Cardiff Economics Working Papers

Working Paper No. E2021/10

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Huw Dixon, Jeremy Franklin and Stephen Millard

May 2021

ISSN 1749-6010

Cardiff Business School
Cardiff University
Colum Drive
Cardiff CF10 3EU
United Kingdom
t: +44 (0)29 2087 4000
f: +44 (0)29 2087 4419
business.cardiff.ac.uk

Sectoral shocks and monetary policy in the United Kingdom

Huw Dixon[(1)], Jeremy Franklin[(2)] and Stephen Millard[(3)]

Abstract

In this paper, we examine the extent to which monetary policy should respond to movements in sectoral inflation rates. To do this we construct a Generalised Taylor model that takes specific account of the sectoral make-up of the consumer price index (CPI). We calibrate the model for each sector using the UK CPI microdata. We find that a policy rule that allows for different responses to inflation in different sectors outperforms a rule which just targets aggregate CPI, as does a rule that responds only to non food and energy inflation. However, we find that the optimal sectoral rule only leads to a small *absolute* improvement in terms of extra consumption.

Key words: CPI inflation, Sectoral inflation rates, Generalised Taylor economy

***JEL* classification**: E17, E31, E52

Word Count: 10,153 words

(1) Cardiff Business School. Email: DixonH@cardiff.ac.uk
(2) Bank of England. Email: Jeremy.franklin@bankofengland.co.uk
(3) Bank of England, Centre for Macroeconomics and Durham University Business School. Email: Stephen.millard@bankofengland.co.uk

The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England. The authors are extremely grateful to Conor Sacks and Kun Tian for research assistance, and to Haroon Mumtaz for his help and advice on the empirical section and for sharing the Matlab code from his previous work. The authors wish to thank seminar participants at the Universities of Cardiff, Durham, Manchester and Singapore and participants at the Association of Southern European Economic Theorists Conference in Limassol, November 2012, the Royal Economic Society Annual Conference in London, April 2013 and the Anglo-French Macroeconomics workshop at Marseilles, December 2014 and the ICMAIF Crete 2015 for useful comments. Any errors and omissions, of course, remain the fault of the authors.

1 Introduction

A key question for monetary policy makers is how to deal with 'relative price' shocks: that is, movements in individual prices that do not reflect aggregate inflationary pressure but that can, as a result of nominal rigidities, lead to temporary changes in inflation. As an example, in November 2017, inflation rose above 3% driven by increases in oil prices, combined with rises in the price of food and other imports resulting from the depreciation of sterling following the vote to leave the European Union. The question as to how monetary policy should respond to these particular relative price shocks was discussed in Carney (2018) and the more general question of how monetary policy should respond to supply shocks (including relative price shocks) in Carney (2017). In both cases, he argued that it was appropriate to adopt a policy that was more expansive than would be implied by a conventional Taylor rule, where the central bank would react to high inflation whatever the cause.

This paper develops a framework to integrate sectoral shocks – which lead to relative price movements – into a model of the UK economy in order to examine more closely the implications of these shocks for inflation and the conduct of monetary policy. More specifically, we seek to link together sectoral shocks in the consumer price index (CPI) data to the behaviour of the economy at the aggregate level. This will enable us to address several questions about the causal links between the aggregate and sectoral levels, though in this paper we concentrate on the practical policy issue of how monetary policy should respond to sectoral shocks. There are several papers that model sectoral shocks in the United States including Mackowiak *et al.* (2009) and Boivin *et al.* (2009), and in the United Kingdom, including Ellis *et al.* (2009). Boivin *et al.* (2009) find using US data that most of the fluctuations in monthly sectoral inflation rates are due to sector-specific factors. Ellis *et al.* (2009) arrive at a similar result using quarterly data. In addition, they find that while sectoral inflation fluctuations are persistent in the raw data, this persistence is due to common macro components and not to the sector specific disturbances. The sector-specific shocks themselves are much less persistent. Therefore, the overall picture is one in which many sectoral prices fluctuate considerably in response to sector specific shocks, but respond sluggishly to aggregate macro shocks, such as monetary policy. As argued by Mackowiak *et al.* (2009), this could be due to the fact that firms focus mainly on what is going on in their sector, and pay rationally little attention to the macro factors.

The key innovation of this paper is to link the 12 CPI *Classification Of Individual Consumption by Purpose* (COICOP) sectors directly into a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model of the UK economy. We achieve this by using the UK CPI microdata for the period 1996-2006 to calibrate a Generalized Taylor (GT) Economy for each of the 12 COICOP sectors. To do this we estimate the cross-sectional distribution of durations within each CPI sector using the Hazard function as in Dixon and Le Bihan (2012). Thus, for each CPI sector we have the proportion of prices in that sector that have a duration of up to one month, one to two months and so on. This can then be represented by a 12-quarter GT model, which enables us to trace the effect of a shock in a particular CPI sector. The GT model of pricing is then embedded into the macroeconomic model. The United Kingdom is a much more open economy than the United States. To capture the openness of the United Kingdom, we have used a version of the model of Harrison *et al.* (2011) and Millard (2011). Furthermore, we are able to separate food and energy out of the CPI sectors: these are both sectors where prices are largely determined outside the United Kingdom and have had a significant impact on inflation in specific periods.

The policy issue on which we focus is how monetary policy should respond to sectoral shocks. We find that a policy rule allowing for different responses to inflation in different sectors outperforms a rule targeting only aggregate CPI, as does a rule responding only to non-food and energy inflation. In this paper, we look at two measures: one that strips out the most volatile components of CPI inflation from the index and a second that strips out that part of CPI inflation that can be thought of as being 'external' to the United Kingdom, leaving only 'domestically-generated inflation' (DGI). Eusepi *et al.* (2011) show that targeting a particular measure of core inflation approximates quite well a policy that targets their CONDI, and such a policy of targeting core inflation is, obviously, easier to explain to the general public. Aoki (2001) shows that a policy of targeting DGI is optimal in an open-economy context. Our approach is similar to his but our model is more general. In particular, in his model CPI was an aggregate of 'sticky' domestic prices and 'flexible' import prices; ie, imports were assumed to be 'final goods'. As a result, 'core' and 'DGI' inflation were equivalent. In our model, imports are both final goods and inputs into domestic production, removing the link between the prices of domestically-produced goods and the domestic component of inflation. In addition, we have multiple sectors with different degrees of price stickiness, which Kara (2010) showed overturned Aoki's result that you should always target the stickiest sector.

In our model, we look at simple rules in which the central bank alters interest rates in response to movements in aggregate and sectoral inflation rates and output relative to its trend. Although our approach is similar to that of Eusepi *et al.* (2011) and Aoki (2001), there are some differences, which we feel means our approach adds to this literature. In particular, we follow Aoki in using an open-economy model whereas Eusepi *et al.* used a closed-economy model; this distinction is likely to be important in the UK context given how open the UK economy is. We have a more complete and realistic input-output structure in our model than both Aoki and Eusepi *et al.*; again, we think it is particularly important to model this given the impact of movements in world energy, intermediates and food price inflation on UK CPI inflation over the past few years and we do not think it immediately obvious that the results of Aoki will go through in this more realistic setting. And finally, we model each of the 12 NFE COICOP sectors as GT economies whereas Eusepi *et al.* use a single measure of price stickiness for each sector and Aoki lumps all the domestic producers together in one 'sticky-price' sector. Again, we think it is instructive to examine whether the results of Aoki and Eusepi *et al.* go through in our more realistic setting.

Our results support the view that since aggregate CPI inflation is simply an arithmetic average of sectoral inflation rates, it must be better to allow policy to respond directly to each sectoral inflation rate. This view must be true in the sense that freely optimizing over the sectoral rates will be better than optimizing over a linear combination such as the arithmetic average. In that vein, Eusepi *et al.* (2011) construct what they call a 'Cost-of-nominal-distortions index' (CONDI) that weights different sectoral inflation rates in such a way as to minimise the welfare costs of price stickiness in each sector. They argue that stabilising this price index is near optimal. Of course, the practical policy issue is whether the improvement is significant or not, particularly given how hard it would be to explain such a target. Kara (2010) found the improvement to be quite small in a simple model calibrated using US data.

The paper is structured as follows. In the next section, we carry out an empirical analysis of sector-specific shocks in the United Kingdom using the approach of Ellis *et al.* (2009). Given these empirical results, we then construct a theoretical model in Section 3 that can be used to think about the interaction of sectoral and aggregate shocks and sectoral and aggregate inflation. Section 4 discusses how we calibrate the model and Section 5 presents some results that validate our use of the model. Section 6 analyses the implications for monetary policy and Section 7 concludes.

2 Sector-specific shocks in the United Kingdom: An empirical analysis

In this section we investigate the empirical properties of quarterly sectoral inflation in the UK over the period 1988-2017. We disaggregate to the 12 COICOP sector level, with three additional sectors created by splitting COICOP 1 into *Food* (1.1) and *Non-Alcoholic Beverages* (1.2), splitting up COICOP 7 into *Transport ex Fuels and Lubricants* (7.1, 7.2.1, 7.2.3, 7.2.4 and 7.3) and *Fuels and Lubricants* (7.2.2) – henceforth *Petrol* – and also by splitting COICOP 4 into *Housing and Water* (4.1-4.4) and *Electricity, Gas and Other Fuels* (4.5) – henceforth *EGF*. We have split off *Food, Petrol* and *EGF* because the prices of these goods will more directly reflect potential external shocks coming from world food and energy prices than other COICOP categories. Whilst the official CPI data broken down by the categories listed above only goes back to 1996, we have constructed data back to 1988 using ONS experimental COICOP CPI data and adjusting RPI data to split out *Food* and *EGF*. We first consider some stylised facts for this data. Next, we estimate a dynamic factor model to decompose each sectoral inflation rate into a macro component and a sector-specific shock and analyse some of the key features of these shocks, which we use later to motivate our modelling approach.

Table A shows the standard deviation, and first-order autocorrelation coefficients, of headline CPI inflation and the sectoral inflation rates, together with the average sectoral rate, in the columns headed 'Data'. Headline CPI inflation is a weighted mean of the sectoral inflation rates, so the variance of headline inflation can be seen as a pooled variance. Insofar as the sectoral inflation rates are uncorrelated with each other, we would expect the variance of sectoral inflation rates to be much larger than the variance of headline inflation and this is indeed what we find. While the sectoral inflation rates are volatile, they are not very persistent. We can see that only *EGF*, *Clothing and footwear*, *Non-Alcoholic Beverages* and *Recreation and Culture* have half-lives extending beyond one quarter. And we can also note that aggregate inflation is not that persistent with an AR(1) coefficient of only 0.53.

While the raw data on sectoral inflation gives us some indication of what sectoral shocks might look like, it is not complete. Some of the variation in sectoral inflation will come from common macroeconomic shocks, and some from sector-specific shocks, and we will want our theoretical model to match up with this. Following Ellis *et al.* (2009), Boivin *et al.* (2009) and Bernanke *et al.* (2005), we use a dynamic factor model to decompose each sectoral inflation rate. This approach uses a large dataset of economic indicators, including the sectoral inflation rates, and estimates a set of principal components which best summarise the information in that dataset. These principal components, or common factors (C_t), are then regressed against the sectoral inflation rates (X_{it}) in order to estimate a set of factor loadings, Λ. The sector-specific shocks are thus modelled as the residual in the equation:

$$X_{it} = \Lambda C_t + e_{it} \qquad \textbf{(1)}$$

Our dataset comprised around 350 macroeconomic UK data series from 1997Q1 to 2017Q4. This included inflation rates for the 15 sectors listed above, a range of aggregate and disaggregated activity measures such as GDP, consumption and industrial production, various price indicators including CPI, RPI and PPI, money and asset price data, and a number of variables that depend on external conditions, ensuring that the common components took into account the impact of foreign shocks, which are likely to be important for the United Kingdom.[1] Where appropriate each series was seasonally adjusted, log-differenced to induce stationarity and normalised. In this application we selected the first eight principal components to make up $\mathbf{C}_t$; however the results are not particularly sensitive to the number of principal components chosen.

There are two key features of the estimated sector-specific shocks worth noting. First, sector-specific shocks are more important in explaining sectoral inflation rates than macroeconomic (common) factors. Table B shows the proportion of the variance of each sectoral inflation rate that can be explained by the common factors, in the column headed 'Data'. Whilst around 79% of the variance of headline CPI inflation can be explained by common factors, they only explain an average of around 36% across the sectors. However, there is heterogeneity across sectors: in three sectors the common factors are more important. Ellis *et al.* (2009) use the disaggregated consumption deflator rather than sectoral CPI series, and data from 1977 to 2006, but obtain similar results. They find that around 81% of the variation in headline CPI can be explained by macroeconomic factors and an average of around 50% across the disaggregated consumption deflator. Boivin *et al.* (2009) use monthly US data from 1976 to 2005 and find around 77% of the Personal Consumption Expenditure (PCE) can be explained by macroeconomic factors compared to around an average of 12% across the disaggregated PCE.

Second, sector-specific shocks exhibit very little persistence and behave similar to 'white noise' processes. In fact, it is the macroeconomic component that is generating most of the persistence found in the sectoral inflation rate. Table C presents the sum of coefficients in estimated AR(4) models for

[1] A list of all the variables we use is available on request.

the macroeconomic component and sector-specific shock for each sector. In ten out of 15 sectors the sectoral shocks are white noise; in the remaining five sectors there is statistical evidence of some autocorrelation, but only in *Recreation and Culture* do we find that the sector-specific shock is highly persistent. These results are consistent with Ellis *et al.* (2009) and Boivin *et al.* (2009) who both also find that sector-specific shocks exhibit little or no persistence. The macroeconomic factors on the other hand do generate significant persistence in some sectors (in nine sectors the sum of AR(4) coefficients exceeds 0.5), whilst in others they do not (in one sector there is no statistically significant macro induced persistence). We use this stylised fact to justify assuming white-noise shocks in our model for all the non food and energy sectors apart from 'Recreation and culture' where we assume an AR(1) process. We also use the estimated standard deviations for these shocks when we come to calibrate the standard deviations of the sectoral shocks within our model, adjusting the estimated standard deviations to account for the fact that they will be smaller than the standard deviations of the shocks given staggered price-setting.

A critique of this methodology has been made by De Graeve and Walentin (2011), who argue that, although all sources of noise in equation **(1)** have been attributed to the sector-specific shock, in practice, some of this noise comes from 'measurement error' or sources such as sales and product substitutions occurring in the CPI data collection process. Furthermore, these factors have little persistence. Whilst sales are clearly important, it is not clear that we should ignore them when it comes to explaining inflation: whilst individual sales might sometimes be 'random', the pattern and structure of sales forms an enduring part of pricing behaviour and we believe that they should not be edited out.

3 The Theoretical Model

In this section we briefly outline our model of the United Kingdom, which is based on Harrison *et al.* (2011) and Millard (2011). The idea is to construct a model that we can use to analyse movements in sectoral inflation rates and the appropriate monetary policy response to them. In order to do this, we need a model in which there are a number of sectors and where the sectoral inflation rates of the model correspond in a meaningful way to the sectoral inflation rates that we observe in the real world.

