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Can a Real Business Cycle Model without price and wage stickiness explain UK real exchange rate behaviour? *

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Abstract

This paper establishes the ability of a Real Business Cycle model to account for UK real exchange rate behaviour. The model is tested by the method of indirect inference, bootstrapping the errors to generate 95% confidence limits for a time-series representation of the real exchange rate, as well as for various key data moments. The results suggest RBC models can explain real exchange rate movements.

JEL Classification : E32, F31, F41

Key Words : Real Exchange Rate, Productivity, Real Business Cycle, Bootstrap, Indirect Inference

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1 Introduction

The continuous strength of the dollar over the 1990s fuelled interest in the relationship between productivity and exchange rates. As US productivity surged in the second half of the 1990s, the dollar began its climb against all the major currencies of the world. This has led to a large body of literature analysing the links between the real exchange rate and productivity. The conventional view of the impact of a productivity shock on an economy is that the real exchange rate will depreciate, in order to permit the extra output to be sold on world markets. However, this is at odds with the empirical findings of currency appreciation after a productivity spurt (for discussion of the dollar’s real appreciation in the 1990s see Tille et al., 2001; Corsetti et al., 2004; Bailey et al., 2001; Schnatz et al., 2003; Meredith, 2001). In this paper we explore the ability of a Real Business Cycle (RBC) model—along the lines of McCallum (1989) and Backus et al. (1994)—to account for the real exchange rate’s behaviour, using UK experience as our empirical focus. First, we find that a deterministic productivity growth shock generates a real depreciation in steady state equilibrium but on impact undershoots this substantially and may even create an appreciation, as part of its business cycle effect—with some weak similarities to the type of behaviour found for the dollar in the 1990s. Second, we show that the RBC alone when perturbed by the model shocks found empirically can reproduce the univariate properties of the real exchange rate—by implication there is no necessary case here to add nominal rigidity.

We define the real exchange rate conventionally as the ratio of foreign consumer prices to home consumer prices, converted into a common currency. A large body of evidence (originating with Engel, 1993) finds that the variation of this ratio is almost entirely dominated by the ratio of home-produced relative to foreign-produced traded goods, the terms of trade. A large number of studies have examined movements in the real exchange rate. They find that they exhibit swings away from various definitions of ‘purchasing power parity’ (PPP) by which is meant the longer-run equilibrium value of Q . Such an equilibrium is akin to the ‘natural rate’ of output or unemployment in a general equilibrium macroeconomic model and it may move over time for a variety of reasons. Many studies have found definite evidence of reversion to PPP but very slow reversion. More recently studies that have allowed for non-linear adjustment (such that as the real exchange rate moves further away from PPP the pressures of goods market arbitrage become stronger) have found that the speed of reversion is much greater, and becomes of similar order to that for other macro variables such as output and inflation—for an early result of this sort see Michael et al. (1997).

One can think of these studies as final form equations of Q , where unspecified shocks to the economy, from demand and supply, stochastically disturb Q away from some smoothly-moving trend. Macroeconomic models that could in principle produce such a final form range from, on the one hand, models with a high degree of nominal rigidity to, at the other extreme, real business cycle models with no stickiness—henceforth RBC models.

In this paper we explore the ability of an RBC model to account for the behaviour of Q , using UK experience as our empirical focus. Our argument will be that the RBC alone, without price stickiness, can reproduce the univariate properties of Q . We do not rule out the possibility that adding a degree of nominal rigidity could also contribute. However our concern is to establish the basic ability of the flexprice RBC model to provide explanatory power. In this respect we depart from much work which has accounted for real exchange rate movements in terms of price stickiness—originally Dornbusch (1976) and more recently Chari, Kehoe and McGrattan (2002) who tested a sticky-price two-country model of the US and EU by comparing simulated moments with their data counterparts; Le et al. (2009, 2010) examine alternatives with varying degrees of stickiness and find that data variances (including that of the real exchange rate) are better matched with only a small degree of it, even though all versions of these models are strictly rejected by the data overall. Rather to our surprise there is little work examining the flexprice RBC model, only the McCallum and Backus et al. papers cited above; however their empirical tests were rather limited and our aim here is to use econometric tests based on indirect inference that were not in use at that time. Unlike the two papers by Le et al. above, we restrict our formal testing focus to the real exchange rate alone, we use raw data, mostly non-stationary, and our model for the UK as a medium-sized open economy treats rest-of-world consumption and real interest rate as exogenous.

Thus the aim of this paper is to extend the testing of flexprice RBC models for their real exchange rate properties by first using a previously-unused test procedure based on statistical inference and second applying it to unfiltered UK post-war data. The paper is organised as follows. In section 2 we set out the

real business cycle model. In section 3 we calibrate the model to UK quarterly data and show the results of a productivity shock. Section 4 establishes the facts of the real exchange rate, Q ; it is integrated of order 1 and can be fitted well by an ARIMA process. In section 5 we explain the method of indirect inference and formally test the model statistically on the real exchange rate data. Section 6 we conclude that Q behaviour in fact can be explained using an RBC model with no nominal rigidity.

2 The Model

Consider a home economy populated by identical infinitely lived agents who produce a single good as output and use it both for consumption and investment; all variables are in per capita terms. It coexists with another, foreign, economy (the rest of the world) in which equivalent choices are made; however because this other country is assumed to be large relative to the home economy we treat its income as unaffected by developments in the home economy. We assume that there are no market imperfections. At the beginning of each period t , the representative agent chooses (a) the commodity bundle necessary for consumption, (b) the total amount of leisure that it would like to enjoy, and (c) the total amount of factor inputs necessary to carry out production. All of these choices are constrained by the fixed amount of time available and the aggregate resource constraint that agents face. During period t , the model economy is influenced by various random shocks.

In an open economy goods can be traded but for simplicity it is assumed that these do not enter in the production process but are only exchanged as final goods. The consumption, C_t in the utility function below, is composite per capita consumption, made up of agents consumption of domestic goods, C_t^d and their consumption of imported goods, C_t^f . We treat the consumption bundle as the numeraire so that all prices are expressed relative to the general price level, P_t . The composite consumption utility index can be represented as an Armington (1969) aggregator of the form

$$C_t = \left[\omega (C_t^d)^{-\rho} + (1 - \omega) \varsigma_t (C_t^f)^{-\rho} \right]^{\left(\frac{-1}{\rho}\right)} \quad (1)$$

where ω is the weight of home goods in the consumption function, ρ , the elasticity of substitution is equal to $\frac{1}{1+\rho}$ and ς_t is a preference error.

