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Footfall for finer food forecasts

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Abstract

Short shelf-lives and low-to-moderate profit margins are common characteristics among fresh-food products, leading to short replenishment cycles and frequent stock-outs. Forecasting of such products is challenged particularly by stock-outs; customers walk past empty shelves, their demand failing to register as sales. We develop an analytical approach for estimating total demand from data on sales and footfall, i.e. the daily customer count. Our investigation reveals that consideration of footfall does not significantly improve forecasts in steady state, but it enables the automatic re-introduction of discontinued products when footfall rises. Additionally, it enables forecasting of the customer conversion rate across stores.

Keywords: Forecasting, Footfall, Newsvendor.

Introduction

Retailers, shopping centres, and airports are tracking your location. How you arrived, where you went, how fast you walked, and how you exited the facility can all be ascertained via the Wifi and Bluetooth connections that we may not be actively using, but have left turned on (Michael and Clarke, 2016). Ignoring the issues involved in this invasion of our private lives, we wonder whether such information could have useful Operations Management consequences. Based on an analysis of using footfall (no. of customer visits) data inside a newsvendor replenishment decision, the answer is yes, but not in the way one might first imagine.

Demand information is required to set inventory levels and to determine order quantities. Retailers may only estimate actual demand from sales data, which does not represent real demand as sales are limited by on-shelf availability (Conrad, 1976). Demand occurring during periods of stock-out is unobservable to some extent. Ignoring this censored demand will bias the forecast as it based on incomplete information, thus inventory levels will be lower than if forecasts were based on true demand (Beutel and Minner, 2012). Its negative implications on profitability has been reported as an increase of 15-30% in future lost sales and 11% less profit (Conlon and Mortimer, 2012). Nevertheless, even a small correction for the censored sales can aid performance (Besbes and Muharremuglo, 2013).

Existing approaches to estimating censored sales involve maximum likelihood estimates (MLE) using historical sales data under assumptions of a Poisson distribution (Conrad, 1976), or for a normal distribution (Nahmias, 1994). However, Tan & Karabati (2004) revealed this estimation procedure works well only when a small fraction of lost sales is present. An alternative approach, the minimax regret principle has been proposed, combining quantitative (mean, mode, median, variance) and qualitative (symmetricity, unimodality, shape) information of demand distribution to achieve its best performance (Perakis and Roels, 2008). Besbes and Muharremoglu (2013) proposed an exploration-exploitation method that used over-ordering to explore the demand distribution, and find exploration to be especially important for integer demand–the main challenge encountered was to define the upper bound for over-ordering. Bensoussan and Guo (2015) argue that large exploration orders are particularly costly for perishable and low-margin products, as large orders create large quantities of unsold product. Besbes and Muharremoglu's model uses an indicator of lost sales availability to tackle this challenge. However, the indicator is only able to indicate a need of upward exploration; it does not help to scale the magnitude. Jain et al., (2015) considered the timing of stock-outs, indicating that this caused a highly variable estimate when the stock-out occurred early on the day. Geurts and Kelly (1986) advocated forecast improvements by gathering and exploiting information on promotions, competitor actions, and weather.

We investigate how footfall data, i.e. the number of customer visits to the store, can be used to improve our estimate of demand in stock-out situations. Footfall is a convenient way to build uncensored forecasts since the information is now readily available. Consider, for example, the information on Google.com, where user location data is used to generate an hour-by-hour graphic of the number of customers in a store, highlighting to internet users the busy times and the average time spent in store. Retailers themselves have developed mobile apps, often linked to their loyalty scheme that can track not only purchase history and store visits, but also customer routes within a store. Retailers also monitor sales data to notify store managers when products have stopped selling. The root cause for no sales could be a genuine stock-out, or it could be because the product has not been moved out of the unloading bay/backroom, and is not accessible to the consumer. We are also aware of at least one retailer, who backfills sales data with the expected sales during periods of stock-outs before making forecasts and replenishment decisions (Potter and Disney, 2010).