Given that Harrison *et al.* (2011) describe their model in full detail, in what follows we concentrate on those areas in which our model is different compared with theirs.[2] There are two main differences. First, we model food consumption and production. Specifically, in the model of Harrison *et al.* households consume utilities, petrol and non-energy; in our model, they consume utilities, petrol, food and non food and energy (NFE). To keep things relatively simple, we follow Catao and Chang (2010) and assume that all food is imported. Second, we split NFE into 12 different goods, whose production differs only as a result of sector-specific productivity shocks and differences in the distribution of price stickiness across firms in each sector. We use the Generalised Taylor (GT) model of Dixon and Kara (2010) to generate these distributions of price stickiness as we think that simply using the median or mean duration of price changes ignores the distribution of durations, which we argue may have an effect on how monetary policy should respond to inflation in particular sectors. This contrasts both with the approach of Carvalho (2006), who uses only the overall distribution of price changes rather than 12 separate distributions, one per sector, and Aoki (2001) who has only two sectors, one with completely flexible prices and one with prices all sharing the same expected duration. The result of these two alterations are that we are left with a model in which households consume 15 different goods, where these goods correspond to the 15 COICOP sectors we examined in Section 2, above. This means we can use the model to assess whether a central bank would want to set monetary policy with respect to sectoral inflation rates, where we can equate the sectors in our model with UK sectors (ie, examine the response to sectoral inflation rates actually observed by the Bank of England).

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[2] The complete equation listing can be found in the Online Technical Appendix.

As is the case in the Harrison *et al.* (2011) model, we assume that the domestic economy imports oil (which is combined with labour and capital to produce petrol), wholesale gas (which is combined with labour and capital to produce 'utilities', ie, what we denoted as *EGF* in Section 2), and other intermediate inputs (ie, non oil, gas and food imports). These intermediates are combined with labour, capital, petrol and utilities to produce output of the 12 NFE consumption goods. The capital stock is composed of NFE goods. The demand side of the economy is more standard with consumption and investment being driven by real interest rates and the central bank setting the nominal interest rate according to a Taylor rule.

3.1 Households

As in Harrison *et al.* (2011), households maximise utility subject to a budget constraint. They get utility out of consuming the bundle of 15 goods and leisure. We assume that households own the capital stock and that they make decisions about capital accumulation and utilisation. These assumptions, standard in the business cycle literature, are made in order to simplify the firms' decision problem. Aggregate consumption, c, is composed of consumption of food (which, as we said earlier, is imported), c_f, petrol, c_p, utilities, c_u, and NFE, c_n. The consumption aggregator is given by:

$$c_t = \kappa_c \left(\psi_f c_{f,t}^{1-\frac{1}{\sigma_c}} + \psi_e \left(\left(\psi_p c_{p,t}^{1-\frac{1}{\sigma_p}} + (1-\psi_p) c_{u,t}^{1-\frac{1}{\sigma_p}} \right)^{\frac{\sigma_p}{\sigma_p - 1}} \right)^{1-\frac{1}{\sigma_c}} + (1-\psi_f-\psi_e) c_{n,t}^{1-\frac{1}{\sigma_c}} \right)^{\frac{\sigma_c}{\sigma_c - 1}} \quad \textbf{(2)}$$

We set the price of NFE to be our numeraire. The aggregate price level (relative to the numeraire), p, is defined as the minimum amount of expenditure required to obtain one unit of consumption:

$$p_t c_t = c_{n,t} + p_{f,t} c_{f,t} + p_{p,t} c_{p,t} + p_{u,t} c_{u,t} \quad \textbf{(3)}$$

Where p_f is the price of food (relative to the numeraire), p_u is the relative price of utilities and p_p is the relative price of petrol. Solving this minimisation problem gives the relative demand for each of the goods in terms of their relative prices.

Unlike Harrison *et al.* (2011), we assume a perfectly-competitive spot labour market in which each household takes wages and prices as given. So, real wages, *w*, will equal the marginal rate of substitution between leisure and consumption:

$$\frac{w_t}{p_{c,t}} = \kappa_h c_t^{\frac{1}{\sigma_c}} h_t^{\frac{1}{\sigma_h}} \qquad \textbf{(4)}$$

where *h* is total hours worked.

3.2 Non food and energy producing firms

The representative NFE producing firm uses labour, capital, intermediate imported goods, petrol and utilities to produce output. The production function is described in Harrison *et al.* (2011), and so will not be repeated here. The key point to note is that except for the sector-specific shocks, which we describe below, real marginal cost, μ, is common across all firms – ie, all firms in all the 12 sectors – producing NFE goods: they all share the same technology and factor prices. We do not attempt to construct a structural model of the NFE sector itself over and above the basic structure of the GT model, which can be thought of as 'duration' sectors superimposed on the CPI sectors within the NFE.

We set up each of the COICOP sectors constituting the NFE sector as in the GT model of Dixon and Kara (2010). Firms in each of the twelve NFE COICOP sectors are divided up into *K* 'duration' subsectors, where subsector $k= 1, \dots K,$ denotes those firms whose prices change every *k* periods. We first note that the optimal flexible price in any subsector will simply be a (time-varying) mark-up over marginal cost in that subsector, where we assume that this mark-up is the same across the entire NFE sector and reflects monopolistic competition in that sector.

We further assume that, after factors of production have already been allocated, the COICOP sectors experience shocks that will cause relative prices to move. Real marginal cost within a COICOP sector will be given by μe^{ε_k} where ε_k is the *relative* shock in COICOP sector *k*. (We can think of these shocks as temporary shocks to productivity or competitive pressure in sector *k*, which would temporarily affect the desired mark-up within that sector.) Given our empirical results, we assume that these shocks are white noise, ie, $E_t \varepsilon_{k,t+j} = 0 \forall j \geq 1$, except in the case of *Recreation and culture* where we assume that the shock follows an AR(1) process. Furthermore we assume that the shocks are uncorrelated across COICOP sectors. Note that we are assuming that there are 12 sectoral shocks: one per sector. In effect, this is because we are looking at the shocks as relative to the *NFE* sector as a whole. Clearly there is an adding up restriction, so there is no 'sector wide' *NFE* shock included in the model, as seems appropriate since we are treating *NFE* as the numeraire. An alternative methodology would have been to include a sector-wide *NFE* shock and then allow for 12 sector-specific shocks that add up to zero (in effect 11 independent shocks). These two approaches are of course linked: we can think of the shocks ε_k in terms of the mean shock (the sector wide element) and the deviation from mean. Conceptually, a technological improvement, or a change in competitive pressure, in *Clothing and Footwear* does not in itself imply that other sectors should get better or worse. However, *NFE* as a whole will experience a technological improvement if the shocks across the COICOP sectors tend to be more positive than negative.

Denote the price set by a firm in GT duration subsector *k* of COICOP sector *z* that is able to reset its price in period *t* by $x_{z,k,t}$. This price will be defined implicitly by the equation:

$$q_{z,k,t}x_{z,k,t}^{-\eta}\left(x_{z,k,t}(1+\tau_n)-\frac{\eta}{\eta-1}\mu_t e^{\varepsilon_{z,t}}\right)$$

$$+E_t\sum_{i=1}^{k-1}\frac{1}{\prod_{j=1}^{i}(1+i_{t+j-1})}q_{z,k,t+i}x_{z,k,t}^{-\eta}\left(x_{z,k,t}(1+\tau_n)-\frac{\eta}{\eta-1}\mu_{t+i}e^{\varepsilon_{z,t+i}}\prod_{j=1}^{i}(1+\pi_{t+j})\right)=0 \qquad \textbf{(5)}$$

Where π denotes the inflation rate in the NFE sector and τ_n is a production subsidy that ensures an efficient level of output in the steady state. The real reset price, $x_{z,k}$ will be eroded by inflation (since it is the nominal price that is kept constant). This is captured by the inflation terms in equation **(5)**: higher expected future inflation will raise the real reset price. Note that in the GT model, as in the simple Taylor model, when it sets its price the firm knows exactly how long the price will last. Whilst some firms who expect their price to last for many periods will be far-sighted, firms who expect the price to last just one month will be myopic in their pricing decision. This contrasts with the Calvo model, where firms face a distribution of probabilities over possible durations for their price, so have to be more forward looking on average when it comes to setting their price.

Note that the sectoral shock $\varepsilon_{z,t}$ is not the same as the sectoral shock estimated in section 2. The sectoral shock here can be regarded as how much the nominal price in the sector would respond *if the price were perfectly flexible*. In the data, since prices are sticky in the NFE sectors, what we observe is a partial muted response. The theoretical shocks in **(5)** need to be considerably larger in terms of variance in order to be consistent with the shocks we observe in the inflation data.

Hence, the average price prevailing in GT subsector k of COICOP sector z (relative to the numeraire) will be given by:

$$p_{z,k,t}=\left(\frac{1}{k}\left(x_{z,k,t}^{1-\eta}+\sum_{j=1}^{k-1}\frac{1}{\prod_{i=0}^{j-1}(1+\pi_{t-i})}x_{z,k,t-j}^{1-\eta}\right)\right)^{\frac{1}{1-\eta}} \qquad \textbf{(6)}$$

Averaging these prices will result in the overall price of COICOP sector *z*:

$$p_{z,t} = \left(\sum_{k=1}^{K} \gamma_{z,k} \, p_{z,k,t}^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{7}$$

And, finally, the price of NFE, the numeraire, will be given by:

$$1 = \left(\sum_{z=1}^{12} \gamma_z \, p_{z,t}^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{8}$$

Whilst we have used the GT framework to allow us to model different levels of nominal rigidity across the COICOP sectors using the microdata, these sectors are otherwise identical except for the sector specific shocks. Clearly, there will be substantial real differences between the sectors that we have not modelled: eg, *Hotels and Restaurants* have a different technology and market structure to *Communications*. These unmodelled factors might affect the inflation we observe in the data but not in the model simulations.

3.3 *Other firms*

As we said earlier, producers of NFE goods combine intermediate imports, petrol, utilities, labour and capital to produce their output. Following Harrison et al. (2011), we first assume that 'Value-added' producers use labour, *h*, and capital, *k*, to produce value-added, *V*:

$$V_t = e^{\varepsilon_{a,t}} \left(\alpha_v \left(k_{t-1} z_t \right)^{1-\frac{1}{\sigma_v}} + (1-\alpha_v)(h_t)^{1-\frac{1}{\sigma_v}} \right)^{\frac{\sigma_v}{\sigma_v - 1}} \tag{9}$$

The capital effectively used in production depends on the intensity of capital utilisation, z, and ε_a represents a shock to productivity.

Petrol, q_p, is produced using inputs of crude oil, I_o, and value-added, V_p, according to a simple Leontieff production function:

$$q_p = \min\left(\frac{I_o}{1-\psi_{qp}}, \frac{V_p}{\psi_{qp}}\right) \quad \textbf{(10)}$$

The motivation for this choice of production function is that it is not clear how adding more and more workers to a given amount of oil could physically increase the amount of petrol that can be produced from it. Firms in this sector are also assumed to be monopolistically competitive and subject to nominal rigidities in their price-setting. Following Calvo (1983), we assume that firms in this sector are able to optimally change their price in any given quarter with probability 1-χ_p. The resulting New Keynesian Phillips Curve (NKPC) is:

$$\pi_{p,t} = \beta E_t \pi_{p,t+1} + \frac{(1-\chi_p)(1-\beta\chi_p)}{\chi_p}\hat{\mu}_{p,t} \quad \textbf{(11)}$$

where π_p represents the inflation rate for petrol prices and $\hat{\mu}_p$ denotes the log-deviation from the efficient steady state of real marginal cost in this sector, where we have assumed a production subsidy that ensures efficient steady-state output of petrol.

Output of utilities, q_u, is produced using inputs of gas, I_g, and value-added, V_u, again according to a simple Leontieff production function:

$$q_u = \min\left(\frac{I_g}{1-\psi_u}, \frac{V_u}{\psi_u}\right) \tag{12}$$

Firms in this sector are again assumed to be monopolistically competitive and subject to nominal rigidities in their price-setting. We assume that firms in this sector are able to optimally change their price in any given quarter with probability 1-χ_u. The resulting NKPC is:

$$\pi_{u,t} = \beta E_t \pi_{u,t+1} + \frac{(1-\chi_u)(1-\beta\chi_u)}{\chi_u}\hat{\mu}_{u,t} \tag{13}$$

where π_u represents the inflation rate for utility prices and $\hat{\mu}_u$ denotes the log-deviation from the efficient steady state of real marginal cost in this sector, where we have assumed a production subsidy that ensures efficient steady-state output of utilities.

3.3 Monetary and fiscal policy

Monetary policy is assumed to follow a Taylor rule with the central bank responding to deviations of inflation from target and value-added from flexible-price value-added, defined as what value-added would be in an economy identical to the one we consider except that prices were completely flexible.

The fiscal authority is assumed to buy only NFE and to have the same preferences across these goods and services as consumers. It gives each of the firms a production subsidy in order to eliminate the welfare distortions resulting from monopolistic competition. Finally, it raises revenue via lump-sum taxes on consumers. When the government's budget constraint is combined with the households' budget constraint and the definition of firms' profits, we obtain the market clearing condition for NFE output:

$$q_t = c_{n,t} + k_t - \left(1 - \delta - \frac{\chi_z}{1+\phi_z}\left(z_t^{1+\phi_z} - 1\right)\right)k_{t-1} + \frac{\chi_k}{2k_{t-1}}\left(k_t - \left(\frac{k_{t-1}}{k_{t-2}}\right)^{\varepsilon_k} k_{t-1}\right)^2 + c_g e^{\varepsilon_{g,t}} + X_{n,t} \tag{14}$$

where q is output of NFE goods, $c_g e^{\varepsilon_g}$ denotes government spending with ε_g being a government spending shock, and X_n denotes exports of NFE.

3.4 Foreign sector

We model the United Kingdom as a small open economy. Following Harrison *et al.* (2011), we assume that there is an infinitely elastic supply of oil and gas on world markets available at exogenous world prices. The domestic prices of oil and gas are determined by the law of one price. Net trade in oil and gas will be determined as the difference between the demands for oil and gas and the United Kingdom's exogenous endowments of each of these goods. UK NFE exporters are assumed to face a downward-sloping demand curve for their goods, subject to an exogenous shock to world demand for NFE goods.

UK food and NFE import prices, on the other hand, take time to adjust to purchasing power parity. So, we assume the following NKPCs for food prices and for import prices ex food and energy:

$$\pi_{f,t} = \frac{\iota_{pf}}{1+\beta\iota_{pf}}\pi_{f,t-1} + \frac{\beta}{1+\beta\iota_{pf}}E_t\pi_{f,t+1} + \frac{(1-\xi_{pf})(1-\beta\xi_{pf})}{(1+\beta\iota_{pm})\xi_{pf}}\left(\varepsilon_{p_f} - \hat{s}_t - \hat{p}_{f,t}\right) \tag{15}$$

$$\pi_{m,t} = \frac{\iota_{pm}}{1+\beta\iota_{pm}}\pi_{m,t-1} + \frac{\beta}{1+\beta\iota_{pm}}E_t\pi_{m,t+1} + \frac{(1-\xi_{pm})(1-\beta\xi_{pm})}{(1+\beta\iota_{pm})\xi_{pm}}\left(\varepsilon_{p_{mf}} - \hat{s}_t - \hat{p}_{m,t}\right) \tag{16}$$

where π_f is the rate of inflation of food prices, π_m is the rate of inflation of NFE import prices, s denotes the real exchange rate, p_m denotes import prices, ε_{p_f} is a shock to world food prices and ε_{p_m} is a shock to the world price of our imports, and 'hats' denote log-deviations from trend.

4 Calibration

In this section, we briefly discuss how we calibrate our model.[3] The idea is to calibrate the model so that it matches UK data in enough detail to allow us to provide a quantitative answer to the question of the extent to which monetary policy makers should respond to movements in sectoral inflation rates.