The consumer maximises this composite utility index, given that an amount \widetilde{C}_t has been chosen for total expenditure, with respect to its components, C_t^d and C_t^f subject to $\widetilde{C}_t = p_t^d C_t^d + Q_t C_t^f$. where p_t^d is the domestic price level relative to the general price level and Q_t ¹ is the foreign price level in domestic currency relative to the general price level (the real exchange rate). The resulting expression for the home demand for foreign goods is

$$\frac{C_t^f}{C_t} = [(1 - \omega)\varsigma_t]^\rho (Q_t)^{-\rho} \quad (2)$$

We also note that:

$$1 = \omega^\rho (p_t^d)^{\rho\rho} + [(1 - \omega)\varsigma_t]^\rho Q_t^{\rho\rho} \quad (3)$$

Hence we can obtain the logarithmic approximation:

¹we form the Lagrangean $L = \left[\omega (C_t^d)^{-\rho} + (1 - \omega) (C_t^f)^{-\rho} \right]^{\left(\frac{-1}{\rho}\right)} + \mu (\widetilde{C}_t - \frac{P_t^d}{P_t} C_t^d - \frac{P_t^f}{P_t} C_t^f)$. Thus $\frac{\partial L}{\partial C_t} = \mu$; also at its maximum with the constraint binding $L = \widetilde{C}_t$ so that $\frac{\partial L}{\partial C_t} = 1$. Thus $\mu = 1$ - the change in the utility index from a one unit rise in consumption is unity. Substituting this into the first order condition $0 = \frac{\partial L}{\partial C_t^f}$ yields equation (2). $0 = \frac{\partial L}{\partial C_t^d}$ gives the equivalent equation: $\frac{C_t^d}{C_t} = \omega^\rho (p_t^d)^{-\rho}$ where $p_t^d = \frac{P_t^d}{P_t}$. Divide (1) through by C_t to obtain $1 = \left[\omega \left(\frac{C_t^d}{C_t}\right)^{-\rho} + (1 - \omega) \left(\frac{C_t^f}{C_t}\right)^{-\rho} \right]^{\left(\frac{-1}{\rho}\right)}$; substituting into this for $\frac{C_t^f}{C_t}$ and $\frac{C_t^d}{C_t}$ from the previous two equations gives us equation (3).

$$\log p_t^d = - \left(\frac{1-\omega}{\omega} \right)^\sigma \log(Q_t) - \frac{1}{\varrho} \left(\frac{1-\omega}{\omega} \right)^\sigma \log \zeta_t + \text{constant} \quad (4)$$

In a stochastic environment a consumer is expected to maximise expected utility subject to the budget constraint. Each agent's preferences are given by

$$U = \text{Max} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right], \quad 0 < \beta < 1 \quad (5)$$

where β is the discount factor, C_t is consumption in period 't', L_t is the amount of leisure time consumed in period 't' and E_0 is the mathematical expectations operator. Specifically, we assume a time-separable utility function of the form

$$U(C_t, 1 - N_t) = \theta_0 (1 - \rho_0)^{-1} \gamma_t C_t^{(1-\rho_0)} + (1 - \theta_0) (1 - \rho_2)^{-1} \zeta_t (1 - N_t)^{(1-\rho_2)} \quad (6)$$

where $0 < \theta_0 < 1$, and $\rho_0, \rho_2 > 0$ are the substitution parameters; and γ_t, ξ_t are preference errors. This sort of functional form is common in the literature for example McCallum and Nelson (1999a). Total endowment of time is normalised to unity so that

$$N_t + L_t = 1 \text{ or } L_t = 1 - N_t \quad (7)$$

Furthermore for convenience in the logarithmic transformations we assume that approximately $L = N$ on average.

The representative agent's budget constraint is

$$C_t + \frac{b_{t+1}}{1+r_t} + \frac{Q_t b_{t+1}^f}{(1+r_t^f)} + p_t S_t^p = (v_t)N_t - T_t + b_t + Q_t b_t^f + (p_t + d_t) S_t^p \quad (8)$$

where p_t denotes the real present value of shares (in the economy's firms which they own), $v_t = \frac{W_t}{P_t}$ is the real consumer wage (w_t , the producer real wage, is the the wage relative to the domestic goods price level; so $v_t = w_t p_t^d$). Households are taxed by a lump-sum transfer, T_t ; marginal tax rates are not included in the model explicitly and appear implicitly in the error term of the labour supply equation, ζ_t . b_t^f denotes foreign bonds, b_t domestic bonds, S_t^p demand for domestic shares and $Q_t = \frac{P_t^f}{P_t}$ is the real exchange rate.

In a stochastic environment the representative agent maximizes the expected discounted stream of utility subject to the budget constraint. The first order conditions with respect to C_t, N_t, b_t, b_t^f and S_t^p are (where λ_t is the Lagrangean multiplier on the budget constraint):

$$\theta_0 \gamma_t C_t^{-\rho_0} = \lambda_t \quad (9)$$

$$(1 - \theta_0) \zeta_t (1 - N_t)^{-\rho_2} = \lambda_t v_t \quad (10)$$

$$\frac{\lambda_t}{1 + r_t} = \beta E_t \lambda_{t+1} \quad (11)$$

$$\frac{\lambda_t Q_t}{(1 + r_t^f)} = \beta E_t \lambda_{t+1} Q_{t+1} \quad (12)$$

$$\lambda_t p_t = \beta E_t \lambda_{t+1} (p_{t+1} + d_{t+1}) \quad (13)$$

Substituting equation (11) in (9) yields :

$$(1 + r_t) = \left(\frac{1}{\beta} \right) E_t \left(\frac{\gamma_t}{\gamma_{t+1}} \right) \left(\frac{C_t}{C_{t+1}} \right)^{-\rho_0} \quad (14)$$

Now substituting (9) and (11) in (10) yields

$$(1 - N_t) = \left\{ \frac{\theta_0 C_t^{-\rho_0} v_t}{(1 - \theta_0) \zeta_t} \right\}^{\frac{-1}{\rho_2}} \quad (15)$$

Substituting out for $v_t = w_t p_t^d$ and using (4) equation (15) becomes

$$(1 - N_t) = \left\{ \frac{\theta_0 C_t^{-\rho_0} \left[\exp \left(\log w_t - \left(\frac{1-\omega}{\omega} \right) \sigma \left(\log Q_t + \frac{1}{2} \log \zeta_t \right) \right) \right]}{(1 - \theta_0) \zeta_t} \right\}^{\frac{-1}{\rho_2}} \quad (16)$$

Substituting (11) in (13) yields

$$p_t = \left(\frac{p_{t+1} + d_{t+1}}{(1 + r_t)} \right) \quad (17)$$

Using the arbitrage condition and by forward substitution the above yields

$$p_t = \sum_{i=1}^{\infty} \frac{d_{t+i}}{(1 + r_t)^i} \quad (18)$$

i.e. the present value of a share is discounted future dividends.

To derive the uncovered interest parity condition in real terms, equation (11) is substituted into (12)

$$\left(\frac{1 + r_t}{1 + r_t^f} \right) = E_t \frac{Q_{t+1}}{Q_t} \quad (19)$$

In logs this yields

$$r_t = r_t^f + \log E_t \frac{Q_{t+1}}{Q_t} \quad (20)$$

Thus the real interest rate differential is equal to the expected change in the real exchange rate. Financial markets are otherwise not integrated and are incomplete².

2.1 The Government

The government finances its expenditure, G_t , by collecting taxes on labour income, τ_t . Also, it issues debt, bonds (b_t) each period which pays a return next period.

The government budget constraint is:

$$G_t + b_t = T_t + \frac{b_{t+1}}{1 + r_t} \quad (21)$$

where b_t is real bonds

2.2 The Representative Firm

Firms rent labour and buy capital inputs, transforming them into output according to a production technology. They sell consumption goods to households and government and capital goods to other firms. The technology available to the economy is described by a constant-returns-to-scale production function:

$$Y_t = Z_t N_t^\alpha K_t^{1-\alpha} \quad (23)$$

²As noted by Chari et al (2002), assuming complete asset markets imposes the condition that the real exchange rate equals the ratio of the two continents' marginal utilities of consumption at all times. This implies that the expected log change in the real exchange rate equals the expected log change in this ratio, i.e. the real interest differential — the real UIP condition again. Thus the conditions are in practice similar: under complete markets the real exchange rate exactly moves with relative consumption whereas under incomplete it is only expected to do so, so that random walk shocks can drive them apart.

where $0 \leq \alpha \leq 1$, Y_t is aggregate output per capita, K_t is capital carried over from previous period ($t - 1$), and Z_t reflects the state of technology.