Hosking (2011) considers the connection between customer visit data and customer purchasing preference and improvement on demand forecasting leaving formulation of a mathematical model as an open question. Hosking (2011) indicates that research on using footfall for estimating censored demand is not yet well developed. In retail management, footfall applications have mostly been developed for marketing purposes. Retail applications include: segmenting the customer base (Liu, 2008), understanding recent and potential customer requirements (Liu and Ju, 2008), identifying seasonal demand patterns (Newing, et al., 2013) and shopping recommendations (Rao and Shalini, 2016).

In this paper, we analytically develop an alternative forecasting model based on footfall and sales. We use the case of a chain of Scandinavian retailer for context, but simulate for large random datasets generated with parameters representative of the case. The next section provides some background information to this case.

Case: Bread sales in a Scandinavian retailer

Our study is motivated by recent research we have conducted with a Scandinavian retailer. The retailer has several hundred stores which are supplied with fresh bread from ten production facilities owned by the same supplier. The stores are categorized into five different formats from supermarkets, discount stores, to premium range stores. A full range of fresh bread and bakery products is delivered to the stores each day. Any remaining bread in the end of the day is written off, with no salvage value or penalty over the cost of materials. Each store sells about 30 diferent types of fresh bread: loaf/white bread, medium brown bread, brown bread and extra brown bread. About 80% of products are low volume with only a few units sold per day. The profit margin of bread in Scandinavia is moderately low which leads to a low target availability that varies between products but is approximately 30%. Principally, products sold out during the day are not replenished until a new delivery arrives the next day.

Using experience, the store manager decides how much to order based on the previous weeks sales and the current season. Sales information from the retailers loyality scheme is available but not used for replenishment decisions. Two factors make the decision complex. First, only sales information is known for the store manager and not the actual demand since lost sales are unobserved. There is a high frequency of stock-outs making it challenging to achieve accurate ordering. Second, the substitution level of bread varies and is high for some types and low for other types–these challenges, illustrated in Figure 1, complicate the calculation of replenishment quantities that strike a good balance between profit, service, and waste. To estimate the footfall data in this study, sales information from stores available as POS data registered on an hourly basis in the enterprise information system can be used.

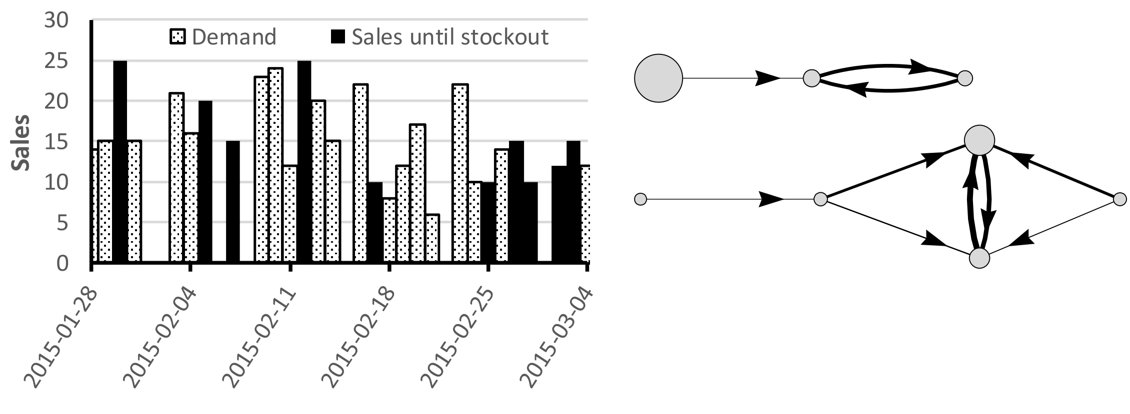

Figure 1. Left: Illustration showing that true demand is not always observable through sales. Right: Demand substitution for two product categories, with the average sales given by node size, and substitution probability given by arc thickness.