When calibrating those parameters within our model that can be found in most standard macroeconomic models, we almost always followed the values used in Harrison *et al.* (2011) and so refer the interested reader to that paper for a complete discussion of where these values come from.[4] The only exceptions to this approach were the parameters governing the Taylor rule, where we used the original values in Taylor (1993) for the responses to inflation and output deviations:

$$i_t - \left(\frac{1-\beta}{\beta}\right) = 1.5\pi_{c,t} + 0.125\left(\hat{y}_t - \widehat{y_{FP,t}}\right) + \varepsilon_{i,t} \qquad \textbf{(17)}$$

When calibrating the process for imported price inflation, we followed Harrison *et al.* (2011). For food inflation we simply assumed that there was no exogenous persistence and that food importers reset their prices every six months.

We used the appropriate CPI weights (those applied in 2016 based on expenditure shares in 2015) to give us the weights of each of our sectors in consumption. We set a number of other parameters so

[3] Further details of the data used, and the calculations we made, to obtain our calibrated parameters can be found in the Online Technical Appendix.
[4] We report values for these parameters in Table A4.1 in the Online Technical Appendix.

that, in steady state, the model would generate spending and input shares that match those found in the 2015 *Supply and Use Tables* (SUTs).[5]

In order to use the model to analyse how monetary policy should respond to movements in sectoral inflation rates, we need the model to match the stylised facts on sectoral inflation presented in Section 2, as well as stylised facts about aggregate inflation and output. In order to do this, we need to calibrate the processes driving the 19 exogenous shocks in our model: aggregate productivity, monetary policy, government spending, the world prices of oil, gas, food, and intermediate imports and real marginal cost in each of the 12 NFE sectors. The results reported in Table C in Section 2 suggest that we can reasonably model 11 of the 12 sectoral shocks as white noise, with the exception being the shock to *Recreation and culture*. We model this shock as an AR(1) process with an AR(1) coefficient equal to 0.63, the estimated first-order autocorrelation for the shock to *Recreation and culture*. As we said earlier, the sectoral shocks of our model are not the same as the sectoral shocks estimated in section 2. So, in calibrating the standard deviations of these shocks, we need to multiply up the standard deviations of the estimated sectoral inflation shocks coming from Equation **(1)** by scaling factors for each sector that depend on their relative stickiness.[6] These scaling factors allow for the fact that prices in stickier sectors are less able to respond to shocks.

Our remaining shocks were all assumed to follow AR(1) processes of the form:

$$\varepsilon_t = \rho \varepsilon_{t-1} + \upsilon_t \qquad \textbf{(18)}$$

where ν is a white-noise process. For the world shocks we used quarterly data from 1996 Q1 to 2016 Q4 on world food prices, world oil prices, world gas prices and the world price of UK NFE imports.

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[5] The equations governing the steady state of the model are laid out in the Online Technical Appendix and the implied parameters are shown there in Table A4.3.
[6] The calculation of these scaling factors is described in the Online Technical Appendix and our calibrated standard deviations for the shocks are shown there in Table A4.4.

For the domestic shocks we used quarterly UK data from 1996 Q1 to 2016 Q4 on the nominal interest rate, CPI inflation, GDP, total hours worked, the capital stock, and real government consumption.[7]

For the sectoral GT weights, we used an updated version of the data in Dixon and Tian (2017), adjusted for the splitting off of *Fuel and lubricants* from *Transport*, *Utilities* from *Housing and water* and *Food* from *Food and Non-alcoholic beverages*. The sectoral GT weights are based on the cross-sectional distributions of completed price-spell lengths, which are depicted in Chart 1. The duration in quarters is on the horizontal axis. The vertical axis is the proportion of prices in the sector which have that duration. We have excluded education, since its spike at four quarters dominates too much. Note that there is a local maximum at 12 quarters for most categories. This is because all durations longer than 12 quarters are included in this. The mean duration across the *NFE* sectors using CPI weights is 4.35 quarters.

As we can see, the share of ‘flexible’ prices (1 quarter duration) in each sector can be quite large. These prices will respond immediately to any shock in that sector. However, in almost all sectors (with only the exception of *Education*), there is a long tail of prices which have a duration beyond two years. Note that we have ‘split off’ flexible parts of the CPI from NFE: *Food* from *Food and Non-Alcoholic Beverages, Fuels and lubricants* from *Transport,* and *Gas, Electricity and other Fuels* from *Housing and Water*. This accounts for the big reduction in the share of flexible prices in these three categories as compared to the results reported in Dixon and Tian (2017). We can see that there is considerable heterogeneity across sectors. The arithmetic mean durations vary quite a lot as well: the longest are *Housing and water*, *Health* and *Transport*, with means of above five quarters and the lowest are *Alcoholic Beverages* (2.8 quarters) and *Communications* (3.13 quarters). This heterogeneity is of course common in CPI data. (See, for example, Bils *et al.* (2012).)

[7] The Online Technical Appendix contains a description of the data used to calibrate these shock processes. The first-order autoregressive coefficients, ρ, and the variances of the white noise processes, σ_v are shown there in Table A4.5.

5 Model validation

In this section, we solve and simulate our model and assess its ability to match the stylised facts about the inflation data presented in Section 2. Table A reports the asymptotic standard deviations of aggregate and sectoral inflation given our model calibration and the first-order autocorrelation coefficient of quarterly aggregate and sectoral inflation rates as implied by the model and compares these with their data analogues.

We first consider the implications of our model for the relative volatility and persistence of quarterly sectoral and aggregate inflation. Recall that in the data, aggregate inflation was less volatile than inflation in almost all of our 15 sectors. As can be seen, in the model aggregate inflation is less volatile than inflation in all 15 sectors. The volatility of aggregate inflation is a little lower than in the data, whereas the average volatility of sectoral inflation rates more or less matches that in the data. As in the data, aggregate inflation has some persistence in this model, whereas sectoral inflation rates have little or no persistence.

Table B shows the proportion of variance in sectoral inflation rates that results from aggregate shocks – which, importantly, include shocks to the world prices of *oil, gas* and *food* as well as *imported intermediates* – and compares this with the proportion of UK sectoral inflation rates explained by the 'common factors' we found in the UK data. Leaving aside *food, petrol* and *utilities*, which are explained entirely by aggregate shocks to the world prices of food, oil and gas, respectively, Table B suggests that sectoral shocks are important in explaining inflation in all sectors. On average, they explain about 68% of variation in sectoral inflation in the model, similar to the roughly 64% of variation in sectoral inflation rates that they explain in the data. In the model, about 87% of the variance of aggregate inflation results from aggregate shocks as opposed to about 79% in the data: both the model and data suggest that aggregate inflation volatility is predominantly driven by aggregate shocks.

Finally, we examine the effects of a one percentage point monetary policy shock within our model on aggregate and sectoral inflation rates. Our results are shown in Chart 2. As can be seen, although aggregate inflation drops immediately, by around 0.3 percentage points, and then rises straight back to its steady-state rate, there is much heterogeneity among sectoral inflation rates. In particular, the behaviour of inflation in the 'education' sector reflects the fact that some prices can change immediately but are then fixed for a year; the result is that there is a drop in inflation in this sector of about 0.13 percentage points, which persists for a year, before the inflation rate in this sector bounces back to just above its steady-state rate. For the other sectors, the more flexible ones in which a greater proportion of prices change every quarter – such as 'alcohol and tobacco' – see larger initial falls in inflation and the less flexible ones – such as 'health' and 'education' – see lower falls in inflation.

6 Implications for monetary policy

In this section, we investigate the implications of relative price shocks for optimal monetary policy. In particular, we investigate whether monetary policy should respond to such shocks or should follow the approach of looking through them and responding only to aggregate shocks. There are typically two approaches to optimal stabilisation policy in the literature. One relies on computing the fully optimal 'Ramsey' policy, the other relies on optimal simple rules (OSR). Here, we use the OSR approach as OSRs have been shown to be robust and close to the optimal rules in many models. (See, eg, Taylor and Williams (2011).)

Before using *dynare* to numerically derive the optimal simple rule, we first derive a loss function for the central bank. Here, we follow Rotemberg and Woodford (1998) and take a second-order approximation of the consumers' utility function around the (non-stochastic) steady state. However, unlike these, and subsequent, authors, we are unable to convert a loss function in terms of consumption and labour supply volatility into one with just inflation and output volatility. This results from the fact that not all domestic output is consumed or exported, since the economy also produces capital and energy, used as an intermediate input, in our model.

Now, the central bank's goal is to maximise the unconditional expectation of the present discounted value of the representative consumer's current and future streams of utility. This is equivalent to maximising the unconditional expectation of the representative consumer's period utility function:

$$U_t = \frac{c_t^{1-\frac{1}{\sigma_c}} - 1}{1 - \frac{1}{\sigma_c}} - \frac{\kappa_h}{1 + \frac{1}{\sigma_h}} h_t^{1+\frac{1}{\sigma_h}} \tag{19}$$

We take a second order approximation of *U* around the steady state to get:

$$U_t - U \approx c^{-\frac{1}{\sigma_c}} \left(c_t - c - \frac{1}{2\sigma_c c} (c_t - c)^2 \right) - \kappa_h h^{\frac{1}{\sigma_h}} \left(h_t - h + \frac{1}{2\sigma_h h} (h_t - h)^2 \right) \tag{20}$$

Note that we can approximate any variable *x* using $x_t = x\left(1 + \widehat{x_t} + \frac{1}{2}\widehat{x_t}^2\right)$. So, expressing all terms as log-deviations from their steady state and dropping all terms of order 3 or higher gives us:

$$U_t - U \approx c^{1-\frac{1}{\sigma_c}} \left(\widehat{c_t} + \frac{1}{2}\left(1 - \frac{1}{\sigma_c c}\right) \widehat{c_t}^2 \right) - \kappa_h h^{1+\frac{1}{\sigma_h}} \left(\widehat{h_t} + \frac{1}{2}\left(1 + \frac{1}{\sigma_h h}\right) \widehat{h_t}^2 \right) \tag{21}$$

Now, given our Taylor rule and the shocks in our model, monetary policy is neutral in the long-run and does not affect the unconditional expectation of consumption or hours. To demonstrate this, note first that we consider only Taylor-rules that are stable and give a unique path to equilibrium. We denote this set $\boldsymbol{\tau}$. Any Taylor rule in this set $\boldsymbol{\tau}$ will give rise to a unique set of impulse response functions (IRFs), that is, a solution for the endogenous variables conditional on a one-off shock to one of the error terms holding all other error terms equal to zero. Taylor rules in $\boldsymbol{\tau}$ will all share the same steady-state, but generate different IRFs. Second, note that since the dynamics are linearized, they are symmetric and proportional, in the sense that the IRF for a shock of magnitude ε is the same in absolute terms as the IRF for a shock -ε, but with the opposite sign. The IRF gives the expected path of consumption and hours worked conditional on a particular realisation of the shock. This will hold for each and every Taylor rule in $\boldsymbol{\tau}$.

Now, the first-order effects of consumption and labour supply on welfare are (by definition) a linear function of the path of consumption. Hence, the welfare effects of a shock ε are exactly equal in absolute value but with the opposite sign for shock -ε. And, the unconditional expectation is effectively the sum of all IRFs across all shocks, weighted by their probability. Hence, for any shock that is symmetrically distributed around zero, the expected value of output is zero. This can be interpreted, effectively, as meaning that if we have no information about the realisation of shocks, then the expected output in any period is zero (the steady state).

These steps are really just a way of saying that the first-order welfare effect is zero, since the expectation of a linear function of consumption and hours worked is the welfare at the expected value of consumption and hours worked. However, it is the combination of linear dynamics and the linearity of the first order effects which means that the expected value of (first order) welfare is exactly the same as welfare at the expected value of consumption and leisure.

That is:

$$\lim_{T\to\infty}\frac{\widehat{c_t}}{T} = \lim_{T\to\infty}\frac{\widehat{h_t}}{T} = 0 \qquad \textbf{(22)}$$

The second order effects, however, do not cancel out. Different Taylor rules will lead to different variances of consumption and labour supply, despite having the same mean. The concavity of the utility function captured by the second-order effect means that even though the IRFs of two shocks sum to zero, the sum of the welfare effects will be negative. The unconditional expectation of second-order welfare will therefore be negative.

So, given the first-order terms in equation **(21)** disappear in the limit as T tends to infinity, we have:

$$E(U_t - U) \approx \frac{1}{2}\left(1 - \frac{1}{\sigma_c c}\right) c^{1-\frac{1}{\sigma_c}} E\left(\hat{c}_t^{\,2}\right) - \kappa_h \frac{1}{2}\left(1 + \frac{1}{\sigma_h h}\right) h^{1+\frac{1}{\sigma_h}} E\left(\hat{h}_t^{\,2}\right) \tag{23}$$

For this section of the paper, we shall use numerical methods to analyse different monetary policy rules in terms of which is optimal, ie, minimises the unconditional expectation of the deviations of utility from its (non-stochastic) steady-state value, given by equation **(23)**, subject to the log-linearised equations that characterise our model.

Given our calibration this results in the loss function:

$$\frac{E(U_t - \bar{U})}{\bar{U}_c \bar{c}} = \frac{E(U_t - \bar{U})}{\bar{c}^{1-\frac{1}{\sigma_c}}} = -0.9334 Var(\hat{c}_t) - 2.0097 Var\left(\hat{h}_t\right) \tag{24}$$

Where $\bar{U}_c$ is the steady-state marginal utility of consumption and we have followed Erceg *et al.* (2000) by scaling the welfare deviation by $\bar{U}_c \bar{c}$ so as to express these welfare losses as a fraction of long-run equilibrium consumption.

We consider the following simple policy rules, which differ in terms of the inflation variable:

$$i_t - i = \theta_\pi \pi_{c,t} + \theta_y \hat{y}_t \tag{25}$$

$$i_t - i = \theta_\pi \pi_t + \theta_y \widehat{y_t} \qquad \textbf{(26)}$$

$$i_t - i = \theta_\pi \pi_t + \sum_{j=1}^{15} \theta_j \pi_{j,t} + \theta_y \widehat{y_t} \qquad \textbf{(27)}$$

$$i_t - i = \theta_\pi \pi_{V,t} + \theta_y \widehat{y_t} \qquad \textbf{(28)}$$

Equation **(25)** represents a standard Taylor rule, in which the central bank responds to aggregate CPI inflation and the deviation of value-added output from its steady-state value. Equation **(26)** considers a Taylor rule in which the central bank responds to NFE inflation. Equation **(27)** is similar, except that we allow the central bank to respond separately to inflation in each of our 15 COICOP sectors. Equation **(28)** considers a rule in which the central bank responds to the rate of inflation of the competitive price of value-added.

These different rules enable us to focus on how monetary policy should respond to sectoral shocks. There are at least three possible views here. The first, embodied in the CPI rule, is that policy should not respond directly to sectoral shocks at all but should just react to aggregate CPI inflation since sectoral price movements only matter indirectly through their effect on the aggregate inflation measure. The second view is that since aggregate CPI inflation is simply a particular average of sectoral inflation rates (weighted by expenditure shares), it must be better to allow policy to respond directly to each sectoral inflation rate in an unconstrained manner.

The third view, embodied in the 'NFE' and 'value-added' rules, is that monetary policy should concentrate on a measure of the underlying rate of inflation, ie, a measure of inflation that strips out the 'noise' induced by relative price movements and, so, provides information on the outlook for inflation over the medium term.

Such measures – sometimes referred to as measures of ‘core’ inflation – involve removing certain items from the CPI index or using statistical methods to try and extract the ‘persistent’ or underlying trend component, as discussed in Stock and Watson (2016). We can think of our NFE rules as the central bank targeting ‘core’ inflation, where our definition of core inflation is based on excluding the most volatile components of CPI inflation from the index. We can note that food inflation is much more volatile in our model than it is in the data, where excluding it would make less sense. The competitive price of value-added in our model corresponds to ‘domestically-generated’ inflation (DGI), ie, the proportion of inflation that is unconnected to import prices and the exchange rate. The argument for targeting DGI is that if inflation were rising (or falling) as a result of a one-off movement in the exchange rate leading to a temporary increase (decrease) in import price inflation, policy makers might want to ‘look through’ this rise (fall) when setting monetary policy. That is, to achieve the target rate of inflation for CPI in the medium term without excess volatility in output, policymakers would do better to respond only to movements in DGI, since the difference between CPI inflation and DGI can show the extent to which movements in CPI inflation are being driven by import price inflation and the exchange rate.

