Interbank Contagion: An Agent-based Model Approach to Endogenously Formed Networks

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Abstract

The potential impact of interconnected financial institutions on interbank financial systems is a financial stability concern for central banks and regulators. In examining how financial shocks propagate through contagion effects, we argue that endogenous individual bank choices are necessary to properly consider how losses develop as the interbank lending network evolves. We present an agent-based model to endogenously reconstruct interbank networks based on 6,600 banks' decision rules and behaviors reflected in quarterly balance sheets. We compare the results of our model to the results of a traditional stationary network framework for contagion. The model formulation reproduces dynamics similar to those of the 2007-09 financial crisis and shows how bank losses and failures arise from network contagion and lending market illiquidity. When calibrated to post-crisis data from 2011-14, the model shows the U.S. banking system has reduced its likelihood of bank failures through network contagion and illiquidity, given a similar stress scenario.

Keywords: Interbank lending market, Agent-based simulation, Contagion, Financial networks, Financial crisis
JEL Classification: D85, G17, G21, L14

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1. Introduction

The experience of the 2007-09 financial crisis suggests that existing models of systemic risk may not adequately capture how latent fragility and shocks propagate among banks, particularly in a dynamic market environment. Analyzing the impact of an idiosyncratic shock or macroeconomic stress requires a model that can apply balance sheet constraints and bank interactions to show how the financial system may evolve.

Since the crisis, banking regulators have expanded their focus from microprudential risks to systemwide characteristics of networks and risks within them (Hałaj and Kok, 2013). Of particular interest is the interconnectedness of financial institutions because of the unknown probability of contagion among banks. Additionally, though less discussed in the network literature, endogenous network formation in which banks decide whether to dissolve or form new financial relationships in response to counterparty risks is emerging.

This issue was seen in the U.S. interbank lending market, one of the most immediate sources of liquidity for banks, during the crisis. Afonso et al. (2011) show that the interbank market behaved with a heightened concern for counterparty risk that reduced liquidity and increased the cost of financing for weaker banks. Banks overall were less likely to lend liquid assets to each other. Large banks, which play a central role in this market, increased their liquidity buffers (Berrospide, 2012), forcing medium and small banks to look for new sources of liquidity.

As a result of these events, several network-based representations of interbank lending and borrowing have begun to be incorporated into regulators' stress testing methods (Bank of Korea, 2012; European Central Bank, 2013; Anand et al., 2014; Martinez-Jaramillo et al., 2014). These models generally use an Eisenberg and Noe (2001) framework where credit or payment obligations between firms create interlocking balance sheets. Examining the impact of an exogenous shock on a set of balance sheets allows one to compute the extent to which the initial loss in asset values cascades through the system, possibly leading to further defaults.

However, this method of interbank contagion modeling has two major shortcomings: (1) most models tackle the problem with a highly stylized stationary structure of obligations; and (2) the assumption of some type of optimal decisions in bank lending and borrowing is broadly applied across all banks in a homogenous manner. The reality is that banks are performance-driven, and their decisions are individually optimal but collectively suboptimal (Acharya, 2009). Banks must adapt to market changes to achieve their performance objectives, often by forming and dissolving interbank relationships. As a result, the overall financial system evolves in complex ways. This characteristic is largely absent in a pure network optimization setting and, thus, may overestimate or underestimate how losses propagate as a network of obligations changes due to stress.

This paper uses historical financial data from the U.S. Federal Financial Institutions Examination Council (FFIEC) to build an agent-based model (ABM) to represent all banks at a 1:1 scale of the U.S. banking system. By incorporating bank lending and borrowing behaviors based on balance sheet statistics of individual banks and general behavior patterns from the empirical findings, we derive a network of bilateral exposures for overnight debt (federal funds), short-term debt, and long-term debt. This reconstructed interbank exposure network uses agent-driven decisions that are compared with and validated against the existing empirical findings, as well as other existing interbank network construction algorithms (Upper and Worms, 2004; Anand et al., 2015).

The model is additionally validated by calibrating it to the pre-crisis FFIEC data and running Monte Carlo simulations. The simulations demonstrate that modeled bank failures follow similar dynamics and outcomes as those seen in 2007-09. The analysis introduces systematic shocks that cause a correlated collapse of asset holdings across the system in the ABM to induce system contagions. Finally, the model is recalibrated with post-crisis banking data, and the simulated results of running a similar shock are compared to those of the pre-crisis results.

The first contribution this paper makes to the current literature is by examining the impact that endogenous network formation, through individual bank performance objectives, has on contagion. We compare the outcome of incorporating endogenous network formation to a stationary network through an Eisenberg-Noe style clearing model. The results demonstrate how individual bank choices help the interbank lending system become more resilient to counterparty defaults and contagion.

This paper makes a second contribution by presenting a model for stress testing the banking system that incorporates indirect losses from contagions driven by insolvency and illiquidity. The power of this methodology is demonstrated by examining how the banking system performs before and after 2007-09 under a shock similar to that of the financial crisis, and how different aspects of the shock propagate defaults. The methodology also provides a platform to test how new regulations, such as Basel III and the Dodd-Frank Consumer Protection and Wall Street Reform Act, have impacted network structure in a dynamic environment. As new policies have constrained balance sheet choice, we find the new network equilibrium and whether it has improved financial stability.

The paper is structured as follows. Section 2 reviews current literature related to systemic risk and interconnectedness, interbank networks topology, and extrapolation techniques. Section 3 discusses U.S. banking financial data used in this study. Section 4 summarizes the methodology used to construct the agent model and incorporate autonomous behaviors of the agents. Section 5 explains the validation and robustness of the model. Section 6 presents model experiments and results. Finally the paper concludes in Section

7 by assessing the results and the methodology's contributions.

2. Background

This section delves into four key aspects of modeling interconnectedness in the U.S. banking system: (1) modeling interconnectedness as it relates to systemic risk, (2) the topology of the interbank networks for short and longer term lending characterized by previous studies, (3) current methods and practices for recovering network structure, and (4) ABM as a method for endogenously determining how networks form under stress.

2.1. Systemic Risk and Endogenous Networks

Among the many factors contributing to the crisis of 2007-09, the role of the growing interconnectedness of the global financial system is perhaps the least well understood (Glasserman and Young, 2015). Pioneering works by Allen and Gale (2000) and Eisenberg and Noe (2001) highlighted the importance of financial interconnectedness and systemic risk and the crisis exposed the fact that regulators and market participants had limited information to examine financial networks and identify risk channels.

Many models have highlighted how interbank network data could be used to examine the spread of contagion (Wells, 2004; Iori et al., 2006; Elliott et al., 2014; Acemoglu et al., 2015a). However, little work has considered how financial network structures evolves as market participant preferences change, defaults occur, or new policies are enacted. The answer to the question of how to use strategic network formation can be traced to seminal works of Jackson and Wolinsky (1996) and Bala and Goyal (2000). This literature focuses on how agents trade off the costs and benefits of creating links with one another and characterizes the set of networks that are formed in equilibrium. More recent works by Acemoglu et al. (2015b) have looked at how endogenous network formation can impact systemic risk, and Gofman (2016) has developed these themes by calibrating network formation based on network features seen in agent trading decisions.

2.2. Interbank Network Topology

The interbank network's structure is of interest to central banks and regulators concerned with bank bilateral exposures and the implications they pose in periods of stress. Research so far has focused on the overnight funding market because of data accessibility. Boss et al. (2004), Iori and Gabbi (2008), and Roukny et al. (2014) investigated the interbank market in Austria, Italy, and Germany, respectively, and discovered similar network features of the banking system in those countries. These features include: (1) sparsity and short average distance among nodes, (2) heterogeneous degree count among nodes that follows a power law distribution, (3) small clustering, and (4) small world properties. Fewer studies have looked at the total network including overnight transactions, short-term loans, and long-term loans in the aggregate due to the lack of data.

Cont et al. (2013) investigated the Brazilian banking system based on balance sheets with complete interbank exposures. Their findings suggest that connectivity properties of the total network are consistent with those of overnight transaction networks. This similarity is due in part to the preference seen in lending practices between large and small banks. Cocco et al. (2009) documents that smaller banks, which normally have higher default risk, tend to rely on large banks when borrowing funds. Large banks prefer to borrow funds with familiar counterparties to reduce interest payments. Though this may create similar network features between the loan maturity networks, the combination of loan types is an important determinant of interbank lending liquidity (Bargigli et al., 2015).

2.3. Interbank Network Extrapolation

Though interbank networks are seen as fundamental channels for systemic risk, in practice, most interbank networks remain unobserved because interbank loans are generally arranged over the counter and data are not centrally collected in most countries. As a result, several methods have been developed to approximate the network with available data. These methods do so by estimating networks from balance sheet lending and borrowing. The predominate approach is the Maximum Entropy method that has a simple risk-sharing mechanism that implicitly assumes perfect competition, i.e. all banks are equally willing to accept an equal share of risk (Upper and Worms, 2004). However, interbank networks have been sparse, because interbank activity is based on relationship banking (Cocco et al., 2009). Smaller banks are limited by the number of linkages they can maintain (Craig and Von Peter, 2014), as it is costly to manage a large and diversified set of lending and borrowing relationships.

As a result, many different algorithms have been used to manage linkage formation by including optimizing features for different network measures.[2] The Basel Committee on Banking Supervision (2015) compared many of these algorithms and found the Minimum Density (Anand et al., 2015) to be one of the most accurate estimators for interbank networks. This said, all these methods optimize linkage formation at the network level rather than at the bank level.

