
Strike*Shock1	β5	-1.679	0.1600	-10.4918	-.--	-.--	-.--	-0.657	0.2867	-2.2916	-2.490	0.2658	-9.3679
Strike*Shock2	β6	-2.496	0.0897	-27.8292	-2.994	0.1026	-29.1813	-4.156	0.0823	-50.4982	0.452	0.0836	5.4067
Strike*ATMVol	β7	-96.526	2.3326	-41.3818	-44.109	2.0069	-21.9784	-22.411	1.7137	-13.0776	-7.141	1.6182	-4.4129
Strike^2	β8	4.547	0.1473	30.8669	5.082	0.1358	37.4218	4.411	0.2187	20.1692	5.652	0.1317	42.9157
Strike^2*Time	β9	-20.470	1.6384	-12.4937	1.847	1.2639	1.4614	-17.584	1.3549	-12.9781	-8.763	0.8932	-9.8108
Strike^2*Time^2	β10	46.894	6.0827	7.7094	-52.302	5.2860	-9.8945	32.372	5.1336	6.3059	15.837	3.3913	4.6699
Strike^2*Crash	β11	-.--	-.--	-.--	-.--	-.--	-.--	1.499	0.0901	16.6371	0.350	0.0384	9.1146
Strike^2*Shock1	β12	1.538	0.1209	12.7192	0.870	0.1129	7.7100	1.226	0.1651	7.4258	-0.505	0.0928	-5.4418
Strike^2*Shock2	β13	-1.325	0.0427	-31.0595	0.167	0.0532	3.1427	-0.757	0.0535	-14.1495	-0.318	0.0355	-8.9577
Strike^2*ATMVol	β14	-.--	-.--	-.--	-17.698	1.2902	-13.7169	-22.729	1.0935	-20.7856	-5.578	0.6412	-8.6987
Strike^3	β15	0.134	0.0156	8.6174	0.342	0.0122	27.9705	0.151	0.0141	10.7092	0.214	0.0064	33.4375
Crash	β16	-.--	-.--	-.--	-.--	-.--	-.--	0.819	0.2431	3.3690	-.--	-.--	-.--
Shock1	β17	-0.401	0.3070	-1.3064	-0.772	0.3235	-2.3874	-.--	-.--	-.--	2.134	0.3722	5.7335
Shock2	β18	-.--	-.--	-.--	-0.703	0.1801	-3.9036	-0.278	0.1599	-1.7386	-2.552	0.2496	-10.2244
ATMVol	β19	9.895	3.2354	3.0583	16.471	3.7509	4.3911	12.233	3.4196	3.5773	10.087	3.8118	2.6463
Time	β20	16.737	3.3321	5.0229	-.--	-.--	-.--	-11.485	10.6026	-1.0832	46.468	14.4177	3.2230
Time^2	β21	-.--	-.--	-.--	-.--	-.--	-.--	240.888	81.9159	2.9407	-321.256	113.0303	-2.8422
Time^3	β22	-185.076	46.1445	-4.0108	70.187	18.1272	3.8719	-700.835	192.7889	-3.6352	789.732	268.8714	2.9372
			(Observations) (8248)			(Observations) (8588)			(Observations) (9058)			(Observations) (9528)	
		R-Squared	0.8098		R-Squared	0.8631		R-Squared	0.8151		R-Squared	0.9016	
		Durbin-Watson	0.8882		Durbin-Watson	0.9224		Durbin-Watson	1.1456		Durbin-Watson	1.0426	

Table 8.2b Ordinary Least Squares Regression Results for four Fixed Income Options

FACTOR		D-MARK			POUND			YEN			S-FRANC		
		COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic
INTERCEPT	α	101.667	0.4569	3.6484	100.214	0.4792	0.4466	103.295	0.3773	8.7327	102.402	0.6149	3.9063
Strike	$\beta 1$	3.933	0.2422	16.2363	-1.002	0.1947	-5.1464	1.247	0.2256	5.5271	2.85	0.3645	7.8189
Strike*Time	$\beta 2$	1.045	0.4019	2.6000	1.929	0.4824	3.9988	-3.428	1.8318	-1.8715	2.177	1.7708	1.2294
Strike*Time^2	$\beta 3$	-.--	-.--	-.--	-.--	-.--	-.--	10.701	6.8146	1.5702	-4.231	6.5244	-0.6485
Strike*Crash	$\beta 4$	-0.595	0.1283	-4.6420	-0.741	0.1401	-5.2891	-1.052	0.5256	-2.0009	-.--	-.--	-.--
Strike*Shock1	$\beta 5$	-1.929	0.1986	-9.7125	-.--	-.--	-.--	1.809	0.5099	3.5470	-2.126	0.2859	-7.4362
Strike*Shock2	$\beta 6$	0.552	0.0598	9.2217	0.333	0.0670	4.9701	1.830	0.0870	21.0383	-.--	-.--	-.--
Strike*ATMVol	$\beta 7$	-15.234	1.1390	-13.3751	4.471	1.0472	4.2695	-6.647	1.3472	-4.9339	-7.234	1.3241	-5.4633
Strike^2	$\beta 8$	7.838	0.1227	63.8766	9.154	0.5791	15.8073	6.340	0.1033	61.3452	8.367	0.1655	50.5559
Strike^2*Time	$\beta 9$	-16.273	0.6849	-23.7600	-14.431	0.9325	-15.4756	-12.943	0.6957	-18.6034	-19.269	0.9969	-19.3289
Strike^2*Time^2	$\beta 10$	36.311	2.6070	13.9284	31.282	3.6792	8.5024	24.408	2.7085	9.0114	53.228	3.7284	14.2764
Strike^2*Crash	$\beta 11$	0.156	0.0641	2.4366	-1.254	0.0782	-16.0358	-0.750	0.1926	-3.8948	0.159	0.1138	1.3972
Strike^2*Shock1	$\beta 12$	-1.087	0.0968	-11.2276	-1.176	0.5676	-2.0719	0.576	0.1838	3.1336	-1.876	0.1237	-15.1657
Strike^2*Shock2	$\beta 13$	-0.127	0.0343	-3.6884	0.391	0.0365	10.7123	0.266	0.0413	6.4547	0.342	0.1084	3.1550
Strike^2*ATMVol	$\beta 14$	-17.166	0.6302	-27.2368	-20.62	0.5570	-37.0197	-12.820	0.6716	-19.0906	-19.014	0.7583	-25.0745
Strike^3	$\beta 15$	-0.089	0.0091	-9.7300	0.032	0.0086	3.7209	-0.169	0.0081	-21.0004	-0.068	0.0100	-6.8000
Crash	$\beta 16$	0.522	0.1912	2.7314	0.517	0.1815	2.8485	3.956	0.6039	6.5510	1.878	0.4488	4.1845
Shock1	$\beta 17$	0.532	0.3495	1.5215	-.--	-.--	-.--	-4.215	0.5737	-7.3476	-0.599	0.3974	-1.5073
Shock2	$\beta 18$	-0.714	0.1180	-6.0456	-0.283	0.1408	-2.0099	1.664	0.1620	10.2706	-2.566	0.4262	-6.0206
ATMVol	$\beta 19$	-13.729	2.2351	-6.1427	-10.127	2.1022	-4.8173	-19.256	2.6602	-7.2384	-25.721	2.6020	-9.8851
Time	$\beta 20$	-.--	-.--	-.--	19.445	9.6800	2.0088	-3.350	0.9182	-3.6487	48.242	8.2293	5.8622
Time^2	$\beta 21$	-.--	-.--	-.--	-223.217	74.9091	-2.9798	-.--	-.--	-.--	-285.325	64.6848	-4.4110
Time^3	$\beta 22$	-.--	-.--	-.--	651.221	175.7895	3.7046	-.--	-.--	-.--	-.--	-.--	-.--
			(Observations) (11079)			(Observations) (9190)			(Observations) (12998)			(Observations) (11834)	
		R-Squared	0.8565		R-Squared	0.8832		R-Squared	0.8634		R-Squared	0.8006	
		Durbin-Watson	1.403844		Durbin-Watson	1.53857		Durbin-Watson	1.33727		Durbin-Watson	1.4904	

Table 8.2c Ordinary Least Squares Regression Results for four Foreign Exchange Options

FACTOR		S&P			FTSE			DAX			NIKKEI		
		COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic
INTERCEPT	α	89.233	0.7955	-13.5349	100.268	0.7246	0.3699	95.78	1.3047	-3.2345	102.5044	0.6796	3.6851
Strike	$\beta 1$	4.529	0.2692	16.8239	-2.573	0.4581	-5.6167	-8.214	0.6445	-12.7448	-7.4774	0.4338	-17.2378
Strike*Time	$\beta 2$	-67.386	2.7542	-24.4666	-18.094	3.6298	-4.9848	-51.966	4.9104	-10.5828	-48.458	4.0730	-11.8975
Strike*Time^2	$\beta 3$	157.642	10.6270	14.8341	43.154	14.4804	2.9802	98.528	19.7032	5.0006	126.4631	16.0947	7.8575
Strike*Crash	$\beta 4$	-10.355	0.2298	-45.0609	-5.497	0.4777	-11.5072	-.--	-.--	-.--	-.--	-.--	-.--
Strike*Shock1	$\beta 5$	-.--	-.--	-.--	-.--	-.--	-.--	0.845	0.4496	1.8794	26,292	0.1652	159162.1769
Strike*Shock2	$\beta 6$	-5.440	0.1550	-35.0968	-0.524	0.3793	-1.3815	-1.824	0.2936	-6.2125	1.6951	0.2063	8.2155
Strike*ATMVol	$\beta 7$	-15.608	0.8206	-19.0202	10.885	1.9505	5.5806	13.415	2.8920	4.6387	-5.4825	1.1686	-4.6915
Strike^2	$\beta 8$	6.021	0.1238	48.6349	8.793	0.1990	44.1859	7.209	0.3196	22.5563	3.647	0.2039	17.8827
Strike^2*Time	$\beta 9$	-12.228	1.1718	-10.4352	-4.550	1.3503	-3.3696	-16.757	2.2762	-7.3618	-9.0871	1.6347	-5.5589
Strike^2*Time^2	$\beta 10$	13.997	4.5609	3.0689	-6.259	5.5506	-1.1276	30.066	10.2100	2.9448	17.652	6.7327	2.6218
Strike^2*Crash	$\beta 11$	0.488	0.1015	4.8079	-1.270	0.2743	-4.6300	-.--	-.--	-.--	-.--	-.--	-.--
Strike^2*Shock1	$\beta 12$	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	1.115	0.0862	12.9305
Strike^2*Shock2	$\beta 13$	-0.558	0.0591	-9.4416	-1.346	0.2360	-5.7034	-1.17	0.1642	-7.1255	0.1274	0.0957	1.3311
Strike^2*ATMVol	$\beta 14$	-4.982	0.3816	-13.0556	-8.369	0.7930	-10.5536	-10.922	1.5421	-7.0825	-3.2348	0.6042	-5.3542
Strike^3	$\beta 15$	0.778	0.0106	73.3962	0.407	0.0122	33.3607	0.337	0.0209	16.1244	0.3445	0.0183	18.8457
Crash	$\beta 16$	12.355	0.7867	15.7048	7.413	1.4643	5.0625	-.--	-.--	-.--	-.--	-.--	-.--
Shock1	$\beta 17$	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-4.9701	0.7027	-7.0726
Shock2	$\beta 18$	-.--	-.--	-.--	-5.536	1.4907	-3.7137	2.837	0.9598	2.9558	3.0946	0.7604	4.0697
ATMVol	$\beta 19$	-26.61	1.6507	-16.1204	-.--	-.--	-.--	-16.202	5.6026	-2.8919	-12.6433	2.6091	-4.8459
Time	$\beta 20$	49.065	10.2094	4.8059	-.--	-.--	-.--	69.334	14.9221	4.6464	-.--	-.--	-.--
Time^2	$\beta 21$	-448.941	81.9542	-5.4779	-239.238	34.0490	-7.0263	-543.429	123.5274	-4.3993	-.--	-.--	-.--
Time^3	$\beta 22$	1218.097	196.9912	6.1835	1134.95	137.0847	8.2792	1280.062	300.4180	4.2609	-.--	-.--	-.--
			(Observations) (12387)			(Observations) (6980)			(Observations) (2768)			(Observations) (3525)	
		R-Squared	0.9618		R-Squared	0.8958		R-Squared	0.9491		R-Squared	0.9319	

Table 8.6a Ordinary Least Squares Regression Results for four Stock Index Options Including Contracts as Dummy Variables

FACTOR		BUND COEFFICIENT	BUND Standard Error	BUND T-Statistic	BTP COEFFICIENT	BTP Standard Error	BTP T-Statistic	GILT COEFFICIENT	GILT Standard Error	GILT T-Statistic	USTB COEFFICIENT	USTB Standard Error	USTB T-Statistic
INTERCEPT	α	95.766	0.5521	-7.6689	96.667	0.5127	-6.4996	97.601	0.5448	-4.4035	99.076	1.4743	-0.6267
Strike	β1	5.630	0.2466	22.8268	0.075	0.2411	0.3115	2.801	0.3467	8.0790	-0.459	0.3053	-1.5035
Strike*Time	β2	-25.607	2.5353	-10.1001	-32.462	2.9599	-10.9674	-10.528	2.1180	-4.9707	-4.177	0.4899	-8.5264
Strike*Time²	β3	83.956	9.1897	9.1358	116.500	11.0687	10.5252	29.111	7.8402	3.7130	-.--	-.--	-.--
Strike*Crash	β4	-.--	-.--	-.--	-.--	-.--	-.--	-1.634	0.1411	-11.5804	0.570	0.1168	4.8793
Strike*Shock1	β5	-1.785	0.1657	-10.7699	-.--	-.--	-.--	-0.587	0.2880	-2.0382	-2.784	0.2423	-11.4894
Strike*Shock2	β6	-2.443	0.0855	-28.5731	-2.903	0.1023	-28.3863	-4.172	0.0791	-52.7434	0.591	0.0743	7.9521
Strike*ATMVol	β7	-98.131	2.2708	-43.2147	-46.802	1.9659	-23.8073	-21.403	1.6508	-12.9652	-9.314	1.4476	-6.4341
Strike²	β8	4.726	0.1557	30.3591	4.989	0.1378	36.1958	4.545	0.2076	21.8931	5.740	0.1137	50.5059
Strike²*Time	β9	-20.411	1.5535	-13.1391	1.477	1.4514	1.0173	-18.134	1.2952	-14.0009	-8.612	0.5785	-14.8875
Strike²*Time²	β10	45.491	5.7631	7.8935	-49.954	5.7707	-8.6566	34.103	4.9083	6.9480	14.798	2.3403	6.3231
Strike²*Crash	β11	-.--	-.--	-.--	-.--	-.--	-.--	1.424	0.0764	18.6387	0.318	0.0446	7.1380
Strike²*Shock1	β12	1.443	0.1154	12.5076	0.881	0.1270	6.9394	1.172	0.1715	6.8338	-0.612	0.0848	-7.2204
Strike²*Shock2	β13	-1.210	0.0564	-21.4653	0.198	0.0522	3.7847	-0.722	0.0510	-14.1569	-0.247	0.0316	-7.8140
Strike²*ATMVol	β14	-2.896	1.4789	-1.9582	-17.336	1.2728	-13.6199	-22.920	1.0324	-22.2007	-5.869	0.5941	-9.8791
Strike³	β15	0.140	0.0148	9.4659	0.342	0.0120	28.3887	0.153	0.0135	11.3333	0.230	0.0056	41.1449
Crash	β16	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	12.860	0.9941	12.9362
Shock1	β17	-3.447	0.7355	-4.6864	-1.221	0.5022	-2.4306	-.--	-.--	-.--	1.154	1.0301	1.1203
Shock2	β18	2.705	0.5986	4.5190	-0.095	0.3700	-0.2573	4.917	0.7806	6.2990	4.165	0.9082	4.5861
ATMVol	β19	55.044	8.2502	6.6718	25.104	5.0502	4.9709	10.931	4.3693	2.5018	-137.600	5.8762	-23.4164
Time	β20	13.612	3.1668	4.2983	-3.788	3.3438	-1.1327	-10.260	10.1334	-1.0125	-.--	-.--	-.--
Time²	β21	-.--	-.--	-.--	-.--	-.--	-.--	221.118	78.3538	2.8220	-.--	-.--	-.--
Time³	β22	-136.332	43.9324	-3.1032	129.161	47.1655	2.7385	-627.851	184.6605	-3.4000	110.549	19.2515	5.7424
			(Observations) (8248)			(Observations) (8588)			(Observations) (9058)			(Observations) (9528)	
		R-Squared	0.8305		R-Squared	0.8706		R-Squared	0.8315		R-Squared	0.9252	

Table 8.6b Ordinary Least Squares Regression Results for four Fixed Income Options Including Contracts as Dummy Variables

	D-MARK			POUND			YEN			S-FRANC		
	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic
α	104.272	0.6011	7.1058	101.358	0.5399	2.5153	95.758	1.3217	-3.2093	101.822	1.3237	1.3764
β1	3.963	0.2498	15.8633	-0.818	0.1912	-4.2782	1.280	0.2224	5.7546	2.325	0.3727	6.2383
β2	1.135	0.3937	2.8824	2.030	0.4682	4.3358	-3.676	1.7830	-2.0618	2.600	1.7175	1.5138
β3	-.--	-.--	-.--	-.--	-.--	-.--	10.422	6.6311	1.5717	-6.736	6.3283	-1.0644
β4	-0.379	0.1268	-2.9879	-0.736	0.1381	-5.3295	-0.955	0.5116	-1.8666	-.--	-.--	-.--
β5	-2.246	0.2114	-10.6224	-.--	-.--	-.--	1.821	0.4953	3.6757	1.622	0.2943	5.5114
β6	0.520	0.0592	8.7685	0.178	0.0678	2.6254	1.814	0.0848	21.3851	-.--	-.--	-.--
β7	-14.736	1.1202	-13.1543	3.135	1.0468	2.9948	-7.994	1.3456	-5.9410	-7.554	1.3530	-5.5831
β8	7.816	0.1208	64.6832	9.174	0.5564	16.4881	6.358	0.1012	62.8150	8.134	0.1636	49.7188
β9	-16.647	0.6666	-24.9731	-14.123	0.8943	-15.7922	-12.952	0.6772	-19.1252	-19.088	0.9636	-19.8090
β10	37.525	2.5355	14.7998	29.494	3.5293	8.3569	24.666	2.6347	9.3617	51.978	3.6051	14.4179
β11	0.185	0.0572	3.2381	-1.202	0.0754	-15.9416	-0.814	0.1899	-4.2853	0.101	0.1101	0.9173
β12	-1.187	0.0943	-12.5835	-1.268	0.5453	-2.3253	0.554	0.1813	3.0533	-1.715	0.1241	-13.8195
β13	-0.130	0.0297	-4.3697	0.384	0.0329	11.6717	0.323	0.0402	8.0180	0.358	0.1050	3.4095
β14	-16.371	0.6143	-26.6506	-20.397	0.5332	-38.2539	-12.780	0.6559	-19.4846	-18.451	0.7393	-24.9574
β15	-0.085	0.0089	-9.6178	0.030	0.0083	3.6145	-0.165	0.0079	-20.9202	-0.065	0.0098	-6.6327
β16	-.--	-.--	-.--	0.755	0.2545	2.9666	6.195	0.7969	7.7740	2.807	0.6990	4.0157
β17	-0.523	0.4062	-1.2874	-.--	-.--	-.--	1.673	1.0348	1.6162	-4.098	0.9089	-4.5087
β18	-.--	-.--	-.--	-.--	-.--	-.--	1.556	0.2540	6.1260	1.482	0.6067	2.4427
β19	-23.301	2.8556	-8.1598	-17.770	2.5731	-6.9061	-28.302	3.3745	-8.3870	-33.359	3.6349	-9.1774
β20	-.--	-.--	-.--	15.952	9.2868	1.7177	-2.539	0.9042	-2.8079	44.982	7.9649	5.6475
β21	-.--	-.--	-.--	-194.080	71.8575	-2.7009	-.--	-.--	-.--	-244.764	62.6896	-3.9044
β22	-.--	-.--	-.--	599.469	168.5887	3.5558	-.--	-.--	-.--	421.266	150.1412	2.8058
		(Observations) (11079)			(Observations) (9190)			(Observations) (12998)			(Observations) (11834)	
	R-Squared	0.865		R-Squared	0.8928		R-Squared	0.8717		R-Squared	0.8146	

Table 8.6c Ordinary Least Squares Regression Results for four Foreign Exchange Options Including Contracts as Dummy Variables

FACTOR		S&P			FTSE			DAX			NIKKEI		
		COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic
INTERCEPT	α	98.016	0.4273	-4.6431	97.952	0.7247	-2.8260	97.301	0.7038	-3.8349	98.6092	0.7030	-1.9784
Strike	$\beta 1$	7.164	0.3264	21.9485	-2.177	0.5387	-4.0412	-9.486	0.6939	-13.6706	-6.8104	0.4561	-14.9318
Strike*Time	$\beta 2$	-75.795	2.9550	-25.6497	-35.191	3.9894	-8.8211	-54.503	4.9858	-10.9316	-47.9097	4.5044	-10.6362
Strike*Time^2	$\beta 3$	176.963	11.2529	15.7260	106.756	16.2766	6.5589	103.017	10.0424	10.2582	132.1116	17.3611	7.6096
Strike*Crash	$\beta 4$	-11.523	0.2765	-41.6745	-5.426	0.5401	-10.0463	-.--	-.--	-.--	-.--	-.--	-.--
Strike*Shock1	$\beta 5$	-.--	-.--	-.--	-.--	-.--	-.--	0.841	0.5185	1.6220	3.5266	0.1643	21.4644
Strike*Shock2	$\beta 6$	-6.172	0.1525	-40.4721	-0.946	0.3911	-2.4188	-1.922	0.3064	-6.2728	1.4019	0.2136	6.5632
Strike*ATMVol	$\beta 7$	-18.674	0.8609	-21.6913	12.008	2.0072	5.9825	22.63	2.9872	7.5757	-11.8623	1.1581	-10.2429
Strike^2	$\beta 8$	6.359	0.1693	37.5605	8.747	0.2665	32.8218	6.752	0.4641	14.5486	3.8856	0.2553	15.2197
Strike^2*Time	$\beta 9$	-14.962	1.3492	-11.0895	-10.821	1.6751	-6.4599	-20.815	2.3243	-8.9554	-8.8546	2.4810	-3.5690
Strike^2*Time^2	$\beta 10$	20.206	5.2515	3.8477	14.828	6.9147	2.1444	41.124	9.9742	4.1230	20.1672	10.0001	2.0167
Strike^2*Crash	$\beta 11$	0.439	0.1392	3.1537	-1.073	0.3609	-2.9731	-.--	-.--	-.--	-.--	-.--	-.--
Strike^2*Shock1	$\beta 12$	-.--	-.--	-.--	-.--	-.--	-.--	0.233	0.4369	0.5333	1.1834	0.0974	12.1499
Strike^2*Shock2	$\beta 13$	-0.66	0.0684	-9.6491	-1.291	0.2928	-4.4092	-0.912	0.2299	-3.9669	-0.0308	0.1151	-0.2676
Strike^2*ATMVol	$\beta 14$	-4.650	0.4578	-10.1573	-8.553	0.8645	-9.8936	-10.829	1.6659	-6.5004	-4.0819	0.6892	-5.9227
Strike^3	$\beta 15$	0.819	0.0122	67.1311	0.434	0.0138	31.4493	0.355	0.0249	14.2570	0.3665	0.0223	16.4350
Crash	$\beta 16$	0.603	0.2020	2.9851	2.308	0.4329	5.3315	-.--	-.--	-.--	-.--	-.--	-.--
Shock1	$\beta 17$	-.--	-.--	-.--	-.--	-.--	-.--	-0.191	0.3532	-0.5408	0.3411	0.2096	1.6274
Shock2	$\beta 18$	0.418	0.1424	2.9354	-0.701	0.3252	-2.1556	-0.16	0.1957	-0.8176	0.3014	0.2504	1.2037
ATMVol	$\beta 19$	-0.513	1.0515	-0.4879	0.895	1.7106	0.5232	1.34	2.4422	0.5466	0.7954	1.3867	0.5736
Time	$\beta 20$	43.379	9.2752	4.6769	0.300	16.0381	0.0187	58.237	13.7725	4.2285	17.9129	14.6714	1.2209
Time^2	$\beta 21$	-426.994	73.6741	-5.7957	-236.418	130.0931	-1.8173	-440.532	108.4722	-4.0612	-44.053	119.0214	-0.3701
Time^3	$\beta 22$	1180.246	176.3265	6.6935	1104.776	318.0436	3.4737	994.62	256.0559	3.8844	-80.7508	287.3514	-0.2810
			(Observations) (12387)			(Observations) (6980)			(Observations) (2768)			(Observations) (3525)	
		R-Squared	0.9567		R-Squared	0.8865		R-Squared	0.9471		R-Squared	0.9078	

Table 8.7a Weighted Least Squares Regression Results for four Stock Index Options