It is assumed that $f(N, K)$ is smooth and concave and it satisfies Inada-type conditions i.e., the marginal product of capital (or labour) approaches infinity as capital (or labour) goes to 0 and approaches 0 as capital (or labour) goes to infinity.

$$\begin{aligned} \lim_{K \rightarrow 0} (F_K) &= \lim_{N \rightarrow 0} (F_N) = \infty \\ \lim_{K \rightarrow \infty} (F_K) &= \lim_{N \rightarrow \infty} (F_N) = 0 \end{aligned} \quad (24)$$

The capital stock evolves according to:

$$K_t = I_t + (1 - \delta) K_{t-1} \quad (25)$$

where δ is the depreciation rate and I_t is gross investment.

In a stochastic environment the firm maximizes the present discounted stream, V , of cash flows, subject to the constant-returns-to-scale production technology and quadratic adjustment costs for capital,

$$MaxV = E_t \sum_{i=0}^T d_{it}^i [Y_t - K_t(r_t + \delta + \kappa_t) - (w_t + \chi_t)N_t - 0.5\xi(\Delta K_{t+i})^2] \quad (26)$$

subject to the evolution of the capital stock in the economy, equation (25). Here r_t and w_t are the rental rates of capital and labour inputs used by the firm, both of which are taken as given by the firm. The terms κ_t and χ_t are error terms capturing the impact of excluded tax rates and other imposts or regulations on firms' use of capital and labour respectively. The firm optimally chooses capital and labour so that marginal products are equal to the price per unit of input. The first order conditions with respect to K_t and N_t^d are as follows:

$$\xi(1 + d_{1t})K_t = \xi K_{t-1} + \xi d_{1t} E_t K_{t+1} + \frac{(1 - \alpha) Y_t}{K_t} - (r_t + \delta + \kappa_t) \quad (28)$$

$$N_t = \frac{\alpha Y_t}{w_t + \chi_t} \quad (29)$$

2.3 The Foreign Sector

From equation (2) we can derive the import equation for our economy

$$\log C_t^f = \log IM_t = \sigma \log(1 - \omega) + \log C_t - \sigma \log Q_t + \sigma \log \varsigma_t \quad (34)$$

Now there exists a corresponding equation for the foreign country which is the export equation for the home economy

$$\log EX_t = \sigma^F \log(1 - \omega^F) + \log C_t^F + \sigma^F \log Q_t + \sigma^F \log \varsigma_t^F \quad (37)$$

Foreign bonds evolve over time to the balance payments according to the following equation

$$\frac{Q_t b_{t+1}^f}{(1 + r_t^f)} = Q_t b_t^f + p_t^d EX_t - Q_t IM_t \quad (40)$$

Finally there is good market clearing:

$$Y_t = C_t + I_t + G_t + EX_t - IM_t \quad (41)$$

3 Calibration & Deterministic Simulation

The model is calibrated with the values familiar from earlier work—see Kydland and Prescott, (1982), Obstfeld and Rogoff (1996), Orphanides (1998), Dittmar et al. (1999), McCallum and Nelson (1999a, 1999b), McCallum (2001), Rudebusch and Svensson (1999), Ball (1999) and Batini and Haldane (1999); the Appendix gives a full listing. Thus in particular the coefficient of relative risk aversion (ρ_0) is set at 1.2 and the substitution elasticity between consumption and leisure (ρ_2) at unity. Home bias (ω, ω^F) is set high at 0.7. The substitution elasticity between home and foreign goods (σ, σ^F) is set at 1 both for exports and for imports, thus assuming that the UK’s products compete but not sensitively with foreign alternatives; this is in line with studies of the UK (see for example Minford et al., 1984).

Before testing the model stochastically against Q behaviour, we examine its implications in the face of a sustained one-off rise in productivity, the shock that will most affect Q in the model. We wish to see whether the model can qualitatively explain large cyclical swings in Q. Figure 1 shows the model simulation of a rise of the productivity level by 12% spread over 12 quarters and occurring at 1% per quarter (the increase in the whole new path is unanticipated in the first period and from then on fully anticipated)—in other words a three-year productivity ‘spurt’.

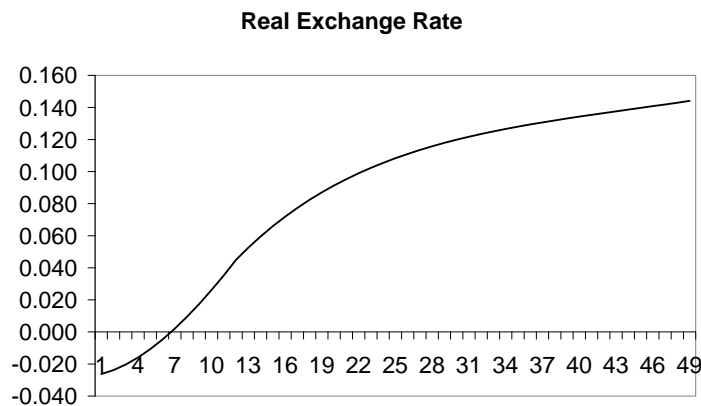


Figure 1: Change in Real Exchange Rate (Q) after a 1% increase in productivity each quarter for twelve quarters

The logic behind the behaviour of the real exchange rate, Q, can be explained as follows. The productivity increase raises permanent income and also stimulates a stream of investments to raise the capital stock in line. Output however cannot be increased without increased labour supply and extra capital, which is slow to arrive. Thus the real interest rate must rise to reduce demand to the available supply while real wages rise to induce extra labour and output supply. The rising real interest rate violates Uncovered Real Interest Parity (URIP) which must be restored by a real appreciation (fall in Q) relative to the expected future value of the real exchange rate. This appreciation is made possible by the expectation that the real exchange rate will depreciate (Q will rise) steadily, so enabling URIP to be established consistently with a higher real interest rate. As real interest rates fall with the arrival on stream of sufficient capital and so output, Q also moves back to equilibrium. This equilibrium however represents a real depreciation on the previous steady state (a higher Q) since output is now higher and must be sold on world markets by lowering its price. (Fig. 2)

