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Comparing behavioural and rational expectations for the US post-war economy*

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Abstract

The banking crisis has caused a resurgence of interest in behavioural models of expectations in macroeconomics. Here we evaluate behavioural and rational expectations econometrically in a New Keynesian framework, using US post-war data and the method of indirect inference. We find that after full reestimation the model with behavioural expectations is strongly rejected by the data, whereas the standard rational expectations version passes the tests by a substantial margin.

Key words: behavioural expectation, rational expectation, bank crisis, indirect inference

1 Introduction

Since the banking crisis of 2007 there has been a resurgence of interest in macroeconomic models embodying expectations-formation other than rational expectations. Evidence of biases in expectations, of herd behaviour and of chart-following has been found by a number of researchers in behavioural economics-for example, Kagel and Roth (1995), McCabe (2003), Camerer et al. (2005) and Della Vigna (2009). Kirman (2011) and De Grauwe (2010) have suggested that such behaviour can be found at the macroeconomic level also (they reject the 'rational learning' models of Sargent (1993) and Evans and Honkapohja (2001), in which for many cases learning converges on rational expectations). Accordingly in this paper we examine how far a model of this behavioural type can account for US business cycle behaviour over the past few decades including the recent crisis period; and we compare its performance with that of a standard New Keynesian model. Our (indirect inference) procedure asks whether each model can match US business cycle behaviour, as described by the variances of

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the three main variables, output, inflation and interest rates, and a VAR embodying their inter-relationships. The match is gauged by a Wald statistic that has a well-defined distribution, enabling us to assess the statistical significance of fit. To enable each model to achieve its best possible performance, we allow its model coefficients to be reestimated and only perform the final tests after this has been done.

The models are identical in form, conforming to a standard New Keynesian model, with a forward-looking IS curve, a Phillips Curve, and a Taylor Rule governing interest rates. The only difference lies in expectations-formation; in the standard model these are rational expectations whereas in the alternative ('behavioural') version they are determined by groups of speculators who follow 'fundamentalist' and 'extrapolative' expectations patterns, as set out by de Grauwe (2010). While initially we calibrate these models with typical parameters found in the New Keynesian literature and we report these results in passing, the results we attach importance to are after reestimation (by indirect estimation) to allow each model to get as close as possible to the data, within the bounds set by its theory.

It might well be thought, given the events of recent years, that the standard model would perform badly over the recent post-war period, while the behavioural version would do well. However, we find exactly the opposite: the behavioural version is strongly rejected by the data (including the crisis period), while the standard version is not rejected at the usual significance levels. This surprising result is of some importance to the macroeconomics debate of the current time and so we feel it deserves to be properly exposed to a broad economist audience.

In the rest of this paper, we first explain the models (section 2); we then set out our testing and reestimation procedure (section 3); we turn next to our results, first on calibrated (section 4) and then on reestimated parameters (section 5); section 6 concludes.

2 The Two Models

The behavioural model is a stylized DSGE model similar to the model in De Grauwe (2010). It includes a standard aggregate demand equation, an aggregate supply function, and a policy rule equation, as follows:

$$\tilde{Y}_t = \tilde{E}_t \tilde{Y}_{t+1} - a_1 (R_t - \tilde{E}_t \pi_{t+1}) + \varepsilon_{1t} \tag{1}$$

$$\pi_t = b_1 \tilde{Y}_t + \beta \tilde{E}_t \pi_{t+1} + k \varepsilon_{2t} \tag{2}$$

$$R_t = (1 - c_1)(c_2 \pi_t + c_3 \tilde{Y}_t) + c_1 R_{t-1} + u_t \tag{3}$$

where $\tilde{Y}_t$ is the output gap, π_t is the rate of inflation, R_t is the nominal interest rate, and ε_{1t}, ε_{2t}, and u_t are the demand error, supply error and policy error respectively. These errors are assumed to be autoregressive processes with the coefficients calculated from the sample estimates. Equation 1 is the aggregate demand equation with as the expectations operator in the behavioural model where the tilde above $\tilde{E}$ refers to expectations that are not formed rationally. The aggregate demand function is standard, which is determined by the expectation of output gap in the next period and real interest rate. Equation 2 is the aggregate supply function, which can be derived from profit maximization by individual producers. The supply curve can also be interpreted as a New Keynesian Phillips Curve, which is a function of output gap and expected inflation in the next period. Equation 3 includes a lagged interest rate in Taylor's (1993) original interest rate rule to achieve smoothing of interest rate reactions over time.