FACTOR		BUND			BTP			GILT			USTB		
		COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic
INTERCEPT	α	98.283	0.4632	-3.7068	99.073	0.4838	-1.9167	97.951	0.4966	-4.1261	94.536	0.7641	-7.1509
Strike	$\beta 1$	5.648	0.2495	22.6373	0.277	0.2624	1.0541	3.262	0.3006	10.8516	-1.781	0.3187	-5.5883
Strike*Time	$\beta 2$	-20.851	2.6915	-7.7470	-33.880	3.0765	-11.0125	-9.625	2.3546	-4.0877	-4.848	1.9550	-2.4798
Strike*Time^2	$\beta 3$	70.959	9.4434	7.5141	121.743	11.2009	10.8690	24.734	8.3542	2.9607	4.672	7.7973	0.5992
Strike*Crash	$\beta 4$	-.--	-.--	-.--	-.--	-.--	-.--	-1.750	0.1371	-12.7644	1.150	0.1275	9.0196
Strike*Shock1	$\beta 5$	-1.060	0.1472	-7.2011	0.312	0.2010	1.5507	-0.738	0.2033	-3.6301	-2.513	0.2379	-10.5633
Strike*Shock2	$\beta 6$	-3.017	0.0849	-35.5359	-2.858	0.1002	-28.5230	-4.476	0.0769	-58.2055	0.363	0.0748	4.8529
Strike*ATMVol	$\beta 7$	-110.165	2.4876	-44.2857	-52.500	2.2190	-23.6594	-24.348	1.7280	-14.0903	-3.553	1.5636	-2.2723
Strike^2	$\beta 8$	4.536	0.1624	27.9310	5.110	0.1550	32.9706	3.856	0.2117	18.2145	5.433	0.1284	42.3131
Strike^2*Time	$\beta 9$	-18.613	1.7685	-10.5247	-2.130	1.7812	-1.1957	-14.166	1.5040	-9.4189	-9.795	0.8253	-11.8684
Strike^2*Time^2	$\beta 10$	38.696	6.4074	6.0393	-40.689	6.9472	-5.8568	24.263	5.5309	4.3868	18.781	3.3376	5.6271
Strike^2*Crash	$\beta 11$	-.--	-.--	-.--	-.--	-.--	-.--	1.740	0.0944	18.4322	0.365	0.0522	6.9923
Strike^2*Shock1	$\beta 12$	1.489	0.1072	13.8899	0.717	0.1397	5.1346	1.057	0.1621	6.5207	-0.266	0.0923	-2.8819
Strike^2*Shock2	$\beta 13$	-1.093	0.0585	-18.6838	0.135	0.0577	2.3345	-0.764	0.0521	-14.6641	-0.262	0.0319	-8.2132
Strike^2*ATMVol	$\beta 14$	-2.567	1.6986	-1.5112	-14.723	1.4223	-10.3513	-20.355	1.1601	-17.5459	-5.663	0.6576	-8.6116
Strike^3	$\beta 15$	0.109	0.0171	6.3743	0.339	0.0143	23.6713	0.178	0.0152	11.7105	0.224	0.0061	36.7213
Crash	$\beta 16$	-.--	-.--	-.--	-.--	-.--	-.--	0.400	0.1807	2.2136	0.088	0.3169	0.2777
Shock1	$\beta 17$	-0.061	0.1926	-0.3167	-0.544	0.2355	-2.3117	0.175	0.2515	0.6958	0.859	0.3496	2.4571
Shock2	$\beta 18$	-0.413	0.1178	-3.5059	-0.289	0.1177	-2.4588	-0.351	0.1032	-3.4012	-2.768	0.2187	-12.6566
ATMVol	$\beta 19$	8.218	3.1056	2.6462	6.293	2.5832	2.4361	8.494	2.3694	3.5849	15.406	3.9805	3.8704
Time	$\beta 20$	18.658	10.1229	1.8431	0.859	10.9150	0.0787	-0.462	9.0296	-0.0512	45.467	12.7148	3.5759
Time^2	$\beta 21$	-41.365	76.0048	-0.5442	27.743	83.3248	0.3330	92.570	67.5988	1.3694	-290.319	102.3172	-2.8374
Time^3	$\beta 22$	-41.598	174.9851	-0.2377	-54.371	194.1634	-0.2800	-281.394	155.5423	-1.8091	674.256	250.0382	2.6966
			(Observations) (8248)			(Observations) (8588)			(Observations) (9058)			(Observations) (9528)	
		R-Squared	0.8072		R-Squared	0.862		R-Squared	0.8142		R-Squared	0.9014	

Table 8.7b Weighted Least Squares Regression Results for four Fixed Income Options

FACTOR		D-MARK COEFFICIENT	D-MARK Standard Error	D-MARK T-Statistic	POUND COEFFICIENT	POUND Standard Error	POUND T-Statistic	YEN COEFFICIENT	YEN Standard Error	YEN T-Statistic	S-FRANC COEFFICIENT	S-FRANC Standard Error	S-FRANC T-Statistic
INTERCEPT	α	100.474	0.4466	1.0616	98.873	1.2585	-0.8955	101.773	0.4083	4.3424	99.173	0.5023	-1.6464
Strike	β1	3.738	0.2759	13.5469	-0.459	3.0876	-0.1487	1.588	0.2261	7.0234	2.515	0.3545	7.0945
Strike*Time	β2	3.285	1.7940	1.8308	4.710	1.9945	2.3615	-4.608	2.0345	-2.2649	2.567	1.8157	1.4138
Strike*Time^2	β3	-9.183	6.4728	-1.4187	-11.322	7.1714	-1.5788	13.270	7.4169	1.7892	-5.564	6.7736	-0.8214
Strike*Crash	β4	-0.793	0.1249	-6.3491	-0.622	0.1365	-4.5568	-0.538	0.5448	-0.9875	0.298	0.5036	0.5917
Strike*Shock1	β5	-1.956	0.2119	-9.2312	-0.367	3.0817	-0.1191	1.261	0.5331	2.3654	-1.906	0.2920	-6.5274
Strike*Shock2	β6	0.637	0.0590	10.7949	0.210	0.0641	3.2761	1.992	0.0876	22.7397	-0.409	0.4964	-0.8239
Strike*ATMVol	β7	-12.988	1.1591	-11.2050	2.092	1.0206	2.0498	-7.562	1.3708	-5.5165	-5.188	1.2288	-4.2220
Strike^2	β8	8.039	0.1432	56.1355	9.541	1.8238	5.2314	6.541	0.1183	55.2916	8.646	0.1738	49.7468
Strike^2*Time	β9	-17.705	1.0480	-16.8938	-15.867	1.0728	-14.7903	-11.994	1.0537	-11.3827	-16.198	1.0642	-15.2208
Strike^2*Time^2	β10	41.456	3.8876	10.6637	33.084	4.1197	8.0307	18.967	3.9784	4.7675	40.190	4.0463	9.9325
Strike^2*Crash	β11	0.261	0.0691	3.7771	-0.985	0.0865	-11.3873	-0.420	0.2325	-1.8065	0.279	0.2115	1.3191
Strike^2*Shock1	β12	-1.015	0.1096	-9.2564	-1.568	1.8218	-0.8607	0.283	0.2234	1.2668	-1.864	0.1338	-13.9312
Strike^2*Shock2	β13	-0.212	0.0360	-5.8944	0.369	0.0377	9.7878	0.469	0.0441	10.6349	0.174	0.2078	0.8373
Strike^2*ATMVol	β14	-18.530	0.6929	-26.7424	-21.485	0.5971	-35.9822	-14.491	0.7308	-19.8290	-21.698	0.7558	-28.7087
Strike^3	β15	-0.091	0.0101	-9.0198	0.018	0.0098	1.8367	-0.209	0.0091	-22.9670	-0.081	0.0103	-7.8641
Crash	β16	0.121	0.1368	0.8816	0.25	1.284	0.1947	3.144	0.5677	5.5381	2.343	0.4368	5.3640
Shock1	β17	0.446	0.2911	1.5311	0.483	1.1972	0.4034	-3.615	0.5543	-6.5217	0.350	0.3603	0.9714
Shock2	β18	-0.489	0.0798	-6.1303	-0.314	0.0958	-3.2777	0.946	0.1178	8.0306	-2.861	0.4257	-6.7207
ATMVol	β19	-7.296	1.5762	-4.6291	-4.692	1.479	-3.1724	-10.942	1.9159	-5.7112	-3.560	1.7168	-2.0736
Time	β20	12.643	6.9280	1.8249	19.785	8.3731	2.3629	14.087	8.3244	1.6923	32.938	6.6320	4.9665
Time^2	β21	-67.523	52.5637	-1.2846	-156.282	62.7263	-2.4915	-147.870	63.8306	-2.3166	-186.882	52.5238	-3.5580
Time^3	β22	92.798	123.1584	0.7535	397.706	144.2809	2.7565	391.407	151.6714	2.5806	338.558	128.9438	2.6256
			(Observations) (11079)			(Observations) (9190)			(Observations) (12998)			(Observations) (11834)	
		R-Squared	0.8563		R-Squared	0.8826		R-Squared	0.8629		R-Squared	0.7992	

Table 8.7c Weighted Least Squares Regression Results for four Foreign Exchange Options

FACTOR		NIKKEI			BUND			YEN		
		COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic
INTERCEPT	α	38.325	0.3266	-188.8280	42.072	0.2737	-211.6478	69.312	0.3778	-81.2282
Strike	β1	-5.860	0.5512	-10.6313	4.611	0.3099	14.8790	1.234	0.2691	4.5857
Strike*Time	β2	-47.573	4.8165	-9.8770	-25.703	2.6815	-9.5853	-3.663	1.9740	-1.8556
Strike*Time^2	β3	114.857	19.2898	5.9543	82.138	9.6815	8.4840	7.977	7.3438	1.0862
Strike*Crash	β4	-.--	-.--	-.--	-.--	-.--	-.--	-1.233	0.6015	-2.0499
Strike*Shock1	β5	2.036	0.2329	8.7416	-2.847	0.2194	-12.9763	2.064	0.5849	3.5288
Strike*Shock2	β6	1.268	0.3107	4.0810	-1.712	0.1318	-12.9894	1.873	0.1117	16.7681
Strike*ATMVol	β7	-8.700	1.5413	-5.6447	-66.744	3.2218	-20.7164	-6.276	1.6045	-3.9115
Strike^2	β8	5.123	0.2322	22.0629	5.552	0.1566	35.4534	6.161	0.1159	53.1579
Strike^2*Time	β9	-13.939	2.0655	-6.7486	-25.032	1.5615	-16.0307	-14.578	0.9369	-15.5598
Strike^2*Time^2	β10	114.857	19.2898	5.9543	62.408	5.8559	10.6573	33.700	3.6554	9.2192
Strike^2*Crash	β11	-.--	-.--	-.--	-.--	-.--	-.--	-0.853	0.2090	-4.0813
Strike^2*Shock1	β12	0.492	0.0917	5.3677	1.225	0.1091	11.2282	0.501	0.1992	2.5151
Strike^2*Shock2	β13	0.485	0.1076	4.5066	-0.836	0.0662	-12.6284	0.231	0.0474	4.8734
Strike^2*ATMVol	β14	-6.302	0.6578	-9.5807	-13.592	1.6504	-8.2356	-10.339	0.7445	-13.8872
Strike^3	β15	0.333	0.0211	15.7670	0.179	0.0159	11.2579	-0.160	0.0087	-18.3908
Crash	β16	-.--	-.--	-.--	-.--	-.--	-.--	4.016	0.7228	5.5562
Shock1	β17	-.--	-.--	-.--	-.--	-.--	-.--	-3.900	0.6848	-5.6951
Shock2	β18	-1.013	0.5439	-1.8624	-0.739	0.2595	-2.8478	1.869	0.2110	8.8578
ATMVol	β19	-9.019	2.5314	-3.5629	45.294	5.8964	7.6816	-34.018	3.2270	-10.5417
Time	β20	28.684	5.2505	5.4631	59.657	13.4699	4.4289	39.241	9.9501	3.9438
Time^2	β21	-312.943	76.9361	-4.0676	-297.977	103.8156	-2.8703	-381.373	78.8261	-4.8382
Time^3	β22	-.--	-.--	-.--	414.547	240.3999	1.7244	938.776	187.6342	5.0032
			(Observations) (3524)			(Observations) (8247)			(Observations) (12997)	
		R-Squared	0.8490		R-Squared	0.7509		R-Squared	0.8263	
		Durbin-Watson	1.80624		Durbin-Watson	1.844168		Durbin-Watson	1.950257	

Table 8.8 Generalized Least Squares Regression Results for three Selected Options Markets

FACTOR		ALL STOCKS			S&P			FTSE			DAX			NIKKEI		
		COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic
INTERCEPT	α	97.202	0.1907	-14.6738	97.163	0.4823	-5.8822	98.027	0.3849	-5.1260	98.2797	0.4978	-3.4562	98.115	0.6739	-2.7972
Strike	β1	3.590	0.2267	15.8374	4.725	0.2804	16.8509	-2.062	0.4670	-4.4154	-8.1672	0.6239	-13.0912	-6.731	0.4883	-13.7846
Strike*Time	β2	-42.325	1.7431	-24.2816	-66.526	2.9015	-22.9281	-22.797	3.4891	-6.5338	-50.35	4.9292	-10.2146	-47.974	4.8015	-9.9915
Strike*Time²	β3	93.393	6.6123	14.1240	159.673	11.2096	14.2443	56.272	13.4858	4.1727	89.7201	19.7946	4.5325	125.628	18.9691	6.6228
Strike*Crash	β4	-9.653	0.2357	-40.9581	-9.925	0.2373	-41.8247	-5.547	0.4840	-11.4607	-.--	-.--	-.--	-.--	-.--	-.--
Strike*Shock1	β5	1.804	0.1740	10.3672	-.--	-.--	-.--	-.--	-.--	-.--	0.8529	0.4175	2.0431	3.252	0.1776	18.3108
Strike*Shock2	β6	-3.51	0.1147	-30.5962	-6.083	0.1553	-39.1693	-0.518	0.3890	-1.3316	-2.0974	0.2883	-7.2761	1.213	0.1929	6.2882
Strike*ATMVol	β7	-8.798	0.6653	-13.2249	-17.17	0.8358	-20.5432	9.58	1.9842	4.8281	14.406	2.9011	4.9657	-9.103	1.2607	-7.2206
Strike²	β8	6.659	0.0970	68.6849	5.899	0.1296	45.5170	8.963	0.1926	46.5369	7.2216	0.2958	24.4146	4.031	0.2067	19.5017
Strike²*Time	β9	-6.763	0.2359	-28.6738	-12.295	1.2359	-9.9482	-6.491	0.4485	-14.4727	-13.8675	1.9932	-6.9576	-12.205	2.3352	-5.2265
Strike²*Time²	β10	-.--	-.--	-.--	15.557	4.8132	3.2322	-.--	-.--	-.--	17.1435	9.1866	1.8661	29.648	9.5672	3.0989
Strike²*Crash	β11	-.--	-.--	-.--	0.681	0.1064	6.4004	-1.359	0.2802	-4.8501	-.--	-.--	-.--	-.--	-.--	-.--
Strike²*Shock1	β12	-1.507	0.0672	-22.4323	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	1.17	0.0775	15.0968
Strike²*Shock2	β13	-0.281	0.0466	-6.0275	-0.717	0.0624	-11.4904	-1.317	0.2396	-5.4967	-1.3096	0.1474	-8.8847	-.--	-.--	-.--
Strike²*ATMVol	β14	-5.612	0.2726	-20.5839	-4.200	0.3970	-10.5793	-9.147	0.7184	-12.7325	-11.3916	1.4880	-7.6556	-4.172	0.5318	-7.8451
Strike³	β15	0.506	0.0074	68.3784	0.801	0.0111	72.1622	0.388	0.0125	31.0400	0.3286	0.0211	15.5862	0.316	0.0207	15.2657
Crash	β16	-.--	-.--	-.--	1.465	0.2700	5.4259	2.921	0.4515	6.4695	-.--	-.--	-.--	-.--	-.--	-.--
Shock1	β17	3.26	0.1723	18.9194	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
Shock2	β18	-0.705	0.1371	-5.1422	0.441	0.1984	2.2228	-0.886	0.3528	-2.5113	-.--	-.--	-.--	-.--	-.--	-.--
ATMVol	β19	-.--	-.--	-.--	-1.835	1.0970	-1.6727	-.--	-.--	-.--	4.0932	3.0059	1.3617	-.--	-.--	-.--
Time	β20	-.--	-.--	-.--	54.822	10.7763	5.0873	-.--	-.--	-.--	1.7553	1.4661	1.1973	48.428	19.0513	2.5420
Time²	β21	-158.922	15.5973	-10.1891	-654.37	86.3968	-6.4166	-291.477	33.2399	-8.7689	-.--	-.--	-.--	-275.02	156.3578	-1.7589
Time³	β22	691.545	62.0277	11.1490	1538.426	207.5037	7.4140	1306.872	132.2727	9.8801	-.--	-.--	-.--	436.426	381.8629	1.1429
S&P	β23	1.782	0.1129	15.7797	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
S&P*Strike	β24	-3.197	0.1286	-24.8697	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
S&P*Strike²	β25	-1.291	0.0795	-16.2492	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
FTSE	β26	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
FTSE*Strike	β27	2.371	0.1149	20.6425	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
FTSE*Strike²	β28	0.988	0.0576	17.1409	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
Nikkei	β29	1.039	0.1790	5.8035	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
Nikkei*Strike	β30	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
Nikkei*Strike²	β31	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
DAX	β32	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
DAX*Strike	β33	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
DAX*Strike²	β34	0.279	0.0650	4.2950	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
			(Observations) (30784)			(Observations) (12387)			(Observations) (6980)			(Observations) (2768)			(Observations) (3525)	
		R-Squared	0.9082		R-Squared	0.9573		R-Squared	0.8875		R-Squared	0.9472		R-Squared	0.9084	

Table 8.9a Ordinary Least Squares Regression Results for four Stock Index Options Compared to All Stock Index Options

FACTOR		ALL BONDS			BUND			BTP			GILT			USTB		
		COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic
INTERCEPT	α	96.583	0.2851	-11.9860	98.144	0.4259	-4.3580	98.471	0.3713	-4.1179	97.224	0.5885	-4.7171	93.853	0.8332	-7.3776
Strike	β1	2.487	0.1465	16.9710	5.534	0.2575	21.4938	-0.044	0.2465	-0.1793	2.895	0.3527	8.2081	-0.969	0.3532	-2.7435
Strike*Time	β2	-16.285	1.2880	-12.6437	-26.088	2.6754	-9.7511	-32.213	3.0246	-10.6502	-9.662	2.2113	-4.3694	-4.085	2.1397	-1.9091
Strike*Time^2	β3	46.323	4.8133	9.6239	85.086	9.6950	8.7763	113.758	11.3120	10.0563	25.352	8.1773	3.1003	0.947	8 0677	0.1174
Strike*Crash	β4	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-1.585	0.1442	-10.9917	0.781	0.1262	6.1886
Strike*Shock1	β5	-1.168	0.09255	-12.6224	-1.679	0.1600	-10.4918	-.--	-.--	-.--	-0.657	0.2867	-2.2916	-2.490	0.2658	-9.3679
Strike*Shock2	β6	-1.851	0.04626	-40.0195	-2.496	0.0897	-27.8292	-2.994	0.1026	-29.1813	-4.156	0.0823	-50.4982	0.452	0.0836	5.4067
Strike*ATMVol	β7	-33.604	0.8452	-39.7584	-96.526	2.3326	-41.3818	-44.109	2.0069	-21.9784	-22.411	1.7137	-13.0776	-7.141	1.6182	-4.4129
Strike^2	β8	5.455	0.0655	83.2824	4.547	0.1473	30.8669	5.082	0.1358	37.4218	4.411	0.2187	20.1692	5.652	0.1317	42.9157
Strike^2*Time	β9	-9.458	0.4583	-20.6395	-20.470	1.6384	-12.4937	1.847	1.2639	1.4614	-17.584	1.3549	-12.9781	-8.763	0.8932	-9.8108
Strike^2*Time^2	β10	14.046	1.7708	7.9317	46.894	6.0827	7.7094	-52.302	5.2860	-9.8945	32.372	5.1336	6.3059	15.837	3.3913	4.6699
Strike^2*Crash	β11	0.137	0.0339	4.0483	-.--	-.--	-.--	-.--	-.--	-.--	1.499	0.0901	16.6371	0.350	0.0384	9.1146
Strike^2*Shock1	β12	-.--	-.--	-.--	1.538	0.1209	12.7192	0.870	0.1129	7.7100	1.226	0.1651	7.4258	-0.505	0.0928	-5.4418
Strike^2*Shock2	β13	-0.311	0.0216	-14.3889	-1.325	0.0427	-31.0595	0.167	0.0532	3.1427	-0.757	0.0535	-14.1495	-0.318	0.0355	-8.9577
Strike^2*ATMVol	β14	-10.510	0.4209	-24.9698	-.--	-.--	-.--	-17.698	1.2902	-13.7169	-22.729	1.0935	-20.7856	-5.578	0.6412	-8.6987
Strike^3	β15	0.220	0.0042	52.5914	0.134	0.0156	8.6174	0.342	0.0122	27.9705	0.151	0.0141	10.7092	0.214	0.0064	33.4375
Crash	β16	1.149	0.1914	6.0033	-.--	-.--	-.--	-.--	-.--	-.--	0.819	0.2431	3.3690	-.--	-.--	-.--
Shock1	β17	1.394	0.1479	9.4266	-0.401	0.3070	-1.3064	-0.772	0.3235	-2.3874	-.--	-.--	-.--	2.134	0.3722	5.7335
Shock2	β18	-1.262	0.0975	-12.9417	-.--	-.--	-.--	-0.703	0.1801	-3.9036	-0.278	0.1599	-1.7386	-2.552	0.2496	-10.2244
ATMVol	β19	8.312	1.9674	4.2250	9.895	3.2354	3.0583	16.471	3.7509	4.3911	12.233	3.4196	3.5773	10.087	3.8118	2.6463
Time	β20	3.279	0.6659	4.9250	16.737	3.3321	5.0229	-.--	-.--	-.--	-11.485	10.6026	-1.0832	46.468	14.4177	3.2230
Time^2	β21	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	240.888	81.9159	2.9407	-321.256	113.0303	-2.8422
Time^3	β22	-.--	-.--	-.--	-185.076	46.1445	-4.0108	70.187	18.1272	3.8719	-700.835	192.7889	-3.6352	789.732	268.8714	2.9372
USTB	β23	-1.154	0.1506	-7.6620	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
USTB*Strike	β24	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
USTB*Strike^2	β25	0.618	0.0276	22.4129	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
GILT	β26	-0.187	0.1364	-1.3732	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
GILT*Strike	β27	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
GILT*Strike^2	β28	-0.260	0.0372	-6.9998	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
BTP	β29	-0.483	0.1195	-4.0452	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
BTP*Strike	β30	-3.113	0.0609	-51.1366	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
BTP*Strike^2	β31	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
BUND	β32	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
BUND*Strike	β33	-1.943	0.0694	-28.0010	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
BUND*Strike^2	β34	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
			(Observations) (35422)			(Observations) (8248)			(Observations) (8588)			(Observations) (9058)			(Observations) (9528)	
		R-Squared	0.8620		R-Squared	0.8098		R-Squared	0.8631		R-Squared	0.8151		R-Squared	0.9016	

Table 8.9b Odinary Least Squares Regression Results for four Fixed Income Options compared to All Fixed Income Options

FACTOR		ALL FX			D-MARK			POUND			YEN			S-FRANC		
		COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic
INTERCEPT	α	102.665	0.1408	18.9319	101.667	0.4569	3.6484	100.214	0.4792	0.4466	103.295	0.3773	8.7327	102.402	0.6149	3.9063
Strike	β1	-.--	-.--	-.--	3.933	0.2422	16.2363	-1.002	0.1947	-5.14638	1.247	0.2256	5.5271	2.850	0.3645	7.8189
Strike*Time	β2	1.112	0.2202	5.0499	1.045	0.4019	2.6000	1.929	0.4824	3.998756	-3.428	1.8318	-1.8715	2.177	1.7708	1.2294
Strike*Time^2	β3	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	10.701	6.8146	1.5702	-4.231	6.5244	-0.6485
Strike*Crash	β4	-0.718	0.0808	-8.8765	-0.595	0.1283	-4.6420	-0.741	0.1401	-5.28908	-1.052	0.5256	-2.0009	-.--	-.--	-.--
Strike*Shock1	β5	0.367	0.0939	3.9135	-1.929	0.1986	-9.7125	-.--	-.--	-.--	1.809	0.5099	3.5470	-2.126	0.2859	-7.4362
Strike*Shock2	β6	0.792	0.0381	20.7750	0.552	0.0598	9.2217	0.333	0.067	4.970149	1.830	0.0870	21.0383	-.--	-.--	-.--
Strike*ATMVol	β7	-4.476	0.4892	-9.1489	-15.234	1.1390	-13.3751	4.471	1.0472	4.269481	-6.647	1.3472	-4.9339	-7.234	1.3241	-5.4633
Strike^2	β8	7.087	0.0569	124.4595	7.838	0.1227	63.8766	9.154	0.5791	15.80729	6.340	0.1033	61.3452	8.367	0.1655	50.5559
Strike^2*Time	β9	-14.230	0.3603	-39.4932	-16.273	0.6849	-23.7600	-14.431	0.9325	-15.4756	-12.943	0.6957	-18.6034	-19.269	0.9969	-19.3289
Strike^2*Time^2	β10	31.524	1.3920	22.6461	36.311	2.6070	13.9284	31.282	3.6792	8.502392	24.408	2.7085	9.0114	53.228	3.7284	14.2764
Strike^2*Crash	β11	-0.224	0.0353	-6.3342	0.156	0.0641	2.4366	-1.254	0.0782	-16.0358	-0.750	0.1926	-3.8948	0.159	0.1138	1.3972
Strike^2*Shock1	β12	-0.518	0.0459	-11.2846	-1.087	0.0968	-11.2276	-1.176	0.5676	-2.07188	0.576	0.1838	3.1336	-1.876	0.1237	-15.1657
Strike^2*Shock2	β13	0.338	0.0186	18.1803	-0.127	0.0343	-3.6884	0.391	0.0365	10.71233	0.266	0.0413	6.4547	0.342	0.1084	3.1550
Strike^2*ATMVol	β14	-16.759	0.3132	-53.5008	-17.166	0.6302	-27.2368	-20.62	0.557	-37.0197	-12.820	0.6716	-19.0906	-19.014	0.7583	-25.0745
Strike^3	β15	-0.075	0.0042	-18.0703	-0.089	0.0091	-9.7300	0.032	0.0086	3.72093	-0.169	0.0081	-21.0004	-0.068	0.0100	-6.8000
Crash	β16	-.--	-.--	-.--	0.522	0.1912	2.7314	0.517	0.1815	2.848485	3.956	0.6039	6.5510	1.878	0.4488	4.1845
Shock1	β17	-.--	-.--	-.--	0.532	0.3495	1.5215	-.--	-.--	-.--	-4.215	0.5737	-7.3476	-0.599	0.3974	-1.5073
Shock2	β18	-0.158	0.0594	-2.6591	-0.714	0.1180	-6.0456	-0.283	0.1408	-2.00994	1.664	0.1620	10.2706	-2.566	0.4262	-6.0206
ATMVol	β19	-17.318	1.1378	-15.2197	-13.729	2.2351	-6.1427	-10.127	2.1022	-4.81733	-19.256	2.6602	-7.2384	-25.721	2.6020	-9.8851
Time	β20	-.--	-.--	-.--	-.--	-.--	-.--	19.445	9.68	2.008781	-3.350	0.9182	-3.6487	48.242	8.2293	5.8622
Time^2	β21	-.--	-.--	-.--	-.--	-.--	-.--	-223.217	74.9091	-2.97984	-.--	-.--	-.--	-285.325	64.6848	-4.4110
Time^3	β22	-.--	-.--	-.--	-.--	-.--	-.--	651.221	175.7895	3.70455	-.--	-.--	-.--	-.--	-.--	-.--
BP	β23	-0.829	0.0737	-11.2464	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
BP*Strike	β24	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
BP*Strike^2	β25	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
JY	β26	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
JY*Strike	β27	1.430	0.0421	33.9644	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
JY*Strike^2	β28	0.265	0.0218	12.1595	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
SF	β29	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
SF*Strike	β30	0.125	0.0467	2.6704	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
SF*Strike^2	β31	-0.187	0.0236	-7.9393	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
DM	β32	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
DM*Strike	β33	0.361	0.0440	8.2036	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
DM*Strike^2	β34	0.106	0.0229	4.6365	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
			(Observations) (45101)			(Observations) (11079)			(Observations) (9190)			(Observations) (12998)			(Observations) (11834)	
		R-Squared	0.8484		R-Squared	0.8565		R-Squared	0.8832		R-Squared	0.8634		R-Squared	0.8006	

Table 8.9c Ordinary Least Squares egression Results for four Foreign Exchange Options compared to All Foreign Exchange Options