Our results are shown in Table .D In order to gain intuition for the performance of our different rules, we report the volatilities of domestic output and inflation implied by each of our optimal simple rules. The rule in which the central bank responds differently to inflation rates in the different sectors outperforms the other rules (as it must), although the improvement is not as large as might have been expected: 14.6% relative to the Taylor rule, but only 0.0006% of Pareto optimal consumption relative to the Taylor rule. These results are in line with both Eusepi *et al.* (2011) and Kara (2010). Our results also suggest that a rule based on NFE inflation outperforms the Taylor rule by 7.2%. The least good rule is that based on DGI; the Taylor rule outperforms it by about 0.8%. Again the differences between these rules in welfare space are all extremely small: using a standard Taylor rule results in a welfare loss equivalent to only 0.0003% of steady-state consumption relative to the rule based on NFE inflation. Interestingly, if we had applied a ‘standard’ loss function based on output and inflation volatility with more than half the weight on inflation, the rule taking account of all the sectoral inflation rates would perform worse than the standard Taylor rule and the rule based on NFE inflation as it results in much more inflation volatility.

In order to understand the significance of the numbers in the optimal policy rules, we can express them in terms of the implied weights on each sector. In the case of the standard Taylor rule, the implied weight on each sector is simply the Taylor coefficient on inflation (1.4578) times the CPI weight of that sector. All of the implied sectoral weights are blown up by the same proportion: since the CPI weights themselves add up to 1, the implied weights add up to 1.4578. These implied weights are reported in the first column of Table E. For the rule targeting NFE inflation, we put zero weight on three sectors (*food*, *petrol* and *utilities*) and adjust the CPI weights of the remaining sectors so that they add up to one: they will still have the same relative values as in the simple Taylor rule. The coefficient on the Taylor rule for NFE inflation is 1.4835. These coefficients are in the second column of the table. The third column gives the corresponding implied weights for the optimal sectoral rule. These are the sum of two things: the coefficient on aggregate NFE (1.4823) times the sectoral weights in NFE, added to the coefficient on the sectoral inflation rate. In the fourth column of Table E, we express the percentage deviation of the coefficients for the optimal sectoral rule from the implied coefficients for the simple Taylor rule.

If we look at Table E, we can note that the sectoral rule implies putting less weight on all sectors except food, relative to the Taylor rule. The big differences are for *Non-alcoholic beverages, Health, Utilities* and, especially, *Education*. In fact, for *Education* the reduction is large enough to take the overall weight below zero. Recall that the only difference between the NFE COICOP sectors is the distribution of durations. But, contrary to the results of Eusepi *et al.* (2011), there is no evidence that the optimal sectoral rule is trying to target sectors with more fixed prices and put less weight on those with flexible prices. The reason for this is possibly that whilst the proportions of price-durations in the one to three quarter range are much larger in the flexible sector, there is still a fat tail of long durations, as can be seen from Chart 3. This illustrates the importance of modelling the entire distribution of price stickiness in each sector rather than using a single measure of price stickiness for each sector as in Eusepi *et al.*

7 Conclusions

In this paper, we have developed an open-economy model which allows us to sensibly explore the question of how sectoral shocks fit into the inflation story for the United Kingdom and how optimal monetary policy should deal with sectoral shocks. The novelty of the paper lies in the fact that we model the COICOP components of the CPI as our ‘sectors’. Furthermore, we use the CPI price microdata to directly calibrate the nominal rigidity within each sector using the Generalised Taylor model.

We first examined the data on sectoral inflation rates and found that the sectoral rates have much bigger variances than aggregate CPI inflation, which can be seen as a pooled variance. When we broke down the raw sectoral inflation shocks into sector-specific and aggregate components, we found that the persistence we observe in aggregate inflation comes mainly from the effect of the aggregate factors with sectoral shocks being white noise.

Using our model, we analysed optimal simple monetary policy rules, where the central bank could respond to aggregate inflation, sectoral inflation rates, NFE inflation (CPI excluding the volatile elements of food and energy) and DGI. We found that the optimal rule in which interest rates respond to sectoral inflation rates leads to an improvement over a rule in which interest rates only respond to aggregate inflation. The gain is large in relative terms, albeit small in small in absolute terms. We further found that a rule based on NFE inflation also performed slightly better than a rule based on aggregate CPI inflation whereas a rule based on DGI performed slightly worse. Although these results were basically in line with those of the previous literature – specifically Aoki (2001) and Eusepi *et al.*

(2011) – our paper added to this literature by showing that those results went through in a more realistic – and so useful for actual policy makers – setting, something that was not guaranteed ahead of time. In addition, this added realism enabled us to show that the relative weights on different NFE sectors were not necessarily tied to the mean duration of prices in those sectors – as in Eusepi *et al.* – but reflected the entire distribution of price stickiness across firms in those sectors.

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Tables

Table A	% Standard deviations		Inflation persistence.	
	Model	**Data**	**Model**	**Data**
Headline CPI	**0.23**	**0.36**	**0.42**	**0.53**
Sector average	**0.93**	**1.09**	**-0.02**	**0.38**
Food	1.58	1.08	0.28	0.45
Non-Alcoholic Beverages	0.69	0.90	-0.30	0.55
Alcoholic Beverages and Tobacco	0.27	0.70	-0.25	0.31
Clothing and Footwear*	0.75	1.05	-0.30	0.63
Housing and Water	0.36	0.46	-0.07	0.41
Electricity, Gas and Other Fuels	0.83	2.96	0.37	0.58
Furniture, household equipment and maintenance	0.51	0.56	-0.20	0.44
Health	0.29	0.35	0.08	-0.01
Transport (ex Fuel and Lubricants)	0.47	0.74	-0.15	0.25
Fuel and lubricants	4.07	3.78	0.17	0.29
Communication	0.86	1.05	-0.32	0.46
Recreation and Culture	0.28	0.40	0.30	0.56
Education	2.28	1.68	0.00	0.08
Restaurants and hotels	0.27	0.25	0.22	0.34
Miscellaneous Goods and Services	0.41	0.45	-0.11	0.39

Table B: Proportion of variance explained by aggregate shocks		
	Model	**Data**
Headline CPI	**0.87**	**0.79**
Sector average (ex petrol, food and utilities)	**0.32**	**0.36**
Food	1.00	0.54
Non-Alcoholic Beverages	0.14	0.39
Alcoholic Beverages and Tobacco	0.23	0.30
Clothing and Footwear*	0.16	0.54
Housing and Water	0.34	0.29
Electricity, Gas and Other Fuels	1.00	0.49
Furniture, household equipment and maintenance	0.32	0.41
Health	0.47	0.33
Transport (ex Fuel and Lubricants)	0.20	0.33
Fuel and lubricants	1.00	0.67
Communication	0.11	0.30
Recreation and Culture	0.85	0.17
Education	0.01	0.09
Restaurants and hotels	0.66	0.32
Miscellaneous Goods and Services	0.34	0.18

Table C: Inflation persistence estimates		
	Macro component	**Sector-specific shock**
Headline CPI	**0.56**	-
Sector average	0.52	0.16
Food	0.84	-
Non-Alcoholic Beverages	0.62	0.32
Alcoholic Beverages and Tobacco	0.48	-
Clothing and Footwear*	0.58	0.28
Housing and Water	0.40	-
Electricity, Gas and Other Fuels	0.78	0.46
Furniture, household equipment and maintenance	0.53	-
Health	-	-
Transport (ex Fuel and Lubricants)	0.62	-
Fuel and lubricants	0.49	-
Communication	0.48	0.31
Recreation and Culture	0.57	0.96
Education	0.42	-
Restaurants and hotels	0.27	-
Miscellaneous Goods and Services	0.70	-
** The 2010 price collection methodology change is likely to affect estimates of persistence in this category.*		

Table D: Optimal simple rules

	Standard Taylor rule	Policy responding to sectoral inflation rates	Policy responding to NFE inflation	Policy responding to DGI
θ_{π}	1.4578	1.4823	1.4835	1.5036
θ_{y}	0.0284	0.0124	0.0195	-0.0053
$\theta_{\text{Non alcoholic beverages}}$	-	-0.0140	-	-
$\theta_{\text{Alcohol and tobacco}}$	-	-0.0103	-	-
$\theta_{\text{Clothing and footwear}}$	-	-0.0158	-	-
$\theta_{\text{Housing and water}}$	-	-0.0194	-	-
$\theta_{\text{Household goods}}$	-	-0.0137	-	-
θ_{Health}	-	-0.0200	-	-
$\theta_{\text{Transport (ex petrol)}}$	-	-0.0213	-	-
$\theta_{\text{Communication}}$	-	-0.0127	-	-
$\theta_{\text{Recreation and culture}}$	-	-0.0140	-	-
$\theta_{\text{Restaurants and hotels}}$	-	-0.0167	-	-
$\theta_{\text{Miscellaneous}}$	-	-0.0170	-	-
$\theta_{\text{Education}}$	-	-0.0557	-	-
θ_{Petrol}	-	-0.0053	-	-
$\theta_{\text{Utilities}}$	-	-0.0216	-	-
θ_{Food}	-	0.0353	-	-
Standard deviations (per cent)				
Consumption	0.52	0.50	0.54	0.60
Total hours	0.25	0.21	0.20	0.14
Value-added output	0.73	0.72	0.73	0.86
CPI inflation	0.27	0.33	0.31	0.34
Loss	$3.7750*10^{-5}$	$3.2240*10^{-5}$	$3.5041*10^{-5}$	$3.8066*10^{-5}$
Improvement relative to Taylor rule benchmark	-	14.60%	7.18%	-0.84%

Table E: Sectoral and aggregate effects.

	CPI	NFE	Sectoral	%(3-1)/1	Mean	Mode
Non-alcoholic beverages	0.0175	0.0211	0.0038	-78.332	4.02	1
Alcohol and tobacco	0.0612	0.0740	0.0520	-15.078	2.81	1
Clothing and footwear	0.1034	0.1251	0.0894	-13.519	3.33	1
Housing and water	0.1238	0.1498	0.1066	-13.911	5.21	1
Furniture, household equipment and maintenance	0.0859	0.1040	0.0738	-14.183	3.71	1
Health	0.0408	0.0493	0.0215	-47.277	5.57	1
Transport excluding fuels and lubricants	0.1763	0.2132	0.1581	-10.327	5.12	2
Communication	0.0466	0.0564	0.0347	-25.487	3.13	1
Recreation and culture	0.2156	0.2608	0.2054	-4.736	4.08	1
Education	0.0364	0.0440	-0.0186	-151.191	4.00	4
Restaurants and hotels	0.1792	0.2167	0.1656	-7.563	4.74	1
Miscellaneous goods and services	0.1398	0.1691	0.1253	-10.399	4.51	1
Food	0.1326	0	0.1702	28.387		
Electricity, gas and other fuels (utilities)	0.0510	0	0.0303	-40.608		
Fuels and lubricants (petrol)	0.0466	0	0.0421	-9.612		

Charts

Chart 1: Sectoral hazard rate distributions

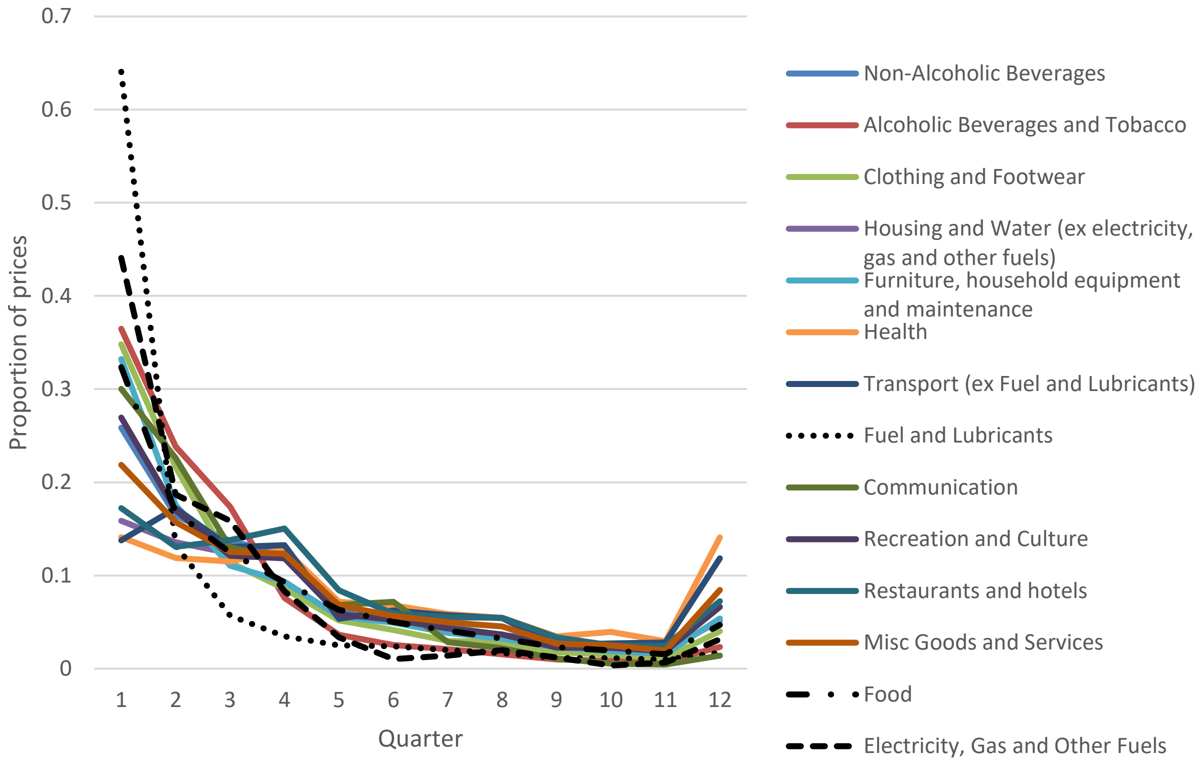
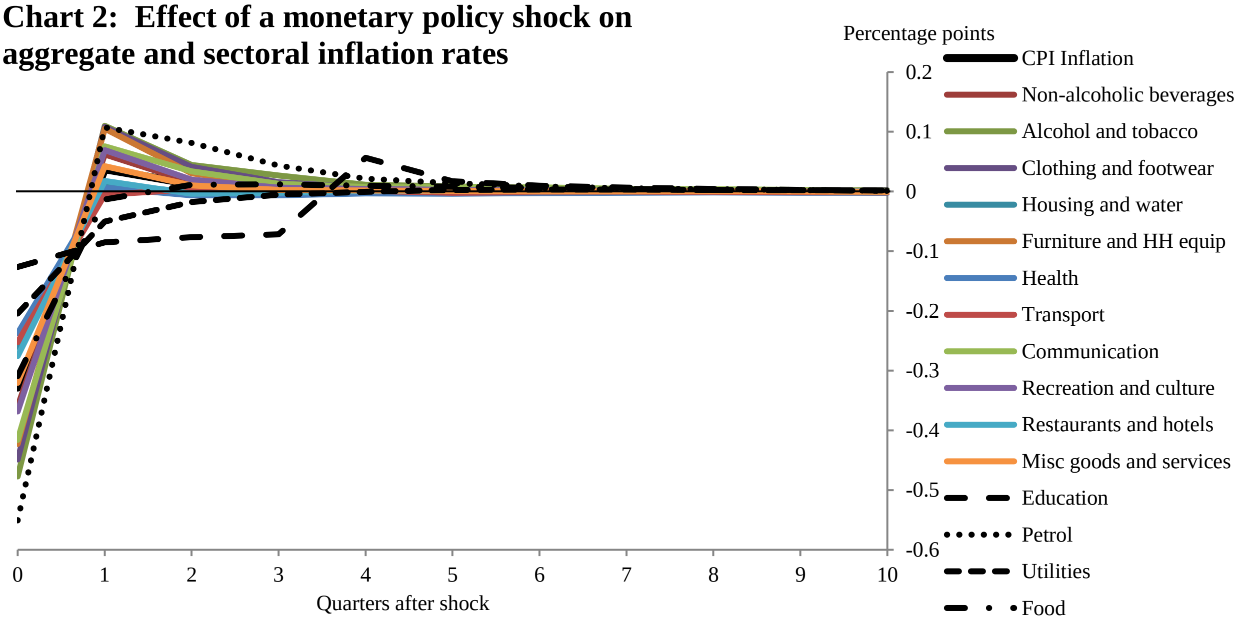


Chart 2: Effect of a monetary policy shock on aggregate and sectoral inflation rates



Technical Appendices

Appendix 1: Derivation of the model and complete equation listing

In this appendix we derive and list all the equations of the model.