The difference between the behavioural and rational expectations model lies in expectations formation. The expectation term in the behavioural model, $\tilde{E}$ is the weighted average of two kinds of forecasting rule. One is the fundamental forecasting rule, by which agents forecast the output gap or inflation at their steady state values. The other one is the extrapolative rule, by which individuals extrapolate most recent value into the future. Thus:

$$\tilde{E}_t^f \tilde{Y}_{t+1} = 0 \tag{4}$$

$$\tilde{E}_t^e \tilde{Y}_{t+1} = Y_{t-1} \tag{5}$$

$$\tilde{E}_t^{tar} \pi_{t+1} = \pi^* \tag{6}$$

$$\tilde{E}_t^{ext} \pi_{t+1} = \pi_{t-1} \tag{7}$$

Equation 4 and 5 are the forecasting rules for the output gap, while Equation 6 and 7 are the equivalents for inflation. The steady state output gap is zero, while the inflation target in the Taylor Rule is the steady state inflation rate, π^*.

In De Grauwe (2010), it is assumed that the market forecast is the weighted average of the fundamentalist and extrapolative rules. Equation 8 is the market forecast for the output gap, while Equation 9 is for inflation.

$$\tilde{E}_t \tilde{Y}_{t+1} = \alpha_{f,t} * 0 + \alpha_{e,t} Y_{t-1} = \alpha_{e,t} Y_{t-1} \tag{8}$$

$$\tilde{E}_t \pi_{t+1} = \beta_{tar,t} \pi^* + \beta_{ext,t} \pi_{t-1} \tag{9}$$

where $\alpha_{f,t}$ and $\alpha_{e,t}$ are the probabilities that agents will use a fundamentalist and extrapolative rule for forecasting the output gap, $\beta_{tar,t}$ and $\beta_{ext,t}$ are the equivalents for inflation. These probabilities sum to one:

$$\alpha_{f,t} + \alpha_{e,t} = 1 \tag{10}$$

$$\beta_{tar,t} + \beta_{ext,t} = 1 \tag{11}$$

These probabilities are defined according to discrete choice theory (see Anderson, de Palma, and Thisse 1992 and Brock and Hommes 1997), which analyses how individuals determine different choices. Agents' utilities are given by the negative of the forecast performance (measured by the squared forecast error) of the different rules as follows:

$$U_{f,t} = -\sum_{k=1}^{\infty} \omega_k (Y_{t-k} - \tilde{E}^{f}_{t-k-1} \tilde{Y}_{t-k})^2 \tag{12}$$

$$U_{e,t} = -\sum_{k=1}^{\infty} \omega_k (Y_{t-k} - \tilde{E}^{e}_{t-k-1} \tilde{Y}_{t-k})^2 \tag{13}$$

$$U_{tar,t} = -\sum_{k=1}^{\infty} \omega_k (\pi_{t-k} - \tilde{E}^{tar}_{t-k-1} \pi_{t-k})^2 \tag{14}$$

$$U_{ext,t} = -\sum_{k=1}^{\infty} \omega_k (\pi_{t-k} - \tilde{E}^{ext}_{t-k-1} \pi_{t-k})^2 \tag{15}$$

where $U_{f,t}$ and $U_{e,t}$ are the utilities for the output gap of the fundamentalists and extrapolators, respectively; while $U_{tar,t}$ and $U_{ext,t}$ are the equivalents for inflation; ω_k are geometrically declining weights, defined as

$$\omega_k = (1 - \rho)\rho^k \tag{16}$$

where ρ, between zero and one, is the memory coefficient.

The probabilities of the fundamentalist and extrapolator in forecasting output are given by the relative utility of their forecasts:

$$\alpha_{f,t} = \frac{\exp(\gamma U_{f,t})}{\exp(\gamma U_{f,t}) + \exp(\gamma U_{e,t})} \tag{17}$$

$$\alpha_{e,t} = \frac{\exp(\gamma U_{e,t})}{\exp(\gamma U_{f,t}) + \exp(\gamma U_{e,t})} \tag{18}$$

while the probabilities of the inflation targeting rule and extrapolative rule are

$$\beta_{tar,t} = \frac{\exp(\gamma U_{tar,t})}{\exp(\gamma U_{tar,t}) + \exp(\gamma U_{ext,t})} \tag{19}$$

$$\beta_{ext,t} = \frac{\exp(\gamma U_{ext,t})}{\exp(\gamma U_{tar,t}) + \exp(\gamma U_{ext,t})} \tag{20}$$

where γ is defined as the 'intensity of choice', assumed to be one in De Grauwe (2010); it measures the degree to which the deterministic component of utility determines actual choice.