FACTOR		ALL MARKETS			ALL STOCKS			ALL BONDS			ALL FX		
		COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic
INTERCEPT	α	97.949	0.1989	-10.3096	97.202	0.1907	-14.6738	96.583	0.2851	-11.9660	102.665	0.1406	18.8319
Strike	β1	-.--	-.--	-.--	3.590	0.2267	15.8374	2.487	0.1465	16.9710	-.--	-.--	-.--
Strike*Time	β2	-18.290	0.7844	-23.3160	-42.325	1.7431	-24.2816	-16.285	1.2880	-12.6437	1.112	0.2202	5.0499
Strike*Time²	β3	52.228	2.9485	17.7136	93.393	6.6123	14.1240	46.323	4.8133	9.6239	-.--	-.--	-.--
Strike*Crash	β4	-0.146	0.0592	-2.4670	-9.653	0.2357	-40.9581	-.--	-.--	-.--	-0.718	0.0808	-8.8765
Strike*Shock1	β5	-2.578	0.0677	-38.0685	1.804	0.1740	10.3672	-1.160	0.09255	-12.6224	0.367	0.0938	3.9138
Strike*Shock2	β6	-1.127	0.0311	-36.2147	-3.51	0.1147	-30.5962	-1.851	0.04626	-40.0196	0.792	0.0381	20.7750
Strike*ATMVol	β7	-8.847	0.4056	-21.8148	-8.798	0.6653	-13.2249	-33.604	0.8452	-39.7584	-4.476	0.4892	-9.1489
Strike²	β8	6.203	0.0459	135.2005	6.659	0.0970	68.6649	5.455	0.0655	83.2824	7.087	0.0569	124.4596
Strike²*Time	β9	-10.614	0.3721	-28.5277	-6.763	0.2359	-28.6738	-9.458	0.4583	-20.6396	-14.230	0.3603	-39.4932
Strike²*Time²	β10	19.884	1.4330	13.8754	-.--	-.--	-.--	14.046	1.7708	7.9317	31.524	1.3920	22.6461
Strike²*Crash	β11	0.381	0.0253	15.0593	-.--	-.--	-.--	0.137	0.0339	4.0483	-0.224	0.0353	-6.3342
Strike²*Shock1	β12	-1.558	0.0303	-51.3652	-1.507	0.0672	-22.4323	-.--	-.--	-.--	-0.518	0.0459	-11.2846
Strike²*Shock2	β13	-.--	-.--	-.--	-0.281	0.0466	-6.0275	-0.311	0.0216	-14.3869	0.338	0.0186	18.1803
Strike²*ATMVol	β14	-8.009	0.1921	-41.6897	-5.612	0.2726	-20.5839	-10.510	0.4209	-24.9698	-16.759	0.3132	-53.5006
Strike³	β15	0.209	0.0031	67.4194	0.506	0.0074	68.3784	0.220	0.0042	52.5914	-0.075	0.0042	-18.0703
Crash	β16	-0.257	0.1012	-2.5400	-.--	-.--	-.--	1.149	0.1914	6.0033	-.--	-.--	-.--
Shock1	β17	3.608	0.1058	34.1150	3.26	0.1723	18.9194	1.394	0.1479	9.4266	-.--	-.--	-.--
Shock2	β18	-0.568	0.0472	-12.0237	-0.705	0.1371	-5.1422	-1.262	0.0975	-12.9417	-0.158	0.0594	-2.6591
ATMVol	β19	-6.696	0.6464	-10.3626	-.--	-.--	-.--	8.312	1.9674	4.2250	-17.318	1.1378	-15.2197
Time	β20	20.209	3.9331	5.1381	-.--	-.--	-.--	3.279	0.6659	4.9250	-.--	-.--	-.--
Time²	β21	-207.346	31.2253	-6.6403	-158.922	15.5973	-10.1891	-.--	-.--	-.--	-.--	-.--	-.--
Time³	β22	570.224	74.7317	7.6303	691.545	62.0277	11.1490	-.--	-.--	-.--	-.--	-.--	-.--
BOND	β23	-1.414	0.0847	-16.6863	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
BOND*Strike	β24	1.245	0.1020	12.2118	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
BOND*Strike²	β25	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
STOCK	β26	-0.576	0.0980	-5.8770	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
STOCK*Strike	β27	-4.118	0.1506	-27.3530	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
STOCK*Strike²	β28	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
FX	β29	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
FX*Strike	β30	4.231	0.0972	43.5422	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
FX*Strike²	β31	0.544	0.0302	18.0312	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
USTB	β32	-0.484	0.1125	-4.3018	-.--	-.--	-.--	-1.154	0.1506	-7.6620	-.--	-.--	-.--
USTB*Strike	β33	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
USTB*Strike²	β34	0.770	0.0224	34.3750	-.--	-.--	-.--	0.618	0.0276	22.4128	-.--	-.--	-.--
GILT	β35	0.172	0.0918	1.8743	-.--	-.--	-.--	-0.187	0.1364	-1.3732	-.--	-.--	-.--
GILT*Strike	β36	0.547	0.0601	9.1000	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
GILT*Strike²	β37	-.--	-.--	-.--	-.--	-.--	-.--	-0.260	0.0372	-6.9998	-.--	-.--	-.--
BTP	β38	-.--	-.--	-.--	-.--	-.--	-.--	-0.483	0.1196	-4.0452	-.--	-.--	-.--
BTP*Strike	β39	-3.163	0.0625	-50.6080	-.--	-.--	-.--	-3.113	0.0609	-51.1366	-.--	-.--	-.--
BTP*Strike²	β40	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
BUND	β41	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
BUND*Strike	β42	-0.785	0.0655	-11.9847	-.--	-.--	-.--	-1.943	0.0694	-28.0010	-.--	-.--	-.--
BUND*Strike²	β43	0.223	0.0304	7.3476	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
S&P	β44	2.281	0.1061	21.6000	1.782	0.1129	15.7707	-.--	-.--	-.--	-.--	-.--	-.--
S&P*Strike	β45	-2.830	0.1118	-25.3036	-3.107	0.1266	-24.8897	-.--	-.--	-.--	-.--	-.--	-.--
S&P*Strike²	β46	-1.586	0.0434	-36.5646	-1.291	0.0795	-16.2492	-.--	-.--	-.--	-.--	-.--	-.--
FTSE	β47	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
FTSE*Strike	β48	3.672	0.1109	33.1258	2.371	0.1149	20.6425	-.--	-.--	-.--	-.--	-.--	-.--
FTSE*Strike²	β49	1.207	0.0279	43.2927	0.968	0.0676	17.1409	-.--	-.--	-.--	-.--	-.--	-.--
Nikkei	β50	2.519	0.1705	14.7785	1.039	0.1790	5.8035	-.--	-.--	-.--	-.--	-.--	-.--
Nikkei*Strike	β51	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
Nikkei*Strike²	β52	-0.159	0.0532	-2.9882	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
DAX	β53	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
DAX*Strike	β54	-0.382	0.1420	-2.6901	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
DAX*Strike²	β55	-.--	-.--	-.--	0.279	0.0650	4.2950	-.--	-.--	-.--	-.--	-.--	-.--
BP	β56	-1.508	0.1042	-14.4684	-.--	-.--	-.--	-.--	-.--	-.--	-0.829	0.0737	-11.2464
BP*Strike	β57	-0.725	0.0517	-14.0341	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
BP*Strike²	β58	0.061	0.0334	1.5265	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
JY	β59	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
JY*Strike	β60	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	1.430	0.0421	33.9644
JY*Strike²	β61	0.084	0.0294	3.1961	-.--	-.--	-.--	-.--	-.--	-.--	0.266	0.0218	12.1596
SF	β62	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
SF*Strike	β63	0.433	0.0517	8.3833	-.--	-.--	-.--	-.--	-.--	-.--	0.125	0.0467	2.6704
SF*Strike²	β64	-0.241	0.0318	-7.5620	-.--	-.--	-.--	-.--	-.--	-.--	-0.187	0.0236	-7.8393
DM	β65	-0.157	0.0972	-1.6147	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--
DM*Strike	β66	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	0.361	0.0440	8.2036
DM*Strike²	β67	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	0.106	0.0229	4.6365
			(Observations) (111307)			(Observations) (30784)			(Observations) (35422)			(Observations) (45101)	
		R-Squared	0.8671		R-Squared	0.9082		R-Squared	0.8620		R-Squared	0.8484	

Table 8.10 Ordinary Least Squares Regression Results for All Option Markets compared to the three Asset Classes

	80	82	84	86	88	90	92	94	96	98	100	102	104	106	108	110	112	114	116	118	120
3	20.0148	18.0150	16.0152	14.0154	12.0160	10.0171	8.0211	6.0368	4.1091	2.3680	1.0671	0.3782	0.1139	0.0319	0.0109	0.0047	0.0026	0.0018	0.0013	0.0010	0.0008
7	20.0349	18.0352	16.0360	14.0375	12.0411	10.0515	8.0811	6.1659	4.3862	2.8569	1.6735	0.8753	0.4078	0.1749	0.0741	0.0336	0.0162	0.0087	0.0055	0.0038	0.0027
10	20.0512	18.0521	16.0541	14.0581	12.0674	10.0931	8.1587	6.3083	4.6138	3.1660	2.0177	1.1885	0.6458	0.3317	0.1639	0.0798	0.0394	0.0213	0.0128	0.0085	0.0056
14	20.0712	18.0726	16.0767	14.0868	12.1096	10.1648	8.2837	6.5115	4.9040	3.5230	2.4020	1.5506	0.9475	0.5530	0.3073	0.1649	0.0868	0.0460	0.0259	0.0147	0.0084
17	20.0900	18.0928	16.1002	14.1178	12.1572	10.2431	8.4037	6.6826	5.1248	3.7806	2.6719	1.8116	1.1757	0.7354	0.4426	0.2569	0.1437	0.0791	0.0442	0.0248	0.0146
21	20.1112	18.1175	16.1318	14.1609	12.2217	10.3384	8.5489	6.8813	5.3729	4.0599	2.9656	2.0951	1.4328	0.9483	0.6060	0.3714	0.2226	0.1322	0.0787	0.0480	0.0293
24	20.1286	18.1388	16.1599	14.2009	12.2796	10.4230	8.6636	7.0291	5.5476	4.2479	3.1588	2.2877	1.6146	1.1067	0.7338	0.4723	0.2953	0.1833	0.1141	0.0715	0.0449
28	20.1624	18.1814	16.2168	14.2785	12.3846	10.5637	8.8405	7.2460	5.8047	4.5413	3.4663	2.5817	1.8771	1.3335	0.9254	0.6292	0.4196	0.2766	0.1828	0.1200	0.0769
31	20.1880	18.2120	16.2561	14.3318	12.4606	10.6621	8.9670	7.3977	5.9785	4.7278	3.6581	2.7713	2.0556	1.4928	1.0624	0.7403	0.5096	0.3456	0.2313	0.1550	0.1032
35	20.2186	18.2525	16.3119	14.4058	12.5563	10.7825	9.1123	7.5627	6.1595	4.9161	3.8496	2.9654	2.2449	1.6709	1.2247	0.8813	0.6293	0.4425	0.3091	0.2158	0.1483
38	20.2368	18.2747	16.3421	14.4527	12.6224	10.8668	9.2096	7.6811	6.2929	5.0625	4.0041	3.1109	2.3844	1.7957	1.3318	0.9779	0.7099	0.5069	0.3554	0.2466	0.1693
41	20.2647	18.3123	16.3898	14.5145	12.7030	10.9657	9.3280	7.8145	6.4422	5.2202	4.1647	3.2723	2.5381	1.9366	1.4604	1.0907	0.8047	0.5839	0.4167	0.2944	0.2087
45	20.2940	18.3519	16.4436	14.5851	12.7888	11.0767	9.4689	7.9800	6.6221	5.4195	4.3715	3.4715	2.7231	2.1109	1.6129	1.2154	0.9062	0.6690	0.4881	0.3505	0.2498
48	20.3315	18.4003	16.5050	14.6574	12.8769	11.1832	9.5930	8.1219	6.7832	5.5848	4.5359	3.6423	2.8860	2.2577	1.7448	1.3319	1.0071	0.7526	0.5606	0.4133	0.3008
52	20.3656	18.4425	16.5589	14.7270	12.9635	11.2873	9.7103	8.2554	6.9262	5.7363	4.6931	3.7953	3.0359	2.4006	1.8744	1.4525	1.1124	0.8445	0.6364	0.4753	0.3518
55	20.4073	18.4934	16.6208	14.8026	13.0534	11.3901	9.8265	8.3835	7.0740	5.9001	4.8616	3.9574	3.1824	2.5322	1.9975	1.5646	1.2134	0.9337	0.7128	0.5392	0.4038
59	20.4615	18.5596	16.7014	14.9016	13.1705	11.5289	9.9873	8.5620	7.2629	6.0924	5.0560	4.1547	3.3778	2.7181	2.1682	1.7164	1.3456	1.0495	0.8119	0.6220	0.4736
62	20.5176	18.6228	16.7757	14.9839	13.2679	11.6437	######	8.7092	7.4218	6.2618	5.2290	4.3230	3.5373	2.8692	2.3072	1.8385	1.4528	1.1402	0.8947	0.6976	0.5399

Table 9.1 Monte Carlo generated Call Option Prices using a Student-t distribution with constant volatility.

	80	82	84	86	88	90	92	94	96	98	100	102	104	106	108	110	112	114	116	118	120
3	#N/A	#N/A	#N/A	#N/A	0.3895	0.3434	0.3065	0.2781	0.2614	0.2490	0.2434	0.2494	0.2614	0.2763	0.3001	0.3295	0.3635	0.4021	0.4379	0.4750	#N/A
7	#N/A	0.3786	0.3493	0.3216	0.2974	0.2790	0.2648	0.2554	0.2510	0.2497	0.2491	0.2499	0.2514	0.2553	0.2630	0.2750	0.2888	0.3049	0.3247	0.3455	0.3665
10	0.3607	0.3353	0.3145	0.2946	0.2775	0.2666	0.2595	0.2544	0.2514	0.2509	0.2507	0.2511	0.2518	0.2547	0.2591	0.2650	0.2725	0.2835	0.2972	0.3127	0.3272
14	0.3124	0.2929	0.2808	0.2715	0.2631	0.2586	0.2558	0.2538	0.2522	0.2518	0.2517	0.2519	0.2525	0.2541	0.2558	0.2582	0.2614	0.2661	0.2733	0.2805	0.2876
17	0.2929	0.2793	0.2709	0.2652	0.2609	0.2592	0.2572	0.2558	0.2544	0.2539	0.2536	0.2541	0.2547	0.2560	0.2575	0.2591	0.2606	0.2628	0.2666	0.2708	0.2766
21	0.2811	0.2719	0.2659	0.2604	0.2573	0.2555	0.2555	0.2548	0.2538	0.2531	0.2528	0.2532	0.2540	0.2550	0.2558	0.2559	0.2568	0.2588	0.2617	0.2660	0.2704
24	0.2755	0.2689	0.2637	0.2593	0.2565	0.2553	0.2551	0.2545	0.2533	0.2520	0.2516	0.2524	0.2537	0.2548	0.2554	0.2559	0.2563	0.2579	0.2605	0.2638	0.2674
28	0.2702	0.2680	0.2652	0.2618	0.2590	0.2577	0.2566	0.2560	0.2554	0.2551	0.2550	0.2552	0.2557	0.2563	0.2570	0.2579	0.2590	0.2605	0.2630	0.2656	0.2675
31	0.2698	0.2664	0.2637	0.2609	0.2594	0.2579	0.2572	0.2565	0.2560	0.2556	0.2554	0.2557	0.2562	0.2568	0.2575	0.2581	0.2593	0.2606	0.2620	0.2641	0.2662
35	0.2689	0.2661	0.2640	0.2611	0.2590	0.2572	0.2561	0.2548	0.2538	0.2529	0.2527	0.2534	0.2543	0.2554	0.2566	0.2576	0.2592	0.2606	0.2624	0.2646	0.2664
38	0.2644	0.2615	0.2600	0.2590	0.2578	0.2562	0.2545	0.2536	0.2528	0.2521	0.2521	0.2523	0.2533	0.2541	0.2550	0.2564	0.2580	0.2591	0.2599	0.2608	0.2618
41	0.2634	0.2617	0.2599	0.2589	0.2581	0.2564	0.2548	0.2537	0.2530	0.2523	0.2522	0.2525	0.2535	0.2542	0.2553	0.2568	0.2582	0.2591	0.2598	0.2605	0.2620
45	0.2593	0.2588	0.2578	0.2571	0.2559	0.2550	0.2543	0.2535	0.2526	0.2524	0.2525	0.2525	0.2530	0.2539	0.2546	0.2552	0.2560	0.2569	0.2578	0.2583	0.2589
48	0.2601	0.2600	0.2591	0.2577	0.2565	0.2557	0.2550	0.2544	0.2540	0.2535	0.2533	0.2538	0.2543	0.2548	0.2554	0.2559	0.2567	0.2574	0.2585	0.2596	0.2603
52	0.2588	0.2581	0.2572	0.2560	0.2549	0.2542	0.2532	0.2528	0.2522	0.2517	0.2517	0.2520	0.2526	0.2532	0.2537	0.2545	0.2552	0.2560	0.2570	0.2578	0.2586
55	0.2597	0.2588	0.2578	0.2567	0.2556	0.2546	0.2536	0.2531	0.2531	0.2532	0.2532	0.2532	0.2533	0.2535	0.2541	0.2550	0.2559	0.2568	0.2577	0.2585	0.2592
59	0.2599	0.2591	0.2581	0.2572	0.2561	0.2554	0.2547	0.2543	0.2541	0.2539	0.2539	0.2541	0.2544	0.2547	0.2551	0.2558	0.2564	0.2573	0.2581	0.2588	0.2595
62	0.2620	0.2603	0.2592	0.2577	0.2568	0.2564	0.2561	0.2557	0.2556	0.2557	0.2557	0.2558	0.2558	0.2561	0.2565	0.2569	0.2573	0.2579	0.2590	0.2601	0.2610

Table 9.2 Volatilities implied by the Monte Carlo generated Call Option Prices using a Student-t distribution with constant volatility.

	80	82	84	86	88	90	92	94	96	98	100	102	104	106	108	110	112	114	116	118	120
3	#N/A	#N/A	#N/A	#N/A	#N/A	141.07	125.90	114.22	107.38	102.28	100.00	102.46	107.38	113.51	123.28	135.33	149.30	#N/A	#N/A	#N/A	#N/A
7	#N/A	#N/A	140.20	129.12	119.38	112.01	106.30	102.53	100.78	100.23	100.00	100.32	100.94	102.51	105.60	110.41	115.95	122.41	130.35	138.69	147.11
10	143.86	133.75	125.46	117.49	110.70	106.32	103.48	101.46	100.27	100.07	100.00	100.13	100.43	101.57	103.34	105.69	108.67	113.06	118.54	124.73	130.49
14	124.12	116.37	111.56	107.86	104.51	102.72	101.62	100.83	100.21	100.04	100.00	100.09	100.33	100.97	101.64	102.58	103.87	105.73	108.57	111.42	114.24
17	115.51	110.13	106.84	104.57	102.88	102.20	101.44	100.86	100.31	100.13	100.00	100.19	100.43	100.97	101.55	102.17	102.75	103.64	105.12	106.79	109.09
21	111.19	107.55	105.14	102.98	101.75	101.06	101.04	100.76	100.37	100.09	100.00	100.15	100.47	100.86	101.16	101.22	101.58	102.34	103.49	105.18	106.94
24	109.48	106.88	104.79	103.05	101.94	101.47	101.36	101.12	100.67	100.16	100.00	100.29	100.81	101.25	101.48	101.68	101.87	102.48	103.51	104.85	106.28
28	105.96	105.09	103.99	102.66	101.56	101.04	100.64	100.38	100.14	100.03	100.00	100.08	100.25	100.51	100.78	101.14	101.55	102.14	103.11	104.14	104.90
31	105.63	104.30	103.22	102.15	101.55	100.98	100.70	100.43	100.21	100.04	100.00	100.12	100.32	100.54	100.81	101.06	101.51	102.02	102.56	103.37	104.23
35	106.41	105.31	104.46	103.31	102.51	101.79	101.34	100.83	100.45	100.10	100.00	100.26	100.63	101.05	101.55	101.94	102.58	103.14	103.84	104.72	105.43
38	104.86	103.71	103.12	102.71	102.27	101.61	100.95	100.61	100.25	100.00	100.00	100.07	100.47	100.79	101.15	101.71	102.31	102.78	103.09	103.45	103.83
41	104.44	103.77	103.07	102.66	102.35	101.68	101.05	100.62	100.31	100.03	100.00	100.14	100.52	100.81	101.25	101.82	102.37	102.75	103.00	103.30	103.88
45	102.69	102.48	102.10	101.83	101.36	100.98	100.70	100.41	100.05	99.97	100.00	100.01	100.22	100.57	100.85	101.09	101.40	101.77	102.09	102.29	102.56
48	102.68	102.63	102.30	101.73	101.25	100.93	100.67	100.45	100.26	100.07	100.00	100.19	100.38	100.58	100.82	101.04	101.34	101.60	102.06	102.47	102.77
52	102.83	102.53	102.18	101.72	101.30	100.99	100.61	100.43	100.19	100.02	100.00	100.12	100.35	100.60	100.79	101.14	101.42	101.73	102.09	102.44	102.75
55	102.56	102.22	101.81	101.38	100.94	100.55	100.16	99.97	99.95	99.99	100.00	100.01	100.02	100.11	100.35	100.72	101.05	101.41	101.78	102.09	102.35
59	102.39	102.05	101.66	101.33	100.89	100.61	100.33	100.17	100.09	100.01	100.00	100.10	100.20	100.32	100.51	100.79	101.02	101.37	101.69	101.93	102.21
62	102.48	101.81	101.36	100.80	100.44	100.28	100.14	100.02	99.99	99.99	100.00	100.03	100.06	100.16	100.32	100.46	100.63	100.85	101.29	101.72	102.09

Table 9.3 Standardized implied volatilities using a Student-t distribution with constant volatility.

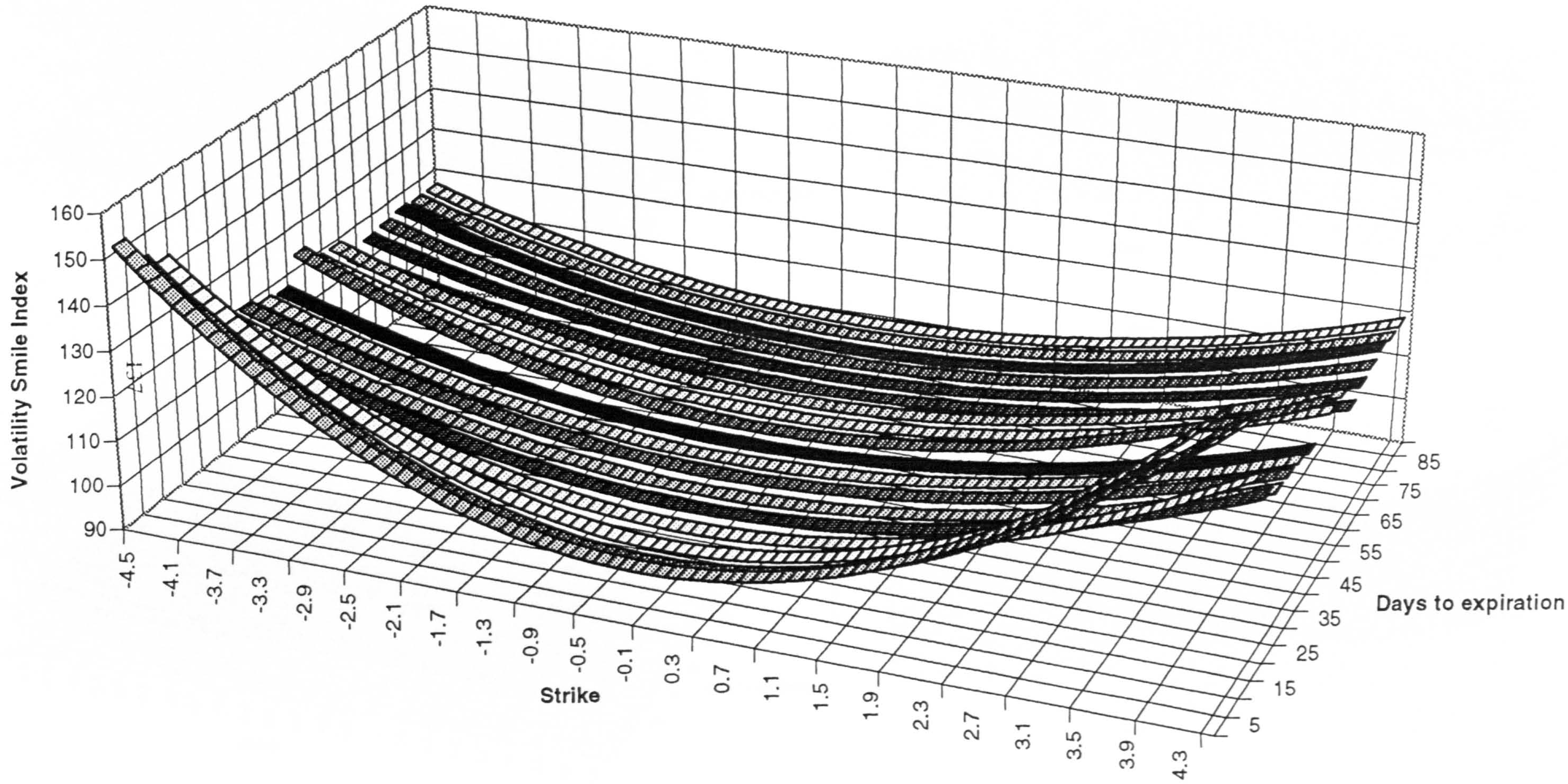

Figure 9.1 Standardized Volatility Smiles using a Student-t distribution with constant volatility

