

A personal history of Hawkes processes

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1 Introduction

I was very honoured to be asked to contribute something on the history of Hawkes processes to a special issue of a journal produced by such an illustrious Institute. At first it was suggested that I might answer a few questions, but I thought that I would like to say a bit more: the result is this article which covers more topics as well as including answers to those questions. My work on this topic falls distinctly into two parts: the early work and a late return starting in 2012. The latter would never have happened without my colleague, Professor Jing Chen, who has contributed so much. I therefore asked her to be co-author. The first part of this article has been written by me (Alan) and Jing is the main author of the second part, but we both contributed to each section.

1.1 Background

I graduated in Mathematics from King's College London in 1960 and moved to the Statistics Department in University College London (UCL) to study a one-year postgraduate diploma in Statistics. At the end of that year I started as a research student under the direction of Professor Maurice Bartlett, working on queueing theory applied to problems in road traffic. One year later I was appointed to a lectureship in the department and continued my research alongside teaching and other duties. My thesis was completed in 1965: the external examiner was Professor David Cox who, amongst many of his important original works, introduced a well-known point process model called the Cox process.

On completion of my thesis, I published two more queueing theory papers then decided that it was time to broaden my research area. I published another four papers on a variety of topics. UCL provided excellent facilities for research. London University had seminars that enabled one to meet scholars from other colleges in London (such as Cox and David Brillinger) and distinguished visitors from abroad, including Jerzy Neyman, to name just a few who made contributions to point process theory. Cox and Lewis [1] published an interesting monograph with some theoretical models, but mostly concentrating on data analysis. Most of all, I was stimulated by a paper by my PhD supervisor, Bartlett [2], in which he introduced what soon became known as the Bartlett spectrum for a point process. I decided that point processes might provide an interesting area to concentrate on for a while.

2 Some exciting point processes

In this section I discuss the time, 1971 – 1974, when those five early papers on Hawkes processes, [3] – [7], were written. Of course, they were not called Hawkes processes at that time but self-exciting and mutually-exciting point processes. What I have to say includes answers to three of the five questions that I mentioned were posed to me.

48 **Question 1:** At the time when the Hawkes process was proposed, what point process
49 models were popular? What were the main difficulties of applying Hawkes models and
50 other models?

51 When I moved to Durham University In 1969 I had already begun to think seriously about
52 point processes. There was some interest in point processes at that time but, it seemed to me
53 then, not many models apart from the obvious Poisson, renewal, semi-Markov processes
54 (Hawkes, 1970, [8] was on the latter topic) together with Neyman-Scott, Bartlett-Lewis and
55 Cox processes. I was influenced particularly by Bartlett so that I thought maybe I could
56 introduce a new model and find its Bartlett spectrum.

57 Nowadays I have a much broader definition of point processes to include anything that
58 models streams of events, such as queueing theory, continuous-time Markov chains (i.e.
59 Markov processes with discrete states in continuous time), neuron firing, epidemic processes,
60 systems reliability etc.

61 The main difficulties in those days were computing and availability of ample accurate data.
62 Computers were physically large and remote, though small in capacity and slow in speed.
63 Software was limited in scope. Today you can have an incredibly powerful machine on your
64 desk and a huge range of software, much of it freely available as the result of work of scholars
65 from around the World. The developments in computer hardware and software have also made
66 it much easier today to get large amounts of reliable data, for example in physiology or
67 recording transactions on financial markets every few milliseconds.

68
69 **Question 2:** How did the process come out in your idea? This process appeared before
70 I was born. I wonder many researchers who are younger than I might be interested in
71 the answer to this question.

72 I do not have a clear memory of exactly how or why things turned out the way they did.
73 The motivation was purely mathematical, although some possible applications were mentioned,
74 particularly epidemic processes. Many people begin articles on Hawkes processes saying that
75 they were originally invented to describe earthquake sequences. This is clearly not true,
76 although the interesting paper by Vere-Jones [9] was cited in Hawkes [3]. At this time I acquired
77 a PhD student and I thought that he should do something practical, so I set him to work on
78 analysing earthquake data. Hawkes [5] introduced marked exciting processes and included the
79 remark "A marked process model is being fitted to earthquake data by L. Adamopoulos at
80 Durham".

81 This came to fruition in Hawkes and Adamopoulos [6], which was indeed the first
82 application of Hawkes processes to any practical subject: but earthquakes were certainly not on
83 my mind when those processes were first introduced. It was not a particularly good paper,
84 because we knew little about earthquakes, but Seismologists felt that it had interesting
85 possibilities. Since they do know about earthquakes they, particularly the famous Japanese
86 experts, have been able to do much better with sensible exciting kernels and developing the
87 well-known ETAS models, including important spatio-temporal versions (Ogata, 1998, [10]).
88 These have also been taken up by people predicting criminal activity, see for example Mohler
89 et al, 2011, [11].

