Business Cycles, Velocity and Asset Prices in a Wicksellian Banking Time Economy

by

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A Thesis in Fulfilment of the Requirements for the Degree of
Doctor of Philosophy

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Abstract

This thesis collects three interrelated pieces of theoretical work, which are connected to each other in the sense of being rooted in an analysis and examination of a specific type of monetary general equilibrium model which is of a cash-in-advance *nature*. All of the three contributions extend the usual *quantity-theoretic* cash-in-advance (CIA) constraint to a more general *exchange* constraint, in that they include the possibility of allowing the representative household to pay for the consumption good using a (self-)produced alternative means-of-exchange, *costly credit*.

The first two pieces extend the cash costly-credit *production*-based monetary RBC framework to include habit persistence in consumption and adjustment costs to investment. Most of the stochastic dynamic analysis emphasizes the role of a goods-sector productivity structural shock *only*, so the thesis focuses on telling a story of a "Wicksellian" Banking time economy, in which it is predominantly this shock *alone* - and it's effect on the Wicksellian equilibrium real rate of interest - that is driving both real and nominal variables in the economy. The growth rate of money is assumed to be of deterministic "k-percent" Friedman-type nature, so as to allow a more focused analysis of the endogenous variation in the *demand* for money.

While the first piece discusses how the chosen modeling framework can successfully account for some factually observed measures related to consumption-money velocity, the second piece uses a similar model setup, but instead discusses the conditional behaviour of key real and nominal variables over the business cycle. Money balances, real quantities, real and nominal rates, as well as (expected) inflation rate series move conditionally over a Solow residual-driven business cycle, so as to closely mimick some of the salient features of a stylized monetary business cycle. Noteably, the additional introduction of credit production shocks allow the artifical economy to closely mimick the breakdown of a stable money demand relationship which is such a pertinent feature of the U.S. monetary business cycle in post-1980 data.

Finally, the third piece deviates marginally from the first two contributions in that it constitutes a discussion of a labour-only economy. Here, credit production is de-centralized and produced subject to a debt- (or collateral-) requirement. Specifically, the decentralized financial intermediary is assumed to retain an amount of short-term government debt equal in value to credit on it's balances sheet, which it eventually pays out as a dividend to the household, reimbursing the latter for the cost of credit.

The money market rate (obtained on a one-period saving deposit) is generally lower than the usual intertemporal risk-free rate, where the difference is always equal to the banking wage bill, which varies endogenously over the business cycle. This model setup and the implied banking wage tax levied on short-term saving deposits can help to explain some of the unconditional as well as conditional behaviour of the low risk-free, the equity premium, and the unconditional shape of the term structure of interest rates.

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Dedication

No man is an island.
Seneca

Before embarking on the journey, thinking about pursuing graduate studies at PhD level at first glance feels equivalent to departing on a potentially marvelous journey of curiosity-quenching and illumination-seeking activities. Thus the first years of coursework are spent in anticipation and with the incubation of - perhaps at times slightly lofty - ever greater growing expectations. But like in Hemingway's the *Old Man and the Sea* you quickly begin to appreciate the caprices and vagaries associated with this undertaking with all its setbacks, slow periods of aimless and despairing drifting, but also the tormenting periods in which progress can't be fast enough and the surfaces begin to ripple in agitation - not to mention the times of self-imposed isolation the researcher eventually forces himself to go through to reach some sort of a finish line whose precise definition remains constantly unclear to all parties to the process - or to help myself to the Hemingway analogy again - to calm oneself and one's surroundings in hope to catch that one big fish.

Doubtlessly, distractions have played their role too, some pleasant and thought-provoking and then again others of less inspiring nature. But that is judgment grounded in uncertain foresight and immature hindsight. The true significance of any experience, I suspect, will only reveal itself as the past recedes, old skins are shed and a new challenges present themselves. At the time of this writing we have by all appearances emerged from what initially looked like a second Great Depression and the traumatic jolts of this experience seem to have passed as normality sets in - but what normality? It has indeed been an exciting *finale* to the completion of a PhD degree in Macroeconomics & Finance and the *Great Recession* of 2007 is sure to influence the profession for a long time to come. Soul-searching and analyses of this - by any statistical means - highly significant event are ongoing and conclusions and policy-making recommendations are only slowly beginning to emerge.

I by no means am tempted to declare myself an old man at the end of all of this, but I certainly would hope to have grown wiser as a result of going through with it all in spite of it all and in spite of my now former past self. I would like to thank my family which has provided me with an incredible amount of support throughout this exciting but also difficult period of my

scholastic life. I would also like to thank my supervisor, Prof. Max Gillman, who has spared a considerable amount of his precious time and has always be fair and persevering with me. I wish him all the best for his future endeavors and hope to be able to pay back some of the endowments invested into me by acting in accordance with the *Benthamian* principle of the maximisation of social welfare and so make my contribution to this - with as few negative externalities as possible - felt as well, in whatever ways I cannot yet possibly foresee. I would also extend my gratitude to Prof. Patrick Minford under whose tutelage I learned the basics of macroeconomics and who has inspired me tremendously to go on and on and keep on living out my inquisitive nature uninhibitedly. My thanks must also go to Prof. Kent Matthews under whose role as head of department I have been given the pleasure of gaining my first experience in self-organized teaching, an experience which I hope I will be able to build on during the my future career.

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Chapter 1

Introduction

"This tension between two incompatible ideas: that changes in money are neutral units changes, and that they *induce* movements in employment and production in the same direction, has been at the centre of monetary theory at least since Hume wrote". (Robert Lucas, 1995 Nobel lecture, emphasis added).

1.1 Introductory Comments

The present thesis comprises an analysis of a set of issues spelled out using the theoretical framework of general equilibrium monetary macroeconomics in a closed economy with a particular focus on explicitly modeling the market for liquidity using a Clower-Lucas (see Clower, 1967; Lucas, 1982) cash-in-advance-type (CIA) constraint. The title of this dissertation ought to be of informative nature regarding it's content and thus suggests the following individual sub-themes which will be emphasised throughout, but also dealt with individually in the chapters containing the main results of this thesis:

1. Quantity-Theoretic Monetary General Equilibrium Modeling
2. The Velocity of Money
3. Monetary Business Cycles
4. Asset Pricing in General Equilibrium

Based on the preceding list, the following succinct introduction shall serve the purpose of reviewing or introducing some *basic ideas*[1] with regards to the items mentioned, and relate them to the novel ideas which are to follow in the largely self-contained paper-style main chapters. In particular, this introduction should also convey clearly some overarching themes or common building blocks which are employed throughout and which link individual chapters together so as to result in a, so I hope, cohesive piece of work providing new insights which are derived from and thus well-connected to the existing body of literature relevant to each of the individual sub-themes.

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[1] Given the paper-style format of the main chapters - which already include literature reviews as part of their introductions - I have decided to collect most of the current and very up-to-date subject-relevant contributions in the *Conclusion*, as that last section also serves the purpose of discussing possible directions of future research.

Before looking at the individual sub-themes and how they are interconnected in the present thesis, I will first turn to the common building block which is emphasised and employed throughout and related to the previously mentioned modeling of money demand in general equilibrium, based on a cash-in-advance type explicit modeling of the market for liquidity (demand). The standard specification of the Clower-Lucas cash-in-advance constraint is typically modeled in terms of an inequality as part of an intertemporal constrained maximisation problem - given by:

$$M_t \geq P_t c_t \tag{1.1}$$

where the evolution of (or the supply of) nominal money balances M_t is then typically specified along the lines of some exogenously determined stochastic law of motion based on some *predetermined* beginning-of-period level of money balances, fully determined at $t-1$, *plus* some lump-sum tax (or transfer) governing the growth rate of money balances into the end-of-period, given by t. The idea is very simple and intuitive: prior to setting off to purchase the consumption good, the representative household is required to hold sufficient cash balances *in advance*[2] so as to be able to carry out the purchases, which is a realistic assumption rooted in the observation that money - in spite of it's return-dominated nature vis-a-vis other assets - still appears to be valued within an intertemporal equilibrium framework and is therefore held for the liquidity services it provides in it's role as a perfect and highly liquid means-of-exchange.

The imposition of a monetary sector of this kind onto the canonical real business cycle model (see Kydland and Prescott, 1982; Hansen, 1985), leads to monetary frictions affecting *real variables* in form of the inflation-tax distorting the marginal

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[2] instead of assuming cash-in-advance (CIA), Carlstrom and Fuerst (2001) explore the alternative assumption of cash-when-i'm-done (CWID).

rate of substitution between consumption and leisure (see Cooley and Hansen, 1989; Walsh, 2003), which are however quantitatively small when compared to the effects on real variables stemming from the usual persistent shocks embodied in the exogenously modeled law of motion of the Solow residual, which is traditionally the main exogenous driving process in the RBC literature. This particular property has been established in quantitative simulation studies of production-based CIA monetary real business cycle models in Cooley and Hansen (1995).

Money in this framework is both part of net wealth, as it appears in the representative household's budget constraint, but also valued[3] for it's role as a means-of-exchange providing the required liquidity services to purchase *real* consumption (for which *real* money balances are required to be held in Leontief fashion), as it appears in the household's cash-in-advance, or more generally, liquidity or exchange constraint. Typically then, for such a particular setup, the first-order condition with respect to consumption, money, bonds, and leisure result in the following characteristic conditions of optimality (see also Walsh, 2003, ch.3):

$$
\begin{aligned}
U_c' &= \lambda_t + \mu_t; \\
(1+i_t) &= E_t\left[(1+r_t)(1+\pi_{t+1})\right]; \\
\frac{U_c'}{U_l'} &= \frac{1+i_{t-1}}{w_t}; \\
i_{t-1} &= \frac{\mu_t}{\lambda_t};
\end{aligned}
$$

where λ_t and μ_t are the marginal valuation (or shadow prices) in terms of (marginal) utility of one extra unit of (real) wealth[4] and one extra unit of the (real) liquidity service, respectively, U_c' and U_l' are the marginal utility of consumption and leisure

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[3] The value of money is equal to $1/P$, where P is the money price of the consumption good.

[4] provided that the budget constraint has been spelled out in *real* terms, by dividing through by P_t. Otherwise λ_t would be the shadow price of one extra dollar.

and r_t, π_t and i_{t-1} are the current-period net real, inflation and the beginning-of-period net nominal rate, respectively. This set of equations therefore illustrates some of the characteristic features of a canonical CIA monetary real business cycle model, which can be summarised as follows:

1. The marginal value of consumption is equated to the marginal value of wealth *and* the marginal value of liquidity.
2. The Fisher equation generally holds, thus the nominal rate equals the real rate times future expected inflation.
3. At the margin, the consumption-leisure trade-off is equal to the relative price of the two, where the monetary friction (or tax) enters via the opportunity cost of holding money to purchase the consumption good.

This framework and it's subsequent extensions[5], which has enjoyed some degree of popularity as a way of modeling the demand for money within a general equilibrium framework during the 1980s and early 1990s, more recently appears to have been dropped in favour of alternative ways of modeling the demand for money in general equilibrium, such as shopping-time models (see Brock, 1974; McCallum and Goodfriend, 1987), in which money and shopping time are functionally combined to produce the transaction services required to purchase consumption, or more direct Baumolean (see Baumol, 1952) resource cost-based transaction cost specifications, such as in Marshall (1992) or Bansal and Coleman (1996)[6]. In particular, it could be argued that the conclusions drawn from a carefully conducted simulation study

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[5] Of which the most notable one is the Prescott-Lucas-Stokey (see Prescott, 1987; Stokey and Lucas, 1987) extension to define preferences over a cash-credit choice, so as to possibly permit a more realistic modeling of consumption velocity.

[6] Most approaches typically rely on the introduction of a monetary *friction* of some kind. References to the more recent search-theoretic models of money demand, such as in Wang and Shougong (2006), can be found in the conclusion of this thesis.

of the seminal preference-based cash-credit model (see Hodrick et al., 1991) and it's failure of being able to model the second moments of both consumption velocity *and* real and nominal rates arguably may have persuaded many to consider alternative formulations of money demand, resulting in a disfavouring of the cash-in-advance approach.

However, the disadvantage which may arise from modeling money using the aforementioned alternative approaches to the seminal CIA approach, is the more or less ad-hoc way in which functional forms of shopping-time or transaction cost functions can be calibrated such as to make a particular model "fit the facts" better. The matter of fact is simply that one cannot hope to find realistic real-world counterparts to theoretical concepts - such as a "transaction cost function" or a "shopping-time specification" - so as to discipline or restrict oneself to a range of calibrated parameters, based on empirical micro (-panel) studies, relevant for the parametrisation of such money demand functions. But it is arguably exactly this feature which has made micro-founded macro-modeling such an attractive proposition, as it provides a disciplining "corset" in terms of the parameter values one can plausibly use in calibrating deep-structure parameters reasonably, leading to a more rigorous and unified development of the subject[7].

It should also be noted at this point, that the rise in popularity of the so-called new neoclassical synthesis (NNS) class of models (see inter alia Yun, 1996; McCallum and Nelson, 1999; Lawrence J. Christiano and Evans, 2005; Canzoneri et al., 2007; Smets and Wouters, 2007), which emphasises in particular the existence of nominal rigidities in the goods and labour markets, modeled theoretically using so-called Calvo contracts (see Calvo, 1983), has led to a general de-emphasis of the explicit

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[7] A good example for this "discipline" in calibration, is given by the parameter of relative risk aversion in iso-elastic utility functions. Evidence on intertemporal gambles clearly places values of this beyond, say, 3 into the realm of implausibility.

modeling of money supply and demand quantities within a quantity-theoretic framework in such models, and instead has tended to focus on a Taylor (Nominal Interest Rate) Rule (see Taylor, 1993) specification describing the operational conduct of monetary policy, in place of ascribing this role to the supply of some monetary aggregate. A sluggish inflation process obtained from a New-Keynesian Philips curve with both forward-looking and backward-looking[8] inflation components and direct (but state-contingent) control over the nominal rate in such models means that the real rate can be manipulated in some state-contingent fashion such as to affect the intertemporal consumption decision of a representative household embodied by the consumption Euler equation (or the intertemporal IS-curve, as some refer to it nostalgically). Often, such models are therefore either derived cashlessly, or using some short-cut money-in-the-utility-function (MIUF) specification (see Sidrauski, 1967), given the exogenous specification of the Taylor Rule, simply imply a corresponding process for the supply of money *residually*.

1.1.1 A CIA generalisation: Endogenous costly credit-money switching

The present study adopts the cash-in-advance monetary friction assumption throughout, but generalises it to take on a broader view based on using a *liquidity* or *exchange* constraint, in which money may not be the only means of exchange available to carry out consumption. Specifically, I adopt the assumption that consumption can be purchased using either money balances or a costly produced credit service (such as an American Express Card), where the latter is either self-produced in a credit Yeoman-economy (see Gillman, 1993; Benk et al., 2005; Gillman and Ke-

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[8] this latter backward-looking inflation component stems from monopolistic intermediate goods firms who cannot change their price, instead index their prices to past inflation.

jak, 2007), or provided and paid for in a decentralised credit-banking economy (see Gillman and Kejak, 2008). This approach, which is motivated by the financial intermediation literature (see Sealey and Lindley, 1977; Clark, 1984; Hancock, 1985), gives rise to an alternative liquidity constraint, given by:

$$M_t + P_t f_t \geq P_t c_t \tag{1.2}$$

which, upon normalising by the (relative) money price of the consumption good P_t, and expressing current money balances as predetermined money plus a net transfer, can also be written as:

$$\frac{m_{t-1}}{1+\pi_t} + v_t + f_t \geq c_t \tag{1.3}$$

where $m_{t-1} = M_{t-1}/P_{t-1}$ and v_t is the real transfer (or lumps-sum tax) governing the growth rate of the stock of money. The key building block to modeling the liquidity market that way is the specification of the credit production function, which is of constant returns-to-scale type in labour and deposits[9] made with a financial intermediary, where total deposits are equal to total liquidity, which in turn equals the level of consumption, i.e.

$$d_t = m_t + f_t = c_t \tag{1.4}$$

where d_t are the level of total real deposits held by the financial intermediary producing the credit service. Given the aforementioned set of assumptions, the credit

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[9] This specification can, and has been generalised to also include physical capital in credit production as well (see Gillman and Kejak, 2008)

production function is specified as:

$$f_t = e^{v_t} A_f \left(n_{f,t}\right)^{\rho} \left(c_t\right)^{1-\rho} \tag{1.5}$$

which upon normalzing by c_t can also be written as a production function of the relative share of credit used in purchasing the consumption good, which is of *decreasing* returns-to-scale type exhibiting a convex upward-sloping marginal cost schedule (see Gillman and Kejak, 2008). Also, v_t is a credit-shock variable with mean $\mu_v = 0$ and a standard error of σ_v. The normalised credit-share production function is thus given by:

$$\frac{f_t}{c_t} = f_t^* = e^{v_t} A_f \left(n_{f,t}^*\right)^{\rho} \tag{1.6}$$

where $n_{f,t}^* = n_{f,t}/c_t$ is equal to the total amount of banking time spent in the production of credit per level of deposits (or consumption). Using the Yeoman-version of a banking time economy, as in Benk et al. (2005), the liquidity constraint, from hereon onwards assumed to be always strictly binding[10], can then be written as follows:

$$\begin{aligned}\frac{m_{t-1}}{1+\pi_t} + v_t &= \left(1 - f_t^*\right) c_t \\ &= \left[1 - e^{v_t} A_f \left(n_{f,t}^*\right)^{\rho}\right] c_t \end{aligned} \tag{1.7}$$

Given this setup[11] of the market for liquidity, which contains both money and costly credit, a recurring central idea emphasised throughout this thesis is that a *unique exchange equilibrium* between money and credit can only exist due to the diminishing

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[10] which is always the case as long as the nominal rate of interest is strictly positive.

[11] My notation slightly differs here from, say, Benk et al. (2005), as they define the share of consumption paid for in cash as $a_t = (1 - f_t^*)$

return nature of the credit-share production function *and* that this equilibrium is determined by the observation that the price of credit (determining credit production) at the margin has to equal the opportunity cost of otherwise holding money, which equals the current beginning-of-period net nominal rate of interest. This means that a unique exchange equilibrium between money and costly credit is determined by:

$$p_t^f = \frac{\mu_t}{\lambda_t} = i_{t-1} \tag{1.8}$$

where p_t^f is the price paid for using credit, and the inter-sectoral labour market optimality condition of

$$w_t = \alpha \frac{y_t}{n_{g,t}} = \rho i_{t-1} \frac{f_t}{n_{f,t}} \tag{1.9}$$

where that latter equates the marginal (revenue) products of labour in the goods sector with that in the credit-banking sector producing the credit liquidity service. This modeling of the liquidity market will be assumed throughout and thus generalises the standard cash-in-advance literature to a more general market for liquidity services, in which the representative household has access to a portfolio of liquidity supply in terms of money and credit, and varies the composition of this portfolio endogenously in response to changes in the price of credit, which I call the *indirect* price channel of credit production, or in response to changes of the marginal cost schedule of credit production due to shocks to the credit sector's total factor productivity, which I call the *direct* marginal cost channel of credit production. A key feature of this particular setup is that the decision of how much money or how much credit to use is governed by an explicit optimisation decision carried out by the representative household-shopper, who responds optimally to changes in the marginal price of using credit, which equals the marginal cost of holding money balances.

Cash-in-advance models, such as the original pure exchange versions given by Lucas (1982); Svensson (1985); Giovannini and Labadie (1991) or the production-based versions described in Cooley and Hansen (1989, 1995); Walsh (2003) are essentially *monetary* extensions of the *real business cycle* paradigm. To understand or to better place into context the former, a discussion of the latter paradigm may prove useful.

1.2 The Real Business Cycle Agenda

In the late 1960s and early 1970s, the emerging consensus in the macroeconomics profession appeared to be best embodied by the Neoclassical-Keynesian Synthesis, based in part on the Hicksian IS-LM framework of aggregate demand (or perhaps even simpler - a mere quantity-theoretic representation of the latter) and some Philips curve relationship between inflation and unemployment, suggesting the view of a viable and policy-relevant short-run trade-off between the two. It was typically believed that the government could - and should - actively seek to smooth out welfare-reducing fluctuations of the business cycle by means of some optimal choice of a combination of monetary and fiscal policy. However, already in the late 1960s and early 1970s, this consensus became increasingly under attack by the monetarist school of thought largely influenced by the works of Milton Friedman, who proposed a natural rate of unemployment and also famously questioned the operational feasibility of monetary policy as a policy tool for stabilisation due to "the long and variable lags" (see Friedman, 1961) with which monetary policy affects *real* variables of the economy.

This view questioned the operational feasibility of the *active management* of the business cycle, as changes in monetary policy - deliberately conducted or otherwise - may simply "arrive too late or untimely" and may therefore even result in the aggravation of output volatility. Owing to this, Friedman proposed a passive "k-percent" steady growth rate of the money supply (see Friedman, 1960) as he believed the latter aggregate to be the main driving force behind business cycle fluctuations[12]. These views also influenced the debate over how to bring down double-digit rates of inflation so prevalent in many industrialised countries during the late 1970s and early 1980s, by means of concerted disinflationary policies, which in Friedman's view would have to be carried out in "gradualist" fashion so as to avoid the large recessions (given some assumed or model-implied "sacrifice ratio") he thought would be associated with too drastic a reduction in the growth rate of the money supply.

The oil crises of the early and late 1970s, the spectre of stagflation, the breakdown of the stable short-run trade-off implied by the Philips curve, and the failure of Keynesian-type demand side policy responses - often "optimally" computed within some large-scale econometric models based on a set of ad-hoc reduced form equations and various equilibrium conditions - led the profession into a crisis and called into question the use of such models for the evaluation of macroeconomic stabilisation policy. Through a series of seminal contributions to the literature (see Lucas, 1972, 1973, 1976), Robert Lucas brought about a paradigmatic shift in the discipline of Macroeconomics which up until then had sought to answer policy-related questions regarding the optimal conduct of monetary and fiscal policy using large-scale econometric models based on a set of equilibrium relationships, which were generally not derived directly from some explicit representative agent decision problem. The main building blocks resulting from these and other philosophically related contributions which began surfacing throughout the 1980s and early 1990s, giving rise to the New Classical[13] Revolution in Macroeconomics, can (albeit, non-exhaustively)

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[12] One noteworthy model allowing for money (inflation) to cause real disturbances, is the Friedman-Phelps money illusion "workers misperception" model (see Friedman, 1968; Phelps, 1968)

[13] rational expectations monetary models of Lucas' "island-story" class are sometimes categorised as "New Classical: Mark I", whereas the real business cycle paradigm as "New Classical: Mark II"

be summarised as follows:

1. Incorporating Rational (model-consistent) Expectations into Macroeconomic Models.
2. The Lucas Critique: Estimated coefficients of simple reduced-from equations are not policy-invariant.
3. The Lucas-Island Model: Explicitly deriving the Philips Curve from a representative agent problem.
4. The RBC Agenda: Using micro-founded, complete-markets intertemporal (Arrow-Debreu) artificial economies (DSGE models) based on deep-structure parameters and maximising behaviour.
5. The Sargent-Wallace Policy Ineffectiveness Proposition: Stabilisation through systematic policy responses is futile and perhaps even counter-productive.
6. The pervasiveness of the principle of "Ricardian Equivalence" in such intertemporal models.
7. A new emphasis on supply-side reforms and also growth-enhancing policies, instead of active business cycle stabilisation.

Regarding the 4th item on this list, the RBC agenda found it's beginnings as a result of the pioneering work by Kydland and Prescott (1982), who used the neoclassical growth model (see Ramsey, 1928; Solow, 1956) as a basis for developing a framework of models which have become known as dynamic stochastic general equilibrium (DSGE) models. Although this stochastic intertemporal microfounded framework based on maximising behaviour is philosophically related to Lucas' earlier contributions, it is very distinct in it's own right in that it generally emphasises the propagation of business cycles due to *productivity shocks* in form of the (persistent modeling of the) Solow residual (total factor productivity) and that it - in it's most basic form - lacks any formal role for a fiat money means-of-exchange.

Therefore, the canonical RBC model, exemplified by Kydland and Prescott's original "time-to-built" formulation, or other versions such as the "indivisible labour" RBC model by Hansen (1985), does not contain any mechanism by which a monetary aggregate *causes* real business cycle fluctuations. Given the enormous influence of "A Monetary History of the U.S." (Friedman, 1971) on the macroeconomics profession, the idea that business cycles can be - or *must* primarily be - *caused by* "demand-side" shocks originating from monetary disturbances (or some formalisation of monetary policy) has arguably become a pervasive belief among many macroeconomists. An early literature building on the RBC modeling paradigm sought to address this issue by a "reverse causation" argument of endogenously determined (broader) monetary aggregates, so as to preserve the validity of the "real" (i.e. supply-side) shock story of such models (see King and Plosser, 1984; Barro, 1989).

1.2.1 The RBC Agenda: Challenges?

The original RBC paradigm's remarkable success of capturing various aspects of factually observed *real* business cycles [14] is summarised in Rebelo (2005), which however also reviews some of this particular paradigm's possible shortcomings. Unsurprisingly, much of the criticisms directed at the RBC school of thought centres around the significance of the Solow residual (or TFP series) on the behaviour of business cycles fluctuations. Most problematically, some empirical work appears

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[14] For a recent defence/retrospective discussion/summary of criticisms of this paradigm, see in particular Gavin and Kydland (1999); Rebelo (2005)

to indicate that estimated TFP series may in fact not be truly exogenous, as they appear to be correlated with military spending (see Hall, 1988) or indicators of monetary policy (see Evans, 1992). Also, as shown in Burnside et al. (1993), variable labour effort (labour hoarding) can also lead to a wedge being driven between TFP and some "true" technology shock[15]. In general, it appears as if over the last two decades or so, the literature has gradually moved away from the idea of the "always-pareto-optimal" RBC paradigm in which business cycles are largely driven by ***persistent*** modeling of the ***exogenously*** specified productivity-shock law of motion, towards a paradigm emphasising an "impulse-propagation" framework, characterised by strong endogenous within-model sources of persistence (say, of output and inflation).

1.3 Money Demand & Velocity in General Equilibrium

A discussion of (the theoretical implementation of) money demand in general equilibrium models naturally paves the way for a simultaneous discussion of velocity measures as well - whether it be consumption or income velocity - as a theory of proportional money demand relative to the aforementioned two quantities defines those velocity measures directly. The prototypical cash-in-advance model defines the demand for real money balances to be directly related to the current level of

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[15] For other such "wedges" which may cast doubt on the significance of some truly exogenous technology shock driving business cycles - such as variable capital utilisation (see Basu, 1996; Burnside et al., 1996) - see again Rebelo (2005). However, in King and Rebelo (2000) variable capacity utilisation coupled with the indivisible labour assumption leads to a model in which much smaller shocks to productivity (or TFP) can propagate into large and persistent output shocks. Furthermore, the latter study also reduces the model's implausible reliance on periods of "technological regress" to account for many business cycle troughs. Further on this issue, Gali (1999) marks the beginning of a series of structural VAR studies casting further doubt on the role of productivity shocks in business cycles.

expenditure on the consumption good, which leads to the counterfactual prediction of a *consumption*-money velocity of unity. This theoretical prediction has in theory been shown to be sensitive to timing assumptions. In Lucas (1982), individuals can acquire money after observing the state of the economy and before purchasing the consumption good, whereas in Svensson (1985), goods trade occurs before assets trade, introducing a precautionary demand for money which in principle can lead to a consumption-money velocity measure bigger than unity, as the CIA constraint may not always bind in the latter case. However, Hodrick et al. (1991) demonstrate in a simulation study employing both timing assumptions, that the CIA constraint almost always binds in practice, thus making irrelevant in practice this subtle difference in timing and the precautionary demand for money obtained in that way.

The development of the *preference-based* cash-credit model (see Prescott, 1987; Stokey and Lucas, 1987) allowed for a more plausible model prediction of the *average* or expected mean of consumption-money velocity in such models, as it allowed only a fraction of the consumption good to be subject to the cash-in-advance constraint, whereas the residual fraction or share of consumption was defined to be a credit good which could be acquired without holding cash. Formally, the setup was given as:

$$\begin{aligned} U &= U\left(c_{1,t}, c_{2,t}\right) \\ c_t &= c_{1,t} + c_{2,t} \\ M_t &\geq P_t c_{1,t} \end{aligned} \tag{1.10}$$

Therefore, utility depends (typically assuming some smooth, but imperfect substitutability) on the level of consumption of the cash good, $c_{1,t}$ and the level of consumption of the credit good, $c_{2,t}$, where both add up to give total consumption and only the former is subject to holding sufficient cash in advance. In this setup, the average money-consumption velocity is determined by optimally setting the marginal rate of substitution between the two types of goods equal to their relative price, which is given by:

$$\frac{U_1'\left(c_{1,t}, c_{2,t}\right)}{U_2'\left(c_{1,t}, c_{2,t}\right)} = 1 + i_{t-1} \tag{1.11}$$

As a specific example, taken from Cooley and Hansen (1995), who define preferences in the cash and credit goods to be logarithmic and additively separable, given by:

$$U\left(c_{1,t}, c_{2,t}\right) = \alpha \log c_{1,t} + (1-\alpha) \log c_{2,t} \tag{1.12}$$

then, using the equilibrium condition above of equating total consumption to the sum of the cash and the credit good and substituting out for the credit good in the marginal rate of substitution expression between the two types of goods, one obtains:

$$\frac{c_{1,t}}{c_t} = \frac{m_t}{c_t} = \frac{\alpha}{1 + (1-\alpha) i_{t-1}} \tag{1.13}$$

This particular specification from Cooley and Hansen (1995), who in contrast to the *endowment* economy specification chosen by Hodrick et al. (1991) formulate a production-based cash-credit real business cycle model, serves as a good example to illustrate the intuition behind the negative findings of Hodrick et al. regarding the successful joint modeling of the second moments of interest rates - both nominal and real - and the consumption-money velocity. For a chosen benchmark steady state benchmark calibration of $\bar{\imath} = 0.10$, say, $\alpha = 0.84$ and $\overline{m/c} = 4.5$, the interest rate elasticity of consumption-money velocity, given by:

$$\frac{\partial\left(c_{1,t}/c_t\right)}{\partial i_t}\frac{i_{t-1}}{\left(c_{1,t}/c_t\right)}=-\frac{\alpha\left(1-\alpha\right)}{\left[1+\left(1-\alpha\right)i_{t-1}\right]^2}\frac{i_{t-1}}{\left(c_{1,t}/c_t\right)}\approx 0.003 \tag{1.14}$$

which is very small indeed and demonstrates this particular shortcoming regarding the realistic modeling of the volatility of consumption-money velocity within such a framework forcefully. *Shopping-time* model specifications of money demand, on the other hand, such as for instance employed in den Haan (1995) given by:

$$v_t=\omega_1 c_t\left(m_t/c_t\right)^{-\omega_2/(1-\omega_2)} \tag{1.15}$$

where v_t is the amount of "shopping time" used up in purchasing the consumption good, can always arbitrarily be specified (in terms of ω_1 and ω_2) so as to obtain sufficiently volatile artificial consumption-money velocity series from a particular model at hand. However, it is particularly the curvature parameter ω_2 which plays a crucial role in determining the sensitivity of consumption-money velocity with respect to inflation. In particular setting ω_2 very close to 1 makes consumption-normalised money demand (consumption-money velocity) very rigid or insensitive, whereas lowering this value in opposite direction (towards 0) increases this sensitivity (and this the volatility of consumption-money velocity). But the basic argument to be outlined and further elaborated on in the Conclusion is that such money demand specifications allow for arbitrary - "reverse-engineered" - calibrated values for parameters such as ω_2, so as to allow a model to better capture certain facts (such as the money velocity of consumption). In contrast, the credit production function used throughout all of the theoretical chapters of this thesis is functionally grounded in empirically estimable counterparts taken from the banking literature and can therefore also be calibrated realistically[16]

1.4 Asset Pricing in General Equilibrium

The starting point for asset pricing in general equilibrium can be traced back to the development of the consumption-based asset pricing model, often also referred to as the consumption-based capital asset pricing model, or in short CCAPM (see Merton, 1971; Breeden, 1979; Lucas, 1978). In contrast to the classical capital asset pricing model, or short CAPM (see Sharpe, 1964; Lintner, 1969)[17], it's consumption-based sibling proxies the representative agent's marginal valuation of wealth directly using marginal utility of consumption, instead of proxying it by using a broad measure of the entire market's current return. The CCAPM posits that any asset in an infinite-horizon general equilibrium intertemporal problem can be priced using Lucas' asset pricing equation, given by[18]:

$$1 = \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \left(1 + r^i_{t+1}\right) \right] = E_t \left[m_t^{t+1} \left(1 + r^i_{t+1}\right) \right] \tag{1.16}$$

where $m_t^{t+1} = \beta \left[u'(c_{t+1}) / u'(c_t) \right]$ is typically referred to as the one-period ahead ***stochastic discount factor*** or also as the one-period ahead *pricing kernel*. This definition is clearly justified as the above expression amounts to saying that one unit of wealth invested into an asset i with uncertain arrival of cash flow (in real terms, i.e. in terms of the units of the real consumption good) in period $t+1$, should therefore have an expected gross real return, which upon discounting by the state-contingent

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[16] However, as shown in den Haan (1995), modeling one aspect correctly, such as consumption-money velocity - may again lead to the problem of capturing another aspect of the same model incorrectly, such as the volatility of nominal rates of interest, which in den Haan is too low.

[17] which is famously put into question regarding it's empirical testability by Roll's critique, claiming mean-variance tautology and unobservability of the market's return (see Roll, 1977)

[18] Here, I discuss some standard familiar specification of this literature. But as shown in Bohn (1991), monetary RBC (or pure exchange) economies may also include the *nominal rate of interest* as part of the stochastic discount factor.

stochastic discount factor, should exactly be equal in value of that one unit of real wealth. This asset pricing equation is the most general formulation and allows any financial asset i to be priced off in the usual way. However, given Lucas' asset pricing equation, a couple of specific examples are perhaps worth pointing out in more detail, such as the *risk-free* rate on the one hand, as well as the treatment of the nominal rate of interest (leading a discussion of the general equilibrium version of the Fisher equation and the inflation risk premium)[19].

1.4.1 Real Bonds and the Real Risk-Free Rate

Including a real bond into the representative household's budget constraint allows for the derivation of the risk-free rate, which represents a special case, as the return on such an asset, say denominated as $\left(1+r_{t+1}^f\right)$, in period $t+1$ is known with perfect certainty in advance, given the current-period equilibrium quotation of the price of that asset, which promises to pay exactly one unit of the consumption good in period $t+1$. Based on equation (1.16), the risk-free rate can therefore be taken out of the brackets in expectations, thus leading to the expression:

$$\left(1+r_{t+1}^f\right)=\left\{\beta E_t\frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)}\right\}^{-1}=\left\{E_t m_t^{t+1}\right\}^{-1} \tag{1.17}$$

This return is indeed a very special case, also because in many countries the trading of inflation-indexed bonds still represents an innovation or may be completely absent, and it has also been claimed that such assets may be traded in ”thin markets“, and are arguably never ”perfectly“ inflation-indexed, as time lags and complexities inherent in appropriately measuring inflation are always present. Related to this, many empirical studies of the market for bonds, which typically revolve around

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[19] An accessible discussion of these and some other issues is provided in Carmichael (1998)

such issues as the *ex-ante* expected real return, the *ex-post* actual return and the validity of the Fisher equation, routinely use data on *nominal* short-term debt simply because a larger sample can be considered. Such issues still amount to the empirical reality of making the real risk-free rate on some hypothetical real bond essentially an *unobservable* variable, which typically has to be proxied by collecting data on nominal returns instead and computing the *ex-ante* real return using some way of empirically measuring current-period inflation expectations. It is this issue I will turn to next in my discussion of the general equilibrium derivation of the (real) return on nominal bonds.

1.4.2 Nominal Bonds, Fisher Relationship and the Inflation Premium

In contrast to the real risk-free rate obtained on some hypothetical asset promising the certain delivery of one unit of the consumption good in period $t+1$, the ex-post realised *real* return obtained from nominal bonds, which also gives rise to the general equilibrium derivation of the Fisher relationship, is not certain. It is only certain in nominal terms. Denominating the *nominal* return on the nominal bond as $\left(1+r^{N}_{t+1}\right)$ and the *real* return of a nominal bonds as $\left(1+r^{r,N}_{t+1}\right)$, I obtain:

$$1 = \beta E_t\left[\frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)}\frac{\left(1+r^{N}_{t+1}\right)}{\left(1+\pi_{t+1}\right)}\right] \tag{1.18}$$

Which can be expanded to give (see Giovannini and Labadie, 1991):

$$E_t\left(1+r^{r,N}_{t+1}\right) = \left(1+r^{f}_{t}\right) - \left(1+i_t\right)\frac{cov_t\left(\frac{c_{t+1}}{c_t},\left(1+\pi_{t+1}\right)^{-1}\right)}{E_t\left[\frac{c_{t+1}}{c_t}\right]} \tag{1.19}$$

This expression shows how risk-aversion and uncertainty over the (joint conditional behaviour of the) inflation rate and the stochastic discount factor (i.e. the state of the world) can lead to the introduction of an inflation risk-premium, which generally distorts the standard Fisher equation relating nominal rates of interest to the real rate and the expectations of the future rate of inflation.

1.4.3 The Term Structure of Interest Rates

This section is aimed at introducing some basic concepts and theoretical contributions regarding the (general equilibrium macroeconomics version of) the term structure of interest rates. Given this particular focus on general equilibrium macroeconomics derivations of prices paid for or generally yields obtained from *zero-coupon* bonds (which are typically also discussed in *real* as opposed to *nominal* terms (see Backus et al., 1989)), it turns out that the evolution of some one-period - usually consumption-based - stochastic discount factor will matter. As discussed in Cochrane (2005, ch.19), although term structure models can also be derived using some purely *statistical* specification for the law of motion of yields and then further analysed using some factor analysis of yield movements, it is well-known that this can easily lead to a representation of yields allowing for arbitrage opportunities.

To preclude this possibility, the literature has focused on the formulation of - or indeed the maximising-behaviour implied *derivation* of - a *positive* stochastic one-period discount factor[20], which can be solved forward so as to price (or equivalently also so as to obtain conditional as well as unconditional yields) of *real zero-coupon* bonds.[21]. Typically then, a further distributional assumption is employed, specifying

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[20] The discount factor existence theorem states that given the existence of a positive discount factor, absence of arbitrage is obtained.

[21] Discussion of *real* - i.e. in terms of the consumption good - bonds abstracts from complications arising from inflation and expected inflation risk. The *zero-coupon* assumption of bonds of various maturity further simplifies derivations, as this financial instrument simply implies the promise of paying one unit of the consumption good in some future period, without intermittent coupon-

the discount factor to be log-normally distributed (see Campbell, 1986), so as to obtain closed-form solutions framed within the so-called *log-normal bond pricing model.* As shown in standard textbook treatments, such as Sargent and Ljungqvist (2004, ch.13.8,p.399) who mostly focus on a discrete time treatment, the price of some τ-period bond is given by:

$$p_{\tau,t} = \beta^{\tau} E_t \left[\frac{\lambda_{t+\tau}}{\lambda_t} \right] \tag{1.20}$$

where λ_t measures the marginal utility (or shadow price) of one extra unit of wealth (or income) in period t. In simple non-monetary endowment (or alternatively also fully specified production) economies, one can then relate λ_t back to some specification of marginal utility, which - as an example - for some standard iso-elastic utility specification separable in consumption and leisure given by:

$$U(c_t, l_t) = \frac{c_t^{1-\eta_1} - 1}{1-\eta_1} + \Psi \frac{l_t^{1-\eta_2} - 1}{1-\eta_2} \tag{1.21}$$

would then imply the following for the consumption-based derivation of the price of a τ-period real bond:

$$p_{\tau,t} = \beta^{\tau} E_t \left[\frac{c_{t+\tau}}{c_t} \right]^{-\eta_1} \tag{1.22}$$

Using the notation:

$$m_t^{t+\tau} = \beta^{\tau} E_t \left[\frac{c_{t+\tau}}{c_t} \right]^{-\eta_1} \tag{1.23}$$

to denominate the τ-ahead stochastic discount factor, in discrete time, the yield-to-

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payments (or certain cash flows)

maturity $\tilde{R}_{\tau,t}$ is then defined as:

$$\tilde{R}_{\tau,t} = \left[m_t^{t+\tau}\right]^{-\frac{1}{\tau}} \tag{1.24}$$

which - following den Haan (1995) - for small interest rates can also be approximated by using the continuous-time analogue, given by:

$$\tilde{R}_{\tau,t} = -\frac{\ln\left(m_t^{t+\tau}\right)}{\tau} \tag{1.25}$$

then, using the assumption of log-normality of the (consumption-based) stochastic discount factor[22], the *conditional* yield-to-maturity for a τ-maturity bond is given by:

$$\begin{aligned} \tilde{R}_{\tau,t} &= -\ln\beta - \frac{-\eta_1 E_t \ln\left(c_{t+\tau}/c_t\right) + (1/2)\left(\eta_1\right)^2 \operatorname{Var}\ln\left(c_{t+\tau}/c_t\right)^{ue}}{\tau} \\ &= R^* - \frac{-\eta_1 E_t \ln\left(c_{t+\tau}/c_t\right) + (1/2)\left(\eta_1\right)^2 \operatorname{Var}\ln\left(c_{t+\tau}/c_t\right)^{ue}}{\tau} \end{aligned} \tag{1.26}$$

where $\operatorname{Var}\ln\left(c_{t+\tau}/c_t\right)^{ue}$ represents the variance of the *unexpected* or unpredictable component of the (either endogenously determined or exogenously specified)[23] law of motion of the (log of the) stochastic discount factor. A discussion of the *conditional* behaviour of the term structure of interest rates is provided in Sargent and Ljungqvist (2004, ch. 2.7.2), which however also follows the ”reverse-engineering“ idea by Backus and Zin (1994) of the time series properties of the stochastic discount factor so as to obtain a *downward-sloping* yield curve on average or *unconditionally*. The conditional behaviour (illustrated by, say, the response of the entire yield curve due to an unexpected shock to the law of motion of the one-period discount factor)

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[22] which within this particular context amounts to saying that *consumption growth* itself is log-normally distributed.

[23] which will crucially depend on whether a pure exchange endowment or a production based economy is assumed.

essentially obeys the expectations hypothesis of the term structure, which implies stronger responses of short rates but more dampened responses of rates at the longer horizon (as they are simply "averages" of chained-together current and future values of the one-period SDF, which following the shock slowly decays back to it's steady state value.

The *unconditional* or average slope of the term structure within this particular framework is therefore given by:

$$\begin{aligned} E_{\infty}\tilde{R}_{\tau,t} =& - \ln\beta + \eta_1 \ln(\bar{c}) - \frac{(1/2)\,(\eta_1)^2 \operatorname{Var}\ln(c_{t+\tau}/c_t)^{ue}}{\tau} \\ =& R^{*} + \eta_1 \ln(\bar{c}) - \frac{(1/2)\,(\eta_1)^2 \operatorname{Var}\ln(c_{t+\tau}/c_t)^{ue}}{\tau} \end{aligned} \tag{1.27}$$

The discussion in den Haan (1995) but also elsewhere (see Sargent and Ljungqvist, 2004; Cochrane, 2005) shows that although the denominator in the variance term in the above expression implies a *rise* in the yield as the maturity increases, a *faster* rise in the numerator - which occurs for a *positively* serially correlated SDF[24] - overall leads to a counterfactual *downward-sloping* theoretically implied yield curve, on average. Thus the theory- (or model-) implied term premium puzzle emerges. The following key points summarise the findings of this particular line of research based on the intertemporally maximising representative agent model:

1. both in pure exchange (see Backus et al., 1989)[25] and also production based (see den Haan, 1995) economies, as both the either exogenously specified or endogenously optimally determined growth rate of consumption is *positively* autocorrelated, the model-implied average yield curve slopes counterfactually down.

2. even in the case of a counterfactually *negatively* autocorrelated stochastic dis-

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[24] and which is what is generally observed in *quarterly* U.S. data of consumption growth

[25] These authors obtain their results from a Mehra and Prescott two-state Markov chain stochastic pure exchange economy setup.

count factor, relatively large levels of risk-aversion are needed to obtain somewhat realistic results.

3. the slope effect obtained following the above given line of argument is of *constant* type, so the significant *curvature* effect (or convexity) observed at the short-end cannot be captured that way.

4. As pointed out by den Haan (1995), *monthly* consumption growth data can be found to exhibit *negative* serial autocorrelation, thus in principle allowing for an upward-sloping average yield curve.

5. *However*, uncontroversially assuming consumption growth to be *mildly* positively autocorrelated or even i.i.d[26] and/or risk-aversion to be low (for instance logarithmic) leads to a theory-implied counterfactually downward-sloping yield curve, whose slope is however quantitatively so small, that taking a "flat" yield curve to constitute the theory's *quantitative* prediction in some approximate *ultimate sense* appears to be an uncontroversial claim or verdict (such is assumed for example in Bansal and Coleman (1996)).

All in all, it can be argued that the general equilibrium term structure literature (based on the basic RBC framework) in some sense faces the same difficulties when it comes to explaining stylized facts as does the equity premium literature, in that, given the time series properties of the aggregate consumption process, *consumption-based* explanations lead to "second-order" risk-adjusted returns of the "Campbell-Cochrane" paradigm-type which are, first of all, too small[27] and secondly, in the particular case of the GE term structure theory, also *directionally* incorrect, as the theory counterfactually implies that representative investors view

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[26] Backus et al. (1989) show that i.i.d. specification of the growth rate of consumption implies a strictly flat term structure on average (i.e. yield curve).

[27] for realistic levels of risk-aversion and relatively standard utility function specifications.

longer-term bonds as consumption-risk (assuming a positively autocorrelated consumption growth process) hedges, thus leading to a risk-adjusted driving down of their returns (i.e a premium price paid) vis-a-vis short-term bonds (see den Haan, 1995). Notwithstanding the theoretically important risk considerations implied by von Neumann-Morgenstern expected utility functions exhibiting curvature, for all intents and purposes assuming a classical "pure" expectations hypothesis of the term structure within the representative agent framework appears to be a very good approximation indeed, implying a completely flat *unconditional* (i.e. "on average") yield curve.

1.5 Solving dynamic stochastic general equilibrium models

This section's purpose constitutes a brief and necessarily non-exhaustive overview over some of the *popular* solution techniques which exist for solving (typically approximations of) DSGE models, which usually consist of a *non-linear* system of expectational difference equations coupled with some initial and transversality condition(s). This field within macroeconomics is an independent and very active research area in it's own right. The purpose of carrying out this discussion is to eventually discuss the class of perturbation methods which has become a very popular approach to obtaining approximate solutions of such models around some steady state, based on the notion of some unique (invariant) subspace (see Sargent and Ljungqvist, 2004, ch. 2.6.1), and which is employed in solving all of the models discussed in the present thesis. Contributions related to these (initially restricted to 1st-order linear approximate) methods of solving DSGE models are notably given by Blanchard and Kahn (1980); King and Watson (2002); Klein (2000), which have recently been extended to 2nd order approximate linear methods (see Sims, 2000; Collard and Juillard, 2001; Schmitt-Grohe and Uribe, 2004)[28].

Early ways of solving so-called RBC dynamic stochastic general equilibrium models (or indeed their more simple pure exchange counterparts) - which typically don't possess analytical closed-form solutions, given realistic modeling assumptions of shocks and/or modeling of the depreciation rate of physical capital[29] - often took the non-linearities which such frameworks presented very serious by devising computationally intensive numerical fixed-point algorithms, operationalized by the dynamic programming framework within which such models could be cast. That way, fairly accurate representations of the so-called optimal policy functions could be obtained, which constitute the solution to the dynamic programming problem.

Given the a-priori unpredictability of future structural shocks (due to the typically normal i.i.d distributional assumption of the errors driving the shock processes), such optimal policy functions are therefore a manifestation of the "open-loop" character of such solutions, as opposed to the "closed-loop" type solutions which are obtained when all future shocks are known with perfect certainty in advance (the perfect foresight case). The problem with such *discrete state-space* approaches [30] is that the so-called "curse of multi-dimensionality" "bites" fairly quickly as one considers increasing the state-space to include more and more shocks (or else endogenous state variables). The problem at hand can quickly become unmanageable both in terms of computation time and/or programming time invested into setting

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[28] in fact, the algorithms provided by Schmitt-Grohe and Uribe (2004) and also the C++ version of Dynare, are capable of solving models using the perturbation approach based on k-order approximations for $k > 2$. So far, in practice the literature has only begun using 2nd-order accurate solution methods.

[29] pure exchange economies may have closed-form solutions, as long as shocks are i.i.d. Production-based RBC models require full within-period depreciation, logarithmic utility *and* either no shocks or i.i.d. shock specifications for closed-form solutions to exist.

[30] which were considerably improved in accuracy by moving from simple two- or multiple-state parsimonious Markov representations of the exogenous law of motion of structural shocks by adopting more realistic quadrature methods instead (see Tauchen and Hussey, 1991)

up the code[31].

Therefore, an entire "approximation" body of literature has emerged centred around methods of which some early ones are summarised (and compared to each other) in Taylor and Uhlig (1990)[32], some of which are the parametrised expectations approach (see den Haan and Marcet, 1990), the linear-quadratic approximation approach (see Christiano, 1990) and the Value Function Grid approach (see also Christiano, 1990). More recently, a literature involving the solution of problems involving *occasionally-binding* constraints has also emerged (see and reference therein Gomme, 1998). By and large, I don't believe it would be too controversial to claim that the current state of affairs regarding this particular issue - and particularly due to the rise in popularity of software packages such as Dynare, but also before that, Harald Uhlig's toolbox[33](see Uhlig, 1995) - perturbation methods have become incredibly popular among the profession, simply because of the simplicity and timeliness with which *linear* systems of expectational difference equations can be solved.[34].

In this thesis I follow the tradition of the class of perturbation methods, and in particular, I employ a method which is most general in that it provides the solution to so-called *singular* linear systems of expectational difference equations (see Klein, 2000), and which uses a *triangularization* technique based on the Schur decomposition, instead of the well-known *diagonalization* Jordan decomposition employed in the seminal work by Blanchard and Kahn (1980), which is only applicable to *non-singular* systems. A computational problem described in and first addressed by King

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[31] Although it seems, with rising computational power (multi-core processors), better software and an increasingly IT-literate profession, such methods may soon be favoured again instead of approximate solutions.

[32] A more up-to-date "race" between *linear* solution methods is provided in Anderson (2006)

[33] which uses an undetermined coefficients approach.

[34] Another method - or set of methods - which may however increase in popularity revolves around the use of *Chebyshev polynomials* - which are part of the class of projection methods - to approximate unknown (policy) functions

and Watson (2002), is that as DSGE models become increasingly complex (in terms of numbers of equations describing optimal behaviour and market-clearing), it may also become increasingly difficult to eliminate (by hand) so-called *intra*-temporal conditions, which are responsible for the singularity of such systems of expectational difference equations. Those authors show that, although in principle such singular systems can be *reduced* by appropriate elimination of variables so as to arrive at a non-singular state-space representation - as long as one actually exists and is unique - this can become quickly burdensome to do by hand and may also be an error-prone process to undertake. Therefore, one needs to understand the algorithms devised by Klein (2000) and King and Watson (2002) as numerical *computer-assisted* methods of dealing with such system reduction problems automatically.[35]

Throughout this dissertation I employ the method described in Klein (2000), which for the models considered exhibits the "beauty of simplicity" in that the (log-)linearised model around (the log of) it's steady state can very parsimoniously be represented in the canonical form given by[36]:

$$\mathrm{A}E_t \begin{bmatrix} \mathrm{z}_{t+1} \\ \mathrm{x}_t \\ \mathrm{y}_{t+1} \end{bmatrix} = \mathrm{B} \begin{bmatrix} \mathrm{z}_t \\ \mathrm{x}_{t-1} \\ \mathrm{y}_t \end{bmatrix} + \epsilon_{t+1} \tag{1.28}$$

Then, following Klein (2000), the solution (the recursive law of motion) itself can

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[35] An often-cited example which demonstrates system reduction "by hand" is King et al. (1988), who reduce a canonical RBC model's conditions of optimality to a system containing only capital and the shadow value of wealth.

[36] more complicated "higher-order" autoregressive formulations, can always be reduced to a first-order form using the familiar "stacking-up" and re-defining of variables tricks which are known from the VAR literature discussing the "first-order" companion form of higher-order systems. This issue arises for instance in models exhibiting habit persistence in consumption, where past consumption enters as an endogenous state variable. Regarding this issue see also Uhlig (1995)

also very parsimoniously be expressed as:

$$\begin{bmatrix} \mathbf{y}_t \end{bmatrix} = \mathbf{F} \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_{t-1} \end{bmatrix} \tag{1.29}$$

and for the evolution of the state of the system:

$$\begin{bmatrix} \mathbf{z}_{t+1} \\ \mathbf{x}_t \end{bmatrix} = \mathbf{P} \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{t+1} \\ \mathbf{0} \end{bmatrix} \tag{1.30}$$

The obtained recursive laws of motion of both the endogenous and exogenous state variables, as well as the control (or "jump") variables - which are functionally related to the current state of the system - are used in this thesis to produce impulse responses as well as simulations from the solved theoretical models discussed and analysed throughout[37].

