The binomial-match, outcome uncertainty, and the case of netball

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Abstract. We introduce the binomial-match as a model for the bivariate score in a paired-contest. This model is naturally associated with sports in which the restart alternates following a goal. The model is a challenger to the Poisson-match, a pair of independent Poisson random variables whose means are related to the strengths of the competing teams. We use the binomial-match primarily to study the relationship between outcome uncertainty and scoring-rate, particularly for high values of the scoring-rate. Netball has a high scoring-rate and motivates our model development. In the binomial-match framework, we also evaluate rule-variations, and study tactical play in netball. Our analysis suggests that the binomial-match is not a better forecaster than the Poisson-match, but it is better for representing outcome uncertainty and evaluating rule-variations and tactics. In general, we find that the binomial-match implies greater outcome uncertainty than the Poisson match, for a given scoring-rate, and that an alternating-restart is a good rule for reducing the frequency of tied outcomes. For netball in particular, we show that starting the final quarter with possession in a close, balanced match may confer a significant advantage.

Keywords: sport, outcome uncertainty, competitive balance, forecasting, netball, tactics.

1. Introduction

1.1. *Aim and motivation*

The aim of this paper is to study the relationship between outcome uncertainty and scoring-rate in a paired-contest, particularly when the scoring-rate is high. Scarf et al. (2019) showed that the scoring-rate in international rugby union has more than doubled since 1960, while in soccer it has remained broadly static (Baker and McHale, 2018). Then, in the same study, the authors were interested to know if rugby union has become more predictable as a result. To answer this question requires a model for the score in the sport. The model was the Poisson-match. An evaluation of how outcome uncertainty depends on scoring-rate is not possible by directly analysing results because team strengths are a confounding factor. The authors concluded that if one accepts the Poisson-match as a good model for rugby union then the sport is less competitive than it used to be. This is an issue (increasing scoring-rate) that has not been addressed by administrators; we will return to this point in more detail below.

Then, one might be curious about outcome uncertainty in ball-sports with high scoring-rates like basketball and netball. Two natural questions arise. What is a good model for such sports? Given such a model, how does outcome uncertainty depend on scoring-rate? In this paper, we answer these two questions.

1.2. *Outcome uncertainty and its importance*

In the professional era, sports are seeking to increase revenue by making their products more attractive to consumers (Beech and Chadwick, 2013). Innovative formats (Cannonier et al., 2015), modifications to scoring rules (Percy, 2009; Arias et al., 2011), and new sports (Heino, 2000) are indicative. In this competitive, broadcaster-led environment, it is important to study the drivers of consumer-demand for sport, factors such as: the exhibition of athletic skill (Trail et al., 2003); excitement (Mutz and Wahnschaffe, 2016); tribal attachments (Cova, 2007); human narratives (Lines, 2001); social interaction (Kim et al., 2013); and participation (Grix and Carmichael, 2012). It is hypothesised that outcome uncertainty (Rottenburg, 1956), and thereby suspense and surprise (Ely et al., 2015; Bizzozero et al., 2016), are important factors. The media put it more simply: “predictability is the death of sport” (Booth, 2013). Much research (e.g. Forrest and Simmons, 2002; Borland and Macdonald, 2003; Alavy et al., 2010; Buraimo and Simmons, 2015; Hogan et al., 2013, 2017; Kuchar and Martin, 2016; Schreyer et al., 2017) has considered the hypothesis, and sports ruling-bodies (administrators) do indeed seek to manage outcome uncertainty: for instance, via the draft in north American sports (Szymanski, 2003); and through the appropriate distribution of revenues from broadcaster-rights in European soccer leagues (Szymanski and Késenne, 2004).

For a given sport, two confounded factors contribute to outcome uncertainty. These are competitive balance (strength variation) and the design of the contest. Thus, for a given set of competitors (fixed strength variation), the outcome of a contest, whether a single match or tournament with many competitors, depends on the format of the contest. Many examples give support to this notion (e.g. Csató, 2021). In soccer, one can compare the FA Cup with the football leagues in England and Wales (17 different winners and runners up of the former in the last 20 years, 7 for the latter). A knockout tournament is more uncertain than a round-robin (Scarf et al., 2009). Seeding decreases outcome uncertainty (Scarf and Yusof, 2011; Norman, 2015). The winner of a series of matches is more predictable than the winner of an individual contest in the series, as reflected in the odds in betting markets for test-match series in rugby union and one-day internationals in cricket, for example.

1.3. *Outcome uncertainty and the Poisson-match*

Outcome uncertainty depends on the length of a contest. This is evident in a Poisson-match (Maher, 1982), an idealised contest between two teams, in which the teams each have a constant propensity to score and scores occur purely at random at the respective propensities. The propensities to score (called the scoring-rates) quantify the strengths of the competitors and hence quantify competitive balance. If strengths and hence scoring-rates are equal then the contest is perfectly competitively balanced. If scoring-rates are unequal, then the longer the contest the greater is the difference in the expected scores and the more certain is the outcome. Equivalently, the higher the scoring-rate, the more certain is the outcome. By way of an aside, one can observe here the subtle distinction between competitive balance (strength variation) and outcome uncertainty (the extent to which an outcome cannot be known in advance), although many works exist that use outcome variation to measure competitive balance (e.g. Utt and Fort, 2002; Jessop, 2006; Owen et al., 2007; Manasis et al., 2013).

Scarf et al. (2019) show that when one team is twice as strong as the other (the scoring-rate ratio is two) the "2-1 game" is the most unpredictable. Soccer approximates to a 2-1 game (Heuer et al., 2010; Baker and McHale, 2018). International rugby union approximates to a 10-5 game (noting that the average points value of a score in rugby union 3.5, equating to 35-18); in the 60s rugby union was a 6-3 game. A high-scoring Poisson-match is more predictable than a low-scoring Poisson-match, and it is perhaps no coincidence that the most popular sport in the world is the most unpredictable.

This has implications for the design of sporting contests. Thus, increasing scoring-rates may offer more of what consumers want (more goals) but also less of what they want (fewer upsets, fewer close, tense matches), so rule-changes that increase scoring-rates (e.g. Friesl et al., 2017, 2020) may not be a "good thing", overall. If video-assistant refereeing (FIFA, 2019) is neutral with respect to goal-scoring in soccer, so that better decision-making does not change the scoring-rate, then administrators have managed an unintended consequence (Kendall and Lenten, 2017). Decision-making by video replay has existed for some time in cricket and rugby, and in aside, it would be interesting to study whether such innovation has modified outcome uncertainty.

1.4. *A new model for a high-scoring contest*

The relationship between outcome uncertainty and scoring-rate implied by the Poisson-match suggests that the Poisson-match is not a good model for high-scoring sports. Otherwise, such sports would be necessarily predictable. Therefore, new models must be sought. Thus, this paper, motivated by netball, proposes the binomial-match.

We choose netball because it has a very high scoring-rate and it is simpler to model than basketball. These two sports have much in common: passing from hand to hand and scoring by "shooting" the ball into the hoop or basket. Broadly stated, netball is a version of basketball played mostly but not exclusively by women. In netball, "dribbling" and contact between an offensive player and defensive opponent are not permitted, a goal counts one point, and possession at the restart alternates regardless of which team scores (*standard* rule). In basketball, the conceder restarts with possession (*catch-up* rule).

There are no modelling studies of netball. For basketball, there exist many quantitative studies (e.g. Wolfers, 2006; Glickman and Sonas, 2015; Lopez and Matthews, 2015; Neudorfer and Rosset, 2018), some of which consider Poisson scores (Merritt and Clauset, 2014; Ruiz and Perez-Cruz, 2015; Martín-González et al., 2016). However, the "three-point basket", a scoring-rule innovation introduced to NBA in 1979, and penalty shots imply three scoring-modes (Baker and McHale, 2013), which makes modelling scores in basketball more difficult, not least because, in the extensive data that exist for basketball results, the numbers of each type of score in a match are typically not recorded. There are also similarities between netball and handball, and there exist some studies of scoring in the latter (e.g. Dumangane et al., 2009; Meletakos and Bayios, 2010; Groll et al., 2020). In handball, the scoring is simple, like netball, but restarts use the *catch-up* rule.

1.5. *Growth of women's sport*

The importance of women's sport and the media coverage of it are growing, particularly for soccer, rugby and cricket (Petty and Pope, 2019). For netball, growth is rather weak (Vann et al., 2015), with interest confined broadly to Commonwealth countries and major domestic tournaments to Australia, New Zealand and the UK. One might speculate that its apparent lack of appeal is because it is relatively uncompetitive and the high-scoring rate, while offering audiences frequent goals, does not offer much by way of suspense and surprise. Or perhaps it is that the major sports are investing in their women's games regardless of the quality-hypothesis (Buraimo et al., 2009) and hence consumer demand. These are critical matters for the development of netball, as administrators of the sport attempt to manage sponsorship, broadcasting rights, labour controls, tournament design, and rule innovation. These considerations further underpin the importance of this paper.

1.6. *Restart rules and tactics*

The binomial-match provides a framework for evaluating innovative rules (e.g. Wright, 2014) and tactics (e.g. Wright, 2009; Percy, 2015) in a high-scoring sport. This is because it specifically models the restart. Restart rules have been widely studied (Brams et al., 2018), but mostly in "service" sports (Wright, 1988; Pollard and Barnett, 2006). Therefore, we briefly discuss restart variations, and also a tactical question that has important consequences for the management of slow-play (time-wasting).