S&P-500
Whole Period

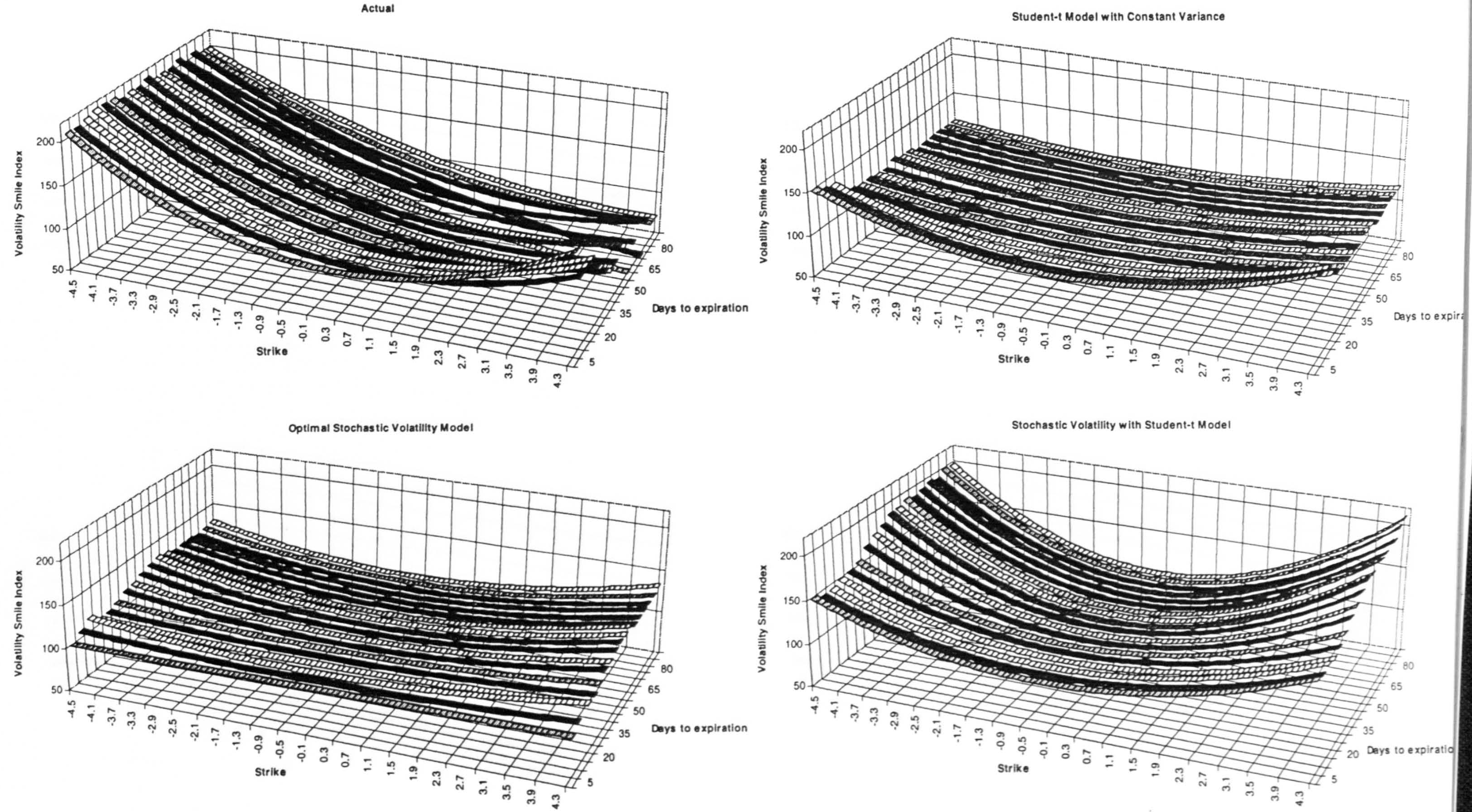

Figure 9.2a Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

FTSE-100
Whole Period

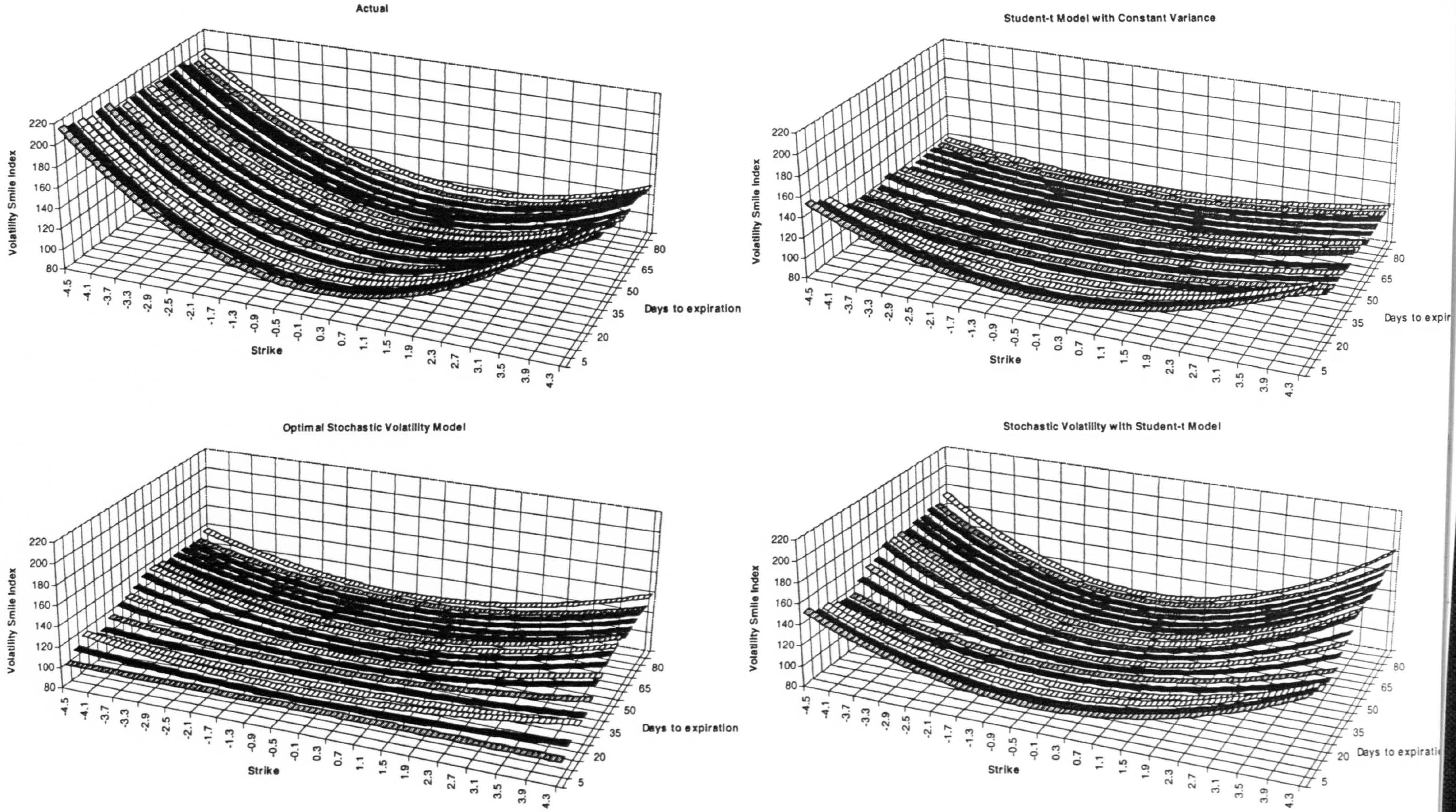

Figure 9.2b Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compare to the Actual

Nikkei-225
Whole Period

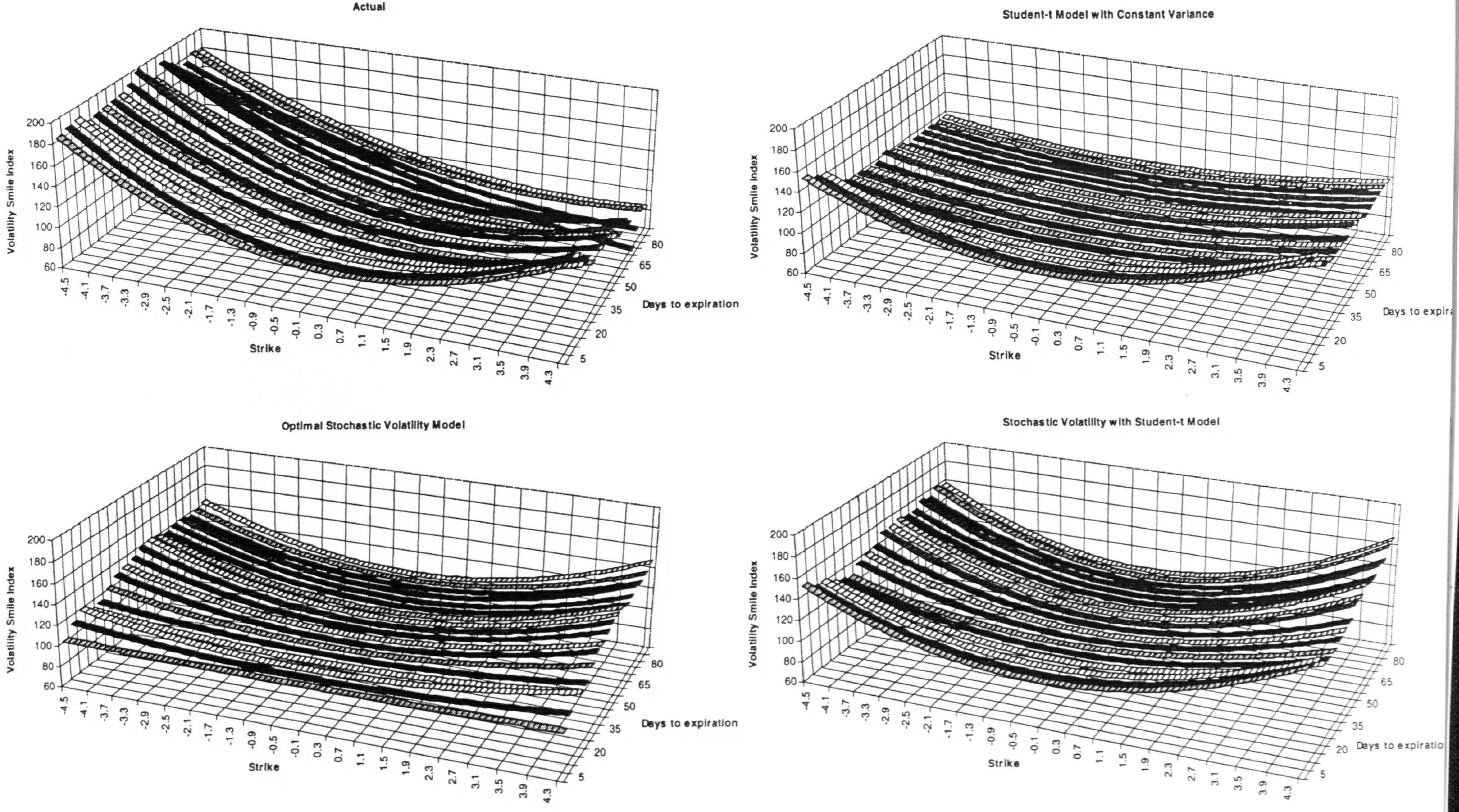

Figure 9.2c Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

DAX
Whole Period

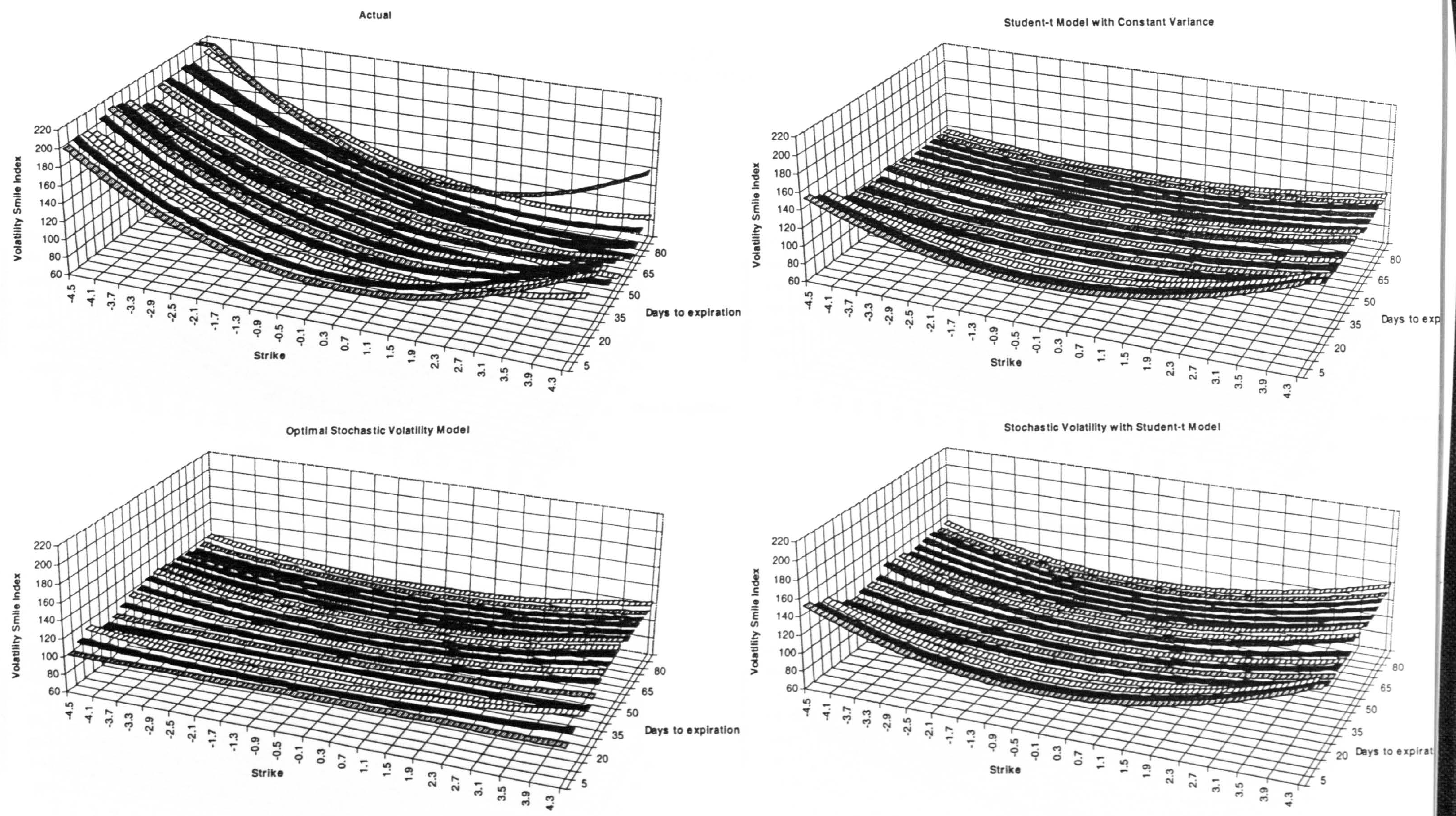

Figure 9.2d Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compare to the Actual