1.5.1 PyMacLab

As part of the research efforts invested into the completion of the present thesis and the work contained therein, I coded an object-oriented program (written in the high-level programming language Python) which allows me to obtain the matrices A and B to a model *without* having to log-linearise the non-linear first order conditions by hand, as is done for instance in Uhlig (1995). Instead, very similar to Dynare, my program takes the original non-linear first-order conditions of optimality - which are summarised in a text file - and the model-implied steady state, and then proceeds by *analytically* differentiating the non-linear system of equations w.r.t. future and current states and controls, so as to obtain the Jacobian of the original non-linear

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[37] Paul Klein also circulates a very accessible "cookbook" to his method, which is circulating on the internet. It is currently available at `http://www.ssc.upenn.edu/~vr0j/ec714-98/cookbook.pdf`

system, which is then evaluated at the (log of the) steady state of the system. The program then proceeds by using Klein's routines - which I have also translated into Python [38] - to obtain the solution of the model given by the matrices F and P, which are used to compute impulse-responses as well as simulations, where the latter may often then also be HP-filtered within the same computational environment. The software has been tested and compared - using various models - to Harald Uhlig's toolbox, so as to make sure that the results obtained are *identical* across software solutions used in solving such models based on the class of perturbation methods. A very similar piece of software - which also analytically differentiates the non-linear first-order conditions of optimality, but using the symbolic toolbox provided by Matlab - has recently been made available by Schmitt-Grohe and Uribe[39].

1.6 U.S. Monetary Business Cycle Data

Finally, this last section of the introduction to this thesis is aimed at presenting some recent (monetary) business cycle data of the United States, so as to motivate the theoretical contributions to follow. In particular, I will mostly raise some questions at this juncture, questions which I believe are at the core of modern (but also past) macroeconomic controversy. All series graphed - except for the federal funds rate - are in logs and are de-trended using the Hodrick-Prescott filter(see Hodrick and Prescott, 1997), so as to extract the business cycle component of each relevant series only[40]. Also, at this stage I have chosen not to summarise evidence obtained by running vector autoregressions - and perhaps to use some sort of identification scheme to identify structural shocks and obtain impulse responses - as this would

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[38] using the Python matrix computation add-on library Numpy.

[39] Their code is available at `http://www.econ.duke.edu/~uribe/2nd_order.htm`

[40] The usual disclaimer applies that this "industry standard" of filtering business cycle data is not without it's critiques (see Cogley and Nason, 1995). Notice also that, following de-trending, the series have been multiplied by 100, so as to express them directly in percentage deviations.

have already led to a picture of the facts based on some set of assumptions related to the identification scheme chosen [41]. A by now "classic" empirical introduction to the field - highlighting many of the "stylized facts" and controversies surrounding it - is provided in Walsh (2003).

Starting with graph (4.1), this illustrates some standard stylized business cycle facts of the U.S. This graph perhaps serves best in capturing the answer to the question of why the real business cycle agenda has been so successful and enormously influential within the literature. It simply shows time series behaviour of real consumption, real investment and real output at business cycle frequency[42], which is well captured by standard non-monetary RBC models. Notice however, that aggregate consumption in the data appears to be slightly too pro-cyclical (i.e. it displays what in the literature has been called the "excess sensitivity" property (see Campbell and Mankiw, 1990, 1991)) to conform to the theory-implied Friedman permanent-income hypothesis consumption smoothing objective. However, incorporating habit persistence in consumption into such models can generally lead to a more auto-correlated aggregate consumption process, better capturing the factually observed consumption process that way. Finally, investment is much more volatile in general than both output and consumption, which is also well captured by simulated data obtained from standard RBC models.

Graph (4.2) shows the federal funds rate, real output and the rate of inflation calculated from using the consumer price index. The funds rate is simply graphed in levels and has also been de-meaned. The graph serves well in capturing the often differing role and interpretation which has been attached to the behaviour of the

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[41] The identification of a so-called "liquidity" effect - a fall in the nominal rate of interest following an (unexpected) increase in the money supply - often also rests on making such identifying restrictions within VARs.

[42] The series are from the St. Louis Fed, and their codes are PCNDGC96(CONS), FPIC1(INV), GDPC1(GDP). The series are seasonally adjusted, the base year is 2000. The range is 1959Q3-2008Q2.

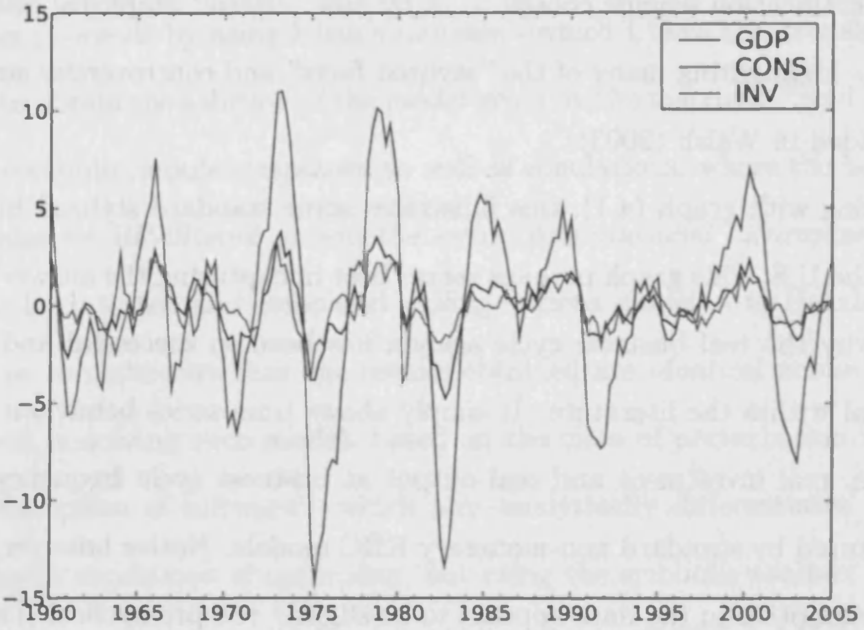

Figure 1.1: U.S. Business Cycle

inflation rate series. Inflation appears to be very persistent in the data[43] and also *eventually* pro-cyclical, so does this mean that prices (wages) ought to be modeled in some "sticky" fashion outside of any quantity-theoretic framework? Is the causation of Keynesian flavour, meaning that excess demand eventually causes upward pressure on prices? Or is the relationship the other way round, i.e. classical, in that inflation (money growth) causes cyclical swings in economic activity, which cannot however by systematically exploited? Or maybe there is a *real* business cycle story driving the picture, with the money market modeled within some quantity-theoretic framework using a money demand function which endogenously responds to interest-rate changes in the goods-market?[44] What about the contemporaneous

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[43] but *varyingly* so, as described and interpreted in Minford et al. (2006)

[44] Related to this is the highly relevant issue raised by Robert Lucas in his 1995 Nobel lecture: "This tension between two incompatible ideas: that changes in money are neutral units changes, and that they induce movements in employment and production in the same direction, has been at the centre of monetary theory at least since Hume wrote".

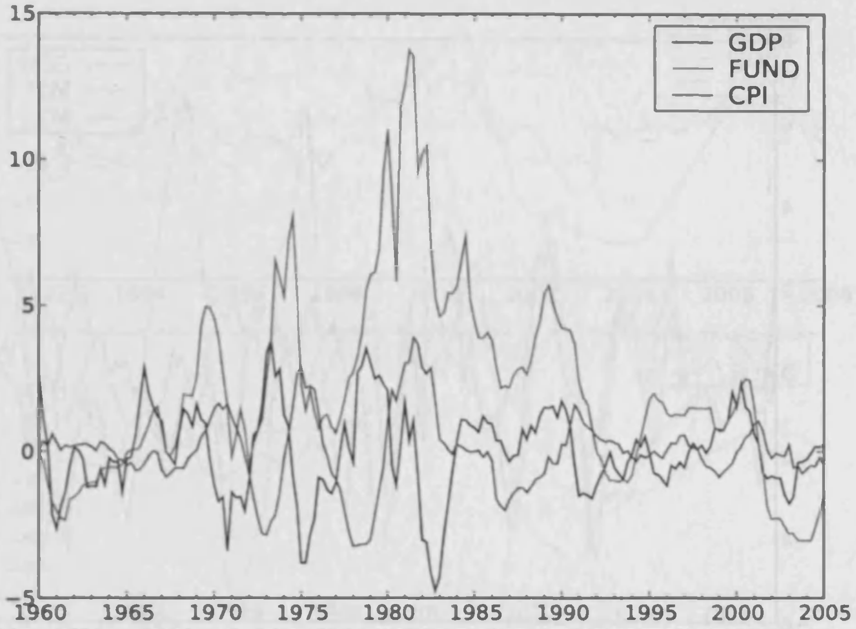

Figure 1.2: U.S. Business Cycle

counter-cyclicality of price with economic activity? Is this feature to be understood as indicative of the predominance of supply-side shocks? Also, ought theoretical models describe the implementation of monetary policy using some interest-rate rule, in that way marginalising the direct role of monetary aggregates in this regard? Is the "leading inverted indicator"(see Boldrin et al., 2001) property of interest rates with regards to fluctuations in economic activity the outcome of a purposeful and deliberate (albeit unsystematically and thus unexpected)[45] manipulation of interest rates? Or again, is there an equilibrium real business cycle story behind interest rates (linking nominal rates to real rates via expected inflation and the Fisher equation) whereby central banks may simply be tracking a somehow "model-equilibrium" implied Wicksellian rate? Graphs (4.3) and (4.4) serve to highlight perhaps one of the

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[45] Mainstream macromodels - which usually assume rational expectations - typically focus on the response of economic activity but also of nominal variables to *unexpected* shocks to monetary policy, i.e. the money supply or a Taylor Rule.

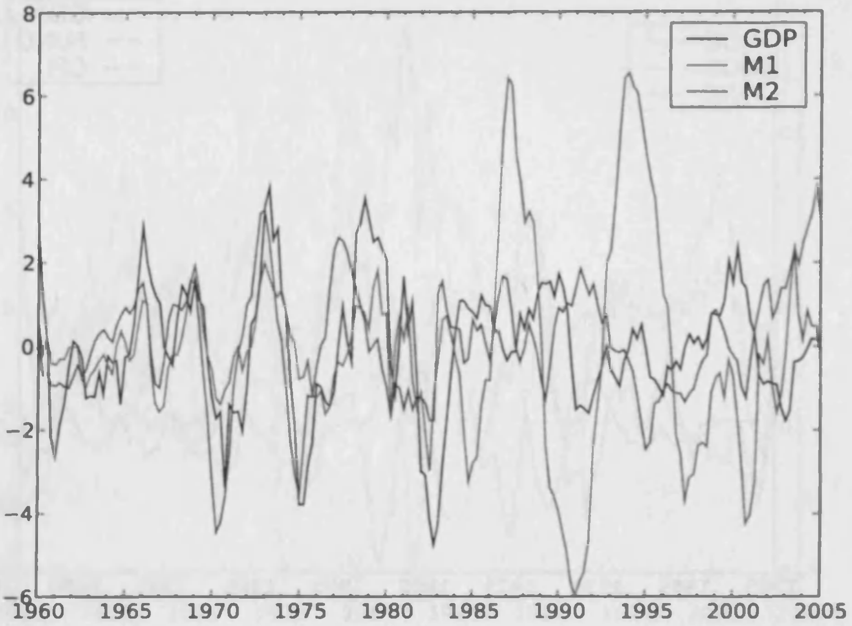

Figure 1.3: U.S. Monetary Business Cycle

key stylized facts underlying many discussions within monetary macroeconomics, exemplified by the highly influential studies on this issue collected in Friedman (1971). Evident from both graphs is the relatively tight (and mainly leading) relationship between some narrow monetary aggregates - such as M1 or M2 - and economic activity, a relationship which remained fairly stable during the period before the late 1970s, when it eventually broke down in periods thereafter. It is also interesting to notice that output appears to be to some extent more contemporaneously procyclical with money than consumption, whose response appears to lag changes in money by more periods. This feature is particularly evident in the trough occurring between 1970-1971. The breakdown of a stable money demand relationship (and the linked increased instability of velocity) have gone hand in hand with a de-emphasis of theoretical as well as practical considerations of so-called "optimal" money supply growth rate rules (i.e. trying to control some monetary aggregate directly), which

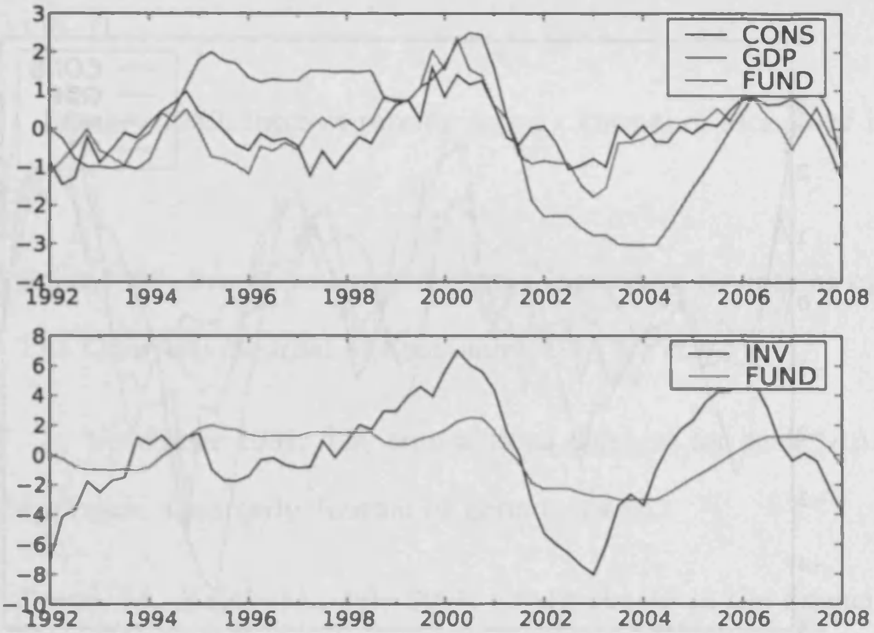

Figure 1.4: U.S. Monetary Business Cycle

have been replaced with a framework focusing on inflation targeting and a more direct control of money demand (or liquidity) via an interest-rate rule, implying an endogenously engineered response of money supply so as to maintain stability in the short-run interest rate target. The graphs do not only reveal the post-1980 instability of some money (demand) relationship, but also the increased volatility of the M1 money stock. Finally, graph (4.5) again summarises some further evidence regarding the fund rate, real consumption, real gdp and real investment, highlighting the series' more recent joint variation through time, by plotting them from 1992Q1-2008Q2. This concludes the introduction to this thesis and sets the stage for the three theoretical paper-style chapters to follow, where each of them are introduced by some non-technical abstract-style introduction.

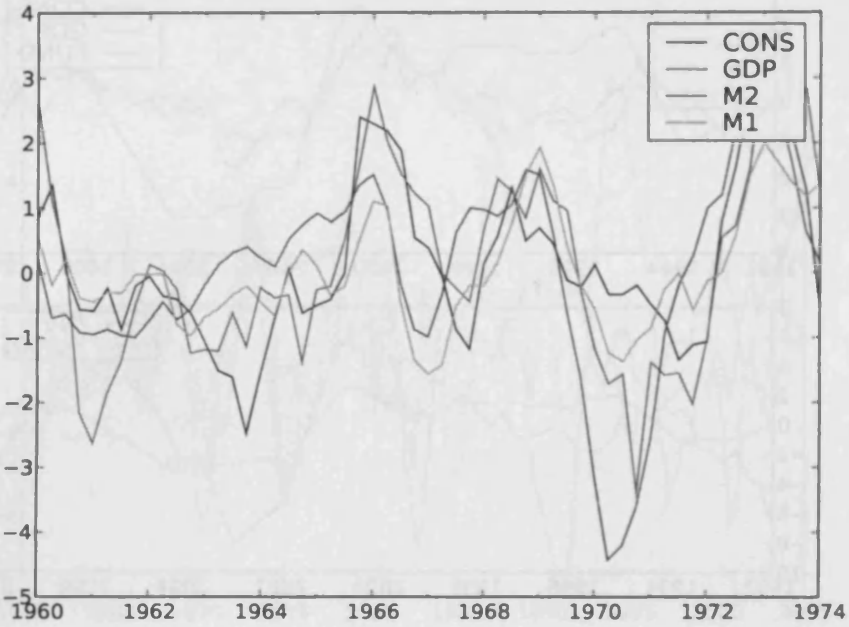

Figure 1.5: U.S. Monetary Business Cycle

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Chapter 2

Consumption Velocity in a Banking Time Model

A standard cash-in-advance-type monetary real business cycle is developed, in which both money and produced (as well as paid-for) credit can be used to shop for the consumption good. The question which is posed is to see whether productivity shocks alone (i.e. disregarding money supply and credit production shocks) can lead to sufficient volatility in *consumption*-money velocity (as opposed to *output*-money velocity). What is found is that for the standard model, this measure - the coefficient of variation - is too small relative to the data. Adding habit persistence in consumption raises (approximately doubles) this volatility measure, by introducing a *phase shift* between the evolution of consumption and money - which is also observed in the data, in which some measure of broad money typically *leads* consumption. But only by the additional introduction of adjustment costs to investment, is the volatility of consumption-money velocity raised sufficiently so as to be in agreement with U.S. data. Even in this case, interest rate volatility - although raised in general - is still too low when compared with the data.

2.1 Introduction

A discussion of velocity within the canonical real business cycle model framework(see Kydland and Prescott, 1982; Hansen, 1985; King et al., 1988) requires a theory explaining why the representative agent would want to hold a return-dominated asset in the first place, ideally by specifying a *purposeful* and *plausible* role for money, without sacrificing the tractability of such models, while perhaps making some concessions to the level of depth of microfoundations attained to preserve that tractability. While research into modeling seriously the microfoundations of money demand in general equilibrium is still an ongoing, very controversial and inconclusive agenda, the last couple of decades have seen the development and popularisation of arguably three "first-generation" theories of money demand in general equilibrium, the Sidrauski money-in-the-utility-function approach (Sidrauski, 1967), the interrelated shopping-time and transactions cost approaches (Baumol, 1952; Tobin, 1956; Barro, 1976; McCallum, 1983; Bansal and Coleman, 1996), and the Clower-Lucas cash-in-advance approach (Clower, 1967; Lucas, 1982; Svensson, 1985; Stokey and Lucas, 1987; Cooley and Hansen, 1989).

While the recent rise in popularity of the new neoclassical synthesis generation of GE models (see Lawrence J. Christiano and Evans, 2005; Canzoneri et al., 2007a; Smets and Wouters, 2007) has seen a concurrent de-emphasising of the significance of modeling money in some purposeful way at all[1], it is also interesting to observe how of all of the three mentioned "first-generation" theories of GE money demand, arguably the most plausible and theoretically robust - the cash-in-advance role of money - to some it may seem is perhaps closest in failing one of the toughest tests of all, the test of time, as some of it's predictive shortcomings have led many to

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[1] Either money is completely absent, or MIUF in combination with a Taylor Rule implies a corresponding money supply rule *residually*.

pursue the alternatives or to devise new ways of modeling money demand in general equilibrium alltogether.

One of such shortcomings of early formulations of the cash-in-advance model, typically spelled out in a simple Lucas-exchange endowment economy framework, was it's prediction that *consumption*-money velocity is always fixed at unity. While a subtle modification of the information set available to the representative agent (see Svensson, 1985) opened up the possibility of a precautionary money demand component, meaning that the cash-in-advance constraint in theory would not always bind and money balances beyond those required for consumption would be held, this avenue was quickly dismissed, as in simulation-based experiments, the CIA constraint was found to be binding almost always in practice (see Hodrick et al., 1991). As a consequence of this finding, cash-in-advance models are now routinely analysed and discussed *assuming* a strictly binding cash-in-advance constraint.

In order to make possible a realistic modeling of the *average* velocity of *consumption* velocity within the cash-in-advance paradigm, Prescott (1987) and Stokey and Lucas (1987) developed the *cash-credit* model, in which *preferences* over a cash-in-advance and a credit good were specified (thus leading to a multi-good barter economy), and the relative price of the cash good vis-a-vis the credit good was in the usual way related to the opportunity cost of holding money, thus making this relative price equal the net nominal rate of interest. In such a model, the average level of consumption velocity is therefore fully characterised by the optimality condition of setting the marginal rate of substitution between the cash and the credit good equal to the relative price between the two:

$$\frac{\partial U\left(c_{m,t}, c_{c,t}\right)/\partial c_{m,t}}{\partial U\left(c_{m,t}, c_{c,t}\right)/\partial c_{c,t}} = 1 + i_{t-1} \tag{2.1}$$

where I have defined $c_{c,t}$ to be current level of the credit good and $c_{m,t}$ to be current level of the money (or cash) good. While this allowed such models - for suitably calibrated preference parameters - to correctly match the empirically observed *average* velocity of consumption, it became quickly apparent that matching the observed *volatility* of consumption velocity required interest rates to be implausibly volatile at the same time (see Hodrick et al., 1991), either by assuming too high a level of relative risk aversion or by adopting habit persistence in consumption, both of which lead to too volatile interest rates *in endowment economies*[2]. It is interesting to note that this specific shortcoming of the predictive failure regarding velocity in particular has led many to adopt either shopping time or transaction cost functions to motivate money demand in their models instead (see inter alia Marshall, 1992; den Haan, 1995; Bansal and Coleman, 1996; Auray et al., 2005).

Intuitively, as the average level of consumption velocity is characterised by the point of tangency between a relatively smooth utility function and a downward-sloping relative price schedule given by the nominal rate of interest, dramatic volatility in the slope of that price schedule is needed to attain significantly different and dispersed loci of tangency. Indeed for the comparatively small perturbations seen in both the real and nominal rates in practice, period-by-period loci of tangency are all contained within some small neighbourhood and thus consumption velocity does not vary sufficiently through time. Although the setup of the model therefore essentially amounts to too small an interest rate elasticity of consumption velocity, this elasticity is typically not independent of the *level* of interest rates, which means that conducting such analysis by calibrating the model at business cycle frequency (quarterly) or a more medium- to long-term frequency (annually) can lead to differences in results obtained.

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[2] The same is typically not the case in production-based RBC models, in which the representative agent can use saving and labour to smooth marginal valuation.

This sensitivity of results in relation to velocity due to the chosen time horizon is also examined by Hodrick et al. (1991), who report results based on both quarterly and annual specification of models. Gillman and Kejak (2007), on the other hand, whose model is a fully specified production-based RBC model with physical and human capital, also calibrate their cash costly-credit model using a quarterly time horizon, but include a shock to credit production productivity which serves as a further channel to explain variability in *income* velocity within their framework - they do not discuss any numerical results pertaining to the variability of *consumption* velocity directly[3]. The importance of *money shocks* in explaining velocity within their framework is possibly also a result of specifying credit innovations to be highly contemporaneously correlated with money shock innovations. This suggests that their results are predominantly driven by the obtained high credit-shock elasticity of velocity and the influence of variation in this credit shock *alone.*

Closely related to this last point, an approximate conceptual analogy can be drawn between the role of *credit shocks* in the *technology-based* cash costly-credit model (see Gillman, 1993; Benk et al., 2005; Gillman and Benk, 2007) on the one hand, and how the analogous counterpart of the same shock could be viewed as a *preference shock* in the desirability of the credit good relative to the cash good in the seminal *preference-based* cash-credit model (see Stokey and Lucas, 1987), on the other. To my knowledge, the latter approach has never been explored, and I would find it surprising if it ever had been, as equipping a simple preference-based cash-credit model with preference shocks to the credit and cash goods would not *explain* the variability in velocity, but through the exogenous specification of such preference shocks, instead essentially amount to assuming it trivially.

This analogy is however only approximate, as in the former case the level and

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[3] Although, they do of course point out that variability in consumption velocity is a component of of the overall variability in income velocity.

variability of velocity depend on the intersection of a horizontal price schedule (the net nominal rate of interest) with a convex upward-sloping marginal cost schedule in credit production (see Gillman and Kejak, 2008), and variability in both the price and the marginal cost schedule affect the variability of velocity, whereas in the latter case the above-discussed changes in loci of tangency matter. What will be of importance in the present study is also the distinction that can be drawn between the two cases with regards to the interest rate elasticity of consumption velocity, and how this elasticity varies with the level of the interest rate, and generally differs in the cash costly-credit from the cash credit model.

On the other hand, a discussion of *income* or *output* velocity within the cash-in-advance paradigm and a fully specified production-based real business cycle model with physical capital and investment presents less of a challenge, as it is typically only consumption which is modeled subject to the liquidity constraint. As demonstrated theoretically by Cooley and Hansen (1989) and re-emphasised by Gillman and Kejak (2007), in such models *income* velocity is therefore trivially different from unity and also exhibits the observed pro-cyclicality seen in U.S. data. Intuitively, the Friedman permanent income-implied consumption-smoothing property (see Friedman, 1957) also leads to smooth behaviour of money demand, whereas productivity shocks and endogenous variation in the leisure-labour trade-off lead to much more volatile *income* fluctuations around a smooth *consumption* (money demand) trend, resulting in the pro-cyclicality of income velocity.

Here, money supply shocks are of little significance (for explaining the variability of velocity measures) and interest-rate implied means-of-exchange switching is either absent (if there is no credit good), or for the above-discussed reasons quantitatively unimportant in explaining much of the volatility seen in income velocity, as interest rates in *production-based* fully specified real business cycle models are typically even smoother than in endowment economies, since the representative household has a larger menu of choice variables (saving, leisure-labour) at his disposal to smooth his marginal valuation through time (see den Haan, 1995; Jermann, 1998).

This discussion therefore makes clear how within this framework, *income* velocity varies sufficiently and pro-cyclically[4], due to the *investment* velocity component alone (investment jumps, but money demand due to consumption demand hardly moves at all), primarily driven by productivity shocks and permanent income-implied consumption smoothing. Clearly then, attempts of modeling *consumption* velocity successfully within this framework would run into the same difficulties already discussed above, related to insufficiently volatile substitution between alternative means-of-exchange.

The present study is complementary to Gillman and Kejak (2007) and related to Hodrick et al. (1991) in the sense that it tries to examine to what extent a decentralised cash costly credit based on Gillman and Kejak (2008), exhibiting the same upward-sloping marginal cost schedule in credit production as in Gillman and Kejak (2007), is capable of explaining the variability of *consumption* velocity but without resorting to either credit or money growth rate shocks. The present study therefore ignores or holds fixed the direct effects from credit shocks causing shifts in the position of the marginal cost schedule, and instead focuses exclusively on the price- (interest rate-) channel affecting velocity in this class of models. It is the focus on this last point which relates the present study to Hodrick et al. (1991). Holding *both* the credit *and* the money growth rate shocks fixed, the present study thus emphasises a purely goods-productivity driven Wicksellian determination of consumption velocity, in which endogenous money demand and it's response to the real rate of interest matters.

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[4] But as pointed out and improved on by Gillman and Kejak (2007) within their framework, the model discussed by Cooley and Hansen (1989) shows *too much* procyclicality of velocity.

There is a strong motivation for conducting such an experiment, since in practice velocity varies also sufficiently at business cycle frequency (quarterly), which begs to be explained using models calibrated and simulated, and with structural shocks mattering most *at business cycle frequency*. In as far as the quantitative analysis conducted by Gillman and Kejak (2007) - although also based on a calibration using a *quarterly* time horizon, and where positive results are obtained predominantly through the inclusion of credit production shocks - can be understood as an analysis focusing on *institutional* shocks embodied by episodes of financial deregulation which may matter less at business cycle frequency, then the observed volatility of consumption velocity measured *quarterly* still appears to pose a theoretical challenge, as financial-deregulatory credit shocks arguably play a lesser role at shorter frequencies, whereas productivity shocks do. It is this last point which justifies a more in-depth study of the extent to which the goods productivity shock-driven *price-channel* alone is capable of explaining the variability in *consumption* velocity, as nominal interest rate volatility induced through goods productivity shocks may matter more at business cycle frequency[5].

To this end, a baseline decentralised credit model is presented exhibiting the same convex upward-sloping marginal cost schedule in credit production as in Gillman and Kejak (2007). I abstract from human capital and endogenous growth, which is however crucial to Gillman and Kejak's analysis to identify credit and money shocks using data, as they affect growth in opposite ways. I show how the baseline model, calibrated and simulated *without* either exogenous credit or money growth rate shocks and with a realistically low steady state nominal rate, cannot account for the observed variability in *consumption* velocity, as too little variability in the real and nominal rate (through little variable inflation expectations) leaves the price channel

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[5] Gillman and Kejak's focus is on the variability of *income* velocity, instead. But novel results are primarily obtained by endogenising the variability of the *consumption* velocity component.

ineffective and the endogenous share of credit in consumption insufficiently variable. I demonstrate, using appropriate simulation graphs, how in the baseline model, credit production moves almost one-for-one with consumption over the business cycle, producing too little variability in the credit share and thus money velocity .

Then, I add habit persistence in consumption, as in Constantinides (1990) (which is of internal relative habit type), which immediately results in some degree of disentanglement of consumption (deposits) from credit (and therefore also more variable consumption velocity). Essentially, simulation graphs reveal that strong habit in consumption introduces a smooth hump-shaped response of consumption to productivity shocks, while preserving and enhancing a strong contemporaneous endogenous switch between means-of-exchange, leading to a *phase shift* in the frequency domain between credit and consumption (deposits)[6].

This effect alone is however not strong enough to account for the variability of consumption velocity seen in the data. Only by adding adjustment costs to investment can the model both disentangle consumption (deposits) from credit production (through the habit-induced phase shift) on the one hand, and do so quantitatively sufficiently through an increased volatility in the real rate of interest, on the other, to make consumption velocity vary sufficiently enough so as to match observed variability. A key finding is that the required volatility in real rates is however nowhere near as unrealistically dramatic as in Hodrick et al. (1991), who report interest rate volatility figures of around 30% (in standard deviations) in order to obtain realistic velocity variability, quite the contrary, variability of real and nominal rates is still below the level of volatility observed in the data.

Introducing habit persistence *and* adjustment costs to investment into a cash-in-advance (or alternatively, here, an exchange-in-advance) model has, to my knowl-

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[6] Aggregate consumption now turns in to a smoothly evolving endogenous state variable, whereas endogenous credit-money switching retains it's "jump variable" nature.

edge, not been done before, whereas the introduction of habit only into such models is not new. In particular, Auray et al. (2005) is very close in spirit to the approach taken here, in that they also study Cooley and Hansen's prototypical monetary RBC model, also add relative habit but introduce endogenous variation in velocity through a transactions cost function as in Marshall (1992) and Carlstrom and Fuerst (2001) instead of using costly credit, as in Gillman (1993); Benk et al. (2005); Gillman and Benk (2007). They show that such types of models suffer from indeterminacy (no stable saddle-path solution) for already fairly low values of the relative habit parameter *and* an increasing net real resource cost of using money, introduced through the transactions cost function.

A surprising - but given the aforementioned authors result, very intuitive - complementary result I obtain in the present cash costly-credit model, is that indeterminacy disappears alltogether, regardless of the strength of relative habit chosen. The reason for this is that the present model provides the representative household with an alternative means-of-exchange to escape the *cash component* of the exchange-in-advance constraint. Crucial to obtaining global determinacy in the cash costly-credit model, is that - although using credit also distorts the margin between consumption and leisure - the representative household's net cost of using credit is zero, as the cost of credit is re-distributed back in terms of the banking wage bill and the return on it's deposits, which feature in credit production[7].

The contribution of the paper is therefore twofold. Firstly, in as far as credit shocks are of institutional nature and should matter less at quarterly horizons, I explore to what extent the price-channel alone, driven by business cycle frequency shocks to the goods sector productivity alone, can explain the variability seen in

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[7] Auray et al. (2005) prove that equipping the representative household with a costless means-of-exchange alternative, makes indeterminacy disappear. The present model provides such a costless alternative in terms of credit, thus exhibiting global determinacy.

consumption velocity. This investigation is thus complementary to Gillman and Kejak (2007), whose results are also driven by their model's high consumption velocity elasticity of credit shocks. Using a similar argument as in Jermann (1998), Hornstein and Uhlig (2000) and Boldrin et al. (2001)[8], I show how a combination of habit persistence *and* adjustment costs to investment is required to make the price (interest rate) channel sufficiently variable enough so as to induce sufficient variability in credit production relative to consumption (deposits).

Credit production overshooting relative to more autocorrelated and smoothly evolving consumption (deposits) is obtained[9], where the latter feature combined with adjustment costs increases the volatility of the real rate and thus (for given inflation expectations) of the price of credit leading to the former phenomenon. Assuming strong habit persistence is important, as it introduces a *phase shift* in the frequency domain between credit and consumption, as consumption responds more sluggishly to productivity shocks than credit.

Solving and simulating the model over a whole range of habit persistence parameters, I also demonstrate that indeterminacy is not a problem, as credit is a costless means-of-exchange in terms of net wealth. Secondly, in contrast to Hodrick et al. (1991), whose cash-credit model exhibits a very low interest elasticity of consumption velocity for the reasons discussed above (thus requiring extremely high interest rate volatility to explain volatility in velocity, close to 30% standard deviation), I demonstrate how the present model requires much less variability in the interest rate in order to induce enough variability in consumption velocity so as to match up with the data.

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[8] Interest Rates in canonical RBC models exhibit very little volatility. The three references provide theoretical frameworks using habit persistence *and* inelastic "q-theory" supply of physical capital, to raise the volatility of interest rates.

[9] "Overshooting" here is meant in a percentage change from steady state sense, *not* in an absolute sense, as credit - being a means of exchange for consumption (deposits), can never overshoot beyond consumption in absolute terms.

The remainder of the paper is structured as follows. Section 2 describes the baseline de-centralised credit-banking model, and using representative simulations from the solved model, illustrates it's inability to capture the observed variability in *consumption* velocity. Section 3 extends the model to include habit persistence and illustrates how this changes the behaviour of the baseline model. Section 4 extends the baseline-habit model to also include adjustment costs to investment, which raises the volatility in both real and nominal rates, as demonstrated by Jermann (1998). In similar fashion to Hodrick et al. (1991), a sensitivity analysis is conducted, based on simulations of the final extended model using combinations of range of parameter values related to habit persistence and investment adjustment costs. Section 5 discusses the results obtained, section 6 concludes.

2.2 Decentralised Credit-Banking: A Baseline Model

The representative agent economy is a standard monetary cash-in-advance real business cycle model (Lucas, 1982; Stokey and Lucas, 1987), which is only modified by adding a further means-of-exchange, credit, which is costly produced by a decentralised financial intermediary (FI) by use of a two factor CRS Cobb-Douglas production function. Following Gillman and Kejak (2008), deposits are created from the total exchange liquidity used in the model for carrying out consumption both in terms of money and credit, which means that consumption and (real) deposits can be used interchangeably:

$$d_t \equiv c_t \tag{2.2}$$

The same amount of deposits (or equivalently the level of consumption) is then used as an input factor to credit production in combination with banking time, which is a credit production specification motivated by the financial intermediation literature (see Hancock, 1985; Clark, 1984). In principle, physical capital could also feature as another input factor in credit production, but is omitted for simplicity.

2.2.1 The financial intermediary

In contrast to Gillman and Kejak (2008) but similar to Benk et al. (2005), physical capital is only used in the goods production sector, whereas credit production is CRS Cobb-Douglas in labour only and deposits. As in Benk et al. (2005), the economy is modeled such as to assume zero growth. The credit production function is therefore given by:

$$f_t = e^{v_t} A_f \left(n_{f,t}\right)^{\rho} \left(d_t\right)^{1-\rho} \tag{2.3}$$

where $n_{f,t}$ represents the fraction of labour time spent in the credit production sector, v_t is the credit shock (which throughout the paper is held fixed at it's steady state value) and A_f is the total factor productivity parameter in credit production, which may generally differ in steady state from the same parameter in the goods production sector, analogously given by A_g. It should be pointed out at this stage that f_t represents a produced stock variable of costly credit, as opposed to money, which is modelled as a flow variable. The credit production function in the *level* of credit can alternatively be written as a decreasing returns-to-scale deposit-normalised credit share production function (see Gillman and Kejak, 2008) given by:

$$f_t^{*} = \frac{f_t}{d_t} = e^{v_t} A_f \left(\frac{n_{f,t}}{d_t}\right)^{\rho} \tag{2.4}$$

Gillman et al. (2007) show in an endogenous growth, Yeoman-version of the same credit economy, how appropriate parametrisation of the diminishing returns parameter ρ leads to a convex upward-sloping marginal cost schedule, which for a given price of credit (equal to the net nominal interest rate), translates into elasticities of money demand of variable size (depending on the *level* of calibrated variables) with respect to key variables, such as the nominal (net) rate of interest and the shock to credit TFP. The properties of the credit production function leading to this and other results is also discussed in more depth in the decentralised steady state discussion provided in Gillman and Kejak (2008). Preference-based cash-credit models, on the other hand, as discussed in Cooley and Hansen (1995) or Stokey and Lucas (1987) exhibit a uniformly much lower interest rate elasticity and abstract from a credit-production sector which may be subjected to shocks. This same low interest elasticity of preference-based cash-credit models is also documented quantitatively in Hodrick et al. (1991).

As in Gillman and Kejak (2008), the financial intermediary is assumed to operate competitively and sets the price of deposits before the household decides how much of the deposits to hold, as with mutual banks. The bank has to obey a solvency restriction, where assets have to be equal to liabilities, given by:

$$P_t f_t + M_t = P_t d_t \tag{2.5}$$

where f_t and M_t are the beginning-of-period stocks of credit and money, respectively. Also, the liquidity constraint implies that cash sourced from the bank for shopping has to be backed by deposits:

$$P_t d_t \geq P_t c_t \tag{2.6}$$

The above two constraints collapse into a single one when credit production is zero and deposits only consist of cash balances held with the FI. The financial intermediary is assumed to be profit-maximising, and it's labour-demand can therefore be obtained by solving the problem:

$$\max_{n_{f,t}} R_{f,t} d_t = p_t^f f_t - w_t n_{f,t} \tag{2.7}$$

where in particular p_t^f is the price in terms of the consumption good the household is paying to the FI per unit of credit used, which in an equilibrium has to equal the cost of otherwise using money, which is the net nominal rate of interest, or $p_t^f = i_{t-1}$. The optimisation problem results in the standard first-order condition of setting the real wage equal to the marginal (revenue) product of labour in credit-production:

$$w_t = p_t^f e^{v_t} A_f \rho \left(n_{f,t}\right)^{\rho-1} \left(d_t\right)^{1-\rho} \tag{2.8}$$

since the model determines the price of credit endogenously by setting it equal to the opportunity cost of using the alternative means of exchange (money), given by the net nominal rate of interest, the above condition can also be re-stated as:

$$w_t = i_{t-1} \frac{f_t}{n_{f,t}} \tag{2.9}$$

The value of credit production due to deposits (consumption) is then re-distributed back to the representative household in form of a dividend per-unit of deposits (which are equal to consumption). Since the un-normalised value (i.e. the total revenue share due to deposits in credit production) is given by $R_{f,t} = f_t \left(1 - \rho\right) i_{t-1}$, the normalised dividend or return is thus given by:

$$\tilde{R_{f,t}} = \frac{R_{f,t}}{d_t} = \left(\frac{f_t}{d_t}\right)(1-\rho)\, i_{t-1} = f_t^*\,(1-\rho)\, i_{t-1} \tag{2.10}$$

Since this term is paid out per unit of deposits and therefore per unit of consumption, the model will exhibit an exchange cost of consumption different from standard cash-in-advance models, which is an average exchange cost distorted by the cost of producing credit (see Gillman and Kejak, 2008), to be discussed in more detail in the following section describing the representative household's optimisation problem.

2.2.2 The goods firm

The firm producing aggregate output y_t is spelled out in decentralised fashion and is also assumed to be the owner of the stock of physical capital. It maximises the net present value of cash flows remitted back to the household in form of dividend payments made on equity holdings, where discounting is carried out such as to respect the stochastic discount factor of the household. The firm's problem is therefore formulated as:

$$\max_{n_{g,t},k_t} E_t \sum_{k=0}^{\infty} \frac{\beta^k \lambda_{t+k}}{\lambda_t} \left\{ y_{t+k} - w_{t+k} n_{g,t+k} - i_{t+k}^k \right\} \tag{2.11}$$

where $n_{g,t}$ is the fraction of time spent in goods production and i_t^k is the amount of investment into physical capital, which the firm pays for exclusively from retained earnings. The technology employed in producing aggregate output is given by a standard constant returns-to-scale Cobb-Douglas production function, given by:

$$y_t = e^{z_t} A_g \left(n_{g,t}\right)^{\alpha} \left(k_{t-1}\right)^{1-\alpha} \tag{2.12}$$

where A_g is the (steady state) goods sector total factor productivity parameter, z_t the corresponding productivity shock, and k_{t-1} the predetermined level of physical capital employed in production. Investment in physical capital i_t^k satisfies the following constraint:

$$i_t^k = k_t - (1-\delta)\, k_{t-1} \tag{2.13}$$

Maximisation of the firm's problem then yields the usual first-order conditions of employing up to the point at which the wage rate equals the marginal product of goods labour, and of installing more physical capital up to the point where the marginal cost today is equal to the discounted future return in terms of the future marginal product of capital net of depreciation. The former condition implies:

$$w_t = \alpha e^{z_t} A_g \left(n_{g,t}\right)^{\alpha-1} \left(k_{t-1}\right)^{1-\alpha} = \alpha \frac{y_t}{n_{g,t}} \tag{2.14}$$

the latter condition related to the optimal amount of physical capital implies (the consumption Euler equation):

$$\begin{aligned}
\lambda_t =& \beta E_t \lambda_{t+1} \left[(1-\alpha)\, e_{z_{t+1}} A_g \left(n_{g,t+1}\right)^{\alpha} \left(k_t\right)^{-\alpha} + (1-\delta)\right] \\
=& \beta E_t \lambda_{t+1} \left[(1-\alpha) \frac{y_{t+1}}{k_t} + (1-\delta)\right] \\
=& \beta E_t \lambda_{t+1} \left[r_{t+1}^k + (1-\delta)\right]
\end{aligned} \tag{2.15}$$

where r_{t+1}^k is the future expected marginal product (or marginal net return exclusive of depreciation) of the current level of installed units of physical capital, decided upon optimally in period t.

2.2.3 The household

The representative household receives it's only non-financial income from selling it's labour endowment to the goods and credit sector at the equilibrium real wage. Since physical capital is assumed to be owned by the goods firm and the optimal investment decision left to the latter, besides receiving it's wage bill, the household also holds money and a vector of financial assets, which may include a risky stock in the firm and inflation-indexed real bonds, but possibly also other assets[10]. Coupled with the vector of assets are corresponding price and dividend vectors, given by p_t^a and d_t^a, respectively, where as an example the "dividend" of the inflation-indexed (or real) bond is just equal to 1, i.e. paying the representative household one unit of the consumption good[11]. Utility is derived from following function:

$$U_t\left(c_t, l_t\right) = \log c_t + \Psi \log l_t \tag{2.16}$$

where I have assumed the representative household's utility to be separable in it's two arguments, consumption c_t and leisure l_t, and logarithmic in both consumption and leisure. The household's budget constraint is therefore given by:

$$w_t\left(n_{g,t} + n_{f,t}\right) + a_{t-1}'\left(p_t^a + d_t^a\right) + \frac{m_{t-1}}{\left(1+\pi_t\right)} + v_t \tag{2.17}$$

$$+ \tilde{R}_{f,t}c_t \geq c_t + a_t' p_t^a + m_t + p_t^f f_t \tag{2.18}$$

where w_t is the real wage, $n_{g,t}$ the amount of labour time spent in goods production and $n_{f,t}$ the amount of time spent in credit production. Notice that the total time

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[10] I follow the notation chosen by Jermann (1998).

[11] This is an all-encompassing notation chosen so as to be able to price all assets within one notational framework, where the dividend paid for the real short-term risk-free government bond is just equal to 1 unit of the consumption good. The same convention is used, for the sake of simplicity, in (Jermann, 1998)

endowment of the representative household is normalised to one, translating into the following time constraint:

$$1 - l_t = n_{g,t} + n_{f,t} \tag{2.19}$$

Using credit incurs a cost in terms of a price charged per unit of credit, so that the total cost from using credit f_t is given by $p_t^f f_t$. The household also receives a dividend payment from the financial intermediary in form of a return per unit of deposit, translating into a total payout due to deposits held with the FI given by $\tilde{R}_{f,t} c_t$. This dividend distorts the usual marginal rate of substitution between consumption and leisure so as to be different from an otherwise standard CIA model. Using the first-order conditions with respect to consumption and leisure results in the steady state relationship:

$$MRS_{c,l} = \frac{\Psi c_t}{l} = \frac{1 + \tilde{i}}{w} \tag{2.20}$$

where $1 + \tilde{i}$ is given by:

$$\begin{aligned}(1 + \tilde{i}) &= 1 + i - f^* (1 - \rho) i \\ &= 1 + (1 - f^*) i + f^* \rho i \end{aligned} \tag{2.21}$$

which shows that the distorted exchange cost affecting the marginal rate of substitution between consumption and leisure equals an average exchange cost which endogenously varies with the share of credit used in consumption (see Gillman and Kejak, 2008, 2005). Further, m_{t-1} is the real value of predetermined money balances held at the beginning of the period and v_t represents some real-valued lump-sum tax governing the growth rate of the money supply. The household purchases the consumption good subject to an exchange constraint, given by[12]:

$$\frac{m_{t-1}}{(1+\pi_t)} + v_t + f_t \geq c_t \tag{2.22}$$

where both money and the credit exchange service can be used in conjunction to pay for the consumption good. For a positive nominal rate of interest, the constraint always binds in a strict sense, which is assumed throughout. Notice that by defining the multiplier on the budget constraint to be equal to λ_t and the multiplier on the liquidity constraint to be μ_t, taking first-order conditions with respect to credit f_t, results in:

$$p_t^f = \left(\frac{\mu_t}{\lambda_t}\right) = i_{t-1} \tag{2.23}$$

demonstrating that in the decentralised credit-banking model, in equilibrium the price of credit has to equal the opportunity cost of otherwise using money, which equals the net nominal rate of interest, i_{t-1}. The growth rate of the money supply has a deterministic and could in principle also be given some random component, and could thus be defined as:

$$m_t = \Theta_t \frac{m_{t-1}}{(1+\pi_t)} = (\Theta^* + e^{u_t} - 1) \frac{m_{t-1}}{(1+\pi_t)} \tag{2.24}$$

where u_t represents the stochastic component of the money supply growth rate, which is modeled assuming a log-normally distributed autoregressive process of order one, as is the productivity shock in the goods sector. However, throughout I am going to set this shock equal to it's steady state value, meaning that throughout is

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[12] Throughout I am going to assume a *fixed* "k-percent" Friedman-type growth rate rule of the money supply, meaning that $v_t = \bar{v} \quad \forall \quad t$

set to $u_t = 0$, and thus assume that:

$$m_t = \frac{\Theta^*}{1+\pi_t} m_{t-1} \tag{2.25}$$

where Θ^* represents the exogenously specified deterministic gross growth rate of the nominal money supply. Therefore, I am going to assume a Friedman-type "k-percent" deterministic growth rate of the nominal supply of money Friedman (1960). Notice also that although, for purposes of comparison, the present model has been described as also containing a shock to the productivity of the credit sector, as in Gillman and Kejak (2007), throughout this paper I am going to set this shock equal to it's steady state value and keep it fixed in simulations. The vector of shocks can thus be summarised as $s_t = [z_t, u_t, v_t]$, where in simulations this vector assumes the shocks to be modeled in logs and to be autoregressive of order one with innovations normally distributed and zero off-diagonal variance-covariance matrix. Therefore, formally, the structural shocks affecting the stochastic off steady state characteristics of the model can be summarised in VAR form as:

$$s_{t+1} = \Phi s_t + \epsilon_{t+1} \tag{2.26}$$

where Φ is a $3x3$ matrix containing the autocorrelation parameters specified along the diagonal of Φ and $\epsilon \sim (0, \Omega)$. Although coefficients of autocorrelation for the structural credit productivity shock as well as the money growth rate shock may be formally be specified, they will not matter in practice, as the structural credit and money shock innovations $\epsilon_{v,t}$ and $\epsilon_{u,t}$ will always be set equal to zero. Since the economy contains no growth trend, and all variables have been expressed normalised by dividing by the relative price of money P_t, the definition of the equilibrium can be set up in recursive form. Denoting the state of the economy as $s_t = [k_{t-1}, m_{t-1}, z_t, u_t, v_t]$, and with $\beta \in (0,1)$, the representative household's optimisation problem can be written in recursive form as:

$$V(s) = \{\log c_t + \Psi \log l_t + \beta EV(s')\} \tag{2.27}$$

The model is solved by symbolically differentiating the first-order conditions, market-clearing identities and exogenously specified laws of motion with respect to current endogenous states, future exogenous states and future endogenous jump (control) variables, as well as with respect to pre-determined past period endogenous states, current exogenous states and current period jump variables, thus obtaining the Jacobian of the system. Before differentiating, all variables (except for rates) will have been expressed in logs. The Jacobian can then be evaluated at the (log) steady state, split into matrices A and B, which can be used to solve for the recursive law of motion using the Schur decomposition as documented in Klein (2000) and Klein and Gomme (2008). I therefore solve for the recursive law of motion using a first-order perturbation method, where the local approximation is taken around the log steady state of the system (except for rates, which are in levels), so that the matrices describing the solution to the system typically contain elasticities and thus percentage changes. The resulting stationary recursive laws of motion are expressible as:

$$\mathbf{X}_t = P\mathbf{X}_{t-1} \tag{2.28}$$

$$\mathbf{Y}_t = F\mathbf{X}_{t-1} \tag{2.29}$$

where the matrices P and F contain the elasticities and describe the solution to the system and vectors $\mathbf{X}$ and $\mathbf{Y}$ contain the endogenous and exogenous states, and the endogenous control (jump) variables, respectively. A stable (non-explosive) solution requires all elements in P to lie within the unit circle, so as to make the evolution of the endogenous and exogenous states behave according to some stationary process.

2.2.4 Credit Production: The Source of Variability in Money Velocity

Before discussing the choice of calibrated values for relevant variables determining the steady state and off-steady state locally approximated dynamics, it will be instructive to discuss the source of variability in consumption velocity within the decentralised credit-banking model. A similar analysis is also carried out in Gillman and Kejak (2007), who discuss and derive the interest rate elasticity of velocity using the same credit production function. Here, instead of focusing on consumption-money velocity directly (given by $M_t/P_tc_t = m_t/c_t$, I conduct my discussion using the inverse of relevant velocity measures (which are the means-of-exchange *shares* in consumption), as volatility in the inverse of a velocity measure translates into volatility of that velocity measure itself. Therefore, substituting the implied labour demand in the credit production sector back into the credit production function, I obtain

$$f_t^* = (A_f e^{v_t})^{\frac{1}{1-\rho}} \left(\frac{\rho i_{t-1}}{w_t}\right)^{\frac{\rho}{1-\rho}} \tag{2.30}$$

which can be log-linearised around the log of steady state variables, to give:

$$\hat{f}_t^* \equiv \hat{f}_t - \hat{c}_t = \left(\frac{1}{1-\rho}\right)\hat{v}_t + \left(\frac{\rho}{1-\rho}\right)\hat{i}_{t-1} - \left(\frac{\rho}{1-\rho}\right)\hat{w}_t \tag{2.31}$$

Setting the credit shock equal to zero, which is assumed throughout the present paper, and recalling that $\hat{i}_{t-1} = \hat{p}_t^f = \hat{\mu}_t - \hat{\lambda}_t$, the above equation can be written as:

$$\begin{aligned} \hat{f}_t^* \equiv \hat{f}_t - \hat{c}_t &= \left(\frac{\rho}{1-\rho}\right)\left(\hat{p}_t^f - \hat{w}_t\right) \\ &= \left(\frac{\rho}{1-\rho}\right)\left(\hat{\mu}_t - \hat{\lambda}_t - \hat{w}_t\right) \end{aligned} \tag{2.32}$$

Notice then, since $\hat{f}_t^*$ is the log-deviation of the share of credit used in purchasing consumption, then $\hat{m}_t^* \equiv \hat{m}_t - \hat{c}_t = \left(1 - \frac{\bar{c}}{\bar{m}}\right)\hat{f}_t^*$ represents the log-deviation of the share of money used in purchasing the consumption good. Sufficient variability in the latter defined share, $\hat{m}_t^*$, translates into sufficient variability of it's inverse $(\hat{m}_t^*)^{-1} = V_t^m$ which is equal to money velocity.