Households

The representative household consumes four final goods and supplies labour to the firms. It is also assumed to own the capital stock and make decisions about capital accumulation and utilisation. This assumption, now standard in the business cycle literature, is done in order to simplify the firms' decision problem. The representative household's problem is then to maximise utility subject to their budget constraint. Mathematically:

Maximise $E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\frac{1}{\sigma_c}}}{1-\frac{1}{\sigma_c}} - \kappa_h \frac{h_t^{1+\frac{1}{\sigma_h}}}{1+\frac{1}{\sigma_h}} \right)$

Subject to

$$b_t + \frac{b_{f,t}}{s_t} + k_{j,t} = \frac{1+i_{t-1}}{1+\pi_t} b_{t-1} + \frac{1+i_{f,t-1}}{1+\pi_t} \frac{b_{f,t-1}}{s_t} + \left(1 - \delta - \frac{\chi_z}{1+\phi_z}\left(z_t^{1+\phi_z} - 1\right)\right) k_{j,t-1}$$
$$- \frac{\chi_k}{2k_{t-1}} \left(k_{j,t} - \left(\frac{k_{t-1}}{k_{t-2}}\right)^{\varepsilon_k} k_{j,t-1} \right)^2 + w_t h_h + w_{k,t} z_t k_{t-1} - p_t c_t - \frac{\chi_{bf}}{2} \left(\frac{b_{f,t}}{s_t}\right)^2$$
$$+ p_{o,t}\overline{O} + p_{g,t}\overline{G} + \Pi_t + T_t$$

Where c denotes consumption, h denotes total hours worked, b denotes (end-of-period) holdings of domestic government bonds (expressed in units of NFE goods, which we use as the numeraire), b_f denotes (end-of-period) holdings of foreign government bonds (expressed in units of NFE goods), k_j denotes the representative household's (end-of-period) capital stock, and k is the economy-wide capital stock (in equilibrium, equal to k_j), i is the domestic nominal interest rate, π is the rate of inflation of NFE goods, i_f is the foreign nominal interest rate, z is capital utilisation (whose steady-state value is normalised to unity), w is the real wage (in units of NFE), w_k is the real rental paid on capital, p is the consumer price index (price of aggregate consumption relative to the price of NFE), p_o is the relative (to NFE) price of oil, p_g is the relative (to NFE) price of gas, $\overline{O}$ is the economy's fixed endowment of oil, $\overline{G}$ is the economy's fixed endowment of gas, Π is total corporate sector profits (returned to the households lump sum) and T is a lump sum transfer from the government to the household sector.

The first order conditions determine the household's choice of aggregate consumption and labour supply:

$$c_t^{-\frac{1}{\sigma_c}} = \beta(1+i_t)E_t \frac{c_{t+1}^{-\frac{1}{\sigma_c}}}{1+\pi_{c,t+1}} \tag{A1}$$

$$\frac{w_t}{p_{c,t}} = \kappa_h c_t^{\frac{1}{\sigma_c}} h_t^{\frac{1}{\sigma_h}} \tag{A2}$$

$$E_t \frac{1+i_t}{1+\pi_{t+1}}\left(1+\chi_k\left(\frac{k_t}{k_{t-1}}-\left(\frac{k_{t-1}}{k_{t-2}}\right)^{\varepsilon_k}\right)\right)$$
$$= 1-\delta-\frac{\chi_z}{1+\phi_z}E_t\left(z_{t+1}^{1+\phi_z}-1\right)+w_{k,t+1}z_{t+1}+\chi_k\left(\frac{k_{t+1}}{k_t}-\left(\frac{k_t}{k_{t-1}}\right)^{\varepsilon_k}\right)\left(\frac{k_t}{k_{t-1}}\right)^{\varepsilon_k}$$
(A3)

$$\chi_z z_t^{\phi_z} = w_{k,t} \quad \textbf{(A4)}$$

Finally, we also obtain the modified uncovered interest parity condition:

$$E\frac{1}{s_t}+\frac{\chi_{bf}b_{f,t}}{s_t^2}=\frac{1+i_{f,t}}{1+i_t}E_t\frac{1}{s_{t+1}} \quad \textbf{(A5)}$$

The portfolio adjustment cost term, $\frac{\chi_{bf}b_{f,t}}{s_t^2}$, ensures that the net foreign asset position of the economy is pinned down, thus closing the open-economy model by ensuring that the model has a steady-state solution (in this case with zero net foreign assets). Further, we assume, without loss of generality, that the supply of domestic government bonds is zero in all periods; that is, the government balances its budget via the lump-sum transfer, *T*, on consumers.

Aggregate consumption is composed of consumption of food (which is imported), c_f, petrol, c_p, utilities, c_u, and 'non food or energy' (NFE), c_n. The consumption aggregator is given by:

$$c_t = \kappa_c\left(\psi_f c_{f,t}^{1-\frac{1}{\sigma_c}}+\psi_e c_{e,t}^{1-\frac{1}{\sigma_c}}+\left(1-\psi_f-\psi_e\right)c_{n,t}^{1-\frac{1}{\sigma_c}}\right)^{\frac{\sigma_c}{\sigma_c-1}} \quad \textbf{(A6)}$$

Where c_e denotes consumption of 'energy':

$$c_{e,t} = \left(\psi_p c_{p,t}^{1-\frac{1}{\sigma_p}}+\left(1-\psi_p\right)c_{u,t}^{1-\frac{1}{\sigma_p}}\right)^{\frac{\sigma_p}{\sigma_p-1}} \quad \textbf{(A7)}$$

The numeraire is NFE. We define the consumer price index as the minimum level of expenditure required to obtain one unit of the consumption good. That is, we solve the problem:

Minimise $p_{c,t}c_t = c_{n,t}+p_{p,t}c_{p,t}+p_{u,t}c_{u,t}+p_{f,t}c_{f,t}$ **(A8)**

Subject to equations **(A6)** and **(A7)**.

The first-order conditions for this problem imply the following inverse demand functions for food, utilities and petrol:

$$\frac{\kappa_c^{\frac{1-\sigma_c}{\sigma_c}}}{\psi_f}\left(\frac{c_t}{c_{f,t}}\right)^{-\frac{1}{\sigma_e}} = \frac{p_{c,t}}{p_{f,t}} \quad \textbf{(A9)}$$

$$\frac{\kappa_c^{\frac{1-\sigma_c}{\sigma_c}}}{\psi_e}\left(\frac{c_t}{c_{e,t}}\right)^{-\frac{1}{\sigma_c}}\frac{1}{1-\psi_p}\left(\frac{c_{e,t}}{c_{u,t}}\right)^{-\frac{1}{\sigma_p}} = \frac{p_{c,t}}{p_{u,t}} \quad \textbf{(A10)}$$

$$\frac{1-\psi_p}{\psi_p}\left(\frac{c_{u,t}}{c_{p,t}}\right)^{-\frac{1}{\sigma_p}} = \frac{p_{u,t}}{p_{p,t}} \quad \textbf{(A11)}$$

Where p_f is the price of food relative to NFE, p_c is the price of the aggregate consumption good relative to NFE, p_u is the price of utilities relative to NFE and p_p is the price of petrol relative to the NFE numeraire.

Non food and energy producing firms

The representative NFE producing firm, firm *j*, say, is assumed to have the following production function for output *q*:

$$q_{j,t} = \kappa_q\left(\left(1-\alpha_q\right)\left(\left(1-\phi_q\right)B_{j,t}\right)^{\frac{\sigma_q-1}{\sigma_q}} + \alpha_q\left(\phi_q e_{j,t}\right)^{\frac{\sigma_q-1}{\sigma_q}}\right)^{\frac{\sigma_q}{\sigma_q-1}} \quad \textbf{(A12)}$$

Firm *j*’s output is produced from two intermediates: energy e_j and a bundle of intermediate output, B_j, produced from value-added V_j, and intermediate imported goods, M_j according to the simple Cobb-Douglas function:

$$B_{j,t} = V_{j,t}^{1-\alpha_B} M_{j,t}^{\alpha_B} \quad \textbf{(A13)}$$

The energy input in this sector is produced by a Leontief production function so that:

$$e_{j,t} = \min\left(\frac{I_{j,p,t}}{\psi_n}, \frac{I_{j,u,t}}{1-\psi_n}\right) \quad \textbf{(A14)}$$

where $I_{j,p}$ is the input of petrol and $I_{j,u}$ is input of utilities.

On the assumption that prices are flexible, the cost minimisation problem for the representative NFE firm, firm j, will be:

Minimise $\quad p_{vc,t}V_{j,t} + p_{m,t}M_{j,t} + p_{p,t}I_{j,p,t} + p_{u,t}I_{j,u,t}$

Subject to $\quad q_{j,t} = \kappa_q e^{-\varepsilon_{j,t}}\left(\left(1-\alpha_q\right)\left(\left(1-\phi_q\right)V_{j,t}^{1-\alpha_B}M_{j,t}^{\alpha_B}\right)^{\frac{\sigma_q-1}{\sigma_q}} + \alpha_q\left(\phi_q \min\left(\frac{I_{j,p,t}}{\psi_n}, \frac{I_{j,u,t}}{1-\psi_n}\right)\right)^{\frac{\sigma_q-1}{\sigma_q}}\right)^{\frac{\sigma_q}{\sigma_q-1}}$

Solving this problem, and integrating over all NFE firms, implies the following demand curves for value-added, imports and energy:

$$p_{vc,t} = \mu_t\left(1-\alpha_q\right)\left(1-\alpha_B\right)\left(\left(1-\phi_q\right)\kappa_q\right)^{\frac{\sigma_q-1}{\sigma_q}}\left(\frac{q_t}{B_t}\right)^{\frac{1}{\sigma_q}}\frac{B_t}{V_{n,t}} \qquad \textbf{(A15)}$$

$$p_{m,t} = \mu_t\left(1-\alpha_q\right)\alpha_B\left(\left(1-\phi_q\right)\kappa_q\right)^{\frac{\sigma_q-1}{\sigma_q}}\left(\frac{q_t}{B_t}\right)^{\frac{1}{\sigma_q}}\frac{B_t}{M_{n,t}} \qquad \textbf{(A16)}$$

$$\psi_n p_{p,t} + \left(1-\psi_n\right)p_{u,t} = \mu_t\alpha_q\left(\phi_q\kappa_q\right)^{\frac{\sigma_q-1}{\sigma_q}}\left(\frac{q_t}{e_t}\right)^{\frac{1}{\sigma_q}} \qquad \textbf{(A17)}$$

where μ is real marginal cost, p_{vc} is the 'competitive' price of value-added (ie, the marginal cost of producing it) and p_{m} is the price of imported intermediates. Notice that since the production of NFE has constant returns to scale, average and marginal cost are equal.

An important point to note is that except for the sector-specific shocks, which we describe below, real marginal cost, μ, is common across all firms in the NFE sector: they all share the same technology and factor prices. We do not attempt to construct a structural model of the NFE sector itself over and above the basic structure of the GT model, which can be thought of as 'duration' sectors superimposed on the CPI sectors within the NFE.

As stated in the main text, we set up each of the COICOP sectors constituting the NFE sector as in the GT model of Dixon and Kara (2010). Firms in each of the twelve NFE COICOP sectors are divided up into K 'duration' subsectors, where sub-subsector $k= 1, \ldots K,$ denotes those firms whose prices change every k periods. We first note that the optimal flexible price in any sub-subsector will simply be a mark-up over marginal cost in that sub-subsector, where we assume that this mark-up is the same across the entire NFE sector and reflects monopolistic competition in that sector.

We further assume that, after factors of production have already been allocated, the COICOP subsectors experience relative productivity shocks (that will cause relative prices to move). Hence, marginal cost within a COICOP subsector will be given by $\mu_t e^{\varepsilon_k}$ where ε_{k} is the *relative* productivity shock in COICOP subsector k. Given our empirical results, we assume that these shocks are white noise, ie, $E_t \varepsilon_{k,t+j} = 0 \forall j \geq 1$ and furthermore we also assume that they are uncorrelated across COICOP sectors. Note that we are assuming that there are 12 sectoral productivity shocks: one per sector. In effect, this is because we are looking at the shocks as relative to the *NFE* sector as a whole. Clearly there is an adding up restriction, so there is no 'sector wide' *NFE* productivity shock included in the model, as seems appropriate since we are treating *NFE* as the numeraire. An alternative methodology would have been to have included a sector-wide *NFE* productivity shock and then allowed for 12 sector-specific productivity shocks that added up to zero (in effect 11 independent shocks). These two approaches are of course linked: we can think of the shocks ε_k in terms of the mean productivity shock (the sector wide element) and the deviation form mean. Conceptually, a technological improvement in *Clothing and Footwear* does not in itself imply that other sectors should get better or worse. However, the *NFE* as a whole will experience a technological improvement if the shocks across the COICOP sectors tend to be more positive than negative.

So, consider the profit maximisation problem for a firm that can reset its price in GTE subsector *k* of COICOP sector *z*. We can write this problem as:

Maximise $q_{z,k,t}\left(x_{z,k,t}(1+\tau_n) - \mu_t e^{\varepsilon_{z,t}}\right) + E_t \sum_{i=1}^{k-1} \frac{1}{\prod_{j=1}^{i}(1+i_{t+j-1})} q_{z,k,t+i}\left(\frac{x_{z,k,t}(1+\tau_n)}{\prod_{j=1}^{i}(1+\pi_{t+j})} - \mu_{t+i} e^{\varepsilon_{z,t+i}}\right)$

Subject to $q_{z,k,t+i} = x_{z,k,t}^{-\eta} q_{t+i}$ for $i = 0 \ldots k-1$

Where $x_{z,k}$ is the firm's reset price relative to the aggregate price of NFE and τ_n is a production subsidy that ensures an efficient level of output in the steady state. Once set, the firm's price stays fixed for *k* periods (by definition) and so its price relative to the aggregate NFE price will be falling by the rate of NFE inflation, π.