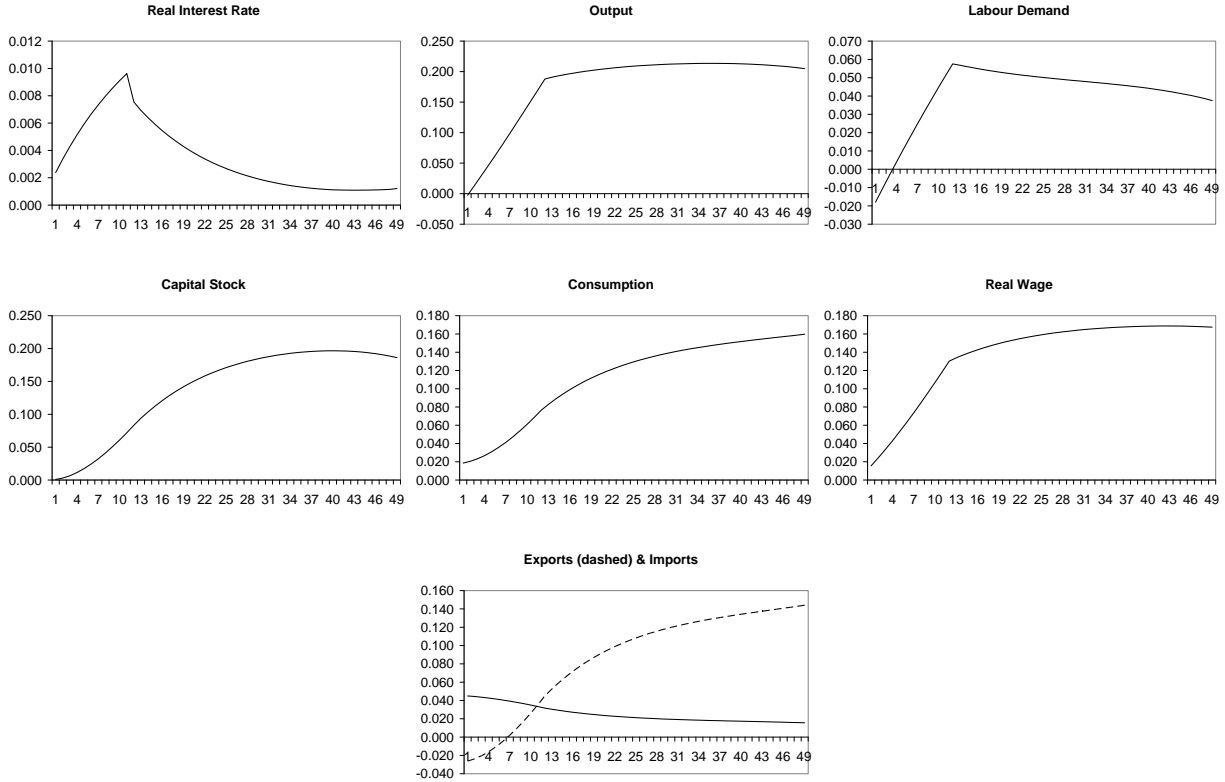


Figure 2: Plots of a 1% Productivity increase each quarter for twelve quarters

4 Data Patterns for Q

In this section we estimate univariate processes for the real exchange rate. The path of Q is presented in Figure 3. The Q data used is the ratio of other OECD to UK consumer prices adjusted for the nominal exchange rate, where the nominal exchange rate is the sterling effective exchange rate. We have used data from 1959 up to 2007.

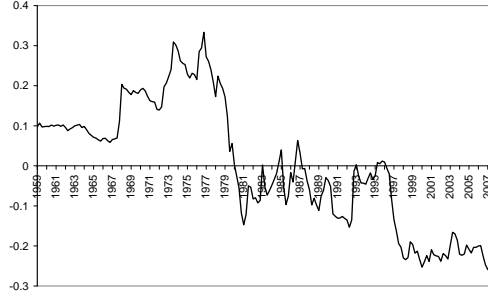


Figure 3: Historical Real Exchange Rate (Q)

Our first observation is that the real exchange rate appears to be non-stationary, as indeed it is found to be in a large body of literature³. Using both the Augmented Dickey Fuller test and Phillips-Perron test, we find that Q is an $I(1)$ series. Table 1 reports the results. The series in levels fails to reject the null hypothesis of non-stationarity at the 1% level of significance, using both the ADF and the PP test statistic. When we test with the first difference form of the series we can easily reject the null, again at 1%.

Unit Root Tests			
	Levels	First Difference	With Trend
ADF Test Statistic	-1.278168 (0.6396)	-11.52616 (0.0000)	-2.702816 (0.2367)
PP Test Statistic	-1.073441 (0.7261)	-11.49443 (0.0000)	-2.530455 (0.3132)

Note: Values in parentheses are p-values

Table 1: Test for Non-Stationarity of the Real Exchange Rate

Having established the non-stationarity of the series we now proceed to estimate the best fitting $ARIMA$ process to the real exchange rate, using data from 1959:1 to 2007:4. Table 2 summarises our results. Clearly the results indicate that Q is a highly persistent series. An $ARIMA(3,1,3)$ describes the data best by quite a large margin as revealed in the full table of results (Table 2)⁴.

5 The model tested against the data by the method of indirect inference

Ultimately we can only settle whether our model is consistent with the facts by asking whether it could have generated the patterns of Q we find in the actual data. Meenagh et al. (2008) explain how our procedure is derived from the method of indirect inference⁵. This method uses an ‘auxiliary model’ to describe the

³See for example Alquist and Chinn (2002).

⁴For the $ARIMA(1,1,2)$ and $ARIMA(3,1,1)$ the estimated MA process was noninvertible so we ignore these cases.

⁵The following is adapted from their explanation. Let $x_t(\theta)$ be an $m \times 1$ vector of simulated time series dependent on the $k \times 1$ parameter vector θ and let y_t be the actual data. We assume that $x_t(\theta)$ is generated from a structural model. We assume that there exists a particular value of θ given by θ_0 such that $\{x_t(\theta_0)\}_{s=1}^S$ and $\{y_t\}_{t=1}^T$ share the same distribution, where $S = cT$ and $c \geq 1$. Thus the null hypothesis is $H_0 : \theta = \theta_0$.

Let the likelihood function defined for $\{y_t\}_{t=1}^T$, which is based on the auxiliary model, be $\mathcal{L}_T(y_t; a)$. The maximum likelihood estimator of a is then

$$a_T = \arg \max \mathcal{L}_T(y_t; a)$$

The corresponding likelihood function based on the simulated data $\{x_t(\theta_0)\}_{s=1}^S$ is $\mathcal{L}_T[x_t(\theta_0); \alpha]$. Let

$$\alpha_S = \arg \max \mathcal{L}_T[x_t(\theta_0); \alpha]$$

Define the continuous $p \times 1$ vector of functions $g(a_T)$ and $g(\alpha_S)$ and let $G_T(a_T) = \frac{1}{T} \sum_{t=1}^T g(a_T)$ and $G_S(\alpha_S) = \frac{1}{S} \sum_{s=1}^S g(\alpha_S)$. We require that $a_T \rightarrow \alpha_S$ in probability and that $G_T(a_T) \rightarrow G_S(\alpha_S)$ in probability for each θ . If $x_t(\theta)$ and y_t are stationary

	C	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)	\bar{R}^2	AIC
ARIMA(1,1,0)	-0.0016	0.179						0.027	-4.319
ARIMA(1,1,1)	-0.0016	-0.099			0.293			0.027	-4.313
ARIMA(1,1,2)	-0.0036	0.896			-0.791	-0.270		0.112	-4.399
ARIMA(1,1,3)	-0.0016	-0.859			1.086	0.114	-0.142	0.040	-4.316
ARIMA(2,1,0)	-0.0016	0.193	-0.075					0.027	-4.309
ARIMA(2,1,1)	-0.0016	0.146	-0.066		0.048			0.022	-4.299
ARIMA(2,1,2)	-0.0015	-1.628	-0.766		1.814	0.970		0.070	-4.344
ARIMA(2,1,3)	-0.0015	-1.674	-0.808		1.925	1.170	0.107	0.072	-4.341
ARIMA(3,1,0)	-0.0016	0.194	-0.075	0.000				0.022	-4.294
ARIMA(3,1,1)	-0.0024	1.152	-0.244	0.049	-1.046			0.104	-4.376
ARIMA(3,1,2)	-0.0015	-1.562	-0.627	0.085	1.815	0.970		0.072	-4.336
ARIMA(3,1,3)	-0.0026	-0.691	0.768	0.726	0.835	-0.827	-0.973	0.085	-4.345

Table 2: ARIMAs for Q

	Coefficient	Std Error
AR(1)	-0.690618	0.055256
AR(2)	0.768467	0.044813
AR(3)	0.726163	0.053759
MA(1)	0.835008	0.018443
MA(2)	-0.827373	0.014870
MA(3)	-0.973378	0.016875
\bar{R}^2	0.085357	
S.E. of regression	0.027070	

Table 3: Best Fitting ARIMA

data—such as our time-series representation here—and estimates the parameters of the structural model of interest as those under which this model can replicate the behaviour of the auxiliary model most accurately according to a criterion of ‘closeness’. The method can also be used to evaluate the closeness of a given model; in effect this arrests the method before estimation proceeds further. This is relevant as here when we are interested in the behaviour of structural models whose structure is rather precisely specified by theory.