Contribution

Our main contribution is the development and evaluation of a demand estimation model that considers footfall in the absence of true demand observations. We show that footfall data is of little use when demand is stationary. Nevertheless, footfall data together with sales allows the forecasting of a customer conversion rate, the purchase probability per customer. This can be used for causal forecasting, and allows inferences about potential sales to be made for other stores, despite differing footfall. Another merit is that a footfall-based forecast combined with a newsvendor policy can decide to re-introduce discontinued products based on significant footfall increases. This is impossible for a forecast purely based on sales that is connected to a conventional newsvendor model. This is a timely contribution given that footfall is available from point-of-sales data, or even from mobile location tracking data now readily available on websites such as Google.com.

Model development: Augmenting censored sales with footfall data

The end result of a forecast is a point estimate of future demand. Here we are not concerned with point estimates, but finding the parameters of the underlying distributions, as this is needed for calculating order quantities. For simplicity of exposition and calculation, we seek the most likely parameter values for the distributions of sales and footfall. This section starts with a demand model based on footfall and a conversion chance, and moves on to the estimation of the underlying parameters, and how this is converted to an order quantity.

Demand, sales, and footfall

Let the integer t represent business days, and assume that footfall f_t follows a Poisson process with mean λ. Each customer has a fixed probability p of selecting the product we consider (through a Bernoulli trial). For a known footfall, this results in hourly demand following a binomial distribution $d_t|f_t \sim \text{Binomial}(x_t, p)$. When the footfall is unknown, we require the marginal distribution of d_t, which is not binomial but follows a Poisson distribution with the parameter $\psi = p\lambda$,

$$d_t \sim \varphi_D(n) = \sum_{n=0}^{\infty} \varphi_P(n)\varphi_B(n,p) = \frac{e^{-\psi}\psi^x}{x!}, \quad (1)$$

where φ_D is the probability mass function (PMF) of demand, φ_P is the Poisson PMF, and φ_P is the binomial PMF. Demand differs from observed sales, as sales are limited by in-store availability. Sales are limited by the stock on-hand at the start of the day, o_t. Then the observed sales in that hour are $s_t = \min(o_t, d_t)$, i.e. the minimum of the stock on-hand at the start at the start of the day, and the daily demand. In other words, sales are demand data censored by o_t.

The probability distribution of d_t is a censored distribution, having the cumulative distribution function

$$\varphi(x|o) = \begin{cases} \varphi(x) & x < o; \\ 1 - \Phi(x-1) & o \leq x. \end{cases} \quad (2)$$

Effectively, the PMF of sales is the same as that of demand when demand is less than the stock on-hand, and for sales equaling the stock on-hand, the probability is the sum of all demand probabilities being equal to or greater than the on-hand inventory.

Demand and footfall estimation

Our estimates will be the ones that are most likely, based on the observed data. We achieve this through maximum likelihood estimation (MLE),

$$\hat{\theta} = \arg\max_{\theta} \prod_{i} \varphi(x_i | o_i), \tag{3}$$

where θ is an arbitrary distribution parameter, implicitly used in the PMF. With Poisson-distributed footfall, the MLE is simply the average of past footfall observations, i.e. $\hat{\lambda} = \bar{f}$ (Box and Tiao, 1973, p. 40). Interestingly, footfall does not aid demand estimation when demand is observed in its entirety:

Proposition 1. Footfall data does not aid parameter estimation when demand is fully observed.

Proof. As footfall is fully observed and Poisson distributed, we know its MLE to be $\hat{\lambda} = \bar{f}$, and similarly with complete demand observations with unknown footfall, $\hat{\psi} = \bar{d}$. For binomial demand with observed footfall, the MLE is $\hat{p} = \bar{d}/\bar{f}$. Finally, observe that $\hat{\psi} = \bar{d} = \bar{f}\hat{p}$. ■

The proof of Proposition 1 immediately leads to another insight.

Proposition 2. Demand data alone cannot be used to generate MLE's for footfall and conversion rate, as there are an infinite number of solutions.