Bund
Whole Period

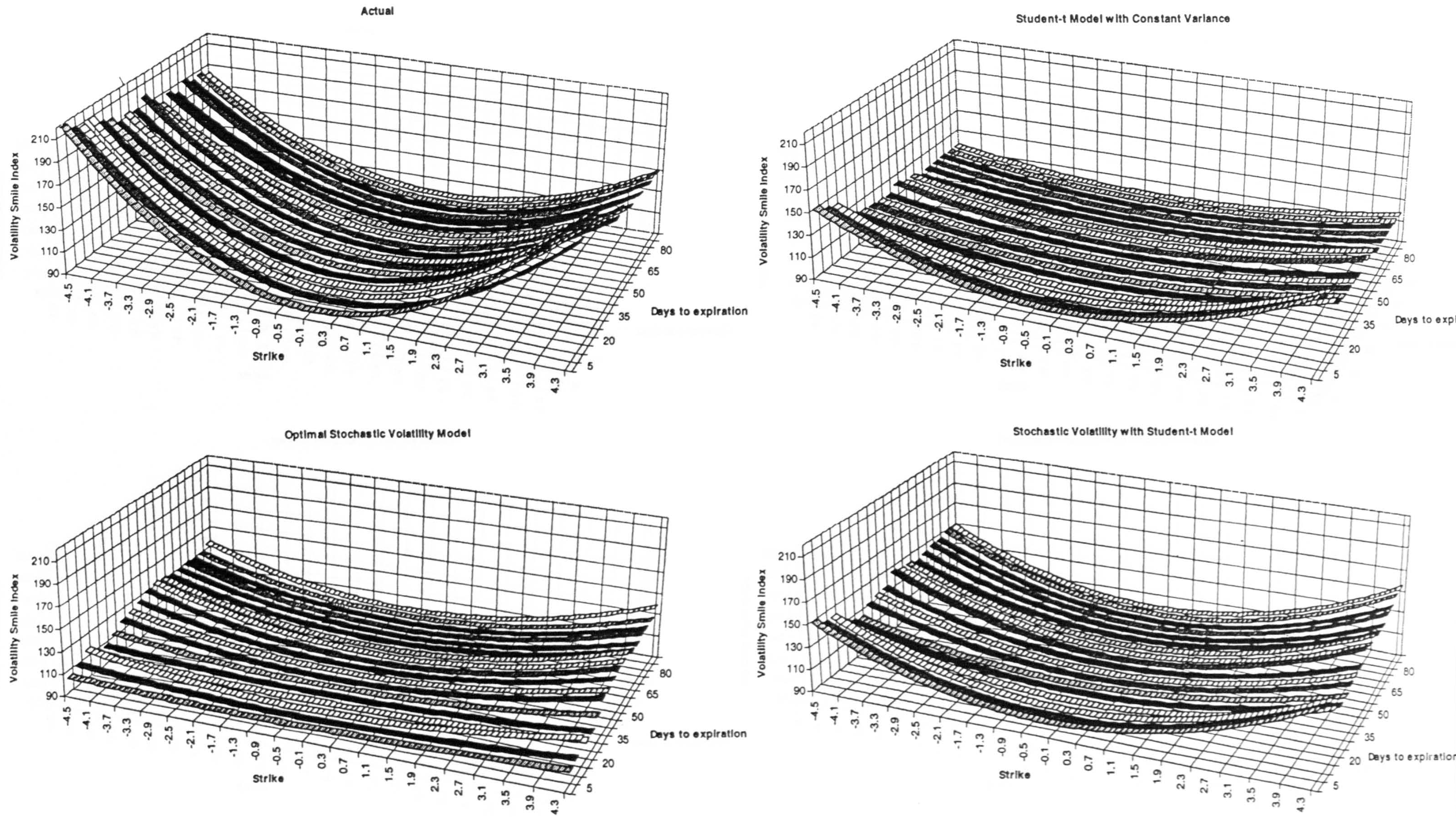

Figure 9.3a Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

Gilt
Whole Period

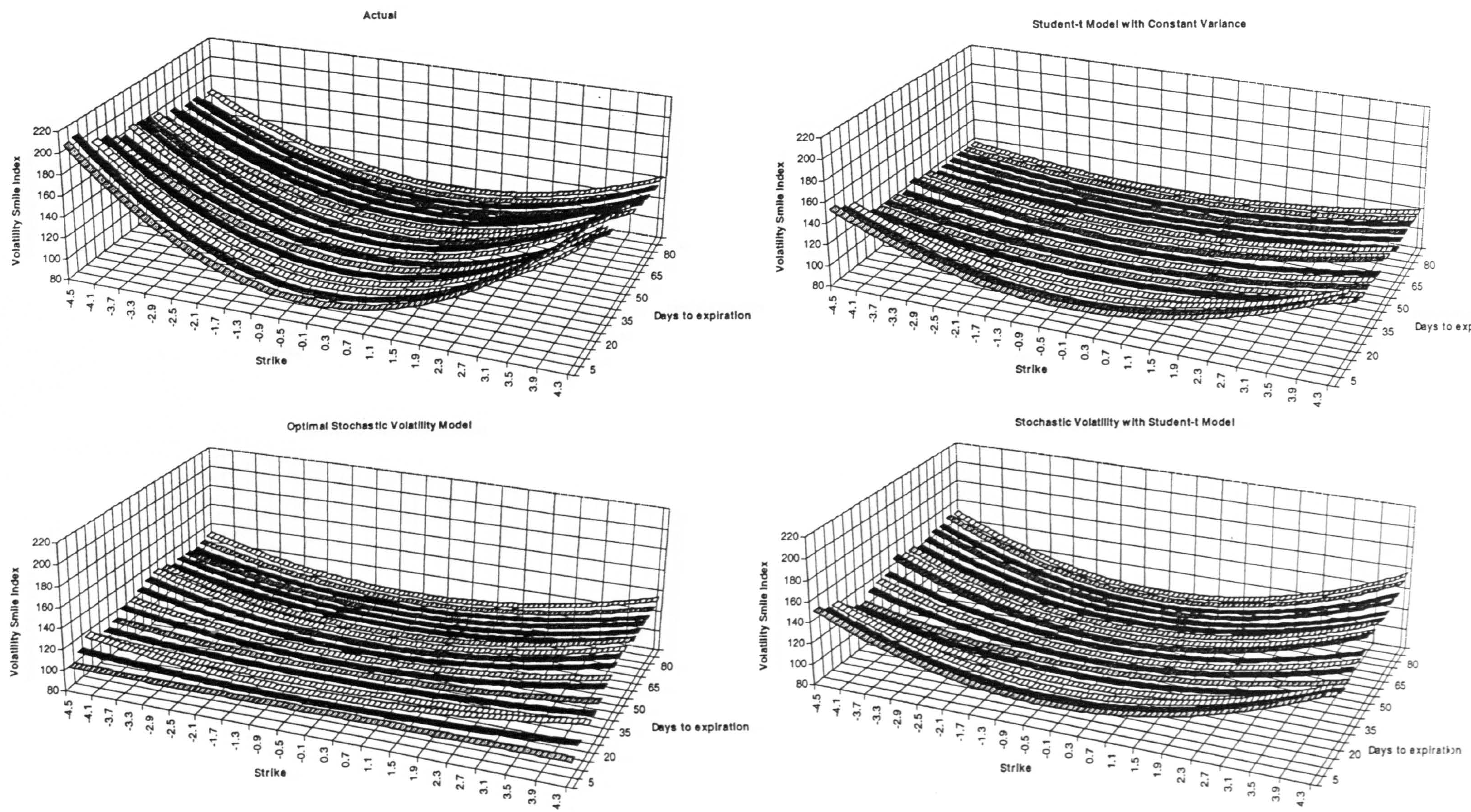

Figure 9.3b Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

BTP
Whole Period

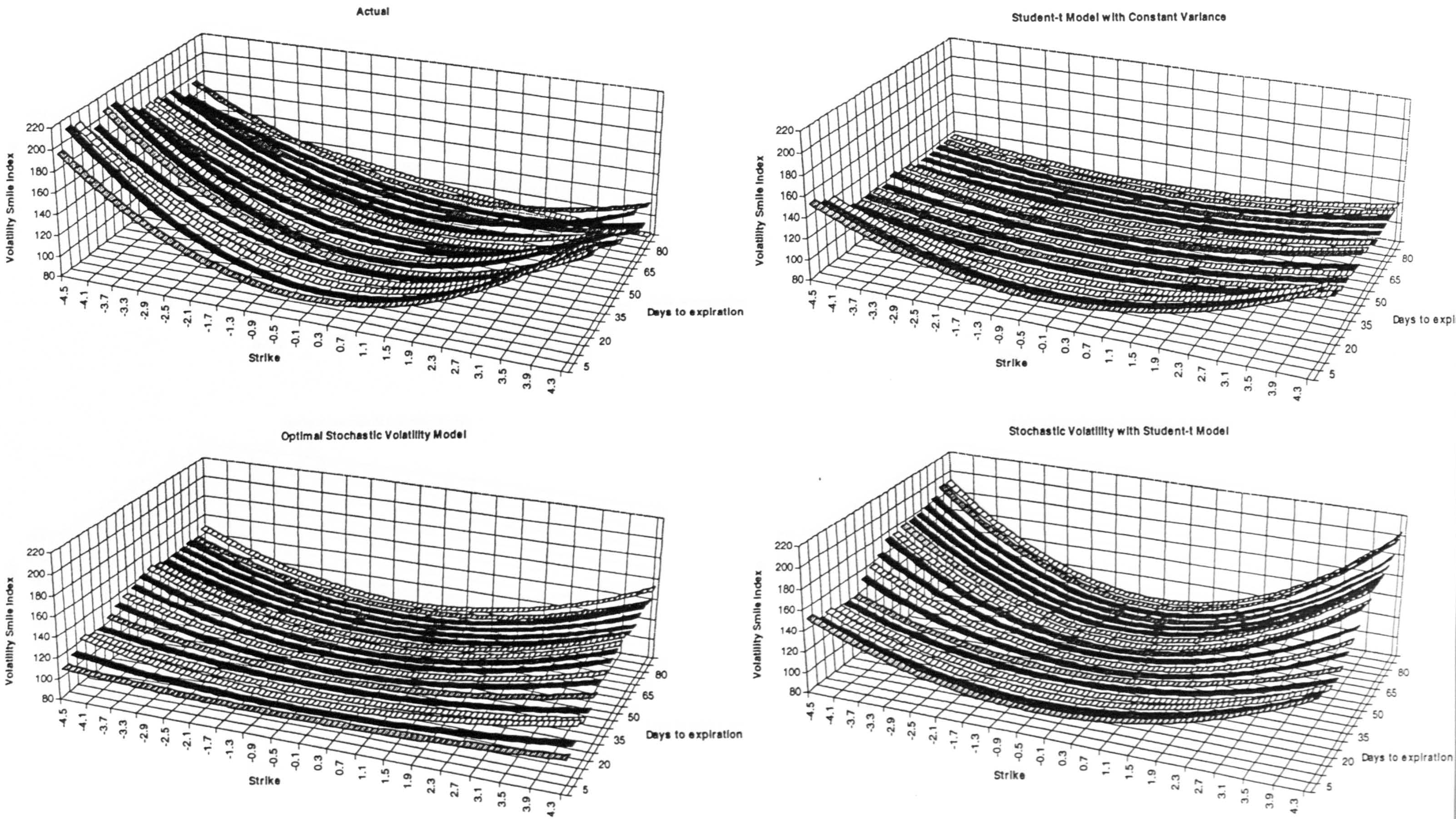

Figure 9.3c Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

US T-Bond
Whole Period

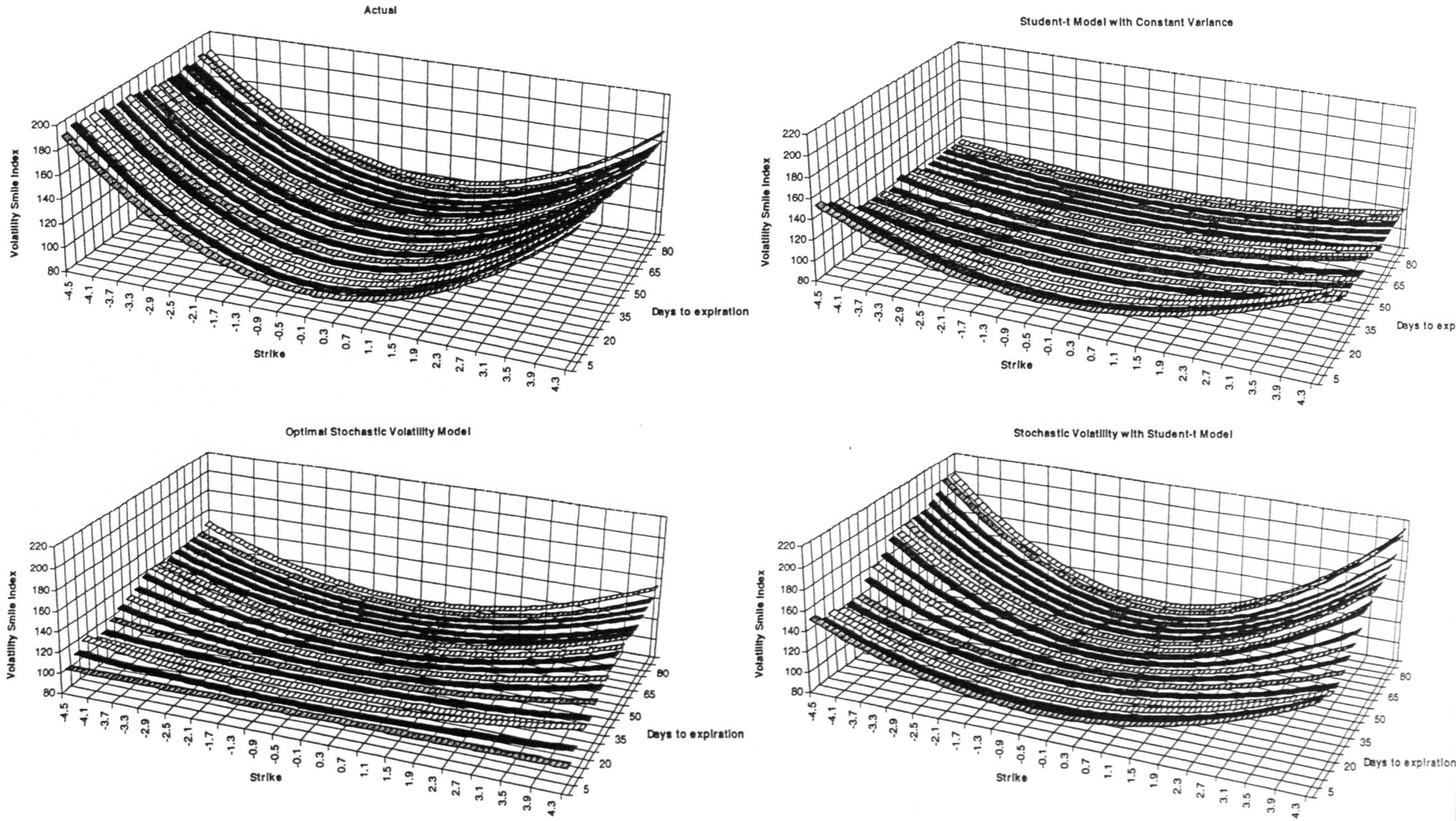

Figure 9.3d Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

D-Mark
Whole Period

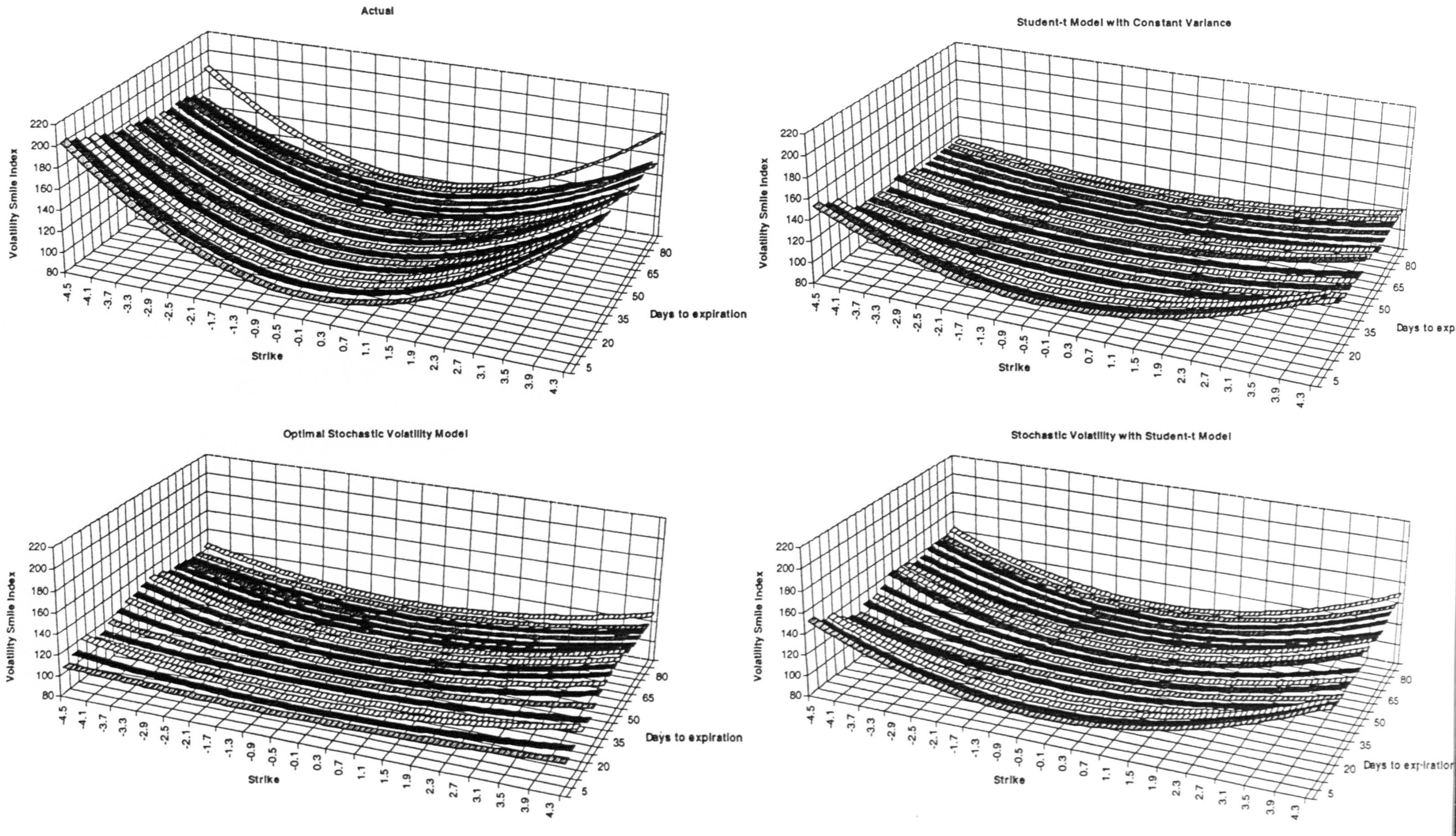

Figure 9.4a Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

J-Yen
Whole Period

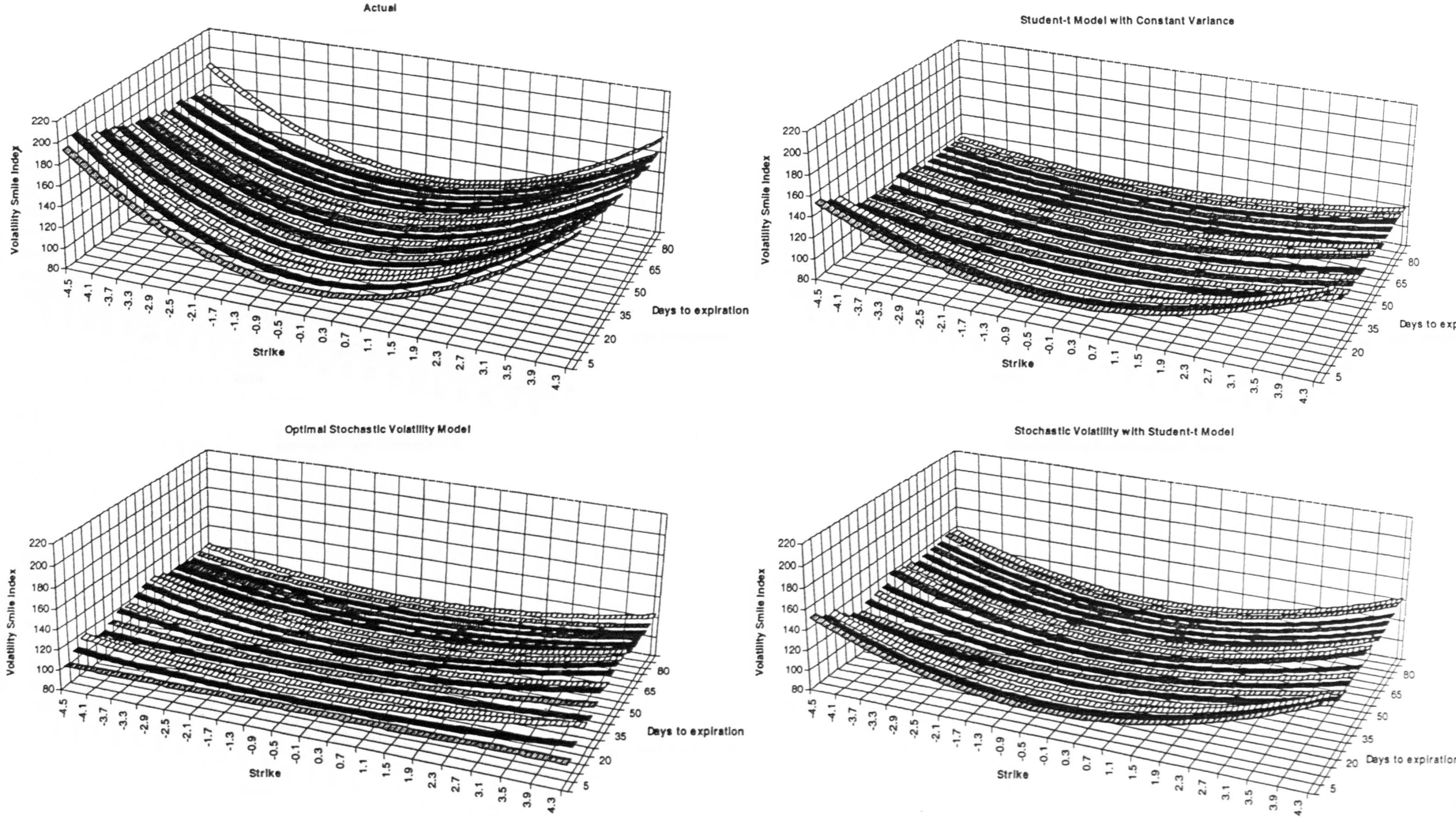

Figure 9.4b Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

B-Pound
Whole Period

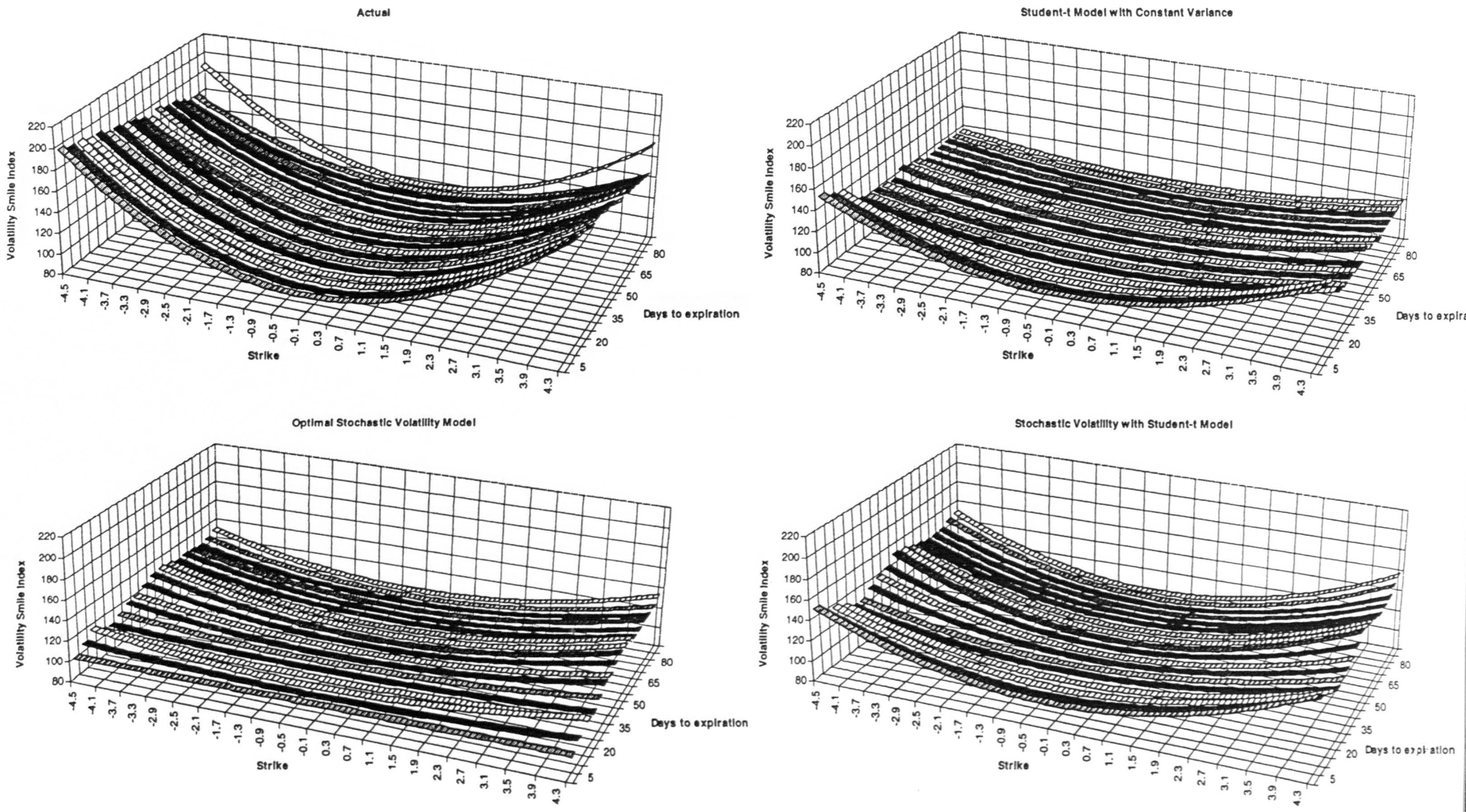

Figure 9.4c Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

S-Franc
Whole Period

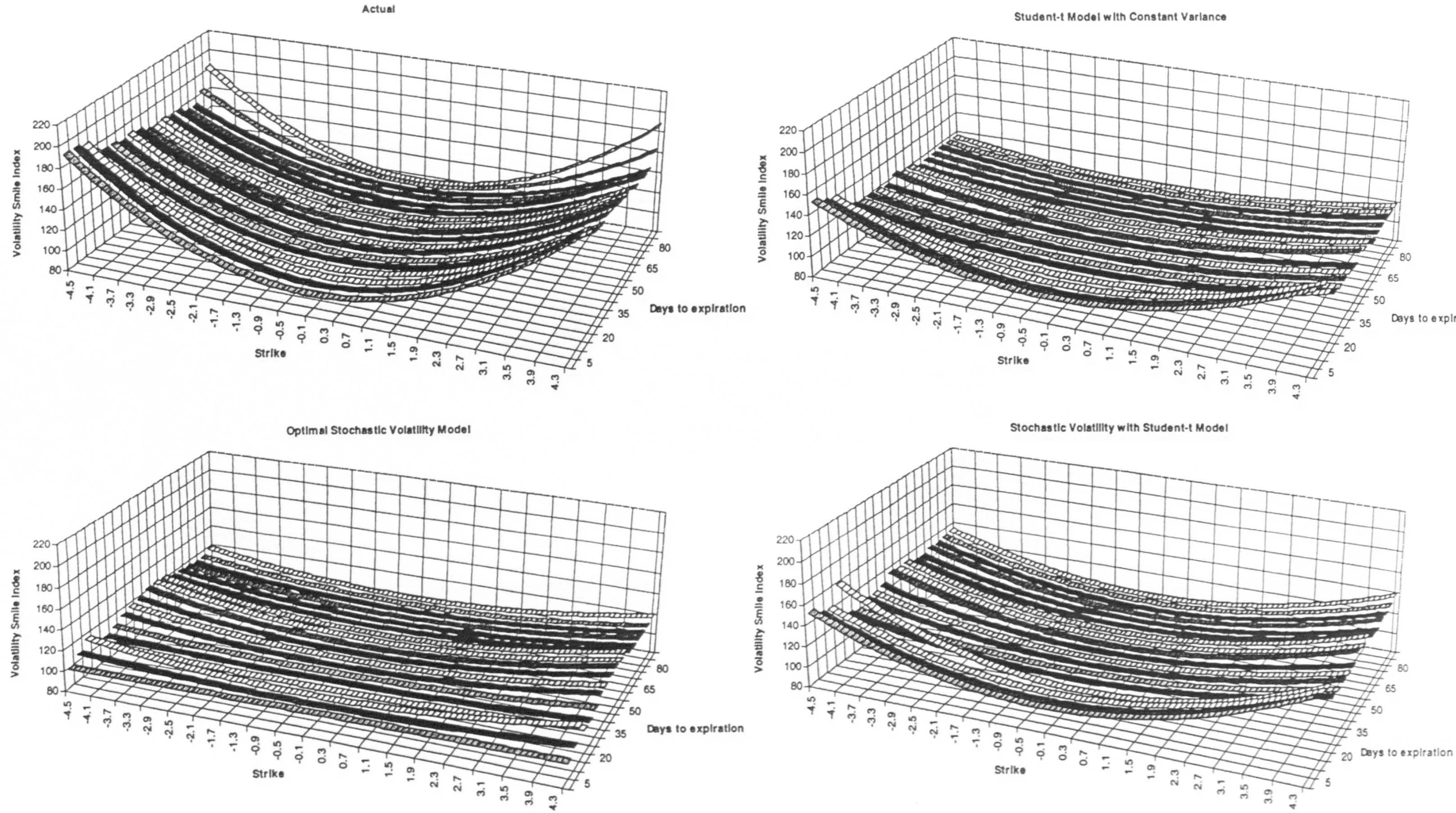

Figure 9.4d Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

S&P-500
First Period

