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Optimization of Engineering Design Problems Using Atomic Orbital Search Algorithm

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ABSTRACT In this paper, optimum design of engineering problems is considered by means of the Atomic Orbital Search (AOS), a recently proposed metaheuristic optimization algorithm. The mathematical development of the algorithm is based on principles of quantum mechanics focusing on the act of electrons around the nucleus of an atom. For numerical investigation, 20 of well-known constrained design problems in different engineering fields are considered; some of which have been benchmarked by the 2020 Competitions on Evolutionary Computation (CEC 2020) for real-world optimization purposes. Statistical results including the best, mean, worst and standard deviation of multiple optimization runs are reported for the AOS algorithm. These results are compared to similar data from previous metaheuristic algorithms found in the literature to establish the efficiency and usefulness of the AOS. It is concluded that the AOS has acceptable behavior in dealing with all the considered constrained optimization problems while the maximum difference of about 40% between the best optimum values of the AOS and other approaches is noted for the robot gripper benchmark problem.

INDEX TERMS Atomic Orbital Search; Engineering Design; Competition on Evolutionary Computation; Constrained Optimization

I Introduction

Optimization is a process of maximizing or minimizing a predefined objective function which may be subjected to multiple design constraints. This is relevant to decision-making and to engineering design across disciplines and stakeholders. For example, chief executive officers aim to maximize the overall profit from investments in engineering construction and infrastructure. Further, practicing engineers aim to minimize resources and materials used in designing components, structures, or processes. In this regard, optimization is a ubiquitous approach to facilitate rationalized decision-making and engineering design. Indeed, inventory, production, machine learning, design procedures and machine scheduling are some of the important problems addressed by optimization in engineering fields.

The two most important facets of optimization are the solution algorithms and the mathematical formulation of the optimal design problem. Optimization algorithms should be conceptualized properly by an established mathematical model to support computationally efficient optimization solutions. Additionally, mathematically rigorous formulations or numerical descriptions of the optimal design problems are also required. The latter facet is addressed based on the physics of engineering problems and on developments in computer science. However, the development of efficient optimization algorithms leading to improved optimal solutions for complex problems is a field of open research. Whilst gradient-based optimization methods have been utilized for many years for the purpose, they are known to have numerous deficiencies which led to the birth and pursue of metaheuristic optimization algorithms. The latter algorithms involve an iterative procedure in which an optimum solution is sought by conducting some random perturbations and search loops which are defined by drawing inspiration from the lifestyle of different leaving creatures (bio-inspired) or other physics-based concepts. Some of the most well-known metaheuristics optimization algorithms are the Genetic Algorithm (GA) [1], Ant Colony Optimization (ACO) [2], Particle Swarm Optimization (PSO) [3], Imperialistic Competitive Algorithm (ICA) [4], Firefly Algorithm (FA) [5], Whale Optimization Algorithm (WOA) [6], Symbiotic Organisms Search (SOS) [7], Ray Optimization Algorithm (ROA) [8], Flower Pollination Algorithm (FPA) [9], Earthworm optimization algorithm (EWA) [10], Crystal Structure Algorithm (CryStAl) [11], Material Generation Algorithm (MGA) [12], Heat Transfer Search (HTS) algorithm [13], Teaching Learning Based Optimization (TLBO) algorithm [14], Passing vehicle search (PVS) algorithm [15], Group Teaching Optimization (GTO) algorithm [16], Aquila Optimizer (AO) [17], Capuchin Search Algorithm (CSA) [18], Archimedes Optimization Algorithm (AOA) [19], and the Chaos Game Optimization (CGO) algorithm [20 and 21]. It also should be noted that some of the standard algorithms have been improved or hybridized for specific applications [22 to 34].

Besides, some of the other challenges in optimization of engineering design problems can be mentioned as the epsilon constraint based HTS algorithm for optimization of multi-objective engineering design problems [35], Layout optimization of wind farms with an improved version of TLBO [36], design optimization of engineering problems by a hybrid approach of TLBO and the Neural Network Algorithm (NNA) [37], Symbiotic Organisms Search (SOS) algorithm for optimum design of multi-objective constrained engineering problems [38], Bayesian optimization (BO) for optimum design of engineering design problems, optimum design of real-world problems by Seagull Optimization Algorithm (SOA) [39] and the Black Widow Optimization (BWO) algorithm for optimization purposes in engineering applications [40].

In this paper, optimum design of engineering problems is considered by means of the Atomic Orbital Search (AOS), which is a recently proposed metaheuristic algorithm by Azizi [41]. This algorithm is developed based on the quantum-based atomic model which follows principles of quantum mechanics governing the act of electrons around the nucleus of an atom. For numerical investigation, 20 of the well-known constrained design problems in different engineering fields are considered, some of which have been benchmarked by the 2020 Competitions on Evolutionary Computation as CEC 2020 [42] for real-world optimization purposes. For statistical investigation, 25 independent optimization runs are conducted by considering 200000 objective function evaluations to evaluate the statistical results including the best, mean, worst and standard deviation while the results of other algorithms are also provided from the literature for conducting a comparative study.

II Atomic Orbital Search (AOS) Algorithm

a) Physical Motivation

In this section, the AOS algorithm is presented in detail focusing on the inspirational concept of the approach alongside its mathematical model. This algorithm is inspired by the principles of quantum mechanics and the atomic orbital model, proposed by Erwin Schrodinger. In this model, electrons are assumed to move in waves with uncertain location instead of orbiting in set paths around the nucleus. In this regard, clouds of probability called orbitals are defined based on the probability of electron location. In the atomic theory developed based on quantum mechanics, an atomic orbital represents the wave-like behavior of electrons in atoms by means of a mathematical function. This mathematical function is utilized for calculating the probability of finding any electron in any specific region around the nucleus of an atom. In other words, the atomic orbital represents specific physical regions or spaces surrounding the nucleus which are probable locations of electrons (Fig. 1A). In Fig. 1B a snapshot of an atom is illustrated in which the electrons are moving around the nucleus by changing their instant positions with a wave-like behavior. In this setting, the electrons behave like a cloud of charge which instantly change their position over time. As presented in Fig. 1C, the positions of electrons around the nucleus are not deterministically defined so the location of electrons around nucleus is defined by means of probability density diagrams. The space around nucleus of an atom is divided into spherical concentric thin imaginary layers with specific radius of r to measure the probability of electrons being located at any specific distance from the nucleus (Fig. 1D). Since the volume of each specific layer increases faster than the probability density of that layer (Fig. 1E), the total probability of detecting any electron in the outer imaginary layers is higher than detecting it in the inner ones.

According to the atomic orbital model, electrons in the ground state of energy are located within imaginary layers around the nucleus. For each imaginary layer with radius, r, a quantum number, n, is assigned which represents the energy level of the electrons positioned in that layer. The layers with higher n values represent the orbitals with larger r values and higher energy levels while the layers with smaller n values correspond to lower energy levels with smaller r values. The electrons in the cloud of charge around nucleus are excited by the interactions with other particles, moving into magnetic fields and also by acts of photons (lights) which result in energy emission or absorption in the atom. In this regime, some binding energy is determined for each electron which represents the amount of energy required for removing the electron from its orbital. Considering the quantum staircase analogy, movement of electrons between different orbitals are possible, resulting in changes to their energy levels. In this regard, if an electron absorbs an amount of energy less than the electron binding energy, it will undergo a transition to an outer orbital with higher energy value. Besides, if an electron emits an amount of energy more than the electron binding energy, it will be repositioned in an inner orbital with lower energy value. The schematic representation of quantum staircase analogy in atoms is illustrated in Fig. 2.

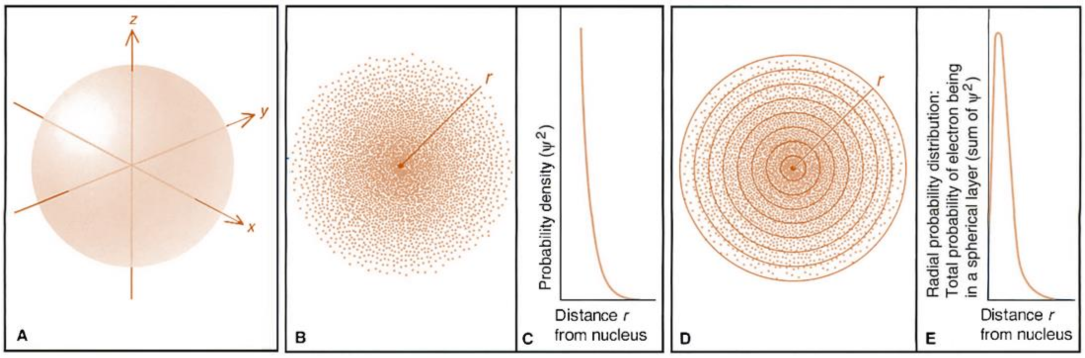


Fig. 1. Atomic orbital model and electron density configuration.

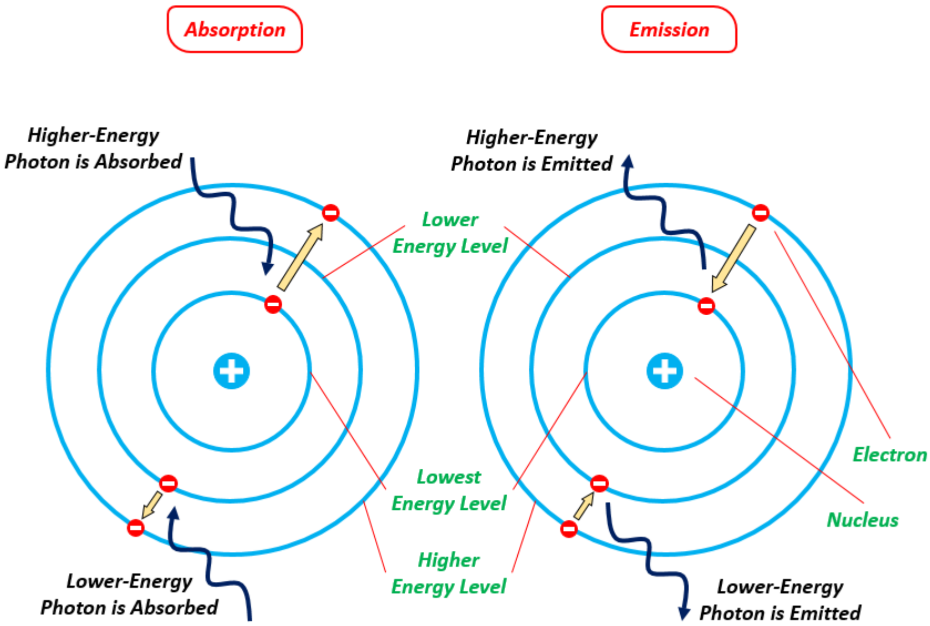


Fig. 2. Schematic representation of atomic quantum staircase analogy.

b) Mathematical Model

The AOS algorithm is inspired by the previously mentioned principles of atomic orbital model in which the emission and absorption of energy by atoms alongside the electron density configuration are in perspective. As the first step, several solution candidates, $\mathbf{X}$, are considered which correspond to the position of electrons around the nucleus of the atom. The solution candidates are taken as the cloud of electrons around the nucleus of an atom while the search space is defined as a spherical space, divided into concentric imaginary layers. Mathematically, this is written as

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_i \\ \vdots \\ X_m \end{bmatrix} = \begin{bmatrix} x_1^1 & x_1^2 & \cdots & x_1^j & \cdots & x_1^d \\ x_2^1 & x_2^2 & \cdots & x_2^j & \cdots & x_2^d \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ x_i^1 & x_i^2 & \cdots & x_i^j & \cdots & x_i^d \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ x_m^1 & x_m^2 & \cdots & x_m^j & \cdots & x_m^d \end{bmatrix}, \quad \begin{cases} i = 1,2,\ldots,m. \\ j = 1,2,\ldots,d. \end{cases} \quad (1)$$

where X_i is the i-th solution candidate (electron) in the search space (electron cloud around nucleus of atom); m is the total number of solution candidates or electrons in the search space; $x_{i,j}$ is the j-th decision variable of the i-th solution candidate; d is the dimension of the considered problem.

