A Parametric Model of Byzantine Church Domes

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***Abstract* - This paper attempts to analyze, through the use of a flexible parametric model, the majority of cases of dome designs of the Byzantine period. Through interactive experimentation and a review of literary sources and case studies, it theorizes on the variety of refinements that were used in order to achieve the uniformly luminous effect present in most Byzantine church domes over a period of a thousand years. The parametric computer model employs a physics simulation engine and animation methods to trace the path of a sunray as it enters and reflects within a Byzantine dome. Through parametric variation and simulation, the method has the potential to produce endless dome designs as well as accurately derive their qualitative and quantitative lighting characteristics.**

***Keywords* – Byzantine domes, lighting simulation, parametric models, natural lighting.**

I. Introduction

In many Byzantine domed churches an interesting phenomenon can be observed. The dome appears to be uniformly illuminated (Fig. 1) even though logically it could not be because the dome windows are at the lower part of the dome and sunlight rays always point downwards. In order to understand this phenomenon, we have attempted several studies and simulations [1]-[5], of which the one appearing in this paper constitutes a new development.

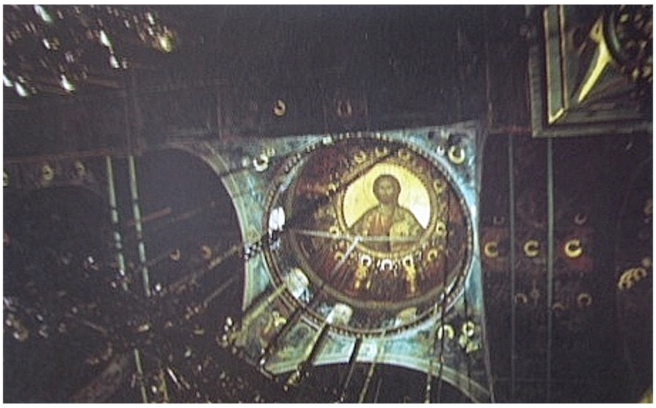

Fig. 1. Dome of the Iveron monastery church, Mount Athos.

II. The original dome of Hagia Sophia

Following an extensive investigation of the original dome of Hagia Sophia of Istanbul, designed by Anthemius of Tralles in 532 A.D., it has been discovered that the dome had been designed as a combination of two curved mirrors (Fig. 2, 3, 4) each one of which employed a different type of curvature [1], [2].

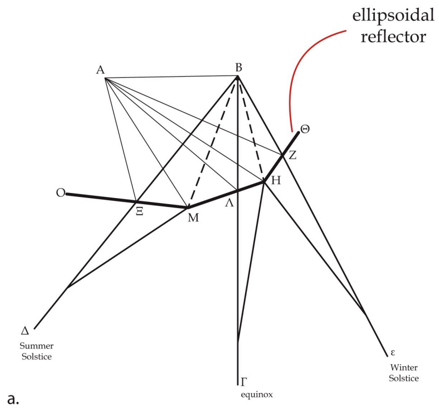

Fig. 2. Anthemius's ellipsoidal reflector.

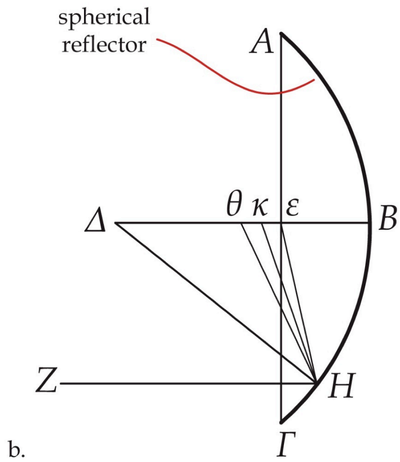

Fig. 3. Anthemius's spherical reflector (i.e. the shallow dome itself).

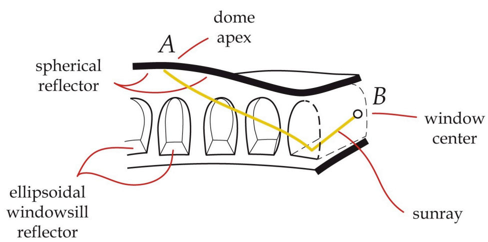

Fig. 4. Sketch showing the location of the reflectors.