2.4. Agent-Based Modeling in Interbank Networks

As an alternative to the static network theoretic-based approach of Allen and Gale (2000) and Eisenberg and Noe (2001), ABM offers flexibility and enhances fidelity to model financial networks dynamics by incorporating individual agent decisions. By its

[2] Alternative methods suggested in the literature include Anand et al. (2015), Baral and Fique (2012), Battiston et al. (2012), Tarashev et al. (2011), Hałaj and Kok (2013), and Mastrandrea et al. (2014)

definition, ABM is a simulation framework comprised of autonomous agents with interacting behaviors, connections between agents, and an exogenous environment (Macal and North, 2010). In contrast to statistical and mathematical models, ABMs have advantages in replicating real social phenomena, adaptive agent behaviors, and information diffusion among agents (Macy and Willer, 2002; Gilbert and Terna, 2000). These features provide an ideal platform for modeling endogenous network formation through behavior-based rules.

ABMs have been used for systemic risk evaluation in the past (Streit and Borenstein, 2009; Bookstaber et al., 2014). Within the banking system more specifically, ABMs have been applied on top of network topologies to explore contagion risk among banks (Georg, 2013; Ladley, 2013). In addition, further extension has replicated multi-layered network structures hinging on multiple types of interbank loans. Kok and Montagna (2013) and Hałaj and Kok (2014) investigated contagion risk among large EU banks and discovered nonlinearities in the shock propagation.

3. Data

U.S. national banks, state member banks, insured state nonmember banks, and savings associations are required to submit quarterly financial reports to the FFIEC known as the *Federal Financial Institutions Examination Council Reports of Condition and Income*.[3] The balance sheet and income statements disclosed on the form show each bank's business model and lending-borrowing practices. The data sample used in this paper covers 14 years, from March 2001 to December 2014, and includes reports from just over $10,000$ active and failed banks. The specific balance sheet items used in this study are tabulated in Table 1. These items are used to help derive the interbank market structure, discussed in Section

[3] In the case of bank holding companies, the data represents only balance sheet information associated with the commercial bank part of the company.

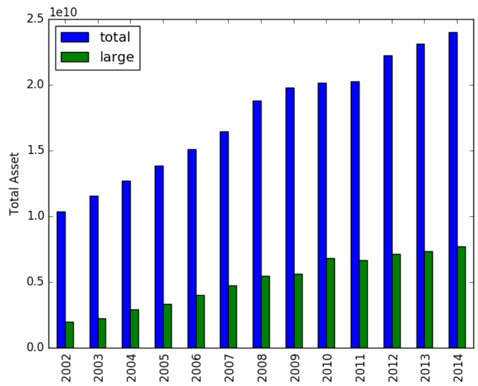
4. As indicated in the 3rd column of Table 1, the long-term lending and borrowing items consist of both domestic and foreign institutions. Based on our analysis, we find less than 20% of the long-term loans are between domestic and foreign banks. We therefore choose the average proportion for the long-term loans in our modeling process. As such we make the simplifying assumption that all lending is domestic when we later use the data in the model so as to create a closed interbank system of obligations.

Table 1: Balance Sheet Entries Collected from Call Reports

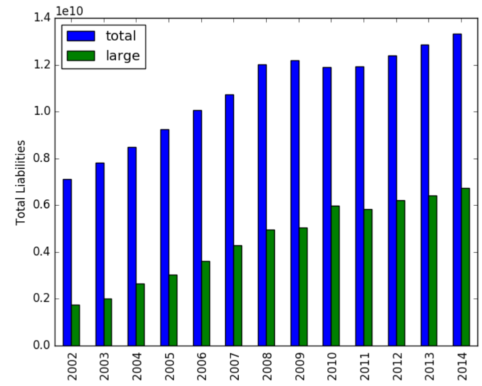
	Entries	Descriptions	Counterparties
Assets	Cash and balance due	Cash items in process of collection	Domestic and foreign
		Balances due from depository institutions	Domestic and foreign
	Overnight lending	Federal funds sold and reverse repurchase	Domestic
	Short-term lending	Federal securities sold under agreements to repurchase	Domestic
	Long-term lending	Loans to depository institutions and acceptances	Domestic and foreign
Liabilities	Overnight borrowing	Federal funds purchased and repurchase agreements	Domestic
	Short-term borrowing	Federal securities purchased under agreements to resell	Domestic
	Long-term borrowing	Other borrowings with a remaining maturity of next repricing date of one year or less	Domestic and foreign

Notes: This table shows the bank balance sheet items used in this study. *Source*: Federal Financial Institutions Examination Council Reports of Condition and Income.

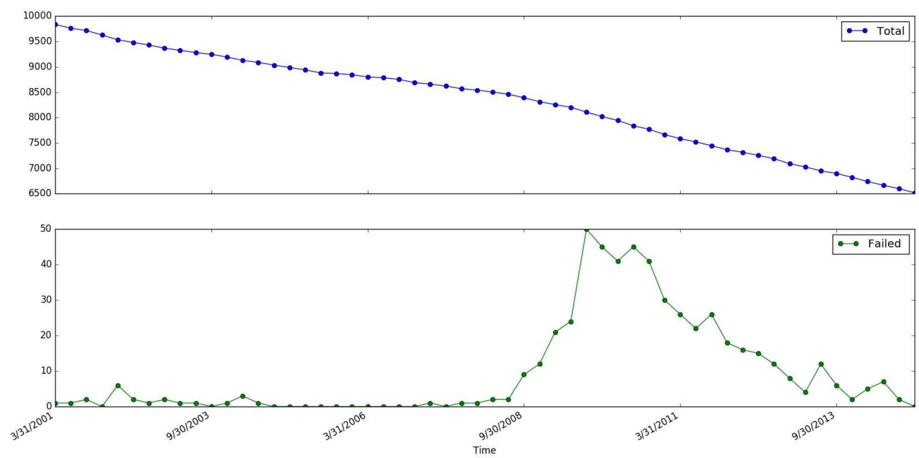
Figure 1 provides aggregate bank balance sheet statistics for the sample period. Figures 1a and 1b plot assets and liabilities held by all U.S. banks and by the 10 largest U.S. banks, showing steady growth on both sides of the balance sheet with exception of a short period



(a) Total Assets of Banks



(b) Total Liabilities of Banks



(c) Number of Banks and Bank Failures

Figure 1: Bank Sample Plots: Assets, Liabilities, and Number.

Notes: The top two bar charts show the total assets and liabilities of all banks in the sample and the largest 10 banks for the first quarter of each year. The bottom two line charts plot the numbers of banks and the number of failed banks in each quarter. *Source*: Federal Financial Institutions Examination Council Reports of Condition and Income.

after the 2007-09 crisis.

Figure 1c shows the total number of banks, and the number of banks that failed. In contrast to an increase in the aggregate balance sheet of the banks, there is a steady decrease in the number of banks mostly due to consolidation. By the end of the analysis period in 2014, the total number of banks had decreased by nearly a third. There is little impact on the trend of this decrease throughout the crisis. However, the number of bank failures, which occur when a bank is unable to meet its obligations to depositors and lenders, substantially increased beginning in the third quarter of 2008, peaked in mid-2010, and slowly decreased through 2014.

3.1. Interbank Lending Markets

Because interbank lending markets fund the most immediate liquidity demands of banks, a source of concern during the financial crisis, bank regulators are interested in monitoring these markets. When stress rises in these markets, it can lead to insufficient bank liquidity and inadequate allocation of capital and risk sharing between banks (Afonso et al., 2014). The FFIEC data show interbank lending on an overnight, short-term, and long-term basis by the amount of federal funds, federal securities, and interbank loans each institution has on its balance sheet.

How banks use interbank markets depends on their liquidity needs. Table 2 shows the average percentage of a bank's balance sheet that each lending and borrowing activity represents during different three-year periods. Banks on average use the overnight market to lend and use the short-term market to borrow.

Considering how these markets have changed in terms of bank balance sheets pre-, during- and post- crisis, there is a noticeable decrease in how important these markets are on both sides of the balance sheet. Both short-term and long-term lending and borrowing in the post-crisis period are half of what they were prior to the crisis. Overnight borrowing is one-fourth of its pre-crisis size and overnight lending has declined marginally.

