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Estimating risk-neutral freight rate dynamics: A nonparametric approach

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Abstract
We present a new method for estimating the unobservable drift of the risk-neutral spot freight rate process from Forward Freight Agreements (FFA) prices in the absence of a closed-form solution and demonstrate robustness via numerical simulations. Moreover, we conduct empirical experiments involving estimation of standard parametric models and a nonparametric model using Baltic Exchange data. We find that our nonparametric approach yields the lowest FFA pricing errors across maturities. Finally, we estimate the market price of risk, analyze its behavior in-sample and out-of-sample and observe that, when estimated using our nonparametric approach, it evolves consistently with the indices under study.

KEYWORDS
finite differences, forward freight agreements, freight rates, market price of risk, nonparametric estimation, risk-neutral drift

JEL CLASSIFICATION
C14, G13, G17

1 | INTRODUCTION

Maritime transportation is an essential link in the global trade. Shipping provides the most efficient way of moving commodities and goods over large distances and 90% of the global volume of trade is transported by sea (UNCTAD, 2020). The shipping industry caters for the transportation of finished and semifinished goods in container boxes, the transportation of dry-bulk commodities, such as iron ore, grains, and coal, and the transportation of liquid cargoes, including crude oil and natural gas. By far the largest segment by cargo weight is the dry-bulk sector which accounts for 50% of all commodities transported by sea. The cost of moving commodities is the freight rate and represents the total cost payable for the provision of service of seaborne transportation or, equivalently, the total revenue received by the owner of the ship.

Freight rates are notoriously volatile which necessitates the use of forward contracts, known as Forward Freight Agreements (FFA), for the management of price risk. The dry-bulk FFA market provides a convenient and effective way for traders to hedge their freight rate risks, reaching an equivalent trading volume of 1.6 billion tonnes of cargo for 2020 which

roughly corresponds to 50% of the global world seaborne trade in coal, iron ore, and grains. Therefore, the accurate evaluation of FFA contracts is very important. Our paper presents a novel technique for estimating the risk-neutral drift of the spot freight rate process aiming to result in more precise FFA prices. Moreover, the estimated risk-neutral drift allows us to obtain the market price of risk, which is unobserved, but gives important insights into the market.

The market for FFA was initially used by shipping market participants only—that is, shipowners, operators, and trading houses—to hedge their freight risks, but is now changing rapidly with the increasing participation of commodity index traders, investment banks, and hedge funds. A number of factors have contributed to this transformation. The main one is that the market for freight derivatives provides a convenient way for nonshipping players to gain access into the dry-bulk shipping industry without being exposed to the complexities of physical freight operations. Forward contracts on shipping freight rates, that is, the FFA, are cash-settled trade in liquid and transparent markets and are cleared via clearinghouses. The second factor is the greater awareness of the role and contribution of shipping to the world economy: the shipping industry is what makes globalization possible and is essential for the distribution of raw materials and commodities globally.

The third factor is the growth in dry-bulk freight rates since the early 2000s, fueled by increases in demand for raw materials from resource-poor emerging economies coupled with a prolonged period of underinvestment in new fleet capacity in the 1990s. Finally, the shipping industry is cyclical and highly volatile, with the freight rates being among the most volatile asset classes, which are stylized features attributed to a combination of inelastic demand along with varying levels of elasticity and significant construction lags on the supply side. Stopford (2009) advocates that the shipping supply curve consists of two regimes: for low levels of demand, the supply curve is perfectly elastic as spare tonnage with similar operational costs is readily available. However, as demand increases, vessels with higher operational costs, typically older and less efficient vessels, enter the supply schedule. This continues to the point where no spare capacity is available and the fleet is fully utilized. In this case, supply can only increase if vessels speed up operations, which comes at the expense of higher operational costs as bunker consumption is convex in speed, or when newly built ships are delivered to the market which is subject to construction lags.

FFA contracts belong to the wider family of average-price forward contracts: they are cash-settled against the average spot freight rate over the trading days of the settlement month. Average-price contracts provide an effective mechanism against high volatility and market manipulation of the underlying spot price, since the average of daily freight rates has lower volatility and is less exposed to extreme market movements compared with daily values. At the same time, pricing average-price derivative contracts is complicated and some of those complexities are addressed in this paper.

There have been numerous studies looking at the pricing and volatility characteristics of the FFA market. For a description of those contracts and their characteristics, refer to Alizadeh and Nomikos (2009). The drivers of the FFA market and its volatility are discussed in Lim et al. (2019) and for details on the pricing of those derivatives see as well Tvedt (1997), Kavussanos and Nomikos (1999), Koekebakker et al. (2007), Nomikos et al. (2013), and Moutzouris and Nomikos (2019). In the literature, there are several models for the freight market assuming a particular stochastic spot freight rate process, like, Tvedt (1997), Koekebakker et al. (2007), Prokopczuk (2011), and Kyriakou et al. (2017). However, parametric approaches impose restrictions on the freight rate dynamics that are often difficult to justify; for instance, Adland (2003) concludes that "the restrictions imposed on stochastic freight rate models are often inconsistent with both economic theory and empirical data." This has led quite often to the application of nonparametric diffusion models for the freight rate dynamics with promising results as in Adland (2003) and Adland and Cullinane (2006).

In this paper, we use a nonparametric approach to estimate the price of risk in shipping markets from traded derivative prices. Our approach is model-independent and versatile as it can be applied to general parametric or nonparametric underlying stochastic processes. To the best of our knowledge, this is an issue that has yet to be addressed in the literature and is important in its own right. From a theoretical perspective, knowing the price of risk allows us to change from the physical $\mathcal{P}$ to the risk-neutral $\mathcal{Q}$ measure, which is necessary for pricing derivatives based on no-arbitrage arguments. As such, the findings of this paper have implications for the pricing of all the derivative instruments in the shipping markets. From a practical perspective, the market price of risk represents the extra return for accepting some level of risk (Kolos & Ronn, 2008) and allows us to derive important insights into the market. As such, we can answer interesting questions about the degree of risk aversion and behavior of market participants and how these vary with market conditions, thus contributing to the recent literature on the risk attitudes in the shipping industry (Giamouzi & Nomikos, 2021).

The structure of this paper is as follows. Section 2 describes the FFA pricing setup and shows some existing models in the literature. Section 3 presents some preliminary numerical experiments. Section 4 describes the data and

empirical results, whereas Section 5 presents the procedure for extracting the market price of risk and its analysis. Section 6 concludes the paper.

2 | MODEL SETTING

In this section, we adopt a diffusion process for the dynamics of the freight rates and show some models for pricing FFA contracts. Then, we derive a new result which allows us to estimate the drift of the freight rate process under the risk-neutral measure.

Let $(\Omega, \mathcal{F}, \{\mathcal{F}\}_{t\geq 0}, \mathcal{P})$ be a complete filtered probability space, which satisfies the usual conditions, where $\{\mathcal{F}\}_{t\geq 0}$ is a filtration (e.g., see Protter, 2005; Shreve, 2004). Therefore, there exists a risk-neutral measure $\mathcal{Q}$ equivalent to $\mathcal{P}$. We assume that, under the measure $\mathcal{Q}$, the spot freight rate evolves according to the general model

$$dS(t) = \mu^{\mathcal{Q}}(S(t))dt + \sigma(S(t))dW^{\mathcal{Q}}(t),\tag{1}$$

where $\mu^{\mathcal{Q}}(S)$ and $\sigma(S)$ are, respectively, the drift and volatility of the process, and $W^{\mathcal{Q}}$ is the standard Brownian motion. We assume that the functions $\mu^{\mathcal{Q}}$ and σ satisfy suitable regularity conditions as follows (see Oksendal, 2003):

1. Functions $\mu^{\mathcal{Q}}$ and σ are measurable and there exists a constant C such that, for all $x \in \mathbb{R}$,

$$|\mu^{\mathcal{Q}}(x)| + |\sigma(x)| \leq C(1 + |x|).$$

2. There exists a constant D such that, for all $x, y \in \mathbb{R}$,

$$|\mu^{\mathcal{Q}}(x) - \mu^{\mathcal{Q}}(y)| + |\sigma(x) - \sigma(y)| \leq D|x - y|.$$

The FFA is a cash-settled financial contract which gives its owner the average spot price over the settlement period $[T_1, T_N]$, with $T_1 < T_2 < \cdots < T_N$, where T_1 and T_N are, respectively, the first and last trading days in a month. We denote by $F(t, S; T_1, ..., T_N)$ the FFA price. As the cost of entering the FFA is zero, $F(t, S; T_1, ..., T_N)$ must satisfy the expected present value equation conditional on the information at time t

$$E^{\mathcal{Q}}\left[e^{-r(T_N-t)}\left(\frac{1}{N}\sum_{i=1}^{N}S(T_i) - F(t, S; T_1, ..., T_N)\right)\middle| S(t) = S\right] = 0,$$

where $E^{\mathcal{Q}}$ represents the conditional expectation under the $\mathcal{Q}$ measure. Therefore, the FFA price at time t is given by

$$F(t, S; T_1, ..., T_N) = \frac{1}{N}\sum_{i=1}^{N}E^{\mathcal{Q}}[S(T_i)|S(t) = S],\tag{2}$$

where the terms in the sum represent the conditional expectations under the risk-neutral measure on the information at time t. If $T_k \leq t < T_{k+1}$, for $k \in \{1, ..., N-1\}$, the right-hand side of (2) equals

$$\frac{1}{N}\left(\sum_{i=1}^{k}S(T_i) + \sum_{i=k+1}^{N}E^{\mathcal{Q}}[S(T_i)|S(t) = S]\right).$$