A random initialization procedure is employed for determining the initial positions of the electrons around the nucleus. Following the atomic model, each electron has a specific state of energy which is defined as the objective function of the solution candidates to be minimized. Therefore, the electrons with lower energy levels are represented by solution candidates with better (lower) values of the objective function while the solution candidates with worse (higher) values of objective function are utilized for electrons with higher energy levels. The following notation is introduced accordingly

$$\mathbf{E} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_i \\ \vdots \\ E_m \end{bmatrix}, \qquad i = 1,2,\ldots,m. \qquad (2)$$

where $\mathbf{E}$ is the vector of objective function values; E_i is the energy level of i-th solution candidates; m represents the total number of solution candidates or electrons in the search spac

To represent the imaginary layers around nucleus mathematically, a random integer number, n, is assigned corresponding to the number of spherical imaginary layers, L, around the nucleus of atom. The imaginarily created layers represent the wave-like behavior of electrons around nucleus while the layer with smallest radius, L_0, indicate the nucleus location and the rest, L_i the location of electrons. These aspects are presented in Fig. 3.

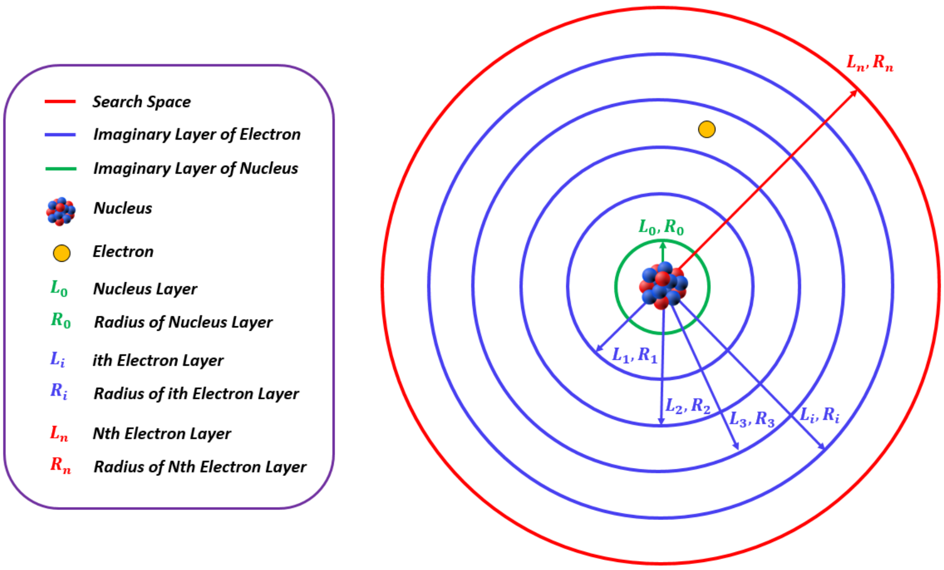


Fig. 3. Schematic presentation of imaginary layers around nucleus.

Based on the quantum-inspired atomic model, the instant locations of electrons are represented by an electron probability density diagram. This is mathematically modeled using a Probability Density Function (PDF). The latter is a mathematical function which specifies the probability of a variable value to lie within a predefined range. The PDF is used for distributing the solution candidates to the imaginary layers around nucleus. To this end, a sorting process is conducted in which the solution candidates with better objective function values (higher PDF values) are positioned in the inner layers with lower energy levels while the candidates with worse objective function values (lower PDF values), are located in the outer layers. In this regard, any of the Weibull, normal, logistic or Kernel PDF can be adopted for this purpose. Herein, the log-normal Gaussian distribution function is utilized. The position determination for electrons (solution candidates) with a log-normal Gaussian distribution function is schematically illustrated in Fig. 4. In this distribution, the overall existence probability of the electrons in the second layer (L_1 to L_2) is higher than the first layer (L_0 to L_1) which represents the real wave-like behavior of the electrons in the quantum-based atomic model.

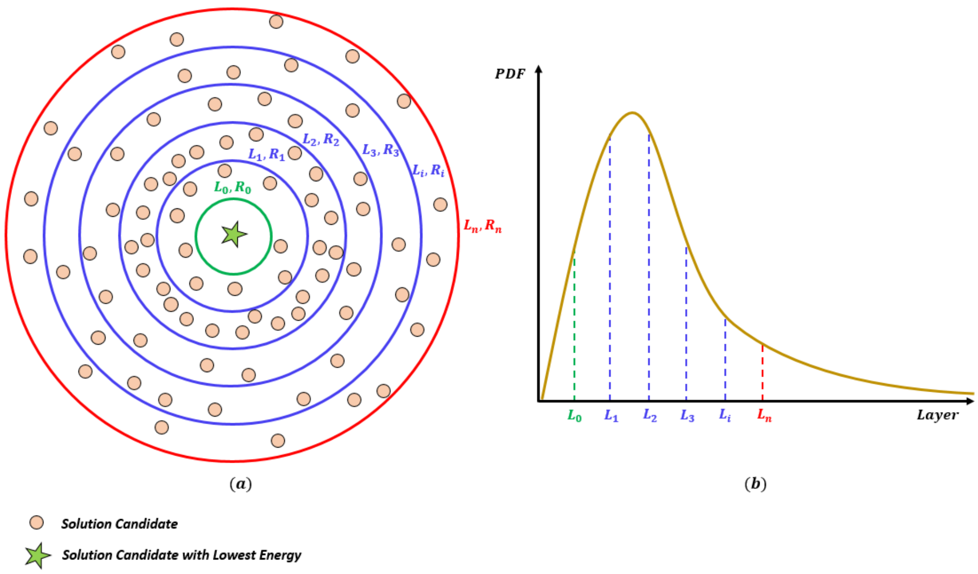


Fig. 4. Position determination of electrons (solution candidates) with PDF distribution.

Using the above position determination process for the electrons, the solutions candidates are distributed in different layers. The vector $\mathbf{X^k}$ containing the candidates in n different layers and their objective function $\mathbf{E^k}$ values are represented as follows

$$\mathbf{X^k} = \begin{bmatrix} X_1^k \\ X_2^k \\ \vdots \\ X_i^k \\ \vdots \\ X_p^n \end{bmatrix} = \begin{bmatrix} x_1^1 & x_1^2 & \cdots & x_1^j & \cdots & x_1^d \\ x_2^1 & x_2^2 & \cdots & x_2^j & \cdots & x_2^d \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ x_i^1 & x_i^2 & \cdots & x_i^j & \cdots & x_i^d \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ x_p^1 & x_p^2 & \cdots & x_p^j & \cdots & x_p^d \end{bmatrix}, \quad \begin{cases} i = 1,2,\ldots,p. \\ j = 1,2,\ldots,d. \\ k = 1,2,\ldots,n. \end{cases} \quad (3)$$

$$\mathbf{E^k} = \begin{bmatrix} E_1^k \\ E_2^k \\ \vdots \\ E_i^k \\ \vdots \\ E_p^n \end{bmatrix}, \quad \begin{cases} i = 1,2,\ldots,p. \\ k = 1,2,\ldots,n. \end{cases} \quad (4)$$

where X_i^k is the i-th candidate positioned in the k-th layer; n is the total number of imaginarily layers; p shows the number of candidates in the k-th layer; d represents the dimension for considered problem; E_i^k represents the vales of objective function for the i-th candidate positioned in the k-th layer. The best candidate in the k-th layer is considered as the electron with lowest levels of energy, LE^k, and the global best of all solution candidates represents the electron with lowest energy level, LE, at the nucleus location (Azizi 2020).

According to the principles of the atomic orbital model, the electrons are taken to be in the ground state of energy level. The concept of binding state in quantum-based atomic model represents the fact that electrons are not affected by others in this state. This attribute is mathematically modeled by considering the independency of solution candidates in the search space. In addition, the binding energy represents the energy amount that is required to move an electron to a different layer. To this end, the concepts of binding state and binding energy are mathematically modeled by considering the mean values of the position vectors and the objective function values of the solution candidates. For each of the considered imaginary layers, the binding state and binding energy are calculated as

$$BS^k = \frac{\sum_{i=1}^{p} X_i^k}{p}, \qquad \begin{cases} i = 1,2,\dots,p. \\ k = 1,2,\dots,n. \end{cases} \tag{5}$$

$$BE^k = \frac{\sum_{i=1}^{p} E_i^k}{p}, \qquad \begin{cases} i = 1,2,\dots,p. \\ k = 1,2,\dots,n. \end{cases} \tag{6}$$

where BS^k is the binding state and BE^k is the binding energy of the k-th imaginary layer.

Since the overall energy level of an atom is evaluated by considering the binding state and binding energy of all the electrons, the mathematical presentation of the mean values of the position vectors and the objective function of the solution candidates in the entire search space are written as

$$BS = \frac{\sum_{i=1}^{m} X_i}{m}, \qquad i = 1,2,\dots,m. \tag{7}$$

$$BE = \frac{\sum_{i=1}^{m} E_i}{m}, \qquad i = 1,2,\dots,m. \tag{8}$$

In the quantum atomic model, electrons with different energy states change their location and move between different layers with different states of energy. This phenomenon is due to the act of photons to electrons as well as to interactions with other particles and magnetic fields. Herein, this phenomenon is utilized for updating the solution candidates during the optimization process in the mathematical model of the AOS algorithm. Specifically, the position of the solution candidates placed in the imaginary spherical layers is updated by considering the absorption or emission of photons alongside other interactions with particles, while accounting for the energy level of electrons and the binding energy of the imaginary layers.