1.7. *Structure of the paper*

We first describe netball and netball data. Then, we present models of the bivariate score in a paired-contest, including the Poisson-match, the binomial-match, and how they parameterise the strengths of competitors (Section 3). Then, we describe estimation for the binomial-match (Section 4) and the results (Section 5). Our study of the association between outcome uncertainty and scoring-rate follows in Sections 6.1 and 6.2. We compare the forecasting performance of the two models in Section 6.3. Restart variations and our tactical question are studied in Sections 6.4 and 6.5.

2. About netball and the data

Two teams of seven players each compete on a rectangular court with elevated goals (rings) at each end. Players have specific roles, which restrict their movement to particular parts of the court. Players pass the ball from hand to hand. A player can only take one step when in possession, and must release the ball within three seconds of receiving it. At elite level, play is fast. A point (goal) is scored when a player (in a designated scoring role) shoots the ball through the ring. The match is played in four quarters of 15 minutes each. A centre pass (CP) starts each quarter and restarts play following each goal. The end of each quarter, signaled by a hooter, can occur at any point in play. Tournament rules vary regarding matches tied at the end of normal time. The first CP is decided by the toss of a coin. Subsequent CPs alternate between the teams, regardless of which team has scored or whether the hooter interrupts play. This restart-rule is a distinct feature of netball, and is important for the model development in Section 3.2.

We use data of 364 matches from five seasons (2014–2018) of the UK Superleague (NetballSL, 2021; Scoreboard, 2021). In 2014, eight teams played a home and away round robin (HARR) followed by knock-out play-off between the four top teams (4KO). The same eight teams played the same tournament format in 2015 and 2016. In 2017, the tournament was expanded to ten teams—with the addition of Sirens, Wasps and Severn Stars and the loss of Yorkshire—and the tournament design was unchanged HARR+4KO. This was repeated in 2018, although the HARR was incomplete—some teams only played 17 matches. In the play-offs (4KO), first place in the HARR plays at home against fourth and second plays third. Teams with equal points in the HARR are separated using score-difference.

The data are the bivariate scores for each match (Figure 1). From 2015 onwards, in the HARR, matches tied at the end of normal time used overtime to determine a result (win or loss). In 2014, there were five drawn matches among the 56 played (9%). For 2015 onwards, there was no information about whether overtime occurred in a match or not.

We can see (Figure 2) that netball is high-scoring. The mean home team score is 52.7 and the mean away team score is 50.9. The total score appears Poisson. A chi-squared goodness-of-fit test supports this: $X^2 = 5.04$ for 8 intervals (<90, 91−95,…,116−120, >120) compared to the critical value at 10% significance, $\chi^2_{6,0.90} = 10.6$. There is a slight negative association in home and away scores but no association between total score and score difference (Figure 2).

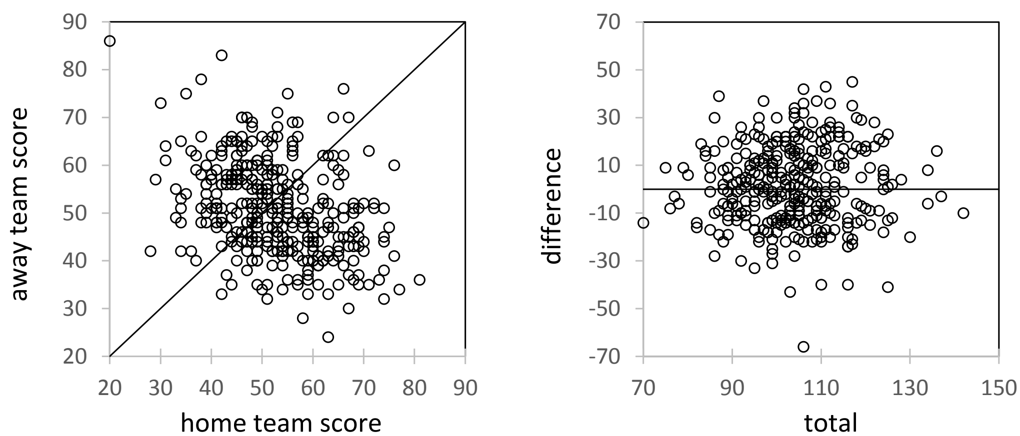


Figure 1. Match scores in UK Superleague 2014-2018. Left: home team score vs away team score. Right: total score versus score difference.

3. Bivariate models for netball scores

In this section, we present the models, beginning here with ideas and notation common to the models. Then, in Section 3.1, the Poisson-match is briefly described. What is new in the paper (the binomial-match) is described in Section 3.2. Section 3.3. describes a non-parametric model, which is a comparator that is useful for judging goodness-of-fit of the parametric models. Section 3.4 is a digression about modelling possession sequences.

We model the final score (X_1, X_2) in a match between team 1, at home, and team 2. The teams have attack and defence strengths α_i and β_i respectively, $i = 1, 2$, so that team 1 attacks with effective strength $\mu_1 = \delta\alpha_1 / \beta_2$, where δ is home advantage, and team 2 attacks with effective strength $\mu_2 = \alpha_2 / \beta_1$. In a simple model $\beta_i = r\alpha_i, i = 1, 2$, so that

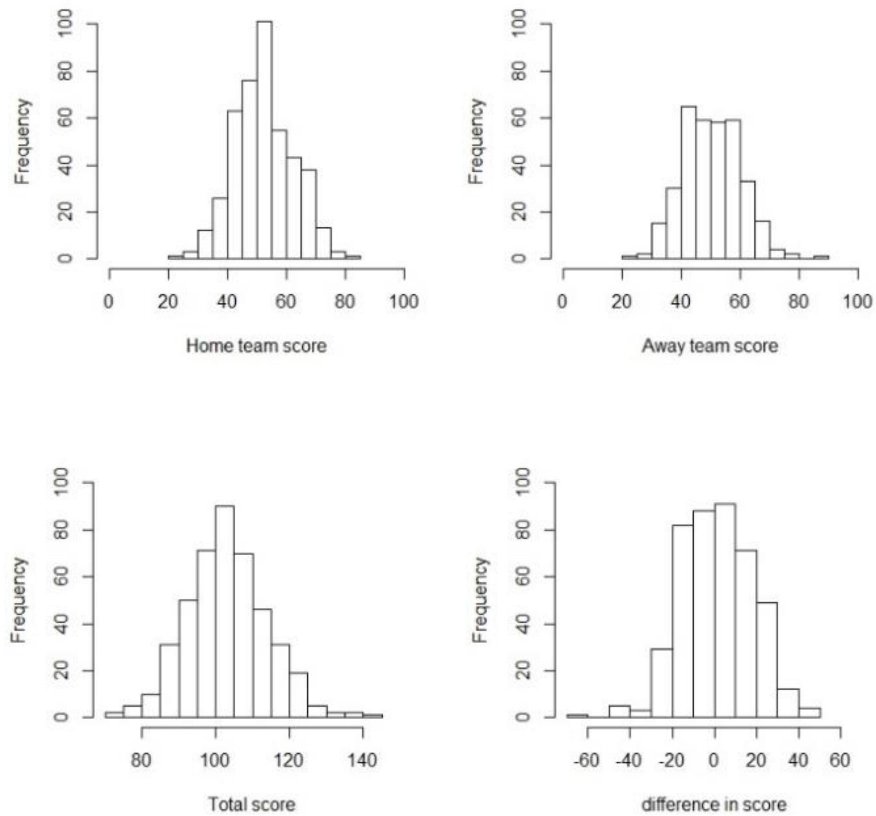


Figure 2. Histograms of scores in UK Superleague 2014-2018.

$$\mu_1 = \delta\alpha_1 / r\alpha_2,\ \ \mu_2 = \alpha_2 / r\alpha_1. \tag{1}$$

Strengths are relative in (1), so we require one constraint on the strength-parameter space. We set $\alpha_2 = 1$, so that strength is measured relative to one particular team.

3.1. *Poisson-match*

This model is due to Maher (1982). Goals-scored X_1 and X_2 are independent Poisson variates with means μ_1 and μ_2, respectively, as in (1). We will call this model a *Poisson-match.* Note, this Poisson model can be written $\Pr(X_1, X_2 | N)\Pr(N)$, where N is the total score and is itself Poisson distributed, and $\Pr(X_1, X_2 | N)$ is the binomial probability that the j th goal is scored by team 1 rather than team 2: ${}^{N}C_{X_1} s^{X_1}(1-s)^{X_1}$ with $s = \mu_1 / (\mu_1 + \mu_2) = \delta\alpha_1^2 / (\delta\alpha_1^2 + \alpha_2^2)$. Hence, conditional on N, we obtain the Bradley-Terry model (Bradley and Terry, 1952; Dewart and Gillard, 2019; Baker and Scarf, 2020). The parameter r appears only in $\Pr(N)$, thus quantifying the overall scoring-rate.

Note also that the mean total score, $\mu = \delta\alpha_1 / \beta_2 + \alpha_2 / \beta_1$, inflates when a strong side play a weak side, since if $\delta\alpha_1 > \alpha_2$ and $\beta_1 > \beta_2$ then $\mu = \delta\alpha_1 / \beta_2 + \alpha_2 / \beta_1 > 2(\delta\alpha_1 + \alpha_2)/(\beta_1 + \beta_2) = \bar{\mu}$, where $\bar{\mu}$ is the mean total score when two equal teams of average attack strength $(\delta\alpha_1 + \alpha_2)/2$ and average defence strength $(\beta_1 + \beta_2)/2$ play each other. This inflation is apparent to a slight extent in Figure 2.