Equation 17-18 show that the probability of fundamentalists increases as the forecast performance of the fundamental rule improves relative to the extrapolative rule. Similarly with inflation, Equation 19-20, where we can interpret the weight on the target in the inflation forecasting rule as a measure of the central bank's credibility in inflation targeting. These mechanisms driving the selection of the rules introduces a dynamic element to the model, rather like adaptive expectations in the old NeoKeynesian Synthesis models.

Dealing with the infinite sum in Equation 12 to 15, we can transform them into recursive representation of the sum, so that model can be solved. Then Equation 12-15 can be transformed by the following:

$$U_{f,t} = -(1-\rho)\rho(Y_{t-1})^2 - \rho U_{f,t-1} \tag{21}$$

$$U_{e,t} = -(1-\rho)\rho(Y_{t-1} - Y_{t-3})^2 - \rho U_{e,t-1} \tag{22}$$

$$U_{tar,t} = -(1-\rho)\rho(\pi_{t-1} - \pi^*)^2 - \rho U_{tar,t-1} \tag{23}$$

$$U_{ext,t} = -(1-\rho)\rho(\pi_{t-1} - \pi_{t-3})^2 - \rho U_{ext,t-1} \tag{24}$$

The solution method to the behavioural model is obtained by substituting the expectation formation of Equation 8 and 9 into Equation 1 and 2, therefore the model becomes

$$\tilde{Y}_t = \alpha_{e,t} Y_{t-1} - a_1(R_t - \beta_{tar,t}\pi^* - \beta_{ext,t}\pi_{t-1}) + \varepsilon_{1t} \tag{25}$$

$$\pi_t = b_1\tilde{Y}_t + \beta(\beta_{tar,t}\pi^* + \beta_{ext,t}\pi_{t-1}) + k\varepsilon_{2t} \tag{26}$$

$$R_t = (1-c_1)(c_2\pi_t + c_3\tilde{Y}_t) + c_1 R_{t-1} + u_t \tag{27}$$

with the definition for the probabilities in Equation 12-20. This model is a pure backward model, which can be solved in an overlapping sequence for each set of innovations.

The stylized DSGE model with rational expectation is defined as Equation 1-3 except that the expectations are formed rationally. This RE version of the model can be solved in the standard way; we use Dynare (Juillard 2001) for this.

3 The Testing Procedure

Indirect Inference provides a framework for judging whether a model with a particular set of parameters could have generated the behaviour found in a set of data. The procedure provides a statistical criterion for rejecting the model as the data generating mechanism.

Indirect inference has been well known in the estimation literature, since being introduced by Smith (1993); see also Gregory and Smith (1991, 1993), Gourieroux *et al.* (1993), Gourieroux and Montfort (1995) and Canova (2005).

In indirect estimation the behaviour of the data is first described by some atheoretical time-series model such as a Vector Auto Regression, the 'auxiliary model'; then the parameters of the structural model are chosen so that this model when simulated generates estimates of the auxiliary model as close as possible to those obtained from actual data. It chooses the structural parameters that can minimise the distance between some function of these two sets of estimates. In what follows we give a brief account of the method; a full account, together with Monte Carlo experiments checking its accuracy and power and comparing it with other methods in use for evaluating DSGE models, can be found in Le, Meenagh, Minford and Wickens (LMMW, 2011 and 2012).

The test is based on the comparison of the actual data with the data simulated from the structural model through an auxiliary model. We choose a VAR as our auxiliary model and base our tests on the VAR coefficients and also the variances (of the variables in the VAR). The reason for choosing a VAR as the auxiliary model is that a DSGE model like the ones here have as their solution a restricted vector autoregressive-moving-average (VARMA), which can be closely represented by a VAR. The VAR captures the dynamic inter-relationships found in the data between the variables of the model. The test statistic is based on the joint distribution of the chosen descriptors- here the VAR coefficients and the variances. The null hypothesis is that the macroeconomic model is the data generating mechanism.