90 **Question 3:** On statistical inference, in my impression, spectrum, generating
91 functional and moment-based methods were the main techniques used in point
92 process data analysis. Likelihood based method (MLE) was just introduced. Is this
93 true?

94 I think that is true. The methods most in use were described in Cox and Lewis [1] while
95 Lewis et al (1969, [12]) provided useful computer programs (SASE IV) to apply those methods.
96 Computer-intensive MCMC Bayes methods, such as Mertropolis-Hastings, were in their

97 infancy but (as far as I can recall) had not yet become common tools for analysing point
98 processes. Hawkes and Adamopoulos [6] introduced "a spectral likelihood" method that relied
99 on the idea that the ratio of the sample spectrum (sometimes called periodogram) and the
100 theoretical Bartlett spectrum computed at various well-chosen frequencies were approximately
101 distributed as i.i.d exponential random variables.

102 I moved from Durham to Swansea university in 1974 and about 1978 David Vere-Jones
103 visited Swansea to give a talk in which he described the now well-known likelihood formula
104 for Hawkes processes published by Osaki (1979, [13]) following a result of Rubin (1972,[14])
105 for general point processes. I believe Vere-Jones had some part in encouraging Osaki to work
106 on that problem. The EM algorithm for maximum likelihood was not generally known before
107 the notable work of Dempster et al (1977, [15]): now it is often used on the branching process
108 representation of the Hawkes process [7].
109

110 **3 The middle years**

111

112 *Question 4:* As many people have noticed, you gave up to continue your work in self-
113 exciting process for many years, what are the main reasons? During these years what
114 were your main research focuses?

115 Throughout my academic career I have always thought of myself primarily as an applied
116 probabilist. I have not been particularly interested in developing mathematical theory for its
117 own sake. I like to work on a concrete problem that has some practical importance. Those self-
118 exciting processes did not seem to have generated all that much excitement: according to
119 Google Scholar, over the next 30 years the first Hawkes paper [3] received on average about
120 1.5 citations per year: nowadays it gets about 200 citations per year. I did not feel that I had
121 sufficient knowledge to make a practical contribution to, say, earthquakes or epidemics. But I
122 found something much more interesting to do.

123 I had known pharmacologist, David Colquhoun, from my time at UCL. I had made a small
124 contribution to a book he was writing, on biostatistics [16] by helping him with the chapter on
125 stochastic processes. Also Hawkes [8] was written in response to a question he asked me about
126 how often you might expect to see extra-large spikes when observing a series of spikes in a
127 nerve system. The paper generalised it and abstracted it out of all recognition, but that was the
128 question he wanted answered.

129 Then he started asking me questions about minute electric currents passing through ion-
130 channels, an essential part of the complex communication system that carries electrical signals
131 around the body, without which we would be unable to feel, think, move or do anything at all.
132 They are also important sites of drug action. For example, local anaesthetics close channels so
133 that no signal can pass and therefore the patient feels no pain. I did not know much about ion
134 channels, but David did and was able to communicate what he needed to know and to
135 understand the mathematics that I produced.

136 So began a wonderful collaboration between us that lasted for about 40 years. David is
137 immensely talented: he became professor of Pharmacology at UCL, a Fellow of the Royal
138 Society (FRS) who worked with three Nobel prize-winners. Such a nice man and good friend,
139 it was a great pleasure to work with him.

140 Together we pioneered the stochastic modelling of single-channel dynamics which came
141 to be indispensable to the worldwide community of pharmacologists, physiologists and
142 biophysicists who were thereby enabled to extract detailed understanding of the experimental
143 results that they routinely measure. I regard this as the most important work that I ever did,
144 having practical importance for our understanding of the bodies of people, animals and even
145 plants. And the mathematics was nice too! We published 21 papers, of which 10 of the most

146 important are included in the bibliography [17]–[26]. I should also mention that my colleague,
147 Dr Assad Jalali, also made a significant contribution to solving the important problem of time
148 interval omission ([20], [21], [27]), when some open or shut channel periods may not be
149 observed because they last for such a short time; less than about 25 μ s. Under this topic I also
150 note my only previous publication in Japanese [28].

151 I also published 14 papers on systems reliability. I was not so motivated by this topic,
152 although some of the mathematics was similar to that used for describing ion-channels. I was
153 mainly drawn into this by students or former students of mine who were interested in that
154 subject and wanted my help from time to time. The bibliography includes 8 of these papers,
155 [29] – [36], mostly in collaboration with Professor Lirong Cui, from Beijing Institute of
156 Technology, and some of his research students.