Table 2: Interbank Lending and Borrowing as a Percentage of the Balance Sheet

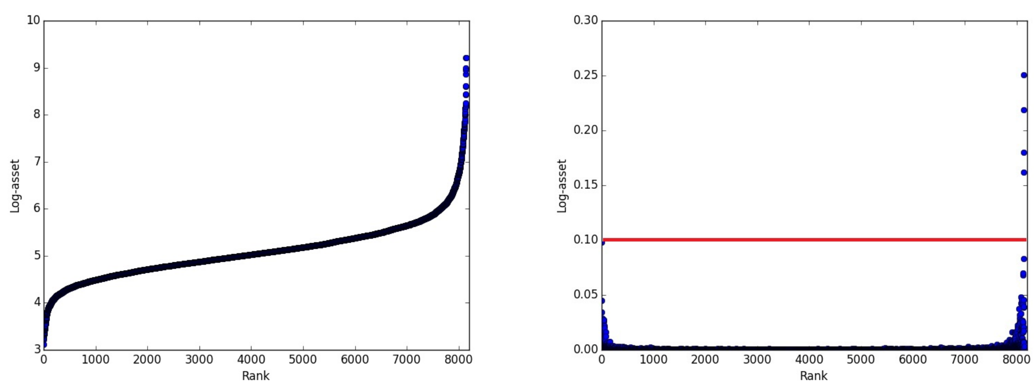
Year	Overnight		Short-term		Long-term	
	Asset	Liability	Asset	Liability	Asset	Liability
Pre-Crisis						
2002	5.13	0.53	0.15	1.06	0.11	0.32
	(7.19)	(3.37)	(1.93)	(3.19)	(1.73)	(3.64)
2004	4.62	0.60	0.17	1.07	0.11	0.28
	(7.30)	(3.60)	(2.45)	(3.28)	(1.97)	(3.50)
2006	4.44	0.79	0.14	1.11	0.09	0.21
	(8.16)	(4.56)	(2.19)	(3.36)	(1.70)	(2.74)
Crisis						
2007	5.51	0.61	0.18	1.14	0.08	0.13
	(8.96)	(4.39)	(2.58)	(3.12)	(1.62)	(1.97)
2008	5.33	0.69	0.16	1.16	0.09	0.15
	(8.42)	(4.44)	(2.50)	(3.12)	(1.67)	(2.28)
2009	3.23	0.45	0.12	1.05	0.10	0.25
	(5.63)	(3.64)	(1.63)	(2.74)	(1.86)	(2.69)
Post-Crisis						
2010	2.57	0.28	0.09	0.99	0.09	0.17
	(2.13)	(3.00)	(1.51)	(2.64)	(1.73)	(2.20)
2012	2.13	0.14	0.10	0.87	0.09	0.10
	(4.75)	(2.04)	(1.56)	(2.45)	(1.78)	(1.92)
2014	1.56	0.16	0.06	0.76	0.09	0.05
	(4.09)	(1.94)	(1.11)	(2.17)	(1.86)	(0.55)

Notes: This table shows the mean and standard deviation (in parentheses) of the percentage that balance sheet interbank lending and borrowing contribute to assets and liabilities. *Source*: Federal Financial Institutions Examination Council Reports of Condition and Income.

3.2. Large and Small Banks

It is well-established in the current literature that the interbank lending market is a mix of two types of banks: small retail banks that need to borrow or lend, and large banks that act as intermediaries to the flow of lending and borrowing needs (Carter et al., 2004; Cocco et al., 2009; Santos and Cont, 2010; Afonso et al., 2014). Previous research by Afonso et al. (2014), has distinguished these two groups using bank asset sizes and has shown that large banks generally have lower liquidity and higher leverage than small banks. This categorization has also been confirmed by various theoretical models in terms of interbank network structure where large banks form the core while small banks attach to the system as peripheries (Upper, 2011; Choromański et al., 2013; Lux, 2015).

Using this method, we separate banks into a large bank group or a small bank group, based on total assets on their balance sheets over time. First, banks are ranked by asset size, as shown in Figure 2, and then split into the two groups by looking at the differences of logarithmic total assets between two adjacent banks in the ranking. All banks above a threshold of 0.10, depicted by the red line in Figure 2b, are considered large. For example, four banks would be categorized as large according to Figure 2b.



(a) Log(Asset) of Banks

(b) Difference of Log(Asset)

Figure 2: Bank Categorization by Asset Value.

Notes: This figure shows the total U.S. bank asset distributions from March 2001 to December 2014.
Source: Federal Financial Institutions Examination Council Reports of Condition and Income.

We use quarterly financial reports from 2001 to 2014 to separate banks into large and small types. Some banks switch between the two groups in different time periods, but four banks consistently appear in the large bank group: Bank of America, Citibank, J.P.

Morgan Chase Banks, and Wells Fargo Bank.

There is a distinct observable difference in how the two groups behave in the interbank market. Large banks borrow and lend more than small banks. In terms of overnight lending, large banks lend 40 percent more than small banks. Large banks also borrow over five times more than small banks, meaning that large banks prefer to borrow from small banks, which is consistent with the empirical findings of Cocco et al. (2009). In the short-term market, large banks lend just over six times more than small banks but do similar amount of borrowing. In the long-term market, large banks do eight times more lending and borrowing than their small bank counterparts. In the following section we will use these differences to structure bank lending and borrowing preferences.

Table 3: Interbank Lending and Borrowing: Large and Small Banks

Type	Overnight		Short-term		Long-term	
	Asset	Liability	Asset	Liability	Asset	Liability
Large	7.2	2.52	0.92	2.09	1.25	1.99
	(13.47)	(3.29)	(1.05)	(2.33)	(1.93)	(1.68)
Small	5.33	0.49	0.15	2.09	0.14	0.28
	(7.25)	(3.37)	(1.98)	(3.09)	(2.08)	(3.61)

Notes: This table shows the mean and standard deviation (in parentheses) of the percentage of the balance sheet interbank lending and borrowing. *Source*: Federal Financial Institutions Examination Council Reports of Condition and Income.

4. Model

This section presents an ABM approach to simulate the U.S. interbank lending system. The model we introduce here follows a traditional clearing vector of lending obligations with interlocking bank balance sheets. However, it also incorporates an evolving interbank network of obligations based on individual bank agent preferences on lending and borrowing. The remainder of this section covers the design of the interbank clearing network and describes how bank balance sheets and objectives are used to construct the interbank network.

4.1. Interbank Clearing Network

We consider an *interbank lending system*, populated by n risk-neutral banks that can only lend to other banks. Following the Eisenberg-Noe framework (2001), each bank is a distinct entity, or node, participating in a clearing network. Each bank, i, has a balance sheet made up of assets, A_i, and liabilities, L_i, represented in Table 4. On the assets side, bank i has interbank loans it made to k bank which include overnight market, ON_{ki}, short-term, ST_{ki}, and long-term, LT_{ki} maturities, as well as cash and cash equivalents, C_i, and other assets, OA_i.[4] On the liabilities side, bank i has interbank loans it has received from j banks in the overnight market, ON_{ij}, short-term, ST_{ij}, and long-term, LT_{ij} market, as well as equity, E_i, and other liabilities, OL_i.

Each bank i in the system may have up to three nominal liabilities to bank j in the system, through $\{ON_{ij}, ST_{ij}, LT_{ij}\}$. These liabilities represent the loan obligations between pairs of banks in the interbank lending system. We assume that all interbank loans are non-negative and that no bank can make a loan to itself. Thus the system can be expressed as (ON, ST, LT, C, OL, OA), where ON, ST, LT represents matrices of obligations between banks and C, OL, OA are vectors that represent cash, other liabilities, and other assets held by the banks.

Each period in the system, banks have to repay some proportion of their $\{ON_{ij}, ST_{ij}, LT_{ij}\}$. Let P_i represent the amount of each loan obligation bank i has to repay to all other banks in the system in the given period. Let $P = (P_1, P_2, ..., P_n)$ represent the vector of *total*

[4]Cash equivalents include Federal Reserve bank deposits and deposits held at other banks.

Table 4: Description of the Bank's Balance Sheet

Assets, A_i		**Liabilities,** L_i	
Overnight lending: federal funds, $\sum_{k \neq i}^{n} ON_{ki}$	Interbank lending	Overnight borrowing: federal funds, $\sum_{j \neq i}^{n} ON_{ij}$	Interbank borrowing
Short-term lending: federal securities, $\sum_{k \neq i}^{n} ST_{ki}$		Short-term borrowing: federal securities, $\sum_{j \neq i}^{n} ST_{ij}$	
Long-term lending: loans due from banks, $\sum_{k \neq i}^{n} LT_{ki}$		Long-term borrowing: loans due to banks, $\sum_{j \neq i}^{n} LT_{ij}$	
Cash and balance due, C_i		Other liabilities, OL_i	
Other assets, OA_i		**Equity,** E_i	

Notes: This description of a bank i's balance sheet focuses on major bank lending and borrowing channels, i.e. overnight, short-term, and long-term markets. The rest of the balance sheet is expressed as into cash or other assets and liabilities. The notations introduced here for the balance sheet will be used throughout this paper. *Source*: Authors' model.

loan payment banks have across all three types of loans.

$$\bar{P}_i = \sum_{j=1}^{n} P_{ij}^{ON} + \sum_{j=1}^{n} P_{ij}^{ST} + \sum_{j=1}^{n} P_{ij}^{LT}, \tag{1}$$

and the *relative payment liability* of bank i to bank j is

$$\Pi_{ij} = \left(P_{ij}^{ON} + P_{ij}^{ST} + P_{ij}^{LT}\right) / \bar{P}_i. \tag{2}$$

where the *relative payment liability matrix* $\Pi = (\Pi_{ij})$ is row substochastic, that is for every i, $\sum_{j \neq i} \Pi_{ij} \leq 1$.

Given that the FFIEC data set is quarterly, we will be modeling periods in the transition of $\{ON, ST, LT\}$ over quarters. We make the following assumptions about repayment size. Overnight debts are repaid in full, such that $P_{ij}^{ON} = ON_{ij}$. Short-term loans are nearly all made for less than three months (Sheldon et al. (1998)), such that we assume $P_{ij}^{ST} = U(99\%, 100\%)ST_{ij}$. Long-term loans usually are repaid in less than one year, or within four quarters, so we will assume 75 percent of outstanding loans continue to exist. Thus we make $P_{ij}^{LT} = U(25\%, 100\%)LT_{ij}$ for long-term debts.