To facilitate the mathematical representation of the position updating process in the AOS algorithm, a randomly generated number, φ, uniformly distributed in the range of [0,1], is assigned to each electron to represent the probability of action of photons or other interactions. To distinguish between different interactions on electrons, the photon rate, PR, parameter is introduced to represent the probability of different interactions on electrons. For $\varphi \geq PR$, the act of photons on the electrons becomes possible. In this case, the energy level, E_i^k, for the i-th electron or solution candidate, X_i^k, in the k-th layer is compared to the binding energy of the k-th layer, BE^k. If $E_i^k \geq BE^k$, the solution candidates (electrons) emit some amount of energy (photon). Depending on the energy, the electron could reach the binding state, BS, of the atom or even the lowest state of energy, LE, in the atom. The position updating step for this case is written as

$$X_{i+1}^k = X_i^k + \frac{\alpha_i \times (\beta_i \times LE - \gamma_i \times BS)}{k}, \qquad \begin{cases} i = 1,2,\dots,p. \\ k = 1,2,\dots,n. \end{cases} \tag{9}$$

where X_i^k is the current and X_{i+1}^k is the updated i-th solution candidate (electron position) at the k-th imaginary layer; α_i, β_i and γ_i are uniformly distributed random numbers in the range of [0,1] which govern the amount of emitted energy.

On the antipode, if $E_i^k < BE^k$, the energy level of the i-th solution candidate in the k-th layer is lower than the binding energy of the considered layer so energy absorption becomes probable. In this case, the solution candidates (electrons) absorb some amount of energy (photon). Depending on the energy, the electron could reach the binding state of the k-th layer, BS^k, or even the lowest state of energy, LE^k, of the considered layer. The position updating step for this case is written as

$$X_{i+1}^k = X_i^k + \alpha_i \times (\beta_i \times LE^k - \gamma_i \times BS^k), \qquad \begin{cases} i = 1,2,\dots,p. \\ k = 1,2,\dots,n. \end{cases} \tag{10}$$

For $\varphi < PR$, the absorption or emission of photons on electrons are not likely so moving into magnetic fields or interactions with other particles are in perspective. In this case, the position updating step for the solution candidates is written as

$$X_{i+1}^k = X_i^k + r_i, \qquad \begin{cases} i = 1,2,\dots,p. \\ k = 1,2,\dots,n. \end{cases} \tag{11}$$

where r_i is a random number uniformly distributed in the range of [0,1].

```
Procedure Atomic Orbital Search (AOS) Algorithm
    Determine initial positions of solution candidates (X_i) in the search space with m candidates
    Evaluate fitness values (E_i) for initial solution candidates
    Determine binding state (BS) and binding energy (BE) of atom
    Determine candidate with lowest energy level in atom (LE)
    while Iteration < Maximum number of iterations
        Generate n as the number of imaginary layers
        Create imaginary layers
        Sort solution candidates in an ascending or descending order
        Distribute solution candidates in the imaginary layers by PDF
        for k=1: n
            Determine binding state (BS^k) and binding energy (BE^k) of the kth layer
            Determine the candidate with lowest energy level in the kth layer (LE^k)
            for i=1:p
                Generate φ, α, β, γ
                Determine PR
                if φ ≥ PR
                    if E_i^k ≥ BE^k
                        X_{i+1}^k = X_i^k + (α_i × (β_i × LE − γ_i × BS)) / k
                    else if E_i^k < BE^k
                        X_{i+1}^k = X_i^k + α_i × (β_i × LE^k − γ_i × BS^k)
                    end
                else if φ < PR
                    X_{i+1}^k = X_i^k + r_i
                end
            end
        end
        Update binding state (BS) and binding energy (BE) of atom
        Update candidate with lowest energy level in atom (LE)
    end while
end Procedure
```

Fig. 5. Pseudo-code of the AOS algorithm.

Further to the above updating steps, the boundary violation of solution candidates alongside the termination criterion are also considered in the mathematical model of the AOS algorithm. In this regard, a flag is implemented in the AOS in which a boundary control for violating decision variables is determined while a predefined number of objective function evaluations or iterations can be utilized as termination criteria. In Fig. 5, the pseudo-code of AOS algorithm is provided while the flowchart of the algorithm is presented in Fig. 6.

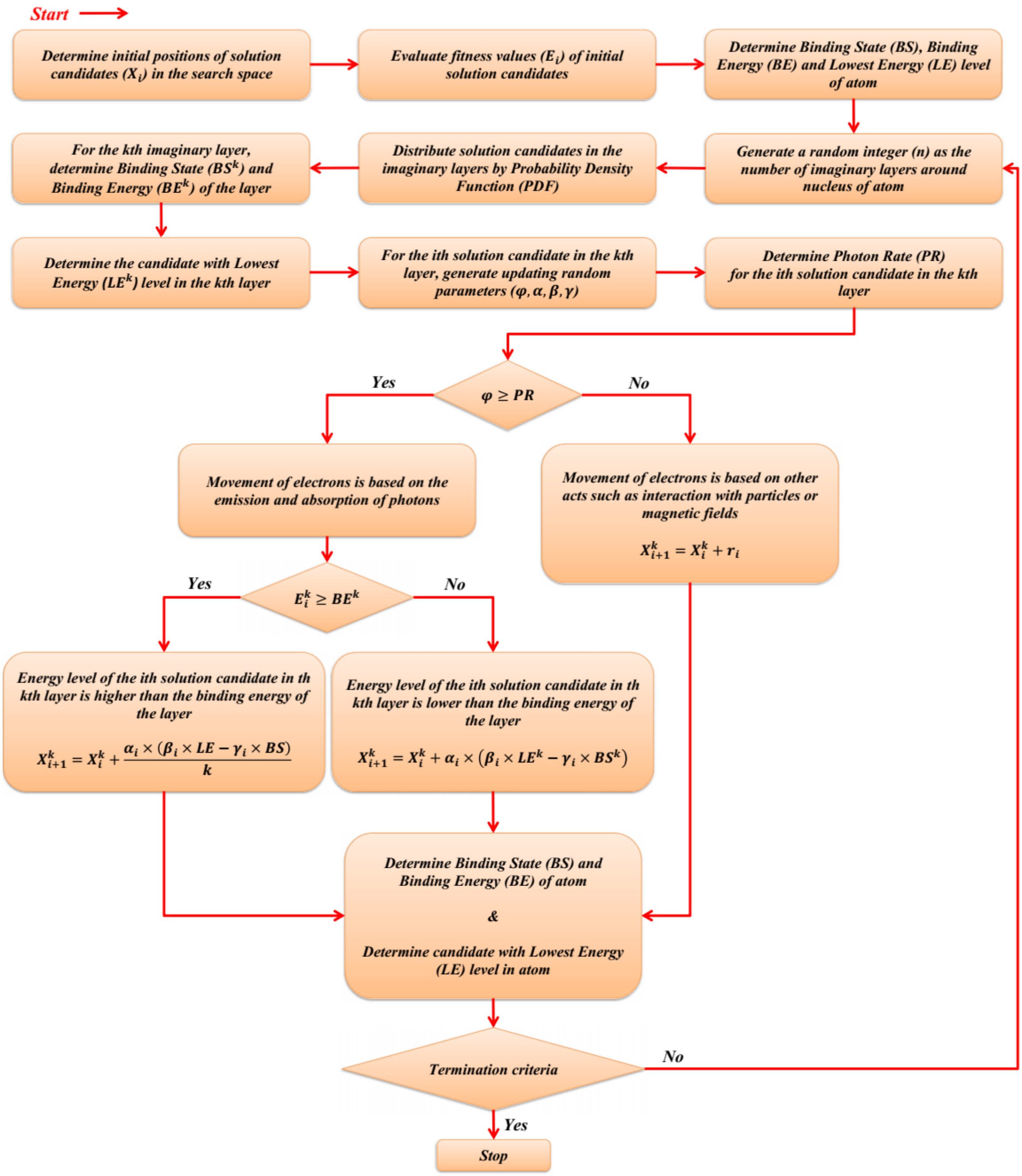


Fig. 6. Flowchart of the AOS algorithm.

III Benchmark Engineering Design Problems

Constraint optimization problems emerge naturally in optimal engineering design in which precise handling of design constraints must be accounted for in minimizing/maximizing the objective function. In this regard, the AOS algorithm is herein applied to 20 well-known constrained design problems in different engineering fields are considered, some of which being benchmarked by the 2020 Competitions on Evolutionary Computation as CEC 2020 for real-world optimization purposes. In Table 1, a brief description of these design provided is provided.

Table 1. Basic characteristics of the considered engineering design problems.

No.	Name	D	g	h	Formulation
F_1	*Industrial Refrigeration System*	14	15	0	*Andrei [43]*
F_2	*Three-Bar Truss*	2	3	0	*Gandomi et al. [44]*
F_3	*Planetary Gear Train*	9	10	1	*Savsani and Savsani [45]*
F_4	*Step-Cone Pulley*	5	8	3	*Rao [46]*
F_5	*Robot Gripper*	7	7	0	*Rao et al. [47]*
F_6	*Hydro-Static Thrust Bearing*	4	7	0	*Rao et al [47]*
F_7	*Four-Stage Gear Box*	22	86	0	*Kumar et al. [42]*
F_8	*Ten-Bar Truss*	10	3	0	*Yu et al. [48]*
F_9	*Rolling Element Bearing*	10	9	0	*Gupta et al [49]*
F_{10}	*Gas Transmission Compressor*	4	1	0	*Kumar et al. [42]*
F_{11}	*Tension/Compression Spring-Case 2*	3	8	0	*He et al. [50]*
F_{12}	*Gear Train*	4	1	1	*Zelinka and Lampinen [51]*
F_{13}	*Himmelblau's Function*	5	6	0	*Himmelblau [52]*
F_{14}	*Topology Optimization*	30	30	0	*Sigmund [53]*
F_{15}	*Steel I-Shaped Beam*	4	2	0	*Gandomi et al [44]*
F_{16}	*Piston Lever*	4	4	0	*Gandomi et al [44]*
F_{17}	*Corrugated Bulkhead*	4	6	0	*Gandomi et al [44]*
F_{18}	*Cantilever Beam*	5	1	0	*Gandomi et al [44]*
F_{19}	*Tubular Column*	2	6	0	*Gandomi et al [44]*
F_{20}	*Reinforced Concrete Beam*	3	2	0	*Gandomi et al [44]*

D: *Dimensions*
g: *Number of inequality constraints*
h: *Number of equality constraints*

IV Numerical Investigation

The results of the numerical study including the best optimum values of the AOS and other alternative algorithms alongside results statistics including the mean, worst and standard deviation are presented in this section. A simple penalty approach is considered as the constraint handling approach in dealing with these constraint problems.

a) Industrial Refrigeration System

This engineering design problem considers the optimum design of an industrial refrigeration system which has 14 design variables (x_1 ~ x_{14}) and 15 inequality design constraints. The complete mathematical formulation of this problem is presented by Andrei [43]. The best results of the AOS algorithm are presented in Table 2 alongside results from other optimization approaches. In addition, the mean, worst and standard deviation statistics for the AOS and alternative algorithms are provided in Table 3. It is seen that AOS is able to provide improved best and statistical results than the other metaheuristic approaches which represents the capability of the algorithm in dealing with difficult optimization problems.

Table 2. Best results of different approaches for the industrial refrigeration system problem.