157 There were also 15 papers of little lasting importance on a miscellaneous collection of
158 topics.

159

160 **4 Return to Hawkes processes**

161 In 2012 I got to know Jing Chen, who persuaded me to return to the subject of Hawkes processes,
162 which she assured me was a "hot topic in finance" and invited me to work with her on that
163 subject. I knew nothing about finance but once again I found myself working with a talented
164 specialist in their field who understood the mathematics sufficiently well to guide my efforts.
165 At the same time I thought I might try to catch up with all the new mathematics and the very
166 broad range of applications of Hawkes processes. This proved to be an impossible task: the
167 original paper [3] now gets over 200 citations per year.

168 As Jing motivated and organised almost everything we did on Hawkes processes in
169 Finance (workshops, journal special issues...), she will continue the story.

170 **4.1 Recent history of Hawkes processes and Finance (Jing Chen)**

171 In 2010 I joined Swansea School of Management as a lecturer in Finance. In 2011 I realized
172 that Hawkes processes were becoming important tools in Finance research, particularly in high
173 frequency trading (HFT) and market microstructure. I also realized that Emeritus Professor
174 Alan Hawkes, in my department, was actually the author of Hawkes processes. One of the
175 important features of HFT is that the events (e.g. trades and quotes) occur at extremely high
176 speed (in milliseconds) and, often, in clusters. This provides a natural environment to apply
177 Hawkes processes. This, indeed, has proved to be the case and the early Hawkes process
178 applications in finance are to understand how limit order book activities related to bids and asks
179 that would move trading prices (See Bowher, 2007 [37] and Large, 2007 [38]). In insurance, a
180 few researches also appeared using Hawkes processes to model the clusters in arrivals of
181 insurance claims (see Stabile & Torrisi, 2010 [39] and Dassios & Zhao, 2011, [40]). The idea is
182 to consider the claim process as a "dynamic contagion process" that exhibits the self- or
183 mutual-exciting characteristics. Such applications are often found highly useful in credit risk
184 modelling as they ultimately relate to the classic problem of the probability of ruin

185 These early applications of Hawkes processes are fascinating, and I hesitated no more to
186 convince Alan that Hawkes processes will be hugely important and helpful in many more areas
187 in Finance. In 2012, we started to work together, Alan teaching me Stochastic processes —
188 especially Hawkes processes - and Statistics, while I taught him Finance. Part of his initial
189 training in Finance was to check the Mathematics and Statistics in several papers that I was
190 submitting to Finance journals.

191 Our first research idea that we worked on was to incorporate a Hawkes jump process into
192 a GARCH financial model. Having developed a theoretical model, we needed a method of

193 identifying jumps in a series of financial returns. We started using one of the most popular
194 methods: we found that it was sometimes very poor at identifying jumps, especially when the
195 jumps showed a contagious property. Therefore, we devised a new method that would behave
196 well in those circumstances. Unfortunately, we have so far found it difficult to persuade quality
197 journals to publish what we think are quite important results — but we keep trying. Our method
198 of jump detection was, however, well received when presented in a talk to the Commodities
199 Futures Trading Commission (CFTC), a US government financial regulatory body in
200 Washington DC. Eventually, we returned to our planned jump-GARCH model, using our new
201 methods of detection, and are close to completion. We hope, through our model, to understand
202 how intra-day behaviour, such as occurrence of jumps, is related to volatility. Subsequently,
203 traders may be able to get more accurate forecasts of volatility at the end of each day, which
204 are practically achievable and feasible for their routine decision making.

205 Meanwhile, I persuaded the journal *Quantitative Finance* to publish a special issue on
206 Hawkes processes, and they asked me to be its chief editor. The issue was published in early
207 2018. It included three papers written by Chen and Hawkes with various other co-authors, [41],
208 [42] and [43], and a review paper by Hawkes [44].

209 First, we model order flows in a financial market through a birth-death-immigration
210 Hawkes model in Khashanah, Chen and Hawkes [41]. This is opposite to most literature using
211 positively exciting processes: we model trades and quotes in a mutual-exciting bi-variate setting
212 so that one stream of events occurs at a decreasing rate due to interactions with the other type
213 of events.

214 Second, Chen, Hawkes, Scalas and Trinh [42] compare three typical information criteria
215 for choosing from a collection of possible exciting kernels: Akaike's information criterion
216 (AIC), Bayesian information criterion (BIC) and the Hannan-Quinn criterion (HQ).

217 In the meanwhile, behaviour finance research has been prevailing. Especially, it is clear
218 that news sentiment exists and drives market prices to move. Yang, Liu, Chen and Hawkes [43]
219 applied a mutually-exciting Hawkes process to understand how positive/negative price and
220 sentiment movements interact. We find that, in particular post the 2008 financial crisis, the
221 news sentiment that reflects traders' belief about the market has become more dominant in
222 leading the market to fluctuate.