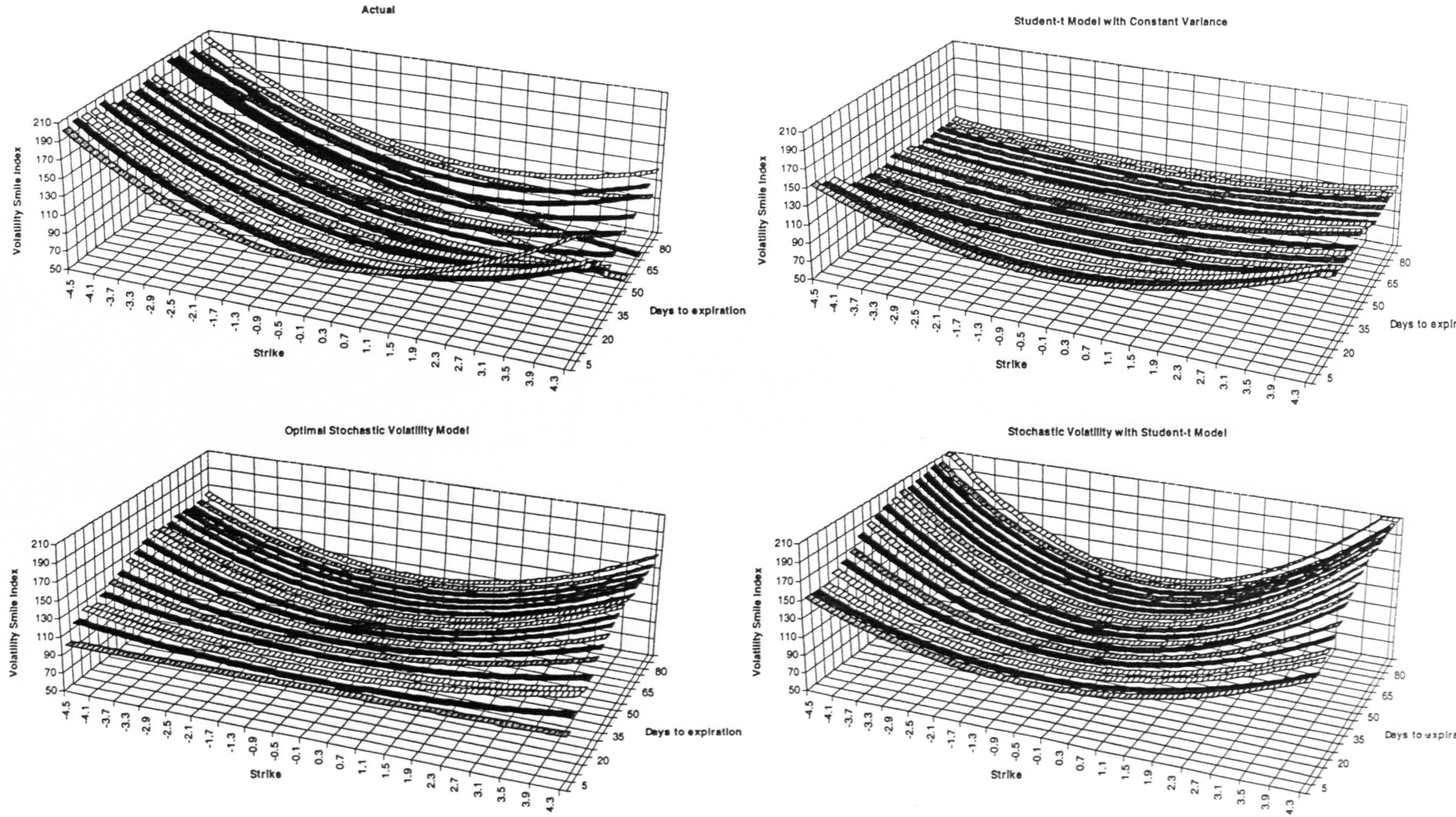

Figure 9.5a Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

FTSE-100
First Period

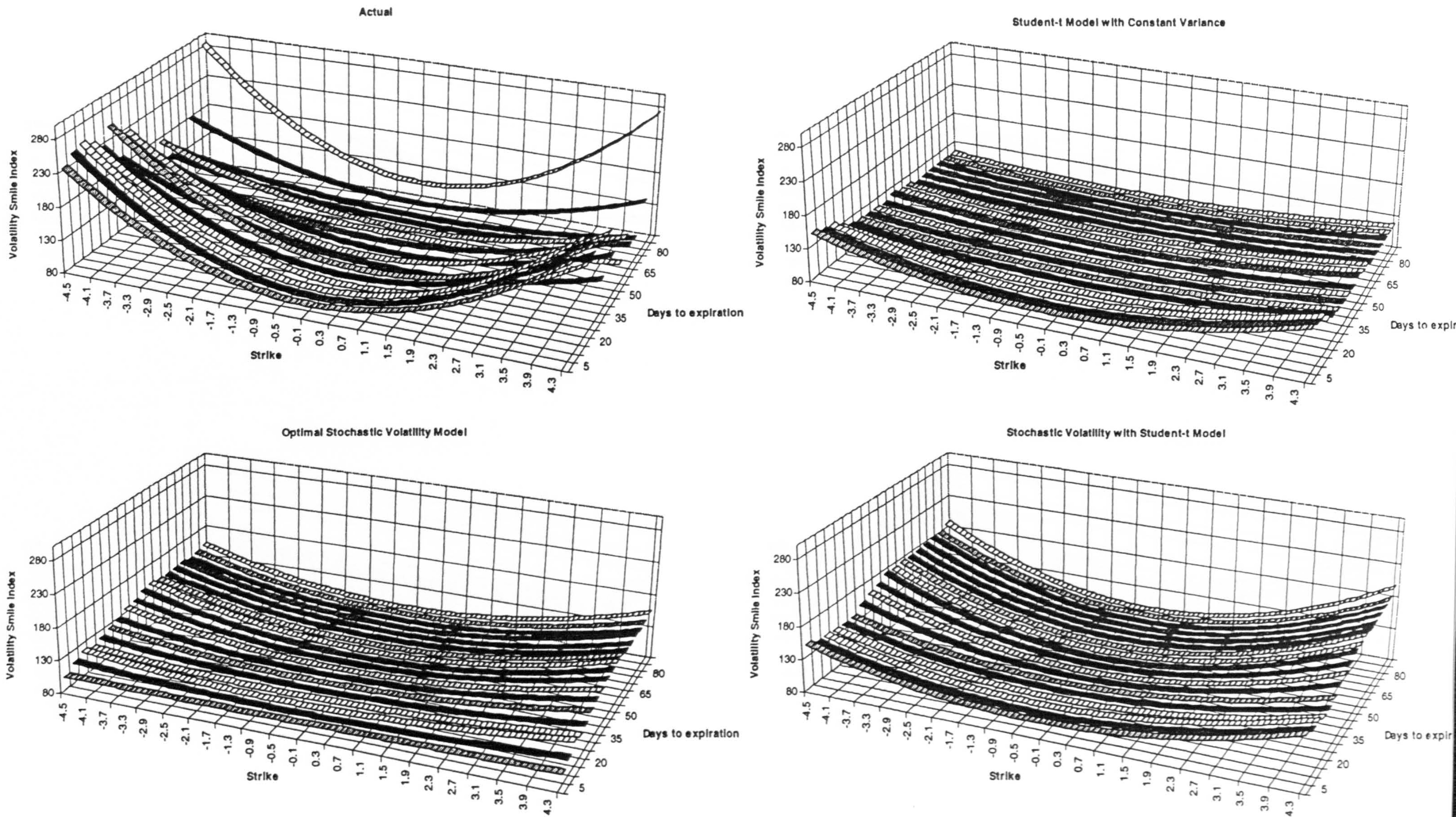

Figure 9.5b Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

Nikkei-225
First Period

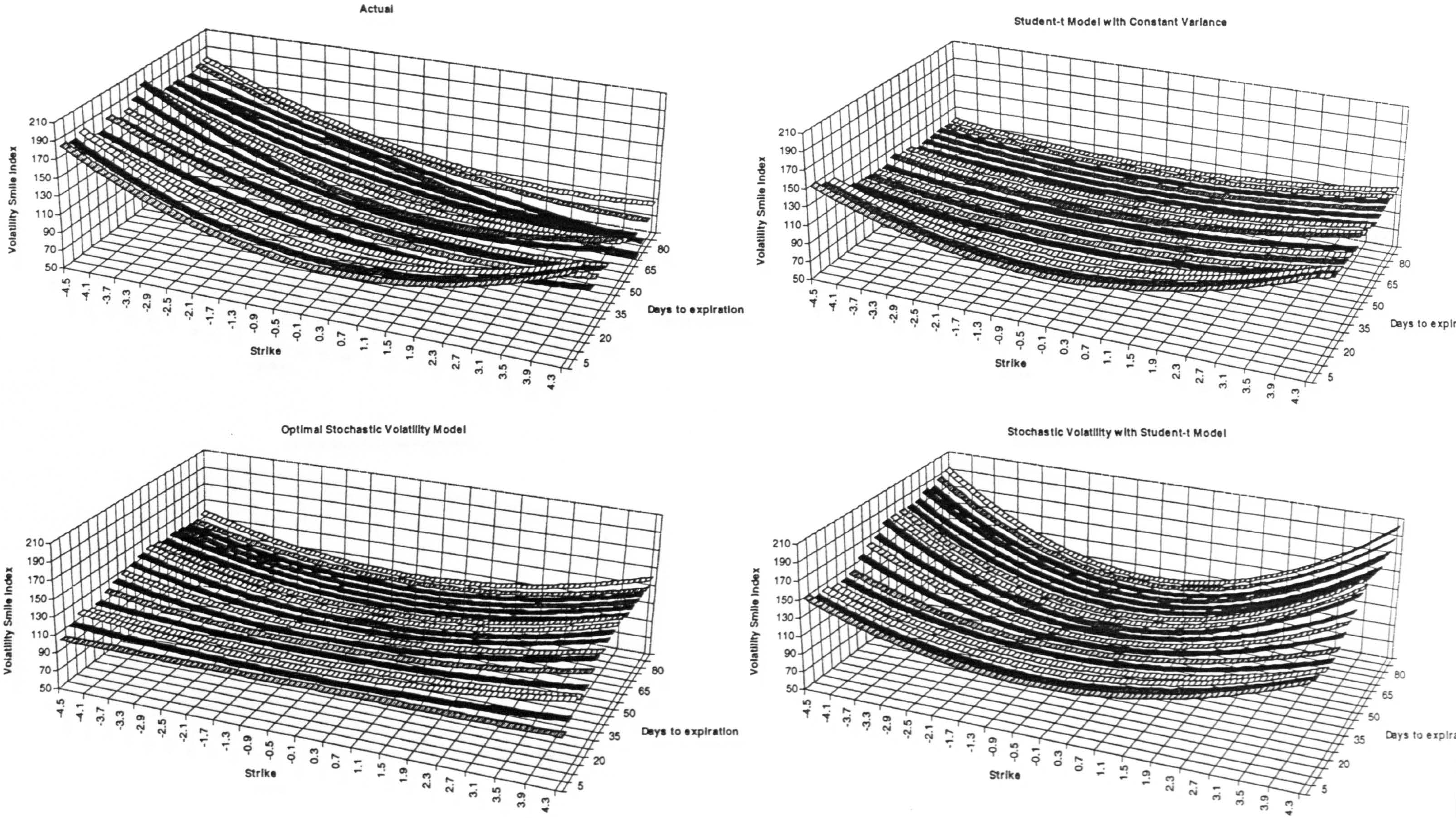

Figure 9.5c Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

DAX
First Period

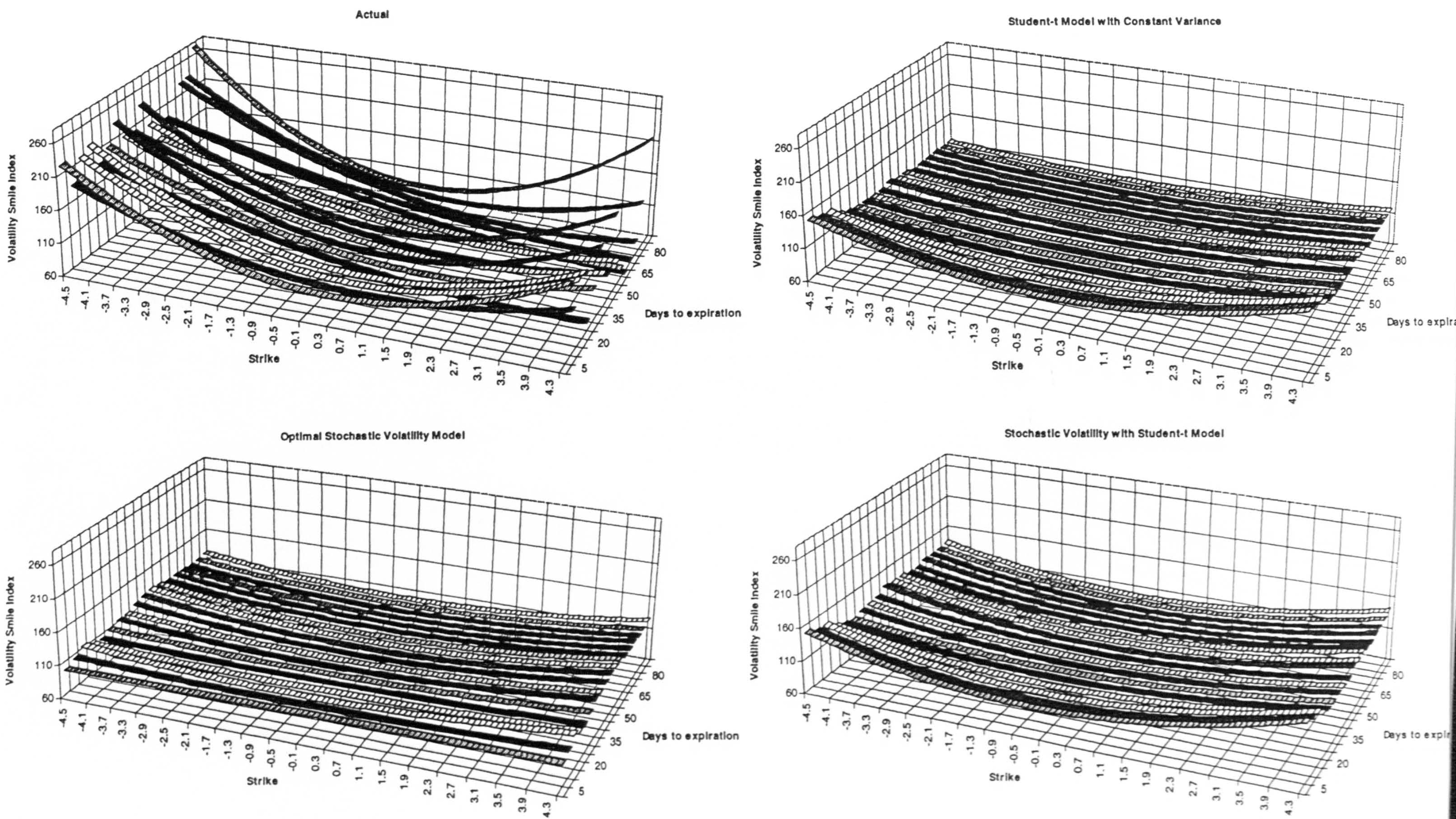

Figure 9.5d Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

Bund
First Period

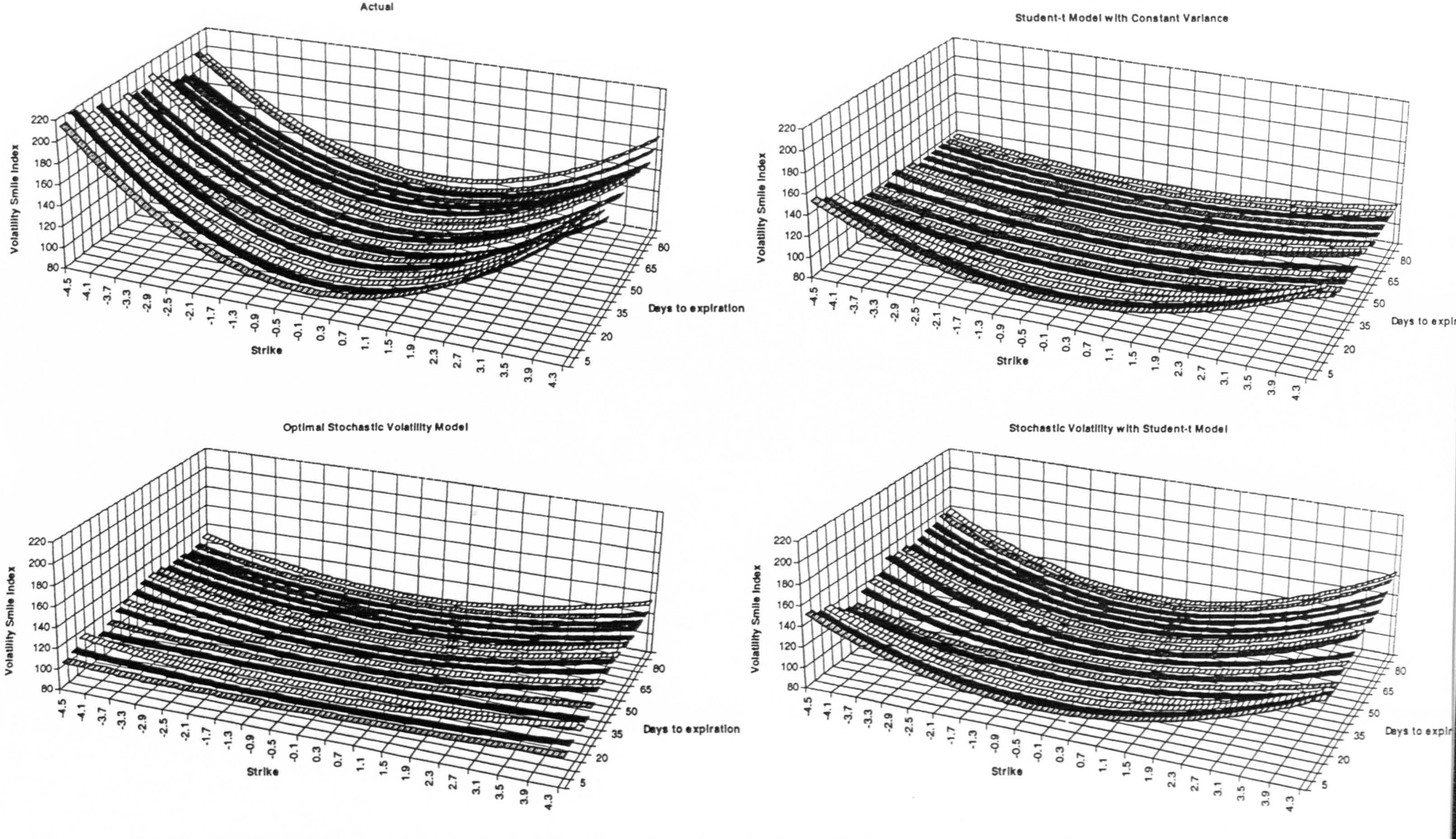

Figure 9.6a Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

Gilt
First Period

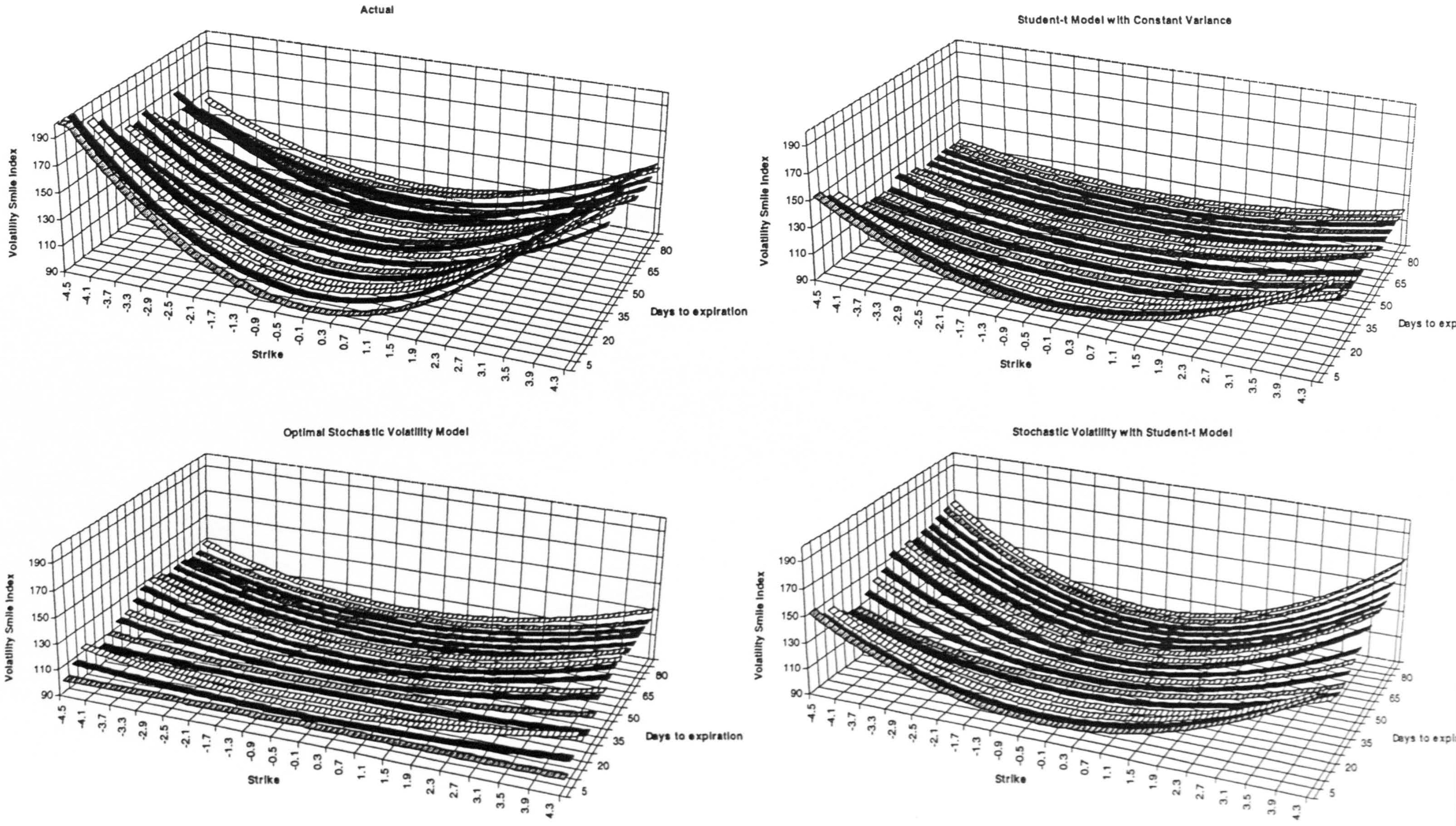

Figure 9.6b Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

BTP
First Period

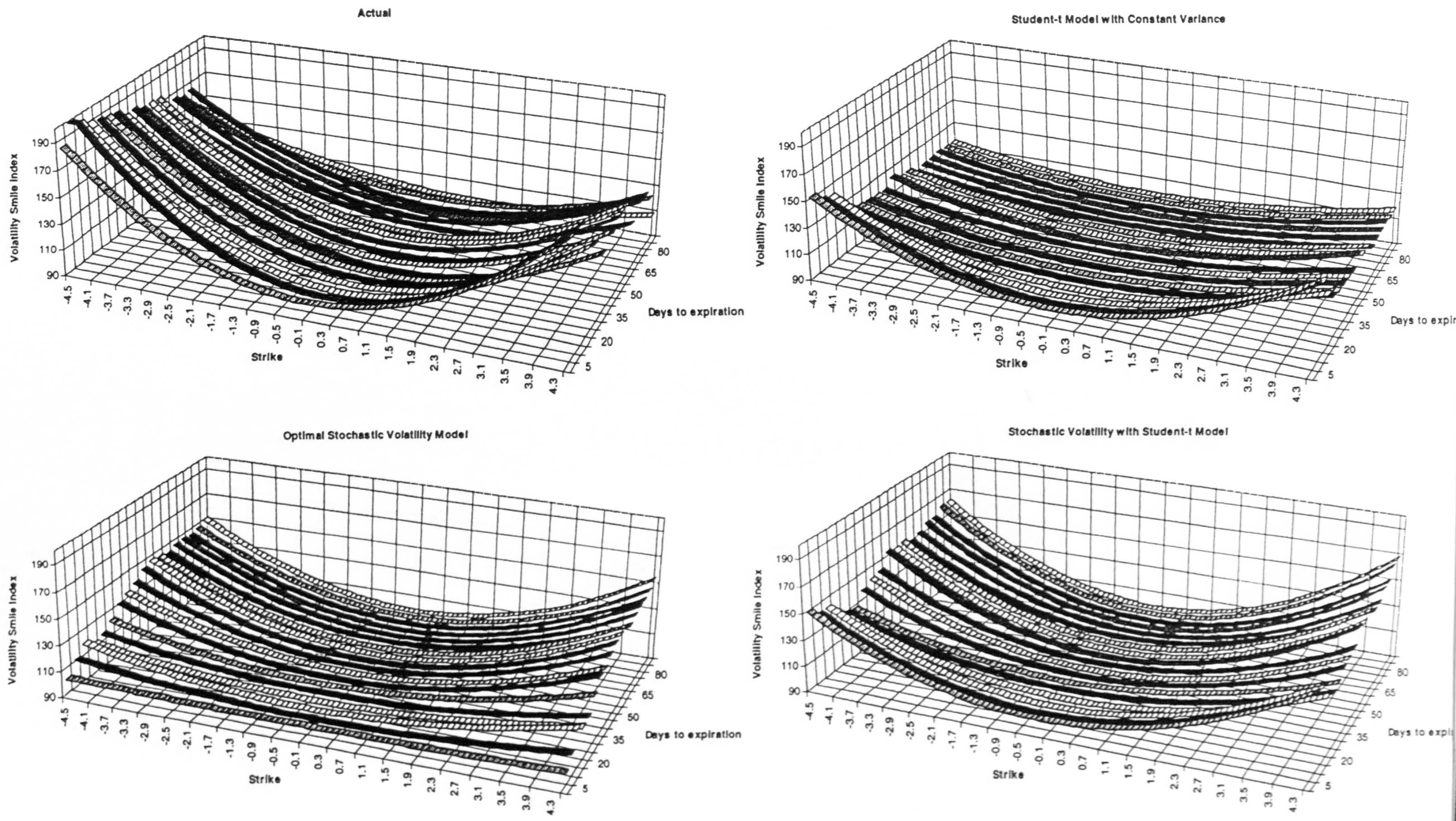

Figure 9.6c Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

US T-Bond
First Period

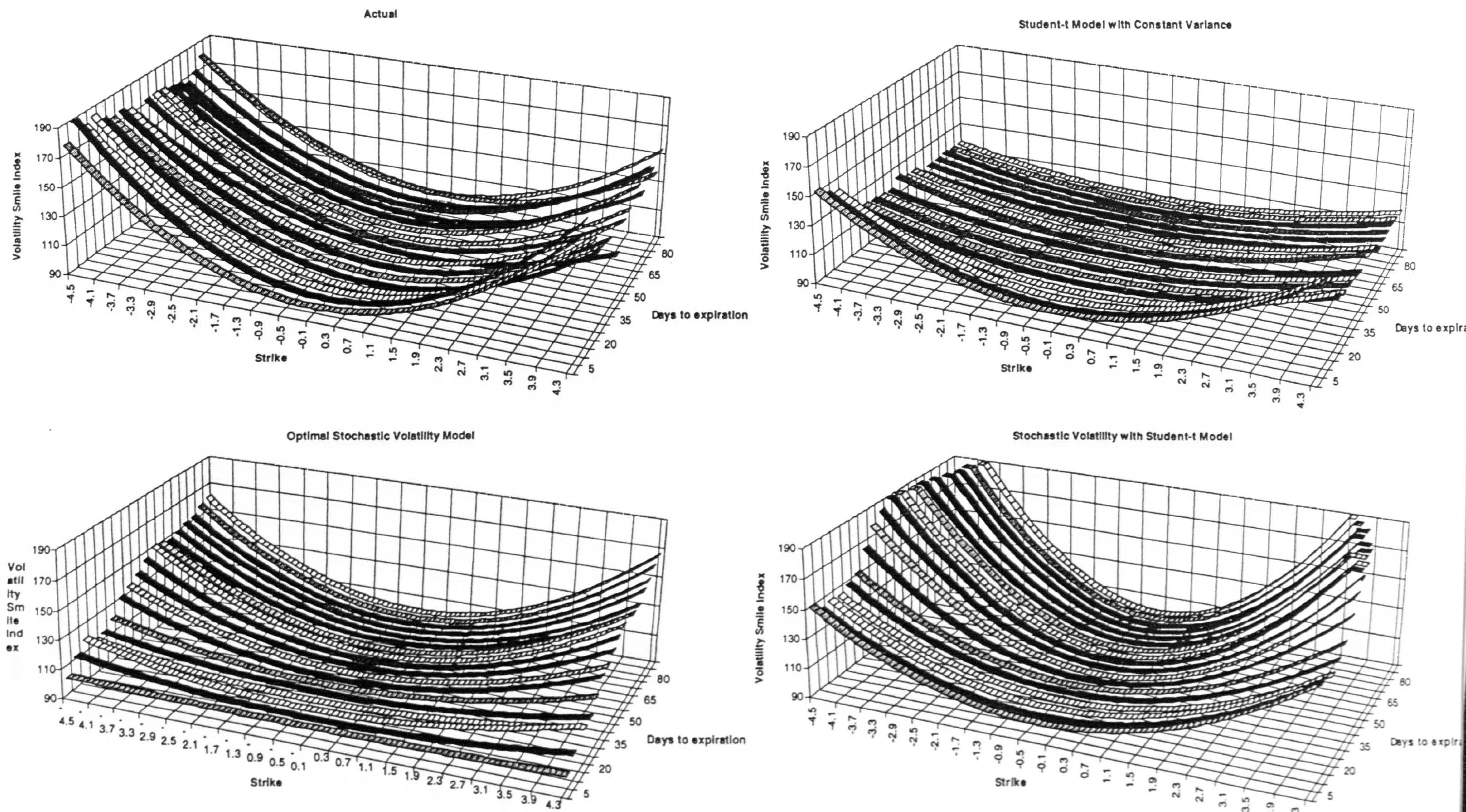

Figure 9.6d Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

D-Mark
First Period

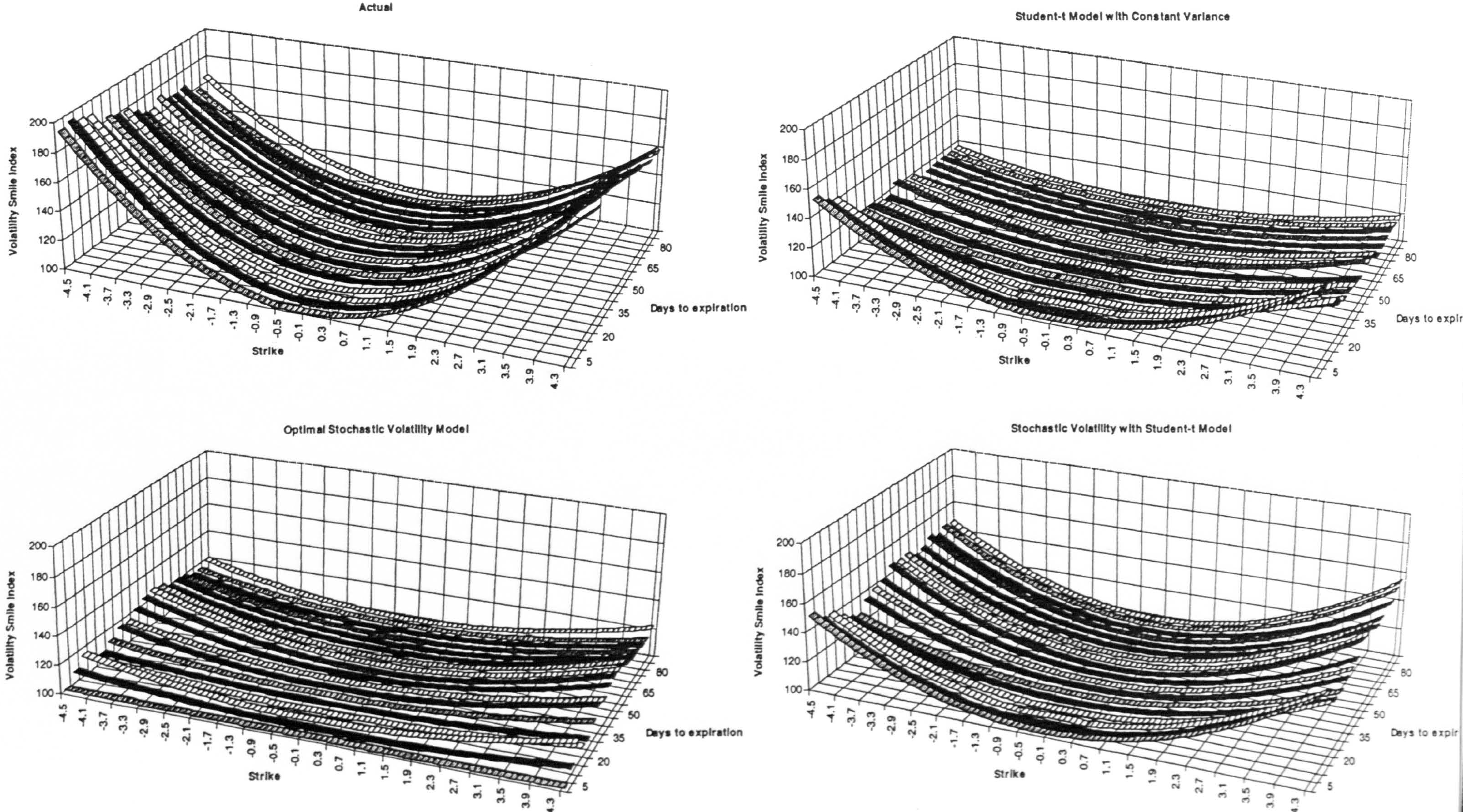