	Andrei [43]	Present Study (AOS)
Best	0.032213008	0.032213001
x_1	0.001	0.001
x_2	0.001	0.001
x_3	0.001	0.001
x_4	0.001	0.001
x_5	0.001	0.001
x_6	0.001	0.001
x_7	1.524	1.524
x_8	1.524	1.524
x_9	5	5
x_{10}	2	2
x_{11}	0.001	0.001
x_{12}	0.001	0.001
x_{13}	0.0072934	0.007293401
x_{14}	0.0875558	0.087555832

Table 3. Statistical results for the industrial refrigeration system problem considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
(IUDE) Kumar et al. [42]	*0.0322*	*0.0322*	*0.0322*	*4.91E-18*
(MAES) Kumar et al. [42]	*0.0322*	*0.0340*	*0.0445*	*4.09E-03*
(LSHADE) Kumar et al. [42]	*0.0322*	*0.0323*	*0.0325*	*1.11E-04*
Present Study (AOS)	*0.032213*	*0.032351*	*0.032555*	*0.003146*

IUDE: *Improved Unified Differential Evolution Algorithm*
MAES: *Matrix Adaptation Evolution Strategy*
LSHADE: *Linear Success-History based Adaptive Differential Evolution*

b) Three-Bar Truss

The total weight optimization of a three-bar truss structure is considered in this design example in which the objective function is formulated by determining the minimum required cross-sectional areas for the truss bars. This engineering design problem has two design variables including the cross-sectional areas of the oblique bars (A_1) and straight bar (A_2) while there are only three inequality design constraints. In Fig. 7, a schematic presentation of this constraint design problem is shown. Gandomi et al. [44] provides the related mathematical formulations.

In Table 4, the best result of multiple optimization runs for the AOS and other algorithms in dealing with the three-bar truss problem are presented in which the optimum design variables and constraints are also provided. Most of the recently developed metaheuristics are capable of finding a similar optimum value; however, the AOS algorithm has also the ability of providing the so far best found optimum solution in this case. The statistical results of different approaches for this problem are also presented in Table 5 for comparative purposes. It is obvious that the AOS algorithm provides much better statistical results than previous approaches.

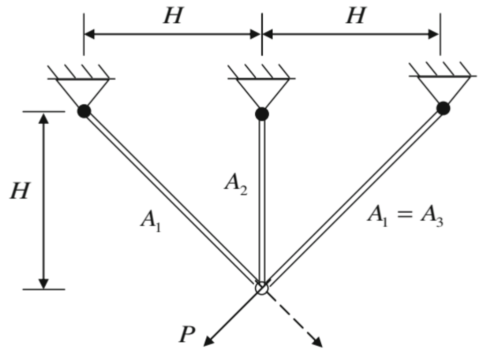


Fig. 7. Visualization of the three-bar truss problem.

Table 4. Best results of different approaches for the three-bar truss problem.

	Gandomi et al. [44]	*Ray & Liew [54]*	*Zhang et al. [55]*	*Garg [56]*	*Present Study (AOS)*
Best	*263.97156*	*263.8958466*	*263.8958434*	*263.8958433*	*263.8958433*
A_1	*0.78867*	*0.7886210370*	*0.7886751359*	*0.788676171219*	*0.7886751359*
A_2	*0.40902*	*0.4084013340*	*0.4082482868*	*0.408245358456*	*0.4082482866*
$g_1(x)$	*-0.00029*	*-8.275E-9*	*-2.104E-11*	*-1.587E-13*	*0*
$g_2(x)$	*-0.00029*	*-1.46392765*	*-1.46410161*	*-1.4641049*	*-1.4641016195*
$g_3(x)$	*-0.73176*	*-0.536072358*	*-0.5358983*	*-0.535895*	*-0.5358983805*

Table 5. Statistical results for the three-bar truss problem considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
Gandomi et al. [44]	*263.97156*	*264.0669*	*NA*	*0.00009*
Ray & Liew [54]	*263.8958466*	*263.9033*	*263.9033*	*1.26E-2*
Zhang et al. [55]	*263.8958434*	*263.8958436*	*263.8958498*	*9.72E-7*
Garg [56]	*263.8958433*	*263.8958437*	*263.8958459*	*5.34E-7*
Present Study (AOS)	*263.8958433*	*263.8958435*	*263.8958453*	*8.26E-9*

c) Planetary Gear Train

In this engineering design problem, the optimization of maximum errors in the gear ratio of the planetary gear train in the automobiles is considered. There are nine design variables including six integer variables for the number of teeth in the gears (N_1, N_2, N_3, N_4, N_5 *and* N_6) and three discrete design variables considering the modules of the first (m_1) gear, the number of planet gears (P), and the modules of the second (m_2) gear. This problem has ten inequality and one equality design constraints. In Fig. 8, a schematic presentation of this constraint design problem is prepared while Savsani and Savsani [45] provides the related mathematical formulations.

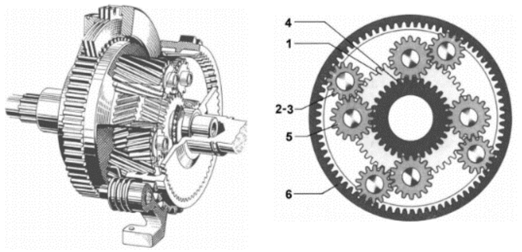


Fig. 8. Visualization of the planetary gear train problem.

In Table 6 and Table 7, the best and statistical results of the different metaheuristic algorithms including the AOS algorithm are presented for the planetary gear train problem. By comparing the best and statistical results of different approaches, it is demonstrated that the AOS performs better than previous algorithms in dealing with this complex engineering design problem with different continues and discrete design variables.

Table 6. Best results of different approaches for the planetary gear train problem.

	Savsani & Savsani [45]	*Present Study (AOS)*
Best	*0.525588*	*0.52325*
N_1	*34*	*40*
N_2	*25*	*21*
N_3	*33*	*14*
N_4	*32*	*19*
N_5	*23*	*14*
N_6	*116*	*69*
P	*4*	*3*
m_1	*2.5*	*1*
m_2	*1.75*	*2*
$g_1(x)$	*NA*	*-77*
$g_2(x)$	*NA*	*-73*
$g_3(x)$	*NA*	*-122*
$g_4(x)$	*NA*	*-0.5*
$g_5(x)$	*NA*	*-12.35490039*
$g_6(x)$	*NA*	*-15.82818888*
$g_7(x)$	*NA*	*-2.896913326*
$g_8(x)$	*NA*	*-780.4549698*
$g_9(x)$	*NA*	*-17*
$g_{10}(x)$	*NA*	*-17*
$h(x)$	*NA*	*-77*

Table 7. Statistical results for the planetary gear train problem considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
Rao & Savsani [57] (PSO)	*0.53*	*0.5361934*	*NA*	*NA*
Rao & Savsani [57] (ABC)	*0.525769*	*0.5272922*	*NA*	*NA*
Zhang et al. [55]	*0.525589*	*0.525589*	*NA*	*NA*
Savsani & Savsani [45]	*0.525588*	*0.53063*	*NA*	*NA*
Present Study (AOS)	*0.52325*	*0.529848233*	*0.537058824*	*0.003894295*

PSO: *Particle Swarm Optimization*
ABC: *Artificial Bee Colony*

d) Step-Cone Pulley

In this engineering design problem, the total weigh optimization of a step-cone pulley is considered in which there are five design variables for the width of the pulley (w) and the diameters of the steps in the pulley (d_1, d_2, d_3, *and* d_4). This problem has three equality and eight inequality design constraints. In Fig. 9, a schematic presentation of this constraint design problem is prepared while Rao [46] have provided the related mathematical formulations.

For the step-cone pulley problem, the best results of different optimization runs considering the AOS and other alternatives are presented in Table 8 while the statistical results are provided in Table 9. It is found that the AOS is capable of providing outstanding best and statistical results in dealing with this problem. It also should be noted that the AOS provides lower values for the mean, worst and standard deviation of the results.

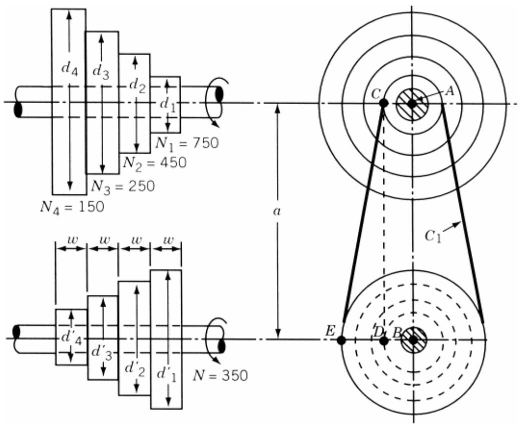


Fig. 9. Visualization of the step-cone pulley problem.

Table 8. Best results of different approaches for the step-cone pulley problem.

	(TLBO) Rao et al. [47]	*(WOA) Yildiz et al. [58]*	*(WCA) Yildiz et al. [58]*	*(MBA) Yildiz et al. [58]*	*Present Study (AOS)*
Best	*16.63451*	*16.6345213*	*16.63450849*	*16.6345078*	*16.08558875*
d_1	*40*	*40*	*40*	*40*	*38.40665412*
d_2	*54.7643*	*54.764326*	*54.764300*	*54.764300*	*52.85751197*
d_3	*73.01318*	*54.764326*	*54.764300*	*54.764300*	*70.44556099*
d_4	*73.01318*	*54.764326*	*54.764300*	*88.428419*	*84.51666791*
w	*73.01318*	*85.986297*	*54.764300*	*85.986242*	*89.98813622*

TLBO: *Teaching-Learning Based Optimization*
WOA: *Whale Optimization Algorithm*
WCA: *Water Cycle Algorithm*
MBA: *Mine Blast Algorithm*

Table 9. Statistical results for the step-cone pulley problem considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
(TLBO) Rao et al. [47]	*16.63451*	*24.0113577*	*74.022951*	*0.34*
(WOA) Yildiz et al. [58]	*16.6345213*	*20.93829477*	*24.8488259*	*3.3498*
(WCA) Yildiz et al. [58]	*16.63450849*	*17.53037682*	*18.83302997*	*0.9229*
(MBA) Yildiz et al. [58]	*16.6345078*	*16.702535*	*18.3237145*	*0.2627*
Present Study (AOS)	*16.08558875*	*16.29548945*	*16.80334816*	*0.177212917*

e) Robot Gripper

The robot gripper problem is one of the difficult engineering design problems in which the difference of the minimum and maximum force in the gripper is sought to be minimized by considering the displacement ranges of the gripper. This problem has seven design variables including the geometric properties of the robot while there are also seven inequality design constrains in the problem definition. In Fig. 10, a schematic presentation of this constraint design problem is prepared while Rao et al. [47] provide the related mathematical formulations.

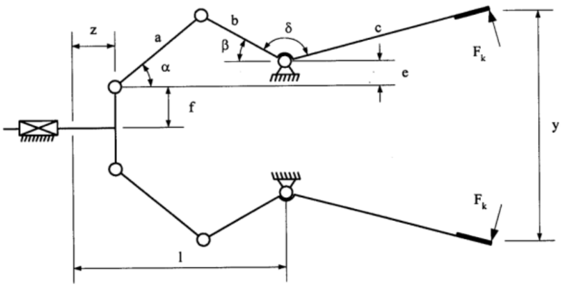


Fig. 10. Visualization of the robot gripper problem.