3.2. *Binomial-match*

This is our new model of a bivariate score (X_1, X_2). Now, $N \sim \mathrm{Po}(\pi)$, $Y_1 \sim B(N, p_1)$ and $Y_2 \sim B(N, p_2)$ independently given N, and

$$X_1 = Y_1 + N - Y_2,\ X_2 = Y_2 + N - Y_1. \tag{2}$$

We call this a *binomial-match.* π encodes the overall scoring-rate, and p_1 and p_2 encode the strengths of the two competitors in this game. This model describes an alternating sequence of Bernoulli trials in which teams take turns (1212…) and a success is recorded for 1 not only when 1 succeeds but also when 2 fails. This is essentially a "penalty shoot-out" in which every shot results in a "goal", "one for us if we score, one for them if we miss". At each trial, each team has its own constant success probability (p_1 and p_2) and trial outcomes are independent. In a simple binomial-match, the number of trials, $2N$, is a fixed, even integer. In an unbalanced binomial-match, the total number of trials can be odd, whence team 1 may get one more attempt than team 2. We omit the mathematical details of this model. In this paper, we model the outcome of a game of netball using an unbalanced binomial-match that has a random number of trials.

For this contest, the mean values of X_1 and X_2 are given by

$$\mu_1 = E(X_1) = \pi(p_1 + 1 - p_2),\ \mu_2 = E(X_2) = \pi(-p_1 + 1 + p_2).$$

We will use these later for finding $\Pr(X_1 > X_2)$ as a function of μ_1 for this model. Also,

$$\mathrm{var}(X_1) = \pi\left(1 + 3p_1 - p_2 - 2p_1p_2\right),$$

$$\mathrm{var}(X_2) = \pi\left(1 - p_1 + 3p_2 - 2p_1p_2\right),$$

and

$$\mathrm{corr}(X_1, X_2) = \frac{1 - p_1 - p_2 + 2p_1p_2}{\sqrt{\left(1 + 3p_1 - p_2 - 2p_1p_2\right)\left(1 - p_1 + 3p_2 - 2p_1p_2\right)}}.$$

See Appendix 1 for derivations of these results. When $p_1 = p_2 = p$, we have

$$\mathrm{corr}(X_1, X_2) = (1 - 2p + 2p^2)/(1 + 2p - 2p^2),$$

which is 1 when $p = 0$ and $p = 1$, has a minimum value of 1/3 when $p = 1/2$, and takes the value 0.5 when $p = 0.789$. This value of *p* is typical of netball at the very highest level (see Appendix 2). Thus, the structure of the binomial-match induces a positive correlation between the scores X_1 and X_2.

Netball possesses a structure similar to the binomial-match because the centre pass (CP) (the restart after a goal) alternates. Thus, let us define a *play* in netball as the sequence of events (passes, turnovers, etc) beginning with a CP and ending at the subsequent goal. Then, if team 1 starts a play then either it scores on this play or the other team scores on the play. Either way, team 2 starts the subsequent play, and so on. This is exactly the structure of the alternating trials in the binomial-match. Netball departs slightly from the binomial-match because the hooter, which calls an end to a quarter, may interrupt a play. Nonetheless, only a relatively small number of plays are interrupted by the hooter. This is because; a) the hooter may sound between a goal and the start of the next play; and b) the typical ratio of hooters to goals is 1/25.

In the binomial-match, we set

$$p_1 = 1 - \exp(-\delta\alpha_1 / \beta_2), \; p_2 = 1 - \exp(-\alpha_2 / \beta_1), \tag{3}$$

so that the "play-success" probabilities are functions of the team strengths. We will see later that typical values of p_i are 0.6 to 0.8. Other functional forms for the parameterisation of p_i might be tried. Also, other discrete distributions (with over- or under-dispersion relative to Poisson) might be tried (e.g. Baker and Kharrat, 2018).

3.3. *Distribution-free model*

We also set up a non-parametric model, which we use to benchmark the forecast-accuracy and implicit goodness-of-fit of the above models. Here, the total number of goals scored by team i in all matches to date, and the total number of goals scored in all matches to date in which team i played, are calculated. At the start of each season these goals-scored are reduced to 25% of their current value. Thus, $K_{i,t}$ is the discounted total number of goals scored by team i in all matches up to time t and $T_{i,t}$ is the discounted total number of goals scored in all matches up to time t in which team i played. If at time t team i has not played any matches, we set $K_{i,t} = 0.025$ and $T_{i,t} = 0.05$; these values are close to zero but avoid division by zero (division overflow) when predicting the first result for a new team, whence $K_{i,t} / T_{i,t} = 1/2$.

Now suppose team 1 plays team 2 at time t. We assume that the strengths of 1 and 2 in this match are $K_{1,t} / T_{1,t}$ and $K_{2,t} / T_{2,t}$, respectively. Denote these strengths by A_1 and A_2, dropping the subscript t to simplify the notation, so that $A_i = K_{i,t} / T_{i,t}, \quad i = 1,2$. In this way, the model ignores home advantage and the identities of the opponents of a team played to date. Next, we assume that a prediction for the outcome of the match 1 vs 2 is $(cA_1 / A_2, cA_2 / A_1)$ for some constant c. Thus, the predicted score of each team is proportional to its effective strength. We can evaluate c because, in a match with N goals, what is not scored by team 1 is scored by team 2 and vice versa. Thus $cA_1 / A_2 + cA_2 / A_1 = N$, and so $c = NA_1A_2 / (A_1^2 + A_2^2)$, and so the forecast score is

$$\left\{ \frac{NA_1^2}{(A_1^2 + A_2^2)}, \frac{NA_2^2}{(A_1^2 + A_2^2)} \right\},$$

and team 1 wins if

$$\frac{NA_1^2}{(A_1^2 + A_2^2)} > \frac{NA_2^2}{(A_1^2 + A_2^2)},$$

otherwise team 2 wins. Notice also that this model forecasts no ties.

For forecasting, we set N equal to the mean goals per match, calculated over the entire dataset without discounting.

3.4. *Modelling possession sequences*

Refinement of the binomial-match is possible given possession data (e.g. Appendix 2). First define q_1 (and q_2) as the probability that team 1 (2) scores given that it is in possession, and note that the single alternative to not scoring is losing possession, with probability $1 - q_1$ ($1 - q_2$). Then, the probability that team 1 (2) ultimately scores on the play they start, p_1 (p_2), is the sum of an alternating geometric series.

That is, team 1 ultimately scores on the play they start if they score on their first possession, with probability q_1, or on their second, with probability $(1-q_1)(1-q_2)q_1$, and so on. Thus

$$p_1 = \frac{q_1}{1-(1-q_1)(1-q_2)} = \frac{q_1}{q_1+q_2-q_1q_2},$$

and

$$p_2 = \frac{q_2}{1-(1-q_1)(1-q_2)} = \frac{q_2}{q_1+q_2-q_1q_2}.$$

Note that

$$q_1 = \frac{p_1+p_2-1}{p_2}, \quad q_2 = \frac{p_1+p_2-1}{p_1},$$

and that

$$p_1+p_2 = \frac{1}{1-q_1q_2/(q_1+q_2)} \geq 1. \tag{4}$$

Notice that (4) follows because, under the model, if team 1 does not score from a play then team 2 must, although in reality a small number of possession sequences end with the hooter rather than a goal ($\sim 1\%$).

Then, using possession data, one could estimate strengths using the specification (3).

4. Estimation

This section discusses technical matters about the estimation for the binomial-match. Strength variation over time (Section 4.2), emergence of new teams (Section 4.3), and detail of the parameterisation (Section 4.4) require full discussion for the model to be used in practice. Nonetheless, readers whose focus is the model and its use could move directly to Section 5.

4.1. *Vanilla likelihood*

Denote the parameter vector by ϑ. We develop the likelihood for an unbalanced binomial-match, with N_1 trials for team 1 and N_2 for team 2, and $N = N_1 + N_2$. Consider the likelihood $L(\vartheta, \mathbf{x}_{12})$ for a single match k in which the score is $\mathbf{x}_{12} = (X_1 = x_1, X_2 = x_2)$. Let us first condition on $N_1 = n_1$ and $N_2 = n_2$. Then

$$\begin{aligned}
&L_k(\vartheta, \mathbf{x}_{12} | n_1, n_2) \\
&= \Pr(X_1 = x_1, X_2 = x_2 | N_1 = n_1, N_2 = n_2) \\
&= \Pr(X_1 = x_1 | N_1 = n_1, N_2 = n_2) \\
&= \sum_{y_1=\max(0,x_1-n_2)}^{\min(x_1,n_1)} \binom{n_1}{y_1} p_1^{y_1}(1-p_1)^{n_1-y_1} \binom{n_2}{n_2-x_1+y_1} p_2^{n_2-x_1+y_1}(1-p_2)^{x_1-y_1},
\end{aligned} \tag{5}$$

because: (i) $x_2 = n_1 + n_2 - x_1$; (ii) team 1 scores $X_1 = x_1$ goals in total if they score y_1 from the n_1 plays they start and $x_1 - y_1$ from plays the other team starts, the latter being equivalent to team 2 scoring $n_2 - x_1 + y_1$ goals from the plays they start; (iii) $Y_1 \sim B(n_1, p_1)$ and $Y_2 \sim B(n_2, p_2)$, independently, conditional on $N_1 = n_1$ and $N_2 = n_2$; and (iv) if $x_1 < n_2$ then the least team 1 score from their own plays is 0 and the most is x_1, otherwise it is $x_1 - n_2$ and n_1 respectively.