In the literature, there are some very well-known models for pricing FFA contracts that we will use as test problems in Section 3. Koekebakker et al. (2007) assume that the spot freight rate follows a geometric Brownian motion:

$$dS = \alpha S dt + \sigma S dW^{\mathcal{P}}.\tag{3}$$

Then, under the $\mathcal{Q}$ measure,

$$dS = \alpha^{\mathcal{Q}} S dt + \sigma S dW^{\mathcal{Q}}, \tag{4}$$

where $\alpha^{\mathcal{Q}} = \alpha - \phi\sigma$ and ϕ is the market price of risk coefficient. This fact yields a closed-form solution for the FFA price given by

$$F(t, S; T_1, ..., T_N) = S \frac{e^{\alpha^{\mathcal{Q}}(T_N - t)}}{N} \frac{e^{-\alpha^{\mathcal{Q}} N \Delta} - 1}{e^{-\alpha^{\mathcal{Q}} \Delta} - 1} \tag{5}$$

(see Koekebakker et al., 2007 for more details); we dub this Model 1. This model is basic and provides a very simple and manageable expression for the FFA price.

Nevertheless, the freight market is a nontradable transportation service hence its dynamics looks quite different from a geometric Brownian motion. For instance, Tvedt (1997) accounts for mean reversion in the spot freight rate process using

$$dS = k(m - \ln S) S dt + \sigma S dW^{\mathcal{P}} \tag{6}$$

and, under the $\mathcal{Q}$ measure,

$$dS = k(m^{\mathcal{Q}} - \ln S) S dt + \sigma S dW^{\mathcal{Q}}, \tag{7}$$

where $m^{\mathcal{Q}} = m - \phi\sigma$; we call this Model 2. This model has been used widely in commodity, energy, and freight markets (e.g., see Schwartz, 1997; Tvedt, 1997). The futures price at time t with maturity time $T \geq t$ is given by

$$\widetilde{F}(t, S; T) = E^{\mathcal{Q}}[S(T)|S(t) = S],$$

which, from Schwartz (1997), for the spot process (7) is equal to

$$\widetilde{F}(t, S; T) = \exp\left\{ e^{-k(T-t)} \ln S + \hat{\alpha}\left(1 - e^{-k(T-t)}\right) + \frac{\sigma^2}{4k}\left(1 - e^{-2k(T-t)}\right) \right\}, \tag{8}$$

where $\hat{\alpha} = m^{\mathcal{Q}} - \sigma^2/(2k)$. Then, the FFA price with settlement period $[T_1, T_N]$, where $T_1 < T_2 < \cdots < T_N$, is given by the arithmetic average of the futures prices with maturities $\{T_i\}_{i=1}^N$,

$$F(t, S; T_1, ..., T_N) = \frac{1}{N} \sum_{i=1}^{N} \widetilde{F}(t, S; T_i), \tag{9}$$

where $\widetilde{F}$ is given by (8).

In both models, the market price of risk coefficient is assumed to be constant resulting in a closed-form solution for the FFA price. However, in the literature, there is neither empirical evidence nor consensus about which stochastic process and market price of risk function are the best to model the spot freight rates and this can easily lead to model misspecification. Moreover, there has been a recent tendency to use nonparametric methods to identify and estimate the functions of the stochastic processes (see Stanton, 1997 in bond pricing; Gómez-Valle & Martínez-Rodríguez, 2016 in commodity markets; or Adland, 2003; Adland & Cullinane, 2006 in freight markets). Nonparametric techniques do not impose a specific parametric structure and, as such, reduce potential misspecification errors.

We describe briefly how the nonparametric estimation works. Suppose a data set consisting of n pairs of observations $\{(S_i, Y_i)\}_{i=1}^n$. We consider a model of the type

$$Y_i = m(S_i) + \varepsilon_i, \quad i = 1, ..., n, \tag{10}$$

where $m(S)$ is an unknown function and ε represents the random error in the observations or the variability from sources not included in S. The error terms $\{\varepsilon_i\}_{i=1}^n$ are assumed to be independent and identically distributed with mean

equal to zero and finite variance. For example, a candidate function for $m(S)$ is the Nadaraya–Watson (NW) estimator (see Härdle, 1990) given by

$$\hat{m}(S) = \frac{\sum_{i=1}^{n} K_h(S - S_i) Y_i}{\sum_{i=1}^{n} K_h(S - S_i)}, \tag{11}$$

where $K_h(S - S_i) = K((S - S_i)/h)$, K is the Gaussian kernel and h the bandwidth parameter.

To apply this nonparametric estimation technique to the FFA price model, we first estimate the volatility of the stochastic process (1). As the volatility is not affected by the change of measure, we can use the spot freight rate process under the physical measure,

$$dS(t) = \mu(S(t))dt + \sigma(S(t))dW^{\mathcal{P}}(t), \tag{12}$$

and spot freight rate market data. We approximate the volatility in (12) based on the Euler discretization

$$\sigma^2(S(t)) = \frac{1}{\Delta} E\left[(S(t + \Delta) - S(t))^2 | S(t) = S\right] + O(\Delta), \tag{13}$$

which for discrete equidistant time points $\{t_i\}_{i=1}^{n}$, subject to spacing Δ, $S_i = S(t_i)$, $(S_i, Y_i) = (S_i, (S_{i+1} - S_i)^2)$ and by considering the NW estimator, gives

$$\hat{\sigma}^2(S) = \frac{1}{\Delta} \frac{\sum_{i=1}^{n-1} K_h(S - S_i)(S_{i+1} - S_i)^2}{\sum_{i=1}^{n-1} K_h(S - S_i)} \tag{14}$$

(for more details, see Fusai & Roncoroni, 2008; Stanton, 1997).

Finally, we need to estimate the risk-neutral drift. Usually, in the literature, this is inferred from the FFA closed-form solution. An alternative method is to estimate the drift under the $\mathcal{P}$ measure using the spot freight rate observations and, then, the market price of risk from the FFA closed-form solution. However, this is not always possible, especially in nonparametric models, where an explicit price is not known. We solve this problem by proving a result which links the risk-neutral drift with the directional derivative of the FFA price with respect to the time points in the settlement period.

Theorem 1. *Let S be the spot freight rate evolving according to process (1) and $F(t, S; T_1, ..., T_N)$ the FFA price given by (2) where $t \leq T_N$. Then,*

$$D_{\mathbf{v}} F(t, S; T_1, ..., T_N) = \frac{1}{N} \sum_{i=1}^{N} E^{\mathcal{Q}}[\mu^{\mathcal{Q}}(S(T_i)) | S(t) = S], \tag{15}$$

where $D_{\mathbf{v}} F$ is the derivative of F with respect to $\mathbf{T} = [T_1, ..., T_N]$ in the direction $\mathbf{v} = (1, ..., 1)$.

Proof. From (1), we have for $i = 1, ..., N$ and $\delta > 0$ that

$$S(T_i + \delta) - S(T_i) = \int_{T_i}^{T_i + \delta} \mu^{\mathcal{Q}}(S(z))dz + \int_{T_i}^{T_i + \delta} \sigma(S(z))dW^{\mathcal{Q}}(z).$$

Summing across $i = 1, ..., N$ and dividing by N, we obtain

$$\frac{1}{N} \sum_{i=1}^{N} S(T_i + \delta) - \frac{1}{N} \sum_{i=1}^{N} S(T_i) = \frac{1}{N} \sum_{i=1}^{N} \int_{T_i}^{T_i + \delta} \mu^{\mathcal{Q}}(S(z))dz + \frac{1}{N} \sum_{i=1}^{N} \int_{T_i}^{T_i + \delta} \sigma(S(z))dW^{\mathcal{Q}}(z).$$

We calculate the conditional expectation under the $\mathcal{Q}$ measure of the above equality and from (2) and for the Itô integral being a martingale, we get

$$F(t, S; T_1 + \delta, ..., T_N + \delta) - F(t, S; T_1, ..., T_N) = \frac{1}{N} \sum_{i=1}^{N} \int_{T_i}^{T_i + \delta} E^{\mathcal{Q}}[\mu^{\mathcal{Q}}(S(z))|S(t) = S] dz.$$