What we now do is to fit the model, as calibrated above, to the available data for the UK, and derive from this fit the behaviour of the productivity and preference shocks. These are therefore the shocks implied jointly by the model and the data—under our null hypothesis that the model holds. The shocks constitute the stochastic part of the model; the shocks are a mixture of I(0) (consumer preference, world consumption, capital demand and export/import shocks) and I(1) (productivity, labour supply and labour demand) processes; we estimate them as autoregressive processes respectively in levels and first differences. We then generate the sampling variability within the model by the method of bootstrapping the random components

and ergodic then these hold *a.s.*, see Canova (2005). It then follows that on the null hypothesis, $E[g(a_T) - g(\alpha_S)] = 0$.

Thus, given an auxiliary model and a function of its parameters, we may base our test statistic for evaluating the structural model on the distribution of $g(a_T) - g(\alpha_S)$ using the Wald statistic

$$[g(a_T) - g(\alpha_S)]'W[g(a_T) - g(\alpha_S)]$$

where $W = \Sigma_g^{-1}$ and Σ_g is the covariance matrix of the (quasi) maximum likelihood estimates of $g(\alpha_S)$ which is obtained using a bootstrap simulation. The auxiliary model is a time-series model—here a univariate ARIMA—and the function $g(\cdot)$ consists of the impulse response functions of the ARIMA. In what follows we specialise the function $g(\cdot)$ to (\cdot) ; thus we base the test on a_T and α_S , the ARIMA parameters themselves. Notice that though Q and its bootstrap samples are I(1) processes, in the test they are stationarised through the ARIMA estimation.

Non-rejection of the null hypothesis is taken to indicate that the dynamic behaviour of the structural model is not significantly different from that of the actual data. Rejection is taken to imply that the structural model is incorrectly specified. Comparison of the impulse response functions of the actual and simulated data should then reveal in what respects the structural model differs.

For further details of indirect inference methods in estimation see also see Smith (1993), Gregory and Smith (1991,1993), Gouriouex et al. (1993), Gouriouex and Monfort (1995) and Canova (2005).

of these processes as a vector to preserve any contemporary correlations. This allows us to generate a large number of pseudo-samples of Q . The model itself would also generate a unit root even with stationary shock processes because of the wealth effect on consumption of net foreign assets, which evolve as a random walk (see Minford et al., 2006, appendix for details of this mechanism). Overall therefore the bootstrapped samples are non-stationary, in conformity with the non-stationary behaviour of Q in the data sample. We then run an *ARIMA* for Q on all these pseudo-samples to generate the distribution of the *ARIMA* parameters. In our final step we compare the estimated parameters for Q with this distribution, using a Wald statistic: this tests whether we can reject the RBC model at the 95% level of confidence on the basis of the complete set of *ARIMA* parameters; we would do this if the *ARIMA* parameters lay outside the 95% confidence limits generated by the bootstrap process.

We begin the comparison with the data by looking at the some measures of the model’s overall fit.

5.1 Some measures of overall model behaviour

5.1.1 Variance decomposition

First we looked at the variance decomposition of the key variables and the shocks. We do this on the assumption that they are independent. Note however that when we do the overall bootstrap drawings we draw the shocks by time vectors to allow for any correlations between them. Of course we have no basis on which to allocate variances between two or more perfectly correlated shocks, in the absence of a model for the shocks themselves. Thus our variance decomposition can be thought of as an allocation for that percent of the shock variation that is not accounted for by other shocks.

We look at the variables in levels here because we are interested in how far the different shocks explain the variation over a sample of the size we have here. It is a key part of the argument of this paper that the real exchange rate’s behaviour over a typical sample is greatly influenced by productivity and other supply shocks, which drive Q in a cyclical but also trended way as illustrated above. As can be seen Q ’s variance is contributed mainly by the general productivity shock (43%) and by the external shocks (27%) with 18% coming from labour supply and another 10% coming from the factor demand (labour and capital demand) shocks. Consumption (C) and output (Y) are both determined by productivity and labour supply shocks, while external shocks dominate net exports (NX) and factor demand shocks dominate the real interest rate (r). (Table 4)

		VARIABLES				
		r	Y	C	NX	Q
S	Consumer Preference	4.388	0.525	1.346	2.815	0.940
H	Productivity	6.740	56.080	52.036	23.961	43.443
O	External	3.283	6.920	3.970	57.967	26.883
C	Labour Supply	3.880	25.339	28.110	9.488	18.240
K	Factor Demand	81.710	11.135	14.538	5.769	10.493
S	TOTAL	100	100	100	100	100

Table 4: Variance Decomposition

5.1.2 Model and data cross-correlations

Second, we compared correlations at increasing lags between pairs of variables, using our bootstrap method, in order to give some idea of whether the model is capable of capturing relationships between variables more broadly. The variables were differenced to ensure stationarity and capture business cycle frequency relationships. Thus in Figure 4 we show (thick red line) the correlations in the data and the 95% bounds (thin blue lines) generated by the model; these are calculated from the model-bootstrap distribution of each correlation. It can be seen that the data-based correlations lie fairly consistently within the model’s 95% bounds. In particular the autocorrelation structure of Q and its cross-correlation with output, Y , lie comfortably within the 95% bounds. Net exports (NX), consumption (C) and real interest (r) autocorrelations also lie largely within, as do the cross-correlations between output and real interest rates and between output

and net exports. Some minor discrepancies apart, the model does appear to capture some basic dynamic facts, especially given that it has not been carefully tuned to match the facts generally.