Now notice that when $x_1 + x_2$ is odd either team 1 start the one extra play or team 2 start the one extra play, and so either $n_1 = (x_1 + x_2 + 1)/2$ and $n_2 = (x_1 + x_2 - 1)/2$ or $n_1 = (x_1 + x_2 - 1)/2$ and $n_2 = (x_1 + x_2 + 1)/2$. We do not observe which occurs in this case. We assume each case is equally likely, although in reality the winning team is more likely to have started the extra play.

On the other hand, when $x_1 + x_2$ is even, $n_1 = n_2 = (x_1 + x_2)/2$.

In aside, this odd-even condition is not strictly true for netball because of the complicated interaction between the hooter and restarts (CPs). Nonetheless, it will provide a close approximation to reality because other scenarios occur with low probability, and so, while they could be coded in the likelihood, would make little difference to estimates.

Let I_{e} be the indicator function for the event $x_1 + x_2$ is even. Then we have that

$$L_k(\vartheta, \mathbf{x}_{12}) = f_{X_1+X_2}(n)\left\{I_{\mathrm{e}} L_k(\vartheta, \mathbf{x}_{12} \,|\, n_1 = (x_1 + x_2)/2, n_2 = n_1) + \right.$$
$$\left.(1 - I_{\mathrm{e}})\{\tfrac{1}{2} L_k(\vartheta, \mathbf{x}_{12} \,|\, n_1 = (x_1 + x_2 + 1)/2, n_2 = n_1 - 1) + \tfrac{1}{2} L_k(\vartheta, \mathbf{x}_{12} \,|\, n_1 = (x_1 + x_2 - 1)/2, n_2 = n_1 + 1)\}\right\},$$

where $f_{X_1+X_2}(n)$ is a suitably specified probability function for the total score, and noting that $n = x_1 + x_2 = n_1 + n_2$. The likelihood function for a single match is then fully specified when p_1 and p_2 are defined using (3).

For the Poisson-match, the likelihood for a match in which team 1 plays team 2 is

$$L_k(\vartheta, \mathbf{x}_{12}) = (x_1!x_2!)^{-1} \exp(-\mu_1 - \mu_2)\mu_1^{x_1}\mu_2^{x_2}.$$

For a tournament with scores $\mathbf{x}$, we assume matches are independent, so that

$$L(\vartheta, \mathbf{x}) = \prod_{k=1}^{M} L_k(\vartheta, \mathbf{x}_{k(1),k(2)}),$$

where $k(1)$ is the index of the home team in match k and $k(2)$ is the index of the away team in match k and M is the total number of matches.

4.2. *Discounting of results of past matches*

The Dixon-Coles approach (Dixon and Coles, 1997) for dealing with variation in strength over time is to exponentially down-weight past matches, so that at a time t ago, a match only counts as $\exp(-\xi t)$ of a match. With ξ in units of year^{-1}, ξ^{-1} is the number of years' data effectively used. We used block discounting, so that matches in the current season are not discounted, and the matches in the season t years ago are discounted by $\exp(-\xi t)$. Thus, entire seasons were discounted geometrically. The optimum value of ξ cannot be found by simply maximising the likelihood function, because the effective amount of data changes with ξ. However, ξ can be estimated without using a holdout sample: with a weighted log-likelihood ℓ, the expected value of the log-likelihood on fresh data is $\ell - p$, where p is the number of parameters estimated. Rescaling this adjusted log-likelihood gives $C = (M / \sum_{i=1}^{M} w_i)(\ell - p)$, where M is the number of matches and w_i the weight on each match, as the criterion to be minimised.

4.3. *Shrinkage of strengths*

Similarly to Baker and McHale (2017) who model women's tennis, an empirical Bayes approach was used to shrink the strengths α_i and β_i. Thus, the term

$$\Delta\ell = -(1/2)\sum_{i=1}^{m} \frac{(\ln(\alpha_i\beta_i) - \mu)^2}{\sigma^2}$$

was added to the log-likelihood. When $\beta_i = r\alpha_i$, this reduces to

$$\Delta\ell = -2\sum_{i=1}^{m} \frac{(\ln(\alpha_i) - \mu)^2}{\sigma^2}.$$

Here μ and σ^2 are notionally the mean and variance of the (log-) strength of an "unknown" team (no games played to date). The parameter μ is estimated as the sample mean of the log-strengths, $\hat{\mu} = \sum_{i=1}^{m} \ln(\alpha_i)/m$. If team k is unknown, $\Delta\ell$ is the only place that its strength appears in the log-likelihood, and the maximum likelihood estimate of α_k is $\hat{\alpha}_k = \hat{\mu}$, so that its log-strength is estimated as the mean log-strength of "known" teams. However, technically, it is best to omit teams from the analysis completely until they have played at least one game (otherwise, strictly, $\hat{\mu} = \sum_{i=1}^{m} \ln(\alpha_i)/m$ is undefined), and this was done.

Logarithms were used in $\Delta\ell$ because $\ln(\alpha_i)$ is more normally-distributed than α_i. The likelihood retains its invariance to changes of scale, $\alpha_i \to \eta\alpha_i$, all i.

If the full prior term for $\ln(\alpha_i) \sim N(\mu, \sigma^2)$ is used (with the addition of $-(1/2)\ln(\sigma^2)$ to $\Delta\ell$), the method would lead to $\sigma^2 = 0$ and $\alpha_i = \alpha$ for all i, which is useless.

The only means to estimate the prior variance σ^2 without using a holdout sample is to use an empirical Bayes method. We estimate σ (rather than σ^2) by maximising $f(\mathbf{x})$, the marginal probability of the observed scores, i.e.

$$f(\mathbf{x}) = \frac{\int \exp(\ell(\mathbf{x}|\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta}}{(2\pi\sigma^2)^{(m-1)/2}},$$

where $\theta_i = \ln(\alpha_i)$, all i, and $\boldsymbol{\theta} = (\theta_1, ..., \theta_m)$, all i, and noting that the power $m-1$ arises because there are $m-1$ free parameters α_i, and also noting that m is the number of "known" teams and not the total number of teams in the complete dataset. Expanding the profile likelihood for $\boldsymbol{\theta}$ using Laplace's approximation,

$$\ell(\boldsymbol{\theta}) \simeq \ell(\hat{\boldsymbol{\theta}}) - (1/2)(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})^{\mathrm{T}} \mathbf{D}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})$$

and integrating, we have that

$$\ln(f(\mathbf{x})) = \ln(\ell(\hat{\boldsymbol{\theta}})) - (1/2)\ln|\mathbf{D}| - (1/2)(m-1)\ln(\sigma^2),$$

where $\mathbf{D}$ is the negative of the Hessian of the profile likelihood for $\boldsymbol{\theta}$, $(\mathbf{D})_{ij} = \partial\ell(\boldsymbol{\theta})/\partial\theta i\partial\theta_j$. In this procedure, other parameters δ and r are held at their estimated values. It is simplest to maximise the likelihood for various values of σ and to then choose the value that minimises the marginal probability $f(\mathbf{x})$. An alternative procedure is to use a holdout sample and minimise forecast error.

4.4. *Computation*

For the binomial-match, the factorisation of the likelihood (5) allows it to be computed in two parts. For δ, r and $(\alpha_1, ..., \alpha_2)$, the computation finds the logarithms of the factorials just once, initially. Then, given p_1 and p_2 the logarithms t_i of the j terms $\exp(t_i)$ in the log-likelihood conditional on n_1 and n_2

are computed and their maximum t_0 found. The log-likelihood is $\ell = \ln\{\sum_{i=1}^{j} \exp(t_i)\}$. The "log-sum-exp" trick is used to compute $\ell = t_0 + \ln\{\sum_{i=1}^{j} \exp(t_i - t_0)\}$. This avoids what can be a huge rounding error or even an overflow, caused by the addition of numbers of widely varying size. We now have an approximation (the logarithm of the largest likelihood component) and the correction to it, which is the logarithm of the sum of unity and $j-1$ negative exponentials.

The distribution (density function) of the total score in a match, $f_{X_1+X_2}$ in (5), is specified by a Poisson distribution with mean λ. The (profile) log-likelihood is then maximised accordingly.

5. Results of the estimation for netball

UK Superleague matches were played in rounds at weekly intervals, but typically spread over an extended weekend (Friday through to Monday). Rather than fit the model daily, for faster computation, matches were fitted every 5 days. Also, they were fitted from 2015 onwards, so that at least one season was available for the first set of estimates. We ignore variation in the duration of matches because there was no information in the dataset about overtime (an additional 14 minutes in matches tied at the end of normal time). Approximately, then, about 9% of matches were 25% longer, so we perhaps overestimate the mean total score (at the end of normal time) by approximately 2%.

With $f_{X_1+X_2}(n)$ specified with a single parameter λ and $m=11$ teams, we have 13 parameters in total for the binomial-match, and 12 for the Poisson-match, noting that the strength parameters have a single constraint (Table 1). By convention, the strength estimates were ordered by size and we set $\alpha_2 = 1$ because the strongest team (Bath) were not competing from the start; they entered in 2017.