The test statistic for this joint distribution is a Wald statistic Following the notation of Canova (2005), y_t is defined as an $m \times 1$ vector of observed data $(t = 1, ..., T)$ and $x_t(\theta)$ is an $m \times 1$ vector of simulated data with S observations from the model, θ is a $k \times 1$ vector of structural parameters from the model. We set $S = T$, because we want to compare simulated data and actual data using the same size of sample. y_t and $x_t(\theta)$ are assumed to be stationary and ergodic. The auxiliary model is $f[y_t, \alpha]$, where α is the vector of descriptors. Under the null hypothesis $H_0 : \theta = \theta_0$, the auxiliary model is then $f[x_t(\theta_0), (\theta_0)] = f[y_t, \alpha]$. The null hypothesis is tested through the $q \times 1$ vector of continuous functions $g(\alpha)$. Under the null hypothesis, $g(\alpha) = g(\alpha(\theta_0))$. a_T is defined as the estimator of α using actual data and $\alpha_S(\theta_0)$ as the estimator of based on simulated data for θ_0. Then we have $g(a_T)$ and $g(\alpha_S(\theta_0))$.The simulated data is obtained by bootstrapping N times of structural errors, so there are N sets of simulated data. We can calculate the bootstrapped mean by $\overline{g(\alpha_S(\theta_0))} = \frac{1}{N}\sum_{k=1}^{N} g_k(\alpha_S(\theta_0))$. The Wald statistic ($WS$) using the bootstrapped distribution of $g(a_S) - \overline{g(\alpha_S(\theta_0))}$ can be specified as

$$WS = (g(a_T) - \overline{g(\alpha_S(\theta_0))})'W^{-1}(\theta_0)(g(a_T) - \overline{g(\alpha_S(\theta_0))})$$

where $W(\theta_0)$ is the variance-covariance matrix of the bootstrapped distribution of $g(a_S) - \overline{g(\alpha_S(\theta_0))}$. Here we use a, the descriptors themselves, as $g(a)$.

The testing procedure involves three steps. The first step is to back out the structural errors from the observed data and parameters of the model. If the model equations have no future expectations, the structural errors can be simply calculated using the actual data and structural parameters. If there are expectations in the model equations, we calculate the rational expectation terms using the robust instrumental variables methods of McCallum (1976) and Wickens (1982); we use the lagged endogenous data as instruments and hence use the auxiliary VAR model as the instrumental variables regression. The errors are treated as autoregressive processes; their autoregressive coefficients and innovations are estimated by OLS. [1]

Secondly, these innovations are then bootstrapped and the model is solved by Dynare. The innovations are repeatedly drawn by time vector to preserve any contemporaneous correlations between them. By this method we obtain N (usually set at 1000) sets of simulated data, or bootstrap samples. These represent the sampling variation of the data implied by the structural model.

Finally, we compute the Wald statistic. By estimating the VAR on each bootstrap sample, the distribution of the VAR coefficients and data variances is obtained, the α. Thus, the estimates of α from the data and the model estimates can be compared. We examine separately the model's ability to encompass the dynamics (the VAR coefficients) and the volatility (the variances) of the data. We show where in the Wald bootstrap distribution the Wald based on the data lies (the Wald percentile). We also show the Mahalanobis Distance based on the same joint distribution, normalised as a t-statistic, as an overall measure of closeness between the model and the data.[2]

We use a VAR(1) as the auxiliary model. With a VAR(1), α contains 12 elements, the 9 VAR coefficients and the 3 data variances. This number of descriptors provides a strong requirement for the structural model to match. Raising the VAR order would increase the number of VAR coefficients (eg with a VAR(2) the number would double to 18, making 21 elements in α in total); the requirement of the test arguably becomes excessive, since we do not expect our structural models to replicate data dynamics at such a high level of refinement.

[1] The idea of using these backed-out errors is that they should be consistent with the model and the data: otherwise the model being tested could be considered rejected by the data at the structural stage. As noted by LMMW (2012), an alternative way to estimate the errors in equations with rational expectations terms is to use the model (including the lagged errors) to generate the expectations and iterate to convergence but in Monte Carlo experiments the LIML method is slightly more accurate (if we knew the true model including the true ρs, then we could back out the exact errors by using the model to solve for the expectations; but of course we do not).