The remainder of the loans bank i will owe to bank j we term $\{Q_{ij}^{ON}, Q_{ij}^{ST}, Q_{ij}^{LT}\}$, where $Q_{ij}^{ON} = ON_{ij} - P_{ij}^{ON}$, $Q_{ij}^{ST} = ST_{ij} - P_{ij}^{ST}$, and $Q_{ij}^{LT} = LT_{ij} - P_{ij}^{LT}$. Let Q_{ij} represent the remainder of all loan obligations bank i will have to bank j.

$$Q_{ij} = Q_{ij}^{ON} + Q_{ij}^{ST} + Q_{ij}^{LT}, \tag{3}$$

The value of the equity of bank i is given by total loans given to borrower less repayments of loans from lenders. In other words, the value of bank i's equity is

$$E_i = \sum_{j=1}^{n} \Pi_{ij}^{T} P_j - P_i + \sum_{j=1}^{n} Q_{ji} - \sum_{j=1}^{n} Q_{ij} + C_i - OL_i + OA_i. \tag{4}$$

Bank i may run into two critical conditions that will cause the size of the payment obligation P_i to be less than the total payment expected $\bar{P}_i$. This occurs when either (1) the bank is insolvent because its equity, E_i, is less than 0 or (2) the bank is illiquid because it does not have enough C_i and incoming payments to make full payment $\bar{P}_i$. In the event that bank i suffers either condition, it will default on its interbank borrowing. Bank i's lender k, will write down payment P_{ik} *pro rata* quantity given the net assets of bank i are insufficient to meet its obligations.[5]

Let $P_{ik}^{*} \leq \bar{P}_{ik}$ denote the realized liability repayment of the loan from i to k. Note that when all of i's counterparties pay in full, that is $P_i^{*} = \bar{P}_i$ for all k, then there is no stress

[5]In the event of bank i's failure, Q_i is assumed to returned in full to lender banks and bank i will receive back $\sum_{k=1}^{n} Q_{ki}$ from its lends at the end of the period.

propagated to bank i. If $P_i^* < \bar{P}_i$ the difference must be borne by i, such that

$$P_i^* = \min\left[\sum_{k=1}^{n} \Pi_{ik}^T P_k^* + C_i, \sum_{k=1}^{n} \Pi_{ik}^T P_k^* + \sum_{k=1}^{n} Q_{ki} - \sum_{k=1}^{n} Q_{ik} + C_i - OL_i + OA_i, \bar{P}_i\right]. \quad (5)$$

Through following the Eisenberg-Noe algorithm, we can compute the extent to which losses in loan obligations cascade through the system, possibly leading to further defaults. Given any vector $P \in \mathbb{R}^{2n+2}$ such that $0 \leq P_{ij} \leq \bar{P}_{ij}$ for all $0 \leq i, j \leq n$, let $\Phi(P)$ be the mapping defined by expression

$$\Phi(P)_i = \bar{P}_i \wedge \left[\sum_{k=1}^{n} \Pi_{ik}^T P_k^* + \sum_{k=1}^{n} Q_{ki} - \sum_{k=1}^{n} Q_{ik} + C_i - OL_i + OA_i\right]_+ \wedge \left[\sum_{k=1}^{n} \Pi_{ik}^T P_k^* + C_i\right]_+^{6} \quad (6)$$

The economic interpretation of Φ is that $\Phi(P)$ represents the total funds that will be applied to satisfy debt obligations, assuming that banks receive inflows specified by P from their debt claims on other banks. As Φ is monotone non-decreasing and bounded, hence by Tarski's theorem it has at least one fixed point for the clearing vector (Tarski et al., 1955).

4.2. Interbank Network Endogenous Formation

Because stresses and shocks are not instantaneous, a bank will tend to make new relationships and dissolve others for optimal behavior. The interbank lending system is built on individual bank lending and borrowing preferences and we represent those preferences by using a set of target financial ratios based on a bank's balance sheet. Because bank i has several different lending and borrowing channels to select from, it uses a combination of ratios that, when maintained in unison, keep constant its interbank lending and borrowing

[6] In general, $x \wedge y \wedge z$ denotes the minimum of the three real numbers x, y, and z. The $[]_+$ represent for all values zero or greater.

preferences for overnight, short-term, and long-term debts (see Table 5). A bank also uses the equity multiplier, the ratio of its total assets to its equity, to control its balance sheet for the degree of leverage desired.

Table 5: Bank Balance Sheet Ratio

Equity Multiplier	$\frac{E_i}{A_i}$
Overnight Lending, Borrowing	$\frac{\sum_{k \neq i}^{n} ON_{ki}}{A_i}$, $\frac{\sum_{j \neq i}^{n} ON_{ij}}{L_i}$
Short-term Lending, Borrowing	$\frac{\sum_{k \neq i}^{n} ST_{ki}}{A_i}$, $\frac{\sum_{j \neq i}^{n} ST_{ij}}{L_i}$
Long-term Lending, Borrowing	$\frac{\sum_{k \neq i}^{n} LT_{ki}}{A_i}$, $\frac{\sum_{j \neq i}^{n} LT_{ij}}{L_i}$

Notes: This table lists all the features of the balance sheet that bank i targets in determining how to allocate its lending and borrowing demand from period to period.
Source: Authors' model.

In each period, t, a bank evaluates its current ratios against its target ratios to determine how much it needs to lend and/or borrow. For example, if bank i's current overnight lending-to-asset ratio is lower than its target, it will want to find a borrower to lend to in the overnight market. Likewise, if bank i's current overnight borrowing-to-liability ratio is lower than its target, it will want to find a lender to borrow from in the overnight market. Once the bank reaches all its targets, it will no longer want to lend or borrow in any of the three markets.

Two types of banks are in the model – large and small – and they are differentiated in two ways. First, large and small banks have different balance-sheet characteristics and interbank lending practices that are important to capture in constructing their balance sheets, as discussed in Section 3.2.[7] Second, large banks are intermediaries for lending and borrowing, which makes them attractive to banks looking for a correspondent.

If a bank needs to lend or borrow in ON, ST, or LT during a period, it goes through a scoring system to determine with whom to do this new activity. This procedure is done by assigning two scores to each bank: a size score, S^{s}, and a relationship score, S^{r}. The size score is meant to capture the preference of banks to do business with larger banks with more assets. This is calculated here as a bank's assets less existing counterparties' average assets:

$$S^{\mathrm{s}}_{i,j}(t) = \log A_j(t) - \frac{\sum_{k,k\neq i} \log A_k(t-1)\mathbb{I}_{ik}(t-1)}{\sum_{k,k\neq i} \mathbb{I}_{ik}(t-1)}, \tag{7}$$

$$\mathbb{I}_{i,k}(t) = \begin{cases} 1, & \text{if } i \text{ and } k \text{ are have a relationship at period } t, \\ 0, & \text{otherwise.} \end{cases}$$

where $S^{\mathrm{s}}_{i,j}(t)$ is the size score of bank j evaluated by bank i in period t, A_j is the total assets of bank j, and $\mathbb{I}_{i,k}(t)$ is a binary variable for keeping track of previous debt obligations.

The relationship score captures a bank's tendency to keep existing relationships. In each model period, this score decreases according to a decaying function and increases if new loans are formed. For each of the three lending channels $\{ON, ST, LT\}$ a different relationships score is calculated:

$$S^{\mathrm{r}}_{i,j,\{ON,ST,LT\}}(t) \begin{cases} \log\{ON_{ij}(t), ST_{ij}(t), LT_{ij}(t)\}, & \text{if } i \text{ and } j \text{ have a loan} \\ \eta S^{\mathrm{r}}_{i,j,\{ON,ST,LT\}}(t-1), & \text{else if } t > 0, \\ \{0,0,0\}, & \text{otherwise.} \end{cases} \tag{8}$$

[7]We split the data sample across large and small banks to ensure that when we parametrize the models through sampling, the data is drawn from similar bank distributions.

where $S^{\mathrm{r}}_{i,j,\{ON,ST,LT\}}(t)$ is the relationship score of bank j evaluated by bank i, in period t, $\{ON_{ij}(t), ST_{ij}(t), LT_{ij}(t)\}$ is any new loans bank i received from bank j in period t, and η is the memory decaying parameter, which we set to a default value of 0.9. Finally, a bank uses the two scores in combination, $S^{\mathrm{c}}(t)$, to rank from whom it wants to borrow (see Equation 9).

Each bank, knowing its borrowing target, first sends one borrowing request at a time to each large bank in order to obtain its desired funding. If the borrowing target is not fulfilled by large banks, the bank will then send one request at a time to each small bank with which it has a previous lending relationship, in order of largest to smallest $S^{\mathrm{r}}_{i,j,\{ON,ST,LT\}}(t)$. Finally if the targeted amount is still not fulfilled, the borrowing bank contacts other small banks, in order of largest to smallest $S^{\mathrm{s}}_{i,j}(t)$.[8] A bank that is unable to fulfill its target after contacting all potential lenders may have a liquidity default on its balance sheet if it does not have enough equity.

When a bank receives a borrowing request, it must decide two things: (1) whether to provide new loans to requesting borrowers, and (2) how much to lend. Two primary factors affect bank lending preferences. Each bank that has not met its target for ON, LT, or ST will lend following a similar scoring system described in Equation 9 with respect to its potential borrowers. Accordingly, a bank chooses to lend by going through each request until its lending target is satisfied or there are no more requests to fill.[9]

$$S^{\mathrm{c}}_{j,i,\{ON,ST,LT\}}(t) = \omega S^{\mathrm{s}}_{j,i}(t) + (1-\omega) S^{\mathrm{r}}_{j,i,\{ON,ST,LT\}}(t), \tag{9}$$

and $S^{\mathrm{c}}_{j,i,\{ON,ST,LT\}}$ is the score that lender j assigns to borrower i. $S^{\mathrm{c}}_{j,i,\{ON,ST,LT\}}$ is the

[8]During this process banks are selected (at random) to send a request to borrow from next available borrower, according to this preference algorithm.