This means that the credit share has to be sufficiently variable in order to obtain sufficient variability in money consumption velocity. The choice of calibrating the labour share parameter ρ in credit production, which matters in the present discussion as it affects the elasticity of the credit share with respect to $\hat{v}_t$, $\hat{i}_{t-1}$ and $\hat{w}_t$, has varied in studies conducted thus far. For instance, Benk et al. (2005) base their calibration of $\rho = 0.21$ on a time series estimate conducted by Gillman and Otto (2005). Gillman and Kejak (2007) obtain a lower calibrated value at $\rho = 0.13$, and show how it can be obtained using financial industry data, Scheffel (2008) also calibrates the credit labour share value to 0.21 so as to match the model's predictions of asset prices. Also, related to this Gillman and Kejak (2008) prove how a value of $\rho < 0.5$ is required for the marginal cost schedule to be convex.

In the present study, I will calibrate $\rho = 0.18$, so as to be comparable to previously chosen calibrations. As previous studies have indicated that a realistic range of this value appears to be $0.1 < \rho < 0.25$, this leads to the direct consequence that credit shocks can potentially have much stronger effects on money velocity than either changes in the price of credit or the wage rate. Using $\rho = 0.18$ as a benchmark case, leads to a credit share elasticity with respect to $\hat{v}_t$ equal to $\eta_v = \left(\frac{1}{1-\rho}\right) = 1.22$, whereas the same elasticity for the price of credit and the wage rate is $\eta_i = \eta_w = \left(\frac{\rho}{1-\rho}\right) = 0.22$, much lower. Disregarding credit (and money growth rate) shocks alltogether and only focusing on how goods productivity shocks can affect money velocity through the price (and indirectly also the wage) channel, requires the price-wage (or interest-wage) ratio to be volatile enough, which as I will show can be achieved by making the real (an for given inflation expectations) and thus also the nominal rate more volatile.

Indeed, as I will demonstrate, combining habit persistence in consumption with adjustment costs to investment as in Jermann (1998), leads to a more volatile behaviour of the representative household's marginal valuation, $\hat{\lambda}_t$, a consequently more volatile stochastic discount factor and thus also a more volatile behaviour of the real rate of interest. As the volatility of marginal valuation also affects the volatility of the credit share (and thus money velocity), this - for some given conditional behaviour of the wage rate - can potentially enhance the effect of the price channel alone on consumption velocity. Before turning to the baseline model's simulation evidence, and later one similar evidence from extended version of the baseline model, I will summarise the calibration of the model in the section which follows.

2.2.5 Steady State Calibration

The above table summarises the chosen baseline calibration - where in anticipation of extensions to the baseline model using habit in consumption and adjustment costs to investment - the table already contains calibrated values for parameter values relevant for the extensions as well. Turning attention to the calibrated parameters, first of all, the discount factor β, the labour share in goods production α, the steady

Table of benchmark calibrated Parameters			
β =0.99	discount factor	ρ =0.18	credit labour param.
α =0.64	goods labour param.	f^* =0.31	credit-to-cons ratio
A_g=1.0	TFP goods	A_f=1.204	TFP credit
l =0.7	leisure	n_f=0.00044	credit labour
Θ =1.0125	money g.	n_g=0.29956	goods labour
b =0.8	habit pers.	κ =3.0	cap. adj. cost
ϕ_u=0.70	AR money g. shock	ϕ_z=0.95	AR goods shock
ϕ_v=0.95	AR credit shock	ϵ_z =0.0075	s.d goods shock
ϵ_u =0.0(set to 0)	s.d. moneyg shock	ϵ_v =0.00(set to 0)	s.d credit shock

Table 2.1: Baseline Calibration

state total labour-leisure trade-off l and $n_f + n_g$ and the steady state growth rate of money, are all set to standard values familiar from calibration exercises of similar models conducted elsewhere. Specifically then, labour in goods production receives approximately 2/3 of the value of production in form of the goods production wage bill and leisure l is 70% of the total time endowment. Also, by calibrating the discount factor $\beta = 0.99$, I obtain an annualised steady state real rate of 4%, and by setting the quarterly gross growth rate in the money supply $\Theta = 1.0125$, I obtain an annualised steady state growth rate of inflation of 5%.

For the credit-banking sector, I chose calibrated values for the steady state share of credit $f^* = 0.31$ to be very close to the same value chosen by Benk et al. (2005) (who set this equal to 0.3) and the labour share parameter $\rho = 0.18$, which is slightly less than what the same authors choose (they set this equal to 0.21). Given the TFP value in the goods sector of $A_g = 1.0$, finding the root of a non-linear system of equations residually determines $n_f = 0.00044$ (which is very close to Benk et al.'s obtained 0.00049) and $A_f = 1.204$ (compared to Benk et al.'s obtained value of 1.422). Structural shock autocorrelation parameters are also chosen in standard fashion and are set to $\pi_u = 0.70$ and $\phi_z = 0.95$, with standard deviations of $\epsilon_u = 0.01$ and $\epsilon_z = 0.0075$, respectively, all of which are as in Benk et al. (2005).

Notice that, although the autocorrelation parameters on the credit and money growth shocks are formally calibrated, the standard deviation of the innovation is set to zero for both cases, so as to effectively eliminate those shocks in simulations of the solved model. The habit persistence parameter was calibrated to $b = 0.8$ as in Jermann (1998) and Constantinides (1990). Finally, using a specification of the adjustment cost function to investment from Canzoneri et al. (2007a), the relevant parameter $\kappa = 3.0$, which is much smaller than Canzoneri et al.'s chosen value of 8.0, thus calibrating the model on the conservative side regarding investment adjustment costs. Next, I am going to discuss simulation results obtained from the baseline model.

2.2.6 Simulation results

In this section I am going to present simulation evidence from the calibrated baseline model. Evidence is presented in two different ways. First of all, graphs of a representative simulation run are provided so as to allow visual inspection and verification of key properties. All graphs also show standard errors (of percentage deviations)[13] - and correlation coefficients - from the representative one-off simulation, so as to convey clearly the volatility of various simulated time series. To emulate a typical quarterly post-war sample size, the simulation length is always fixed at $n = 200$, all simulated series are hp-filtered prior to graphing and computing relevant statistics. Also, shocks to productivity have been drawn once and then kept fixed in graphs across *all* extended versions of the baseline model (inclusive of the baseline model itself), so as to make simulation graphs directly comparable across model versions.

Secondly, tables with key statistical measures computed from simulations of the model with habit persistence and adjustment costs to investment are provided, which

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[13] where interest rates are quarterly deviations.

are not based on one, but 1000 simulations so as to obtain expected values and standard errors of statistical measures. The simulation length is, as before, held fixed at $n = 200$. Tables are only computed for the final extended version of the baseline model, as for appropriate choices of the parameter space, this model version nests all other models versions. The main focus throughout is to examine if sufficient variability in (money-) consumption velocity can be obtained without requiring too volatile interest rate series (both real and nominal), but other key statistics are also examined. Regarding the definition of "sufficient" variability in consumption velocity, I use the reported sample means calculated by Hodrick et al. (1991), who measure variability in velocity using the coefficient of variation statistic to capture this measure[14].

For annual data, they obtain a coefficient of variation approximately equal to 0.46 with a standard error of 0.0097, and for quarterly data corresponding values of approximately 0.4 with standard error 0.006. A secondary concern, also examined by Hodrick et al. (1991), is whether this can be achieved with statistics of other key variables lying within plausible ranges as well, where my approach is more focused here mainly emphasising the joint volatility of interest rates. So with regards to this, I take the computed sample value of the percentage standard deviation of the real risk-free rate in Mehra and Prescott (1985) as a benchmark, which is also reported in Jermann (1998), and is equal to 5.76%. So a model calibrated quarterly needs to obtain a value for the coefficient of variation in consumption velocity of 0.4 or more *and* a standard deviation of the real rate in the neighbourhood of 5.76%[15] in order to successfully jointly capture the second moments of consumption velocity and real

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[14] This is defined as the ratio of the standard deviation to the mean, so it is essentially equal to a % standard deviation, which has the advantage of being scale-free.

[15] Actually, in principle assuming a much lower variability in real rates is permissible, as the latter reported figure of Mehra and Prescott (1985) is based on the variability of *ex-post* realised real rates, as opposed to their *ex-ante* expected counterparts, using *expectations* of inflation.

interest rates.

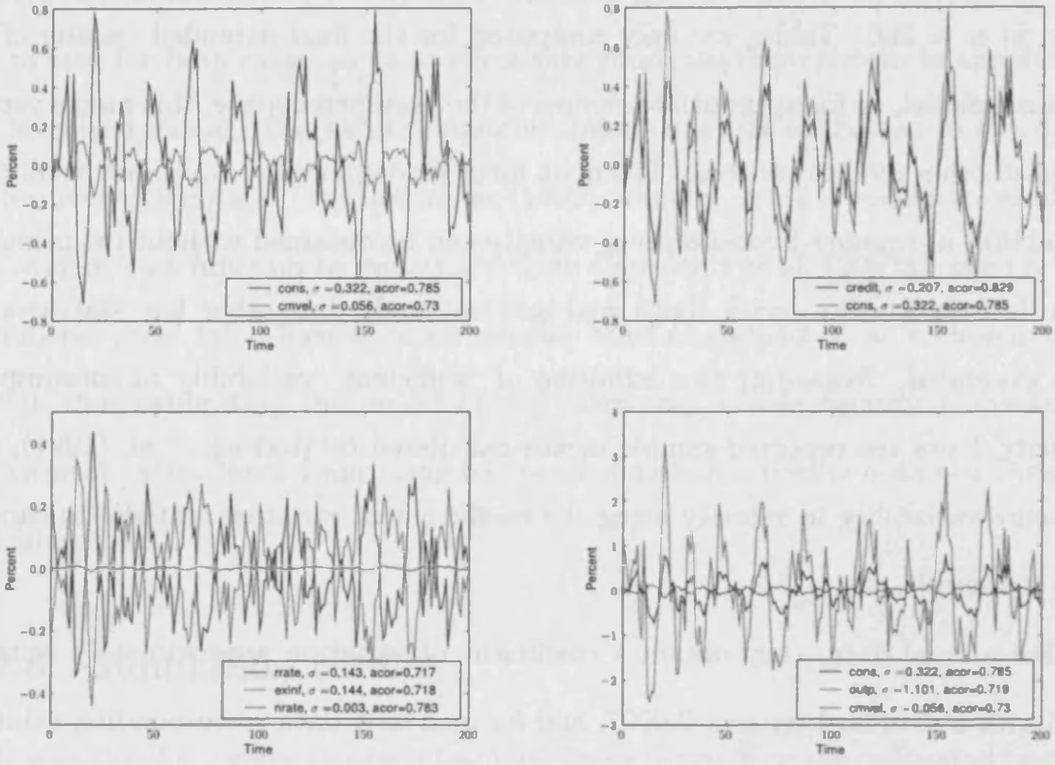

Figure 2.1: Baseline Model

Figure (4.1) documents the behaviour of various key variables of the baseline credit-banking model described in the theoretical section, using graphs of one representative simulation run. First of all, as is usual for standard (cash-in-advance monetary) real business cycle models containing no additional frictions, the stochastic discount factor (and thus the real rate) is conditionally relatively high whenever (expected) consumption is higher than on average. As usual, since this coincides with lower current period marginal valuation vis-a-vis expected future marginal valuation (thus leading to a conditionally low intertemporal marginal rate of substitution), the conditional real interest rate is high, so as to prevent the household from borrowing against the expected future rise in consumption[16].

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[16] (see Uhlig, 1995, p.15).

More importantly, the top and bottom left-hand quadrants illustrate that the baseline model's variability in the nominal rate of interest is very low, thus also leading to very little variability in the price of credit. This in turn leaves the variability in the share of credit in consumption very low, thus resulting in a very low variability of consumption velocity. The nominal rate turns out to be so dampened in its movement relative to the real rate, as increases in consumption (and thus liquidity demand) is not entirely covered by increases in credit, and thus has to be partially also met by increasing real money balances via *a fall* in (expected) inflation. In this particular case, inflation expectations moving in opposite direction to the real rate, leave the nominal rate fairly invariant. A similar situation occurs in the model with habit persistence, where I discuss this property in more detail.

2.3 Adding habit persistence

In this section I am going to investigate the model's properties which are obtained by adding habit persistence in consumption, which is of relative habit type as in Constantinides (1990), and not of "keeping-up-with-the-Joneses" type, as in Abel (1990). Doing so amounts to a modification of the utility function of the representative household, thus affecting the stochastic as well as the steady state expression for the marginal utility of consumption. The modified utility function including habit persistence is thus given by:

$$U_t = \log\left(c_t - bc_{t-1}\right) + \Psi \log l_t \tag{2.33}$$

The marginal utility with respect to current consumption will therefore also include a term taking into account how the choice of current consumption affects next period's marginal utility. Therefore:

$$U_{c,t}=\frac{\partial U_t}{\partial c_t}=\frac{1}{(c_t-bc_{t-1})}-b\beta\frac{1}{(E_tc_{t+1}-bc_t)} \tag{2.34}$$

Habit persistence eliminates the time-separability in consumption, as current margina valuation not only depends on current consumption, but on current consumption relative to some fraction of last period's level of consumption. Notice that although this creates a dynamic smoothing objective for consumption[17], the steady state value of marginal valuation *and* the marginal rate of substitution between consumption and leisure are practically unaltered. This follows from writing down the steady state version of the marginal utility of consumption:

$$\bar{U}_c=\frac{(1-\beta b)}{(1-b)\,\bar{c}}\approx=\frac{1}{\bar{c}} \tag{2.35}$$

which for $\beta = 0.99$ can be approximated by the steady state of the marginal utility of consumption for the logarithmic case

Figure (4.2) illustrates the conditional behaviour of key variables of the credit-banking model with habit persistence in consumption. The top right-hand quadrant diagram documents how the representative household's smoothing objective (relative to the previous level of consumption) leads to an endogenous consumption process which is highly autocorrelated and much smoother compared to the baseline model's prediction, in which consumption responds in proportional fashion to productivity shocks hitting the goods production sector. Due to habit persistence, the same quadrant also reveals how consumption is less volatile, whereas credit becomes more volatile, in particular relative to consumption. In general, compared to the baseline model, the level of credit becomes to some extent disentangled from the

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[17] i.e. the evolution of consumption will optimally be more autocorrelated (see den Haan, 1995).

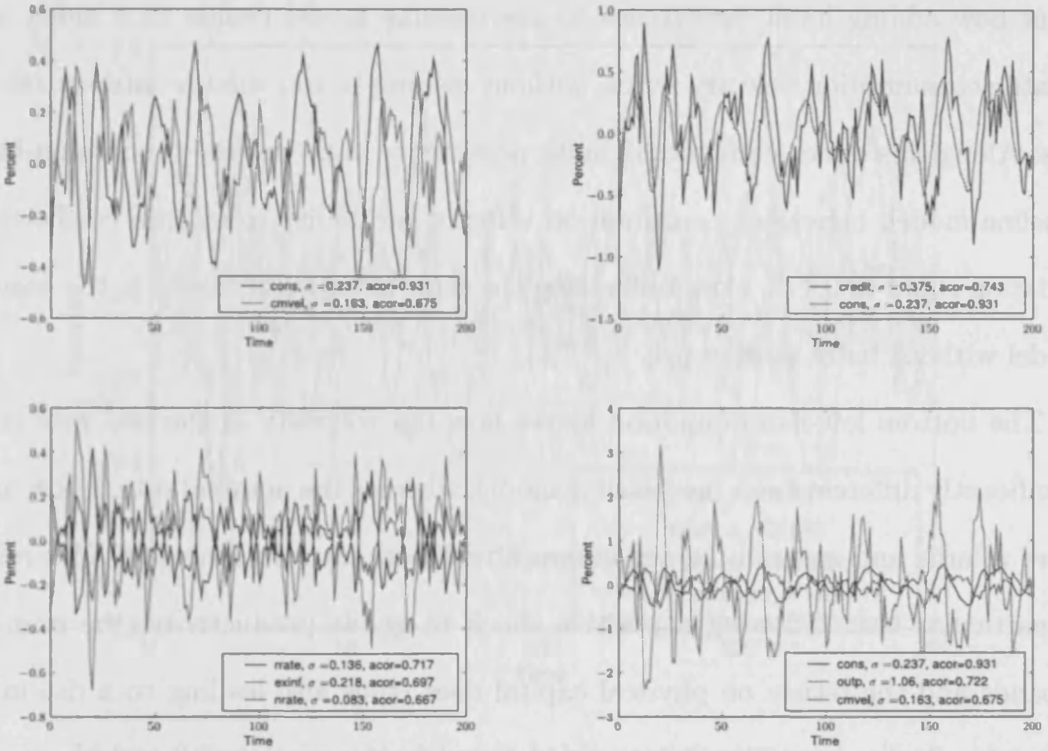

Figure 2.2: Baseline Model with habit persistence

endogenous consumption process, and since past consumption works as a "drag" on current consumption, current consumption reacts less responsively to current period productivity shocks.

The top left-hand quadrant shows consumption-money velocity graphed against the level of consumption. In spite of the already mentioned inertia displayed in the endogenous consumption process, which results in velocity leading consumption slightly, the graph still displays that consumption-money velocity is counterfactually counter-cyclical with both consumption (but less so than with output, due to consumption's habit-induced inertia) and output. The counter-cyclicality of velocity displayed by the baseline model with habit persistence only thus runs counter evidence from the U.S. documented by Hansen (1985) and Gillman and Kejak (2007). At the same time both the top left-hand and bottom right-hand quadrants document how adding habit persistence to the baseline model results in a much more volatile consumption velocity series, without leading to too volatile interest rate series. Along this dimension, adding habit persistence improves the production-based baseline model, increasing consumption velocity variability to reach a coefficient of variation equal to 0.173, close to trebling the same measure obtained in the baseline model without habit persistence.

The bottom left-hand quadrant shows how the volatility of the real rate is not significantly different from the baseline model, whereas the nominal rate is now much more volatile and generally moves in opposite direction to the real rate. This means in particular that following a positive shock to goods productivity, the economy expands and the return on physical capital rises (thus also leading to a rise in the real rate of interest). The higher productivity in the goods sector relative to the credit production sector leads to a movement of labour to the former, thus resulting in a drop of credit production.

Less credit produced means more money demanded for some level of consumption purchases, which requires pre-determined money balances to be adjusted upwards by a drop of inflation below it's steady state value and convergence of the latter from below. But this implies a fall in inflation expectations which - through the Fisher equation - is strong enough so as to result in a nominal rate moving in opposite direction to the real rate. In order to illustrate this last point better, figure (4.3) shows the co-movement of consumption, the nominal and real rate, and expected inflation, which through the Fisher relationship will move so as to equate the two rates in real terms, adjusting the nominal rate by inflation.

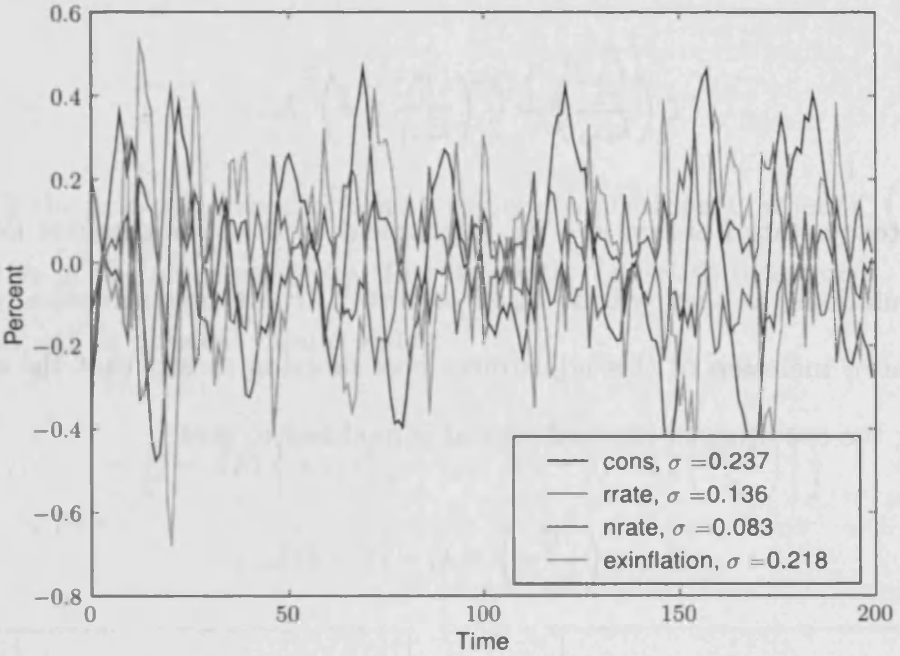

Figure 2.3: Baseline Model with habit persistence, prod. shock only

2.4 Adding adjustment costs to investment

In this section I am going to investigate the model's properties which are obtained by adding also adjustment costs to investment to the baseline model in addition to habit persistence in consumption. Adjustment costs have been studied by Eisner and Strotz (1963), Prescott and Lucas (1971), Hayashi (1982), Baxter and Crucini (1993) and Jermann (1998). They typically also feature in many models of the new neoclassical synthesis type, such as Lawrence J. Christiano and Evans (2005) and Canzoneri et al. (2007b). This modification requires a discussion of the modified problem of the decentralised aggregate output producing firm. First of all, notice that I use an adjustment cost function as in Canzoneri et al. (2007a). This function is given by:

$$\zeta\left(\frac{i_t^k}{k_{t-1}}\right)=\frac{\kappa}{2}\left(\frac{i_t^k}{k_{t-1}}-\delta\right)^2 k_{t-1} \tag{2.36}$$

Since in steady state I obtain $\bar{i^k}=\delta\bar{k}$, adjustment costs will not matter for steady state calculations, as they will be equal to zero[18]. However stochastically (or off steady state), inclusion of this adjustment cost function means that the equation describing the evolution of physical capital is modified to give:

$$i_t^k-\zeta\left(\frac{i_t^k}{k_{t-1}}\right)=k_t-(1-\delta)\,k_{t-1} \tag{2.37}$$

The firm's problem is then setup differently, where first-order conditions with respect to capital *and* investment have to be taken. The firm's modified problem is thus stated as:

$$\begin{aligned}\max_{n_{g,t+k},k_{t+k}} E_t\sum_{k=0}^{\infty}\frac{\beta^k\lambda_{t+k}}{\lambda_t}\Bigg\{ & y_{t+k}-w_{t+k}n_{g,t+k}-i_{t+k}^k \\ & +\frac{\xi_{t+k}}{\lambda_t}\left[i_{t+k}^k-\zeta\left(\frac{i_{t+k}^k}{k_{t+k-1}}\right)-k_{t+k}+(1-\delta)\,k_{t+k-1}\right]\Bigg\}\end{aligned} \tag{2.38}$$

where the multiplier on the firm's capital accumulation constraint is equal to marginal utility in steady state only, i.e. $\bar{\lambda}=\bar{\xi}$. Notice that the ratio of the marginal value of installed physical capital to the marginal value of one extra unit of wealth is equal to Tobin's q, i.e. $q_t=\xi_t/\lambda_t$. Taking first-order conditions with respect to investment i_t^k and the end-of-period physical capital stock k_t, results in the following conditions

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[18] This property does not only hold for the average adjustment costs, but also the derivatives with respect to capital and investment.

of optimality, which differ from the baseline model:

$$\lambda_t = \xi_t \left[1 - \zeta_i' \left(\frac{i_t^k}{k_{t-1}} \right) \right] \tag{2.39}$$

which is the first-order condition with respect to investment, where $\zeta_i' \left(\frac{i_t^k}{k_{t-1}} \right)$ is the derivative of the adjustment cost function with respect to investment. Optimality with respect to physical capital yields:

$$\xi_t = \beta E_t \left\{ \lambda_{t+1} r_{t+1}^k + \xi_{t+1} \left[(1 - \delta) - \zeta_k' \left(\frac{i_{t+1}^k}{k_t} \right) \right] \right\} \tag{2.40}$$

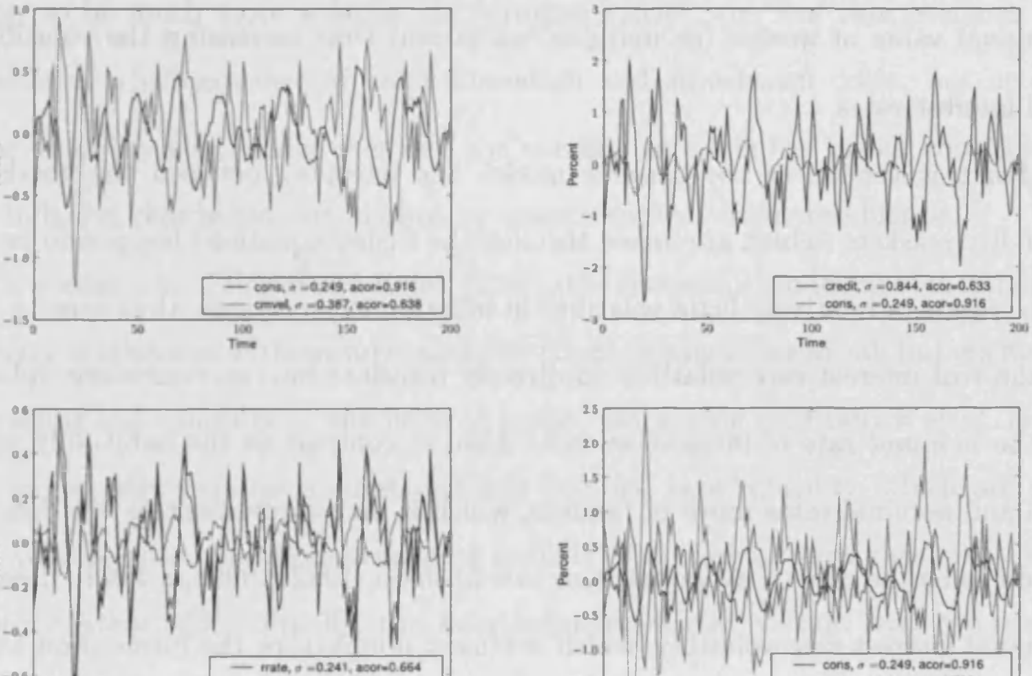

Figure 2.4: Baseline Model with habit persistence & adjustment costs

Figure (4.4) summarises key results of the baseline model with habit and adjustment costs to investment using graphs of a representative simulation run. Adding adjustment costs to investment to the model which already contains habit in consumption has two well-known effects documented by Jermann (1998). Firstly, it raises the volatility of consumption in comparison to the habit-only model, as the representative household faces a more inelastic supply of the physical capital storage technology, i.e. higher demand in savings (consumption smoothing) goes hand in hand with higher adjustment costs incurred from equating savings to investment, thus resulting in consumption smoothing to be less successfully implemented (or to be frustrated by high values of q_t in times when the household would want to save more). Secondly, combining habit persistence (which makes the representative household care more about volatility in the *absolute level* of consumption) with the inelastic supply of investment opportunities leads to a more volatile series of the marginal value of wealth (or marginal valuation) thus increasing the volatility in real interest rates.

For this version of the baseline model, the interplay between the goods and liquidity markets (which are linked through the Fisher equation) happen to be such as to result in relatively little volatility in inflation expectations, thus causing most of the *real* interest rate volatility to directly translate into an equivalent volatility in the *nominal* rate of interest as well. Also, in contrast to the habit-only model, real and nominal rates move in tandem, which is an improvement as this has been found to be the case in various studies (see Mishkin, 1982, 1990b,a, 1992). Real and nominal interest rate volatility, which is almost doubled for the former and trebled for the later in comparison to the habit-only model, thus leads to an increased volatility of the *price-channel* in credit production, which is the main focus of the present study.

For a given fixed upward-sloping marginal cost schedule in credit production, the increased volatility in the price of credit leads to a larger variability in the production of credit relative to consumption (or deposits) and thus also to a more volatile consumption velocity process, which now possesses a coefficient of variation close to 0.4, thus making this version of the original baseline model capable of successfully explaining the variability in consumption velocity observed in the data. The key difference between the baseline model and this version incorporating habit and adjustment costs, can be discovered by comparing the respective model variants' top right-hand quadrants, which graph consumption and the level of credit.

Whereas the volatility of consumption is approximately the same, credit production now overshoots[19] consumption in percentage terms. As in Jermann (1998), here the very rigidity or inertia in consumption coupled with adjustment costs of investment leads to an increased volatility in the price of credit, which causes the latter to be much more volatile. In Jermann (1998), the *real* rate becomes more volatile as a consequence of habit formation and adjustment costs, but here the same volatility also carries over into the nominal rate, via the Fisher equation and the inflation rate behaviour implied by *countercyclical* credit production.

In contrast to Gillman and Kejak (2007), the successful modeling of consumption velocity is obtained without any shocks to credit productivity at all, but exclusively by raising the volatility of the price of credit. Improving on Hodrick et al. (1991), the model only requires modest real and nominal rate volatility, which are much less volatile than the aforementioned authors' reported interest rate volatility of approximately 30%. Finally, the habit-adjustment-cost variant also reverses the negative finding of the habit-only model, in which velocity was found to be countercyclical, which is now found to be pro-cyclical again, as observed in the data.

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[19] actually, more precisely it *undershoots* following a positive shock to productivity, and it is real money balances which are now procyclical and overshooting, as observed in the data. But due to the inert behaviour of consumption, relative to this series, overshooting sometimes is apparent.

2.5 Sensitivity analysis of perturbation of the parameter space

In what follows, I present a sensitivity analysis based on a large number of simulations and over some range of calibrated parameters of interest, where I wish to focus in particular on the degree of habit persistence (the parameter b), and the extent to which adjustment costs to investment matter (the parameter κ). In particular, I choose a parameter space for b equal to $b = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9]$ and for

κ equal to $\kappa = [0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0]$ which amounts to simulating the model over a 9×12 grid. For each grid point, expected values and standard errors of relevant statistics are computed based on 1000 simulations of the model, setting the simulation length equal to a typical postwar quarterly sample size of $n = 200$. All simulated series are filtered using the Hodrick-Prescott filter prior to calculating the statistics. The first row in each cell is always the coefficient of variation of simulated consumption-money velocity (with standard errors), while the second row is always the corresponding volatility of the real rate[20] of interest (with standard errors). It is clear that - unlike in Jermann (1998), who does not incorporate a labour-leisure choice - although real rate volatility rises as physical capital supply becomes more inelastic *and* habit persistence increases, it stays well below the unrealistically high values reported in Hodrick et al. (1991). This is because endogenous responses in labour help to dampen the volatility of the stochastic discount factor (or equivalently the marginal valuation of wealth).

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[20] The interest rate volatility has been *annualised*.

Habit Adj. Cost	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.5	0.071 (.005) 0.051 (.004)	0.091 (.005) 0.024 (.004)	0.114 (.007) 0.049 (.004)	0.140 (.009) 0.044 (.004)	0.168 (.010) 0.040 (.004)	0.193 (.013) 0.038 (.004)	0.218 (.015) 0.034 (.003)	0.228 (.020) 0.038 (.003)	0.209 (.021) 0.06 (.002)
1.0	0.077 (.006) 0.011 (.001)	0.103 (.006) 0.016 (.001)	0.134 (.007) 0.022 (.002)	0.169 (.010) 0.028 (.002)	0.204 (.012) 0.035 (.003)	0.237 (.015) 0.041 (.004)	0.271 (.020) 0.048 (.005)	0.288 (.024) 0.053 (.005)	0.271 (.024) 0.057 (.005)
1.5	0.082 (.006) 0.045 (.004)	0.114 (.006) 0.054 (.005)	0.150 (.008) 0.063 (.005)	0.191 (.011) 0.075 (.005)	0.231 (.014) 0.085 (.006)	0.274 (.018) 0.096 (.007)	0.311 (.024) 0.106 (.008)	0.338 (.027) 0.110 (.009)	0.322 (.029) 0.117 (.010)
2.0	0.085 (.006) 0.051 (.005)	0.121 (.007) 0.082 (.006)	0.163 (.009) 0.098 (.007)	0.208 (.012) 0.112 (.007)	0.254 (.016) 0.127 (.008)	0.300 (.020) 0.140 (.010)	0.344 (.025) 0.154 (.012)	0.375 (.031) 0.164 (.014)	0.358 (.032) 0.162 (.015)
2.5	0.088 (.006) 0.090 (.008)	0.127 (.008) 0.105 (.008)	0.172 (.010) 0.123 (.008)	0.221 (.012) 0.142 (.008)	0.273 (.017) 0.162 (.011)	0.321 (.021) 0.177 (.013)	0.371 (.027) 0.194 (.015)	0.408 (.036) 0.206 (.019)	0.395 (.036) 0.202 (.019)
3.0	0.090 (.006) 0.105 (.009)	0.132 (.008) 0.124 (.009)	0.180 (.010) 0.145 (.010)	0.233 (.014) 0.168 (.011)	0.286 (.016) 0.189 (.012)	0.343 (.021) 0.211 (.014)	0.395 (.026) 0.228 (.016)	0.433 (.032) 0.24 (.018)	0.420 (.040) 0.232 (.023)
3.5	0.092 (.006) 0.117 (.009)	0.136 (.008) 0.139 (.010)	0.186 (.010) 0.164 (.010)	0.241 (.014) 0.189 (.012)	0.300 (.019) 0.215 (.014)	0.356 (.022) 0.238 (.015)	0.415 (.030) 0.260 (.019)	0.459 (.037) 0.273 (.022)	0.444 (.038) 0.266 (.022)
4.0	0.094 (.006) 0.128 (.010)	0.140 (.008) 0.152 (.011)	0.191 (.011) 0.179 (.011)	0.247 (.014) 0.208 (.012)	0.311 (.020) 0.238 (.014)	0.376 (.024) 0.267 (.018)	0.431 (.030) 0.286 (.020)	0.476 (.038) 0.300 (.024)	0.460 (.038) 0.278 (.023)
4.5	0.095 (.006) 0.135 (.011)	0.141 (.008) 0.162 (.010)	0.197 (.010) 0.195 (.012)	0.256 (.015) 0.226 (.015)	0.319 (.020) 0.258 (.018)	0.384 (.023) 0.286 (.018)	0.445 (.030) 0.308 (.022)	0.493 (.040) 0.321 (.027)	0.480 (.041) 0.299 (.026)
5.0	0.095 (.006) 0.140 (.010)	0.145 (.008) 0.173 (.011)	0.200 (.012) 0.206 (.013)	0.262 (.015) 0.242 (.016)	0.326 (.020) 0.276 (.018)	0.393 (.025) 0.306 (.020)	0.456 (.035) 0.329 (.026)	0.503 (.040) 0.339 (.028)	0.495 (.044) 0.316 (.028)
5.5	0.096 (.006) 0.146 (.012)	0.145 (.009) 0.180 (.013)	0.203 (.011) 0.217 (.013)	0.266 (.015) 0.254 (.014)	0.335 (.022) 0.294 (.020)	0.403 (.026) 0.325 (.022)	0.475 (.035) 0.354 (.030)	0.521 (.038) 0.360 (.026)	0.505 (.044) 0.33 (.036)
6.0	0.010 (.006) 0.154 (.012)	0.148 (.009) 0.189 (.014)	0.208 (.010) 0.229 (.013)	0.272 (.016) 0.270 (.016)	0.340 (.021) 0.308 (.020)	0.412 (.026) 0.343 (.022)	0.481 (.034) 0.369 (.027)	0.530 (.044) 0.375 (.032)	0.518 (.047) 0.344 (.032)

Table 2.2: Parameter Sensitivity Analysis of Variability of Velocity and Real Rates

2.6 Conclusion

The observed variability of consumption velocity, modeled using a simple preference-based cash-credit model (see Stokey and Lucas, 1987), has been found to be impossible to explain jointly with plausible variability of *real* interest rates as implied by the same model (see Hodrick et al., 1991). In that model, sufficient variability in the former requires too high variability in the latter. Therefore, successfully modeling the second moments of the two variables jointly appears to be an impossibility in that particular framework. Since velocity is determined by a point of tangency between a downward-sloping relative price schedule (determined by the *nominal* rate of interest) and a smooth utility function in the cash and credit good, large fluctuations in that relative price are needed in order to induce sufficient variability in velocity by sufficiently dispersing that locus of tangency through time. This theoretical failure is therefore a direct consequence of the low nominal interest rate elasticity of velocity in the preference-based cash-credit model. With very little variability in expected inflation - a typical outcome of flex-price models - sufficient variability in the *nominal* rate is - through the Fisher relationship - induced primarily through variability in the *real* rate, which is required to be too high in comparison with empirical evidence on observed rates to explain consumption velocity.

A decentralised version of a cash costly-credit model (see Gillman, 1993; Benk et al., 2005; Gillman and Benk, 2007) determines the *average* of and variability in consumption velocity instead through the intersection (and for variability through the dispersion of that point of intersection) of a convex upward-sloping marginal cost schedule in credit production and the price of credit, which equals the net nominal rate of interest. This results in a different interest-rate elasticity of consumption velocity, compared to the preference-based cash-credit model, which is based on the technology specification of credit production. In contrast to Gillman and Kejak (2007), who focus on modeling the variability in *income* velocity primarily through the marginal cost channel (the credit shock channel) and the resulting high *credit shock* elasticity of consumption velocity, the present study has exclusively emphasised the variability of the *price channel* in determining *consumption* velocity variability. It is this focus which likens the present study to Hodrick et al. (1991).

The primary focus is to examine quantitatively using simulation evidence, whether the model is capable of exhibiting sufficient variability in consumption velocity without relying on too volatile interest rate behaviour, both real and nominal. As in Jermann (1998), increasing interest rate volatility is induced in a standard monetary RBC model using a combination of habit persistence in consumption *and* adjustment costs to investment. The second moment of consumption velocity are well-matched, which constitutes a significant improvement over earlier findings of Hodrick et al. (1991) using a simple cash-credit model. Both the second moments of the real and nominal rates of interest - although increased through the habit-adjustment-cost framework - remain relatively low, and are nowhere as unrealistically high as in Hodrick et al. (1991)[21].

Finally, successfully obtained stable saddle-path solutions over a large range of habit-persistence parameters indicates global determinacy of the model, making the cash costly-credit model superior in this regard vis-a-vis simple cash-only or cash-credit models, which have both been found to exhibit real indeterminacy (see Auray et al., 2005). The reason for this is that costly credit provides the representative household with an alternative means-of-exchange to escape the cash-only component

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[21] One reason why in the present model, interest rate volatility remains low compared to Jermann (1998), is because a labour-leisure choice is incorporated, which is absent in Jermann (1998). See also Lettau and Uhlig (2000) on how endogenous variation in labour can dampen fluctuations in the real rate, in spite of other rigidities such as habit formation and capital adjustment costs.

of the exchange constraint *costlessly* in a net wealth sense, as the cost of using credit is re-distributed back in form of the banking wage bill and the dividend return on deposits held with the financial intermediary.

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Chapter 3

A *Real* Monetary Business Cycle?

A standard cash-in-advance monetary real business cycle is developed in which the representative agent can self-produce a second means of exchange, credit, using labour. Two further additional assumptions are imposed which deviate from a canonical discussion of a monetary cash-in-advance real business cycle. The representative household exhibits habit persistence in consumption *and* the goods-producing sector can only invest subject to some convex adjustment cost to investment, thus leading to an inelastic supply of physical capital and conditional variation in Tobin's q over the business cycle. Credit production locally implies an LM-type demand for real money balances, inelastic supply of physical capital leads to the inverted indicator behaviour of *real* rates of interest. Interestingly though, under the maintained assumption of a deterministic "k-percent" Friedman-type growth rate of the money supply, many of the economy's *nominal* variables' conditional behaviour is similar to that observed in U.S. data.

3.1 Introduction

Typical macroeconomic models used in policy-making during the late 70s, 80s and early 90s often combined the IS-LM framework, modeling aggregate demand, with a Phillips-type aggregate supply function and some backward-looking inflation expectations formation mechanism, dubbed the neoclassical synthesis. The *actual* inflation process emerging in such models was determined by the interplay of an exogenously specified Friedman-type monetary growth rate rule, perhaps with a bit of randomness, and an LM-type money demand function in some scaling variable related to expenditure, such as output, and in the nominal rate of interest, where this latter feature was often either motivated by some optimal portfolio allocation argument between money and bonds, or a Baumolean resource cost (or inventory cost) shoeleather story. The derivatives of this money demand function were therefore positive in the expenditure scaling variable, but negative in the opportunity cost nominal interest rate variable. Using dots to denote instantaneous changes, equilibrium in the money market was given by:

$$\dot{m} - \dot{p} = \eta_y \dot{y} - \eta_R \dot{R} \tag{3.1}$$

where η_y and η_R are the money demand elasticities with respect to the expenditure scaling variable and the nominal rate of interest, respectively. Like in any other market of exchange, in the money market - which is linked to the goods market - too the observed ex-post actual and ex-ante expected values of *real* money balances actually held in form of some narrowly defined monetary aggregate, the observed inflation rate, the nominal rate of interest and expenditure variable are all *endogenously* determined *simultaneously.*

A more modern-age general equilibrium framework which perhaps comes closest in replicating this argument by modeling the existence of a market for money - or for liquidity in general - explicitly, is epitomised by the cash-in-advance model (see Lucas, 1982; Svensson, 1985; Stokey and Lucas, 1987; Cooley and Hansen, 1989, 1995), in which money balances are predetermined beginning-of-period net wealth and actual money balances available for expenditure on the *consumption* good depend also on some net transfer (or lump-sum taxation) by the government, determining the growth rate of money and in standard cash-in-advance models typically also the rate of inflation (the change in the money price of the consumption good). This money market is, for the maintained assumption of strictly positive nominal rates, modeled by a strictly binding cash-in-advance constraint, in nominal terms given by:

$$M_{t-1} + T_t \equiv M_t = P_t c_t \tag{3.2}$$

or in real terms, given by:

$$\frac{m_{t-1}}{1+\pi_t} + \tau_t \equiv m_t = c_t \tag{3.3}$$

where $\tau_t = T_t/P_t$. Notice that the original cash-in-advance formulation absent of a preference-based explanation of a credit good, as in Stokey and Lucas (1987), implies a counterfactual money-consumption velocity of unity. The current-period net transfer is typically modeled such as to imply an increase of money balances at some (random) growth rate, thus leading to:

$$\tau_t = \Theta_t \frac{m_{t-1}}{1+\pi_t} \equiv \left(\Theta^* + e^{u_t} - 1\right) \frac{m_{t-1}}{1+\pi_t} \tag{3.4}$$

Using this, and recalling the well-known inflation tax result leading to a distortion of the marginal rate of substitution between consumption (the cash good) and leisure (see Cooley and Hansen, 1989), the cash-in-advance constraint can also be written as:

$$\frac{m_{t-1}(1+\Theta_t)}{1+\pi_t}=c_t(\tilde{z}_t,\pi_t) \tag{3.5}$$

where I have pointed out, that in general equilibrium endogenous consumption depends (negatively) on the inflation tax and in general also on some other variables which I have summarised here in the catch-all variable $\tilde{z}_t$. In spite of using a "catch-all-variable" here to make this point more general, it is well-known that in a two-shock (money and productivity) prototypical monetary real business cycle model as in Cooley and Hansen (1989, 1995), it is mostly the goods productivity shock, to which consumption reacts proportionately, albeit in Friedman-type consumption smoothing fashion. In such a prototypical monetary CIA business cycle model with production and physical capital, in which the representative household has a strong consumption smoothing objective, consumption varies very little[1], so that *exogenously modeled money supply* growth rate shocks with some persistence lead to almost one-for-one adjustments in the unexpected component of inflation due to the unexpected money growth rate innovation, leaving very little adjustment in the expected inflation component along the expected transitional path back to the steady state, which is only due to modeling the money growth rate process with some persistence.

White-noise money growth rate innovations would thus, ceteris paribus, leave conditional inflation expectations over the business cycle completely unaffected following a positive innovation to money growth (see also Walsh, 2003, chp.3). Notice also that in such models, following a, say, positive *productivity* shock without a

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[1] depending on the specification of the utility function, where in particular iso-elastic specification with high relative risk aversion makes the consumption process particularly smooth, as risk-aversion across states also implies risk-aversion across time.

corresponding money supply growth rate accommodation in upward direction, both actual and expected inflation typically fall below their steady state values, as predetermined real money balances need to be adjusted upwards such as to provide the household with sufficient real money balances to buy the consumption good, whose demand has risen proportional to the productivity shock. In other words, given insufficient growth in money supply to keep up with the growth in money demand (or consumption demand), the growth rate of the money price of goods has to fall sharply below it's steady state value, in order to establish equilibrium in the money market in the period of the shock, only to grow at an increasing rate so as to converge to the steady state inflation rate from below.

Early formulations of cash-in-advance models spelled out in endowment environments (see Lucas, 1982; Svensson, 1985; Stokey and Lucas, 1987; Giovannini and Labadie, 1991) or in complete production-based real business cycle models (see Cooley and Hansen, 1989, 1995), typically proceed by modeling the growth rate of *the supply of* money balances in some *stochastic exogenously* specified fashion, using data of some narrowly defined monetary aggregate, say M0 or M1, to determine the evolution of the structural money growth rate shock. Although common practice then, in hindsight and in contrast to my earlier discussion of the LM-type money demand function I find this way of empirically estimating a model-implied *exogenously* specified money supply growth rate very puzzling.

Surely, even the most narrow definition of a monetary aggregate as *observed factually*, ought to be understood to some extent also as an endogenously determined level of money balances willingly and in some optimal portfolio-based sense held by the public, according to some well-defined *money demand* function. Indeed, it may very well be worth considering to take a complete opposite stance to the above described common practice of calibrating the *exogenous* evolution of the growth rate of the money supply *stochastically* and instead entertain the assumption of a completely *deterministic* Friedman-type growth rate rule in the *supply* of nominal money balances instead with no exogenously specified disturbances at all, and let an *endogenously* varying *money demand* function from a model explain the observed variation in real money balances held, leaving conditional inflation to vary so as to establish equilibrium between the supply of and demand for real money balances[2]. This means that one may entertain modeling the exogenously specified law of motion for the narrow monetary aggregate as:

$$m_t = \frac{m_{t-1}}{1+\pi_t} + \bar{v} = \frac{m_{t-1}\left(1+\Theta^*\right)}{1+\pi_t} \tag{3.6}$$

This is the approach I will take in this paper by formulating an endogenous-velocity production-based monetary real business cycle model, in which the representative household can self-produce a credit service, which in conjunction with money, can be used to pay for the consumption good. The specification of self-produced credit follows Kejak and Gillman (2005); Gillman and Kejak (2008) and instead of the preference-based cash-credit framework of Stokey and Lucas (1987), is a technology-based and thus Baumolean resource-based story of credit production and thus also of endogenous *money demand*, resulting from total *liquidity* demand (consumption) minus endogenous variation in credit demand (supply).

To model the exogenously specified law of motion of the supply of real balances of the narrowly defined monetary aggregate as a completely deterministic Friedman-type money growth rate rule must be understood in this context as a thought experiment to figure out how well, given this assumption, endogenous variation in the demand for real balances *alone* can produce a monetary business cycle which

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[2] This argument is of course not novel and follows the strand of literature prevalent in the real business cycle school of thought emphasising endogenous money and reverse causation.

successfully captures the salient features of some set of real and nominal stylized facts. Given the well-known theoretical framework of the, presumably endogenously varying, money multiplier linking changes in the monetary base to the actual supply of money balances, I do not wish to convey the impression that a "rock-steady" deterministic supply of real money balances reflects reality.

More to the point is the observation that, like in any other market of exchange, *observed* quantities (money balances actually held) and prices (say, inflation and the nominal rate of interest) are merely the outcome of the interaction of some underlying theories about demand and supply of money. The purpose of this paper is to study the endogenous variation of key real and nominal quantaties and prices over the business cycle, assuming the interaction of a purely deterministic Friedman-type growth rate rule of money supply with the endogenous responses of an LM-type money demand due to productivity-driven variation in a Wicksellian real rate of interest. In contrast to this, Freeman and Kydland (2000) are using a transaction cost motivated demand for money and deposits, and given this, focus on the endogenous variation of M1 through the endogenous determination of the money multiplier.

It is exactly this last point on the interaction of money supply and demand which puts into question the common practice of specifying the *exogenous* law of motion of some money *supply* growth rate rule with the corresponding process growth rate shock by regressing some AR1 process to the *observed* growth rate series of, say, M2 and *more importantly*, by obtaining the time series properties of some exogenously specified structural money supply growth rate shock by proceeding in that particular fashion. In all likelihood, given a maintained assumption of a relatively stable evolution of the supply of a narrow monetary aggregate, what one traces out by carrying out such regressions, is actually the response of money balances held due to *money* *demand*, which presumably varies, in a more or less stable and predictable fashion, over the business cycle in state-contingent fashion. It is exactly this view which is taken in the present paper, which endogenises velocity to obtain an LM-type money demand function derived from a microfounded theory of self-produced credit, which in contrast to the seminal *preference-based* cash-credit model (see Stokey and Lucas, 1987), is instead based on some *technolog-based* Baumolean resource cost view of credit production (see also Gillman and Kejak, 2008).

The model presented is of cash-in-advance real business cycle type, similar to Cooley and Hansen (1989, 1995), but the household has a *portfolio* of total liquidity supply, composed of predetermined cash plus a current transfer *and* self-produced credit, available at its disposal to meet its total liquidity demand (which equals consumption, as is usual[3]). Similar to Jermann (1998) and Boldrin et al. (2001), I model the supply of physical capital to be inelastic by including adjustment costs to investment. I also include habit persistence in consumption which is of internal first-difference type as in Constantinides (1990) and which has also been employed by Jermann (1998) and Boldrin et al. (2001).

Inelastic physical capital supply changes the canonical RBC model's prediction in as far as, following a positive productivity shock, interest rates *fall* as the investment boom occurs and the economy expands. Via the Fisher equation I obtain a falling nominal rate as well. Modeling the consumption process as a smoothly evolving endogenous state variable, makes total liquidity demand very smooth, which for some given state-contingent evolution of the household's liquidity supply portfolio, leads to highly persistent inflation *expectations*. The *actual* process of inflation, involving both unexpected and expected components, turns out to exhibit high-frequency variation resulting from to the unexpected component, which in turn is due

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[3] I do not model investment to be carried out subject to liquidity services, as in Stockman (1981).

to the erratic portfolio reallocation between credit and money over the business cycle. Assuming that the central bank would eliminate such high-frequency occurrences of inflation variability by inelastically supplying money balances would make the highly persistent low-frequency expected inflation component matter relatively more in the determination of the actual inflation process.

Since interest rates fall as the economy expands, and credit production implies an LM-type money demand, money balances held optimally within the liquidity portfolio move endogenously procyclically over the business cycle, as observed in U.S. data (see King and Watson, 1996), and also *lead* consumption. Following a sudden fall in interest rates, the household's self-produced level of credit falls, and money demand shoots up residually, which for a given supply of money has to be accommodated by the inflation rate, which first drops sharply. In spite of the highly persistent and procyclical inflation expectations which are obtained, the initial unexpected sudden drop in inflation can be interpreted as and explanation of the robust *price puzzle* found in structural VARs, in which inflation first falls, following an expansionary innovation to monetary policy (see Sims, 1992). Simulated time series based on the goods production productivity shock alone imply a highly stable conditionally procyclical money demand over the business cycle, which by introducing direct shocks to velocity in form of credit productivity shocks, can be broken down so as to realistically mimic this apparent feature observed in post-1980 U.S. data.

All results assume a Friedman-type deterministic growth rate of the nominal balances of money supply (see Friedman, 1960), making productivity-driven responses of a state-contingent demand for real money balances function to a Wicksellian real rate of interest the main driving element in the economy. Impulse-responses from innovations in productivity lead to endogenous responses of real and nominal variables, which are generally indistinguishable from responses obtained by shocking the Taylor Rule in a prototypical New Keynesian model. What emerges then is a *real* monetary business cycle, in which productivity-driven responses in the goods market and corresponding responses in the Wicksellian real rate of interest propagate into the financial (liquidity market) in almost unidirectional fashion, with very little feedback of the financial markets back into the goods market[4].

The remainder of the paper is organised as follows. Section 2 describes the Wicksellian banking time model consisting of a representative household, who buys consumption subject to a liquidity constraint, using a portfolio of total liquidity supply composed of predetermined money plus a current (constant) transfer and self-produced credit, and a decentralised physical capital-owning goods producing firm. The specification of exogenous shocks assumes a deterministic growth rate rule of the money supply throughout, i.e. money evolves according to some deterministic Friedman-type money growth rate rule. Following the discussion of the steady state and solution method, section 3 discusses the behaviour of the model implied by impulse responses and simulations obtained from the reduced form solution of the model. Section 4 provides a discussion, section 5 concludes.