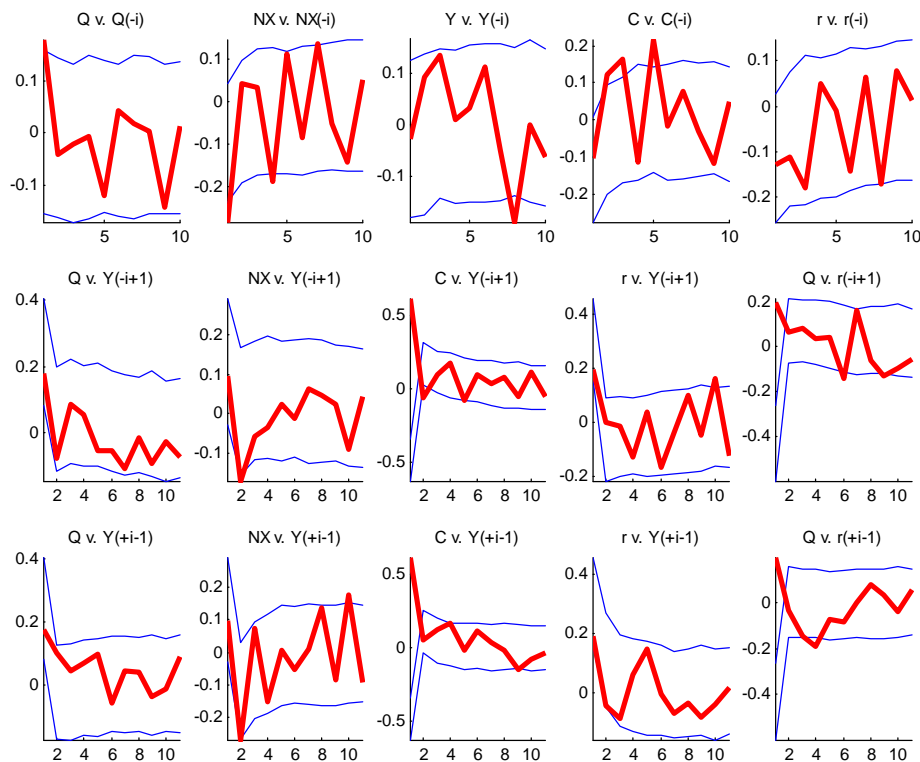


Figure 4: Cross-Correlations

5.1.3 The Q data and bootstrap samples

Third, we look at the bootstrap samples for Q. Our tests are for the model’s dynamic behaviour (or lagged transmission of shocks). However we would also like the model to match the variance of the data for Q as a minimum requirement. Fortunately this is the case. We find that the variance of Q in the data sample is 0.0251 in levels; the 95% bounds for the bootstrap samples are 0.0046 (lower) and 0.0709 (upper) with a mean bootstrap variance of 0.0588.⁶ In differences, the data variance is 0.00079, the lower 95% bound is 0.00047 and the upper 95% bound is 0.00087. Thus the data variance lies inside the 95% bounds. We show in Figure 5 some typical Q bootstraps in levels (with no deterministic trend) against the data to illustrate.

We turn finally to the implications of the bootstrap samples for the model’s fit with the data. We run *ARIMA* regressions on all the pseudo-samples to derive the implied 95% confidence intervals for all the coefficients⁷. Finally we compare the *ARIMA* coefficients estimated from the actual data to see whether they lie within these 95% confidence intervals; the overall test of the model lies in the Wald statistic that is computed for the joint parameter distribution. This statistic is expressed as the percentile of the joint distribution in which the data-estimated parameters fall. A related statistic is the Normalised Mahalanobis

⁶Though Q is nonstationary and therefore has infinite variance in an infinitely long sample, for a finite sample the variance is also finite.

⁷We discarded as uninformative all regressions with *AR* or *MA* roots outside the unit circle, leaving 659 that we used here. An alternative procedure would be to use ML estimation throughout on the *ARIMAs* with the roots constrained inside the unit circle. But this lies beyond our scope here.

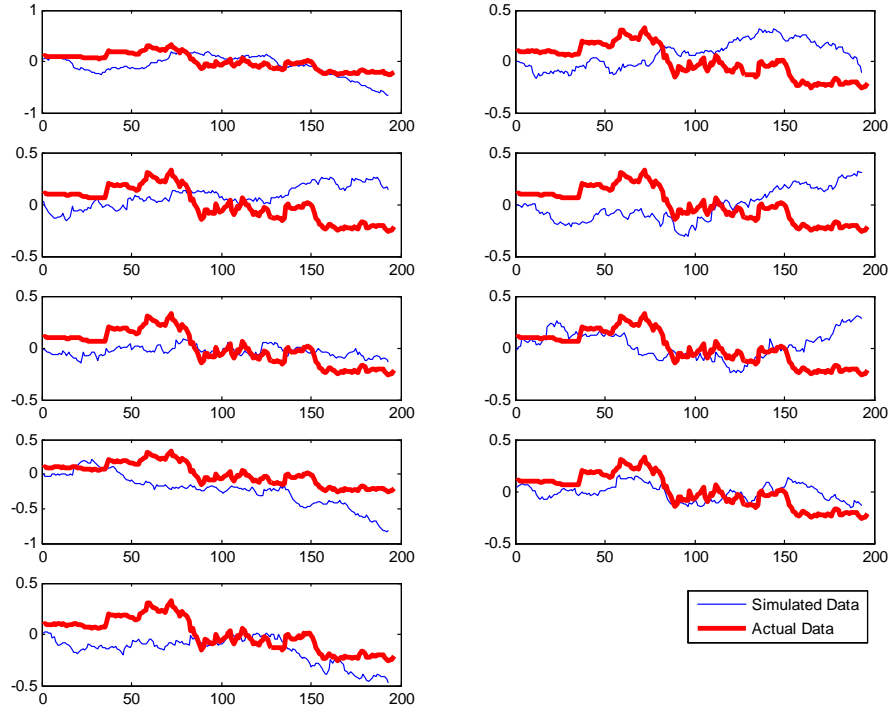


Figure 5: Comparing Historical Q with Model Generated Pseudo Q- levels

distance; this is given by $[a_T - \alpha_S]'W[a_T - \alpha_S]$ where $W = \Sigma_a^{-1}$ and Σ_a is the covariance matrix of the AR and MA parameters α_S derived from the bootstrap distribution and a_T are the data-estimated values. Thus this again measures where in the joint distribution the data-estimated parameters are found; we normalise it as a normal variate whose value is 1.645 at the 95% percentile; a number less than this therefore indicates acceptance of the null.

Table 5 summarises the results of this exercise for the best ARIMA representation of the data (3,1,3).

		Actual	Lower	Upper	Wald	Normalised Mah. Dist
ARIMA(3,1,3)	AR(1)	-0.690618	-0.997065	1.562262	94.2097	1.5370
	AR(2)	0.768467	-1.362501	0.809060		
	AR(3)	0.726163	-0.909547	0.908941		
	MA(1)	0.835008	-1.609619	1.051643		
	MA(2)	-0.827373	-0.870092	1.418858		
	MA(3)	-0.973378	-0.972193	0.972520		

Table 5: Table Caption

What we see here is that the $ARIMA(3, 1, 3)$ that fits the data best lies within the 95% bounds on the Wald statistic, and all but one of the parameters lie inside their individual bounds. The Wald test reflects the covariances between these parameters which influence the joint test but not the individual ones. Hence our null hypothesis that real exchange rate behaviour is consistent with the RBC framework is accepted.

When we look at the implied impulse response of ΔQ to a unit shock in the data-estimated $ARIMA(3, 1, 3)$ (Figure 6), we see that the effect drops off rapidly ending in a small persistent negative; thus we can see

that Q is not greatly different from a random walk in behaviour. The Figure also shows the 95% bounds for the $ARIMA(3,1,3)$ which in line with our Wald test easily encompass the estimated response.