Now, dividing by δ and letting δ approach 0, we obtain (15). □

We implement this result to estimate $\mu^{\mathcal{Q}}(S)$ based on the following steps. First, we approximate the left-hand side of (15) using a finite difference. More specifically, for fixed t and S, we define a new function $G(\delta) = F(t, S; \mathbf{T} + \delta \mathbf{v})$. Then,

$$D_{\mathbf{v}} F(t, S; \mathbf{T}) = \lim_{\delta \to 0} \frac{F(t, S; \mathbf{T} + \delta \mathbf{v}) - F(t, S; \mathbf{T})}{\delta} = \lim_{\delta \to 0} \frac{G(\delta) - G(0)}{\delta} = G'(0).$$

Second, a second-order difference approximation yields

$$
\begin{aligned}
G'(0) &= \frac{-3G(0) + 4G(\delta) - G(2\delta)}{2\delta} \\
&= \frac{-3F(t, S; T_1, ..., T_N) + 4F(t, S; T_1 + \delta, ..., T_N + \delta) - F(t, S; T_1 + 2\delta, ..., T_N + 2\delta)}{2\delta}.
\end{aligned}
\tag{16}
$$

Finally, given $E^{\mathcal{Q}}[\mu^{\mathcal{Q}}(S(T_i))|S(t) = S] \simeq \mu^{\mathcal{Q}}(S(t))$, we get the following estimate of function $\mu^{\mathcal{Q}}(S)$:

$$
\begin{aligned}
\mu^{\mathcal{Q}}(S) &\simeq \frac{1}{N} \sum_{i=1}^{N} E^{\mathcal{Q}}[\mu^{\mathcal{Q}}(S(T_i))|S(t) = S] \\
&\simeq \frac{-3F(t, S; \mathbf{T}) + 4F(t, S; \mathbf{T} + \delta \mathbf{v}) - F(t, S; \mathbf{T} + 2\delta \mathbf{v})}{2\delta},
\end{aligned}
\tag{17}
$$

where the second approximation is due to (15) and (16).

We use as the observations Y_i in (10) the right-hand side of (17), with the shortest equidistant maturities, and the spot freight rates as S_i. Then, any technique can be applied to estimate the risk-neutral drift, such as the nonparametric NW estimator (11). An important merit of this result is that it can be applied to estimate the risk-neutral drift directly from FFA prices in the market, independently of any assumptions one makes about the underlying model or the estimation technique used. We will call *dRNP* this approach based on the directional derivative of the FFA price and the risk-neutral process. This allows us to evaluate derivatives without estimating either the market price of risk or the physical drift, hence reduces potential model misspecification.

In the rest of this paper, this approach, implemented with nonparametric estimation, will be referred to as the *Nonpar* model. Its main advantage over the previous parametric models (Models 1 and 2) is that it uses fully functional procedures to identify and estimate the functions of the stochastic processes appearing in the most recent literature (see, among others, Adland, 2003; Adland & Cullinane, 2006; Bandy & Phillips, 2006). Therefore, it does not impose any arbitrary restrictions to either the functions of the spot freight rate processes or the market price of risk.

3 | NUMERICAL SIMULATIONS

To illustrate the precision of our proposed approach, in this section we carry out numerical experiments with test problems to study the robustness and efficiency of our estimation method in Theorem 1. We test the approximation in problems with exact solutions to calculate and analyze the errors. As we are not testing here the model realism, but rather an approximate against an exact solution, we do not use real market data (except only for the parameter estimates mentioned next from previous research).

We investigate our method for Models 1 and 2 in Section 2 as test problems. We conduct Monte Carlo simulations of the spot rate based on (3) and (6), respectively. We calculate FFA prices for each model from (5) and (9), respectively, for different values of the market price of risk coefficient. We then analyze the error from misspecifying the risk-neutral drift, in particular, by setting this equal to the drift under the $\mathcal{P}$ measure; we refer to this as the risky process (*RP*) assumption.

First, we use the parameter values obtained by Gómez-Valle et al. (2020) for (3), that is, $\alpha = 0.0041$ and $\sigma = 0.3738$; and Gómez-Valle and Martínez-Rodríguez (2021) for (6), that is, $k = 0.4041$, $m = 6.8805$, and $\sigma = 0.3740$. Both

estimations were made using Baltic Dry Index (BDI) data (Baltic Exchange, 2020) from January 2013 to January 2019 as a proxy for the spot freight rate. The BDI is a composite daily freight index that records freight rates for transporting dry-bulk commodities, like, iron ore, coal, grain, and other agricultural commodities, across 20 different shipping routes. We simulate 5000 spot freight rate daily trajectories for these parameter values.

Second, to estimate the risk-neutral drift, we need FFA prices for three short equidistant maturities, such as 1–3 months. We compute these using the previous parameters, the closed-form solutions (5) and (9), and various values of the market price of risk coefficient $\phi \in \{0, \pm 0.02, \pm 0.05, \pm 0.2\}$. We then estimate the risk-neutral drifts for both parametric models by means of Theorem 1, the second-order approximation (17) and nonlinear least squares, using, respectively, for Models 1 and 2

$$\alpha^{\mathcal{Q}} S_i \simeq \frac{-3F_1(t_i, S_i) + 4F_2(t_i, S_i) - F_3(t_i, S_i)}{1/6},$$

$$k\left(m^{\mathcal{Q}} - \ln S_i\right) S_i \simeq \frac{-3F_1(t_i, S_i) + 4F_2(t_i, S_i) - F_3(t_i, S_i)}{1/6},$$

where $F_j(t_i, S_i)$ is the price of the FFA with maturity j months for observed spot freight rate S_i, $i = 1, ..., n$. The volatility parameter is estimated, as it is not affected by the change of measure, using the simulated spot rate process under the measure $\mathcal{P}$.

We consider the mean absolute percentage error (MAPE) and the root mean square percentage error (RMSPE) for a given maturity τ,

$$MAPE_\tau = \frac{1}{n} \sum_{i=1}^{n} \left| 1 - \frac{F_{i,\tau}^{\theta}}{F_{i,\tau}^{M}} \right| \quad \text{and} \quad RMSPE_\tau = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left| 1 - \frac{F_{i,\tau}^{\theta}}{F_{i,\tau}^{M}} \right|^2},$$

where $F_{i,\tau}^{\theta}$ and $F_{i,\tau}^{M}$ are, respectively, the estimated and simulated FFA prices for maturity τ. In particular, we consider, here, $\tau = 1, 2, 3, 6,$ and 12 months. We also denote by $MAPE$ and $RMSPE$ the corresponding errors based on all maturities:

$$MAPE = \frac{1}{n \, n_\tau} \sum_{i=1}^{n} \sum_{j=1}^{n_\tau} \left| 1 - \frac{F_{i,j}^{\theta}}{F_{i,j}^{M}} \right| \quad \text{and} \quad RMSPE = \sqrt{\frac{1}{n \, n_\tau} \sum_{i=1}^{n} \sum_{j=1}^{n_\tau} \left| 1 - \frac{F_{i,j}^{\theta}}{F_{i,j}^{M}} \right|^2}.$$

In Tables 1 and 2, we report the errors from comparing the FFA prices obtained from the simulations of each of Models 1 and 2 with those calculated based on the *RP* misspecified risk-neutral drift and the *dRNP* risk-neutral drift of Section 2. We find that pricing FFA contracts using the *dRNP* approach yields very small errors. More specifically, these do not exceed 10^{-3} in the case of Model 2, whereas in the case of Model 1 these appear to be as low as 10^{-8} with some

TABLE 1   Mean absolute percentage errors (MAPEs) and root mean square percentage errors (RMSPEs) when pricing FFA contracts under Model 1

	RP		*dRNP*	
ϕ	*MAPE*	*RMPSE*	*MAPE*	*RMPSE*
−0.20	2.4092×10^{-2}	3.5411×10^{-2}	8.5903×10^{-5}	1.2709×10^{-4}
−0.05	5.9064×10^{-3}	8.7247×10^{-3}	6.1678×10^{-6}	9.1251×10^{-6}
−0.02	2.1793×10^{-3}	3.2224×10^{-3}	1.2350×10^{-6}	1.8271×10^{-6}
0	3.2248×10^{-4}	4.7714×10^{-4}	1.0599×10^{-8}	1.5679×10^{-8}
0.02	2.8380×10^{-3}	4.2020×10^{-3}	3.7920×10^{-7}	5.6103×10^{-7}
0.05	6.6371×10^{-3}	9.8370×10^{-3}	4.0321×10^{-6}	5.9655×10^{-6}
0.20	2.6111×10^{-2}	3.8893×10^{-2}	7.7622×10^{-5}	1.1485×10^{-4}

Abbreviations: dRNP, directional derivative risk-neutral process; FFA, Forward Freight Agreements; RP, risky process.