Proof. Under the assumption of Poisson arrivals and a Bernoulli conversion rate, any combinations of f and p, such that $\bar{d} = fp$, are equally likely to generate a sequence of sales. ■

In practical terms, Proposition 2 means that a footfall of 50 and a conversion rate of 2% is just as likely to generate a sequence of sales as a footfall of 100 and a conversion rate of 1%. In order to estimate the conversion rate, we need footfall data, sales, and receipts. Returning to the demand estimation problem, we have cases with or without footfall observations:

When footfall is unknown, we seek the MLE of demand, $\hat{\psi}_h = \arg\max_{\psi} \prod_t \varphi(x_{t,h} | o_{t,h})$. To find $\hat{\psi}_h$, we must use numerical methods due to the Gamma functions present. Our estimate of lost sales merely considers the fully observed demand and on the frequency of stock-outs at various stocking levels. The addition of footfall data permits us to estimate this potential demand with knowledge of the conversion rate (p).

When footfall is known, sales data are augmented with actual customer visits, and we seek the distribution of demand knowing sales and footfall. As we know demand to be generated via Bernoulli trials for each customer, the probability of demand is binomially distributed with f as a parameter. This changes the demand support range from infinite to finite; providing f as an upper bound on demand. Therefore, in periods when $o_t = f_t = s_t$, $d_t = f_t$, the demand is known *despite* the occurrence of a stock-out. This gives the following likelihood estimator for p,

$$L(p) = \begin{cases} \varphi_B(s|f,p) & s < o \cup s = f \\ 1 - \Phi_B(s-1|f,p) & s = o \cap s \neq f, \end{cases} \tag{4}$$

where Φ_B is the cumulative distribution function of the binomial distribution. $L(p)$ can also be expressed as

$$L(p) = \begin{cases} (1-p)^{f-s} p^s \binom{f}{s} & s < o \cup s = f \\ 1 - Q(s, \lambda\, p) - \dfrac{e^{-\lambda}(\lambda p)^s}{s!} & s = o \cap s \neq f, \end{cases} \tag{5}$$

where Q is the regularized Gamma function. Since λ is unknown, we must replace it with $\hat{\lambda}_t$. Maximizing the product of these likelihoods gives the estimate of the customer conversion rate. The next section discusses how these estimates generate an order quantity through the traditional newsvendor model.

The newsvendor model

The newsvendor model identifies a profit-maximizing order quantity (Churchman, et al., 1957), stating that the probability of ending the day with positive stock should equal the profit margin m.

$$Pr(d \leq o^*) = m; \tag{6}$$

$$o^* = \Phi^{-1}(m), \tag{7}$$

where Φ^{-1} is the inverse of the estimated cumulative distribution function of demand. The process of determining the order quantity from sales and footfall appears in Figure 2.

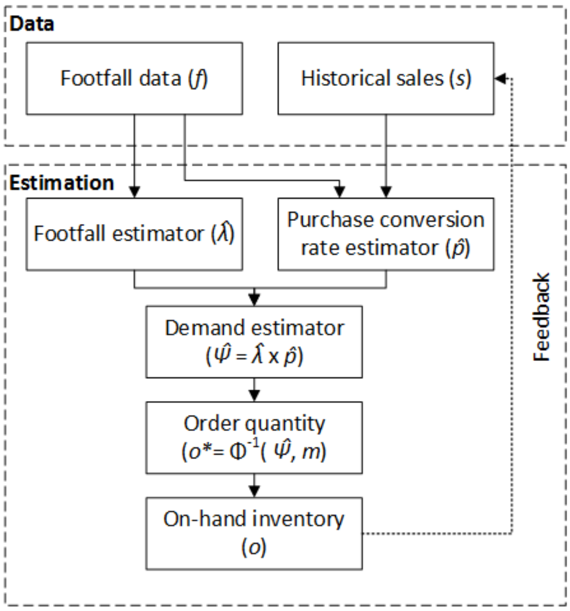

Figure 2. Schematic of the footfall-based demand estimation model.