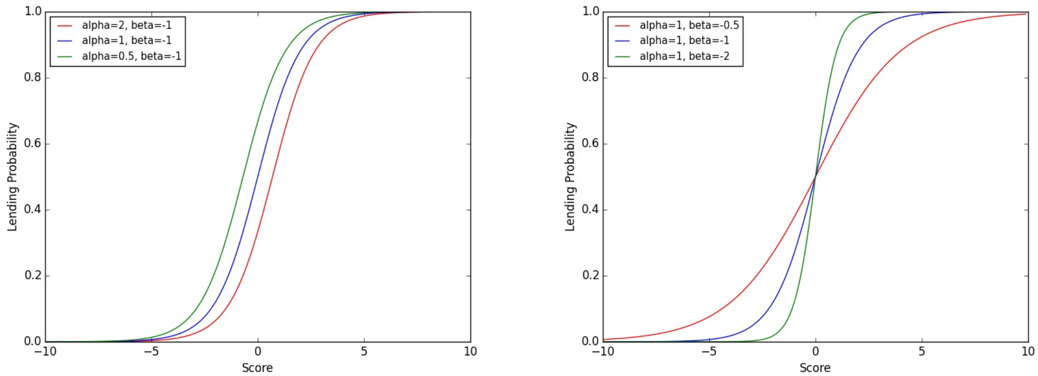
[9]These scores evolve with time as the balance sheets of banks do.

weighted average of the relationship score and size score of bank i. Equal weights are set to these scores ($\omega = 0.5$). However, a lending bank does not agree to every borrowing request, even if it has the capacity, and uses an S-shaped function, $P(S^{c}_{j,i})$, to assess the chance that lending bank j settles new debts to borrowing bank i, where

$$P(S^{c}_{j,i,\{ON,ST,LT\}}(t)) = \frac{1}{1 + \alpha \times \exp\left(\beta \times S^{c}_{j,i,\{ON,ST,LT\}}(t)\right)}; \tag{10}$$

where $P(S^{c}_{j,i,\{ON,ST,LT\}})$ is the probability that i lends to j, and α and β are two parameters that control the intercept and slope, respectively. In this function, α is a positive real number. The larger the number, the lower probability of lending to a bank scoring 0 (see Figure 3a). To represent different preferences of large banks and small banks, values are chosen from the uniform distribution $U(0.3, 0.5)$ for large banks and from the uniform distribution $U(0.9, 1.1)$ for small banks. This approach allows more lending from large banks to small banks. β is a negative real number, and the larger it is the slower the probability moves from 0 to 1 (see Figure 3b). In other words, a larger β means a tighter lending policy such that fewer borrowers get loans. Default values are chosen for banks from the uniform distribution $U(-1.1, -0.9)$.

A lending bank follows a uniform distribution to determine the fraction it wants to lend from its available lending limit. The lower value between the one determined by the lending bank and requested by the borrowing bank is set as the new debt size.



(a) Sigmoid function sensitivity to α **(b)** Sigmoid function sensitivity to β

Figure 3: Lending Probability Determined by Sigmoid Function. Source: Authors' analysis.

5. Model Validation & Robustness Test

Validation exercises confirm that the model produces an interbank market resembling the real market based on individual bank decisions on lending and borrowing. The model is first validated based on bank balance sheet ratios and interbank lending network properties by comparing its results to those empirically observed using 2001-06 data. Secondly, the model's network topology features are compared to those observed in other papers. We then preform some robustness testing of the parameter selections used in the network formation. Lastly, the ABM network formation performance is compared to other algorithmic methods in selecting network linkages and creating stylized facts.

5.1. Bank Balance Sheets Validation

Banks make lending and borrowing decisions based on many different factors, but this study focuses on two aspects: risk and behavior. Balance sheet information is used to measure bank decisions. Two ratios are used to indicate risk: the liquidity ratio and the leverage ratio. Other two ratios are defined to measure the interbank lending and borrowing behaviors. All four ratios are defined in Equations (11, 12, 13, and 14)

$$\text{Leverage Ratio} = \frac{A_i}{E_i} \tag{11}$$

$$\text{Liquidity Ratio} = \frac{C_i}{A_i} \tag{12}$$

$$\text{Interbank Lending Ratio} = \frac{\sum_{k=1,k\neq i}^{n} ON_{ki} + LT_{ki} + ST_{ki}}{A_i} \tag{13}$$

$$\text{Interbank Borrowing Ratio} = \frac{\sum_{j=1,j\neq i}^{n} ON_{ij} + LT_{ij} + ST_{ij}}{L_i} \tag{14}$$

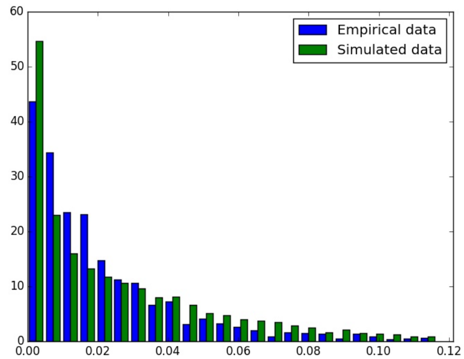
Given these measures, the model is first initialized based on 2001 financial data. The distribution of the four selected ratios is validated according to simulation data of 20 quarters and empirical data from 2001 to 2006 (see Figure 4). A comparison of the distributions of the four observed versus simulated ratios shows that from a balance sheet perspective, the simulation closely resembles real bank lending and borrowing behaviors.

5.2. Network Properties Validation

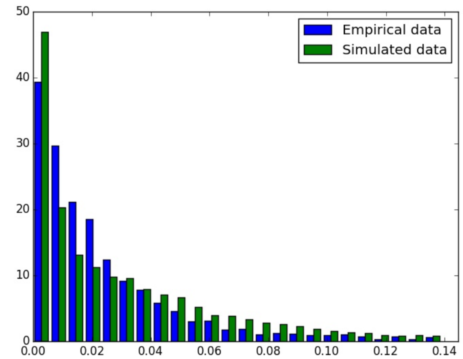
We perform a network properties comparison of the simulated overnight lending market and the U.S. federal funds market.[10] Bech and Atalay (2010) evaluated 6,600 banks' transactions using 2006 Federal Fund market data and documented the empirical network structures. Here, 100 simulations are conducted with the same number of agents (see Table 6) and compared to the findings from Bech and Atalay (2010).

A comparison of two networks, based on the same set of statistics as in Section 5.4, shows a good overall match in the three average aggregate statistics. The clustering coefficient shows the weakest match, suggesting that the model may have a stronger propensity

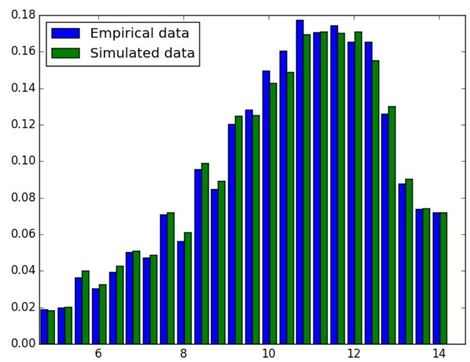
[10]This is the only U.S. lending market category the authors are aware of that has had an empirical network analysis.



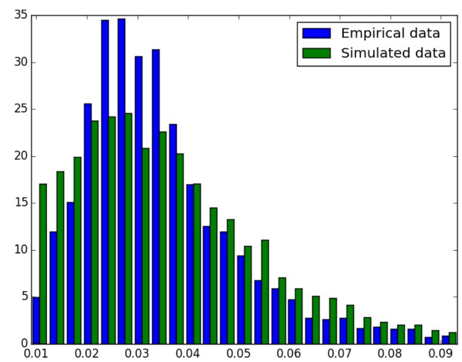
(a) Interbank Lending Ratio



(b) Interbank Borrowing Ratio



(c) Leverage Ratio



(d) Liquidity Ratio

Figure 4: Bank Categorization Through Asset Value.

Notes: The figure shows the comparison of ratio distribution (histogram) between the real bank financial data and the simulated results. The leverage ratio and the liquidity ratio are used as measures of bank risk. Interbank lending and borrowing ratios are used to quantify interbank lending practices.
Source: Federal Financial Institutions Examination Council Reports of Condition and Income; Authors' model.

to form lending relationships between large and small banks than the real market does.

Table 6: U.S. Federal Funds Market Interbank Network Property Comparison

	Average In-Degree	Average Out-Degree	In-Clustering Coefficient	Out-Clustering Coefficient	Power Law
U.S. Federal Funds Market	9.30	19.10	0.10	0.28	2.00
Model (100 simulations)	10.39	17.14	0.03	0.21	1.94

Notes: This table lists the key network measures between the real U.S. federal funds market and the simulation results. For the In-Degree and Out-Degree measure we used the GSCCD - giant strongly connected component reported in Bech and Atalay (2010) for the reason that it reflects the interbank market mostly. *Source*: Bech and Atalay (2010); Authors' calculations.