TABLE 2 Mean absolute percentage errors (MAPEs) and root mean square percentage errors (RMSPEs) when pricing FFA contracts under Model 2

ϕ	RP		dRNP	
	MAPE	RMPSE	MAPE	RMPSE
−0.20	2.2087×10^{-1}	2.6259×10^{-1}	6.5746×10^{-4}	9.4679×10^{-4}
−0.05	2.1892×10^{-1}	2.6024×10^{-1}	7.7965×10^{-4}	7.7965×10^{-4}
−0.02	2.1852×10^{-1}	2.5980×10^{-1}	5.1600×10^{-4}	7.4533×10^{-4}
0	2.1826×10^{-1}	2.5950×10^{-1}	4.9928×10^{-4}	7.2235×10^{-4}
0.02	2.1799×10^{-1}	2.5921×10^{-1}	4.8235×10^{-4}	6.9930×10^{-4}
0.05	2.1760×10^{-1}	2.5878×10^{-1}	4.5659×10^{-4}	6.6466×10^{-4}
0.20	2.1582×10^{-1}	2.5676×10^{-1}	3.2095×10^{-4}	4.9447×10^{-4}

Abbreviations: dRNP, directional derivative risk-neutral process; FFA, Forward Freight Agreements; RP, risky process.

discrepancies observed depending on the value of the market price of risk coefficient. Expectedly, the errors increase considerably when the FFA contracts are priced under the *RP* drift, in particular, by a factor of 10^2 up to 10^3, or even 10^4 in some cases, compared with our *dRNP* approach. Tests based on alternative parameter values yielded similar results. It is worth noting that, even when the market price of risk is assumed to be equal to zero (i.e., $\phi = 0$), the errors from the *RP* drift remain high. In this case, the parameters of the stochastic process under the $\mathcal{P}$ measure are estimated using the spot freight rate observations in the market which can lead to possible misspecification in the estimation of the drift (see Adland, 2003; Stanton, 1997). Our results confirm the importance of an accurate estimation of the risk-neutral drift for an accurate pricing of the freight derivatives, therefore, in what follows, we study further the *dRNP* drift approach.

4 | DATA AND EMPIRICAL APPLICATION

In this section, we estimate the functions of the risk-neutral spot freight rate process and price FFA contracts with different maturities for the dry-bulk freight rates. We focus on the Capesize, Panamax, and Supramax dry-bulk carriers which are the most popular contract categories and jointly account for more than 95% of the total derivatives volume in the dry-bulk FFA market. In addition, this is the oldest market for freight derivatives with continuous price data going back to at least 2006. At the larger end of the scale, Capesize carriers have a carrying capacity of 180,000 dead-weight (dwt) tons and are the largest vessel category in the dry-bulk sector. Capesize vessels are primarily employed for the transportation of iron ore from South America and Australia to the Far East (primarily China) and coal from North America, Australia, and South Africa to the Far East and North Europe. The name is attributed to the fact that this type of vessel is too large to transit the Panama canal, hence it has to navigate around Cape Horn or Cape of Good Hope. Panamax carriers have a capacity of about 74,000 mt dwt and, as the name implies, are the largest-sized vessels that can transit the Panama Canal. Finally, Supramax vessels have a cargo-carrying capacity of 58,000 mt dwt and are more versatile than the other vessel categories in terms of commodities they carry and the routes they serve. They are also equipped with cranes and, as such, can operate in ports where there is not sufficient infrastructure. According to Clarksons Shipping Intelligence Network (SIN), as of October 2020 the Capesize, Panamax, and Supramax sectors accounted, respectively, for 40%, 25%, and 23% of the total cargo-carrying capacity of the dry-bulk fleet. In addition, these vessels are almost exclusively used for the transportation of the so-called "major bulks," including commodities, such as iron ore, grain, coal, bauxite, and alumina and phosphate, and jointly account for about 25% of the overall global volume of trade. Therefore, these vessel types are representative of the overall bulk fleet and their dynamics.

Our key variables are the spot and forward freight rates for the Capesize, Panamax, and Supramax sectors known, respectively, as the Baltic Capesize Average of five key time-charter routes (BCI 5TC), the Baltic Panamax Average of four key time-charter (BPI 4TC), and the Baltic Supramax Average of five time-charter routes (BSI 5TC), which are published by the Baltic Exchange (2020) on a daily basis.[1] The sample period for our data is from January 2006 to

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[1]Although the Baltic Exchange Indices are known as the Average of Time-Charter routes, their constituent routes are in fact trip-charter as opposed to time-charter. Time-Charter refers to hiring a ship for a period of time, usually between 6 months and 3 years. Trip-Charter, on the other hand, refers to hiring a vessel for the duration of a specific voyage.

December 2019. We also consider the FFA rates that price the forward value of the index. These contracts are available at any given point in time for various maturities up to 7 years ahead. A particular issue when using forward prices is that individual contracts expire, complicating the empirical analysis. As such, we create synthetic forward contracts with fixed maturities. For example, we create a 30-day-ahead synthetic contract using contracts that expire in M and L days, where $M < 30 < L$:

$$F_{30} = F_L \frac{30 - M}{L - M} + F_M \frac{L - 30}{L - M}. \tag{18}$$

We use Equation (18) to construct daily forward contracts with maturities of up to 2 years ahead, which are then used in our empirical application.

In Figure 1, we present the daily time plot of the BCI, BPI, and BSI from January 2006 to December 2019. It is important to note that the three indices have a very similar behavior, although the BCI has, in general, the highest values, especially during the period 2006–2010. We note a persistent rally in freight rates during the period 2006–2008, which culminates with the indices reaching their peaks in July 2008. For example, following the financial crisis, the BCI dropped by more than 95% from a peak of 230,000 \$/day in July 2008 to a low value of 5000 \$/day by December of the same year. Since then, the market exhibits very high volatility with large swings, which is typical of the behavior of dry-bulk freight rates.

We estimate all the functions of the spot risk-neutral process with data from January 2006 to December 2017 and keep data from January 2018 to December 2019 to analyze the results for robustness, that is, we divide our data in the in-sample period (January 2006–December 2017) and out-of-sample period (January 2018–December 2019).

First, using the in-sample data, we estimate the drift and volatility of the process under the $\mathcal{Q}$ measure in (1) as established in Section 2. When applying (17) to estimate the risk-neutral drift, we require three FFA prices with the shortest equidistant maturities; here, we use prices with maturities of 1–3 months. As the BCI and BPI reached very high values, especially in the period 2007–2008 (see Figure 1), we choose to work on the log-scale, that is, use as observations $\ln S_i$ instead of S_i in (11) to achieve more accurate nonparametric estimations of $\mu^{\mathcal{Q}}(S)$ in (17). For the BSI, we use the original spot freight rate S_i observations as these are lower than BCI and BPI's. Also, to improve the estimation of the risk-neutral drift of the BSI, we use the Local Linear Regression (LLR), instead of the NW estimator, to reduce the bias at the boundaries (see Loader, 1999 for more details); however, this increases considerably the computational cost and does not always provide better results than NW.

Figure 2 exhibits the scatter plots of the observed BSI and the logarithm of BCI and BPI over the in-sample period against the approximation (17) and the nonparametrically estimated risk-neutral drift. We observe that there are only a few very low and high spot rate observations. Moreover, the high values are very scattered, especially for the BCI.

For comparison, we also estimate the risk-neutral drift and volatility of the parametric spot freight benchmark Models 1 and 2. We apply the usual methodology to estimate the parameters via their corresponding closed-form solutions (5) and (9) and nonlinear least squares. The volatility of Model 1 is estimated from the discretization of the spot freight rate process under the $\mathcal{P}$ measure as in Section 3. Table 3 reports the estimated parameters for the BCI, BPI,
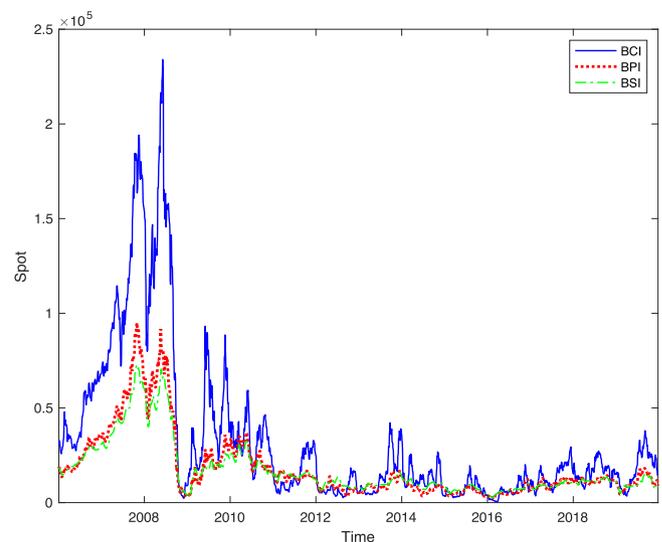
FIGURE 1 Daily BCI, BPI, and BSI from January 2006 to December 2019. BCI, Baltic Capesize Average; BPI, Baltic Panamax Average; BSI, Baltic Supramax Average [Color figure can be viewed at wileyonlinelibrary.com]

TABLE 3 Estimated parameter values under the Q measure for Models 1 and 2 for the BCI, BPI, and BSI

Parameters	BCI		BPI		BSI	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
α^Q	−0.0928	–	−0.0458	–	−0.0502	–
σ	0.9297	0.0062	0.4326	2.2751	0.2456	2.0536
k	–	0.0953	–	0.0696	–	0.0442
m^Q	–	8.9591	–	9.6749	–	7.9771

Abbreviations: BCI, Baltic Capesize Average; BPI, Baltic Panamax Average; BSI, Baltic Supramax.

