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Testing and Estimating Models Using Indirect Inference

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Abstract

In this short article we explain how to test an economic model using Indirect Inference. We then go on to show how you can use this test to estimate the model.

1   Introduction

Indirect Inference provides a classical statistical inferential framework for testing a model. The aim is to compare the performance of the auxiliary model estimated on the simulated data derived from the model, with the performance of the auxiliary model when estimated from the actual data. In practice we use a VAR as the auxiliary model, but you could also use IRFs and moments. If the structural model is correct then its predictions about the time series properties of the data should match those based on actual data. We choose a VAR as the auxiliary model because the solution to a log-linearised DSGE model can be represented as a restricted VARMA model, and this can be closely represented by a VAR.[1] A levels VAR can be used if the shocks are stationary.

2   Model Evaluation by Indirect Inference

The method of evaluating a model by Indirect Inference was introduced in Le et al. (2012). The criterion we use when evaluating the model is the Wald test of the differences between the vector of relevant VAR coefficients from simulated and actual data. If the DSGE model is correct then it should produce simulated data that is similar to the actual data, and therefore the VAR estimates on the simulated data will not be significantly different from the VAR estimates on the actual data. From the actual data we get the VAR parameters β^a, and from the simulations we get N sets of VAR parameters β^i (for $i = 1..N$), from which we calculate $\bar{\beta}$. The Wald statistic that we calculate is:

$$W = \left(\beta^a - \bar{\beta}\right)' \Omega^{-1} \left(\beta^a - \bar{\beta}\right) \tag{1}$$

where

$$\Omega = cov\left(\beta^i - \bar{\beta}\right) \tag{2}$$

In essence we are measuring the distance the actual VAR parameters are from the average of the simulated VAR parameters.

2.1   Implementation of the Wald test by bootstrapping

The following steps summarise how to implement the Wald test by bootstrapping:

Step 1: Calculate the errors of the economic model conditional on the data and parameters.
Step 2: Derive the simulated data by bootstrapping
Step 3: Compute the Wald statistic.

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[1] See Canova (2005), Dave and DeJong (2007), Del Negro and Schorfheide (2004, 2006) and Del Negro et al. (2007a,b) as well as the comments by Christiano (2007), Gallant (2007), Sims (2007), Faust (2007) and Kilian (2007).

Step 1: Calculating the errors The number of independent structural errors is taken to be less than or equal to the number of endogenous variables. Using the data and the parameters we can calculate the structural error. If the equation doesn't have any expectations then the errors are simply backed out from the equation and the data. If the equation has expectations in it we need to estimate the expected values. To do this we use the robust instrumental variables methods of McCallum (1976) and Wickens (1982), with the lagged endogenous data as instruments. In practice we estimate a VAR of all the expected variables and use this to calculated the expectations.

Step 2: Simulated the data In some DSGE models many of the structural errors are assumed to be generated by autoregressive processes. If they are, then we need to estimate them. Once we have the disturbances and persistence of the shocks we can simulate the data by bootstrapping these disturbances. We bootstrap by time vector to preserve any simultaneity between them, and solve the resulting model using Dynare (Juillard, 2001). To obtain the N bootstrapped simulations that we need we repeat this process, drawing each sample independently.

Step 3: Compute the Wald statistic We estimate the auxiliary model — a VAR(1) — using both the actual data and the N samples of simulated data. We then calculate the Wald statistic using Equation 1. The bootstrap distribution of the Wald statistic can be found by substituting each β^i for β^a in Equation 1.

The choice of variables and the order of the VAR is up to you. The Wald test is a strict test, so by increasing the order of the VAR makes the test more stringent, so in practice we use a VAR(1). You can use all the variables in the VAR, or a subset of variables to see what combinations of parameters the model can fit.

For the model to fit the data at the 95% confidence level we want the Wald statistic for the actual data to be less than the 95th percentile of the Wald statistics from the simulated data. The Wald statistics from the simulated data come from a χ^2 distribution with degrees of freedom equal to $k-1$, where k is the number of parameters in β. To make it easier to understand whether the model hasn't been rejected by the data we transform the Wald for the actual data into a t-statistic using the formula and scale it so that if the Wald was equal to the 95th percentile from the simulated data we would get a Transformed Wald of 1.645.

$$T = 1.645 \left(\frac{\sqrt{2W^a} - \sqrt{2k-1}}{\sqrt{2W^{95}} - \sqrt{2k-1}} \right) \tag{3}$$

where W^a is the Wald statistic on the actual data and W^{95} is the Wald statistic for the 95th perentile of the simulated data.

3   Model Estimation by Indirect Inference

As mentioned earlier, the Wald statistic measures the distance between the data and the model. Therefore to estimate the model parameters we can use any minimising algorithm to minimise the Wald for the actual data. The function to minimise takes the coefficients as an input and then does Steps 1–3 above, giving the Wald as the output. I've chosen to use the Simulated Annealing algorithm supplied in Matlab, as I have found that it finds a global minima. In practice it is better to minimise the Transformed Wald because it is easier to see if we have found a set of parameters where the model isn't rejected, as we are just looking to see if we find a Transformed Wald less than 1.645.

4   Example Code

Example code that estimates a simple 3-equation model can be found at `www.patrickminford.net/IIestimation`. In each file there are comments explaining each step. A description of the files is as follows.

act_data.mat This file contains the data. In this case a matrix where each column corresponds to output gap, inflation and the interest rate.

starting_coef.mat This file has the calibrated coefficients we are using as a starting point for the search algorithm.

NKsmall.mod This is the Dynare model file. We only want to use Dynare to get the first order approximation of the model, so I've changed the options in stoch_simul so that nothing is printed out.

CalcWald.m This is the function that takes the coefficients as an input, calculates the structural errors, bootstraps the model and calculates the Transformed Wald as an output. This file on its own can be used to test the model.

run_CalcWald_SA This file will use a Simulated Annealing algorithm to estimate the model. It loops over the SA algorithm using the previous best solution as the new starting point as a way of ensuring we find the minima.

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