Once the errors and their autoregressive coefficients (ρ) are estimated, they become part of θ_0 and are fixed for the testing process therefore. In indirect estimation the search algorithm finds the structural parameters, the backed-out errors and the ρs that jointly get closest to the α found in the data. If they are also not rejected by these α, then we may treat this model as the data generating mechanism.

[2] The Mahalanobis Distance is the square root of the Wald value. As the square root of a chi-squared distribution, it can be converted into a t-statistic by adjusting the mean and the size. We normalise this here by ensuring that the resulting t-statistic is 1.645 at the 95% point of the distribution.

The steps above detail how a given model, with particular parameter values, is tested. These values would typically be obtained in the first place by calibration. However, the power of the test is high and the model will be rejected if the numerical values chosen for the parameters are inaccurate. Therefore, to test a model fully one needs to examine its performance for all (theoretically permissible) values of these parameters. This is where we introduce Indirect Estimation; in this we search for the numerical parameter values that minimise the Wald statistic and then test the model on these values. If it is rejected on these, then the model itself is rejected, as opposed merely to its calibrated parameter values. We discuss details of this further below.

4 Data, Calibration and Calibrated Results

4.1 Data

We apply the models to quarterly US data from 1981Q4 to 2010Q4 on the output gap ($\tilde{Y}_t$), the inflation rate (π_t), and the interest rate (R_t) [3], collected from Federal Reserve Bank of St. Louis. The data include the recent financial crisis as far availability permits.

The output gap ($\tilde{Y}$) is defined by the percentage gap between real GDP and potential GDP, for which we use the HP filter. Inflation (π) is defined as the quarterly change in the log of the CPI. The interest rate is the federal funds rate, expressed as a fraction per quarter. π_t and R_t are linearly detrended. Figure 1 displays the resulting data; Table 1 also gives the ADF test results, which show that they are all stationary.

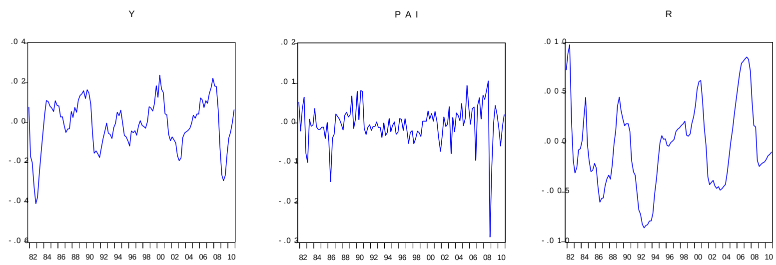

Figure 1: Time Paths of $\tilde{Y}$, π, $\hat{R}$

[3] To calculate the lagged variables $U_{f,t}$, $U_{e,t}$, $U_{tar,t}$, $U_{ext,t}$, we go back to 1970Q2.

Variable	τstatistics	status
$\tilde{Y}$	-2.137881	stationary
π	-8.313042	stationary
$\hat{R}$	-4.300952	stationary

Table 1: ADF Test Results

4.2 Calibration

Table 2 shows the calibrated parameters used for the two models. The first part of the table shows the parameters that are common to both models, following Minford and Ou (2010); the second part shows the parameters individual to each model- the values of γ and ρ follow De Grauwe (2010). The structural errors backed out from model and data all are autoregressive; their AR(1) parameters are shown as $\rho_i(i = 1, \text{demand}; 2, \text{supply}; 3, \text{policy})$.

BF/RE	Parameters	Definitions	Values
BF/RE	a_1	real interest rate elasticity on output gap	0.50
	b_1	coefficient of output gap on inflation	2.36
	π^*	inflation target	0
	β	discount factor	0.99
	κ	coefficient of supply shock on inflation	0.42
	c_1	interest rate persistence parameter	0.8
	c_2	policy preference on inflation	2.0
	c_3	policy preference on output gap	0.1
BF	γ	intensity of choice parameter	1
	ρ	memory parameter	0.5
	ρ_1	autoregressive coefficient for demand error	0.69
	ρ_2	autoregressive coefficient for supply error	0.84
	ρ_3	autoregressive coefficient for policy error	0.18
RE	ρ_1	autoregressive coefficient for demand error	0.89
	ρ_2	autoregressive coefficient for supply error	0.86
	ρ_3	autoregressive coefficient for policy error	0.18