5.3. Model Robustness

The validity of an ABM model is largely determined by the assumptions and choices of model parameters. In our simulation, we categorize the model parameters into either an empirical statistic based or an agent characteristic based group. Most of our model parameters are determined by empirical statistics derived from banks' balance sheet data and empirical findings from the existing literature. These parameters include initial interbank network parameters, banks' risk and performance ratios, large and small bank behavior preference ratios, etc. The effect of these parameters on the robustness of the model is all manifested through repeated simulation runs through drawing random samples from the empirical distributions. Hence, the rest of the model robustness test depends on the sensitivity analysis of the model parameters that control agents' characteristics. The primary parameters falling into the this category are α and β in the likelihood function of bank lending decisions (see Equation 10). The parameter selection rule for our simulation is matching stylized facts to a real interbank network. As to demonstrate the robustness of our model to the parameter setting, we evaluate the network degree of the whole interbank market, a combined view of all overnight, short-term, and long-term lending. In particular, we test α for large and small banks from -10.0 to 10.0, β for large and small banks from

0.0 to 2.0. We change one parameter in each experiment and keep other parameters at the original settings.

Table 7: Robustness Test of α

α	Large Banks			Small Banks		
	Degree	In-degree	Out-degree	Degree	In-degree	Out-degree
-10	22.54	18.06	11.76	22.62	18.00	11.80
	(0.84)	(0.69)	(0.55)	(0.84)	(0.69)	(0.55)
-5	22.35	18.00	11.65	22.52	17.97	11.75
	(0.85)	(0.70)	(0.54)	(0.85)	(0.70)	(0.55)
0	22.76	17.98	11.88	22.54	17.92	11.76
	(0.86)	(0.70)	(0.56)	(0.85)	(0.70)	(0.55)
5	22.61	18.02	11.78	22.34	17.85	11.64
	(0.85)	(0.71)	(0.55)	(0.84)	(0.70)	(0.54)
10	22.67	18.25	11.81	20.82	17.03	10.94
	(0.84)	(0.69)	(0.56)	(0.81)	(0.66)	(0.54)

Notes: This table lists average network degree and standard deviations (in parentheses) for different α.

We notice that the network degree does not change dramatically when turning or shifting the S-shaped function (10). In general, we find the interbank linkages are similar to what we obtain from the original parameter settings (see Table 7 and Table 8). We note that for small banks, we do see some decreases in degree in the network as we increase α away from our original settings, though insignificant. This is not surprising as a high α represents increased risk aversion in lending as banks tighten lending policies and fewer lending relationships form.

In addition, the network degree is lower for small banks when β is around 0.0. This is not unexpected as β controls the slope of function, and therefore, influences banks' lending decisions to stop discriminating based on size and relationship scores. As a result, banks cannot optimize their lending formation according to self-preference when β is extremely close to zero. We also examine the other interbank network properties and power-law fit,

Table 8: Robustness Test of β

β	Large Banks			Small Banks		
	Degree	In-degree	Out-degree	Degree	In-degree	Out-degree
0.0	22.41	18.10	11.65	20.61	17.45	10.71
	(0.86)	(0.73)	(0.55)	(0.75)	(0.63)	(0.46)
0.5	22.61	18.01	11.79	22.50	18.06	11.72
	(0.84)	(0.70)	(0.55)	(0.84)	(0.69)	(0.54)
1.0	22.54	17.92	11.76	22.54	17.92	11.76
	(0.85)	(0.70)	(0.55)	(0.85)	(0.70)	(0.55)
1.5	22.50	17.94	11.74	22.58	17.99	11.78
	(0.84)	(0.70)	(0.55)	(0.84)	(0.69)	(0.55)
2.0	22.57	18.01	11.77	22.55	18.02	11.76
	(0.84)	(0.69)	(0.55)	(0.84)	(0.69)	(0.55)

Notes: This table lists average network degree and standard deviations (in parentheses) for different β.

we do not see dramatic change neither. Overall, our model outcomes are robust with regard to the changes of these agent characteristic parameters as well.

5.4. Interbank Network Formation Comparison

The interbank lending networks are generated based on bank agent lending and borrowing behaviors using FFIEC balance sheet data. The results are compared with two established interbank network reconstruction approaches from previous studies: Maximum Entropy (ME) and Minimum Density (MD) methods. Both set certain optimization rules inferring interbank exposures from observable marginals. However, as mentioned earlier, the results generated from these two methods reflect a global optimization approach to network formation in contrast to the model presented here.

Following this paper's earlier methodology, repeated simulations are run with parameters based on 6,600 U.S. banks' financial data from 2001 to 2006. In each of the 30 simulation runs, the interbank network topology is initialized using the ME algorithm and the simulation model runs until the network properties stabilize at a steady state, allowing the calculation of the interbank network properties. The initial bank networks are also constructed using both ME and MD methods, and interbank network properties are computed. The three descriptive measures of network topology are evaluated, i.e. degree distribution, clustering, and average path of the networks, generated by all three approaches. Additionally, the power law exponent of the degree distribution is assessed to examine the characteristics of the reconstructed networks. Results show the model sits between the ME and MD methods (see Table 9).

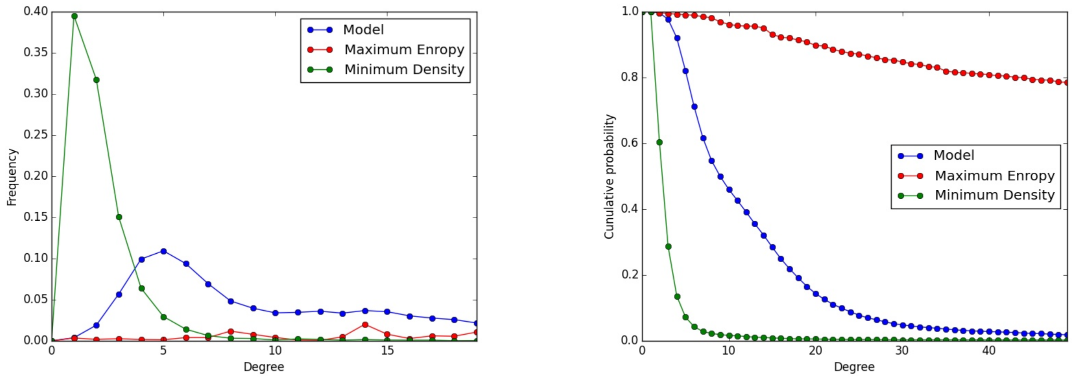
Table 9: Comparison of Network Properties (average of 30 simulations)

	Average degree	Clustering coefficient	Power law	Average path
Maximum Entropy	476.73	0.80	2.31	1.93
Model	14.78	0.36	2.39	2.11
Minimum Density	2.71	0.02	3.14	4.89

Notes: This table shows the network properties generated by the three methods for 6,600 U.S. banks. For the Maximum Entropy and Minimum Density methods, these properties are generated offline using bank balance sheets, and the average numbers are presented accordingly. For the ABM, these properties are based on the average of 30 simulations. *Source*: Authors' calculations.

The degree distribution presents differences of network connections more clearly. In networks generated by the model, the majority of agents create less than 10 links, and very few agents create as few as 1 or 2 links. This can be observed from the probability density function (PDF) in Figure 5. On the other hand, the ME method distributes interbank exposures so widely that the degree measure typically is meaningless. The MD method generates comparatively fewer links, which is evident from Figure 5. However, the average degree generated by MD is at the lower end with a value of 2.71, while the average degree generated by ME is at the higher end with a value of 476.73.

The clustering coefficient, the propensity of nodes to form cliques, is informative. The



(a) PDF Degree Distribution of Banks **(b)** CDF Degree Distribution of Banks

Figure 5: Comparison of Degree Distribution.

Notes: This figure displays the degree distribution in both probability density function (PDF) and cumulative density function (CDF). The degree distribution from the ABM is based on 30 simulations and is represented by the blue line. In the CDF plot, it sits between the Maximum Entropy and Minimum Density methods.
Source: Authors' calculations.

local clustering coefficient averages the probabilities that two neighboring nodes are connected (Jackson, 2008). The MD method produces a value of 0.02 and gives the appearance that local clustering cannot be found, meaning the MD method tends to generate star-like networks, while the ME method seems to be at the other extreme with a high number of links creating a nearly complete network. That suggests that both ME and MD methods fail to preserve local clustering. The ABM produces a middle ground that is also close to results obtained in a study of the German interbank network (Anand et al., 2015).