The designs of these two mirrors were found in the architect's own extant writings [6]. The hypothesis we have proposed is twofold: first, that Anthemius's design for the mirrors was meant for the dome of Hagia Sophia, and second, that they ought to be combined in a certain way in order to generate the impression described by Procopius, the historian eye-witness of the effect (Fig. 5). This hypothesis can be considered largely proven in terms of geometric, topographic and textual evidence [1]. However, it may still require archaeological evidence produced by future excavations, in order for it to be considered fully proven. Within the philosophical and artistic realm, this discovery provided evidence of the outstanding aesthetic sensibility reached by Byzantine civilization but it also opened up some further research areas.

Fig. 5. Computer model of the original dome with the two mirrors in place.

III. The case of other Byzantine Churches

The most persistent questions posed, whenever these findings had been presented in academic fora, were the following: “Hagia Sophia is a very special, rather unique case. It is the highest and most celebrated monument of the Byzantine world. May the lighting method employed in it be generalized? How would ordinary domes of lesser church examples achieve a similar visual effect? If they did not employ the same method how did they achieve the constantly and uniformly luminous effect?” These questions were reasonable enough and called for a more systematic investigation.

Any attempt to answer these questions tends to become particularly complex. The reason is that one is not investigating a single building, which one can study exhaustively from a number of different aspects. Instead, one has to investigate many different buildings of various eras often designed by unknown architects and built by differently skilled workers. Therefore, in order to tackle the enormity of the problem one needs to carefully plan a strategy.

One way of approaching the problem would be to investigate each Byzantine church dome separately as a case study. This would necessitate the employment of a large team who would survey in detail each church dome, which, in turn, would provide us with detailed data. This data, in order to be reliable for our purpose, should be of a high quality, comparable to the survey provided by the Robert Van Nice team for Hagia Sophia [7]. From this data one would then attempt to derive a prototypical scheme, which might present some common elements with other church domes. These elements would then have to be grouped in various typologies. It is understandable from the outset that this procedure would be very lengthy and costly and would require not only great precision of measurement but also a profound understanding of what exactly one is measuring by all surveying teams involved. One cannot rely on ordinary mechanical measurements. Instead, these measurements are particularly delicate and necessitate special survey equipment, which is hard and costly to acquire. Moreover, the individuals needed to conduct these surveys should be specialized, have a very good understanding of solid geometry and catoptrics, as well as of the specific problem to be resolved. The satisfaction of all these conditions would necessitate extensive funding.

IV. Parametric Modeling

A different way of approaching the problem would be through parametric modeling. Such a model could be based on the theoretical understanding of the way in which light moves and the basic form elements and their arrangement in a church dome. In contrast to the afore-mentioned procedures, such a model would facilitate experimentation with a large number of theoretical test cases. This model would have to be constructed as to be able to incorporate all the different variables that could possibly influence the luminous dome effect. By changing some of the variables that are basic and easy to measure one could discover what other variables should be changed in order to receive the luminous dome effect. Essentially, one would have to pose the problem as follows: “What do domes usually look like and what does it take to make them luminous?” assuming the stance, as it were, of the designer.

Following the construction of the parametric model one could then resort to surveys of existing domes. In typical surveys of Byzantine churches, published in various specialized articles, books, and dissertations some basic, easily measurable variables are already available. Thus, one would have a significant amount of elementary measurements, (i.e. dome height, drum height, number and dimensions of dome windows) to begin with, before even considering travelling to the actual monument site. One would then have to observe whether the dome of the monument is luminous, using existing photographs, and would manipulate the subtler variables of the parametric model in order to achieve the effect. Once the luminous effect had been achieved in the model one would have already a fair estimate of the variables that affect this visual impression. As a result, only those variables of the actual form would have to be measured that made sense in terms of the model. Strategically, this would be the most cost effective procedure. If, against all precautions, one fails to verify the manner in which the dome was actually designed then one would have to rethink the parametric model.