3.2 The credit model

The representative agent economy is a standard monetary cash-in-advance real business cycle model (Cooley and Hansen, 1989, 1995), but is extended as in Kejak and Gillman (2005) to allow for endogenous variations in consumption velocity through the use of produced credit. The representative agent derives utility from consumption, which is of internal first-differenced habit type, as in Constantinides (1990),

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[4] The only feedback from the financial market back to the goods market is the consumption-leisure distortion through the (expected) inflation tax.

and leisure according to a separable utility function, which is initially specified to be iso-elastic in consumption and leisure, and given by:

$$U(c_t, c_{t-1}, l_t) = \frac{(c_t - bc_{t-1})^{1-\eta_1} - 1}{1-\eta_1} + \Psi \frac{l_t^{1-\eta_2} - 1}{1-\eta_2} \tag{3.7}$$

Utility therefore over the representative agent's entire lifetime (with infinite horizon) is given by:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t - bc_{t-1})^{1-\eta_1} - 1}{1-\eta_1} + \Psi \frac{l_t^{1-\eta_2} - 1}{1-\eta_2} \right], \qquad 0 < \beta < 1 \tag{3.8}$$

The consumer can purchase the consumption good using either money or costly produced (using banking time) credit. Denote the share of the consumption good purchased with credit $f_t^* \in [0,1)$, then the representative household's liquidity constraint is given by[5]:

$$\frac{m_{t-1}}{1+\pi_t} + v_t \geq (1 - f_t^*)\, c_t \tag{3.9}$$

where $f_t^* = f_t/c_t$ is the share of total credit used in purchasing the consumption good, m_{t-1} are real beginning-of-period predetermined money balances, $1+\pi_t$ is the inflation rate and v_t represents the governments lump-sum taxation determining the rate of money growth on the economy. Therefore, v_t satisfies:

$$v_t = \Theta_t \frac{m_{t-1}}{1+\pi_t} = (\Theta^* + e^{u_t} - 1) \frac{m_{t-1}}{1+\pi_t} \tag{3.10}$$

where Θ_t is the growth rate of money which is further decomposed into a deterministic Friedman-type constant steady-state growth rate of money Θ^* and a random

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[5] Kejak and Gillman (2005) specify the share of consumption paid for in cash directly using the variable a_t. I specify the credit share directly, instead, so that my exposition is related to theirs as follows: $(1 - f_t^*) = a_t$.

component given by $e^{u_t} - 1$. Notice that randomness in the money growth rate is introduced by u_t, which follows an autoregressive process of order one[6]:

$$u_t = \phi_u u_{t-1} + \epsilon_{ut}, \qquad \epsilon_{ut} \sim N\left(0, \sigma_{\epsilon u}^2\right), \quad 0 < \phi_u < 1 \tag{3.11}$$

Credit Production is subject to a Cobb-Douglas production function, which is constant returns-to-scale in labour and consumption. This specification is motivated by the financial intermediation literature (see Sealey and Lindley, 1977; Clark, 1984; Hancock, 1985) and also by a de-centralised version of the same model in which consumption equals deposits held by a financial intermediary (see Gillman and Kejak, 2008). Credit production is therefore given by:

$$f_t = e^{v_t} A_f \left(n_{f,t}\right)^{\rho} \left(c_t\right)^{1-\rho} \tag{3.12}$$

While the total value of real credit is constant returns-to-scale in banking-time $n_{f,t}$ and consumption (deposits) c_t, the production of the credit share f_t^* is therefore a decreasing returns-to-scale function in consumption-normalised banking-time only, and thus given by:

$$f_t^* = \frac{f_t}{c_t} = e^{v_t} A_f \left(\frac{n_{f,t}}{c_t}\right)^{\rho} = e^{v_t} A_f \left(n_{f,t}^*\right)^{\rho} \tag{3.13}$$

where $n_{f,t}^* = n_{f,t}/c_t$ is the banking-time spent over total consumption (or deposits). Due to decreasing returns in the production of the credit share f_t^*, the representative household, who self-produces credit, faces an upward-sloping marginal cost curve in credit production, where f_t^* is determined by the intersection of the marginal cost

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[6] I specify theoretical randomness in money growth only for purposes of comparison with the literature and with Kejak and Gillman (2005). Throughout this paper the money growth shock will be set equal to it's steady state in simulations, meaning $\Theta_t = \Theta^* \quad \forall \quad t$.

curve with the opportunity cost of using the other means-of-exchange, money, which equals the (net) nominal rate of interest. Analogously to the money growth rate innovation, total factor productivity in producing credit is also random, due to v_t, which also follows an autoregressive process of order one, and is given by[7]:

$$v_t = \phi_v v_{t-1} + \epsilon_{vt}, \qquad \epsilon_{vt} \sim N\left(0, \sigma_{\epsilon v}^2\right), \quad 0 < \phi_v < 1 \tag{3.14}$$

Further, the representative household can spent her time endowment (which is normalised to 1) by taking leisure, by self-producing credit or by working in the decentralised goods production firm. This means that the following time constraint needs to be obeyed at all times:

$$1 - l_t = n_{g,t} + n_{f,t} \tag{3.15}$$

The decentralised goods firm uses labour and physical capital to produce output y_t and is also assumed to own the physical capital and optimally invest from retained earnings. Additionally, I assume investing is subject to a quadratic adjustment cost function, specified as in Canzoneri et al. (2007), which is given by:

$$\zeta\left(\frac{i_t^k}{k_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{i_t^k}{k_{t-1}} - \delta\right)^2 k_{t-1} \tag{3.16}$$

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[7] This shock is also set equal to it's steady state value in simulations, except for the section discussing the model's ability to mimic the breakdown of a stable money demand function.

The decentralised firm's optimisation problem, which is solved subject to the household's discount factor, is then formulated as:

$$\max_{n_{g,t+k},k_{t+k},i_t^k} E_t \sum_{k=0}^{\infty} \frac{\beta^k \lambda_{t+k}}{\lambda_t} \Bigg\{ y_{t+k} - w_{t+k} n_{g,t+k} - i_{t+k}^k + \frac{\xi_{t+k}}{\lambda_t} \left[i_{t+k}^k - \zeta\left(\frac{i_{t+k}^k}{k_{t+k-1}}\right) - k_{t+k} + (1-\delta)\, k_{t+k-1} \right] \Bigg\} \tag{3.17}$$

where the multiplier on the firm's capital accumulation constraint is equal to marginal utility in steady state only, i.e. $\bar{\lambda} = \bar{\xi}$. This means that Tobin's q is given by the ratio of $q_t = \xi_t / \lambda_t$, which is equal to one in steady state, but varies over the business cycle due to the adjustment costs and the capital gains or losses of installed physical capital. Taking first-order conditions with respect to investment, end-of-period physical capital and goods sector labour hired from the household, leads to the following conditions of optimality:

$$\lambda_t = \xi_t \left[1 - \zeta_i' \left(\frac{i_t^k}{k_{t-1}} \right) \right] \tag{3.18}$$

which is the first-order condition with respect to investment, where $\zeta_i' \left(\frac{i_t^k}{k_{t-1}} \right)$ is the derivative of the adjustment cost function with respect to investment. Optimality with respect to physical capital yields:

$$\xi_t = \beta E_t \left\{ \lambda_{t+1} r_{t+1}^k + \xi_{t+1} \left[(1-\delta) - \zeta_k' \left(\frac{i_{t+1}^k}{k_t} \right) \right] \right\} \tag{3.19}$$

Finally, the first-order condition of optimality with respect to goods production labour yields the usual condition of:

$$w_t = \alpha \frac{y_t}{n_{g,t}} \tag{3.20}$$

3.2.1 Steady State Calibration

Table of benchmark calibrated Parameters			
β =0.987	discount factor	ρ =0.22	credit labour param.
α =0.64	goods labour param.	f^* =0.30	credit-to-cons ratio
η_1 =1.00	curv. param. cons.	η_2 =1.00	curv. param. leisure
A_g=1.0	TFP goods	A_f=1.461	TFP credit
l =0.7	leisure	n_f =0.00061	credit labour
Θ =1.0125	money g.	n_g =0.29939	goods labour
b =0.8	habit pers.	κ =2.0	cap. adj. cost
ϕ_u =0.70	AR money g. shock	ϕ_z =0.90	AR goods shock
ϕ_v =0.95	AR credit shock	$\sigma_{\epsilon z}$=0.0075	s.d goods shock
$\sigma_{\epsilon u}$=0.01	s.d. moneyg shock	$\sigma_{\epsilon v}$=0.01	s.d credit shock

Table 3.1: Baseline Calibration

The above table summarises the baseline calibration of the banking time model. The calibration of the model is carried out to be in line with standard values used in the literature hitherto. In particular, I calibrate the discount factor at $\beta = 0.987$, slightly below the usual 0.99 found otherwise in the literature, where my value implies an annualised steady state value of the real rate of interest of roughly 5.2%. I then choose a steady state growth rate of the money supply equal to 1.0125 quarterly, resulting in an annualised steady state rate of inflation equal to 5%, where the two aforementioned calibrated values together imply an annualised steady state value of the nominal rate of interest equal to 10.2%. Calibrated values for the goods production sector are standard. I choose a labour share in production equal to $\alpha = 0.64$, steady state total factor productivity in the goods sector to be normalised at $A_g = 1.0$ and the steady state amount of leisure to be $l = 0.7$, implying a steady state of *total* labour residually equal to 0.3, which has to be allocated between goods and credit production according to the labour market equilibrium condition of equating marginal (revenue) products of labour:

$$w = \alpha \frac{y}{n_g} = i\rho \frac{f}{n_g} \tag{3.21}$$

I calibrate the steady state share of credit production $f^* = 0.3$, which follows Kejak and Gillman (2005) and roughly matches the observed steady state long-run behaviour of consumption velocity in U.S. data. Also, following Kejak and Gillman (2005), who obtain their value of the labour share in credit production equal to 0.21 from an empirical time series study conducted by Gillman and Otto (2005), I calibrate this value at $\rho = 0.22$, which is only slightly above theirs. Given the above set of calibrated parameters, I use a nonlinear equation solver and residually obtain $A_f = 1.461$ and $n_f = 0.00061$, which is very close to the corresponding values of 1.422 and 0.00049 obtained by Kejak and Gillman (2005). The calibration of the habit persistence parameter in consumption is as in Constantinides (1990) and fixed at $b = 0.8$. I calibrate the investment adjustment cost parameter $\kappa = 2.0$, which is only one-quarter of the calibration chosen in Canzoneri et al. (2007), so as to model only a small amount of frictions regarding the supply of physical capital. The shock processes in the model are also parametrised along standard values. In particular, I model the persistence in the goods sector total factor productivity shock to be equal to $\phi_z = 0.90$, which implies a slightly lower degree of persistence than the usually chosen value of 0.95, with a standard deviation of the iid innovations equal to $\sigma_{\epsilon z} = 0.0075$ (as compared to 0.00721 Cooley and Hansen (1989)). The autoregressive parameters for the money supply growth rate shock u_t and the credit total factor productivity shocks v_t are 0.6 and 0.95, respectively, where the former choice follows Kejak and Gillman (2005) and the latter follows the common calibration for productivity persistence found in the literature. The corresponding standard deviations for the iid shock innovations are calibrated at $\sigma_{\epsilon v} = 0.01$ and $\sigma_{\epsilon v} = 0.015$. However, it is important to emphasise at this point, that the chosen values describing the evolution of the money growth rate shock are completely irrelevant for the present study, as throughout the paper u_t will *never* receive any shocks. Notice also that I do actually include the exogenous law of motion of the money supply growth rate shock into the state of the model, but here since shocks are set to zero, even the calibrated autoregressive parameter ϕ_u does not play any role, as the first-order conditions of the model never involve any forward-looking prediction of $E_t m_{t+1}$ or $E_t u_{t+1}$. Also, for the majority of my discussion, the exogenously specified evolution of the credit productivity shock v_t is also assumed to receive no shocks in form of iid innovations on it's law of motion, except for an extension towards the end, in which I discuss the model's ability to mimic the break-down of a stable money demand function. Again, I actually do include the law of motion of v_t when solving the model, but since it never gets shocked and no forecasts of $E_t f_{t+1}$ or $E_t v_{t+1}$ are involved, the specification of the autoregressive root ϕ_v does not matter.

3.2.2 Solution & Competitive Equilibrium

The model's first-order conditions of optimality, market equilibrium identities and non-linear specification of the shock processes are linearised by taking a first-order Taylor series expansion around the (log of the) steady state. I obtain a linear system of expectational difference equations given by:

$$\mathbf{A}E_t \begin{bmatrix} \mathbf{z}_{t+1} \\ \mathbf{x}_t \\ \mathbf{y}_{t+1} \end{bmatrix} = \mathbf{B} \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_{t-1} \\ \mathbf{y}_t \end{bmatrix} \tag{3.22}$$

where $\mathbf{z}_t = [z_t, u_t, v_t]'$ is a vector containing the current-period exogenous structural shocks, $\mathbf{y}_t = [w_t, \pi_t, y_t, l_t, n_{g,t}, n_{f,t}, i_t^k, \lambda_t, \mu_t, \xi_t, f_t, R_t, I_t, q_t]$ is a vector containing the current-period endogenous control, or "jump" variables and $\mathbf{x}_{t-1} = [m_{t-1}, k_{t-1}, c_{t-1}]$ is a vector containing the endogenous states of the system. All variables collected in vectors are now log deviations from steady state. Since I have not reduced the system by hand, as is done in King et al. (1988) and also shown in Uhlig (1995), so as to eliminate intra-temporal equations, the matrix **A** is generally going to be singular, disallowing direct use of the BK diagonalization technique (see Blanchard and Kahn, 1980). So instead I use the triangularization technique developed in Klein (2000) to solve for the reduced-form solution of the system which is given by the recursive law of motion of the system:

$$\begin{bmatrix} \mathbf{y}_t \end{bmatrix} = \mathbf{F} \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_{t-1} \end{bmatrix} \tag{3.23}$$

and for the evolution of the state of the system:

$$\begin{bmatrix} \mathbf{z}_{t+1} \\ \mathbf{x}_t \end{bmatrix} = \mathbf{P} \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{t+1} \\ \mathbf{0} \end{bmatrix} \tag{3.24}$$

3.3 Results

The purpose of this section is to analyse and discuss simulated evidence from the artificial banking time model. To this end, the reduced-form solution of the linearised rational expectations model (i.e. the recursive law of motion) is used to produce impulse responses and simulated time series. Since the model is a prototypical monetary cash-in-advance RBC model, as discussed in Cooley and Hansen (1995), most of my attention will be focusing on the economy's response to an innovation in the goods sector total factor productivity parameter.

For most of my simulation results, I will first consider holding the credit production productivity shock fixed at it's steady state value, so as to disallow direct production-based shocks to velocity measures implied by this shock affecting the position of the marginal cost curve in credit production. In order to better capture the historically observed disassociation of monetary aggregates from the business cycle in post-1980s data of the U.S., U.K. and other countries, in a separate simulation exercise I will consider this direct shock again.

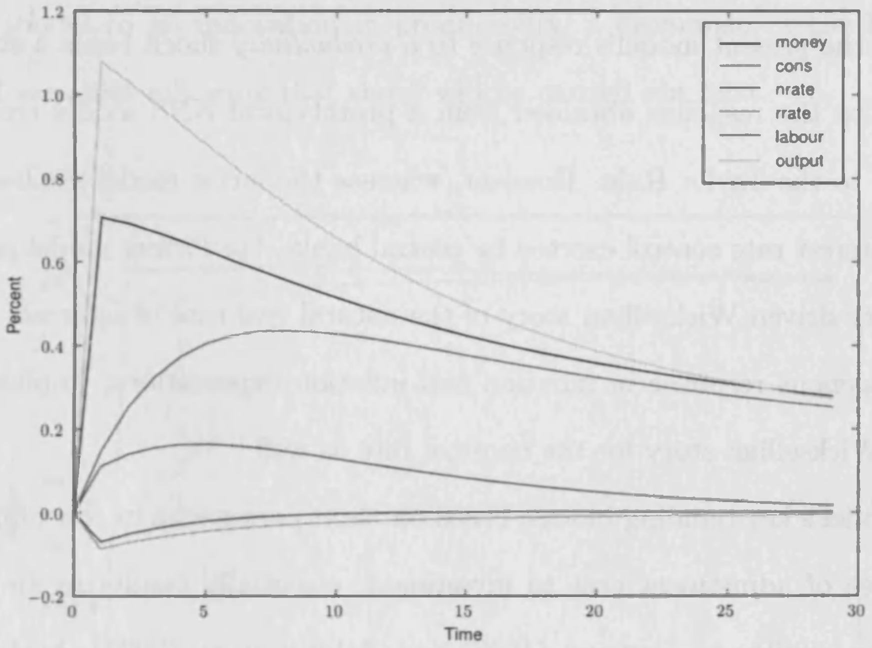

Figure 3.1: 1% innovation in productivity

Figure (4.1) illustrates the economy's response to a 1% innovation in the goods sector total factor productivity. Many of the observed responses are well-known from the canonical RBC framework, such as a Hansen RBC model with divisible labour Hansen (1985). However, three features of the present *monetary* RBC model stand out and beg explaining. In contrast to a standard RBC model (which does not contain a monetary sector and is thus incapable of being informative about nominal variables), following a shock to productivity, both the real and nominal variables fall and real money balances held rise. Further more, the real rate falls by more than the nominal rate, implying a rise in inflation *expectations*, through the Fisher equation.

Incidentally, this latter set of effects is also a typical property of models of the new neoclassical synthesis school, in which the shock producing such responses is however assumed to affect nominal rates directly through some innovation to a Taylor Rule implying an unexpected shock to monetary policy (a monetary expansion). Therefore, the present model's response to a *productivity* shock bears a striking resemblance to the response obtained from a prototypical NNS model receiving an innovation to the Taylor Rule. However, whereas the latter model implies a direct nominal interest rate control exerted by central banks, the former model produces a productivity-driven Wicksellian story of the natural real rate of interest, which for some endogenous response in inflation and inflation expectations, implies a corresponding Wicksellian story for the nominal rate as well.

The model's key building blocks, based on habit persistence in consumption and some degree of adjustment cost to investment, essentially results in an economic environment similar to Jermann (1998) and Boldrin et al. (2001), both of which are studies emphasising the explanation of asset pricing regularities. Boldrin et al. however, discuss how their *real* business cycle model (which does not contain a monetary sector) can account surprisingly well for various business cycle facts and represents an improvement over the canonical RBC model. Boldrin et al. also discuss how the improved picture results from a combination of habit persistence, inelastic physical capital supply and input factor market rigidities (i.e. some degree of labour market immobility as well).

An important element missing in their story disallowing a *direct* comparison with NNS models based on nominal rigidities is the incorporation of a story for money demand so as to be able to model nominal variables as well. The present study fills this gap through the inclusion of a cash-in-advance type constraint and interest-elastic credit production, globally implying a Cagan-type money demand function with a non-linearly *falling* interest-rate elasticity, but around some local steady state (i.e. some fixed calibrated nominal rate of interest) a simple LM-type money demand function with a *fixed* interest rate elasticity. Before elaborating more fully on the underlying mechanisms leading to the observed impulse response of the present model to an innovation in productivity, a discussion of the behaviour of nominal variables following that shock will be carried out next.

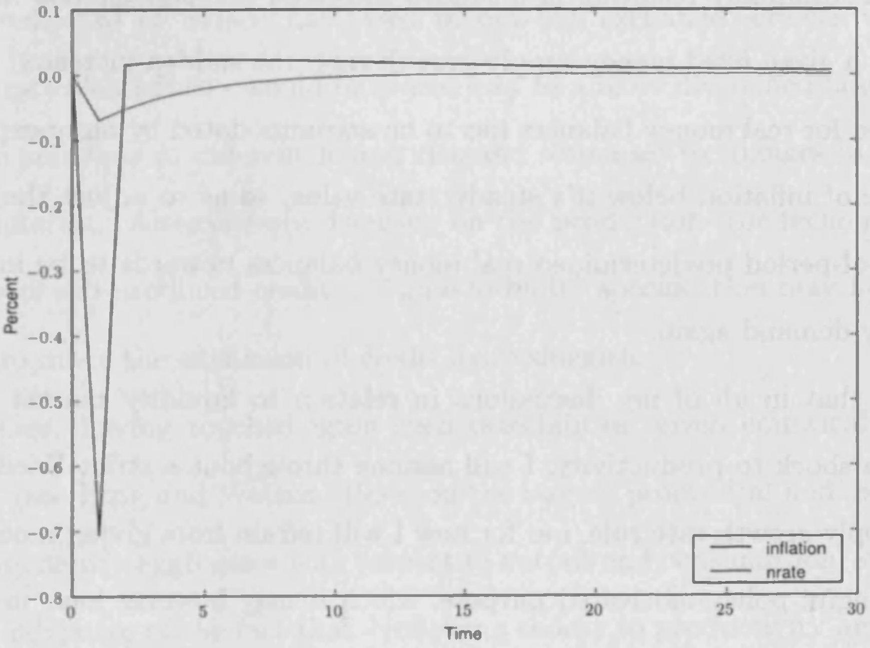

Figure 3.2: 1% innovation in productivity

3.3.1 Interest Rates and Liquidity Portfolio Reallocation

Figure (4.2) illustrates the banking time model's response of the nominal rate of interest and the inflation rate to an innovation in productivity. A standard (monetary) RBC model will typically exhibit a *rise* in both real and nominal rates following a shock to productivity, followed by an investment boom. The investment boom occurs here as well, but it coincides with a fall in both rates. This has also been observed by Boldrin et al. (2001). I will defer a discussion of this particular detail until later, when a full narrative of the model's mechanism will be presented. Focusing on the inflation response, figure (3.2) reveals a sudden unexpected drop in the inflation rate. Given a fixed Friedman-type growth rate of the money *supply*, a sudden drop in the nominal rate implies also a sudden drop in the level of self-produced credit, thus residually resulting in a sudden increased demand for real money balances. For a given fixed money supply growth rate, the sudden increased growth in the demand for real money balances has to be accommodated by an unexpected fall in the rate of inflation below it's steady state value, so as to adjust the *supply* of beginning-of-period predetermined real money balances upwards to be in line with real money demand again.

Notice that in all of my discussions in relation to liquidity market responses following a shock to productivity, I will assume throughout a strict Friedman-type money supply growth rate rule, i.e. for now I will refrain from giving money supply any meaningful policy-motivated purpose, which it may however have in the usual state-contingent fashion. Having said that, although this sharp *unexpected* drop in inflation below it's steady state value appears drastic, clearly a central bank would rush to dampen such drastic drops by inelastically supplying sufficient money (through open market operations) such as to counteract any looming high-frequency volatility in the rate of inflation (of course, in practice this is done at the level of the high-powered monetary base, and not to dampen high-frequency volatility in inflation, but rather in the nominal rate). In as far as as such a response, though partially succeeding in dampening sudden inflationary swings, may still occur too late, it would still lead to an initial fall of the rate of inflation following a shock to productivity and an expansion of the economy. The negative response of *actual* inflation in the banking time model could therefore possibly be interpreted as an explanation of the so-called *price puzzle*, which has been found in analyses of impulse-responses obtained from identified VARs (see Sims, 1992; Eichenbaum, 1992).

Another perspective on this is to realise that the model provides the representative household only with a very simplified liquidity portfolio, in which nominal interest rate movements lead to strong substitution effects from credit to money and vice-versa. In the real world, other means of exchange - perhaps other production-based credit-like or indeed fiat-based money-like exchange services with varying interest rate elasticities - would of course lead to a more diversified liquidity supply portfolio and thus to different money demand responses to changes in the nominal rate of interest. Alternatively, focusing on the production- (or technology-) based property of self-produced credit, a "time-to-built" specification may be introduced such as to make the expansion of credit more sluggish.

However, having touched upon such possibilities, given empirical evidence of the U.S. (see King and Watson, 1996) on the largely procyclical and *leading* role of narrow monetary aggregates with respect to output and consumption, such evidence may be indicative of the fact that - following shocks to productivity and immediate *real* Wicksellian changes of interest rates - the reallocation of means-of-exchange within a broader portfolio of liquidity supply in response to this appears to occur in contemporaneous fashion, whereas goods markets respond late. This evidence is compatible with the view that financial markets react fairly quickly to Wicksellian (productivity-driven) interest rate changes, thus putting into question some of the above entertained modifications to introduce some degree of sluggishness into the liquidity market. In the model presented, goods markets lag in consumption, clearly due to habit persistence, but may also - so far absent in this model - generally lag in output as well, either by modeling labour as less mobile, or perhaps, as in Cochrane (1988, 1993); Belo (2007); Jermann (2006), by allowing the firm to intertemporally smooth the arrival of an underlying productivity shock into the *effective* productivity shock based on the current period's valuation implied by the household's discount factor.

The important point here is that in the model presented, liquidity markets, both in quantities and prices, react contemporaneously to productivity-driven movements of a Wicksellian real rate of interest (and via the Fisher equation, corresponding movements of the nominal rate as well), whereas goods markets respond slightly sluggishly because of habit persistence in consumption. Notice that although output moves essentially contemporaneously with the productivity shock, as labour market real rigidities are absent, labour still partially expands in hump-shaped fashion, in as far as the maximum response of labour occurs 2 quarters after the shock. Given a humped-shaped consumption pattern and persistence in the productivity shock, labour first optimally jumps up discretely, but then also exhibits a hump-shaped segment of further smooth expansion.

It is also important to note at this point that the above obtained result contradicts Boldrin et al. in their view that the general equilibrium investment adjustment cost framework in Jermann (1998) *always* produces counterfactual *counter-cyclical* labour. I have found this to be sensitive to calibration. In particular, modeling physical capital supply *too* inelastically and/or the representative agent too risk-averse can easily lead to counter-cyclical labour. None of the models sluggish goods market quantity (consumption and labour) responses are due to sluggish price and/or wage responses and although output rises contemporaneously, in principle labour markets could also be modeled sluggishly so as to permit a more hump-shaped response in output too (see Boldrin et al., 2001). But what about the *expected transitional* response of inflation following the first-period unexpected sharp drop due to the reallocation of the household's portfolio of liquidity supply? It is this issue I will turn to next.

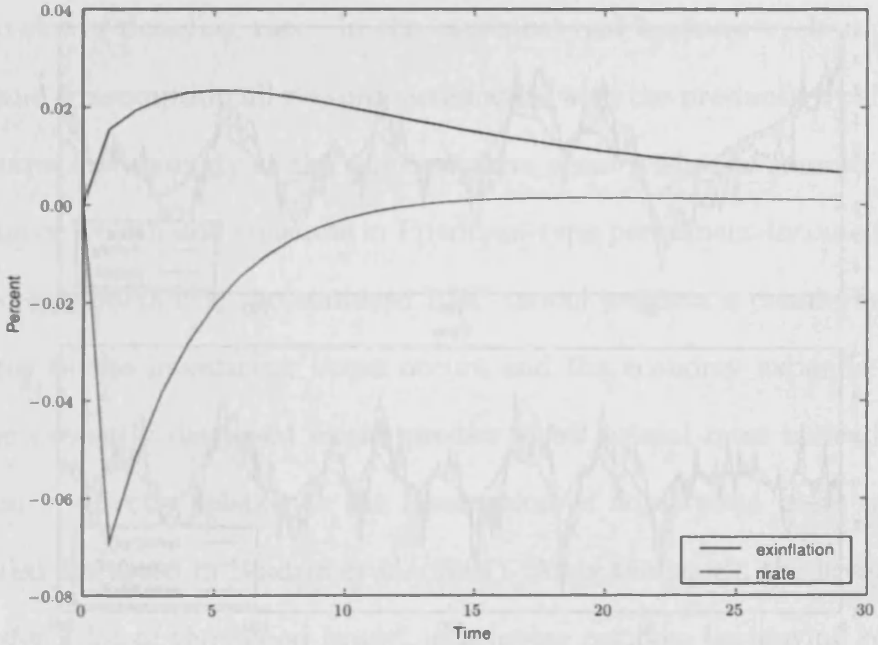

Figure 3.3: 1% innovation in productivity

Following the described unexpected first-period sharp response in inflation, equating liquidity demand (consumption) and the erratic portfolio reallocation of liquidity supply (credit and real money balances) in that period, the expected transitional path of inflation back to steady state follows a pattern illustrated in figure (4.3). As interest rates (both real and nominal) begin to rise again, self-produced credit slowly picks up as well and real money balances demanded therefore falls slowly residually through the credit production-implied LM-type money demand function. Since along the expectational path, real money balances demanded adjust back to their steady state value from above, for a given deterministic steady state Friedman-type growth rate rule of money supply, the adjustment of the aforementioned has to come from above-steady state inflation rate which are gradually falling, so as to match money supply with the endogenously smoothly falling demand in real money balances.

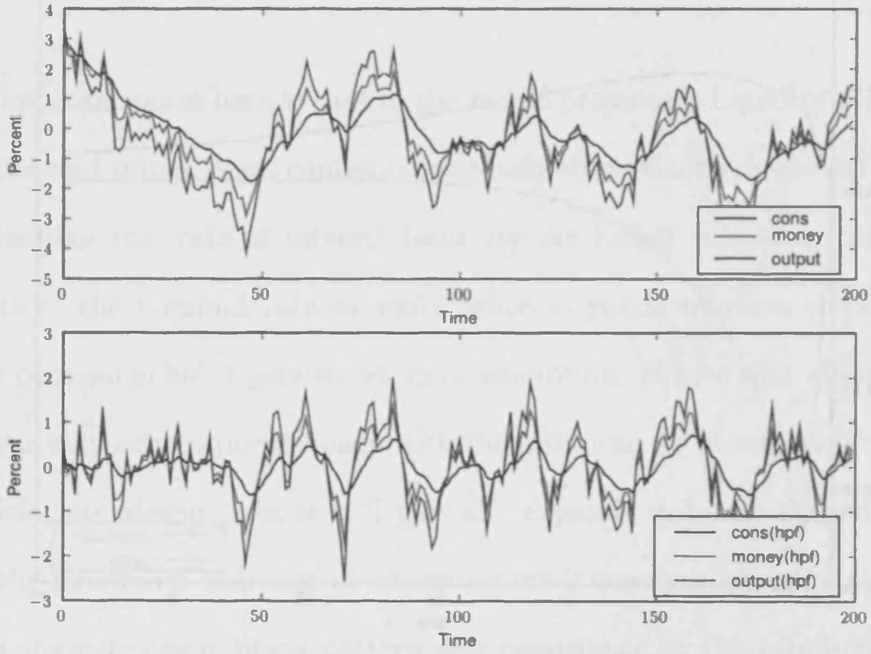

Figure 3.4: Simulation Evidence: Productivity Shocks

3.3.2 A real monetary business cycle: A narrative

This section is going to provide a narrative of the mechanisms at work in the banking time economy, following a shock to the innovation in productivity. This will be conducted in an informal style, through which I wish to make a convincing argument for the relatively successful way the model can model the observed real-world facts.

What I hope to convey here is that the model provides a Wicksellian banking time story, in which key real and nominal variables move in intuitive fashion over the business cycle and in which productivity-driven changes in the goods market affect the financial (or liquidity) market in almost uni-directional fashion, with very little feedback of the financial markets back to the goods market. So what is the *real* story behind the Wicksellian banking time model's monetary business cycle?

Following the productivity shock, the sun starts to shine and through the persistence of the Solow residual is projected to carry on shining in expectation, albeit at some slowly decaying rate. In the canonical real business cycle model, output, labour and consumption all rise proportionately with the productivity shock, but the latter series less strongly as the representative agent wishes to smooth his marginal valuation of wealth and consume in Friedman-type permanent-income implied fashion. More importantly, the standard RBC model predicts a counterfactual *rise* in real rates as the investment boom occurs and the economy expands. Why then, does the presently discussed model predict a *fall* in real rates instead? This phenomenon is directly related to the assumption of adjustment costs to investment and is also discussed in Boldrin et al. (2001). After the shock, the household wishes to transfer a lot of the "good times" into future periods, by moving resources into the physical capital storage technology, or in plain words by trying to save via the capital markets. With an inelastically modeled supply of the same, the household quickly drives up Tobin's q leading to capital gains of installed physical capital, a bull market ensues and stock markets rally! This is clearly shown in the last simulated series of figure (4.6), in which Tobin's q moves procyclically with output and countercyclical with real and nominal rates.

But now, through the household's enormous desire to save - which is even enhanced because of the even smoother optimal consumption pattern implied by habit persistence - it quickly becomes too costly to keep on buying capital and instead the household just has to accept a rather suboptimal[8] implementation of his consumption projection through time, meaning that he will "overeat" today relative to tomorrow and carry on doing so even in subsequent periods. This means that the marginal valuation of wealth will always be lower in any period relative to the following period, which through the first-order condition for real bonds implies a fall in the real rate below it's steady state value and a gradual rise back to the steady state from below. Notice that in principle the household could try and dampen the initial fall in the marginal valuation of wealth, $\hat{\lambda}_t$, simply by working less and taking more leisure.

But this does not occur here, because through habit persistence the household's projected (or future expected) appetite is of hump-shaped nature and is thus projected to expand with a very high root - more than 0.9 for the solved reduced form's autoregressive coefficient of consumption on it's past value. So if the representative agent were to take leisure now in the initial periods following the shock, it may dampen the fall in $\hat{\lambda}_t$ now, but it would come at the expense of not having enough physical capital in future periods, acquired through today's wage bill, to satisfy it's projected future increasing appetite. Notice that the procyclical *rise* in labour is very sensitive to calibration; modeling physical capital supply *too* inelastically by raising κ will lead to countercyclical labour in the initial periods following the shock, which would then eventually rise above steady state, but only much later, certainly too late to account for the stylized facts which clearly show labour to be procyclical (see King and Watson, 1996).

For reasons already discussed in the section on interest rates and liquidity portfolio reallocation, the sudden drop in the real and nominal rate (due to inelastic

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[8] *suboptimal* relative to a standard RBC model's consumption response. Of course, given the increased cost of physical capital, the consumption response is optimal.

capital, the stock market rally and "suboptimally"[9] overeating today relative to tomorrow compared to perfectly elastic capital supply in the standard RBC model), cause a sudden drop in self-produced credit and given current (and future projected hump-shaped increases of) total liquidity demand due to consumption, demand for real money balances jumps up. For a given deterministic Friedman steady state growth rate of money supply, for a much higher one-off rise in real money demand, the inflation rate has to - unexpectedly - drop sharply below steady state to adjust money supply sufficiently. But along the expected projected path, inflation has to converge from above steady state, as gradually rising real and nominal rates imply a shift away from money and back into self-produced credit, so that money balances have to be gradually adjusted downwards again through inflation rates above steady state.

The picture that emerges is one of procyclical *endogenously* determined demand for real money balances, since inelastic capital markets lead to a *fall* in real and nominal rates following the shock to productivity. Self-produced credit and it's comparatively high interest elasticity lead to a - in terms of local dynamics around some steady state - LM-type money demand function for real money balances along implying sudden shifts in the portfolio of liquidity between credit and money. Figure (4.4) shows a simulation of real money balances, consumption and output over the business cycle, where the top graphs is unfiltered and the bottom is hp-filtered to remove high-frequency components. What the simulation clearly shows is that in a world of a stable LM-type money demand function, *endogenously* determined real money balances move closely together with consumption and output, and also lead consumption, because of the latter's slow response due to habit persistence. Including labour market rigidities, as in Boldrin et al. (2001) or using some other

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[9] Given adjustments costs to investment, the chosen consumption profile is of course optimal. In absence of the same costs, a smoother consumption process were chosen.

mechanism could also lead to a slower response of output, thus making endogenous money demand also lead output.

But here money neither causes output nor consumption! Following the productivity shock and a sudden fall in the Wicksellian real rate of interest, the representative household's liquidity portfolio experiences a sudden reallocation from credit to money, financial markets move fast! But the goods market's response may generally lag the same sudden drop in the productivity shock-induced real rate, here however only in consumption which exhibits habit persistence. Further below, I will also discuss and show in simulations, how the model is in principle capable of modeling the well-documented breakdown of the tight relationship between real money demand and both consumption and output, by introducing or "switching back on" the direct credit production productivity shock v_t, so as to model direct shocks to velocity.

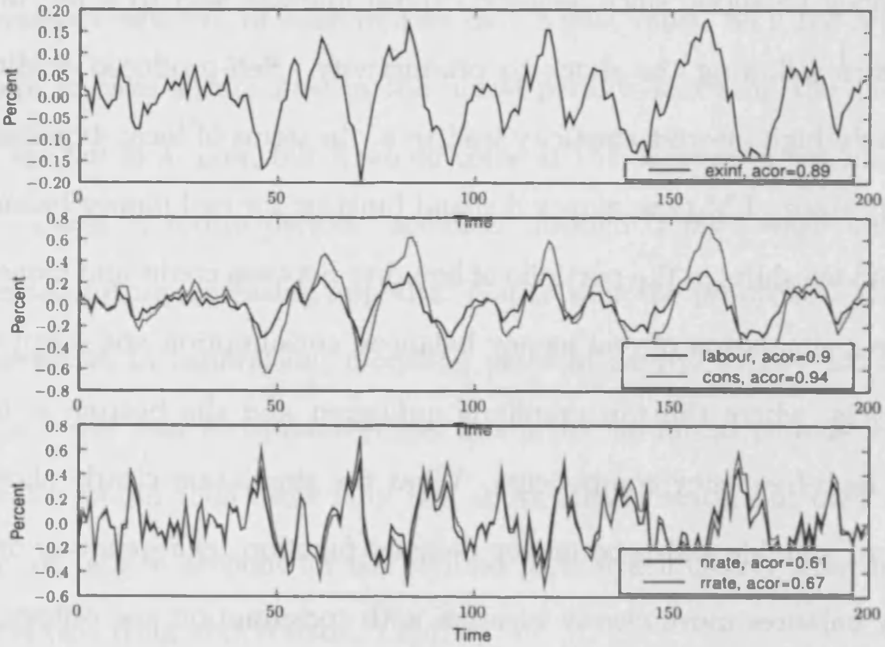

Figure 3.5: Simulation Evidence: Productivity Shocks

There are also other feature revealed by simulations deserving of mention. First of all, the model is capable of exhibiting highly procyclical and very persistent inflation expectations, which are even slightly lagging expansions in output. Also, interest rates are inverted indicators and real rates move by more than nominal rates following a productivity shock. Such responses of real and nominal variables have often been associated with an innovation to *monetary policy* as implied by a negative innovation on a Taylor nominal interest rate rule. Here however, all real and nominal results are driven by a productivity-driven fall in the Wicksellian real rate of interest; the causation from goods to financial (or liquidity) markets simply happens to be such as to make the overall picture which emerges observationally equivalent to one which would follow a Taylor rule-type expansionary monetary policy innovation in New Keynesian models incorporating price and wage rigidities.

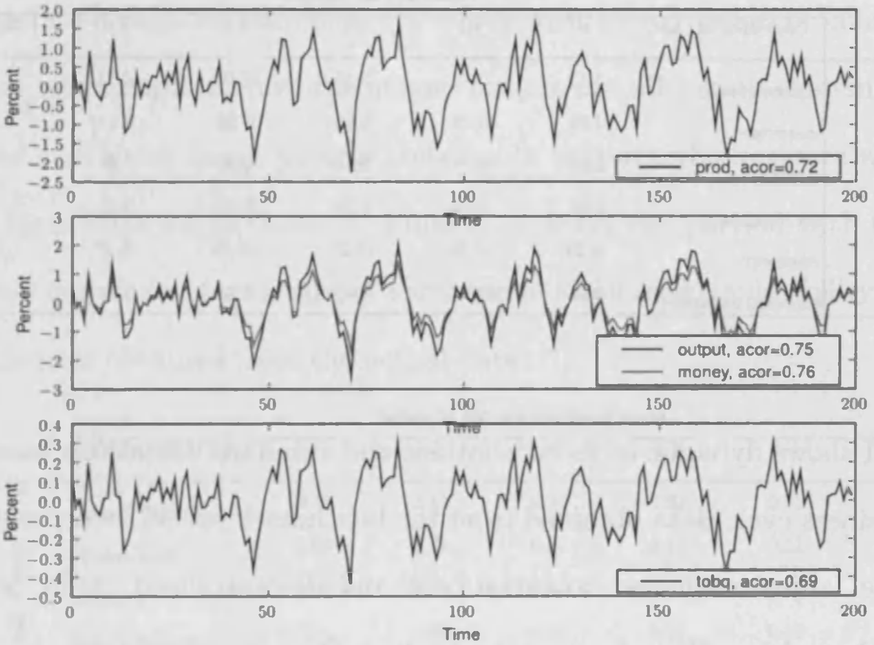

Figure 3.6: Simulation Evidence: Productivity Shocks

3.3.3 Simulation Evidence

The following section contains a discussion of simulation evidence in the form of autocorrelation tables[10] which serve to illustrate the changes in cyclical behaviour which occur whenever a number of features or assumptions of the full model are relaxed. In particular, the reported simulation evidence serves to highlight the change in the model's behaviour occuring when, first of all, adjustment cost of capital is removed, and the secondly, when in addition to this habit persistence in consumption is also excluded from the full benchmark model's specification. This is of particular interest and relevance in as far as it is helps to compare and contrast the dynamics of the full benchmark model with the alternative preference-based cash-credit specification.

Table 1: Full Model

	Correlation at Lag	t	t-1	t-2	t-3	t-4	SD(%)
Full Model	consumption(t)	0.73	0.43	0.16	-0.03	-0.01	0.34
	investment(t)	0.98	0.76	0.56	0.38	0.12	3.12
	output(t)	1.00	0.74	0.51	0.31	0.13	0.71
	(total) labour(t)	0.87	0.59	0.34	0.13	0.01	0.08
	inflation(t)	-0.39	-0.34	-0.30	-0.29	-0.27	0.52
	expected inflation(t)	0.95	0.67	0.43	0.22	0.01	0.12

Table 1 shows dynamic cross-correlations and standard deviations based on simulated business cycle data obtained from the benchmark model incorporating habit persistence, adjustment cost to investment and de-centralized credit production. All series have been filtered using the Hodrick-Prescott filter prior to calculating relevant business cyle statistics. The first point to take note of is that in this goods-productivity shock model, the standard deviation of inflation and also the nominal rate of interest (not shown in the table) is too small relative to the same measure

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[10] As a result of the approximate symmetry of cross-correlations for leads & lags of the simulated models, I only present cross-correlations at lags.

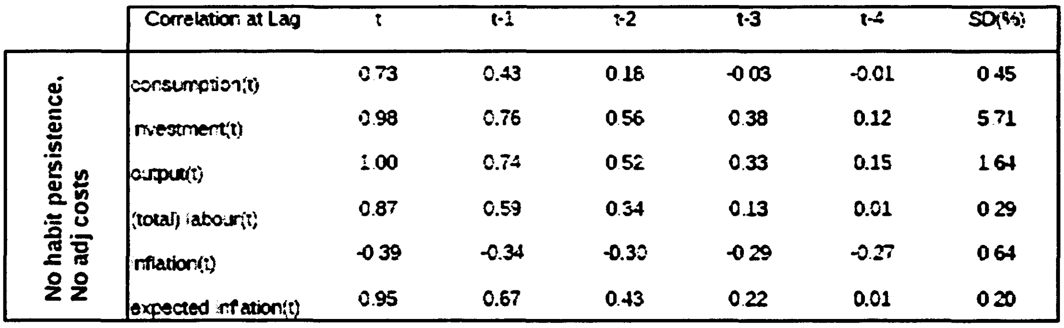
observed in U.S. data (which was also filtered using the same method as the simulated data). In the absence of money supply shocks, the only way prices (and thus inflation) can - or rather are forced to - adjust is through the variable money demand channel which is activated by variation in the nominal rate of interest. As this rate of interest is equivalent to the price of credit the consumer has to pay in order to avoid the inflation tax associated with shopping with money instead, changes in that price lead to re-allocations in the consumer's portfolio of means-of-exchange. As a result of the latter's low degree of variation, money demand and thus inflation remains similarly subdued. Further more, the full benchmark model is similarly also not able to replicate realistically the dynamic behaviour of inflation and output commonly associated with a Phillips curve, as contemporaneous output is not in any way positively correlated with past values of inflation. In fact the full model instead exhibits a negative correlation of ouput with lagged values of inflation. Apart from that, consumption, investment and output are all highly contemporaneously correlated exhibiting lag structures (relative to output) which square well with the stylized facts reported in Table 5. Finally, in direct comparison with filtered U.S. data, cross correlations with output show the model's reasonable ability to replicate similar metrics obtained from the actual data.

Table 2: No adjustment cost

	Correlation at Lag	t	t-1	t-2	t-3	t-4	SD(%)
No adj. Costs	consumption(t)	0.67	0.53	0.42	0.32	0.21	0.54
	investment(t)	0.89	0.73	0.53	0.42	0.12	5.34
	output(t)	1.00	0.73	0.51	0.31	0.13	1.45
	(total) labour(t)	0.81	0.49	0.31	0.11	0.03	1.12
	inflation(t)	-0.41	-0.35	-0.27	-0.25	-0.19	0.67
	expected inflation(t)	0.88	0.61	0.39	0.20	0.01	0.21

Table 2 provides evidence of simulated time series from the model with adjustment costs to investment removed. As expected the standard deviation of investment rises somewhat, as positive productivity shocks can now more easily be transferred into future periods without driving up the cost of installed investment too much. Similarly, the variation of consumption falls somewhat, although not by too much as habit persistence in consumption leaves this aggregate relatively volatile. The volatility of labour and output rises in similar fashion, as the availability of a cheaper intertemporal storage induces the representative agent to increase labour in the period of the shock, which naturally also pushes up output in the same direction. Apart from this, many of the other statistics remain relatively stable in their relationship with one another. Note that although the cross-correlation of inflation with output has not changed much, the general behaviour of money demand changes once adjustment costs to investment are removed, as the interest rate now loses it's inverted indicator feature and responses of the demand for real money balances now turn counter-cyclical instead.

Table 3: No adjustment costs, no habit persistence



	Correlation at Lag	t	t-1	t-2	t-3	t-4	SD(%)
No habit persistence, No adj costs	consumption(t)	0.73	0.43	0.16	-0.03	-0.01	0.45
	investment(t)	0.98	0.76	0.56	0.38	0.12	5.71
	output(t)	1.00	0.74	0.52	0.33	0.15	1.64
	(total) labour(t)	0.87	0.59	0.34	0.13	0.01	0.29
	inflation(t)	-0.39	-0.34	-0.30	-0.29	-0.27	0.64
	expected inflation(t)	0.95	0.67	0.43	0.22	0.01	0.20

Table 3 present cross-correlations obtained from the model without adjustment costs to investment as well as no habit persistence in consumption. For the baseline specification, consumption is now smoother - as it reverts to the standard RBC's Friedman consumption-smoothing property - and investment turns correspondingly somewhat more volatile. As a result of abstracting from habit persistence, labour also turns more volatile responding more strongly to productivity shocks which in turn also makes output more volatile overall. Most of the nominal side to the model, again, continues to exhibit similar characteristics as was the case with simulations from the previous two versions of the model.

Table 4: Preference-based cash credit model

	Correlation at Lag	t	t-1	t-2	t-3	t-4	SD(%)
No habit persistence, No adj cost, Preference credit good	consumption(t)	0.73	0.43	0.18	-0.03	-0.01	0.53
	investment(t)	0.98	0.76	0.56	0.38	0.12	5.90
	output(t)	1.00	0.07	0.44	0.24	0.13	1.69
	(total) labour(t)	0.98	0.64	0.36	0.15	0.05	0.35
	inflation(t)	-0.14	0.04	0.05	0.04	0.01	1.23
	expected inflation(t)	-0.13	0.03	0.05	0.04	0.01	0.59

Finally, table 4 summarizes simulation results obtained from a solved preference-based cash-credit model in which the consumption good is split into two separate cash and a credit consumption goods and demand for the former is motivated by placing it directly into the utility function and creating demand for it in that particular way. What stands out is that the preference cash-credit model is somewhat more successful at explaining the volatility of inflation than the alternative cash costly-credit model. What needs to be borne in mind however is that the simulation evidence computed for the preference-based model employs money supply growth rate shocks as well, which are completely absent in the alternative model specification considered here. Also, as a result of the lack of other features such as habit persistence, adjustment costs to investment and costly credit production, the preference-based model is quiet - or perhaps closer to the point - incapable of displaying features such as the inverted indicator feature of interest rates, variations in Tobin's q, procyclical money demand, excess sensitivity of consumption to output as well as highly persistent inflation expectations. As a result of the deliberate parsimony[11] of shocks hitting the model some features of the full benchmark model are not as successful in replicating some of the stylized business cycle facts as the preference-based model. An interesting question to be posed but not answered at this stage is to see how well the model could be fitted to the data if all of the three possible shocks were to be considered jointly.

Table 5: U.S. Business Cycle Data

	Correlation at Lag	t	t-1	t-2	t-3	t-4	SD(%)
U.S. Data (1962–2004)	consumption(t)	0.76	0.63	0.54	0.41	0.32	1.06
	investment(t)	0.68	0.56	0.45	0.38	0.32	4.69
	output(t)	1.00	0.81	0.64	0.42	0.23	1.76
	(total) labour(t)	0.61	0.48	0.37	0.28	0.20	1.15
	inflation(t) – CPI	0.12	0.24	0.31	0.36	0.31	1.12
	expected inflation(t)	n/a	n/a	n/a	n/a	n/a	n/a

3.3.4 The breakdown of the stable money demand function

This section is going to demonstrate, how the discussed banking time model is in principle capable of mimicking the well-known breakdown of a fairly close association between real money balances and both output and consumption observed in U.S. data (see King and Watson, 1996). Thus far, simulation and impulse response evidence has only focused on telling a "real" story of the monetary business cycle, in as far as only shocks to goods productivity were considered. Combined with habit persistence in consumption and - more crucially - adjustment costs to investment, the model showed some success in mimicking the salient features of the U.S. monetary business cycle. Inverted indicator interest rates led to liquidity portfolio reallocation in favour of real money balances over the business cycle (i.e. making real money procyclical). As a results of sudden liquidity portfolio reallocation due to sudden

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[11] some authors often motivate simplicity of model structure by appealing to Occam's razor. A similar argument could be put forth here with regards to the number of shocks considered.

changes in the Wicksellian real rate of interest, a price puzzle ensued (a sharp drop in actual inflation below it's steady state value), followed by a procyclical and highly persistent convergence of expected inflation from above it's steady state value. All results were obtained by assuming a deterministic Friedman-type constant growth rate of the nominal money supply. Countercyclical interest rates and a well-defined LM-type demand for real money balances (due to credit production's *positive* interest rate elasticity) led to the latter's intuitive behaviour over the business cycle as implied by the model. Money, output an consumption were shown to exhibit a very close contemporaneous relationship. So how then, could the model also explain the breakdown of this relationship which is such a striking feature of post-1980's data?

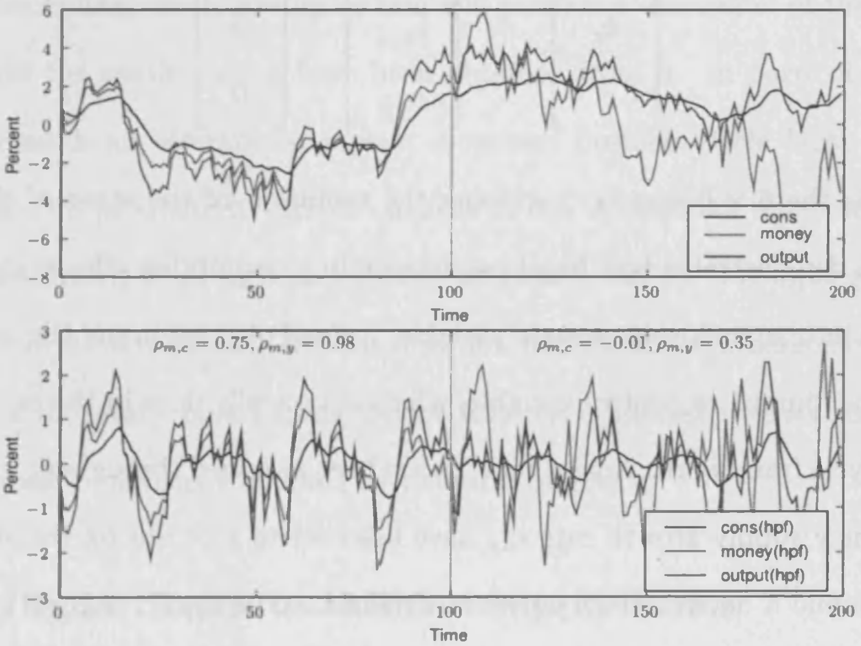

Figure 3.7: Simulation Evidence: Productivity Shocks & Credit Shocks

The answer to this question lies with the shock to credit production total factor productivity, v_t, which could be thought of as a direct shock to velocity, as it disturbs or re-positions the upward-sloping marginal cost curve in credit production, leading to changes in the share of credit over consumption that way for any given price (i.e. the nominal rate) of credit. Figure (4.7) illustrates a simulation of the model, which contains exactly the same shocks to productivity as the ones used in producing figure (4.4) but only up to time period 100. As an illustrative exercise, beginning from period 101 onwards[12], I have fed also credit production shocks into the exogenous-endogenous state system described by:

$$\begin{bmatrix} z_{t+1} \\ u_{t+1} \\ v_{t+1} \\ m_t \\ k_t \\ c_t \end{bmatrix} = \mathbf{P} \begin{bmatrix} z_t \\ u_t \\ v_t \\ m_{t-1} \\ k_{t-1} \\ c_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{z,t} \\ \epsilon_{u,t} \\ \epsilon_{v,t} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.25}$$

where $\mathbf{P}$ is the 6×6 matrix describing the evolution of the state of the system, which also demonstrates how habit persistence in consumption effectively turns the latter also into an endogenous state variable, instead of it assuming the nature of an endogenous "jump" or control variable, which it typically does in the canonical real business cycle framework. Notice that, as has been assumed throughout, innovations to the money supply growth rate $\epsilon_{u,t}$ have been set to zero during the simulations, so as to mimic a deterministic constant Friedman-type money supply growth rate rule.