The best results of the AOS and other approaches for the considered robot gripper problem are presented in Table 10 while the optimum design variables and design constraints are also provided for comparative purposes. In Table 11, the statistical results of different approaches considering multiple optimization runs are also presented. It is concluded that the AOS provides outstanding results than the other metaheuristics. The maximum difference between the best results of the AOS and other algorithms is about 40%.

Table 10. Best results of different approaches for the robot gripper problem.

	(TLBO) Rao et al. [47]	*Present Study (AOS)*
Best	*4.247643634*	*2.54383687*
a	*150*	*149.9973899*
b	*150*	*149.880236*
c	*200*	*200*
d	*0*	*0*
e	*150*	*149.9954554*
f	*100*	*100.9429469*
δ	*2.339539113*	*2.297394124*
$g_1(x)$	*-28.09283911*	*-49.99999477*
$g_2(x)$	*-21.90716089*	*-5.23E-06*
$g_3(x)$	*-33.64959994*	*-49.99996461*
$g_4(x)$	*-16.35040006*	*-3.54E-05*
$g_5(x)$	*-79 999.998*	*-79737.112*
$g_6(x)$	*-9.8E-11*	*-36.02117726*
$g_7(x)$	*-0.00001*	*-0.943046876*

Table 11. Statistical results for the robot gripper problem considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
(ABC) Rao et al. [47]	*4.247644*	*5.086611*	*6.784631*	*0.07*
(TLBO) Rao et al. [47]	*4.247644*	*4.93770095*	*8.141973*	*0.56*
Present Study (AOS)	*2.54383687*	*2.791745357*	*3.143355667*	*0.226323642*

ABC: *Artificial Bee Colony*
TLBO: *Teaching-Learning Based Optimization*

f) Hydro-Static Thrust Bearing

In this engineering design problem, the optimum configuration of bearing power loss in the hydro-static thrust bearing system is considered in which four design variables including the recess radius (R_0), bearing step radius (R), flow rate (Q) and the oil viscosity (μ) with seven inequality design constraints are considered in the problem formulation. In Fig. 19, a schematic presentation of this constraint design problem is prepared while Rao et al. [47] have provided the related mathematical formulations.

Table 12 and Table 13 provide the best and statistical results of multiple optimization runs for the AOS and other approaches in dealing with the hydro-static thrust bearing design problem. The data demonstrate that the AOS has the ability of providing better results than the other metaheuristics while it yields better statistical results as the mean of runs, worst run and standard deviation values.

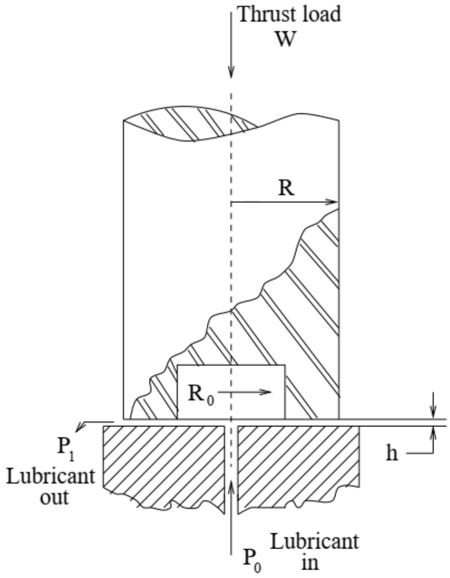


Fig. 11. Visualization of the hydro-static thrust bearing problem.

Table 12. Best results of different approaches for the hydro-static thrust bearing problem.

	Siddall [59]	*Deb & Goyal [60]*	*Coello [61]*	*Rao et al. [47]*	*Present Study (AOS)*
Best	2288:2268	2161.4215	1950.2860	1625.44276	1621.926212
R	7.155	6.778	6.271	5.9557805026	5.968100069
R_0	6.689	6.234	12.901	5.3890130519	5.402028631
μ	8.321E-06	6.096 E-06	5.605E-06	0.0000053586	5.36E-06
Q	9.168	3.809	2.938	2.2696559728	2.267705635
$g_1(x)$	-11086.7430	-8329.7681	-2126.86734	-0.0001374735	-63.57841887
$g_2(x)$	-402.4493	-177.3527	-68.0396	-0.0000010103	-3.930479341
$g_3(x)$	-35.057196	-10.684543	-3.705191	-0.0000000210	-0.039093072
$g_4(x)$	-0.001542	-0.000652	-0.000559	-0.0003243625	-0.000324394
$g_5(x)$	-0.466000	-0.544000	-0.666000	-0.5667674507	-0.566071438
$g_6(x)$	-0.000144	-0.000717	-0.000805	-0.0009963614	-0.000996358
$g_7(x)$	-563.644401	-83.618221	-849.718683	-0.0000090762	-1.865244618

Table 13. Statistical results for the hydro-static thrust bearing problem considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
Şahin et al. [59]	1625.46467	1627.744198	1650.698747	3.815546973
Rao & Waghmare [60]	1625.44271	1796.89367	2104.3776	0.21
Rao et al. [61]	1625.44276	1797.70798	2096.8012	0.19
Present Study (AOS)	1621.926212	1752.413561	1831.449755	23.6285497

g) Four-Stage Gear Box

In this design example, the weight optimization of a gear box with four stage is considered which has 22 design variables for determining the positions of the gear and pinion, number of teeth and blank thickness with 88 design constraints. The complete mathematical formulation of this problem is presented in [42]. In Table 14, the best results of the AOS algorithm considering multiple optimization runs are presented in which the design variable are also provided for clarification. The statistical results for the AOS and some other metaheuristics are also presented in Table 15 for comparative purposes. It can be concluded that the AOS algorithm provides competitive best and statistical results in dealing with the four-stage gear box as a complex engineering design problem.

Table 24. Best results of different approaches for the four-stage gear box problem.

	Present Study (AOS)
Best	37.4042245
x_1	18
x_2	43
x_3	19
x_4	41
x_5	18
x_6	32
x_7	19
x_8	41
x_9	1
x_{10}	1
x_{11}	1
x_{12}	1
x_{13}	2
x_{14}	5
x_{15}	3
x_{16}	4
x_{17}	5
x_{18}	6
x_{19}	4
x_{20}	3
x_{21}	4
x_{22}	5

Table 25. Statistical results for the four-stage gear box problem considering different approaches.

Approaches	Best	Mean	Worst	Std-Dev
(IUDE) Kumar et al. [42]	35.4	39.1	45.6	3.62
(MAES) Kumar et al. [42]	60.7	57.8	19.9	68.5
(LSHADE) Kumar et al. [42]	36.5	40.3	54.2	5.52
Present Study (AOS)	37.4042245	52.83708891	90.81422082	11.89354773

IUDE: *Improved Unified Differential Evolution Algorithm*
MAES: *Matrix Adaptation Evolution Strategy*
LSHADE: *Linear Success-History based Adaptive Differential Evolution*

h) Ten-Bar Truss

The weight optimization of a truss structure with ten structural elements is considered in this design example which has ten design variables for the cross-sectional areas of structural bars (A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}) with three inequality constraints. In Fig. 19, a schematic presentation of this constraint design problem is shown while Yu et al. [48] provide the related mathematical formulations.

In Table 16, the best results of multiple optimization runs for different metaheuristics including the AOS algorithm in dealing with the ten-bar truss design example are presented. Regarding the fact that this example is one of the well-known real-size design examples in the structural optimization field, there is a challenging competition in finding the optimum weight of this truss structure. By comparing the best results of AOS to the reported results of other alternatives, it is concluded that AOS provides outstanding optimum values. In addition, the statistical results of the AOS algorithm including the mean, worst and standard deviation of multiple optimization procedures are also provided in Table 17 for having a valid judgment.

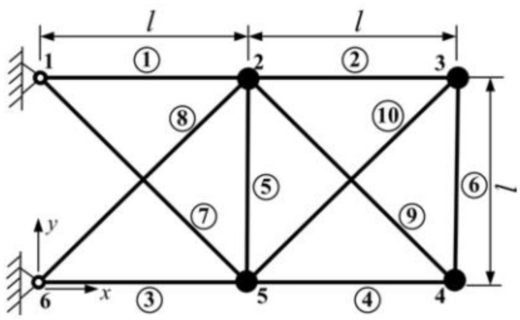


Fig. 12. Visualization of the ten-bar truss example.

Table 16. Best results of different approaches for the ten-bar truss example.

	Yu et al. [48]	Lamberti & Pappalettere [62]	Baghlani & Makiabadi [63]	Kaveh & Zolghadr [64]	Present Study (AOS)
Best	544.7	534.57	530.76	529.25	525.6788438
A_1	36.380	35.148	35.494	39.569	34.8119633
A_2	12.941	13.169	14.777	16.740	15.30794832
A_3	35.764	37.69	36.203	34.361	34.78346867
A_4	18.314	19.556	15.387	12.994	13.71838609
A_5	3.002	1.087	0.6451	0.645	0.782401649
A_6	5.433	4.844	4.5896	4.802	4.666928874
A_7	20.989	18.314	23.211	26.182	25.61578707
A_8	24.14	27.415	24.561	21.260	22.17289405
A_9	9.753	12.562	12.482	11.766	11.71022039
A_{10}	18.102	12.106	12.324	11.392	13.66557392

Table 17. Statistical results of the AOS algorithm for the ten-bar truss problem.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
Present Study (AOS)	*525.6788438*	*534.4838193*	*590.8453285*	*8.652827447*

i) Rolling Element Bearing

In the rolling element bearing design example, the optimum tuning of the load-carrying capacity rolling element bearing system is considered in which a total number of five design variables including the ball diameter (D_b), inner raceway curvature coefficient (f_i), total number of balls (Z), pitch diameter (D_m), the outer raceway curvature coefficient (f_0) and the specific design parameters of the system (K_{Dmin}, K_{Dmax}, ε, e, ζ) with nine inequality design constraints are considered in the problem definition. In Fig. 13, a schematic presentation of this constraint design problem is shown while Gupta et al. [49] provide the related mathematical formulations.

In Table 18, the best results of the AOS and other metaheuristic algorithms are presented for the rolling element bearing design example alongside the optimum design variables. The statistical results including the mean of runs, worst run and standard deviation of multiple optimization runs are also provided in Table 19 for competitive purposes. Based in the results, it is concluded that the AOS is capable of providing very competitive results among other approaches.

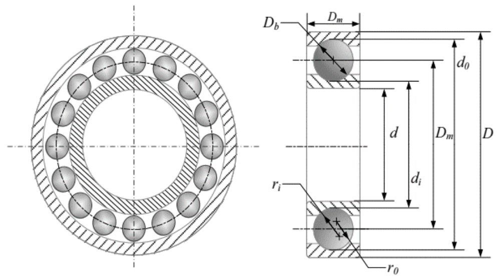


Fig. 13. Visualization of the rolling element bearing problem.