Table 1. Maximum likelihood estimates for models fitted at end-2018, with strengths (α_i) relative to team 2, Loughborough ($\alpha_2 = 1$). Here: defence strength is proportional to attack strength ($\beta_i = r\alpha_i$); δ is the home advantage parameter (equation 3); λ is the mean of the total score.

			binomial-match		Poisson-match	
parameter			estimate	s.e.	estimate	s.e
λ			104.3	0.9		
δ			1.049	0.025	1.035	0.019
r			1.065	0.259	0.0197	0.0003
α_i	1	Wasps Netball	1.056	0.036	1.043	0.027
	3	Manchester Thunder	0.973	0.032	0.980	0.025
	4	Team Bath	0.921	0.032	0.936	0.024
	5	Saracens Mavericks	0.906	0.031	0.928	0.024
	6	Severn Stars	0.876	0.031	0.903	0.024
	7	Surrey Storm	0.867	0.030	0.896	0.024
	8	Strathclyde Sirens	0.833	0.030	0.872	0.024
	9	Team Northumbria	0.780	0.027	0.829	0.022
	10	Celtic Dragons	0.714	0.025	0.774	0.021
	11	Yorkshire Jets	0.810	0.073	0.841	0.062

The discounted likelihood method yields $\xi = 1.35$. Thus, for models fitted in year j the matches in year $j-1$ were discounted by $\exp(-1.35) \approx 0.25$. Therefore, for the model fitted immediately after round 1 in 2015, the effective number of seasons of data fitted was $1/10 + 1/4 = 0.35$, and for the final model fitted (end 2018), it was $1 + (1/4) + (1/4)^2 + (1/4)^3 + (1/4)^4 = 1.33$. The empirical Bayes procedure gave the optimum value of σ as 0.25. The Poisson-match and binomial-match models were compared. The Poisson-match fits slightly better than the binomial, with a mean log-likelihood of -509.45 for, compared to -510.90 for the binomial-match, and noting here that for a fair comparison only that part of the log-likelihood for δ, r and $(\alpha_1, ..., \alpha_2)$ was used. The mean home-away difference is 1.772 goals, standard error on the mean 0.871. This is significant with $p = 0.047$ (Wilcoxon test), $p = 0.043$ (t-test). There is no significant correlation of the home-away difference with time ($\rho = 0.015$, $p = 0.77$), so there is no evidence that home advantage is evolving.

In aggregate, the correlation between scores is observed to be negative, presumably because of the variation in strength across teams. For repeated matches between constant-performance teams, the Poisson-match predicts zero correlation, as scores are independent Poisson random variables. As we discussed in Section 3.2, the binomial-match implies a positive correlation between scores in repeated matches, with value depending on p_1 and p_2.

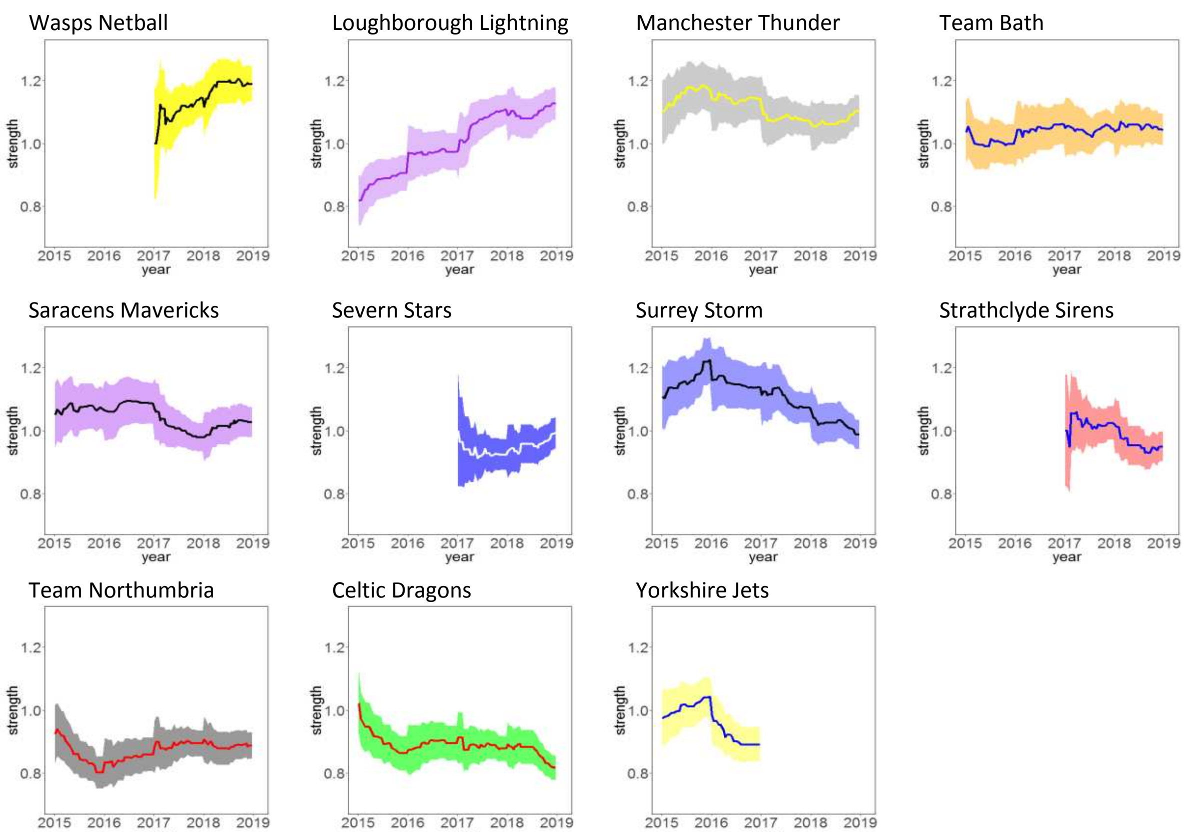


Figure 3. Ribbon plots of strength estimates $\hat{\alpha}_i$ over time (ribbon is +/- 2 s.e.) for the fitted binomial-match. The colours used here correspond approximately to the respective team colours.

Figure 3 shows the development of strengths over time for all teams. Here we rescale the strengths so that $\sum \log(\hat{\alpha}_i) = 0$. Otherwise, using the constraint $\alpha_2 = 1$ that we use in the parameter estimation step would imply a fixed strength for team 2 over all time. It is noticeable in some of these plots that the largest strength changes tend to occur at the boundaries of the seasons.

Roughly, we can see that strength variation over time is similar to strength variation within teams at a particular time, suggesting inter-season outcomes that are reasonable uncertain. Within-match uncertainty of outcome is considered next. Note, some examples of (p_1, p_2) implied by these estimates are: (0.77, 0.47) when Wasps (strongest) at home play Celtic Dragons (weakest); (0.63,0.63) when $p_1 = p_2$ (equal effective strengths); (0.61,0.63) when Loughborough at home play Wasps (c.f. Appendix 2).

6. Implications of the model

6.1. *Outcome uncertainty calculation*

Suppose team 1 plays team 2 in a binomial-match. An approximation P_1 to $\Pr(X_1 > X_2)$ can be used when the number of goals scored is large. Ignoring the possibility that N_1 and N_2 are different, (2) implies that team 1 beats team 2 if $Y_1 > Y_2$. Since Y_1 and Y_2 are binomial, when the number N of scores (equivalently plays) is large, $Y_i \sim N[(N/2)p_i, (N/2)p_i(1-p_i)]$, $i = 1, 2$, and so

$$Y_1 - Y_2 \sim N[N(p_1 - p_2)/2, N\{p_1(1-p_1) + p_2(1-p_2)\}/2].$$

Therefore, conditional on N, the probability that $Y_1 > Y_2$ is approximately

$$\Phi(\frac{(N/2)^{1/2}(p_1 - p_2)}{\{p_1(1-p_1) + p_2(1-p_2)\}^{1/2}}),$$

where $\Phi(.)$ is the normal distribution function. Also, N, the total score, is approximately Poisson distributed, and so $X = 2N^{1/2}$, the variance-stabilised variate, is approximately $N[2\lambda^{1/2}, 1]$, where λ is the mean total score. Notice that X tends to normality faster than N. Therefore, relaxing the conditioning, we have

$$P_1 = (1/\sqrt{2\pi})\int_{-\infty}^{\infty} \Phi(\frac{(\lambda/2)^{1/2}(p_1 - p_2)}{\{p_1(1-p_1) + p_2(1-p_2)\}^{1/2}}) \exp(-(x - 2\lambda^{1/2})^2/2) \mathrm{d}x. \tag{6}$$

This can be written as

$$P_1 = (1/\sqrt{2\pi})\int_{-\infty}^{\infty} \Phi(\kappa x) \exp(-(x - \nu)^2/2) \mathrm{d}x, \tag{7}$$

where $\kappa = 2^{-3/2}(p_1 - p_2)\{p_1(1-p_1) + p_2(1-p_2)\}^{-1/2}$ and $\nu = 2\lambda^{1/2}$. The integral in (7) can be most easily evaluated using a probabilistic argument. It is the probability that a r.v. $X \sim N[\nu, 1]$ exceeds a r.v. $Z \sim N[0, 1/\kappa^2]$, that is, $\Pr(X - Z > 0)$. Now $X - Z \sim N[\nu, 1 + 1/\kappa^2]$ so that, approximately,

$$P_1 = \Phi(\nu/\sqrt{1 + 1/\kappa^2}) = \Phi(2\lambda^{1/2}/\sqrt{1 + 1/\kappa^2}).$$

In the argument above, it is implied that $\kappa > 0$. It can be shown that a formula valid for any κ is

$$\Phi(2\lambda^{1/2}\kappa/\sqrt{1+\kappa^2}).$$

Notice that as $p_1 \to p_2$, $\kappa \to 0$ and $P_1 \to 1/2$. Also, as $\lambda \to \infty$, $P_1 \to 1$ for any $p_1 > p_2$.