The graph also reports correlation coefficients of money with consumption and output in simulated periods $0-100$ on the one hand, and the same coefficients of lin-

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[12] Strictly speaking, if one treated the simulated sample as a model-implied counterpart to the real data, one would only want to feed in credit shocks in the last quarter of that sample. For illustrative purposes I choose a 50/50 divide.

ear association for periods 101 – 200 in which the credit shock has been added. It is thus clear to see, how the tight correlation which exists in the simulated sub-period without the credit shock, which results from "riding up and down" a stable LM-type money demand function over the business cycle, breaks down or is less strong in the simulated sub-period with credit shocks. Notice that the nature of such shocks which affect the endogenously determined technology-based liquidity portfolio composition between money balances and credit can be thought of as stemming from financial deregulation, an interpretation which has also been chosen and studied elsewhere (see Kejak and Gillman, 2005; Gillman and Kejak, 2007).

3.4 Discussion

Before concluding, the following section will provide a discussion of the model presented and the results which have been obtained from it. In particular, since the model presents an attempt to explain monetary business cycle facts using a *real* story based on productivity-driven changes in the Wicksellian real rate of interest, the lessons drawn from such an experiment clearly call for a comparison with the current *theoretical* consensus embodied by new neoclassical synthesis models and the conduct of monetary policy using Taylor Rule type interest rate-setting behaviour and *practical* consensus embodied by inflation-targeting.

3.4.1 Taylor Rules vs. Money Supply Rules

The current consensus of theory-informed monetary policy is embodied by the prescriptions emanating from models of the so-called new neoclassical synthesis, or New Keynesian school of thought emphasising in particular the existence of nominal rigidities in the goods and/or labour markets, operationalised theoretically through the incorporation of so-called Calvo contracts (see Calvo, 1983). As monopolistic intermediate goods firms are only allowed to adjust their prices in each period with some fixed probability, they have to base their current-period optimal price-setting decision using some forecast of future inflation, which will also affect their future nominal marginal cost. Thus, the purely forward-looking micro-founded Phillips curve emerges[13].

Due to the assumption of price stickiness *and* the additional assumption that those firms which cannot adjust their prices in any given period index their prices to past inflation, such models are also capable of exhibiting a very persistent inflation process, which is therefore "built-in institutionally", so to speak (see inter alia Yun, 1996; McCallum and Nelson, 1999; Lawrence J. Christiano and Evans, 2005; Canzoneri et al., 2007; Smets and Wouters, 2007). With price-indexation, a Phillips curve incorporating both backward- and forward-looking inflation emerges. Also, a short-run trade-off emerges again between inflation and output, which in this micro-founded utility maximisation-based framework does not lead to a political-economy argument of opportunistic exploitability, as this would reduce welfare of the representative household.

In such a framework, optimal monetary policy is typically specified by a Taylor Rule nominal interest rate setting description, which - if operated optimally and given the sluggish inflation process - has to track the flex-price Wicksellian real rate of interest as closely as possible, so as to minimise welfare-harming distortions in the goods and labour market. In other words, the current view is such as to describe *current-period* inflation dynamics, in some sense, to be under little control by policy makers and indeed, given the myriad of shocks hitting an economy at all times,

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[13] Which, among other, has the unfortunate feature that announced and fully credibly anticipated monetary expansions cause recessions, as firms increase their prices ahead of time (see Mankiw, 2001). Also, Calvo price setting means that with very low probability some firm(s) may *never* be able to change their price!

to almost take on a life of it's own due to it's institutionalised nature given to it through the Calvo price setting framework coupled with price indexation.

So given sluggish inflation, and perfect control over the nominal rate as the main policy instrument, the central bank can control the real rate of interest as it sees fit, but perhaps would optimally want to do so in a way so as to closely track the flex-price Wicksellian rate of interest. Feasible interest rate rules, in an operational sense, as discussed for instance in Canzoneri et al. (2007), vary for instance in their definition of how to define the output gap (current minus steady state vs. current minus flex-price output) in a Taylor Rule, but may perhaps target other variables alltogether, such as nominal wage inflation (see also Canzoneri et al., 2007).

The Wicksellian banking time model, which I have described and analysed in this paper tells a different story of the world. In particular, the real rate of interest is an equilibrium price which ensues as a result of productivity shocks in the goods market, a "bull market" embodied in capital gains due to increases in Tobin's q, and the household's "overeating" today relative to tomorrow due to the increased cost of transferring wealth into future periods via the capital markets. "Overeating" today relative to tomorrow means a lower marginal valuation of wealth today versus tomorrow, which leads to a fall in the household's stochastic discount factor as implied by it's first-order condition with respect to real bonds. There are no nominal rigidities, but a procyclical and highly persistent inflation process develops nevertheless, simply due the the persistence in liquidity demand embodied by modeling consumption to exhibit habit persistence.

Since the money supply has been assumed to be of a Friedman constant growth rate type throughout, and most variation in nominal variables has been associated with the evolution of total liquidity demand (consumption) on the one hand and endogenous velocity (credit production) on the other (both of which were productivity-driven through the Wicksellian rate of interest), it would of course be natural to ask at this point whether money supply could be given some optimal state-contingent, and operationally feasible specification. Linked to this consideration is the question of what criterion the central bank would want to target in endogenously varying the money supply growth rate rule in feedback fashion. Also related to this is the usual questionable assumption of the central bank's *direct* control over the narrow monetary aggregate, or whether the model should not be amended to include some specification of the evolution of monetary base so as to link this to M1 via a general equilibrium formulation of the money multiplier, provided by the financial accelerator framework developed by Bernanke et al. (1999).

The answer to the criterion question would probably point to reducing the margin-distorting effect of the inflation-tax between consumption and leisure, whereas the question of an *operationally feasible* money supply feedback function would probably point to the late Milton Friedman's famous argument related to the "long and variable lags of the effects of monetary policy" Friedman (1961). However, in the current model the problem would not lie in the long and variable lags with which monetary policy may affect the state of the economy, but conversely in the long and variable lags with which money supply changes, embodied by an *operationally feasible* money supply feedback function, would *react* to the current state of the economy. The question of an optimal state-contingent money supply response is beyond the scope of this paper, but endogenous money supply functions in similar flex-price models have already been examined (see Gavin and Kydland, 1999).

3.4.2 Cash-in-advance, Habit Persistence and Real Indeterminacy

It is interesting to note at this point, that it has been shown that combining *standard production-based cash-in-advance* models as in Cooley and Hansen (1989, 1995) with habit persistence can easily lead to real indeterminacy, thus such models have no stable saddle-path solutions (see Auray et al., 2005). However, the same authors show, that real indeterminacy disappears completely when the household has access to a second means-of-exchange - allowing him to escape the cash-part of the liquidity constraint that way - which is *costless in a net wealth sense.* In particular, Auray et al. show that by endogenising velocity using a transactions cost technology as in Marshall (1992) or Carlstrom and Fuerst (2001), does not eliminate the problem of real indeterminacy, as the consolidated real resource constraint of the household still contains the net cost embodied by the transaction cost. The present model's way of endogenising consumption velocity is accomplished through self-produced credit, which is therefore costless in a net wealth sense and thus provides a framework in which real indeterminacy as described in Auray et al. (2005) does not pose a problem.

3.4.3 The Price Puzzle in identified SVARs

In some sense, complementary to the analysis of *calibrated* artificial economies by means of studying simulations and impulse-responses obtained from reduced-form solutions of approximated non-linear rational expectations systems, is the structural VAR (sVAR) literature, which instead is based on *estimation* of the coefficients of unrestricted VARs using data and some *non-unique* orthogonalisation method to identify the "true" structural shocks (see Sims, 1986; Bernanke, 1986; Blanchard and

Quah, 1989)[14]. This approach has been employed in particular to empirically evaluate the importance of monetary policy shocks in the determination of endogenous response of business cycle quantities and prices (see Leeper et al., 1996; Christiano et al., 1999; Uhlig, 2005) and, due to the non-uniqueness of identification, is not without it's critics (see Rudebusch, 1998; Chari et al., 2005).

One part of this literature specifically concerning itself with the analysis of the importance of monetary policy shocks (and thus concerning itself empirically with the monetary transmission mechanism) based on impulse responses, has led to the identification of a very robust phenomenon, called *the price puzzle* (see Sims, 1992). Contrary to many economists' a-priori expectations about the response of the economy to a, say, positive shock to monetary policy, impulse responses from estimated and identified VARs show an initial *fall* in the price level, following the positive innovation to the short-term nominal rate[15].

Assuming the central bank tracks a completely productivity-driven Wicksellian real rate of interest fairly closely (by setting the nominal rate "correctly" for some given endogenous evolution of the (expected) rate of inflation), then the present model is capable of providing an explanation for this puzzle, which is rooted in the initial response of inflation due to the sudden reallocation of the household's liquidity portfolio in favour of real money balances (due the the LM-type demand for real money balances implied by credit production), which has to be accommodated by a sudden drop in actual inflation given the maintained assumption of an underlying "passive" Friedman-type constant money supply growth rate rule, followed by convergence from above (see figure (4.2)).

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[14] Many different orthogonalisation schemes have been employed, ranging from the "standard" Cholesky decomposition, to schemes based on long-run restrictions.

[15] This analysis of course assumes that the federal funds rate correctly identifies the instrument of monetary policy. Other studies have employed a narrow monetary aggregate, such as M1 (see Sims, 1972; Eichenbaum, 1992)

Of course, the model at present due to it's parsimonious nature is still very stark in the responses of inflation it produces following a positive shock to productivity, in that the initial drop is of unexpected "one-off" nature, followed by an immediate jump of inflation above steady state and gradual and persistent convergence from above. However, if one were to entertain the view that the central bank actually engaged in "real-time" open-market operations, eliminating "high-frequency" erratic shocks to money demand (and thus inflation) by *inelastically* supplying money so as to partially dampen this, a less stark picture would emerge. Also, as already discussed above, the market for liquidity in the present model is also very parsimonious in structure, as it consists of a simple liquidity portfolio comprising only two means-of-exchange, money and credit and a simple inert evolution of total liquidity demand, modeled by habit persistence in consumption. Contemplating a microfounded view of a more complex and diversified portfolio, and smooth state-contingent substitution between such means-of-exchange, for some given liquidity demand, presumably would lead to a less pronounced variation in the unexpected component of inflation as implied by the model.

3.4.4 Investment Adjustment Cost & Endogenous Labour

Boldrin et al. (2001) modify a canonical RBC model to include various sources of real rigidities, such as inelastic supply of physical capital, inter-sectoral labour market rigidities and lagged responses of labour to changes in sectoral productivity. More importantly, they also compare their model to the model setup used in Jermann (1998), which employs the habit-persistence-adjustment cost framework in a similar way as is done in the present paper. The former authors conclude that Jermann's framework *always* leads to countercyclical labour following a one-off shock to productivity. This is indeed true if one decides to calibrate adjustment costs to investment to rise very fast as investment (demand) increases. Figure (4.8) demonstrates this by plotting the economy's response to a one-off shock to the productivity innovation, but for a much higher calibrated adjustment cost parameter, set to $\kappa = 10.0$ instead of the benchmark calibration of $\kappa = 2.0$. For such an inelastic supply of physical

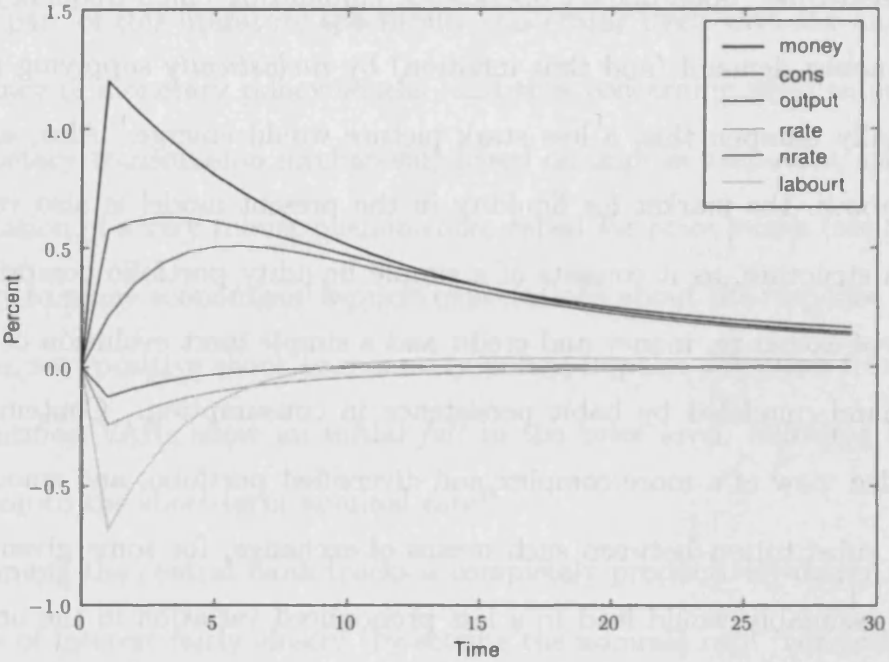

Figure 3.8: 1% innovation productivity, $\kappa = 10.0$

capital, following the positive shock to productivity, the saving objective of the representative household is frustrated to an even greater extent, leading to the outcome that the household decides to smooth it's marginal valuation of wealth by taking more leisure and thus working less ("labourt" is the *total* amount of labour). This of course would make the predictions of the model much worse in that labour turns strictly countercyclical - and as a result - would also lead to much smaller volatility of output in general, as the positive shocks to the Solow residual process are dampened by a substitution away from the usual "make hay while the sun shines" effect towards taking more leisure instead. But as I have shown, assuming only moderate amounts of adjustment costs to investment still results in the inverted indicator effect of interest rates, while preserving the procyclical nature of endogenous labour.

3.4.5 An extension using the Stockman assumption

The model discussed thus far represents a general equilibrium discussion of a cash-in-advance model mimicking some of the features found in the fairly traditional IS-LM framework in that it describes a general equilibrium version of the traditional money demand function which is functionally relating the demand for real money balances to a scaling variable such as consumption or income on the one hand, and the nominal rate of interest on the other. The present model can be likened to this traditional framework even further by making not only consumption but also investment subject to the cash-in-advance constraint, which is the assumption made in (Stockman, 1981, see). Combined with the physical capital adjustment cost assumption and habit persistence in consumption, the ensuing counter-cyclicality of interest rates with the simulated business cycle would help the model attain a greater degree of realism in that both consumption and investment would be affected by productivity shock-induced changes to the real rate of interest. This in turn would lead to the desirable effect of permitting both investment and consumption to be overshot by money holdings in response to falling interest rates bringing about a re-allocation of exchange means from credit to money. The resulting Wicksellian business cycle economy would then constitute a general equilibrium variant of the traditional IS-LM framework, but with the difference that the main shock driving the business cycle would occur in the goods production sector and money demand would fluctuate endogenously.

3.5 Conclusion

I have described and solved a monetary real business cycle banking time model, in which self-produced credit (see Kejak and Gillman, 2005; Gillman and Kejak, 2007) with a positive interest rate elasticity *locally* leads to an LM-type demand for real money balances. *Globally*, for different steady state calibrations of the nominal rate of interest, the same money demand function is of Cagan-type (see Cagan, 1956), as higher nominal rates imply a falling interest-rate elasticity (see Gillman and Kejak, 2008). The model thus follows in spirit the cash-in-advance literature but endogenises velocity by introducing a second, self-produced, means-of-exchange which is a perfect complement to money in this sense.

Contrary to common practice in the cash-in-advance literature, I do not model the growth rate of the money supply *stochastically*, but instead assume a Friedman-type constant growth rate rule which is completely deterministic. I justify this decision by showing that the model's *endogenous demand for real money balances* is such as to vary intuitively over the business cycle so as to reproduce closely the co-movement of real balances, consumption and output, and the rate of inflation, which has been documented in many studies (see inter alia Friedman, 1971; King and Watson, 1996).

A key building block required for obtaining this result is to model the supply of physical capital inelastically, by incorporating a quadratic adjustment cost term in the investment process. This has already been shown elsewhere (see Boldrin et al., 2001) to improve the canonical real business cycle framework in as far as it enables an inverted indicator modeling of real interest rates. Intuitively, following a shock to productivity the household wants to smooth it's marginal value of wealth and thus exhibits a strong demand for saving in the periods following the shock. With inelastic capital however, the implementation of this consumption smoothing (saving) objective is quickly frustrated, as capital gains and a "bull market" (increases in Tobin's q) make investing increasingly expensive. The household thus chooses, in some loosely speaking sense, to "overeat" in all periods after the shock relative to tomorrow, leading to a fall in the real rate, as implied by the stochastic discount factor derived from the first-order condition with respect to real bonds.

As the nominal rate - through the Fisher equation - is found to fall as well, the household optimally reallocates it's liquidity supply portfolio in accordance with the LM-type money demand function implied by credit production, favouring real money balances over credit procyclically over the business cycle. Therefore, the household is found to vary the composition of it's portfolio of liquidity supply in response to productivity-driven changes in the nominal rate - which for given inflation expectations and the Fisher equation - derives from the Wicksellian real rate of interest. With habit persistence in consumption, the goods market, in some sense, reacts more sluggishly whereas the financial (liquidity) market reacts instantaneously to interest rate movements. The model's endogenous behaviour off steady state is almost unidirectional from the goods market to the financial market. The only way responses in the financial (liquidity) market feed back into the goods market is through the (expected) inflation tax affecting the marginal rate of substitution between consumption and leisure, which is however small quantitatively (see Cooley and Hansen, 1995).

Following a positive shock to productivity, the sudden reallocation of the household's liquidity portfolio in favour or real money balances - for a given deterministic supply of real balances - requires equilibrium in the market for real money balances to be established through a sudden drop in the inflation rate below it's steady state value, to adjust money supply to be in line with money demand. However, along the subsequent expected projected path back to the steady state, interest rates converge gradually from below, credit is thus gradually favoured again and the demand for real money balances has to fall gradually, residually, while in the meantime because of consumption habit, total liquidity demand is gradually expanding in hump-shaped fashion. Again, for a deterministically given supply of real money balances, the inflation rate has to adjust endogenously so as to equate money demand and supply along the transitional expected path, meaning that inflation after the sudden drop, jumps back up above steady state so as to converge gradually from above.

I have discussed how this behaviour of actual and expected inflation is in some sense capable of giving a partial explanation for the so-called price puzzle observed in impulse responses from identified VARs following a monetary expansionary shock, with the caveat that such studies employ non-unique identification schemes of innovations to structural shocks. The model has been shown to exhibit highly persistent *expected* inflation rate time series in simulations, which are also procyclical. By introducing productivity shocks in credit production which ought to be understood as shocks embodying financial deregulation (see Kejak and Gillman, 2005; Gillman and Kejak, 2007), I have shown how the model is in principle capable of reproducing the breakdown of a stable demand function for real money balances, which is a striking feature of U.S. post-1980s data. Since I have taken a constant deterministic money growth rate rule as my maintained assumption throughout, I believe a natural question one may ask next is whether the current model provides a framework to determine, in some sense, an optimal state-contingent money supply growth rate response, and whether one would want to model this assuming direct control over the monetary aggregate, or only the monetary base, linking the former via the financial accelerator mechanism (see Bernanke et al., 1999) to the latter.

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Chapter 4

Asset Pricing in a Banking Time Model

A standard cash-in-advance type monetary real business cycle is developed, in which the representative agent has access to a second means of exchange (or liquidity), credit, which is produced in decentralized fashion by a separate banking sector. The paper follows the idea that credit production is subject to retaining a share of short-term government debt equal to the amount of credit on the bank's balance sheet, which is then re-distributed back to the household in form of a lump-sum dividend payment equal to the cost of credit. Therefore, the actual payout (and thus also return) on short-term debt (i.e. the risk-free rate) is partially paid out in form of a return obtained on a short-term saving deposit *and* the banking wage bill, thus residually leading to a *lower* return on that deposit. Since the degree of this distortion is linked to the endogenous nature of credit production (which in turn is affected by inflation), non-standard results regarding the unconditional shape of the term structure of interest rates and conditional and unconditional behaviour of the equity premium are obtained.

4.1 Introduction

Ever since the development of the consumption-based general equilibrium version of the CAPM model (Merton, 1971; Breeden, 1979; Lucas, 1978), the majority of contributions to the literature studying asset prices within this framework have focused on determining ex-ante expected asset returns in terms of risk premia derived from undiversifiable systematic risk. This approach of studying risk premia implied by the covariance of an asset's return with a typically preference-based stochastic discount factor (SDF) - or alternatively, the beta representation involving a correlation coefficient times a unique market price of risk[1] - has established itself as the standard way of studying asset prices in general equilibrium, not exclusively due to, but also because of the striking resemblance to earlier approaches in finance, most notably the standard CAPM (Sharpe, 1964; Lintner, 1969) of pricing assets, which instead of using some consumption-related measure, typically uses the market-portfolio's return as a way of proxying current marginal value of wealth.

In spite of the relative success of general equilibrium models in explaining the behaviour of aggregate quantities, it has proven immensely difficult to accomplish the same regarding asset prices, where any such failure of matching up theory with financial data stylized facts has typically been labelled a "puzzle". Two of such puzzles are the equity premium puzzle (Mehra and Prescott, 1985) and the closely related risk-free rate puzzle (Weil, 1989), on the one hand, and the term premium puzzle (Backus et al., 1989; Donaldson et al., 1990), on the other. Whereas the equity premium puzzle documents the *quantitative* failure of the consumption beta model to explain the observed excess return risky stocks earn over the risk-free rate,

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[1] Using Cochrane's notation, $p = E(mx)$ can always be expressed as $E(R^i) = R^f + \left(\frac{cov(R^i,m)}{var(m)}\right)\left(\frac{var(m)}{E(m)}\right)$ which is just $E(R^i) = R^f + \beta_{i,m}\lambda_m$, the beta representation (Cochrane, 2005)

the low risk-free rate puzzle asks why, given a historical long-term annual growth rate of consumption in U.S. data of roughly 2%, the observed historical average real risk-free rate has been only approximately 1%. In a recent empirical study based on various utility specifications and using data on consumption and inflation, Canzoneri et al. (2007a) compare the theory-implied (i.e. consumption Euler equation-implied) CCAPM rates with the observed ex-post money market rates, only to find that they typically bear little resemblance conditionally and that they exhibit a positive spread unconditionally, the low risk-free rate again. They also find that the spread between the two rates is directly related to the stance of monetary policy. Kocherlakota (1996) emphasises how the low-risk free rate can really be viewed as a puzzle arising from the tension which emerges from explaining the two phenomena of the low risk-free and the high equity returns *simultaneously*. Also, in contrast to much of literature's recent emphasis placed on unconditional excess returns typically derived from first-order conditions using log-normal distributional assumptions about returns, Giovannini and Labadie (1991) show how in dynamic simulations of theoretical ex-ante bond and stock returns obtained from a monetary endowment economy with standard power utility, both rates move conditionally almost in identical fashion together, leading to the striking result that only fluctuations in the SDF (or equivalently the marginal utility of wealth) represent the underlying common factor driving movements in both rates, whereas conditional movements of risk premia appear to play little or no significance in this regard, the equity premium puzzle again. There now exists a sizeable literature trying to explain the high equity premium, whose review would be beyond the scope of this paper. A recent and very comprehensive survey of the equity premium literature is provided by Mehra and Prescott (2003). Other good discussions of the equity premium are also contained in Cochrane (2007), Cochrane (2005) and Campbell (2000).

The term premium puzzle, on the other hand, pertains to the unconditional yield curve of government-issued bonds, which in post-war U.S. data is upward-sloping, both for the real[2] and the nominal yield curve (see Fama, 1990; den Haan, 1995). What is also of importance is that the unconditional yield curve is typically much steeper at the short- than the long-end, so it is also highly convex on average[3]. In contrast to this, bond yields derived from standard general equilibrium models obey a generalised, risk-adjusted version of the pure expectations hypothesis of the term structure of interest rates (see Backus et al., 1989; Donaldson et al., 1990; den Haan, 1995). Risk-neutral investors or deterministic settings imply a completely flat yield curve "on average", risk-averse agents facing uncertainty and a *positively* autocorrelated process for the stochastic discount factor, imply an unconditionally downward-sloping yield curve. Within the latter set of assumptions, Backus et al. (1989) also show that an independently evolving stochastic discount factor also implies a flat yield curve on average. Typically, general equilibrium as well as a-theoretical "affine" one-factor models approximate the nominal yield curve by simply modeling it's real counterpart, in order to make valuation of yields tractable and to avoid theoretical concerns over how money demand ought to be motivated on theoretical grounds. Labadie (1994) and den Haan (1995) also show that care needs to be taken in specifying the endowment process *in levels* in a simple Lucas exchange economy. Difference-stationary specifications lead to small persistence in expected consumption growth and positively autocorrelated consumption growth

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[2] Using evidence from UK inflation-indexed bonds, Seppala (2000) recently argued that the *real* term structure for the UK is downward-sloping, so he asserts the standard RBC model's predictions are correct. However, in a series of studies, Mishkin (1982, 1990b,a, 1992) found that real and nominal interest rates move in tandem, contradicting this view.

[3] Campbell et al. (1997), using the McCulloch and Kwon (1993) U.S term structure data base, find an average spread of the 10-year zero-coupon log yield over the one-month TB yields of 137 basis points. Also, the average yield spread over the one-month TB is 33 BP at three months, 77 BP at one year and 96 BP at two years. There is very little further change in average yields after two years.

(leading to the counterfactual downward-sloping term structure on average), whereas trend-stationary specification leads to the opposite (an upward-sloping term structure on average, but with a counterfactual dynamic behaviour of the SDF using power utility[4]). Also, regardless what type of autocorrelation for the SDF is either assumed or endogenously obtained from within a model, for standard power utility with low risk aversion, the slope would be relatively constant and thus not exhibit the stronger curvature effects at the short end seen in the data[5], as well as quantitatively small thus in an approximate sense leading to a practically flat yield curve unconditionally[6]. Related to this last point, Hansen and Jagannathan (1991) argue that the *observed* slope of the term structure must imply high volatility of the stochastic discount factor due to high Sharpe ratios in the bond market which result from small average bond term premia coupled with low term premia volatility. However, Campbell (2000) points out that "high Sharpe ratios of this sort [...] are of course highly sensitive to transactions costs or *liquidity services* [emph. added] provided by Treasury bills".

A large majority of explanations put forth in an attempt to resolve the equity premium or term premium puzzle are typically derived from simple *endowment* economies, in which output is perishable and governed by an exogenously specified process, thus through market clearing also determining the level of consumption in each period. The habit persistence literature (see Constantinides, 1990; Abel, 1990) has been a particular focus of attention, as habit persistence in consumption

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[4] However, if the discount factor is equal to some power of expected consumption growth, both Donaldson et al. (1990) and den Haan (1995) discuss how observed consumption growth can be either positive or *negative*, depending on whether quarterly or monthly consumption data is analysed, thus raising concerns over aggregation bias.

[5] This is a typical feature of simple *one-factor* affine term structure models to which standard general equilibrium models with power utility typically reduce to. Backus et al. (1998) provide a good survey of discrete-time term structure models demonstrating the lack of convexity in one-factor models.

[6] This fact is also emphasised by (Bansal and Coleman, 1996, p.1148) who also call the theoretical term structure in standard models "essentially flat".

can potentially alter the stochastic discount factor in ways to induce more volatility in marginal utility, and thus lower the degree of risk-aversion required to obtain sufficiently large risk premia. Equally, other utility function specifications, such as Epstein-Zin preferences (Epstein and Zin, 1991) have also been explored, as they allow disentangling of the elasticity of intertemporal substitution from the coefficient of relative risk aversion. However, as recently emphasised by Cochrane (2007) and Mehra and Prescott (2003), the current state of affairs is such that none of the contributions made hitherto have reduced macro-finance's reliance on assumptions of fairly large levels of risk aversion in order to explain the equity premium, whereas a broad consensus view on the parameter of relative risk-aversion appears to place this value at a plausible maximum of five, and perhaps closer to one, i.e. logarithmic specification (see Kocherlakota, 1996, p.52). Indeed, to draw an analogy to another popular literature, just as research in the New Keynesian literature for a period of time has asked "How much rigidity [in price and wage contracts] do we need?", apparently the general equilibrium asset pricing literature has and still remains asking itself "How much risk-aversion do we need?"

Assuming sufficiently high levels of risk-aversion and adopting new utility functions has resulted in some degree of success in explaining stylized asset pricing facts from within *endowment* economies. However, regarding both the equity premium and the term premium, den Haan (1995) and Jermann (1998) demonstrate how fully specified general equilibrium models with non-trivial production and a physical capital storage technology (in which consumption and dividends are endogenously determined), allow the representative agent to more successfully implement her consumption smoothing objective (i.e. allowing consumption to react endogenously in order to smooth the volatility in the marginal value of wealth), thus eliminating many positive results obtained from simple endowment economies, in particular such which have been obtained in combination with habit persistence. In fact, Jermann (1998) is only able to preserve a sufficiently large equity premium by reducing physical capital's effectivity as a storage technology by introducing adjustment costs to investment (and thus making the supply of physical capital inelastic) in addition to incorporating habit persistence in consumption. Similarly, Boldrin et al. (2001) also combine habit formation with real rigidities in the productive sector, by adding a capital-goods production sector with decreasing returns (leading to an inelastic supply of capital that way) and disallowing labour to react to current-period shocks.

One problem which is shared by both of the aforementioned production-economy based explanations of the equity premium (and indeed in general with other models using habit persistence in consumption), is that the increased volatility in the marginal value of wealth, which stems from the non-separable nature of such utility specifications, also raises the volatility of real interest rates to implausibly high levels and as a result also alters the behaviour of aggregate quantities in non-trivial ways.[7]. Tallarini (2000) modifies an environment similar to the production general equilibrium model studied by Jermann to include Epstein and Zin non-expected utility. Although he cannot account for the equity premium, he improves on the risk-free rate puzzle and the market price of risk (or equivalently, the Sharpe ratio). But the main result of his paper is to show that there is a real possibility to modify simple general equilibrium models such as to improve asset pricing predictions, leaving aggregate quantity dynamics *practically unaltered*, something which could not be said for Jermann (1998) and Boldrin et al. (2001)[8].

The present paper follows the tradition of the above-mentioned literature on asset

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[7] Within an *endowment* economy framework, Campbell and Cochrane (1999) have recently proposed a solution to the interest rate volatility problem, by specifying a nonlinear habit utility function, in which the "intertemporal substitution" effect - which is the culprit for implausibly high variation in interest rates - is just offset by a "precautionary savings" effect.

[8] However, (Cochrane, 2007, p. 297) argues against the possibility of ever obtaining a pure "separation theorem" of quantity and price dynamics.

pricing within fully specified production economies, embodied by Jermann (1998); Boldrin et al. (2001); Tallarini (2000), whose focus is primarily on "second-order" *risk-induced* arguments related to undiversifiable systematic risk, and McCallum and Goodfriend (2007); Canzoneri and Diba (2005); Canzoneri et al. (2008), whose focus is instead on "first-order" certainty-equivalent arguments typically related to implicit liquidity returns of short-term debt. Building on previous work by Gillman and Kejak (2008) and Benk et al. (2005), this paper develops, calibrates and dynamically analyses a monetary general equilibrium model, which is essentially of cash-in-advance type, but is modified by the addition of a de-centralised micro-founded banking sector, whose credit production specification is motivated by the financial intermediation literature (Clark, 1984; Hancock, 1985).

The banking sector acts as a financial intermediary, using labour and deposits to produce a credit service which can be used in conjunction with money to pay for the consumption good. Further, the bank is also holding government debt of all maturities, which are translated one-for-one into equivalent bond-backed saving deposits of various maturity held in turn by the representative household. Closely resembling an argument related to the liquidity-providing role of short-term government debt developed by Bansal and Coleman (1996), credit is assumed to be produced subject to a collateral requirement, meaning that a *share* of the economy-wide supply of short-term (nominally) riskless nominal debt is retained on the banking sector's balance sheet in order to back up the aggregate amount of credit produced.

In contrast to Bansal and Coleman, the banking sector's payout on it's collateral is re-distributed back to the household in form of a dividend payment equal in value to the total cost of using credit. *Instead*, the total payout[9] on the credit-backing collateral *share* of economy-wide short-term debt (and thus the total payout on the

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[9] A discussion of the "total payout" on (a share) of debt is possible in this model, as short-term debt is assumed, as in Bansal and Coleman (1996) to be in net *positive* supply.

equivalent share of the one-period saving deposit) is *replaced* by the total revenue from credit production *minus* the banking wage bill, which upon normalising by credit (or equivalently by the share of economy-wide short-term debt equal in value to credit) implies a return on that share equal to the price (or average revenue) of credit minus the average product (or average cost) paid out to the consumer in form of the credit-normalised banking wage bill. The lower than usual residual return on the collateral share of debt which equals the per-credit normalised revenue share of deposits in credit production constitutes a no-arbitrage equilibrium vis-a-vis the higher (nominal) return on capital represented by the CCAPM rate as defined in Canzoneri and Diba (2005), as the household is compensated in form of the banking wage bill equal to that difference.

The model presented here thus offers a micro-founded financial intermediation approach, driving a cost-wedge derived from liquidity (credit) production between the CCAPM and the money market rate, which is in contrast to Canzoneri and Diba (2005), who obtain a similar return wedge by placing (a function of) bonds in an ad-hoc way directly into the cash-in-advance constraint, and McCallum and Goodfriend (2007), by specifying a loan-management function, in which bonds are more effective as collateral vis-a-vis physical capital, obtaining the interest rate wedge that way. Also, McCallum and Goodfriend (2007) cannot account for variations in velocity, since in their model loans and the monetary base (i.e. two distinct "high-powered" exchange bases) are lumped together through the financial accelerator framework (see Bernanke et al., 1999) to give a measure for broad money entering the cash-in-advance constraint as the only effective means of exchange. In contrast, the model presented here is capable of exhibiting endogenous variation in velocity, as a unique exchange equilibrium between money and produced credit is obtained through diminishing returns to labour only in the credit sector and the resulting intersection between the cost of holding money and a convex upward-sloping marginal cost schedule in credit production (see Gillman and Kejak, 2008).

Through the key mechanism described above, the model is qualitatively capable of simultaneously explaining the low risk-free rate, the high equity premium and a term structure which is on average upward-sloping and convex in shape. Novel asset pricing results are obtained by distorting *deterministic mean* returns directly in a certainty-equivalent framework sense through endogenous cost-driven effects of the banking sector, rather than through the usual *risk-induced* variations of ex-ante returns, related to the undiversifiability of systematic risk. However, as I will discuss further below, "first-order" certainty-equivalent "market-driven" asset price distortions (such as those in Bansal and Coleman (1996), Canzoneri and Diba (2005) and Canzoneri et al. (2008) and the model presented here) can potentially be combined trivially with second-order risk-induced asset price distortions stemming from undiversifiable systematic risk, as long as the former distort returns (or prices) of traded assets such as to be *visibly* different in equilibrium from the usual CCAPM rates in ways which are directly compensated (or hedged) elsewhere, leading to distortions which do not constitute undiversifiable systematic risk but are completely hedged *by construction*[10].

The model improves on Bansal and Coleman (1996) along two dimensions. First, the model's results are more transparently driven by the distortive effects of a micro-founded banking sector, instead of appealing to an essentially ad-hoc transactions-cost function (McCallum, 1983). Secondly, combined with the usual expectations hypothesis of the term structure present in such models, the term structure results do

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[10] The model presented here satisfies this condition, as endogenous cost-driven variations of the low risk-free rate are always perfectly hedged by equivalent compensating variations in the banking wage bill. Related to this, Coeurdacier et al. (2007) explain the equity home bias puzzle through the hedging function of the domestic goods production wage bill, when households face re-distributive shocks.

no require multi-period bonds to provide liquidity services and - directly related to this last point - the model is capable of explaining a term structure which is steeper at the short-end than the long-end. Further, simulations reveal a positive correlation between the nominal rate and velocity and a negative correlation between the ex-post real risk-free rate and inflation. Finally, in line with recent theoretical (Canzoneri and Diba, 2005; Canzoneri et al., 2007b) and empirical (Canzoneri et al., 2007a) evidence of a systematic link between monetary policy and the spread between a theory-implied CCAPM and the observed ex-post money market rate[11], the model is capable of linking monetary tightening to a fall in this spread and vice-versa.

In order to permit a direct comparison with Bansal and Coleman (1996), I present theoretical asset pricing results using two different modeling frameworks, one which follows Canzoneri et al. (2008) in which the nominal CCAPM rate of interest is in the usual way *endogenously* determined through the standard Fisher relationship and the money supply process is *exogenously* specified, and Bansal & Coleman's modeling technique, in which the nominal rate of interest is assumed to be an essentially fixed state-contingent government target (or alternatively, exogenously specified process) and the money supply process is, for a given endogenous real rate of interest determined from the model, *endogenously* implied through the Fisher equation. Using the latter approach, I show how the credit-banking model presented here is capable of producing asset pricing results which are functionally equivalent to those in Bansal & Coleman, and in particular how this permits simultaneously lowering the risk-free rate and raising the equity premium.

I show how the cost-driven distortive effects of the banking sector are essentially driven by a partitioning of the return on a fraction of short-term debt into a lower

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[11] In similar fashion to Canzoneri and Diba (2005), the model explains this spread by providing a *theoretical* framework in which both the CCAPM and the money market rates (and thus their spread) are determined from within a model.

than usual risk-free rate and a compensating payout in form of the banking wage bill paid out to the representative household, which constitutes a no-arbitrage position vis-a-vis the standard CCAPM return, as the former two always add up to the latter. Further, I argue that standard uncertainty-induced risk premia results (typically embodied by covariances of the stochastic discount factor and the cash flow of an asset in terms of the consumption good) stemming from the undiversifiability of systematic risk in stochastic environments are preserved, since the only additional uncertainty the representative household faces is the ex-ante ignorance over how much of the return he receives will be in form of his short-term saving deposit and how much residually in form of the banking wage bill. Since the two always add up to the standard CCAPM rates (both real and nominal), this type of return uncertainty is not of systematic risk type and in fact perfectly hedged or insured and thus will not upset standard stochastic ex-ante *effective* returns, once the return in form of the banking wage bill is added back towards the *visibly* lower return on the short-term saving deposit.

The remainder of the paper is structured as follows. Section 2 outlines the setup of the model which is populated by a household, a firm and and a credit-banking sector. Section 3 discusses how the model is capable of altering asset pricing results obtained from standard (monetary) real business cycle models. A direct comparison with Bansal and Coleman (1996) is presented and in particular, the model's *stochastic* asset pricing implications are discussed using Bansal&Coleman's modeling technique. In preparation for dynamic analysis of the first-order approximate solution of the model contained in section 5, section 4 studies the steady state properties of the model in it's entirety, instead of exclusively focusing on asset pricing results. In similar spirit to Canzoneri et al. (2008), the dynamic analysis in the section thereafter focuses on impulse responses as well as correlations obtained from simulations of the model solved for it's recursive law of motion. Section 6 provides a discussion of the results and finally, section 7 concludes.

4.2 The economic environment

In what follows I am going to write down an essentially standard cash-in-advance real business cycle model (Lucas, 1982; Stokey and Lucas, 1987), which is only modified by adding a further means-of-exchange, credit, which is costly produced by a decentralised financial intermediary (FI) by use of a two factor CRS Cobb-Douglas production function, whose specification is motivated by the financial intermediation literature (see Clark, 1984; Hancock, 1985). Following Gillman and Kejak (2008), deposits are created from the total exchange liquidity used in the model for carrying out consumption both in terms of money and credit, which means that consumption and (real) deposits can be used interchangeably:

$$c_t \equiv d_t \tag{4.1}$$

Further, in line with Canzoneri and Diba (2005)[12], I will refer to the standard derivations of prices for real and nominal bonds as (where my derivations assume a constant rate of economic growth equal to γ):

$$p_{1,t}^{b,N} = E_t\left[\frac{\beta\lambda_{t+1}}{\gamma\lambda_t\left(1+\pi_{t+1}\right)}\right] = m_{t,N}^{t+1} \tag{4.2}$$

and

$$p_{1,t}^{b} = E_t\left[\frac{\beta\lambda_{t+1}}{\gamma\lambda_t}\right] = m_t^{t+1} \tag{4.3}$$

as the CCAPM prices of short-term nominal and real bonds, respectively. $m_{t,N}^{t+1}$ and m_t^{t+1}, on the other hand, are the the corresponding one-period nominal and real

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[12] They only define CCAPM *returns*, but returns can of course always alternatively be expressed in terms of prices as well.

CCAPM stochastic discount factors from period t to $t+1$. Notice also, that the above expressions can also be expressed in terms of CCAPM *real* returns on nominal and real bonds, where the real return on the real bond is simply the inverse of the real discount factor:

$$1 + r_{1,t} = \left[E_t \left(\frac{\beta \lambda_{t+1}}{\gamma \lambda_t} \right) \right]^{-1} \tag{4.4}$$

and the real-valued return on the nominal bond, given by $1 + i^r_{1,t}$, is defined as:

$$\begin{aligned} 1 + i^r_{1,t} &= (1 + i_{1,t})\, E_t \left[(1 + \pi_{t+1})^{-1} \right] \\ &= (1 + r_{1,t}) - (1 + i_{1,t}) \frac{Cov\left(\frac{\lambda_t}{\lambda_{t+1}}, (1 + \pi_{t+1})^{-1} \right)}{E_t\left(\frac{\lambda_t}{\lambda_{t+1}} \right)} \end{aligned} \tag{4.5}$$

where the covariance term reflects the inflation risk-premium driving a wedge between the pure Fisher equation relationship (see Giovannini and Labadie, 1991). Notice that it may sometimes be convenient in analytical computations to set this covariance between the expected marginal value of wealth and inflation equal to zero, so that for given inflation expectations, modeling of nominal rates of returns can be approximated by modelling real counterparts[13]. Referring to the above *standard* concepts in this explicit fashion is necessary, as the present model will provide an alternative definition for the equivalent risk-free real and nominal price for bonds, given by:

$$\tilde{p}^b_{1,t} = \tilde{m}^{t+1}_t \tag{4.6}$$

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[13] see Gibbons and Ramaswamy (1993) for an application of this assumption in the Cox-Ingersoll-Ross model of the term structure.

and

$$\tilde{p}^{b,N}_{1,t} = \tilde{m}^{t+1}_{t,N} \tag{4.7}$$

respectively, which will generally differ from the standard definitions given above. The derivation of these distorted short-term real and nominal discount factors (and their corresponding rates) and analysis of their endogenous behaviour with respect to various factors, such as monetary policy, is the key contribution of this paper. Further, in order to aid derivations of the CCAPM (or purely intertemporal) rates, the household's budget constraint will also contain net balances of one-period nominal *virtual* bonds, which are not thought to be traded in reality and correspond to the usual nominal bonds in standard cash-in-advance models in which the absence of an FI eliminates distortions to the analogous risk-free payout on the corresponding short-term saving deposit. The net balances of these virtual nominal short-term bonds as they appear on the consumer's budget constraint are given by:

$$\frac{(1+i_{t-1})}{(1+\pi_t)}b^*_{t-1} - b^*_t \tag{4.8}$$

In similar fashion to Bansal and Coleman (1996), I assume that the financial intermediary needs to retain a share of the short-term government debt equal in value to credit as collateral on it's balance sheet. Instead of paying out the usual return on that share (i.e. the net nominal CCAPM rate), the FI re-distributes the earnings on it's collateral back to the representative household in form of a dividend equal in value to the original cost of purchasing the credit service. However, as I will show, the representative household still obtains the same *effective* return on the short-term saving deposit, as it would in an undistorted cash-in-advance model. However, since part of this return is re-distributed back in form of the banking wage bill, the *visibly* obtained residual return on the short-term saving deposit will reflect this cost and thus be lower.

It is this arrangement which leads to the result of a lower than usual risk-free rate (and a convex upward-sloping average term structure), which I will discuss in more detail further below. The model is set up such as to allow only trade in nominally-denominated government debt, however shadow prices and returns of inflation-indexed bonds can of course be derived in the usual way. Before specifying the various sectors characterising the economic environment, I will first discuss how short-term debt is modeled to be in positive net supply, in terms of a government-targeted debt ratio, similar to Bansal and Coleman (1996) and Canzoneri et al. (2008)[14].

4.2.1 Modeling the supply of short-term debt

The results derived below require short-term government debt to be modeled in positive net supply, as a share of the short-term government bond will affect asset pricing results in the economy. Here, I will briefly discuss my strategy of doing so, which employs the idea of a proportional supply of short-term debt, relative to deposits. Therefore, I define the variable:

$$\tilde{\eta}_{t-1} = \frac{\tilde{b}_{1,t-1} / (1+\pi_t)}{d_t} \tag{4.9}$$

to be the *pre-transfer* beginning-of-period proportional amount of debt, relative to real deposits, before any fiscal government adjustment has taken place and $\tilde{b}_{1,t-1}$ as the corresponding *pre-transfer* pre-determined beginning-of-period amount of short-term (one-period) government nominal debt (in terms of beginning-of-period prices).

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[14] The former specify an exogenous process for the value of total bond issue, the latter also work with debt ratios.

As will be shown, the model's results are derived from some notion of how a share of this proportional debt will be retained by the FI and issued instead as a dividend payment on retained collateral. Since credit in the model is an endogenously determined variable, the share of retained government debt is also endogenously determined. In order to keep the analysis tractable and the intuition simple, my strategy will be to keep the supply of proportional debt fixed in all time periods.

Notice though that debt is pre-determined at the beginning of the period $t-1$, but deposits (or consumption) and inflation are endogenously determined from within the model at the end of the period t. In order to keep the proportional supply of debt fixed in all time periods, the following timing convention will hold. After the revelation of all shocks and the resolution of uncertainty, the government can move first and, knowing the full structure of the economy (and thus the outcome of the competitive equilibrium prices and quantities), and thus also the level of end-of-period deposits and the level of inflation, uses fiscal transfers to adjust the level of pre-determined debt such as to perfectly obtain a fixed debt-to-consumption ratio. After that all other agents in the economy move and the competitive equilibrium is obtained. This timing convention amounts to the following specification of the proportional supply of short-term government debt:

$$\eta_{t-1} = \bar{\eta} = \frac{b_{1,t-1}/\left(1+\pi_t\right)}{d_t} \tag{4.10}$$

where $b_{1,t-1}$ is the corresponding beginning-of-period amount of short-term debt *after* the fiscal adjustment to keep in line with the endogenously determined level of deposits and inflation has taken place[15]. This simplifying assumption allows me to focus only on the way a *share* of this fixed proportional supply of debt influences

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[15] The main idea here is to adjust the absolute size of debt before agents trade, so that after they willingly hold the debt, they still incur the end-of-period erosion through inflation.

relevant results in the model. In particular, as I will show below, the share of consumption paid for in credit - where the absolute level of credit is given by f_t and the corresponding share by f_t^* - is a well-defined (production) function bounded between $[0,1]$ and is defined as:

$$f_t^* = \frac{f_t}{d_t} \tag{4.11}$$

Therefore, as long as the government incurs sufficient short-term proportional (relative to deposits) liabilities relative to the proportional (relative to deposits as well) production of credit[16], i.e

$$\bar{\eta} = \frac{b_{1,t-1}/\left(1+\pi_t\right)}{d_t} > \frac{f_t}{d_t} \tag{4.12}$$

for fixed $\bar{\eta}$ it is then always possible to define a share s_t^b, which as long as the above fiscal liability condition holds is bounded between $[0,1]$ and defines the proportional production of credit relative to the proportional amount of short-term debt. Because the proportionality factor is given by deposits for both supply of debt and production of credit, s_t^b defines the proportion of credit relative to debt directly, i.e.

$$s_t^b = \frac{f_t^*}{\eta} = \frac{f_t/d_t}{\frac{b_{1,t-1}}{1+\pi_t}/d_t} = \frac{f_t}{b_{1,t-1}/\left(1+\pi_t\right)} \tag{4.13}$$

Notice that s_t^b represents the share of short-term debt which is retained by the financial intermediary as collateral for credit, and I may sometimes wish to refer to it as the *debt utilisation rate* due to credit production or simply the banking sector's debt utilisation rate. Next, I will discuss the optimisation problem of the financial intermediary, which acts on behalf of the representative household and

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[16] Canzoneri et al. (2008) also require sufficient debt to be issued relative to demand in order for an equilibrium to be attained, see footnote 11 in their paper.

thus discounts current and future items on it's balance sheet using the household's stochastic discount rate.