Table 18. Best results of different approaches for the rolling element bearing example.

	(TLBO) Rao et al. [47]	*(ABC) Yildiz et al. [58]*	*(GWO) Yildiz et al. [58]*	*(ALO) Yildiz et al. [58]*	*Present Study (AOS)*
Best	*81859.74*	*85428.2495*	*85529.0830*	*85546.6377*	*83918.49253*
D_m	*21.42559*	*125.6599*	*125.7090*	*125.718*	*125*
D_b	*125.7191*	*21.40862*	*21.42316*	*21.425242*	*21.875*
Z	*11*	*11*	*11*	*11*	*10.77700905*
f_i	*0.515*	*0.515*	*0.515*	*0.515*	*0.515*
f_0	*0.515*	*0.515*	*0.529322*	*0.5157018*	*0.515*
K_{Dmin}	*0.424266*	*0.427166*	*0.420867*	*0.4541646*	*0.476110618*
K_{Dmax}	*0.633948*	*0.668849*	*0.633296*	*0.6464928*	*0.658142645*
ε	*0.3*	*0.3*	*0.300224*	*0.3000122*	*0.3*
e	*0.068858*	*0.071386*	*0.02*	*0.0638003*	*0.02*
ζ	*0.799498*	*0.6*	*0.619432*	*0.6107592*	*0.618242202*

TLBO: *Teaching-Learning Based Optimization*
ABC: *Artificial Bee Colony*
GWO: *Grey Wolf Optimizer*
ALO: *Ant Lion Optimizer*

Table 19. Statistical results for the rolling element bearing problem considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
(TLBO) Rao et al. [47]	*81859.74*	*81438.987*	*80807.8551*	*0.66*
(ABC) Yildiz et al. [58]	*85428.2495*	*85121.7544*	*83859.0851*	*362.57*
(GWO) Yildiz et al. [58]	*85529.0830*	*83395.0849*	*43543.4508*	*8224.5*
(ALO) Yildiz et al. [58]	*85546.6377*	*84032.8636*	*73872.8164*	*3121.8*
Present Study (AOS)	*83918.49253*	*82175.21266*	*83826.38337*	*23.38511*

j) Gas Transmission Compressor

In this engineering design problem, the design optimization of a gas transmission compressor is considered which has four design variables with one inequality design constraint. The complete mathematical formulation of this problem is presented by Kumar et al. [42]. The best results of the AOS algorithm in dealing with this problem are presented in Table 20 while the statistical results for different approaches are also provided in Table 21 for comparative purposes. Since the results of other metaheuristics were not provided with accurate digits, it can be concluded that the results of the AOS is somehow better than the results of other metaheuristics.

Table 20. Best results of different approaches for the gas transmission compressor problem.

	Present Study (AOS)
Best	*2964895.417*
x_1	*50*
x_2	*1.178283953*
x_3	*24.59259097*
x_4	*0.388353075*
$g(x)$	*0*

Table 21. Statistical results for the gas transmission compressor problem considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
(IUDE) Kumar et al. [42]	*2.96E+06*	*2.96E+06*	*2.96E+06*	*6.59E-10*
(MAES) Kumar et al. [42]	*2.96E+06*	*2.96E+06*	*2.96E+06*	*0.00E+00*
(LSHADE) Kumar et al. [42]	*2.96E+06*	*2.97E+06*	*2.97E+06*	*1.23E+03*
Present Study (AOS)	*2964895.417*	*2965102.327*	*2966483.832*	*251.8360974*

IUDE: *Improved Unified Differential Evolution Algorithm*
MAES: *Matrix Adaptation Evolution Strategy*
LSHADE: *Linear Success-History based Adaptive Differential Evolution*

k) Tension or Compression Spring-Case 2

This problem is an extension of the tension or compression spring while the difference between this case and the standard version of this problem is in the objective functions and the design variables. In this case, the volume minimization of the required steel wire for a helical tension or compression spring is considered while three continuous, discrete and integer design variables (d, D, N) are considered for problem definition with a total number of eight inequality design constraints. The mathematical formulation and comprehensive description of this constraint example is provided by He et al. [50].

In Table 22, the best result of different approaches for the case 2 of tension or compression spring problem are provided in which the optimum values for the design variables and design constraints are also presented. It can be concluded that the AOS algorithm achieves better results than the other alternative algorithms. The statistical results of the AOS algorithm including the mean of runs, worst run and standard deviation of multiple optimization runs are also included in Table 23 for a comparison.

Table 22. Best results of different approaches for the tension or compression spring (Case 2).

	Lampinen & Zelinka [65]	*Deb & Goyal [60]*	*Sandgren [66]*	*He et al. [50]*	*Present Study (AOS)*
Best	*2.65856*	*2.665*	*2.7995*	*2.65856*	*2.615360373*
d	*0.283*	*0.283*	*0.283*	*0.283*	*7.200436705*
D	*1.223041010*	*1.226*	*1.180701*	*1.223041010*	*1.364635836*
N	*9*	*9*	*10*	*9*	*0.2905583*
$g_1(x)$	*-1008.8114*	*-713.510*	*-54309*	*-1008.8114*	*-44.67223896*
$g_2(x)$	*-8.9456*	*-8.933*	*-8.8187*	*-8.9456*	*-9.407128275*
$g_3(x)$	*-0.083*	*-0.083*	*-0.08298*	*-0.083*	*-0.0905583*
$g_4(x)$	*-1.777*	*-1.491*	*-1.8193*	*-1.777*	*-1.635364164*
$g_5(x)$	*-1.3217*	*-1.337*	*-1.1723*	*-1.3217*	*-1.696599054*
$g_6(x)$	*-5.4643*	*-5.461*	*-5.4643*	*-5.4643*	*-5.464216405*
$g_7(x)$	*0*	*0*	*0*	*0*	*0*
$g_8(x)$	*0*	*-0.009*	*0*	*0*	*-0.000161721*

Table 23. Statistical results of the AOS algorithm for the tension or compression spring (Case 2).

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
Present Study (AOS)	*2.615360373*	*2.64371161*	*2.863796184*	*0.042854835*

l) Gear Train

In this design problem, the optimization of a compound gear train is considered in which the overall ratio of the gears is to be minimized. There are four design variables for the number of teeth in the gears of the system (z_d, z_b, z_a, z_f) with only one inequality design constraint. In Fig. 14, a schematic presentation of this constraint design problem is shown while Zelinka and Lampinen [51] provide the related mathematical formulations.

In Table 24, the best results of the AOS and some other metaheuristic alogirthms in dealing with the gear train design problem are presented alongside the optimum design variables. Since the main aim of this problem is to reach to a lower ratio of the gears, the AOS is capable of providing the lowest possible minimum value for this ratio in the optimization process. In addition, the statistical results of different approaches are presented in Table 25 in which the superiority of the AOS algorithm in obtaining better mean of multiple runs, worst run and standard deviation results are seen.

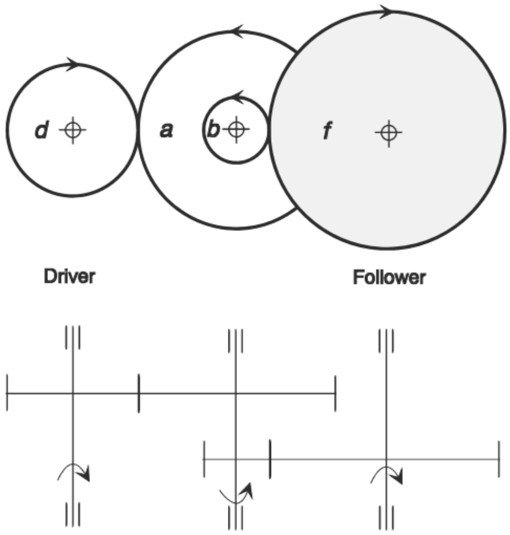


Fig. 14. Visualization of the gear train problem.

Table 34. Best results of different approaches for the gear train problem.

	Gandomi et al. [43]	*Loh & Papalambros [67]*	*Kannan & Kramer [68]*	*Sandgren [66]*	*Present Study (AOS)*
Best	2.701E-12	2.7E-12	2.146E-08	5.712E-06	2.29E-19
z_d	19	19	13	18	16.17108014
z_b	16	16	15	22	14.24826982
z_a	43	43	33	45	39.40873922
z_f	49	49	41	60	40.52327337

Table 35. Statistical results for the gear train problem considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
Gandomi et al. [43]	2.7009E-12	1.9841E-9	2.3576E-9	3.5546E-9
Loh & Papalambros [67]	2.7E-12	2.7E-12	2.7E-12	2.2122E-28
(CPKH) Wang et al. [69]	2.22E-16	2.22E-16	8.5E-09	7.96E-22
(ABC) Wang et al. [69]	2.92E-15	3.18E-15	8.5E-09	9.81E-10
Present Study (AOS)	2.29E-19	6.25E-15	9.06E-14	1.26E-14

CPKH: *Chaotic Particle Swarm Krill Herd*
ABC: *Artificial Bee Colony*

m) Himmelblau's Function

Himmelblau's function is a well-known nonlinear benchmark constraint optimization problem which has been utilized as test function for performance evaluation of different novel and improved metaheuristic algorithms. This problem has five design variables with six inequality constraints while the complete mathematical presentation of this problem is provided by Himmelblau [52]. In Table 26, the best results of different metaheuristic algorithms are provided for evaluating her overall performance of the AOS algorithm in which the optimum design variables and design constraints are also included. It is seen that the AOS yields acceptable results in dealing with this problem. Statistical results of different optimization runs including the mean of results, worst run and standard deviation are also presented in Table 27 for comparison.

Table 26. Best results of different approaches for the Himmelblau's function.

	Runarsson & Yao [70]	*Himmelblau [52]*	*Gen & Cheng [71]*	*He et al. [50]*	*Present Study (AOS)*
Best	-30665.539	-30373.949	-30183.576	-30665.539	-30665.539
x_1	78	78.62	81.49	78	78
x_2	33	33.44	34.09	33	33
x_3	29.995256025682	31.07	31.24	29.995256025682	29.99525603
x_4	45	44.18	42.2	45	45
x_5	36.775812905788	35.22	34.37	36.775812905789	36.77581291
$g_1(x)$	-92	-91.7927	-91.7819	-92	-92
$g_2(x)$	-98.8405	-98.8929	-99.3188	-98.8405	-11.15949969
$g_3(x)$	-20	-20.1316	-20.0604	-20	-8.840500309

Table 27. Statistical results of AOS algorithm for the Himmelblau's function.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
Present Study (AOS)	-30665.539	-30638.19946	-30317.71871	66.71554068

n) Topology Optimization

Herein, the material layout optimization of a simply supported structural element in dealing with a predefined set of loadings is considered. This problem has 30 design variables which considers the geometric configuration of the element with 30 inequality design constraints. In Fig. 15, a schematic presentation of the problem is shown while Sigmund [53] provides the related mathematical formulations.