Table 2: Calibration of Behavioural and Rational Expectation Model

4.3 Test Results Based on Calibration

Our auxiliary model is the VAR(1), Equation 28,

$$\begin{bmatrix} \tilde{Y}_t \\ \pi_t \\ R_t \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{21} & \beta_{31} \\ \beta_{12} & \beta_{22} & \beta_{32} \\ \beta_{13} & \beta_{23} & \beta_{33} \end{bmatrix} \begin{bmatrix} \tilde{Y}_{t-1} \\ \pi_{t-1} \\ R_{t-1} \end{bmatrix} + \Omega_t \tag{28}$$

The VAR's nine coefficients represent the dynamic properties found in the data. We also look at the volatility properties as indicated by the variances.

We consider these two properties both separately and together, calculating Wald statistics for each. We show these as the percentile where the data Wald lies in the Wald bootstrap distribution.

4.3.1 Behavioural Model

Table 3 shows the VAR estimates on the actual data and also the 95% bounds of the VAR estimates from the 1,000 bootstrap samples. It shows that five out of nine parameters lie outside the 95% bootstrapped bounds. They are the coefficients of the lagged interest rate on output, and of lagged inflation and lagged interest rates on inflation and interest rates. It is not surprising therefore that overall the model is strongly rejected by the dynamic properties of the data.

Categories	**Actual VAR Coefficients**	**95% Lower Bound**	**95% Upper Bound**	**IN /OUT**
β_{11}	0.9145	0.7558	0.9319	IN
β_{21}	0.0205	-0.1187	0.0369	IN
β_{31}	-0.2214	-0.2041	-0.0148	OUT
β_{12}	0.0554	-0.1792	0.3909	IN
β_{22}	0.1214	0.9468	1.1706	OUT
β_{32}	0.1413	-0.7642	-0.3567	OUT
β_{13}	0.0336	-0.0583	0.1758	IN
β_{23}	-0.0073	0.3638	0.4656	OUT
β_{33}	0.8849	0.4953	0.6697	OUT
Wald (Dynamics)	100%			

Table 3: Dynamic Properties of Behavioural Model Based on Calibration

Table 4 shows the volatility properties of the data and the behavioural model. The table shows that only the output variance can be captured by the model. The variances of inflation and interest rate in the data are far below the range of the 95% model bounds. Jointly the model -generated bounds on the variances are closer to the data, with the Wald percentile at 96.4%, indicating marginal rejection at 95%; this can be reconciled with the rejections of the two variances on their own by noting that the variance values generated by the model will be highly correlated; hence the lower 95% bound of the joint distribution will lie well below the individual 95% bounds of inflation and interest rates.

Nevertheless, when one combines the dynamic and volatility properties, the behavioural model is strongly rejected, with an overall Wald of 100%.

4.4 The Rational Expectations Model

Table 5 shows the test findings for the RE model. On its dynamic properities the model is marginally rejected, with a Wald of 95.6%. It is therefore fairly

Categories	Actual Variances	95% Lower Bound	95% Upper Bound	IN /OUT
var(y)	0.1584	0.0768	0.2512	IN
var(π)	0.0238	0.2270	0.8546	OUT
var(r)	0.0183	0.1605	0.5726	OUT
Wald (Volatility)	96.4%			
Overall Wald	100%			

Table 4: Volatility and Full Properties of Behavioural Model Based on Calibration

close to the data; individually, only one out of nine parameters lies outside the 95% bootstrapped bounds- the coefficient of the lagged interest rate on output.

Categories	Actual VAR Coefficients	95% Lower Bound	95% Upper Bound	IN /OUT
β_{11}	0.9145	0.7143	0.9197	IN
β_{21}	0.0205	-0.3961	0.0963	IN
β_{31}	-0.2214	-0.2133	0.3020	OUT
β_{12}	0.0554	-0.0748	0.0779	IN
β_{22}	0.1214	0.1187	0.4813	IN
β_{32}	0.1413	-0.0620	0.3252	IN
β_{13}	0.0336	-0.0249	0.0471	IN
β_{23}	-0.0073	-0.0221	0.1614	IN
β_{33}	0.8849	0.7916	0.9481	IN
Wald (Dynamics)	95.6%			

Table 5: Dynamic Properties of Rational Expectation Model Based on Calibration

Turning to the volatility properties, Table 6 shows that the model is not rejected by the data, with a Wald at 26.6%; individually, all the three variances lie well inside their 95% bounds.