4.2.2 The financial intermediary

The financial intermediary acts in a decentralised fashion as a producer of the credit exchange service demanded by the representative household and is also assumed to be profit maximising, sharing the economy-wide discount factor, given by $\frac{\beta^k \lambda_{t+k}}{\lambda_t}$. It produces credit using a CRS technology in labour and deposits created by the household, which is given by:

$$f_t = e^{v_t} A_f \left(\kappa_{t-1} n_{f,t}\right)^{\rho} \left(d_t\right)^{1-\rho} \tag{4.14}$$

where κ_t is a labour-augmenting exogenously specified parameter evolving according to:

$$\kappa_t = \gamma \kappa_{t-1} \tag{4.15}$$

thus determining the exogenously specified growth rate of the economy. Notice that it will often be convenient to express the credit-production function as a deposit- (or consumption-) normalised equivalent credit-share production function, which exhibits decreasing returns in deposit-normalised augmented labour and is given by:

$$f_t^* = \frac{f_t}{d_t} = e^{v_t} A_f \left(\frac{\kappa_{t-1} n_{f,t}}{d_t}\right)^{\rho} \tag{4.16}$$

Notice that this specification - assuming appropriately parametrised values for ρ, implies a convex marginal cost schedule in credit production (see Benk et al., 2005; Gillman and Kejak, 2008), which given a certain price of credit (which will turn out to be the net nominal CCAPM rate) leads to a unique exchange equilibrium between money and credit. Further, in a stochastic environment with shocks to credit productivity (and thus to credit production's marginal cost schedule) and shocks leading to variation in the (net) nominal CCAPM rate of interest, the economy will exhibit variation in (money-consumption) velocity, which will be positively correlated with the nominal CCAPM rate (or the price of credit). The FI is assumed to be the conduit for all liquidity supplied to the consumer. Besides providing the produced credit, the FI is also assumed to carry out optimal portfolio decisions on behalf and as instructed by the representative household. This means that the FI holds beginning-of-period money balances and receives instructions over how much much of current wealth be used to acquire end-of-period money balances, i.e.

$$\frac{m_{t-1}}{(1+\pi_t)} - m_t \tag{4.17}$$

where m_t represent end-of-period t real money balances. Government taxes or transfers are not modeled explicitly. However, given the government's endogenously determined fiscal policy to hit the fixed debt-to-deposit ratio and the corresponding interest payment obligations, appropriate helicopter-money lump-sum taxation can always be chosen independently in state-contingent fashion such as to implement a steady state money growth rate Θ^* with some random component embodied by the shock parameter u_t. The stochastic nominal money growth rate is thus given by

$$\Theta_t = (M_t/M_{t-1} - 1) = (\Theta^* + e^{u_t} - 1) \tag{4.18}$$

where Θ_t is the growth rate of money and Θ^* is it's stationary counterpart. Since the economy is growing at the steady state growth rate γ, in order to obtain a particular steady state target level of *inflation*, the monetary authority has to set the growth rate of money above the exogenous economic level of growth by that inflation target, so in steady state:

$$1 + \pi = \frac{1 + \Theta^*}{\gamma} \tag{4.19}$$

which for a positive steady state target inflation rate $(1 + \pi) > 1$ implies setting $1 + \Theta^* > \gamma$. The money balances on the bank's balance sheet are part of it's liabilities, as the representative household (frictionlessly) sources money balances from ATM machines with equivalent electronic balances (which are in turn linked to non-interest paying current account balances appearing on the representative household's budget constraint). These ATM electronic balances are sourced from current accounts (D_t^c) and therefore appear on the bank's balance sheet as follows:

$$-\frac{d_{t-1}^c}{(1 + \pi_t)} + d_t^c \tag{4.20}$$

where the bank's liquidity restriction given by $m_t = d_t^c \quad \forall t$ implies:

$$\frac{m_{t-1}}{(1 + \pi_t)} - m_t - \frac{d_{t-1}^c}{(1 + \pi_t)} + d_t^c = 0 \tag{4.21}$$

where d_t^c are current-period current account balances in terms of the consumption good, from which withdrawals from ATM machines are sourced. Moreover, The FI is also the holder of nominal government debt balances of all maturity, which as I will show will translate into one-for-one nominal balances of equivalent saving accounts, which will be part of the representative household's budget constraint. The receipts of government nominal debt balances net of new purchases will therefore appear in the FI's balance sheet as follows:

$$\frac{1+i_{1,t-1}}{(1+\pi_t)}b_{1,t-1}+\sum_{j=2}^{n}\frac{1+i_{j,t-1}}{(1+i_{j-1,t})\,(1+\pi_t)}b_{j,t-1}-b_{1,t}-\sum_{j=2}^{n}b_{j,t} \tag{4.22}$$

Notice that the FI willingly holds all government debt on behalf of the representative household at prices implied by the (nominal) stochastic discount factor consistent with the one from the representative household. Instead of viewing the FI as the portfolio optimiser acting on behalf of the representative agent one may also view the FI as the channel through which the government "floats" it's debt structure, thus commissioning the FI to convert bond holdings into equivalent saving deposits held in turn by the representative household. From this perspective, and as a specific example only focusing on the one-period nominal short-term debt (which is modeled in net positive supply), the FI has an obligation to the monetary authority (or the government), to float *all of the supply* and to pay the interest as implied by the equilibrium nominal stochastic discount factor. This perspective will be relevant in further discussions related to the derivation of the risk-free (nominal and real) rate. Finally, closely resembling a liquidity argument originally developed by Bansal and Coleman (1996), credit balances, given by f_t have to be backed up by a retained share of the short-term government debt equal to that value on it's balance sheet, which will serve as collateral for credit and will be defined as Ω_t^c. Since the collateral has to be retained by the FI, the net CCAPM return on this collateral multiplied by the amount of collateral (equal to credit) is re-distributed back to the representative household as a dividend payment from the financial intermediary. This implies:

$$f_t = s_t^b \frac{b_{1,t-1}}{(1+\pi_t)} \equiv \Omega_t^c \tag{4.23}$$

where Ω_t^c is the collateral. This implies for the re-distributed dividend Π_t^f:

$$\Pi_t^f = \Omega_t^c i_{t-1} = \Omega_t^c \left(\frac{\mu_t}{\lambda_t}\right) = f_t \left(\frac{\mu_t}{\lambda_t}\right) = f_t p_t^f \tag{4.24}$$

Where the last equality emphasises the fact that the cost of credit is re-distributed back to the household in lump-sum fashion in form of a dividend payment from the FI. Notice also that equation (4.23) can alternatively be expressed in terms of proportions (relative to deposits) and then solved for the share variable, s_t^b:

$$s_t^b = \frac{f_t/d_t}{\frac{b_{1,t-1}}{(1+\pi_t)}/d_t} = \frac{f_t^*}{\bar{\eta}} \tag{4.25}$$

which emphasises the endogenous determination of this share and how this affects the (proportional) share of short-term government debt which is retained by the FI on it's balance sheet and whose payout is finally re-distributed in terms dividend payment. Of course the discussion thus far then begs the question: If the FI has the obligation to pay interest on *all* of the amount of short-term debt as commissioned by the government (and thus also the equivalent short-term saving deposit held by the representative agent), but retains some share of this short-term bond as collateral for credit production whose value (i.e. collateral time the net CCAPM return) is re-distributed as a dividend payment, how is it going to fulfil this obligation?[17] It can only do so by paying out the revenue from credit production instead (which is price times quantity, i.e. the net nominal CCAPM rate times the amount of credit equal to the retained debt-share as collateral, or $i_{t-1}f_t = i_{t-1}s_t^b\frac{b_{1,t-1}}{(1+\pi_t)}$), whose amount exactly equals that share of short-term government debt which is retained as collateral. However, part of this revenue is paid out directly as the banking wage

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[17] equivalently, this is the same as asking how the financial intermediary can honour paying out the return on government debt if a share of the short-term debt, i.e. of the one-period bond, is held on to by the bank in form of collateral

bill, which upon normalising by $f_t = s_t^b \frac{b_{1,t-1}}{(1+\pi_t)}$ to convert it into a return, equals the average product of banking labour time[18]:

$$\rho\left(\frac{\mu_t}{\lambda_t}\right) = \rho i_{t-1} = \frac{w_t n_{f,t}}{s_t^b \frac{b_{1,t-1}}{(1+\pi_t)}} \tag{4.26}$$

Thus implying a proportional banking time cost defined over the entire supply of short term debt given by[19]:

$$s_t^b \rho\left(\frac{\mu_t}{\lambda_t}\right) = s_t^b \rho i_{t-1} = \frac{w_t n_{f,t}}{\frac{b_{1,t-1}}{(1+\pi_t)}} \tag{4.27}$$

Since the return on the retained share of short-term debt (and the equivalent share of the short-term saving deposit) is partially paid out in terms of the average product of banking labour time, it follows that the residual *visible* earned rate of return on the short-term saving deposit itself, net of that banking labour cost, has to be lower and, given a fixed deposit-proportional supply of short-term nominal debt $\bar{\eta} = \frac{b_{1,t-1}}{(1+\pi_t)}/d_t$, that endogenous variation in the share of credit used in purchasing consumption $f_t^* = f_t/d_t$ affecting s_t^b, will cause variations in this cost distortion. The FI's balance sheet solvency restriction that assets equal liabilities is given by:

$$P_t f_t + M_t = P_t d_t \tag{4.28}$$

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[18] To better understand the validity of this expression, recall that $f_t = s_t^b \frac{b_{1,t-1}}{(1+\pi_t)}$. Therefore the above can also be written as $i_{t-1}\rho f_t = w_t n_{f,t}$ which results from Cobb-Douglas specification of credit production.

[19] Notice that in my derivations I typically abstract from including the stationary growth rate κ and assume that all growing variables have already been normalised by κ and thus converted to stationary equivalents.

The balance sheet liquidity constraint is that money withdrawn by the consumer is covered by deposits:

$$P_t d_t \geq M_t^h \tag{4.29}$$

which would hold with strict equality if no credit were produced. The FI's objective is to maximise it's discounted stream of current and future profits:

$$\begin{aligned}
\max_{n_{f,t}} E_0 \sum_{k=0}^{\infty} \frac{\beta \lambda_{t+k}}{\lambda_t} \Bigg\{ & \\
& p_{t+k}^f f_{t+k} - w_{t+k} n_{f,t+k} \\
& + \left(1 + i_{1,t+k-1}\right)\left(1 - s_{t+k}^b\right) \frac{b_{1,t+k-1}}{\left(1 + \pi_{t+k}\right)} + \sum_{j=2}^{n} \frac{1 + i_{j,t+k-1}}{\left(1 + i_{j-1,t+k}\right)\left(1 + \pi_{t+k}\right)} b_{j,t+k-1} \\
& - b_{1,t+k} - \sum_{j=2}^{n} b_{j,t+k} + \Omega_{t+k}^c i_{t+k-1} - \Pi_{t+k}^f \\
& - \frac{1 + \tilde{i}_{1,t+k-1}}{\left(1 + \pi_{t+k}\right)} d_{1,t+k-1}^s - \sum_{j=2}^{n} \frac{1 + \tilde{i}_{j,t+k-1}}{\left(1 + \tilde{i}_{j-1,t+k}\right)\left(1 + \pi_{t+k}\right)} d_{j,t+k-1}^s \\
& + d_{1,t+k}^s + \sum_{j=2}^{n} d_{j,t+k}^s \\
& + \frac{m_{t+k-1}}{\left(1 + \pi_{t+k}\right)} - m_{t+k} \\
& - \frac{d_{t+k-1}^c}{\left(1 + \pi_{t+k}\right)} + d_{t+k}^c \Bigg\}
\end{aligned} \tag{4.30}$$

Notice therefore that the earnings on the retained share of debt as collateral equal in amount to credit are given by:

$$\left(\frac{\mu_t}{\lambda_t}\right) f_t = i_{t-1} f_t = s_t^b \frac{b_{1,t-1}}{\left(1 + \pi_t\right)} i_{t-1} \equiv \Omega_t i_{t-1} = \Pi_t^f \tag{4.31}$$

The return on the share of debt which is retained is then simply replaced by the revenue from credit production, which upon normalising by that share of debt (or alternatively by credit, since they are the same in amount), results in a partitioned payout in form of the banking wage bill and a residually visibly lower return on that share of the short-term saving deposits, i.e.:

$$\frac{MP_{n_{f,t}} \times n_{f,t}}{f_t} = \frac{w_t n_{f,t}}{f_t} = \frac{w_t n_{f,t}}{s^b_{1,t-1}\frac{b_{1,t-1}}{(1+\pi_t)}} = p^f_t \rho = i_{t-1}\rho \qquad (4.32)$$

where $MP_{n_{f,t}}$ is the marginal product of banking labour which upon multiplication with the amount of banking labour, represents the return in form of the banking wage bill and

$$\frac{MP_{d,t} \times d_t}{f_t} = \frac{MP_{d,t} \times d_t}{s^b_{1,t-1}\frac{b_{1,t-1}}{(1+\pi_t)}} = p^f_t\,(1-\rho) = i_{t-1}\,(1-\rho) \qquad (4.33)$$

represents the residual payout on the short-term saving deposit, which relates to the revenue creation from deposits in credit production. $MP_{d,t}$ is the marginal product of deposits in credit production and $MP_{d,t} \times d_t$ therefore the total revenue share of credit production due to deposits. This makes clear how the share of the short-term saving deposit thus commands a return which equals the net nominal CCAPM rate (or the price of credit) *minus* the average cost paid out in form of the banking wage bill. Therefore, the model predicts a money market rate paid out to the representative household's short-term saving deposit which equals the usual CCAPM pure intertemporal rate, *minus* the salary the household takes home from his activity as a banker, whose business it is to produce an exchange credit service evading the exchange cost.

4.2.3 The household

The representative household derives utility in standard fashion from a momentary utility function in consumption and leisure:

$$U_t = U(c_t, l_t) \tag{4.34}$$

where later on, I typically may want to consider a specific parametrisation which is additively separable and logarithmic in both consumption and leisure. An important reason for doing so is to emphasise that significant asset price distortions can be obtained as in Canzoneri and Diba (2005) and Canzoneri et al. (2008) in terms of "first-order" market-driven effects instead of the usual "second-order" risk-induced arguments which typically rest on assumption of high risk aversion and/or non-standard utility function specifications. Throughout the paper I will therefore assume the following specification for utility:

$$U_t = \log c_t + \Psi \log l_t \tag{4.35}$$

The household's only non-financial endowment is labour time, which she can supply to both the goods producing firm producing the consumption good and the financial intermediary producing exchange credit, or partially use up by taking leisure. This endowment is normalised to one and therefore translates into the following constraint:

$$1 - l_t = n_{g,t} + n_{f,t} \tag{4.36}$$

where $n_{f,t}$ is the amount of time spend in producing the credit exchange service and $n_{g,t}$ is the amount of time spend working in the goods-producing sector, both of which are remunerated by paying the household the equilibrium wage rate. Deciding on the optimal level of consumption, the household needs to obey an exchange condition in form of a cash-in-advance constraint, which is modified to allow not only money balances to provide the required liquidity services, but also a costly credit exchange service. The total amount of nominal liquidity translates into an equivalent nominal value of deposits held with the FI. Expenditure will be sourced from the total deposits, either by withdrawal of the household's cash balances from an ATM machine, which are in turn connected to corresponding current accounts, given by D_t^c or residually by using costly credit exchange services, where the money (or, residually, credit) velocity measures will crucially also depend on the price of credit. The exchange constraint is thus given by:

$$P_t d_t \equiv D_t^c + P_t f_t \geq P_t c_t \tag{4.37}$$

or, by dividing through by the current price level, defining $d_t^c = D_t^c / P_t$ and with a positive CCAPM nominal rate resulting strict equality we have:

$$d_t \equiv d_t^c + f_t = c_t \tag{4.38}$$

Using the definition of the per unit of deposits credit production function, f_t^*, which is the inverse of credit-deposit velocity (or alternatively, the inverse of credit-consumption velocity), enables me to re-write the exchange constraint in implied money demand form, as a function of the inverse of deposit-credit velocity and total deposits (being identically equal to consumption):

$$d_t^c = (1 - f_t^*)\, d_t \equiv (1 - f_t^*)\, c_t \tag{4.39}$$

On choosing the optimal level of consumption, the household creates (real-valued) deposits with a financial intermediary, which are then taken as a given input factor to producing credit.

$$d_t \equiv c_t \tag{4.40}$$

Finally, as the credit exchange service is produced by a decentralised financial intermediary, the price of this credit service is explicitly spelled out (see Gillman and Kejak, 2008), and in a unique no-arbitrage exchange equilibrium between money and credit, has to be equal to the cost of using money in carrying out transactions, which is the usual CCAPM net nominal market interest rate, as defined in Canzoneri and Diba (2005). The household's budget constraint is thus given by:

$$\begin{aligned}
& w_t\left(n_{g,t}+n_{f,t}\right)+\Pi_t^f+\frac{d_{t-1}^c}{(1+\pi_t)} \\
& +\frac{(1+i_{1,t-1})}{(1+\pi_t)}b_{1,t-1}^{*}+\sum_{j=2}^{n}\frac{1+i_{j,t-1}}{(1+i_{j-1,t})(1+\pi_t)}b_{j,t-1}^{*} \\
& +\frac{(1+\tilde{i}_{1,t-1})}{(1+\pi_t)}d_{1,t-1}^{s}+\sum_{j=2}^{n}\frac{1+\tilde{i}_{j,t-1}}{(1+\tilde{i}_{j-1,t})(1+\pi_t)}d_{j,t-1}^{s} \\
& \geq c_t+p_t^f f_t+\sum_{j=1}^{n}b_{1,t+j}^{*}+\sum_{j=1}^{n}d_{1,t+j}^{s}
\end{aligned} \tag{4.41}$$

where in particular, $D_{1,t}^s$ equals the amount of dollars held in a nominal short-term debt (or saving) deposit with the financial intermediary (which the financial intermediary in turn backs up one-for-one by an equal amount of nominal government bonds), earning the household the current period nominal risk-free interest rate of $1+\tilde{i}$, which, as will be shown, is different from the usual nominal CCAPM interest rate, $1+i_t$, due to the way short-term debt is partially used as credit-backing collateral. Notice that $d_t^c = D_t^c/P_t$ represents the corresponding level of the real-valued saving deposit. As in Canzoneri and Diba (2005), I have included a virtual nominal bond, given by $B^{*}_{1,t-1}$, in order to aid the derivation of the standard CCAPM nominal return from the household's side[20]. As they do, I do not assume that such bonds are actually held or traded by the household directly in this economy (although they *will* be traded or held by the financial intermediary). As mentioned above, the household needs to pay a price for using the credit exchange service in conjunction with money balances, which is given by $p_t^f = \mu_t/\lambda_t = i_{t-1}$. Subject to her budget and exchange constraint, the representative household maximises her life-time utility over an infinite horizon by choosing an optimal recursive policy function in order to maximise:

$$V\left(m_{t-1}, s_t\right) = \max_{c_t, m_t} E_0 \left\{ U\left(c_t, l_t\right) + \beta V'\left(m_t, s_{t+1}\right) \right\} \tag{4.42}$$

where s_t is some vector of structural shocks, whose exogenously specified law of motion will be specified further below.

4.2.4 The goods-producing sector

The discussion of the goods-producing firm is standard. The goods-producing firm is maximising the present discounted value of current and future dividend streams, whereby it only has to optimally decide on labour demand. The production technology of the firm is given by a standard CRS production function, which is typically assumed to be of Cobb-Douglas specification (or in absence of physical capital, just linear in labour):

$$y_t = e^{z_t} A_g F\left(\kappa_{t-1} n_{g,t}\right) = e^{z_t} A_g \kappa_{t-1} n_{g,t} \tag{4.43}$$

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[20] One may think of these virtual nominal bond holdings shadowing the equivalent nominal government bond holdings of the financial intermediary.

where A_g is some stationary total productivity factor and κ_t is the same labour-augmenting exogenously specified technological progress specified in credit production, governing the steady state trending growth path of the economy. The firm's objective is maximised using a discount factor equivalent to that of the representative household and is given by:

$$\max_{n_{g,t}} \sum_{k=0}^{\infty} \frac{\beta^k \lambda_{t+k}}{\lambda_t} \left\{ y_{t+k} - w_{t+k} n_{g,t+k} \right\} \tag{4.44}$$

where λ_t is the current period multiplier on the household's budget constraint. Optimising with respect to goods labour leads to the usual (after de-trending) condition of optimality equal to:

$$w_t = e^{z_t} A_g \tag{4.45}$$

4.2.5 Equilibrium, Government Financing constraint, Shocks

After netting out financial asset positions on the one hand, and the price of credit times credit minus the re-distributed dividend payment on collateral from the financial intermediary, on the other, we can write the social resource constraint as follows:

$$c_t = n_{g,t} w_t = y_t \tag{4.46}$$

Also, as already discussed above, the government implements a steady state growth rate of the money supply equal to Θ^* which also contains a random component. Further, the government is assumed not to engage in any Ponzi-game regarding the management of it's debt. The government financing constraint is given by:

$$M_t + V_t - M_{t-1} = (1 + i_{t-1}) B_{1,t-1} - B_{1,t} \tag{4.47}$$

Notice that since the proportional amount of nominal debt is adjusted at the beginning of the period such as to implement a constant real-valued debt-to-consumption ratio $\bar{\eta}$, the level of debt varies in state-dependent fashion, and so do the debt interest payment obligations of the government. However, given this circumstance, the government can always vary V_t independently in state-dependent fashion such as to implement any desirable money growth rate with some random component, given by Θ_t. All shocks behave according to some log-normal autoregressive process of order one. The vector of shocks is given by $s_t = [z_t, u_t, v_t]'$, where the shocks are goods productivity, money growth rate and credit productivity, respectively, whose law of motion can be summarised in VAR form as:

$$s_{t+1} = \Phi s_t + \epsilon_{t+1} \tag{4.48}$$

where Φ is a 3x3 matrix with the autocorrelation parameters specified along the diagonal of Φ and $\epsilon \sim (0, \Omega)$.

4.3 Asset Pricing in the Credit-Banking Model

This section is going to describe how the distortive effects of the banking sector (embodied by the average cost in terms of the banking wage bill) will affect asset pricing results in the model. The key intuition underlying the derived asset pricing formulae (and in particular the low risk-free rate) is that the usual return on a *share* of short-term debt equal to the banking sector's debt-utilisation rate (or collateral requirement relative to debt supply) is instead paid out as a dividend. The household's return on the equivalent share of it's short-term saving deposit is instead equal to (or replaced by) the price of credit minus the average cost of producing credit, which is being paid out in form of the banking wage bill. First, I am going to demonstrate how, using Bansal and Coleman's modeling technique, - they *exogenously* specify a nominal interest rate target (see Coleman, 1996; Bansal and Coleman, 1996) - which allows for simple closed-form derivations of *stochastic* asset pricing results, their and my results regarding the risk-free rate and the equity premium are functionally equivalent. Following this, I am going to follow Canzoneri and Diba (2005) and show how using the usual modeling strategy applied to standard monetary general equilibrium models (in which money supply is exogenously modelled and the nominal rate endogenously determined), the banking sector's distortive effects creates a wedge between the CCAPM and the risk-free rate (as defined by Canzoneri and Diba) and how this, through the usual expectations hypothesis of the term structure of interest rates, produces an upward-sloping term structure, which is steeper at the short-end.

4.3.1 A comparison with Bansal and Coleman

Bansal and Coleman's theoretical as well as numerical results obtained from simulations of their estimated model are based on a modelling technique, which essentially amounts to holding *fixed* (or alternatively, *exogenously* specifying the law of motion of) the *nominal* rate of interest. In order to preserve a convincing general equilibrium framework, they then proceed by assuming that, given a *fixed* nominal interest rate target, the monetary authority *endogenously* delivers a state-contingent money supply growth rate, such as to produce an (expected) state-contingent inflation rate which is such as not to be in violation with the usual Fisher equation derived from within the model. Therefore in Bansal and Coleman, the nominal rate is *exogenously*, the real rate in the usual way endogenously and the money supply also *endogenously* determined such as to satisfy the Fisher equation[21]. This is in contrast with dynamic treatments of monetary general equilibrium models such as standard cash-in-advance models described in textbooks such as Walsh (2003), in which typically the real rate is endogenously determined from within the model, the (expected) inflation rate is essentially driven by the exogenously specified money supply growth rate and the nominal rate is also endogenously obtained from the Fisher relationship between the real rate and expected inflation. As will be clear, Bansal and Coleman's modeling strategy regarding the interplay between the real, nominal and inflation rate coupled with a simplified assumption about the distribution of shocks allows closed-form solutions for the risk-free rate and equity premium under uncertainty. Notice that Bansal and Coleman introduce a role for money (as well as other means of exchange, such as credit and checkable deposits) through an ad-hoc transaction cost function, which extends the transaction cost literature (Baumol, 1952; Tobin, 1956; Barro, 1976; McCallum, 1983). They also show how their results (regarding velocity), which are driven by the *technology* specification of this function (and thus the marginal rate of transformation between cash and credit goods), closely resemble results obtained from Stokey & Lucas' cash-credit model, in which results are instead driven by the *preference* specification of the utility function over the cash and credit goods (and thus the marginal rate of substitution between cash and credit goods). Velocity and asset pricing results in the model presented here are also technology-driven in the sense of being dependent on the specification of the credit-production function and the resulting stable money-credit equilibrium driven by the convex marginal cost schedule. Bansal and Coleman's key result regarding

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[21] This technique was first employed in Coleman (1996) to theoretically explore reverse causation from output to money, hence the necessity for a framework in which money is endogenous.

the risk-free rate stems from equation (22) in their paper, which I reproduce here:

$$\begin{aligned}\frac{1-\xi_2(pc)+\xi_3(pc)}{1-\xi_2(pc)}\frac{u_1(c)}{1+\xi_1(pc)}\left[1-\xi_2(pc)\right]q \\ =\beta E_s\left[\frac{u_1(c')}{1+\xi_1(p'c')}\frac{1-\xi_2(p'c')}{\Pi'}\right]\end{aligned} \tag{4.49}$$

where the ξ_i are derivatives of the transaction cost function with respect to different means of exchange. What is most important in this expression is the term:

$$\frac{1-\xi_2(pc)+\xi_3(pc)}{1-\xi_2(pc)} \tag{4.50}$$

which is smaller than one as long as bond-backed checkable deposits are used in purchasing the consumption good, where $\xi_3(pc) < 0$ is the marginal product in the "production" of transactions services due to bond-backed checkable deposits and pc is the consumption velocity of cash. Notice that, similar to Stokey and Lucas's cash-credit model (Stokey and Lucas, 1987), in which velocity is determined by equating the nominal interest rate with the marginal rate of substitution derived from the preference specification of the cash and credit goods, velocity in Bansal & Coleman's model is also pinned down in current and future periods, once the nominal rate of interest is assumed to be held *fixed* at some target level. Using this modelling strategy and by assuming that consumption growth $\varrho' = \frac{c'}{c}$ is identically and independently distributed, that the (gross) inflation rate is fixed at some state-contingent target $\bar{\Pi}$ (implying an expected inflation rate which does not violate the Fisher relationship), and that utility is of CRRA type and given by $U(c) = \frac{c^{1-\tau}}{1-\tau}$, Bansal and Coleman's equation (22) can be simplified to give:

$$\frac{1-\xi_2(pc)+\xi_3(pc)}{1-\xi_2(pc)}q = \frac{\beta}{\bar{\Pi}}E\left[\varrho^{-\tau}\right] \tag{4.51}$$

implying a *real* risk-free rate of interest equal to:

$$\frac{1}{\beta E\left[\varrho^{-\tau}\right]}\frac{1-\xi_2(pc)+\xi_3(pc)}{1-\xi_2(pc)} \tag{4.52}$$

which, as long as short-term debt is providing liquidity service, embodied by the term $\xi_3 < 0$, implies a real risk-free rate which is lower than the standard rate, given by $1/\beta E\left[\varrho^{-\tau}\right]$. Notice that the model presented in this paper permits an equivalent representation of the risk-free rate, but instead of relying on an argument based on the *marginal* product in (the production of) transactions costs due to short-term debt (which backs up checkable deposits), here it is the *average* cost paid out in form of the banking wage bill which drives down the risk-free rate. Also, in contrast to Bansal and Coleman, in the model presented, measures of relative supply of short-term debt (given by the debt-to-deposits ratio η), and the credit demand-linked (and thus inflation-dependent) debt utilisation rate $s^b_{1,t}$, which essentially represents the banking sectors collateral demand backing up the produced credit service, matter. In general therefore, for a fixed relative supply, inflation-induced increases in the use of credit (and thus increases in money velocity as less money is used), lead to an increase of the debt utilisation rate $s^b_{1,t}$, which will also affect asset price results. The derivation of the nominal risk free rate in the model is therefore as follows:

$$1+\tilde{i}_t = E_t\left\{1+\left[s^b_{1,t+1}\left(1-\rho\right)+\left(1-s^b_{1,t+1}\right)\right]i_t\right\} \tag{4.53}$$

which shows that the one-period nominal risk-free rate consists of (an expectation over) an endogenously moving average, in which the share of short-term debt (or the equivalent share of the short-term saving deposit) backing up credit, given by

$s^{b}_{1,t+1}$ commands a rate of return which is net of the average cost paid out in terms of the banking wage bill[22], whereas the residual share of economy-wide debt, which does not serve as collateral, given by $\left(1 - s^{b}_{1,t+1}\right)$ commands the usual undistorted net nominal CCAPM rate. To simplify notation, let me define:

$$\Upsilon^{a}_{t} \equiv s^{b}_{1,t}\rho\left(\frac{\mu_t}{\lambda_t}\right) \equiv s^{b}_{1,t}\rho p^{f}_{t} \tag{4.54}$$

and

$$\Upsilon^{b}_{t} \equiv \frac{1}{1 + s^{b}_{1,t}\rho\left(\frac{\mu_t}{\lambda_t}\right)} \equiv \frac{1}{1 + s^{b}_{1,t}\rho p^{f}_{t}} \tag{4.55}$$

Notice then, since $i_t = p^{f}_{t}$ is of the order of a (quarterly) net rate and that $0 < s^{b}_{1,t} < 1$ and $0 < \rho < 1$, thus making $s^{b}_{1,t}\rho p^{f}_{t}$ of the order of a (quarterly) net rate, it follows that, for some variable of the order of a gross (quarterly) rate $1 + i_t$:

$$(1 + i_t) - \Upsilon^{a}_{t} \approx (1 + i_t)\,\Upsilon^{b}_{t} \tag{4.56}$$

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[22] To understand that this is an (expected) average cost, recall that in the model $E_t\left(\mu_{t+1}/\lambda_{t+1}\right) = i_{t+1} = E_t\left(p^{f}_{t+1}\right)$ meaning that the net nominal CCAPM rate is also equal to the expected price, and thus expected total average product, of credit.

Using the above definitions, the gross nominal risk-free rate can thus be written as:

$$
\begin{aligned}
1+\tilde{i}_t & \\
&= E_t\left\{1+\left(1-\rho s^b_{1,t+1}\right) i_t\right\} \\
&= (1+i_t) - E_t\left(s^b_{1,t+1}\rho p^f_{t+1}\right) \\
&= (1+i_t) - E_t\left(\Upsilon^a_{t+1}\right) \\
&\approx (1+i_t)\, E_t\left(\Upsilon^b_{t+1}\right)
\end{aligned}
\tag{4.57}
$$

Within Bansal and Coleman's modeling framework, within which I wish to place my results in order to allow for a direct comparison, both the nominal CCAPM and the inflation rate are assumed to be fixed state-contingent targets, which in turn implies (using some further restrictions placed on the model economy presented here, which I will discuss below) that current and future Υ^a and Υ^b are fixed and thus known in advance. Following Bansal and Coleman, I therefore get:

$$
\begin{aligned}
1+\tilde{i}_t = & \\
&= \frac{1+i}{1+\rho s^b_1 p^f} \\
&= \frac{\gamma(1+\bar{\pi})}{\beta} E_t\left[\frac{\lambda_t}{\lambda_{t+1}}\right]\left[\frac{1}{1+\rho s^b_1 p^f}\right] \\
&= \frac{\gamma(1+\bar{\pi})}{\beta} E_t\left[\frac{\lambda_t}{\lambda_{t+1}}\right]\left[\Upsilon^b\right]
\end{aligned}
\tag{4.58}
$$

implying in particular a real risk-free rate given by:

$$
\frac{\gamma}{\beta} E_t\left[\frac{\lambda_t}{\lambda_{t+1}}\right]\left[\Upsilon^b\right] \tag{4.59}
$$

which after substituting out for the marginal utility of wealth in terms of marginal utility and the usual cash-in-advance relative cost of consumption (the gross nominal interest rate), gives:

$$\frac{\gamma}{\beta}E_t\left[\frac{(1+i)\,U_c(c,l)}{(1+i')\,U_c(c',l')}\right]\left[\Upsilon^b\right] \tag{4.60}$$

If I now appeal to the same modeling strategy of Bansal & Coleman and assume fixed nominal interest rate and inflation rate targets delivered by the monetary authority, specify consumption growth $\varrho' = \frac{c_{t+1}}{c_t}$ to be identically and independently distributed and utility to be of CRRA type in consumption (and additively separable in consumption and leisure), such as:

$$U(c,l) = \frac{c_t^{1-\tau}-1}{1-\tau} + A\log(l_t) \tag{4.61}$$

the above formulae reduces to:

$$\frac{\gamma}{\beta^* E\left[\varrho^{-\tau}\right]}\left[\Upsilon^b\right] \tag{4.62}$$

where $\beta^* = \beta\gamma^{1-\tau}$ is the growth-adjusted impatience factor (see Jermann, 1998). For an economy with no growth (as was assumed by Bansal and Coleman), the above reduces to:

$$\frac{1}{\beta E\left[\varrho^{-\tau}\right]}\left[\Upsilon^b\right] \tag{4.63}$$

where $\Upsilon^b < 1$, which shows the functional equivalence between their and my results for the low risk-free rate, when viewed from their modeling assumption. Notice that in order to fully emulate an environment equivalent to Bansal and Coleman's (which is, in contrast to the model here, a simple exchange economy), assuming a fixed nominal CCAPM rate of interest and fixed rate of inflation is not quite sufficient, as this will not restrict the credit-banking model enough to make next period's debt utilisation rate s_1^b (and therefore next period's value of Υ^b) known with perfect certainty. This share's future value will only be known with certainty, as long as future credit production is known with certainty. Fixing the nominal CCAPM rate already accomplishes fixing the *price* of credit, but two more conditions have to be imposed in order to make next period's level of credit production known with certainty. First, it has to be assumed that there are no shocks to credit production (which would shift the marginal cost schedule of credit production) and secondly, that labour between the credit and goods sector is completely immobile. This last requirement is necessary, since shocks to the goods production sector alone would lead to labour movement between sectors in order to equate the marginal products and thus the wage rate. As I have shown above, the debt utilisation rate can also be equivalently expressed as a function of the credit-banking cost (in terms of the banking wage bill) relative to the economy-wide short term debt in the economy (which, relative to deposits or consumption is always fixed). Therefore, the above additional restrictions essentially amount to holding next period's banking wage bill fixed, thus making next-period's debt-utilisation rate constant. Finally, to replicate Bansal and Coleman shock distribution for the discount factor (which they use to illustrate their analytic results, in numerical exercises shock processes are autoregressive of order one), the productivity shock on the goods production function would have to be identically and independently distributed.

Although the risk-free rate thus obtained is functionally equivalent to Bansal & Coleman's low risk-free rate, the intuition is of a different sort. Whereas in Bansal & Coleman the term $\frac{1-\xi_2(pc)+\xi_3(pc)}{1-\xi_2(pc)}$ is responsible for driving down the risk-free rate, which is directly related to the production specification of the ad-hoc transaction cost function, in the model presented here it is the partitioning (into the banking wage bill and residually the return on the saving deposit) of the payout received on the collateral share of short-term debt which leads to a lower return. Notice that both here and in Bansal & Coleman, variations in inflation, velocity and asset prices are linked together. Also, in contrast to Stokey and Lucas (1987) whose velocity results depend on preference specifications, both here and in Bansal & Coleman velocity results are essentially technology-driven, with the difference that here the technology is transparently modelled in terms of a micro-founded theory of financial intermediation, whereas Bansal & Coleman's specification is based on an ad-hoc transaction cost function. Notice that the obtained ”reduced” risk-free rate can exist in the absence of arbitrage, as the household is compensated in return in form of the banking wage bill. The degree to which this banking time cost can affect the risk-free rate depends on the relative production of credit to deposits on the one hand, and the relative supply of debt to deposits, which has been assumed to be a government target which is held fixed in each period. As in Bansal and Coleman (1996), whenever such considerations of proportional supply of debt matter, the standard fashion of modeling debt to exist in net zero supply have to abandoned and replaced with some notion of specific supply-modeling. Notice also that in the model presented here, the banking wage bill generated by credit production drives down the return on the short-term saving deposit. This production function was given by:

$$f_t = e^{v_t} A_f \left(\kappa_{t-1} n_{f,t}\right)^{\rho} c_t^{1-\rho} \qquad \text{where} \qquad c_t \equiv d_t \tag{4.64}$$

Bansal & Coleman's specification of their *transactions cost* function, which may be thought of as the transactions cost literature's exchange cost analogue to the exchange cost (given by the net nominal interest rate) in the cash-in-advance literature, is given by:

$$\Psi\left(c, c_1, c_2\right) = \bar{\Psi} c^{\alpha} l^{1-\alpha} \qquad \text{where} \qquad l = \left(c_1^{\omega} + \kappa c_2^{\omega}\right)^{1/\omega} \tag{4.65}$$

Comparing the two functions and noticing that c_2 is the fraction of consumption goods paid for with the *bond-backed* checkable deposits in Bansal & Coleman, this demonstrates the close analogy between their approach and the approach taken here. In their specification, bond holdings also yield a transactions service return in terms of marginally adding to the total value of such transaction services, which could equivalently expressed in terms of labour foregone[23] (which they hint at, by defining the transaction cost function in that particular way, using $l = \left(c_1^{\omega} + \kappa c_2^{\omega}\right)^{1/\omega}$). In contrast, in the model presented here, the transaction cost share due to the use of credit is *equal* to the value of some share of beginning-of-period short-term bond holdings, whose return paid out in terms of a dividend from it's use as collateral, and instead replaced by the total return of credit production, which is partitioned into the banking wage bill and *residually* into the return on the short-term saving deposit. Also, in contrast to Bansal & Coleman, here proportional supply and the credit-induced banking sector's demand of debt matter, as they define the bond utilisation rate, $s_{1,t}^{b}$, whereas in their approach, the total value of the bond issue is always equal to checkable deposits.

After demonstrating, within their specific modelling strategy, how the risk-free rate is affected by the liquidity role of short-term debt, Bansal & Coleman proceed

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[23] On page 1140, Bansal & Coleman state that purchasing the consumption good incurs a transaction cost in terms of foregone output or in terms of *time devoted* to the *production* of consumption goods. In the credit-banking model, the credit share of consumption is also the outcome of a productive process, involving labour.

by including in standard fashion a risky asset into their model and show how the same term affecting the risk-free rate, $\frac{1-\xi_2(pc)+\xi_3(pc)}{1-\xi_2(pc)}$, also affects the equity premium and the non-parametric HJ bound (Hansen and Jagannathan, 1991). Because of the functional equivalence between their and my results, I refer the reader to the straightforward derivation provided in their paper and state for completeness the equity premium and the modified HJ bound[24]. Assuming an identically and independently distributed growth rate of dividends given by χ, the equity premium (defined as the ratio of the expected gross return to equities over the gross return to bonds) is thus modified to be defined as follows:

$$\frac{E\left[\gamma^{-\varrho}\right]E\left[\chi\right]}{E\left[\gamma^{-\varrho}\chi\right]}\left[1+\rho s_1^b p^f\right] \tag{4.66}$$

which depends in the usual way on the covariance of the growth in marginal utility with the growth in dividends, but rises proportionately with the term $\left[1+\rho s_1^b p^f\right]$. Therefore, the same term which lowers the risk-free, also raises the equity premium. Similarly, Bansal & Coleman show how the HJ-bound given by:

$$\frac{\sigma(k)}{E(k)} \geq \frac{E(\zeta)}{\sigma(\zeta)} \tag{4.67}$$

where k is equal to the intertemporal marginal rate of substitution and ζ equals any excess return, is easier to satisfy (for equity) once the liquidity-providing role of the short-term bond is taken into account. In the model presented here, the same argument can be made by taking into account how here the effect of the banking wage bill distorts the risk-free rate. Analogously to Bansal & Coleman, it can thus be shown that, with regard to equities, the HJ bound imposes restrictions on the

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[24] Another reason for placing less emphasis on this derivation is the suspicion that B&C's derivation of the high equity premium is an artifact of their peculiar modeling assumption based on reverse causation and endogenous money.

excess return according to:

$$\zeta_e' = R_e' - \left[1 + \rho s_1^b p^f\right](1 + i) \tag{4.68}$$

Therefore, as long as credit is produced and the bond-utilisation rate s_1^b is different from zero, the average excess return $E(R_e')$ is smaller than the observed equity premium, given by $R_e' - (1 + i)$, by an amount related to the average cost in producing the debt-backed credit service. This feature tends to lower the Sharpe ratio on ζ_e' and therefore makes it easier for the above bound to be satisfied.

4.3.2 Stochastic and Steady State Asset Price Analysis

This section is going to explore the implications for stochastic as well as steady state asset pricing results in the credit-banking model. Notice that from here on onwards, I will take the dynamics of the model to be based on the usual interpretation of the Fisher relationship, and not the one employed by Bansal and Coleman (1996) which was motivated by a study of reverse causation from output to money. Also, for the discussion of stochastic asset pricing results, it will be convenient to restate the problem first in terms of prices and then to use continuous time formulae to convert back to net returns[25]. Further, notice that for a given ex-post realised rate of inflation, the cost-term driving down the nominal risk-free rate would also imply a reduction in the short-term real rate by the same amount. Therefore, in order to abstract from inflation and reduce cluttering of my analytical results with products or sums of log inflation rates, I will only consider shadow prices (and returns) of inflation-indexed bonds, also aiding comparison with previous results from the literature focusing on asset pricing results of real bonds. Of course, having

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[25] This technique is also used by den Haan (1995). Cochrane (2005, p. 15) emphasises the interchangeability of price-based and return-based asset price derivations.

said that, it is important to highlight the fact that no matter whether one considers real or nominal rates of return, the credit-banking related cost-wedge in form of the proportional banking wage bill over total short-term debt is always a function of the net *nominal* CCAPM rate, as the price of credit equals the opportunity cost of money holding money.

The low risk-free rate

The low nominal risk-free rate obtained above implies for a short-term inflation-indexed bond's real return, or the equivalent inflation-indexed short-term saving deposit's return:

$$1+\tilde{r}_{1,t}=(1+r_{1,t})\,E_t\left(\Upsilon^b_{t+1}\right)=\frac{\gamma}{\beta}\frac{E_t\lambda_{t+1}}{\lambda_t}\left[E_t\left(\Upsilon^b_{t+1}\right)\right] \tag{4.69}$$

The above return derivation of the low real risk-free rate, implies the following for the price of the same financial asset:

$$\tilde{p}^b_{1,t}=(1+\tilde{r}_{1,t})^{-1}=\frac{\beta}{\gamma}\frac{\lambda_t}{E_t\lambda_{t+1}}\left[E_t\left(\Upsilon^b_{t+1}\right)\right]^{-1} \tag{4.70}$$

Defining $\Lambda_{t+1}=\frac{\lambda_t}{E_t\lambda_t}=\frac{c_{t+1}(1+i_t)}{c_t(1+i_{t-1})}$ and assuming this variable to be log-normally distributed (which it would be, if consumption and nominal interest rates are log-normal, an assumption also used by Bohn (1991)), then using continuous time formulae as in den Haan (1995) implies:

$$\begin{aligned}\tilde{r}_{1,t} &= -\ln\left(\tilde{p}^{b}_{1,t}\right)\\ &= \left[-\ln\beta^{*} + \ln\left(E_t\Upsilon^{b}_{t+1}\right)\right] - \ln\left[E_t\Lambda_{t+1}\right] - \frac{1}{2}Var\left[\ln\left(\Lambda_{t+1}\right)\right]\\ &= \left[-\ln\beta^{*} - E_t\Upsilon^{a}_{t+1}\right] - \ln\left[E_t\Lambda_{t+1}\right] - \frac{1}{2}Var\left[\ln\left(\Lambda_{t+1}\right)\right]\\ &= \left[\bar{r} - E_t\Upsilon^{a}_{t+1}\right] - \ln\left[E_t\Lambda_{t+1}\right] - \frac{1}{2}Var\left[\ln\left(\Lambda_{t+1}\right)\right] \qquad (4.71)\end{aligned}$$

where $\ln\beta^{*} = \ln\left(\frac{\beta}{\gamma}\right)$. Upon substituting out for the marginal value of wealth, the above expression can be written as:

$$\begin{aligned}\tilde{r}_{1,t} = &\left[\bar{r} - E_t\Upsilon^{a}_{t+1}\right] + \left[E_t\Delta c_{t+1} + \Delta i_t\right]\\ &- \frac{1}{2}Var\left(E_t\Delta c_{t+1}\right) - \frac{1}{2}Var\left(\Delta i_t\right) - Cov\left(E_t\Delta c_{t+1}, \Delta i_t\right) \qquad (4.72)\end{aligned}$$

This expressions is equal to the *conditional* value of the risk-free rate as implied by conditional variations in the stochastic discount factor. Observing that the model's first-order conditions have been divided through by κ_{t-1} such as to make all variables stationary, taking unconditional expectations implies $E_{\infty}\Delta c_{t+1} = 0$ and $E_{\infty}\Delta i_t = 0$, and the *unconditional* value of the same expression is equivalent to:

$$E_{\infty}\tilde{r}_{1,t} = \left[\bar{r} - E_{\infty}\Upsilon^{a}_{t+1}\right] - \frac{1}{2}Var\left[\ln\left(\Lambda_{t+1}\right)\right] \qquad (4.73)$$

The above conditional and unconditional expressions for the risk-free rate generalise standard derivations of the this rate in barter economies along two dimensions. First of all, since exchange-in-advance (in form of either credit or money) translates into the usual exchange cost of the net CCAPM nominal rate of interest, the stochastic discount factor given by the marginal rate of substitution in consumption reflects this cost in marginal utilities of consumption in the current and next period. This of course then means that upon expansion of the expectation of the log-normally distributed SDF, the variance of the net CCAPM interest rate and it's covariance with consumption growth matter as well (see Bohn, 1991). The intuition for the negative effect of the volatility of the SDF on the unconditional return of a risk-free bond is well known and relates the the increased demand for savings today when valuation of tomorrow's state is relatively volatile (see Jermann, 1998). Secondly, and more importantly, there is a "first-order" market-driven cost effect in form of the partitioning of the *effective* return of the bond into the residually lower return on the short-term saving deposit, given by $\tilde{r}^b_{1,t}$ and the banking wage bill. This effect is embodied by the conditional expectation term:

$$E_t \Upsilon^a_{t+1} = E_t \left(s^b_{t+1} \rho p^f_{t+1} \right) = E_t \left(s^b_{t+1} \rho i_t \right) = \frac{w_{t+1} n_{f,t+1}}{b_{1,t} / E_t \left(1 + \pi_{t+1} \right)} \tag{4.74}$$

which is subtracted from the *pure deterministic* rate given by $\bar{r} = -\log\left(\beta^*\right)$. Credit-banking asset price distortions do not produce "second-order" risk-induced effects in form of covariances embodying systematic risk. The intuition for this is that although the investor does not know ex-ante how much of his payout on the short-term saving deposit will be subtracted and paid out instead in form of his wage bill he receives in his activity as a banker, the variation of the low-risk free rate due to that effect is perfectly hedged by construction. All that is required to establish the valuation of the deterministic part of his short-term saving deposit's return, is to subtract the expected payout in form of the expected proportional cost given by the future expected banking wage bill (which is directly related to the future expected level of credit production). Of course, ex-post this valuation may turn out to be wrong, say, because of an unexpectedly large level of credit production (implying a higher banking wage bill and an ex-post lower return on the short-term saving deposit), but such unexpected outcomes and the resulting unexpected variation in the deterministic return component of the risk-free rate is perfectly offset by compensating unexpected variations in the ex-post realised banking wage bill. Finally, in line with Canzoneri and Diba (2005), the model still allows the definition of the purely intertemporal rate affecting the consumption Euler equation, which is defined in the usual way as:

$$r_{1,t} = \bar{r} + [E_t \Delta c_{t+1} + \Delta i_t] - \frac{1}{2} Var\left[\ln\left(\Lambda_{t+1}\right)\right] \tag{4.75}$$

with an unconditional mean of:

$$E_{\infty} r_{1,t} = \bar{r} - \frac{1}{2} Var\left[\ln\left(\Lambda_{t+1}\right)\right] \tag{4.76}$$

Of course, the only element distinguishing this intertemporal rate from it's low-risk free return counterpart which the representative household earns on the short-term saving deposit is the cost-distortion term due to credit production.

Notice that we could also think of a risky asset (equity) whose valuation would depend on the uncertain flow of future dividend payments. In the credit-banking model presented here, this dividend could be endogenously determined as the the firm's revenue minus it's wage bill, i.e. $div_t = e^{z_t} y_t - w_t n_{g,t}$[26] or proxied by simply setting it equal to consumption, $div_t = c_t$. Since utility is assumed to be logarithmic and additively separable, it is then a well-known fact that the price of this risky asset, given by p_t^{eq}, is proportional to the current dividend payout in the following

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[26] This would cause the dividend to be equal to the productivity innovation in the goods sector (see Rouwenhorst, 1995; Jermann, 1998).

way:

$$p_t^{eq} = \frac{\beta}{1-\beta} div_t \tag{4.77}$$

Defining the gross real return of this risky asset in the usual way as as the expected future price and dividend payment divided by the current purchase price:

$$r_t^{eq} = \frac{E_t \left(p_{t+1}^{eq} + div_{t+1} \right)}{p_t^{eq}} \tag{4.78}$$

the model would then also produce a condition of optimality for this asset in the usual way as:

$$1 = E_t \left[\frac{\gamma}{\beta} \Lambda_{t+1} r_t^{eq} \right] \tag{4.79}$$

which leads to the well-known derivation of an excess return of the risky asset over the *purely intertemporal CCAPM rate* given by:

$$r_t^{eq} - r_{1,t} = -\frac{Cov \left(\Lambda_{t+1}, r_t^{eq} \right)}{E_t \Lambda_{t+1}} \tag{4.80}$$

This means that the conditional excess return of the risky asset over the *low risk-free* rate on the short-term saving deposit is therefore given by[27]:

$$r_t^{eq} - \tilde{r}_{1,t} = E_t \Upsilon_{t+1}^a - \frac{Cov \left(\Lambda_{t+1}, r_t^{eq} \right)}{E_t \Lambda_{t+1}} \tag{4.81}$$

This result shows how the equity premium in the credit-banking model defined as the excess return of the risky asset over the cost-distorted risk-free rate obtained on

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[27] The covariance between stock returns and some measure of the marginal value of wealth, like consumption, is negative. Consumption typically rise (lowering marginal valuation) when stock markets rise. Therefore $-Cov \left(\Lambda_{t+1}, r_t^{eq} \right) > 0$

the short-term saving deposit is given in the usual way by the risk-adjustment due to systematic risk of receiving a low dividend in times of already low consumption plus the expected payout in form of the future expected banking wage bill. The latter factor will crucially depend on the expectation of future credit production *and* the future expected price of credit, $p_{t+1}^f = \mu_{t+1}/\lambda_{t+1}$.Given the stochastic specification of the low risk-free rate and the excess return of a risky asset over this rate, the steady state real risk-free rate ignores risk-induced adjustments between the risky and the purely intertemporal rate and is thus given by:

$$\begin{aligned} 1+\tilde{r} =& \frac{\gamma}{\beta}\left[\frac{1}{1+s^b\rho p^f}\right] \\ =& \frac{\gamma}{\beta}\Upsilon^b \\ =& \frac{\gamma}{\beta} - \Upsilon^a \\ =& \frac{\gamma}{\beta} - s^b\rho p^f \end{aligned} \tag{4.82}$$

which is different from the usual steady state real rate in standard models given by γ/β, and as long as credit production is positive, *lower* than the standard rate, due to average cost incurred from producing the debt-backed credit. Therefore, the ratio of the real CCAPM over the real risk-free rate (and thus the steady state equity premium) is given by:

$$\frac{1+r}{1+\tilde{r}} = 1 + \rho s^b p^f \tag{4.83}$$

or

$$r - \tilde{r} = \rho s^b p^f = \frac{w_t n_f}{b_1/(1+\pi)} \tag{4.84}$$

which again demonstrates the simple intuition that the wedge driven between the real risk-free and the real CCAPM rates is related to the proportional banking time cost in terms of the banking wage bill over the total amount of short-term debt.