223 Another half dozen excellent papers are also published in this special issue, addressing
224 various theoretical and practical issues such as high-dimensional, non-linear Hawkes model for
225 limit order book (Achab et al., [45]; Lu & Abergel, [46]), dark pool trading (Gao et al., [47]),
226 financial jumps and co-jumps (Calcagnile et al., [48]), liquidity/illiquidity spillover (Schneider
227 et al., [49]) and constant proportion portfolio insurance (Buccioli & Kokholm, [50]).

228 To further advance the applications of Hawkes processes in Finance, we are currently
229 editing another special issue of *The European Journal of Finance*. Many more important issues
230 in finance such as volatility clustering, financial jumps, financial networks, portfolio
231 optimisation etc. will be discussed. This will also include a review by Hawkes (2021, [51]) on
232 Hawkes jump-diffusions in Finance.

233 We have also promoted Hawkes processes by a series of three Workshops in Cardiff (2017),
234 Swansea (2018) and Stevens Institute in Hoboken, New Jersey (2019). The Swansea meeting
235 was a 2-day affair to celebrate Alan's 80th birthday. David Colquhoun talked about Alan's work
236 on ion channels. Most of the other talks were about Hawkes processes in Finance. Main
237 speakers were Mathieu Rosenbaum, Frédéric Abergel. Steve Hardiman and Judith Rousseau
238 (Paris); Valérie Chavez-Demoulin and Matthias Kirchner (Switzerland); Khaldoun Khashanah
239 and Steve Yang (New Jersey); Fabrizio Lillo (Italy); Mark Tippet (Australia), Lirong Cui (China),
240 Anton Merlushkin (Credit Suisse, London), Enrico Scalas (Sussex, UK) and, of course, Alan
241 Hawkes. It was a great birthday!

242 Another strand of development in Hawkes processes and their applications is to consider
243 using more complex kernels.: Chen, Hawkes and Scalas (2020, [52]) introduce a Mittag-Leffler
244 type kernel to replace the classic ETAS models' powerlaw kernel. This can take advantage of
245 the well-studied Laplace transform of the Mittag-Leffler function, thus providing a simpler
246 calculation of some properties of such a Hawkes model. Cui, Hawkes and Yi (2020, [53])
247 develop a method for finding moments of properties of some Hawkes processes.

248 We also continue the research in news sentiment and price behaviour. Two papers by Liu,
249 Yang, Chen and Hawkes (2020) do not involve Hawkes processes at all, but are based on
250 entropy. [54] introduces entropy-based measures to quantify information flows driven by prices
251 or news, and thus identify different types of trading behaviours and market regimes. Based on
252 11 years of news and market data, we find that the financial market has been dominated by
253 different information flows before and after the double crises period (the 2008 liquidity and
254 euro-zone debt crises). The responses to information flows are well studied in finance literature
255 of Granger causality studies. [55] again uses entropy to further find traders' responses to news
256 have become much more pre-dominant during the crisis and, therefore, the more effective price
257 discovery process will need to adopt both price- and news-driven information.
258

259 **5 The present and the future of Hawkes processes**

260 And now the final question.

261 *Question 5:* How do you think about recent developments and applications of Hawkes
262 models? How do you expect future development of Hawkes models will be?

263 It has been amazing to see the volume and scope of applications of Hawkes processes over the
264 last five years, considering that they were virtually ignored for about 30 years. The traditional
265 users, the Seismologists, are still active. Among the more recent users we find plenty of activity
266 in Finance, Social media and Mathematical theory, with some regular work also in
267 Neuroscience; crime and acts of violence. We suppose that the frequency of publications using
268 Hawkes processes will die down eventually but, like the Poisson process, it will never entirely
269 fade away: it is just too useful. The processes are flexible in choice of base process, and exciting
270 kernels, with marks and spatial effects if needed, and interaction with exogenous processes.
271 They can also be modified in various ways to suit particular circumstances. We expect an
272 increasing number of researches in various fields of study will continue to make ingenious use
273 of these properties.

274 It is interesting to note that, despite the suggestion in [3], that these processes might be
275 useful for modelling disease, and the obvious implications of the title ETAS, Epidemiologists
276 have not made a great deal of use of Hawkes processes. The arrival of COVID-19 has brought
277 a sudden spurt of interest. We do not refer to any particular publication of this type in the
278 bibliography as it is too early to decide which are the important ones. It is interesting to note
279 that the branching ratio, often called R , is a regular part of the daily news reporting of the
280 pandemic.

281 For the general use of Hawkes processes we would like to see development of a coherent
282 set of model-fitting techniques with well-tested, freely available, software. Bayesian methods
283 show particular promise, combining time-varying properties and model-learning features.
284

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