A. *Geometric principles of the parametric model*

Having thoroughly introduced the topic and the reasons for undertaking such a study the fundamental geometric ideas on which the model was built can now be explained.

A real dome is not a simple construct, as one might think, imagining it in abstract geometrical terms. Especially, the Byzantine church domes vary considerably. From the point of view of solid geometry, they are made up of various parts. Often, there is an upright standing tube-like part, which may be cylindrical or polygonal, called the “drum”. On top of it a roughly semispherical dome is placed. On the drum several openings are pierced which differ in number. They are six or eight or twelve but they may go up to as many as forty. What we outlined above is an elementary overall geometry. However, the closer one looks at that geometry the more variations one discovers to this basic scheme.

Below, certain general observations and principles are outlined which lie behind the construction of the model. First, the drum is not always of the same height and its proportions are related to the scale of the dome ensemble (i.e., the diameter of the dome in plan) as well as to the total height. The general proportioning rule is that a greater dome radius corresponds to a lower height of drum. This rule (which is not absolute) has not yet been established as a strict mathematical proportional relationship.

In some cases, the drum disappears completely. This is the case, for instance, of the dome of the church of Hagia Sophia in Istanbul (Fig. 5), while in the church of Saints Sergius & Bacchus – which served as a model for the design of the former - the drum is quite low. In the church of Hagia Eirini, of the same city, while a substantial drum is visible from the outside, in the interior the very smooth curvature of the dome blends in with the drum and makes it invisible as if it were part of it. This difference is visible in most churches of later periods as well. While the drum is clearly articulated as a form from the exterior, it looses all articulation and blends in with the curvature of the dome as it is seen from the inside. Architecturally, the exterior and the interior seem as belonging to two different aesthetic approaches. This phenomenon requires a special investigation, which, however, is not the focus of this paper. What we might retain from this observation for now is that for some reason this sense of transition from one geometric form to another, so that the composite form appears unified, as seen from the inside, constituted an essential goal.

Second, the windows piercing the drum depend on the drum's height. The taller the drum the greater the height of the windows and the slenderer their proportions become. However, as we have already mentioned, the drum in some cases may not exist at all. In these cases, the window height acquires a proportional relationship to the circular segment of the dome. Therefore, in the parametric model the possibility of inserting windows on the dome curvature should be incorporated.

Third, it has been noticed that the dome may not be necessarily semispherical. What has been observed in Hagia Sophia and other churches is that other types of curvatures may have also been employed. It appears that in Hagia Sophia a very shallow curvature was employed which was so distinguishable that it acquired a special name in Greek language; it was called "a shield on plumes" in an attempt to express the shallowness of its curvature ("saucer dome" in English). Such shallow curvatures have been observed in various other churches as well, e.g. in Hagia Sophia of Thessaloniki. Moreover, similar shallow curvatures were not limited to the dome but were employed in vertical wall surfaces as well. Such a subtly curved vertical surface has been observed above the main gate of the church of Hagia Sophia, which seems to be responsible for the concentration of light so that the image of Christ present on that surface becomes suddenly lit at a certain distance [8]. Shallow curvatures, in general, seem to be connected to the impression of a uniformly lit dome. In fact, the way in which this is achieved geometrically is explained in one of the problems examined in Anthemius's extant writings found in the *Fragmentum Mathematicum Bobiense*. What Anthemius tells us, expressed of course in terms of solid geometry and catoptrics, is that a shallow curvature, as long as it is made of reflective material, has the ability to trap light and distribute it uniformly onto its surface. This particular characteristic would render the shallow curvature invaluable to a dome designer. Therefore, our parametric model should have the capability of incorporating a shallow curvature within the dome.