While the order quantity from the newsvendor model is optimal when one knows the true distribution of demand, problems arise when using estimated demand. One such case is when the retailer has started selling a new product, and purely by the randomness of demand faces an unlucky streak of sales in the first week–say {2,1,0,1,2,1} being complete observations over six days. Updating the demand estimate gives $\hat{\psi} = 1$, from which the newsvendor solution gives an optimal order quantity of $o_7 = \Phi^{-1}_{\hat{\psi}=1}(0.3) = 0$, making the next order zero, which effectively *discontinues* the product. The threshold determining if this should occur depends on the estimated demand and the profit margin, occurring at $\psi' = \max_\theta\left(\theta \middle| \{\Phi^{-1}_\theta(0.3) = 0\}\right) = 1.204$. An estimate below this will terminate replenishment, and is illustrated in Figure 3. For conventional estimates based only on sales there is no recovery, but footfall-based estimates can re-stock a previously discontinued product. This results from the footfall estimates being updated even without any product being sold–sales are used only to update the conversion rate estimate $\hat{p}$, which is frozen as long as no replenishment takes place. When footfall increases such that $\psi' < \bar{f}\hat{p}$, ordering is resumed. Such a recovery is impossible with likelihood estimates based purely on sales.

Proposition 3. (a) Newsvendor models based purely on estimates from sales data will never recover after discontinuing a product. (b) Newsvendor models using estimates from footfall data have a positive probability of re-stocking a discontinued product.

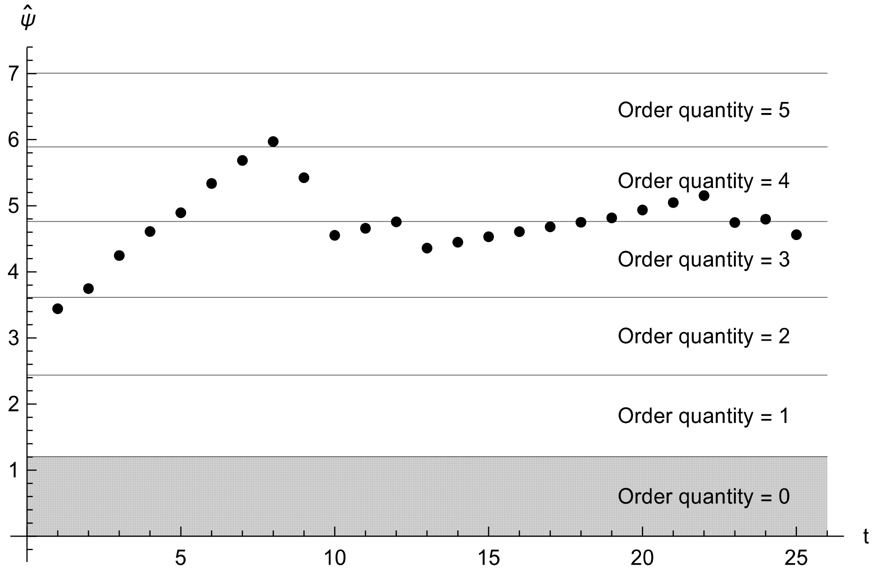

Figure 3. Thresholds that map the demand estimate to an order quantity. Note the zero-order-quantity zone in grey where replenishment terminates.

Numerical results: The value of information

To illustrate the footfall estimation, we consider a store of moderate size with on average 800 visits per day, and a typical product with a long-run average demand of 4, implying $p = 0.5\%$. To match the target service level of 30%, we postulate a unit cost of NOK 20 and a sales price of NOK 28.57. Since both the conventional estimates and the footfall ones converge, the differences will be most noticeable in the first few periods, therefore we simulate a sequence of 25 days, doing 50k replications. To ensure that the model is initialized properly, the estimator is given prior information (a random Poisson observation each for demand and footfall, replaced with unity if either value is zero) to determine the first order. Figure 4 contains graphs showing the average profit realized per period, the mean squared error of the estimator, and the percentage of orders that were of the optimal value $o^* = 3$. Observe that the footfall results are virtually indistinguishable from the ones not using footfall. This largely depends on the integer effect of the optimal safety stock–the whole range of estimators between 3.62 and 4.76 are rounded to the optimal order quantity 3, so that once a satisfactory precision of the forecast is achieved, further improvements yield little or no benefit.