Figure 9.7a Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

J-Yen
First Period

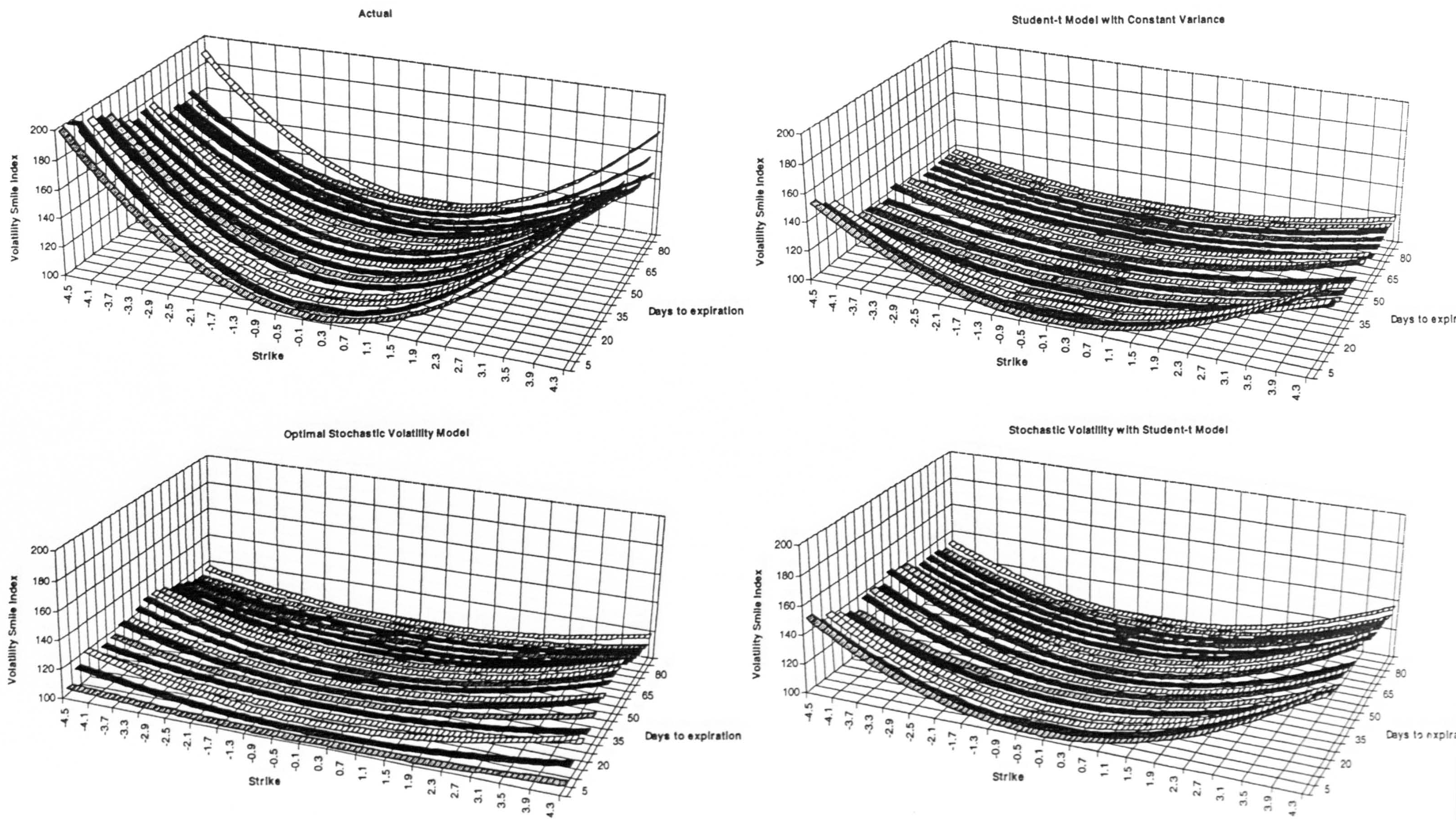

Figure 9.7b Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

B-Pound
First Period

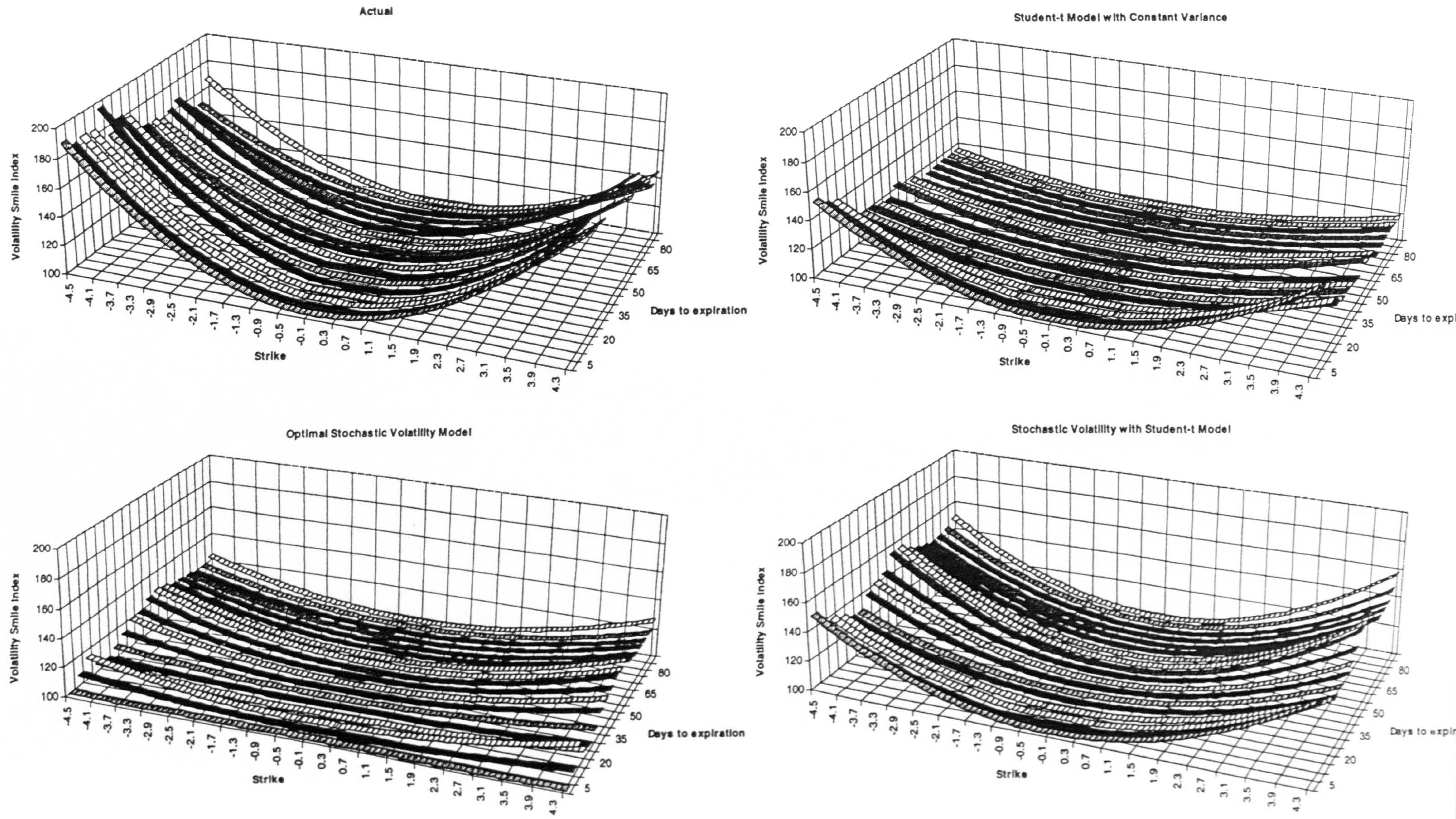

Figure 9.7c Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

S-Franc
First Period

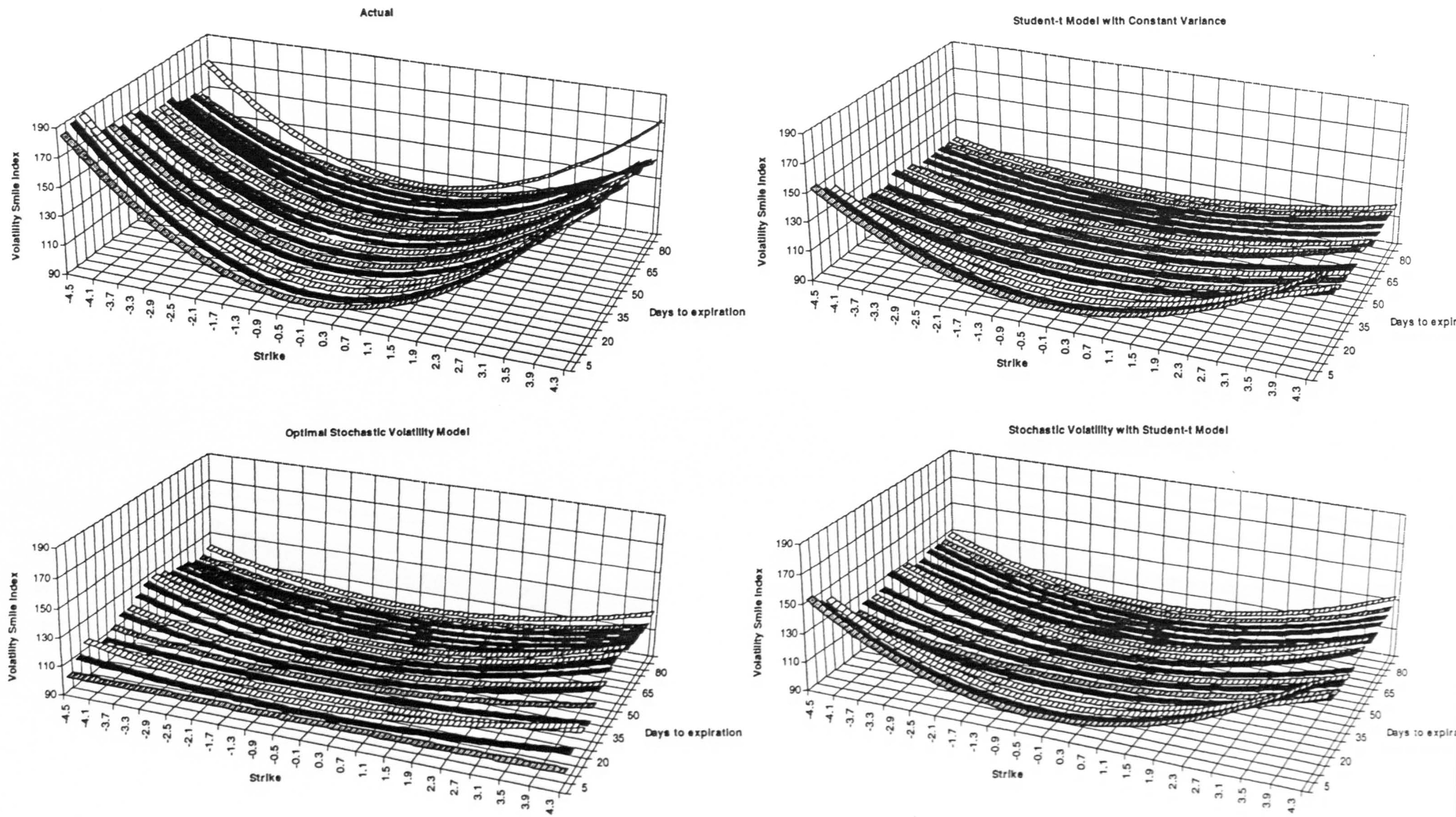

Figure 9.7d Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

S&P-500
Second Period

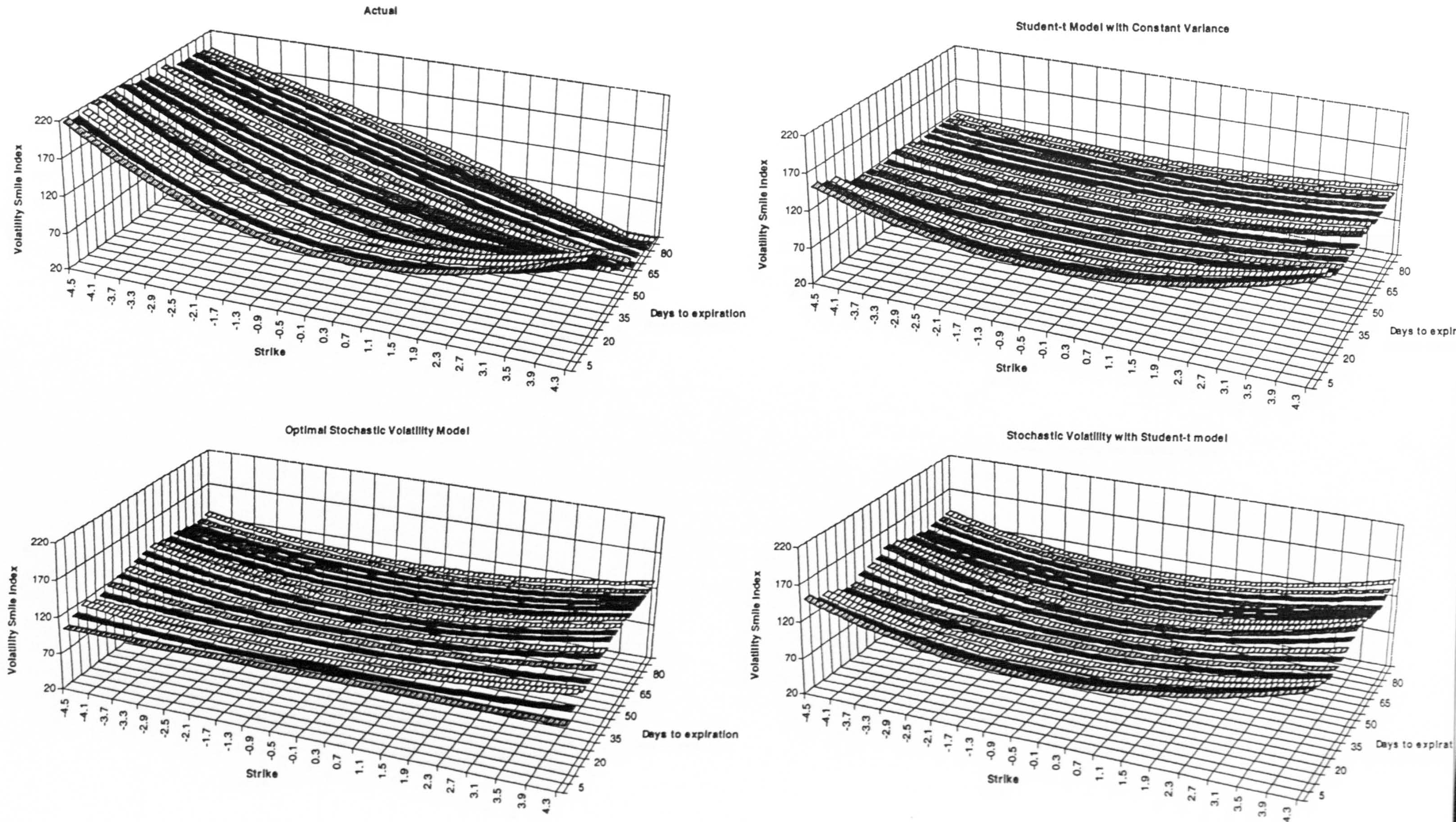

Figure 9.8a Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

FTSE-100
Second Period

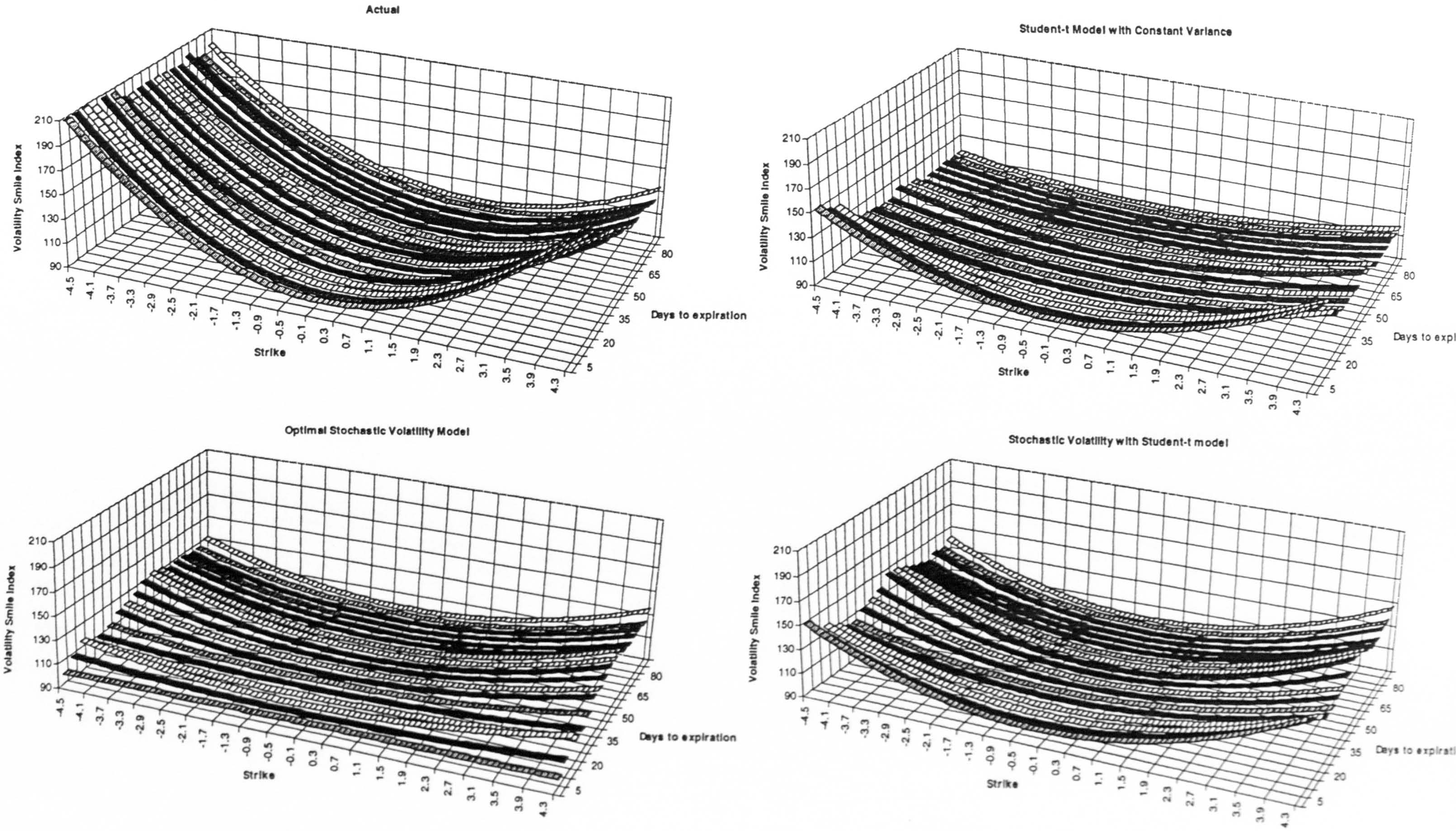

Figure 9.8b Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

Nikkei-225
Second Period

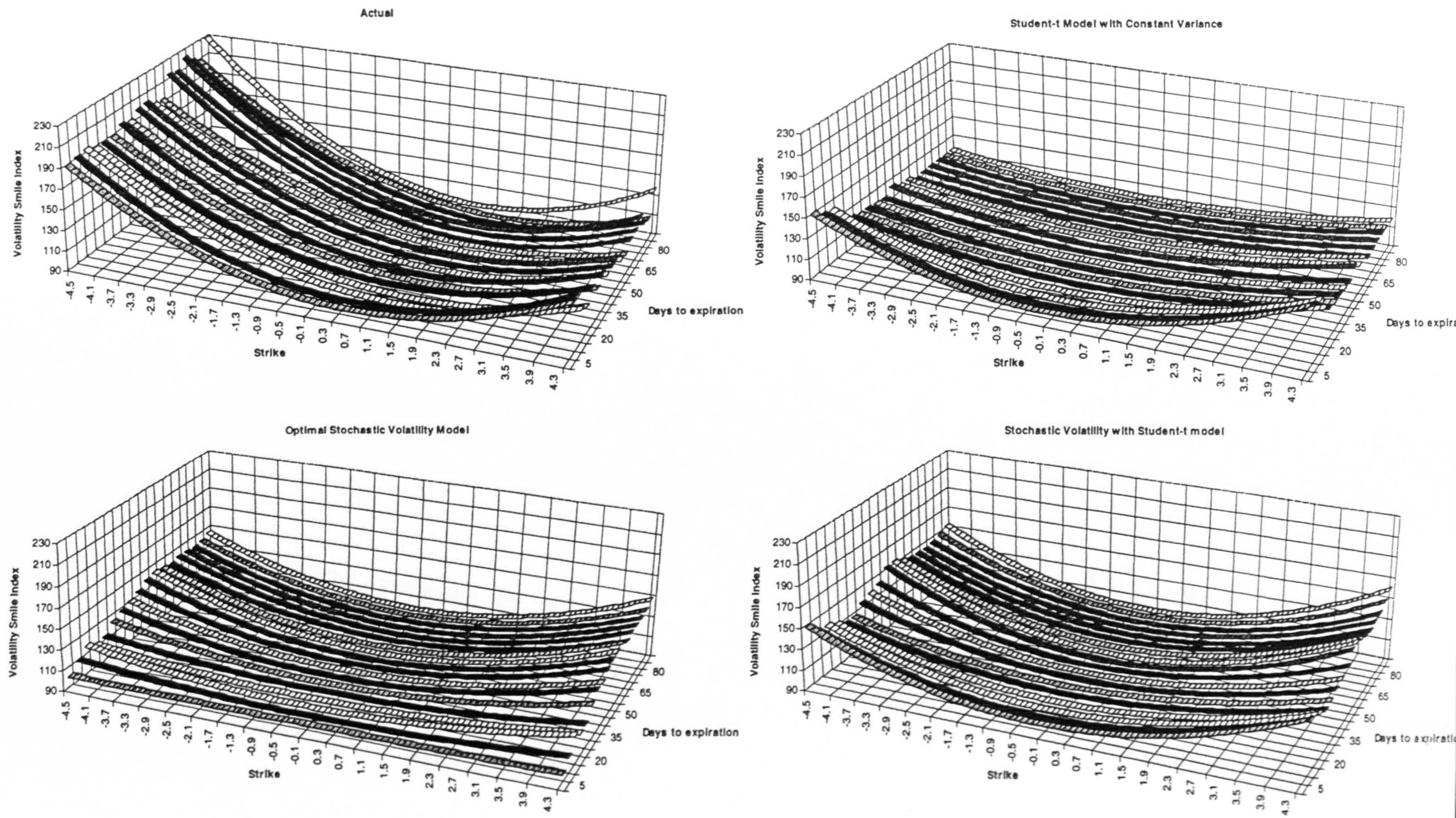

Figure 9.8c Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

DAX
Second Period

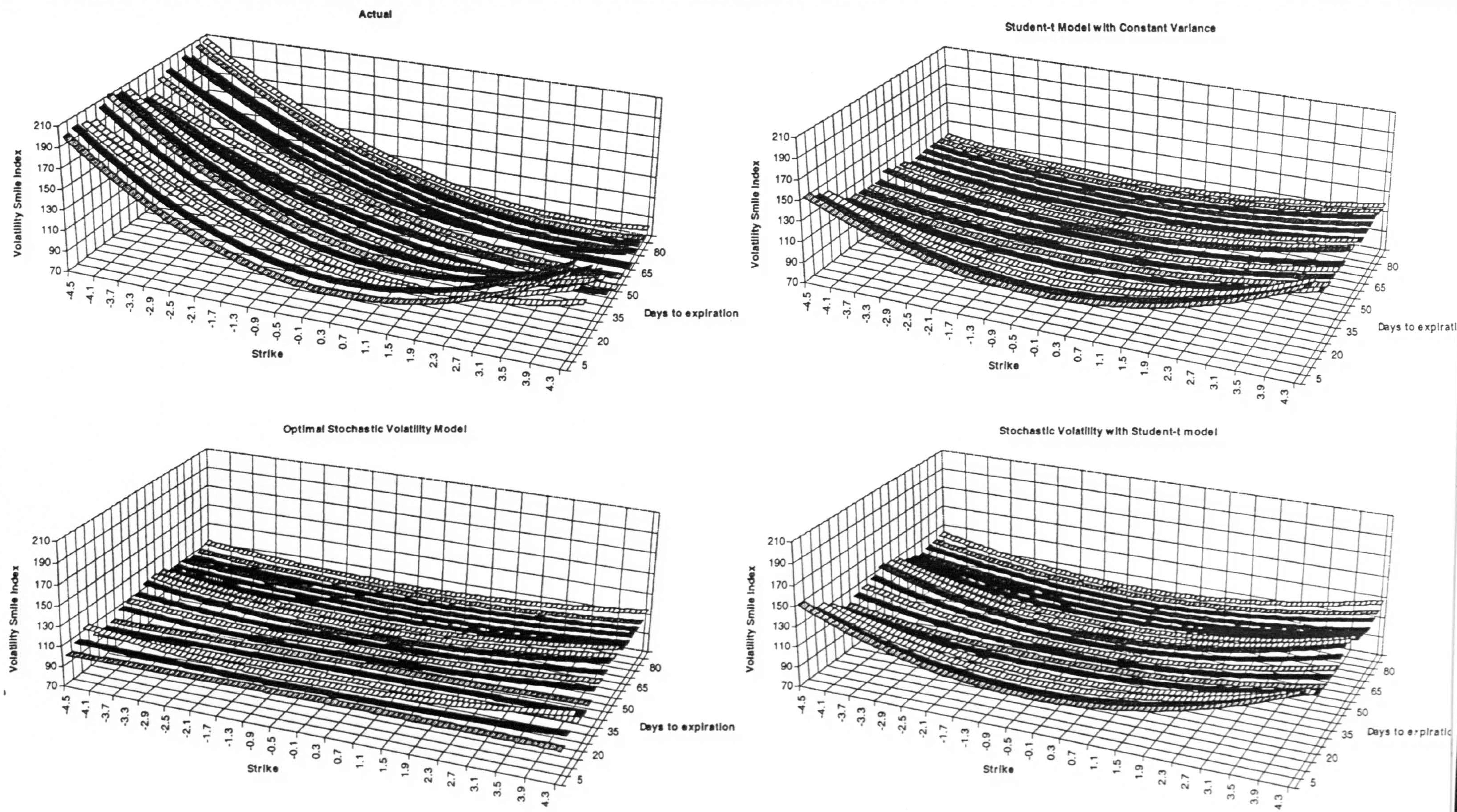

Figure 9.8d Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

Bund
Second Period

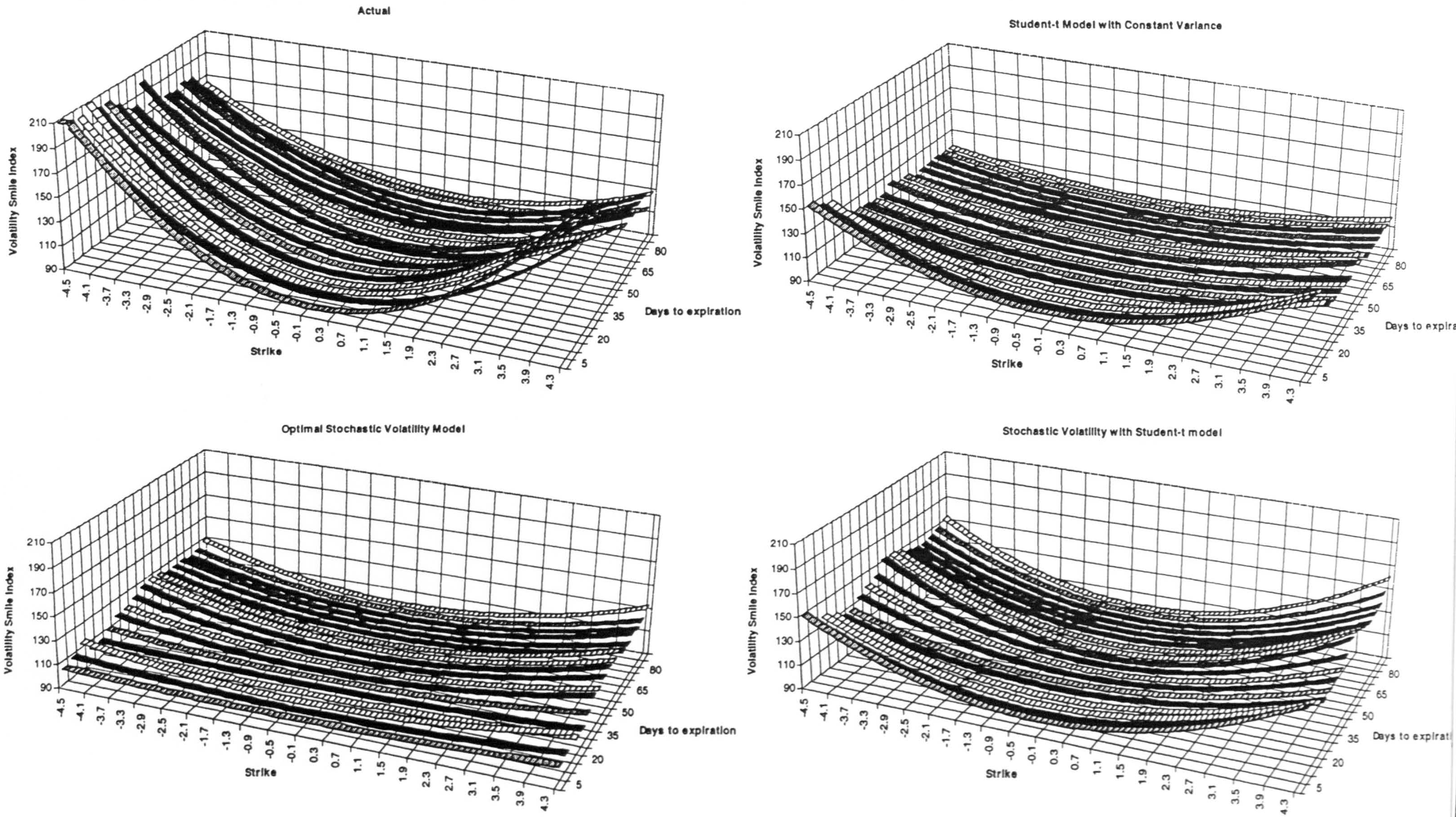