The average path – the average number of steps along the shortest paths for all possible pairs of network nodes – measures how efficient borrowers are at finding lenders through the network. Empirical studies find that the average path in interbank networks is between 2 and 3 in length (Boss et al., 2004; Bargigli et al., 2015). The MD method generates a relatively large number (4.89), while the ME method is lower than the observed empirical

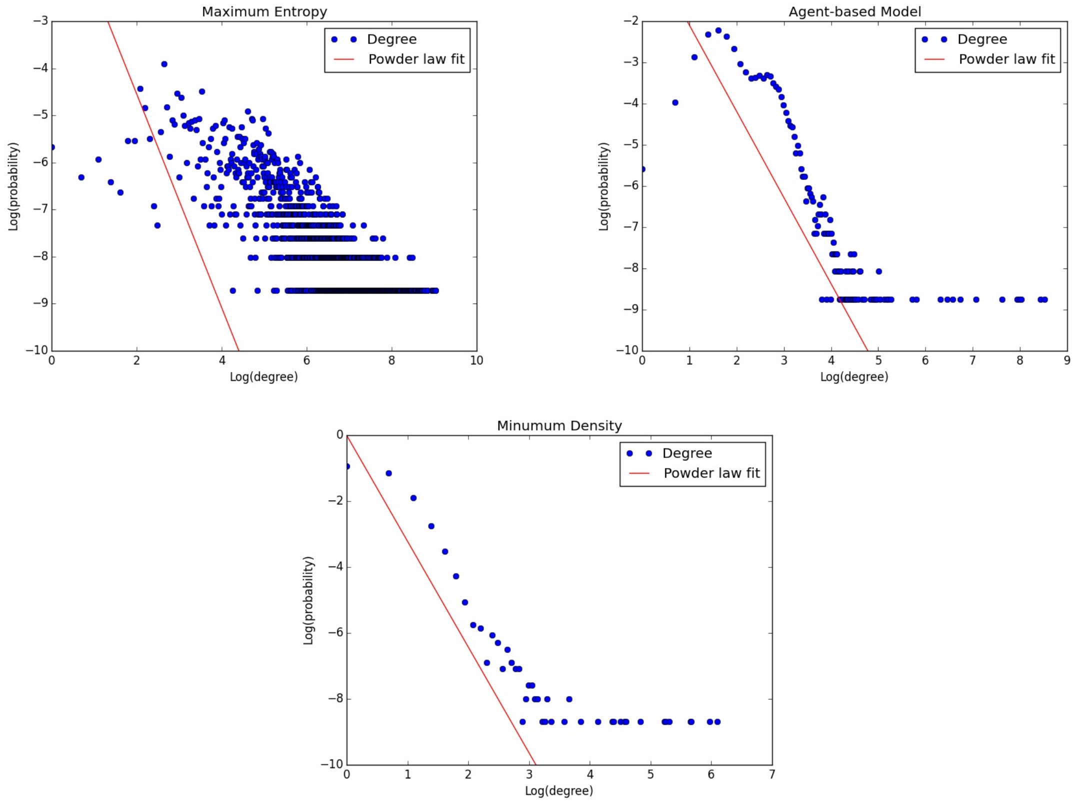


Figure 6: Comparison of Power Law Fit.

Notes: These figures show log-log power law fitting of the degree distribution of the interbank networks generated by the Maximum Entropy, Agent-based Model, and the Minimum Density methods. *Source*: Authors' calculations.

range. The ABM here produces an average of 2.11, which is within the range documented for Austrian and German interbank networks.

Lastly the power law degree distribution exponent of the networks are generated by the three methods and a linear regression in a log-log plot of the cumulative distribution is used to obtain the power-law exponent (like the ones reported for the degree distribution) for the networks generated by the three methods. The ME method has a power law exponent of 2.31; the MD method, 3.14; the agent-based method, 2.39. The ME and agent-based methods produce values consistent with a scale-free network structure which range between 2 and 3 Choromański et al. (2013), while the MD appears on the high end. However, when additionally examining the logarithm degree distributions in Figure 6, we find the ME method produces an interbank network closer to a complete network, and does not fit the constant power exponent.

Overall, as Anand et al. (2015) pointed out, the true network structure should lie between the results from the ME and MD methods. This experiment demonstrates the model produces a reasonable network structure that is well-bounded by the established ME and MD methods.

6. Model Experiments

This section examines the informativeness of the ABM in replicating the impact of stress. A stress similar to the 2007-09 crisis is applied to bank balance sheets to see how effectively the model can match the number of actual banks that failed. We examine the influence of the endogenous formation versus a stationary network form of the model to decipher the interaction of contagion and bank failure. Last, we examine the impact of post-crisis reforms to bank balance sheets and how banks would fare under a similar stress event.

6.1. The 2007-09 Financial Crisis

The ABM simulates the banking system dynamics and allows for the discovery of potential contagion of bank failures due to exogenous shocks. We run an experiment to replicate the 2007-09 crisis and show a simulated market response. From 2003 to mid-2007, banks increased their debt burden from rising home prices. When the housing bubble burst, it triggered a domino effect of bank defaults leading into the financial crisis.

As to provide a similar series of shocks to bank balance sheets in the model, we reevaluate the portion of the balance sheet of banks that were made up of real estate assets using quarterly the House Price Index (HPI).[11] For this exercise we divide the total OA of each bank into real estate, OA^R, and non-real estate assets, OA^{NR}, proportional to the 2006 empirical distribution of real estate loans to other assets. We then create a weighted portfolio for OA^R based a quarterly mortgages vintages $Q(t)$, turn over in real estate loans using an exponentially decaying function over time (Leland and Toft (1996)), and finally normalize it to the size of real estate sales in each quarter, $N(t)$. Lastly we depreciate the percentage of the portfolio by the average delinquency rate, $D(t)$, of real estate loans per quarter and the average percentage cost of foreclosure during the crisis of 25 percent.[12] At each period t, banks write down their real estate loans according to the change in HPI, ΔHPI, following equation (15).

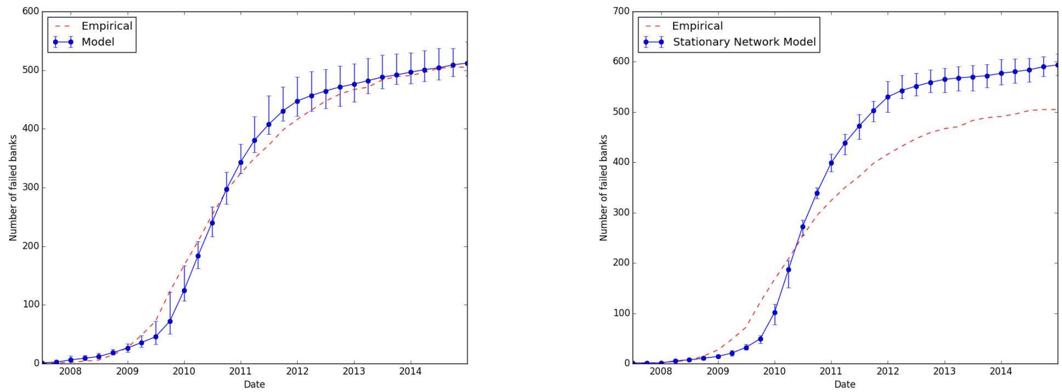
$$OA^R(t) = \Delta HPI(t) \left(\sum_{v=t-1}^{t-40} \frac{Q(v)e^{v/4}}{N(v)} (1 - 0.25D(t-1)) \right) \tag{15}$$

The exogenous shocks are triggered for 29 quarters, corresponding to the U.S. housing price drop from 2007 Q1 through 2014 Q1[13], and each quarter the number of bank failures is recorded. The experiment is run 30 times and then plotted against real bank failures from 2007 Q2 to 2014 Q4 reported by the FFIEC (see Figure 7 (a)). The results show a sudden increase in bank failures during the housing price crash and a recovery period after 2011 that closely resembles the actual bank failures.

[11] Source: Federal Housing Finance Agency

[12] Source: CoreLogic Market Trends and CoreLogic Listing Trends

[13] We assume for simplicity of the model that banks are forced to reevaluate their real estate loans on a quarterly basis.



(a) Endogenous Network Model

(b) Stationary Network Model

Figure 7: Validation of Failed Banks in the 2007-09 Financial Crisis.

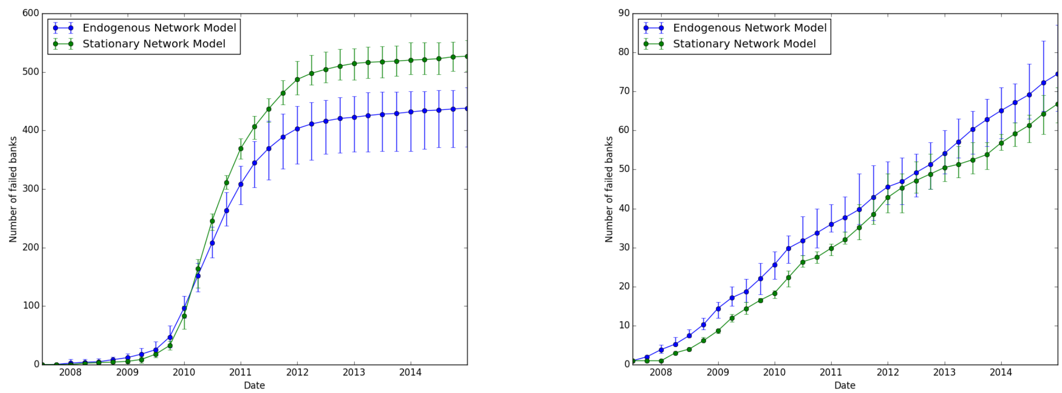
Notes: This figure shows the simulation of bank contagion during the 2007-09 financial crisis. The red dotted line represents the cumulative number of bank failures from 2007 Q2 to 2014 Q4. The blue line represents the average number of failed banks from 30 simulation runs with bars representing 95% confidence interval.*Source*: Federal Financial Institutions Examination Council Reports of Condition and Income

6.2. Endogenous Networks and Contagion

We next consider how the endogenous formation of the network affects the overall results. To tease out this component of the model, we compare the model to a stationary network, similar to the original Eisenberg-Noe model, using just the formulation discussed in section 4.1. To do a fair comparison of the two models, we start both models off with the same sets of values for (ON, ST, LT, C, OL, OA). Then we impose the stress as discussed in Section 6.1 on both models.

Figure 7 (b) plots the stationary model's results over 29 periods. The results of simulation, for the given shock scenario, suggest the stationary network model overestimates the amount of contagion and bank failures in the interbank network system by nearly 20 percent on average. This difference suggests the endogenous dynamics of the network may provide robustness to the interbank lending system.