The first-order condition for this problem implies:

$$q_t x_{z,k,t}^{-\eta}\left(x_{z,k,t}(1+\tau_n) - \frac{\eta}{\eta-1}\mu_t e^{\varepsilon_{z,t}}\right)$$
$$+E_t \sum_{i=1}^{k-1} \frac{1}{\prod_{j=1}^{i}(1+i_{t+j-1})} q_{t+i} x_{z,k,t}^{-\eta}\left(x_{z,k,t}(1+\tau_n) - \frac{\eta}{\eta-1}\mu_{t+i} e^{\varepsilon_{z,t+i}} \prod_{j=1}^{i}\left(1+\pi_{t+j}\right)\right) = 0 \quad \textbf{(A18)}$$

This equation implicitly defines the optimal reset price at time *t*. The optimal reset price equates the expected net present values of marginal cost with marginal revenue over the duration of the contract. The last term in brackets is the period *t+i* marginal revenue and marginal cost measured in period *t* prices. The weights vary with period specific outputs and elasticity; if these were constant, you could divide through the sum and they would drop out.

Hence, the average price prevailing in GT subsector *k* of COICOP sector *z* (relative to the numeraire) will be given by:

$$p_{z,k,t} = \left(\frac{1}{k} \left(x_{z,k,t}^{1-\eta} + \sum_{j=1}^{k-1} \frac{1}{\prod_{i=0}^{j-1} (1+\pi_{t-i})} x_{z,k,t-j}^{1-\eta} \right) \right)^{\frac{1}{1-\eta}} \qquad \textbf{(A19)}$$

And the average price prevailing across sector *z* will be given by:

$$p_{z,t} = \left(\sum_{k=1}^{K} \gamma_{z,k} p_{z,k,t}^{1-\eta} \right)^{\frac{1}{1-\eta}} \qquad \textbf{(A20)}$$

Finally, the price of non-food and energy (the numeraire) will be given by:

$$1 = \left(\sum_{z=1}^{12} \gamma_z p_{z,t}^{1-\eta} \right)^{\frac{1}{1-\eta}} \qquad \textbf{(A21)}$$

Value-added sector

Perfectly-competitive 'value-added' producers use labour and capital to produce value-added, *V*, according to the production function:

$$V_t = e^{\varepsilon_{a,t}} \left((1-\alpha_v) h_t^{\frac{\sigma_v - 1}{\sigma_v}} + \alpha_v (z_t k_{t-1})^{\frac{\sigma_v - 1}{\sigma_v}} \right)^{\frac{\sigma_v}{\sigma_v - 1}} \qquad \textbf{(A22)}$$

The term in *z* shows that the capital effectively used in production depends on the intensity of capital utilisation and ε_a represents a shock to productivity; recall that capital utilisation is decided by the consumers so firms can only choose the total capital input, zk_{-1}. The representative firm's problem is to choose capital and labour input so as to maximise its profit subject to this production function, where its profit will be given by:

$$p_{vc,t} V_t - w_t h_t - w_{k,t} z_t k_{t-1}$$

The first-order conditions for this problem imply the following demand curves for capital and labour:

$$\frac{w_t}{p_{vc,t}} = e^{\left(1-\frac{1}{\sigma_v}\right)\varepsilon_{a,t}} (1-\alpha_v) \left(\frac{V_t}{h_t}\right)^{\frac{1}{\sigma_v}} \qquad \textbf{(A23)}$$

$$\frac{w_{k,t}}{p_{vc,t}} = e^{\left(1-\frac{1}{\sigma_v}\right)\varepsilon_{a,t}} \alpha_v \left(\frac{V_t}{z_t k_{t-1}}\right)^{\frac{1}{\sigma_v}} \qquad \textbf{(A24)}$$

Petrol producers

Petrol, q_p, is produced using inputs of crude oil, I_o, and value-added, V_p. We assume a simple Leontieff production function:

$$q_{p,t} = \min\left(\frac{I_{o,t}}{1-\psi_{qp}}, \frac{V_{p,t}}{\psi_{qp}}\right)$$

Cost minimisation implies:

$$I_{o,t} = (1-\psi_{qp})q_{p,t} \quad \textbf{(A25)}$$

$$V_{p,t} = \psi_{qp} q_{p,t} \quad \textbf{(A26)}$$

$$\mu_{p,t} = \frac{\psi_{qp} p_{vc,t} + (1-\psi_{qp}) p_{o,t}}{p_{p,t}} \quad \textbf{(A27)}$$

Where p_o denotes the price of oil (relative to NFE) and μ_p denotes real marginal cost in this sector.

Petrol producers are assumed to operate in a monopolistically competitive market and face price rigidities a la Calvo (1983). Specifically, they are able to optimally change their price in any given quarter with probability 1-χ_p. Now, each period those producers that can change their price will set it so as to maximise their expected profit subject to their demand curves and the fact that they may not be able to change their price for a long while. The expected profit for a firm (say firm *j*) that can set its price in period *t* will be given by:

$$E_t \sum_{s=0}^{\infty} (\beta\chi_p)^s \left(\frac{p_{p,j,t}}{p_{p,t+s}}(1+\tau_p) - \mu_{p,t+s}\right)\left(\frac{p_{p,j,t}}{p_{p,t+s}}\right)^{-\eta_p} y_{t+s}$$

Where $p_{p,j}$ is the price (relative to the numeraire, NFE) set by petrol producer *j* and τ_p denotes a production subsidy, which offsets the distortion caused by monopolistic competition in the petrol sector.

So, the first-order condition for a price-changing firm (in this case, firm *j*) will be given by:

$$E_t \sum_{s=0}^{\infty} (\beta\chi_p)^s \left(\left(1+\tau_p - \eta_p\left((1+\tau_p) - \mu_{p,t+s}\frac{p_{p,t+s}}{p_{p,j,t}}\right)\right)\right)\left(\frac{p_{p,j,t}}{p_{p,t+s}}\right)^{-\eta_p} \frac{y_{t+s}}{p_{p,t+s}} = 0 \quad \textbf{(A28)}$$

The aggregate petrol price will be given by

$$p_{p,t} = p_{p,t-1}^{\chi_p} p_{p,j,t}^{1-\chi_p}$$

Taking a first-order Taylor series expansion of these two equations around a zero steady-state inflation rate gives the New Keynesian Phillips curve (NKPC) for this sector:

$$\pi_{p,t} = \beta E_t \pi_{p,t+1} + \frac{(1-\chi_p)(1-\beta\chi_p)}{\chi_p}\hat{\mu}_{p,t} \tag{A29}$$

where π_p represents the inflation rate for petrol prices.

Utilities producers

Utilities producers are exactly analogous to petrol producers. Output of utilities, q_u, is produced using inputs of gas, I_g, and value-added, V_u. We again assume a simple Leontieff production function:

$$q_u = \min\left(\frac{I_g}{1-\psi_u}, \frac{V_u}{\psi_u}\right)$$

Cost minimisation implies:

$$I_{g,t} = (1-\psi_u)q_{u,t} \tag{A29}$$

$$V_{u,t} = \psi_u q_{u,t} \tag{A30}$$

$$\mu_{u,t} = \frac{\psi_u p_{vc,t} + (1-\psi_u)p_{g,t}}{p_{u,t}} \tag{A31}$$

Where p_g denotes the price of wholesale gas (relative to NFE) and μ_u denotes real marginal cost in this sector.

Firms in this sector are again assumed to be monopolistically competitive and subject to nominal rigidities in their price-setting: they are able to optimally change their price in any given quarter with probability 1-χ_u. Going through an analogous argument to that shown for the petrol sector, we can derive the NKPC for this sector as:

$$\pi_{u,t} = \beta E_t \pi_{u,t+1} + \frac{(1-\chi_u)(1-\beta\chi_u)}{\chi_u}\hat{\mu}_{u,t} \tag{A32}$$

where π_u represents the inflation rate for utility prices and $\hat{\mu}_u$ denotes the log-deviation from steady state of real marginal cost in this sector.

Monetary and fiscal policy

Monetary policy is assumed to follow a Taylor rule with the central bank responding to deviations of inflation from target and value-added from flexible-price value-added, y_{FP}:

$$i_t - \left(\frac{1}{\beta} - 1\right) = \theta_{rg}\left(i_{t-1} - \left(\frac{1}{\beta} - 1\right)\right) + \left(1 - \theta_{rg}\right)\left(\theta_\pi \pi_{c,t} + \theta_y\left(\widehat{y_t} - \widehat{y_{FP,t}}\right)\right) + \varepsilon_{i,t} \quad \textbf{(A33)}$$

where ε_i is a monetary policy shock. Flexible-price value-added is defined as what value-added would be in a flexible-price version of the model given the estimated values of the shocks.

The government is assumed to buy only NFE goods and to have the same preferences across these goods as consumers. It collects lump-sum taxes from each of the firms in order to eliminate the welfare distortions resulting from monopolistic competition. Any remaining budget shortfall is met via lump-sum taxes on consumers or budget surplus returned to consumers via lump-sum transfers.

We can write its budget constraint as:

$$b_t = \frac{1+i_{t-1}}{1+\pi_t} b_{t-1} + c_g e^{\varepsilon_{g,t}} + T_t - \tau_n q - \tau_p q_p - \tau_u q_u \quad \textbf{(A34)}$$

Where q, qp and qu denote the steady-state levels of output in the non food and energy, petrol and utilities sectors, respectively, c_g denotes steady-state government spending and $\varepsilon_{g,t}$ is a government spending shock.

Foreign sector

We assume that the United Kingdom is a small open economy. Hence, world prices are exogenous. We assume that oil and gas prices adjust immediately to their world prices:

$$p_{o,t} = \frac{e^{\varepsilon_{p_o,t}}}{s_t} \quad \textbf{(A35)}$$

$$p_{g,t} = \frac{e^{\varepsilon_{p_g,t}}}{s_t} \quad \textbf{(A36)}$$

where ε_{p_o} is a shock to world oil prices and ε_{p_g} is a shock to world gas prices and we have normalised the steady-state world prices of oil and gas to unity.

UK food and non food and energy import prices, on the other hand, take time to adjust to purchasing power parity.[8] We assume the following NKPCs for food prices and for import prices ex food and energy:

$$\pi_{f,t} = \frac{\iota_{pf}}{1+\beta\iota_{pf}}\pi_{f,t-1} + \frac{\beta}{1+\beta\iota_{pf}}E_t\pi_{f,t+1} + \frac{(1-\xi_{pf})(1-\beta\xi_{pf})}{(1+\beta\iota_{pm})\xi_{pf}}\left(\varepsilon_{p_f} - \hat{s}_t - \hat{p}_{f,t}\right) \quad \textbf{(A37)}$$

$$\pi_{m,t} = \frac{\iota_{pm}}{1+\beta\iota_{pm}}\pi_{m,t-1} + \frac{\beta}{1+\beta\iota_{pm}}E_t\pi_{m,t+1} + \frac{(1-\xi_{pm})(1-\beta\xi_{pm})}{(1+\beta\iota_{pm})\xi_{pm}}\left(\varepsilon_{p_{mf}} - \hat{s}_t - \hat{p}_{m,t}\right) \quad \textbf{(A38)}$$

[8] The underlying assumption here is that UK importers of food and imported intermediate goods excluding food, oil and gas face 'Calvo' frictions in their ability to set prices.

where π_f is the rate of inflation of food prices, π_m is the rate of inflation of non food and energy import prices, ε_{p_f} is a shock to world food prices and ε_{p_m} is a shock to the world price of our imports.

Finally, we assume the following demand function for our exports of non-energy goods, x_n:

$$x_{n,t} = \kappa_x x_{n,t-1}^{\psi_x} \left(s_t^{-\eta_x} \bar{x}\right)^{1-\psi_x} \quad \textbf{(A39)}$$

Where ψ_x is captures the idea that foreign preferences exhibit a form of 'habit formation' and $\overline{x}$ denotes 'world demand' (assumed to be exogenous and constant).

Market clearing

We close the model with the following market-clearing conditions (in addition to equations **(A8)** and **(A34)**):

$$V_t = V_{n,t} + V_{u,t} + V_{p,t} \quad \textbf{(A40)}$$

$$q_{p,t} = c_{p,t} + I_{p,t} \quad \textbf{(A41)}$$

$$q_{u,t} = c_{u,t} + I_{u,t} \quad \textbf{(A42)}$$

$$\overline{O} = I_{o,t} + X_{o,t} \quad \textbf{(A43)}$$

$$\overline{G} = I_{g,t} + X_{g,t} \quad \textbf{(A44)}$$

$$q_t = c_{n,t} + k_t - \left(1 - \delta - \frac{\chi_z}{1+\phi_z}\left(z_t^{1+\phi_z} - 1\right)\right) k_{t-1} + \frac{\chi_k}{2k_{t-1}}\left(k_t - \left(\frac{k_{t-1}}{k_{t-2}}\right)^{\varepsilon_k} k_{t-1}\right)^2 + c_g e^{\varepsilon_{g,t}} + X_{n,t} \quad \textbf{(A45)}$$

Where X_n denotes exports of non food and energy. We assume that the economy does not export food, petrol or utilities.

Finally, combining the consumers' and government's budget constraints with the definition of profits in each sector implies the balance of payments equation:

$$\frac{b_{f,t}}{s_t} - \frac{b_{f,t-1}}{s_t(1+\pi_t)} = X_{n,t} + p_{g,t}X_{g,t} + p_{o,t}X_{o,t} - p_{m,t}M_{n,t} - p_{f,t}c_{f,t} + \frac{i_{f,t-1}}{(1+\pi_t)}\frac{b_{f,t-1}}{s_t} - \frac{\chi_{bf}}{2}\left(\frac{b_{f,t}}{s_t}\right)^2 \quad \textbf{(A46)}$$

Where X_o denotes exports of oil, X_g denotes exports of natural gas.

These equations complete the description of the model.

Appendix 2: The efficient steady state

In this annex, we first define the conditions under which a zero-inflation steady state is efficient and then show that we can obtain an efficient steady state in our decentralised economy by appropriately setting the various taxes and subsidies we introduced.

Consider a social planner who maximises the households' period utility function, subject to the aggregate resource constraint and market clearing in the housing and labour markets. Note that since the cost of foreign bond holdings is assumed to be a deadweight loss, then in the efficient steady state, foreign bond holdings will be zero and the economy will run a zero trade balance. Given that, we can write the social planner's problem as:

Maximise $\frac{c^{1-\frac{1}{\sigma_c}}}{1-\frac{1}{\sigma_c}} - \kappa_h \frac{h^{1+\frac{1}{\sigma_h}}}{1+\frac{1}{\sigma_h}}$

Subject to $c = \kappa_c \left(\psi_f c_f^{1-\frac{1}{\sigma_e}} + \psi_n c_n^{1-\frac{1}{\sigma_e}} + (1-\psi_f-\psi_n)\kappa_e \left(\psi_p c_p^{1-\frac{1}{\sigma_p}} + (1-\psi_p) c_u^{1-\frac{1}{\sigma_p}} \right)^{\frac{\sigma_p}{\sigma_p-1}} \right)^{\frac{\sigma_e}{\sigma_e-1}}$

$$c_n + X_n = q = q(h_n, k_n, M_n, I_p, I_u)$$

$$c_p + I_p = \frac{\bar{O} - X_O}{1-\psi_{qp}}$$

$$c_u + I_u = \frac{\bar{G} - X_g}{1-\psi_u}$$

$$p_f c_f + p_m M_n = X_n + p_O X_o + p_G X_G$$

Let λ_n, λ_p, λ_u, λ_f be the Lagrange multipliers on the non food and energy, petrol and utilities resource constraints and the balanced trade constraint, respectively. Then the first-order conditions will imply:

$$c^{-\frac{1}{\sigma_c}} \frac{\kappa_c^{\frac{\sigma_e-1}{\sigma_e}}}{\psi_n} \left(\frac{c}{c_n}\right)^{\frac{1}{\sigma_e}} = \lambda_n \quad \textbf{(B1)}$$

$$c^{-\frac{1}{\sigma_c}} \frac{\kappa_c^{\frac{\sigma_e-1}{\sigma_e}}}{\psi_f} \left(\frac{c}{c_f}\right)^{\frac{1}{\sigma_e}} = \lambda_f p_f \quad \textbf{(B2)}$$

$$c^{-\frac{1}{\sigma_c}} \frac{\kappa_c^{\frac{\sigma_e-1}{\sigma_e}}}{1-\psi_n-\psi_f} \left(\frac{c}{c_e}\right)^{\frac{1}{\sigma_e}} \frac{\kappa_e^{\frac{\sigma_p-1}{\sigma_p}}}{1-\psi_p} \left(\frac{c_e}{c_u}\right)^{\frac{1}{\sigma_p}} = \lambda_u \quad \textbf{(B3)}$$

$$c^{-\frac{1}{\sigma_c}} \frac{\kappa_c^{\frac{\sigma_e-1}{\sigma_e}}}{1-\psi_n-\psi_f} \left(\frac{c}{c_e}\right)^{\frac{1}{\sigma_e}} \frac{\kappa_e^{\frac{\sigma_p-1}{\sigma_p}}}{\psi_p} \left(\frac{c_e}{c_p}\right)^{\frac{1}{\sigma_p}} = \lambda_p \quad \textbf{(B4)}$$

$$\kappa_h h^{\frac{1}{\sigma_h}} = \lambda_n q'(h_n) \quad \textbf{(B5)}$$

$$\frac{\lambda_f}{\lambda_n} p_m = q'(M_n) \qquad \textbf{(B6)}$$

$$\frac{\lambda_u}{\lambda_n} = q'(I_u) \qquad \textbf{(B7)}$$

$$\frac{\lambda_p}{\lambda_n} = q'(I_p) \qquad \textbf{(B8)}$$

These equations imply the familiar conditions that the marginal rate of substitution between consumption of the non food and energy good and consumption of leisure, food, utilities and petrol is equal to the marginal rate of transformation between, respectively, labour, imports, utilities and petrol and output.