FIGURE 3 Estimated risk-neutral drifts (including confidence bands for the *Nonpar* model) for the BCI (top), BPI (middle), and BSI (bottom). BCI, Baltic Capesize Average; BPI, Baltic Panamax Average; BSI, Baltic Supramax Average [Color figure can be viewed at wileyonlinelibrary.com]
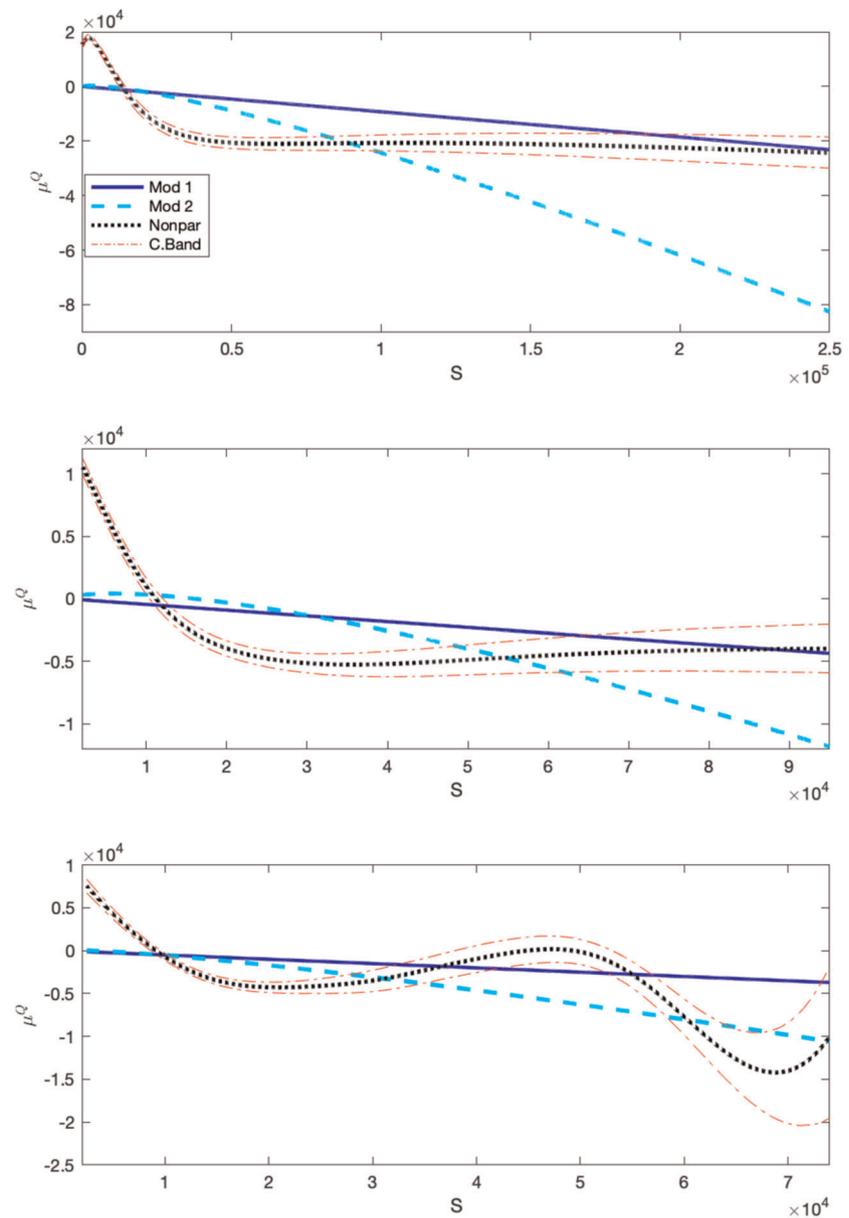
and BSI. Figure 3 shows for each index the estimated risk-neutral drifts $\mu^Q(S)$ of the two parametric models and the nonparametric model together with its associated confidence bands obtained by bootstrap (see Martínez & Martínez, 2016). In all the cases, we observe a decreasing trend, but the behavior of the three models varies. In Models 1 and 2, the shape is determined by the functions imposed to the risk-neutral drifts, whereas in the *Nonpar* model this is determined by the behavior of the BCI, BPI, and BSI observations in the sample period. The nonparametric risk-neutral drifts of the BCI and BPI are smoother than the BSI's. (The smoothness of the nonparametric estimation depends on the bandwidth h as well as the magnitude of the independent variables $\ln S_i$ or S_i.)

Finally, we price FFA contracts under all the models and for maturities from 1 to 24 months. For Models 1 and 2, we use their corresponding closed-form solutions (5) and (9). For the *Nonpar* model, we use, by definition, $F(t, S; T_1, ..., T_N) = \sum_{i=1}^{N} \widetilde{F}(t, S; T_i)/N$ with $\widetilde{F}$ satisfying the partial differential equation for the futures price

$$\widetilde{F}_t + \mu^Q(S)\widetilde{F}_S + \frac{1}{2}\sigma^2(S)\widetilde{F}_{SS} = 0 \qquad (19)$$

TABLE 4 Mean absolute percentage errors (MAPEs) and root mean square percentage errors (RMSPEs) for Models 1, 2, and *Nonpar* for the Capesize sector

Maturity	1 month	3 months	6 months	12 months	24 months	Aggregate
In-sample MAPE						
Model 1	0.1381	0.2312	0.3195	0.3458	0.4092	0.2921
Model 2	0.1367	0.2246	0.3049	0.3061	0.3069	0.2580
Nonpar	0.1219	0.1959	0.2655	0.2654	0.2611	0.2209
In-sample RMSPE						
Model 1	0.1855	0.3009	0.4135	0.4280	0.4868	0.3815
Model 2	0.1837	0.2930	0.3984	0.3831	0.3808	0.3388
Nonpar	0.1828	0.2739	0.3737	0.3669	0.3460	0.3140
Out-of-sample MAPE						
Model 1	0.1120	0.1812	0.3305	0.3441	0.3476	0.2617
Model 2	0.1109	0.1761	0.3181	0.3168	0.2960	0.2418
Nonpar	0.0899	0.1259	0.1820	0.1426	0.1493	0.1369
Out-of-sample RMSPE						
Model 1	0.1391	0.2182	0.3884	0.4092	0.4211	0.3324
Model 2	0.1377	0.2120	0.3737	0.3785	0.3642	0.3059
Nonpar	0.1132	0.1519	0.2173	0.1788	0.1777	0.1696

Note: In-sample period, January 2006–December 2017; out-of-sample period, January 2018–December 2019.

with final conditions $\widetilde{F}(T_i, S; T_i) = S$ for $i = 1, ..., N$; we solve (19) using the Crank–Nicolson finite difference method (e.g., see Duffy, 2006). To obtain the FFA prices in the out-of-sample period, we use spot freight rates in the out-of-sample period and parameters and nonparametric functions estimated using the in-sample data.

Table 4 reports the in-sample and out-of-sample FFA pricing errors for all the different cases for the Capesize sector. First, we compute the $MAPE_\tau$ and $RMSPE_\tau$ for maturities $\tau = 1$, 3, 6, 12, and 24 months. (Additional maturities between 1 and 24 months led to similar conclusions. This is also valid for all three indices studied in the paper.) We find that Model 1 is the worst-performing and that *Nonpar* yields the lowest errors. As a measure of the aggregate performance of each model, we report also the *MAPE* and *RMSPE* computed across all maturities (last column). The superiority of *Nonpar* is also maintained both in-sample and out-of-sample. Table 5 reports the relevant FFA pricing errors for the Panamax sector. We observe that in the in-sample period and for maturities up to 3 months, *Nonpar* results in the lowest errors, but they are slightly higher for longer maturities from 6 to 24 months; it also yields the lowest out-of-sample errors. The relationship between the errors of Models 1 and 2 is similar to that in the Capesize sector. Finally, Table 6 presents the FFA pricing errors for the Supramax sector. *Nonpar* takes the lead once more both in-sample and out-of-sample, whilst the relationship between the errors of Models 1 and 2 remains unaffected.