The Term Structure of Interest Rates

Restating the representative household's choice of (inflation-indexed) bonds (and undistorted virtual bonds) in her budget constraint in terms of prices rather than returns, i.e.

$$\begin{aligned}
&\ldots + d^s_{1,t-1} + \sum_{j=2}^{n} \tilde{p}^b_{j-1,t} d^s_{j,t-1} - \sum_{j=1}^{n} \tilde{p}^b_{j,t} d^s_{j,t} \ldots \\
&\ldots + b^*_{1,t-1} + \sum_{j=2}^{n} p^b_{j-1,t} b^*_{j,t-1} - \sum_{j=1}^{n} p^b_{j,t} b^*_{j,t} \ldots
\end{aligned} \tag{4.85}$$

Then, the first-order conditions for subsequent inflation-indexed bonds imply the following formula for the price of the j-th period inflation-indexed bond:

$$\begin{aligned}
\tilde{p}^b_{j,t} &= \left(\frac{\beta}{\gamma}\right)^j \left[\frac{E_t \lambda_{t+j}}{\lambda_t}\right] \left[E_t \Upsilon^b_{t+j}\right]^{-1} \\
&= \left(\frac{\beta}{\gamma}\right)^{j-1} \left[\frac{\beta}{\gamma} \left(E_t \Upsilon^b_{t+j}\right)^{-1}\right] \left[\frac{E_t \lambda_{t+j}}{\lambda_t}\right]
\end{aligned} \tag{4.86}$$

The above expression thus shows how the cost-distortion in form of the banking wage bill only affects the expected tail-end one-period return of any j-period bond. For bonds with shorter maturity, the distortion will have a disproportionately larger effect on the average yield than for bonds with very high maturity. This argument clearly explains the intuition behind the convexity of the term structure in the credit-banking model. Using the same logic as for the one-period real risk-free rate, taking the negative log of the above price for a j-th period bond, the expression can can be written in terms of the net *holding period* return for a j-th period bond:

$$\begin{aligned}\tilde{r}_{j,t} &= (j-1)\,\bar{r} + \left(\bar{r} - E_t\Upsilon^a_{t+j}\right) \\ &\quad + \left[E_t\Delta c_{t+j} + E_t\Delta i_{t+j-1}\right] - \frac{1}{2}Var\left[\ln\left(\frac{\lambda_t}{E_t\lambda_{t+j}}\right)\right]\end{aligned} \tag{4.87}$$

by dividing through by j we can find the average yield of a j-period bond:

$$\begin{aligned}\hat{\tilde{r}}_{j,t} &= \left(\frac{j-1}{j}\right)\bar{r} + \left(\frac{1}{j}\right)\left(\bar{r} - E_t\Upsilon^a_{t+j}\right) \\ &\quad + \frac{\left[E_t\Delta c_{t+j} + E_t\Delta i_{t+j-1}\right] - \frac{1}{2}Var\left[\ln\left(\frac{\lambda_t}{E_t\lambda_{t+j}}\right)\right]}{j}\end{aligned} \tag{4.88}$$

The above expression for the *conditional* yield of inflation-indexed bonds of various maturity embeds the derivation of the low risk-free rate by setting $j = 1$. Also, the *unconditional* or average yield is given by:

$$E_\infty\hat{\tilde{r}}_{j,t} = \left(\frac{j-1}{j}\right)\bar{r} + \left(\frac{1}{j}\right)\left(\bar{r} - E_\infty\Upsilon^a_{t+j}\right) - \frac{\frac{1}{2}Var\left[\ln\left(\frac{\lambda_t}{E_t\lambda_{t+j}}\right)\right]}{j} \tag{4.89}$$

Notice that the analogous *virtual* term structure related to the undistorted fictitious bonds again ignores the distortive cost term and it's implied conditional yield expression is therefore given by:

$$\hat{r}_{j,t} = \bar{r} + \frac{\left[E_t\Delta c_{t+j} + E_t\Delta i_{t+j}\right] - \frac{1}{2}Var\left[\ln\left(\frac{\lambda_t}{E_t\lambda_{t+j}}\right)\right]}{j} \tag{4.90}$$

and it's unconditional or average yield expression by:

$$E_{\infty}\hat{r}_{j,t} = \bar{r} - \frac{\frac{1}{2}Var\left[\ln\left(\frac{\lambda_t}{E_t\lambda_{t+j}}\right)\right]}{j} \tag{4.91}$$

What is important here is that the cost-distorted lower return of the tail end on any j-period bond has a disproportionately larger effect on bonds with smaller maturity than bonds for which j is very large. Indeed for a bond for which $j \to \infty$, the effect of the reduced tail-end return asymptotically disappears, meaning that the steady state average yield for the limiting bond is equal to to the steady state real CCAPM rate:

$$\begin{aligned}
&(1+\hat{r}_j)_{j\to\infty} \\
&= \lim_{j\to\infty}\left\{\left[\frac{\gamma}{\beta}\right]^{\frac{j-1}{j}}\left[\frac{\gamma}{\beta} - E_t\Upsilon^a_{t+j}\right]^{\frac{1}{j}}\right\} \\
&= \frac{\gamma}{\beta}
\end{aligned} \tag{4.92}$$

Notice however the following drawback implied by this result. Calibrating the CCAPM rate (by choosing a sufficiently low enough impatience factor) such as to match it with the average return obtained on equity as observed in the data, means that one automatically also pins down the average yield of the long-term (limiting) bond to the same average return. However, in the data, the real return on a short-term bond (i.e. the risk free money market rate) is roughly equal to 1%, whereas the premium of a long-term bond above the risk-free rate is typically only equal to 1% (thus earning an average yield of roughly 2%). Mehra and Prescott (2003) raise exactly the same concern, by concluding that "[Bansal & Coleman's] model implies that there should be a significant yield differential between T-bills and long term government bonds that presumably do not have a significant transaction service component". However, in Bansal & Coleman's particular modeling framework, they clearly show how this argument may not be valid, as roughly the same liquidity argument responsible for lowering the usual stochastic risk-free also simultaneously raises the equity premium by proportionately more. But this argument may indeed be an artifact of their particular modeling strategy, in which money supply is endogenously determined. In steady state the above expressions regarding the term structure can be simplified to give:

$$1+r_j=\left(\frac{\gamma}{\beta}\right)^j \tag{4.93}$$

which is the undistorted steady state holding period return for the *virtual* term structure implying an undistorted per-period average yield which is just equal to the steady state real interest rate:

$$1+\hat{r}_j=(1+r_j)^{1/j}=\frac{\gamma}{\beta} \tag{4.94}$$

where $1+\hat{r}_j$ represents the j-period real bond's average yield. Including multi-period saving deposits into the household's budget constraint which are backed up by corresponding multi-period government bonds and taking into account the distortive effect of the banking sector on the short-term risk free rate, implies the following steady state average yield on a j-period bond:

$$1+\hat{r}_j=\left\{\left[\frac{\gamma}{\beta}\right]^{j-1}\left[\frac{\gamma}{\beta}\left(\Upsilon^b\right)\right]\right\}^{1/j} \tag{4.95}$$

where $\Upsilon^b<1$ for as long as credit is produced in the economy.

4.4 The Steady State

In the following section I am going to describe the steady state levels of endogenous variables, that result after de-trending all growing variables by dividing through by κ_{t-1} and thus obtaining a non-trending stationary equilibrium. The steady state can be summarised by the following set of equations:

$$1+\pi=(1+\Theta)/\gamma \tag{4.96}$$

$$1+r=\frac{\gamma}{\beta} \tag{4.97}$$

$$1+i=\frac{\gamma}{\beta}(1+\pi) \tag{4.98}$$

$$p^f=\left(\frac{\mu}{\lambda}\right)=i \tag{4.99}$$

$$\frac{f}{c}=f^*=A_f^{\frac{1}{1-\rho}}\left(\frac{\rho}{w}\right)^{\frac{\rho}{1-\rho}}i^{\frac{\rho}{1-\rho}} \tag{4.100}$$

$$\frac{m}{c}=m^*=(1-f^*) \tag{4.101}$$

$$MRS_{c,l}=\frac{l}{\Psi c}=\frac{1+i}{w} \tag{4.102}$$

$$A_g=i\rho\frac{f}{n_f}=w \tag{4.103}$$

$$1+\tilde{i}=1+\left[\left(1-s_1^b\rho\right)i\right] \tag{4.104}$$

$$1+\tilde{r}=\frac{1+\tilde{i}}{1+\pi} \tag{4.105}$$

$$s_1^b=\frac{f^*}{\bar{\eta}} \tag{4.106}$$

First of all, equation (4.96) in the usual way sets the steady state rate of inflation equal to the growth rate of the money supply adjusted for the exogenously specified economy-wide economic growth rate. The steady state real CCAPM rate is just equal to the inverse of the pure impatience factor, adjusted for growth, which is given by equation (4.97). Then, given some calibrated values of the discount factor β and the exogenously specified growth rate of the economy γ, using the Fisher equation

(4.98), I residually obtain the standard CCAPM nominal rate of interest. Equation (4.99) shows that the price of using the credit exchange service in equilibrium has to be equal to the cost of otherwise holding money, which is the net CCAPM interest rate. Given this price of credit and the first-order conditions of optimality of the FI with respect to labour, substituting the implied labour factor demand back into the credit production function gives equation (4.100), which is the steady state value of the inverse of credit-deposit (or credit-consumption) velocity. Residually from this, the inverse of the money-consumption velocity is thus also defined and given by equation (4.101). As shown by equation (4.102), the usual exchange cost embodied by the (net) CCAPM nominal rate will thus affect the marginal rate of substitution between consumption and leisure in the usual way. Turning attention to the productive sectors in the economy, perfectly mobile labour between the two sectors results in a condition given by equation (4.103), which just means that the marginal *revenue* products of labour in each sector have to be equal to one common equilibrium wage rate. Equations (4.104), (4.105) and (4.106) embody the key results of this paper and show how the nominal (and inflation-adjusted real) risk-free rates paid out on the short-term saving deposit held by the representative household are below the usual CCAPM nominal (and real) rates, due to the average cost (or average product) in collateral-backed credit production paid out in form of the banking wage bill, which crucially depends on the debt utilisation rate s_t^b *and* the price of credit, which in turn defines the proportional banking time cost over the total amount of short-term debt available in the economy.

Table of benchmark calibrated Parameters			
$1+r$=$1.03^{(1/4)}$	real. CCAPM rate	ρ =0.21	credit labour param.
$\frac{f}{c}$ =0.25	credit-to-cons ratio	A_g=1.0	TFP goods
γ =$1.02^{(1/4)}$	g. rate	l =0.7	leisure
n_f =0.0003	credit labour	n_g=0.2992	goods labour
Θ =$1.05^{(1/4)} \times \gamma$	money g.	ϕ_z=0.95	AR goods shock
ϕ_u =0.60	AR money g. shock	ϕ_v=0.95	AR credit shock
η_y =0.40	debt-to-deposit ratio	ϵ_z =0.65	s.d goods shock
ϵ_u =0.01	s.d. moneyg shock	ϵ_v =0.75	s.d credit shock

Table 4.1: Baseline Calibration

4.5 Calibration & Dynamic Analysis

In the following section, I am going to motivate and describe the baseline calibration on which steady state and dynamic analyses obtained from the solved model are based. As the mechanics underlying the de-centralised credit-banking model presented here are very similar to Benk et al. (2005), in which credit is directly produced by the household, similar steady state results are obtained. However, reflecting the asset pricing approach taken in this paper, the calibration to follow differs from standard treatments such as in Benk et al. (2005) in the way the usual real steady state interest rate (which in this model is equal to the real CCAPM rate) is chosen. Typically, standard calibration exercises of models with zero growth set the impatience factor $\beta = 0.99$, thus obtaining a *quarterly* real steady state risk-free rate equal to 1% (and thus an annual of 4%). However, the risk-free rate in the model discussed here is going to be below the usual CCAPM rate, where in particular the cost term $\frac{1}{1+s^{b}\rho p^{f}}$ affecting the money market rate received on the short-term saving deposit is going to play a key role in this respect. Owing to this latter argument, I calibrate the real CCAPM rate to be a somewhat lower annualised 3%[28] The steady state money supply growth rate is calibrated such as to imply an annual inflation

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[28] Tallarini (2000) argues in the same way when calibrating the impatience factor in his Epstein-Zin production general equilibrium model, where he also calibrates β so as to imply a theoretical risk-free rate which is closer to the one observed in the data.

rate of 5%, the economy's steady state exogenous growth rate to equal 2%, implying an annualised nominal CCAPM rate of 8% and an impatience factor $\beta = 0.9975$. The goods sector's steady state total productivity term is set to $A_g = 1.0$ and leisure and *total* labour are in the usual way calibrated to $l = 0.7$ and $n_f + n_g = 0.3$. Then I proceed in similar fashion to Benk et al. (2005) and Gillman and Kejak (2005) and calibrate the degree of diminishing returns in the credit sector $\rho = 0.21$, which is the U.S. time-series estimate obtained for this parameter in Gillman and Otto (2005). Also, similar to Benk et al. (2005), the steady state share of credit used in purchasing the consumption good (i.e. the inverse of credit-consumption velocity) is fixed at a value of $\frac{f}{c} = 0.25$, which is somewhat lower than the chosen 0.3 by the aforementioned authors. My choice is motivated by Canzoneri et al. (2007b), who use a calibrated value for the *debt-to-deposit ratio* in U.S. banking institutions (that is debt which is held by those institutions relative to deposits) equal to 25%. Assuming, as is the case in the model presented here, that the banking sector's holdings of debt is equal in value to the amount of credit produced and that deposits are equal to the level of consumption, the calibrated value for $f^{*} = \frac{f}{d} = \frac{f}{c} = 0.25$ is obtained. Although Benk et al. (2005) base their calibration of the share of credit used in purchasing consumption on observable long-run velocity of some monetary aggregate, my choice based on the level of intra-bank debt reflecting the collateral requirement assumption of credit in the model, roughly results in the same calibrated value and thus makes this calibration more robust, as both perspectives yield roughly the same value. The calibrated values residually imply a leisure utility preference parameter $\Psi = 2.29$, and steady state banking time share of $n_f = 0.0003$, which is within close range of values obtained by Benk et al. (2005), who obtain a value of 0.00049. Also residually obtained then are a goods sector labour share equal to $n_g = 0.2993$ and a banking sector total productivity term $A_f = 1.05$, where the latter differs from Benk et al. (2005) obtained value of 1.422, which is likely due to their inclusion of physical capital into their model, which is absent here, but also their higher calibrated value for f^*. Based on the chosen baseline calibration, figures (4.1) and (4.2) show the theoretical steady state average yield curve, which has been calculated based on the steady state formulae provided in the preceding asset pricing section. The periods are in quarters and the yields represent annualised values. The reduced one-period return, due to the cost-distortion term $\frac{1}{1+s_1^b\rho p_f}$, is perpetuated throughout the entire term structure, thus implying a decreasing effect on yields of higher maturity, as only their tail-end one-period return is affected by this. Notice therefore, how the

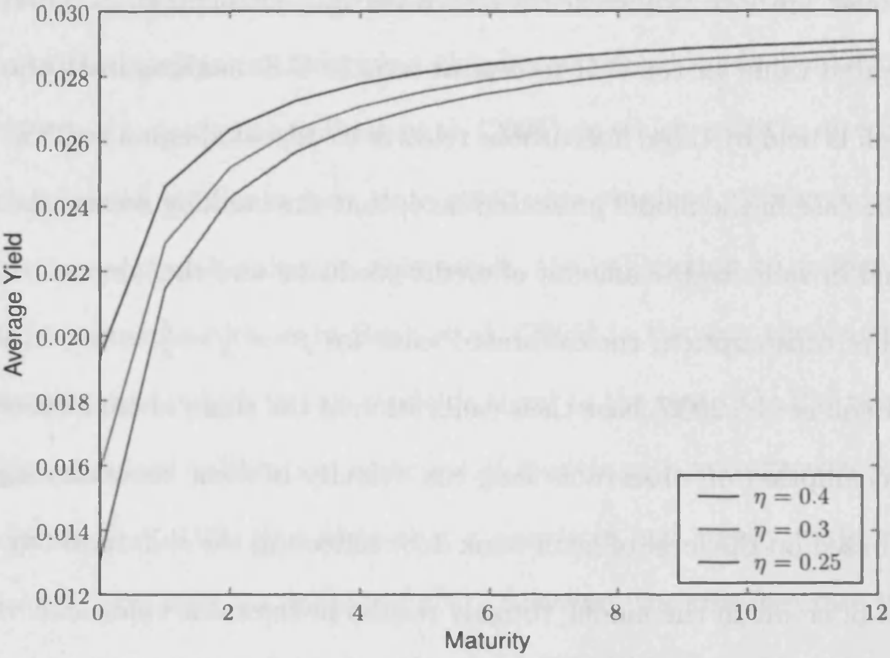

Figure 4.1: Theoretical Average Yield Curve (for $\rho = 0.21$)

two graphs show how the average yield structure also depends on the calibration of the degree of diminishing returns in credit production, ρ, but also on the calibration of the amount (or supply) of short-term debt relative to consumption (or in this model, deposits). Clearly, as figure (4.1) demonstrates, increasing the amount of relative short-term debt available in the economy, makes the distortive cost-effect of the banking sector matter relatively less (since the steady state share of credit is calibrated at a fixed level of 0.25). In contrast, increasing the degree of diminishing returns in the credit sector, for a given price of credit, increases the banking wage bill and thus lowers the residual payout received on the short-term saving deposit net of that cost.

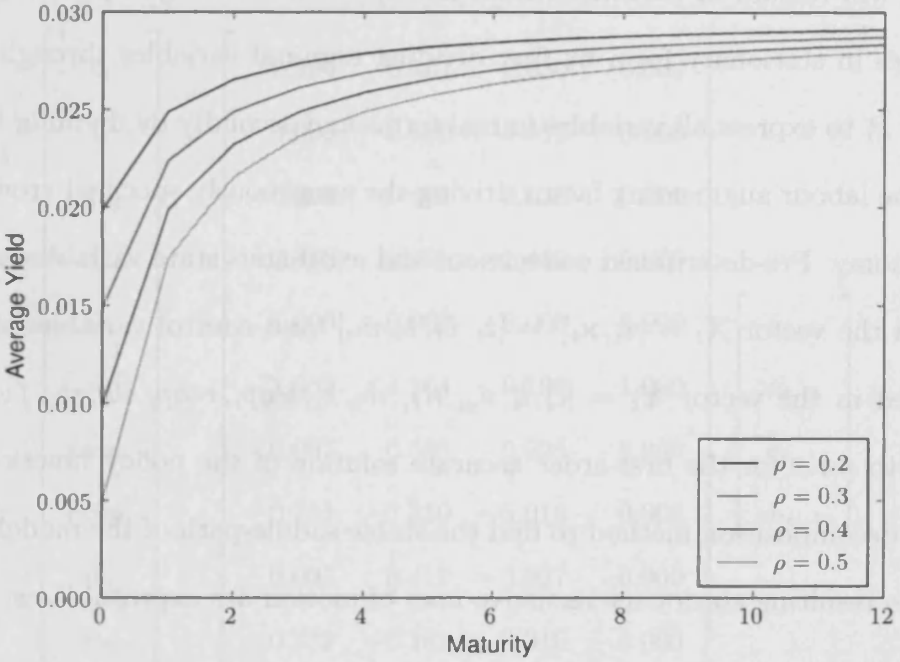

Figure 4.2: Theoretical Average Yield Curve (for $\eta = 0.40$)

4.5.1 Dynamic Analysis

In this section I am going to present a dynamic analysis of the model, based on impulse response graphs as well as calculated contemporaneous correlations obtained from simulations of the solved model. A competitive equilibrium for this economy consists of a set of allocations $\{c_t, l_t, n_{g,t}, n_{f,t}, \pi_t, fv_t, mv_t, k_t, M_t\}_{t=0}^{\infty}$, a set of prices $\{w_t, , icap_t, rcap_t, ib_t, rb_t\}_{t=0}^{\infty}$, exogenous shock processes $\{z_t, u_t, v_t\}$, money supply process and initial condition M_{-1} such that given the prices, shocks and government transfers, the allocations solve the consumers utility maximisation problem, solve the firms profit maximisation problem and the goods, labour and money markets clear. By log-linearising the de-trended non-linear first-order conditions around the steady state I obtain a (singular) linear rational expectations system of equations, where variables will typically be in terms of log-deviations from steady state (but rates will remain in deviations from levels)[29]. De-trending implies expressing all variables in stationary form by first dividing nominal variables through by the price level P_t to express all variables in real terms and secondly by dividing through by κ_{t-1}, the labour augmenting factor driving the exogenously specified growth rate in the economy. Pre-determined endogenous and exogenous state variables are summarised in the vector $\mathbf{X}_t = [\mathbf{z}_t, \mathbf{x}_t]' = [\hat{z}_t, \hat{u}_t, \hat{v}_t, \hat{m}_t]'$ and control variables similarly summarised in the vector $\mathbf{Y}_t = [\hat{c}_t, \hat{l}_t, \hat{n}_{g_t}, \hat{n}_{f_t}, \hat{w}_t, \hat{\pi}_t, \widehat{icap}_t, \widehat{rcap}_t, \widehat{ib}_t, \widehat{rb}_t, \widehat{fv}_t, \widehat{mv}_t]'$. I proceed to solve for the first-order accurate solution of the policy function using the Schur decomposition method to find the stable saddle-path of the model (Klein, 2000). The resulting stationary recursive laws of motion are expressible as:

$$\mathbf{X}_t = P\mathbf{X}_{t-1} \tag{4.107}$$

$$\mathbf{Y}_t = F\mathbf{X}_{t-1} \tag{4.108}$$

which are used to produce impulse-responses to describe relevant effects and also later on to simulate the model and analyse contemporaneous correlations between

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[29] I do not log-linearise explicitly, but actually symbolically differentiate the set of first-order, market clearing conditions and exogenous laws of motion (which are all modified to express variables in logs) w.r.t. to future and current states and controls to obtain the Jacobian of the system, which I evaluate at the (log) steady state. The Jacobian can then be split into matrices A containing partial derivatives w.r.t. future controls and current states, and B containing partial derivatives w.r.t. current controls and past pre-determined states, which can be solved for the recursive laws using the Schur decomposition (see Klein and Gomme, 2008).

variables. The solved system is thus given by:

$$\begin{bmatrix} \hat{z}_t \\ \hat{u}_t \\ \hat{v}_t \\ \hat{m}_t \end{bmatrix} = \begin{bmatrix} 0.95 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.60 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.95 & 0.00 \\ 1.04 & -3.35 & -1.97 & 0.00 \end{bmatrix} \begin{bmatrix} \hat{z}_{t-1} \\ \hat{u}_{t-1} \\ \hat{v}_{t-1} \\ \hat{m}_{t-1} \end{bmatrix} \tag{4.109}$$

and

$$\begin{bmatrix} \hat{c}_t \\ \hat{l}_t \\ \hat{n}_{gt} \\ \hat{n}_{f_t} \\ \hat{w}_t \\ \hat{\pi}_t \\ \widehat{icap}_t \\ \widehat{rcap}_t \\ \widehat{ib}_t \\ \widehat{rb}_t \\ \widehat{fv}_t \\ \widehat{mv}_t \end{bmatrix} = \begin{bmatrix} 0.994 & -0.461 & 0.030 & 0.000 \\ 0.003 & 0.180 & -0.012 & 0.000 \\ -0.006 & -0.461 & 0.038 & 0.000 \\ 0.353 & 40.69 & -1.452 & 0.000 \\ 1.000 & 0.000 & 0.000 & 0.000 \\ -1.033 & 4.264 & 0.195 & 1.000 \\ 0.007 & 0.583 & -0.035 & 0.000 \\ -0.231 & -3.350 & -0.016 & 0.000 \\ 0.005 & 0.417 & -0.027 & 0.000 \\ -0.232 & -0.181 & 0.010 & 0.000 \\ 0.133 & -8.511 & -0.678 & 0.000 \\ -0.039 & 2.821 & 0.225 & 0.000 \end{bmatrix} \begin{bmatrix} \hat{z}_{t-1} \\ \hat{u}_{t-1} \\ \hat{v}_{t-1} \\ \hat{m}_{t-1} \end{bmatrix} \tag{4.110}$$

The above recursive laws of motion for the endogenous states and control variables are used in the usual way to compute impulse-response graphs and correlations between variables based on the simulated time-series obtained from subjecting the model to goods sector productivity, money growth rate, and credit productivity shocks.

4.5.2 Impulse Responses

In this section I am going to present the model's behaviour in response to one-off shocks, where I particularly wish to focus on monetary growth rate as well as credit productivity shocks. The impulse responses obtained from money supply growth innovations demonstrate how monetary policy shocks are capable of conditionally increasing the wedge between the nominal CCAPM and nominal risk-free rates, given by:

$$\Upsilon_t^b = \frac{1}{1 + s_t^b \rho p_t^f} \tag{4.111}$$

but only in as far as such shocks lead to higher expected inflation, thus raising the nominal CCAPM rate and therefore also the price of credit. For money shocks, this also leads to a rise in the share of credit, increasing the banking wage bill overall, due to price and quantity effects. It is well-known from standard cash-in-advance models (see Walsh, 2003), that the price effect (i.e. primarily through changes in expected inflation) is only possible when the exogenous process for the money growth rate shock is modeled with some degree of persistence. White noise money growth shocks, on the other hand, would never change inflation expectations, but only lead to one-off variations in the unexpected component of inflation (i.e. inflation forecast errors), leaving inflation expectations unaltered.

Also, analysis of credit productivity innovations reveals that increases in the share of credit alone do not necessarily lead to an increase of the wedge between the nominal CCAPM and the nominal risk-free rates, since a fall in the price of credit (through a fall in inflation expectations, thus lowering the nominal CCAPM rate) may offset the quantity effect sufficiently enough in order to lead to a conditional fall in the banking wage bill, closing the conditional gap between the nominal

CCAPM and the nominal risk-free rate. This case is obtained for the one-off credit productivity innovation.

Monetary Shocks

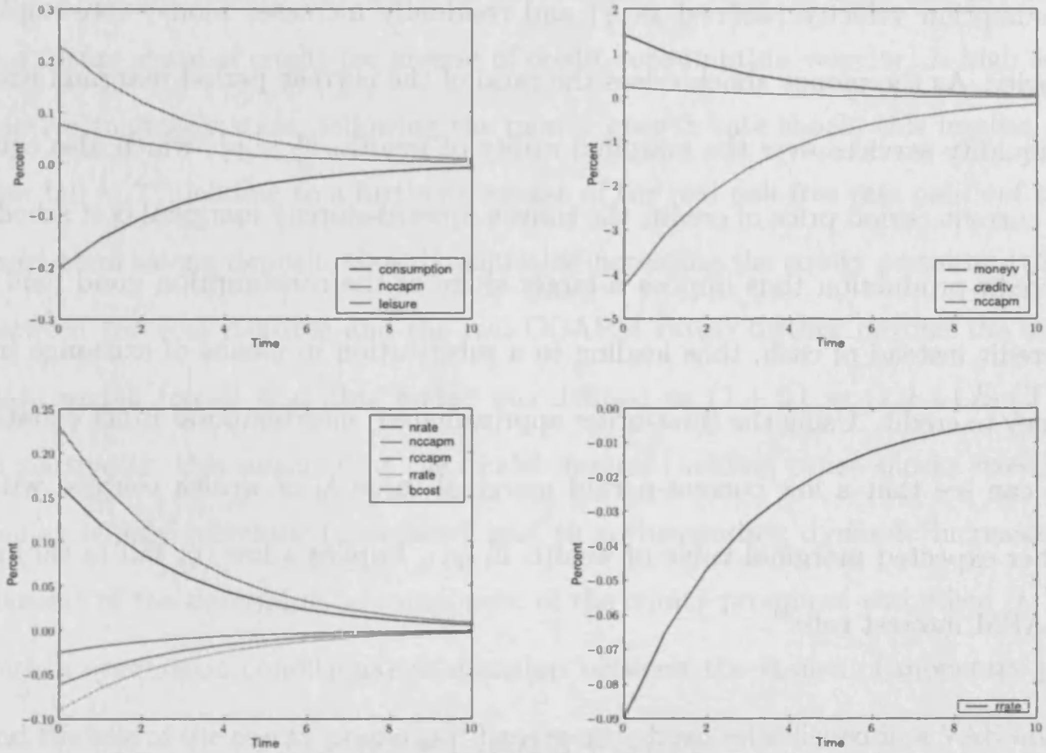

Figure 4.3: Response to 1 percent innovation in money growth

When shocked with 1% standard error innovation in the money supply growth rate process, regarding the nominal CCAPM rate and consumption, the model exhibits a behaviour which is equivalent to a standard cash-in-advance model explained in standard textbook treatments, such as Walsh (2003). Increases in the money supply growth rate lead to an increase in (expected) inflation, thus raising the nominal CCAPM rate through the Fisher relationship. This increases the exchange cost of consumption in the usual inflation-tax way (Cooley and Hansen, 1989), thus lowering the level of consumption and leading to a substitution effect towards more leisure.

Notice that, although not shown here, although total labour residually has to fall, there is a shift of labour from the goods to the credit sector, thus leading to decrease in goods labour and an increase in banking labour. The top right hand quadrant in figure (4.3), shows how the increase in the nominal CCAPM rate lowers credit-consumption velocity (defined as $\frac{c}{f}$) and residually increases money-consumption velocity. As the money shock raises the ratio of the current period marginal utility of liquidity services over the marginal utility of wealth, $\frac{\mu_t}{\lambda_t} = p_t^f$, which also equals the current-period price of credit, the convex upward-sloping marginal cost schedule in credit-production thus implies a larger share of the consumption good paid for in credit instead of cash, thus leading to a substitution in means of exchange from money to credit. Using the (first-order approximated) intertemporal Euler equation, one can see that a low current-period marginal value λ_t of wealth coupled with a higher expected marginal value of wealth $E_t\lambda_{t+1}$, implies a low (or fall in the) real CCAPM interest rate[30]:

$$\begin{aligned} &E_t\left[r_{1,t} + E_t\Delta\lambda_{t+1}\right] \approx 0 \Leftrightarrow \\ &E_t\left[r_{1,t} - \Delta c_{t+1} - \Delta i_t\right] \approx 0 \end{aligned} \tag{4.112}$$

The bottom-right quadrant of figure (4.3) illustrates how a lower marginal valuation today vis-a-vis a higher expected marginal valuation in future periods, through the dynamic Euler equation implies a modest fall in the *real* CCAPM rate. Although following the shock, expected consumption growth is *positive*, implying a fall in the real CCAPM rate, nominal interest rates are falling making the term $\Delta i_t < 0$. As the latter effect dominates the former, marginal valuation turns out to be low today and expected to rise, thus leading to a fall in the real rate. The bottom-left quadrant il-

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[30] see Uhlig (1995)

lustrates the response of return measures as well as the cost-distortion term $\frac{1}{1+s^b_{1,t}\rho p^f_t}$ (called bcost in the graph), responsible for driving a wedge between the nominal CCAPM and nominal bond rate, thus for a given (expected) rate of inflation, residually implying a lower real return on the bond-backed saving deposit. Notice that since both the current and expected future price of credit is high and the current and future share of credit (or inverse of credit-consumption velocity) is high as well relative to steady state, following the money growth rate shock, this implies a further fall in Υ^b_t, leading to a further decrease of the real risk-free rate paid out on the short-term saving deposit, thus dynamically increasing the equity premium (the gap between the real risk-free and the real CCAPM rates) further beyond the steady-state wedge (recall that this wedge was defined as $\left(1+\tilde{i}_t\right) = \left(1+i_t\right) E_t\left(\Upsilon^b_{t+1}\right)$). In particular, this means that the model implies (holding other shocks fixed) that money supply increases (decreases) lead to corresponding dynamic increases (decreases) of the deterministic component of the equity premium, embodied by Υ^b_{t+1}. Such a systematic conditional relationship between the stance of monetary policy and the size of the equity premium[31] has recently been established in a VAR analysis by Canzoneri et al. (2007a). Whereas their analysis defines the behaviour of the equity premium as the conditional behaviour of the difference between a model-implied CCAPM rate and the observed risk-free money market rate, the model presented here explains endogenous variation in this gap theoretically through the distortive (cost-driven) behaviour of a micro-founded banking sector. In summary, a money growth rate shock raises current and expected inflation, translating into increases of the current and expected future price of credit. This leads to an expansion of the credit sector, an increase in money velocity and a residual fall in credit velocity,

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[31] In their empirical analysis, they do not explicitly call this the equity premium, but just an interest rate spread between the CCAPM and money market rate. However, the idea of calling and calibrating this according to the equity premium is entertained in Canzoneri and Diba (2005).

as the representative household reacts to the increased inflation tax by using more credit instead of money balances, which are now taxed more heavily. As both the price and the share of credit rise, the proportional cost-driven distortive effect on the short-term saving deposit rises, thus increasing the wedge between the nominal CCAPM and the nominal bond rate, for given inflation expectations, implying a fall in the ex-ante *real* risk-free rate. As consumption falls, more leisure is taken and although *total* labour falls, banking time actually increases at the expense of less time spent in the goods production sector.

4.5.3 Credit Shocks

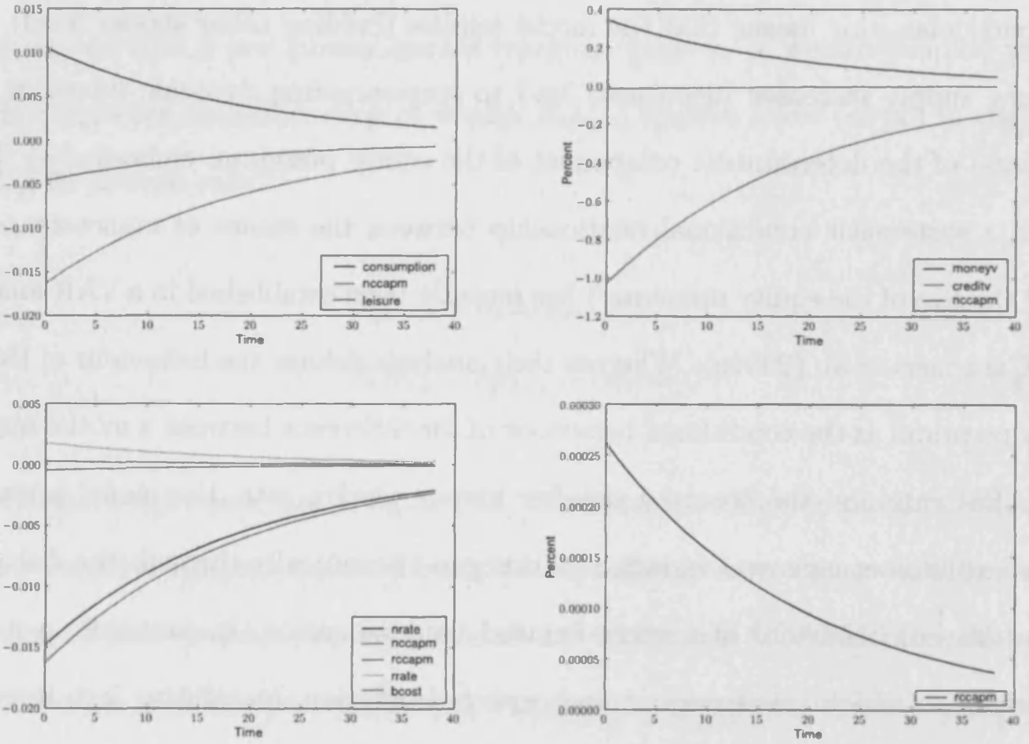

Figure 4.4: Response to 1 percent innovation in credit productivity

Focusing first on the top-right hand quadrant of figure (4.4), which summarises the responses to a 1% innovation in credit productivity, the responses of credit and money velocity are qualitatively similar to those obtained from a money growth rate shock. But whereas the shock to money growth increased the price of credit, thus leading to a higher credit share that way, here increasing the productivity of the credit sector lowers the marginal cost of producing credit for any given level of credit (and for any given price of credit), thus leading to a higher use of credit that way. Where the two figures differ, is shown in the top-left hand quadrant, which shows how the nominal CCAPM rate (and thus price of credit) falls, thus leading to a higher level of consumption and a substitution effect away from labour towards leisure. Notice that, although the bottom-right hand quadrant shows how the real CCAPM rises, the fall in the nominal CCAPM rate is due to a fall in the expected rate of inflation, which follows from an initially large spike in current inflation, due to the sharp fall in the demand for money balances. This large spike in inflation after a credit shock, followed by convergence from below it's steady state value (implying a fall in the expected rate of inflation), has also been documented by Benk et al. (2005) in their analysis of a similar credit model. Notice that although leisure increases and thus total labour has to fall residually, the labour spent in the goods sector actually increases, whereas banking time falls. This movement of labour from the banking to the goods sector is primarily due to the falling relative price of credit. Recall that the labour market equilibrium condition between the two sectors was given by:

$$w_t = i_{t-1}\rho \frac{f_t}{n_{f,t}} = \frac{y_t}{n_{g,t}} \tag{4.113}$$

Therefore, is the fall in the price of credit, given by the net nominal CCAPM rate i_{t-1} is stronger relative to the increase in credit production, then following the shock, the marginal revenue product of labour in the credit sector falls below the one in the goods sector. Therefore, labour will move from the former to the latter sector until the marginal products are equalised at some common wage rate. In spite of the fall in banking time, more credit relative to deposits (consumption) can be produced, due to the boost in credit productivity alone. Regarding return measures, the responses in the risk-free rate paid on the short-term saving deposit are quite different when compared to those obtained from the money growth shock. Notice that although the share of credit used in purchasing the consumption good has increased, typically implying a fall in the term responsible for lowering the risk-free rate below the CCAPM rate, given by $\Upsilon_t^b = \frac{1}{1+s_t^b \rho p_t^f}$, as the top-left hand quadrant shows, the term actually rises (so it's denominator must be falling). This is because the falling price (cost) of credit, p_t^f, more than outweighs the increase in the debt utilisation rate s_t^b through higher credit production, thus leading to an effective increase in this term. This means that following a credit shock, the conditional gap between the CCAPM and risk-free rate paid on the short-term saving deposit actually falls. In summary, a positive shock to credit productivity increases the share of credit, but also lowers the price of credit through the falling nominal CCAPM rate. Due to the falling exchange cost, consumption rises and less leisure is taken. The negative price effect of credit is strong enough to induce a shift of labour from the credit to the goods sector, implying a fall in banking time and an increase in goods production labour. Also, regarding the conditional determination of return measures, the same price effect is strong enough to outweigh the velocity effect on the distortionary cost term, thus for a given expected inflation rate, leading to a temporary increase in the real risk-free rate, and a temporary fall in the steady state gap between the CCAPM and the risk-free rate paid out on the saving deposit.

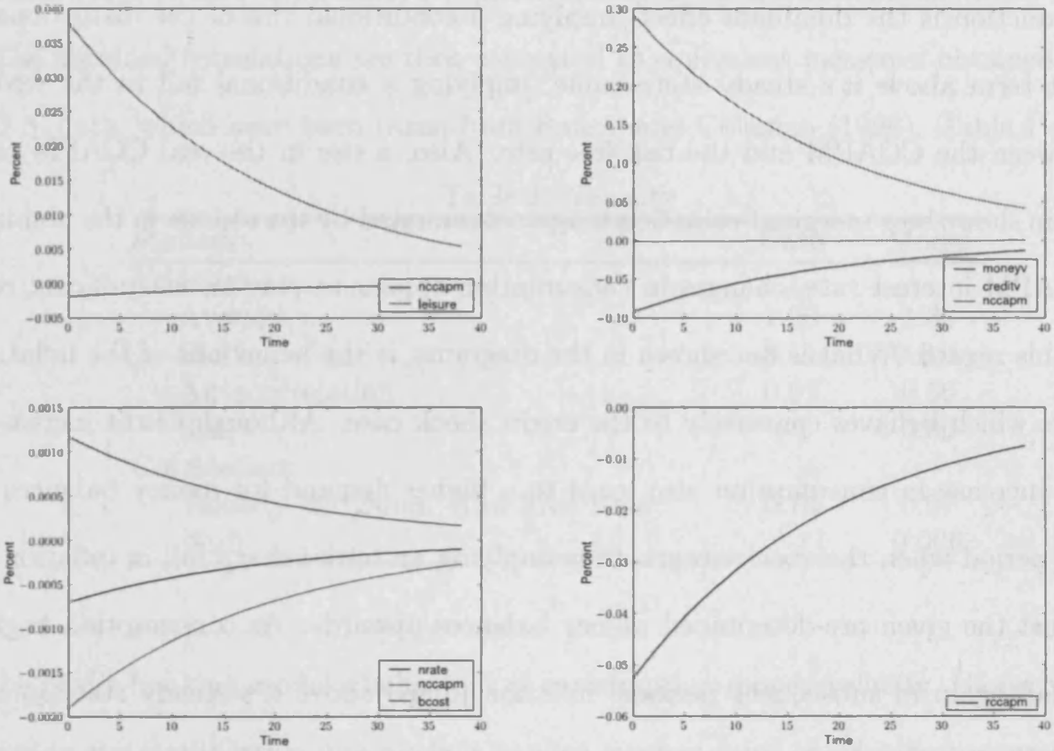

Figure 4.5: Response to 1 percent innovation in goods productivity

4.5.4 Goods Shocks

Following a 1% standard error to the goods sector productivity, the top left-hand quadrant shows a modest increase in the nominal CCAPM rate as well as a quantitatively similarly small substitution effect towards leisure (Consumption therefore rises almost one-for-one with the rise in goods productivity, but is omitted in the graph, in order to better illustrate the modest increases in the other two variables). In spite of the increase in the nominal CCAPM rate, leisure is taken such as to imply a fall in both goods and labour time. As revealed by the top right-hand quadrant, the fall in banking labour time leads to a fall of the share of credit used in consumption and a residual rise in the money share, thus implying a corresponding increase in credit and decrease in money velocity, respectively. The bottom left-hand quadrant illustrates how in spite of the modest increase in the price of credit, the fall in credit production is the dominant effect, implying a conditional rise of the distortionary cost term above it's steady state value, implying a conditional fall in the wedge between the CCAPM and the risk-free rate. Also, a rise in the real CCAPM rate again shows how marginal valuation is again dominated by the change in the nominal CCAPM interest rate, changes in consumption appear to play an insignificant role in this regard. What is not shown in the diagrams, is the behaviour of the inflation rate, which behaves conversely to the credit shock case. Although credit increases, the increase in consumption also leads to a higher demand for money balances in the period when the shock occurs, thus implying an initial sharp fall in inflation to adjust the given pre-determined money balances upwards. As consumption begins to fall again in subsequent periods, inflation jumps above it's steady state in the period after the shock only to converge to it's long-run level from above. In summary, a goods sector productivity shock leads to a modest increase in the real and nominal CCAPM rate, implying a modest increase in the price of credit. There is a modest substitution effect towards leisure, implying a modest fall in total labour, where this time both banking time and goods production time fall. This leads to less credit being produced and more money being held. The distortionary cost term rises, thus leading to an fall in the gap between the CCAPM rate and the risk free rate, conditionally reducing the equity premium.

4.5.5 Simulation Analysis

This section is going to analyse the simulated time series from the solved model and, similar to Bansal and Coleman (1996), in particular focus on correlations of velocity and ex-post asset returns with measures of monetary policy. In order to make simulations comparable to historical post-war quarterly time series data, a simulation length of 200 was chosen, where each time series is hp-filtered. Standard errors (where applicable) are generated by repeating simulations 1000 times. The obtained correlations are then compared to equivalent measures obtained from U.S. data, which have been taken from Bansal and Coleman (1996). Table 1 shows the credit-banking model's behaviour of consumption-money velocity. Based on the steady state calibration, the model's implied average value of velocity measure compare favourably with the equivalent measure observed in U.S. data and represents an improvement over standard cash-in-advance models which typically exhibit a velocity value of unity. Also, in contrast to the credit-cash model by Stokey and Lucas (1987), in which velocity measures different from unity and a positive relationship to the nominal interest rate is obtained and whose approach is based on a simple preference specification argument (in which cash and credit goods are imperfectly substitutable), velocity in the credit-banking model is primarily determined by variations in the *price* (equalling the net nominal rate) of credit on the one hand, and the credit-productivity induced *shifts* in the convex marginal cost schedule of the credit sector, on the other. Therefore, velocity is not preference-, but instead technology-driven and credit is purchased in a de-centralised market in which the intersection of the price of credit (which in a unique exchange equilibrium between use of cash and credit has to equal the opportunity cost of cash, the net nominal CCAPM rate i_t) and the upward-sloping marginal cost curve determined by the

Table 1: Velocity

Statistic	Data	Model
Velocity:		
Average	1.20	1.33
Std.	0.11	0.021
Autocorrelation	0.97	0.56
Std.	(-)	0.06
Correlation:		
Velocity and Nom. Risk-Free Rate	0.74	0.97
Std.	(-)	0.006

degrees of diminishing return parameter ρ, determine velocity.

The model does fairly well in capturing the autocorrelation of velocity as well as the contemporaneous correlation with the nominal rate of interest. Notice that in simulation results not reported here, the autocorrelation of velocity is strongly linked to how persistently money supply growth rate shocks are modeled. The model is less successful in capturing the observed volatility of velocity, which is not surprising given the findings of Hodrick et al. (1991), who study the variability of velocity in a preference-based cash-credit model, only to find that high levels of risk-aversion are needed, in order to make interest rates more volatile, leading to sufficient variability in velocity that way. The model presented here also exhibits low variability in the nominal rate, due to low variability of the stochastic discount factor (or the real rate) coupled with low variability in inflation expectations, which is common for flex-price models, in which a large proportion of money supply growth innovations directly translate into unexpected inflation forecast errors in the period of the shock, leaving little left to be captured by inflation expectations. Notice that, in spite of the credit-banking model's second potential channel affecting volatility - shocks to credit productivity - it appears that given the baseline calibration, variability in the price of credit seems to matter far more for the determination of the variability of velocity.

Table 2 illustrates the model's time series characteristics of the ex-post low risk-free rate and compares this to historical equivalent measures from the U.S. Through the credit-banking cost distortion, the low risk-free rate obtained on short-term saving deposits can be calibrated such to to be much closer to the historically observed low risk-free rate of approximately 1%. The model does very well in capturing the observed standard deviation of the ex-post real return on the short-term saving deposit. More importantly, there is a strong negative correlation between this real rate and the realised rate of inflation. But this results is hardly surprising, as the ex-post rate is constructed by subtracting the ex-post realised rate of inflation from the ex-ante nominal risk-free rate. Therefore, all of the inflation forecast errors (which are very large) are contained in the ex-post rate, such as to produce a very high correlation of this rate with the ex-post inflation rate (which contains the same inflation forecast errors).

What is more interesting however, is that the model is capable of producing a strongly negative correlation between the *ex-ante* real rate and the *ex-ante* expected rate of inflation, which is also seen in U.S. data and has been found to be robust through various studies (see Huizinga and Mishkin, 1984; Summers, 1984). The intuition for why this is the case is straightforward. As inflation expectations rise, so does the current and future price of credit (through the rise in the current and future expected nominal CCAPM rate which is largely driven by inflation expectations), leading to a current and future expected expansion of the credit sector. This however increases the future expected proportional payout of the short-term saving deposit in form of the future expected banking wage bill, leading to a residually lower real risk-free return. Notice that although this implies an apparent unconditional as well as conditional violation of the Fisher equation as measured by *observable*

Table 2: Real Risk-Free Rate

Statistic	Data	Model
Ex-Post Real Rate:		
Average	1.12%	1.95%
Std.	3.27	2.31
Correlation:		
Ex-Post Real Rate and Inflation	-0.68	-0.99
Std.	(-)	0.001
Ex-Ante Real Rate and Exp. Inflation	-0.34	-0.98
Std.	(-)	0.001

Note:The ex-post risk-free rate is defined as $1 + i_{1,t} - \pi_{t+1}$

cost-distorted money market returns and inflation, once the banking wage bill is taken into account, the *effective* Fisher relationship is not violated unconditionally (naturally, it will however never hold exactly ex-post conditionally, because of errors in inflation expectations, but also never ex-ante unconditionally, because of the inflation risk-premium).

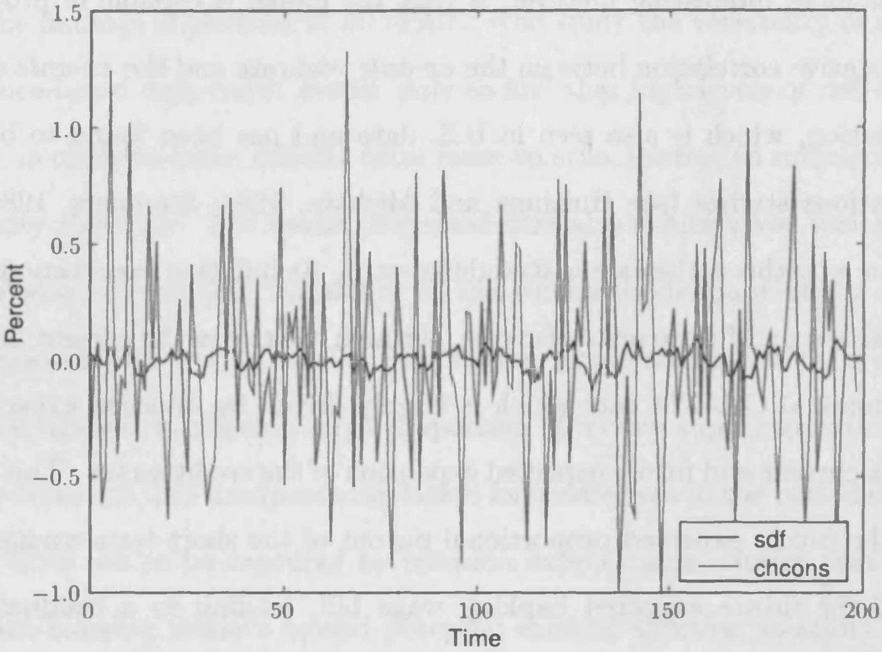

Figure 4.6: %Δ of Log Stochastic Discount Factor and Consumption

Figure (4.6) illustrates a representative simulation for the endogenously determined (from within the model) % change in consumption *growth* on the one hand, and the % change in the log stochastic discount factor of the credit-banking model, on the other. Notice that the latter is given by:

$$\log(\Lambda_{t+1}) = \log E_t \left[\frac{\gamma}{\beta} \frac{\lambda_{t+1}}{\lambda_t} \right] \tag{4.114}$$

which differs from the usual log discount factor of endowment barter economies, which is just equal to the (expected) change in consumption, whereas here marginal valuation also depends on the nominal CCAPM rate of interest. Fitting an AR1 process to a representative simulation of consumption growth results in an autocorrelation coefficient equal to -0.16 with a standard error of 0.45, making this measure close to i.i.d. The relevant log stochastic discount factor for the economic environment discussed here has a positive autocorrelation coefficient equal to 0.62 with a standard error of 0.03. Regarding the stochastically implied average yield curve of the term structure of interest rates, this implies a slightly downward-sloping yield curve due to the cumulative effect of positive Jensen's inequality terms (or variance terms), which are subtracted from the deterministic mean return (see Backus et al., 1989; den Haan, 1995; Cochrane, 2005). However, it is well-known that this risk-adjustment of yield returns due to the hedging role of long-term bonds when the representative household faces growing future volatility of valuation decreases with ever less risk-averse representative agents, making this effect quantitatively very small for logarithmic specification of preferences (see den Haan, 1995). In any case, the quantitative effect of the "first-order" cost-distortion leading to the downward-sloping convex-shaped of the *deterministic* component of average yields will outweigh the previously mentioned "second-order" risk-induced effect causing the yield curve to be slightly downward-sloping, leading overall to a downward-sloping convex-shaped yield curve, stochastically[32].

Another feature of the credit-banking model which sets it somewhat apart from standard cash-in-advance models is that inflation forecast errors, though of course through rational expectations on average zero, will however generally be much larger on average. This is illustrated in figure (4.7). The reason for this lies in the en-

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[32] Cochrane (2005, ch.19,p.361) shows how a log discount factor with autocorrelation coefficient of $\rho = 0.9$ and a standard error of $\sigma_\epsilon = 0.02$ results in a downward-sloping yield curve with a quantitatively very small slope, leading to an essentially flat yield curve.

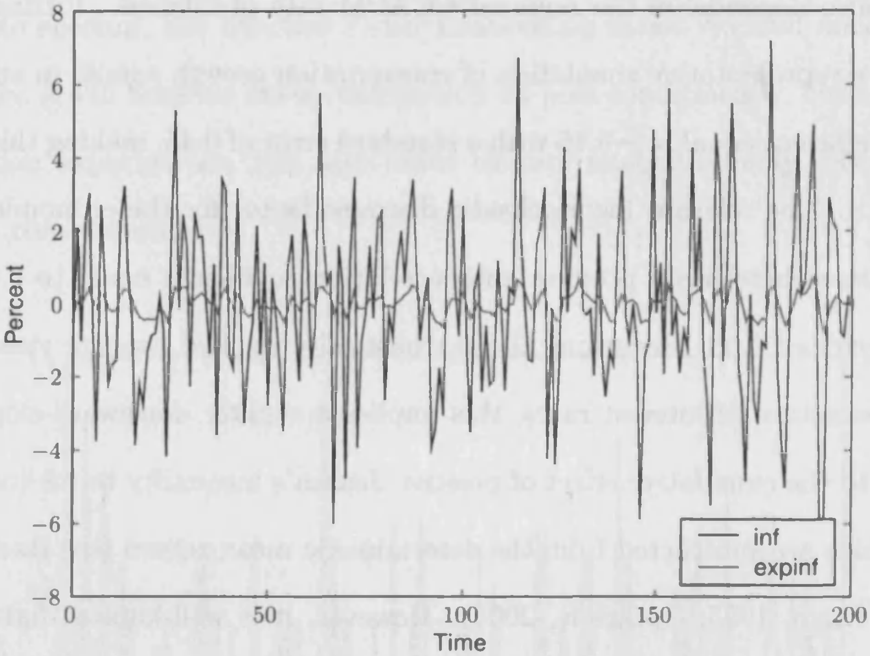

Figure 4.7: Ex-Ante Expected and Ex-Post Realised Inflation

dogenous variation of velocity measures (through the endogenous variation in credit production). Since money balances are pre-determined, shocks leading to an imbalance between nominal money supply and money demand, require an endogenous response in actual inflation in order to restore monetary equilibrium in real terms. If, during the same time (say, following an unexpected money growth shock), credit expands as well, then nominal pre-determined money balances have to experience and even stronger adjustment in real terms through inflation in order to establish an equilibrium between the total supply of exchange means (i.e. money *and* credit) and money demand, given by the current level of consumption. Therefore, adding credit supply to a liquidity market (given by the cash-in-advance constraint) leads to much stronger variation in actual inflation relative to expected inflation than otherwise found in standard CIA models. Of course, additional shocks to credit productivity further increase the uncertainty about future expected inflation[33].