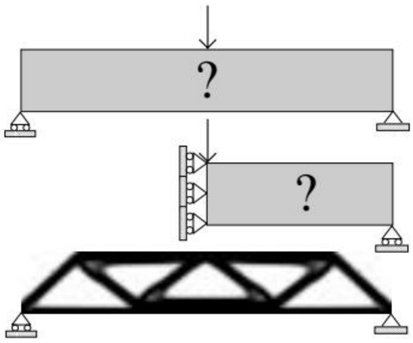


Fig. 15. Visualization of the topology optimization problem.

In Table 28, the best result of the AOS algorithm is provided alongside the optimum design variables. Statistical results of different metaheuristic algorithms based on different optimization runs including the mean of results, worst run and standard deviation values are reported in Table 29. It is found that the AOS algorithm provides improved statistical results compared to other approaches.

Table 28. Best AOS result for the topology optimization problem.

	Present Study (AOS)
Best	2.639346497
x_1	1
x_2	1
x_3	1
x_4	1
x_5	1
x_6	1
x_7	1
x_8	1
x_9	1
x_{10}	1
x_{11}	1
x_{12}	1
x_{13}	1
x_{14}	1
x_{15}	1
x_{16}	1
x_{17}	1
x_{18}	1
x_{19}	1
x_{20}	1
x_{21}	1
x_{22}	1
x_{23}	1
x_{24}	1
x_{25}	1
x_{26}	1
x_{27}	1
x_{28}	1
x_{29}	1
x_{30}	1

Table 29. Statistical results for the topology optimization problem considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
(IUDE) Kumar et al. [42]	2.64	2.64	2.64	4.44E-16
(MAES) Kumar et al. [42]	2.65	2.65	2.65	8.64E-03
(LSHADE) Kumar et al. [42]	2.64	2.64	2.64	1.03E-15
Present Study (AOS)	2.639346497	2.639346497	2.639346497	1.33227E-15

IUDE: *Improved Unified Differential Evolution Algorithm*
MAES: *Matrix Adaptation Evolution Strategy*
LSHADE: *Linear Success-History based Adaptive Differential Evolution*

o) Steel I-Shaped Beam

In this design example, the minimization of vertical displacement in a simply-supported steel I-shaped beam is considered in which there are four design variables including the width of the flanges (*b*), height of the web (*h*), thickness of the web (t_w), and the thickness of the flanges (t_f) with two inequality design constraints. In Fig. 16, a schematic presentation of this constraint design problem is prepared while Gandomi et al. [44] provide the related mathematical formulations.

The best results of different optimization algorithms including the AOS algorithm are presented in Table 30 while the optimum design variables are also included. In addition, statistical results of different optimization runs are also provide in Table 31 for having a valid comparative investigation. It is seen that the AOS yields improved results in dealing with this kind of complex optimization problem.

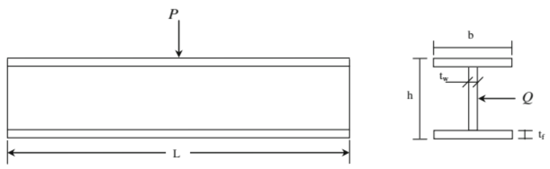


Fig. 16. Visualization of the steel I-shaped beam problem.

Table 30. Best results of different approaches for the steel I-shaped beam problem.

	(ARSM) Wang [72]	*(I-ARSM) Wang [72]*	*(MATLAB) Wang [72]*	*(CSA) Gandomi et al. [44]*	***Present Study (AOS)***
Best	*0.0157*	*0.131*	*0.0131*	*0.0130747*	*0.01307412*
h	*80*	*79.99*	*80*	*80*	*80*
b	*37.05*	*48.42*	*50*	*50*	*50*
t_w	*1.71*	*0.9*	*0.9*	*0.9*	*0.9*
t_f	*2.31*	*2.4*	*2.32*	*2.3216715*	*2.321792097*

ARSM: *Adaptive Response Surface Method*
I-ARMS: *Improved Adaptive Response Surface Method*
MATLAB: *Matrix Laboratory Optimization Approach*
CSA: *Cuckoo Search Algorithm*

Table 31. Statistical results for the steel I-shaped beam problem considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
Gandomi et al. [44]	0.0130747	0.0132165	0.01353646	0.0001345
Present Study (AOS)	0.01307412	0.013178898	0.013814045	0.000155511

p) Piston Lever

In his problem, the volume optimization of the required oil in the piston lever is considered to optimally tune the position of the piston. There are four design variables in this problem including the *H*, *B*, *X* and *D* which represent the position of the piston with only four inequality design constraints. In Fig. 17, a schematic presentation of this constraint design problem is prepared while Gandomi et al. [44] provide the related mathematical formulations.

In Table 32, the best results of AOS and other metaheuristic algorithms are presented while the statistical results including the mean of the results, worst run and standard deviation of multiple optimization runs are provided in Table 33. By comparing the results, it is found that the AOS outranks the other approaches.

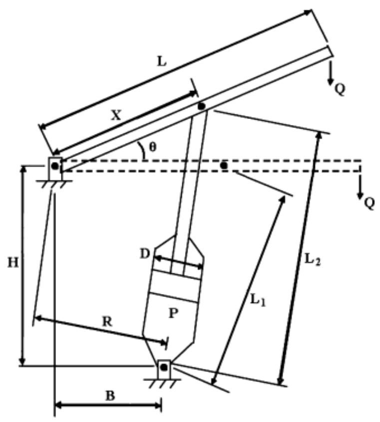


Fig. 17. Visualization of the piston lever problem.

Table 32. Best results of different approaches for the piston lever problem.

	Gandomi et al. [44]	*Present Study (AOS)*
Best	*8.4271*	*8.419142742*
H	*0.05*	*0.05*
B	*2.043*	*2.042112482*
X	*120*	*119.951727*
D	*4.0851*	*4.084004492*

Table 33. Statistical results for the piston lever problem considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
(HPSO) Gandomi et al. [44]	*162*	*187*	*197*	*13.4*
(GA)(Gandomi et al. [44]	*161*	*185*	*216*	*18.2*
(DE) Gandomi et al. [44]	*159*	*187*	*199*	*14.2*
(CSA) Gandomi et al. [44]	*8.4271*	*40.2319*	*168.5920*	*59.0552*
Present Study (AOS)	*8.419142742*	*33.7412759*	*60.66498628*	*93.46674724*

HPSO: *Hybrid Particle Swarm Optimization*
GA: *Genetic Algorithm*
DE: *Differential Evolution*
CSA: *Cuckoo Search Algorithm*

q) Corrugated Bulkhead

In this problem, the weight minimization of a corrugated bulkhead in tankers is considered. The problem has 4 design variables including the width (b), length (l), depth (h) and thickness (t) of the bulkhead with 6 inequality constraints. The problem is mathematically presented by Gandomi et al. [44]. Table 34 reports the best results of the AOS and other metaheuristic algorithms including the optimum design variables while the statistical results are also provided in Table 35 for comparative purposes. It is seen that AOS is competitive.

Table 34. Best results of different approaches for the corrugated bulkhead problem.

	Gandomi et al. [44]	*Present Study (AOS)*
Best	*5.894331*	*6.84295801*
b	*37.1179498*	*57.69230769*
h	*33.0350210*	*34.14762035*
l	*37.1939476*	*57.69230769*
t	*0.7306255*	*1.05*

Table 35. Statistical results for the corrugated bulkhead problem considering different approaches.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
Gandomi et al. [44]	*5.894331*	*5.988257*	*6.126749*	*0.064360*
Present Study (AOS)	*6.84295801*	*7.060808377*	*7.066936186*	*0.000649111*

r) Cantilever Beam

In this design example, the weight minimization of a cantilever beam with 5 stepped hollow square sections is considered. There are 5 design variables including the width of the beam in different cross sections (x_1, x_2, x_3, x_4 *and* x_5) with only one inequality design constraints. Gandomi et al. [44] provide the related mathematical formulations.

In Table 36, the best results of different metaheuristic alongside the results of AOS are presented for comparative purposes. The optimum design variables are also provided for clarity. By comparing the obtained results of the AOS to the results of other algorithms, it is concluded that the AOS provides improved results. For completeness, the AOS statistical results are presented in Table 37.

Table 36. Best results of different approaches for the cantilever beam problem.

	(MMA) Gandomi et al. [44]	*(GCA-I) Gandomi et al. [44]*	*(GCA-II) Gandomi et al. [44]*	*(CSA) Gandomi et al. [44]*	*Present Study (AOS)*
Best	*1.34*	*1.34*	*1.34*	*1.33999*	*1.339956366*
x_1	*6.01*	*6.01*	*6.01*	*6.0089*	*6.016165407*
x_2	*5.3*	*5.3*	*5.3*	*5.3049*	*5.308902645*
x_3	*4.49*	*4.49*	*4.49*	*4.5023*	*4.494577659*
x_4	*3.49*	*3.49*	*3.49*	*3.5077*	*3.501505539*
x_5	*2.15*	*2.15*	*2.15*	*2.1504*	*2.152508461*

MMA: *Method of Moving Asymptotes*
GCA: *Generalized Convex Approximation*
CSA: *Cuckoo Search Algorithm*

Table 37. Statistical results for the cantilever beam problem considering AOS algorithm.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
Present Study (AOS)	*1.339956366*	*1.351954573*	*1.491711377*	*0.02499743*

s) Tubular Column

In this problem, the material and construction cost optimization of a tubular column is sought. There are three design variables including the average column section thickness (t) and average diameter of the column section (d) and six inequality design constraints. In Fig. 18, a schematic presentation of this problem is shown while Gandomi et al. [44] provide the related mathematical formulations.

In Table 38, the best results of different metaheuristics including the AOS algorithm are presented for comparative purposes while the statistical results are also presented in Table 39 based on multiple optimization runs. It is found that the AOS performs better in this problem.

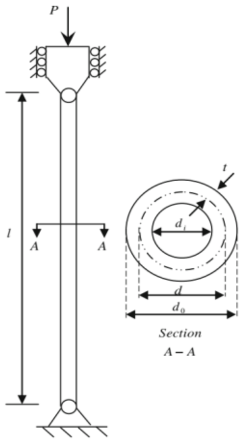


Fig. 18. Visualization of the tubular column problem

Table 38. Best results of different approaches for the tubular column problem.

	Hsu & Liu [73]	Rao [46]	Gandomi et al. [44]	Present Study (AOS)
Best	25.5316	26.5323	26.53217	26.53137828
d	5.4507	5.44	5.45139	5.451152962
t	0.292	0.293	0.29196	0.291966716
$g_1(x)$	-7.8E-05	-0.8579	-0.0241	-3.64E-06
$g_2(x)$	0.1317 *	0.0026 *	-0.1095	-2.47E-06
$g_3(x)$	-0.6331	-0.8571	-0.6331	-0.633105141
$g_4(x)$	-0.6107	0	-0.6106	-0.610631931
$g_5(x)$	-0.3151	-0.75	-0.3150	-0.314990412
$g_6(x)$	-0.6350	0	-0.6351	-0.635041605

* Violated Sets

Table 39. Statistical results for the tubular column problem considering different approaches.