To understand why the endogenous model has fewer bank failures than the stationary model, we separate the bank failures by types: insolvency, or illiquidity. Figure 8 plots the two models' failure types across the stress scenario. Insolvency makes up the majority of the failures and is also where a substantive difference can be seen in how the two models performed. The finding suggests that by reevaluating lending counterparties, banks not only reduce their own counterparty risk but also the systemic risk coming from contagion.



(a) Insolvency Failure   **(b)** Illiquidity Failure

Figure 8: Comparison of Failed Banks from the Stationary and Endogenous Network Models

Notes: The number of illiquidity and insolvency failures obtained from the stationary and endogenous network model simulations. The blue and green lines represent average of 30 simulation runs, and the vertical bars represent 95% confidence intervals. *Source*: Author's model using Federal Financial Institutions Examination Council Reports of Condition and Income.

6.3. Pre-Crisis versus Post-Crisis Interbank Networks

To analyze if the model is able to provide insights on regulatory reform impacts, we examine how new policies and regulations have influenced bank lending preferences and indirectly changed the robustness and resilience of the interbank system. Our aim is to understand if the post-crisis banking system is in better condition than the pre-crisis system. To answer this question, we set up an experiment to examine the impact on the interbank exposures and financial contagion in the post-crisis era.

The model is calibrated with bank financial data from March 2011 to December 2014. In addition to changes in interbank lending and borrowing ratios presented in Table 2, other ratios also shifted after the financial crisis (see Figure 9). In particular, the balance due from the Federal Reserve Banks (FRB) is 10 times bigger than that in 2001 as the FRB injected more liquidity into the system. The figure also shows the overnight lending ratio dropped 50 percent as banks reduced their balance sheet ratios after the crisis. The latter should lead to a more robust and resilient interbank network structure. The recalibrated model follows the same bank decision rules and activity procedures as in the previous experiment, but with post-crisis data.

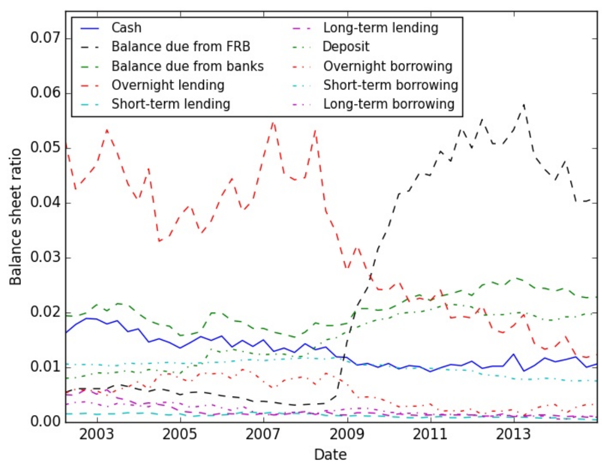


Figure 9: Average Balance Sheet Ratios (2002-14).

Notes: This figure shows average balance sheet ratios each quarter from 2002 to 2014. *Source*: Federal Financial Institutions Examination Council Reports of Condition and Income.

In comparing the network properties between pre-crisis and post-crisis models, Table 10 shows a number of network topological changes in the overnight market but few in the short-term and long-term markets. The overnight network remains a scale-free network with similar power law exponents and the average path length. Both the average degree and clustering coefficient are reduced to one-third of the pre-crisis level, indicating a much lower number of interbank connections. Overall, the post-crisis overnight interbank network appears sparse compared to the pre-crisis period. For the short-term and long-term debt markets, the networks remain relatively sparse, but the clustering coefficients increase slightly, which indicates that although banks tend to have fewer connections, the tendency to form tightly knit groups has increased in the post-crisis era. Therefore, the post-crisis network structure reduces the chance of transmitting shocks to the rest of the system when one bank fails. However, the contagion may become more concentrated as the result of the sparse connections.

Table 10: Interbank Network Topology

	Average degree	Clustering coefficient	Power law	Average path
Overnight				
Pre-Crisis	14.78	0.36	2.39	2.11
Post-Crisis	5.33	0.13	2.45	3.09
Short-term				
Pre-Crisis	1.04	0.43	2.44	2.30
Post-Crisis	1.04	0.53	2.29	2.21
Long-term				
Pre-Crisis	2.42	0.40	2.14	2.44
Post-Crisis	2.42	0.57	2.15	2.28

Notes: The table presents the two balance sheet driven models of pre- and post-crisis banks using 6,600 representative U.S. banks. The Overnight market is the only one where a substantive difference in the network structure can be seen by looking at the four network statistics. *Source*: Authors' model.

Shocks' in the post-crisis network showed the number of failed banks dropping from 500 to 370, a 25 percent decrease at a steady state (see Figure 10). This result is consistent with the network topology analysis that suggests higher stability in a post-crisis network. Both the pre-crisis and post-crisis bank failure curves share the same inflection point, yet the post-crisis failure slope is much smaller than the pre-crisis one. It shows that at the beginning of the contagion, the post-crisis shock transmission rate is higher than the pre-crisis scenario. Toward the end of the contagion, the post-crisis shock transmission rate is smaller than the pre-crisis scenario.

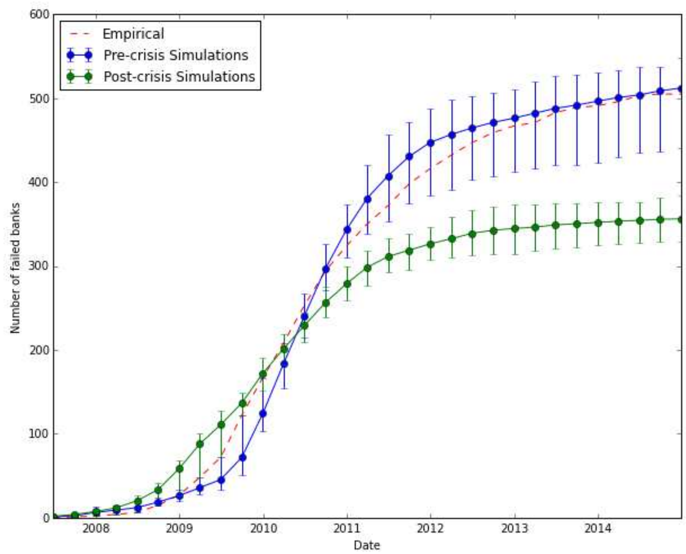


Figure 10: Number of Failed Banks from the Pre-crisis and Post-crisis Simulations.

Notes: The number of empirically observed failed U.S. banks during the 2007-09 financial crisis period vs. the average number bank failures obtained from model simulations using pre-crisis and post-crisis balance sheet data. The blue and green lines represent average of 30 simulation runs, and the vertical bars represent 95% confidence intervals. *Source*: Author's model using Federal Financial Institutions Examination Council Reports of Condition and Income.

This shift in the bank failure pattern can be explained by the network topological changes in the post-crisis era. The post-crisis network has a greater concentration of exposures with fewer links. This means that a large loss transmitted by a given link is more likely to exceed the capital of the lending bank and cause its default. At the same time, the concentration effect is balanced by the fact that the scope of contagion is somewhat limited by the sparsity of the network. A lower number of linkages also reduces the channels allowing the propagation of losses. At the beginning of the contagion, the post-crisis network suffers more bank failures than the pre-crisis network. Toward the end of the contagion, the post-crisis network has fewer bank failures than the pre-crisis network. This is consistent with the observation of Allen and Gale (2000) that complete networks tend to have less contagion effects early, while incomplete networks generated higher contagion effects quickly. Overall, the post-crisis network is more resilient to the same types of shocks, the contagion rate is relatively mild and slower, and bank failures are reduced by 25 percent compared with the pre-crisis network.

7. Conclusion

This paper presents an agent-based approach, using balance sheet data from 6,600 banks, to model the U.S. interbank lending market. This dynamic model incorporates quarterly financial data reported to the FFIEC that reflects actual bank behaviors and performance-based decisions to endogenously reconstruct interbank networks.

We evaluated the model against an established clearing network contagion method under a stationary network form, and show how adding bank-level lending behavior produces results closer to the actual bank contagion that occurred in 2007-09. In one exercise, the model was calibrated to banks' balance sheets in the pre-crisis period of 2001-06, then correlated real estate loan loss shocks were added to the system. The results suggest that network models without the endogenous feature of agent choice overestimate propagation of losses when estimating losses under traditional clearing methods of static networks. Our model shows contagion risk while conditioning the reformation of the network based on bank level behavior preferences observed through bank balance sheet.

As bank balance sheets vary with time, the results of the model calibrated to different period of time also reflect the changes of the entire interbank lending market as the result of the individual bank level behavior changes. As a second exercise, we calibrated the model with post-crisis data from 2011-14, examined the network property differences, and compared the contagion effect with pre-crisis results. We find that in the post-crisis era, banks have fewer counterparty exposures as shown by a sparser network interbank structure than before the crisis. Furthermore, the post-crisis era network is more resilient to correlated asset write-down shocks and has fewer bank failures.

Overall, the methodology presented here is an alternative tool to better understand the contagion impact and network transitions in a bank network. The model provides a vehicle for bank regulators to stress test the interbank system by examining the severity of outcomes. It also could allow regulators to test the impact of new regulations and policies on market microstructure and network relationships, while being mindful to address concerns of the Lucas critiques through incorporating individual optimization.

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