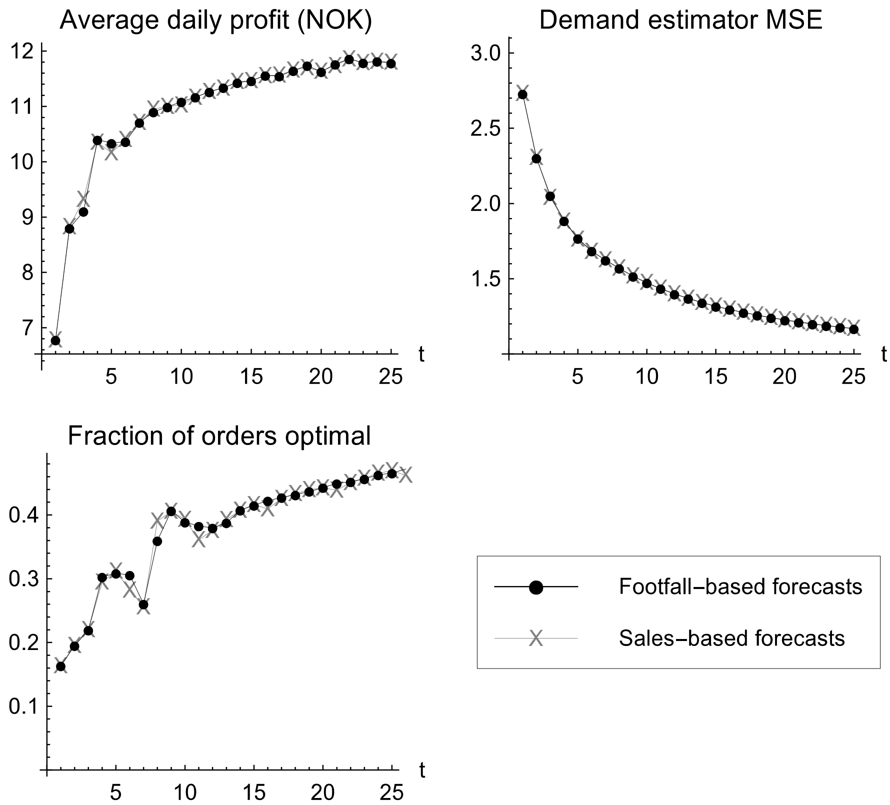

Figure 4. Performance of estimators using footfall data, or sales only. The outcomes are for practical purposes identical.

Managerial implications and concluding remarks

We have shown that maximum likelihood estimates of demand are improved when they account for footfall. Although the improvement is small, the improvement matters when true demand is close to the thresholds determining how much to order. A key contribution is the integration of a causal forecasting method with censored demand–although the likelihood estimation proceeds as usual, the crux is the identification of the likelihood function, which is the conditional distribution of sales given demand parameters (not demand data) and footfall data.

The efficacy of the method depends on where the thresholds are that convert the continuous demand estimate to a discrete order quantity. Favourable conditions have a mean demand centered between these thresholds, as this makes for a quicker convergence to the optimum. Unfavourable conditions have thresholds close to the demand mean, making it more difficult to identify the optimal order quantity. As the thresholds depend on demand and profit margin, not much can be done about this, except for manipulating the price to shift the thresholds.

More important than the cost improvement, our footfall mechanism contains an auto-restart mechanism. This property deserves more attention is the number of breads in our case company with extremely low demand volumes is rather high and the auto-restart mechanism will be particularly useful here. With reference to the case company, this study is a proof-of-concept. Practical applications will face the additional challenges of estimating footfall data from POS-data, estimating the type of demand distribution from censored data, and identifying a demand distribution that is *conditional* upon footfall.

While the binomial distribution allows for a natural interpretation, one must make sure that it fits the actual sales data.

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