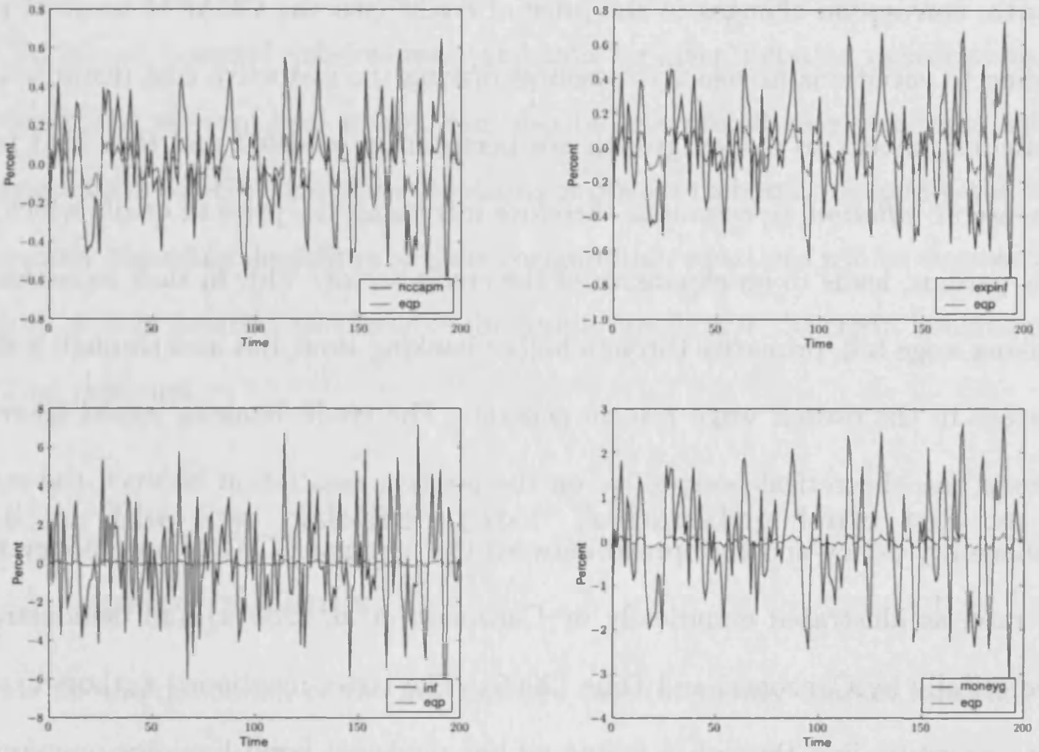

Figure 4.8: Simulated conditional %Δ in equity premium ($eqp_t = i_{1,t} - \tilde{i}_{1,t}$)

Figure (4.8) illustrates how variations in monetary policy indicators affect the cost-wedge between the nominal CCAPM and the nominal risk-free rate (and thus the equity premium) over the business cycle and how the stance of monetary policy is positively correlated with this wedge. The top left and right hand, and the bottom right hand quadrant graphs are essentially all linked through the stochastic money supply growth process and corresponding changes in inflation expectations, also affecting the nominal CCAPM rate through the Fisher relationship. The same positive relationship between the equity premium and actual inflation also holds, however with a less stronger association for the reasons discussed above regarding larger inflation forecast errors. The simulations confirm the results derived in the

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[33] This point is also discussed in Gillman et al. (2007), section 3.5.2: Effects of Shocks on Inflation.

theoretical section and clearly show the link between unexpected shocks to money growth, correspond changes in the price of credit (via the CCAPM nominal rate) leading to variations in velocity measures driving the distortive cost distortion. In particular, shocks to money growth are persistently modeled and thus lead to an increase in *inflation expectations*, therefore increasing the price of credit which, ceteris paribus, leads to an expansion of the credit sector. This in turn increases the banking wage bill, primarily through higher banking time, but also through a slight increase in the overall wage rate in general. The credit-banking model therefore offers a new theoretical perspective on the positive association between the stance of monetary policy and the spread between the nominal CCAPM and money market rate, as illustrated empirically by Canzoneri et al. (2007a) and demonstrated theoretically by Canzoneri and Diba (2005). The latter mentioned authors explain this systematic link through a falling ad-hoc modelled bond liquidity premium as the issuing of bonds increases in an open-market operation reducing the amount of money. The credit-banking model, on the other hand, links increases in money growth, through their effect on inflation expectations, to a rise in the price of credit and credit production, thus resulting in a larger proportion of short-term debt's return to be paid out in form of the wage the representative household takes home in his activity as a banker. The model therefore provides a micro-founded theoretical explanation of this effect based on de-centralised credit production motivated by the financial intermediation literature.

4.6 Discussion

Before concluding, this section's purpose is to briefly related the results obtained from the credit-banking model to relevant themes of the existing literature. In particular, I will discuss how the distortive cost-effect due to credit production (and directly related to the banking wage bill) driving a wedge between the nominal CCAPM and nominal risk-free rate (and thus for given inflation expectations, also between the relevant real rates), can also be interpreted as a tax (or a subsidy, depending on whether one refers to returns or prices of a bond). Secondly, the results relevancy regarding the failure of Euler consumption equations will be discussed, and finally, a more detailed discussion of the equity premium in the credit banking model will be provided.

4.6.1 The low risk-free rate: A banking time tax on the price of bonds?

Thus far the discussion of the low risk-free rate (or money market rate) has been spelled out in terms of a partitioned payout on a short-term saving deposit, which was backed up one-for-one by an equivalent amount of short-term government debt by the financial intermediary. The relevant result describing this was given by:

$$\begin{aligned}
&(1+\tilde{r}_{1,t}) \\
&= (1+r_{1,t}) - E_t\left[\Upsilon^a_{t+1}\right] \\
&\approx (1+r_{1,t})\, E_t\left[\Upsilon^b_{t+1}\right] \qquad (4.115)
\end{aligned}$$

where $E_t\left[\Upsilon^b_{t+1}\right] = E_t\left[1/1 + s^b_{t+1}\rho p^f_{t+1}\right]$ is the term responsible for driving down the real risk-free rate. Therefore, writing the above results out in full, it is clear that one could alternatively view this as a banking time distortion of real CCAPM rate, thus resulting in the lower risk-free rate obtained on the short-term saving deposit:

$$(1+\tilde{r}_{1,t}) = (1+r_{1,t})\, E_t\left[\frac{1}{1+s^b_{t+1}\rho p^f_{t+1}}\right] \tag{4.116}$$

where the distortion is equal to the future expected proportional payout of the return on the short-term bond in terms of the banking wage bill:

$$\begin{aligned} E_t\left[\frac{1}{1+s^b_{t+1}\rho p^f_{t+1}}\right] &\approx E_t\left[-s^b_{t+1}\rho p^f_{t+1}\right] \\ =E_t\left[-\frac{i_{t+1}\rho f^*_{t+1}}{\tilde{\eta}}\right] &= E_t\left[-\frac{w_{t+1}n_{f,t+1}}{b_{1,t}/\left(1+\pi_{t+1}\right)}\right] \end{aligned} \tag{4.117}$$

Alternatively, the distortion of the CCAPM real return related to the future banking wage bill can also be viewed as a *tax* on the price of the financial asset commanding that return:

$$\begin{aligned} \tilde{p}^b_{1,t} &= (1+\tilde{r}_{1,t})^{-1} = p^b_{1,t}\left[E_t\Upsilon^b_{t+1}\right]^{-1} \\ & p^b_{1,t}E_t\left(1+s^b_{t+1}\rho p^f_{t+1}\right) \end{aligned} \tag{4.118}$$

Besides the usual Tobin effects which are often cited as one of the factors responsible for affecting the Fisher relationship, the banking time tax on short-term debt implies a distortion of the Fisher equation implied by ***observable*** money market rates and measures of expected inflation affecting this relationship both in a steady state long-run, but also conditionally over the business cycle. The credit-banking model discussed here makes this relationship crucially depend on two factors: firstly, the ***price*** of credit embodied by the net nominal CCAPM rate which typically also directly influences the level of production of credit[34], and secondly $\tilde{\eta} = \left[b_{1,t-1}/\left(1+\pi_t\right)\right]/c_t$

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[34] Although, I have shown in the impulse response analysis, that a shock to credit productivity can lead to a fall in the price and a rise in the quantity of credit, where the former has outweighed the latter in it's effect on the conditional equity premium.

the proportional amount or *supply* of short-term debt circulating in the economy and how this relates to the proportional amount of debt distorted, which is captured by the proportional *supply* of credit $f_t^* = f_t/c_t$. Therefore, both the unconditional average of $s_t^b = f_t^*/\bar{\eta}$ and it's conditional behaviour over the business cycle are crucial in understanding the degree to which the banking time tax can affect the return (or price of) on short-term debt.

4.6.2 Euler Equation Rates and Money Market Rates

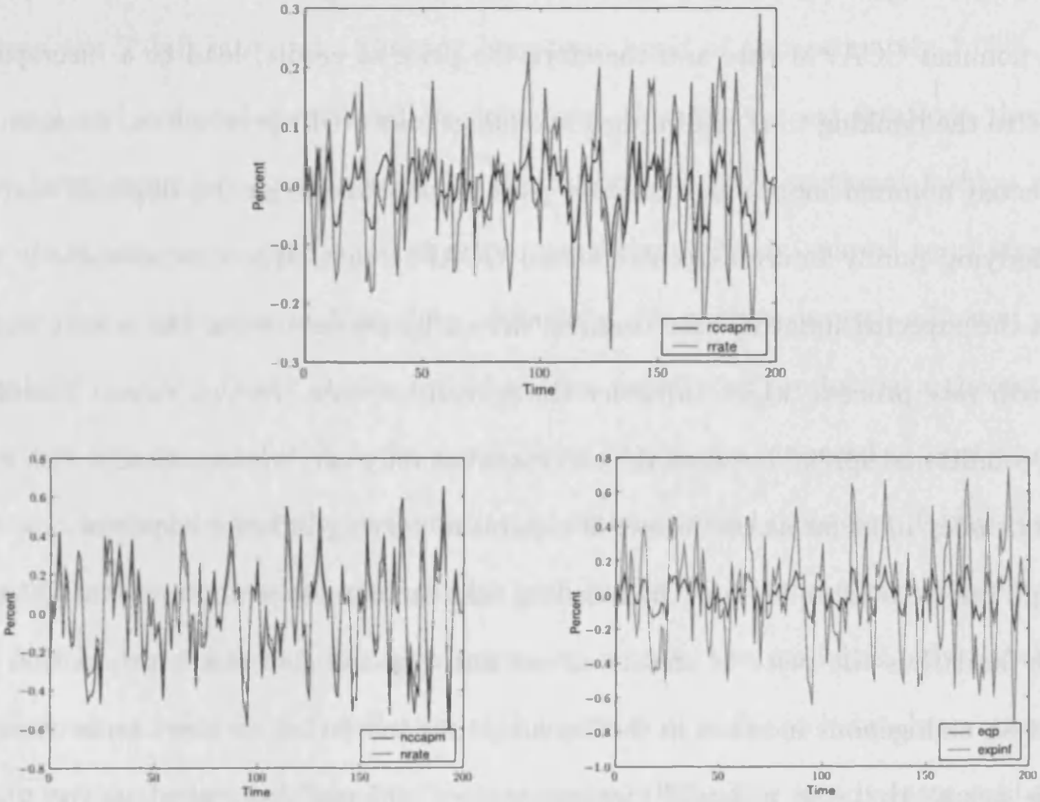

Figure 4.9: Simulated conditional %Δ in interest rate spreads (real and nominal)

As pointed out by Canzoneri et al. (2007a), there exists a sizeable literature documenting the empirical failure of consumption Euler equation regressions based on the behaviour of aggregate consumption and observable money market rates. This is problematic for models discussing optimal monetary policy in a fashion implying *equivalence* of the observed nominal money market rate and the CCAPM rate integral to the consumption Euler equation. Further more, Canzoneri et al.'s empirical study suggests that there exists a *systematic* link between the spread of the two rates and monetary policy, and how this fact confirms certain central banker's as well as academics unease about models of monetary policy embodied by the new neoclassical synthesis in which the role of money has been marginalised (and monetary policy is modeled by empirical Taylor rules) and how financial intermediaries are not modeled at all. Figure (4.9) clearly shows how changes in expected inflation (thus raising the nominal CCAPM rate and therefore the price of credit) lead to a discrepancy (due to the banking time tax implied by endogenous credit production) between the observed nominal money market rate paid on short-term saving deposits and the underlying purely intertemporal nominal CCAPM rate. The crucial point is that it is the *expected* inflation rate channel, driven by persistence in the money supply growth rate process, which influence the spread between the two rates. Therefore, the conditional spread between the two rates can only vary systematically with monetary policy in as far as the former is capable of varying inflation expectations. The graph can be best understood by recalling that an increase in the *nominal* CCAPM rate (and thus the price of credit) causes and expansion of credit production and thus an endogenous increase in the banking time tax levied on short-term deposits. This means that the *nominal* market rate is "buffered" compared to the underlying nominal CCAPM rate, it moves less in either way as the nominal CCAPM either falls or rises, due to the banking time tax. As the top graph reveals, for a given inflation expectation as implied by the Fisher relationship between the real and nominal CCAPM rates, the ex-ante *real* rate on the short-term saving deposit typically move more than the purely real intertemporal rate and sometimes they even appear to be de-linked in their movements. Notice that, although not pursued in this paper, this de-linked nature of the real money market and the real CCAPM rate and the uncertainty over it could even be increased by modeling a government debt-to-deposit target which is only imperfectly met each period, such as to make $\eta_t = \rho_\eta \eta_{t-1} + \epsilon_{\eta,t}$ an exogenous state variables as well, incorporating expected and unexpected variation in the proportional supply of debt as well.

4.6.3 The decline of the Equity Premium (Puzzle) ?

The credit-banking model creates an interest rate distortion (or differential) between the T-bill rate and a limiting long-term bond of approximately 1.05%, both in real and nominal terms and by doing so - through the expectations theory of the term structure - propagates this distortion in a cross-sectional fashion across bonds of various maturity and thus produces the convexly shaped term structure of average yields seen in U.S. data. Similarly, for a given growth-adjusted deterministic discount factor β, the model is thus capable of producing a theoretically low risk-free rate at 1.95% to be much closer to the one observed in post-war U.S. data. The model-theoretic premium return on long-term bonds over T-bills (i.e. the money market or risk-free rate) is also seen in the data to be approximately equal to 1.05%, but regarding the high return on equity, the model is only capable in contributing towards the resolution of this puzzle in as far as it has been successful in reducing the low risk-free rate by that same 1.05%. The steady state calibration of the model reflects a compromise between fitting unconditional returns of equity and bonds. In particular, I have chosen to slightly over-estimate both the average real return on the T-bill rate (at 1.95% versus the roughly 1% seen in U.S. data) and the average real return on a long-term bond (at approximately 3% versus the roughly 2.3% seen in U.S. data). However, the theoretical *spread* between short and long rates is correctly fitted. Notice therefore that in steady state, the return of a long-term limiting bond quickly approaches the CCAPM rate implied by the Euler equation, where the latter ought to be understood as the model's approximate counterpart of the return on equity. This implies a steady state return on equity equal to 3%, and a steady state return on the short-term saving deposit of 1.95%[35]. Based on early results of the equity premium literature, this may seem only a small contribution towards explaining the return differential between risky and (cost-distorted) money market rates. However, the current asset pricing literature appears more and more in favour of a view claiming an initial over-estimate of this return differential and it is not uncommon to encounter views which place the value of the equity premium to be as low as 2% – 3%. One reason why the *true population* equity risk premium as sampled from many different stock and bond exchanges may be lower, is due to survivorship bias implied by observable U.S. stock and bond returns (see Brown et al., 1995). Also, some authors have argued that, given the large historical fluctuations in stock returns and the relatively short amount of data available, one may view the post-war experience as an unrepresentative spell of luck (in terms of high equity returns vis-a-vis the risk-free rate) and indeed, given the sample's variability, an equity premium of 2% – 3% is still within range of a 95% confidence interval[36]. Indeed, stronger than expected *economic growth* and thus also *dividend growth* affecting stock returns may explain the unusually high and unexpected equity risk-premium observed over such a long period of time (see Cochrane, 2007, p.266). Therefore, contrary to the view put forth in Kocherlakota (1996), the direction the current consensus appears to take is to de-emphasise excessively high stock market returns of the past. Indeed, in spite of recent downturns, recently observed

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[35] The quarterly spread is calculated as $ips^b = 0.02 \times 0.21 \times (0.25/0.4) = 0.002625$. This implies an annual spread of 0.0105 or 1.05%

[36] Lettau et al. (2006) examine the role of a fall in macroeconomic risk leading to the fall of the equity premium in the 1990s.

stock market prices well above historical levels, broader stock market participation and the corresponding decline in the return on equity could be taken as evidence supporting the view that much of the historically observed excess return of equity over the risk-free rate was unexpectedly and unrepresentatively high. However, calibrating the real CCAPM rate at values of 3% – 4% and viewing this as the model's approximate return on capital (i.e. the risky rate), still leaves both the unconditional as well as *systematic* conditional variation of the low risk-free rate to be explained. The model presented here puts forth a theory of the risk-free or money market rate earned on short-term savings deposits, which is based on the endogenous variation of a banking time tax.

4.7 Conclusion

The equity premium and the complementary risk-free rate puzzle (Mehra and Prescott, 1985; Weil, 1989), the related failure of theory-implied consumption Euler equation regressions and the apparent inequality of observed money market rates and theory-implied Euler consumption equation rates (Canzoneri et al., 2007a), as well as the term premium puzzle (Backus et al., 1989), are all indicative of a hole in our understanding of how such return measures ought to be derived within a general equilibrium framework. A promising avenue contributing towards filling this gap is to devise ways of modeling financial intermediation more explicitly (see McCallum and Goodfriend, 2007; Canzoneri et al., 2008; Gillman and Kejak, 2008), thus opening up possibilities to distort such return measures by better understanding what roles such financial intermediaries may play and what implications for relevant measures might ensue as a result.

Building on previous steady state analysis work conducted by Gillman and Kejak (2008) and stochastic dynamic analysis by Benk et al. (2005), I have described and solved a model of essentially cash-in-advance nature, which was modified by incorporating a de-centralised credit-banking sector, serving the dual role of conduit for liquidity in terms of money and a produced credit exchange service, and of being the sole point-of-sales outlet for saving deposits of various maturity held by the representative household (which are internally backed up one-for-one by corresponding government bonds). The model is capable of driving a cost-related wedge (in form of the banking wage bill) between both the nominal and real CCAPM rates and the corresponding nominal and real rates obtained on the short-term saving deposit, thus lowering the *deterministic* component of the stochastic risk-free rate beyond the usual (growth-adjusted) real rate defined by the inverse of the representative household's impatience factor. The mechanism underlying the derivation of this wedge is motivated by the distortive cost effects produced by a micro-founded banking sector based on the theory of financial intermediation (Hancock, 1985; Clark, 1984). In contrast to Bansal & Coleman, I show how the reduced short-term money market rate is perpetuated through the term structure via the expectations theory, thus leading to a convex upward-sloping term structure with a much steeper slope at the short- than the long end, as only any j-period bond's tail-end return is affected by the banking sector's cost distortion, implied by the banking wage bill.

The key mechanism driving steady state, as well as dynamic results asset pricing results, is that some share of economy-wide short-term debt equal in value to the credit exchange service is retained as credit-backing collateral within the banking sector and instead re-distributed in form of a dividend payment back to the household at the end of the period. Instead, the representative household receives on this share of the short-term saving deposit the per unit-of-credit normalised revenue generated by the deposits (or average product of deposits, equal to $(1-\rho)\,i_t$), which equals the price of credit residual of the average product (cost) paid out to banking labour in terms of the banking wage bill. Since asset pricing results depend on the magnitude of the bond utilisation rate $s^b_{1,t}$, relative supply and relative credit-production induced demand of short-term debt matters. In as far as positive money supply growth rate innovations can lead to an increase in the nominal CCAPM rate (primarily by affecting the expected rate of inflation) and thus the price of credit, the credit-banking model experiences an expansion of the credit sector and thus a proportionately larger cost distortion in form of a banking time tax on the return of the short-term saving deposit, as the bank's debt-utilisation rate rises. This leads to a conditional widening of the gap between the nominal CCAPM and the nominal market rate. The paper therefore puts forth a new perspective on the systematic link between the stance of monetary policy and the spread between the money market and the CCAPM rate as implied by the consumption Euler equation, as described empirically by Canzoneri et al. (2007a) and explored theoretically by Canzoneri and Diba (2005). Finally, the model is also capable of generating velocity above unity and a positive correlation of this with the nominal rate of interest (both the intertemporal nominal CCAPM and the nominal market rate), and a negative correlation between the ex-post real rate and inflation, but more importantly, also a negative correlation between the *ex-ante* real rate and the *ex-ante* expected rate of inflation.

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Chapter 5

Conclusion

“Inflation is taxation without legislation” (Milton Friedman)

5.1 Concluding Remarks

In writing my PhD dissertation, I have been motivated by a personal ambition of wanting to explore both the role of money in stochastic dynamic general equilibrium frameworks and the determination of asset prices/returns within the same framework, which - ever since it's first seminal appearance (see Kydland and Prescott, 1982) - has become the unifying standard in modern-day macroeconomic research. Of course, whereas the latter reference embodies the seminal work of transporting the neoclassical growth model (see Solow, 1956; Ramsey, 1928) with physical capital into a stochastic intertemporal setting, other simple endowment-type "pure exchange" stochastic intertemporal models pre-date this (see Lucas, 1978), where the latter-cited contribution also represents a seminal contribution to the beginnings of asset pricing in stochastic dynamic general equilibrium frameworks (see also Merton, 1971; Breeden, 1979).

The introduction of money (demand) into such frameworks has been a difficult undertaking and it can be argued that the current state is far from the theoretical rigour in terms of microfoundations one would hope to obtain eventually. In order to obtain a positive demand for money balances within such Arrow-Debreu economies (such as the RBC model framework) money needs to have a positive value in equilibrium. Money-in-the-utility (see Sidrauski, 1967) - in spite of the often cited isomorphism arguments of Feenstra (1986) relating it to other approaches - is arguably too simplistic and ad-hoc, and is particularly peculiar in that it translates into positive utility, even when the representative household is not consuming (or taking leisure) at all! However, I believe that the unifying or overarching theme of my PhD thesis, which is epitomised by a microfounded banking theory of credit (card services) production embedded within a standard cash (or liquidity) in advance constraint, is good step forward in terms of formalising theoretically the sort of liquidity means-switching one would think occurs in the real, financially highly deregulated and innovated world. In particular, my section labeled "A real monetary business cycle" discusses how such a setup can allow for the formal modeling of a classical LM-type money (or indeed, Cagan-type) demand function for money balances.

Shopping time and general transaction cost theories (see Baumol, 1952; Tobin, 1956; Barro, 1976; McCallum, 1983; McCallum and Goodfriend, 1987; den Haan, 1995; Bansal and Coleman, 1996; Gavin and Kydland, 1999) explaining positive demand for money provide another collection of money demand frameworks, which due to their functional specification *generality* can often successfully capture certain measures related to monetary economies, such as simulated time series properties of various velocity measures. But it can be argued that this very *functional generality* and the model builder's essentially unguided and undisciplined fashion of choosing (calibrating) various parameters (say, in shopping time models) so as to simultaneously match moments of variables such as income velocity and other measures (for examples, see den Haan (1995); Bansal and Coleman (1996)), eliminates the rigour or discipline in calibration which arguably make *micro*founded models such an attractive proposition. Clearly, even though such approaches have resulted in some degree of success in allowing monetary RBC or endowment models to better fit certain nominal facts, what features of the economy are exactly or *explicitly* embodied by such functions is not really answered, which results in the arbitrariness in calibration. In general, it can be argued that by proceeding in this fashion, little is added to the efforts of pushing the boundaries of *micro*founded theoretical developments regarding money in general equilibrium.

As I have argued throughout, the cash-in-advance literature (see Lucas, 1982;

Stokey and Lucas, 1987; Clower, 1967) of modeling the demand for cash balances in general equilibrium - of some set of "1st generation" general equilibrium theories of money demand - is arguably still the theoretically most convincing and rigorous way of doing so, as it embodies the idea of modeling a market for liquidity explicitly through a cash-in-advance constraint, which typically assumes that cash is held in Leontief fashion with consumption only (see inter alia Lucas, 1982; Svensson, 1985; Giovannini and Labadie, 1991) or also with investment (Stockman, 1981) or with traded assets (Lucas, 1990). Here, money balances are typically held in advance (i.e. the CIA assumption, which is in contrast to the cash-when-i'm-done, or CWID assumption, explored by Carlstrom and Fuerst (2001)) for consumption purchases and thus do not only yield utility in terms of their marginal valuation of net wealth (say equal to λ_t), but also in terms of the marginal valuation derived from the liquidity services they provide (say equal to μ_t), thus resulting in a wedge driven between the marginal utility of consumption and the marginal utility of wealth, which also results in a distortion (embodied by the nominal rate of interest) due to the inflation tax present in such models (see Cooley and Hansen, 1989).

A canonical monetary endowment (Lucas, 1982; Giovannini and Labadie, 1991) or production-based RBC (Cooley and Hansen, 1995; Walsh, 2003) model has been shown to exhibit various shortcomings regarding it's predictions obtained from simulation exercises, such as the failure to account realistically for the moments of various velocity measures (consumption-money or income-money velocity) (see inter alia Hodrick et al., 1991; Cooley and Hansen, 1995), the failure in such flex-price models to capture a more smoothly and autocorrelated time series process of the rate of inflation and the related failure of being unable to realistically replicate the time series process of observed nominal rates of interest, which typically also exhibit some degree of positive autocorrelation and are also more volatile than implied by

CIA-type flex-price models. Intuitively, in such models following shocks to either (the growth rate of) money supply - which is typically exogenously specified - or (the growth in) money demand, which is directly related to the (growth of the) variables for which cash needs to be held in advance - typically only consumption - then in the period following the shock (the unexpected component), the price level adjusts strongly and therefore so does inflation, typically resulting in very small responses of inflation along it's expected trajectory (or expected inflation) (see Cooley and Hansen, 1995; den Haan, 1995; Walsh, 2003).

Through the standard Fisher equation - relating nominal rates to real rates and expected future inflation - which typically features in such models, the nominal rate then also inherits the low volatility of the model-implied expected inflation series. This "low-volatility" problem (see den Haan, 1995) is further exacerbated by the fact that production-based RBC models exhibit very little real interest rate volatility, as the household - with endogenous labour and physical capital storage technology - has sufficient choice variables on it's menu to smooth it's marginal valuation of income, which through the stochastic discount factor interpretation of real rates, also leads to smoothness of the latter (see den Haan, 1995; Lettau and Uhlig, 1997, 2000; Hornstein and Uhlig, 2000).

Such and other shortcomings have resulted in a state of affairs which throughout the mid-1990s into today's time have led to a theoretical landscape of monetary general equilibrium economics *within certain standard modeling frameworks* [1], which either often favours more ad-hoc and very general functional representations of money demand - such as shopping time or other transaction cost approaches - at the expense of marginalising the arguably more "theoretically robust" formulation

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[1] where I am here referring exclusively to models based on an explicit *quantity-theoretic* monetary framework, such as CIA-models, which notably excludes the so-called New Neoclassical Synthesis class of models.

of the CIA assumption, or which promotes the developing of completely new - and perhaps better microfounded - approaches to money demand, which are often of search-theoretical nature (see Kiyotaki and Wright, 1989; Trejos and Wright, 1995; Shougong, 1995; Kiyotaki and Wright, 1993; Kiyotaki and Moore, 2002; Wang and Shougong, 2006), or else other nature (see also Wallace and Kocherlakota, 1998; Kocherlakota, 1998; Krueger and Kocherlakota, 1999; Kocherlakota, 2002).

5.1.1 Monetary Business Cycles

In spite of this latter aforementioned very provocative strand of literature regarding the role of money, to my knowledge a serious integration of such approaches into standard production-based real business cycle frameworks has not occurred so far, partly also because of the necessity of modeling multiple (heterogeneous) agents or other complexities, which cannot easily be embedded into the standard representative agent framework. However, in Lucas (1990) the idea of a representative family or cohort is introduced, which allows for within-period heterogeneity - but due to pooling of resources at the end of each period - also period-to-period homogeneity, preserving all the simplifying abstractions of the representative agent framework. Related to this, recently a "segmented-markets" (traders vs. non-traders) approach has been formulated to better capture the liquidity effect from open-market operations (increases in the stock of money supply) and in general to develop a microfounded quantity-theoretic model of monetary equilibrium (see in particular Alvarez et al., 2001).

However, segmented markets models (or the related limited participation models) appear to be rooted in a very stark assumption of separating markets of liquidity (and bonds) in some arbitrary fashion, allowing only a fraction (or a certain type of agent, like a financial intermediary (see inter alia Fuerst, 1992; Christiano, 1991)), to absorb liquidity injections. This is almost akin to the arbitrariness of new neoclassical synthesis models' assumption of price stickiness due to a random probability of being able to reset prices or not. In as far as one would want to continue emphasising the equilibrium principle of markets in an aggregate sense, and share a belief that economic markets should share a basic law of physics, embodied in the well-known phrase "nature abhors a vaccum", then the very idea of segmented markets is certainly incompatible with this. Related to the limited participation literature is often also the assumption that firms need to borrow in order to finance the wage bill (i.e. the working capital assumption). Again, it is difficult to envisage a state of affairs in which firms' balance sheets are highly geared reflecting an aggregate amount of borrowing equal to the value-added of the labour input.

Moreover, the segmentation of markets of some measure of *liquidity* is particularly difficult to accept, since in an integrated world without capital controls, multinational private and financial organisations and some concept of no-arbitrage, one would think that "liquidity vacua" (with appropriately priced-in risk) are filled incredibly quickly by market forces. For whatever reasons (perhaps more significant transportation and transactions costs), segmenting *goods* markets may be a more plausible modeling assumption, but doing the same to markets for liquidity (which can be transferred electronically) as a key model assumption describing the evolution of *quarterly* data appears hard to swallow. The fact that time series properties of inflation and the nominal rate of interest typically suggest sluggish adjustment (or inertia) further questions the theoretical approach of segmented market models, as they typically only exhibit short-lived liquidity effects and thus fairly insignificant (or completely absent) propagation mechanisms regarding the nominal side of the economy. In other words, the idea of *persistent* "liquidity vacua" is even less plausible than that of relatively short-lived ones.

Instead, what I hope I have been able to sketch in my chapter titled "A *real* monetary business cycle", is that a promising avenue of modeling the nominal side of a general equilibrium model with some degree of success is to start again with the simple assumption of the "venerable" cash-in-advance constraint as a very explicit quantity-theoretic way of modeling the market for money (or more general, the market for *liquidity*), but to add a second means-of-exchange - self-produced credit - which results in an LM-type demand for money, meaning that money demand does not only depend on some scaling variable, such as consumption or income, but also on the nominal rate of interest. Building on work by Jermann (1998) and Boldrin et al. (2001), I have incorporated habit persistence in consumption and adjustment costs to investment, so as to obtain a smoother (or more autocorrelated/inert) evolution of the consumption process and an inelastic supply of physical capital. Adding these theoretical extensions to a canonical monetary business cycle model (see Cooley and Hansen, 1995; Walsh, 2003) - driven by productivity and eventually credit shocks only - but with *self-produced credit*, results in highly persistent simulated time series process of liquidity demand, inflation expectations and nominal rates of interest, while making the demand for money procyclical and both the real and nominal rates of interest countercyclical (inverted indicators). Throughout a deterministic "k-percent" money growth rate rule is assumed (see Friedman, 1960), leaving any questions regarding the role of (state-contingent "optimal") money supply process to be explored in the future.

5.1.2 Velocity

Related to the successful modeling of a monetary business cycle is almost always the successful modeling of some measure of velocity. Here, and in particular when one analyses this issue in a production-based RBC model, it is crucial to distinguish between consumption-money and income-money velocity, which due to the residual investment-money velocity component typically have different theoretical model-implied simulated time series properties. This distinction also matters in the simulated time series properties of *income* velocity studied in Cooley and Hansen (1995), which exhibits volatility, as the demand for money - due to the cash-in-advance assumption - is tightly linked to the Friedman-type permanent income-determined smooth endogenous evolution of consumption, whereas income itself (which through market-clearing is of course equal to consumption *plus investment*) responds proportionately much stronger to productivity shocks.

The observed time series properties of consumption-money velocity, on the other hand, have been found to be very difficult to explain in cash-in-advance type models, where in particular the careful simulation study carried out by Hodrick et al. (1991) springs to mind. Here, the preference-based cash-credit model is employed, so as to enable the successful modeling of the 1st moment of consumption-money velocity, while some measure of the 2nd moment (they use the coefficient of variation) is found to be too low compared to what one observes in the data, and also simultaneously successfully modeling the 2nd moments of consumption velocity *and* interest rates is impossible within this framework, as too high a volatility of the former is required to explain (some amount of) volatility of the latter [2]. Further, this study is also conducted within a monetary pure exchange economy, in which interest rate volatility can be increased trivially by increasing the coefficient of relative risk aversion and/or introducing habit persistence in consumption. For the reasons I have already outlined above, production-based RBC models with a storage technology, endogenous consumption and labour, will not allow for such a modeling approach of raising interest rate volatility in that fashion. In general, any attempt to model

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[2] For a recent *search-theoretic examination of this issue, see (Wang and Shougong, 2006)*

this measure successfully is rendered almost impossible by, firstly, too low volatility of interest rates (both real and nominal), and secondly, too low an interest-elasticity of consumption-money velocity.

What I hope to have shown in the chapter titled "Consumption Velocity in a Banking-Time Model", is that a banking-time economy with self-produced credit, in which the time series properties of consumption-money velocity are not determined by some preference-based, but rather some production-based explanation rooted in an upward-sloping convex marginal cost schedule (see Gillman and Kejak, 2008), much different interest-elasticities of consumption velocity are obtained. This is also observed by Gillman and Benk (2007) who are using a very similar model setup to revisit the issue of explaining the volatility of income-money velocity. My approach differs in that I abstract from endogenous human-capital growth, assume a "k-percent" deterministic money supply growth rate rule, and also abstract from direct shocks to credit production productivity. Just as in "A *real* monetary business cycle", my approach is in some sense "Wicksellian" in that I consider only productivity shock-induced variations in interest rates (both real and nominal) - which represent the *indirect* price channel in credit production - to explain the volatility in consumption velocity. By introducing habit persistence in consumption, I obtain a *phase shift* of the evolution of consumption relative to money demand, which already results in some success of increasing the variability of velocity. But only by adding q-theory motivated adjustment costs to investment (see Eisner and Strotz, 1963; Lucas, 1967; Hayashi, 1982), does the variability of consumption velocity rise sufficiently so as to match up with the data.

5.1.3 Money in GE - What's on the horizon?

By entertaining speculations about the role of money in dynamic stochastic general equilibrium models, one could of course review the *current consensus* state of affairs in the literature and then engage in some exercise in extrapolation, predicting future developments with regards to this issue that way. Bizarrely, doing so could indeed lead to the provocative prediction that specifying some role for a monetary aggregate in some quantity-theoretic fashion within the DSGE modeling framework may be abandoned alltogether! Michael Woodford's much-lauded "Interest and Prices" is a particularly note-worthy case in point, in which he describes optimal *monetary* policy using an interest-rate policy rule in a *cashless* economy (see Woodford, 2003, ch.3). Here, I am of course referring to the current popularity of models belonging to the so-called "New Neo-Classical Synthesis", or short NNS, which have nostalgically been likened to the venerable IS-LM framework (see McCallum and Nelson, 1999; Hicks, 1937). Let me summarise at this juncture some of the integral building blocks comprising such models (a current "state-of-the-art" model of this sort is described in Lawrence J. Christiano and Evans (2005)), so as to be able to better contrast them with cash-in-advance type real business cycle models, such as in Cooley and Hansen (1995) or with costly-credit (banking-time specification) as in Gillman and Benk (2007).

Of paramount importance regarding the development of this "Keynesian-flavoured" class of DSGE models, was the formalisation and introduction of a monopolistically competitive *price-setting* intermediate goods producing layer (see Blanchard and Kiyotaki, 1985) and the supplementing assumption that such intermediate goods firms would only be able to reset their own price (relative to the aggregate price level) with some fixed probability in each period (see Calvo, 1983), giving rise to a "smart"[3] forward-looking Phillips curve. Drawing on empirical evidence reported and theoretical considerations entertained by Fuhrer and Moore (1995), a "backward-looking" inflation component was introduced by allowing firms which were not allowed to reset their price in any period, to peg the evolution of their price (partially or fully) to lagged inflation. Combining the derivation of the New Keynesian Phillips curve in that way with the intertemporal consumption Euler equation (the intertemporal IS-curve) and specifying monetary policy by means of an empirically-motivated Taylor Rule (see Taylor, 1993) obeying the so-called "Taylor Principle" (sufficient weight of nominal interest rate responses to inflation, so as to result in a stable rational expectations saddle-path solution) completes the by now well-known simple "three-equation" version of a prototypical NNS model. Of course, the state-of-the-art CEE model (Lawrence J. Christiano and Evans, 2005) goes further, by including also:

1. nominal rigidity in the labour-market (Calvo-style wage setting) (see Erceg et al., 2000) [4]
2. variable capacity utilisation of the physical capital stock
3. the "working-capital" assumption regarding payment of the wage-bill
4. habit-persistence in consumption
5. and adjustment costs to investment (q-theory).

Lawrence J. Christiano and Evans clearly state that the first three items on the above list are important for explaining the impulse-responses of prices and output

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[3] Perhaps not so smart after all, as Gregory Mankiw emphasises by pointing out that fully anticipated monetary expansions imply a contraction of activity within this framework Mankiw (2001).

[4] which has proven to be the far more important rigidity in terms of keeping conditional expansions of marginal cost in check, following an unexpected fall in the nominal rate.

from an (essentially arbitrarily) identified VAR, whereas the latter two items are needed to better fit the same responses of consumption and investment obtained from that same VAR.

At this point it may also be useful to critically appraise the role of identified VARs (often also called structural VARs or sVARs) in the theory-led part of macroeconomic research. It is somewhat ironic to observe that *structural* VARs are susceptible to the same or a similar kind of criticism which the original VAR literature sought to deal with, namely that related to "incredible restriction" in macroeconomic research (see Sims, 1980). It seems as though the "incredible restrictions", which were once imposed on the deterministic part of a simultaneous equation model, have now been replaced by arguably equally incredible restrictions imposed on the variance-covariance matrix of estimated VARs so as to "identify" true structural and thus exogenous shocks, perhaps through the recursive ordering scheme implied by the Cholesky decomposition or other kinds of identifying restrictions, which may also be of long-run nature (see Blanchard and Quah, 1989). Sometimes one cannot help but think that sVARs have been employed to create a picture (in terms of impulse-responses) from identified structural shocks, so as to fulfil economists' a-priori expectations or beliefs about the effects of, say, a monetary policy ***innovation*** on the behaviour of the economy (see Eichenbaum, 1992). It is also arguably unfortunate how the entire debate focusing on policy *innovations* implies that the deterministic (or fully expected) endogenous response of (an either Taylor-rule implied or money supply) policy feedback rule alone, abstracting from any consideration of policy "surprises", consequently appears to be *irrelevant* within the empirical modeling framework of sVARs (see also Walsh (2003) on this debate).

Noteworthy is also the fact that Lawrence J. Christiano and Evans are able to capture an initial *fall* in inflation following an innovation to monetary policy (an unexpected monetary expansion via the Taylor Rule) followed by a subsequent rise in the latter. The *contemporaneous counter-cyclicality* of prices, as for instance summarised empirically in Gavin and Kydland (1999), has often been used by RBC proponents that emphasise the prevalence of productivity-driven *supply-side* shocks, instead of demand shocks, which within more simplified models - excluding, say, variable capacity utilisation and the wage bill working capital assumption - typically produce an *immediate and thus contemporaneous* rise in inflation following a shock to the demand side of the economy. Lawrence J. Christiano and Evans sometimes also frame their argument by referring to an *exogenous* money supply shock as an alternative measure of monetary policy within their model, but really what such models typically imply is that the interest rate rule embodies monetary policy, whereas some MIUF-implied money demand function then requires an *endogenous* and thus residually implied behaviour of money supply so as to be consistent with the behaviour of that very nominal interest rate rule.

It is perhaps this last point which best illustrates the contrasting mechanisms and difference in underlying assumptions of such NNS models, on the one hand, and cash-in-advance type monetary RBC models, on the other. Whereas in cash-in-advance type models one typically always finds the classical Fisher equation cropping up from the manipulation of various first-order conditions of optimality, NNS models imply *exogenous* (but of course in state-contingent fashion) control over the nominal rate with some random unexpected component in form of some interest rate setting rule. But more importantly, the definition of *inflation* within these two distinct frameworks is philosophically completely distinct! Indeed, cash-in-advance type models define inflation as the change in the (relative) *money price* of the consumption good whose evolution is determined in some quantity-theoretic fashion through an *explicit* modeling of the market for money (or more generally, the market for *liquidity*).

NNS models' definition or determination of inflation is not linked to any concept of a market for money (or liquidity) resulting in some embodiment of the quantity theory of money, but in contrast, define this measure completely through the "optimal" price setting behaviour of intermediate goods firms and wage setters facing Calvo-type price-setting restrictions. Essentially, inflation in the NNS framework describes the evolution of the rigid setting of a *relative* price which is not in any way directly related to some meaningful concepts of money demand and supply relationships which interact in some market for liquidity, determining inflation that way. Further, any discussion of financial intermediation or banks, as well as the modeling of various types of interest rates, is absent in this class of models, as noted by Goodfriend (2005).

In what follows, I briefly wish to trace out what I consider to be an emerging "leading" frontier of research in relation to a general *re-emphasis* of monetary aggregates, financial intermediation or banks and the modeling of various interest rate measures. This is a subjective view which I have developed during my own explorations into the frontiers of current research in relation to money in general equilibrium. I have already mentioned one recent significant contribution in this regard by Alvarez et al. (2001), which employs the segmented-markets framework and the trader-shopper cohort framework. Another strand of literature (see Gavin and Kydland, 1999; Freeman and Kydland, 2000; Dittmar et al., 2005; Kydland and Henriksen, 2005) mostly emphasises simple RBC-style flex-price models but typically makes the supply of some broader measure of money endogenous or else also shows how inflation persistence can ensue even in flex-price environments.

Another strand of recent literature, whose inception can perhaps be traced back to Goodfriend (2005) - and regarding the "special" role of short-term debt in this literature as far back as Keynes (1936); Friedman (1969); Bansal and Coleman (1996)

- and which has been "followed up" by contributions of McCallum and Goodfriend (2007) and Çanzoneri and Diba (2005); Canzoneri et al. (2007, 2008), emphasises the role of financial intermediaries or banks, or else also alternative uses (often assuming a liquidity-providing function) of government short-term debt (bonds), and the ensuing existence of various interest rate measures within the DSGE framework (such as for example drawing a distinction between a purely intertemporal CCAPM and a distorted risk-free rate).

However, as I will argue below, particularly the last line of research emphasising some form of financial intermediation and the special role of short-term debt, can also be directly related to the general equilibrium literature of asset pricing (with particular relevance to the "low risk-free rate"), thus representing a theoretical effort of integrating quantity-theoretic monetary business cycle models with asset pricing considerations. It is this issue of asset pricing - and also how I have chosen to conduct a theoretical discussion related to this in my chapter "Asset Pricing in a Banking Time Model" - to which I wish to turn next.

5.1.4 Asset Prices

Asset Pricing in General Equilibrium involves a rich literature, which is often described by or categorised into a set of "puzzles", meaning that standard RBC model-implied asset prices/returns do not match up with their empirical counterparts, sometimes by large orders of magnitude. As pointed out by Cochrane and Hansen (1992), due to the tight theoretical link between the consumption process (governed by the intertemporal IS-type consumption Euler equation) and the theory-implied return on physical capital (and perhaps also other assets modeled in the economy), observed asset prices should be exploited by business cycle researchers as a source of guidance in model-building, so as to eventually produce microfounded models which are successful along the aggregate quantities dimension *as well as* the price dimension. The most well-known “twinned” puzzles - I call them so as they are often mentioned in tandem and since obtaining a way to explain one often also helps in explaining the other - are the (in)famous “equity premium” puzzle (see Mehra and Prescott, 1985) and the related “low risk-free rate” puzzle (see Weil, 1989). Although more “exotic” assets such as options, can also be priced from within a representative agent-based general equilibrium framework, the literature has traditionally focused on the (real) return on a risky asset, the (real) return on very short-term debt (i.e. some risk-free rate) and the derivation of the (real) return on government (essentially default-free) bonds of higher maturity, leading directly to a discussion of the term structure of interest rates.

Risk-averse agents facing uncertainty over the amount and valuation of future cash flows derived from holding assets can be shown to be willing to pay risk-premia for the price of acquiring assets reflecting this. Regarding the equity premium, this risk-premium is typically related to some covariance term between the return on the risky asset and the behaviour of the consumption-based stochastic discount factor - falling (expected) returns during (expectations of) “hard times” and thus a higher marginal valuation of more consumption (i.e. more “appetite“) imply a lower price (and thus a higher return) to reflect that risk (see Cochrane, 2005). This microfounded view of using a consumption-based discount factor to capture the representative household-investor’s marginal valuation of wealth thus represents a general equilibrium analogue to the traditional CAPM model, in which a general broad market’s portfolio’s return serves this purpose.

The problem with or Achilles heel of this consumption-based approach of measuring marginal valuation ("good times and bad times“) is that the time series properties of aggregate consumption data suggest that households live in a fairly "safe" and certain world regarding this measure of marginal valuation. Although stock returns typically decline in contemporaneous fashion with consumption, thus leading to a "qualitative" or "directional" success of this theory regarding the equity premium, the *quantitative* failure of this theory (and thus the emergence of the puzzle) stems from the low volatility of consumption and the corresponding low volatility of the stochastic discount factor for standard *power* utility functions implying *realistic* specification of risk-aversion regarding intertemporal gambles of wealth.

As I have discussed and surveyed in "Asset Pricing in a Banking Time Model", the literature has largely proceeded by somehow increasing the perceived riskiness of the representative agent by the adoption of various different utility function specifications, both employed within pure exchange and production-based RBC models, with varying success. Cochrane (2007) remarks that the current state of affairs is such that it has thus far not been accomplished to write down a theoretical model which does not require some arguably unrealistically high calibrated value of the coefficient of relative risk aversion in order to explain observed risk premia. Related to this is the "theoretical tension" which appears to exist between the derivation of the risk-free rate and the risk-premium, as they are intimately linked to each other (see Kocherlakota, 1996; Cochrane, 2005).

The literature on general equilibrium (but also more "a-theoretical" or statistically motivated formulations of) asset pricing - due to the expectations which typically have to be taken over some future cash flow and valuation - often employs the distributional assumption of log-normality of return measures (or also log-normality of the one-period discount factor), so as to be able to obtain *exact closed-form* solutions to asset pricing phenomena (see Campbell, 1986). Essentially, this is a distributionally simplifying assumption so as to be able to deal with

Jensen's inequality terms which typically crop of when taking expectations of products of future expected variables. This is particularly useful when applied to the modeling of the term structure of interest rates - using the log-normal model - in which the pricing of multi-period bonds requires a "chaining together" of current and future one-period bonds, leading to a "piling up" of such Jensen's inequality terms (Cochrane, 2005; Sargent and Ljungqvist, 2004; den Haan, 1995, see).

Regarding this last point, a discussion of the *unconditional* behaviour (or shape) of the term structure of interest rates (i.e. the unconditional, "on average" yield curve), is similarly affected by risk-considerations faced by the representative investor-household, which are directly related to the above-mentioned Jensen's inequality terms. Here, the term premium puzzle arises (see Backus et al., 1989; den Haan, 1995) which for a positively autocorrelated process of the (log of the) consumption-based discount factor typically implies a *mildly downward-sloping*[5] shape of the yield curve, as representative household-investors perceive longer-term bonds as hedges against consumption risk, and are thus willing to accept a lower price, leading to a lower average return (or yield) (see den Haan, 1995). Also, this standard general equilibrium derivation of the average yield curve implies no curvature effect (so it is a constant slope effect) and, again, for low risk aversion and standard power utility functions, risk premia which are so small, that one could essentially treat the theoretical term structure as "flat", approximately obeying the classical pure version of the expectations theory.

Motivated by the analysis of this and other issues by Bansal and Coleman (1996), the section "Asset Pricing in a Banking-Time Model" represents an attempt to model the distortive effects of a financial intermediary - employing a share of short-term debt as collateral - on the risk-free rate of interest, leading to a lower risk-free rate

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[5] Again, this depends on assumption made about the volatility and auto-correlatedness of the discount factor. For more on this, and an explicit example, see Cochrane (2005)[ch.19]

than otherwise implied by the "standard" purely intertemporal CCAPM rate, as so defined by Canzoneri and Diba (2005), with a conditional positive correlation of this interest rate gap with the stance of monetary policy. The distortion arises from the fact that a share of the total payout on short-term debt is actually received in form of a banking wage bill - directly related to the proportional level of credit production - thus leading to the conditional and endogenously (as credit production depends on inflation-induced tax avoidance of holding money balances) determined interest rate gap that way. An upward-sloping unconditional term structure is obtained, as it is only the return on the one-period, short-term debt instrument, which receives this distortion. Proxying the return on the risky asset simply by employing the return on capital (as is also done in Lettau (2003)) or in other words by the CCAPM rate, leads to a conditionally varying equity premium related to the banking time distortion determining the banking wage bill.

5.1.5 Asset Pricing in GE - What's on the horizon?

The current forefront of general equilibrium asset pricing is a very active research area indeed. An excellent and very up-to-date survey is contained in Cochrane (2007). Much of the theoretical results follow what some may like to refer to as the "Campbell-Cochrane" paradigm, which epitomises the view of obtaining differences in ex-ante traded prices of various assets by considering differences in some measure of association between a stochastic discount factor and the future expected cash flows across assets. This is therefore clearly an approach firmly rooted in the (classical CAPM-finance) view of pricing assets based on some "second-order"[6] concept of undiversifiable risk.

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[6] I use the term "second-order" so as to distinguish the undiversifiable risk argument from a "first-order" market-driven distortive argument of explaining asset prices or returns, such as in Bansal and Coleman (1996).

Clearly then, it is not surprising that in response to the "equity-premium/low risk-free rate" puzzles, much research has focused on how different utility function specifications (thus affecting the marginal utility of consumption) can possibly solve some of these puzzles. Related to this is also a sub-strand of the literature which attempts to derive from micro-foundations particular aspects of the Fama-French three-factor model (see Fama and French, 1988). Some contributions within this line of research are given by Lettau and Ludvigson (2004) and Xing (2008). The former use a conditional model of the stochastic discount factor employing a "cay" variable, the consumption-to-wealth ratio, to capture conditional changes (capturing conditional changes in risk-aversion that way). The latter alternatively re-interprets the value premium using a q-theory adjustments costs to investment approach.

Some recent and influential ideas regarding the equity premium/risk-free rate frontier of research have been put forth by Campbell and Cochrane (1999) and also by Yogo (2006, 2008), where the former use some non-linear habit specification to explain asset price stylized facts (in particular a simultaneous explanation of the equity premium and the low risk-free rate) whereas the latter uses state-dependent utility by introducing consumption of durable goods. Also noteworthy is the resurrection of an old idea (see Rietz, 1988) by Barro (2005), which re-emphasises the idea of "rare disasters" again, resulting in a longer and fatter lower tail to the return distribution. This idea - which is spelled out using a closed-economy Lucas endowment economy, has recently also been extended to a two-economy setup (see Copeland and Zhu, 2007), in which the possibility of international diversification reduces the size of the equity premium obtained in Barro's version.

The forefront of research in relation to the term structure of interest rates is still largely influenced by the popularisation of the affine (or latent variable) term-structure model strand of literature, which was initiated by Duffie and Rui

(1996); Dai and Singleton (2000) and has recently taken on a more "macro-finance twist" (see Wu and Rudebusch, 2008; Diebold et al., 2005) by combining the modeling of the short-term rate via a microfounded macromodel (typically of NNS type so as to contain a Taylor Rule) and the modeling of the return (or yield) on bonds of higher maturity using canonical affine (linear) term structure models. To my knowledge, a particularly strong focus of seriously modeling the (conditional and average) yield curve by referring only to a fully specified microfounded general equilibrium model - as in Bansal and Coleman (1996) - has not emerged[7], clearly also because of the fact that "deriving" the classical three factors (or latent variables) - level, slope, curvature - which appear to explain most of the conditional behaviour of yield curves, from microfoundations appears to represent a difficult task. More important however regarding this issue is also the fact that the underlying "one-factor" explanation of the general equilibrium version of the term structure of interest rates, embodied by the conditional behaviour of the stochastic discount factor (in turn usually determined by the conditional behaviour of aggregate consumption) is simply too stark or oversimplified a representation to seriously model the term structure through time in such models. However, *conditionally* varying specifications of the general equilibrium implied discount factor, such as in Bansal and Yaron (2004) and Campbell and Cochrane (2000), as well as the sort of "first-order" market distortions as discussed in Bansal and Coleman (1996); Canzoneri and Diba (2005) and in this thesis' section "Asset Pricing in a Banking Time Economy", could perhaps be combined to relate the purely a-theoretical latent factors to macroeconomic fundamentals.

As I have already mentioned above, a very provocative possibility is to see more "first-order" distortive explanations of asset prices or a combination of the latter with the usual "second-order" undiversifiable risk explanations embodied by the

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[7] with the notable exceptions of Wachter (2006).

usual risk-adjusting covariance terms, to explain both unconditional and conditional characteristics of the low risk-free rate, the equity premium and the term structure, derived from various functional specifications in relation to the special role of short-term government bonds, such as in Bansal and Coleman (1996); Canzoneri and Diba (2005); Canzoneri et al. (2008); McCallum and Goodfriend (2007). This could permit the joint explanation of various asset pricing facts using simple standard iso-elastic power utility functions, instead of having to rely on some combination of high risk-aversion and/or "non-standard" utility functions such as habit-persistence, Epstein-Zin or other non-separable specifications, such as those obtained from the incorporation of durable consumption goods.

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