5 | MARKET PRICE OF RISK

In this section, we study the market price of risk, which is essential for pricing derivatives based on no-arbitrage principles. The market price of risk is not observed, rendering its estimation quite nontrivial. Derivative pricing models with known closed-form solutions are usually used to infer the market price of risk from market quotes of traded derivatives. However, model misspecification can lead to inaccurately estimated market price of risk. A potential way about this problem is to implement, instead, a nonparametric model approach; the main challenge there is the absence of an explicit solution for any derivative price that could be used to this end, therefore

TABLE 5 Mean absolute percentage errors (MAPEs) and root mean square percentage errors (RMSPEs) for Models 1, 2, and *Nonpar* for the Panamax sector

Maturity	1 month	3 months	6 months	12 months	24 months	Aggregate
In-sample MAPE						
Model 1	0.0767	0.1268	0.1620	0.2115	0.2852	0.1756
Model 2	0.0758	0.1236	0.1541	0.1823	0.2369	0.1553
Nonpar	0.0706	0.1167	0.1611	0.2152	0.2430	0.1650
In-sample RMSPE						
Model 1	0.1027	0.1679	0.2110	0.2638	0.3437	0.2360
Model 2	0.1015	0.1634	0.2016	0.2362	0.2949	0.2103
Nonpar	0.0930	0.1519	0.2117	0.2817	0.3154	0.2315
Out-of-sample MAPE						
Model 1	0.0653	0.1033	0.1566	0.1849	0.1851	0.1404
Model 2	0.0646	0.0994	0.1500	0.1765	0.2278	0.1450
Nonpar	0.0519	0.0723	0.0980	0.1185	0.1750	0.1054
Out-of-sample RMSPE						
Model 1	0.0919	0.1363	0.2118	0.2657	0.2693	0.2088
Model 2	0.0909	0.1322	0.2058	0.2471	0.2505	0.1952
Nonpar	0.0690	0.0899	0.1416	0.1854	0.2282	0.1553

Note: In-sample period, January 2006–December 2017; out-of-sample period, January 2018–December 2019.

TABLE 6 Mean absolute percentage errors (MAPEs) and root mean square percentage errors (RMSPEs) for Models 1, 2, and *Nonpar* for the Supramax sector

Maturity	1 month	3 months	6 months	12 months	24 months	Aggregate
In-sample MAPE						
Model 1	0.0541	0.0919	0.1291	0.1601	0.2230	0.1332
Model 2	0.0539	0.0909	0.1206	0.1375	0.1963	0.1201
Nonpar	0.0514	0.0856	0.1078	0.1326	0.1661	0.1100
In-sample RMSPE						
Model 1	0.0727	0.1219	0.1690	0.2079	0.2784	0.1858
Model 2	0.0724	0.1202	0.1605	0.1836	0.2422	0.1655
Nonpar	0.0681	0.1083	0.1473	0.1792	0.2120	0.1532
Out-of-sample MAPE						
Model 1	0.0371	0.0813	0.1274	0.1508	0.1427	0.1060
Model 2	0.0371	0.0818	0.1302	0.1705	0.2331	0.1293
Nonpar	0.0366	0.0769	0.1115	0.1271	0.0979	0.0876
Out-of-sample RMSPE						
Model 1	0.0569	0.1093	0.1712	0.1941	0.1845	0.1498
Model 2	0.0568	0.1091	0.1694	0.1976	0.2520	0.1688
Nonpar	0.0492	0.0917	0.1319	0.1378	0.1117	0.1065

Note: In-sample period, January 2006–December 2017; out-of-sample period, January 2018–December 2019.

prohibiting the estimation of the market price of risk. Our proposed approach is able to remove this major block, as we explain next.

Following Weron (2008), the market price of risk of the spot freight rate is given by

$$\lambda(S) = \mu(S) - \mu^{\mathcal{Q}}(S),$$

which, therefore, requires estimation of both drifts under the $\mathcal{P}$ and $\mathcal{Q}$ measures. $\mu^{\mathcal{Q}}(S)$ can be estimated using the closed-form model solution for any traded freight derivative or, if this is unknown as in the case of a nonparametric model, via Theorem 1. Here, we follow the same approach and use the same data as in Section 4; refer to Figure 3 for the outcome of this estimation. Under the $\mathcal{P}$ measure, $\mu(S)$ can be inferred from the observed spot freight rate series. In particular, we adopt a similar nonparametric approach to the volatility (recall Equations 13 and 14) based now on the first moment from an Euler discretization of (12), that is,

$$\mu(S) = \frac{1}{\Delta} E[S(t + \Delta) - S(t)|S(t) = S] + O(\Delta),$$

and, from the NW estimator (11), we get

$$\hat{\mu}(S) = \frac{1}{\Delta} \frac{\sum_{i=1}^{n-1} K_h(S - S_i)(S_{i+1} - S_i)}{\sum_{i=1}^{n-1} K_h(S - S_i)}.$$

Finally, for Model 1 we use maximum likelihood estimation as in Gómez-Valle et al. (2020), whereas for Model 2 we transform first (6) to an Ornstein–Uhlenbeck process and then use least squares as in Gómez-Valle and Martínez-Rodríguez (2021). Table 7 reports for Models 1–2 and each index the resulting parameter estimates under the $\mathcal{P}$ measure.

On the basis of the estimation procedure delineated above, Figure 4 shows the estimated market price of risk $\lambda(S)$ for each model and for the observed values S of each index. We observe varying patterns for the different models. More specifically, Model 1 exhibits a linear and increasing market price of risk, whereas for Model 2 this is slightly nonlinear and increasing for BPI and BSI, but decreasing for BCI. These patterns reflect the parametric structures imposed by Models 1 and 2 (see Equations 3–4 and 6–7). On the other hand, the nonparametric market price of risk is nonlinear in all cases. We also observe in Figure 4 that the changes of the nonparametric market price of risk are quite perceptible especially in regions of very low and high observations. This behavior can be attributed to boundary bias (see Loader, 1999) which is very common in nonparametric estimation, especially when there is scarcity of very high or low observations as in our case (see Härdle et al., 2004).

Figure 5 shows the market price of risk from 2009 to 2017. (We do not present results for the period 2006–2008 which includes the global financial crisis; the high volatility and values of freight rates in that period resulted in not accurate estimates for the nonparametric model.) Models 1 and 2 return always positive market prices of risk; as these models assume a constant market price of risk coefficient, the sign of the market price of risk will depend on this and remain unchanged over time. Without such a restriction, the nonparametric market price of risk has instead both positive and negative values and this is frequently observed also in other financial markets,

TABLE 7 Estimated parameter values under the $\mathcal{P}$ measure for Models 1 and 2 for the BCI, BPI, and BSI using in-sample data (2006–2017)

| Parameters | BCI | | BPI | | BSI | |
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
α	0.3884	–	0.0525	–	−0.0171	–
σ	0.9297	0.9300	0.4326	0.4326	0.2456	0.2456
k	–	0.3318	–	0.1368	–	0.0614
m	–	10.9983	–	9.8856	–	9.2644

Abbreviations: BCI, Baltic Capesize Average; BPI, Baltic Panamax Average; BSI, Baltic Supramax Average.

FIGURE 4 Market price of risk for Models 1, 2, and *Nonpar* as a function of the BCI (top), BPI (middle), and BSI (bottom). BCI, Baltic Capesize Average; BPI, Baltic Panamax Average; BSI, Baltic Supramax Average [Color figure can be viewed at wileyonlinelibrary.com]
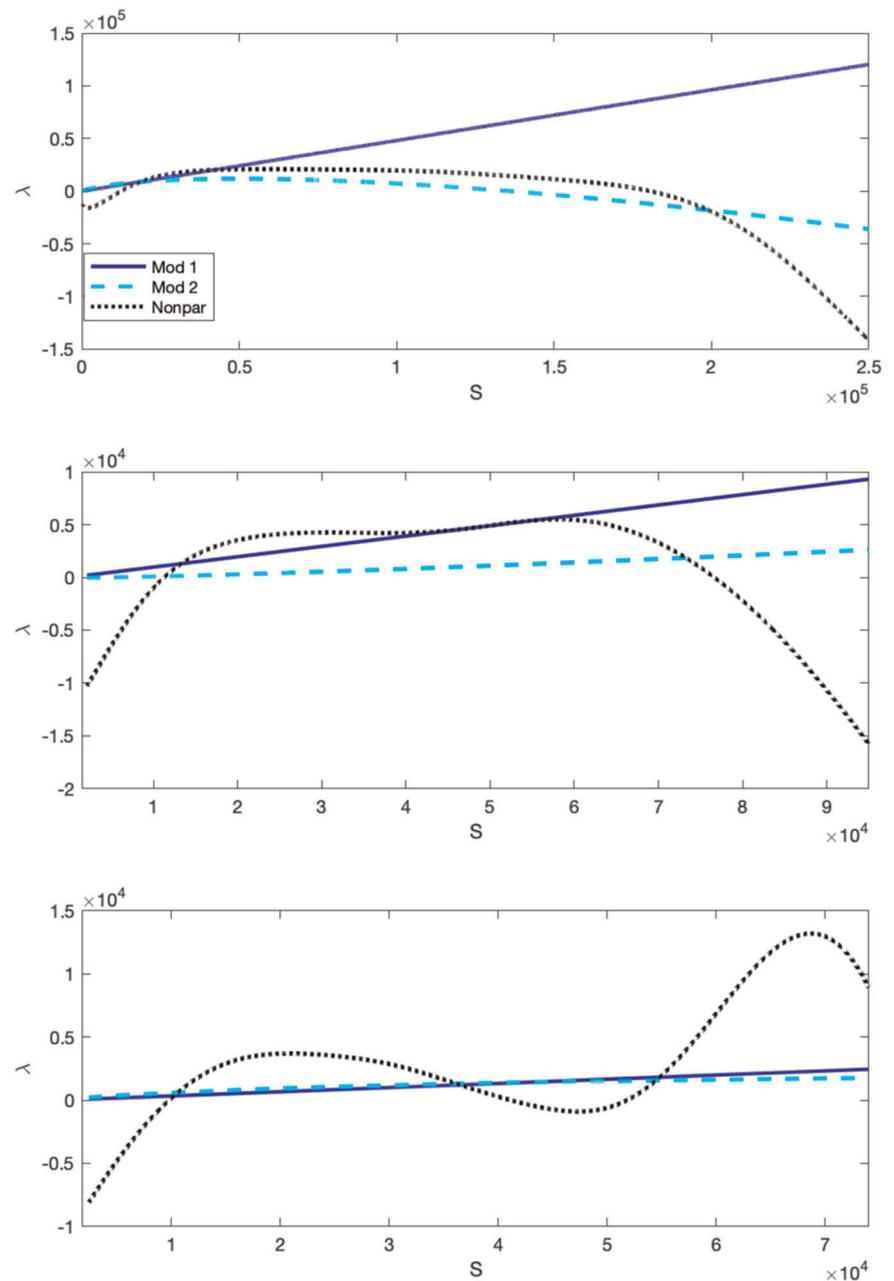
such as in the electricity market (e.g., see Weron, 2008) and the fixed-income market (see Ahmad & Wilmott, 2007). Furthermore, this result supports the assertion that the risk premium in the freight market is time-varying and depends on the prevailing state of the freight market (see Adland & Cullinane, 2005). Indeed, by super-imposing on the plots in Figure 5 the observed BCI, BPI, and BSI values, we see that, in general, the nonparametric market price of risk follows the movement pattern of the corresponding index more closely than the parametric models.