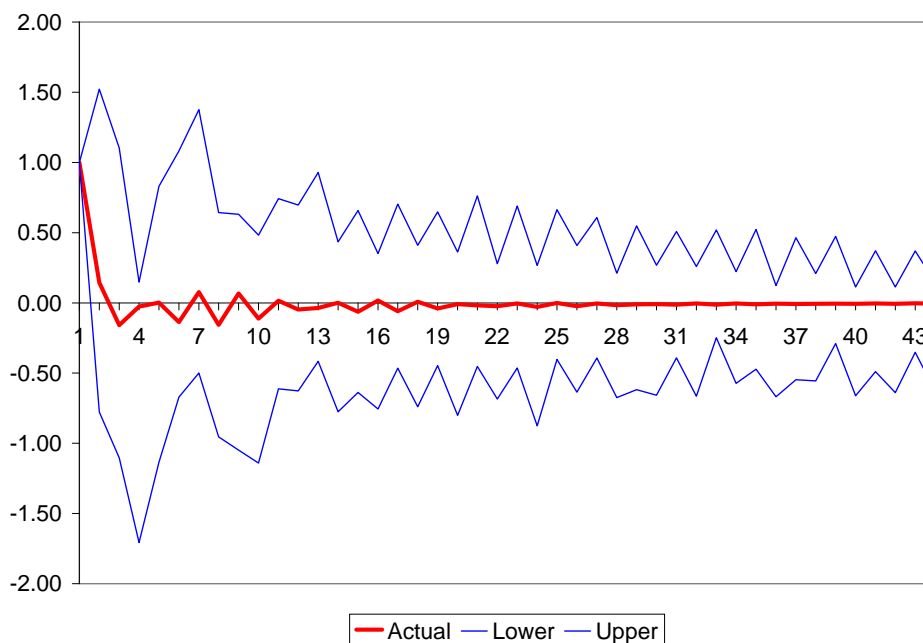


Figure 6: Impulse Response Function of ΔQ and 95% Bounds

5.2 The role of different shocks in the explanation of real exchange rate movements

We have seen above that of all the shocks entering the model, productivity contributes the largest share of the variance of Q (the real exchange rate) according to the model, while the next biggest share comes from labour supply; of the rest most comes from the shocks to factor demand and from external shocks. Consumer preference shocks are of no importance. Plainly in testing the model overall we ask how it can generate Q behaviour in the presence of all shocks. However we can also ask how far this explanation derives from the model's behaviour in response to particular shocks—a decomposition of the model's explanatory success by shock source. Thus in turn we close down all shock sources other than one and repeat our indirect inference exercise with that shock alone. Thus we are asking: 'if this shock alone were perturbing the model would the model's behaviour look like that of the data?'

If we focus on the three main shock sources we can see that neither on its own can account for the data well. If productivity alone is shocked the Wald for the $ARIMA(3, 1, 3)$ is 99.8; if labour supply alone it is 100. If external shocks alone are entered the Wald falls to 95.6, only marginally rejected; with factor demand shocks alone the Wald is 99.0. When all shocks are included, however, the Wald falls to 94.2, an acceptance of the model overall. Notice that the overall Wald is not an average of all the different shocks weighted by their share in the variance decomposition of Q ; this is because the shocks are not independent. As noted above the variance decomposition can only decompose the effects of that part of the shocks that is uncorrelated with others.

Thus while it is tempting to think of this model as driven 'effectively' by productivity, this would be inaccurate. The other shocks are important in explaining Q . There does not appear to be any justification in this model for restricting the menu of shocks among those implied by the model and the data (Table 6).

		Wald	Normalised Mah. Dist
S	Consumer Preference	100.0	5.3
H	Productivity	99.8	16.2
O	External	95.6	1.8
C	Labour Supply	100.0	10.2
K	Factor Demand	99.0	2.6
S	TOTAL	94.2	1.5

Table 6: Individual Shocks

6 Conclusion

This paper establishes the ability of a Real Business Cycle model to account for UK real exchange rate behaviour. The model is tested by the method of indirect inference, bootstrapping the errors to generate 95% confidence limits for a time-series representation of the real exchange rate, as well as for various key data moments. The results suggest RBC models can explain real exchange rate movements.

The model ascribes around two thirds of real exchange rate variation to productivity and labour supply shocks but other shocks are also important contributors. To account for the data satisfactorily a full menu of shocks is required. Thus the work here does not support the idea of restricting the menu of shocks artificially.

The model implies that the response of the real exchange rate to a productivity shock is to depreciate in the long run but in the short run to undershoot this substantially and even to appreciate. This is a weak echo of the large impact appreciation found for the dollar in some studies.

There are limitations to our study that need examining in future work. We have not fully examined the model's ability to replicate the broader behaviour of the economy; it could be for example that adding a degree of nominal rigidity could be useful in that task as suggested in the open economy literature. Furthermore we have addressed the data's non-stationarity by examining changes in the real exchange rate, and not its level: in ongoing work we are examining alternative ways of using the original data. However our concern here has been a preliminary one, given that the level of the exchange rate is notoriously difficult to model: to establish that an RBC model without nominal rigidity cannot be dismissed empirically as a theory of real exchange rate changes.

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7 Appendix: Listing of the RBC Model

Behavioural Equations

(1) Consumption C_t ; solves for r_t :

$$\begin{aligned} (1 + r_t) &= \frac{1}{\beta} E_t \left(\frac{C_t}{C_{t+1}} \right)^{-\rho_0} \left(\frac{\gamma_t}{\gamma_{t+1}} \right) \\ \log(1 + r_t) &= r_t = -\rho_0 (\log C_t - E_t \log C_{t+1}) + \log \gamma_t - E_t \log \gamma_{t+1} + c_0 \end{aligned}$$

Here we use the property that for a lognormal variable x_t , $E_t \log x_{t+1} = \log E_t x_{t+1} - 0.5\sigma_{\log x}^2$. Thus the constant c_0 contains the covariance of $(-\rho_0 \log C_{t+1})$ with $(\log \gamma_{t+1})$.

(2) UIP condition:

$$r_t = r_t^F + E_t \log Q_{t+1} - \log Q_t + c_1$$

where r^F is the foreign real interest rate.

Note that equations (1) and (2) are combined.

(3) Production function Y_t :

$$\begin{aligned} Y_t &= Z_t N_t^\alpha K_t^{1-\alpha} && \text{or} \\ \log Y_t &= \alpha \log N_t + (1 - \alpha) \log K_t + \log Z_t \end{aligned}$$

(4) Demand for labour :

$$\begin{aligned} N_t &= \left(\frac{\alpha Y_t}{w_t(1 + \chi_t)} \right) && \text{or} \\ \log N_t &= c_2 + \log Y_t - \log w_t + \chi_t \end{aligned}$$

(5) Capital :

$$\begin{aligned} \xi(1 + d_{1t})K_t &= \xi K_{t-1} + \xi d_{1t} E_t K_{t+1} + \frac{(1 - \alpha) Y_t}{K_t} - (r_t + \delta + \kappa_t) \text{ or} \\ \log K_t &= c_3 + \zeta_1 \log K_{t-1} + \zeta_2 E_t \log K_{t+1} + (1 - \zeta_1 - \zeta_2) \log Y_t - \zeta_3 r_t - \zeta_3 \kappa_t \end{aligned}$$

(6) The producer wage is derived by equating demand for labour, N_t , to the supply of labour given by the consumer's first order conditions:

$$\begin{aligned} (1 - N_t) &= \left\{ \frac{\theta_0 C_t^{-\rho_0} \left[\exp \left(\log w_t - \left(\frac{1-\omega}{\omega} \right)^\sigma (\log Q_t + \frac{1}{\rho} \log \varsigma_t) \right) \right]}{(1 - \theta_0) \xi_t} \right\}^{\frac{-1}{\rho_2}} && \text{or} \\ \log(1 - N_t) &= -\log N_t = c_4 + \frac{\rho_0}{\rho_2} \log C_t - \frac{1}{\rho_2} \log w_t + \frac{1}{\rho_2} \left(\frac{1 - \omega}{\omega} \right)^\sigma \log Q_t \\ &\quad + \frac{1}{\rho_2} \left(\frac{1 - \omega}{\omega} \right)^\sigma \log \varsigma_t + \frac{1}{\rho_2} \log \xi_t \end{aligned}$$

where Q_t is the real exchange rate, $(1 - \omega)^\sigma$ is the weight of domestic prices in the CPI index.