Fourth, it is understandable that a ray of the sun passing through a dome window cannot go upwards toward the dome unless it is somehow reflected. The reason for that is that the sun as soon as it rises starts sending its rays downward toward the earth. Therefore, the rays may not travel directly into the dome since the dome windows lie at the lower part of the dome. The sunrays may reach the interior surface of the dome only if reflected on a surface at the lower part of the dome window. Anthemius especially designed such a surface for this position and it worked extremely well as it had been proven in an older simulation [4]. However, this design, which was an ellipsoidal reflector placed on the dome windowsill produced a surface that sloped inward. For the case of Hagia Sophia, which was a dome without a drum, this windowsill slope was a reasonable solution. For a dome with a drum, however, the same principle would produce a slightly curved but outward sloping windowsill. We have found evidence of such treatment on the dome windowsills of the Monastery of Iveron church, which has a particularly bright and uniformly lit dome (Fig. 1). Therefore, our model should have the capability of manipulating the windowsills by changing their slope and curvature.

Fifth, if not planned carefully, the reflected ray, once it reached the dome, would most probably hit it once and then is reflected downward to the floor. The point on which it hit would vary depending on the position of the sun in the sky. This means that a luminous point in the dome would appear at different points while the rest of the dome would remain dim. The only way to create a uniformly and constantly luminous dome would be to trap the light in it. This can happen if two conditions are met. First, as already mentioned, one can employ a shallow reflective surface for the interior of the dome itself. Second, one can design a windowsill with a slope so as to send the rays into the dome such that further reflections continue to remain entrapped within the dome. Through these observations and syllogisms, we have reached a point to expect that most ordinary churches may incorporate not one but two curvatures within the dome. The shallowness of the dome may vary depending on the geometry of the windows and the height of the drum. The upper curvature of the dome may have to be deeper when a drum exists and be proportional to its height.

B. Computer Implementation

In order to investigate these hypotheses, we created a computer script within the Autodesk 3ds Max 3D modeling and animation software that generates a three-dimensional solid model of a dome, drum and windows using multiple input parameters (Fig. 6).

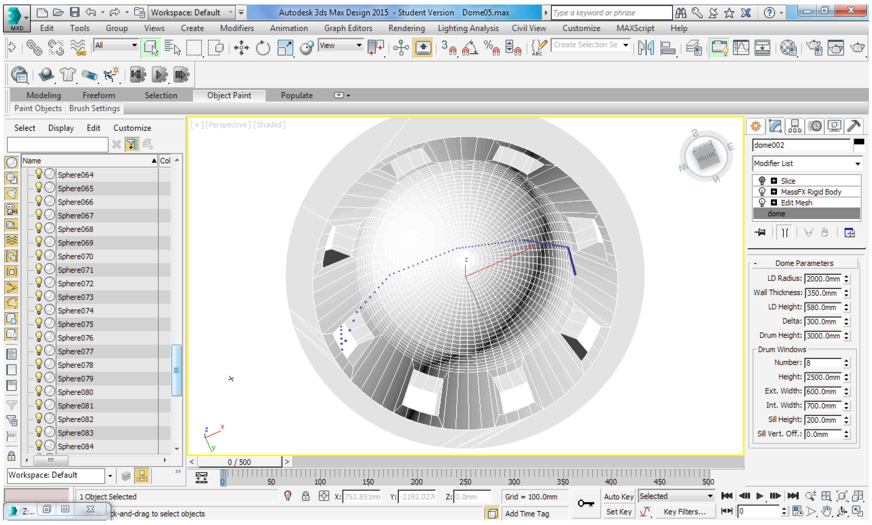

Fig. 6. Screen capture of the software environment and the input variables on the right-hand side.

Once the dome has been created, we used the software's ability to simulate the physics of collision and reflection to visualize and animate a representation of the sun's rays as they hit the windowsill and reflect multiple times inside the dome. By varying the input parameters and re-running the physics simulation, we were able to derive the appropriate parameters that can achieve the desired luminous affect through the entrapment of light within the dome.

The parametric model presented here has been constructed with the capability of representing two different dome curvatures above the drum. The dome can acquire any radius and wall thickness. The lower and upper curvatures of the dome can vary in terms of their vertical height; thus creating different composite curvatures while maintaining a consistent geometric relationship between them. Any number of windows, of any width and height, can be incorporated into the drum or into the lower curvature of the dome. The windowsill can be made to slope inward or outward. It was thought that the most visually effective method to present the idea of the multiple reflections within the dome was to show a small sphere that represents and follows the direction of a sunray. The sphere impacts the windowsill and reflects into the dome area immediately above the window through which it entered and starts travelling along the dome surface. Here, two cases are presented one with a high and one with a low drum.