Finally, Figure 6 shows the estimated market price of risk in the out-of-sample period (2018–2019). As before, the nonparametric market price of risk follows very closely the movements of the observed index and takes both negative and positive values. On the contrary, for Models 1 and 2 this barely changes and is always positive.

In summary, in the in-sample as well as the out-of-sample periods, we observe that the nonparametric market price of risk is negative when the spot freight rate drops, that is, when investors need extra return for taking risk (e.g., see Wilmott, 1998). This is consistent with the view that when freight rates are low, long hedgers are
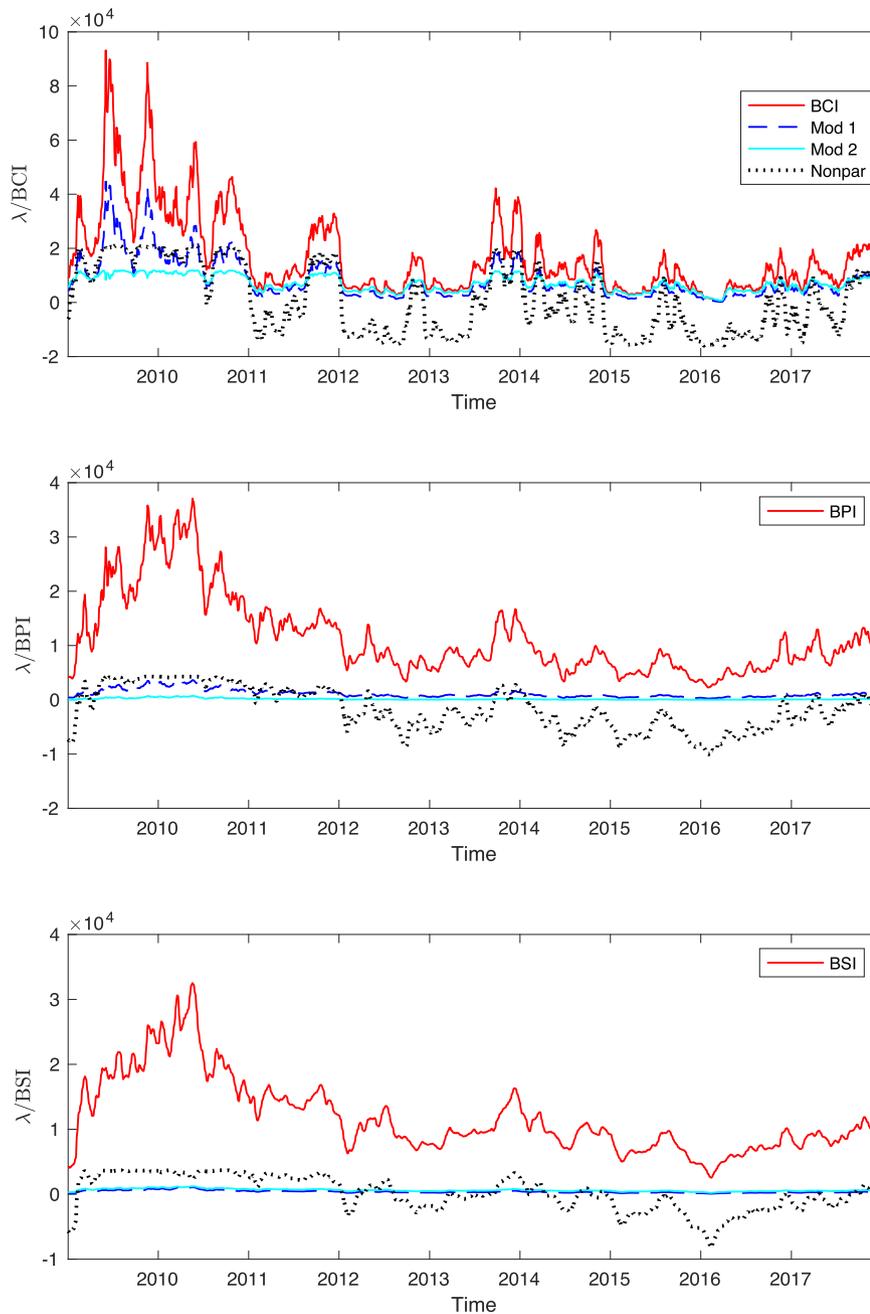
FIGURE 5 BCI (top), BPI (middle), and BPI (bottom) and the corresponding estimated market prices of risk for Models 1, 2, and *Nonpar* over the in-sample period (2006–2017). BCI, Baltic Capesize Average; BPI, Baltic Panamax Average; BSI, Baltic Supramax Average [Color figure can be viewed at wileyonlinelibrary.com]

willing to pay a premium to lock in favorably low freight rates; similarly, short hedgers are willing to accept a discount to lock in high freight rates. Overall, its qualitative behavior is similar to that of the corresponding index.

6 | CONCLUSIONS

In this study, we propose a novel nonparametric approach for estimating the drift of risk-neutral freight rate dynamics. We prove a result which links the risk-neutral drift of the spot freight rate with the directional derivative of FFA prices. This development, jointly with some approximations, allows us to estimate the risk-neutral drift and even the market price of risk, without knowledge of a closed-form solution of any freight derivative, using only readily available market data, such as spot freight rates and FFA prices. Moreover, the estimated risk-neutral drift and market price of risk can

FIGURE 6 BCI (top), BPI (middle), and BPI (bottom) and the corresponding estimated market prices of risk for Models 1, 2, and *Nonpar* over the out-of-sample period (2018–2019). BCI, Baltic Capesize Average; BPI, Baltic Panamax Average; BSI, Baltic Supramax Average [Color figure can be viewed at wileyonlinelibrary.com]
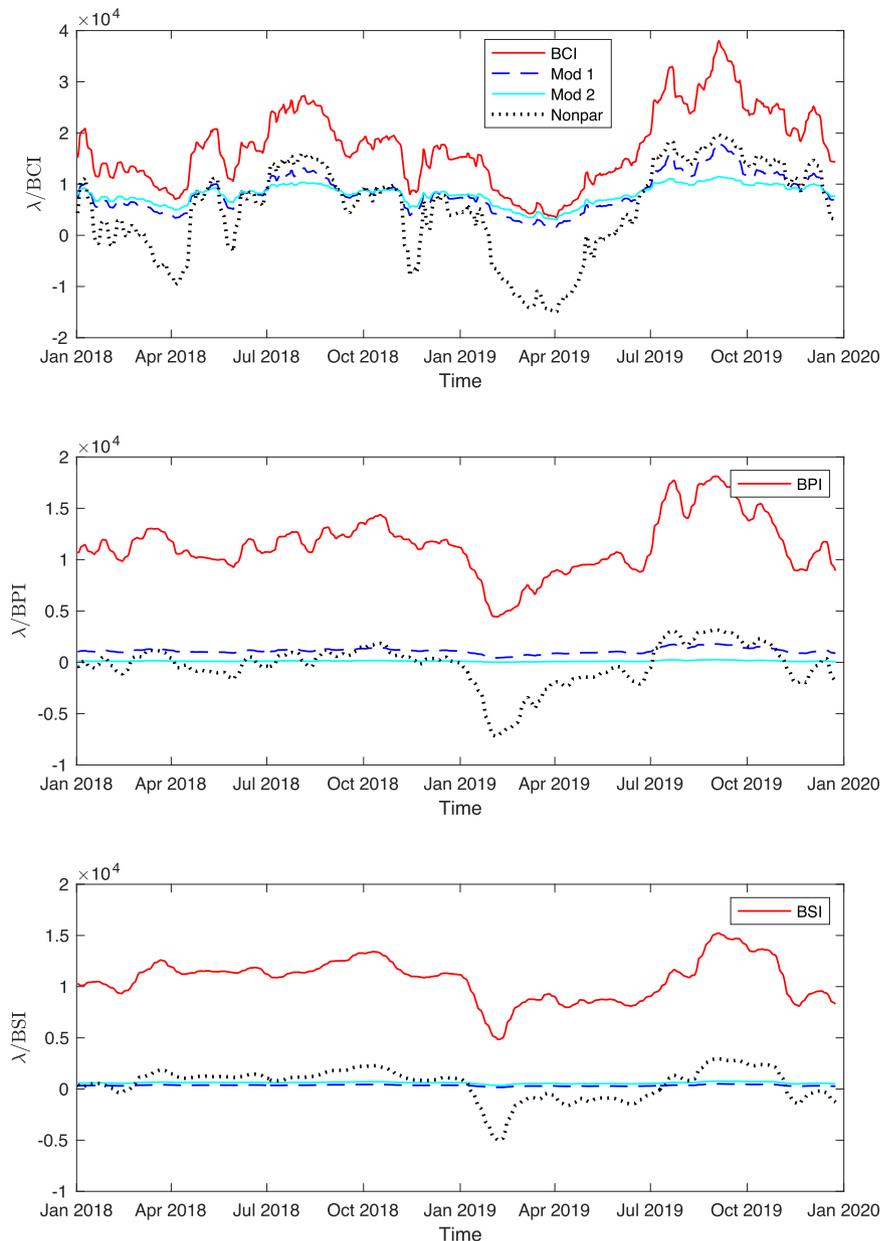
be used to price any other freight derivative as they determine the equivalent martingale measure that is necessary for pricing under no-arbitrage arguments.