(7) Imports IM_t :

$$\log IM_t = \sigma \log(1 - \omega) + \log C_t - \sigma \log Q_t - \sigma \log \varsigma_t$$

(8) Exports EX_t :

$$\log EX_t = \sigma^F \log(1 - \omega^F) + \log C_t^F + \sigma^F \log Q_t - \sigma^F \log \varsigma_t^F$$

Budget constraints, market-clearing and transversality conditions:

(9) Market-clearing condition for goods:

$$Y_t = C_t + I_t + G_t + EX_t - IM_t$$

where investment is :

$$I_t = K_t - (1 - \delta)K_{t-1}$$

and we assume the government expenditure share is an exogenous process. Loglinearised using mean GDP shares, this becomes

$$\log Y_t = 0.77 \log C_t + 6.15(\log K_t - \log K_{t-1}) + 0.3 \log G_t + 0.28 \log EX_t - 0.3 \log IM_t$$

(10) Evolution of b_t ; government budget constraint:

$$b_{t+1} = (1 + r_t)b_t + PD_t$$

(11) Dividends are surplus corporate cash flow :

$$\begin{aligned} d_t \bar{S}_t &= Y_t - N_t^s w_t - K_t(r_t + \delta) \\ d_t &= \frac{Y_t - N_t^s w_t - K_t(r_t + \delta)}{\bar{S}_t} \end{aligned}$$

(12) Market-clearing for shares, S_{t+1}^p :

$$S_{t+1}^p = \bar{S}_t .$$

(13) Present value of share :

$$p_t = E_t \sum_{i=1}^{\infty} \frac{d_{t+i}}{(1 + r_t)^i}$$

where d_t (dividend per share), p_t (present value of shares in nominal terms).

(14) Primary deficit PD_t :

$$PD_t = G_t - T_t$$

(15) Tax process T_t designed to ensure convergence of government debt to transversality condition:

$$T_t = T_{t-1} + \gamma^G \frac{(PD_{t-1} + b_t r_t)}{Y_{t-1}}$$

(16) Evolution of foreign bonds b_t^f :

$$\frac{Q_t b_{t+1}^f}{(1 + r_t^f)} = Q_t b_t^f + EX_t - Q_t IM_t$$

(17) Evolution of household net assets A_{t+1} :

$$A_{t+1} = (1 + r_{At})A_t + Y_t - C_t - T_t - I_t$$

where r_{At} is a weighted average of the returns on the different assets.

(18) Household transversality condition as $T \rightarrow \infty$:

$$\Delta\left(\frac{A_T}{Y_T}\right) = 0$$

(19) Government transversality condition $T \rightarrow \infty$:

$$\Delta\left(\frac{b_T}{Y_T}\right) = 0$$

Values of coefficients

Coefficient	Value—Single equation
α	0.70
β	0.97
δ	0.0125
ρ_0	1.20
θ_0	0.50
γ^G	0.05
ρ_2	1.00
ω	0.70
ρ	-0.50
ω^f	0.70
h	0.80
ρ_3	-0.50
σ	1
σ_F	1
Coefficient	Value—Single equation
$\zeta_1, \zeta_2, \zeta_3$	0.5, 0.475, 0.25

Model solution methods

The model is solved in the loglinearised form above using a projection method set out in Minford et al. (1984, 1986); it is of the same type as Fair and Taylor (1983) and has been used constantly in forecasting work, with programme developments designed to ensure that the model solution is not aborted but re-initialised in the face of common traps (such as taking logs of negative numbers); the model is solved by a variety of standard algorithms, and the number of passes or iterations is increased until full convergence is achieved, including expectations equated with forecast values (note that as this model is loglinearised, certainty equivalence holds). Terminal conditions implement the transversality conditions (implying current account balance) at the terminal date. The method of solution involves first creating a base run which for convenience is set exactly equal to the actual data over the sample. The structural residuals of each equation are either backed out from the data and the model when no expectations enter as the values necessary for this exact replication of the data; or, in equations where expectations enter, they are estimated using a robust estimator of the entering expectations as proposed by McCallum (1976) and Wickens (1982), using instrumental variables; here we use as instruments the lagged variables in univariate time-series processes for each expectational variable. The resulting structural residuals are treated as the error processes in the model and together with exogenous variable processes, produce the shocks perturbing the model. For each we estimate a low-order ARIMA process to account for its autoregressive behaviour. The resulting innovations are then bootstrapped by time vector to preserve any correlations between them. Two residuals only are treated as non-stochastic and not bootstrapped: the residual in the goods market-clearing equation (the GDP identity) and that in the uncovered interest parity (UIP) condition. In the GDP identity there must be mis-measurement of the component series: we treat these measurement errors as fixed across shocks to the true variables. In the UIP condition the residual is the risk-premium which under the assumed homoscedasticity of the shocks perturbing the model should be fixed; thus the residuals represent risk-premium variations due to perceived but according to the model non-existent movements in the shock variances. We assume that these misperceptions or mismeasurements of variances by agents are fixed across shocks perturbing the model — since, although these shocks are being generated by the true variances, agents nevertheless ignore this, therefore making these misperceptions orthogonally.

To obtain the bootstraps, shocks are drawn in an overlapping manner by time vector and input into the model base run (including the ARIMA processes for errors and exogenous variables). Thus for period 1, a vector of shocks is drawn and added into the model base run, given its initial lagged values; the model is solved for period 1 (as well as the complete future beyond) and this becomes the lagged variable vector for period 2. Then another vector of shocks is drawn after replacement for period 2 and added into this solution; the model is then solved for period 2 (and beyond) and this in turn becomes the lagged variable vector for period 3. Then the process is repeated for period 3 and following until a bootstrap simulation is created

for a full sample size. Finally to find the bootstrap effect of the shocks the base run is deducted from this simulation. The result is the bootstrap sample created by the model's shocks. We generate some 1500 of such bootstraps.

To generate the model-implied joint and individual distributions of the parameters of the ARIMA estimated on the data, we carry out exactly the same estimation on each bootstrap sample. This gives us 1500 sample estimates which provide the sampling distribution under the null of the model. The sampling distribution for the Wald test statistic, $[a_T - \alpha_S]'W[a_T - \alpha_S]$, is of principal interest. We represent this as the percentile of the distribution where the actual data-generated parameters jointly lie. We also compute the value of the square root of this, the Mahalanobis distance, which is a one-sided normal variate; we reset this so that it has the 95% value of the variate at the same point as the 95th percentile of the bootstrap distribution (which is not necessarily normal). This 'normalised Mahalanobis Distance' we use as a measure of the distance of the model from the data under the bootstrap distribution. Its advantage is that it is a continuous variable representation of the theoretical distribution underlying the bootstrap distribution — which is made finite by the number of bootstraps.