When one combines the dynamics and volatility, Table 6 shows that the model is not rejected, with an overall Wald percentile of 90.4%.

We bring all these results together in Table 7. It can be seen that, if we use our calibrated parameter values, only the rational expectations model fails to be rejected overall by the behaviour found in the data. However, it could be that this conclusion depends critically on the parameter values chosen and that the calibrated ones give a misleading impression. We accordingly now turn to the reestimation of these parameters.

Categories	Actual Variances	95% Lower Bound	95% Upper Bound	IN /OUT
var(y)	0.1584	0.0595	0.2265	IN
var(π)	0.0238	0.0150	0.0349	IN
var(r)	0.0183	0.0108	0.0443	IN
Wald (Volatility)	26.6%			
Overall Wald	90.4%			

Table 6: Volatility and Full Properties of Rational Expectation Model Based on Calibration

Wald	**BF Model**	**RE Model**
Dynamics	100%	95.6%
Volatility	96.4%	26.6%
Overall	100%	90.4%

Table 7: Comparison of Behavioural and Rational Expectation Model Using Calibration

5 Indirect Inference Estimation

The main idea of indirect inference as an evaluation method is to see if the chosen parameter set θ_0 could have generated the actual data. However, if it cannot do so, another set of parameters could possibly have done so. If no set of parameters can be found under which the model fails to be rejected, then the model itself is rejected. Models that are already unrejected may also get closer to the data with alternative parameters. We now use indirect estimated variables in each equation to obtain the set of parameters that maximises the chances of the model passing the test- in other words minimises the overall Wald statistic. For this purpose we use a powerful algorithm due to Ingber (1996) based on Simulated Annealing in which search takes place over a wide range around the initial values, with optimising search accompanied by random jumps around the space.

Table 8 and 9 show the estimation results for behavioural and rational expectation models respectively. For both models, apart from β (time preference) which is held fixed on a priori grounds, all the parameters are allowed to vary as required by each model. For the behavioural model, the estimated parameters are in Table 8. The IS, Phillips Curve and Taylor Rule parameters need to vary generally by more than 40%, which implies that the original calibrated values were substantially at variance with the data's requirements. The parameters of expectation formation, γ and ρ, vary little however, suggesting that the problem lies with the expectations scheme itself and not with its parameter values. The autoregressive coefficients of the errors also vary little, implying that the parameter changes largely offset each other in their effects on the left-hand-side variable in each equation; nevertheless the changes by affecting the model's transmission processes can change its implied behaviour substantially.. .

Table 9 shows the equivalent results for the rational expectations model. Here the parameters do not generally change so much; only two parameters change more than 40%, the effect of the output gap on inflation and the policy reaction to inflation, both of which increase sharply over the calibrated values.

Parameters	**Estimates**	**Calibration**	**Variation**
a_1	0.7358	0.50	47%
b_1	3.4324	2.36	45%
k	0.5980	0.42	42%
c_1	0.4336	0.8	46%
c_2	2.9230	2.0	46%
c_3	0.0560	0.1	44%
γ	1.0397	1	4%
ρ	0.5304	0.5	6%
ρ_1	0.69	0.69	0%
ρ_2	0.85	0.84	1%
ρ_3	0.16	0.18	11%

Table 8: Estimation of Behavioural Model

Parameters	**Values**	**Calibration**	**Variation**
a_1	0.4307	0.50	14%
b_1	3.5046	2.36	49%
k	0.2935	0.42	30%
c_1	0.8190	0.8	2%
c_2	2.8641	2.0	43%
c_3	0.0804	0.1	20%
ρ_1	0.8849	0.89	1%
ρ_2	0.8677	0.86	14%
ρ_3	0.1736	0.18	4%

Table 9: Calibration of Rational Expectation Model

5.1 Testing Comparison Based on Estimated Parameters

Table 10 and Table 13 show how the test results on these estimated parameters. The behavioural model is still strongly rejected, with six out of twelve parameters still outside the 95% bounds, and while it remains quite close to the data's volatility it is rejected decisively on the dynamics as well as in total, with an overall Wald of 100%.