We now show that by appropriate choices of taxes and subsidies we can achieve the efficient steady state in our decentralised economy. In particular, we need to subsidise the producers of non food and energy, petrol and utilities in order to neutralise the inefficiencies resulting from monopolistic competition in each of these industries.

First, we note that combining equations **(A8)** through **(A11)** with **(B1)** through **(B4)** implies:

$$c^{-\frac{1}{\sigma_c}} = \lambda_n p_c \qquad \textbf{(B9)}$$

$$\lambda_f = \lambda_n \qquad \textbf{(B10)}$$

$$\lambda_n p_u = \lambda_u \qquad \textbf{(B11)}$$

$$\lambda_n p_p = \lambda_p \qquad \textbf{(B12)}$$

Now, combining equations **(A2), (A15)** and **(A23)** implies:

$$\kappa_h h^{\frac{1}{\sigma_h}} = \lambda_n \mu q'(h_n) \qquad \textbf{(B13)}$$

The steady-state version of equation **(A18)** implies:

$$(1 + \tau_n) = \frac{\eta}{\eta - 1} \mu \qquad \textbf{(B14)}$$

So, if we set the production subsidy to firms in the non food and energy sector, τ_n, equal to $\left(\frac{\eta}{\eta-1} - 1\right)$, then steady-state real marginal cost, μ, will equal unity and equation **(B13)** will be equivalent to the efficiency condition given by equation **(B5)**.

Furthermore combining equation **(A13)** with equation **(B10)** will then give:

$$\frac{\lambda_f}{\lambda_n} p_m = q'(M_n) \qquad \textbf{(B15)}$$

That is, the efficiency condition **(B6)**.

Similarly, combining equation **(A14)** with equation **(B12)** gives:

$$\frac{\lambda_p}{\lambda_n} = q'(I_p) \tag{B16}$$

That is, the efficiency condition **(B8)**.

And finally, combining equation **(A15)** with equation **(B11)** gives:

$$\frac{\lambda_u}{\lambda_n} = q'(I_u) \tag{B17}$$

That is, the efficiency condition **(B7)**.

Now, the steady-state versions of equations **(A27)** and **(A28)** imply:

$$(\eta_p - 1)(1 + \tau_p) = \eta_p \mu_p \tag{B18}$$

So, if we set the production subsidy to firms in the petrol sector, τ_p, equal to $\left(\frac{\eta_p}{\eta_p - 1} - 1\right)$, then steady-state real marginal cost in this sector, μ_p will equal unity. Similarly, if we set the production subsidy to firms in the utilities sector, τ_u, equal to $\left(\frac{\eta_u}{\eta_u - 1} - 1\right)$, then steady-state real marginal cost in this sector, μ_u will equal unity.

With these subsidies in place, we can now write out the complete equations for the steady state of our model. We normalise all foreign prices, the CPI, TFP and total hours worked in steady state to all equal unity.

$$1 = \beta(1+i) = \beta(1+i_F) \tag{B19}$$

$$w = \kappa_h c^{\frac{1}{\sigma_c}} \tag{B20}$$

$$1 = \beta(1 - \delta + \chi_z) \tag{B21}$$

$$c = \kappa_c \left((1 - \psi_e - \psi_f) c_n^{1-\frac{1}{\sigma_e}} + \psi_e c_e^{1-\frac{1}{\sigma_e}} + \psi_f c_f^{1-\frac{1}{\sigma_e}} \right)^{\frac{\sigma_e}{\sigma_e - 1}} \tag{B22}$$

$$c_e = \kappa_e \left((1 - \psi_p) c_u^{1-\frac{1}{\sigma_p}} + \psi_p c_p^{1-\frac{1}{\sigma_p}} \right)^{\frac{\sigma_p}{\sigma_p - 1}} \tag{B23}$$

$$c = c_n + c_f + p_u c_u + p_p c_p \tag{B24}$$

$$1 = \kappa_c^{\frac{\sigma_e - 1}{\sigma_e}} \psi_f \left(\frac{c_f}{c} \right)^{-\frac{1}{\sigma_e}} \tag{B25}$$

$$p_u = \frac{\kappa_e^{\frac{\sigma_p - 1}{\sigma_p}} (1-\psi_p) \left(\frac{c_u}{c_e} \right)^{-\frac{1}{\sigma_p}}}{\kappa_c^{\frac{\sigma_e - 1}{\sigma_e}} \psi_e \left(\frac{c_e}{c} \right)^{-\frac{1}{\sigma_e}}} \tag{B26}$$

$$\frac{p_u}{p_p} = \frac{1-\psi_p}{\psi_p} \left(\frac{c_u}{c_p} \right)^{-\frac{1}{\sigma_p}} \tag{B27}$$

$$q = \kappa_q \left((1-\alpha_q)((1-\phi_q)B)^{1-\frac{1}{\sigma_q}} + \alpha_q (\phi_q e)^{1-\frac{1}{\sigma_q}} \right)^{\frac{\sigma_q}{\sigma_q - 1}} \tag{B28}$$

$$B = V_n^{1-\alpha_B} M_n^{\alpha_B} \tag{B29}$$

$$e = \frac{I_p}{\psi_n} = \frac{I_u}{1-\psi_n} \tag{B30}$$

$$p_{vc} V_n = \kappa_q^{1-\frac{1}{\sigma_q}} (1-\alpha_q)(1-\alpha_B)(1-\phi_q)^{1-\frac{1}{\sigma_q}} q^{\frac{1}{\sigma_q}} B^{1-\frac{1}{\sigma_q}} \tag{B31}$$

$$M_n = \kappa_q^{1-\frac{1}{\sigma_q}} (1-\alpha_q) \alpha_B (1-\phi_q)^{1-\frac{1}{\sigma_q}} q^{\frac{1}{\sigma_q}} B^{1-\frac{1}{\sigma_q}} \tag{B32}$$

$$\psi_n p_p + (1-\psi_n) p_u = \kappa_q^{1-\frac{1}{\sigma_q}} \alpha_q \phi_q^{1-\frac{1}{\sigma_q}} q^{\frac{1}{\sigma_q}} e^{\frac{-1}{\sigma_q}} \tag{B33}$$

$$V = \left(1 - \alpha_v + \alpha_v k^{1-\frac{1}{\sigma_v}} \right)^{\frac{\sigma_v}{\sigma_v - 1}} \tag{B34}$$

$$\frac{w}{p_{vc}} = (1-\alpha_v) V^{\frac{1}{\sigma_v}} \tag{B35}$$

$$\frac{\chi_z}{p_{vc}} = \alpha_v \left(\frac{V}{k} \right)^{\frac{1}{\sigma_v}} \tag{B36}$$

$$q_p = \frac{I_o}{1-\psi_{qp}} = \frac{V_p}{\psi_{qp}} \tag{B37}$$

$$p_p = \psi_{qp} p_{vc} + 1 - \psi_{qp} \tag{B38}$$

$$q_u = \frac{I_g}{1-\psi_u} = \frac{V_u}{\psi_u} \tag{B39}$$

$$p_u = \psi_u p_{vc} + 1 - \psi_u \tag{B40}$$

$$q = c_n + \delta k + c_g + X_n \tag{B41}$$

$$X_n = \kappa_x \tag{B42}$$

$$V = V_n + V_u + V_p \tag{B43}$$

$$q_P = c_P + I_P \tag{B44}$$

$$q_u = c_u + I_u \tag{B45}$$

$$\overline{O} = I_O + X_O \tag{B46}$$

$$\overline{G} = I_G + X_G \tag{B47}$$

$$x_n + X_g + X_o = M_n + c_f \tag{B48}$$

Appendix 3: The relationship between the sectoral shocks of the model and the estimated sectoral inflation shocks

In section 2, we make an empirical estimate of the shocks to sectoral inflation under the assumption that the shocks are residuals once we have stripped out the macroeconomic factors. The underlying sectoral shocks give rise to these observed inflation shocks. However, the effect of the sectoral shocks on sectoral inflation will depend on the degree of nominal rigidity. If prices are perfectly flexible, the sectoral price level and hence inflation will reflect these shocks fully. If prices are completely fixed, then the sectoral shocks will not result in any inflation at all, as prices cannot vary.

With a generalised Taylor (GT) economy, there is a simple relationship between the sectoral shocks and the resultant sectoral inflation shocks. Let us consider sector *k*. Within sector *k* there are a proportion of firms γ_{k1} who have flexible prices: therefore from equation **(20)** the price they set at period *t* will fully reflect the sectoral shock $\varepsilon_{k,t}$. There is also a proportion γ_{k1} who set prices for two periods. Half of these will reset their price in period *t*. Since the expected shock in the next period is 0 ($E_t\, \varepsilon_{k,t+1}=0$) it follows that (ignoring discounting) their current reset price will reflect only half of the sectoral shock. Similarly, for the proportion γ_{ki} who set their prices for *i* periods, only a proportion i^{-1} will reset their prices. Since they expect all future shocks to be zero, their reset price will only be affected by i^{-1} of the current sectoral shock $\varepsilon_{k,t}$. Hence the response of prices in sector *k* to the sectoral shock is given by:

$$\Delta P_k = \varepsilon_{k,t} \cdot \sum_{i=1}^{12} \frac{\gamma_{ki}}{i} \tag{C1}$$

With discounting, the appropriate formula is:

$$\Delta P_k = \varepsilon_{k,t} \cdot \sum_{i=1}^{12} \frac{\gamma_{ki}}{i\left(\sum_{j=1}^{i} \beta^{j-1}\right)} \tag{C2}$$

Since the quarterly discount rate is β =0.9925, discounting has little quantitative effect with the maximum price-spell of 12 quarters. Hence, we can see that the sectoral shocks will in general be much larger than the corresponding sectoral inflation shocks. So given our estimates of the size of the inflation shocks, we will need to scale up the size of the sectoral shocks:

Using the GT coefficients for each sector, we obtain the appropriate scaling factor to two decimal places:

Non-Alc bev.	Alcohol	Cloth&F	H&W	Furn	Health	Transport	Comm	Rec&Cult	R&H	Misc	Average
2.15	1.71	1.81	2.75	1.91	2.95	2.78	1.90	2.12	2.62	2.36	2.34
2.17	1.72	1.82	2.79	1.92	2.99	2.82	1.92	2.14	2.65	2.38	

The figures in the first row are the scaling factors with discounting. The second row gives the figures without discounting: as we see, there is a very small quantitative effect if we approximate $\beta = 1$. The average scaling factor in NFE is 4.25 (using CPI weights). Education is missing from our data set. Given that we treat all price-spells as 4 quarters in *Education* this would indicate a scaling factor of 16, the value we use in our analysis.

Appendix 4: Calibration

Table A4.1: Standard parameter values

Parameter	Value	Description
β	0.9925	Discount factor
χ_{bf}	0.001	Cost of adjusting portfolio of foreign bonds
δ	0.013	Depreciation rate
χ_z	0.0206	Scales the effect of capital utilisation on the depreciation rate
ε_k	0.5	Degree of persistence in investment adjustment costs
χ_k	201	Inverse elasticity of investment adjustment costs
ϕ_z	0.56	Inverse elasticity of capital utilisation costs
η	20	Elasticity of demand in the NFE and petrol sectors
η_u	2.25	Elasticity of demand in the utilities sector
σ_h	0.43	Inverse Frisch elasticity of labour supply
σ_c	0.66	Intertemporal elasticity of consumption
σ_e	0.4	Elasticity of substitution between non-energy and energy in consumption
σ_p	0.1	Elasticity of substitution between petrol and utilities in energy consumption
σ_v	0.5	Elasticity of substitution between labour and capital in value-added production
σ_q	0.15	Elasticity of substitution between energy and everything else in non-energyproduction
α_v	0.25	Weight on capital in value-added production
ϕ_q	0.99	NFE production function coefficient
χ_u	0.67	Probability of resetting utility prices
χ_p	0.33	Probability of resetting petrol prices
ξ_{pm}	0.6	Probability of resetting import prices
ι_{pm}	0.17	Indexation in import price setting
ξ_{pf}	0.5	Probability of resetting food prices
ι_{pf}	0	Indexation in food price setting
ψ_x	0.24	Persistence of export demand
η_x	1.5	Elasticity of demand for exports

Steady-state weights and shares

Here, we discuss how we use data on the steady-state shares of various items in consumption and production to calibrate a number of parameters within our model. To start, we use the appropriate CPI weights (those applied in 2016 based on expenditure shares in 2015) to give us the weights of each of our sectors in consumption. These are shown in Table A4.2, below.

We also need the weights of gas in utilities output and oil in petrol output. From the 2015 *Supply and Use Tables* (SUTs), we can note that inputs of 'oil and gas extraction' into 'electricity production and distribution and gas distribution' were worth £18,529 million and into 'coke ovens and refined petroleum products' were worth £9,035 million. The total output of these industries at basic prices was £104,955 million and £19,728 million, respectively. This gives us shares of 0.1765 and 0.4580, respectively. Total value-added of 'oil and gas extraction' at basic prices was £15,309 million. This compares with gross value added at basic prices (GDP) for the whole economy of £1,684,937 million and implies a share of 'oil and gas extraction' output of 0.9086%. If we assume that the relative proportions of ‘oil’ and ‘gas’ equal the relative proportions used as inputs into ‘petrol’ and ‘utilities’, respectively, then we get shares of 0.0030 for ‘oil’ in GDP and 0.0061 for ‘gas’ in GDP.