The case with the double curvature and a high drum has taller drum windows. The windowsill slopes outward fairly steeply (Fig. 7, 8).

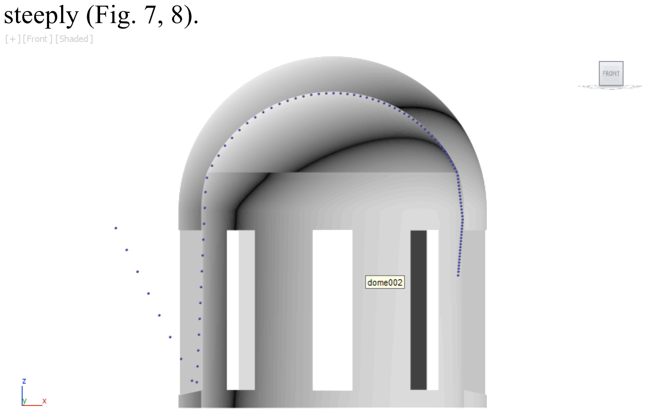

Fig. 7. Cross-section showing the path of the light through a double curvature dome with a tall drum.

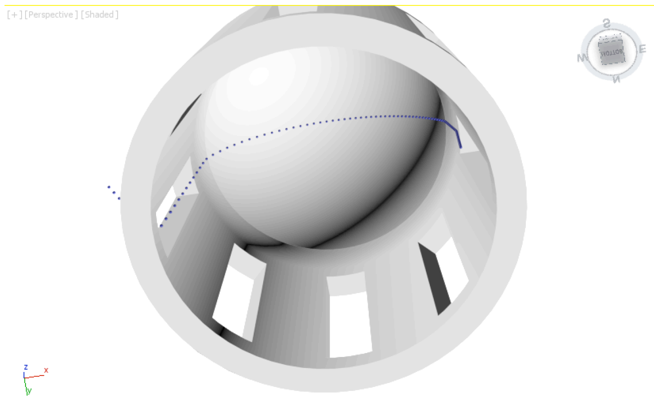

Fig. 8. View from below showing the path of the light through a double curvature dome with a tall drum.

It can be seen in this image that the sphere is reflected in such a way that it washes the entire dome surface. This light behavior guarantees the uniform illumination of the entire dome.

The second case is of a dome with a shorter drum. Here two sub-cases are presented: One with a deeper upper dome (Fig. 9, 10) and another with a shallower upper dome (Fig. 11). Our parametric experimentations have illustrated that the shallower upper dome generates a greater number of reflections that more closely fit the dome's overall curvature.

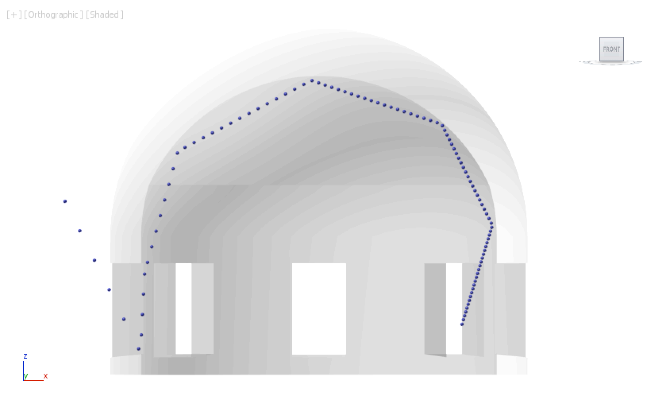

Fig. 9. Cross-section showing the path of the light through a double curvature dome with a deeper upper dome and a short drum.