As N was slightly overdispersed ($\operatorname{var} N = aE(N)$ where $a \simeq 1.2$), a correction for this can be made:

$$P_1 = \Phi(2\lambda^{1/2}\kappa/\sqrt{1+a\kappa^2}). \tag{8}$$

The formula (8) shows how the variability in N shifts the outcome probabilities towards ½ (as a increases). Also, the outcome probabilities are more sensitive to decreasing λ than to increasing λ. More plays do not increase the probability that a team wins by very much, but fewer plays can decrease it. This further contributes to reducing the probability that a stronger team wins as uncertainty in the number of plays increases. Presumably, a strong team should therefore not delay scoring (through play that produces long sequences of possession) and a weak team should do the opposite. In practice, these effects are not likely to be very large.

With a continuity correction to allow for ties, (6) becomes

$$P_1 = (1/\sqrt{2\pi})\int_{-\infty}^{\infty}\Phi(\frac{-0.5+(\lambda/2)^{1/2}(p_1-p_2)}{\{p_1(1-p_1)+p_2(1-p_2)\}^{1/2}})\exp\{-(x-2\lambda^{1/2})^2/2\}\mathrm{d}x. \tag{9}$$

It is difficult to carry the -0.5 term through the calculations that follow (7) and so outcome probabilities that account for ties can be calculated either using a numerical evaluation of (9) or by simulation. When N is small, a simulation of the exact binomial-match might be preferred. Such a context might arise when investigating the probability that one team scores more than the other in a quarter. Interestingly, in lower divisions of UK netball, league-points are awarded for "winning" quarters.

To account for variance-inflation in the total score, one could introduce the variance inflation factor a into (9), or in a simulation one could use a discretised-normal distribution rather than a Poisson distribution, so that $N = \text{round}(S)$ where $S \sim N[\lambda, a\lambda]$, ignoring negative values of N generated in this way.

For the Poisson-match, with a continuity correction and the addition of the variance inflation factor

$$P_1 = \Phi\{\frac{-0.5+\mu_1-\mu_2}{\sqrt{a(\mu_1+\mu_2)}}\}. \tag{10}$$

6.2. *Outcome uncertainty results*

We plot outcome uncertainty (P_1) against the scoring rate μ of the weaker team for the Poisson-match in Figure 4. The relative scoring rate (strength) of the stronger team, ε, is specified using the relative strengths for a range of typical contests, all at a neutral arena (equivalently ignoring home advantage): equal-strength teams play; 2nd best plays 3rd best; median plays best team; and weakest plays best team. Thus, we specify ε in these four cases as 1, $\alpha_2^2/\alpha_3^2 = 1.042$, $\alpha_1^2/\alpha_6^2 = 1.333$, and $\alpha_1^2/\alpha_{11}^2 = 1.816$, respectively.

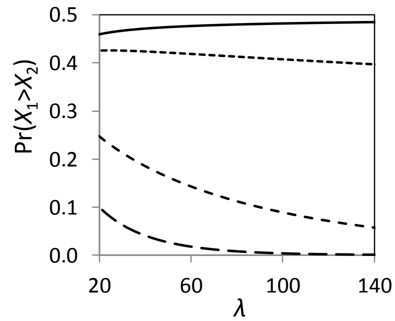
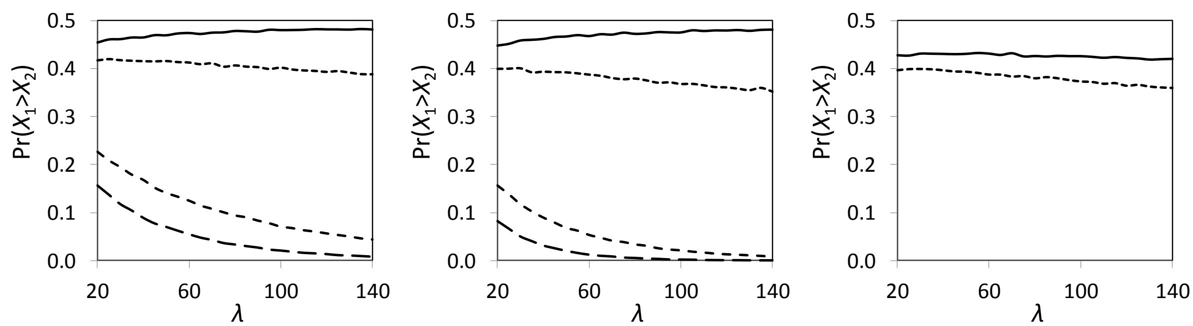


Figure 4. $\Pr(X_1 > X_2)$ for $X_1 \sim \mathrm{Po}(\mu)$ and $X_2 \sim \mathrm{Po}(\varepsilon\mu)$ independent as a function of the total scoring rate $\lambda = (1+\varepsilon)\mu$ for various ε: solid line $\varepsilon = 1$; short dash $\varepsilon = 1.042$ (equivalent to 3rd strongest plays 2nd strongest); medium dash $\varepsilon = 1.333$ (6th (median team) plays strongest); long dash $\varepsilon = 1.816$ (weakest plays strongest).

For the binomial-match, the situation is more complicated because the outcome uncertainty will vary with both the mean total scoring-rate, λ, and with the quality, p, of the teams. In Figure 5a, we plot $\Pr(X_1 > X_2)$ versus λ, for the same scenarios implied by our estimates in Table 1 and as in Figure 4: equal-strength teams play; 3rd best plays 2nd best; median plays best team; and weakest plays best (at neutral arenas). In Figure 5b, we have increased the value of p in the base case (equal strengths) to 0.750, and in the other cases we have increased the values of p by the same relative amount.



(a) (b) (c)

Figure 5. $\Pr(X_1 > X_2)$ for binomial-match (equations 1) as a function of total scoring rate λ: a) solid line $p_1 = p_2 = 0.609$ (equal teams); short dash $p_1 = 0.599, p_2 = 0.619$ (3 v 2); medium dash $p_1 = 0.541, p_2 = 0.678$ (6 v 1); long dash $p_1 = 0.513, p_2 = 0.706$ (11 v 1); b) solid line $p_1 = p_2 = 0.750$; short dash $p_1 = 0.738, p_2 = 0.762$; medium dash $p_1 = 0.666, p_2 = 0.835$; long dash $p_1 = 0.632, p_2 = 0.869$; c) solid line $\tilde{p}_{\mathrm{NZ}} = p_1 = 0.786$, $\tilde{p}_{\mathrm{Aus}} = p_2 = 0.807$; short dash $\tilde{p}_{\mathrm{Lough}} = p_1 = 0.717$, $\tilde{p}_{\mathrm{Wasps}} = p_2 = 0.729$ (from Appendix 2).

We can see overall that for a balanced match the outcome is very uncertain, and for an unbalanced match it is almost certain. Furthermore, at the typical scoring-rate in netball ($\lambda = 102$), approximately 5% of matches would be tied at the end of normal time.

Comparing the two models (Figure 4 vs Figure 5a) we can see that the binomial-match implies greater outcome uncertainty than the Poisson-match, particularly for unbalanced matches. If we regard the binomial-match as the preferred, more plausible model, then netball is more uncertain than the Poisson-match would imply.

Notice also that as the overall standard increases, matches become relatively less uncertain (Figure 5b vs Figure 5a). Figure 5b is representative of a standard of play somewhere between the highest standards at international level and national level.

6.3. *Forecasting netball scores*

The results for forecasting one-step ahead are shown in Table 2. There were 302 match-results forecast because the models were fitted from 2016 onwards. For win forecasts, the percentage of correct forecasts and the Brier score were calculated. The latter is the mean squared error of the forecasts, where the error is the difference between the match-forecast (win-probability) and the actual match-outcome (Brier, 1950). The win-probabilities were calculated using equation (8) for the binomial-match and equation (10) without the continuity correction for the Poisson-match. Figure 6 shows score-difference forecasts, $(\hat{E}(X_1)-\hat{E}(X_2))$, versus actuals x_1-x_2. The mean absolute error of the score-difference forecasts was also calculated (Table 2).

Table 2. Forecast performance, one-round ahead, out-of-sample, from 2016 onwards.

Model	% correct forecasts	Brier score	Mean Absolute Error
Binomial-match	77.15	.1527	9.22
Poisson-match	76.82	.1525	9.17
Distribution-free	78.48		9.51
Home-win forecast	52.98		--

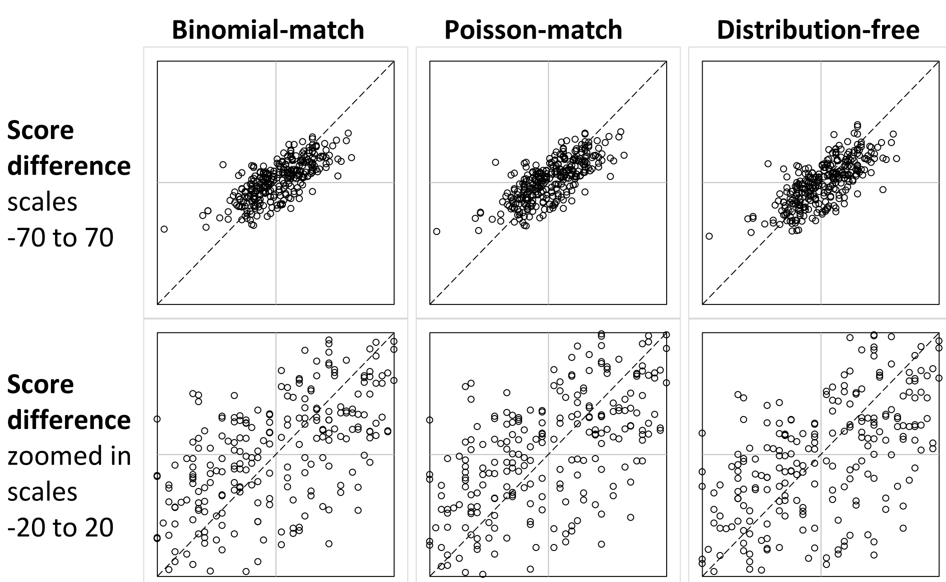


Figure 6. Forecasts ↑ versus actuals →; dashed line is forecast=actual.