Though it is still strongly rejected overall, the behavioural model is now closer to the data. We can see this from the transformed Mahalanobis distance (TM) described above, which is a convenient transformation of the Wald

Categories	**Actual VAR Coefficients**	**95% Lower Bound**	**95% Upper Bound**	**IN /OUT**
β_{11}	0.9145	0.7136	0.9212	IN
β_{21}	0.0205	-0.4512	0.0343	IN
β_{31}	-0.2214	-0.1148	0.1964	OUT
β_{12}	0.0554	-0.0770	0.1309	IN
β_{22}	0.1214	0.4001	0.7728	OUT
β_{32}	0.1413	-0.2115	0.0668	OUT
β_{13}	0.0336	-0.0757	0.2062	IN
β_{23}	-0.0073	0.4943	1.0040	OUT
β_{33}	0.8849	0.2266	0.5977	OUT
var(y)	0.1584	0.0634	0.2336	IN
var(π)	0.0238	0.0220	0.0729	IN
var(r)	0.0183	0.0543	0.1799	OUT

Table 10: Testing Details of Behavioural Model

Wald Percentiles	**Calibration**	**Estimation**
Dynamics	100%	100%
Volatility	96.4%	96.0%
Overall	100%	100%

Table 11: Comparison of Behavioural Expectation Model results under Calibration and Estimation

statistic: it is a normalised t-statistic taking the value1.645 at the 95% Wald percentile. Table 12 shows that the TM for the behavioural model improves materially after estimation.

Tsfmd Mahalanobis	**Calibration**	**Estimation**
Dynamics	32.00	5.55
Volatility	1.98	1.92
Overall	30.01	5.93

Table 12: Comparison TM of Behavioural and Rational Expectation Model Using Estimated Parameters

Table 13 shows that the rational expectations model improves to considerable closeness to the data behaviour. All the individual parameters are now inside their 95% bounds and overall the model would not be rejected at 80% confidence (see Table 14).

Categories	**Actual VAR Coefficients**	**95% Lower Bound**	**95% Upper Bound**	**IN /OUT**
β_{11}	0.9145	0.7277	0.9316	IN
β_{21}	0.0205	-0.3817	0.1688	IN
β_{31}	-0.2214	-0.2566	0.3016	IN
β_{12}	0.0554	-0.0772	0.0756	IN
β_{22}	0.1214	0.0892	0.4276	IN
β_{32}	0.1413	-0.1136	0.2630	IN
β_{13}	0.0336	-0.0252	0.0420	IN
β_{23}	-0.0073	-0.0266	0.1429	IN
β_{33}	0.8849	0.8027	0.9525	IN
var(y)	0.1584	0.0613	0.2514	IN
var(π)	0.0238	0.0119	0.0320	IN
var(r)	0.0183	0.0100	0.0408	IN

Table 13: Testing Details of Rational Expectation Model

Wald Percentiles	**Calibration**	**Estimation**
Dynamics	95.5%	90.0%
Volatility	26.6%	24.2%
Overall	90.4%	79.8%

Table 14: Comparison of Rational Expectation Model under Calibrated and Estimated Parameters

In sum, we can see that while the behavioural model remains rejected overall, the rational expectations model has after estimation lowered the threshold at which it would not be rejected to 80%. It would seem that behavioural expectations are clearly rejected in favour of rational expectations in the context of a standard macroeconomic model.

6 Conclusion

This paper investigates whether behavioural expectations can improve on rational expectations in our understanding of recent macroeconomic behaviour. The banking crisis impelled many economists and commentators to question the standard New Keynesian model with rational expectations; one suggested improvement was that expectations could be formed in a behavioural manner. We have found in our work here that in fact this would be no improvement; indeed the standard model fits the behaviour found in the data, including the crisis period, rather well while the behavioural model is decisively rejected.

This is not to say that the standard model cannot be enriched in some way to improve our understanding of the events surrounding the crisis. In particular, our work makes no attempt to assess the shift in the economy's trend behaviour, as we abstract from trends in the usual way- others argue (eg Le, Meenagh and Minford (2012)), that shifts in trend were an important determinant of the US crisis. Nor does it attempt to model the behaviour of banks and how this was related to the economy in the crisis. Plainly these topics are important ones to investigate. However, our work here suggests that behavioural expectations are not a promising route to account for the banking crisis.

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