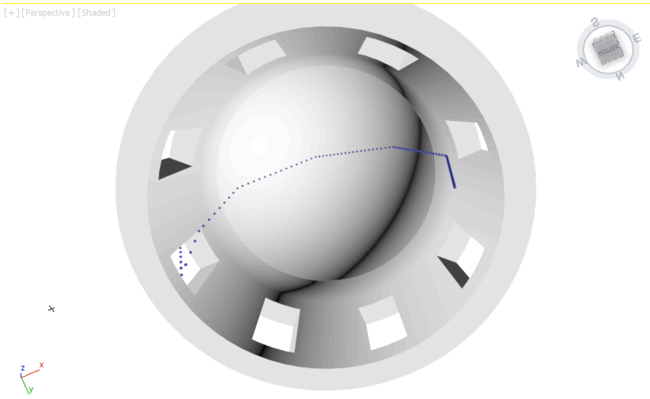

Fig. 10. View from below showing the path of the light through a double curvature dome with a deeper upper dome and a short drum.

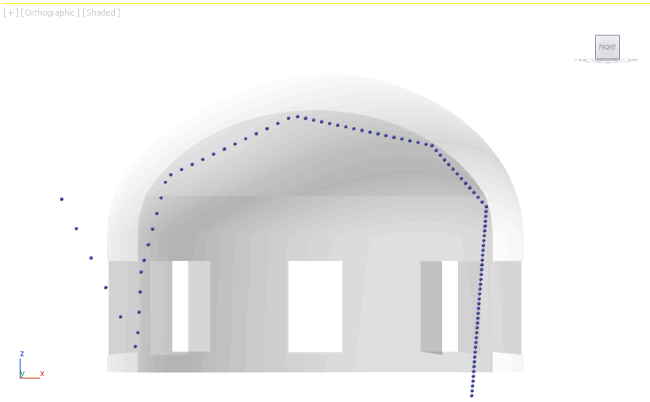

Fig. 11. Cross-section showing the path of the light through a double curvature dome with a shallower upper dome and a short drum.

V. CONCLUSIONS AND FUTURE WORK

Byzantine church domes are paradoxically brighter than all surrounding surfaces. This phenomenon has been investigated in previous work for the case of the dome of Hagia Sophia of Istanbul. The lighting of this dome had been resolved by the combination of two reflectors designed by Anthemius. In order to understand the dome design methods employed in lesser church examples where a luminous dome effect is achieved a parametric model was constructed which gave us the opportunity to experiment with a number of parameters influencing the behavior of light within the dome before we apply it to any particular church case. It has been found that several of the parameters are interrelated as for instance, the height of the drum, the slope of the windowsill and the height of the two curvatures of the dome. It is probable that there is a precise mathematical relationship governing these factors. This is an area to be investigated in future work. Additional areas of future work will be the study of particular church cases through the model as well as the possibilities of engagement of this tool into the study of new building or product designs.

VI. REFERENCES

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VII. CURRICULUM VITAE

Iakovos Potamianos is a Professor in the Faculty of Fine Arts, School of Drama, Aristotle University of Thessaloniki, Greece. He received a Bachelor's degree in Architecture from the Aristotle University of Thessaloniki, a Master of Architecture degree from California Polytechnic State University in San Luis Obispo, and a Ph.D. in Architecture from the University of Michigan. He is a registered architect in Greece and the U.S.A. He has taught in three American and three Greek Universities. Currently he teaches at the School of Drama and the Department of Architecture as well as in two interdisciplinary postgraduate programs at the A.U.Th. He also serves as Director of the Post Graduate Program in Lighting Design of the Hellenic Open University. His work focuses on architectural design, design-teaching systems, issues of space and form and relations to philosophy, perception, and poetics with a major interest in the lighting methods employed in Byzantine churches.

Wassim Jabi is a Senior Lecturer (Associate Professor) at the Welsh School of Architecture, Cardiff University, United Kingdom. He received his Bachelor of Architecture from the American University of Beirut and his Masters and Ph.D. in Architecture from the University of Michigan, Ann Arbor, USA. Dr. Jabi has taught at three U.S. universities and has published widely on the topic of parametric design, digital fabrication, and digital lighting simulation in architecture. He is the author of the book *Parametric Design for Architecture*, Laurence King Publishing, London, 2013.

Dr. Jabi has served as president and member of the Board of Directors of the Association for Computer-Aided Design in Architecture (ACADIA) and is currently a member of the editorial board of the International Journal of Architectural Computing (IJAC).