Table A4.2: 2016 CPI Weights

Sector	Weight (per cent)
Non-alcoholic beverages	1.2
Alcohol and tobacco	4.2
Clothing and footwear	7.1
Housing and water	8.5
Furniture, household equipment and maintenance	5.9
Health	2.8
Transport excluding fuels and lubricants	12.1
Communication	3.2
Recreation and culture	14.8
Education	2.5
Restaurants and hotels	12.3
Miscellaneous goods and services	9.6
Food	9.1
Electricity, gas and other fuels (utilities)	3.5
Fuels and lubricants (petrol)	3.2

We also need the weight of energy in NFE output. We define ‘food’ as the sum of ‘Agriculture’, ‘Fishing’, ‘Meat processing’, ‘Fish and fruit processing’, ‘Oils and fats’, ‘Dairy products’, ‘Grain milling and starch’, ‘Animal feed’, ‘Bakery’, and ‘Other food products’ in the SUTs. Total final demand of the food sector (so defined) was £114,683 million in 2015. Total final demand for all industries was £2,438,268 million. From this, we take out total final demand for food, oil and gas extraction (£14,305 million), utilities (£30,074 million) and petrol (£41,264 million) to get total final demand at purchasers’ prices of the NFE sector of £2,237,942 million.

Now, total intermediate demand for oil and gas extraction was £29,667 million. Of this, the food sector used zero, and the oil and gas extraction sector used £696 million. So, the weight of energy inputs into production of utilities, petrol and NFE will equal 0.0125. Total intermediate demand for utilities was £80,413 million and for petrol was £36,588 million. Of this, the food sector used £2,996 million of utilities and £2,613 million of petrol; the oil and gas extraction sector used £233 million of utilities and £116 million of petrol; the utilities sector used £48,307 million of utilities and £807 million of petrol; and the petrol sector used £951 million of utilities and £3,080 million of petrol. Putting all this together, we get an input of utilities into NFE of £27,926 million and of petrol into NFE of £29,972 million. So the shares are 0.0125 for utilities and 0.0134 for petrol. The ratio of the two is then 1.0733. We next calculate the share of NFE imports in NFE output. Total imports of goods are services were £549,531 million in 2015. Of these, £37,022 million were food, £19,697 million were oil and gas extraction, £19,276 million were petrol and £997 million were utilities. So imports of NFE were £472,539 million. So the share of NFE imports in NFE output was 21.11%. Finally, the 2015 SUTs suggest that final consumption of central and local government was equal to £362,062 and that this consisted entirely of spending on NFE. Hence, the share of government spending in NFE is equal to 16.18%. Given these shares, we can then set our remaining parameters so that the model generates these shares in steady state. The implied parameters are shown in Table A4.3, below.

Table A4.3: Parameter values set to match expenditure and cost shares

Parameter	Value	Description	Comment
κ_x	0.2440	Exports of non-energy	Normalises the exchange rate, *s*, to equal unity in steady state
κ_c	1.3013	Scale parameter on consumption aggregator	Normalises the relative price of consumption, p_c, to equal unity in steady state
κ_h	1.6787	Relative utility of leisure	Normalises total hours, *h*, to equal unity in steady state
α_b	0.2167	Parameter governing share of non-energy imports in non food and energy production	Ensures that the steady-state share of non food and energy imports in non food and energy output is 22.29%.
α_q	0.9907	Parameter governing share of energy in non food and energy production	Ensures that the steady-state share of energy in non food and energy output is 2.9%.
ψ_e	0.0007	Parameter governing share of energy in consumption	Ensures that the steady-state share of energy in consumption spending is 8.7%.
ψ_n	0.4839	Parameter governing share of petrol in non food and energy production	Ensures that the steady-state ratio of petrol to utility input in non food and energy output is 1.3682.
ψ_p	0.1078	Parameter governing share of petrol in consumption	Ensures that the steady-state share of petrol in consumption spending is 4.3%.
ψ_{qp}	0.6698	Parameter governing share of oil in petrol production	Ensures that the steady-state share of oil in petrol production is 0.7592.
ψ_u	0.8888	Parameter governing share of gas in utility production	Ensures that the steady-state share of gas in utility production is 0.3026.
ψ_f	0.0037	Parameter governing share of food in consumption	Ensures that the steady-state share of food in consumption spending is 10.3%.
$\overline{O}$	0.0021	Economy's endowment of oil	Ensures that the steady-state share of oil in GDP is 1.3%.
$\overline{G}$	0.0043	Economy's endowment of gas	Ensures that the steady-state share of gas in GDP is 1.4%.
c_g	0.1411	Government purchases of non food and energy	Ensures that the steady-state share of government spending in non food and energy demand is 18.38%.

Shock processes

To calibrate the processes driving real marginal cost in each of the 12 NFE sectors, we first note that the results reported in Table C in Section 2 suggest that we can reasonably model 11 of the 12 sectoral shocks as white noise, with the exception being the shock to *Recreation and culture*. We model this shock as an AR(1) process with an AR(1) coefficient equal to 0.63, the estimated first-order autocorrelation for the shock to *Recreation and culture*. We then multiply up the standard deviations of the estimated sectoral inflation shocks coming from Equation **(1)** by the scaling factors for each sector that we derived in Appendix 3, above. These scaling factors allow for the fact that prices in stickier sectors are less able to respond to shocks. The resulting standard deviations for the sectoral shocks in our model are shown in Table A4.4, below.

Table A4.4: Standard deviations of sectoral shocks (per cent)	
Non-Alcoholic Beverages	1.51
Alcoholic Beverages and Tobacco	1.01
Clothing and Footwear	1.29
Housing and Water	1.07
Furniture, household equipment and maintenance	0.82
Health	0.83
Transport (ex Fuel and Lubricants)	1.67
Communication	1.67
Recreation and Culture	0.21
Education	25.60
Restaurants and hotels	0.52
Misc Goods and Services	0.94

For the world shocks we used quarterly data from 1996 Q1 to 2016 Q4 on world food prices, world oil prices, world gas prices and the world price of UK NFE imports. To construct a world price index for food, we multiplied the implicit price deflator for UK consumption of imported food, beverages and tobacco (*BQAR*/*BPIA*) by the sterling effective exchange rate index (*ERI*). Similarly, to construct a world price index for UK imports excluding food and energy, we calculated an implicit price deflator in sterling by stripping out imports of food (*BQAR* for values, *BPIA* for volumes) and energy (*BQAT* for values, *BPIC* for volumes) from total imports (*IKBI* for values, *IKBL* for volumes), and then multiplied this deflator by the sterling ERI. We then took logs and HP-filtered the resulting series. Finally we estimated AR(1) processes for the HP-Filtered series.

We assumed the monetary policy shock was white noise. Using quarterly UK data from 1996 Q1 to 2016 Q4 for the nominal interest rate (*AMIH*), CPI inflation and GDP (*ABMM*), we constructed an implied series for the monetary policy shock based on equation **(17)**. Inflation and the interest rate were both demeaned and we used HP-filtered GDP as a measure of the output gap. The standard deviation of this series was equal to 0.0018; we use this value for the standard deviation of the monetary policy shock in our model.

In a similar vein, we used quarterly UK data over the same time period on GDP, total hours worked (*YBUS*) and the capital stock to construct a time series for our productivity shock.[9] Specifically, we used a version of equation **(9)** in which capacity utilisation was always at its steady state together with the calibration in Table A5.1 to obtain:

$$\varepsilon_{a,t} = \widehat{V}_t - 0.75\left(\frac{V}{h}\right)\widehat{h}_t - 0.25\left(\frac{V}{k}\right)\widehat{k_{t-1}} \qquad \textbf{(D1)}$$

In the steady state of our calibrated model $\frac{V}{h} = 1.1587$ and $\frac{V}{k} = 0.5238$. Given these values, and HP-filtered GDP, total hours worked and capital, we constructed ε_a and estimated its AR(1) process.

[9] For a description of how the capital services series we used was constructed see Oulton and Srinivasan (2003).

Finally, to construct the government spending shock, we estimated an AR(1) model using HP-filtered real government consumption (NMRY), over the same time period.

The estimated AR(1) coefficients and standard deviations of the driving processes are shown for each of these shocks in Table A4.5, below.

Table A4.5 Estimated shock processes

Shock	AR(1) coefficient, ρ	Standard deviation of the driving process, σ_v
World food price	0.70	0.0398
World crude oil price	0.75	0.1445
World wholesale gas price	0.62	0.2161
World NFE export price	0.84	0.0156
Monetary policy shock	0.00	0.0018
Productivity	0.72	0.0057
Government spending	0.29	0.0089

Estimating the sectoral hazard functions

These are estimated using UK data from February 1996 to February 2018. Specifically, we use an updated version of the data in Dixon and Tian (2017), adjusted for the splitting off of *Fuel and lubricants* from *Transport*, *Utilities* from *Housing and water* and *Food* from *Food and Non-alcoholic beverages*. The ONS micro price data we use is described by Bunn and Ellis (2012). The derivation of the 12x12 matrix of sectoral duration coefficients γ_{ji} is based on the steady-state identities derived in Dixon (2012). By averaging out over the ten years, issues such as seasonality will wash out. Following Dixon and Le Bihan (2012), we estimate the sectoral hazard rates excluding left-censored spells, and treating right-censored spells as price-changes. The CPI data for education is not available from the ONS: casual empiricism indicates that these prices are set annually, so we have set the share of four-quarter spells equal to one and the rest to zero.

The estimated coefficients are shown in Table A4.6. The modal duration is highlighted in yellow, the median duration is underlined and the arithmetic mean is in the bottom row.[10] The mean duration across the *NFE* sectors using CPI weights is 4.35 quarters. Two factors need to be noted: first the NFE sector accounts for 81% of the CPI and the remaining 19% are mostly flexible prices, so that the mean duration across the entire CPI would be lower; second, the distribution is truncated at 12 quarters which reduces the mean. Overall, the mean estimated by Dixon and Tian (2012) across all sectors excluding education is 10.9 months.

The method used for estimating the hazard function is the non-parametric Kaplin-Meier method (KM). One of the main issues in applying the method to the data is how to deal with censored data. It is best to think of the CPI data as a panel with attrition and replacement. We can see the CPI categories as collections of rows spreading across the 120 months. There are about 600 products and services

[10] The median occurs within the duration specified. Thus, duration *i* means the median duration is between *i*-1 and *i*, hence the first cell in a column that causes the sum of the hazards to exceed 0.5 contains the median.

sampled, with about 100,000 observations per month: each product is sampled in a variety of locations and across different sellers in order to be 'representative'.

Table A4.6: The Sectoral GT weights in NFE

Duration	Non-Alcoholic beverages	Alcoholic beverages	Clothing and Footwear	Housing and Water	Furniture	Health
1	0.258538	0.364759	0.348084	0.158687	0.332254	0.141129
2	0.164464	0.23835	0.216387	0.13578	0.175622	0.118738
3	0.136859	0.173809	0.114029	0.123914	0.110584	0.115163
4	0.123069	0.075701	0.085725	0.129873	0.093401	0.127984
5	0.064396	0.036453	0.05174	0.071711	0.056067	0.071579
6	0.050603	0.025364	0.041523	0.062403	0.049431	0.068069
7	0.044919	0.020769	0.030003	0.056376	0.038479	0.058657
8	0.036083	0.015377	0.028401	0.047687	0.03236	0.054727
9	0.03048	0.009892	0.01777	0.030343	0.023007	0.034519
10	0.019933	0.008122	0.014715	0.029691	0.018941	0.039368
11	0.018816	0.008006	0.01147	0.029484	0.015815	0.029385
12	0.051841	0.023397	0.040153	0.124051	0.054041	0.14068
Mean	4.02	2.81	3.33	5.21	3.71	5.57
Duration	**Transport**	**Communications**	**Education**	**Recreation and Culture**	**Restaurants and Hotels**	**Miscellaneous**
1	0.13765	0.640577	0.0000	0.269477	0.172279	0.218706
2	0.172243	0.132255	0.0000	0.169827	0.130765	0.157369
3	0.130431	0.057103	0.0000	0.121293	0.137958	0.125223
4	0.132647	0.034746	1.0000	0.118622	0.150232	0.123234
5	0.053646	0.024914	0.0000	0.058508	0.084055	0.067948
6	0.062459	0.024163	0.0000	0.052384	0.059117	0.056757
7	0.057418	0.019847	0.0000	0.043088	0.055404	0.049628
8	0.054172	0.016807	0.0000	0.036776	0.054482	0.045321
9	0.025527	0.011325	0.0000	0.022754	0.033704	0.026843
10	0.027099	0.011436	0.0000	0.022228	0.025119	0.025148
11	0.028093	0.009692	0.0000	0.018752	0.024373	0.019332
12	0.118616	0.017135	0.0000	0.06629	0.072515	0.084491
Mean	5.12	3.13	4.00	4.08	4.74	4.51

Within each CPI category there are sequences of price observations for each product. These can be identified as consecutive observations of the price of a particular product at a particular location: this is called a *trajectory*. The choice of products and locations reflects ONS policy, as does the length of trajectory, and both can be treated as an exogenous 'act of God'. Within each trajectory is a sequence of price-spells. For the purpose of this study, we are looking at all price-spells including temporary sales. There are four main types of price-spells in terms of censoring. First there are spells which constitute a whole trajectory. For example, there are some pharmaceutical products in the Health category which have the same price for all 10 years. These are left and right censored: we do not see when they begin or end. Second, there is the first spell in a trajectory of two or more spells. We see the price persist for some time, but do not know when the spell began. This is a left-censored spell.

Thirdly, there is the last spell in the trajectory, which we observe starting and persisting, but not ending. Fourthly, there are the rest of the price-spells which we see beginning, persisting, and ending. These are uncensored spells.

There are different ways of treating censored data which can have a large impact on the results. The 'classic' KM method (developed for analysing the data of medical trials) is to exclude all left-censored spells, include all right censored data, and treat the end of a right censored data as a non-price-change. A price-spell ends with a price-change. In the case of right-censored data, you do not observe that ending: it just falls out of the data because the ONS stopped looking at it. This treatment of right censored spells is not a good one in our context. In effect, we know that for our purposes all price-spells end at 12 quarters. In terms of the steady-state identities, not registering a price-change when the ONS stops looking at it means that implicitly the price-spell extends to 12 quarters. The price has to change sometime and the classic KM treatment will lead to an under-estimate of the hazard for each period.

Censoring can only reduce the observed length of price-spells. As a better alternative to the classic KM method, we make two other assumptions:

(a) We exclude all censored data in estimating the hazard function. This can be justified if we believe that the censored spells and uncensored spells have the same properties. However, the nature of the process of observation means that longer spells are more likely to be censored. This will mean that there is a downward bias in the mean length of spells.
(b) Following Dixon and Le Bihan (2010), we can treat right-censoring as a price-change ('loss is failure' or LIF). This is the opposite extreme to the classic KM assumption and will almost certainly result in an overestimate of the hazard.

We have employed both methods and the results are quantitatively similar, so we followed (b). We illustrate the differences with two estimated hazards at the end of this section.

This gives us the hazard function for each COICOP sector j for months $\tau = 1\ldots35$ $h(j,\tau)$ with $h(j,36)=1$. Following the steady-state identities outlined in Dixon (2012). The survival rate for period τ is defined as the proportion of spells surviving to the end of period τ. By definition, $S(1) = 1$ (no price spell is observed to last less than one month). We then define:

$$S(\tau) = \prod_{i=1}^{\tau-1}(1 - h(j,i)) \qquad \textbf{(D2)}$$

The corresponding monthly cross-sectional distribution (distribution across firms) is then defined by the steady-state relationship:

$$\gamma_{j\tau} = \tau . h(j,t) . S(\tau) . \bar{h} \qquad \textbf{(D3)}$$

where $\bar{h}(j) = 1/(\sum_{k=1}^{36} S(\tau))$. The corresponding quarterly distribution is then obtained by adding up the three months in that quarter. This yields the 12-vector $\gamma_j = (\gamma_{ji})_{i=1\ldots12}$.