Figure 9.9a Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

Gilt
Second Period

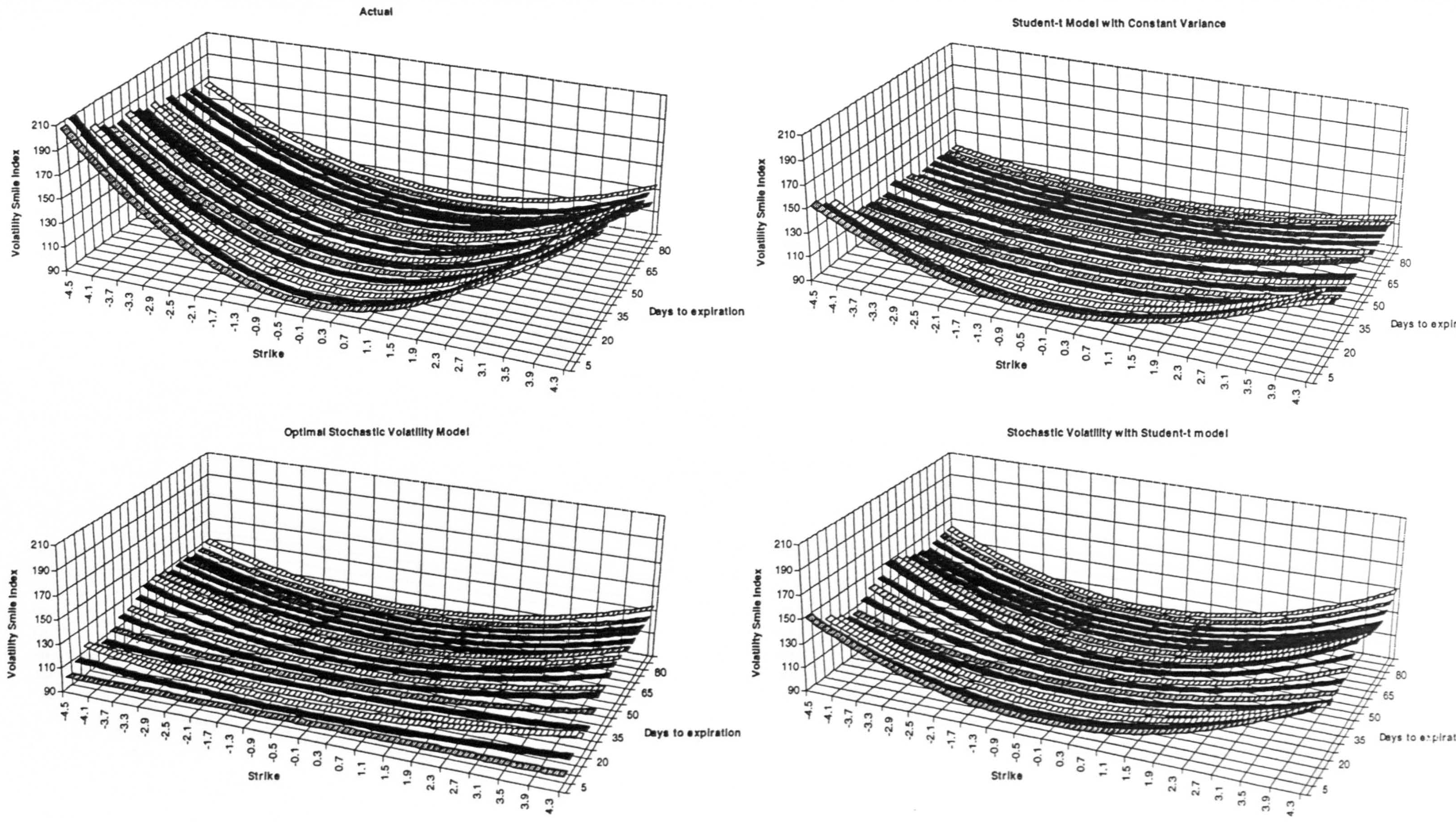

Figure 9.9b Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

BTP
Second Period

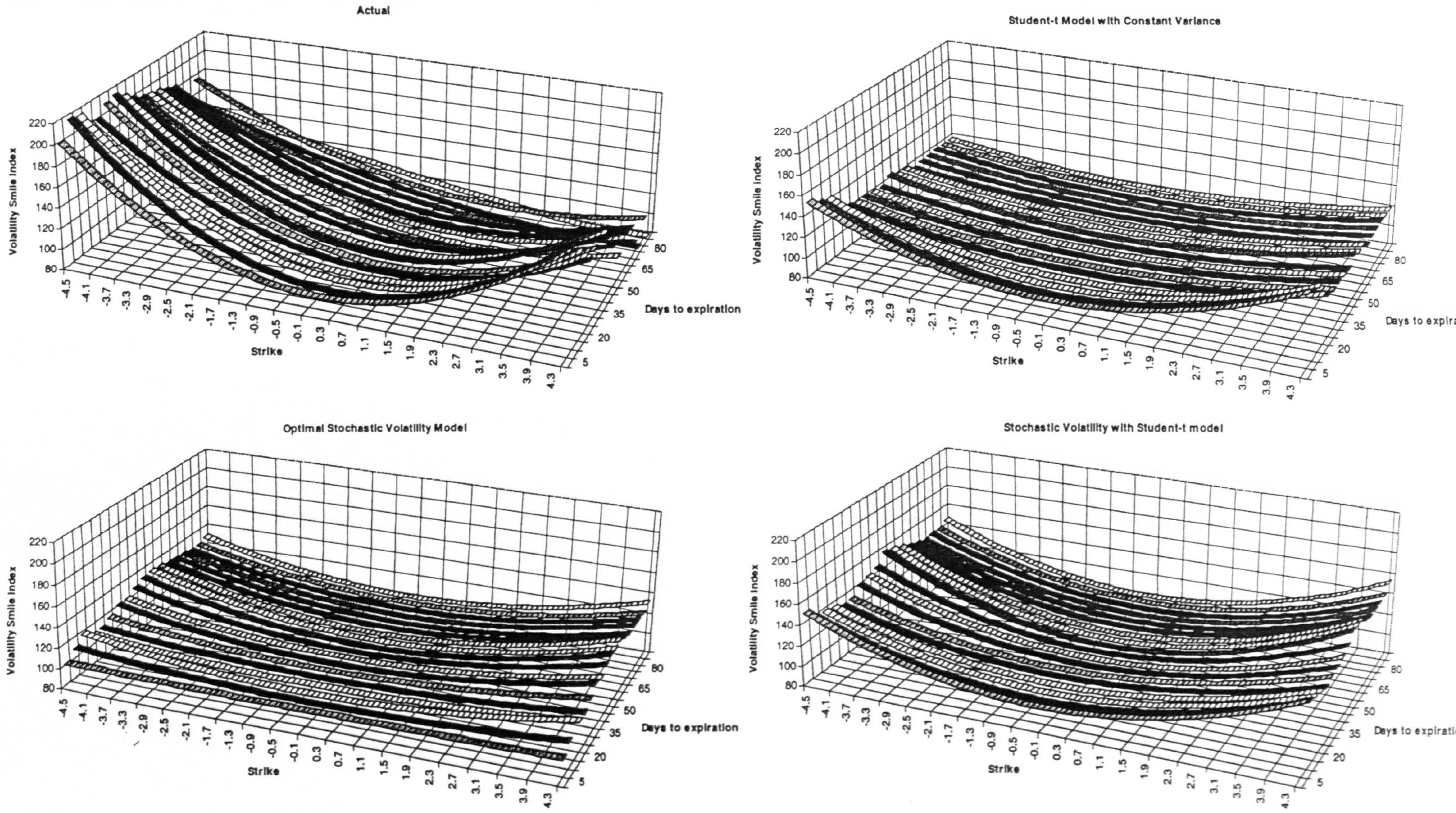

Figure 9.9c Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

US T-Bond
Second Period

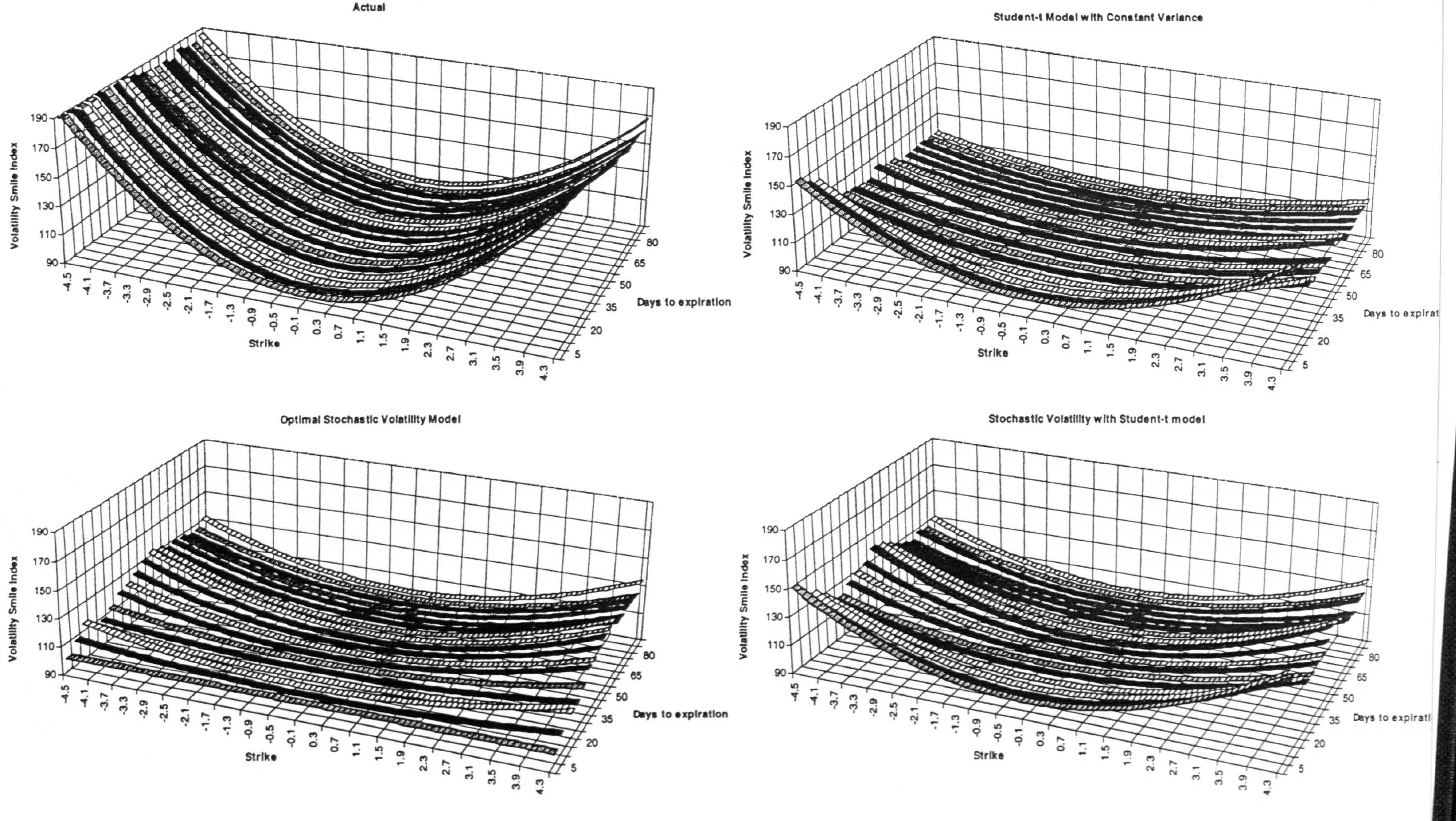

Figure 9.9d Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

D-Mark
Second Period

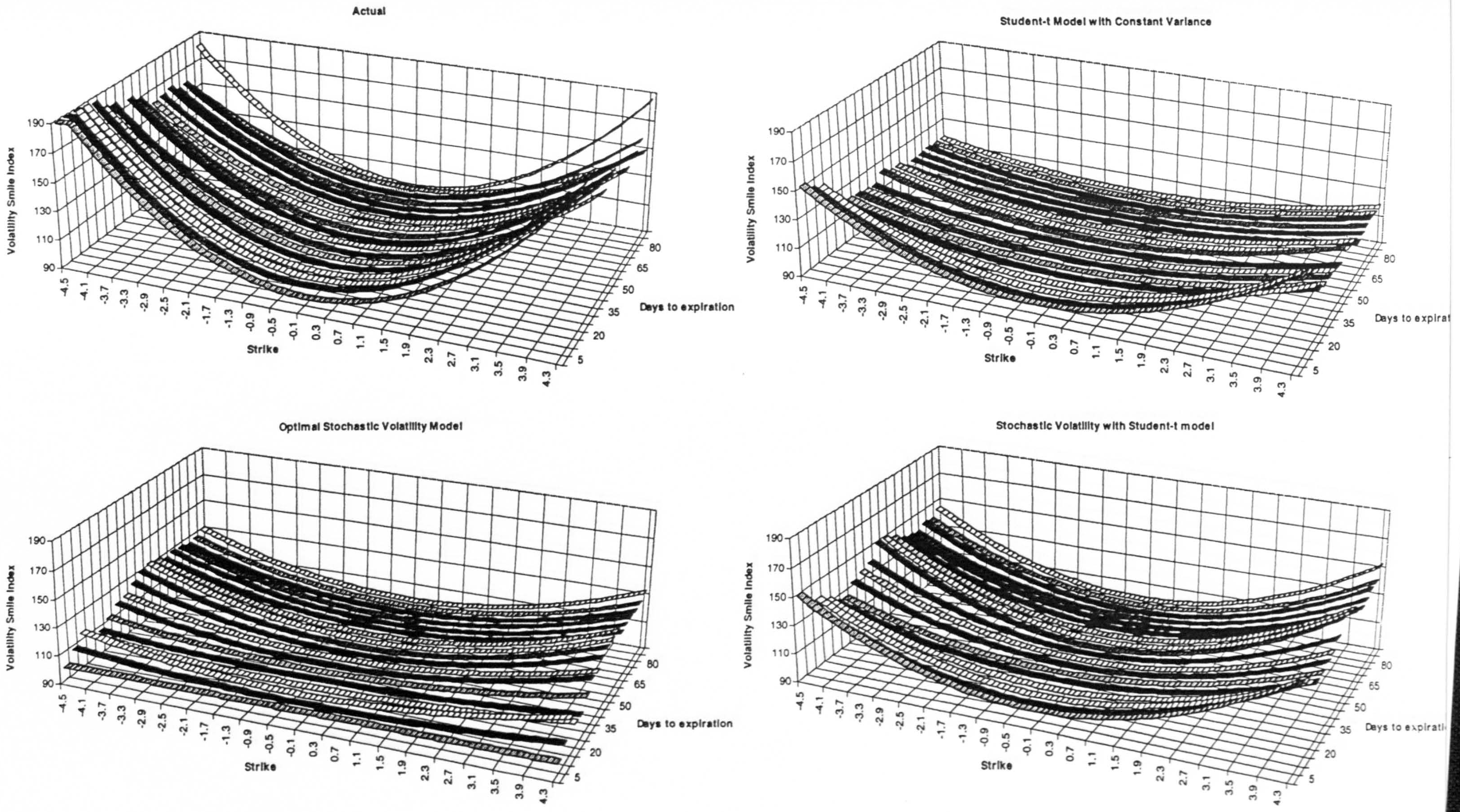

Figure 9.10a Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

J-Yen
Second Period

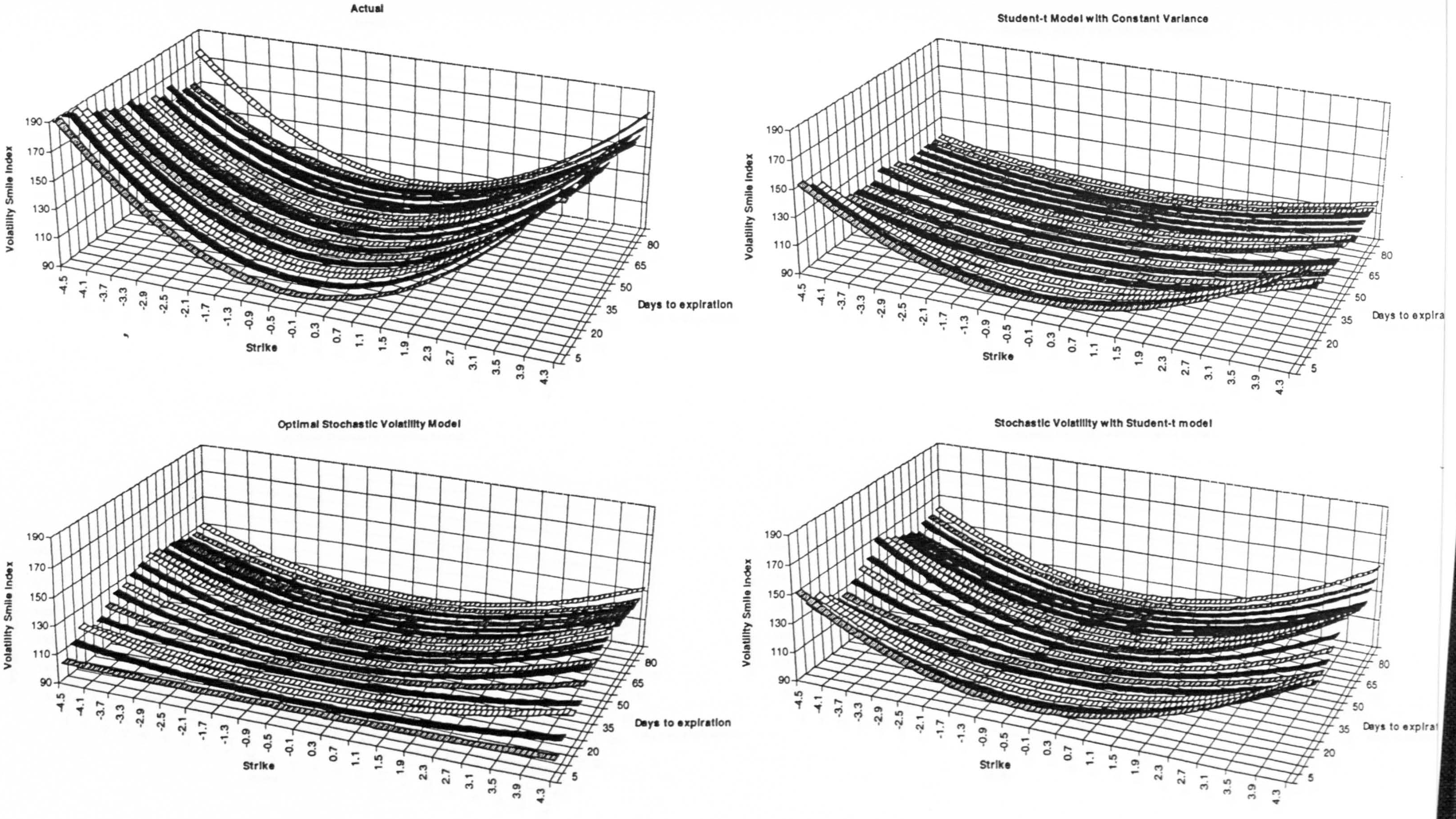

Figure 9.10b Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

B-Pound
Second Period

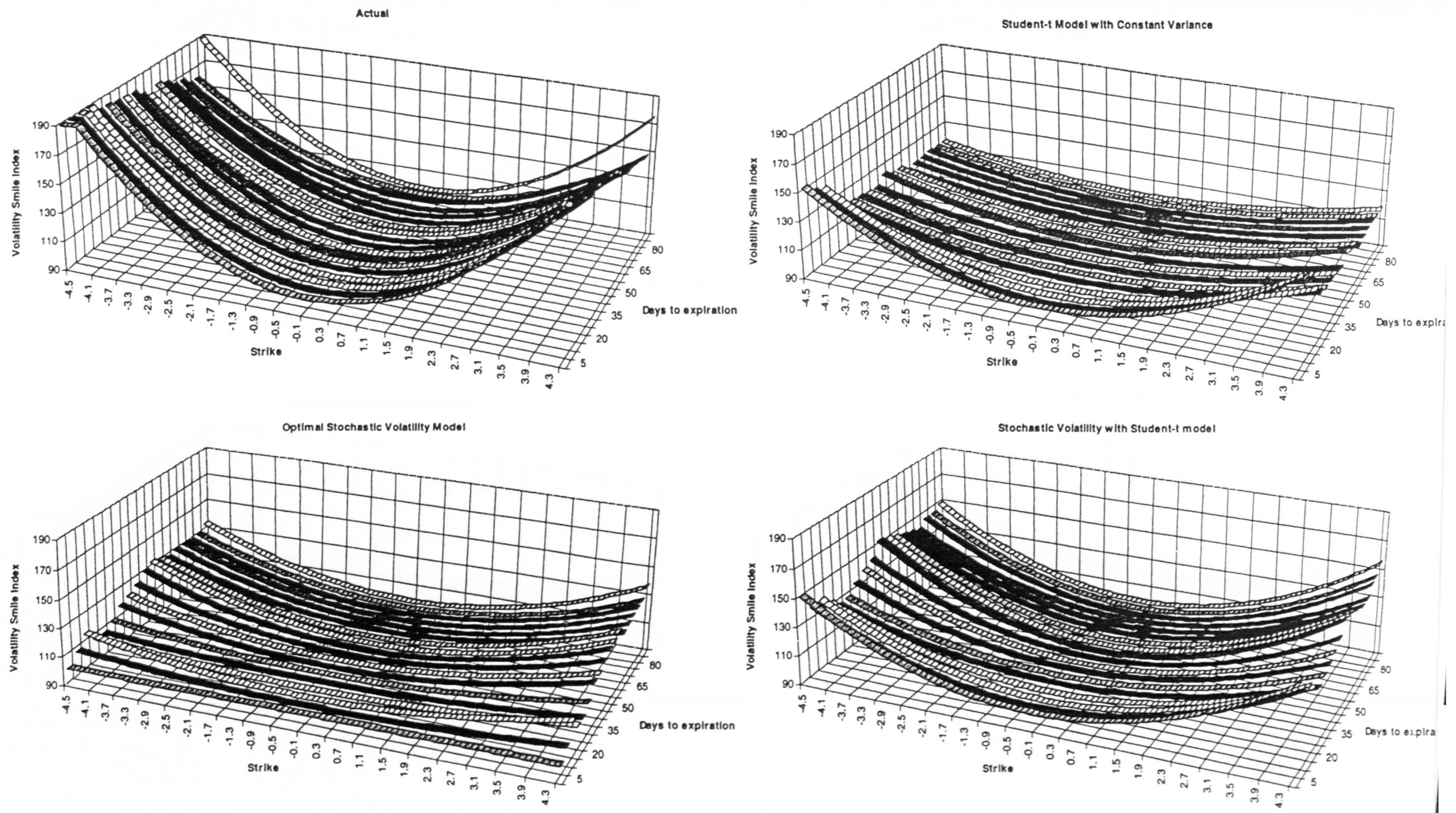

Figure 9.10c Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual

S-Franc
Second Period

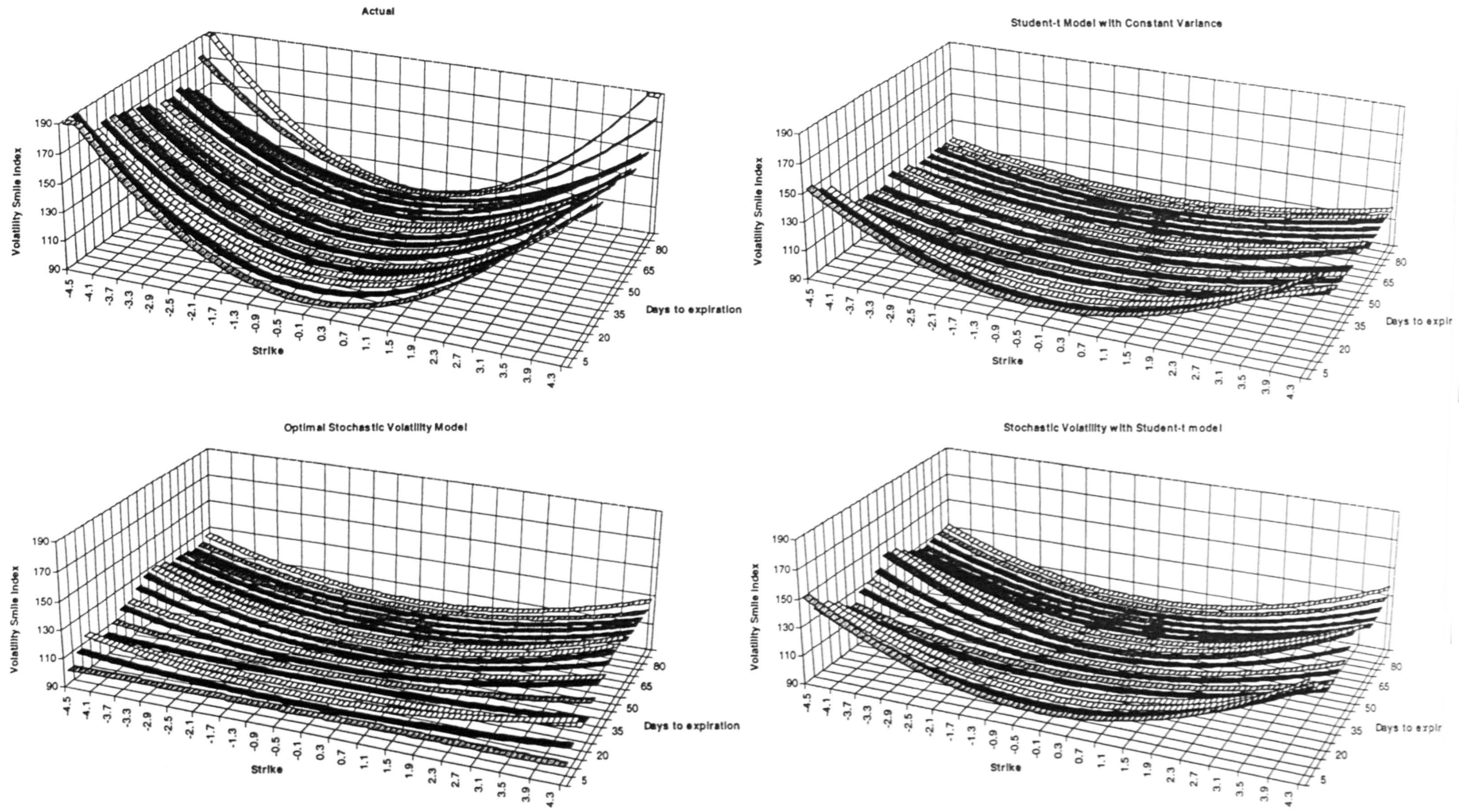

Figure 9.10d Implied Volatility Smile Patterns that are consistent with the three Security Price Models estimated using Unconditional Dispersion Process compared to the Actual