We illustrate the efficiency and robustness of our approach via numerical experiments using two popular parametric models in the literature as test problems. Furthermore, we consider several Baltic indices to calibrate the risk-neutral drift with our approach and price FFA contracts. As a result, we show the superiority of our methodology, implemented by means of a nonparametric technique, for the in-sample and out-of-sample periods. Finally, we obtain the market price of risk and show that this is time-varying and its behavior is closely related to the corresponding Baltic index.

The implications of this study are of potential interest to both researchers and market participants. On the one hand, the precise estimation of the risk-neutral drift of the spot freight rate is necessary to price accurately any freight derivative. On the other hand, the market price of risk, obtained from the drift under the risk-neutral and physical measures, helps us understand the behavior and risk attitudes of market participants. Moreover, it allows us to deduce in the short term the expected risk that market participants must assume.

CONFLICT OF INTERESTS
The authors declare that there is no conflict of interests.

DATA AVAILABILITY STATEMENT
The data that support the findings of this study are available from the Baltic Exchange. Restrictions apply to the availability of these data, which were used under license for this study. Data are available at https://www.balticexchange.com with the permission of the Baltic Exchange.

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REFERENCES
Adland, R. (2003). *The stochastic behavior of spot freight rates and the risk premium in bulk shipping* (Thesis). Department of Ocean Engineering, Massachusetts Institute of Technology.

Adland, R., & Cullinane, K. (2005). A time-varying risk premium in the term structure of bulk shipping freight rates. *Journal of Transport Economics and Policy*, *39*, 191–208.

Adland, R., & Cullinane, K. (2006). The non-linear dynamics of spot freight rates in tanker markets. *Transportation Research Part E: Logistics and Transportation Review*, *42*, 211–224.

Ahmad, R., & Wilmott, P. (2007). The market price of interest-rate risk: Measuring and modelling fear and greed in the fixed-income markets. *Wilmott Magazine*, January, 64–70.

Alizadeh, A., & Nomikos, N. K. (2009). *Shipping derivatives and risk management*. Palgrave MacMillan.

Baltic Exchange. (2020). *Spot and forward freight rates for the Capesize, Panamax and Supramax sectors*. https://www.balticexchange.com

Baltic Exchange. (2020). *Spot freight rates for the Baltic Dry Index (BDI) and spot and forward freight rates for the Capesize, Panamax and Supramax sectors*. https://www.balticexchange.com

Bandy, F. M., & Phillips, P. C. B. (2006). Fully nonparametric estimation of scalar diffusion models. *Econometrica*, *71*, 241–283.

Duffy, D. J. (2006). *Finite difference in financial engineering: A partial differential equation approach*. Wiley Finance, Wiley.

Fusai, G., & Roncoroni, A. (2008). *Implementing models in quantitative finance: Methods and cases*. Springer Finance, Springer-Verlag.

Giamouzi, M., & Nomikos, N. K. (2021). Identifying shipowners' risk attitudes over gains and losses: Evidence from the dry bulk freight market. *Transportation Research Part E: Logistics and Transportation Review*, *145*, 102129.

Gómez-Valle, L., López-Marcos, M. A., & Martínez-Rodríguez, J. (2020). Two new strategies for pricing freight options by means of a valuation PDE and by functional bounds. *Mathematics*, *8*, 620.

Gómez-Valle, L., & Martínez-Rodríguez, J. (2016). Estimation of risk-neutral processes in single-factor jump-diffusion interest rate models. *Journal of Computational and Applied Mathematics*, *291*, 48–57.

Gómez-Valle, L., & Martínez-Rodríguez, J. (2021). Including jumps in the stochastic valuation of freight derivatives. *Mathematics*, *9*, 154.

Härdle, W. (1990). *Applied nonparametric regression*. Econometric Society Monographs. Cambridge University Press.

Härdle, W., Müller, M., Sperlich, S., & Werwatz, A. (2004). *Nonparametric and semiparametric models*. Springer Series in Statistics. Springer-Verlag.

Kavussanos, M. G., & Nomikos, N. K. (1999). The forward pricing function of the shipping freight futures market. *Journal of Futures Markets*, *19*, 353–376.

Koekebakker, S., Adland, R., & Sodal, S. (2007). Pricing freight rate options. *Transportation Research Part E: Logistics and Transportation Review*, *43*, 535–548.

Kolos, S. P., & Ronn, E. I. (2008). Estimating the commodity market price of risk for energy prices. *Energy Economics*, *30*, 621–641.

Kyriakou, I., Pouliasis, P. K., Papapostolou, N. C., & Andriosopoulos, K. (2017). Freight derivatives pricing for decoupled mean-reverting diffusion and jumps. *Transportation Research Part E: Logistics and Transportation Review*, *108*, 80–96.

Lim, K. G., Nomikos, N. K., & Yap, N. (2019). Understanding the fundamentals of freight markets volatility. *Transportation Research Part E: Logistics and Transportation Review*, *130*, 1–15.

Loader, C. (1999). *Local regression and likelihood. Statistics and computing*. Springer-Verlag.

Martínez, W. L., & Martínez, A. L. (2016). *Computational statistics handbook with MATLAB* (3rd ed.). Computer Science and Data Analysis Series.Chapman & Hall/CRC.

Moutzouris, I. C., & Nomikos, N. K. (2019). The formation of forward freight agreement rates in dry bulk shipping: Spot rates, risk premia, and heterogeneous expectations. *Journal of Futures Markets*, *39*, 1008–1031.

Nomikos, N., Kyriakou, I., Papapostolou, N. C., & Pouliasis, P. K. (2013). Freight options: Price modelling and empirical analysis. *Transportation Research Part E: Logistics and Transportation Review*, *51*, 83–94.

Oksendal, B. (2003). *Stochastic differential equations—An introduction with applications*. Universitext, Springer-Verlag.

Prokopczuk, M. (2011). Pricing and hedging in the freight futures market. *Journal of Futures Markets*, *31*, 440–464.

Protter, P. E. (2005). *Stochastic modelling and applied probability* (2nd ed.). Springer-Verlag.

Schwartz, E. S. (1997). The stochastic behavior of commodity prices: Implications for valuation and hedging. *The Journal of Finance*, *52*, 923–973.

Shreve, S. (2004). *Stochastic calculus for finance II—Continuous-time models*. Springer Finance Textbooks. Springer-Verlag.

Stanton, R. (1997). A nonparametric model of term structure dynamics and the market price of interest rate risk. *The Journal of Finance*, *52*, 1973–2002.

Stopford, M. (2009). *Maritime economics* (3rd ed.). Routledge.

Tvedt, J. (1997). Valuation of VLCCs under income uncertainty. *Maritime Policy & Management*, *24*, 159–174.

UNCTAD. (2020). *Review of maritime transport*. https://unctad.org/system/files/official-document/rmt2020_en.pdf. Accessed 2021-06-15.

Weron, R. (2008). Market price of risk implied by Asian-style electricity options and futures. *Energy Economics*, *30*, 1098–1115.

Wilmott, P. (1998). *Derivatives: The theory and practice of financial engineering* (University ed.). John Wiley & Sons Inc.