Approaches	Best	Mean	Worst	Std-Dev
Gandomi et al. [44]	26.53217	26.53217	26.53972	0.00193
Present Study (AOS)	26.53137828	26.53161399	26.60821361	0.001030078

t) Reinforced Concrete Beam

Herein, the cost optimization of a reinforced concrete beam is sought. There are three design variables including the steel area (A_s), beam depth (h) and beam width (b) and two inequality design constraints. In Fig. 20, a schematic presentation of the problem is shown while Gandomi et al. [44] provide the mathematical formulation.

In Table 40, the best results of different approaches including the AOS algorithm are presented. It is seen that the AOS provides better results than other algorithms. For completeness, statistical AOS results from different optimization runs are presented in Table 41.

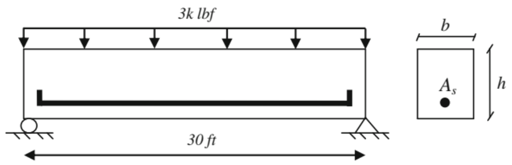


Fig. 20. Visualization of the reinforced concrete beam.

Table 40. Best results of different approaches for the reinforced concrete beam problem.

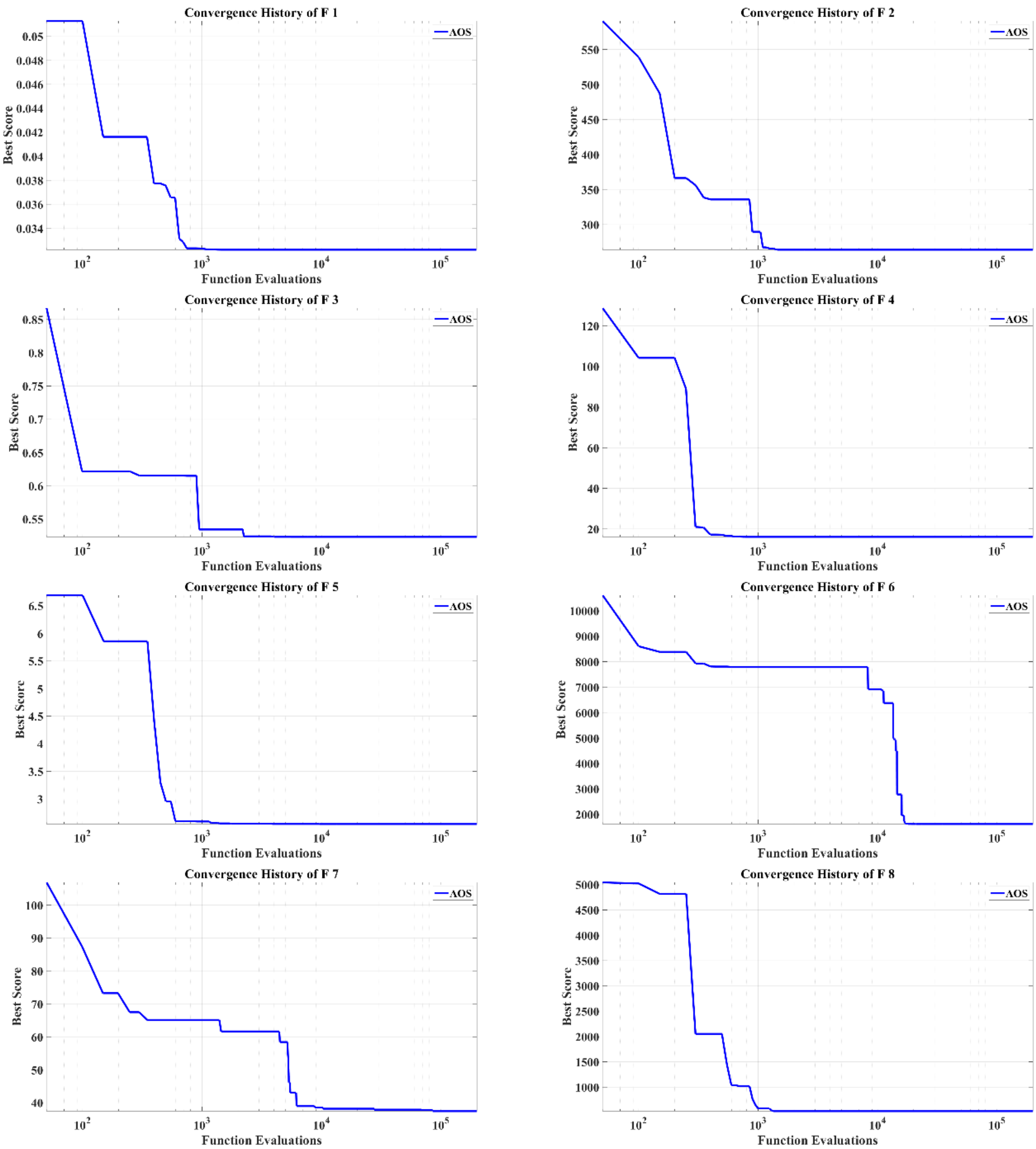
	Amir & Hasegawa [74]	Shih & Yang [75]	Yun [76]	Gandomi et al. [44]	Present Study (AOS)
Best	374.2	362.00648	364.8541	359.2080	359.20800
As	7.8	6.32	6.16	6.32	6.32
b	31	34	35	34	34
h	7.79	8.637180	8.7	8.5	8.5
$g_1(x)$	-4.2012	-0.7745	-3.6173	-0.2241	-0.224094986
$g_2(x)$	-0.0205	-0.0635	0	0	-1.00E-07

Table 41. Statistical results for the reinforced concrete beam problem considering AOS algorithm.

Approaches	*Best*	*Mean*	*Worst*	*Std-Dev*
Present Study (AOS)	*359.20800*	*359.3306872*	*362.2535612*	*0.59614901*

V Convergence History

In this section, the convergence behavior of the AOS algorithm in dealing with the considered constraint optimization problems is presented to demonstrate the convergence trends of the AOS to the optimum values of the objective functions in each of the considered problems. In Fig. 21, these convergence curves are illustrated in which the best results of 25 independent runs are determined for the considered problems.



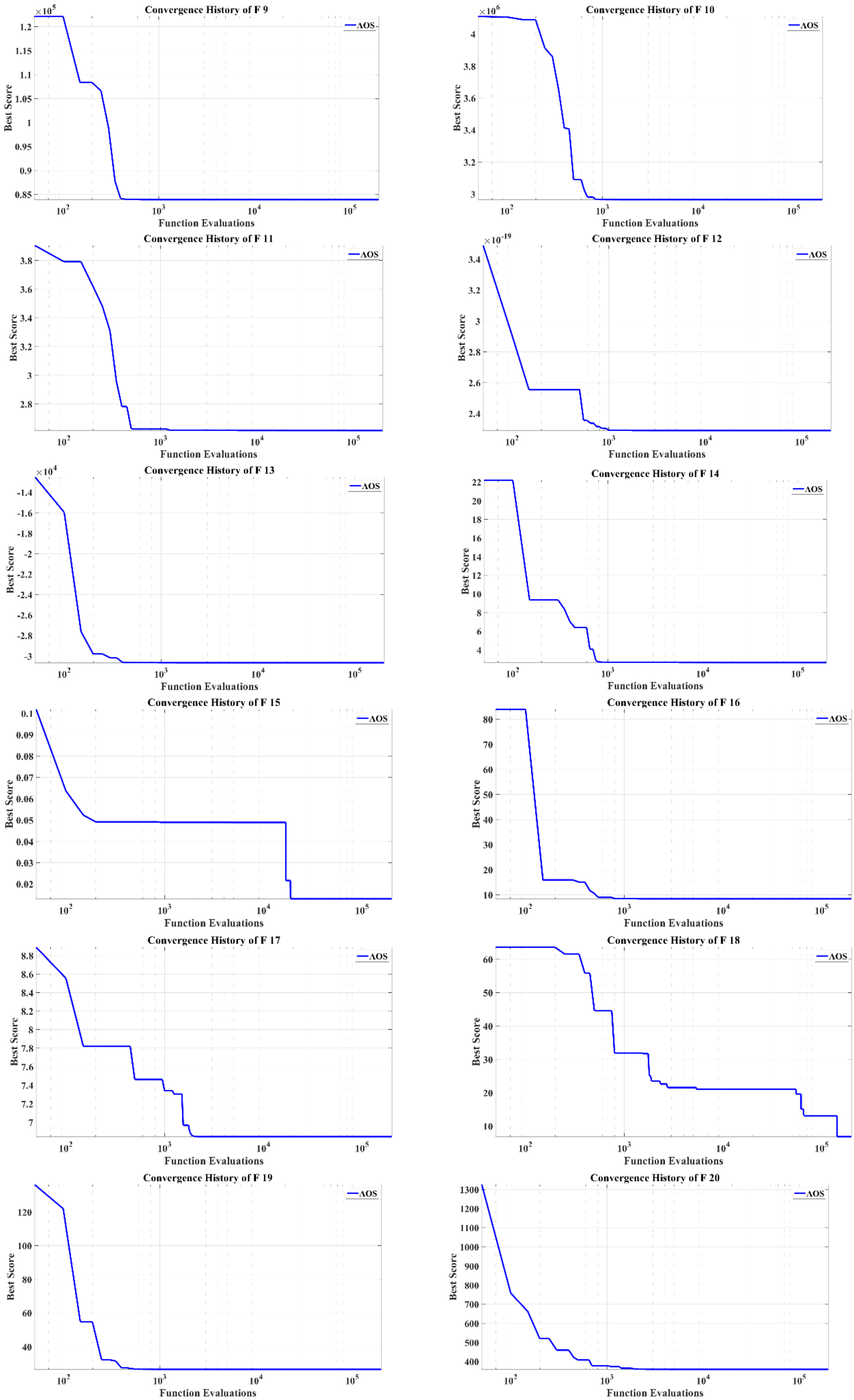


Fig. 21. Convergence history of the AOS for different constraint problems.

VI Conclusions

Optimum design of engineering problems has been addressed in this paper by means of the Atomic Orbital Search (AOS). The inspirational concept of this algorithm stems from the quantum-based atomic model relying on principles of quantum mechanics. For numerical investigation, 20 well-known constrained design problems in different engineering fields have been considered corresponding to real-life optimization benchmark design problems. By evaluating the results of the AOS algorithm in dealing with the considered engineering design problems, it was found that AOS has better performance in most cases as evidenced by comparing to the results of other metaheuristic algorithms from the recent literature. The maximum difference between the best optimum values of the AOS and other approaches are about 40% for robot gripper problem. In addition, the results of the AOS algorithm in dealing with three of the considered design examples including the four-stage gear box problem, rolling element bearing and the corrugated bulkhead are very competitive regarding the results of other approaches. The herein reported results renders the AOS a promising approach to tackle large-scale complex engineering optimization problems such as optimization-driven design of building structures under gravitational, wind, and seismic loads [22,25,30,77,78]. Such applications are left for future work.

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