As can be seen, the Poisson and binomial models perform very similarly, although the binomial model has one more parameter than the Poisson model. However, they both tend to underestimate variability (the actual score-differences tend to be smaller than the forecast score-differences). The benchmark, distribution-free forecast is better on percent-correct, but underperforms on mean absolute error, appearing to underestimate the difference.

A *t*-test shows that match numbers in the season are very similar for correct and incorrect predictions, so there is no learning effect over the season, and the runs test confirmed the randomness of correct and incorrect predictions. The average size of score difference is 15.63 for correct prediction, 7.66 for incorrect, significantly different ($p < 0.001$). Predicted probability of a wrong forecast (the weaker team wins) is the average value of $\max(q, 1-q)$, where q is the probability that team 1 wins. This was 79.42% for the Poisson-match, and 79.05% for the binomial, not far above the observed proportion of correct predictions. This shows that the models are well-calibrated. The distribution-free forecast works well because of the round-robin format, so that matches played to date are somewhat balanced.

6.4. *Centre pass variations*

To illustrate a rule variation, we simulate a simplified match between two typical teams under different restart rules: 1) *standard* rule (alternating restart); 2) *catch-up* rule (conceding team restart); 3) alternating-order *behind-first* rule (trailing team restart or restart alternates if score tied). These rules have been studied, and are so named in the context of penalty shoot-outs (e.g. see Brams and Ismail (2018). Other interesting properties relating to fairness might also be studied (see e.g. Palacios-Huerta, 2012; Csató, 2020; Lambers and Spieksma, 2020).

For each simulated match for each of the three rules, we simulate *N,* the total number of goals, as a Poisson with mean 102.0. Then, for the *standard* rule, we simulate an unbalanced binomial-match at a neutral ground. If *N* is even, teams have an equal number *N/*2 of CPs, otherwise team 1 has one more restart than team 2. Equation (2) determines the score. Under the *catch-up* and *behind-first* rules, the total number of restarts is set to *N,* and the outcome of each restart is simulated sequentially, while applying the respective restart rule. The scores accumulate and the match simulation ends once the *N*-th goal has been scored.

Table 3 presents the results for various p_1 and p_2. Broadly, we can see that the *catch-up* rule offers little improvement over the *standard* rule in the chances of the weaker side. This is a surprise, although when a strong team concedes more goals (than a weak team) it gets more restarts (than under the *standard* rule), perhaps compensating for poor performance. Thus, the *catch-up* rule only appears to increase the probability of a tie. On the other hand, the *behind-first* rule improves the chances of the weaker team considerably, but at the expense of many more matches tied at the end of normal time (37.4% for $p_1 = p_2 = 0.8$). This is perhaps because, for large p_1 and p_2, under the *behind-first* rule, the score-difference throughout a match will nearly always be -1, 0 or +1. Note, the small differences between the win and loss probabilities in the cases with $p_1 = p_2$ are inaccuracies due to the finiteness of the simulations. Also, analogously, a penalty shootout in soccer reaches the sudden death stage with the highest probability if the *behind-first* rule is applied (Csató and Petróczy, 2021).

Table 3: Relative frequencies of outcomes at end of normal time (win, tie and loss) as a function of p_1 and p_2 for different restart rules (simulation of 10^5 matches).

			p_2								
			0.6			0.7			0.8		
			win	tie	loss	win	tie	loss	win	tie	loss
p_1	0.6	*standard*	0.480	0.041	0.480						
		catch-up	0.475	0.049	0.476						
		behind-first	0.415	0.169	0.415						
	0.7	*standard*	0.844	0.023	0.133	0.479	0.044	0.477			
		catch-up	0.837	0.031	0.132	0.471	0.060	0.469			
		behind-first	0.574	0.206	0.220	0.356	0.286	0.358			
	0.8	*standard*	0.986	0.003	0.011	0.867	0.023	0.110	0.478	0.050	0.472
		catch-up	0.984	0.005	0.011	0.859	0.035	0.106	0.461	0.079	0.460
		behind-first	0.680	0.205	0.115	0.475	0.316	0.209	0.315	0.374	0.311

6.4. *A tactical question in netball*

We consider now a tactical question relating to play towards the end of a quarter. The alternating restart (CP) rule implies that it is somewhat important to the take the first CP. This is because the number of CPs could be odd, and if it is, the starting team get the extra CP. We investigate how important using simulation of a binomial-match. The first CP of a match is not in the control of team, but for other quarters (Qs) it is to some extent, because a team may, as a Q-end approaches, attempt to delay scoring until a point at which the opposition have time to start but no time to score; this acts like a turnover.

In Table 4 we consider perfectly balanced matches and outcomes given match-state at the start of Q4. Thus, this analysis for the highest level ($p_1 = p_2 = 0.8$) suggests possession at the final quarter restart confers: a 10% advantage to a team that is 1-up (0.583 vs 0.520); a 14% advantage in a tied match (0.479 vs 0.421); and a 17% advantage to a team that is 1-down (0.380 vs 0.324).

Table 4: Relative frequency of outcomes at the end of normal time (win, tie, loss) for team that takes first CP of Q4 for various $p = p_1 = p_2$ (equal strengths) and for various match-states at start of Q4 (1-ahead, tied, 1-down); 10^5 simulations of a binomial-match with mean total goals of 25.5.

	$p = p_1 = p_2$	win	tie	loss
Team starting 1-up	0.6	0.552	0.077	0.371
	0.7	0.560	0.084	0.356
	0.8	0.583	0.092	0.324
Scores tied	0.6	0.468	0.082	0.450
	0.7	0.473	0.087	0.440
	0.8	0.479	0.100	0.421
Team starting 1-down	0.6	0.390	0.080	0.530
	0.7	0.391	0.086	0.523
	0.8	0.380	0.101	0.520

7. Discussion

We have developed a new model for a paired-contest called a binomial-match. This contest is a sequence of Bernoulli trials with alternating attempts by A and B, and with the novel characteristic that a success is recorded for A not only when A succeeds but also when B fails. This is essentially a "penalty shoot-out" in which the outcome of every shot is a "goal", "one for us if we score, one for them if we miss". At each trial, each competitor has its own constant success probability, and trial outcomes are independent. For example, assuming A goes first, the sequence AABA is success for A, failure for B, failure for A, and failure for B, with probability $p_A(1-p_B)(1-p_A)(1-p_B)$, and the outcome is a 3-1 "win" for A. In a simple binomial-match, the number of trials is a fixed, even integer. In this paper, we model the outcome of a game of netball using a binomial-match that has a random number of trials. Netball and a (standard) penalty shoot-out share the alternating property because in netball the restart following a goal broadly alternates regardless of which team scores. This is an unusual restart rule. Kabaddi is another invasive sport in which points can be won and conceded at each of a sequence of alternating trials, although it is not a ball-sport. Typically, sports use a *catch-up* rule: either the conceder restarts if restarting confers an advantage to the restarter or the scorer restarts if not.

We use the binomial-match to study the relationship between outcome uncertainty and scoring-rate in a general setting. For a given overall scoring-rate, the binomial-match implies greater outcome uncertainty than the Poisson-match, particularly for unbalanced matches. Scores in a binomial-match are strongly, positively correlated, and this correlation moderates the relationship between outcome uncertainty and scoring-rate. To some extent, this supports the notion that the binomial match is a better model than the Poisson-match for a high-scoring sport. Nonetheless, the forecasting exercise that we carry out with the two models is inconclusive. On outcome uncertainty and scoring-rate, we imply a general point about invasive ball-sports (e.g basketball, water polo, handball, all codes of football and hockey, etc.). This is that administrators should be careful of rule-changes that lead to increased scoring-rate, intentional or otherwise. This is because the work in this paper evidences the notion that more scoring will reduce outcome uncertainty, and hence reduce suspense and surprise, even when scores are correlated.

The framework of two types of Bernoulli trial that underlies the binomial-match allows us to study different restart rules in a way that a direct model of the bivariate score would not. Therefore, in this framework, we compare the *standard* alternating restart rule (used in netball) with the *catch-up* rule and *behind-first* rule, in which the trailing team restarts. It appears that the *catch-up* rule increases the probability of a tie but does not increase the probability of a win for the weaker competitor (relative to the standard rule). The *behind-first* does appear to help the weaker side, but at the expense of a much greater tie-probability. So, arguably, the *standard* rule in netball is a tie-minimising rule. Sports that use the *catch-up* rule typically solve the tie-problem by using overtime. The *standard* rule also offers entertainment that the *catch-up* rule does not: dramatic 3-point swings (because a team that scores on turnover tends to score three goals in a row).

We also study a tactical question particular to netball. The binomial-match model implies that it is important to start the final quarter of a very close match in possession, affirming that tactical play to hold possession in the closing stages of the third quarter, and the first and second to a lesser extent, is sensible.

As a model of netball in particular, the binomial-match has limitations. In reality, plays are interrupted by the hooter; strength may vary through a game, depending on the score and the time. The speed of play (quantified by the mean total scoring-rate) may also vary with time and score. Distributional assumptions are also indeed assumptions, although other distributions for total goals, which are under or over dispersed relative to the Possion, could be used. Some of these issues may explain, for example, why the frequency of ties is slightly underestimated by the model. In an extended study, more detailed simulation is possible, and given suitable data within-match time-varying parameterisations could be studied.

There is also potential for further research in other sports. Handball shares some of characteristics of netball e.g. a high scoring-rate (~60 goals per match) and a high probability that the restarting team scores next (~2/3) (Meletakos and Bayios, 2010). Handball however uses a *catch-up* restart; hence its potential for development of a new, interesting model. Volleyball, although not an invasive sport, is another high scoring sport with a *catch-up* restart, but the division of matches into sets makes modelling more difficult (Ntzoufras et al., 2021). Lastly, water polo, although lower scoring than netball and handball, might make an interesting study because scores have been shown to be positively correlated (Karlis and Ntzoufras, 2003).

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Appendix 1: Derivation of the means, variances and correlation of scores

Throughout we shall use the general result $E\{g(X_1, X_2)\} = E_N\{E(g(X_1, X_2 | N))\}$ for compound random variables. Thus, we first condition on the value of the compounding random variable, here N, and then in the second step (to relax the conditioning) take the expectation with respect to N. Thus, for the binomial-match we have

$$E(X_1) = E_N(E(X_1|N)) = E_N(E(Y_1 + N - Y_2)) = E_N(Np_1 + N - Np_2) = \lambda(p_1 + 1 - p_2),$$

and (symmetrically) $E(X_2) = \lambda(-p_1 + 1 + p_2)$, because $N \sim \text{Po}(\lambda)$, $Y_1 \sim B(N, p_1), Y_2 \sim B(N, p_2)$ independently given N, and $X_1 = Y_1 + N - Y_2$, $X_2 = Y_2 + N - Y_1$.

Next note that $E(N) = \lambda$ and $E(N^2) = \operatorname{var} N + \{E(N)\}^2 = \lambda + \lambda^2$.

Now $\operatorname{var} X_1 | N = \operatorname{var}(Y_1 + N - Y_2) = \operatorname{var} Y_1 + \operatorname{var} Y_2 = Np_1(1 - p_1) + Np_2(1 - p_2)$ because Y_1 and Y_2 are independent, and so

$$E(X_1^2 | N) = \operatorname{var} X_1 | N + \{E(X|N)\}^2 = Np_1(1 - p_1) + Np_2(1 - p_2) + N^2(1 + p_1 - p_2)^2.$$

Therefore

$$E(X_1^2) = E_N E(X_1^2 | N) = \lambda p_1(1 - p_1) + \lambda p_2(1 - p_2) + (\lambda + \lambda^2)(1 + p_1 - p_2)^2,$$

and so we get after a little manipulation $\operatorname{var} X_1 = E(X_1^2) + \{E(X_1)\}^2 = \lambda(1 + 3p_1 - p_2 + p_1 p_2)$, and $\operatorname{var} X_1 = E(X_1^2) + \{E(X_1)\}^2 = \lambda(1 - p_1 + 3p_2 + p_1 p_2)$ by symmetry.

To obtain the covariance, we first need

$$E(X_1 X_2 | N) = E\{(Y_1 + N - Y_2)(Y_2 + N - Y_1) | N\} = E(Y_1|N)E(Y_2|N) - E(Y_1^2) - E(Y_2^2) + N^2$$

which follows because Y_1 and Y_2 are independent. Then we obtain

$$E(X_1 X_2 | N) = N(p_1^2 + p_2^2 - p_1 - p_2) + N^2(1 - p_1^2 - p_2^2 + 2p_1 p_2).$$

Then we have that

$$\begin{aligned}
\operatorname{cov} E(X_1, X_2) &= E_N E(X_1 X_2 | N) - E(X_1)E(X_2) \\
&= \lambda(p_1^2 + p_2^2 - p_1 - p_2) + (\lambda + \lambda^2)(1 - p_1^2 - p_2^2 + 2p_1 p_2) - \lambda^2(1 + p_1 - p_2)(1 - p_1 + p_2) \\
&= \lambda(1 - p_1 - p_2 + p_1 p_2)
\end{aligned}$$

and so

$$\operatorname{corr}(X_1, X_2) = \frac{1 - p_1 - p_2 + 2p_1 p_2}{\sqrt{(1 + 3p_1 - p_2 - 2p_1 p_2)(1 - p_1 + 3p_2 - 2p_1 p_2)}}.$$

Appendix 2: Two illustrative matches with possession sequences and scores

Tables 5 and 6 show the possession sequences and the evolution of the score in two matches, chosen for illustration. Thus, starting top right in the first quarter, Wasps (W), the away team, started the match with the first centre pass (CP) and were in possession until they scored; there was no possession change prior to their scoring. Then, next, Loughborough (L) started and were in possession until they scored. This pattern continued until the seventh score by L following a W CP and a change of possession to L.

Table 5. Loughborough Lightning (L) vs Wasps Netball (W), 18/3/2019. Final score 60-59. Possession sequences shown (first column), L score (second) and W score (third). There were 60 CPs each, with the possession in one of those (final CP in Q1 by Wasps) interrupted by the hooter and not shown. Conversion proportions are $\tilde{p}_{\text{Lough}} = 0.729$ and $\tilde{p}_{\text{Wasps}} = 0.717$.

Quarter 1			Quarter 2			Quarter 3			Quarter 4		
W	0	1	LW	16	15	L	29	30	LW	40	47
L	1	1	WL	17	15	W	29	31	W	40	48
W	1	2	L	18	15	LW	29	32	L	41	48
L	2	2	WL	19	15	WL	30	32	WL	42	48
W	2	3	LW	19	16	L	31	32	LW	42	49
L	3	3	WL	20	16	W	31	33	W	42	50
WL	4	3	LWL	21	16	LW	31	34	L	43	50
L	5	3	W	21	17	WL	32	34	W	43	51
W	5	4	L	22	17	L	33	34	L	44	51
L	6	4	W	22	18	W	33	35	W	44	52
W	6	5	LWL	23	18	LW	33	36	L	45	52
LW	6	6	W	23	19	WL	34	36	WL	46	52
WLW	6	7	LWLW	23	20	L	35	36	L	47	52
LW	6	8	W	23	21	W	35	37	W	47	53
W	6	9	LWLW	23	22	LW	35	38	L	48	53
L	7	9	W	23	23	W	35	39	W	48	54
WL	8	9	L	24	23	L	36	39	L	49	54
LW	8	10	W	24	24	WLW	36	40	WL	50	54
WL	9	10	L	25	24	LW	36	41	L	51	54
L	10	10	W	25	25	W	36	42	WL	52	54
W	10	11	LW	25	26	L	37	42	L	53	54
LW	10	12	W	25	27	W	37	43	W	53	55
W	10	13	L	26	27	L	38	43	L	54	55
L	11	13	W	26	28	W	38	44	W	54	56
WL	12	13	L	27	28	L	39	44	LW	54	57
L	13	13	W	27	29	W	39	45	WL	55	57
W	13	14	L	28	29	L	40	45	L	56	57
LWL	14	14	W	28	30	W	40	46	WL	57	57
WLWL	15	14							L	58	57
L	16	14							W	58	58
W	16	14							L	59	58
									W	59	59
									L	60	59

Table 6. New Zealand (N) vs Australia (A) in Netball World Cup 2015 final. Final score 55-58. Possession sequences shown (first column), N score (second) and A score (third). Conversion proportions: $\tilde{p}_{\mathrm{NZ}} = 0.786$ and $\tilde{p}_{\mathrm{Aus}} = 0.807$.

Quarter 1			Quarter 2			Quarter 3			Quarter 4		
N	1	0	N	8	16	A	22	31	N	38	43
A	1	1	A	8	17	N	23	31	A	38	44
N	2	1	N	9	17	A	23	32	N	39	44
A	2	2	AN	10	17	N	24	32	A	39	45
N	3	2	NANANA	10	18	A	24	33	N	40	45
A	3	3	A	10	19	N	25	33	A	40	46
NA	3	4	NA	10	20	A	25	34	N	41	46
A	3	5	A	10	21	N	26	34	A	41	47
NAN	4	5	N	11	21	A	26	35	N	42	47
A	4	6	A	11	22	N	27	35	A	42	48
NA	4	7	N	12	22	A	27	36	N	43	48
A	4	8	A	12	23	N	28	36	A	43	49
NANA	4	9	N	13	23	AN	29	36	N	44	49
A	4	10	A	13	24	N	30	36	A	44	50
N	5	10	NA	13	25	A	30	37	NA	44	51
A	5	11	AN	14	25	NANA	30	38	A	44	52
N	6	11	N	15	25	A	30	39	N	45	52
A	6	12	A	15	26	N	31	39	A	45	53
N	7	12	N	16	26	A	31	40	N	46	53
A	7	13	A	16	27	N	32	40	A	46	54
NA	7	14	N	17	27	AN	33	40	N	47	54
A	7	15	AN	18	27	N	34	40	AN	48	54
NA	7	16	N	19	27	AN	35	40	N	49	54
ANANA	7	16	A	19	28	N	36	40	A	49	55
			N	20	28	A	36	41	N	50	55
			A	20	29	N	37	41	A	50	56
			N	21	29	A	37	42	N	51	56
			A	21	30	NA	37	43	A	51	57
			N	22	30	*AN*	37	43	N	52	57
									AN	53	57
									N	54	57
									A	54	58
									N	55	58
									AN	55	58